Anchoring Expectation Formation Proposal

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1 Motivation

Central bankers continuously talk about the need to "anchor" inflation expectations - an issue whose importance the recent post-pandemic inflation surge has once again highlighted. This policy need coincides with the recognition in macroeconomics that, because inflation expectations are key drivers of inflation, modeling inflation dynamics requires a model of expectation formation that tackles deviations from the full-information rational expectations (FIRE) paradigm, while maintaining some degree of model-consistency and optimality. We thus need a model of expectations that can match key empirical facts about inflation expectations without devating too far from the framework of optimizing agents.

2 Optimal Expectations Process with Anchoring and Hump Shapes

In this paper, I propose a theory of inflation expectations that addresses these two concerns. The model of inflation expectations is conceptually simple: it postulates that agents think that the inflation process has a time-varying mean and time-varying variance, and they learn these two unknowns using an optimal technology given the non-linear environment: a particle filter. The model provides optimal and model-consistent expectations for mean inflation and the variance of inflation, yielding a natural metric of unanchoring as the change in expected mean inflation over the change in expected inflation variance. The economic intuition behind this is that expectations unanchor when agents attribute the changes in observed inflation to the inflation mean changing, rather than to the variance of shocks increasing. This notion of unanchoring is conceptually related to those in Carvalho et al. (2023) or Gáti (2023), but is an improvement in that inflation expectations are not formed in an ad-hoc way, but are instead the model-consistent and optimal outcome of a non-linear filtering problem. One the one hand, the optimality of expectations renders the normative analysis of policy more meaningful, and on the other hand, the model-consistency of expectations opens the door to analyzing how communication policy affects anchoring; something which has so far been missing from the literature.

This model of expectations that I dub "anchoring expectation formation" is also able to match a key empirical regularity about expectations documented by Angeletos et al. (2021), namely the hump-shaped response of expectations following a shock. Matching this has proven challenging for the expectations literature because of the tendency of model-consistent, optimal expectations to behave like "jump variables" following a shock, assuming their new, optimal value immediately. Therefore, many models of expectation

formation introduce some ad-hoc sluggishness into expectations, such as the adaptive learning rule in my own earlier work (Gáti, 2023). Even Pfäuti (2025), the paper that to my knowledge comes closest to matching the hump-shaped expectations response while maintaining an optimizing framework for expectations, imposes the ad-hoc feature of an attention threshold, and calibrates it to match the expectations dynamics above and below the threshold. Here, instead, the hump-shaped response of expectations to shocks is an *outcome* of the optimal expectations process. This is because in the aftermath of shocks, agents face an identification problem: was the shock the result of mean inflation changing, or the inflation variance changing? As I illustrate below (Figure 1), this produces situations in which agents' long-run inflation expectations keep going up for several periods after the shock before agents realize that the shock is subsiding and that mean inflation did not increase. This, in turn, also produces hump-shaped inflation responses.

3 Learning about Means and Variances

I plan to embed the learning of means and variances in the standard three-equation New Keynesian (NK) model, which can be summarized as follows.

$$\pi_t = \kappa x_t + \beta \hat{\mathbb{E}}_t \pi_{t+1} + u_t, \tag{1}$$

$$x_t = \hat{\mathbb{E}}_t x_{t+1} - \sigma i_t + \sigma \hat{\mathbb{E}}_t \pi_{t+1} + v_t, \tag{2}$$

$$i_t = \delta_\pi \pi_t + \delta_x x_t + \epsilon_t, \tag{3}$$

where $y_t \equiv (\pi_t, x_t, i_t)'$ are the observables, $s_t \equiv (u_t, v_t, \epsilon_t)'$ are the shocks (cost-push, natural rate and monetary policy, respectively), and the expectations operator $\hat{\mathbb{E}}$ denotes subjective expectations that are optimal given the information structure of the agents.[1] I assume an AR(1) structure for the shocks, so that $s_t = \rho s_{t-1} + \eta e_t$, where ρ is a 3×3 matrix containing the autocovariances on the diagonal, and η is also a 3×3 matrix with the standard deviations of the innovations e_t on the diagonal, so that $e_t \sim \mathcal{N}(\mathbf{0}, \mathbf{1})$. Under rational expectations, the solution of the model can be written in the state-space form:

$$s_t = \rho s_{t-1} + \eta e_t \tag{4}$$

$$y_t = g s_t, \tag{5}$$

where g is a 3×3 matrix, mapping the states to the observables. I assume that agents observe time t realizations of y_t and s_t, and they also know the state evolution equation, Equation 4. Thus, they can form the rational expectation $\mathbb{E}_t\, s_{t+k} = \rho^k s_t$. What they do not know is Equation 5, and instead assume that the law of motion for the observables is

$$y_t = a_t + b_t s_t, \tag{6}$$

where a is 3×1 and b is 3×3. Note that a has the interpretation of long-run means for y_t, and b describes the response of the observables to each of the shocks. For example, $\mu \equiv a(1)$, the first element of a, is mean inflation, while the (1,1) element of b, $\sigma \equiv b(1,1)$, is the response of inflation to the cost-push shock. While for the paper, I plan to work with the general case outlined in Equation 6, for the purpose of this proposal I

[1] Note that unlike in Gáti (2023), I have simply substituted non-rational expectations into the otherwise unchanged NK model. This amounts to the assumption that agents know that they share the same expectation formation process, and I make this assumption to render comparisons with the standard model easier, and to avoid the additional volatility that comes from writing the aggregate equations of the model as including the full term structure of expectations (Preston, 2005), so that dynamics are attributable exclusively to the learning framework.

will focus on the simplified setting where only two things are learned: mean inflation and the response of inflation to cost-push shocks. I assume that agents think of the latter as the standard deviation of cost-push shocks. Thus, for this proposal, I focus on the restricted perceived law of motion (PLM) for inflation of the form

$$\pi_t = \mu_t + \sigma_t u_t, \tag{7}$$

where the agents learn the coefficients (μ_t, σ_t) using a particle filter (Kitagawa, 1996). This choice is motivated by the desire to have an expectation formation process that is both model-consistent and optimal given the non-linearity of the PLM. For the particle filter, I assume that the public entertains AR(1) priors for the evolution of the hidden states (μ_t, σ_t):

$$\begin{bmatrix} \mu_t \\ \sigma_t \end{bmatrix} = \begin{bmatrix} \rho_\mu & 0 \\ 0 & \rho_\sigma \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \sigma_{t-1} \end{bmatrix} + \begin{bmatrix} \tau_\mu & 0 \\ 0 & \tau_\sigma \end{bmatrix} \begin{bmatrix} \varepsilon_t^\mu \\ \varepsilon_t^\sigma \end{bmatrix}, \qquad \varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{1}). \tag{8}$$

Using the calibration outlined in Table 1, I compute impulse responses to a one-standard-deviation cost-push shock. For realized inflation, I compare the impulse responses with those from the rational expectations (RE) version of the model, and for inflation and the estimated expectations variables $\hat{\mu}_t$ and $\hat{\sigma}_t^2$, I also plot an alternative expectation formation mechanism, namely adaptive learning with a constant gain, k. I set the deep parameters of the NK model, $\beta, \kappa, \delta_\pi$ and δ_x, to conventional values. The constant gain k is calibrated to the consensus value I found by surveying the literature that estimates gains (Gáti, 2023). While there is considerable room to calibrate the cost-push shock process and the agents' beliefs on the evolution of the hidden states rigorously, for the purposes of this proposal I did a back-of-the-envelope calibration. I set the cost-push process to be persistent ($\rho_u = 0.7$) but not close to a random walk, and to display small but not minute innovation volatility ($\eta_u = 0.05$). I let the agents entertain beliefs for the inflation mean and standard deviation that are both more persistent ($\rho_\mu = \rho_\sigma = 0.9$) and more volatile. Here I let the market believe that volatility shocks are much more important than trend shocks ($\tau_\mu = 0.1 << \tau_\sigma = 0.6325$), so that outsized trend shocks are not the driver of the private sector's beliefs.

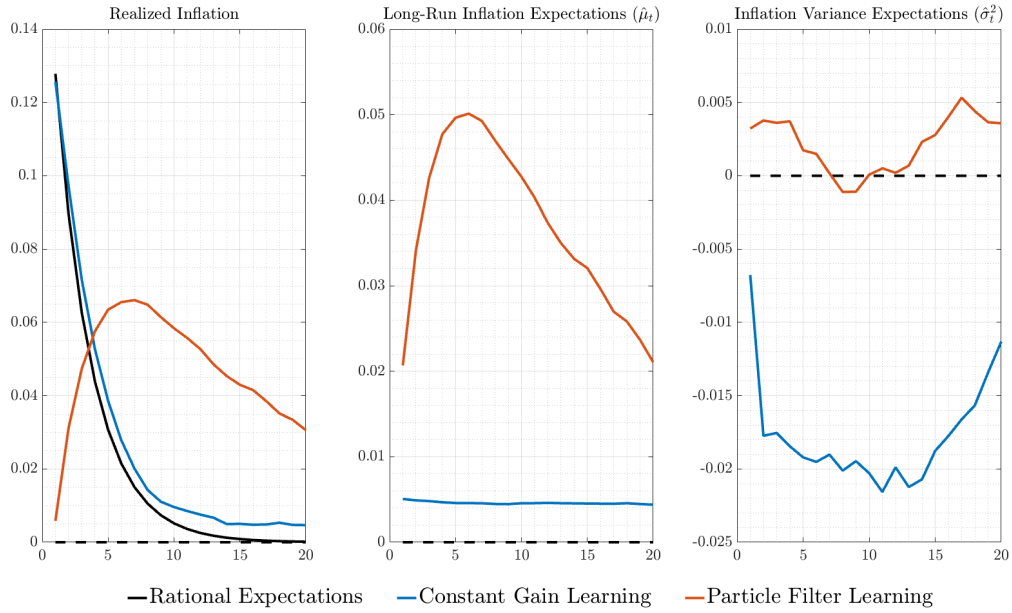
Table 1: Calibration

Parameter	Value	Target
β	0.99	Chari et al. (2000)
κ	0.0842	Expected price duration of two quarters
δ_π	1.5	Taylor (1993)
δ_x	0.5	Taylor (1993)
k	0.05	Gáti (2023)
ρ_u	0.7	Persistent but not a random walk
η_u	0.05	Small but not insignificant
ρ_μ	0.9	Persistent
ρ_σ	0.9	Persistent
τ_μ	0.1	Sizable
τ_σ	0.6325	Considerably higher than τ_μ

Figure 1 shows the response of inflation (left panel) to a one-standard-deviation cost-push shock. The black line corresponds to the RE version of the model, the blue line to the constant gain adaptive learning case, and the red line to the "anchoring expectation formation" case, where agents learn using the particle

filter. For the two learning models, the middle panel shows the evolution of long-run inflation expectations (in deviations from their values in the simulation without the impulse), and the right panel shows the evolution of the expectations for the variance of inflation.

Figure 1: Impulse Responses to Cost-Push Shock (Deviations from Unshocked Values)



Looking first at realized inflation on the left panel, we see that the constant gain learning model displays an inflation path that is very close to that under RE. The only difference is that the constant gain learning model sustains a much more extended convergence back towards the initial inflation level. When we look at the long-run inflation expectation on the middle panel, we see that its dynamics explains the slow convergence of inflation: under constant gain learning, $\hat{\mu}_t$ jumps up and then stays elevated, only decreasing exceedingly slowly. This is because constant gain learning is mechanical: inflation surprises are always passed through to mean and variance expectations with the same weight, the constant gain. This is also the reason for why a constant gain learning model cannot speak to the phenomenon of anchoring.

In the anchoring model, instead, $\hat{\mu}_t$ jumps up on impact, but instead of decreasing back or remaining elevated, it *increases* for about eight periods until it peaks and begins falling again. Looking at the right panel, we see that in the anchoring model, inflation variance expectations, $\hat{\sigma}_t^2$, do not move much. They rise a bit initially, but then fall back down, and then rise again. Clearly, unlike agents in the constant gain learning world, agents who learn with a particle filter initially attribute the cost-push shock to both the inflation mean and variance having increased, but in the subsequent period they attribute more and more to the mean. Because learning is optimal, it gives agents the flexibility to adjust their expectations of mean and variance to better fit the data. Unlike constant gain learners, whose beliefs evolve mechanically after the impulse, particle filter learners use the first few periods after the impulse to revise their beliefs, which explains the slow build-up of their long-run inflation expectations. Because inflation depends on inflation expectations through Equation 1, inflation inherits the hump shape of long-run inflation expectations, displaying an expectations-driven *delayed overshooting*, matching the evidence of Angeletos et al. (2021). Thus, the anchoring expectations model can produce the hump-shaped expectations and inflation pattern observed in the data not in spite of, but *because* expectations are formed in an optimal way.

In terms of the policy, the model offers promising avenues for normative exploration because since expectations are the outcome of an optimization problem given the information structure, counterfactuals are not subject to the Lucas critique. Furthermore, since the market's priors for the evolution of the hidden states matter, one can explore ways in which policy can influence those priors. For example, there could be a role for communication policies, such as the announcement of average inflation targeting, that commit to keeping the inflation mean, μ, constant. The model thus offers insights on the role of not only conventional monetary policy, but also of communication policy in anchoring inflation expectations.

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