Costly Increases in Public Debt when $r < g^*$

By YONGQUAN CAO, VITOR GASPAR, AND ADRIAN PERALTA-ALVA[†]

July 30, 2025

This paper quantifies the economic costs of rising public debt using a deterministic, overlapping-generations model. The model has two assets: public debt and private capital, and the return on private capital exceeds the interest rate on public debt because of a "convenience yield." In an analytical version of the model, we show higher public debt reduces private investment and GDP, regardless of the interest rate on public debt and growth differential ($r - g$). Then, we consider a quantitative version of the model. We find that the 60 percentage point increase in the debt to GDP ratio that occurred since 2000 could have led to up to a 15 percent decrease in capital stock and an 8 percent reduction in GDP. Subsidizing investment through debt financing to prevent crowding out appears challenging, based on US data.

(JEL E43, E62, H63)

I. Introduction

By 2020, a curious anomaly was apparent in global financial markets: despite rising and historically high debt-to-GDP ratios, sovereign bond yields were declining to very low levels (even, in some cases, turning negative in real terms). Olivier Blanchard characterized this landscape as 'unusual' in his book *Fiscal Policy under Low Interest Rates*, where he extensively explores the associated facts and policy implications. While one might be tempted to dismiss these low-interest rates as a historical aberration—especially in light of recent inflationary pressures and high nominal rates—such a dismissal may be premature. Prior to the COVID-19 pandemic, both real and nominal neutral interest rates had been on a declining trajectory for decades. Various structural factors, from sluggish economic growth and demographic changes, as outlined by the IMF (2023), to a global scarcity of safe assets, as discussed by Caballero et al. (2017), contributed to this enduring

[*]Disclaimer: The views expressed in the paper are those of the authors and do not necessarily represent the views of the IMF, its Executive Board, or IMF management. We thank two anonymous referees for their constructive feedback. We are also indebited to Cristina Arellano, Ellen McGrattan, Eric Leeper, Juan P. Nicolini, Mark Aguiar, Tim Kehoe, and seminar participants at the SED 2025 in Copenhagen, the 2025 Midwest Macro Meeting in the Kansas City Fed, the Seventh China International Conference in Macroeconomics in Wuhan, China, the LACEA-LAMES 2023 in Bogota, the IMF, Centro de Investigacion Economica at ITAM, and the University of Miami.

[†]International Monetary Fund, 700 19th St NW, Washington D.C., United States, 20431.

trend. Many of these factors are expected to remain operating going forward. Hence, as inflation reduces, low interest rates may well return.[1]

Our paper addresses two critical questions. First, do increasing levels of public debt result in economic costs in standard overlapping generations models in which the marginal product of capital, m, economic growth, g, and the interest rate on government debt, r, satisfy $m > g > r$—that is, when the economy is dynamically efficient despite $r - g < 0$? Second, what is the plausible magnitude of the negative effects of higher debts? Our analysis confirms that surging public debt crowds out private capital, suppressing long-term output. Quantitatively, we find large costs associated with public debt accumulation.

The negative differential between interest rates and growth has led scholars such as Blanchard (2023) and Blanchard (2019) to argue, using stylized models, that elevated levels of public debt may incur minimal fiscal and social costs. Our research focuses on investigating the conditions under which public debt does impose significant costs, especially in models that are adequate for the quantitative analysis of real-life economies. Such analysis requires realistic life spans, interactions with foreign capital markets, and fiscal systems that can be clearly mapped to the policies of specific countries. In our baseline results, we deliberately set aside any consideration of the potentially productive uses of such debt.[2]

The limitations imposed by resource scarcity and finite production capacity apply to all sectors, including governmental activities. These constraints become particularly apparent when we explicitly consider the endogeneity of the condition $r-g < 0$. Both the real interest rate and economic growth are determined in equilibrium. For the purposes of this study, we operate under the assumption that marginal product of capital is greater than growth. This is consistent with estimates of the risk premium of about 5 percent and estimates of return of capital in the United States (ranging from 10 percent on average for businesses to about 8 percent for all capital as reported in Gomme et al. (2011), and their updated data Gomme et al. (2022)).

It is well understood that high debt levels expose countries to multiple equilibria that include attacks on sovereign debt (see for example Cole and Kehoe (2000) and Aguiar and Amador (2021), for a comprehensive survey on sovereign debt and default issues). We will abstract away from these and other considerations associated to risk. Our study focuses squarely on crowding out, which operates in two main ways. First, assuming households demand for assets is given, if the government increases its debt, then the share of assets available to private capital will decrease. Second, if government debt assumes a larger share of households' assets while offering lower returns compared to private capital due to convenience yield, this results in an overall reduction in their asset returns and dissuade households from saving. In a general equilibrium setting, however, the demand for assets is not given and the returns of different assets will change as debt increases.

Our baseline analyses will be based on a standard overlapping generations model of a closed economy. A key feature of our model is that it includes two assets. The rate of return on private capital will be determined in equilibrium. Government debt delivers a rate of return lower than that of pri-

[1] Other, more temporary factors, might have also payed a role in keeping interest rates low, including quantitative easing policies implemented by major central banks. Determining the overall quantitative effects of the interplay of all these forces is beyond the scope of this paper.

[2] We explore the utilization of such debt to subsidize capital in one of our alternative specifications.

vate capital. In the baseline experiments, this "convenience yield" is exogenous and represents the reward for holding public debt. However, the model can be easily extended to include an endogenous convenience yield (with bonds in the utility function). The main qualitative and quantitative features of the analysis are unchanged in such version of the model (as proven in the appendix). Aguiar et al. (2024), show an alternative way to generate a wedge between private capital and government debt in a model with idiosyncratic risk. The paper also shows that even when the differential of rates of return between private capital and government debt is endogenous, crowding out is a critical force to be dealt with.

To start exploring our research questions, we begin with a stylized model. We consider an economy on a balanced growth path, and focus on parameterizations that allow for an equilibrium with $r - g < 0$. Unexpectedly, debt to GDP increases and stabilizes at a new, higher, level. No further shocks are expected to happen thereafter. We study the impact of the higher level of debt to GDP on key macroeconomic aggregates. In this analytically tractable model, we can transparently illustrate the crowding out mechanism. An important result is that crowding out operates irrespective of whether $r - g$ is positive or negative.

The core of the paper is based on a version of the model designed for the quantitative analysis of the U.S. economy. It incorporates realistic life spans and fiscal policies. We look at the data and identify the key drivers of the rising U.S. debt-to-GDP ratio since 2000, with primary deficits playing a central role due to persistently higher expenditures relative to revenues. Major revenue sources, when measured as a share of GDP, fluctuated without exhibiting a clear trend. The data, however, reveals substantial changes on the expenditure side of the government budget constraint, especially transfers. The model contains a rich set of transfers that helps us to study the effects of such changes in spending.

In the baseline quantitative experiment debt increases because of increased government consumption. We later consider shifts in the composition of expenditures, demographic factors such as population ageing, and analyze both the transitional dynamics and steady-state effects of higher debt-to-GDP ratios.

Our analysis reveals that, over a period where interest rates were low, and expected to remain low, a permanent increase in debt from 60 to 120 percent of GDP–akin to the experience of the United States since 2000– can incur substantial costs. In the long term, we observe a reduction in GDP by around 8%. In the transition phase of the quantitative model, GDP exhibits a monotonous decline, while interest rates rise, even though $r - g$ remains negative. The primary mechanism behind these results is the decline of about 15% in the stock of private capital. This occurs because, in contrast to small changes in saving, individuals need to allocate a larger share of their wealth to government bonds rather than private capital. In equilibrium, the capital stock decreases, leading to a drop in output and an increase in the return on capital. Crowding out is the dominant effect of higher debt to GDP ratios, while the composition of changes in revenues, spending, and demographics play comparatively minor roles.

Crowding out manifests as well as a higher return rate for private capital, which also drives up the interest rate on government debt. We find that the model's elasticity of the interest rate on government bonds in response to changes in the debt-to-GDP ratio, an untargeted moment, is actually lower than the range of empirical estimates found in the existing literature, like Rachel and Sum-

mers (2019) and Krishnamurthy and Vissing-Jorgensen (2012). As we discussed in more detail later, this suggests that our results on the quantitative effects of crowding out may be conservative.

The quantitative analysis includes a set of robustness checks. First, we consider the effects of demographics. Although the ageing of the U.S. population can have important effects on GDP on its own, we find that the effect of GDP of higher debt is mostly due to crowding out. The observed demographic patterns cannot alone explain the increase in debt, nor interact in important ways with crowding out. Second, we extend the model to account for the fact that about 30 percent of the public debt is held by foreign investors in recent decades. Under these conditions, the quantitative impacts are reduced, but remain quite substantial with a permanent decline of 4 percent in GDP. Next, we consider the impact of increasing taxes rather than debt financing, conditional on maintaining the same patterns of expenditures of the baseline case. A later part of the paper analyses the implications of lowering the convenience yield. In this case the elasticity of the model of the interest rate on government debt to changes in debt can be matched to the data. Finally, we find that capital subsidies (financed with the debt issued) cannot be used to substantially reduce the negative effects of higher debt.

The literature on the consequences of public debt is large. Blanchard (2019) illustrates that the interest rate and growth differential has been low. If the future is like the past, he argues, it is possible that debt carries no fiscal cost although public debt can reduce capital accumulation and result in welfare cost. However, our analysis shows that higher debt may change the future relative to the past, with increasing interest rates. He also uses an overlapping generations model and shows that the differences between the interest rate on government debt, the marginal product of capital, and the growth rate of the economy matter for determining whether higher debt is welfare improving or welfare reducing. Our model, is similar in spirit, but includes richer features necessary for quantitative analysis. Under the calibrations we consider, higher debts have significant negative impacts on the economy.

Our work builds on Reis (2021) study, which also explores the macroeconomic impact of rising public debt. Reis focuses on the government's fiscal constraints in scenarios where the interest rate on government debt is lower than economic growth. His insights on resource scarcity parallel our own discussion on $m > g > r$ conditions and the crowding out of private capital. Like Reis (2021), we emphasize the intrinsic fiscal constraints that governments face, but we extend the analysis by calibrating our models to U.S. economic conditions to quantify the long-term GDP effects of higher public debt levels.

Aguiar et al. (2024) show that in an incomplete-markets economy with sizable precautionary saving, the government may be able to issue debt and use the proceeds as a capital subsidy so that *no* agent is worse off and there is no reduction in private capital. A key statistic in their analytics is the semi-elasticity of the real bond rate with respect to the debt ratio. Under their baseline calibration illustration (Epstein–Zin preferences, mark-up $\mu = 1.4$) this elasticity is about 0.5 basis-points per one-percentage-point increase in debt; and the associated interest bill is small enough to fund the required subsidy.

Our framework keeps the same crowding-out mechanism but differs on two quantitative margins. *First*, once intangible capital is added to match U.S. data, the capital stock per dollar of GDP roughly doubles, so the subsidy needed to offset crowding-out scales up one-for-one. *Second*, our

quantitative experiment is guided by the empirical elasticity of the interest on debt. Taken together, we are left with no surplus revenue to finance the subsidy: the policy would be fiscally infeasible under a U.S. calibration. This does not contradict the theoretical insights; it simply shows that, once the elasticity of interest to changes in government debt and the capital stock are disciplined by U.S. data, the conditions for a robust Pareto-improvement are more difficult to be attained.[3]

Brunnermeier et al. (2022) propose a different way to generate a wedge between the interest rate on capital and that on government. The key elements are aggregate risk and a focus on equilibria where government debt forms a bubble. Quantitatively, their findings rely on two key assumptions: that risks rise during recessions, increasing demand for government debt, and that additional revenues from debt issuance are used for private sector subsidies. Under these conditions, their model produces a Laffer curve and higher government debt may even crowd in private capital. In our model, we also consider the effects of a falling convenience yield as debt increases. In our case, crowding out dominates: private capital decreases, and GDP is ultimately reduced. Importantly, in reality, debt issuance was not used for private sector subsidies.

Acharya and Dogra (2022) present a study that aligns closely with the questions explored in our research. They utilize an overlapping generations model, similar to our simple analytical model, but extend their framework to include nominal rigidities and idiosyncratic capital income risk. Importantly, their model features an endogenous convenience yield, in contrast to the exogenous yield considered in our work, allowing for a nuanced analysis of welfare implications. The authors posit that an increased supply of 'safe assets', primarily in the form of government debt, could restore the efficacy of conventional monetary policy in economies constrained by the zero lower bound (ZLB). Notably, Acharya and Dogra (2022) observe that, even in models with endogenous convenience yields, crowding out effects mirror our findings: they are costly and lead to diminished investments in physical capital. These observations hold true especially in economies where the safe interest rate can assume negative values without compromising dynamic efficiency. While their study offers valuable theoretical insights, it remains largely qualitative. In contrast, our research is calibrated meticulously to reflect U.S. economic conditions and policies, aiming to quantify the repercussions on GDP of escalating levels of public debt.

Before turning to the formal set-up, it is worth stating explicitly what our analysis is—and is not—intended to do. We treat the economy as a controlled laboratory: by holding everything but public indebtedness constant we can isolate the pure crowding-out channel and measure its quantitative importance. In reality, many other factors—monetary policy, decreases in the relative price of capital, risk premia, sectoral reallocation—moved jointly with debt. Incorporating all of them in a single environment would obscure the mechanism we wish to highlight. Hence, our experiments should therefore be interpreted as counterfactuals, not as an attempt to fit every contour of post-2000 U.S. GDP or investment data.

[3] We have also explored a calibrated version of our quantitative model abstracting from intangible assets and crowding out effects of public debt are even higher than what is found in our baseline.

II. A simplified model

A. *Economic Environment*

Time is discrete and runs to infinity. Each individual lives for two periods. Hence, individuals born at time t live for dates t and $t+1$ Preferences over consumption , can be represented by the time separable utility function

$$u(c_t^t) + \beta u(c_{t+1}^t),$$

where function $u : \mathbb{R}_+ \to \mathbb{R}$ satisfies standard assumptions, and $0 < \beta < 1$ is the discount factor. We assume exponential population growth at constant rate $n \geq 0$, and an exogenously given population size at period 0, L_0.

$$L_t = (1+n)^t L_0$$

Production is undertaken by competitive firms. The aggregate production function is F and satisfies constant return to scale and other standard assumptions

$$Y_t = F(K_t, L_t).$$

Let $k = K/L$, so that $f(k) = \frac{F(K,L)}{L} = F(k,1)$. From here on, capital letters will denote aggregates, while lowercase letters will represent per capita values. As factor markets are assumed competitive, firms will generate zero profit. So wage is given by $w_t = f(k_t) - f'(k_t)k_t = (1-\alpha)f(k_t)$. The optimization problem of an individual born at time t, is

$$\max u(c_t^t) + \beta u(c_{t+1}^t)$$

subject to:

$$\begin{gathered}(1+\tau_t)c_t^t + s_t^t = w_t l_t^t \\ (1+\tau_{t+1})c_{t+1}^t = R_{t+1}s_t^t \\ l_t^t = 1, l_{t+1}^t = 0 \\ s_{t-1}^t = 0\end{gathered}$$

Individuals can consume, save, pay taxes and work for only the first period and supply one unit of labor inelastically, earning the wage rate w_t. To keep the model simple, we do not endogenize individuals' asset allocation decisions. Instead, we follow Braun and Joines (2015) and assume that individuals save in a fund that allocates a fraction $1-\phi_t$ of savings to government debt and a fraction ϕ_t to firms' capital. Households take the fraction allocated to each asset as given, but they hold rational expectations. The fraction allocated to each asset will be determined in equilibrium. Hence, the value of ϕ_t will be consistent with the values debt and private capital stock observed in equilibrium (which in turn depend on ϕ_t). The gross rate of return to saving, R_t, is thus a weighted average of the rental rate of capital i_t^k and the rate paid by government bonds i_t^b so that

$$\begin{aligned}1 + i_t^k &= f'(k_t) + 1 - \delta \\ R_t &= \phi_t(1+i_t^k) + (1-\phi_t)(1+i_t^b)\end{aligned}$$

We assume $1+i_t^b = 1+i_t^k-\gamma$, with γ the exogenously determined convenience yield. However, in the appendix we present a version of the model with bonds in the utility function and an endogenous convenience yield. We also show that the qualitative features of such a model are the same as those of the simpler model presented here. In particular, crowding out always happens and there is a "Laffer-type" curve for the maximum sustainable deficits in relation to debt to GDP.

Finally, there is a government with an initial debt level B_0 that finances a predetermined stream of expenditures $\{G_t^c\}$, $G_t^c > 0$ for all $t = 0, 1, 2, ...$ Government expenditures do not directly affect the utility function of any private agent of this economy. The government can tax households using only consumption taxes, τ_t [4]. The law of motion for government debt is the following:

$$B_{t+1} = G_t^c + (1+i_t^b)B_t - \tau_t C_t$$

or

$$b_{t+1} = \frac{g_t^c + (1+i_t^b)b_t - \tau_t c_t}{1+n}$$

where C_t is the aggregate consumption across individuals.

B. *Equilibrium*

Total savings in the economy are

$$S_t = L_t s_t^t$$

Market clearing for assets requires

$$K_{t+1} + B_{t+1} = S_t.$$

Consistency of expected and actual returns requires (noting that the private stock of capital depends on ϕ_t through its effects on the cost of capital)

$$\phi_t = \frac{K_t}{K_t + B_t}.$$

An equilibrium, for a given sequence of government debt, and expenditures is a sequence of prices (interest rates and wages), tax rates, allocations (fraction of assets allocated to private capital, consumption and savings), such that individuals solve their optimization problems, taking prices as given, the government budget constraint holds, and markets clear.

C. *Comparative Statistics (steady state)*

We now study the long-run impact of increasing public debt. For simplicity, we assume individuals live for two periods, and their preferences can be represented by the log utility, $u = log(c)$. A steady state is an equilibrium where per capita variables remain constant over time. While exogenous growth can be easily introduced, it does not alter any of our conclusions; in such scenarios, one

[4] A comprehensive and realistic set of direct and indirect taxes will be introduced in the next section

would focus on a balanced growth path where all per capita variables grow at an exogenously given constant rate. [5]

Notice that since the only tax is the consumption tax, and in a steady state such tax is constant over time, taxation does not affect the intertemporal Euler equation of individuals, namely

$$\frac{1}{w-s} = \beta R \frac{1}{Rs}$$

solving for savings yields

$$s = \frac{\beta w}{1+\beta}.$$

For asset markets to clear

$$(1+n)k^* + (1+n)b^* = s = \frac{\beta w}{1+\beta},$$

must hold. The following equation characterizes the steady state of the model

$$(1+n)k^* + (1+n)b^* = \frac{(1-\alpha)\beta f(k^*)}{1+\beta}. \tag{1}$$

This model is as a generalization (to the case of two assets) of standard OLG analyses in models with production. Its graphical representation can be seen in Figure 1, which corresponds to Figure 9.2 in McCandless and Wallace (1991). As is apparent from the diagram, the steady state capital with zero debt, k^{max} is the maximal attainable. Higher levels of debt reduce steady state capital. Hence, the crowding out mechanism holds regardless of assumptions on the interest rate and growth.

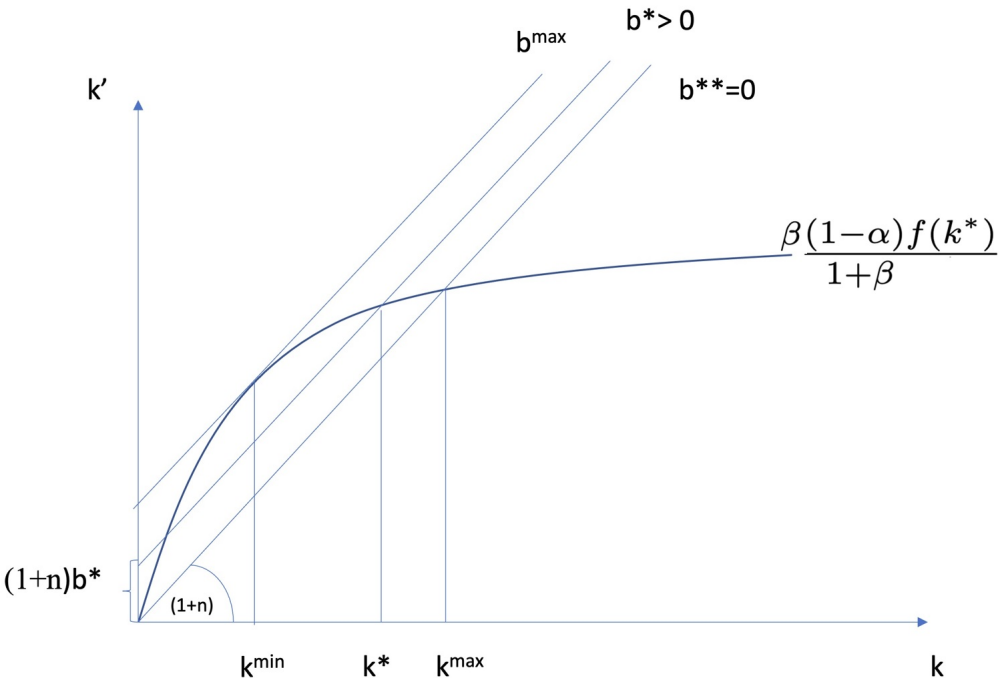
Having established the basics of our analytical framework and its implications for steady-state equilibrium, we now proceed with a more rigorous investigation through a series of lemmas. These lemmas extend the model's insights and provide a more nuanced understanding of the dynamics at play.

When $r-g < 0$ there is a primary deficit that is compatible with a stationary debt to GDP ratio.[6] We are interested in the relation between such a sustainable primary deficit and the steady state public debt ratio. When the debt to GDP ratio is zero the primary deficit compatible with a constant debt ratio is zero as well (even with $r - g < 0$). But increasing debt ratios push interest rates up so that the primary deficit reaches a maximum sustainable level. Subsequently, it declines. When $r - g = 0$, in steady state, the sustainable primary deficit is zero as well. Subsequently, for even larger debt ratios $r - g$ turns positive and constant public debt ratios require primary surpluses. Therefore, the relation between the debt ratio and the primary deficit looks like an inverted U-curve (very much like a Laffer curve – see Figure 2). The difference is that on the horizontal axis there is the debt ratio instead of the tax rate. The debt ratio can be interpreted as analogous to a tax base.

[5] We are not addressing the scenario where a high population growth rate leads to dynamic inefficiency in the economy, as this is not quantitatively relevant to our study for the U.S.

[6] Economic growth is the sum of TFP growth and population growth. In the simplified model, population growth is equal to economic growth when TFP growth is zero, i.e., $n = g$.

FIGURE 1. CHARACTERIZATION OF EQUILIBRIUM IN THE ANALYTICAL MODEL



Lemma 1. *At the stable steady state, higher government debt always crowds out private capital.*

Proof. The stability of the steady state equilibrium requires that $\frac{dk_{t+1}}{dk_t}\Big|_{k=k^*} < 1$, i.e., $\frac{\alpha\beta(1-\alpha)}{1+\beta}k^{\alpha-1} \leq 1+n$. According to 1, under standard calibration (i.e., $\alpha, \beta \in (0,1), n \in (-1,+\infty)$),

$$\frac{\partial k^*}{\partial b^*} = -1 + \frac{\alpha\beta(1-\alpha)}{(1+n)(1+\beta)}k^{*\alpha-1} = \frac{1}{1+n}\left(-(1+n) + \frac{\alpha\beta(1-\alpha)}{1+\beta}k^{*\alpha-1}\right) < 0. \tag{2}$$

So, whenever government bond increases, private capital decreases. □

The conclusion from our analysis this far, is that increasing government debt is costly because it will crowd out private investment. This will happen even when $r-g$ is negative. Further, there exists a maximum amount of per capita debt compatible with the existence of equilibrium.

Lemma 2. *At the stable steady state, there is a maximum amount of government debt, b_{max}, and a minimum amount of capital, k_{min} that the economy can support.*

$$k_{min} = \left(\frac{(1+n)(1+\beta)}{\alpha\beta(1-\alpha)}\right)^{\frac{1}{\alpha-1}} \tag{3}$$

$$b_{max} = -k_{min} + \frac{\beta(1-\alpha)}{(1+\beta)(1+n)}k_{min}^{\alpha} \tag{4}$$

Proof. According to 1, we know $\frac{db^*}{dk^*} = -1 + \frac{\alpha\beta(1-\alpha)}{(1+n)(1+\beta)}k^{\alpha-1}$, which is a decreasing function in k^*, i.e., $\frac{\partial^2 b^*}{\partial k^{*2}} < 0$. Therefore, by setting $\frac{db^*}{dk^*}$ to zero, we get the maximum of the debt that the

economy can support. At the stable steady state, there is a negative relationship between private capital and government bond. So, the corresponding level of capital is the minumum amount of capital, k_{min}, the economy can support. Therefore, $k_{min} = \left(\frac{(1+n)(1+\beta)}{\alpha\beta(1-\alpha)}\right)^{\frac{1}{\alpha-1}}$ and $b_{max} = -k_{min} + \frac{\beta(1-\alpha)}{(1+\beta)(1+n)}k_{min}^{\alpha}$. □

The government budget constraint at the steady state is the following:

$$\begin{aligned}(1+n)b^* &= g^{c*} + (1+i^b)b^* - \tau C^* \\ pd^* &= g^{c*} - \tau C^* = (1+n)b^* - (1+i^b)b^* \coloneqq -(r-g)b^*\end{aligned}$$

where pd is the primary deficit.

Lemma 3. *When $r - g < 0$, there is a maximum of primary deficit the economy can support at the stable steady state.*

Proof. At the steady state, primary deficit, pd, can be characterized as follows:

$$pd = -(r-g)b = -(\alpha k^{\alpha-1} + 1 - \delta - (1+n) - \gamma)b \tag{5}$$

So the first derivative of primary deficit over primary capital is

$$\frac{\partial pd}{\partial k} = -(\alpha k^{\alpha-1} + 1 - \delta - (1+n) - \gamma)\frac{\partial b}{\partial k} - (\alpha(\alpha-1)k^{\alpha-2})b \tag{6}$$

when $b = b_{max}$ and $k = k_{min}$, $\frac{\partial b}{\partial k} = 0$. So,

$$\left.\frac{\partial pd}{\partial k}\right|_{b=b_{max},k=k_{min}} = -(\alpha(\alpha-1)k_{min}^{\alpha-2})b_{max} > 0. \tag{7}$$

When $b = 0$ and $r - g < 0$,

$$\left.\frac{\partial pd}{\partial k}\right|_{b=0} = -(\alpha k^{\alpha-1} + 1 - \delta - (1+n) - \gamma)\frac{\partial b}{\partial k} < 0 \tag{8}$$

The second derivative of primary deficit over primary capital is

$$\begin{aligned}\frac{\partial^2 pd}{\partial k^2} &= -(\alpha(\alpha-1)k^{\alpha-2})\frac{\partial b}{\partial k} - (\alpha(\alpha-1)(\alpha-2)k^{\alpha-3})b \\ &\quad -(\alpha k^{\alpha-1} + 1 - \delta - (1+n) - \gamma)\frac{\partial^2 b}{\partial k^2} - (\alpha(\alpha-1)k^{\alpha-2})\frac{\partial b}{\partial k} < 0\end{aligned} \tag{9}$$

To prove the given equation, we analyze each term individually. For the first term, $-(\alpha(\alpha-1)k^{\alpha-2})\frac{\partial b}{\partial k}$, since $0 < \alpha < 1$, $\alpha(\alpha-1) < 0$, and given $\frac{\partial b}{\partial k} < 0$, the product is positive, making the term negative. For the second term, $-(\alpha(\alpha-1)(\alpha-2)k^{\alpha-3})b$, with $0 < \alpha < 1$, $-\alpha(\alpha-1)(\alpha-2) < 0$, and $b > 0$, $k > 0$, the term is negative. The third term, $-(\alpha k^{\alpha-1} + 1 - \delta - (1+n) - \gamma)\frac{\partial^2 b}{\partial k^2}$, given $r - g < 0$, implies $\alpha k^{\alpha-1} + 1 - \delta - (1+n) - \gamma < 0$, and since $\frac{\partial^2 b}{\partial k^2} < 0$, the term is negative.

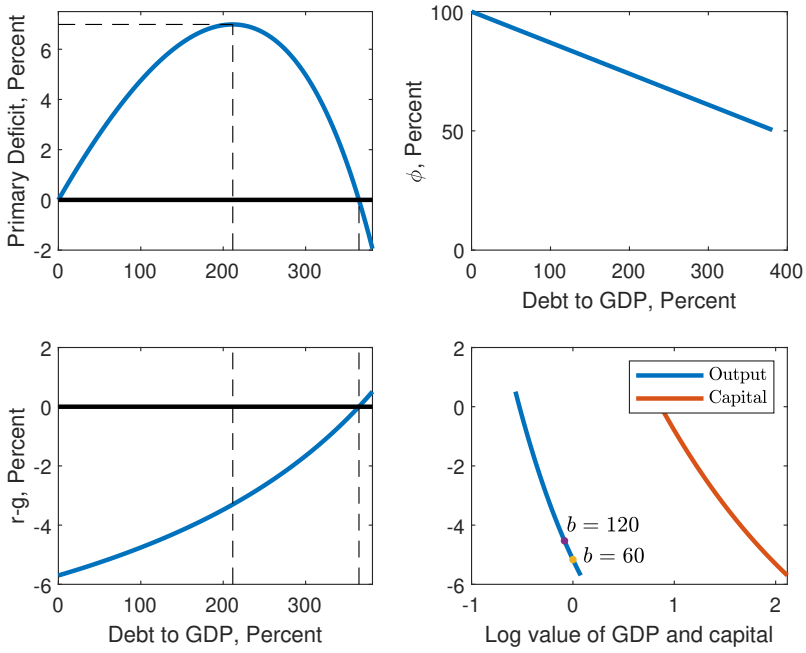
Lastly, for $-(\alpha(\alpha-1)k^{\alpha-2})\frac{\partial b}{\partial k}$, with $0 < \alpha < 1$, $\alpha(\alpha-1) < 0$, and $\frac{\partial b}{\partial k} < 0$, the term is negative. Combining all terms, we conclude that $\frac{\partial^2 pd}{\partial k^2} < 0$.

Therefore, as k increases and b decreases, the first order derivative turn from positive to negative. So, over $b \in (0, b_{max})$, pd is an inverted U-curve in b and there is a maximum of pd which is determined by setting 6 to zero. □

The intuition is that, even though $r - g$ is negative, government debt crowds out private capital and raises the interest rate. As a result, $r - g$ becomes less negative. Consequently, the sustainable deficit in the economy follows is reminiscent of Laffer curve, as a function of government debt.[7] Figure 2, illustrates some of the results in Lemmas 1-3 for a parameterization where a period equals about 40 years, a zero percent growth rate for population, a discount factor $\beta = 0.6$, and a Cobb-Douglas production function with a capital share $\alpha = 0.484$. The convenience yield is set to 4% annually to guarantee $r - g < 0$. As Lemma 3 shows, the maximum deficit is a inverted-U shape as a function of debt to GDP.

In this case, the parameterized economy is on the increasing part of the hump shape curve relating deficits and debt, and the economy can finance a higher deficit. This also translates into declines in output. The negative effects on GDP happen irrespective of whether $r - g$ is negative or positive.

FIGURE 2. RESULTS IN THE ANALYTICAL MODEL



[7] Aguiar et al. (2022) and Mian et al. (2020) also find inverted U curves for the deficit vs debt to GDP relation.

III. The quantitative model

The model that follows is a full general-equilibrium OLG economy, disciplined by both macro aggregates (national accounts, fixed-asset tables) and micro evidence on U.S. taxes, transfers, and life-cycle earnings. All calibrated objects—technology, preferences, demographic structure, and the legal incidence of each fiscal instrument—are held fixed at their data-consistent values, and we then vary only the fiscal policies whose empirical counterparts changed after 2000. This "laboratory" set-up deliberately abstracts from other forces (monetary accommodation, financial-market frictions, global imbalances) so that the crowding-out channel can be measured in isolation. Ours is a tightly calibrated general-equilibrium counterfactual focused on one mechanism.

The model is based on McGrattan and Prescott (2017), which includes a rich set of features in the taxation system (for both, individuals and different types of businesses) that are key to match the U.S. data. The main difference is that we introduce a convenience yield for government debt. To do so, we assume, as in the simple analytical model, that the interest rate on government bonds equals the interest rate on private assets minus an exogenously given value (or sequence of values).

Households have an OLG structure with measure n_t^1 arriving as working-age households at the beginning of date t. The year since entry into the workforce is called *age* and is denoted by j. The measure of age j households at date t is n_t^j The maximum possible age is J. The probability of an age $j < J$ household of any type at date t surviving to age $j + 1$ is $\sigma_t^j > 0$. The n_t^1 are parameters that define the income population at date t, which evolves as follows:

$$n_{t+1}^1 = (1 + \eta_t) n_t^1$$

with initial population distribution $\{n_0^j\}$, given. Parameter η_t is the growth rate of households entering the workforce.

A. *State Vector*

To simplify notation, we use recursive competitive equilibrium language. All stocks are beginning-of-period stocks. The variables that define the aggregate state vector s are as follows:

- (i) $t = 0, 1, 2, \ldots$, is the time period.
- (ii) $\{a^j, n^j\}$ are the assets a^j (net worth) of an age j, and n^j is the measure of these households.
- (iii) B is the government debt.
- (iv) K_{T1} and K_{T2} are aggregate tangible capital stocks for two business sectors (described below).
- (v) K_{I1} and K_{I2} are aggregate intangible capital stocks for two business sectors.

B. *Portfolios and the return of assets*

Individuals are assumed to save in shares of ownership of an asset constituted by a fraction ϕ_t of of claims to the flows from private firms' capital and a fraction $1 - \phi_t$ to government debt, as is also assumed in Braun and Joines (2015) and Kitao (2015). Hence, the rate of return of the only financial asset available is a weighted average of the returns of government debt and private capital. Households take this return as given, and expectations are rational. In the baseline, the capital account is assumed to be closed, and total asset holdings in this composite financial asset must equal government debt and private capital for equilibrium in financial markets to hold. In a later section of the paper we allow foreigners to also hold government debt. In all cases, fraction ϕ_t is endogenously calculated as part of equilibrium, to achieve consistency between household expectations, envisioned portfolio weights, and actual weights of the different asset classes and their returns.

C. *Prices and Policy*

The relevant equilibrium *price* sequences for the households are government debt and private capital interest rates $\{i_t^b, i_t^k\}$ and wage rates $\{w_t\}$.

Policy specifies the following sequences:

- (i) Tax rates $\tau = \{\tau_t^c, \tau_{1t}^d, \tau_{2t}^d, \tau_{1t}^\pi\}$, where c denotes consumption, d distributions from businesses to their owners, and π profits. Sector 2 businesses are not subject to the corporate profit tax and must distribute all their profits to their owners.
- (ii) Net tax schedules $\{T_t^w(\cdot), T_t^r(\cdot)\}$ for those with positive labor income (e.g., workers) and no labor income (e.g., retirees).
- (iii) Government debt $\{B_t\}$.
- (iv) Pure public good consumption $\{G_t\}$.

D. *Taxes and Transfers*

The model is quite detailed in its treatment of personal taxes and government transfers. Here, we describe the key equations that define the tax and transfer schedules for both workers and retirees.

Tax Schedule for Workers

Workers are subject to a progressive tax schedule, which applies to labor income. The net tax schedule for workers, $T^w(y)$, is a piecewise linear function that can be written as:

$$T^w(y) = \beta_i y + \alpha_i,$$

where y represents labor income. The term β_i is the marginal tax rate applied to labor income in a given income interval $[\underline{y}_i, \bar{y}_i]$, and α_i is an intercept term capturing transfers and benefits that do

not depend directly on income, such as employer-provided health insurance and other non-wage compensation.

This tax structure allows for discontinuities, reflecting transfer programs like Medicaid that introduce cliffs at specific income thresholds. The progressive nature of the schedule is determined by the income-dependent marginal tax rates, which increase across income brackets.

Tax Schedule for Retirees

Retirees face a simpler tax and transfer schedule. Since retirees no longer have labor income, they receive net transfers from the government in the form of Social Security, Medicare, and other government benefits. The transfer to retirees, T^r, is defined as:

$$T^r = \text{SS} + \text{Medicare} + \text{Other Transfers}.$$

Unlike the tax schedule for workers, this schedule does not vary with income, reflecting the fact that most government transfers to retirees are based on age or eligibility, rather than income.

Government Spending

Government spending is divided into non-productive spending and transfers. It is financed through a combination of taxes on labor income and debt issuance.

E. *The Household's Problem*

The value function of a household of age $j \in \{1, 2, \ldots, J\}$ satisfies

$$v_j(a, s) = \max_{a', c, \ell \geq 0} \{u(c, \ell) + \beta \sigma_t^j v_{j+1}(a', s')\}$$

subject to

$$(1 + \tau_{ct})c + a'\sigma_t^j \leq \left(\phi_t(1 + i^k) + (1 - \phi_t)(1 + i^b)\right) a + y_t - T_t^j(y_t)$$
$$y_t = w_t \ell \epsilon^k$$
$$s' = F(s).$$

We will assume that individuals save in a fund that allocates a fraction $1 - \phi_t$ of savings to government debt and a fraction ϕ_t to firms' capital and $1 + i_t^b = (1 + i_t^k - \gamma)$, with γ convenience yield. The weighted return on the fund is taken as given by households but they form expectations about its value, and will be ultimately determined in equilibrium. Symbol ℓ denotes the labor services of a household. Households with $j > J_R$ are retired and their ℓ's are zero. The net tax schedule for retirees ($j > J_R$) is $T^j(y) = T^r(0)$ and is equal to the (negative) transfers to retirees since they have no labor income. The net tax schedule for workers ($j \leq J_R$) is $T^j(y) = T^w(y)$ and is equal to their total taxes on labor income less any transfers. The prime denotes the next period value of a variable and $v_{J+1} = 0$. Savings are in the form of an annuity that makes payments to members of a cohort in their retirement years conditional on them being alive. Effectively, the return on savings depends on the survival probability as well as the interest rate. Aggregate labor supply L is

$$L = \sum_j n^j \ell^j \epsilon^j.$$

The equilibrium law of motion of the aggregate state variable, F, is taken as given by the private agents.

F. *Technology*

The U.S. tax code has an explicit treatment of taxation based on legal status of incorporation. To match this fact, the model considers one sector is subject to the corporate income tax and produces intermediate good Y_{1t}, while the other is the non-corporate sector, which also produces intermediate good Y_{2t}. The aggregate production function of the composite final good is

$$Y_t = Y_{1t}^{\theta_1} Y_{2t}^{\theta_2},$$

where the exponents are positive and sum to 1.

The aggregate sectoral production function is Cobb-Douglas with inputs of tangible capital K_{iTt}, intangible capital K_{iIt}, and labor L_{it}:

$$Y_{it} = K_{iTt}^{\theta_{iT}} K_{iIt}^{\theta_{iI}} (\Omega_t L_{it})^{1-\theta_{iT}-\theta_{iI}}$$

for $i = 1, 2$. The labor-augmenting technical level at date t in both sectors is Ω_t, which grows at rate γ, so

$$\Omega_{t+1} = (1+\gamma)\Omega_t.$$

Capital stocks depreciate at a constant rate, so

$$\begin{aligned} K_{iT,t+1} &= (1-\delta_{iT})K_{iTt} + X_{iTt} \\ K_{iI,t+1} &= (1-\delta_{iI})K_{iIt} + X_{iIt} \end{aligned}$$

for $i = 1, 2$, where T and I denote tangible and intangible, respectively, and X is investment. Depreciation rates are denoted as δ and are indexed by sector and capital type. The resource balance constraint is

$$Y_t = C_t + X_{Tt} + X_{It} + G_t,$$

where $X_{Tt} = \sum_i X_{iTt}$ and $X_{It} = \sum_i X_{iIt}$.

G. *Government Budget Constraints*

Some notation must be set up before the law of motion for government debts can be specified. The prices of the intermediate good relative to the final good are p_{1t} and p_{2t}. The accounting profits of Schedule C corporations are given by

$$\Pi_{1t} = p_{1t}Y_{1t} - w_t L_{1t} - X_{1It} - \delta_{1T}K_{1Tt},$$

and distributions to the corporations' owners are

$$D_{1t} = (1-\tau_{1t}^{\pi})\Pi_{1t} - K_{1T,t+1} + K_{1Tt}.$$

Other business distributions to their owners are

$$D_{2t} = \Pi_{2t} = p_{2t}Y_{2t} - w_t L_{2t} - X_{2It} - \delta_{2T} K_{2Tt}.$$

We can now specify the law of motion of government debt:

$$B_{t+1} = B_t + i_t^b B_t + G_t - \sum_j n_t^j T_t^j (w_t \ell_t^j \epsilon^j) - \tau_t^c C_t - \tau_{1t}^{\pi} \Pi_{1t} - \tau_{1t}^d D_{1t} - \tau_{2t}^d D_{2t}.$$

Thus, next period's debt is this period's debt plus interest on this period's debt, plus public consumption, minus tax revenues (net of transfers). Taxes are levied on labor income and consumption, on profits of Schedule C corporations, on distributions of Schedule C corporations to their owners, and on distributions of other business firms to their owners

H. *Equilibrium Conditions*

The equilibrium conditions are as follows:

- (i) Labor, capital, and goods markets clear at each point in time (this includes a value of ϕ_t that makes expectations consistent with the prices and assets emerging in equilibrium).
- (ii) The government budget constraint (and law of motion of government debt) is satistied at each point in time.
- (iii) The household policy functions $\{a' = f_j(s)\}_j$ imply the aggregate law of motion $s' = F(s)$.

IV. Calibration

We follow the calibration strategy of McGrattan and Prescott (2017) and make the model (before changes in policy) consistent with both the national accounts and fixed assets reported by the BEA from 2000 to 2010, as well as the distribution of individual and household incomes reported in the CPS.

A. *Parameters Based on Macro Data*

The work life is of 45 years, and survival probabilities calibrated to match the 2010 life tables in Bell and Miller (2005). Preference parameters are chosen such that the model's labor input and labor share align with CPS data from the United States. Assuming logarithmic preferences given by the function:

$$u(c, \ell) = \log c + \alpha \log(1 - \ell),$$

we set α to 1.185, which implies a fraction of work time of approximately 0.3. The growth rate of labor-augmenting technology is set at 2 percent, consistent with the trend growth in the United States. Table 1 summarizes the calibrated parameter values.

TABLE 1—PARAMETERS OF THE ECONOMY CALIBRATED TO U.S. AGGREGATE DATA

Names	Value
Demographic parameters	
Growth rate of population (η)	0.01
Work life in years	45
Number of workers per retiree	3.93
Preference parameters	
Disutility of leisure (α)	1.185
Discount factor (β)	0.991
Technology parameters	
Growth rate of technology (γ)	0.02
Income share, Schedule C corporations (θ_1)	0.500
Capital shares	
Tangible capital, Schedule C (θ_{1T})	0.182
Intangible capital, Schedule C (θ_{1I})	0.190
Tangible capital, other business (θ_{2T})	0.502
Intangible capital, other business (θ_{2I})	0.095
Depreciation rates	
Tangible capital, Schedule C (δ_{1T})	0.050
Intangible capital, Schedule C (δ_{1I})	0.050
Tangible capital, other business (δ_{2T})	0.015
Intangible capital, other business (δ_{2I})	0.050
Capital tax rates	
Profits, Schedule C corporations (τ_1^{π})	0.330
Distributions, Schedule C corporations (τ_1^{d})	0.144
Distributions, other business (τ_2^{d})	0.382

The choice of tangible capital shares (θ_{1T}, θ_{2T}) and tangible depreciation rates (δ_{1T}, δ_{2T}) ensures alignment between the model's investments and fixed assets with the tangible investments and stocks reported by the BEA and Flow of Funds. The two sectors have tangible capital shares of $\theta_{1T} = 0.182$ and $\theta_{2T} = 0.502$. The annual depreciation rates, $\delta_{1T} = 0.050$ and $\delta_{2T} = 0.015$, are selected to generate investment rates consistent with U.S. data. In addition, we set $\beta = 0.991$ to ensure that the model's predicted division of income into labor and capital aligns with that of the U.S. national accounts.

We set the level of government consumption as a percentage of GDP to 0.044 in the initial steady state. The initial debt parameter, $1 - \phi$, is set to match the U.S. government debt to GDP at the beginning of the sample, resulting in $1 - \phi = 0.6$.

The return on government bonds is lower than the return on private assets due to the convenience yield ($CY = i^k - i^b$). The convenience yield is calibrated to be 4.0 percent, resulting in an interest

rate on government bonds of 0.79 percent. This interest rate broadly aligns with the real interest rate in the early 2000s, which was estimated to be around 0.93 percent.[8]

The capital tax rates are assumed to be the same for all asset holders. However, there are two categories of businesses that are subject to different taxation: Schedule C corporations (that we can refer to simply as corporations) and all other businesses. This is important because in the U.S. data tax rates have changed depending on legal form of organization (with most important declines in the case of corporate businesses).

Schedule C corporations are subject to the corporate income tax. The effective corporate income tax rate, denoted as τ_1^{π}, is approximately 33 percent over the period 2000–2010[9]. In addition to the corporate income tax, investors in these corporations also pay an additional tax on distributions, denoted as τ_1^d. This additional tax rate is 14.4 percent and is applied to distributions in the form of dividends and share buybacks. For other businesses, the tax rate on distributions, denoted as τ_2^d, is 38.2 percent.

In order to keep the calibration of the baseline economy tractable, we assume a one percent growth rate of the population, but impose a stationary population distribution. Tables 2-4 summarize the baseline (pre any shocks or policy changes) values for the main variables of the model, together with model-consistent data (following McGrattan and Prescott (2017)). The calibration of the model results in a very reasonable fit to the data at the macro and sectoral level. It also does a good job fitting key categories of expenditures and tax revenues (See Table 2). It is important to mention that the consumption tax rate is determined residually in order to balance the government budget. In our baseline parameterization, this requires setting the tax rate to 8.2 percent.

[8]Calculated as the average monthly difference between Long-Term Government Bond Yields and CPI inflation in 2005.

[9]This figure is based on the calculation in McGrattan and Prescott (2017)

TABLE 2—KEY FISCAL AGGREGATES (RATIOS TO GDP)

Component	Model	(McGrattan and Prescott, 2017)
Consumption of GS	0.044	0.044
Transfers (Total)	0.353	0.352
Of which NIPA non-defense spending		0.135
To retirees	0.127	0.128
Of which NIPA non-defense spending		0.036
To workers	0.226	0.224
Of which NIPA non-defense spending		0.099 [a]
Interest paid	0.030	0.025
Consumption taxes	0.045	0.075
Labor taxes	0.214	0.223
Dividend taxes	0.004	0.004
Corporate profits taxes	0.026	0.026
Noncorporate profits taxes	0.094	0.094
Deficit=Expend.-Receipts	0.018	0.016

[a]Data is taken from McGrattan and Prescott's additional files at https://users.cla.umn.edu/~erm/data/sr472/Data/MicroData/CPS/unicon/2005/incdist05.out

TABLE 3—AGGREGATE MODEL TARGETS (RATIOS TO GDP)

Component	Value	Revised NIPA (McGrattan and Prescott, 2017)
Consumption (C)	0.746	0.745
Tangible investment (XT)	0.210	0.211
Corporate (XT1)	0.069	0.069
Noncorporate (XT2)	0.141	0.076
GDP (Y-XI)	1.000	1.000
Labor income (wL)	0.584	0.585
Corporate (wL1)	0.356	0.356
Noncorporate (wL2)	0.228	0.229
Capital income (Y-wL-XI)	0.416	0.415
Corporate value added	0.459	0.478
Noncorporate value added	0.465	0.522
Addenda: intangible (XI)	0.134	0.134

TABLE 4—FIXED ASSETS (RATIOS TO GDP)

Component	Value	Revised Fixed Asset Tables (McGrattan and Prescott, 2017)
Tangible capital (KT)	4.093	4.117
Corporate (KT1)	0.881	0.674
Noncorporate (KT2)	3.212	3.443
Intangible capital (KI)	1.711	1.700
Corporate (KI1)	1.142	1.146
Noncorporate (KI2)	0.570	0.554
Total	5.804	5.835

B. *Parameters Based on Micro Data*

The next step involves setting the parameters that are relevant to the model's predicted distributions of income and net taxes. In our model, we assume the existence of one representative family for one age cohort. To calibrate the model, we use labor efficiency units based on productivities derived from CPS family data, which captures the life-cycle income profile. It is important to note that households will not join the model economy until they reach at least 15 years of age.

To parameterize the initial net tax schedules, $T_0^j(\cdot)$, we use piecewise net tax schedule in McGrattan and Prescott (2017). We assume that all workers face the same schedule, $T_0^j(y) = T^w(y)$, $j \leq J_R$. Similarly, all retirees face the same schedule, $T_0^j(y) = T^r(0)$, $j > J_R$, which does not vary with labor income because retirees are assumed to have no labor income. $T^w(y)$ on each AGI income interval $[\underline{y}_i, \bar{y}_i]$, $i = 1, \ldots, I$, are calibrated as follows:

TABLE 5—CURRENT AND FUTURE LABOR INCOME NET TAX SCHEDULES, $T_w^\infty(y) = \alpha_i + \beta_i y$.

Earnings Over	α_i	β_i
0	-11,762	0.059
5,132	-12,819	0.246
11,664	-13,518	0.264
17,418	-14,365	0.293
23,718	-15,211	0.316
29,692	-15,971	0.332
36,351	-17,000	0.349
45,274	-18,503	0.367
58,274	-20,359	0.382
74,560	-22,880	0.396
106,007	-28,810	0.409
191,264	-45,792	0.409

Note: Earnings and net tax function intercepts are reported in 2004 dollars.

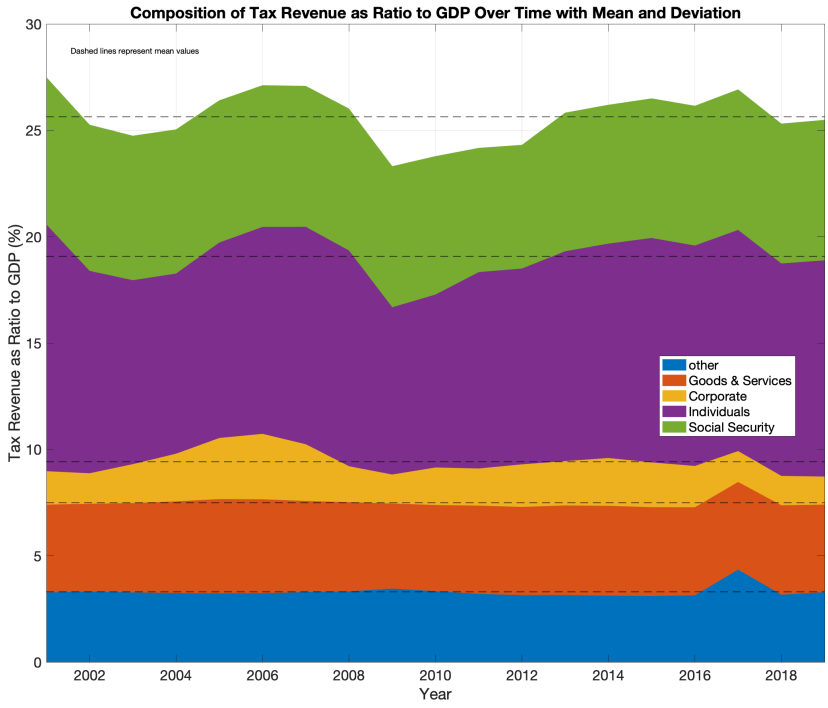
V. Why did public debt to GDP increase in the U.S.?

The U.S. federal debt-to-GDP ratio increased by close to 60 percentage points of GDP since 2000. In the appendix, we present a detailed analysis of the drivers of debt-to-GDP. Since our findings will be key inputs of the quantitative analysis, we summarize the most important ones here.

First, persistent deficits, driven by expenditures outpacing revenues, were the dominant factor in the increase in the debt-to-GDP ratio. Some of the components of the primary balance remained relatively stable, while others changed by substantial amounts. Specifically:

- The sum of tax revenues and social security contributions as a percentage of GDP over the period do not display a statistically significant trend (up or down), and have fluctuated around 25%. Figure 3 illustrates the breakdown of of the total as a percentage of GDP over the period.

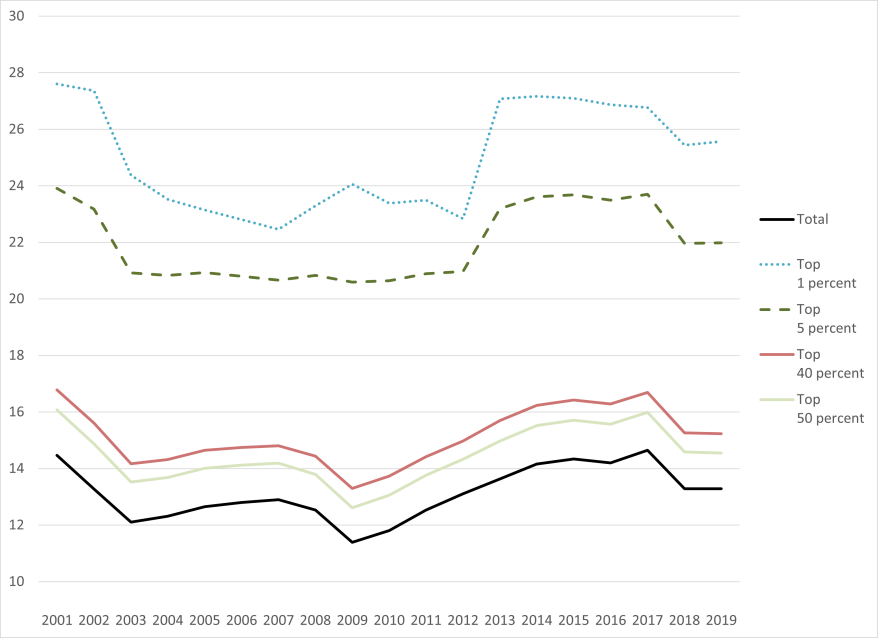
FIGURE 3. COMPOSITION OF TAX REVENUE AND SOCIAL SECURITY CONTRIBUTIONS (AS A RATIO TO GDP OVER TIME (2000-2019))



- Corporate tax revenues do show a statistically significant negative trend as a ratio to GDP. The Tax Cuts and Jobs Act (TCJA) of 2017, for example, lowered the rate from 33 to 21 percent. The overall impact is close to a 1 percentage point of GDP reduction in overall tax revenues during our sample period.

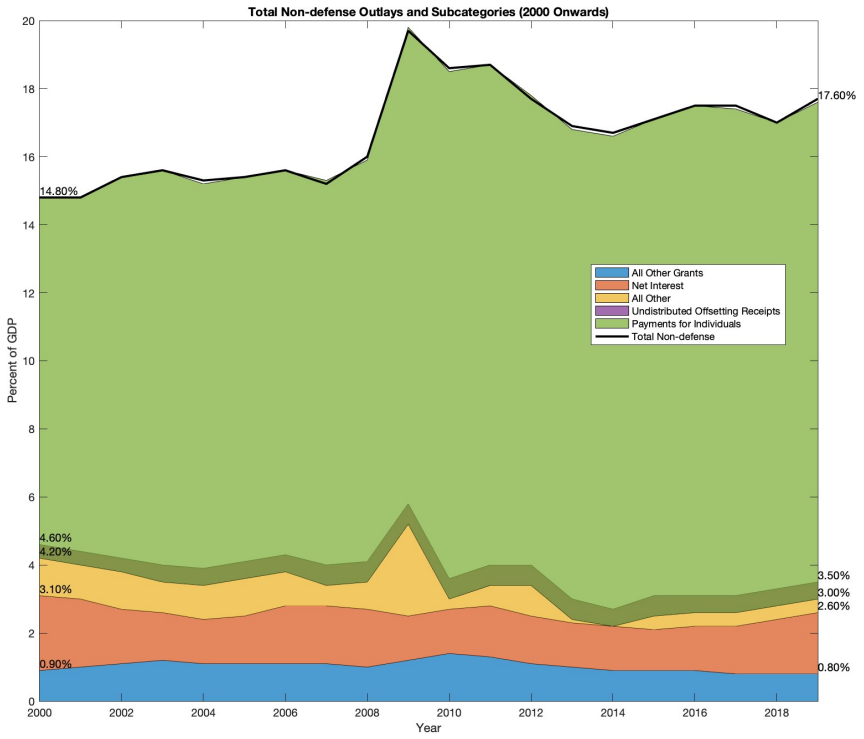
- Personal income tax revenues as percent of GDP do not display any noticeable trend over the sample period. This is in spite of changes in statutory personal income tax rates, particularly in 2001, 2003, and 2017. Revenues remained unchanged despite rate adjustments because the tax changes included base-broadening measures, such as eliminating personal exemptions and capping state and local tax (SALT) deductions. Indeed, data of the Statistics of Income of the IRS, shows effective personal income tax rates have fluctuated around a broadly constant trend over the sample period, even across income brackets. See Figure 4.

FIGURE 4. AVERAGE EFFECTIVE PERSONAL INCOME TAX RATES BY INCOME PERCENTILE (2000-2019)



- In terms of spending, the category of defense was relatively stable over our sample period.
- There were important changes in non-defense spending components, which are illustrated in Figure 5. The most notable is in payments to individuals, with an increase in 4 percent of GDP. Social security payments, other social safety nets and healthcare played the most important roles.

FIGURE 5. COMPONENTS OF NON-DEFENSE SPENDING TO GDP OVER TIME (2000-2019)

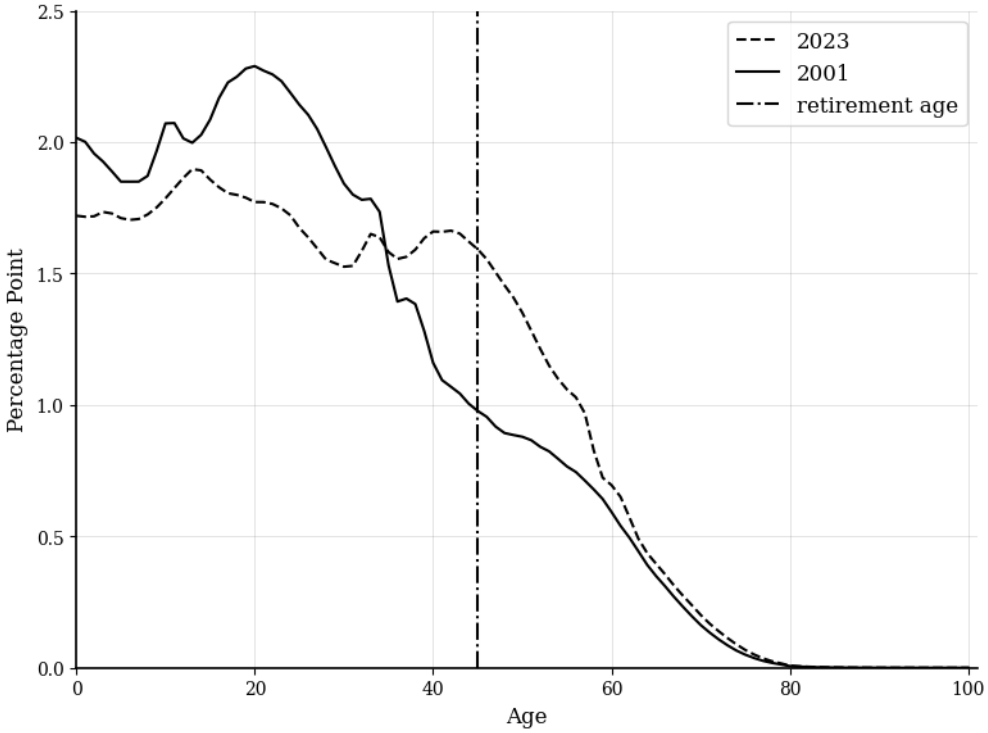


Since changes in payments to individuals are key to understand observed increases in debt, we consider whether such changes could have been driven mechanically by changes in the age distribution of the population. Figure 6 documents the significant transformation in U.S. demographics since 2000. The vertical axis represents the percentage of the population within specific age groups as a ratio of the population over age 20, while the horizontal axis denotes age (mapped to the model, with 0 corresponding to age 20 in reality and 45 aligning with the typical retirement age of 65). In 2001, one can see a noticeable bulge around age 25 (45 in reality). This is due to the "baby boomer generation." By 2023, this has of course shifted to the right as the baby boomers age.

Figure 7 highlights the substantial increase in the proportion of older adults within the population. The percentage of individuals aged 65 and over rose from 16% in 2000 to 21% in

2023—a 31% increase. Concurrently, federal mandatory spending on age-related programs, primarily Social Security, Medicare, and Medicaid, increased from approximately 7.3% of GDP in 2000 to 11.0% in 2023, representing an increase of around 50%, which is significantly higher than the increase in the share of individuals aged 65 and over. The aging population alone, especially through its slow dynamics which contrast with the rapid rise in spending, cannot fully account for the larger expenditures in payment to individuals. Policy changes were also at play.

FIGURE 6. POPULATION DISTRIBUTION OVER AGE 20, 2001 VS 2003

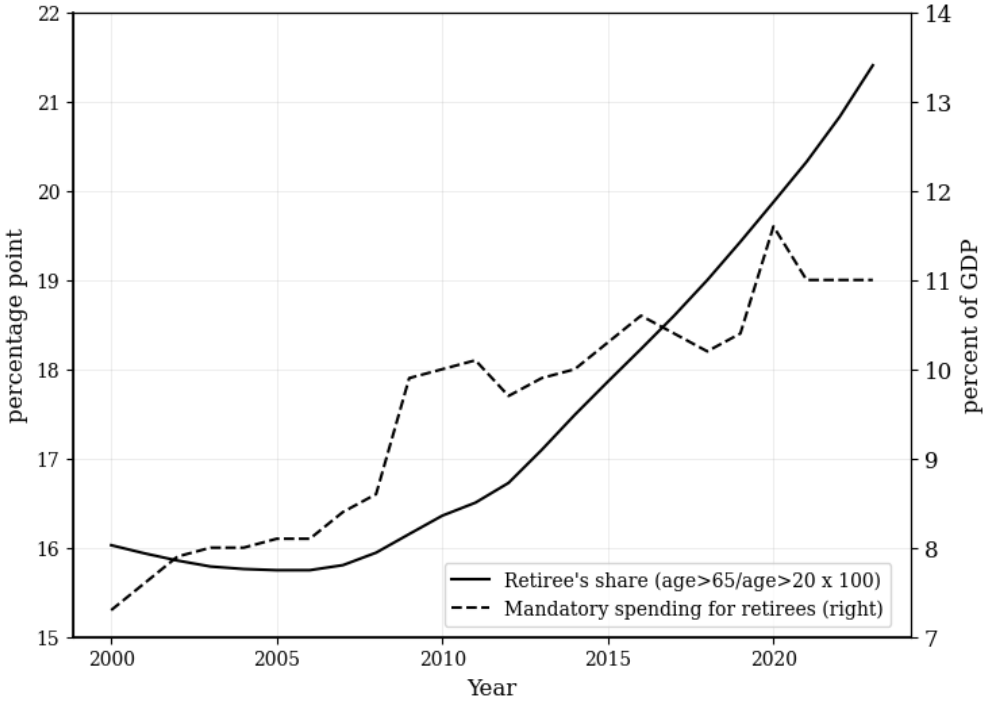


Source: UN Population Database. Population share is based on the percentage relative to the population aged 20 and above, consistent with model specification. Age represents the number of years from the current age to 20, which is consistent with model specification. Age 45 in the model corresponds to age 65 in reality, which is the retirement age.

Hence, policy responses to economic downturns and evolving social needs were key to understand expansions in payments to individuals. Demographic forces alone are not sufficient. Temporary expansions in unemployment insurance and nutrition assistance programs implemented during the GFC persisted beyond the recovery, preventing expenditures from fully reverting to pre-crisis levels. Similarly, incremental expansions in Medicare coverage and Social Security cost-of-living adjustments, coupled with broader Medicaid eligibility following the Affordable Care Act, further increased mandatory spending.

In summary, the rise in the U.S. public debt-to-GDP ratio since 2000 was driven by persistently high deficits from the outset, with expenditures consistently outpacing revenues. Growing trans-

FIGURE 7. RETIREE'S POPULATION SHARE AND RELATED SPENDING (2000-2023)



Source: UN Population Database and CBO. Retiree's share is calculated as the percent of the population aged over 65 relative to the population aged 20 and above, consistent with model specification. Mandatory spending for retirees include social security, medicare and medicaid.

fers—primarily due to rising expenditures on healthcare programs like Medicare and Medicaid, as well as Social Security—intensified the pressure on public finances.

VI. The impact of a permanent increase in public debt

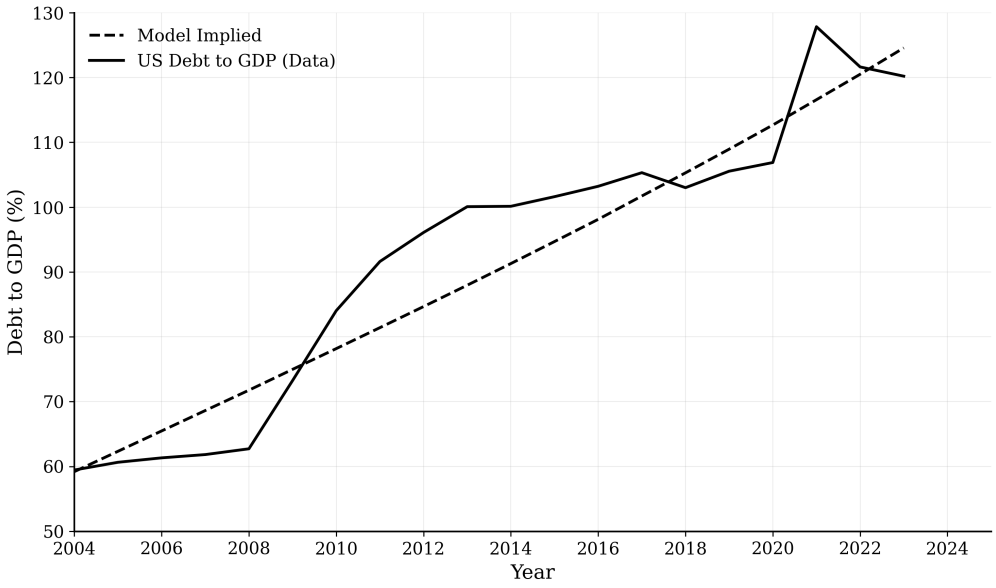
In the previous section, our analysis suggested that demographic factors alone cannot explain the increase in U.S. public debt since 2000. This section focuses on the other key determinants. Specifically, we begin by examining a baseline scenario with a stationary population distribution, where persistent deficits generated by higher government consumption lead to rising debt. We then incorporate the central policy shifts identified as significant. A detailed, model-based assessment of demographic factors is deferred to Section 7, where we confirm that they do not materially alter our main conclusions.

A. *The baseline case with a stationary population distribution*

Our starting counter-factual is deliberately stark. We take the fully calibrated general-equilibrium economy described above and impose a single policy change: from year 1 to year 20 the Federal government purchases an additional 3.5 percentage points of GDP in goods and services that yield

no direct utility. All other fundamentals—technology, demographics, preference parameters, and the detailed tax-transfer system—remain fixed at their data-consistent values. Deficits generated by this surge in "useless" spending are financed one-for-one with new Treasury issuance; from year 21 onward the purchase share reverts to its initial level and a proportional consumption tax stabilises the debt ratio at its new, higher plateau.[10]

FIGURE 8. DEBT PATH OVER 2004-2022



Note: Data refers to the total public debt as a percentage of gross domestic product from FRED, 2004-2022.

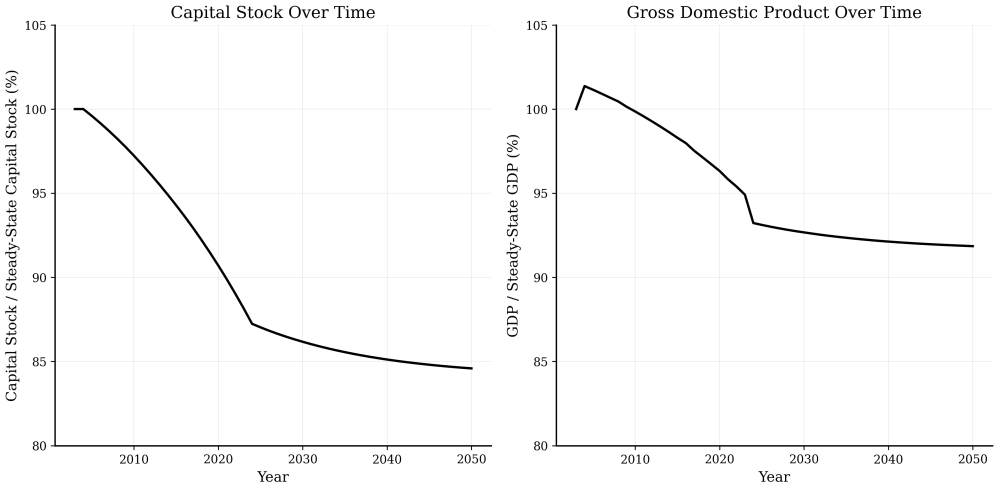
The simulated debt path (Figure 8) mimics the raw trend in U.S. public debt since 2000. We see this set-up as a controlled laboratory in which we can quantify how aggressively private capital and GDP must adjust when the government taps the same pool of household savings.

Over the first twenty years private capital falls by about 12 percent and GDP by just over 4 percent, with long-run declines of 15 and 8 percent, respectively (Figure 9).

According to our model, the observed increase in the debt-to-GDP ratio translates into a rise in the interest rate of 57 basis points. In contrast, the survey by Rachel and Summers (2019) suggests that a one-percentage-point increase in the debt-to-GDP ratio raises interest rates by 3 to 4 basis points. Based on these elasticities, we would expect the interest rate on public debt to increase by 180 to 240 basis points. This is 3 to 4 times larger than our model's baseline predictions. From this perspective, the crowding out effects we find may represent a lower bound of the actual effects. Part of the discrepancy could be due to the erosion of the convenience yield, an issue we will investigate later on.

[10]We hold monetary policy, risk premia, and other macro shocks constant so that any change in real allocations can be traced to the crowding-out channel alone.

FIGURE 9. GROSS DOMESTIC PRODUCT AND CAPITAL OVER THE TRANSITION, BASELINE



B. *The role of increasing transfers*

Payments to individuals were the single fastest–growing spending item after 2000, climbing by about four percentage points of GDP. A natural question is whether a reallocation of the additional deficit from "useless" government consumption (the baseline) to *transfers* changes the macroeconomic consequences of the debt surge. Because the incidence of transfers matters for savings behaviour, we consider first two mutually exclusive cases:

(a) **Transfers to workers**, disbursed proportionally to effective labour units;

(b) **Transfers to retirees**, disbursed proportionally to the size of the retired cohort.

In both experiments the path of the primary deficit is *identical* to that in the benchmark of Subsection 2.1: the government raises the deficit by 3.5 pp. of GDP for twenty years, finances it with debt, and stabilises the ratio thereafter with a consumption tax. The only difference is the *composition* of spending.

Accounting for behavioural responses. In the model workers choose labour, consumption and savings optimally. A transfer to workers reduces the marginal utility of after-tax labour income but simultaneously increases disposable resources, generating

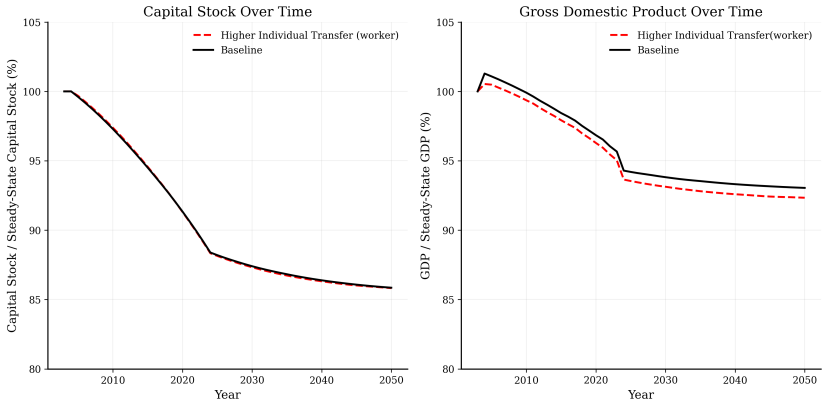
$$\text{(i) a wealth effect} \quad - \quad \text{(ii) a substitution effect.}$$

The former lowers labour supply while the latter raises precautionary savings. Transfers to retirees bypass the labour-leisure decision entirely; instead they alter workers' *intertemporal* savings motive because a larger share of old age consumption is now publicly financed.

Quantitative results

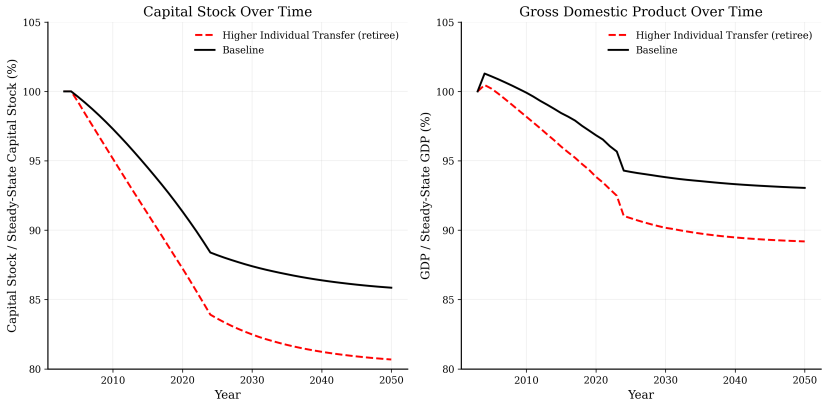
Transfers to workers. In this case, aggregate capital falls by an amount comprable to the benchmark path (Figure 10). The higher precautionary savings of working-age households nearly offset the crowding-out pressure from public debt. GDP nevertheless declines visible more on impact and remains below the benchmark thereafter, driven entirely by the labour-supply wealth effect.

FIGURE 10. CAPITAL AND GDP WITH TRANSFERS TO *workers* (PERCENT OF INITIAL STEADY STATE)



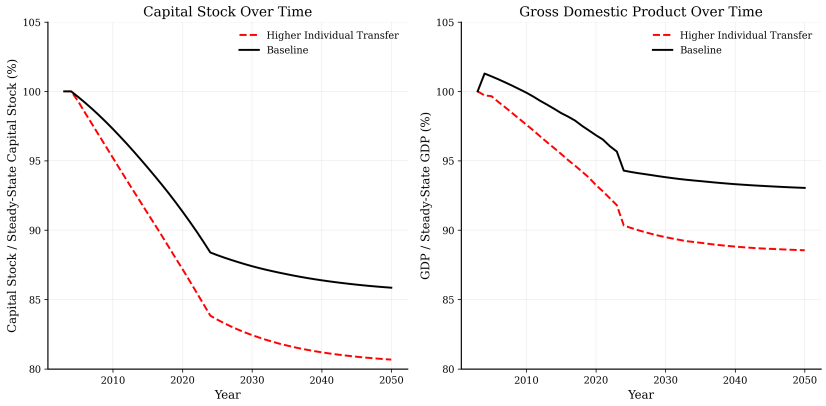
Transfers to retirees. When the same transfer stream is directed to retirees, the macro response is markedly different. By year 20 the capital stock is about 4 pp. *lower* than in the benchmark, and the long-run gap widens by 5 pp. (Figure 11). The mechanism is the suppression of workers' private retirement saving—public benefits reduce the need for self-insurance—so government dissaving is no longer counter-balanced.

FIGURE 11. CAPITAL AND GDP WITH TRANSFERS TO *retirees* (PERCENT OF INITIAL STEADY STATE)



Combined scenario. Because retirees receive roughly one third of transfers in the calibrated U.S. age structure, the aggregate path lies much closer to the retiree experiment (Figure 12). Crowding-out of private capital and the output cost of the debt build-up are much stronger in the long run than the baseline.

FIGURE 12. CAPITAL AND GDP UNDER THE COMBINED TRANSFER PROGRAMME (DEMOGRAPHIC-WEIGHTED AVERAGE)



Implications

Taken together, the experiments show that what matters for macro outcomes is not the scale of the deficit but *how* it is financed: shifting the same increased spending from non-productive purchases to transfers intensifies crowding out whenever those transfers weaken households' incentive to save. The amplification is modest when transfers accrue to workers—higher disposable income raises leisure but also precautionary saving—yet it becomes substantial when the funds go to retirees, whose public benefits almost fully displace private retirement saving. Accordingly, the benchmark that raises "wasteful" government consumption delivers the most conservative estimate of capital and output losses.

VII. Alternative specifications

In this section, we explore alternative specifications to test the robustness and extend the applicability of our main results. The first subsection investigates the impact of demographics. Then we study the effects of foreign ownership of U.S. public debt and illustrate how global demand for safe assets can affect domestic economic outcomes. The third subsection addresses the crucial policy question of financing mechanisms for public spending. Specifically, we compare the long-term effects on GDP between debt financing and tax financing. In this case, the study of welfare provides interesting insights. We then explore the implications of a lower convenience yield, driven by the increase in debt since 2000. Finally, we consider the possibility of using government debt proceeds to subsidize capital investments and counter crowding out.

A. *The Role of Demographics*

In this section, we quantify the effects of demographics. To keep the calibration exercise tractable, we maintain the assumption of a stationary period 0 population distribution and match the macro and micro data for such an economy. Demographic changes are introduced (unexpectedly) as follows. First, we introduce a permanent decline in mortality rates to match those in the data of 2023, which show substantial increases in old-age survival rates. The birth rate is also reduced from 1% to zero.

The labor supply adjusts endogenously in our model: individuals retire at age 65, their productivity evolves over their lifetime, and the number of hours devoted to work each period responds to economic conditions and policy changes. As the population ages, all of these forces will affect the average effective hours worked supplied.

In the first experiment, we hold all policy variables constant (at their period 0 values). As before, after 20 years we start adjusting consumption taxes to guarantee that debt is stable. The effects of demographics on the different categories of spending are summarized in Table 6. There we show the values of key government spending and expenditure categories in period zero, during the transition, and after the economy reaches the new balanced growth path. Demographics matter for government spending in terms of its effects on transfers to retirees. But this happens only gradually, and even after 20 years transfers to retirees have increased by only 2 percent of GDP.

In terms of macroeconomic effects, Figure 13 shows that with demographics only, debt to GDP would increase by about 13 percentage points, while GDP and capital would be reduced by more than 5 percentage points.

Hence, there are fiscal implications of demographics alone, but they are not enough to account for the observed changes in debt.

Our central focus is on understanding how higher public debt crowds out private activity. We now ask whether this crowding out mechanism is affected by demographic change. To this end, we proceed with an approach analogous to a difference-in-differences comparison.

In our earlier analysis, we established the first "difference": starting from a baseline scenario with no demographic change and no policy adjustments, we introduced higher government expenditure

TABLE 6—GOVERNMENT EXPENDITURES AND RECEIPTS (RATIOS TO GDP)

Component	Initial	$t = 5$	$t = 20$	$t = 50$	Final
Consumption of G&S	0.044	0.044	0.044	0.044	0.044
Transfers (Total)	0.353	0.358	0.372	0.406	0.406
To retirees	0.127	0.133	0.147	0.183	0.183
To workers	0.226	0.225	0.225	0.223	0.223
Interest paid	0.004	0.004	0.005	0.004	0.004
Expenditures	0.401	0.406	0.421	0.454	0.454
Tax revenues	0.383	0.383	0.402	0.439	0.439
Consumption taxes	0.045	0.045	0.064	0.100	0.100
Labor taxes	0.214	0.213	0.211	0.210	0.210
Dividend taxes	0.004	0.004	0.005	0.006	0.006
Corporate profits taxes	0.026	0.026	0.027	0.028	0.028
Noncorporate profits taxes	0.094	0.094	0.094	0.094	0.094

(leading to more debt) and compared the resulting outcomes. This initial comparison isolated the effects of policy changes on macroeconomic variables when demographics are held constant.

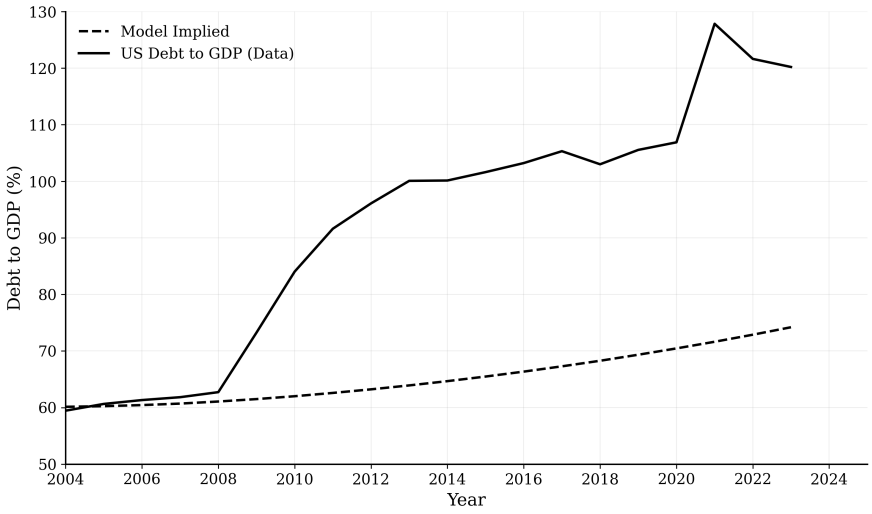
We now generate the second "difference" by introducing observed demographic changes. Specifically, we consider two economies: both have the evolving population distribution, but only one increases government consumption to achieve the same debt-to-GDP path as in the no-demographic case.

By contrasting these two differences, we assess whether demographic changes influence the mechanisms by which rising debt crowds out GDP and investment. In Figure 14, the black line represents our baseline findings from earlier sections, while the red line illustrates the differential effect of crowding out. The main finding is that GDP declines similarly in both cases, indicating that the increase in the debt-to-GDP ratio is the primary driver of the outcome, even when demographic factors are accounted for.

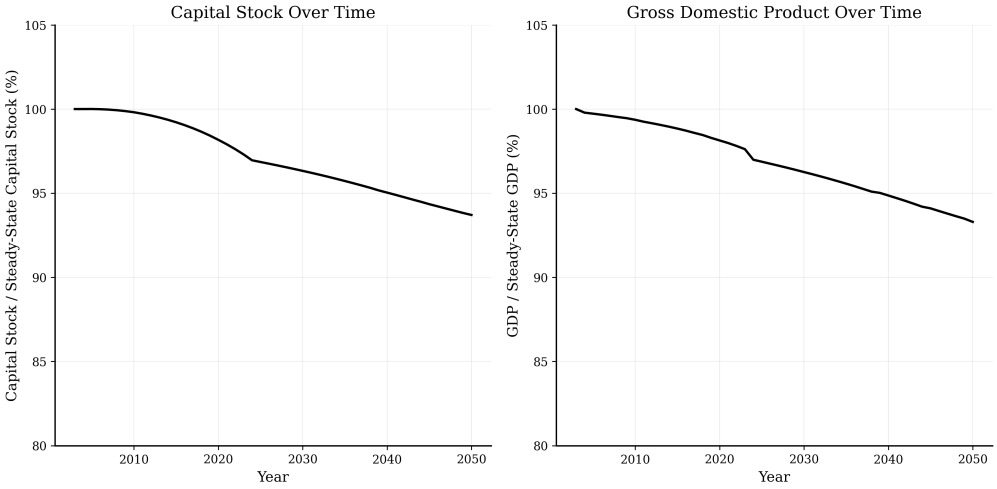
To investigate the distinct impacts of demographic changes on GDP, we repeated the experiment from Figure 14 under two separate conditions: increased longevity and declining birth rates. In the scenario in Figure 15, a fixed retirement age combined with increased longevity results in a longer expected life as a retiree, necessitating higher savings during working years, as evidenced by the red line slightly above the black line.

Conversely, Figure 16 examines the effect of a declining birth rate, where fewer new entrants to the labor market compared to retirees lead to reduced long-term savings. Despite these differences in savings behavior, the overall impact on GDP remains similar to the baseline results, suggesting that while demographic shifts influence individual savings and labor supply, their aggregate effect on GDP with the same debt path aligns with baseline projections.

FIGURE 13. DEMOGRAPHICS ONLY: MACROECONOMIC EFFECTS



(A) DEBT



(B) K AND GDP

Note: Panel (A) shows debt data only. Panel (B) displays combined figures for K and GDP, with demographic considerations.

B. *Foreigners also hold public debt*

To test the robustness of our quantitative findings, we conducted an alternative simulation in which foreign investors hold 30 percent of U.S. government debt, a value on the higher side or recent historical experience (see Figure 17).

FIGURE 14. GROSS DOMESTIC PRODUCT AND CAPITAL OVER THE TRANSITION WITH DEMOGRAPHIC CHANGES

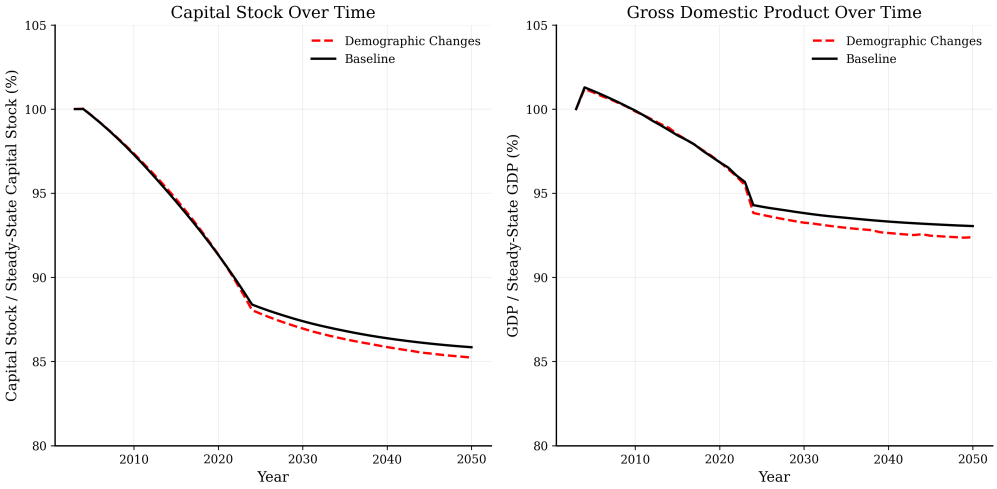
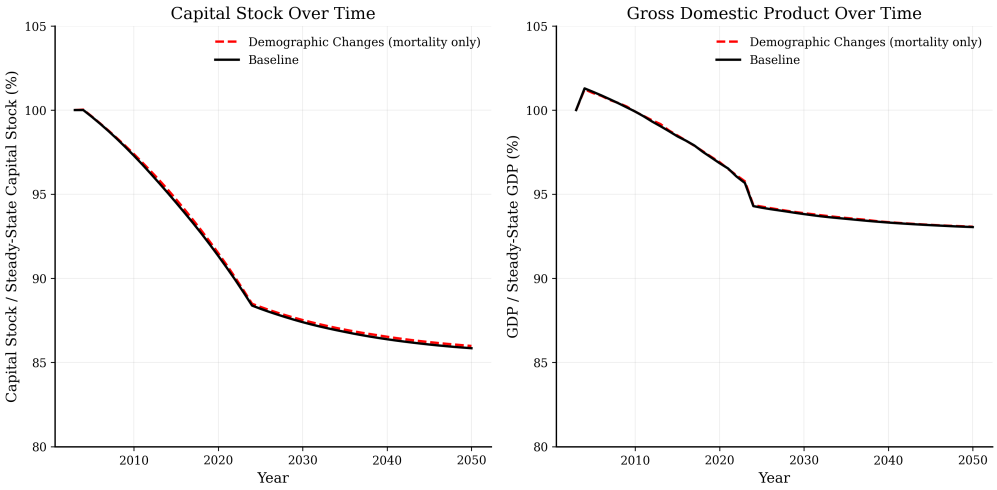


FIGURE 15. GROSS DOMESTIC PRODUCT AND CAPITAL OVER THE TRANSITION WITH DEMOGRAPHIC CHANGES, MORTALITY ONLY



In the model, We assume the foreign investors hold χ fraction of the government debt issued. So, the equilibrium conditions for the domestic capital market is

$$\sum_j n_t^j a_{t+1}^j = V_{I,t+1} + V_{T,t+1} + (1-\chi)B_{t+1}.$$

Where $V_{I,t+1}$ and $V_{T,t+1}$ represent the equity of tangible and intangible equity. For the simplicity, we assume the capital flow is equal to net exports. Therefore, the new resource balance constraint is

$$Y_t = C_t + X_{Tt} + X_{It} + G_t + NX_t,$$

FIGURE 16. GROSS DOMESTIC PRODUCT AND CAPITAL OVER THE TRANSITION WITH DEMOGRAPHIC CHANGES, BIRTH ONLY

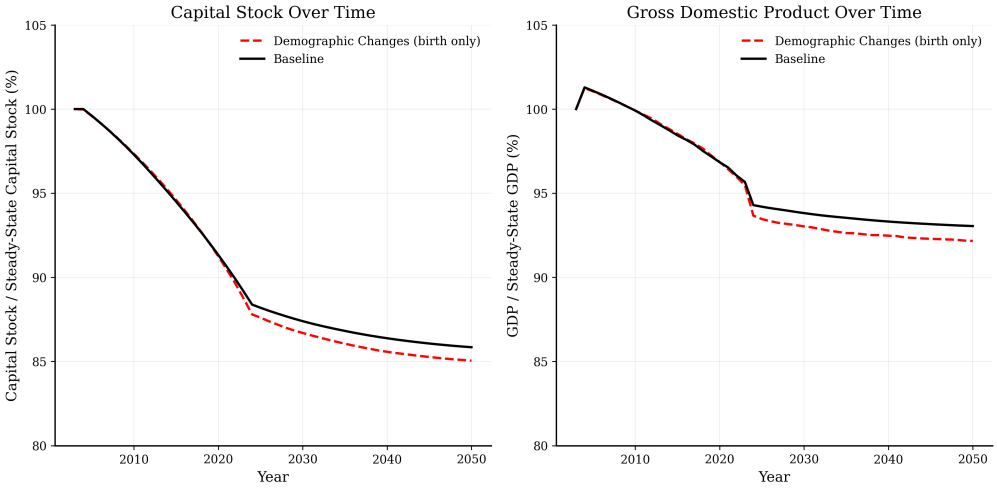
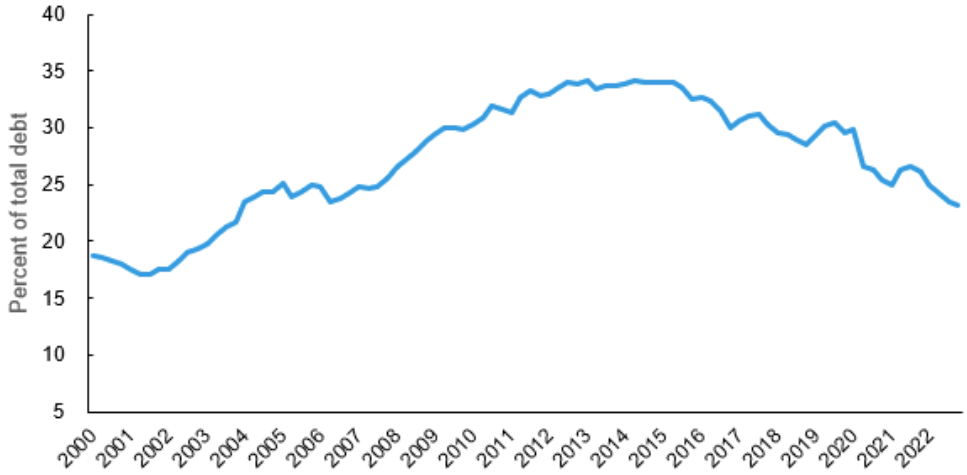


FIGURE 17. FEDERAL DEBT HELD BY FOREIGN AND INTERNATIONAL INVESTORS

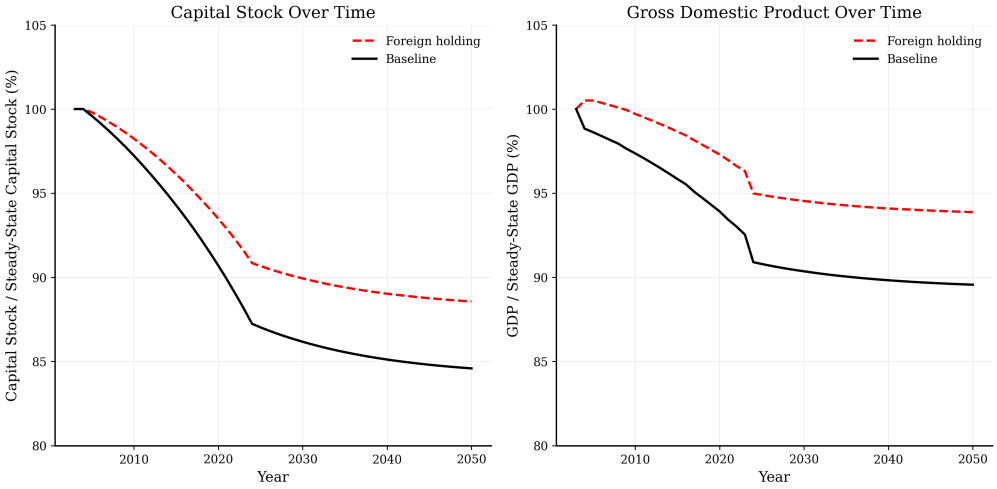


Source: Federal Reserve Economic Data by the Federal Reserve Bank of St. Louis.

At the equilibrium, $NX_t = \chi \left(B_{t+1} - B_t(1 + i_t^b) \right)$.

Under these modified conditions, the consequences for GDP are consistent with our initial findings but exhibit a smaller magnitude (see Figure 18). Specifically, GDP declines by about 4 percent in the initial 20-year period, and by slightly more over the long run. The crowding out effect is mitigated because a portion of newly-issued government debt is absorbed by foreign investors, reducing the domestic impact.

FIGURE 18. CAPITAL STOCK AND GROSS DOMESTIC PRODUCT OVER TIME



These results may still represent a lower bound on the crowding-out effects within open financial markets. This is because, by design, we fix the foreign share at 30 percent regardless of the overall debt level. Data, however, indicate a downward trend in foreign holdings.

Of course this analysis is not a substitute for a more comprehensive model capable of accurately capturing the global demand and supply dynamics of safe assets. However, the insights gleaned from the model underscore the importance of the elasticity of demand for safe assets as a key variable in quantifying the crowding out effect.

C. *Tax Financing vs Debt Financing*

It is interesting to consider, given the spending associated to our baseline experiments, whether debt financing or tax financing would be preferred. Our quantitative model is rich in detail. So, to address this question one has to take a stand on what tax to use. In the baseline calibration with log preferences, consumption taxes appear to be a natural choice. The reason is, as we will show, that consumption taxes do not distort output or capital accumulation. Higher debt simply causes a one to one crowding out of private consumption.

Consider then an arbitrary non-negative sequence $\{\Delta G_t\}_{t\geq 0}$. In period t the government

1. raises the flow of the pure public good from G_t^* to $G_t' = G_t^* + \Delta G_t$;

2. *finances* ΔG_t **exclusively** with the proportional consumption tax, choosing $\tau_{c,t}' > \tau_{c,t}^*$ so that the period budget remains balanced:

$$G_t' \;=\; \tau_{c,t}'\, C_t'.$$

   All other tax instruments and policy variables are left unchanged.

Proposition 1 (Output and capital neutrality of consumption-tax financed spending)**.** *Let*

$$\left\{c^*_{g,j}, \ell^*_{g,j}, a^*_{g,j}, K^*_{iTt}, K^*_{iIt}, w^*_t, i^{k*}_t, i^{b*}_t, \phi^*_t, \tau^*_{c,t}, G^*_t, B^*_t\right\}_{\substack{g\geq 0 \\ j=0,\ldots,J-1 \\ t\geq 0}}$$

be the baseline competitive equilibrium for the quantitative model. Define, for every period t*,*

$$\lambda_t \;=\; \frac{1+\tau^*_{c,t}}{1+\tau'_{c,t}} \;\in\; (0,1).$$

Construct the allocation:

1. ***Consumption:*** *for every household alive at* t *(*$j = t-g$*) set* $c'_{g,t-g} = \lambda_t\, c^*_{g,t-g}$*; for all other* (g,j) *keep* $c'_{g,j} = c^*_{g,j}$*.*
2. ***Labor and assets:*** *keep* $\ell'_{g,j} = \ell^*_{g,j}$ *and* $a'_{g,j} = a^*_{g,j}$ for all (g,j).
3. ***Technology and policy:***

$$K'_{iTt} = K^*_{iTt}, \quad K'_{iIt} = K^*_{iIt}, \quad w'_t = w^*_t, \quad i^{k'}_t = i^{k*}_t, \quad i^{b'}_t = i^{b*}_t, \quad \phi'_t = \phi^*_t.$$

Then, for every $t \geq 0$*,*

$$C'_t = C^*_t - \Delta G_t, \qquad Y'_t = Y^*_t,$$

and the primed objects constitute a competitive equilibrium under the new tax sequence $\{\tau'_{c,t}\}_{t\geq 0}$*.*

The proof is provided in the appendix. The basic intuition is that with the construction of consumption the Euler equations are unaltered and all other equilibrium conditions can be easily verified to hold.

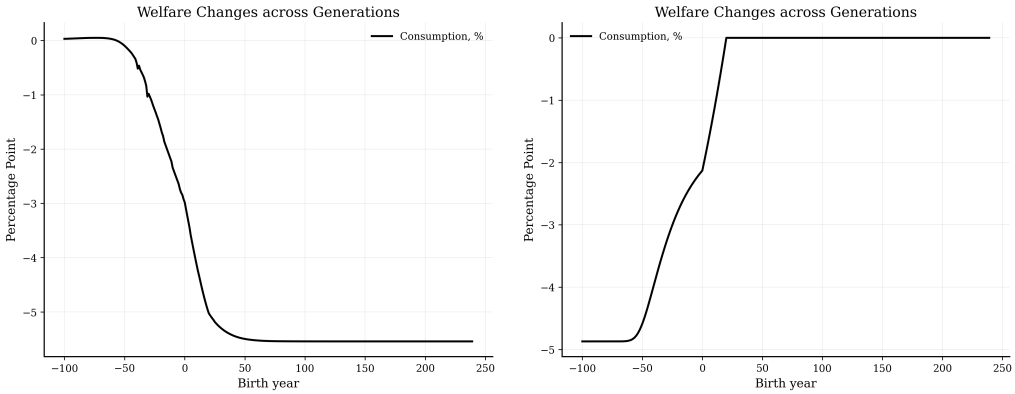
Hence, output would not be negatively affected by crowding out. In this particular experiment, analyzing welfare is still informative, as it highlights the intergenerational trade-offs inherent in different methods of financing the same amount of public expenditure.

Figure 19 summarizes the results. Here, the x-axis represents the birth year of each generation, while the line denotes the change in welfare relative to a baseline scenario in which no changes in spending occur. Welfare is measured as the permanent, constant change in consumption that individuals would require to remain indifferent (in terms of present discounted life-time utility) between the no-spending case and the financing method under consideration. A negative welfare change implies individuals are losing in the new scenario, whereas a positive change suggests they are better off in the new scenario.

The welfare results underscore the reversal of intergenerational trade-offs between the two financing methods. In the baseline scenario, where the expenditure is financed through debt, younger and future generations experience a loss in welfare. This is because the debt repayment is deferred, placing the burden on those born after the debt accumulation begins.

By contrast, when the expenditure is financed through consumption taxes, the burden falls primarily on older generations. The immediate implementation of taxes affects those currently alive, particularly older individuals, who have fewer remaining years to adjust their consumption patterns in response to the tax. Younger generations, having more time to smooth consumption over their lifetimes, are less affected.

FIGURE 19. WELFARE IMPACT UNDER ALTERNATIVE FINANCING SCHEMES

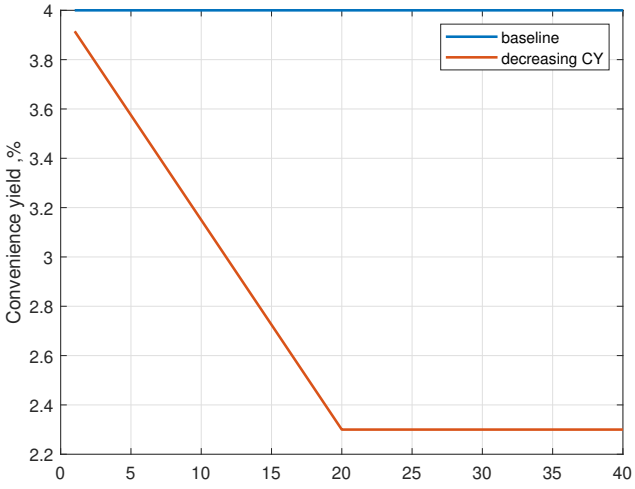


(A) BASELINE WELFARE (B) TAX FINANCING WELFARE

D. *Shrinking Convenience Yield*

Empirical evidence suggests that changes in Treasury supply influence the convenience yield on government debt. According to Krishnamurthy and Vissing-Jorgensen (2012), a one-standard-deviation change in the debt-to-GDP ratio (approximately 20 percentage points) alters the convenience yield by about 44 to 77 basis points. Scaling these estimates linearly for a 60 percentage point increase in the debt-to-GDP ratio suggests a decrease in the convenience yield of roughly 1.3 to 2.35 percentage points.[11]

FIGURE 20. CONVENIENCE YIELD

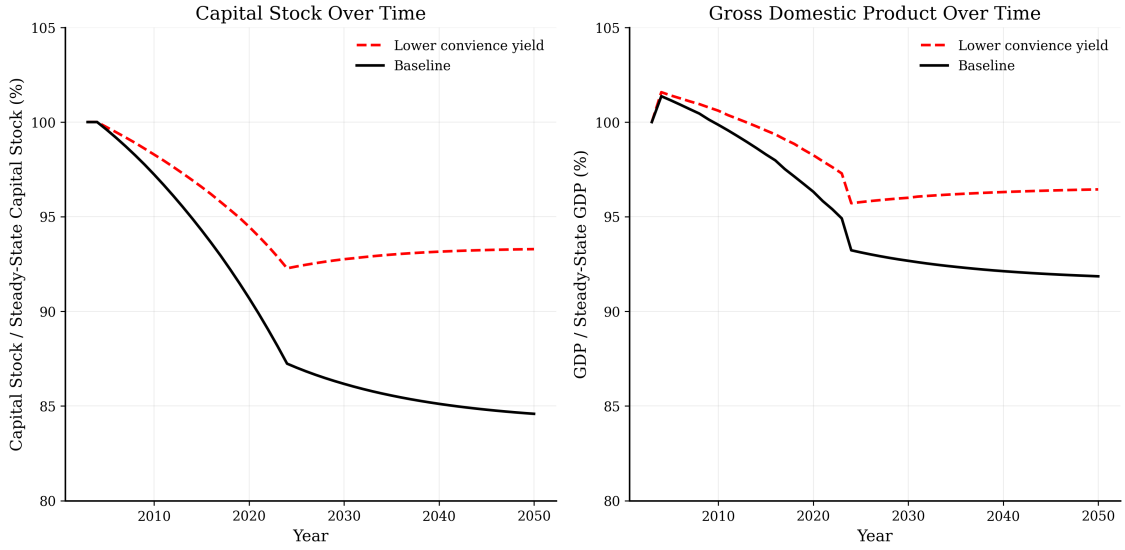


[11] A 20 percentage point change corresponds to 44–77 bps. Multiplying by three yields about 132–231 bps. These are clearly only rough approximations.

We pick a value in the middle of this range and reduce the convenience yield by 1.7 percentage points. Figure 20 shows that this reduction in convenience yield leads to higher interest payments for savers, potentially moderating crowding-out effects. However, as illustrated in Figure 21, the economy still experiences substantial crowding out, with GDP declining by about 5 percent over the long run.

Although the non-linearities in the relationship documented in Figure 1 of Krishnamurthy and Vissing-Jorgensen (2012) suggest that the actual decrease in convenience yield at high levels of debt (like the ones considered here) could be smaller, the conclusion that elevated debt continues to exert negative pressure on economic performance remains robust.

FIGURE 21. CAPITAL STOCK AND GROSS DOMESTIC PRODUCT OVER TIME



Note: The initial steady state GDP is normalized to 100.

E. *Subsidizing Capital*

Finally, we investigate whether issuing government debt can be Pareto improving when combined with a capital subsidy when $r - g$ is negative. The goal is to explore a scenario where we utilize the "free lunch" resulting from the negative $r - g$ to subsidize capital, aiming to achieve the Pareto-improving fiscal policies outlined by Aguiar et al. (2024). The necessary condition derived in that study for the existence of a Pareto-improving policy combination is that the revenue generated by the negative $r - g$ must be substantial enough to subsidize capital sufficiently to recover the old marginal product of capital (MPK).

In Table 7, we present how to check these necessary conditions for our data and calibration, and contrast them with their baseline "Constant-K Policy Transition." There, debt increasesm zero to 60 percent of GDP, the revenue generated by the negative $r - g$ is more than the cost to subsidize capital to recover the old MPK. Their paper considers many other scenarios, including raising debt

from 60 to 80 percent of GDP and also an exploration of the limits of debt in generating Pareto Improvements (concluding up to levels of 1.7 this would be possible).

In our baseline, the cost is more than the revenue. In our model, the capital-to-GDP ratio is twice as high as in their quantitative illustration, as we include both tangible and intangible capital. Therefore, the cost of subsidize capital is more than doubled. We also consider a version of our model where the change in the interest rate of government debt (the case with a reduced convenience yield) matches empirical estimates, and we also cannot find sufficient revenue to generate the Pareto Improvement.

There are three reasons that explain the differences between their simulations and our quantitative analysis. A first key difference is that the authors employ GHH preferences. Under their specification, this causes the savings supply to be very flat, in contrast with our log preference specification. Second, their paper considers positive mark-ups, which makes the supply of assets more responsive to the interest rates. These two factors taken together generate a low elasticity of the interest of government bonds.[12] This is critical to keep the cost of the subsidy low. However, such elasticity is too low relative to empirical estimates for the United States. The third difference between the two models is the presence of intangible assets in our analysis. Intangible assets are a key feature of modern economies. In the United States their value is large, and this feature raises the cost of the needed subsidy. Hence, although under some parameterizations a Pareto Improvement seems possible, it seems elusive for a quantitative analysis of the U.S. economy that matches its total assets and empirically plausible elasticities of the interest rate.

On the other hand, the revenue generated from issuing new debt when $r - g$ is negative is

$$-(r_{\text{new}} - g_{\text{new}})b_{\text{new}} + (r_{\text{old}} - g_{\text{old}})b_{\text{old}}.$$

The revenue in our model is also lower than in their results because our baseline debt starts at 60 percent of GDP. We find the revenue to be 0.75% of GDP,[13] which is insufficient to cover the cost of the required subsidies.

TABLE 7—INVESTIGATING PARETO IMPROVING POSSIBILITIES

Model	Δ r (%)	$\Delta \frac{B}{GDP}$	$\frac{K}{GDP}$	$r_{old} - g_{old}$ (%)	Cost (% GDP)	Revenue (% GDP)
Aguiar et al. (2024)	0.30	0-60	2.60	-1.70	0.78	0.84
Baseline	0.49	60-120	5.80	-2.25	2.84	0.75

Note: Aguiar et al. (2024) shows in their baseline experiment where debt to GDP increases from zero to 60 percent of GDP. In our baseline model, the debt to GDP increases from 60 percent to 120 percent of GDP.

It is, of course, possible to achieve a Pareto improving policy in our model under different parameter values. As hinted in the robustness check with shrinking convenience yield in Section D, we show that reducing the gap between the marginal product of capital (MPK) and the return on bonds will increase the return on the household asset portfolio and boost savings, thereby reducing costs compared to the baseline. Additionally, if the government uses the new financing to lower profit taxes, the decline in capital and output is also smaller (see Figure **??**).

[12] Our appendix discusses in detail the analytics behind these two differences

[13] $-(r_{\text{new}} - g_{\text{new}})b_{\text{new}} + (r_{\text{old}} - g_{\text{old}})b_{\text{old}} = -120\% \times (-1.75) - 2.25 \times 60\% = 0.75\%$

To further elaborate on this point, we conduct an experiment with capital subsidies. What sets this model apart from the one presented in the quantitative section is the inclusion of the subsidy (τ^P), representing a capital return subsidy on the return on capital. We maintain the assumption of a fixed convenience yield on the return on bonds. Consequently, the total subsidies amount to ($\tau^P \sum a$), which must be covered by the government. We fix the consumption tax rate at the level in the new baseline, where the debt-to-GDP ratio is zero.

$$(1+\tau_{ct})c + a'\sigma_t^j \leq (1+\boldsymbol{\tau^P})\left(\phi_t(1+i^b) + (1-\phi_t)(1+i^k)\right)a + y_t - T_t^j(y_t)$$

For purposes of illustration, we consider a different calibration and perform a steady-state comparison under the assumption that the growth rate is 5 percent instead of zero percent. In this scenario, with a 10 percent increase in the debt-to-GDP ratio, we manage to generate a Pareto-improving policy with a lower MPK and higher capital and GDP compared to the baseline. This is because the revenue generated by the negative $r - g$ is sufficient to cover the subsidy, making the Pareto-improving policy feasible.

VIII. Conclusion

This paper examined the macroeconomic costs of increasing levels of public debt within a general equilibrium overlapping generations framework, abstaining from considerations of default and uncertainty. Our focus has been on the crowding out mechanism, captured in a model that incorporates two assets. The key contribution of our paper is to quantify the impact of crowding out in a model adequate for the analysis of the U.S. economy. The point that crowding out can occur regardless of the sign of the interest growth differential has also not been stressed in the literature. In our model this is transparent.

An essential aspect of our analysis lies in the feasibility constraints that all agents, including the government, must adhere to. This adds another layer of complexity to the discussion around the sustainability of public debt. The endogeneity of both the real interest rate (r) and the economic growth rate (g) plays a pivotal role in our analysis. These variables are influenced by policy decisions, and their interaction with elevated public debt levels warrants more scrutiny.

By using a calibrated version of our model tailored to the United States, we uncover that the quantitative impact of increasing public debt can be substantial. In particular, the increase in public debt of about 60 percentage points of GDP since 2000 could lead to a permanent reduction in long-term GDP by about 8 percentage points. We consider alternative specifications of the model economy, allowing for example for external holdings (as in the data) of U.S. debt, or a shrinking convenience yield. The costs of higher debt remain large in those experiments. The elasticity of the interest rate on government debt of the model, an untargeted moment, is within the lower end of the values reported by the empirical literature, adding credibility to our estimates.

While the fiscal costs of public debt may appear manageable in a low-interest-rate environment, we illustrate that the macroeconomic costs may be quite significant. Our study calls for a nuanced understanding of public debt dynamics, especially in light of the endogenous nature of key economic indicators.

A Appendix: Patterns of Debt-to-GDP in the United States: An Analysis since 2000

A. *Introduction*

This Appendix considers the drivers of debt-to-GDP in the United States since 2000. The analysis reveals that the U.S. federal debt-to-GDP ratio increased almost 60 percentage points of GDP over this period.

B. *Cumulative Debt Decomposition*

Our point of departure is a standard period by period debt decomposition identity. We then extend it to study the cumulative change in debt over time. Hence, we provide a decomposition that aggregates the contributions of various factors—interest payments, inflation, economic growth, primary balances, and residuals—over multiple years, providing insights into the long-term drivers of changes in the debt-to-GDP ratio since 2000.

Consider the debt dynamics equation as derived in Escolano (2010), which expresses the change in the debt-to-GDP ratio as a function of interest rates, inflation, real GDP growth, the primary balance, and other residual factors. The equation is given by:

$$d_t - d_{t-1} = \frac{i_t}{1+\gamma_t} d_{t-1} - \frac{\pi_t}{1+\gamma_t} d_{t-1} - \frac{g_t}{1+g_t} d_{t-1} - PB_t$$

where:

- d_t is the debt-to-GDP ratio at time t,
- i_t is the nominal interest rate on government debt,
- γ_t is the nominal GDP growth rate,
- π_t is the inflation rate,
- g_t is the real GDP growth rate, and
- PB_t is the primary balance as a share of GDP.

This equation provides a point-in-time analysis of the factors influencing changes in the debt ratio. To evaluate the cumulative impact of these factors over a longer horizon, we extend this framework by summing the contributions across multiple years.

B..1 Cumulative Decomposition Methodology

The cumulative debt decomposition approach involves aggregating the yearly contributions of each factor over the period of interest. This allows us to capture the long-term effects of fiscal and economic variables on the debt-to-GDP ratio.

The cumulative change in the debt-to-GDP ratio from the initial year to year t can be expressed as:

$$\Delta d_t = \sum_{k=1}^{t} \left(\frac{i_k}{1+\gamma_k} d_{k-1} - \frac{\pi_k}{1+\gamma_k} d_{k-1} - \frac{g_k}{1+\gamma_k} d_{k-1} - PB_k + R_k \right)$$

Were we follow the same notation as before, and R_k accounts for residuals and other factors not captured by the main components.

This cumulative approach provides a comprehensive picture of how various factors interact and contribute to changes in the debt-to-GDP ratio over time, offering a long-term perspective on debt dynamics.

B..2 Empirical Findings

Table 8 summarizes the cumulative contributions of interest payments, inflation, real GDP growth, primary balances, and residuals to the change in the debt-to-GDP ratio since 2000.

TABLE 8—CUMULATIVE CONTRIBUTIONS TO DEBT-TO-GDP RATIO SINCE 2000

Year	$\frac{i_t}{1+\gamma_t} d_{t-1}$	$-\frac{\pi_t}{1+\gamma_t} d_{t-1}$	$-\frac{g_t}{1+\gamma_t} d_{t-1}$	$-PB_t$	R_t	d_t
2000						54.91
2001	3.15	-1.22	-0.52	-1.21	-0.58	54.53
2002	5.95	-2.07	-1.43	0.23	-0.88	56.72
2003	8.49	-3.15	-2.97	3.53	-1.80	59.01
2004	10.89	-4.64	-5.16	6.91	-2.70	60.20
2005	13.43	-6.39	-7.19	9.35	-3.48	60.63
2006	16.05	-8.17	-8.83	11.14	-3.93	61.17
2007	18.85	-9.74	-10.03	12.25	-4.40	61.84
2008	21.57	-10.89	-10.10	15.36	-3.23	67.61
2009	24.19	-11.31	-8.31	25.12	-2.57	82.03
2010	26.77	-12.26	-10.47	33.72	-2.78	89.90
2011	29.55	-14.08	-11.85	42.05	-5.93	94.64
2012	32.16	-15.80	-13.97	48.67	-7.22	98.75
2013	34.56	-17.42	-16.02	52.70	-9.69	99.05
2014	36.94	-19.03	-18.45	55.45	-8.75	101.06
2015	39.20	-19.91	-21.35	57.87	-11.68	99.04
2016	41.64	-20.87	-23.12	60.98	-9.63	103.91
2017	44.07	-22.67	-25.61	64.37	-12.05	103.03
2018	46.72	-24.92	-28.58	68.14	-12.38	103.90
2019	49.44	-26.61	-31.19	72.71	-14.01	105.24
2020	52.05	-27.99	-28.87	87.38	-11.49	125.98
2021	54.52	-33.21	-36.06	99.10	-19.38	119.87
2022	57.12	-40.96	-39.00	104.39	-17.88	118.58

The primary balance emerges as the most significant contributor to the increase in the debt-to-GDP ratio.

Interest payments have also had a substantial impact, contributing approximately 57 percentage points to the cumulative debt increase. The burden of servicing a growing stock of debt, particularly in a low-interest-rate environment, has added to the debt load.

Inflation and real GDP growth provided offsetting effects, reducing the cumulative debt increase by approximately 40.96 and 39.0 percentage points, respectively. These factors taken together add to 79.96 percent of GDP more than counteracting the upward pressure from interest payments. Finally, the residual, which adds up to negative -17.88.

C. *Analysis of Revenues and Expenditures*

Since primary balances were key drivers of the debt to GDP ratio, in this section we analyze the trends in government revenues and expenditures relative to GDP from 2000 to 2019. The data indicate that, despite substantial changes in statutory tax rates, particularly for corporations and high-income individuals, the overall revenue-to-GDP ratio remained relatively constant, fluctuating around 25% (following OECD tax revenue definitions). Expenditures were persistently larger than revenues over the period (thus explaining the persistent deficits), but some important long-lasting increases also took place.

C.1 Revenue Composition

The composition of federal revenues has fluctuated over time, but without any noticeable trend over the period we consider. Individual income taxes, corporate taxes, and social security contributions constituting the largest shares. The data show that:

- **Individual Income Taxes:** These have consistently represented the largest share of federal revenues, averageing around 10% of GDP. Despite changes in marginal tax rates, the overall contribution of individual income taxes remained stable, due to the offsetting effects of base-broadening measures. Indeed, data the the IRS Statistics of Income shows average tax rates, even by income percetile that oscillate without a definite trend.

- **Corporate Taxes:** The contribution of corporate taxes to GDP has declined over time, particularly after the TCJA of 2017, which reduced the corporate tax rate from 35% to 21%. This led to a reduction of almost 1 percentage point in the corporate tax revenue-to-GDP ratio.

- **Social Security Contributions:** These have remained stable, averageing around 6% of GDP. The stability is largely due to the relatively steady growth in employment and wages, which form the base for these contributions.

C.2 Base-Broadening Measures

One of the main reasons for the stability in tax revenues as a percentage of GDP has been the implementation of base-broadening measures, which effectively expanded the tax base by reducing or eliminating deductions, credits, and exemptions. Key measures include:

- **Limiting Itemized Deductions:** The reinstatement of the Pease limitation in 2013 reduced the value of itemized deductions for high-income earners, effectively increasing their taxable income.

- **Cap on State and Local Tax (SALT) Deduction:** The Tax Cuts and Jobs Act (TCJA) of 2017 capped the SALT deduction at $10,000, which limited the ability of taxpayers in high-tax states to reduce their taxable income by deducting state and local taxes. This measure broadened the tax base significantly for high-income earners in these states.

- **Elimination of Personal Exemptions:** The TCJA also eliminated personal exemptions, which previously allowed taxpayers to reduce their taxable income based on the number of dependents. This change further broadened the tax base, particularly affecting families with multiple dependents.

- **Alternative Minimum Tax (AMT) Adjustments:** Although the TCJA increased the exemption amount for the AMT, it maintained the AMT structure, ensuring that high-income earners continued to pay a minimum level of taxes, despite reductions in regular tax rates.

C..3 Spending Patterns

An analysis of government spending patterns since 2000 reveals a significant increase in federal outlays, rising from 17.7% of GDP in 2000 to 20.9% of GDP in 2019. This increase was primarily driven by changes in non-defense spending, with particular emphasis on social safety nets, healthcare, and interest payments.

Total Non-Defense Spending

The bulk of the increase in federal outlays came from non-defense spending, which grew from 14.80% of GDP in 2000 to 17.70% of GDP by 2019. This category includes various subcomponents such as payments for individuals, net interest, and other federal programs.

- **Payments for Individuals (Including Social Security)**: Payments for individuals, which include Social Security and Medicare, saw a substantial increase. This category rose from 10.6% of GDP in 2000 to 14.6% of GDP by 2019. This growth was largely driven by demographic shifts, such as the ageing population and the retirement of the Baby Boomer generation.

- **Net Interest Payments**: Net interest payments on the federal debt accounted for 2.2% of GDP in 2000 but decreased to 1.8% of GDP by 2019.

- **All Other Non-Defense Spending**: Other non-defense spending, which includes various federal programs, saw a decrease from 1.5% of GDP in 2000 to 0.9% of GDP by 2019.

- **All Other Grants**: This category, which includes federal grants not classified elsewhere, slightly decreased from 0.9% of GDP in 2000 to 0.8% of GDP by 2019.

- **Undistributed Offsetting Receipts**: This category saw a small decrease from -0.4% of GDP in 2000 to -0.5% of GDP in 2019.

Defense Spending

In contrast to the significant changes in non-defense spending, defense spending remained relatively stable as a percentage of GDP. National Defense expenditures started at 2.9% of GDP in 2000 and ended slightly higher at 3.2% of GDP by 2019. Despite fluctuations due to military operations in Iraq and Afghanistan in the early 2000s, the overall trend was one of stability.

Summary of Spending Trends

Overall, the data indicates that the growth in federal spending from 2000 to 2019 was driven primarily by increases in non-defense spending, particularly in payments for individuals, including Social Security and healthcare, as well as the changes in interest payments and other categories. These components were the main contributors to the persistent primary deficits observed during this period, which in turn drove the increase in the debt-to-GDP ratio.

TABLE 9—CONTRIBUTIONS TO THE CHANGE IN FEDERAL SPENDING AS % OF GDP

	2000	**2019**	**Change**
Expenditure Category	**(percent of GDP)**		
National Defense	2.90%	3.20%	+0.30%
Non-Defense Spending	**14.80%**	**17.70%**	**+2.90%**
Payments for Individuals (Including Social Security)	10.60%	14.60%	+4.00%
Net Interest Payments	2.20%	1.80%	-0.40%
All Other Non-Defense Spending	1.50%	0.90%	-0.60%
All Other Grants	0.90%	0.80%	-0.10%
Undistributed Offsetting Receipts	-0.40%	-0.50%	-0.10%
Total Outlays	**17.70%**	**20.90%**	**+3.20%**

B Appendix: Proof of Proposition 1

Step 1: Household feasibility. Fix a household from cohort g who is age $j = t - g$ in period t. The baseline period-t budget identity reads

(10)
$$(1+\tau^*_{c,t})\,c^*_{g,j} + a^*_{g,j+1}\sigma^j_t = \Big[\phi^*_t(1+i^{k*}_t) + (1-\phi^*_t)(1+i^{b*}_t)\Big]a^*_{g,j} + w^*_t\ell^*_{g,j}\epsilon^j - T^j_t\Big(w^*_t\ell^*_{g,j}\epsilon^j\Big) + b^*_t.$$

Inclusive expenditure unchanged. By construction $(1+\tau'_{c,t})\,c'_{g,j} = (1+\tau'_{c,t})\,\lambda_t\,c^*_{g,j} = (1+\tau^*_{c,t})\,c^*_{g,j}$, so the left-hand side of (10) is invariant when we keep $a'_{g,j+1} = a^*_{g,j+1}$.

Resources unchanged. Because capital stocks, wages, interest rates, portfolio weights, taxes on labour income, and the accidental-bequest rebate are unaltered, the right-hand side of (10) is identical across baseline and experiment.

Hence the individual budget constraint holds exactly in period t, and—since $a'_{g,j+1} = a^*_{g,j+1}$—it does so recursively for every subsequent age.

Step 2: Household optimality. With log utility the intra- and intertemporal FOCs are

$$(\mathrm{FOC}_1) \qquad \frac{\psi}{\ell_{g,j}} = \frac{w_t}{(1+\tau_{c,t})c_{g,j}},$$

$$(\mathrm{FOC}_2) \qquad \frac{1}{(1+\tau_{c,t})c_{g,j}} = \beta\,\sigma^j_t\,(1+r_{t+1})\frac{1}{(1+\tau_{c,t+1})c_{g,j+1}}.$$

Because $(1+\tau'_{c,t})c'_{g,j} = (1+\tau^*_{c,t})c^*_{g,j}$, both conditions remain valid with $\ell'_{g,j} = \ell^*_{g,j}$ and $a'_{g,j+1} = a^*_{g,j+1}$. For all other ages nothing changes; thus the original choice sequence continues to maximise lifetime utility.

Step 3: Firms and factor prices. Capital and labour inputs are identical to baseline in *both* sectors. Profit maximisation therefore yields the same wages and rental rate,

$$w'_t = w^*_t, \qquad i^{k'}_t = i^{k*}_t.$$

By Assumption **A3** the Treasury yield mechanically satisfies $i^{b'}_t = i^{k'}_t - \gamma_t = i^{b*}_t$, leaving the composite portfolio return and weight unchanged: $R^{a'}_t = R^{a*}_t$ and $\phi'_t = \phi^*_t$.

Step 4: Accidental bequests. Because every household's asset path $\{a'_{g,j}\}$ coincides with the baseline, the pool of assets left by deceased agents and the lump-sum rebate b_t are unchanged.

Step 5: Government budget constraint. Balanced-budget finance implies $G'_t - \tau'_{c,t}C'_t = 0$ in every period. All other cash flows (taxes on distributions, profits, labour income, bond service) coincide with baseline, so the period government budget remains satisfied and the path of public debt $\{B_t\}$ is unchanged.

Step 6: Market clearing and resource feasibility.

- *Financial market:* Households collectively hold $A'_t = A^*_t = B^*_t + K^*_{T1,t} + K^*_{T2,t} + K^*_{I1,t} + K^*_{I2,t}$ by construction, matching outstanding assets.
- *Labour and capital markets:* Inputs $\{L_{it}, K_{iTt}, K_{iIt}\}$ are unchanged, so factor markets clear.
- *Goods market:* Investment is identical to baseline. Therefore

$$Y'_t = C'_t + G'_t + X'_{Tt} + X'_{It} = (C^*_t - \Delta G_t) + (G^*_t + \Delta G_t) + X^*_{Tt} + X^*_{It} = Y^*_t.$$

Step 7: Conclusion. All private and public constraints, first-order conditions, and market clearing relations hold under the primed allocation. By standard arguments the competitive equilibrium is unique, hence the constructed objects *are* the new equilibrium. Aggregate output is unaffected, while aggregate consumption adjusts one-for-one with the public-good increment, completing the proof. □

C Appendix: A model with bonds in the utility function and an endogenous convenience yield

We consider a two-period OLG model where agents derive utility from consumption in both periods and from bond holdings:

$$U = \log(c_{1,t}) + \beta \log(c_{2,t+1}) + \chi \log(b_{t+1})$$

A young agent receives wage income w_t and chooses:

- Consumption when young: $c_{1,t}$
- Capital saving: k_{t+1}
- Bond saving: b_{t+1}

Budget constraints:

$$\begin{aligned} c_{1,t} + k_{t+1} + b_{t+1} &= w_t \quad \text{(young)} \\ c_{2,t+1} &= R^k_{t+1} k_{t+1} + R^b_{t+1} b_{t+1} \quad \text{(old)} \end{aligned}$$

Reparameterization: Total Saving and Portfolio Share

We reparameterize the problem in terms of total saving s_t and the portfolio share θ_t:

$$c_{1,t} + s_t = w_t, \quad \text{where } s_t = \text{total saving}$$

$$k_{t+1} = (1-\theta_t)s_t, \quad b_{t+1} = \theta_t s_t$$

$$c_{2,t+1} = R^k_{t+1}(1-\theta_t)s_t + R^b_{t+1}\theta_t s_t$$

Utility in Terms of s_t and θ_t

Substituting into utility:

$$U(s_t, \theta_t) = \log(w_t - s_t) + \beta \log\Big(s_t \cdot [R^k(1-\theta_t) + R^b\theta_t]\Big) + \chi \log(\theta_t s_t)$$

Grouping terms:

$$U = \log(w_t - s_t) + (\beta + \chi)\log(s_t) + \beta \log\Big(R^k(1-\theta_t) + R^b\theta_t\Big) + \chi \log(\theta_t)$$

To solve for optimal saving, treat θ_t as fixed and differentiate with respect to s_t.

First-Order Condition with Respect to s_t

$$\frac{\partial U}{\partial s_t} = -\frac{1}{w_t - s_t} + \frac{\beta + \chi}{s_t}$$

Setting the FOC to zero:

$$\frac{1}{w_t - s_t} = \frac{\beta + \chi}{s_t} \Rightarrow s_t = \frac{\beta + \chi}{1 + \beta + \chi} w_t$$

$$\boxed{s_t = \frac{\beta + \chi}{1 + \beta + \chi} w_t}$$

There is no interest rate in this saving decision because under log utility, the wealth and substitution effects exactly offset. When $\chi = 0$, the solution collapses to the baseline case without utility from bonds.

Optimal Bond Return

From the FOC with respect to θ_t, the return on bonds satisfies:

$$R^b_{t+1} = R^k_{t+1} \cdot \frac{\beta\theta_t - \chi(1 - \theta_t)}{(\beta + \chi)\theta_t}$$

We can express this more transparently as:

$$\boxed{R^b_{t+1} = R^k_{t+1} \cdot \left(1 - \frac{\chi}{(\beta + \chi)\theta_t}\right)}$$

This implies that:

- A higher χ (preference for bonds) leads to a larger convenience yield (lower R^b)
- A higher θ_t (larger bond holdings) reduces the convenience yield

Firm Problem

Assume a Cobb-Douglas production function with labor normalized to 1:

$$w_t = (1 - \alpha)K_t^{\alpha}, \quad R^k_t = \alpha K_t^{\alpha - 1} + 1 - \delta$$

Steady state

Let n be the population-growth rate and $b \equiv B_t/N_t = B_{t+1}/N_{t+1}$ the per-capita supply of bonds. Capital accumulation:

$$k = \frac{s}{1+n} - b, \qquad s = \frac{\beta+\chi}{1+\beta+\chi}(1-\alpha)k^{\alpha}.$$

Combine:

$$(1+n)\Big(k+b\Big) = \frac{\beta+\chi}{1+\beta+\chi}(1-\alpha)k^{\alpha}.$$

Denote $A \equiv \frac{\beta+\chi}{1+\beta+\chi}(1-\alpha)$.

$$\boxed{(1+n)\Big(k+b\Big) = A\,k^{\alpha}}$$

Comparative statics:

$$\boxed{\frac{dk}{db} = -\frac{1+n}{(1+n) - A\alpha k^{\alpha-1}} < 0.}$$

Comparative Statics with Respect to b

Differentiate both sides with respect to b:

$$(1+n)\frac{dk}{db} + 1 = A\alpha k^{\alpha-1} \cdot \frac{dk}{db}$$

Solving for $\frac{dk}{db}$:

$$\Big[(1+n) - A\alpha k^{\alpha-1}\Big]\frac{dk}{db} = -1 \quad \Rightarrow \quad \boxed{\frac{dk}{db} = -\frac{1}{(1+n) - A\alpha k^{\alpha-1}} < 0}$$

Hence, an increase in per-capita bond supply b reduces the steady-state capital per worker k. This implies:

- $k \downarrow$: capital per worker falls
- $R^k = \alpha k^{\alpha-1} \uparrow$: return on capital rises
- $\theta = \frac{b}{s} \uparrow$: bond share increases
- $R^b = R^k\left(1 - \frac{\chi}{(\beta+\chi)\theta}\right) \uparrow$: bond return rises

Conclusion: The same qualitative effects observed along the transition path also hold in the steady state. Higher bond supply crowds out capital, raises the return on capital, and raises the bond return through both R^k and reducing convenience yield.

3. Maximum sustainable debt

Solving for b gives

(DC-1) $$b(k) = \frac{A}{1+n}\, k^{\alpha} - \frac{n+\delta}{1+n}\, k, \qquad k > 0.$$

[Debt ceiling] $b(k)$ is strictly concave. Its unique maximum occurs at

() $$k_{\min} = \left[\frac{n+\delta}{A\alpha}\right]^{1/(\alpha-1)}, \qquad b_{\max} = b(k_{\min}),$$

and no larger per-capita debt can be sustained in steady state.

Proof. $b'(k) = \frac{A\alpha}{1+n}k^{\alpha-1} - \frac{n+\delta}{1+n}$ and $b''(k) = \frac{A\alpha(\alpha-1)}{1+n}k^{\alpha-2} < 0$. Setting $b'(k) = 0$ yields $k_{\min}$; strict concavity gives the maximum. □

4. Primary deficit and a debt–"Laffer" curve

4.1 Government budget. With a consumption tax τ and residual spending g, the steady-state *primary balance* is

(PB-1) $$\mathrm{PB} = g - \frac{\tau}{1+\tau}c = -(R^b - 1)\, b.$$

4.2 Returns.

$$R^b = R^k\Big[1 - \tfrac{\chi}{(\beta+\chi)\theta}\Big], \quad R^k = 1 - \delta + \alpha k^{\alpha-1}, \quad \theta = \tfrac{b}{s},\ s = Ak^{\alpha}.$$

4.3 Primary deficit as a function of k. Set

(D-1) $$\kappa = \frac{\chi}{\beta+\chi}\, A, \qquad f(k) = \kappa\, R^k(k)\, k^{\alpha}, \qquad D(k) = f(k)\, b(k).$$

Let $k^* > k_{\min}$ be the unique root of $b(k) = 0$ and define $I = (k_{\min}, k^*)$.

4.4 Unimodality of $D(k)$

Lemma 4. *With* $\Lambda(k) = \Big[\ln f(k) + \ln b(k)\Big]'$, *one has* $\Lambda(k_{\min}) > 0$, $\Lambda(k^*) < 0$, *and*

(L-1) $$\Lambda'(k) < 0 \quad \textit{for all } k \in I.$$

Proof. Write

$$f(k) = \kappa\Big[(1-\delta)k^{\alpha} + \alpha k^{2\alpha-1}\Big], \qquad b(k) = \frac{k}{1+n}\Big[Ak^{\alpha-1} - (n+\delta)\Big].$$

Elementary algebra gives[14]

$$\Lambda'(k) = -\frac{\alpha}{k^2} + \frac{\alpha(\alpha-1)(1-\delta)}{kR^k(k)} - \frac{(n+\delta)A\alpha(1-\alpha)}{(1+n)^2}\,\frac{k^{\alpha-2}}{b(k)^2},$$

and each term is strictly negative on I, proving (L-1). Since $b'(k_{\min}) = 0$ and $b(k_{\min}) > 0$, $\Lambda(k_{\min}) = f'(k_{\min})/f(k_{\min}) > 0$; because $b(k^*) = 0$ and $b'(k^*) < 0$, $\Lambda(k^*) < 0$. □

[Debt–Laffer curve] The primary deficit $D : I \to \mathbb{R}_+$ attains a *unique* maximum at some $k^{\#} \in I$. Precisely,

$$D'(k_{\min}) > 0, \quad D'(k^{\#}) = 0, \quad D'(k^*) < 0,$$

and D is strictly increasing on $(k_{\min}, k^{\#})$ and strictly decreasing on $(k^{\#}, k^*)$.

Proof. Because $D(k) > 0$ on I, $D'(k)$ and $\Lambda(k)D(k)$ have the same sign. Lemma 4 shows that Λ is strictly decreasing and crosses zero exactly once; hence D' does the same, giving the stated shape and the unique maximiser $k^{\#}$. □

[14] Derivatives use $0 < \alpha < 1 \Rightarrow \alpha - 1 < 0$ and $b(k) > 0$ on I.

D Appendix: Elasticities of Savings, of the Demand of Assets, and interest rate in models of the Aguiar et al. (2024) type

A. *Recursive preferences and household problem*

We will refer to our version of Aguiar et al. (2024) as AAA. The instantaneous utility is *GHH* in consumption and labour, $x_i(c,n) = c - v_i(n)$, and is embedded in the recursion:

$$V_t^i \;=\; \Big[(1-\beta)\, x_{it}^{1-1/\zeta} + \beta\Big(\mathbb{E}_t[V_{t+1}^{i\,1-\gamma}]\Big)^{\frac{1-1/\zeta}{1-\gamma}}\Big]^{1/(1-1/\zeta)}, \tag{11}$$

where ζ is the intertemporal elasticity of substitution (IES) and γ the coefficient of relative risk aversion.

Financial assets evolve as $a_{i,t+1} = (1+r_t)a_{i,t} + y_{it} - c_{it}$ with $a_{it} \geq \underline{a}$.

B. *The derivative of savings*

Let $\mu_{it} \equiv x_{it}^{-1/\zeta}$ denote marginal utility of the GHH composite under Epstein–Zin preferences (11). Define $\Psi_t^i \equiv \Big(\mathbb{E}_t[V_{i,t+1}^{1-\gamma}]\Big)^{\frac{1/\zeta-1}{1-\gamma}}$. The Euler equation of the households is

$$\mu_{it} = \beta\, \Psi_t^i\, \mathbb{E}_t\Big[V_{i,t+1}^{\frac{1/\zeta-\gamma}{1-\gamma}}\,(1+r_t)\,\mu_{i,t+1}\Big]. \tag{EZ-Euler}$$

Linearisation around the stationary distribution. Following Auclert etal. (2021, "sequence-space Jacobian") let $\hat{z}$ denote a *log-deviation* from the steady state, $\hat{z}_t = z_t/z_t$. Totally differentiating (EZ-EULER) with respect to the permanent interest-rate shift r gives for each individual

$$\hat{\mu}_{it} = \beta(1+r)\Psi^i\mathbb{E}\Big[\Theta_i\,\hat{\mu}_{i,t+1}\Big] \;+\; \beta\Psi^i\,(1+r)\,\hat{r}, \qquad \Theta_i \equiv \frac{V_i^{\frac{1/\zeta-\gamma}{1-\gamma}}\,\mu_{i,t+1}}{\mathbb{E}[V_i^{\frac{1/\zeta-\gamma}{1-\gamma}}\,\mu_{i,t+1}]}. \tag{12}$$

Solve forward, impose transversality, and use the envelope $\hat{c}_{it} = -\hat{\mu}_{it}/\zeta$ (GHH implies unit wealth effect on labour). The individual *marginal propensity to consume* out of r is

$$\frac{\partial c_i}{\partial r} = -\frac{c_i}{\zeta(1+r)} \;-\; \frac{(\gamma - 1/\zeta)}{\zeta} \underbrace{\sum_{s\geq 0} \beta^{s+1}(1+r)^{s+1}\mathbb{E}\Big[\Theta_i^{(s)}\, c_{i,t+s+1}\Big]}_{\Xi_i\,>\,0}.$$

Here $\Theta_i^{(s)}$ is the product of Θ_i along s future steps. The term Ξ_i captures prudence (prudence index $P \equiv \gamma - (1/\zeta)$ for EZ utility) times the stream of discounted covariance between future marginal utilities and consumption.

Asset derivative. Because $a_{i,t+1} = (1+r)a_{it} + y_{it} - c_{it}$ in steady state,

$$\frac{\partial a_i}{\partial r} = \frac{a_i}{1+r} - \frac{1}{1+r}\frac{\partial c_i}{\partial r}.$$

Summing over i and multiplying by population $(1+n)$ gives the aggregate-saving slope

$$A'(r) = \underbrace{\frac{\zeta}{r}A}_{\text{intertemporal-substitution term}} + \underbrace{\left(\gamma - \tfrac{1}{\zeta}\right)\sum_i \Xi_i}_{\text{precautionary term}}. \tag{13}$$

Properties of the precautionary term.

1. $\Xi_i > 0$ whenever there is *any* idiosyncratic risk or a binding borrowing constraint; the series converges because $\beta(1+r) < 1$ in equilibrium.
2. The coefficient $P \equiv \gamma - \frac{1}{\zeta}$ is the index of absolute prudence for Epstein-Zin preferences (Kimball–Weil 1992). Hence
$$\text{precautionary term} > 0 \iff P > 0 \text{ (i.e. } \gamma > 1/\zeta).$$
3. Tighter borrowing limits enlarge Ξ_i because more agents spend a larger fraction of life at the constraint, where $\partial a_i/\partial r$ is highest (Carroll 1997 "consumption ratchet").

Sign condition. Equation (13) implies

$$A'(r) > 0 \text{ iff } \gamma > \tfrac{1}{\zeta} \quad \text{or} \quad \gamma \le \tfrac{1}{\zeta} \text{ but borrowing limit sufficiently tight.} \tag{14}$$

This derivative feeds directly into the asset-market identity $\varepsilon_{r,b} = 1/[A'(r) - K'(r)]$ used throughout the rest of the appendix.

C. *Capital Demand*

Step 1: Capital demand with a mark-up. Firms operate a Cobb–Douglas technology $F(K,L) = K^{\alpha}L^{1-\alpha}$ and face a constant product-market mark-up $\mu > 1$. The FOC is

$$\mu(r+\delta) = \alpha\, k^{\alpha-1}, \qquad \Longrightarrow \qquad k(r) = \left(\tfrac{\alpha}{\mu(r+\delta)}\right)^{1/(1-\alpha)}.$$

Note that:

$$K'(r) = \frac{k}{r} = -\frac{k(r)}{(1-\alpha)(r+\delta)}\,\mu^{-1/(1-\alpha)}. \tag{15}$$

Step 2: Asset-market clearing and $\varepsilon_{r,b}$. In a steady state with the constant-K policy $A(r) = K_0 + B$. Totally differentiating and using $K'(r) = 0$ (capital is *by policy* held fixed):

$$A'(r)\, r = B \quad \Longrightarrow \quad \frac{r}{B} = \frac{1}{A'(r)}.$$

Step 4: Consistency check against the general formula. If we *do not* impose the capital subsidy, the general identity $\varepsilon_{r,b} = 1/[A'(r) - K'(r)]$ applies.

E Saving Supply and Capital Demand in a simplified version of Cao–Gaspar–Peralta (CGP)

We consider a simplified version of the model in our paper. The economy is closed and government bonds provide no direct utility.[15]

A. *Households*

A continuum of cohorts is born each period; each individual lives J periods and supplies one unit of labour when young. Preferences (period utility $\log c$) imply the age-s consumption Euler equation

$$\frac{1}{c_s} = \beta(1+r)\,\frac{1}{c_{s+1}}, \qquad s = 1, \ldots, J-1.$$

Let s_y be the single working period.[16] Budget constraints are

$$c_s + a_{s+1} = (1+r)a_s + \begin{cases} w, & s = s_y, \\ 0, & s \neq s_y, \end{cases} \qquad a_1 = 0,\ a_J = 0.$$

Optimal saving rule. Solving forward yields

$$a_{s_y+1} = \frac{\beta}{1 + \beta + \cdots + \beta^{J-1-s_y}}\, w \;\equiv\; \kappa\, w, \qquad 0 < \kappa < 1. \tag{16}$$

Rate independence. The fraction κ depends on (β, J) but *not* on r; however, the wage w itself is endogenous.

B. *Aggregate Saving Schedule*

Let n be the (constant) growth rate of newborn cohorts. Aggregate saving per effective worker equals

$$A(r) \;=\; (1+n)\,\kappa\, w(r), \qquad w(r) = (1-\alpha)\,k(r)^{\alpha}.$$

[15] The convenience-yield extension in CGP Section D is thus *excluded* here.

[16] Allowing for a full life-cycle earnings profile leaves the algebra unchanged but adds notation.

Differentiating:

$$A'(r) = (1+n)\kappa\, w'(r) = (1+n)\kappa(1-\alpha)\alpha k^{\alpha-1}\,\frac{k}{r}$$
$$= -\,\frac{(1+n)\kappa\alpha k^{\alpha}}{r+\delta}, \tag{17}$$

using $k/r = -k\Big[(1-\alpha)(r+\delta)\Big]^{-1}$ from the firm's FOC.

C. *Capital Demand*

The representative firm chooses K, L under perfect competition ($\mu = 1$):

$$r + \delta = \alpha k^{\alpha-1}, \qquad \frac{k}{r} = -\,\frac{k}{(1-\alpha)(r+\delta)}.$$

Hence

$$K'(r) \;=\; -\,\frac{k}{(1-\alpha)(r+\delta)}.$$

D. *Relative Slopes*

Dividing (17) by the absolute value of (C.) gives

$$\frac{|A'(r)|}{|K'(r)|} = \kappa\,\alpha(1-\alpha)\,k^{\alpha-1}. \tag{18}$$

For CGP's calibration ($\beta \simeq 1$, J large, so $\kappa \approx 0.5$; $\alpha = 0.30$; $k \approx 3$), one obtains $|A'|/|K'| \simeq 0.03$: *aggregate saving is roughly thirty times flatter than capital demand.*

Consequently

$$\frac{\partial r}{\partial b} = \frac{1}{A'(r) - K'(r)} \;\approx\; \frac{1}{-|K'(r)|},$$

F Empirical Benchmarks and What the Two Models Imply

A. *From theory to the statistic the data deliver*

Both AAA and CGP imply the same general form for the semi-elasticity of the long-run real interest rate with respect to the public-debt ratio,

$$\varepsilon_{r,b} \;\equiv\; \frac{\partial r}{\partial b} \;=\; \Big[A'(r) - K'(r)\Big]^{-1}, \tag{19}$$

As discussed in the paper, empirical studies estimate $\varepsilon_{r,b}$ directly; a representative range is

$$2 \text{ bp per pp} \;\le\; \varepsilon_{r,b}^{\text{data}} \;\le\; 4 \text{ bp per pp},$$

B. *What $\varepsilon_{r,b}$ is in AAA*

AAA's calibration targets a constant-K steady state with $B/Y = 0.60$, $k \approx 2.6$ and $r_0 = -1.7\%$. The bottom-left panel of their Figure 3 and the accompanying text (p. 3703) state that the *long-run* elasticity of aggregate assets with respect to the interest rate is 75, and that a 23 percent wealth increase comes with a 30bp rise in r. Hence

$$\varepsilon_{r,b}^{\text{AAA}} \;=\; \frac{30 \text{ bp}}{60 \text{ pp}} \;=\; 0.5 \text{ bp per pp.}$$

C. *What $\varepsilon_{r,b}$ is in CGP*

CGP's baseline experiment (closed economy, no bonds-in-utility) raises debt by 60 pp of GDP and reports a 57bp rise in the bond yield, implying

$$\varepsilon_{r,b}^{\text{CGP}} = \frac{57 \text{ bp}}{60 \text{ pp}} \;\approx\; 0.95 \text{ bp per pp.}$$

(See Cao–Gaspar–Peralta–Alva, Table 2 and Figure 2.)

D. *Fiscal arithmetic of the AAA constant-K policy*

Using AAA numbers in their baseline simulations ($k = 2.6$, $r_0 = -1.7\%$, $\Delta B/Y = 0.60$) the net budget effect of the constant-K policy is

$$\text{Net}(\varepsilon) = \underbrace{-\big[r_0 + \varepsilon\,\Delta B\big]\frac{B'}{Y}}_{\text{seigniorage}} \;-\; \underbrace{\varepsilon\,\Delta B\,k}_{\text{capital subsidy}}\,, \qquad B'/Y = 0.60. \tag{20}$$

TABLE 10—BUDGET BALANCE AT ALTERNATIVE SEMI-ELASTICITIES

$\varepsilon_{r,b}$ (bp/pp)		Δr (bp)	r' (%)	Seign. (% GDP)	Net (% GDP)
0.50	(AAA)	30	-1.40	$+0.84$	$+0.06$
2.00	(data min)	120	-0.50	$+0.30$	-2.82
3.00	(data mid)	180	$+0.10$	-0.06	-4.74
4.00	(data max)	240	$+0.70$	-0.42	-6.66

At AAA's own elasticity the policy just balances; at any data-consistent $\varepsilon_{r,b}$ it runs deficits of 3–7 %GDP.[17]

[17] Steady-state seigniorage values for the constant-K experiment can be cross-checked in Online Appendix D.2, Table 2, which reports $r' = -1.4\%$ when $B/Y = 0.60$ and $K/Y = 2.6$.

E. *Why the gap arises*

- **AAA**: idiosyncratic risk and IES ≈ 1 produce $A'(r) > 0$ so large that the denominator of (19) almost vanishes, while $\mu = 1.4$ flattens $K'(r)$ further. Result: $\varepsilon_{r,b} = 0.5$ bp/pp.

- **CGP**: with a representative agent $A'(r)$ is tiny and *negative*; perfect competition ($\mu = 1$) keeps $K'(r)$ steep. Result: $\varepsilon_{r,b} \approx 0.9$ bp/pp— still below but far closer to the 2–4 bp/pp corridor.

REFERENCES

Acharya, Sushant and Keshav Dogra, "The Side Effects of Safe Asset Creation," *Journal of the European Economic Association*, April 2022, *20* (2), 581–625.

Aguiar, Mark and Manuel Amador, *The Economics of Sovereign Debt and Default*, Princeton University Press, December 2021.

_ , _ , and Cristina Arellano, "Micro Risks and (Robust) Pareto Improving Policies," *American Economic Review*, 2024.

_ , Satyajit Chatterjee, Harold Cole, and Zachary Stangebye, "Self-Fulfilling Debt Crises, Revisited," *Journal of Political Economy*, May 2022, *130* (5), 1147–1183. Publisher: The University of Chicago Press.

Bell, Felicitie C and Michael L Miller, *Life tables for the United States social security area, 1900-2100* number 120, Social Security Administration, Office of the Chief Actuary, 2005.

Blanchard, Olivier, "Public Debt and Low Interest Rates," *American Economic Review*, April 2019, *109* (4), 1197–1229.

_ , *Fiscal Policy under Low Interest Rates*, The MIT Press, 2023.

Braun, R. Anton and Douglas H. Joines, "The implications of a graying Japan for government policy," *Journal of Economic Dynamics and Control*, August 2015, *57*, 1–23.

Brunnermeier, Markus K, Sebastian A Merkel, and Yuliy Sannikov, "Debt as Safe Asset," Working Paper, National Bureau of Economic Research 2022.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas, "The Safe Assets Shortage Conundrum," *Journal of Economic Perspectives*, August 2017, *31* (3), 29–46.

Cole, Harold L. and Timothy J. Kehoe, "Self-Fulfilling Debt Crises," *The Review of Economic Studies*, 2000, *67* (1), 91–116.

Gomme, Paul, B. Ravikumar, and Peter Rupert, "The Return to Aggregate Capital and the Business Cycle," *Review of Economic Dynamics*, April 2011, *14* (2), 262–278.

_ , _ , and _ , "Replication Data and Python Code for "The Return to Aggregate Capital and the Business Cycle"," https://paulgomme.github.io/ July 2022. Data and Python code, updated July 2022.

IMF, "World Economic Outlook (October 2023) -Rate of Interest: Drivers and Implications for Policy," *IMF World Economic Outlook*, 2023.

Kitao, Sagiri, "Fiscal cost of demographic transition in Japan," *Journal of Economic Dynamics and Control*, 2015, *54*, 37–58.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 2012, *120* (2), 233–267.

McCandless, George T and Neil Wallace, *Introduction to Dynamic Macroeconomic Theory: an Overlapping Generations Approach*, Harvard University Press, 1991.

McGrattan, Ellen R. and Edward C. Prescott, "On Financing Retirement with an Aging Population," *Quantitative Economics*, 2017, *8* (1), 75–115. https://onlinelibrary.wiley.com/doi/pdf/10.3982/QE648.

Mian, Atif R, Ludwig Straub, and Amir Sufi, "The Saving Glut of the Rich," Working Paper, National Bureau of Economic Research 2020.

Reis, Ricardo, "The Constraint on Public Debt when r<g but g<m," *CEPR Discussion Papers*, 2021, *15950.*

Łukasz Rachel and Lawrence H. Summers, "On Secular Stagnation in the Industrialized World," Working Paper, National Bureau of Economic Research August 2019.