Proposal: Tariffs and Global Inflation

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Recent policy developments have reignited debates over the inflationary consequences of tariffs. In a 2025 speech, Federal Reserve Chair Jerome Powell acknowledged the considerable uncertainty surrounding the inflationary impact of tariffs, noting that while they are likely to cause a temporary increase in inflation, their persistence and broader macroeconomic effects remain unclear. In response, the Federal Reserve has adopted a "wait-and-see" approach, postponing any reduction in policy rates until there is greater clarity regarding the extent to which tariff policy will translate into sustained inflationary pressures. At a conceptual level, tariffs function as cost-push shocks, raising the price of imported goods and potentially triggering stagflationary outcomes—higher inflation accompanied by lower output (Guerrieri et al., 2025). A central policy question, therefore, is: What determines the magnitude and persistence of the inflationary effects of tariffs, both domestically and globally?

This project seeks to evaluate the dynamic effects of tariffs on global inflation using a three-country dynamic stochastic general equilibrium (DSGE) model, building on Camara, Christiano, and Dalgic (2024). The model incorporates financial frictions, international production linkages, and heterogeneous monetary policy responses across countries. Our central hypothesis is that the inflationary consequences of tariffs are shaped primarily by (i) the persistence of the tariff shock, (ii) the elasticity of substitution between domestic and foreign goods in both the short and long run, and (iii) the stance of monetary policy in the United States and abroad.

To our knowledge, this is one of the first studies to systematically analyze the interaction between tariffs, inflation dynamics, and financial frictions in a multi-country DSGE framework. The project contributes to ongoing policy and academic debates concerning the appropriate monetary policy response to protectionist trade measures. Our analysis builds on the theoretical framework developed by Camara et al. (2024), which we extend to incorporate tariffs and global value chain linkages. The model features three regions: (i) the United States, (ii) an advanced economy (AE), and (iii) an emerging market economy (EME), each characterized by financial frictions and nominal rigidities.

The model includes several key features. First, financial frictions follow Christiano et al. (2011), with balance sheet effects modeled through a costly state verification framework as in Bernanke et al. (1999). Entrepreneurs in EMEs borrow in U.S. dollars, rendering them particularly sensitive to exchange rate fluctuations. Second, tariffs are modeled as exogenous shocks to the price of imported intermediate goods, with both transitory and persistent processes, following Schmitt-Grohé and Uribe (2025). Third, the elasticity of substitution between domestic and foreign goods is endogenous, following Erceg et al. (2006). Fourth, monetary policy follows a Taylor-type rule, allowing for heterogeneity in policy frameworks across countries.

We plan to calibrate the model to match key macroeconomic ratios and trade elasticities. We then simulate the effects of a tariff shock under alternative assumptions about its persistence, the elasticity of substitution, and the monetary policy rules in place in each country. A detailed description of the model is included as an appendix to this proposal.

This project contributes to several strands of literature. First, it builds on recent work by Monacelli (2025), Bianchi and Coulibaly (2025), and Auclert et al. (2025), who emphasize the critical role of monetary policy in mediating the inflationary impact of tariffs. Our framework integrates these insights into a global model with financial frictions and cross-country linkages. Second, we contribute to the literature on monetary policy under supply shocks (Werning et al., 2025; Guerrieri et al., 2025), showing that the inflation-output trade-off is significantly worsened when tariffs are persistent and financial vulnerabilities are present. Third, we highlight the importance of heterogeneous monetary policy responses, in line with findings by Sebnem et al. (2025). Finally, the model is calibrated to reflect key features of the 2025 U.S. tariff episode, including the depreciation of the dollar and the initially muted response of inflation expectations.

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2 Model

2.1 Household in Country j

There are J countries indexed by $j \in [1, 2, ..., J]$, where the $j = 1$ corresponds to the United States. Countries are distinguished by the values of utility and technology parameters, but not by their basic structure. So, here we describe the j^{th} economy.

We suppose that there is a continuum of households indexed by $h \in (0, 1)$. Wages are sticky in the model, following the lead in Erceg et al. (2000).[1] In our model each worker's labor is differentiated in the sense of the intermediate good suppliers in the sticky price model of Calvo (1983). In particular, the h^{th} household is a monopolistically competitive supplier of $\ell_{jt}(h)$ units of labor. Following Calvo (1983) there is a household-specific idiosyncratic shock that determines whether the household can re-optimize its wage. Complete markets ensure that the household experience no income effect from the realization of this shock and for this reason we do not index the non-labor variables in the household problem by h.

The h^{th} representative household's preferences are:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(C_{jt} - b_j \bar{C}_{j,t-1}\right) - \frac{\exp(\tau_j)\, \ell_{j,t+i}(h)^{1+\varphi}}{1+\varphi} - h_{j,t}(\Theta_{jt}) + \varrho_{j,t} \log\left(\frac{S_{jt} M^{\$}_{jt}}{Z_t P_{jt}}\right) \right\}, \tag{1}$$

where $\ell_{j,t}(h)$ denotes employment, $C_{j,t}$ denotes consumption and $\Theta_{j,t}$ denotes the households share of financial wealth held in dollars. The term, $\bar{C}_{j,t-1}$, denotes last period's aggregate consumption. Also, $M^{\$}_t$ denotes end-of-period t holdings of US zero-interest-rate US dollars ('cash'). We assume that the number of households is n_j and express all variables in country j per capita terms. In addition to holding US dollars in cash, households also hold a portfolio of interest bearing bonds in the domestic currency and in dollars. The share of dollars in their portfolio is $\Theta_{j,t}$ and $h_{j,t}(\Theta_{j,t})$ denotes a non-pecuniary cost of deviating from a target, $\Upsilon_{j,t}$, for $\Theta_{j,t}$.[2] In particular,

$$\Theta_{j,t} \equiv \frac{S_{jt} D^{\$}_{j,t}}{S_{jt} D^{\$}_{j,t} + D_{j,t}}, \quad h_{j,t}(\Theta_{j,t}) = \frac{\gamma}{2}\left(\Theta_{j,t} - \Upsilon_{j,t}\right)^2, \Upsilon_{jt} = \Upsilon_j + \gamma_{j,R^{\$}}\left(R^{\$}_{d,t} - R^{\$}\right) + \gamma_{j,\Theta}\left(\bar{\Theta}_{j,t} - \bar{\Theta}_{j,t-1}\right) + e^{\Upsilon}_{jt}. \tag{2}$$

Here, Υ_{jt} denotes the time t target portfolio share of dollars, and Υ_j denotes the value of that target in steady state. We do not explicitly model how this target is determined.[3] The term involving $\bar{\Theta}_{j,t} - \bar{\Theta}_{j,t-1}$ with *portfolio inertia parameter*, $\gamma_{j,\Theta} < 0$, is a reduced form representation of adjustment costs in adjusting portfolios. Such adjustment costs were proposed by Bacchetta et

[1] Although the equilibrium conditions are the same, we follow the slightly different interpretation of wage stickiness described in Christiano et al. (2005).

[2] The variable, h_{jt} should not be confused with h, which indexes the household.

[3] Christiano et al. (2021) derives Υ_j endogenously from hedging considerations.

al. (2022) to explain features of capital flows and asset prices. The object, $e^{\Upsilon}_{j,t}$, is a 'noise' shock to the household's target, which has a first order autoregressive representation. Also, the *risk appetite parameter,* $\gamma_{j,R^\$} > 0$, captures the idea that when $R^\$_{d,t}$ is high people prefer dollar assets, perhaps because of a decrease in 'risk appetite', or 'flight to safety'. The *target portfolio cost parameter* is given by $\gamma > 0$. The variable, $D^\$_{jt}$, denotes the household's end-of-period t holdings of dollar assets which pay $R^\$_{d,t}$ gross interest in dollars in period $t+1$. The variables, S_{jt} and D_{jt} denote the number of LCU per dollar and end-of-period t holdings of LCU domestic assets (here, $S_{U,t} \equiv 1$). The latter pay $R_{jd,t}$ gross interest in LCU in period $t+1$. The household's flow budget constraint is:

$$\begin{aligned} & S_{jt}\left(D^\$_{jt} + M^\$_{jt}\right) + D_{jt} + P^c_{jt}C_{jt} + E_t Q_{jt,t+1}a_{jt,t+1} \quad (3) \\ = S_{jt}R^\$_{d,t-1}D^\$_{jt-1} + R_{jd,t-1}D_{jt-1} + S_{j,t}M^\$_{j,t-1} + W_{jt}(h)\,\ell_{jt}(h) + \Pi_{jt} - T_{jt} + a_{jt-1,t} + \Xi_t(h). \end{aligned}$$

Here, $a_{t,t+1}$ denotes one-period-ahead Arrow securities markets in which the household trades with country j banks (here, we follow Castillo et al. (2024)). Also, $Q_{t,t+1}$ denotes the period $t+1$ state-contingent price of an Arrow security (suitably scaled by probability).[4] For additional discussion of this market, see section (2.3). Households in the US, $j = 1$, also receive a helicopter drop of money, $\Delta M^{\$,supply}_t (\equiv M^{\$,supply}_t - M^{\$,supply}_{t-1})$, from the US government during period period t. People in the other countries do not receive a helicopter drop.

Because the worker has some market power in the model, its wage is on average a markup, $\lambda_j/(\lambda_j - 1)$, over its marginal cost. The probability that the worker can change its wage is $1-\theta_j$. In the probability θ_j case that the worker cannot re-optimize, its wage is indexed to inflation and TFP growth:

$$W_{jt}(h) = W_{jt-1}(h)\,\tilde{\pi}_{w,t}, \quad (4)$$

where the degree of indexation, $\tilde{\pi}_{w,t}$, is given by:

$$\tilde{\pi}_{w,t} \equiv \exp\left(\iota^a_j \Delta a + \left(1-\iota^a_j\right)\Delta a_t\right)\left(\pi^c_{jt-1}\right)^{\iota_j}\left(\pi^c_j\right)^{1-\iota_j},\ \iota^a_j, \iota_j \in [0,1]. \quad (5)$$

Here, π^c_{jt} denotes the gross rate of inflation in the price of consumption from $t-1$ to t and π^c_j is the associated steady state value. This specification implies that if realized inflation rises, then (as long as $\iota_j > 0$) there will be dynamic upward pressure over time on costs, and hence, realized inflation. Also, Δa_t and Δa correspond to aggregate realized and steady state growth rates of TFP, respectively (see sections 2.2.1 and 2.5.5).

Because the idiosyncratic Calvo shock and $W_{jt}(h), \ell_{jt}(h)$ are assumed to be observable and the Calvo shock is iid across $h \in (0,1)$, it is possible to have an insurance market so that in equilibrium, $W_{jt}(h)\,\ell_{jt}(h) + \Xi_t(h)$ is independent of h and and $\int_h \Xi_{jt}(h)\,dh = 0$. Additional

[4]The scaling is conventional. The idea is that $E_t Q_{jt,t+1}a_{jt,t+1}$ is a sum of prices and quantities over all the $t+1$ states of nature, conditional on t. This sum involves no probabilities, only prices and quantities.

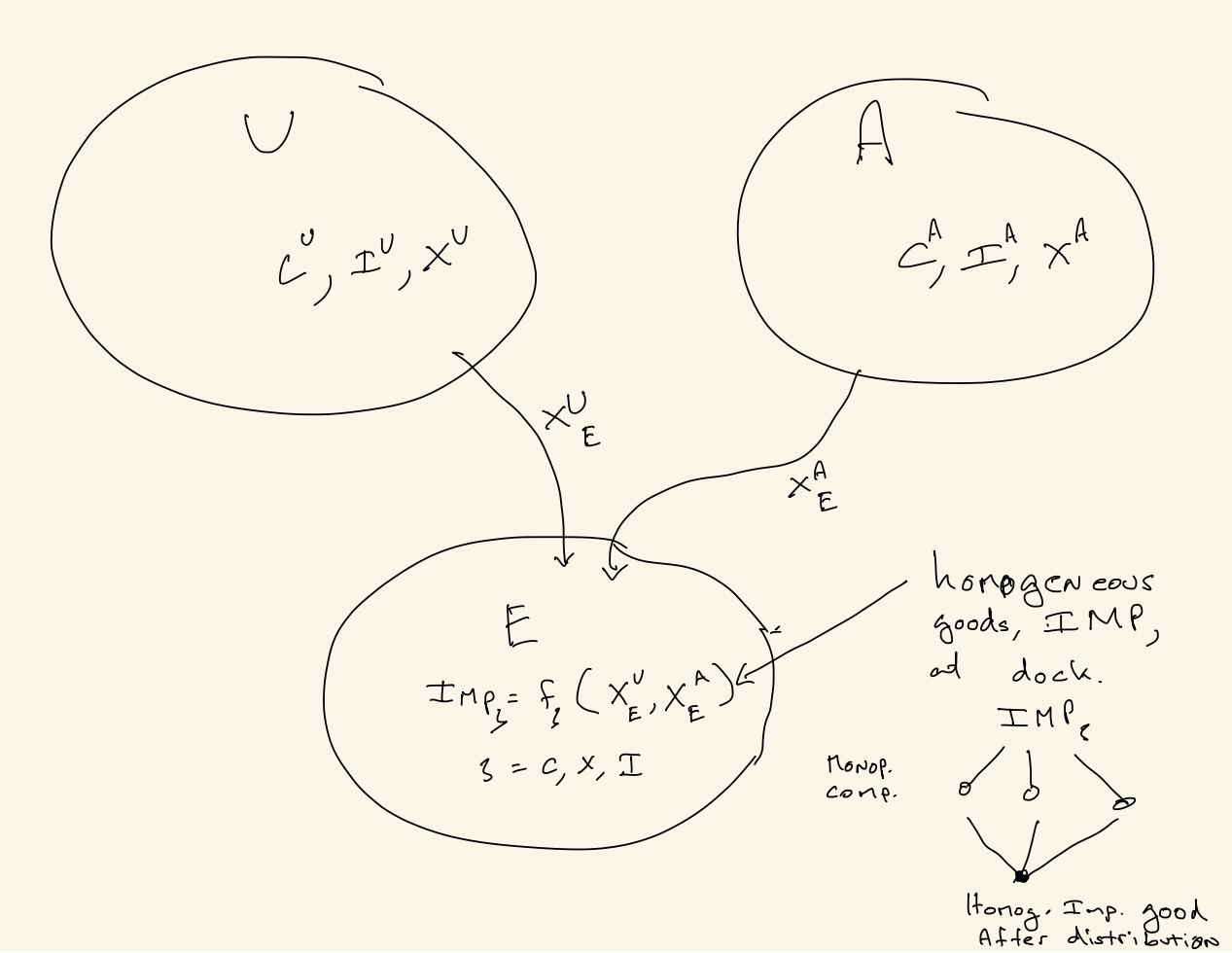
details are provided in Appendix section (A.2).

2.2 Production

Ours is a multi-country general equilibrium model, with three types of country. Broadly, the structure of each economy resembles the structure of Adolfson et al. (2007). That is, each economy produces a homogeneous domestic good using capital and labor. The homogeneous good is then combined with imports from the other countries for production in three sectors: consumption, investment and exports. In evaluating the empirical implications of the model, we think of consumption in the model as corresponding to consumption of non-durable goods plus the services from the stock of durables. Similarly, we think of 'investment' as corresponding to inputs into augmenting business capital as well as household durables.

In subsection 2.2.3 we discuss how imports are taken from the border to each sector in a way that limits pass through from import prices into sectoral prices. Subsection 2.2.4 discusses dominant currency pricing in our model (see Gopinath et al. (2020)). In particular, exports from each country are priced in dollars and those prices are sticky. As in the case of wages in the previous section, all price stickiness in our model is modeled as in Calvo (1983).

Figure 1:



2.2.1 Homogeneous Domestic Goods in Country j

The structure of production in the homogeneous good sector follows the Calvo-style approach standard in NK models. A homogeneous non-tradable good, Y_{jt}, is produced by a representative competitive firm in the j^{th} country by combining a unit measure of differentiated goods. Each of the differentiated goods is produced by a monopolistically competitive firm using domestic capital and labor. This good is an input into the production of domestic consumption, investment and export goods.

The production function for the homogeneous good, Y_{jt}, uses a continuum of domestically-produced intermediate inputs, $Y_{ji,t}$, $i \in [0,1]$ as follows:

$$Y_{jt} = \left[\int_0^1 Y_{ji,t}^{\frac{\varepsilon_j - 1}{\varepsilon_j}} di\right]^{\frac{\varepsilon_j}{\varepsilon_j - 1}}, \varepsilon_j > 1. \tag{6}$$

The representative, competitive firm that produces Y_{jt} takes as given the output price and input prices, P_{jt} and $P_{ji,t}$, $i \in [0,1]$, respectively. The firm chooses $Y_{j,t}$ and $Y_{ji,t}$ to maximize profits and

the first order necessary conditions for optimization are given by:

$$Y_{ji,t} = \left(\frac{P_{ji,t}}{P_{j,t}}\right)^{-\varepsilon_j} Y_{j,t}. \tag{7}$$

Good $Y_{ji,t}$ is produced by a monopolist and its period t profits are given by:[5]

$$\left[P_{ji,t} - MC_{j,t}\right] Y_{ji,t},$$

where $MC_{j,t}$ denotes the marginal cost of production. The intermediate good producer operates a Cobb-Douglas production function in labor and capital services, $\tilde{K}_{ji,t}$ and $\ell_{ji,t}$, respectively:

$$Y_{ji,t} = \tilde{K}_{ji,t}^{\alpha_j} \left(A_{jt}\ell_{ji,t}\right)^{1-\alpha_j}, \; 0 < 1 < \alpha_j. \tag{8}$$

Here, $A_{j,t}$ denotes a technology shock:

$$A_{j,t} = a_{j,t}A_t, \tag{9}$$

xwhere A_t is common across countries and $a_{j,t}$ is specific to country j.

It is useful for us to define a world 'trend' in the model. We define this as a variable, $\mathcal{Z}_t$, which cointegrated with A_t and is a long moving average of past A_t :

$$\mathcal{Z}_t = A_t^{\delta}\mathcal{Z}_t^{1-\delta}$$

where $\delta \cong 1$ and $\delta < 1$. Note that the contemporaneous and near term response of $\mathcal{Z}_t$ to A_t is negligible.

The producer is competitive in the market for inputs, where it takes the wage rate, $W_{j,t}$, and rental rate of capital, $r_{j,t}$, is given. As a result, the producer's marginal cost is the same for all i and is independent of $Y_{ji,t}$:

$$MC_{j,t} = \left(\frac{W_{j,t}}{(1-\alpha_j)\,A_{jt}}\right)^{1-\alpha_j} \left(\frac{r_{j,t}}{\alpha_j}\right)^{\alpha_j} \tag{10}$$

The monopoly producer maximizes discounted profits subject to its demand curve, equation (7), and the Calvo price-setting frictions:

$$P_{ji,t} = \begin{cases} \tilde{P}_{jt} & \text{with probability } 1-\theta_j \\ P_{ji,t-1} & \text{with probability } \theta_j \end{cases}. \tag{11}$$

[5]The assumption that the elasticity of substitution, ε_j, is greater than unity guarantees that the monopoly producer's problem is well-defined.

2.2.2 Production of Final Consumption, Investment and Export Goods in Country j

Let $\varsigma = C, I, X$ where C, I, X denote consumption, investment and exports, respectively. Each good is produced by a representative competitive firm using the modified CES technology. It is convenient for us to express that technology in terms of the scale-of-production, measured by the domestic input, and the mix of inputs, measured by the ratio of foreign to domestic inputs. In particular, production of ς in country j, time t is given by:

$$\varsigma_{j,t} = \Gamma_{\varsigma j,t} \overbrace{\left[\omega_{\varsigma,j}^{\frac{1}{\eta_{\varsigma,j}}} + (1-\omega_{\varsigma,j})^{\frac{1}{\eta_{\varsigma,j}}} \left(\varsigma_{fd,jt}\right)^{\frac{\eta_{\varsigma,j}-1}{\eta_{\varsigma,j}}}\right]^{\frac{\eta_{\varsigma,j}}{\eta_{\varsigma,j}-1}}}^{\tilde{\varsigma}_{j,t}} \varsigma_{dj,t},\ 0 < \omega_{\varsigma,j} < 1, 0 < \eta_{\varsigma,j} \tag{12}$$

for $\varsigma = C, I, X$ and $j = 1, 2, 3$. Also,

$$\varsigma_{fd,jt} = \frac{\varsigma_{fj,t}}{\varsigma_{dj,t}},$$

where $\varsigma_{fj,t}$ denotes the quantity of imported goods used in sector ς by country j. In the above expression, $\tilde{\varsigma}_{j,t}$ is a CES function of the domestic and foreign inputs, where $\varsigma_{dj,t}$ denotes the quantity of the domestic homogeneous good. The term, $\Gamma_{j,t}$, to the right of the equality in equation 12 captures the idea that changing the mix of inputs, $\varsigma_{fd,jt}$, is costly:

$$\Gamma_{\varsigma j,t} = 1 - \frac{\gamma_j^{\varsigma}}{2}\left(\frac{\varsigma_{fd,jt}}{\varsigma_{fd,j,t-1}} - 1\right)^2. \tag{13}$$

Loosely, the model captures that technology is putty-clay in the short run (i.e., input mix is hard to change in the short run), but putty-putty in the longer run. We do not index $\Gamma_{j,t}^{\varsigma}$ by its arguments in order to minimize clutter. Our device, $\Gamma_{\varsigma j,t}$, for reducing the short run elasticity of substitution is an extension of the one proposed in Erceg et al. (2006). We depart from the latter in that the term denominator term, $\varsigma_{fd,j,t-1}$,denotes the firm's own input ratio in period $t-1$. In Erceg et al. (2006) the denominator contains the aggregate, economy-wide, input ratio. Our treatment of the denominator term allows us to capture the response of a firm, as well as inflation and imports to anticipated future tariffs.[6]

The representative firm's problem is:

$$\max_{\left\{\varsigma_{dj,t+i}, \varsigma_{fj,t+i}\right\}_{i=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta \upsilon_{j,t+i} \left[P_{j,t}^{\varsigma} \frac{\varsigma_{j,t+i}}{\varsigma_{dj,t+i}} - P_{j,t+i}^{d} - P_{j,t}^{f} \varsigma_{fdj,t+i}\right] \varsigma_{dj,t+i}. \tag{14}$$

where $\upsilon_{j,t+i}$ for $i \geq 0$ denotes the Lagrange multiplier on the j^{th} country's household period $t+i$ budget constraint (see equation (3)).

[6]For a recent contribution that models the putty-clay versus putty-putty feature of adjustment more explicitly, see Bohr (2024).

All prices sticky in LCU for $\varsigma = C, I$. The domestic input is sticky in LCU because it is composed of the homogeneous domestic goods discussed in section 2.2.1. The imported good is also sticky in LCU because of Calvo-stickiness in the import sector (see section 2.2.3). In the case of exports, $\varsigma = X$, the price of the two inputs and output are all denominated in dollars. The price of exports is sticky in dollars, because of Calvo-stickiness in both the domestic input (see section A.6) and the foreign input (see section 2.2.3).

We can see from equation 14 that for the firm problem to be well defined, it must be that each term in square brackets is zero. If, say, the $(t+i)^{th}$ term were positive, the firm problem is solved by setting $\varsigma_{dj,t+i} = \infty$,a value that violates market clearing in an economy with finite resources. If the term in square brackets were negative then $\varsigma_{dj,t+i} = 0$, which would also violate market clearing because the supply of homogeneous goods is positive.

2.2.3 The Distribution of Imported Goods to the j^{th} Country's Consumption, Investment and Export Sectors

The previous section discussed the demand for imported goods in sectors $\varsigma = C, I, X$ in the j^{th} country. We now describe the supply of imported goods to each sector, ς. Suppliers of foreign goods into country j face Calvo-type frictions. In the case of sectors $\varsigma = C, I$ the Calvo frictions limit pass through from the price of raw imports, which are specified in dollars, into domestic prices. In the case of importers in the export sector, $\varsigma = X$, the Cavo frictions on imports make export prices sticky in dollars.

In all three sectors Calvo frictions take the standard form. Each sector has a representative 'import packer' that purchases a continuum of specialized intermediate import goods from sector ς monopolists. Each of these monopolists produces their specialized good using raw imported inputs from the other two countries.

Import Packers In sector ς a representative competitive import packer purchases differentiated inputs, $\varsigma^h_{fj,t}$, from monopolists, $h \in (0,1)$. The representative import packer then produces $\varsigma_{fj,t}$ using the production function,

$$\varsigma_{fj,t} = \left[\int_0^1 \left(\varsigma^h_{fj,t}\right)^{\frac{\varepsilon_{j\varsigma}-1}{\varepsilon_{j\varsigma}}} dh\right]^{\frac{\varepsilon_{j\varsigma}}{\varepsilon_{j\varsigma}-1}}, \ \varepsilon_{j\varsigma} > 1,$$

taking the output prices, $P_{f\varsigma j,t}$, and input prices, $P^h_{f\varsigma j,t}$, as given. Optimization leads to:

$$\varsigma^i_{fj,t} = \left(\frac{P^h_{f\varsigma j,t}}{P_{f\varsigma j,t}}\right)^{-\frac{1}{\varepsilon_{j\varsigma}}} \varsigma_{fj,t}. \tag{15}$$

As noted above, the prices in this problem are in LCU for $\varsigma = C, I$ and in dollars for $\varsigma = X$. Note that the price of imported goods are different, even for $\varsigma = C, I$. This is because imported goods destined for each of sectors C, X, I use different CES functions of the raw imports obtained from the other countries.

Intermediate Good Importers A specialized good, $\varsigma^h_{fj,t}$, destined for sector ς import packers (see section (2.2.3)) in country j is produced by a monopolist, $h \in (0, 1)$. The inputs of the h^{th} monopolist are exports purchased from the other countries. It is convenient to define the following cyclic indexing scheme. Let J denote the number of countries in the world, so $j \in [1, 2, ..., J]$, where $j = 1$ corresponds to the US. Consider a particular country, $j = \bar{j}$, where $1 < \bar{j} < J$. The $J - 1$ countries with which country $\bar{j}$ trades correspond to $\bar{j} + \ell = [\bar{j} + 1, \bar{j} + 2, ..., J, 1, ..., \bar{j} - 1]$, for $\ell = 1, ..., J - 1$ (note how $\bar{j} + \ell$ wraps around when $\bar{j} + \ell$ exceeds the value J).[7] The US (i.e., $\bar{j} = 1$) trades with countries $\bar{j} + \ell \in 2, ..., J$, for $\ell = 1, ..., J - 1$. Country $\bar{j} = J$ trades with countries $\bar{j} + \ell = [1, ..., J - 1]$ as $\ell = 1, ..., J - 1$.[8]

The h^{th} specialized importer for sector ς and country j produces an import good using the following CES production function over import goods from all other countries:

$$\varsigma^h_{fj,t} = \Gamma^h_{\varsigma j,t} \left[\sum_{\ell=1}^{J-1} \omega_{\varsigma j\ell}^{\frac{1}{\eta_{f\varsigma j}}} \left(X^h_{\varsigma,j,j+\ell,t} \right)^{\frac{\eta_{f\varsigma j}-1}{\eta_{f\varsigma j}}} \right]^{\frac{\eta_{f\varsigma j}}{\eta_{f\varsigma j}-1}}, \tag{16}$$

where $0 < \omega_{\varsigma j\ell} < 1, \eta_{f\varsigma j} > 0$, and

$$\sum_{\ell=1}^{J-1} \omega_{\varsigma j\ell} = 1.$$

Here, $\eta_{\eta_{f\varsigma j}}$ denotes the long run elasticity of substitution between imports from countries, $j + 1$ and $j + 2$ used in sector ς in country j. The term, $\Gamma_{f\varsigma j,t}$, is analogous to the adjustment cost term in equation (13), which captures the idea that the elasticity of substitution may be lower in the short run after a shock.[9]

The price of the imports, $X^h_{\varsigma,j,j+1,t}, ..., X^h_{\varsigma,j,j+J-1,t}$, purchased by the h^{th} importer are denomi-

[7] It is important not to confuse the index, ℓ, with our notation for labor, ℓ_t.

[8] To be exact, suppose $J = 6$, so that the countries are labelled 1, 2, ..., 6. The j^{th} row of matrix A below contains the indexes of the countries with which country j trades. ChatGPT converted this idea into MATLAB code as follows:

```
J = 6;
A = zeros(J, J-1);
for j = 1:J
% Build the row: j+1, j+2, ..., j+J-1 with wraparound
row = mod(j + (1:J-1) - 1, J) + 1;
A(j, :) = row;
end
```

[9] For tractability, the lagged ratio of inputs in the adjustment cost function, $\Gamma_{f\varsigma j,t}$, corresponds to the aggregate ratio, as in Erceg et al. (2006). When it is the firm's own lagged ratio, then the firm has a state variable and the Calvo sticky price mechanism is substantially more complicated (see, e.g., Altig et al. (2011)).

nated in dollars. The price charged by the h^{th} importer for its intermediate good, $P^h_{f\varsigma j,t}$, is set in LCU for the case in which $\varsigma = C, I$. In the case of exporters, $\varsigma = X$, the intermediate good price, $P^h_{f\varsigma j,t}$, is denominated in dollars. The h^{th} firm's time t flow profits t are given by

$$\left[P^h_{f\varsigma j,t} - S_{jt}MC_{f\varsigma j,t}\right]\varsigma^h_{fj,t}, \varsigma = C, I,\ \left[P^h_{fXj,t} - MC_{fXj,t}\right]\varsigma^h_{fj,t} \tag{17}$$

for $j = 1, ..., J-1$. Because the raw imports purchased by the h^{th} importer are price in dollars, the importer's marginal cost, $MC_{f\varsigma j,t}$, is denominated in dollars for $\varsigma = C, I, X$.

The intermediate good importer sets $P^h_{f\varsigma j,t}$ and output, $\varsigma^h_{fj,t}$, subject to its demand curve, (15) and the Calvo price frictions:

$$P^h_{f\varsigma j,t} = \begin{cases} \tilde{P}_{f\varsigma j,t} & \text{with probability } 1-\theta_{f\varsigma j} \\ P^i_{f\varsigma j,t-1} & \text{with probability } \theta_{f\varsigma j}. \end{cases} \tag{18}$$

Here, $0 \leq \theta_{f\varsigma j} < 1$ denotes the probability that an intermediate good firm in country j serving sector ς cannot change its price from the previous period. When $\theta_{f\varsigma j} > 0$ then the price chosen by an intermediate good firm in country j in period t depends on the future states in which its price remains unchanged from its period t value, $\tilde{P}_{f\varsigma j,t}$. The firm that can choose its price in t, does so to optimize the present discounted value of current and future profits. The firm weighs profits in future dates and states by the discount factor of the representative household in country j (recall, we do not allow cross-border ownership of firms).

Tariffs are paid by the intermediate good firms, when they buy raw imports at the border. They enter the firm's cost minimization problem. In particular, $C\left(\varsigma^h_{fj,t}; P^X_{j+\ell,t}, \tau^\varsigma_{j,j+\ell,t}, \ell = 1, ..., J-1\right)$ denote the cost to the h^{th} importer of producing $\varsigma^h_{fj,t}$ units of output:

$$C\left(\varsigma^h_{fj,t}; P^X_{j+\ell,t}, \tau^\varsigma_{j,j+\ell,t}, 1, ..., J-1\right) = \min_{\left\{X^h_{\varsigma,j,j+\ell,t}\right\}_{\ell=1}^{J-1}} \sum_{\ell=1}^{J-1} \tau^\varsigma_{j,j+\ell,t} P^X_{j+\ell,t} X^h_{\varsigma,j,j+\ell,t}, \tag{19}$$

where $\tau^\varsigma_{j,j+\ell,t}$ denotes the tariff levied by country j on imports from country $j+\ell$ that are destined for sector $\varsigma = C, I, X$. Also, $P^X_{j+\ell,t}$ denotes the dollar price of export goods from country $j+\ell$. Marginal cost, $MC_{f\varsigma j,t}$, corresponds to the multiplier in the Lagrangian representation of the h^{th} firm's cost minimization problem.

2.2.4 Exports and the Dominant Currency Paradigm (DCP)

As explained in section 15, a representative final good exporter produces a homogeneous export good in country j. Export goods across countries are different because countries have different technologies. Under the DCP, export prices are not only invoiced in dollars but they are also sticky in dollars. In our model this is the case because exporters' domestic and foreign inputs

are sticky in dollars. We explained the latter in section 15. Here we explain how it is that the domestic input into exports is also sticky in dollars.

The sticky price mechanism is a suitably adapted version of the Calvo-style mechanism described in section 15. The domestic input into exports, $X_{dj,t}$, is produced by a packer as follows:

$$X_{dj,t} = \left[\int_0^1 \left(X^h_{dj,t}\right)^{\frac{\varepsilon_{X^d}-1}{\varepsilon_{X^d}}} dh\right]^{\frac{\varepsilon_{X^d}}{\varepsilon_{X^d}-1}}, \ \varepsilon_{X^d} > 1.$$

The price of $X_{dj,t}$, $P_{dj,t}$, is exogenous to the packer, as are the prices, $P^h_{dj,t}$, of $X^h_{dj,t}$. All these prices are in dollars. The first order optimality condition of the representative packer is given by

$$X^h_{dj,t} = \left(\frac{P^h_{dj,t}}{P_{dj,t}}\right)^{-\frac{1}{\varepsilon_{X^d}}} X_{dj,t}.$$

We now discuss the problem of the monopoly producer of the h^{th} intermediate good, which is essentially the same as the problem described in section 2.2.3. The monopolist's only input is the domestic homogeneous good, so that its marginal cost is simply the price of that good. That is, $MC_{Xdj,t} = P_{j,t}$. The monopolist's price, $P^h_{dj,t}$,is set to optimize the present discounted value of profits (the analog of equation (17)), subject to the Calvo price frictions, i.e., the analog of equation (18). We denote the price stickiness parameter by $\theta_{X^d j}$.

2.2.5 Production of Capital

Capital in country j is produced by competitive capital producers in that country. These producers buy domestically-produced investment goods from the investment sector (see Section 2.2.2) at a given price, $P^I_{j,t}$. Capital accumulation in the model is subject to Christiano et al. (2005) type investment adjustment costs. In period t the representative capital producer uses 'old capital', x, and investment goods, $I_{j,t}$, to produce 'new capital', k:

$$k = x + \left[1 - S^j\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right] I_{t,j},$$

where $S^j(z) = S'^j(z) = 0$ and $S''^j(z) = \kappa^j > 0$, for $z = I_{j,t}/I_{j,t-1}$ along a steady state growth path. Here, S'^j and S''^j denote the first and second derivatives, respectively, and κ^j is a model parameter. The price of a unit of old capital, x, is the same as the price, $P^k_{j,t}$, of a unit of new capital, k. So, costs for the capital producer are $P^k_{j,t}x + P^I_{j,t}I_t$. The representative capital producer solves for sequences of $x_{t+j}, I_{j,t+j}$ to optimize profits,

$$E_t \sum_{i=0}^{\infty} \beta^i \upsilon_{j,t+i} \left\{ P^k_{j,t+i}\left[x_{t+j} + \left(1 - S^j\left(\frac{I_{j,t+i}}{I_{j,t+i-1}}\right)\right) I_{j,t+i}\right] - P^k_{j,t+i}x_{t+i} - P^I_{j,t+i}I_{j,t+i}\right\},$$

where $v_{j,t+i}$ for $i \geq 0$ denotes the multiplier on the j^{th} country's household period $t+i$ budget constraint.

In equilibrium, $k = K_{j,t}$ the total quantity of capital purchased in period t. Also, $x = (1-\delta^j) K_{j,t-1}$, the quantity of capital that was rented out in period t and depreciated by δ^j.

2.3 Finance in the Model

To capture balance sheet effects of exchange rate changes and accelerator effects, we adopt an open economy version of the costly state verification (CSV) adopted in Bernanke et al. (1999).[10] Entrepreneurs acquire capital and rent it to the intermediate good producers in the homogeneous good sector. Entrepreneurs find it desirable to leverage their own resources (net worth) by acquiring funds from a lender ('bank'). After the entrepreneur's capital is acquired and put to work, it experiences an idiosyncratic productivity shock, ω. Ex ante, there is no asymmetric information between bankers and entrepreneurs. Each knows the net worth of the entrepreneur and they know the common distribution, F, from which all entrepreneurs will independently draw ω. Ex post, the realized value of ω is observed by the entrepreneur, but is only observable to the bank at a cost. Banks provide entrepreneurs with menu of standard debt contracts: the entrepreneur receives a loan and must then pays a specified potentially state-contingent interest rate in the next period. If the entrepreneur cannot pay because its ω is too low, then it goes into default and, after being monitored by the bank, the entrepreneur must transfer all its assets to the bank. The interest rate in the standard debt contract is higher than the risk-free rate because loans to entrepreneurs are risky: entrepreneurs who draw a high value of ω must pay enough to cover the costs to the banks of defaulting entrepreneurs. Naturally, entrepreneurs with low net worth ('bad balance sheets') are restricted in the amount of debt they receive. This creates a *balance sheet effects* and these are potentially very large when there is a currency depreciation. To the extent that entrepreneurial loans are financed by dollar liabilities, entrepreneurial net worth falls when the exchange rate depreciates and this reduces their capacity to borrow, forcing them to cut back on the purchase of capital, producing a fall in investment.

2.3.1 Entrepreneurs in country j

We assume that the entrepreneurs in country j are members of the household and infinite-lived. Each entrepreneur has a history of idiosyncratic shocks, which determines its current net worth. To preserve our representative household assumption, we suppose that each household has a large number of entrepreneurs. The entrepreneurs in each household have the same distribution of net worth as in the economy as a whole.

[10] To our knowledge, the first paper to introduce CSV into a closed economy macroeconomics model is Williamson (1987a; 1987b). Another early influential contribution is Carlstrom and Fuerst (1997).

Consider an entrepreneur with end-of-period t net worth, N_t. That entrepreneur goes to a bank and receives a loan contract, $\left(\overline{B}^e_{j,t}, \{Z_{j,t+1}\}\right)$. Here, $\overline{B}^e_{j,t}$ is a quantity of LCU money and $Z_{j,t+1}$ is the LCU gross rate of interest the entrepreneur pays, which potentially depends on the period $t+1$ realized aggregate state of nature. Combining its net worth and loan, the entrepreneur purchases $K_{j,t}$ units of capital:

$$N_{j,t} + \overline{B}^e_{j,t} = P^k_{j,t} K_{j,t}, \tag{20}$$

where $P^k_{j,t}$ is the period t market price of capital in country j.

After purchasing a unit of capital, the entrepreneur's effective capital becomes $\omega K_{j,t}$. The idiosyncratic shock is drawn from a log Normal distribution, F_j, $E\omega = 1$ and $var\left(\log \omega\right) = \sigma^2_{j,t}$. The entrepreneur rents out its effective capital in period $t+1$ at the competitive rental rate, $r_{j,t+1}$. The entrepreneur sells the effective capital left over at the end of period $t+1$ for $\left(1-\delta^j\right)\omega K_{j,t} P^k_{j,t+1}$, where δ^j denotes the capital depreciation rate. So, the entrepreneur that buys K_t units of raw capital in t receives the following income in period $t+1$:

$$\begin{aligned} K_{j,t}\omega r_{j,t+1} + \left(1-\delta^j\right)\omega K_{j,t} P^k_{j,t+1} &= K_{j,t} P^k_{j,t} \omega \left[\frac{r_{j,t+1} + \left(1-\delta^j\right) P^k_{j,t+1}}{P^k_{j,t}}\right] \\ &= K_{j,t} P^k_{j,t} \omega R^k_{j,t+1}, \end{aligned} \tag{21}$$

where $R^k_{j,t+1}$ denotes the rate of return on a unit of effective capital, in LCU units. This rate of return is exogenous to the entrepreneur.

The standard debt contract specifies that the entrepreneur must pay the bank $Z_{j,t+1}\overline{B}^e_{j,t}$ in period $t+1$. Let $\bar{\omega}_{j,t+1}$ denote the cutoff value of ω which gives the entrepreneur just enough income to cover the interest and principal on its debt. That is,

$$\bar{\omega}_{j,t+1} = \frac{Z_{j,t+1}\overline{B}^e_{j,t}}{P^k_{j,t} K_{j,t} R^k_{j,t+1}}. \tag{22}$$

That the cutoff is indexed by $t+1$ reflects that it is a function of the period $t+1$ aggregate state of nature. Entrepreneurs with $\omega < \bar{\omega}_{j,t+1}$ go into default in $t+1$ and must turn over all their resources to the bank, after those resources have been verified by the bank at a cost. The entrepreneur receives full consumption insurance from its household. In exchange, the household expects the entrepreneur to choose a debt contract that maximizes:

$$E_t \upsilon_{j,t+1} \int_{\overline{\omega}_{j,t+1}}^{\infty} \left[P^k_{j,t} K_{j,t} \omega R^k_{j,t+1} - Z_{j,t+1}\overline{B}^e_{j,t}\right] dF_{j,t}\left(\omega\right), \tag{23}$$

where $\upsilon_{j,t+1}$ is discussed in section 2.2.5.

2.3.2 Banks in Country j

The entrepreneur chooses a standard debt contract from a menu that is determined in a competitive market equilibrium. To explain this, we need to examine the circumstances of the banks. For simplicity, consider a representative bank that specializes in making loans to entrepreneurs with net worth, $N_t > 0$. Since the bank has no funds of its own, it must issue liabilities to finance $\overline{B}^e_{j,t}$ units of LCU loaned to entrepreneurs. Following Castillo and Medina (2021), we assume that the bank finances ϕ^j of its borrowing needs in an LCU credit market and $1-\phi^j$ in dollars:

$$B_{j,t} = \phi^j \overline{B}^e_{j,t},\ S_{j,t}B^{\$}_{j,t} = \left(1-\phi^j\right)\overline{B}^e_{j,t}, \tag{24}$$

where $B_{j,t}$ denotes the quantity of LCU borrowed at the gross interest rate, $R^j_{d,t}$. This interest rate must be paid in period $t+1$ and is non-state contingent. Also, $B^{\$}_{j,t}$ denotes dollars borrowed at the non-state contingent dollar rate, $R^{\$}_{d,t}$. Recall that $j=1$ corresponds to the US, so that $S_{1,t} \equiv 1$, $B_{1,t} = B^{\$}_{1,t}$,$R^1_{d,t} = R^{\$}_{d,t}$ and ϕ^1 drops from the model.

The representative bank can borrow as much as it wants at the risk free rates, $R^j_{d,t}$ and $R^{\$}_{d,t}$, and there is no circumstance in which it cannot pay its liabilities. This is because the bank makes loans to a large population of entrepreneurs with net worth, N_t. The population is large enough that the distribution of ω across the bank's borrowers matches the distribution, F. In addition, we follow Castillo et al. (2024) in making the assumption that banks have access to one-period-ahead Arrow securities markets.

Let $A_{jt,t+1}$ denote bank receipts from entrepreneurs, net of monitoring costs and repayment of bank liabilities:

$$\begin{aligned} A_{jt,t+1} = &\left[1-F_j\left(\overline{\omega}_{j,t+1}\right)\right]\overline{B}^e_{j,t}Z_{j,t+1} + \left(1-\mu^j\right)G\left(\overline{\omega}_{j,t+1}\right)P^k_{j,t}K_{j,t}R^k_{j,t+1} \\ &- R_{jd,t}\phi\overline{B}^e_{jt} - s_{jt+1}\left(1-\phi\right)\overline{B}^e_{jt}R^{\$}_{d,t}, \end{aligned} \tag{25}$$

where $s_{j,t+1} \equiv S_{j,t+1}/S_{j,t}$ denotes the rate of depreciation, $F_j\left(\overline{\omega}_{j,t+1}\right)$, denotes the period $t+1$ fraction of non-performing entrepreneurs and

$$G\left(\overline{\omega}_{j,t+1}\right) = \int_0^{\overline{\omega}_{j,t+1}} \omega dF_j\left(\overline{\omega}_{t+1}\right). \tag{26}$$

The first term after the equality in equation (25) corresponds to payments by the $1-F\left(\overline{\omega}_{j,t+1}\right)$ non-defaulting entrepreneurs. The next term corresponds to the resources, net of monitoring costs, recovered from defaulting entrepreneurs. Total monitoring costs, in LCU units, are $\mu^j G\left(\overline{\omega}_{j,t+1}\right)P^k_{j,t}K_{j,t}R^k_{j,t+1}$. Under the assumption of free entry, the ex ante value of banking must be zero:

$$E_t \upsilon_{j,t+1}A_{jt,t+1} = 0. \tag{27}$$

In the presence of complete markets this is the only restriction on profits. In particular, as of period t it is possible for the bank to have a 'deficit', $A_{jt,t+1} < 0$, in one period $t+1$ continuation state and a surplus, $A_{jt,t+1} > 0$, in another period $t+1$ state. Household optimality in the Arrow securities market requires only that the prices, $Q^j_{t,t+1}$, equal $\upsilon_{j,t+1}$. With prices set in that way, households are willing to take a position opposite to the bank so that each period $t+1$ Arrow security market clears in period t, i.e., $A^j_{t,t+1} + a^j_{t,t+1} = 0$ for each state of nature possible in $t+1$ conditional on t.

2.3.3 Equilibrium Contract

We assume that under competition, bankers offer a menu of contracts, $\left(\overline{B}^e_{j,t}, \{Z_{j,t+1}\}\right)$, which satisfy equation (27). So, the equilibrium contract is the one that solves the problem in equation (23) subject to equation (27). In principle the entrepreneur could choose a contract in which $Z_{j,t+1}$ is not contingent on the period $t+1$ state of nature. But, this would come at a cost to the bank since it would force the bank to enter the Arrow security markets and potentially pay a high price for funds in a bad period $t+1$ state in exchange for funds in a good $t+1$ state. This cost to the bank would be passed on to the entrepreneur in the form of a higher non-state $t+1$ contingent interest rate, $Z_{j,t+1}$. In our computations we find that $Z_{j,t+1}$ responds sharply to shocks, so that little use is made of Arrow securities in equilibrium.

We assume that an exogenous fraction, γ^e_j, of an entrepreneur's period t earnings is transferred to its household while the entrepreneur keeps the rest. In addition, each entrepreneur receives a small LCU lump sum transfer from households in the amount, $W^e_{j,t}$. The transfer ensures that even bankrupt entrepreneurs in period t, who receive no income from their standard debt contract, nevertheless still have a small amount of net worth. It is a property of the model that if an entrepreneur has no net worth, then it cannot borrow at all. If $W^e_{j,t} = 0$ for all t then, because all entrepreneurs experience bankruptcy at some point, all entrepreneurs would eventually be unable to borrow. It follows that the aggregate amount of net worth held by all entrepreneurs during period t after period t uncertainty is resolved and production has occurred is given by:

$$N_{j,t} = \gamma^e_j \left[1 - \Gamma_t\left(\overline{\omega}_{j,t}\right)\right] P^k_{j,t-1} K_{j,t-1} R^k_{j,t} + W^e_{j,t}. \tag{28}$$

where $W^e_{j,t}$ denotes a transfer of net worth from households to entrepreneurs. The period t transfer to households from entrepreneurs is $\left(1-\gamma^e_j\right)\left[1 - \Gamma_j\left(\overline{\omega}_{j,t}\right)\right] P^k_{j,t-1} K_{j,t-1} R^k_{j,t} - W^e_{j,t}$ and this term is part of $\Pi_{j,t}$ in the household's budget constraint, equation (3).

2.4 Government Policy

We assume that all three countries conduct monetary policy according to a Taylor rule. We assume that the US ($j = 1$) does not conduct FX operations, but the other two countries ($j = 2, 3$) do.

All three countries have an exogenous government spending rule.

2.4.1 Government Purchases of Goods and Services

We assume that the local government purchases domestic homogeneous goods:

$$G_t^j = g^j \mathcal{Z}_t \tag{29}$$

where $\mathcal{Z}_t$ denotes the trend in world variables:

$$\mathcal{Z}_t = A_t^{\delta} \mathcal{Z}_{t-1}^{(1-\delta)},$$

where

2.4.2 Interest Rate Policy

The government in country j in principle has two policy tools. The first is the traditional Taylor monetary policy rule:

$$\begin{aligned}\log\left(\frac{R_{d,t}^j}{R_d^j}\right) =& \rho_R^j \log\left(\frac{R_{d,t-1}^j}{R_d^j}\right) \\ &+(1-\rho_R^j)\left[r_\pi^j \log\left(\frac{\pi_t^j}{\pi^j}\right) + r_y^j \log\left(\frac{GDP_t^j}{GDP_{t-4}^j}\right) + r_S^j \log\left(\widetilde{S}_{j,t}\right)\right] + \epsilon_{R,t}^j.\end{aligned}$$

Here, $\pi_t^j = P_{j,t}/P_{j,t-1}$ and GDP_t^j denotes country j GDP. Our decision to include π_t rather than π_t^c in the interest rate rule was also motivated by the argument in Egorov and Mukhin (2023) which suggests that this specification of the rule may be welfare-improving. In addition, central banks tend to focus on 'core inflation', a variable that is not sensitive to transitory shocks, and π_t may satisfy that condition because it is relatively insensitive to shocks in the exchange rate. Also, $\tilde{S}_{j,t}$ corresponds to $S_t/\left(\psi^t \bar{S}\right)$, where $\bar{S} > 0$ is a parameter. For $j = 1$ (the US) we set $r_S^1 = 0$ and $S_{1,t} = 1$. We indicate the nonstochatic steady state of a time series variable by deleting its time subscript. Although $r_S^j > 0$, the value assigned to it is small. The idea is that exchange rates wander a lot, but not as much as if they had a unit root, which is what the model would predict in the case, $r_S^j = 0$.

2.4.3 FX Interventions

In addition to the traditional monetary policy, we allow a central bank to conduct FX interventions in the non-US countries, $j = 2, 3$. We assume that the central bank uses a Taylor-type intervention

rule following Castillo and Medina (2021):

$$\frac{F^{\$}_{j,t}}{\overline{F}^{\$}_{j,t}} = \left(\frac{F^{\$}_{j,t-1}}{\overline{F}^{\$}_{j,t-1}}\right)^{\rho_{jfx}} \left(\frac{R^{\$}_{d,t}-1}{R^{\$}_{d}-1}\right)^{-\theta^{j}_{R^{\$}}}, \tag{30}$$

where $F^{\$}_{j,t}$ denotes the country j central bank holdings of dollar assets and $\overline{F}^{\$}_{j,t}$ denotes a target value the central bank's holdings of reserves. Our measure of $\overline{F}^{\$}_{j,t}$ reflects the conventional wisdom about how much reserves a central bank ought to hold (see, e.g., Greenspan (1999)). According to this approach, the period t target for reserves is proportional to the gross dollar liabilities of the country at the start of period t. In our model this is the term in the square brackets:

$$\overline{F}^{\$}_{j,t} = \left(\upsilon^{j,cb}\right)^{1-\vartheta^{j}} \left[R^{\$}_{t-1}B^{\$}_{j,t-1} + \overbrace{\left(P^{X}_{j+1,t}X^{i}_{j,j+1,t} + P^{X}_{j,j+2,t}X^{i}_{j+2,t}\right)/S_{j,t}}^{\text{imports in units of dollars}}\right]^{1-\vartheta^{j}} \left(\overline{F}^{\$}_{j,t-1}\right)^{\vartheta^{j}}, \tag{31}$$

for $j = 2, 3$. The value of imports is taken from equation xx and does not include tariffs paid to the government. In practice, we choose a value for $\upsilon^{j,cb}$ so that the model's steady state is consistent with the average dollar reserves to GDP ratio of a country. We assume that FX interventions are financed by lump sum taxes.

The central bank's FX strategy generates profits and losses on its balance sheet depending on movements in the exchange rate and interest rates. We assume these are rebated in lump-sum form to households. In our model, FX interventions have zero effect without the presence of UIP distortions. For example, the frictions are turned off with $\gamma^{j} = 0$, in which case sterilized FX interventions are exactly undone by households, who willingly accept the dollar assets offered by the central bank in exchange for LCU bonds without any change in market prices. The reason has to do with the fact that the households receive, in lump sum form, any gains or losses that the central bank experiences on its portfolio. For example, if the central bank sells a lot of dollars to households, then household portfolios have a lot more exchange rate risk. They don't mind because their overall exposure to exchange rate risk, which takes into account the transfers from the government, has not changed. When $\gamma > 0$ then households have a particular interest in the composition of the financial portfolio in their direct possession and and then sterilized interventions do have an effect. There is continuity, so that when $\gamma^{j} > 0$ but small, then the household buys nearly all of any amount of dollar assets the central bank sells, and they need only a very small market incentive to absorb those dollars. This could be some combination of a fall in $R_{jd,t}$ or a rise in $R^{\$}_{d,t} + E_t \log S_{j,t+1} - \log S_{j,t}$. Since $R^{\$}_{d,t}$ is exogenous to the SOE, the latter would involve some combination of a fall in $\log S_{j,t}$ or rise in $E_t \log S_{j,t+1}$.

2.5 Market Clearing, Balance of Payments and GDP

2.5.1 Financial Market Clearing

Each country acquires financial claims in its own currency, as well as in dollars. There are two financial markets in which foreigners do not participate. The first is the market for Arrow one-period securities associated with entrepreneurial loans and the market clearing condition is:

$$A^j_{t,t+1} = a^j_{t,t+1}, \tag{32}$$

where $a^j_{t,t+1}$ is in LCU. In the second domestic market for bonds is LCU bonds, household purchase, $D_{j,t}$, bonds and banks sell $B_{j,t}$ such bonds. Thus, in $j = A, E$, the LCU market clearing condition is:

$$D^j_t = B^j_t. \tag{33}$$

Recall that $j = 1$ corresponds to the US and we assume that US has no LCU market and only trades in the dollar market. So, the analog of equation (33) does not exist for $j = 1$. There is a single world market for dollars, so market clearing implies:

$$\sum_{j=1}^{3} n_j \left[D^{\$}_{j,t} + F^{\$}_{j,t} - B^{\$}_{j,t} \right] = 0, \tag{34}$$

where n_j denotes the size of country j and we adopt the normalization, $\sum_j n_j = 1$.

2.5.2 Goods Market Clearing

Supply, Y_t, equals demand in the homogeneous goods market requires (see section :

$$Y_{j,t} = I_{dj,t} + C_{dj,t} + X_{dj,t} + g^j \mathcal{Z}_t + \frac{\overbrace{\mu^j G\left(\bar{\omega}_{j,t}\right) P^k_{j,t-1} K_{j,t-1} R^k_{j,t}}^{\text{homog goods for monitoring by banks}}}{P_{j,t}}, \tag{35}$$

where $I_{dj,t}$ and $C_{dj,t}$ denote homogeneous goods used in the production of investment and consumption, respectively (see Section 2.2.2). Also, $X_{dj,t}$ denotes the quantity of homogeneous goods used to produce the domestic input into exports (see Section 2.2.2) and the next term denotes government purchases (see equation (29)). Finally, the last term corresponds to the homogeneous goods purchased by banks to monitor defaulting entrepreneurs (see Section 2.3.2). The object, $G\left(\overline{\omega}_{j,t}\right)$, is defined in equation (26).

The international trade clearing conditions are:

$$n_j X_{j,t} = \int_0^1 n_{j-1} \left(\sum_{\varsigma=[C,I,X]} X^i_{\varsigma,j-1,j,t} \right) di + \int_0^1 n_{j-2} \left(\sum_{\varsigma=[C,I,X]} X^i_{\varsigma,j-2,j,t} \right) di,$$

for $j = U, A, E$. The term on the left of the equality denotes the total (not, per capita) exports of country j. The first term on the right of the equality indicates the total purchases from country j by all intermediate good importers, $i \in (0,1)$, in country $j-1$. The second term indicates total purchases from country j by importers in country $j-2$.

2.5.3 Balance of Payments

Total imports, imp_{jt}, by country j in period t is given by:

$$imp_{jt} = \int_0^1 \left[\sum_{\varsigma=[C,I,X]} \left(P^X_{j+1,t} X^i_{\varsigma,j,j+1,t} + P^X_{j+2,t} X^i_{\varsigma,j,j+2,t} \right) \right] di.$$

For each country, $j \in (U, A, E)$, the balance of payments is as follows:

$$P^X_{j,t} X_{j,t} - imp_{jt} = F^{\$}_{j,t} - R^{\$}_{d,t-1} F^{\$}_{j,t-1} + D^{\$}_{j,t} - R^{\$}_{d,t-1} D^{\$}_{j,t-1} - B^{\$}_{j,t} + B^{\$}_{jt-1} R^{\$}_{d,t-1} \tag{36}$$

Let $R^{\$}_{d,t} = 1 + r^{\$}_{d,t}$, so that the current account, CA_t, is net exports plus net interest earned from abroad:

$$CA_{jt} = P^X_{j,t} X_{j,t} - imp_{jt} + r^{\$}_{d,t-1} \left(F^{\$}_{t-1} + D^{\$}_{t-1} - B^{\$}_{t-1} \right). \tag{37}$$

Then, we have that (36) can be written as follows:

$$CA_{jt} = F^{\$}_{j,t} + D^{\$}_{j,t} - B^{\$}_{j,t} - \left(F^{\$}_{j,t-1} + D^{\$}_{j,t-1} - B^{\$}_{j,t-1} \right), \tag{38}$$

which states that the current account equals the change in net foreign assets. Since these assets have maturity one period, the change in net foreign assets is not affected by valuation effects.

2.5.4 GDP

Per capita GDP, expressed in terms of purchases of final goods, is:$X^i_{\varsigma,j-1,j,t}$

$$GDP_t = \frac{P^C_{j,t} \left(C_{j,t} + G_{j,t} \right) + P^I_{j,t} I_{j,t} + P^X_{j,t} X_{j,t} - imp_{j,t}}{P^C_{j,t}}.$$

Note that we measure country j GDP in units of that country's consumption goods.

2.5.5 Shocks

The world, aggregate technology shock is assumed to evolve as follows:

$$\Delta a_t = (1-\rho_1)(1-\rho_2)\Delta a + (\rho_1+\rho_2)\Delta a_{t-1} - \rho_1\rho_2 \Delta a_{t-2} + \sigma_{\Delta a} \varepsilon^a_t \tag{39}$$

where $\Delta a_t = a_t - a_{t-1}$, $a_t = \log A_t$ and the variance of ε_t^a is unity. Also, $\rho_1, \rho_2 \in [0, 1)$.To see that A_t and $\mathcal{Z}_t$ are cointegrated we have:

$$z_t \equiv \frac{\mathcal{Z}_t}{A_t} = \frac{A_t^{\delta} \left(\mathcal{Z}_t\right)^{1-\delta}}{A_t} = \left(\frac{z_{t-1}}{\exp\left(\Delta a_t\right)}\right)^{1-\delta}$$

and note that $\log z_t$ is stationary because Δa_t is.

The j^{th} country's component of technology is assumed to evolve like this:

$$\log\left(a_{j,t}\right) = \left(1-\rho_{j1}\right)\left(1-\rho_{j2}\right)\log\left(a_j\right) + \left(\rho_{j1}+\rho_{j2}\right)\log\left(a_{j,t-1}\right) - \rho_{j1}\rho_{j2}a_{j,t-2} + \sigma_{a_j}\varepsilon_t^{a_j},$$

where ρ_{j1}, $\rho_{j2} \in [0, 1)$ and the standard deviation of $\varepsilon_t^{a_j}$ is unity.

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