A Policy Framework for the Fed's Employment Mandate*

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Abstract

We formalize the maximum, optimal and equilibrium levels of employment in a frictional model of unemployment and characterize the long-run inflation rates that are consistent with each employment target.

The Federal Reserve's dual mandate to promote maximum employment and price stability has generated extensive research on optimal monetary policy design. While the literature on inflation targeting has developed sophisticated frameworks for determining appropriate price stability goals, the employment component of the mandate has received less systematic treatment. Most existing work models the employment objective indirectly through output gap stabilization or as activity levels consistent with long-run price stability, effectively subordinating employment considerations to inflation targets. This approach reflects the lack of a clear definition of the concept of "maximum" employment in existing work, where employment outcomes are primarily viewed through the lens of their inflationary consequences.

The resulting gap in the literature is particularly problematic given the Fed's explicit statutory requirement to pursue maximum employment as a co-equal objective with price stability.

*The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Cleveland or the Federal Reserve Board.

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Recent developments in labor search and matching theory provide the analytical tools necessary to address this deficiency by offering rigorous definitions of employment concepts within frictional frameworks. However, the connection between these microeconomic foundations and macroeconomic policy targets remains underexplored. Specifically, the literature has not adequately addressed how different employment objectives—whether maximum, optimal, or equilibrium employment—translate into specific inflation rate targets that would implement these goals.

This paper fills this gap by developing a tractable framework that derives inflation targets directly from employment objectives within a frictional labor market model. We define maximum employment in a frictional model of unemployment as the level associated with the lowest feasible level of unemployment that still ensures voluntary participation in the labor market by employers and households. This level is independent of monetary policy and the inflation rate in the economy. To achieve maximum employment, the planner has to incentivize employers to maximize job creation, which requires maximizing their share of the production surplus, while ensuring that the unemployed are not altogether discouraged from searching for a job. This is tantamount to a labor market allocation where employers are provided with the entirety of the surplus while workers are paid their reservation value, leaving them indifferent between unemployment and employment. The key insight is that maximum employment can only be attained by creating many low-paying jobs.

In a general equilibrium setting, workers recoup some of the surplus as dividends, because households are also the owners of firms. Nonetheless, maximum employment level is not optimal because the job creation is costly in a frictional model. The recruiting costs associated with constantly trying to fill a large number of vacant positions so as to maintain the lowest feasible level of unemployment exceeds the additional value created by those jobs. The optimal level of employment, which maximizes the welfare, better balances the costs and benefits of job creation. Equilibrium employment is distinct from the optimal and maximal levels of employment except in special cases, e.g., equilibrium aligns with maximal employment when employees have no bargaining power against employers, and, hence would have been paid their reservation wage anyways.

In Section **??**, we introduce trend inflation in our model and analyze its effects on equilibrium employment. To that end, we restrict the frequency of wage re-negotiations for ongoing employee-employer pairs, which puts deflationary pressure on real wages in an inflationary environment. Importantly, we allow for wage negotiations when the worker is hired and re-negotiations at random intervals thereafter, or if the real wage is deflated to a point where the workers would prefer unemployment to their job. Under these assumptions, the labor market clears at the hiring margin, since the starting salaries are flexible, and the wage rigidity never results in inefficient separations.[1] We then show that if the space of wage contracts is not restricted, then (long-run)

[1] A comment on Barro's critique, and Hall's version of it which restricts starting wages, also mention Bils and

inflation is neutral, i.e., it does not change equilibrium employment. This is because any real effect that might arise from (anticipated) deterioration of real pay is fully negotiated away at the time of hire by frontloading wage payments.

At sufficiently high inflation levels, however, frontloading of pay requires the employer to compensate the worker more than their marginal product early in their tenure to balance anticipated losses in real wages. Depending on the rate of inflation, the required early compensation can be substantial, straining firms' finances. We therefore consider credit frictions that limit the extent of frontloading that can be financed, thereby, allowing inflation to raise equilibrium employment by eroding worker's share of the surplus.[2] Because such constraints do not bind at low levels of inflation, where, for instance, required frontloading in wage contracts can be handled by firms' internal sources, the model features a non-linear Phillips Curve with inflation raising employment only if it is sufficiently high.

We then ask what levels of inflation are generally consistent with maximal and optimal employment levels. We first show that trying to implement the maximal employment level is prohibitively costly. By contrast, implementing optimal employment is feasible, with the required inflation rate depending on the particular nature of the labor market and the job matching process. The inflation target that implements optimal employment is lower when the marginal value of an additional vacancy in job creation is low, or, equivalently, that of an unemployed worker is high; when the job separation rate from employment to unemployment is low; or, when the real interest rate is low. A number of policy implications emerge. For instance, when the number of job seekers and recruiters are gross complements in job creation and the market is short on unemployed workers relative to vacant positions, the model calls for a lower inflation target. Higher rates of job separation or real interest rates call for higher inflation targets. Complementarity of vacancies and unemployed workers in the matching process therefore points to optimality of procyclical inflation targeting for the Fed.

To be completed:

Because the optimal unemployment rate is linked to labor market fundamentals in our framework, the long-run inflation target consistent with the employment mandate is endogenous to changes in those fundamentals. As such, secular trends in labor market flows, separation rates, real rates call for adjustments to the inflation target. We specify key parameters of the model using historical data on inflation rates, vacancies and unemployment rates, and illustrate the optimal employment rates and the associated inflation targets that would deliver those rates for different sets of labor market fundamentals.

In Section **??**, we estimate the model using long-run data on inflation rates, vacancies and unemployment rates, thereby capturing the empirical Beveridge Curve and Phillips Curve rela-

Chang here.

[2]We considered alternative models of monetary non-neutrality. While these models differ in the particular mechanism with which inflation affects real allocations, they do not change the nature of our conclusions.

tionships in the U.S. Using the estimated model, we compute the optimal employment rates and the associated inflation rate targets that would deliver those rates.

Section **??** discusses alternative modeling assumptions and **??** concludes.

1 Model

1.1 Economic Environment

We consider a continuous-time, infinite-horizon economy with a fixed labor force normalized to unity. The labor force consists of unemployed workers u and employed workers n, such that $n + u = 1$. There are v vacant positions posted by firms seeking to hire workers. The model is characterized by several key parameters. The discount rate is denoted by $r > 0$, while $\delta \in (0, 1)$ represents the exogenous job destruction rate. Unemployed workers receive flow utility $z \geq 0$, which captures unemployment benefits and the value of home production. Firms pay a flow cost $\kappa > 0$ to maintain each vacancy. When matched, each worker produces output $p > 0$. The parameter $\beta \in (0, 1)$ represents the worker's bargaining power in wage negotiations. The matching process is governed by a constant returns to scale matching function $m(u, v)$ that is increasing and concave in the number of job seekers and the number of vacancies available: $m_u, m_v > 0$, $m_{uu}, m_{uv} < 0$, and $m_{uv} > 0$.

Labor market tightness is defined as $\theta = v/u$, representing the ratio of vacancies to unemployed workers. The job-finding rate for unemployed workers is $p(\theta) = m(u, v)/u$, while the vacancy-filling rate for firms is $q(\theta) = m(u, v)/v$. These rates satisfy $p'(\theta) > 0$, $p''(\theta) < 0$, $q'(\theta) < 0$, $q''(\theta) > 0$, and the relationship $\theta p'(\theta) = (1 - \alpha)p(\theta)$.

1.2 Worker Value Functions

The value function for an unemployed worker, denoted U, satisfies the Bellman equation

$$rU = z + p(\theta)[W_0 - U], \tag{1}$$

which states that the flow return to unemployment equals the utility from being unemployed plus the expected capital gain from finding employment. An unemployed worker receives immediate utility z and transitions to employment at rate $p(\theta)$, experiencing a discrete jump in value equal to $W_0 - U$ upon successful matching, where W_0 is the value of a new job to the worker.

The value function for a newly employed worker, W_0, satisfies

$$rW_0 = w + \delta(U - W_0) + \pi[W - W_0]. \tag{2}$$

The flow return to employment consists of a *negotiated* wage payment w minus the expected capital loss from job destruction and from inflation. With probability δ per unit time, the employment relationship dissolves and the worker returns to unemployment, experiencing a discrete drop in value equal to $W_0 - U$. At the inflation rate π, the value of a job to the worker changes from W_0, resulting in a drop in job's value to the worker by $W_0 - W \geq 0$.

The value of an ongoing job to a worker is given by:

$$rW = \underline{\mathrm{w}} + \delta[U - W] + \phi[W_0 - W]. \tag{3}$$

The flow return to the worker from an ongoing employment relationship consists of a lower wage payment $\underline{\mathrm{w}} \leq w$, which is not negotiated, minus the expected capital loss from job destruction plus the expected capital gain from renegotiating the wage rate. At rate ϕ, workers renegotiated their wage, which results in a capital gain of $W_0 - W$.

1.3 Firm Value Functions

The value of maintaining a vacant position, V, is characterized by

$$rV = -\kappa + q(\theta)[J_0 - V] \tag{4}$$

A firm with a vacancy incurs the flow cost κ and successfully fills the position at rate $q(\theta)$. Upon matching with a worker, the firm experiences a discrete increase in value equal to $J_0 - V$, where J_0 represents the value of a newly filled position and satisfies the following Bellman equation:

$$rJ_0 = p - w + \delta(V - J_0) + \pi[J - J_0] \tag{5}$$

The firm's flow return to a newly filled position equals the worker's productivity less of the negotiated wage payment, adjusted for the expected losses from job destruction and expected gains from real wage deflation. When the job is destroyed at rate δ, the firm loses $J_0 - V$. At the inflation rate π, firm's gains from the position rise from J_0 to $J \geq J_0$.

The value of an ongoing employment relationship to the firm is:

$$rJ = p - \underline{\mathrm{w}} + \delta(V - J) + \phi[J_0 - J] \tag{6}$$

The flow return consists of a higher ongoing profit margin $p - \underline{\mathrm{w}}$ mines the expected losses from job destruction and wage renegotiation, which happens at rate ϕ and reduces the value of the job to the firm back to J_0.

1.4 Wage Determination and Free Entry

The model has nine endogenous variables: six values in Equations 1-6, the two wage rates w and $\underline{w}$, and the labor market tightness θ. To the six value equations above, we add three conditions here to define the model equilibrium.

First, we assume that the negotiated wage payment w is determined through Nash bargaining between the worker and firm over the total match surplus $S = (W_0 - U) + (J_0 - V)$. The Nash bargaining protocol maximizes the weighted product of the parties' gains from the match: $w = \arg\max\ (W_0 - U)^{\beta}(J_0 - V)^{1-\beta}$, where $\beta \in [0, 1]$ represents worker's relative bargaining power. We restrain the set of possible wages so that at any point in time the wage rate paid may not exceed a worker's instantaneous productivity: $w \leq p$.[3] The first-order condition for this optimization problem yields the surplus-sharing rule

$$\beta(J_0 - V) = (1 - \beta)(W_0 - U) + \mu, \tag{7}$$

where $\mu \geq 0$ is the Lagrange multiplier associated with the credit constraint. If the credit constraint does not bind ($\mu = 0$), Equation (7) states that the worker's bargaining power determines the division of the total match surplus. Otherwise, $\mu > 0$, and $w = p$.

Ongoing wage rate $\underline{w} \leq w$ is not negotiated. Without loss of generality we assume that $\underline{w}$ leaves the worker indifferent between unemployment and employment.

$$W = U. \tag{8}$$

Finally, we assume that the equilibrium is characterized by free entry of firms into the vacancy posting process, which drives the value of maintaining a vacancy to zero in equilibrium.

$$V = 0. \tag{9}$$

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1.5 Steady-State Equilibrium

The equilibrium is characterized by free entry of firms into the vacancy posting process. This condition drives the value of maintaining a vacancy to zero in equilibrium: $V = 0$. Substituting this condition into the vacancy value function yields $\kappa = q(\theta)J_0$, which states that the flow cost of maintaining a vacancy equals the expected benefit from filling it. From equation (5), the latter is equivalent to:

[3]The no-credit requirement could be extended to allow for some borrowing $w \leq \gamma p$ with $\gamma > 1$. The results would be qualitatively similar.

$$J_0 = \frac{p - w + \pi(J - J_0)}{r + \delta}.$$

The job creation condition emerges from combining the free entry condition with the value function for filled positions, which yields:

$$\frac{p - w + \pi(J - J_0)}{r + \delta} = \frac{\kappa}{q(\theta)}, \tag{10}$$

indicating that the expected cost of hiring a worker equals the present discounted value of the profit stream from the employment relationship. When $\pi = 0$, equation (10) gives the job creation condition in the canonical random search model. When $\pi > 0$, inflation raises profits, ceteris paribus, and encourages firms to post more vacancies since $J \geq J_0$. As we show next, however, the effect of inflation on equilibrium vacancy creation, all things considered, depends on the outcome of wage negotiations.

Consider first an economy without any borrowing constraints on firms: $\mu = 0$. The equilibrium wage rate in that case is:

$$w = \beta p + (1 - \beta) rU + \pi(W_0 - W). \tag{11}$$

The negotiated wage rate is convex combination of productivity and the flow reservation value of an unemployed worker, adjusted for future deflation of the real wage in an inflationary environment. When $\pi = 0$, the model replicates the standard wage setting process in the canonical model. $\pi > 0$ implies higher wages when the worker start their job, implying that workers recapture at least some of the anticipated loss in value from wage deflation. In fact, they capture the entirety of it if starting salaries are unbounded. This can be established by first showing that a worker's loss from deflation is equal to the firm's gain from wage deflation.

Lemma 1. $J - J_0 = W_0 - W$

Because the inflationary process effectively transfers resources from the worker to the firm, we have $J - J_0 = W_0 - W \geq 0$. Therefore the additional gains in firm's flow profits, $\pi(J - J_0)$ in equation (13), is paid out entirely to worker by raising starting salaries by the same amount $\pi(W_0 - W)$. Substituting the wage rate in equation (11) into the job creation condition therefore gives the canonical equilibrium condition for labor market tightness in the random search model.

Proposition 1. *If wage payments are unbounded, then the equilibrium wage rate is given by:*

$$w = \beta p + (1 - \beta) z + \beta \kappa \theta + \pi \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}. \tag{12}$$

Considering the equilibrium valuations of jobs and unemployment to a worker, the negotiated wage reflects a combination of productivity, value of leisure, and compensation for the

deflationary effect of price inflation on real wages. That compensation depends on the strength of the worker's bargaining power in wage negotiations and increases with the rate of inflation in the economy. Note that wages can easily rise above current productivity level if $\beta > 0$ and the inflation rate is high. The faster the real wage deflation, higher the upfront negotiated wage has to be to compensate the worker.

Proposition 2. *Equilibrium level of labor market tightness is given by:*

$$\frac{p-z}{r+\delta} = \frac{\kappa}{q(\theta^*)} \cdot \frac{r+\delta+\beta p(\theta^*)}{(1-\beta)(r+\delta)}. \tag{13}$$

Equation (13) is the standard equilibrium condition in the canonical model and states that, in equilibrium, the discounted value of the flow surplus associated with a job equals the effective cost of creating that job to the firm. That is higher than the firm's expected cost of filling a vacancy. Because such filling costs are sunk by the firm prior to wage negotiations, workers capture some of those during negotiations as long as they have some bargaining power, $\beta > 0$, which raises the effective cost of posting vacancies in equilibrium and discourages firm entry.[4] That in turn results in lower job creation than would occur if workers had no bargaining power.

Importantly, π does not appear in the equilibrium condition above, giving our first main result on monetary neutrality.

Corollary 1. *Suppose that w is unbounded. Then the inflation rate has no real effect on equilibrium allocations.*

The model without credit constraints does not feature a long-run Phillips Curve, because the deflationary pressure on real wage rate is negotiated away. The firm agrees to frontload pay in order to compensate the worker for future loss. Setting $w = p$ gives the inflation level above which the negotiated wage exceeds the contemporaneous productivity level:

$$\underline{\pi} = \frac{1-\beta}{\beta}(r+\delta). \tag{14}$$

Higher levels of inflation require the firm to borrow against future profits to pay its workers. Note that the threshold inflation rate above which this becomes necessary is lower, higher the worker's bargaining power, which determines the amount of inflation-related compensation. Furthermore, higher the effective discount rate for the match, $r+\delta$, lower is the required upfront pay to compensate future losses, and, hence, higher is π.

Next we consider the scenario where the firm is limited in its ability to frontload pay due to credit constraints. Specifically, we assume the highest salary a firm can pay is equal to

[4]This is represented by the $\beta\kappa\theta$ term in the equilibrium wage function.

productivity: $w \leq p$. Therefore, when $\pi \geq \underline{\pi}$, $w = p$, which gives the following job creation condition:

Proposition 3. *In an economy with credit constraints, the equilibrium level of labor market tightness solves equation* (13) *if* $\pi \leq \underline{\pi}$ *and solves the following equation if* $\pi \geq \underline{\pi}$*:*

$$\frac{\pi}{r+\delta+\pi+p(\theta)}\frac{(p-z)}{r+\delta} = \frac{\kappa}{q(\theta)}. \tag{15}$$

Proposition 3 describes a hockey stick shaped Phillips Curve: there is a positive relationship between the inflation rate and equilibrium labor market tightness, and, hence, a negative between inflation and the unemployment rate, at higher levels of inflation.[5] For $\pi < \underline{\pi}$, inflation rate does not affect the real economy.

Proposition (check if phillips curve is convex)**.** *The following are true in equilibrium*

1. $d\pi/d\theta^* > 0$
2. $d^2\pi/d(\theta^*)^2 > 0$ *?*
3. $du^*/d\pi < 0$
4. $d^2u^*/d\pi^2 =?$

In what follows, we focus on the economy with credit constraints and consider different interpretations of the Fed's employment mandate and examine the levels of inflation that are consistent with each interpretation.

2 Employment Mandate and Inflation Targets

2.1 Maximum Employment

Suppose the planner wants to maximize steady-state employment, which is equivalent to minimizing unemployment in our setup. In doing so, the planner must ensure market participation by firms and workers: the firms are willing to operate and post vacancies, the unemployed workers are interested in seeking employment:

$$\max_{w} n = (1-u) \quad \text{subject to } W_0 \geq U \text{ and } J_0 \geq V$$

[5] To see this, note that the left hand side of equation (23) is increasing in π, which requires a rise in θ to rebalance since $p'(\theta) > 0$ and $q'(\theta) < 0$.

At the steady-state equilibrium, where unemployment rate is stable, hires equal separations: $m(u, v) = \delta(1 - u)$, which gives the following unemployment rate:

$$u(\theta) = \frac{\delta}{\delta + p(\theta)}. \tag{16}$$

Equation (16) implicitly describes the Beveridge Curve relationship between vacancies and unemployment. Because $p'(\theta) > 0$, the steady-state unemployment rate is decreasing in labor market tightness. Therefore the planner's problem is equivalent to maximizing $\theta = v/u$, which requires incentivizing vacancy creation as much as possible. That entails delivering the entire surplus from the match to the firm. The lowest value that will not deter unemployed workers from searching for a job in the first place requires $W_0 = U$, which leads to a corner solution to the planner's problem where workers participation constraint binds. The level of maximum attainable employment consistent with the planner's solutions is, therefore, is well-defined as summarized by the following proposition.

Proposition 4. *The maximum attainable labor market tightness solves*

$$\frac{p - z}{r + \delta} = \frac{\kappa}{q(\theta^{max})}, \tag{17}$$

and the corresponding maximum employment rate is:

$$e^{max} = 1 - u^{min} = \frac{p(\theta^{max})}{\delta + p(\theta^{max})} \tag{18}$$

In the planner's allocation, firm takes all of the surplus from the match. Note that the maximum employment level only depends on the fundamental parameters of the economy. It does not depend on the inflation rate, implying that it is a stable target regardless of the adopted monetary stance.

By definition, equilibrium employment is below the maximal employment level so long as $\beta > 0$, i.e., workers capture some of the match surplus: $\theta^* < \theta^{max} \iff \beta > 0$. Recall from Proposition **??** that equilibrium employment is increasing in the inflation rate. It is natural to ask: What level of the inflation rate implements maximum employment? Comparison of equations (23) and (17) reveals, however, that equilibrium employment approaches maximum employment only as a limiting case, when the inflation rate approaches infinity $\pi \to \infty$.[6]

[6]Maximum employment in equilibrium requires $\frac{\pi}{r+\delta+\pi+p(\theta)} = 1$ which can only hold if $\pi = \infty$.

2.2 Optimal Employment

Following **?**, our definition of the optimal unemployment rate maximizes welfare:

$$\max_{u,v} \int_0^\infty e^{-rt}(1-u)p + uz - v\kappa\theta \equiv \min_{u,v} \int_0^\infty e^{-rt}u(p-z) + v\kappa$$

subject to

$$\dot{u} = (1-u)\delta - m(u,v)$$

Note that welfare does not depend on the wage rate because w is a transfer from workers to firms. Total flow utility for a job is, therefore, $p - w + w = p$ including the firm owners. The solution to this optimal control problem gives the well-known Hosios optimality condition, which is stated below:

Proposition 5 (Hosios, 1990)**.** *The optimal tightness level is given by:*

$$\frac{p-z}{r+\delta} = \frac{\kappa}{q(\theta^H)} \cdot \frac{r+\delta+\eta_u^m p(\theta^H)}{\eta_v^m(r+\delta)}, \tag{19}$$

where $\eta_u^m = m_u(u,v)u/m$ and $\eta_v^m = m_v(u,v)v/m$ denote the elasticities of the matching function with respect to the numbers of unemployed workers and vacancies.

The optimality of the equilibrium is paramount to understanding the recipes for monetary policy for a planner that seeks to maximize welfare. Comparing equations (19) and (13) reveals the well-known condition for equilibrium optimality in the canonical, unconstrained economy: $\beta = \eta_u^m = (1-\eta_v^m)$. This ensures that the social marginal return to an additional unemployed worker and an additional vacant position are aligned by the private incentives of job seekers and firms.[7]

The Hosios condition is met for all values of u and v when $m(u,v)$ is given by a Cobb-Douglass function with the relevant elasticities set to β and $1-\beta$. In that case, all equilibria are efficient in the unconstrained economy. Because credit constraints do not bind at low levels of inflation, all levels of inflation $\pi < \underline{\pi}$ ensures that the equilibrium is optimal. Therefore a natural strategy for a central bank that aims to maximize welfare is to maintain a low inflation rate.

However, the requirement that the matching function be of Cobb-Douglas form with elasticities coinciding with relative bargaining powers in wage negotiations is an extremely special case.

[7]The marginal value of an unemployed worker is the product of marginal matches created and the surplus each match generates : $m_u(u,v) \times S$, while the private return equals the product of job-finding probability and the workers' share of the surplus $m(u,v)/u \times \beta S$. Equating the two gives: $\beta = \eta_u^m$. Similarly, $\beta = (1-\eta_v^m)$ follows from social value of vacancies and firms' posting incentives: $m_v(u,v) \times S = m(u,v)/v \times (1-\beta)S$.

More generally, matching elasticities vary with the numbers of unemployed workers and vacancies. In that case the optimality of equilibrium employment depends on where the elasticities stand vis-à-vis the bargaining powers of workers and firms. Moreover, because the inflation is non-neutral only above a certain level, the planner's ability to implement optimal employment whenever the equilibrium deviates from it depends on whether that deviation happens at high or low inflation rates. We go over these cases below.

When $\eta_u^m < \beta$, equilibrium employment is suboptimal. In that case, the planner can raise employment level by targeting an inflation rate above $\underline{\pi}$. Comparing equations (19) and (23) gives the optimal inflation rate that delivers optimality as follows.

Proposition 6. *If $\eta_u^m < \beta$, the welfare maximizing inflation rate is*

$$\pi^* = \frac{1 - \eta_u^m}{\eta_u^m}(r + \delta) > \underline{\pi} \tag{20}$$

The optimal inflation rate requires the planner to follow an active monetary policy by raising π above $\underline{\pi}$. The optimal inflation target is decreasing in the elasticity of the matching function with respect to unemployed workers. If the numbers of unemployed workers and vacancies are gross complements in the matching process, then a low elasticity is associated with a higher unemployment rate relative to vacant positions. In that case the optimal inflation target rises with the unemployment rate suggesting that optimal policy is countercyclical.

When $\eta_u^m > \beta$, worker's retain too low a share of the match surplus and employment is super-optimal: $\theta^H < \theta^* < \theta^{max}$. Because the inflation rate is already low in unconstrained equilibria, an active monetary policy which raises the inflation rate high enough to stimulate employment would worsen welfare by raising employment further above optimality. Therefore the optimal policy calls for a low inflation target $\pi^* < \underline{\pi}$. If inflation happens to be high enough that the economy is constrained when $\eta_u^m > \beta$, then the policy could improve welfare by lowering the inflation target.

3 Short-Run Policy Tradeoffs

4 Quantitative/Empirical Analysis

5 Conclusion

A Mathematical Appendix

This appendix contains the proofs for the propositions and derivations for some of the equations in the main text. We first list the equations that describe the model for convenience. The model is summarized by the following value functions:

$$\begin{aligned} rV &= -\kappa + q(\theta)(J_0 - V) \\ rJ_0 &= p - w + \delta(V - J_0) + \pi(J - J_0) \\ rJ &= p - \underline{\mathrm{w}} + \delta(V - J) + \phi(J_0 - J) \\ rU &= z + p(\theta)(W_0 - U) \\ rW_0 &= w + \delta(U - W_0) + \pi(W - W_0) \\ rW &= \underline{\mathrm{w}} + \delta(U - W) + \phi(W_0 - W) \end{aligned}$$

and the following equilibrium conditions and assumptions:

$$\begin{aligned} V &= 0 \\ W &= U \\ \beta(J_0 - J) &= (1 - \beta)(W_0 - U) + \mu, \end{aligned}$$

where $\mu \geq 0$ is the multiplier on the firm's credit constraint. Corresponding to these 9 equations, the steady state consists of 9 unknowns: 6 values, two wage rates, w and $\underline{\mathrm{w}}$, and labor market tightness, θ.

Proposition (Equation (11))**.**

$$w = \beta p + (1 - \beta) rU + \pi(W_0 - W).$$

Proof. To determine the equilibrium wage rate, consider the following representations for worker's and firm's shares of the total surplus.

$$W_0 - U = \frac{w - rU + \pi(W - W_0)}{r + \delta} \tag{21}$$

$$J_0 - V = \frac{p - w + \pi(J - J_0)}{r + \delta}, \tag{22}$$

where worker's surplus is obtained by subtracting rU from equation (2). Using the Nash bargaining protocol, we have $(1 - \beta)(W_0 - U) = \beta(J_0 - V)$. Solving for w gives:

$$w = \beta p + (1-\beta) rU + \pi(W_0 - W).$$

□

Lemma 1. $J - J_0 = W_0 - W$

Proof. The difference between equations (5) and (6) and the that between (3) and (2) gives:

$$J - J_0 = \frac{w - \underline{w}}{r + \delta + \phi + \pi} = W_0 - W.$$

□

Proposition 1. *If wage payments are unbounded, then the equilibrium wage rate is given by:*

$$w = \beta p + (1-\beta) z + \beta \kappa \theta + \pi \frac{\beta}{1-\beta} \frac{\kappa}{q(\theta)}. \tag{12}$$

Proof. First note that.

$$W_0 - W = W_0 - U = \frac{\beta}{1-\beta} \frac{\kappa}{q(\theta)}.$$

The first equality is by assumption. Second equality uses the Nash bargaining equation along with the free entry condition that implies $V = 0$ and $J_0 = \kappa / q(\theta)$:

$$W_0 - U = \frac{\beta}{1-\beta}(J_0 - V) = \frac{\beta}{1-\beta} J_0 = \frac{\beta}{1-\beta} \frac{\kappa}{q(\theta)}$$

Substituting for $W_0 - U$ from the lemma above in equation 1 gives:

$$rU = z + p(\theta) \frac{\beta}{1-\beta} \frac{\kappa}{q(\theta)}$$

Then, substituting for rU and $W_0 - W$ from the equations above gives the following representation for the wage equation:

$$w = \beta p + (1-\beta) z + \beta \kappa \theta + \pi \frac{\beta}{1-\beta} \frac{\kappa}{q(\theta)}$$

□

Proposition 2. *Equilibrium level of labor market tightness is given by:*

$$\frac{p - z}{r + \delta} = \frac{\kappa}{q(\theta^*)} \cdot \frac{r + \delta + \beta p(\theta^*)}{(1-\beta)(r+\delta)}. \tag{13}$$

Proof. Simply substitute the solution for the wage rate from (12) and $J - J_0 = W_0 - W = \frac{\beta}{1-\beta}\frac{\kappa}{q(\theta)}$ in the job creation condition.

$$\begin{aligned}\frac{p - w + \pi(J - J_0)}{r + \delta} &= \frac{p - \beta p - (1-\beta)rU - \pi(W_0 - W) + \pi(J - J_0)}{r + \delta} \\ &= \frac{(1-\beta)(p - rU)}{r + \delta} \\ &= \frac{(1-\beta)(p - z)}{r + \delta} - \frac{\beta(p(\theta))}{r + \delta}\frac{\kappa}{q(\theta)} = \frac{\kappa}{q(\theta)},\end{aligned}$$

where the last equality follows from the job creation condition. Rearranging terms gives

$$\frac{\kappa}{q(\theta)}\left[1 + \frac{\beta(p(\theta))}{r + \delta}\right] = \frac{\kappa}{q(\theta^*)} \cdot \frac{r + \delta + \beta p(\theta^*)}{(1-\beta)(r + \delta)} = \frac{p - z}{r + \delta}$$

□

Proposition (Equation (14))**.** *The negotiated wage exceeds productivity if the inflation rate is above*

$$\underline{\pi} = \frac{1-\beta}{\beta}(r + \delta).$$

Proof. Setting Nash w to p and solving for π.

$$w^N = p = \beta p + (1-\beta)z + \beta\kappa\theta + \pi\frac{\beta}{1-\beta}\frac{\kappa}{q(\theta)} \tag{23}$$

The equilibrium condition (13), under Nash bargaining is:

$$\frac{p - z}{r + \delta} = \frac{\kappa}{q(\theta^*)} \cdot \frac{r + \delta + \beta p(\theta^*)}{(1-\beta)(r + \delta)}.$$

which equals the condition above under $w = p$ if and only if:

$$\frac{\pi}{r + \delta + \pi + p(\theta)} = \frac{(1-\beta)(r + \delta)}{r + \delta + \beta p(\theta^*)}$$

Solving for π:

$$\begin{aligned}\pi(r + \delta + \beta p(\theta)) &= (1-\beta)(r + \delta)(r + \delta + \pi + p(\theta)) \\ \pi(r + \delta + \beta p(\theta) - (1-\beta)(r + \delta)) &= (1-\beta)(r + \delta)(r + \delta + p(\theta)) \\ \pi\beta(r + \delta + p(\theta)) &= (1-\beta)(r + \delta)(r + \delta + p(\theta)) \\ \underline{\pi} &= \frac{1-\beta}{\beta}(r + \delta)\end{aligned}$$

Therefore equilibrium tightness is determined by equation (13) if $\pi \leq \underline{\pi}$ and by equation (23) otherwise.

□

Proposition 3. *In an economy with credit constraints, the equilibrium level of labor market tightness solves equation* (13) *if* $\pi \leq \underline{\pi}$ *and solves the following equation if* $\pi \geq \underline{\pi}$*:*

$$\frac{\pi}{r+\delta+\pi+p(\theta)}\frac{(p-z)}{r+\delta} = \frac{\kappa}{q(\theta)}. \tag{15}$$

Proof. Setting $w = p$ in the job creation condition (10) gives:

$$\frac{\overbrace{p-p}^{=0}+\pi(J-J_0)}{r+\delta} = \frac{\kappa}{q(\theta)}$$

Subtracting J from J_0:

$$J - J_0 = \frac{w - \underline{\mathrm{w}}}{r+\phi+\pi+\delta}$$

Setting $W = U$ gives $\underline{\mathrm{w}}$:

$$\underline{\mathrm{w}} = z + (p(\theta)-\phi)(W_0 - U) = z + (p(\theta)-\phi)\frac{w-z}{r+\delta+\pi+\phi}$$

where the last equality follows from subtracting rU from rW_0. Subtracting both sides from w gives:

$$w - \underline{\mathrm{w}} = (w-z)\frac{r+\delta+\pi+\phi}{r+\delta+\pi+p(\theta)}$$

Note, when $\phi = p(\theta)$, then $\bar{w} = z$ as expected.

Setting $w = p$ and plugging this back in for $J - J_0$ in the job creation condition gives:

$$\frac{\pi}{r+\delta+\pi+p(\theta)}\frac{(p-z)}{r+\delta} = \frac{\kappa}{q(\theta)},$$

which holds for $\pi > \underline{\pi} > 0$. □

Proposition 4. *The maximum attainable labor market tightness solves*

$$\frac{p-z}{r+\delta} = \frac{\kappa}{q(\theta^{max})}, \tag{17}$$

and the corresponding maximum employment rate is:

$$e^{max} = 1 - u^{min} = \frac{p(\theta^{max})}{\delta + p(\theta^{max})} \tag{18}$$

Proof. The lowest value that will not deter unemployed workers from searching for a job in the first place requires $W_0 = U$. Using equations (2) and (1), the added value to a worker from a job is:

$$W_0 - U = \frac{w - z - \pi(W_0 - W)}{r + \delta + p(\theta)}$$

Setting that zero gives the lowest acceptable wage to the worker: $w = z + \pi(W_0 - W)$. Substituting it back in the job creation condition in (10) gives the maximum attainable labor market tightness:

$$\frac{p - z + \pi(J - J_0 - W_0 + W))}{r + \delta} = \frac{\kappa}{q(\theta)},$$

where $J - J_0 - W_0 + W = 0$ by Lemma 1. □

Proposition 6. *If $\eta_u^m < \beta$, the welfare maximizing inflation rate is*

$$\pi^* = \frac{1 - \eta_u^m}{\eta_u^m}(r + \delta) > \underline{\pi} \tag{20}$$

Proof. First note that $\pi^* > \underline{\pi}$ whenever $\eta_u^m < \beta$. Therefore the equilibrium is constrained and $w = p$. Comparing equations (23) and (19), labor market tightness is optimal if and only if

$$\frac{r + \delta + \pi + p(\theta)}{\pi} = \frac{r + \delta + \eta_u^m p(\theta^H)}{\eta_v^m (r + \delta)}$$

Solving for π gives the condition in the proposition. □