Demand or Supply?
Expectations and Inflation under Confusion *

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Abstract

How do agents respond when the origins of macroeconomic shocks are uncertain? We develop a theory of confusion between aggregate shocks. When inflation signals are more informative, agents mistake negative supply shocks for positive demand shocks—negative confusion. When output signals dominate, positive confusion arises. In New Keynesian models, negative confusion amplifies inflation after supply shocks via strategic price complementarity and muted expectations of real wage declines, while positive confusion reverses both channels and dampens inflation. Estimating the model pre- and post-1990 reveals a shift from inflation- to output-based inference, reconciling three empirical puzzles: (i) the Phillips curve's flattening post-1990 and steepening during the pandemic; (ii) a pronounced "supply view," a negative relations between expected output and inflation; and (iii) the inflation–output disconnect, wherein the two variables appear driven by distinct shocks.

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"It's two blades of a scissors. I'm not sure we would have today's inflation without the pandemic, and I'm not sure we would have it without the overwhelming stimulus we applied."
— Lawrence Summers

1 Introduction

U.S. CPI inflation rose from 1.3 percent (2020Q4) to 9.0 percent (2022Q2), the steepest two-year increase since 1979, despite three decades of evidence that the Phillips curve had flattened. The price surge was accompanied by pervasive uncertainty about its provenance among firms, households, and economists. Two prominent diagnoses illustrate the debate: some economists attributed the rise mostly to *excess demand*, whereas Bernanke and Blanchard (2023) emphasized the central role of *cost shocks*.[1] Confusion about the underlying aggregate shocks of macroeconomic fluctuations is not unique to the pandemic era. During the Great Inflation of the 1970s, Fed Chair Arthur Burns blamed soaring prices on "*special factors—oil, harvest failures, the removal of controls*," arguing that tighter money supply would miss what he viewed as a largely cost-push problem. By contrast, Milton Friedman—and other monetarists—insisted that inflation is "*always and everywhere a monetary phenomenon*". This pattern resembles the post-2020 episode: in both cases, profound uncertainty about the origin of inflation coincided with a steepened Phillips curve.

This paper argues that *confusion* between aggregate shocks has important implications for business cycle fluctuations. Unlike standard models, where agents know the type of aggregate shock but not its magnitude, real-world signals often reflects multiple shocks, making both dimensions uncertain. We show that such confusion gives rise to five novel results: (i) confusion provides a resolution to the weak inflation response following supply shocks typically observed in standard New Keynesian model; (ii) it sheds light on the "inflation-output disconnect puzzle," wherein inflation and output empirically appear driven by distinct shocks (Angeletos et al., 2020); (iii) confusion helps explain the observed flattening of the Phillips curve from 1990 to 2019 (Del Negro et al., 2020; Hazell et al., 2022), as well as its subsequent steepening during the COVID-19 pandemic. Our findings are consistent with Weber et al. (2023), who show that high-inflation periods coincide with more precise inflation signals received by agents. (iv) quantitatively, our framework jointly matches the time series of inflation, real GDP growth, and their corresponding forecasts from professional forecasters. In contrast, standard information-friction models without confusion struggle to match forecasters' beliefs; (v) finally, we document a pronounced "supply view" among professional forecasters prior to 1990 (Candia et al., 2023a), whereby both output and

[1] Prominent voices publicly revised their views as new evidence accumulated. Paul Krugman later admitted that his early-2021 "*Team Relaxed*" view on the $1.9 trillion American Rescue Plan was "*a very bad call*". Treasury Secretary Janet Yellen likewise states that she had been "*wrong about the path inflation would take*," pointing to unanticipated energy shocks and supply bottlenecks she "*didn't fully understand*." The academic literature mirrored this uncertainty: a quick Google Scholar search uncovers at least eight papers published between 2020 and 2023 with titles that explicitly frame the issue as "Demand or Supply?"—for example, *Demand or Supply? Price Adjustment during the COVID-19 Pandemic* (Balleer et. al. 2020); *How Much Do Supply and Demand Drive Inflation?* (Shapiro, 2022).

inflation fluctuations were predominantly attributed to supply shocks. After 1990, although forecasters continued to emphasize supply shocks, their perceptions aligned more closely with actual composition of shocks.

A Theory of Confusion We develop a general theory of confusion between simultaneous demand and supply shocks.[2] Agents observe two signals: a weighted sum of the shocks (the *output signal*) and a weighted difference (the *inflation signal*). The *sign* of confusion depends on which signal is more precise. If the inflation signal dominates, agents mistake demand shocks for negative supply shocks (*negative confusion*); if the output signal dominates, they mistake supply shocks for demand shocks (*positive confusion*).

The degree of confusion is non-monotonic in signal precision. When both signals are either highly precise or highly noisy, confusion is minimal. It peaks when one signal is highly accurate and the other is extremely noisy. The key insight is that when agents observe only one mixed signal, it is fundamentally impossible to disentangle the individual shocks. Partial identification is possible only when the second signal is at least somewhat informative. This contrasts with standard information-friction models (Woodford, 2003), where confusion arises from the blending of aggregate and idiosyncratic shocks and vanishes as idiosyncratic noise dissipates.

We also characterize *higher-order confusion*: agents' beliefs about others' beliefs become even more distorted. Unlike standard models, where higher-order beliefs adjust sluggishly, confusion in our framework is non-monotonic—initially increasing, then declining. This occurs because the noisier signal exerts disproportionately less influence on higher-order beliefs. At sufficiently high orders, both signals lose influence, and confusion fades. The implication is that when agents are confused about fundamentals, their expectations about others' beliefs are even more confused.

Confusion in New Keynesian Model We embed confusion into a stylized New Keynesian model. In standard models, macroeconomic responses are separable by shock. Under confusion, however, the system must be solved jointly. Negative confusion amplifies inflation and output responses to supply shocks, while its effect on demand shocks is ambiguous. The mechanism works as follows. Suppose a negative supply shock hits. If the inflation signal is more informative, firms confuse it with a positive demand shock. Inflation is amplified for two reasons: (i) firms expect demand-driven inflation and raise prices due to strategic complementarity, and (ii) they perceive a muted decline—or even an increase—in real wages, as demand shocks typically raise wages.

In standard models, a negative supply shock lowers real wages, dampening inflation. Information frictions without confusion weaken both the decline in productivity and in wages, limiting the amplification of inflation (Angeletos and La'o, 2010). By contrast, confusion disproportionately weakens the real wage channel, steepening the Phillips curve in response to supply shocks. When

[2] Demand shocks raise both output and inflation; supply shocks raise output but lower inflation, as in standard New Keynesian models (Christiano et al., 2005; Smets and Wouters, 2007; Justiniano et al., 2010). We abstract from confusion between shocks of the same type.

the output signal is more informative, confusion instead attenuates inflation. For demand shocks, the effects of confusion are ambiguous. If firms mistake a positive demand shock for a negative supply shock, they raise prices due to strategic complementarity but may also expect a decline in real wages, which could dampen the inflationary response. The net effect is therefore uncertain. Overall, the Phillips curve becomes both shock-dependent and regime-dependent.

Extending the model to endogenous information, we impose consistency between firms' beliefs and aggregate outcomes (Grossman and Stiglitz, 1980). With only a precise inflation signal, multiple equilibria can arise under demand shocks. One equilibrium features no price adjustment and zero inflation—rendering the signal uninformative. Adding a precise output signal would restores uniqueness . Endogenous information also makes inflation more sensitive to signal precision. A more precise inflation signal increases negative confusion, amplifying inflation. This stronger inflation response feeds back into signal informativeness, creating a self-reinforcing loop.

Quantification We estimate a New Keynesian model with endogenous information and learning, allowing for shifts in signal precision and shock variances before and after 1990. To address the computational challenges of solving equilibrium under confusion, we employ recent development in frequency-domain methods (Rondina and Walker, 2017; Adams, 2019; Han et al., 2022; Huo and Takayama, 2023). Our estimation reveals that prior to 1990, inflation signals were more informative, generating near-maximal negative confusion and substantially amplifying the inflation response to supply shocks. After 1990, output signals became more precise, reversing the direction of confusion and moderating inflation dynamics.

This regime shift flattens the Phillips curve. The on-impact response of inflation to demand shocks falls by 25%, and to supply shocks by over 75%. We estimate the Phillips curve slope using a standard procedure: simulating the model and regressing inflation on the output gap, controlling for actual next-period inflation. Under demand shocks, the slope declines from 0.009 to 0.006; under supply shocks, it drops sharply from 0.060 to 0.015—highlighting both the flattening and shock dependence of the Phillips curve.

We further conduct a historical decomposition of supply and demand shocks that exactly replicate the time series of real GDP and the GDP deflator. Quantitatively, demand shocks account for 81% of output variation after 1990, up from just 41% before; supply shocks remain the dominant driver of inflation in both periods. Our framework reconciles the inflation–output disconnect puzzle (Smets and Wouters, 2007; Angeletos et al., 2020), which had compelled prior models to rely on ad hoc "markup" shocks. Although professional forecasters' expectations are not directly targeted, the estimated shocks closely track their forecasts for both output and inflation. Notably, we uncover a strong pre-1990 *supply view*: most output fluctuations were perceived as supply-driven. While this perception weakens post-1990, it persists.

These patterns are tightly linked. Our model fits both realized and expected data because, despite high estimated precision of inflation signals, large supply shock variance continues to

obscure information about demand. This generates substantial monetary non-neutrality and a much muted *expected* output response to demand shocks. Before 1990, the combination of high supply variance, precise inflation signals, and noisy output signals produces maximal confusion, strongly amplifying the inflation response to supply shocks. Agents thus perceive supply shocks as the dominant source of both output and inflation fluctuations. After 1990, the reversal in signal precision reduces confusion, shifting perceived output variation away from supply shocks and flattening the Phillips curve.

Literature Our paper contributes to three related literatures. First, regarding the Phillips curve, aggregate supply, and the foundations of monetary non-neutrality, we build on the literature that examines the role of information frictions in business cycle dynamics (Lucas, 1972; Woodford, 2003; Mankiw and Reis, 2001; Maćkowiak and Wiederholt, 2009; Hellwig and Venkateswaran, 2009; Angeletos and La'o, 2010; Angeletos and Lian, 2022; Flynn et al., 2023). Our baseline model without explicit nominal rigidity is particularly close to Lucas (1972) in its focus on learning from endogenous objects. We show that confusion about *aggregate shocks* generates regime- and shock-dependent inflation dynamics: the inflation response following negative supply shocks is heightened under negative confusion and becomes more muted under positive confusion. In the calibration, we also find that demand shocks generate larger monetary non-neutrality and imply a flatter Phillips curve, consistent with empirical evidence (Del Negro et al., 2020).

Second, we relate to the literature on Phillips curve flattening, inflation-output disconnect. Del Negro et al. (2020) and Hazell et al. (2022) document post-1990 flattening, while Angeletos et al. (2020) show that shocks driving output differ from those driving inflation. Our model explains these facts via regime- and shock-dependent confusion. Before 1990, agents prioritized inflation signals and confused supply shocks with demand shocks, amplifying inflation. After 1990, attention shifted to output, causing demand shocks to be mistaken for supply, flattening the curve. Moreover, since supply shocks The model also explains why demand shocks yield large output changes with little inflation, while supply shocks cause the reverse—reconciling the disconnect. Unlike standard information-friction models, our mechanism allows for either amplification or attenuation depending on signal salience, generating regime shifts in inflation without changes in policy or underlying model structure.

Third, we speak to recent empirical work on macroeconomic expectations. Weber et al. (2023) show that attention to inflation varies with regimes: when inflation is high, firms pay more attention to inflation. We show the importance of considering signals on output in addition to inflation signals, and how the relative informativeness of these signals can determine the sign and magnitude of confusion. Though our model does not endogenize the choice of information acquisition, we estimate the model for different regimes with high and low inflation, and show how endogenous attention steepens or flattens the Phillips curve. Firm surveys further reveal that managers and households interpret price changes as cost-driven—what Candia et al. (2023b) term a persistent

"supply-side" view. Our framework explains this: when prices are more informative than quantities, firms interpret shocks as supply-driven, consistent with our model's historical decompositions.

Outline In Section 2, we provide a general theory of confusion. In Section 3, we show the general equilibrium consequences of confusion by letting firms be econometricians and price setters simultantously. Section 4 brings a general model into data, and estimate the information friction and confusion before and after 1990, and study its implication for observed inflation, and slope of Phillips curve. Section 5 concludes.

2 A Theory of Confusion

We first study the information extraction problem of an econometrician, who observes output and price inflation and tries to determine the nature of the shocks driving the economy—just as a macroeconomist would. We show that if the econometrician relies on endogenous objects, such as inflation or output, to learn about the underlying shocks, confusion inevitably arises. We then characterize the key properties of this confusion. In the next section, we show that if the econometrician is also a decision maker, such confusion affects the data-generating process and thus the very information the econometrician observes.

We assume, as in reality, that agents do not observe aggregate shocks directly and must form expectations under incomplete information. Aggregate demand and supply shocks have distinct implications for output and inflation: in the standard demand-supply paradigm, a positive demand shock increases both output and inflation, while a positive supply shock raises output but reduces inflation. If agents observe only output and receive no information about inflation, they may misinterpret a demand-driven expansion as a supply-driven one. Conversely, if they observe only inflation but not output, they might mistake a positive demand shock for a negative supply shock. We refer to such misperceptions as *confusion*. For illustration, we focus only on two types of signals—an output signal and an inflation signal—which take the following form:[3]

$$x_{it}^{y} = \omega_{ys} a_t + \omega_{yd} \beta_t + \epsilon_{it}^{y}, \quad x_{it}^{\pi} = -\omega_{\pi s} a_t + \omega_{\pi d} \beta_t + \epsilon_{it}^{\pi}, \tag{1}$$

where all ω are strictly positive, with the interpretation that ω_{ys} and $\omega_{\pi s}$ are output and inflation's general equilibrium impulse responses to a supply shock, while ω_{yd} and $\omega_{\pi d}$ are output and inflation's impulse responses to a demand shock. We take these impulse responses as exogenously given for the purpose of this section but will endogenize them in Section 3.2. The noise terms, with a slight abuse of notation, follow $\epsilon_{it}^{y} \sim \mathcal{N}(0, \sigma_y^2)$ and $\epsilon_{it}^{\pi} \sim \mathcal{N}(0, \sigma_\pi^2)$. In other models, such as

[3] In principle, agents can observe other signals, for example, separate measures for unemployment, wages, output, and so on. However, all these signals boil down to different rotations of output and inflation. To keep the information structure minimal, we abstract from these additional signals. In Section 4, where we lay out the quantitative model, agents have inflation and output signals together with private signals.

island models à la Lorenzoni (2009), these noises are interpreted as random consumption baskets, idiosyncratic unemployment, and so forth. The minus sign on $\omega_{\pi s}$ captures the negative impact of supply shocks on inflation.

This signal structure provides the most parsimonious framework for capturing confusion between aggregate demand and supply shocks. A higher realization of the output signal, x_{it}^{y}, can reflect either an increase in productivity or an increase in aggregate demand. Similarly, a higher realization of the inflation signal, x_{it}^{π}, may result from either a negative productivity shock or a positive demand shock. The mixture of these two signals gives flexibility in relative precision of these two signals that determines the direction of confusion. In the limiting case where the noise variances σ_y and σ_π approach zero, firms are able to perfectly back out supply and demand shocks from the observed signals. For now, we consider exogenous signals, i.e., ω coefficients are taken as exogenously specified. In Section 3, we analyze endogenous signals, i.e., the signal coefficients are required to be aligned with the endogenous impulse responses of inflation and output to unit supply and demand shocks. That is,

$$\omega_{ys} = \tilde{y}_s, \quad \omega_{yd} = \tilde{y}_d, \quad \omega_{\pi s} = \tilde{\pi}_s, \quad \omega_{\pi d} = \tilde{\pi}_d \tag{2}$$

where $\tilde{\pi}_s, \tilde{\pi}_d, \tilde{y}_s, \tilde{y}_d$ denote the equilibrium responses of inflation and output to unit supply and demand shocks.[4] Confusion with endogenous signals thus depends on specific applications. For now, we could start to analyze the signal extraction problem of the econometrician, and the following proposition presents the definition of confusion:

Theorem 1 (Confusion in Belief Formation)**.** *Given the signal structure in (2), the projection matrix from average agents' belief to aggregate shocks,* $\mathbf{P}$*, is given by,*

$$\begin{pmatrix} \bar{E}a \\ \bar{E}\beta \end{pmatrix} = \mathbf{P} \begin{pmatrix} a \\ \beta \end{pmatrix} = \begin{pmatrix} D_s & C_s \\ C_d & D_d \end{pmatrix} \begin{pmatrix} a \\ \beta \end{pmatrix}$$

Where $C_s = C_d = C$ *represent confusion and* $D_s, D_d < 1$*. In addition,*

$$C \propto \frac{1}{\sigma_y^2}\omega_{yd}\omega_{ys} - \frac{1}{\sigma_\pi^2}\omega_{\pi d}\omega_{\pi s}$$

Where ω*s are weights on the shocks in signal process (8), and* σ_y*,* σ_π *are variances of noises in agents' output and inflation signals.*

Proof. See Appendix A □

This proposition establishes that, under the signal structure described in (8), firms' average belief about a demand shock, denoted by C_s, is generally nonzero following a supply shock. Likewise,

[4] By responses to unit shocks, we mean $y_d = \tilde{y}_d\beta, y_s = \tilde{y}_s a, \pi_d = \tilde{\pi}_d\beta, \pi_s = \tilde{\pi}_s a$.

C_d represents firms' belief about a supply shock after experiencing a demand shock. Due to the symmetry of signals, both terms are identical. Importantly, the sign of confusion is determined by the relative informativeness of both signals, given by the product of the weights, $\omega_{yd}, \omega_{ys}, \omega_{\pi d}, \omega_{\pi s}$, or impulse responses in the case of endogenous signals. Intuitively, if inflation responds more to shocks (i.e., if $\omega_{\pi d}, \omega_{\pi s}$ are large), inflation contains more information and agents will learn more from it. Since an increase in inflation can be due to either a positive demand shock or a negative supply shock, learning more from inflation leads agents to confuse a *negative* supply shock with a *positive* demand shock:

$$\bar{E}a = D_s a + \underbrace{C}_{<0}\beta.$$

Conversely, if output is more responsive to shocks (i.e., if ω_{yd}, ω_{ys} are large), agents will extract more information from output. In this case, an increase in output can be due to either a positive demand shock or a positive supply shock, so learning more from output leads agents to confuse a *positive* supply shock with a *positive* demand shock:

$$\bar{E}\beta = \underbrace{C}_{>0} a + D_d \beta.$$

Note that the impulse responses $\omega_{yd}, \omega_{ys}, \omega_{\pi d}, \omega_{\pi s}$ are further weighted by the variance of the noise in the corresponding signals. Since agents naturally learn more from inflation if it is more informative (i.e., σ_π is smaller), and vice versa, the overall pattern follows: agents will confuse a *negative* supply shock with a *positive* demand shock if inflation responds more strongly or if the inflation signal is more informative. Agents will confuse a *positive* supply shock with a *positive* demand shock if output responds more strongly or if the output signal is more informative.

Proposition 1 (Properties of Confusion)**.** ***Part I.*** *For a fixed relative noise ratio $\rho = \frac{\sigma_y^2}{\sigma_\pi^2}$, we have*

$$\lim_{\sigma_y \to 0} C = \lim_{\sigma_y \to \infty} C = 0.$$

Moreover, for given weights ω, when $\rho < \rho^*$, $C > 0$, and it first increases and then decreases in σ_y; when $\rho > \rho^*$, $C < 0$, and the cross-projection first decreases and then increases in σ_y.

Part II. *For given weights ωs, C increases with σ_π, given σ_y fixed; C decreases with σ_y, given σ_π fixed.*

Proof. See Appendix A □

Proposition 1 shows that when both signals are highly precise, firms can fully recover the underlying structural shocks, and thus confusion converges to zero. Conversely, when signals are extremely noisy, firms extract no useful information, and all entries in the projection matrix in Proposition 1 converge to zero. Given the signal weights ω and the relative noise ratio, the magnitude of confusion exhibits a non-monotonic relationship with signal precision: confusion is

maximized at an intermediate level of precision.

In the second part, we consider a different form of monotone comparative statics. Holding the noise level of the output signal fixed, an increase in the noise of the inflation signal reduces the relative informativeness of the inflation signal, shifting the direction of confusion. As $\sigma_\pi \to \infty$ with fixed σ_y, the magnitude of confusion increases. A symmetric result holds when the inflation signal is held constant and the noise in the output signal increases. The underlying intuition follows directly from Proposition 1: agents rely on both signals to identify structural shocks. (i) Fixing the noise variance of the inflation signal, a reduction in the noise variance of the output signal makes agents rely more on the output signal, thereby confusing a positive demand shock with a positive supply shock, or equivalently, C increases; (ii) fixing the noise variance of the output signal, a reduction in the noise variance of the inflation signal makes agents rely more on the inflation signal, thereby confusing a positive demand shock with a *negative* supply shock, or equivalently, C decreases.

To see this more clearly, let us zoom in on two extreme cases:

Corollary 1 (Maximal Confusion)**.** *Given ω and shock-generating processes, if $C > 0$, C is maximized when $\sigma_y \to 0$ and $\sigma_\pi \to \infty$, denoted by C_{max}; If $C < 0$, C is minimized when $\sigma_\pi \to 0$ and $\sigma_y \to \infty$, denoted by C_{min}. They are given by,*

$$C_{min} = -\frac{\sigma_d \sigma_s \omega_{\pi d} \omega_{\pi s}}{\sigma_d^2 {\omega_{\pi d}}^2 + \sigma_s^2 {\omega_{\pi s}}^2}, \quad C_{max} = \frac{\sigma_d \sigma_s \omega_{yd} \omega_{ys}}{\sigma_d^2 {\omega_{yd}}^2 + \sigma_s^2 {\omega_{ys}}^2}$$

Moreover, when $\sigma_a = \sigma_d$ and $\omega = 1$, the maximal confusion and the associated projection matrix are given by,

$$C_{max} = -C_{min} = \frac{1}{2}; \ \mathbf{P} = \begin{pmatrix} \frac{1}{2} & \pm\frac{1}{2} \\ \pm\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Proof. See Appendix A □

This corollary identifies the conditions under which maximal confusion arise. Proposition 1 establishes that the magnitude of confusion becomes large when one signal is less uninformative, given the informativeness of the other. Building on this result, the corollary shows that the magnitude of confusion reaches its maximum when the informativeness of the other signal is also maximized.

In a particularly symmetric case where both shocks have equal weight in agents' priors and that the signal weights are normalized to one—the maximal magnitude of confusion is exactly $\frac{1}{2}$. Moreover, the other two entries of the projection matrix, which capture the dampened beliefs about the respective shocks, also equal $\frac{1}{2}$. This implies that agents attribute half of the variation in the signal to the demand shock and the other half to the supply shock. The sign of confusion is determined by which signal is more informative. In general, confusion is characterized by the following proposition:

Proposition 2 (Confusion and Spectral Decomposition)**.** *The projection matrix* $\mathbf{P}$ *is symmetric and positive definite. The spectral decomposition of* $\mathbf{P}$ *is,*

$$\mathbf{P} = \lambda_1 v_1 v_1^T + \lambda_2 v_2 v_2^T$$

Eigenvalues satisfy $0 \leq \lambda_2 \leq \lambda_1 \leq 1$ *and* $v_1 v_1^\top = \begin{pmatrix} x & \mu \\ \mu & z \end{pmatrix}$ *and* $v_2 v_2^\top = \begin{pmatrix} x & -\mu \\ -\mu & z \end{pmatrix}$*. The confusion is given by,*

$$C = (\lambda_1 - \lambda_2)\mu$$

where, we term μ *eigen-confusion. For given* ω *and shock-generating process, the eigen-confusion is constant for fixed noise ratio* $\frac{\sigma_y}{\sigma_\pi}$*. The difference of eigenvalues* $\lambda_1 - \lambda_2$ *first increases and then decreases in* σ_y*.*

Proof. See Appendix A. □

This proposition provides a deeper decomposition of confusion using the spectral decomposition of the projection matrix. It shows that confusion is proportional to the distance between the two eigenvalues and the *eigen-confusion*. The *eigen-confusion*, defined as the component of confusion in the space spanned by the matrix's eigenvectors, is a generic form of confusion. For given weights and the shock-generating process, it is fixed for a given relative noise ratio of the two signals. In other words, when both signals become noisy, keeping the ratio fixed, only the difference between the eigenvalues changes in the way shown in Proposition 1. The reason is that the eigen-confusion captures the rotation of the projection matrix, which depends only on the relative informativeness of the signals, while the absolute informativeness of the signals, that is, the levels of σ_π and σ_y, determines the overall level of confusion.

Again, to understand this decomposition more clearly, let us look at two extreme examples:

Corollary 2. *For given* ω *and shock-generating processes* $\sigma_a = \sigma_d = \omega = 1$*, in the limit where* $\sigma_\pi \to 0$*, the eigenvalues and* μ *are:*

$$\lambda_1 = 1, \quad \lambda_2 = \frac{1}{\sigma_y^2/2 + 1}, \quad \mu = -\frac{1}{2}$$

Similarly, in the limit where $\sigma_y \to 0$*, the eigenvalues and* μ *are:*

$$\lambda_1 = 1, \quad \lambda_2 = \frac{1}{\sigma_\pi^2/2 + 1}, \quad \mu = \frac{1}{2}$$

Proof. See Appendix A. □

Since the relative noise ratio is always fixed at 0 or ∞ in the limits where σ_π or $\sigma_y \to 0$, the eigen-confusion remains fixed. Importantly, it is fixed at the maximum or minimum level of confusion, as shown in Corollary 1. However, the level of confusion is determined by the levels of σ_y and σ_π. For the case where agents observe inflation perfectly, that is, $\sigma_\pi \to 0$, the second eigenvalue

increases with σ_y: the larger the output signal noise, the lesser the effect of the eigen-confusion. This is intuitive: even though agents maximally confuse a negative supply shock with a positive demand shock, the more precise the output signal is, the more attention agents devote to the output signal, resulting in a smaller absolute value of confusion.

Note that when a signal has smaller and smaller idiosyncratic variance, it becomes increasingly similar to public information. How would an agent then perceive other agents' beliefs about the underlying shocks? The following proposition shows that the higher-order beliefs exhibit interesting dynamics.

Proposition 3 (Higher-Order Confusions)**.** *The h-order average belief* $\bar{E}^h$ *is given by* $\bar{E}^h \begin{pmatrix} \beta \\ a \end{pmatrix} = P^h \begin{pmatrix} \beta \\ a \end{pmatrix}$. *The higher-order confusion is given by,*

$$C_h = (\lambda_1^h - \lambda_2^h)\mu$$

For given ω*s and shock-generating processes,* $|C_h|$ *first increases and then decreases with the order of confusion h. In the limit when* $\sigma_y \to 0$ *or* $\sigma_\pi \to 0$, $|C_h|$ *increases with h. In addition,*

$$\lim_{h\to\infty,\, \sigma_y\to 0} C_h = C_{max}; \quad \lim_{h\to\infty,\, \sigma_\pi\to 0} C_h = C_{min}$$

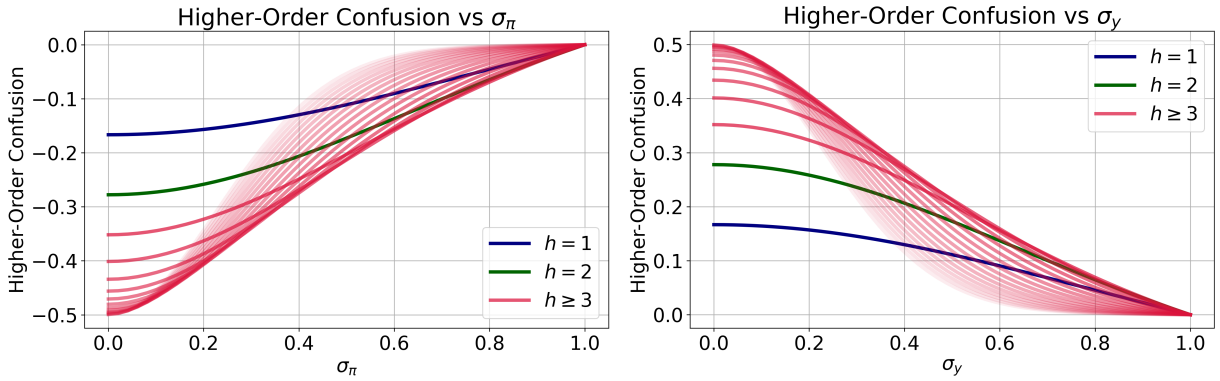
Proof. See Appendix A.

□

The proposition shows that higher-order confusion can be decomposed into two parts. The first depends on the distance between higher-order eigenvalues, which diminishes as the order increases. The second is the eigen-confusion, a component that remains constant across orders. According to Proposition 2, the higher-order confusion can be interpreted as the first-order confusion when agents receive signals with different noise levels, keeping the relative noise ratio fixed.

The magnitude of confusion first increases and then decreases across orders. The initial rise is driven by the rapid decay of the smaller eigenvalue. The eventual decline occurs because of the slower decay of the larger eigenvalue. The intuition is as follows: agents rely on two signals to infer the underlying shocks. However, the less informative signal contributes significantly less to second-order beliefs than the more informative one; agents understand that others are more anchored to the more informative signal, since it behaves more like public information (Morris and Shin, 2002; Angeletos and Pavan, 2007). However, relying increasingly on one signal means it becomes progressively harder for agents to resolve the identification problem, hence amplifying confusion. As the belief order increases further, the influence of both signals diminishes, and confusion naturally dissipates. In the extreme, when one of the noises is zero, the larger eigenvalue becomes one and the confusion never declines. This is because the signal effectively becomes public, shifting the common prior. This implication stands in stark contrast to standard models, where higher-order beliefs respond more sluggishly to aggregate shocks. As we will show in Section 3,

Figure 1: Higher-Order Confusions



Notes: The figure shows the higher-order confusion. The left panel shows the case where $\sigma_d = \sigma_s = \sigma_y = 1$ and we vary σ_π. The right panel shows the opposite.

higher-order confusion contributes significantly to the effect of confusion on general equilibrium responses.

Figure 1 shows the confusion of different orders in two cases. In the first case, we fix $\omega = 1$ and $\sigma_a = \sigma_d = \sigma_y = 1$ and vary σ_π. The second case fixes σ_π and varies σ_y. The magnitude of confusion is largest when $\sigma_\pi = 0$ and decreases over σ_π in the first case, consistent with Proposition 1. Also, in both cases, the magnitude of higher-order confusion initially increases and then declines, consistent with Proposition 3. Notably, even when σ_π or σ_y is small but not exactly zero, the magnitude of higher-order confusion can be substantially larger than the initial few orders. Moreover, the higher-order confusion remains approximately constant even when signal precision is not perfect. These patterns are relevant for estimated signal precisions in Section 4.

3 A Simple New Keynesian Model with Confusion

We now analyze the consequences of confusion in general equilibrium. We present a simple model building on Woodford (2003) in which firms confuse supply and demand shocks. Time is discrete and infinite. The timeline is as follows: In each period, Nature first draws aggregate demand and supply shocks and idiosyncratic shocks. In the morning, firms set prices. Firms cannot set fully state-contingent pricing plans but are allowed to set a pricing schedule conditional on the output and inflation information they will observe.[5]. In the afternoon, households make labor supply and consumption-saving decisions. Firms hire labor and produce to meet the demand. Labor market and good market clear. At date-t, firms have incomplete information about shocks, while households have full information. From next period onward, all agents know the realized

[5]This a specification is similar to the pricing schedule in Vives (2017), which in turn provides one micro-market structure for Grossman and Stiglitz (1980).

shocks.[6]

Household – The representative household is infinitely lived, supplying labor and consuming goods. It can also save in risk-free bonds that pays nominal interest rate. The bond market is in zero net supply. Since household's problem is standard, we delegate it to Appendix A. The log-linearized Euler equation and labor supply condition are given by,

$$y_t = y_{t+1} - \frac{1}{\sigma}(i_t - \pi_{t+1}) + \beta_t \tag{3}$$

$$\eta n_t = -\sigma y_t + w_t - p_t \tag{4}$$

where c_t is aggregate consumption; $\pi_t = p_t - p_{t-1}$ is inflation and p_t is the price index; i_t is the interest rate; n_t is the labor supply; w_t is the nominal wage; η is the inverse labor elasticity; β_t is aggregate discount rate shock. It follows $\beta_t = \rho_d \beta_{t-1} + \epsilon_d$, where $\epsilon_d \sim \mathcal{N}(0, \sigma_d^2)$. An increase in β_t implies a positive aggregate demand shock.

Monetary Authority – The monetary authority follows the Taylor rule, $i_t = \phi \pi_t$.

Firms – The setup of firms draws heavily from Woodford (2003). The final consumption good is consisted of a continuum of differential goods aggregated with CES, i.e., $C_t = \int_0^1 C_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj$, implying the demand curve for each good $C_{jt} = (\frac{P_{jt}}{P_t})^{-\varepsilon} C_t$, where P_t is the price index defined as $P_t = (\int_0^1 P_{jt}^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}$. Each variety is produced by a monopolistic firm with the production function $Y_{it} = A_{it} N_{it}^{1-\alpha}$. Firms are subject to both aggregate and idiosyncratic productivity shocks, i.e., $a_{it} = a_t + \epsilon_{it}$, where ϵ_{it} follows $\mathcal{N}(0, \sigma_\epsilon^2)$ and a_t follows $a_t = \rho_s \beta_{t-1} + \epsilon_s$ where $\epsilon_s \sim \mathcal{N}(0, \sigma_s^2)$. Firm i sets price P_{it} to maximize profit conditional on its information set period by period. At date-t, firm i's problem is,

$$\max_{P_{it}, Y_{it}, N_{it}} E_{it}\left(\frac{U'(C_t)}{P_t}(P_{it} Y_{it} - W_t N_{it})\right)$$
$$\text{s.t.} \quad Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t; \quad Y_{it} = A_{it} N_{it}^{1-\alpha}$$

where $U'(C_t)$ is the marginal utility of consumption, N_{it} is the labor hired, and α measure the decreasing return to scale. The log-linearized first-order condition writes,

$$p_{it} = E_{it}\Big(\varphi(p_t + \frac{1}{\varepsilon} y_t) + (1-\varphi) w_t\Big) - \frac{1-\varphi}{1-\alpha} a_{it} \tag{5}$$

[6] We incorporate future periods rather than assuming i.i.d. aggregate shocks for two reasons. First, it allows for greater generality. When we set shock persistence to be zero, the model essentially becomes static. Second, inflation responses to supply shocks vary significantly with their persistence. We would illustrate how confusion can amplify the inflation response even when supply shock is highly persistent.

where $\varphi = \frac{\alpha}{1-\alpha}/(\frac{\alpha}{1-\alpha} + 1/\varepsilon)$, and w is the nominal wage. As in Woodford (2003), due to decreasing returns to scale and the timing of labor market clearing, the firm must form expectations about both the intercept of its demand curve, $p_t + \frac{1}{\varepsilon}y_t$, and the nominal wage. The log-linearized labor demand is $n_{it} = \frac{1}{1-\alpha}(y_{it} - a_{it})$. Integrate it out and combine it with labor supply condition in equation (4), we obtain the labor market clear condition,

$$w_t - p_t = \sigma y_t + \frac{\eta}{1-\alpha}(y_t - a_t) \tag{6}$$

Firms know the goods and labor markets have to clear, so we can plug equation (6) back to the first-order condition (5) and integrate out over all firms. Further, we can normalize the last period inflation to the steady state value, $p_{t-1} = 0$. Then, $\pi_t = p_t$. The pricing rule is then given by,

$$\pi = \bar{E}(\pi + \psi_y y) - \psi_a(\eta \bar{E}a + a) \tag{7}$$

where $\psi_y = \frac{\varphi}{\varepsilon} + (1-\varphi)(\sigma + \frac{\eta}{1-\alpha})$ and $\psi_a = \frac{1-\varphi}{1-\alpha}$. Parameter ψ_y summarizes the effect of aggregate output on inflation. It increases with the inverse labor elasticity η. Equation (7) shows that changes in inflation arise from three distinct sources. First, the strategic complementarity in pricing implies that firm's pricing strategy must depend on the expected price index. Second, the expected change in output affects price-setting through affecting the real marginal cost due to both decreasing return to scale and inelastic labor supply. Finally, the expected aggregate productivity is part of the expected real marginal cost. The actual aggregate productivity is the result of integration of individual productivity a_{it}.

3.1 Confusion in General Equilibrium

Information Structure With the model setup in place, we are ready to examine the role of confusion in shaping output and inflation responses. To clearly illustrate the connection between the econometrician as an observer, analyzed in length in Section 2, and the econometrician also as a decision maker within the firm, we adopt the same information structure as in Section 2 and examine the effect of confusion in general equilibrium. That is: firms observe two signals, one about output while the other about inflation:

$$x_{it}^y = \omega_{ys} a_t + \omega_{yd}\beta_t + \epsilon_{it}^y, \quad x_{it}^\pi = -\omega_{\pi s} a_t + \omega_{\pi d}\beta_t + \epsilon_{it}^\pi, \tag{8}$$

where ω again represents the impulse responses to shocks. We take these responses as exogenous, just as any individual firm would. In Section 3.2, we will impose that $\omega_{ys} = \tilde{y}_s$, $\omega_{yd} = \tilde{y}_d$, $\omega_{\pi s} = \tilde{\pi}_s$, and $\omega_{\pi d} = \tilde{\pi}_d$. We assume that (i) households possess full information and (ii) from the next period onward, firms also have full information. Overall, the firms' information set at time t is given by $\mathcal{I}_t = \{x_{it}^y, x_{it}^\pi, a_{t-1}, \beta_{t-1}, \mathcal{I}_{t-1}\}$. Standard macroeconomic models typically analyze the equilibrium response to each aggregate shock in isolation. However, when firms confuse one shock for another,

the responses to a given shock can influence the responses to the other. This interdependence implies that solving for the two equilibria jointly is essential. Equipped with Proposition 1, the following proposition characterizes the equilibrium,

Proposition 4. *The following system of equations characterize the joint responses of inflation and output to unit aggregate demand and supply shocks, i.e.,* $\{\tilde{\pi}_s, \tilde{\pi}_d, \tilde{y}_d, \tilde{y}_s\}$,

$$\begin{aligned}
\tilde{\pi}_d &= (1 - D_d)^{-1} \left(\psi_y D_d \tilde{y}_d + C(\tilde{\pi}_s + \psi_y \tilde{y}_s) - \eta \psi_a C \right) \\
\tilde{\pi}_s &= (1 - D_s)^{-1} \left(\psi_y D_s \tilde{y}_s + C(\tilde{\pi}_d + \psi_y \tilde{y}_d) - \eta \psi_a D_s - \psi_a \right) \\
\tilde{y}_d &= -\frac{1}{\sigma} \phi \tilde{\pi}_d + F_d \\
\tilde{y}_s &= -\frac{1}{\sigma} \phi \tilde{\pi}_s + F_s
\end{aligned}$$

where $F_d = \frac{\phi}{\phi - \rho_d}$ *and* $F_s = (1 + \eta) \frac{\psi_a}{\psi_y} \rho_s \frac{\phi - 1}{\phi - \rho_s}$ *captures the effect of demand shock and future consumption. The solution of the inflation responses is given by,*

$$\begin{pmatrix} \tilde{\pi}_s & \tilde{\pi}_d \end{pmatrix} = \underbrace{\mathbf{U}}_{PE} \underbrace{(\mathbf{I} - z\mathbf{P})^{-1}}_{GE}$$

where $z = 1 - \phi \psi_y / \sigma$ *and* $\mathbf{U} = \begin{pmatrix} \psi_y (F_s D_s + F_d C) - \eta \psi_a D_s - \psi_a & \psi_y (F_s C + F_d D_d) - \eta \psi_a C \end{pmatrix}$.

Starting with the demand side, since households have full-information rational expectations, $-\frac{1}{\sigma} \phi \tilde{\pi}$ summarizes the effect of inflation on the real interest rate, while F summarizes their own next period consumption and inflation effect, for supply and demand shock, respectively. F_s is positive because a positive supply shock raises households' real wage, leading to higher demand and thus an increase in output. This rise in real wage outweighs the decline in inflation. Moreover, such effect is larger when the shock is more persistent, implying a higher F_s. Due to this channel, the standard New Keynesian models has long struggled to generate sizable inflation responses to persistent supply shocks.

Moving on to the supply side, compared to the standard incomplete-information framework in Woodford (2003), the key distinction lies in the interaction between inflation and output responses across shocks. Specifically, the combined response of inflation and output to a unit supply shock, $\tilde{\pi}_s + \psi_y \tilde{y}_s$, directly influences the pricing rule in response to a demand shock via the confusion term C, and vice versa. This interdependence arises from the fact that firms cannot separately perceive demand and supply shocks under the specified information structure. In addition, the demand shock also induces confusion about aggregate productivity, which directly affects pricing. Without confusion C, incomplete information produces the same dampening effect on expectations, captured by D_s and D_d, as in Woodford (2003) and Angeletos and La'o (2010).

Regime- and Shock-Dependent Phillips Curve Confusion operates through expected aggregate inflation, $C\tilde{\pi}_s$, and expected real wages, $C(\varphi_y\tilde{y}_s - \eta\varphi_a)$. In regimes where inflation signal is more informative, they are more likely to confuse a *negative* supply shock with a positive demand shock. As a result, both negative supply shocks and demand shocks become more inflationary, with supply shocks having an even stronger effect on inflation.

The mechanism is as follows: (i) Following a negative supply shock, firms tend to raise prices due to price complementarity to perceived demand-driven inflation, as $-C\tilde{\pi}_d > 0$.[7] Expected real wage rises as perceived demand shock drives up the real income, so the firm further raises prices, as $-C\varphi_y\tilde{y}_d > 0$. (ii) By contrast, after a positive demand shock, the firm still raises prices due to price complementarity, $C\tilde{\pi}_s > 0$, but the real wage channel is ambiguous, as income and productivity effects partly offset each other, $C(\varphi_y\tilde{y}_s - \eta\varphi_a)$. The overall effect of confusion on inflation in the case of demand shock case is ambiguous.

In regimes where output signal is more informative, firms confuse a positive supply shock with a positive demand shock. As a result, both positive supply and demand shocks become less inflationary, with the inflationary effect of supply shocks becoming even more muted.

Because of the differential effect of confusion on supply and demand shocks, if an econometrician estimates a standard Phillips curve of the form

$$\pi_t = \kappa(y_t - y_t^*) + \beta\mathbb{E}_t\pi_{t+1}$$

using identified demand and supply shocks in this economy, she would find that the estimated slope based on supply shocks is larger than that based on demand shocks. Notice that in the case of supply shocks, the estimation would also require a precise estimate of potential output.[8] Moreover, in regimes with high inflation, both slopes increase, but the slope estimated using supply shocks rises more. We zero in on this point in Section 4.2.

The reason that we emphasize the shock-dependent slope is to address the long-standing inflation puzzle: standard New Keynesian models have long struggled to generate substantial inflation responses to supply shocks. This limitation is precisely why many DSGE models introduce so-called "cost-push" or "markup" shocks, which shift prices exogenously without entering the firm's optimization problem, where price setting is otherwise constrained by nominal rigidities. Under the standard incomplete information framework, where $\mathbf{P}$ is diagonal and there is no confusion, realizations of supply shocks lead to a dampened perceived shock magnitude and a muted permanent income channel. This combination suppresses the responses of both output and inflation. As a result, while information frictions can generate plausible monetary non-neutrality

[7] Since the impulse responses in Proposition 4 are derived for positive demand and supply shocks, the responses to negative shocks have the same magnitude but opposite sign.

[8] This requirement can be met using simulated data as we know the change in potential output from our model. In reality, the potential output contains measurement error and the mapping from a supply shock to potential output is unknown. Estimating the slope of Phillips curve for supply shocks is not feasible. The literature usually assumes the same slope for all shocks.

on the demand side, they alone imply a counterfactually weak inflation response to supply shocks.[9] We sure can assume different signal precision for different shocks, with more precise signal about supply shocks. The purpose of our analysis is to generate shock-dependent Phillips curve under a unified and realistic information structure. We show that when multiple aggregate shocks are present, and confusion arises, the inflation response can be significantly amplified.

To zero in on the equilibrium outcome, we draw on the second part of Proposition 4. The equilibrium inflation response depends on U, which captures the partial equilibrium effect—specifically, the wage channel discussed above.[10] The fact that firms raise prices under negative confusion simply due to higher perceived aggregate inflation is embedded in the general equilibrium multiplier $(\mathbf{I} - z\mathbf{P})^{-1}$. The strength of this strategic complementarity depends on the Taylor rule coefficient and the elasticity of labor supply. To see the general equilibrium effect of confusion more clearly, consider the extreme cases in Corollary 1: when agents are fully extract information either from inflation or to output. Theorem 2 shows that when agents extract information exclusively from inflation or output, the inflation response to a supply shock is amplified and dampened by $\left(\frac{1}{2}U_1 - \frac{1}{2}U_2\right)$ and $\left(\frac{1}{2}U_1 + \frac{1}{2}U_2\right)$, where $U_1 < 0$ and $U_2 > 0$. Therefore, supply and demand shocks amplify inflation beyond the partial equilibrium effect when agents rely exclusively on inflation signals, but dampen inflation relative to the partial equilibrium effect when they rely exclusively on output.

Theorem 2. *In the rational expectations equilibrium with $|z| < 1$, the general equilibrium is given by,*

$$\mathbf{U}\sum_{h=0}^{\infty} z^h \mathbf{P}^h = \left(\underbrace{U_1}_{PE} + \underbrace{(\tilde{\lambda}_1 + \tilde{\lambda}_2)xU_1 + (\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu U_2}_{GE} \quad \underbrace{U_2}_{PE} + \underbrace{(\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu U_1 + (\tilde{\lambda}_1 + \tilde{\lambda}_2)zU_2}_{GE} \right)$$

where $\tilde{\lambda}_k = \frac{z\lambda_k}{1 - z\lambda_k}$ for $k = 1, 2$. $\mathbf{U} = \begin{pmatrix} U_1 & U_2 \end{pmatrix}$ captures the partial equilibrium effect.

1. *If $\sigma_a = \sigma_d = \omega = 1$, and in the limit where $\sigma_\pi \to 0$ and $\sigma_y \to \infty$:*

$$\mathbf{U}\sum_{h=0}^{\infty} z^h \mathbf{P}^h = \frac{1}{1-z}\left(\underbrace{U_1}_{PE} + \underbrace{(\frac{1}{2}U_1 - \frac{1}{2}U_2)}_{GE} \quad \underbrace{U_2}_{PE} + \underbrace{(-\frac{1}{2}U_1 + \frac{1}{2}U_2)}_{GE} \right)$$

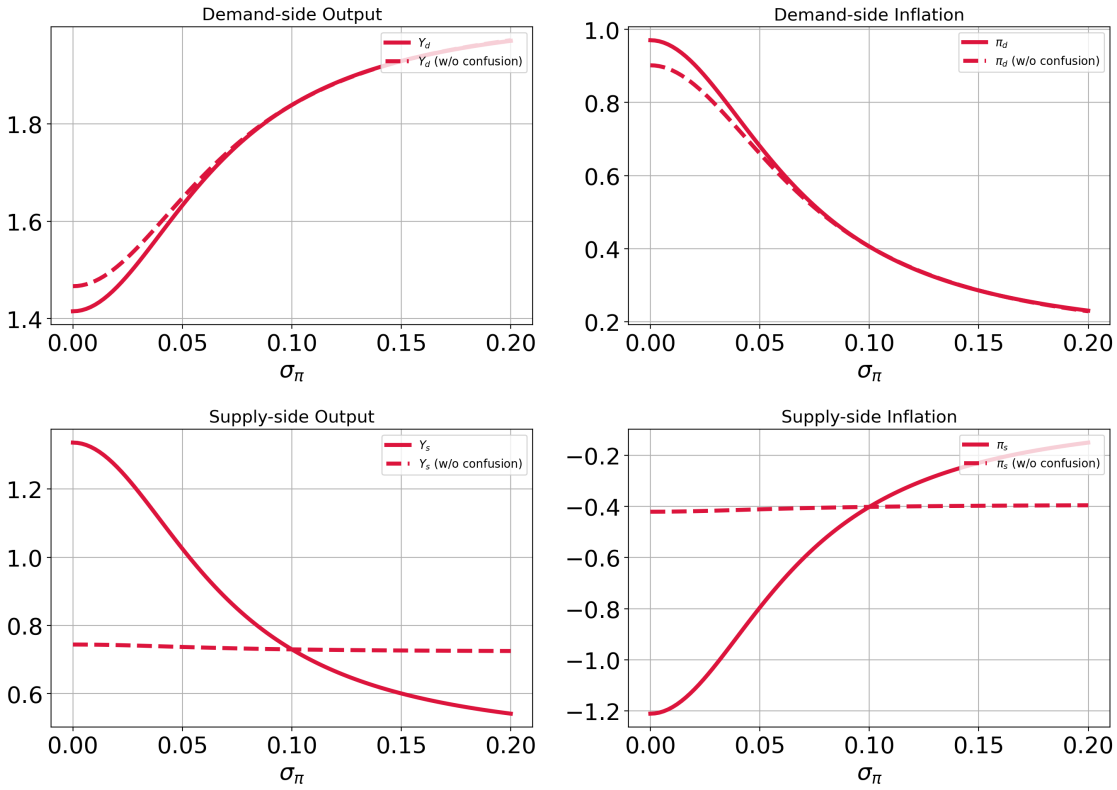
2. *In the limit where $\sigma_y \to 0$ and $\sigma_\pi \to \infty$:*

$$\mathbf{U}\sum_{h=0}^{\infty} z^h \mathbf{P}^h = \frac{1}{1-z}\begin{pmatrix} U_1 + (\frac{1}{2}U_1 + \frac{1}{2}U_2) & U_2 + \frac{1}{2}U_1 + \frac{1}{2}U_2 \end{pmatrix}$$

[9] When firms have no information about the shock, inflation is simply ψ_a, which can be smaller than the full-information response when shock persistence is low.

[10] The partial equilibrium effect is most transparent if both the demand and supply shocks are assumed to be transitory, i.e., $\rho_d = \rho_s = 0$, in which case $\mathbf{U} = \begin{pmatrix} \psi_y C - \eta\psi_a D_s - \psi_a & \psi_y D_d - \eta\psi_a C \end{pmatrix}$ When $C < 0$, all terms in the first entry are negative, while the sign of the second entry is ambiguous—reflecting precisely the wage effect discussed above.

Figure 2: Effect of Confusion on Output and Inflation

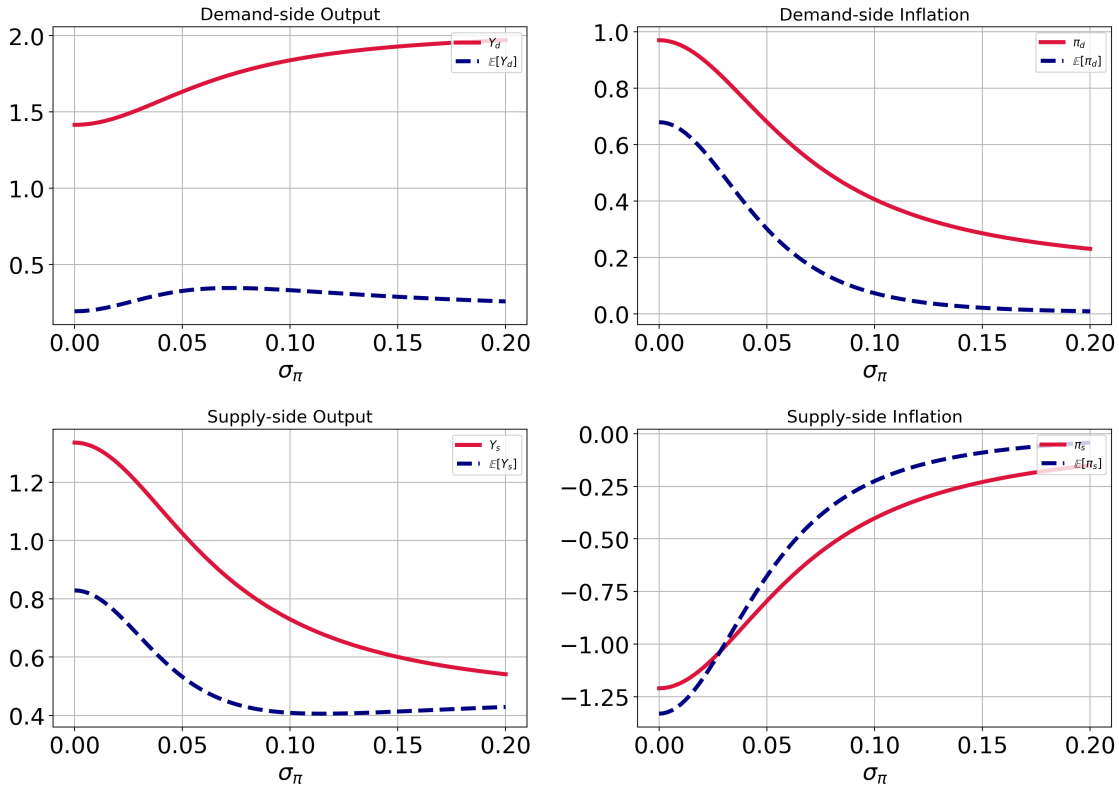


Notes: The figure shows the responses of inflation and output to demand and supply shocks. The top left panel plots the output response after a demand shock for different σ_π. The top right panel plots the inflation response after a demand shock. The bottom left panel plots the output response after a supply shock. The bottom right panel plots the inflation response after a supply shock. The solid lines represent the case with confusion. The dashed lines represent the benchmark case where $C = 0$. ωs are all set to 1. See Appendix A for other standard parameter values used in this exercise.

Since higher-order confusion is highly persistent in the limiting cases, as show in Section 2, the effect of higher-order confusion can be quite large even for intermediate z, e.g., $z = 0.5$, especially in the case either inflation or output information is quite informative. To visualize the equilibrium outcome as we vary σ_π and σ_y and Figure 2 plots out the response of inflation and output to positive demand and supply shock, respectively in the first and second row. We set all ωs to 1, so that the inflation and output signals each reflect half demand and half supply, albeit with opposite signs. σ_y is set to 1 while we vary σ_π. To highlight the role of confusion, we compare our results to the benchmark without confusion. Specifically, we set $C = 0$ while keeping D_s and D_d unchanged. This case is similar to the information structure considered in Woodford (2003) and Angeletos and La'o (2010).

Heightened and Dampened Inflation Response In Figure 2, the top panel plots the inflation and output responses to a unit demand shock as σ_π varies, while the bottom panel shows the responses to a unit supply shock. The solid lines represent the baseline case with confusion; the dashed lines show the no-confusion benchmark where $C = 0$. From Proposition 1, we know that for a

Figure 3: Effect of Confusion on Actual and Expected Output and Inflation



Notes: The figure shows the responses of actual and expected inflation and output to demand and supply shocks. The top left panel plots the output response after a demand shock for different σ_π. The top right panel plots the inflation response after a demand shock. The bottom left panel plots the output response after a supply shock. The bottom right panel plots the inflation response after a supply shock. The solid lines represent the actual responses while the dashed lines represent the the expectation. See Appendix A for other standard parameter values used in this exercise.

given σ_y, agents initially confuse a negative supply shock with a positive demand shock when they extract more information from inflation, and that the magnitude of this confusion is maximized when $\sigma_\pi = 0$. Theorem 2 further shows that when this confusion arises, the inflation response is amplified compared to the case where agents confuse a positive demand shock with a positive supply shock. This effect is again, differential across shocks, and is particularly evident in the supply shock case: when σ_π is close to zero, the inflation response nearly triples, and the output response more than doubles relative to the no-confusion benchmark. As σ_π increases, this effect of *negative* confusion gradually weakens, while the effect of *positive* confusion emerges, since the degree of confusion shrinks and eventually reverses sign when $\sigma_\pi > 0.1$—precisely the point where confusion disappears according to Proposition 1.

Overreacted Expectations In addition, expected inflation tends to overreact relative to actual inflation in the neighborhood of $\sigma_\pi = 0$ due to confusion. Figure 3 shows the responses of expected and actual inflation and output. In this region, the combined effect of confusion and dampening, $|C| + D_s$, exceeds one, implying an amplified response of expected inflation. This stands in stark

contrast to standard models, where average expectations are necessarily attenuated relative to the actual response (Woodford, 2003; Angeletos and La'o, 2010). Notably, confusion can drive expected demand-driven output to zero or even negative. The reason is that firms confuse a positive demand shock with a negative supply shock and therefore believe output should be low, as a negative supply shock pushes output down. By contrast, the expected output following a positive supply shock is much larger due to the high actual output response. In Section 4, we show that because of this mechanism, most of the variation in expected output from survey forecasts is driven by supply shocks, whereas actual output may be driven more by demand shocks.

3.2 Endogenous Information

So far, we have taken the weights in the inflation and output signals as given. In rational equilibrium, however, these weights are themselves endogenous. In this section, we relax this assumption and show that our conclusions continue to hold. We first lay out the conditions for the existence and uniqueness of equilibrium, and then examine how confusion arises when information is endogenous.

Endogenous information considerably increases the challenge of analytically characterizing the equilibrium. Beyond fundamental complementarity, endogenous information also generates an information externality: even in the absence of fundamental complementarity, the fact that one agent's action becomes another agent's signal creates strong and nonlinear strategic complementarities.

Proposition 5. *Suppose firms receive only a signal about inflation, i.e., $\sigma_y \to \infty$. Then, the equilibrium solution for inflation:*

$$\tilde{\pi}_d^* = \begin{cases} 0 \quad or \quad \frac{F_d}{2\phi}\left(1+\sqrt{1-4\frac{\phi}{\psi_y F_d^2}x}\right), & \text{if } x < x^* = \frac{1}{4}\frac{\psi_y F_d^2}{\phi}, \\ 0, & \text{if } x \geq x^*. \end{cases}$$

where $x = \sigma_\pi^2 + (\tilde{\pi}_s + \psi_y \tilde{y}_s - \eta\psi_a)\sigma_d\sigma_s\tilde{\pi}_s$. When $\sigma_y < \bar{\sigma}_y$, there exists unique solution $\tilde{\pi}_d^*$.

Proposition 5 discusses the uniqueness and possible multiplicity of the endogenous information equilibrium when agents observe only inflation, highlighting the forces at play. The first part of this proposition establishes that when firms rely solely on inflation signals, zero inflation always constitutes an equilibrium. This arises from the information externality: if all firms refrain from adjusting prices, the signal conveys no meaningful information about the underlying demand shock, consistent with the initial inaction. This can also be seen from Theorem 1: the magnitude of confusion when there is only an inflation signal depends on the total variation in inflation, $\tilde{\pi}_s\tilde{\pi}_d$. As a result, zero inflation following a demand shock—when the signal is noisy enough—is consistent with the absence of confusion, since any observed change in the price index must be attributed to supply shocks. However, there also exists another equilibrium in which firms coordinate on

changing prices, thereby providing information about the underlying demand shock to others, which in turn self-fulfills the initial adjustment. Finally, when the inflation signal is sufficiently noisy, this nonzero equilibrium ceases to exist because any deviation from zero inflation is too weak to be detected by other firms.

Incorporating an additional output signal can eliminate the zero-inflation equilibrium if the output signal is sufficiently informative. In this case, the solution converges to $\frac{1}{\phi}$ under full information. The intuition is as follows: if firms refrain from adjusting prices after a unit demand shock, output will increase by one. Because firms also observe an output signal, they can infer that a demand shock is likely present. This inference provides a strong incentive to raise prices, pushing the economy out of the zero-inflation equilibrium. By contrast, in Appendix A we show that there is a unique equilibrium following supply shocks even when firms observe only the inflation signal.[11]

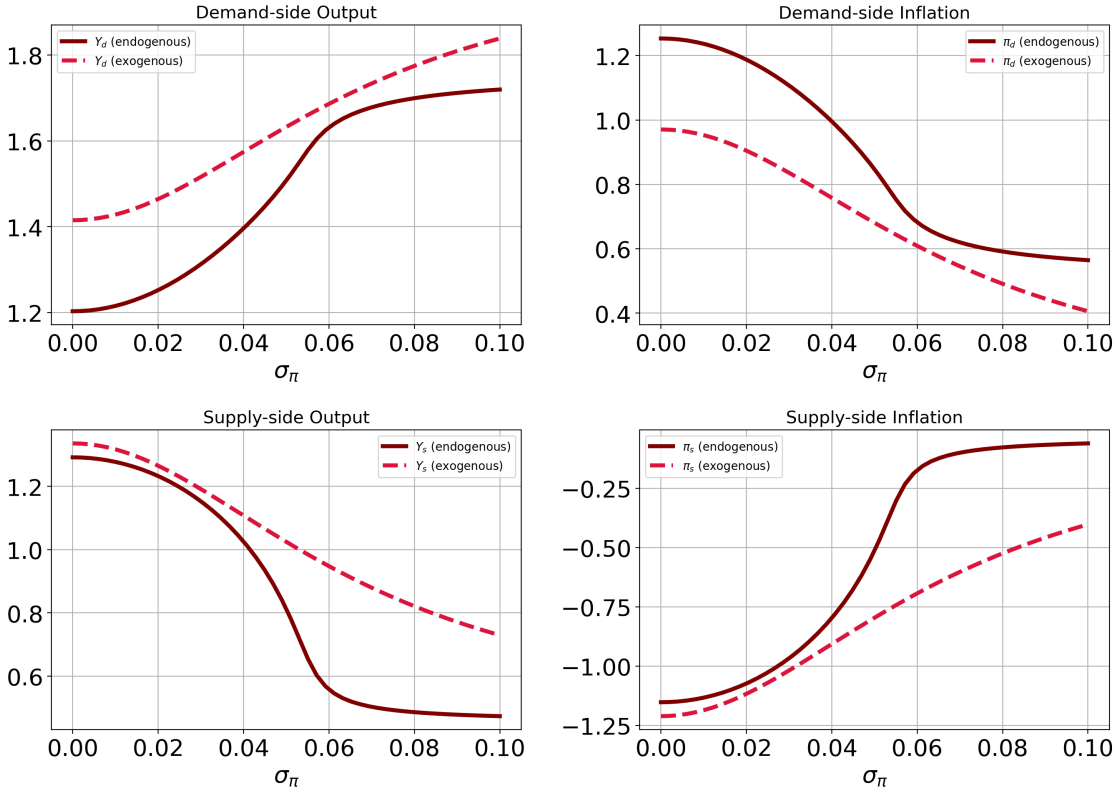
Next, we visually illustrate the role of endogenous information. Because of the information externality inherent in an endogenous information economy, the model exhibits greater sensitivity to σ_π and σ_y: small changes in the variance of the noise can give rise to drastically different responses in inflation and output.

Figure 4 shows the inflation and output responses under endogenous information. The responses to a supply shock resemble those under exogenous information in the neighborhood of $\sigma_\pi = 0$. However, the output response declines more steeply as σ_π increases. This implies that even a small increase in σ_π can have large effects on output and expectations, generating very different implications for the contribution of supply shocks to variation in output and expected output, as we show in Section 4. The mechanism can be seen from the following example: lowering σ_π increases the precision of the inflation signal. A lower σ_π means agents extract more information from the inflation signal, prompting them to become more *negatively* confused. Under endogenous information, this stronger response further increases the informativeness of the inflation signal itself, since the weight $\omega_{\pi d}$ now depends on the actual response. As a result, the degree of confusion rises, leading to even larger inflation responses to supply shocks. Endogenous information thus creates a feedback loop between general equilibrium responses and the informativeness of the observed signals.

Summary There are three main lessons from analyzing the simple general equilibrium model: (i) Whenever agents extract more information from inflation, or when inflation exhibits greater total variation, agents tend to confuse a negative supply shock with a positive demand shock. When agents extract more information from output instead, the reverse occurs. (ii) Demand and supply shocks have differential effects on inflation and output, giving rise to different slopes of the estimated New Keynesian Phillips curve. (iii) When agents rely more on inflation signals, inflation responses are endogenously heightened. When agents rely more on output signals, inflation

[11] The intuition is that firms always adjust prices in response to productivity changes, even if they do not observe any changes in other firms' prices.

Figure 4: Effect of Confusion on Output and Inflation with Endogenous Information



Notes: The figure plot the inflation and output responses to demand and supply shocks under endogenous information. The top left panel plots the output response after a demand shock for different levels of standard deviation of output signal. The top right panel plots the inflation response after a demand shock. The bottom left panel plots the output response after a supply shock. The bottom right panel plots the inflation response after a supply shock. The solid lines represent the case with endogenous signals. The dashed lines represent case with exogenous signals.

responses are endogenously more muted. Next, we estimate a standard New Keynesian model with incomplete information, multiple aggregate shocks, and endogenous learning to examine the effect of confusion on output and inflation dynamics.

4 Quantitative Model

The model is the familiar, standard New Keynesian framework: households supply labor to firms through a competitive labor market, and firms set prices subject to Calvo-style sticky pricing. The key innovation in our setup is that firms learn from observed output and inflation, giving rise to rational confusion.

Households The economy is inhabited by a unit-mass continuum of households indexed by $i \in [0, 1]$. Each household maximizes expected lifetime utility, taking as given the sequence of real

bond prices, wages set in economy-wide markets, and lump-sum profit transfers:

$$\max_{\{C_{i,t+k},N_{i,t+k}\}_{k\geq 0}} \mathbb{E}_{i,t} \sum_{k=0}^{\infty} \Big(\prod_{s=0}^{k-1} \beta_{i,t+s}\Big) \Big[\frac{C_{i,t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{i,t+k}^{1+\eta}}{1+\eta}\Big], \tag{9}$$

where $\sigma > 0$ is the coefficient of relative risk aversion, $\eta > 0$ is the inverse Frisch elasticity of labor supply, and $\beta_{i,t} \in (0,1)$ is a household-specific, possibly time-varying, intertemporal discount factor between t and $t+1$.

The flow budget constraint is

$$C_{i,t} + B_{i,t+1} = R_t B_{i,t} + W_t N_{i,t} + \Pi_t, \tag{10}$$

where $B_{i,t}$ is the real payoff of one-period risk-free bonds purchased in period $t-1$; R_t is the gross real interest rate set by the monetary authority; W_t is the real wage; and $\Pi_t \equiv \int_0^1 \Pi_{j,t}\, dj$ denotes firm profits rebated lump-sum and symmetrically to households. Bonds are in zero net supply, $\int_0^1 B_{i,t}\, di = 0$ for all t.

Final-good consumption is a constant-elasticity-of-substitution (CES) aggregate of the differentiated varieties $j \in [0,1]$:

$$C_{i,t} = \Big(\int_0^1 C_{i,j,t}^{\frac{\epsilon-1}{\epsilon}}\, dj\Big)^{\frac{\epsilon}{\epsilon-1}}, \qquad \epsilon > 1, \tag{11}$$

where $C_{i,j,t}$ is household i's consumption of variety j at time t, and ϵ is the elasticity of substitution across varieties.

Firms A unit-mass continuum of monopolistically competitive firms indexed by $j \in [0,1]$ produces differentiated goods with

$$Y_{j,t} = A_{j,t} N_{j,t}^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{12}$$

where $A_{j,t}$ is firm-specific productivity and $N_{j,t}$ is labor hired at the competitive wage W_t.

Price adjustment is subject to Calvo rigidity: in any period a firm may reset its price with probability $1-\theta$ and must keep its previous price with probability $\theta \in (0,1)$. When allowed to reset, the optimal price $P_{j,t}^*$ solves

$$P_{j,t}^* = \arg\max_P \ \mathbb{E}_{j,t} \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \big[P\, Y_{j,t+k}(P) - W_{t+k} N_{j,t+k}(P)\big], \tag{13}$$

where $Q_{t,t+k}$ is the stochastic discount factor relevant for households, and demand for the firm's

variety while the price P is in place is

$$Y_{j,t+k}(P) = \left(\frac{P}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{14}$$

with P_t the aggregate price index and Y_t aggregate output.

Monetary Authority The central bank sets the real interest rate to in response to inflation and close the output gap, following a standard Taylor rule. In log-linear form:

$$r_t = \rho r_{t-1} + \phi\, \pi_t + (1-\rho)\phi_y \left(y_t - \mathbb{E}_t^{\text{cb}}\, y_t^*\right),$$

where y_t^* denotes potential (frictionless) output. Because Y_t^* is unobservable, the rule depends on the central bank's expectation $\mathbb{E}_t^{\text{cb}}\, y_t^*$.

Equations (9)–(14), together with market-clearing conditions and the central bank's interest-rate rule, fully characterize the equilibrium of the model.

Consumption and Pricing Functions We log-linearize the system around the non-inflationary steady state, and solve a linearized system of equation. It is well discussed in Angeletos and Lian (2018) and Angeletos and Huo (2021) that, with (almost) arbitrary information structure, the consumption and pricing response function from the households block and firms block can be written as:

$$c_{i,t} = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k\, \mathbb{E}_{i,t}[r_{t+k} + \beta_{i,t+k}] + (1-\beta) \sum_{k=0}^{\infty} \beta^k\, \mathbb{E}_{i,t}[c_{t+k}]$$

$$p_{j,t}^* = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k\, \mathbb{E}_{j,t}[mc_{t+k} + p_{t+k}]$$

Expanding the marginal cost, and defining $z_{j,t} = p_{j,t}^* - p_{t-1}$ to remove the unit root in the pricing best response function (where p_{t-1} is the log aggregate price level last period, making $z_{j,t}$ the reset price relative to last period's price level), with $\pi_t = p_t - p_{t-1}$ is inflation. Again, $\psi_y = \frac{\varphi}{\varepsilon} + (1-\varphi)(\sigma + \frac{\eta}{1-\alpha})$ and $\psi_a = \frac{1-\varphi}{1-\alpha}$. ψ_y summarizes the effect of aggregate output on inflation, increases with the inverse labor elasticity η. y_t is log-aggregate output, a_t is log-aggregate productivity, and $a_{j,t}$ is log firm-specific productivity. The parameter η is the inverse Frisch elasticity from the household problem. With full information rational expectation, aggregate consumption and pricing function would collapse to the familiar Euler equation, and New Keynesian Phillips curve. With dispersed information, since law of iterative expectation fails, they could not be written recursively at the aggregate level. However, since individuals are rational, we could re-write them

in recursive form *at the individual level*,

$$c_{i,t} = -\frac{1}{\sigma}\,\mathbb{E}_{i,t}[r_t] + (1-\beta)\,\mathbb{E}_{i,t}[c_t] + \beta\beta_{i,t} + \beta\,\mathbb{E}_{i,t}[c_{i,t+1}], \quad c_t = \int_0^1 c_{j,t}dj$$

$$z_{j,t} = \mathbb{E}_{j,t}\,\pi_t + (1-\beta\theta)\{\psi_y\,\mathbb{E}_{j,t}\,y_t - \eta\psi_a\,\mathbb{E}_{j,t}\,a_t - \psi_a a_{j,t}\} + \beta\theta\,\mathbb{E}_{j,t}\,z_{j,t+1}, \quad \pi_t = (1-\theta)\int_0^1 z_{j,t}dj$$

Solution Concept and Information Structure Agents have incomplete information in the economy. Each agent $g \in \{i, j\}$ (household or firm) observes idiosyncratic shocks to demand, $\beta_{g,t}$, and to productivity, $A_{g,t}$. In addition, agents receive two noisy aggregate signals, from the quantity and price of goods they buy and sell:

$$\tilde{y}_{g,t} \;=\; y_t + \xi^d_{g,t}, \qquad \tilde{\pi}_{g,t} \;=\; \pi_t + \xi^\pi_{g,t},$$

where $\xi^d_{g,t}$ and $\xi^\pi_{g,t}$ are independent, mean-zero noises. We impose that households and firms share the same information structure to reduce parameter dimensionality and because the only available long-span forecast data—the Survey of Professional Forecasters—is a blend of household and firm expectations Reis (2022).[12] All shocks—including the noise in the signals—are assumed independent and normally distributed with mean zero and standard deviations as specified in the calibration table. The model is solved under incomplete-information rational expectations: following Grossman and Stiglitz (1980), agents observe their signals and choose actions that collectively constitute the equilibrium. The interpretation follows Vives (2017): agents plan their pricing and output decisions as functions of aggregate price, aggregate output, and both idiosyncratic and aggregate shocks.

To summarize, every households and firms $g \in \{i, j\}$ learn from their own demand and supply factor, and will also learn from prices, quantity. That is:

$$\mathcal{I}_{g,t} = \{\beta_{g,t}, A_{g,t}, \tilde{y}_{g,t}, \tilde{\pi}_{g,t}\}$$

The central bank is assumed to have full-information rational expectations, and we do not let households or firms learn from the policy rate for two reasons. First, the informational channel of monetary policy lies outside our paper's focus. Prior studies (e.g. Lorenzoni (2009)) inject noise into agents' observations of central bank–targeted inflation (or add other shocks) to weaken the

[12] One interpretation and micro-foundation of such an information structure is the *Lucas island* model. For example, in Lorenzoni (2009), an entrepreneur's random consumption basket gives rise to $\bar{p}_{l,t} = p_t + \xi^p_{l,t}$. From the demand for the entrepreneur's output, $y_{l,t} = y_t + \epsilon\,(p_{l,t} - p_t) + \xi^d_{l,t}$, with $\xi^p_{l,t}$ and $\xi^d_{l,t}$ denoting realizations of the sampling shocks. Agents linearly combine these two signals—along with the fact that entrepreneurs observe the price they set, $p_{l,t}$—and write

$$\tilde{y}_{l,t} = y_t + \xi^y_{l,t}, \quad \tilde{p}_{l,t} = p_t + \xi^p_{l,t},$$

thus obtaining two signals with correlated shocks. However, we prefer to specify two independent information signals—one for inflation and one for output—to highlight the operative forces of confusion: as demonstrated in the simple example, the informativeness of aggregate prices and output is the key driver of rational confusion.

signal in the interest rate; here, to keep our exposition clean and highlight our channels, we avoid further noise. Second, the existence of a "Fed information effect" remains contested Bauer and Swanson (2023). Because it is still unsettled whether real-world agents learn from policy rates, we defer a full endogenous-information treatment of central-bank persuasion to future work.

4.1 Computational Challenges and Solution Strategy

The equilibrium system we seek is in the realm of *dynamic multivariate beauty-contest with endogenous information*. Each household or firm conditions its choice on signals that are themselves functions of aggregate variables, while those aggregates are jointly determined by the full cross-section of expectations. The fixed point therefore lies not only in agents' actions but in the very information sets on which those actions rest.

A distinctive challenge in our framework is the combinatorial blow-up of higher–order expectations with dynamic complementarity. Let the vector of structural shocks be ξ_t and define the discounted fundamental

$$\zeta_t \;\equiv\; (1-\beta L)^{-1}\xi_t, \quad \text{with } L \text{ the lag operator.}$$

Each agent observes a vector of signals, $x_{i,t} \;=\; A(L)\,y_t \;+\; C(L)\,\zeta_{i,t}$, where $A(L)$ and $C(L)$ are one-sided, square-summable lag polynomials; in particular, $A(L)$ measures the (possibly infinite-order) *feedback* from current and past aggregates y_t to the signals agents use to forecast those very aggregates.

Fix a horizon $k \geq 0$. For any chain of dates $t = t_1 < t_2 < \cdots < t_h = t+k$, the relevant belief object is the nested expectation

$$\overline{\mathbb{E}}_{t_1}\Big[\overline{\mathbb{E}}_{t_2}\big[\cdots \overline{\mathbb{E}}_{t_h}[\zeta_{t+k}]\,\big]\Big],$$

where $\overline{\mathbb{E}}_\tau$ denotes the cross-sectional expectation taken at date τ. Even with *exogenous* information this hierarchy already contains k distinct second-order terms, $k(k+3)/2$ third-order terms, and in general $\big[(h-2)k(k+1)/2 + k\big]$ distinct h-th-order terms for every $h \leq k$.

Endogeneity makes the problem markedly harder. Because $A(L) \neq 0$, each layer now depends on beliefs about *future* signals that themselves load on future—yet-unknown—aggregate actions. The conditional-expectation operators therefore become nonlinear, and the equilibrium system, in general, cannot be collapsed into any finite-state VARMA representation (Huo and Takayama). Moreover, feedback through $A(L)$ induces an additional *information complementarity*: agents' actions become correlated not only through strategic motives but also because their signals jointly depend on the aggregate outcome, amplifying fluctuations even when the primitive game lacks strategic complementarity.

To tame these obstacles we employ the *Analytic Policy-Function Iteration* (APFI) framework introduced by Han et al. (2022). First, with the stationary environment, the endogenous aggregate vector y_t always admits an infinite moving-average representation $y_t = \sum_{k=0}^{\infty} \Psi_k\, \varepsilon_{t-k}$, $\mathbb{E}[\varepsilon_t \varepsilon'_{t-\ell}] =$

0 ($\ell \neq 0$). Collecting the MA coefficients into the *transfer function* $\Psi(z) \equiv \sum_{k=0}^{\infty} \Psi_k z^k$, we obtain an element of the Hardy space $H^2(\mathcal{D})$ of matrix–valued analytic functions on the open unit disk $\mathcal{D}$. APFI seeks a fixed point for $\Psi(z)$ by iterating on a rational approximation. In each iteration we posit the assumption that $\Psi(z) = P(z)Q(z)^{-1}$, where $P(z)$ and $Q(z)$ are matrix polynomials of degrees at most (p, q) and $Q(0) = I$. Substituting this rational form into the frequency–domain equilibrium conditions produces a set of analytic identities $\mathcal{G}(z; \Psi(z)) = 0,\ z \in \mathcal{D}$. Following Han et al. (2022) we evaluate these identities on a Chebyshev grid of real points $z_j \in (-1, 1) \subset \mathcal{D}$; conditional expectations under each information set are computed with the Wiener–Hopf formulae, implemented numerically via discrete Fourier transforms. The residuals $\mathcal{G}(z_j; \Psi)$ are projected back onto the rational basis, yielding a linear system for the coefficients of P and Q. Solving this system updates $\Psi(z)$; the procedure iterates until the supremum norm of successive transfer functions on $(-1, 1)$ falls below a preset tolerance. By analytic continuation agreement on the real interval implies equality on the entire disk, so convergence of $\Psi(z)$ certifies that the recovered sequence $\{\Psi_k\}$ solves the original fixed-point problem. Operating in the frequency domain eliminates the need for an explicit state-space and, crucially, separates the computation of conditional expectations from the dimensionality of the information hierarchy. As a result APFI bypasses both the curse of dimensionality and the explosion of higher-order beliefs. In quantitative applications, rational orders as low as $(p, q) = (4, 4)$ deliver accuracy while retaining numerical stability.

4.2 Quantitative Evidence

We now quantify the model's ability to explain U.S. macroeconomic fluctuations and the evolving slope of the Phillips curve. The estimation proceeds in two stages. Stage 1 calibrates preference, technology, and policy parameters to conventional values; Stage 2 estimates four key variance parameters that govern belief dynamics using simulated method of moments (SMM), matching the second moments of output, inflation, and their Survey of Professional Forecasters (SPF) projections. We split the sample at 1990:Q1, in line with Del Negro et al. (2020); Hazell et al. (2022), to study the differential dynamics of distinct regimes in the postwar U.S. economy. The four parameters we estimate are the variances of demand and supply shocks, and the precisions of inflation and output. The underlying philosophy is as follows: the variances of demand and supply shocks govern changes in the exogenous shock processes before and after the 1990s, while the precision of inflation and output—as shown in the simple model section—determines the degree of forecast confusion and its macroeconomic effects. One might suspect that information frictions not arising from endogenous sources, such as cross-firm dispersion in TFP and demand shocks, could also shape the results. To limit such external frictions, we calibrate a very low degree of exogenous information friction—among the lowest in the literature (Lorenzoni, 2009; Melosi, 2017). The resulting estimates reconcile three salient facts: (1) a flat Phillips curve after 1990;(2) large inflation swings driven by supply shocks before 1990; (3) the limited role of demand shocks for inflation variance in both subperiods.

Data, Sample Split, and Target Moments We target four quarterly series: real GDP (y_t), GDP-deflator inflation (π_t), and their three-quarter-ahead SPF mean forecasts ($y^{\text{SPF}}_{t|t+3}$, $\pi^{\text{SPF}}_{t|t+3}$). To be consistent with our log-linear solution around a zero-inflation steady state, we convert the three-quarter-ahead SPF forecasts of GDP growth and inflation back to levels. Each series is then filtered using the one-sided Hamilton filter (Hamilton, 2018) with a horizon of $h = 4$. The SMM objective matches the contemporaneous and first two lag autocovariances of the joint filtered series. Motivated by evidence of a structural break documented in Del Negro et al. (2020); Hazell et al. (2022), we estimate the variance parameters separately over two subsamples: pre-1990 (1971:Q3–1989:Q4) and post-1990 (1990:Q1–2016:Q3).

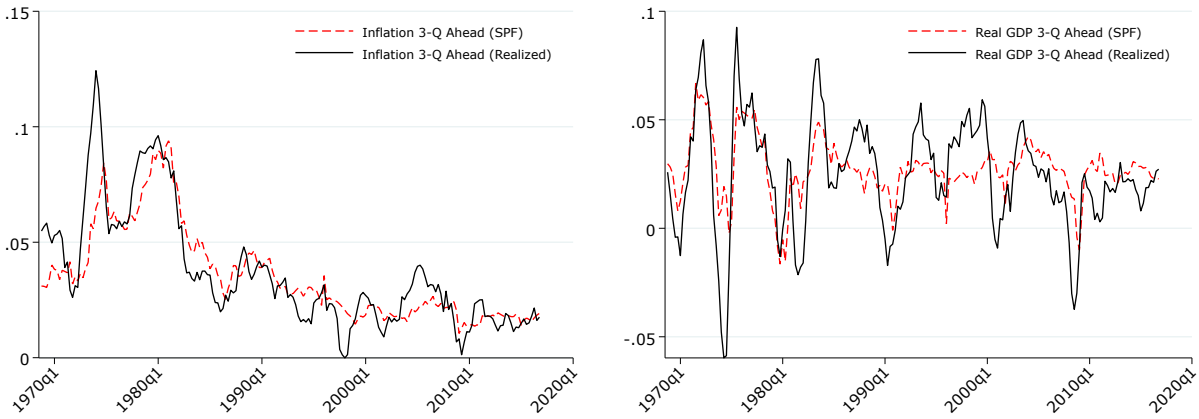
Identification The two free parameters that govern informational frictions are the signal-to-noise ratios for output and inflation, σ_y and σ_π. We identify them using the joint dynamics of four observables: realized output and inflation, $\{y_t,\ \pi_t\}$, and the corresponding SPF forecasts, $\{y^{\text{SPF}}_{t|t+3},\ \pi^{\text{SPF}}_{t|t+3}\}$. Intuitively, large values of σ_y or σ_π render the endogenous information, inflation and output, uninformative, forcing agents to fall back on private information—the canonical damping channel in noisy-information models such as Woodford (2003). That extreme would imply virtually no comovement between expected and realized inflation, which is flatly rejected by the data: SPF forecasts track actual outcomes with an R^2 of 0.78. Figure 5 illustrates this tight covariance.

Instead of matching impulse-response functions (e.g., Christiano et al., 2005), we target the unconditional autocovariance matrix of $(y_{t+3},\ \pi_{t+3},\ y^{\text{SPF}}_{t|t+3},\ \pi^{\text{SPF}}_{t|t+3})'$. In models with confusion, the variances of the underlying shocks enter the state-space solution, so these second-moment restrictions map sharply into σ_y and σ_π, whereas IRFs would identify them only indirectly through higher-order interactions of transition coefficients. Therefore, our simulated-method-of-moments (SMM) estimator minimizes the quadratic form $m(\theta)'Wm(\theta)$ with an identity weighting matrix.[13] The resulting parameters deliver signals that are precise enough to replicate the close link between SPF forecasts and realizations, yet noisy enough to generate the sizable monetary non-neutrality required by our impulse-response targets.

Calibration and Estimation Table 1 lists the parameters fixed *a priori* (Panel A) and those estimated by the simulated method of moments (Panel B). The discount factor β, the inverse elasticity of intertemporal substitution σ, the Taylor-rule feedback coefficient ϕ_π, and other standard parameters are set to the textbook values in Galí (2015). Following Hamilton (2018), all raw series are one-sided Hamilton-filtered ($h = 4$) before moment matching. To prevent exogenous shock persistence from being conflated with internal propagation, we fix the persistence of demand and supply shocks at 0.7. We set the Calvo adjustment parameter to $\theta = 0.75$, in line with micro-level evidence

[13] Using the optimal two-step weighting matrix changes point estimates by less than 2 percent; full diagnostics appear in Table C.1.

Figure 5: Realized and SPF Expected Output and Inflation



Notes: Left (inflation) and right (real GDP) panels contrast the Survey of Professional Forecasters' mean three-quarter-ahead forecast (dashed red) with the corresponding realization (solid black) observed in the same quarter. Quarter-on-quarter growth rates are cumulated into level indices and normalized to 100 in 1968:Q3, so vertical gaps read as percentage forecast errors. Series construction and vintage harmonization follow Bordalo et al. (2020).

on price adjustment (Bils and Klenow, 2004; Nakamura and Steinsson, 2008; Eichenbaum et al., 2011). To isolate the confusion stemming from endogenous learning, we set the standard deviation of idiosyncratic supply and demand shocks to be ten times larger than that of their aggregate counterparts.[14] By calibrating shock persistence, adjustment frequency, and private-information parameters, we allow the data to identify the confusion channel without confounding influences.

Panel B of Table 1 documents the estimation results. Two features stand out. First, the variances of both demand and supply shocks decrease after 1990, consistent with the "Great Moderation" narrative, though the decrease is more significant for demand shocks. Second, the standard deviation of the inflation signal increases markedly (i.e., σ_π rises), while the output signal becomes more precise (i.e., σ_y falls). Note that before the 1990s, σ_π takes on a very small value. This is intuitive when considered alongside Figure 5, where expected inflation closely tracks actual inflation before 1990. Such tight tracking can only be achieved with a highly precise signal about inflation. After 1990, the close link between output and inflation deteriorates slightly, giving rise to a larger estimated σ_π. This reallocation of information precision is central to the flattening of the Phillips curve documented below. Carrying the insight from the simple model forward, one might envision a flattened Phillips curve and dampened inflation responses.

Models Fit Given that our estimation targets the full autocovariance structure of the data, a more intuitive way to assess model performance is to compare the actual and model-implied time series of SPF expectations. We begin by using the observed joint time series of real GDP and inflation to recover the sequence of structural shocks through model inversion. Because the model features

[14] This choice is consistent with estimates in, for example, Melosi (2017), who find idiosyncratic demand shocks to be roughly an order of magnitude more volatile than aggregate demand shocks.

Table 1: Calibration and Estimated Parameters

Calibrated parameters					
Parameter	**Description**	**Value**	**Parameter**	**Description**	**Value**
β	Discount factor	0.998	ϕ_y	Taylor rule (output gap)	0.6
σ	EIS	1	ρ_i	Taylor rule persistence	0.75
η	Frisch labour supply	1	σ_{ai}/σ_a	Idio. demand dispersion	10
α	Decreasing returns to scale	0.33	σ_{di}/σ_d	Idio. supply dispersion	10
ϵ	Elasticity of substitution	6	ρ_d	Persistence of demand shock	0.7
θ	Calvo price adjustment	0.75	ρ_a	Persistence of supply shock	0.7
ϕ_π	Taylor rule (inflation)	1.5			

Estimation: 1971q3–1989q4			Estimation: 1990q1–2016q3		
Parameter	**Description**	**Value**	**Parameter**	**Description**	**Value**
σ_d	Variance of demand shock	0.0331	σ_d	Variance of demand shock	0.0232
σ_a	Variance of supply shock	0.0798	σ_a	Variance of supply shock	0.0790
σ_y	Output Informativeness	0.0828	σ_y	Output Informativeness	0.0413
σ_π	Inflation Informativeness	0.0041	σ_π	Inflation Informativeness	0.0183

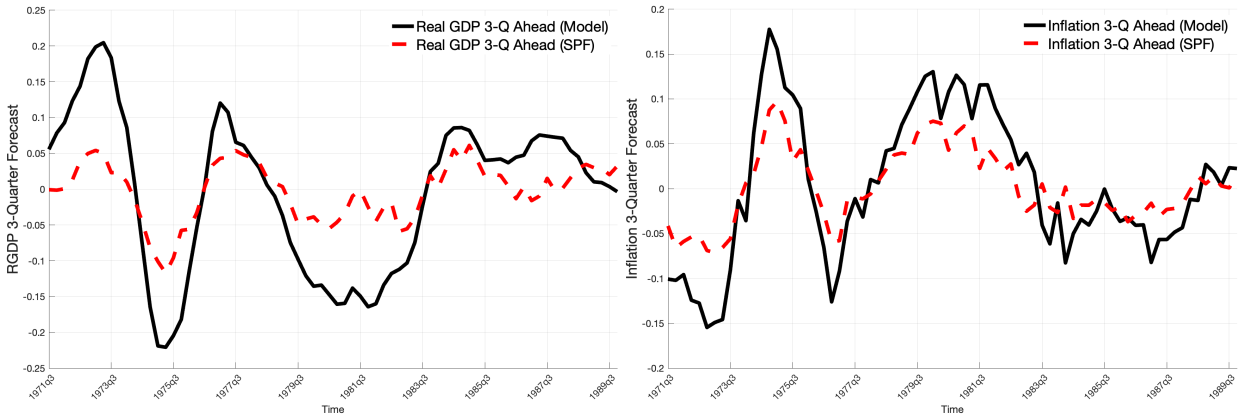
two structural shocks and two observables, this inversion matches the observed time series exactly by construction. We then simulate these inferred shocks forward to produce the model-implied expectations. Importantly, the expectation series is not targeted in identifying the shocks and thus serves as an overidentifying test of model fit. Figures 6 compare the empirical SPF expectations (dots) with those implied by the estimated model (dashed lines).

Both the pre-1990 and post-1990 models exhibit a reasonably good fit, especially given the model's parsimony. However, the post-1990 specification tracks the expectation dynamics more closely. Notably, SPF expectations display substantial variation over time. This feature has two key implications: (i) It poses a challenge for canonical incomplete information models with a single aggregate shock (e.g., Woodford (2003)), which must trade off between signal precision and the responsiveness of expectations.[15] (ii) Replicating the observed volatility in SPF expectations requires that agents process both output and inflation relatively precisely—yet, under informational frictions, this necessarily leads to confusion between shocks. That is, to match the magnitude of expectation fluctuations, agents will rationally confused the source of observed fluctuations.

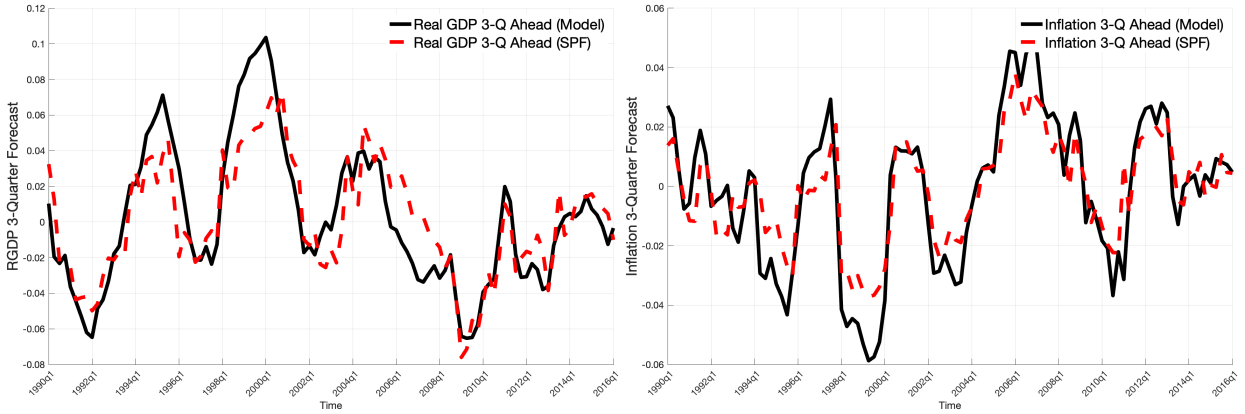
Pre-1990 vs. Post-1990: Heighten and Muted Inflation Responses Figures 7 display the responses to one-percent demand and supply shocks from models estimated using data before and after 1990. Several salient patterns emerge. For demand shocks, both models exhibit a substantial degree of monetary non-neutrality. As shown in Figure A1 in Appendix C.1, the inflation responses in both periods are substantially muted compared to the model with full-information rational expectations

[15] In models with a single aggregate shock and dispersed information, increasing signal precision causes the model to converge to its full-information rational expectations counterpart. But with significant informational frictions, expectations respond only weakly to shocks.

Figure 6: Estimation Fit: Untargeted Time Series



(a) Pre-1990 Model Fit: Untargeted Moments



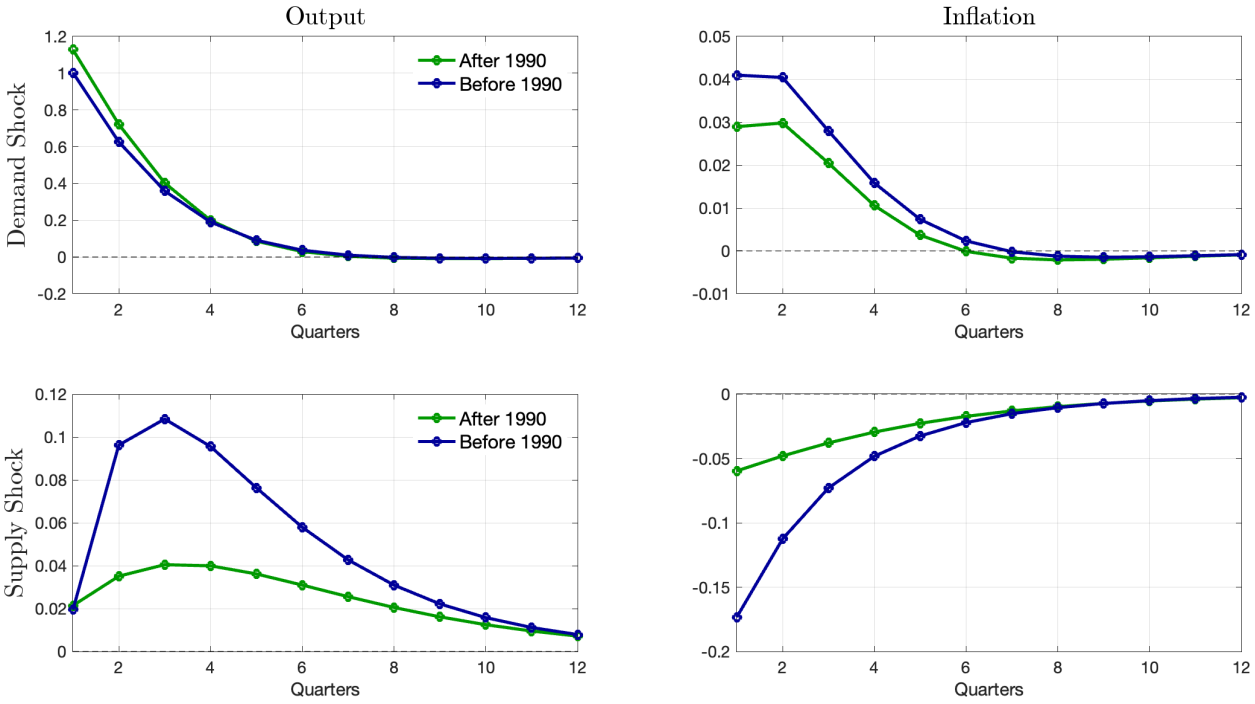
(b) Post-1990 Model Fit: Untargeted Moments

Notes: This figure presents the Hamilton filtered (h = 4) series of real GDP, GDP-deflator inflation three-quarter-ahead SPF forecasts with those implied by the estimated model, respectively. Panel (a) uses the pre-1990 series (1971 Q3–1989 Q4) and the pre-1990 estimated parameters; Panel (b) uses the post-1990 series (1990 Q1–2016 Q3) and the pre-1990 estimated parameters. The actual time series of SPF forecast in all panels are untargeted. All variables are demeaned and scaled so that 0.01 equals one percentage-point deviation from trend.

(FIRE): for demand shocks, the pre-1990 and post-1990 models generate only about 25% and 18% of the inflation response implied by the FIRE benchmark, respectively. However, for demand shocks, the pre-1990 model's inflation response is markedly stronger—approximately 33% greater than in the post-1990 model. The differences are even more pronounced for positive supply shocks: the pre-1990 model exhibits significantly larger responses in both output and inflation—approximately 275% and 310% greater, respectively, than the corresponding post-1990 estimates. These findings reinforce our theoretical prediction that the slope of the Phillips curve can vary considerably across regimes and, crucially, is *shock dependent*.

To understand the mechanisms underlying these patterns, note that prior to 1990, σ_π is estimated to be very small. As illustrated in the simple model, when the inflation signal is highly precise, firms systematically confuse positive demand shocks with negative supply shocks. This mechanism

Figure 7: Impulse Response from Estimated Model

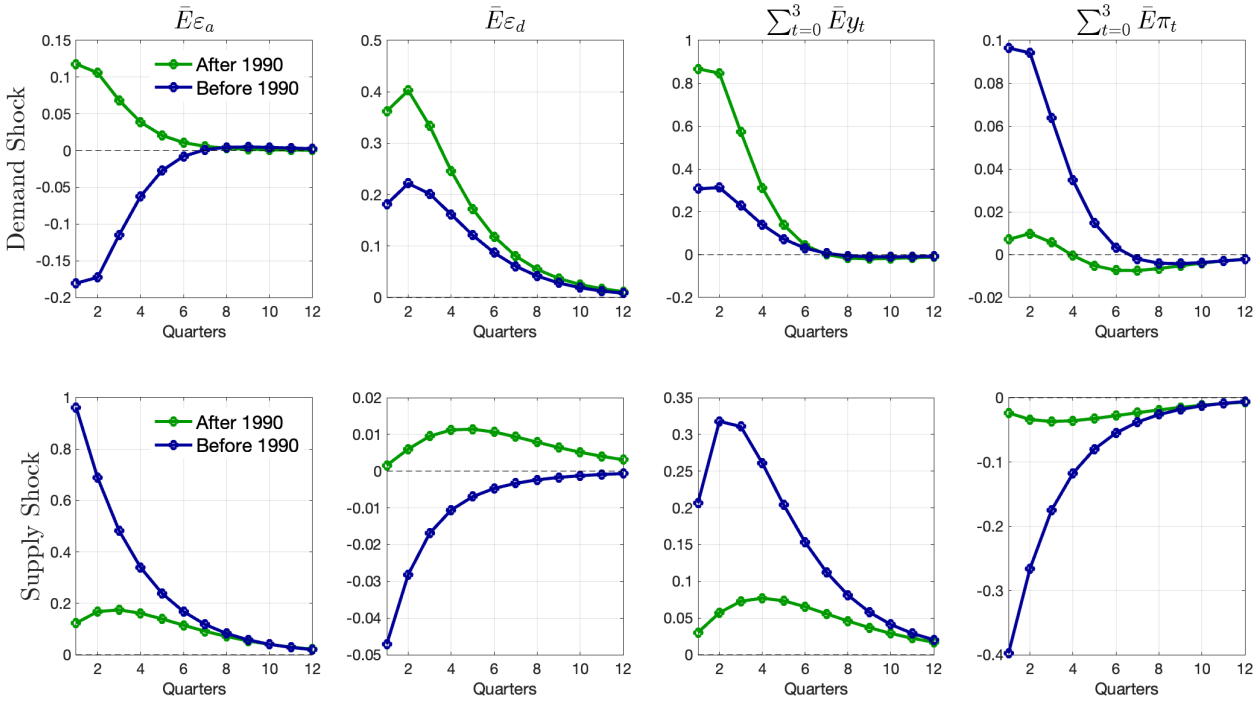


Notes: Impulse response functions (IRFs) for output and inflation are computed using the pre- and post-1990 structural parameters reported in Table 1. The first row presents the impulse response to a 1% innovation in the demand shock, while the second row shows the response to a 1% innovation in the supply shock. Shocks are normalized to a one-percent innovation in aggregate demand or supply, and responses are expressed as percent deviations from the non-inflationary steady state. Horizons are quarterly.

is evident in Figure 8: following an actual demand shock, firms' expectations about the demand shock are positive, while the expected supply shock is negative. As predicted by the theory, such confusion amplifies the inflation response, particularly for supply shocks, relative to the pre-1990 benchmark. After 1990, the pattern reverses: although the increase in σ_π is modest and the decline in σ_y is similarly limited, the shift is sufficient to alter firms' interpretation of shocks. As a result, firms endogenously place greater weight on output signals, which now leads them to confuse a positive demand shock with a positive supply shock. This generates a dampened inflation response for both types of shocks, most notably for supply shocks.

To decompose the channels driving the inflation response to demand shocks, Figure 8 shows that, in the pre-1990 model, expected inflation is significantly higher, accompanied by an expected negative supply shock—both of which push firms to raise prices. This is consistent with the mechanism whereby firms, when confusing a positive demand shock with a negative supply shock, form higher inflation expectations regardless of the shock's true origin, and best respond by raising prices. By contrast, the post-1990 model features much weaker inflation expectations alongside an expected positive supply shock, both of which suppress firms' incentives to increase prices. For supply shocks, the inflation response in the pre-1990 model closely mirrors that of the FIRE benchmark, as seen in Figure A1 and the expected supply shock path in Figure 8. In contrast, the

Figure 8: Impulse Response from Estimated Model: Expectation

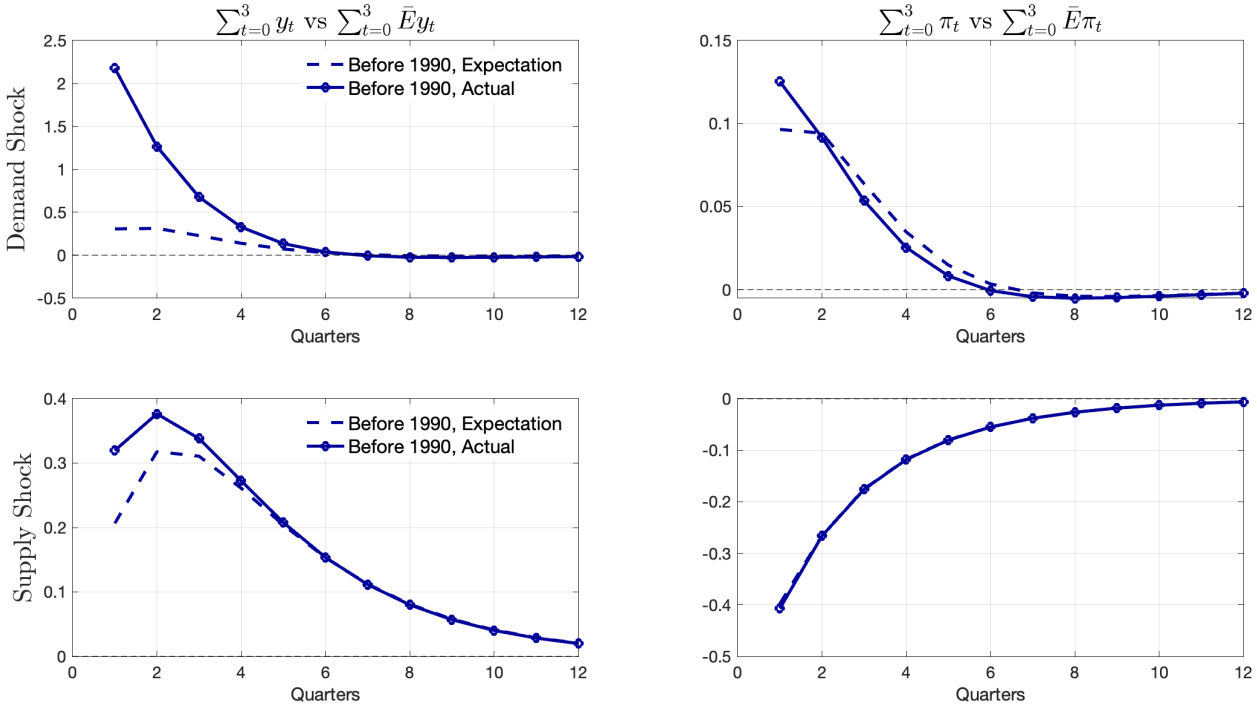


Notes: Impulse response functions (IRFs) for agents' expectations are computed using the pre- and post-1990 structural parameters reported in Table 1. The first row presents the impulse response to a 1% innovation in the demand shock, while the second row shows the response to a 1% innovation in the supply shock. Column 1 reports the average expectation of the aggregate supply shock, Column 2 reports the expectation for the aggregate demand shock, Column 3 shows the cumulative expected output deviation, and Column 4 shows the cumulative expected inflation. Shocks are normalized to a one-percent innovation in aggregate demand or supply, and responses are expressed as percent deviations from the non-inflationary steady state. Horizons are quarterly.

post-1990 model displays a markedly different pattern: both perceived supply shocks and inflation expectations are substantially muted, resulting in a much weaker inflation response.

Delayed Overshooting: Confusion under Dispersed Information A notable implication of the estimated model is its ability to generate the pattern of delayed overshooting documented by Angeletos et al. (2021): following a shock, agents' expectation initially underreact, but subsequently overreact. It is widely believed that dispersed information alone can only dampen expectations (see Angeletos et al. (2021), Angeletos and Lian (2023)). Thus, evidence of overreaction is often taken as indicative of alternative expectation frictions, such as diagnostic expectations. However, **??** reveals a subtler mechanism: following a demand shock, expected inflation initially falls short of realized inflation but eventually overshoots and remains elevated for several periods. This suggests that rational agents facing dispersed information can also exhibit delayed overshooting. The underlying mechanism is as follows. Upon the arrival of a shock, agents, due to dispersed information, form dampened beliefs about the nature of the disturbance. As additional information accumulates, they begin to infer that the economy is undergoing a sizable fluctuation. Because inflation is more informative than output in this regime, agents place greater weight on price signals and confuse

Figure 9: Delayed Overshooting



Notes: The panels plot agents' three-quarter-ahead expectations about output and inflation and their actual values, for the pre-1990 estimated model. The dashed line represents the expectation, while the solid line represents the actual value. The first row presents the impulse response to a 1% innovation in the demand shock, while the second row shows the response to a 1% innovation in the supply shock. Column 1 reports the actual and average expectation of the cumulative expected output deviation, and Column 2 shows the actual and cumulative expected inflation.

the positive demand shock as a negative supply shock. Since negative supply shocks typically generate greater inflationary pressure, agents then overpredict inflation relative to its realization.

Non-Inflationary Demand Shocks and Business Cycle Decomposition Beyond the differential inflation responses and the associated flattening or steepening of the Phillips curve, our model also reconciles a closely related empirical puzzle: especially after the 1990s, the shock that accounts for most of the business-cycle-frequency variation in real macroeconomic variables—such as real GDP and unemployment—generates little inflation and does not resemble standard supply shocks. Conversely, the shock that drives inflation accounts for little variation in output. In other words, inflation dynamics appear to be primarily driven by supply shocks, while real activity is largely driven by demand shocks (Angeletos et al., 2020). This pattern is clearly replicated in our model's post-1990 impulse responses, shown in Figure 7, where the muted inflation response to a demand shock arises from firms' systematic confusion between positive demand and supply shocks.

Quantitatively, the post-1990 model reproduces the variance decomposition documented in Angeletos et al. (2020), in which the primary business cycle shock is nearly orthogonal to inflation dynamics. They find that this dominant shock explains 73.71% of unemployment fluctuations but only 6.96% of inflation variation at business-cycle frequencies. In our model, historical decomposi-

tion of the post-1990 period shows that the identified demand shock accounts for 80.9% of output fluctuations, while explaining 1.41% of inflation variation. These findings underscore the model's capacity to replicate the empirical decoupling of inflation from real activity.

Historical Decompositions Figure 10 presents the historical decomposition of output, inflation, and their expectations before 1990 into demand and supply components. For real GDP, it is immediately apparent that both demand (blue area) and supply (red area) shocks contributed roughly evenly to output fluctuations. Quantitatively, demand and supply shocks account for 41.1% and 58.9% of the variance in output, respectively. In contrast, expectations of real GDP were almost entirely driven by supply shocks, which explain 99.8% of their variance—reflecting a strong *"supply-side"* view of the economy. This asymmetry is consistent with the earlier impulse response analysis: supply shocks elicit a much stronger inflation response, leading firms to place greater weight on inflation information. As a result, firms interpret heightened inflation as indicative of supply shocks—even though demand shocks explain a large portion of the actual variation in output.

However, inflation, and by extension, inflation expectations—appears to be primarily driven by supply shocks. This stems from the *shock-dependent* slopes of the Phillips curve: for demand shocks, the relationship between output and inflation is nearly flat, with a 1% increase in output generating only about a 0.04% rise in inflation. As a result, producing any meaningful inflation response from demand shocks would require implausibly large changes in output. Consequently, most of the observed inflation fluctuations are attributed to supply shocks, further reinforcing their dominant role in shaping inflation expectations.

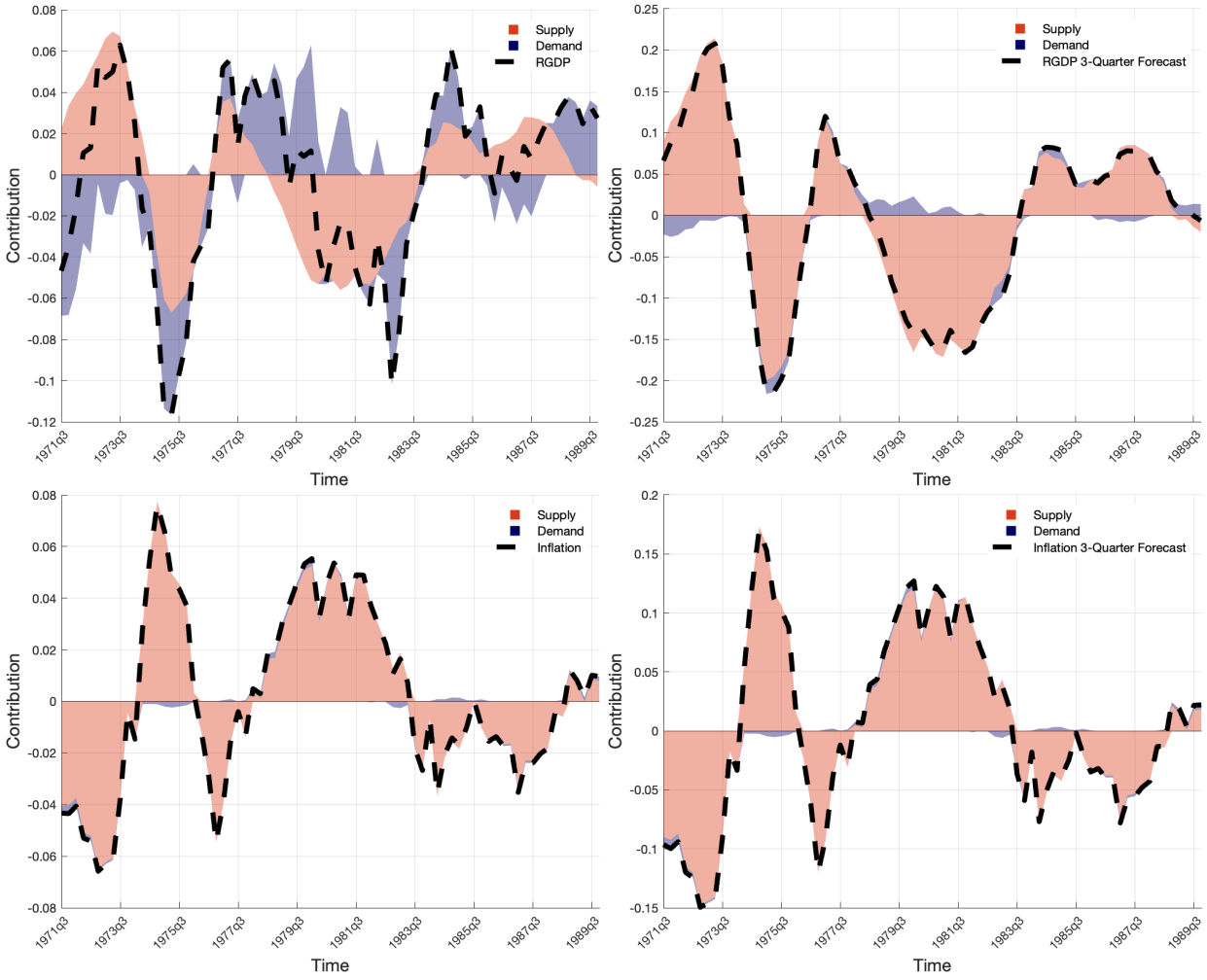
Figure 12 presents the historical decomposition of output, inflation, and their expectations after 1990 into demand and supply components. Actual output fluctuations are now predominantly driven by demand shocks, which account for 80.9% of the variance, marking a notable shift from the more balanced decomposition observed before 1990. While expectations about future output still exhibit a supply-side bias, this bias is substantially weaker than in the earlier period: demand shocks now explain 37.2% of the variation in output expectations, compared to near-zero pre-1990.

Second, inflation remains primarily driven by supply shocks, again due to the *shock-dependent* slopes of the Phillips curve. In the post-1990 model, the slope of the Phillips curve with respect to demand shocks becomes even flatter, with a 1% increase in output generating less than a 0.03% rise in inflation. As a result, observed inflation and inflation expectations continue to be predominantly attributed to supply shocks.

4.3 Shock Dependent Phillips Curve

A central mechanism in our model, the *shock-dependent* Phillips curve, offers a natural resolution to the empirical puzzle highlighted by Angeletos et al. (2020) that, particularly in the post-1990 period, the shock that drives the bulk of business-cycle fluctuations in real activity is disconnected

Figure 10: Historical Decomposition: Before 1990



Notes: Stacked-area plots attribute quarterly fluctuations in real GDP, inflation, and their SPF expectations, before 1990, to the structural demand (blue) and supply (red) shocks recovered via model inversion. The first row shows the results for real GDP and its forecast, while the second row shows the results for inflation and its forecast.

from both TFP and inflation, while the shock that drives inflation exhibits little correlation with output. In our framework, this decoupling arises endogenously from the interaction between dispersed information and agents' inference: demand shocks elicit large output movements but only minimal inflation responses, whereas supply shocks induce strong inflation signals with more muted output effects. This asymmetry stems from the differing slopes of the Phillips curve under each shock type, steep for supply, flat for demand, and leads firms to disproportionately interpret inflation fluctuations as evidence of supply disturbances, even when output variation is predominantly demand-driven.

To further illustrate this mechanism, we examine unconditional comovement patterns between output and inflation in both the data and the model. Figure 13 compares simulated series generated from the estimated pre- and post-1990 models to actual macroeconomic time series. The model

captures the qualitative features of the data well, albeit with slightly lower volatility. Before 1990, both the model and the data exhibit a strongly negative relationship between output and inflation, consistent with the heightened responsiveness of inflation to supply disturbances. After 1990, this relationship flattens considerably, reflecting a regime in which demand shocks dominate output variation, while inflation continues to be driven by supply-side perceptions. The flattening becomes even more evident when we simulate the model conditional on demand shocks alone: the inflation response is nearly flat despite sizable output movements. This confirms the shock-dependent nature of the inflation-output tradeoff emphasized by our theory.

We formalize these patterns by estimating the slope of the Phillips curve conditional on the nature of the shock. Suppose the econometrician has access to well-identified demand and supply shocks as well as a credible measure of the natural level of output. Further, she holds the view that inflation and the output gap are linked through the canonical linearized New Keynesian Phillips Curve. She could then estimate the slope κ using the impulse responses to these structural shocks, following a local projection approach in the spirit of Barnichon and Mesters (2020) and Del Negro et al. (2020):

$$\pi_t = \kappa(y_t - y_t^*) + \beta \mathbb{E}_t \pi_{t+1}.$$

We implement this procedure using simulated data from our model. The resulting estimates, reported in Table 2, reveal a stark asymmetry. Prior to 1990, the Phillips curve slope conditional on demand shocks is only $\kappa_{\text{demand}} = 0.0093$, whereas under supply shocks it rises to $\kappa_{\text{supply}} = 0.0602$—a sixfold difference. After 1990, both coefficients decline significantly: κ_{demand} falls by nearly 50%, while κ_{supply} drops by approximately 75%. The model thus quantitatively replicates the flattening of the Phillips curve documented in recent empirical work, such as the state-level estimates in Hazell et al. (2022). These results underscore two central insights: first, the Phillips curve is intrinsically *shock-dependent*; second, its slope is endogenous to the information structure and varies systematically across regimes, generating time-varying inflation-output trade-offs even in the absence of changes to structural parameters.

Table 2: Reduced-Form NKPC Slopes Estimated on Simulated Data

	κ_{demand}		κ_{supply}		Full-Info RE
	1971–1989	1990–2016	1971–1989	1990–2016	(FIRE)
Estimate	0.0093	0.0058	0.0602	0.0150	0.0633
s.e.	(0.0045)	(0.0031)	(0.0013)	(0.0015)	
R^2	0.336	0.246	0.995	0.910	1

Figure 11: Historical Decomposition: After 1990

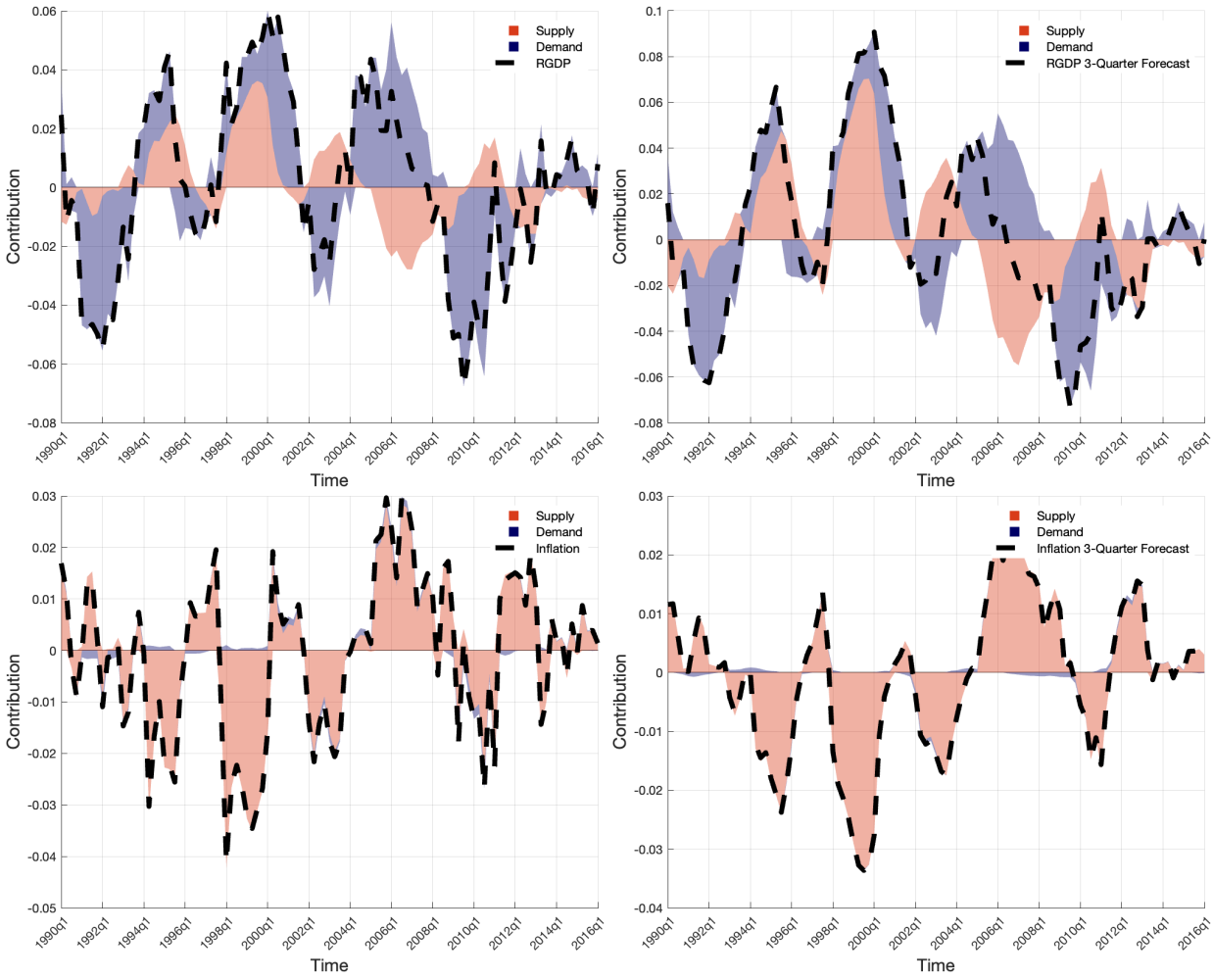
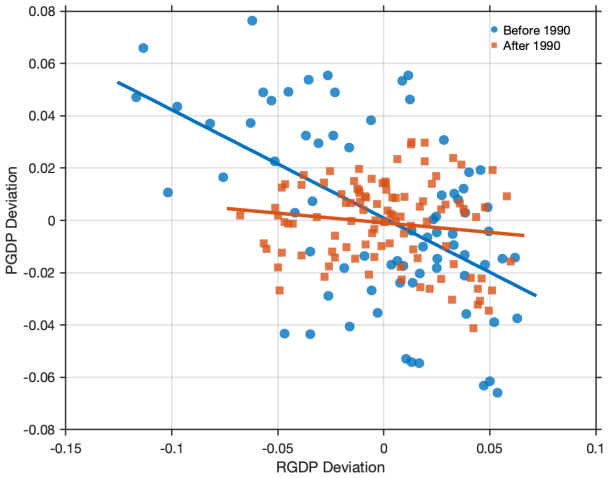


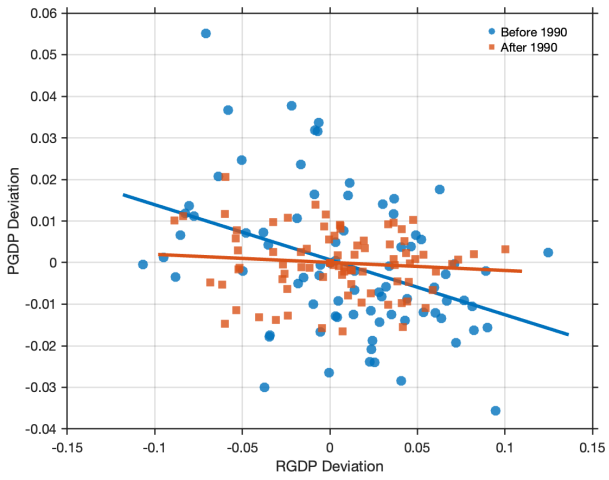
Figure 12: Historical decomposition, 1971:Q3–1989:Q4. Left: output; right: inflation.

Notes: Stacked-area plots attribute quarterly fluctuations in real GDP, inflation, and their SPF expectations, after 1990, to the structural demand (blue) and supply (red) shocks recovered via model inversion. The first row shows the results for real GDP and its forecast, while the second row shows the results for inflation and its forecast.

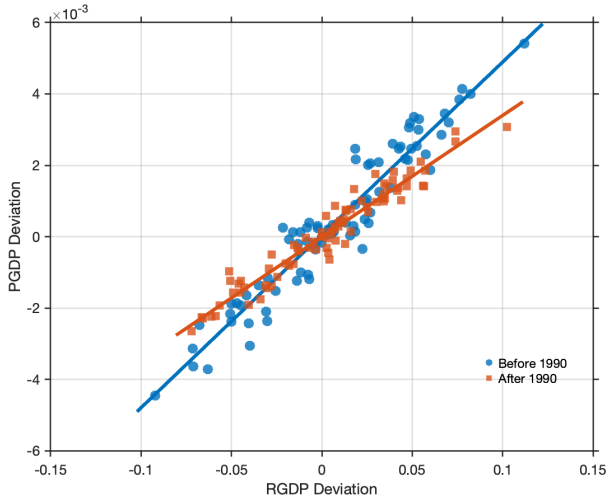
Figure 13: Data v.s. Simulation from Estimated Models



(a) Data: Before and After 1990



(b) Model Simulation: Before and After 1990



(c) Model Simulation: Demand Shock only

Notes: Each scatter plot shows demeaned quarterly pairs of real GDP and inflation. The top panel plots the observed pairs for the periods before and after 1990. The bottom-left panel plots the same pairs from simulations of the pre- and post-1990 estimated models. The bottom-right panel plots the same pairs from simulations of the pre- and post-1990 estimated models using only the demand shock. The simulations use the same number of observations as the observed time series.

5 Conclusion

This paper develops a theory of confusion in macroeconomic expectations, showing how firms' rational inferences about aggregate shocks can systematically confuse supply and demand shocks. By embedding this mechanism in a New Keynesian framework, we demonstrate how confusion gives rise to a regime- and shock-dependent Phillips curve, reconciling several empirical puzzles: the flattening of the Phillips curve since the 1990s, the decoupling of inflation from real activity, and the enduring "supply-side" lens that managers adopt when interpreting price changes.

Quantitatively, we estimate the model on U.S. data split before and after 1990, allowing the informativeness of price and output signals to shift across regimes. We find that before 1990, firms placed more weight on prices, mistaking negative supply shocks for positive demand shocks and thus amplifying inflation responses. After 1990, firms relied more on output, leading them to confuse positive demand shocks with positive supply shocks and thereby flattening the Phillips curve. This endogenous re-weighting of information channels explains why demand shocks produce large output responses but little inflation, while supply shocks remain the dominant driver of inflation fluctuations.

Our results suggest that confusion is not merely a theoretical curiosity but a structural force shaping modern business cycles. In doing so, we provide a unified explanation for the apparent flattening of the Phillips curve, the observed historical decompositions of output and inflation, and the coexistence of muted demand-driven inflation with persistent cost-push narratives in firm surveys. More broadly, we show that the way agents process signals — and the confusion this creates — can generate macroeconomic dynamics that might otherwise be attributed to changing structural parameters or shifts in policy regimes.

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A Details of the Simple Model

Under Construction

B Proofs

Under Construction

C Additional Results for Quantitative Model

Under Construction

C.1 Additional Impulse Responses

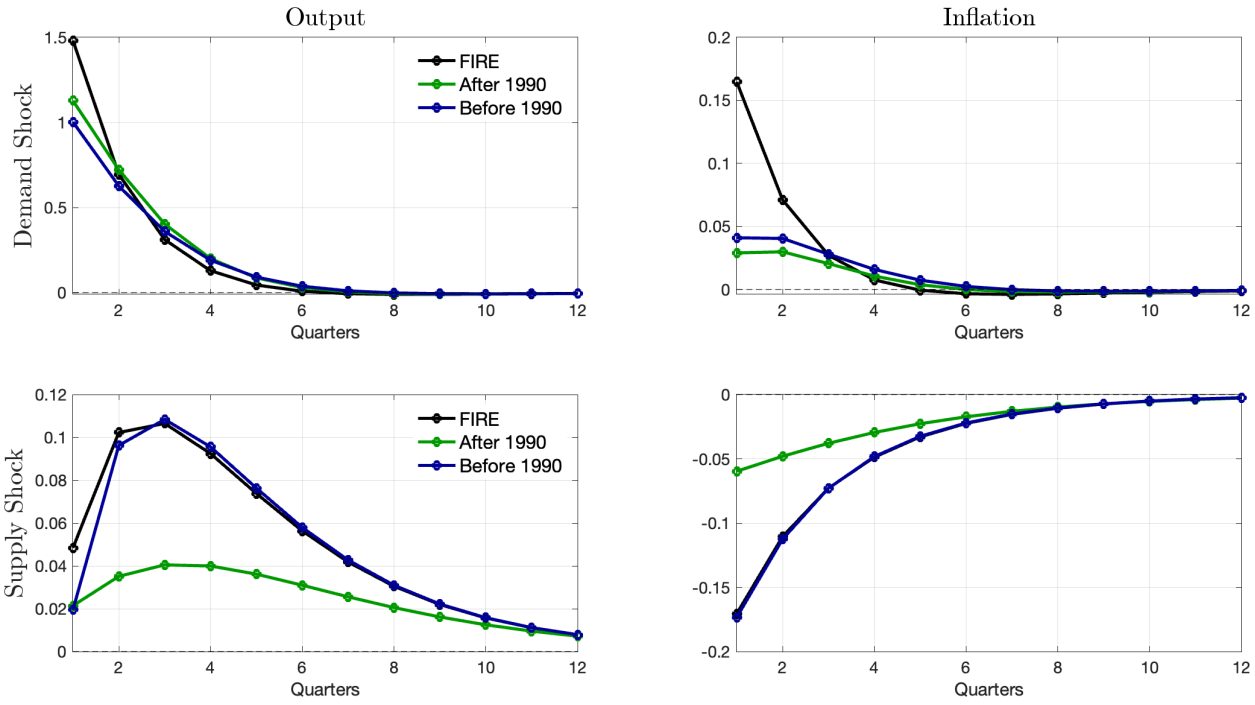


Figure A1: Impulse responses to 1% demand (blue) and supply (red) shocks, 1971:Q3–1989:Q4.

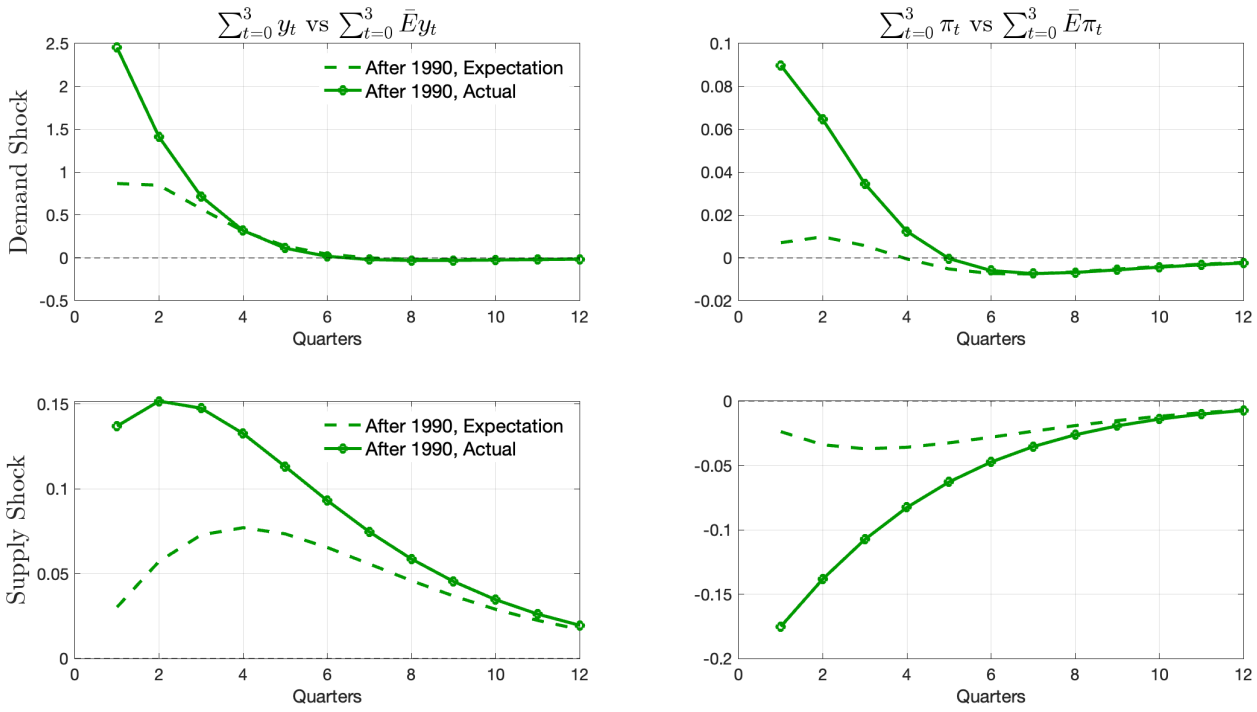


Figure A2: Impulse responses to 1% demand (blue) and supply (red) shocks, 1990:Q1–2016:Q3.