Savings-and-Credit Contracts*

Bernardus van Doornik† Armando Gomes‡
David Schoenherr§ Jānis Skrastiņš¶

June 24, 2025

Abstract

In this paper, we study savings-and-credit contracts (SCC) that mandate a savings period with a default penalty before providing credit. We demonstrate that SCCs mitigate adverse selection and can outperform traditional loan contracts amid information frictions, thereby expanding access to credit. Empirical evidence from a financial product incorporating an SCC design supports our theory. While appearing riskier on observables, we observe lower realized default rates for product participants than for bank borrowers. Further consistent with the theory, a reform that reduces the default penalty during the savings period induces worse selection and higher realized default rates.

JEL Codes: D47, G21, G23, G51, J61.
Keywords: access to credit, contract design, information asymmetry, signaling.

*The views expressed in this paper are those of the authors and should not be cited to reflect the views of neither the Banco Central do Brasil nor Bank for International Settlements. We thank Jon Frost, Brett Green, Sean Higgins, Marcio Kuroski, Tatyana Marchuk, Muhammad Meki, Christopher Palmer, Jan Starmans, Dominik Supera, and seminar participants at the BIS, Banco Central do Brasil, Princeton University, Seoul National University, Sungkyunkwan University, University of São Paulo, Washington University in St. Louis, and conference participants at the BSE Summer Forum on Financial Intermediation and Risk, EFA, and the LBS Summer Finance Symposium for comments and suggestions.

†BIS and Banco Central do Brasil, Setor Bancario Sul Q.3 BL B - Asa Sul, Brasilia DF 70074-900, bernardus.doornik@bcb.gov.br

‡Washington University in St. Louis, One Brookings Drive, St. Louis MO 63130, gomes@wustl.edu

§Seoul National University, Gwanak-gu, 1 Gwanak-ro, Seoul, a081891@snu.ac.kr

¶Washington University in St. Louis, One Brookings Drive, St. Louis MO 63130, jskrastins@wustl.edu

1 Introduction

In this paper, we explore a financial product, Savings-and-Credit Contracts (SCCs), that serves millions of households around the world to gain access to credit in the presence of severe information frictions. SCCs are a contractual arrangement that combine a mandatory savings period during which individuals make an initial deposit with a credit period in which individuals receive credit conditional on making the initial deposit during the savings period and a second down payment. If individuals make the initial deposit but withdraw before making the down payment on the loan, they forfeit a fraction of the accumulated savings, the withdrawal penalty. Prominent examples of SCC-type contracts include Rotating Savings and Credit Associations (RoSCAs) in developing countries or contracts linking mortgages to a mandatory savings period in developed countries (e.g., German "Bausparen").

Although widely used in practice, many of the economic mechanisms that make SCCs successful and widely used in practice remain theoretically underdeveloped. This paper attempts to fill this void in the literature by providing a theoretical and empirical investigation of SCCs. In the first part of the paper, we develop a formal model of SCCs under private information and demonstrate that SCC design plays an important role as a screening device that incorporates not only retrospective information potentially available through formal information systems (e.g., credit scores) but also private information only known to borrowers, such as expected liquidity shocks.

In the model, borrowers commit to making two down payments before receiving credit. If borrowers make the first down payment, but fail to make the second down payment, they are repaid the initial down payment minus a penalty. If borrowers make both down payments, they receive credit to buy the asset, and, if the credit is repaid, the borrowers get to enjoy the value of the asset permanently. Otherwise, the asset is seized by the lender. There are two types of borrowers whose types are not verifiable. Good (low risk) borrowers are less likely to experience income shocks during both the savings and credit period compared with bad (high risk) borrowers. Thus, good types are more likely to be able to make both down payments and to repay credit. This persistence in types is necessary for screening to be informative.

With a standard loan contract, pricing is based on the expected credit risk of all borrowers, which may lead to adverse selection and lower credit supply under information asymmetry (Akerlof, 1970). We show that the SCC design allows the lender to better separate

between good and bad borrowers. While lenders can also screen borrowers using variation in down payments in standard spot loans (e.g., Milde and Riley (1988), Adams, Einav, and Levin (2009), and Einav, Jenkins, and Levin (2012)), in the presence of liquidity constraints, the SCC design is a more efficient separating device. Intuitively, for bad borrowers, committing to the savings period adds additional exposure to default risk, which is costly due to the associated penalty. Put differently, with a standard loan contract, agents save on their own and can walk away if they are hit with a bad income shock before making the down payment. With a mandatory savings period they have to commit some funds earlier, part of which they forego if the income shock hits before the second down payment. While this applies to good and bad types, the expected cost of committing to the savings period is lower for the good type, since they are less likely to default.

The property that allows for screening and separation with both types of contracts is that the single-crossing property holds. That is, the marginal rate of substitution between savings and loan payments (for SCCs) or between down payments and loan payments (for spot loans) is increasing in borrower quality. However, the increase in the marginal rate of substitution is stronger for signaling with costly savings than with a single down payment. This makes SCCs a more effective way to screen borrowers than spot loans.

We characterize the competitive equilibrium when only spot loans are available and when SCCs are offered as well. Offering SCCs increases credit supply compared with a credit market with only spot loan contracts, in particular when borrowers face higher real interest rates or have a higher discount rate, and when there is more unobservable information as measured by the spread between good and bad types. Finally, credit supply increases more with SCC contracts when the default penalty during the savings period is higher, since it facilitates signaling for the good type by making it costlier for the bad type to mimic the behavior of the good type by committing to the savings period.

SCCs also dynamically screen borrowers further improving credit outcomes. Low-type borrowers—who face a higher probability of early withdrawal—are more likely to exit before reaching the credit stage. This selection process filters our riskier borrowers, raising the average quality of borrowers over time. Unlike standard loans, SCCs can benefit from low-type borrowers by extracting funds from them when they withdraw during the savings period and a fraction of their accumulated savings is not returned. This implies a novel U-shaped profit structure on borrower types under which lenders may find it optimal not to screen out low types ex-ante.

We explore the consequences of behavioral agents—borrowers who underestimate their own liquidity risk and mistakenly choose SCC contracts. These agents are more likely to withdraw early, generating penalty revenues for lenders. In a competitive market, these excess profits are competed away, resulting in zero expected profits for lenders. However, this dynamic leads to a welfare gain for high-type borrowers, who are effectively cross-subsidized by the behavioral types. Thus, the presence of behavioral agents enhances the attractiveness and welfare implications of SCCs for strategic high-type borrowers. Interestingly, the dynamics observed in SCC markets resemble those in life insurance markets. Gottlieb and Smetters (2021) show that behavioral agents are widespread in the life insurance sector, where policy lapses are common and lapsers effectively cross-subsidize those who maintain coverage.

Finally, we analyze moral hazard in SCCs by studying borrower effort choice during the saving and credit phases. We find that borrowers exert strictly greater effort to avoid default after receiving the loan than to avoid early withdrawal during the saving phase. This asymmetry arises from two key forces. First, default during the credit phase carries significant reputation costs—such as a damaged credit record or exclusion from future borrowing—which are absent in the saving phase. In contrast, early withdrawal allows partial recovery of deposits, making it less costly and reducing the incentive to exert effort upfront. Second, the required down payment upon receiving the loan heightens the borrower's financial stake, further incentivizing repayment. Together, these forces help explain why default rates during the credit period tend to be lower than withdrawal rates during the savings period in SCCs.

In the second part of the paper, we provide empirical support for the predictions of the model using data on consorcio participants and bank borrowers in Brazil that seek financing to buy a car. Consorcios provide credit to finance durable goods and apply an SCC design. They are used by almost 10m participants in a given year. Every month, participants in a consorcio group make identical contributions, which are then allocated to a subset of participants as credit designated for purchase of an asset. All participants continue their contributions until everybody has been awarded credit. If an individual defaults before obtaining credit (i.e., during the savings period), their savings are returned after discounting a significant penalty. If an individual defaults after obtaining credit, the asset is seized by the group's administrator. When individuals sign up for a group there is no active screening. Anybody with a social security number can sign up. Enforcement operates through physical rather than social collateral. Participants share no social ties and do not live in geographic

proximity.

We start our empirical analysis by comparing observable characteristics of bank borrowers and consorcio participants. Our model suggests that SCCs achieve positive selection based on unobservables. Thus, conditional on controlling for observable characteristics, we should observe lower realized default rates for consorcios than for bank loans. First, we show that on observables consorcio participants appear riskier than bank borrowers. Specifically, consorcio participants are 15 percentage points less likely to be formally employed, 1.7 percentage points more likely to currently be in default on a bank loan, 1.3 percentage points more likely to have defaulted on a bank loan in the last 5 years, and their simulated credit score is 12.4 percent worse. However, conditional on controlling for contract- and borrower-level observable characteristics, we observe an about 5 percentage point lower realized default rate on credit allocated to consorcio participants compared with standard bank loans. Together, this suggests that consorcios extract relatively safer borrowers conditional on observables. This is consistent with the positive selection and screening effects of mandating a costly savings period predicted by the model.

Since consorcios differ from bank loans in other ways, we sharpen the interpretation of the analysis by exploiting a regulatory reform in 2009 that changed the default cost associated with the penalty. Prior to the reform, participants who default during the savings period were returned their savings minus a penalty at the ultimate dissolution of the group. After the reform, participants who default during the savings period are returned their savings minus a penalty when their number is drawn in a lottery. Thus, after the reform, savings are returned after a (about 50 percent) shorter waiting period compared with before the reform. In our model, lowering the expected penalty during the savings period weakens SCCs' ability to separate borrowers and leads to more adverse selection.

Comparing bank and consorcio borrowers around the reform, we find that observable characteristics of consorcio participants relatively worsen compared with bank borrowers. In addition, realized default rates significantly increase for consorcio participants relative to bank borrowers after the reform. This suggests that the penalty plays an important role in improving selection into the SCC, as suggested by the model. It is important to highlight that higher realized default rates *after* providing credit to consorcio participants also rule out moral hazard as the explanation for the effects around the reform. Even if borrowers are more likely to default during the savings period after the reform because default is less costly, we should not see higher default rates *after* credit is allocated, since there is no variation in

moral hazard incentives after receiving credit as a result of the reform.

Finally, we document that withdrawal rates during the savings period are higher than default rates during the credit period for consorcios, consistent with the prediction of the model that moral hazard drops once individuals make the second down payment after which no payments are returned upon default and a default flag is added to their credit report. In addition, we observe that withdrawal rates and default rates on credit are highest at the beginning of a consorcio but decline over time as lower type borrowers are more likely to exit earlier due to liquidity shocks they experience, consistent with the dynamic screening feature of SCCs described in our model.

The idea of linking regular savings to assessing credit risk, as in the SCC design, is widely implemented around the world.[1] For example, the widely available Credit Builder Loans (CBLs) in the U.S. require individuals to commit to regular monthly deposit payments that are reported to credit bureaus as installment loans. Consistent with our model, Burke et al. (2022) find that CBLs exhibit positive selection. While CBLs do not directly offer credit as in a SCC, successfully completing a CBL improves an individual's access to future credit by improving their credit score. This suggests that the ability to make regular monthly payments provides important information about individuals' creditworthiness,[2] as reflected in the high weight on regular on-time payments in credit scoring algorithms. The benefit of the SCC design is that it can directly link access to credit with a savings period and a monetary penalty rather than the indirect channel through a credit registry, which is important in settings without sophisticated credit scoring systems and limited information sharing. Another prominent example relates to mortgage contracts in developed world. Germany's Bausparen requires 3-7 years of savings before accessing favorable mortgage rates (Scholten, 2000). Similar features are incorporated in France's Plan Épargne Logement, Austria's Bausparvertrag, Switzerland's Pillar 3a pension-linked mortgage arrangements, and Japan's Flat 35 program. These mirror traditional Building and Loan Associations in the US and UK, where members contributed regularly before qualifying for home loans.

In developing countries, the same principles of demonstrated financial responsibility apply in rotating savings and credit associations (RoSCAs), which resemble many features of an

[1] While we do not explore the role of SCC design as a commitment device to saving, in principle, it may help individuals who struggle to commit to saving in the absence of strong incentives (Laibson, 1997; Karlan, Ratan, and Zinman, 2014; Ericson and Laibson, 2019).

[2] Similarly, cash flow information can be used by lenders to assess borrowers' creditworthiness (Cao, Garcia-Appendini, and Huylebroek, 2024).

SCC.[3] Defaulting before winning the "pot" in RoSCAs, which is akin to defaulting during the savings period in an SCC, involves substantial costs. Typically, a defaulted individual cannot win the pot in future allocations, and foregoes all prior payments. The insights from our analysis suggest that the expected cost of default during the savings period could help explain why RoSCAs attract low-risk individuals (Levenson and Besley, 1996) and can charge lower interest rates (Kapoor et al., 2011), similar to what we document for consorcios.

The insights from this paper contribute to several strands of literature. The key insight of Spence (1973) is that when agents have different marginal costs of signaling, they can be induced to take actions that reveal their type in equilibrium. However, when Adams, Einav, and Levin (2009) empirically study subprime auto lending in the U.S., they find little variation in down payments, even though subprime borrowers should exhibit large heterogeneity in unobservables. One possible explanation for the lack of screening with spot loans documented in Adams, Einav, and Levin (2009) could be that spot loans are a weak mechanism to separate borrowers. To achieve a significant reduction in the interest rate, the borrower has to make a very large down payment, which may not be feasible due to liquidity constraints or too costly even for good-quality borrowers. In contrast, an SCC design may improve screening by generating a higher wedge in the marginal rates of substitution between early deposits and loan payments for good and bad borrowers.

Many studies on household finance focus on private information about the cost of default, such as credit score effects and social stigma (e.g., Brueckner (2000), Gupta and Hansman (2022)), or on asymmetric information about collateral values (Stroebel, 2016). In addition, the literature explores how lenders can utilize collateral pledging to screen borrowers' private information (Bester (1985), Stiglitz and Weiss (1981), Besanko and Thakor (1987), Chan and Thakor (1987)), by offering a menu of loans with higher or lower collateral pledging linked to lower or higher interest rates. Focusing on private information about the distribution of borrowers' future income, our paper extends the use of down payments or loan size as the relevant sorting device for household financing studied in Milde and Riley (1988), Adams, Einav, and Levin (2009), and Einav, Jenkins, and Levin (2012).

Our paper also relates to the literature examining RoSCAs. Different from consorcios, RoSCAs rely on social rather than physical capital for enforcement (Besley, Coate, and

[3]Similar products exist under different names in a wide range of countries with some variation in their design (e.g., Tanda in Mexico, Stokvel in South Africa, Ajo in Nigeria, Gamiya in Egypt). Zambrano et al. (2023) provide an excellent recent review.

Loury, 1993). As a result, the literature on RoSCAs primarily focuses on the role of social capital in improving screening and enforcement (e.g., Banerjee, Besley, and Guinnane, 1994; Besley and Coate, 1995; Ghatak and Guinnane, 1999; Morduch, 1999; Anderson, Baland, and Moene, 2009), including cases in which RoSCAs are organized through third-party intermediation (Klonner and Rai, 2007; El-Gamal et al., 2014). In contrast, we highlight the positive selection effect of having a default penalty before obtaining credit in a combined savings-and-credit contract. An important limitation for RoSCAs is their narrower scope, since they rely on social capital in tight-knit local communities. By relying on physical collateral, consorcios allow for vastly greater scale. The downside is that enforcement in consorcios requires physical collateral and its efficient seizure and liquidation, which makes it unsuitable in contexts where financing is required for non-durable assets and consumption goods or where seizing collateral is difficult.

Recent developments in fintech have significantly increased lenders' ability to access new types of information to reduce information frictions and expand access to credit (Berg et al., 2019; Berg, Fuster, and Puri, 2022; Fuster et al., 2022). In addition, services such as pay-as-you-go (PAYgo) have become a popular technology in low-income countries that allows the lender to suspend the flow benefits of collateral remotely in case of missed payments (Gertler, Green, and Wolfram, 2024). This can improve enforcement of contracts and facilitate the spread of innovation in contract design that relies on physical collateral to enforce contracts. Thus, technological innovation and innovation in contract design, such as SCCs, are not mutually exclusive approaches to resolve frictions in credit markets, but can be complimentary to expand access to credit and increase borrower welfare. Innovation in contract design provides additional advantages. Unlike backward-looking measures, such as credit scores, which are based on historical data, borrowers' private information is forward-looking and may account for structural changes ignored by predictive models. Incorporating this information can also help avoid perpetuating inequalities by allowing borrowers with low or no credit scores to gain access to credit at fair prices based on their positive, forward-looking private information.

As we show in the model, screening through SCCs is most valuable when borrowers face higher real interest rates, when they have a higher discount rate or when there is more unobservable information. This is likely to apply to financial markets in mid-income and developing countries or to low-income households without formal income or credit history more generally. An active debate in the literature seeks to understand how to improve credit

allocation in such environments, and to expand access to finance while also maximizing returns to credit (Karlan and Zinman, 2011; Attanasio et al., 2015; Augsburg et al., 2015; Banerjee et al., 2015; Crepon et al., 2015; Tarozzi, Desai, and Johnson, 2015; Banerjee et al., 2021; Meager, 2019, 2022; Gertler, Green, and Wolfram, 2024; Bari et al., 2024). While we focus on the screening features of SCCs in this paper, van Doornik et al. (2024a,b) and van Doornik, Schoenherr, and Skrastins (2025) provide evidence that recipients of credit in consorcios also generate improvements in life outcomes such as expanded labor market access, reduced mortality, and higher fertility. This suggests that SCC design can succeed at allocating credit to individuals with high returns.

2 Theory

We develop a parsimonious framework to study the properties of Savings-and-Credit Contracts (SCCs) in a credit market with asymmetric information. In our setting, lenders are uninformed about borrowers' default risk, while borrowers know their own type. The model takes the form of a screening game in which the uninformed party (the lender) moves first by offering a menu of contracts designed to induce self-selection. This contrasts with signaling models, where the informed party initiates the interaction.[4]

We show that SCCs allow lenders to more effectively screen for default risk than traditional loan contracts using down payments, resulting in less adverse selection. After presenting the institutional environment, we characterize the main equilibrium outcomes and welfare implications of SCCs under asymmetric information. We conclude the section by extending the model to explore additional features and theoretical predictions of the contract.

2.1 The Model

The main model unfolds over three periods $t = 0$, $t = 1$, and $t = 2$. Agents (borrowers) seek to purchase an asset and require credit financing. Financing is provided by a competitive credit market composed of risk-neutral lenders. For simplicity, we normalize the lender's interest rate to zero ($r = 0$), implying no intertemporal discounting. Agents are risk-neutral

[4]Classic examples of screening models include Rothschild and Stiglitz (1976) and Riley (1979), while seminal signaling models include Spence (1973); and in the finance literature, Leland and Pyle (1977) and John and Williams (1985).

but more impatient than lenders. Their preferences are represented by a discount factor $\beta = 1/(1+\delta)$, where $\delta > 0$. They choose consumption plans $\{c_t\}_{t=0}^{2}$ to maximize expected utility, $E\left[\sum_{t=0}^{2} \beta^t c_t\right]$, where c_t denotes consumption in period t.

Because agents are more impatient than lenders ($\delta > r$), they prefer to front-load consumption. This makes borrowing attractive and saving costly (see Deaton (1992)). Nevertheless, agents face borrowing constraints, except when credit is obtained through secured lending arrangements.

This section examines two types of secured loan contracts: a standard secured loan and a Savings-and-Credit Contract (SCC). Figure 1 illustrates the SCC timeline. In a standard secured loan, the borrower provides a down payment and receives the loan immediately. In contrast, an SCC requires the borrower to save over two periods before receiving the loan. At $t = 0$, the borrower signs a contract for a future loan, contingent on making deposits at $t = 0$ and $t = 1$. If no liquidity shock occurs and both deposits are made, the loan is disbursed at $t = 1$ and the asset is acquired. If a liquidity shock prevents the second deposit, the initial deposit is refunded, net of a penalty for early withdrawal. The borrower repays the loan at $t = 2$.

Agents are endowed with wealth $w_0 > 0$ in period $t = 0$, and receive an income stream y_t in periods $t = 1$ and 2. The asset provides utility value v to the borrower in period $t = 2$, but must be purchased for cost I in period $t = 1$, with $v > I$. To incentivize the borrower to repay the loan during period $t = 2$, lenders can seize the collateralized asset if they do not receive the loan payment. The value of the collateral to the lender is worth $\vartheta < I$, a fraction of the asset's cost.

Agents face liquidity constraints that prevent them from borrowing in period $t = 0$. Their initial wealth w_0 is insufficient to meet the minimum down payment required at that time. In period $t = 1$, however, agents receive income y_1, which enables them to satisfy the down payment required by lenders and obtain credit. We assume $w_0 + y_1 < I$, so that the borrower cannot purchase the asset outright and must rely on external financing.

Defaults arise when borrowers face liquidity shocks that prevent repayment (Aydin (2024)). These shocks are privately observed and not fully reflected in credit scores or observable borrower characteristics. A key model assumption is that agents' private information regarding liquidity shocks during the saving period positively correlates with liquidity shocks during the credit period.

Formally, lenders observe a measure of a borrower's credit risk –the agent's credit score θ, quantifying the likelihood of *not* suffering liquidity shocks. The credit score can be in the continuum interval $\theta \in [\underline{\theta}, 1]$, with $\underline{\theta} > 0$. A borrower with credit score θ faces a probability $1 - \theta$ of withdrawing early due to a liquidity shock during the saving phase ($t = 1$). Conditional on continuing, they face the same probability $1 - \theta$ of defaulting during repayment ($t = 2$). Thus, better credit scores reflect lower risk of shocks in both phases.

Agents can be of two private "types" –the simpler case that allows us to obtain the main insights from the adverse selection model. A borrower with credit score θ can either be of high type (or good, safe type) $\theta_h(\theta) = \theta$ or a low type (or bad, risky type) $\theta_l(\theta) = \theta(1 - \Phi(\theta))$, with higher likelihood of suffering liquidity shocks. The degree of information asymmetry is $\Phi(\theta) \in (0, 1)$. Agents privately learn their type at the beginning of period $t = 0$. We assume the population consists of a fraction μ of high-type agents and a fraction $1 - \mu$ of low-type agents.

We assume that the information asymmetry measure $\Phi(\theta)$ regarding a borrower's risk increases as the credit score decreases, with derivative $\Phi'(\theta) < 0$. Whenever it is clear from the context, for notational convenience, we suppress the dependency on the observable credit score θ, and refer to the types as θ_l or θ_h, and the degree of information asymmetry as $\Phi = (\theta_h - \theta_l)/\theta_h$.

Agents can finance the asset purchase with a savings-and-credit or a loan contract:

Savings-and-Credit Contracts (SCC): Under an SCC contract, the borrower agrees to make a stream of payments (s, d, p) to the lender. Specifically, the borrower makes an additional initial deposit s in period $t = 0$, the saving period, and agrees to make a further down payment d in period $t = 1$ to receive a secured loan I. If the borrower fails to make the second down payment, they forfeit a fraction $\tau \leq 1$ of the deposit s and does not receive the loan. The secured loan, as specified above, is such that the agent keeps the asset conditional on making the payment p in period $t = 2$, or if the payment is not made, the lender seizes the asset. We denote such an SCC by $x = (s, d, p)$, and let $\mathcal{C}$ be the set of all such contracts.

Loan Contracts (SLC): Under the standard loan contract, the borrower makes down payment d at time $t = 1$, and the lender provides $I - d$ to purchase the asset. Agents promise to pay p in the next period, in which case they enjoy the value of the asset v, or if the payment is not made, the lender seizes the asset, in which case the borrower gets 0 and the lenders get the collateral worth ϑ. We denote a loan contract as $x = (d, p)$ and let $\mathcal{L}$ be

the set of all such loan contracts.

Value Functions The time-0 expected utility of contract $x = (s, d, p)$ to a type θ agent is $U_\theta(x)$ given by,

$$U_\theta(s, d, p) := \theta\beta(\theta\beta(v - p) - d) + (1 - \theta)\beta(1 - \tau)s - s. \tag{1}$$

Note that we are focusing on the incremental contract utility, excluding the utility associated with the income flows w_0, y_1, and y_2. Therefore, the agent's participation constraint is $U_\theta(x) \geq 0$; otherwise, the agent would be better off not choosing the contract at all.

The expected utility takes into account that the agent will only get the loan and make the down payment d if she does not suffer a liquidity shock in period $t = 1$ and that she forfeits a fraction τ of the deposit s in case she suffers a liquidity shock. At $t = 0$, the agent pays s, and the $t = 1$ cash flows, discounted by β, are either $-d$ if the agent is able to make the down payment, which occurs with probability θ, or is $(1 - \tau)s$, since the agent forfeits τs if she receives a bad shock with probability $1 - \theta$. In period $t = 2$, the agent enjoys the value v of the asset and makes a payment of p with probability θ^2 and discounted by β^2.

The lenders' expected profit associated with offering contract $x = (s, d, p)$ to a type θ agent is $\Pi_\theta(x)$ given by,

$$\Pi_\theta(s, d, p) := \theta(\theta p + (1 - \theta)\vartheta - (I - d)) - (1 - \theta)(1 - \tau)s + s. \tag{2}$$

At $t = 0$, the lender receives the deposit s, and the $t = 1$ cash flows are $-(I - d)$ if the agent receives a positive shock because the lender finances the asset costing I after receiving the down payment d or is $-(1 - \tau)s$ if the agent receives a bad shock and the lender returns a fraction $(1 - \tau)$ of the savings s to the agent. Conditional on providing the loan, the $t = 2$ cash flows are either p if the agent receives a positive shock or ϑ after seizing the collateral.

Note that the expected utility and profits associated with a loan $x = (d, p)$ to a type θ agent can be simply denoted as

$$U_\theta(d, p) \equiv U_\theta(0, d, p) \text{ and } \Pi_\theta(d, p) \equiv \Pi_\theta(0, d, p), \tag{3}$$

where we set of the early down payment equal to zero.

Finally, the maximum loan payment p in period $t = 2$ is subject to an upper bound $\bar{p}$

which arises from either income constraints or moral hazard considerations. First, repayment must be feasible given the agent's period-2 income, implying $\bar{p} \leq y_2$. Second, drawing on the moral hazard framework of Tirole (2010), excessively high repayment obligations may distort effort incentives and increase the likelihood of default. In such cases, incentive compatibility may require capping the payment. We discuss an extension of the model with moral hazard in Section 2.8.

Formally, the set of feasible SCC contracts is defined as:

$$\mathcal{C} = \{(s, d, p) : 0 < s \leq w_0,\ d \geq 0,\ p \leq \bar{p}\},$$

and the set of feasible loan contracts is $\mathcal{L} = \{(d, p) : d \geq 0 \text{ and } p \leq \bar{p}\}$.

For notational convenience, we define the null contract $x = \varnothing$ to represent situations in which a borrower does not receive financing and therefore does not purchase the asset. The null contract yields zero utility to the borrower, $U_\theta(\varnothing) = 0$, and zero profit to the lender, $\Pi_\theta(\varnothing) = 0$, for all types θ. Thus selecting the null contract is equivalent to not obtaining financing.

2.2 Full Information Case: Benchmark

We first analyze the benchmark case where the agent's type θ is common knowledge, and there is no adverse selection. Our analysis demonstrates that the optimal contract in the full information case is a loan contract. Intuitively, agents have a higher discount factor than lenders and, therefore, want to borrow as much as possible, and saving is inefficient. Thus, the SCC design, which mandates early down payments, is not optimal under full information whenever borrowers can afford the down payment without saving.

Lemma 1 *The optimal contract under full information—selected from the set of standard loan and SCC contracts—that maximizes the agent's utility subject to the lender break even and participation constraints is characterized by the following program:*

$$\begin{aligned} \max_{x \in \mathcal{C} \cup \mathcal{L}} \quad & U_\theta(x) \\ \textit{subject to } & \Pi_\theta(x) \geq 0 \textit{ (CC)} \\ & U_\theta(x) \geq 0 \textit{ (IR).} \end{aligned}$$

For all types $\theta \geq \widehat{\theta} := \frac{I-\vartheta}{(1-\beta)(\bar{p}-\vartheta)+\beta v}$*, the optimal contract is the loan contract* $x_\theta^\circ = (\underline{d}_\theta, \bar{p})$*, where the down payments is given by* $\underline{d}_\theta := I - (\theta\bar{p} + (1-\theta)\vartheta)$*. For all types* $\theta < \widehat{\theta}$*, no financing is offered.*

We show in the Appendix, that the lenders' break-even constraint (CC) binds, and the unique solution of the full information problem above is a loan with the minimum possible down payment $\underline{d}_\theta$, and the maximum possible future loan payment $p = \bar{p}$.

In the absence of asymmetric information, high-type borrowers make smaller down payments than low types–that is, $\underline{d}_{\theta_h} < \underline{d}_{\theta_l}$. Asset financing with the loan contract $x_\theta^\circ = (\underline{d}_\theta, \bar{p})$ yields zero profit for the lender and utility $\bar{U}_\theta = U_\theta(x_\theta^\circ)$ for the borrower satisfying $\bar{U}_{\theta_h} > \bar{U}_{\theta_l}$.

Borrowers with sufficiently low credit scores ($\theta < \widehat{\theta}$) are credit constrained and do not receive financing. That is, they are better off selecting the null contract, which yields zero utility $U_\theta(\varnothing) = 0$, and zero profit to the lender $\Pi_\theta(\varnothing) = 0$, rather than accepting any feasible secured contract. The threshold credit score $\widehat{\theta}$, below which no financing offered, is decreasing in asset value and the collateral value (v and ϑ).

To simplify our formal analysis, without loss of generality, we assume throughout the remainder of the article that the lowest type is creditworthy; that is, $\underline{\theta} > \widehat{\theta}$.

We conclude that under full information, both types of borrowers optimally finance the asset purchase at $t = 1$ with a standard loan x_θ°. Therefore, expanding the contract space to include SCC contracts does not improve borrower utility under full information.

2.3 Equilibrium Concept under Asymmetric Information

We apply the revelation principle to analyze the screening problem under asymmetric information. The key insight of the revelation principle is that it is sufficient to consider a single contract for each type, ensuring that each type has an incentive to select the contract designated for them.

Consider the following definition of equilibrium under asymmetric information.

Definition 1 *Given a feasible contract set* $\mathcal{X}$ *and the private types* Θ *then:*
(Separating equilibrium) A separating equilibrium is a menu of contracts $x = (x_\theta)_{\theta\in\Theta}$ *, where*

$x_\theta \in \mathcal{X}$, such that $x_\theta \neq x_{\theta'}$ for all $\theta \neq \theta'$, and the following conditions are satisfied:
(i) $U_\theta(x_\theta) \geq U_\theta(x_{\theta'})$, for all $\theta, \theta' \in \Theta$ (the incentive compatibility condition- IC) and;
(ii) $U_\theta(x_\theta) \geq 0$, for all $\theta \in \Theta$ (the individual rationality condition- IR).
(iii) $\Pi_\theta(x_\theta) \geq 0$, for all $\theta \in \Theta$ (the competitive condition -CC).
(Competitive separating equilibrium) A competitive separating equilibrium a separating equilibria in which in the lenders earn zero profits, $\Pi_\theta(x_\theta) = 0$ for all $\theta \in \Theta$.
(Pareto ranking) A separating equilibrium $x = (x_\theta)_{\theta \in \Theta}$ Pareto dominates another separating equilibrium $x' = (x'_\theta)_{\theta \in \Theta}$ if $U_\theta(x_\theta) \geq U_\theta(x'_\theta)$, for all $\theta \in \Theta$, with strict inequality holding for at least one type. A separating equilibrium is Pareto dominant if it Pareto dominates all other separating equilibrium.

The equilibrium we study involves credit policies that are informationally consistent (or separating) given the set of feasible contracts. In a competitive separating equilibrium, lenders offer a menu of contracts such that each borrower type voluntarily selects the contract designed for their type, and the lender breaks even on each contract.

Among all such equilibria, the Pareto-dominant separating equilibrium is the one that yields the highest utility for all borrower types compared to any other separating outcome. This equilibrium represents the most natural and widely adopted solution in our screening environment, consistent with Riley (1979).

Our objective is to characterize this Pareto-dominant competitive separating equilibrium. For brevity, we henceforth refer to it simply as the competitive equilibrium.

2.4 Equilibrium with Loan Contracts

We first study the equilibrium with asymmetric information when only spot loan contracts are feasible, that is $\mathcal{X} = \mathcal{L}$.

The problem is similar to the classic unidimensional screening problem (see Riley (2001) for a survey of the literature). Asymmetric information creates a distortion because the unverifiability of types implies that high type borrowers can no longer be offered a loan with a lower down payment than low type borrowers, as would be possible in a full information scenario. If this were not the case, low type borrowers would mimic high type borrowers, leading to losses for lenders. We characterize the competitive equilibrium below.

Proposition 1 *(Standard credit market separating equilibrium) In a credit market restricted to secured loans ($\mathcal{X} = \mathcal{L}$), and assuming $y_1 \geq d_h^\circ$, the unique competitive equilibrium is characterized as follows: low-type borrowers choose loan $x_l^\circ = (d_l^\circ, p_l^\circ)$, where $p_l^\circ = \bar{p}$ and $d_l^\circ = I - (\theta_l \bar{p} + (1 - \theta_l)\vartheta)$; high-type borrowers choose loan $x_h^\circ = (d_h^\circ, p_h^\circ)$ as defined in equation (4), satisfying $d_h^\circ > d_l^\circ$ and $p_h^\circ < p_l^\circ$. In equilibrium, higher-type borrowers post larger down payments (i.e., lower loan-to-value ratios) and receive lower interest rates than low-type borrowers.*

In the separating equilibrium, the low-type borrower receives their full-information utility, while the high-type borrower makes a larger down payment (i.e., lower LTV) in exchange for a lower interest rate. There are many separating equilibria, but the Pareto dominant one is uniquely defined by the following expressions for the high type's contract:

$$d_h^\circ = d_l^\circ + \frac{\theta_l \beta \left(p_l^\circ - \vartheta\right)\left(\theta_h - \theta_l\right)}{\theta_h - \beta\theta_l} \text{ and } p_h^\circ = p_l^\circ - \frac{\left(p_l^\circ - \vartheta\right)\left(\theta_h - \theta_l\right)}{\theta_h - \beta\theta_l}. \tag{4}$$

The marginal rate of substitution (MRS) quantifies how a borrower trades off a higher down payment for a reduced repayment amount, while holding utility constant. Formally, it is given by:

$$MRS_{d,p}^{\theta} = -\frac{\partial U_\theta\left(x\right)}{\partial d} / \frac{\partial U_\theta\left(x\right)}{\partial p} = -\frac{1}{\theta\beta},$$

which is the slope of the indifference utility curve of type θ in the loan payment/down payment space. When the single-crossing-property holds, the marginal rate of substitution is increasing in the type θ. Indeed, the derivative with respect to type of the $MRS_{d,p}^{\theta}$ is increasing, so higher types have flatter indifference curves.[5]

A separating equilibrium is sustained by the single-crossing property: the indifference curves of high- and low-type borrowers intersect only once, and at the crossing point, the low-type curve is steeper. This ensures incentive compatibility.

[5]The derivative with respect to type of the $MRS_{d,p}^{\theta}$ is increasing: $\frac{\partial MRS_{d,p}^{\theta}}{\partial \theta} = \frac{1}{\theta^2 \beta} > 0$.

2.5 Equilibrium with Savings-and-Credit and Loan Contracts

We now derive our main results (Proposition 2 and 3 and its corollaries), which characterizes the unique competitive equilibrium when lenders can offer long-term contracts with costly savings features in addition to loan contracts, i.e., the contracting space is $\mathcal{X} = \mathcal{C} \cup \mathcal{L}$.

In this setting, lenders screen borrower types using a multidimensional screening contract $x = (s, d, p)$. This problem is more complex than the classic unidimensional screening problem. It resembles the multidimensional screening problem studied by Matthews and Moore (1987), in which a firm, uncertain about a customer's type (high or low), optimally designs product bundles with varying quality, quantity, and price to extract higher profits from higher types (see also survey by Rochet and Stole (2003)).

In the unidimensional screening problem of the previous section, the primary tradeoff involved balancing higher down payments d against lower payments p. However, in the multidimensional setting, lenders can employ both monotonic strategies–higher s and d, and lower p– and non-monotonic strategies, such as higher s, lower d, and either lower or higher p, among other possibilities. The lack of monotonicity presents the main analytical challenge, as demonstrated by Matthews and Moore (1987), who show that the optimal solution may not be monotonic—a feature that will also arise in our setting.

We start our analysis by showing that the single-crossing-property also holds in the multidimensional contracting space $x = (s, d, p)$. The marginal rate of substitution between deposits s and down payments d, or loan payments p, defined as

$$MRS^{\theta}_{s,d} = -\frac{\partial U_{\theta}(x)}{\partial s} / \frac{\partial U_{\theta}(x)}{\partial d} \text{ and } MRS^{\theta}_{s,p} = -\frac{\partial U_{\theta}(x)}{\partial s} / \frac{\partial U_{\theta}(x)}{\partial p},$$

are both are increasing in the type θ (see proof below). This property ensures that high types are willing to make higher deposits payments than lower types in exchange of either lower down payments (d), or lower payments (p).

Lemma 2 *Consider two contracts, $x = (s, d, p)$ and $x' = (s', d', p')$, such that either (i) $s' > s$, $d' = d$, and $p' < p$; or (ii) $s' > s$, $d' < d$, and $p' = p$. Then, for any type θ, if $U_{\theta}(x') = U_{\theta}(x)$, it follows that for all higher types $\theta' > \theta$, $U_{\theta'}(x') > U_{\theta'}(x)$, and conversely, $U_{\theta'}(x') < U_{\theta'}(x)$ for all lower types $\theta' < \theta$.*

This lemma illustrates that SCC contracts allow lenders to screen borrowers in multiple ways beyond the standard mechanism of offering lower interest rates in exchange for higher down payments. One screening strategy (case i) involves substituting higher deposits for lower payments or interest rates, consistent with the separating equilibria analyzed in the previous section. Another strategy (case ii) involves substituting higher deposits for lower down payments. This latter case highlights the informational strength of early deposits: they are a more powerful signal of borrower type than down payments, particularly because deposits made earlier are more burdensome for lower types when penalties are present ($\tau > 0$).

An alternative interpretation relies on the marginal rate of substitution between deposits s and down payments d, which increases in borrower type: $\frac{\partial MRS^{\theta}_{s,d}}{\partial \theta} = \frac{\tau}{\theta^2} > 0$, with no discounting (see lemma's proof). This reinforces the notion that high-type borrowers are more inclined to adopt SCC contracts, leveraging early deposits in place of larger down payments. In doing so, they benefit from the contract's superior screening efficiency in the presence of penalties.

However, a separating equilibrium involving SCC contracts is not necessarily welfare-improving relative to one based solely on standard loan contracts. That is, there is no guarantee that SCC-based separating equilibria cannot be dominated by equilibria that rely only on traditional loan terms. In the next proposition, we characterize the optimal competitive separating equilibria and identify the conditions under which SCC contracts strictly improve agents' welfare compared to standard loan contracts.

We show below that the Pareto-dominant separating equilibrium can be determined by solving the following maximization problem:

$$\begin{aligned} \underset{x=(s,d,p)\in\mathcal{C}\cup\mathcal{L}}{\text{maximize}} \quad & U_{\theta_h}(x) \\ \text{subject to} \quad & U_{\theta_l}(x) \leq U_{\theta_l}(x_l^{\circ}) \text{ (IC)} \\ & \Pi_{\theta_h}(x) \geq 0 \text{ (CC)} \end{aligned}$$

where the high type selects $x_h^* \in \mathcal{C}\cup\mathcal{L}$, solution of the problem, while the low type chooses the same loan contract as in the full information case, $x_l^* = x_l^{\circ} \in \mathcal{L}$. Our analysis demonstrates that both the lender's profit constraint (CC) and the incentive compatibility constraints for the low type (IC) are binding, which yields a unique optimal solution. Due to an increasing marginal rate of substitution, the incentive compatibility and participation constraint for

the high type are not binding, and thus can be ignored, as $U_{\theta_h}(x_h^*) > U_{\theta_h}(x_l^*) > 0$.

Proposition 2 *(Equilibrium with Savings-and-Credit and Loan Contracts) Consider a credit market where borrowers can choose from both SCC and standard loan contracts, $\mathcal{X} = \mathcal{C} \cup \mathcal{L}$. If the combined forces of asymmetric information, impatience, and repayment penalties are sufficiently strong, i.e., $\Phi + \tau + \delta > 1$, the unique competitive equilibrium features the high-type borrower choosing an SCC contract $x_h^* = (s_h^*, d_h^*, p_h^*) \in \mathcal{C}$, as defined in equation 5, and the low-type borrower selects the full-information loan contract $x_l^* = x_l^\circ \in \mathcal{L}$. In contrast, when $\Phi + \tau + \delta \leq 1$, both types select standard loan contracts as in Proposition 1.*

SCC are more advantageous than standard loans only when there is a sufficiently high degree of information asymmetry (Φ), along with high discount rates (δ) and penalties (τ). Proposition 3 below quantifies the resulting welfare gain for high types, showing the precise increase in utility due to more efficient screening under SCC contracts.

The equilibrium down payments and loan payments under SCC contracts are explicitly given by the expressions below. These values are non-monotonic when compared to the optimal loan-only contracts characterized in the previous section. Specifically, the SCC equilibrium satisfies $s_h^* > 0$, $d_h^* < d_h^\circ$, and $p_h^* > p_h^\circ$:

$$\begin{aligned} p_h^* - p_h^\circ &= s_h^* \left[\frac{\theta_h (1-\beta) + \tau\beta (\theta_h - \theta_l)}{\beta\theta_h\theta_l (\theta_h - \beta\theta_l)} \right] > 0 \text{ and} \qquad (5) \\ d_h^* - d_h^\circ &= -s_h^* \left[\frac{\beta\theta_l\theta_h (\theta_h - \beta\theta_l) + \theta_h^2 (1-\beta) + \tau (\beta\theta_h^2 (1-\theta_l) - \beta^2\theta_l^2 (1-\theta_h))}{\beta\theta_l\theta_h (\theta_h - \beta\theta_l)} \right] < 0 \end{aligned}$$

The equilibrium loan payment p_h^* is greater than p_h°, which implies higher loan amounts $L^* = \theta_h p_h^* > L^\circ = \theta_h p_h^\circ$. The equilibrium down payments are in aggregate $s_h^* + d_h^*$, which is less than d_h°, and the aggregate down payment is decreasing with savings s_h^*, while the loan payment p_h^* is increasing. The high type obtains a loan with higher LTV with SCC contracting than with standard loans and the low type continues to choose the same spot loan.

We now characterize the welfare gains to high types when SCC contracts are allowed.

Proposition 3 *(Welfare Gains) When $\Phi + \tau + \delta > 1$, high-type borrowers attain strictly higher equilibrium utility under SCC contracts than under standard loans: $U_{\theta_h}(x_h^*) > U_{\theta_h}(x_h^\circ)$. The resulting welfare gain $\Delta U_{\theta_h}^* = U_{\theta_h}(x_h^*) - U_{\theta_h}(x_h^\circ)$ is given by the closed-form expression:*

$$\Delta U_{\theta_h}^* = s_h^* \times \frac{\delta}{1+\delta}\left(\frac{\Phi}{1-\Phi}\right)\left(\frac{\Phi+\delta+\tau-1}{\Phi+\delta}\right) > 0. \tag{6}$$

This welfare gain is strictly increasing in both the degree of information asymmetry (Φ) and the borrower's discount rate (δ).

High-type borrowers benefit from SCC contracts only when the condition $\Phi + \tau + \delta > 1$ is satisfied. This condition is more likely to hold in underdeveloped credit markets, where information asymmetries, borrowing costs, and impatience are high.

Conversely, when $\Phi + \tau + \delta < 1$, SCC contracts reduce welfare for high types relative to a separating equilibrium with only standard loan contracts: $\Delta U_{\theta_h} < 0$. This situation arises in environments with low information frictions, low discounting, or weaker penalties.

Combining these results with the fact that information asymmetry declines with credit score implies the existence of a threshold credit score θ^* such that borrowers with higher scores prefer loans, while those with lower scores may benefit from SCC contracts.

Corollary 1 *(Equilibrium Threshold Credit Scores) There exists a threshold credit score θ^* such that all borrowers with $\theta > \theta^*$ prefer loan contracts. Among borrowers with $\theta < \theta^*$, only high-type borrowers choose SCC contracts, while low types prefer loans. As a result, there is positive selection into SCC contracts below the threshold.*

The threshold θ^* is implicitly defined by the cutoff $\Phi(\theta^*) = \Phi^* = 1 - \delta - \tau$, since $\Phi(\theta)$ is a strictly decreasing function of credit score. Thus, for $\theta > \theta^*$, $\Phi(\theta) < \Phi^*$, and vice versa. Proposition 2 then implies the corollary.

We also consider how changes in the repayment penalty τ affect utility. A lower penalty increases utility for low types already enrolled in SCC contracts, since $\frac{\partial U_\theta}{\partial \tau} = -(1-\theta)\beta s < 0$. However, for high types choosing new SCC contracts, utility may fall due to adverse selection: $\frac{\partial U_{\theta_h}}{\partial \tau} = s_h^* \times \frac{\delta}{1+\delta}\left(\frac{\Phi}{1-\Phi}\right)\left(\frac{1}{\Phi+\delta}\right) > 0$.

Thus, reducing the penalty τ benefits low types more than high types, weakening SCC's power as a screening mechanism.by the condition $\Phi(\theta^*) = \Phi^* = 1 - \delta - \tau$, since $\Phi(\theta)$ is a continuously decreasing function of θ. It follows that for all $\theta > \theta^*$, we have $\Phi(\theta) < \Phi^*$, and for all for all $\theta < \theta^*$, $\Phi(\theta) > \Phi^*$. By Proposition 2, this implies the corollary regarding the relationship of the equilibrium with credit scores.

Finally, we explore the effects of exogenous changes in the penalty. Although a decrease in the penalty increases the utility of agents who have already selected the SCC contract– formally the derivative of U_θ is $\frac{\partial U_\theta}{\partial \tau} = -(1-\theta)\beta s < 0$– it reduces the utility of high type agents who are choosing a new contract, due to the worsening of adverse selection. Formally, the derivative of the high type utility is $\frac{\partial U_{\theta_h}}{\partial \tau} = s_h^* \times \frac{\delta}{1+\delta}\left(\frac{\Phi}{1-\Phi}\right)\left(\frac{1}{\Phi+\delta}\right) > 0$.

Decreasing the penalty τ improves the utility of low types more than that of high types, making SCC contracts a less effective mechanism for separating high and low types. Formally:

$$-\frac{\partial U_{\theta_l}(s,d,p)}{\partial \tau} = \beta s(1-\theta_l) > \beta s(1-\theta_h) = -\frac{\partial U_{\theta_h}(s,d,p)}{\partial \tau}.$$

Consequently, an exogenous reduction in the penalty worsens the pool of borrowers selecting into SCC contracts, as it increases the incentive for low types to mimic high types.

To further understand result, observe from equation 5 that a lower penalty τ leads to a higher down payment d_h^* and a lower loan payment p_h^*. Intuitively, when the penalty is reduced, it becomes less costly for low types to mimic high types by selecting the SCC contract. To preserve incentive compatibility, the contract must be adjusted—requiring higher up-front payments and offering lower future benefits. This adjustment, in turn, reduces the welfare of high types who are choosing new contracts under the revised screening mechanism.

This reduction in the penalty also leads to a decline in the average credit score and an increase in the default rate among borrowers selecting SCC contracts.

Corollary 2 *(Equilibrium Credit Quality and the Saving Penalty) The threshold credit score θ^* increases with the saving penalty τ. Consequently, a reduction on the saving penalty leads to a decline in the average credit score and an increase in the default rate among borrowers who choose SCC contracts.*

To prove the corollary, we simply observe that $\Phi(\theta^*(\tau)) = 1 - \delta - \tau$, where $\Phi(\theta)$ is a

continuously decreasing function of θ. It follows that $\theta^*(\tau) = \Phi^{-1}(1-\delta-\tau)$, which implies that $\theta^*(\tau)$ is increasing in τ.

The new informative signal, the early down payment, is the key feature that makes SCC contracts more effective than spot loans to screen out agents. This new signal provides lenders a new instrument to screen borrowers less costly, leading to more access to credit. Intuitively, a saving period exposes the agent to an additional period in which (costly) default can occur, which is more costly for the low type agent in expectation. Importantly, this signal is relevant to lenders because it informs them of defaults during the credit period due to the positive correlation between liquidity shocks in the saving and credit period.

In standard loan arrangements, borrowers independently save for the down payment, especially when they are liquidity constrained, and can withdraw if they encounter an adverse income shock before making the down payment, thereby disregarding important information about borrower risk. Conversely, with a mandatory saving period, borrowers must commit funds earlier and forfeit a portion if a liquidity shock occurs before securing the loan. While this requirement applies to all borrowers, good type borrowers incur lower marginal costs due to their reduced likelihood of defaulting during both the saving and credit periods.

The subsequent theoretical sections highlight several additional key properties of SCC contracts, extending beyond their role in generating positive selection.

2.6 Dynamic Screening and Penalty Profits

In this section, we demonstrate how SCC contracts dynamically screen out lower-type borrowers when multiple types select into the contract ex-ante, as would be the case in a pooling equilibrium. This mechanism leads to higher ex-post withdrawal rates and lower loan default rates. We also highlight another interesting unique feature of SCC contracts related to their dynamic screening property: lender profits can increase as borrower types worsen and may remain positive even for very low-type agents. Consequently, lenders may have little incentive to ex-ante exclude lower types from participating in the contract.

A key advantage of SCC contracts lies in their *dynamic screening* property, which naturally filters out riskier borrowers over time. Among applicants selecting SCC contracts, suppose a fraction λ are high types (θ_h) and the remaining $1-\lambda$ are lower types (θ_l). Early-stage withdrawals serve as a selection mechanism: lower-type agents are more likely to exit

the contract before the loan stage, increasing the average quality of the remaining borrower pool. This process results in a withdrawal rate that exceeds the default rate, as many riskier agents drop out before receiving financing. Over time, the pool becomes increasingly composed of high types, causing the default rate to converge to the low default probability of the highest type. Thus, SCC contracts improve credit outcomes by endogenously refining the composition of borrowers as the contract unfolds.

Proposition 4 *(Dynamic Screening Property) Suppose among the agents selecting an SCC contract, a fraction λ are high types θ_h and a fraction $(1-\lambda)$ are low types θ_l, with $\theta_h > \theta_l$. Then the equilibrium withdrawal rate exceeds the default rate due to dynamic screening: lower-type agents are more likely to withdraw early, and are thereby filtered out before receiving the loan. This selection process reduces the average risk among borrowers who proceed to the loan phase, resulting in a lower default rate than the withdrawal rate.*

Specifically, the withdrawal rate is

$$h_w = \lambda\left(1-\theta_h\right) + \left(1-\lambda\right)\left(1-\theta_l\right),$$

and the default rate–i.e., the probability of default conditional on reaching the loan stage is

$$h_d = \frac{\left(1-\theta_h\right)\lambda\theta_h + \left(1-\theta_l\right)\left(1-\lambda\right)\theta_l}{\lambda\theta_h + \left(1-\lambda\right)\theta_l}.$$

Withdrawals occur earlier during the contract than loan defaults. However, higher-type agents have a higher survival rate than lower types. As a result, the fraction of high types in the pool increases over time. After one period, the share of high types increases from λ to $\frac{\lambda\theta_h}{\lambda\theta_h+(1-\lambda)\theta_l}$, and as time progresses, most of the remaining population will consist of high types as this effect compounds. For example, during the n-th period the fraction of high types among those who remain is $\frac{\lambda\theta_h^n}{\lambda\theta_h^n+(1-\lambda)\theta_l^n}$, which converges to one as $n \to \infty$. Consequently, the default rate converges to $\left(1-\theta_h\right)$.

The next result establishes that lender profits under SCC contracts may increase as borrower types decline and are strictly positive for sufficiently low types. In contrast, the lender's profit function under standard loan contracts is always strictly increasing in borrower type.

This outcome is driven by two key forces. First, lenders earn higher penalty revenues from low-type borrowers, who are more likely to withdraw during the saving period. Second, these same low types are less likely to reach the loan stage, thereby reducing the lender's exposure to default risk.

Proposition 5 *(U-Shaped Lender Profit Function) Let $x = (s, d, p)$ denote an SCC contract with $s > 0$ and $\tau > 0$. Then the lender's profit function $\Pi_\theta(x)$ is U-shaped in the borrower's type $\theta \in (0, 1)$. Moreover, suppose that x is an equilibrium contract designed for the high type θ_h, such that $\Pi_{\theta_h}(x) = 0$ and $\frac{\partial \Pi_\theta(x)}{\partial \theta}\Big|_{\theta=\theta_h} > 0$. Then, for all types $\theta < \theta_l(x)$, the lender earns strictly positive profits where $\theta_l(x) < \theta_h$ is the smallest solution to the lenders' breakeven condition $\Pi_\theta(x) = 0$.*

To prove this result, observe that introducing a penalty τ in contracts with early deposits s gives rise to two distinct positive roots in the lender's U-shaped quadratic profit function:

$$\Pi_\theta(x) = (p - \vartheta)\theta^2 - (I - \vartheta - d - s(1 - \tau))\theta + s\tau.$$

Without the penalty τ, the profit function has at most one positive root.

By contrast, the lender's profit function under a standard loan contract $x = (d, p)$ is:

$$\Pi_\theta(x) = (p - \vartheta)\theta - (I - \vartheta - d),$$

which is strictly increasing in the borrower's type θ.

When the SCC profit function has two positive roots, it is strictly decreasing to the left of its minimum and necessarily crosses zero at some $\theta \in (0, 1)$ below this minimum. Therefore, lender profits are strictly positive for all borrower types $\theta < \theta_l(x)$.

This feature implies that lenders may have no incentive to screen out low-type borrowers, even when they are able to do so, since these borrowers remain profitable. In the next section, we show that this is not merely a theoretical possibility, but one that arises endogenously in equilibrium. We then explore its implications for contract design and selection dynamics.

2.7 Equilibrium with Behavioral Types

Gottlieb and Smetters (2021) show that behavioral agents are common in life insurance

markets, which—like savings-and-credit (SCC) contracts—require a stream of payments. Consistent with their findings, our empirical analysis also shows that behavioral agents are prevalent in SCC contracts (see empirical section).

In the remainder of this section we explore implications of extending the model to incorporate behavioral agents who underestimate their liquidity risk and mistakenly choose SCC contracts.

We extend the model to allow for a fraction ε are behavioral (or noise) types who are low types, but who underestimate their liquidity risk. While these behavioral agents face a high probability of experiencing a liquidity shock, they mistakenly believe their shock probability is as low as that of the high types.

Specifically, among agents with credit score θ, a fraction $1-\varepsilon$ consists of the strategic types considered earlier, and fraction ε are the behavioral types $\widehat{\theta}_l$. As before, the strategic agents can be either a high-type θ_h with probability μ, or a low-type θ_l with probability $1-\mu$.

We characterize below the competitive equilibrium with behavioral types. We show that lenders continue to offer two contracts: an SCC contract $\hat{x}_h$ and a loan contract x_l°. The strategic low type self-selects into the loan as before.

However, in this setting, both the high types and the behavioral types self-select into the SCC contract. Due to market competition, lenders earn zero expected profits on both contracts. The behavioral types mistakenly choose the SCC contract and ultimately generate positive profits for lenders through penalties incurred from early withdrawal. These profits, however, are competed away in equilibrium, leading to zero expected profits for lenders overall. As a result, high types are cross-subsidized—lenders incur losses on them. High types ultimately benefit in equilibrium, as they obtain higher utility than they would in a setting without behavioral types.

Proposition 6 *(Equilibrium with Behavioral Types) Consider a competitive credit market in which borrowers can choose between SCC and loan contracts, $\mathcal{X} = \mathcal{C} \cup \mathcal{L}$. Suppose a fraction ε of borrowers are behavioral types $\widehat{\theta}_l$, while the remaining $1-\varepsilon$ are strategic types, either θ_h or θ_l. Assume that $\Phi + \delta + \tau > 1$, and let x_h^* denote the SCC equilibrium contract for high types as defined in Proposition 2. Suppose further that $\widehat{\theta}_l < \theta_l(x_h^*)$, as defined in Proposition 5.*

Then there exists Pareto-dominating competitive separating equilibrium in which lenders offer an SCC contract $\hat{x}_h \in \mathcal{C}$ to both high-type and behavioral borrowers, and offer the loan contract $x_l^\circ \in \mathcal{L}$ to strategic low types. Borrowers self-select into these contracts. Lenders earn zero expected profits in equilibrium. However, they do not screen out the behavioral types, who generate strictly positive profits, $\Pi_{\widehat{\theta}_l}(\hat{x}_h) > 0$, and thereby cross-subsidize the high types, $\Pi_{\theta_h}(\hat{x}_h) = -\frac{\varepsilon}{(1-\varepsilon)\mu}\Pi_{\widehat{\theta}_l}(\hat{x}_h) < 0$. As a result, high-type borrowers attain strictly higher utility in this equilibrium than in the separating equilibrium without behavioral types: $U_{\theta_h}(\hat{x}_h) > U_{\theta_h}(x_h^*)$.

Due to market competition, lenders earn zero expected profits in equilibrium:

$$(1-\varepsilon)\,\mu\Pi_{\theta_h}(\hat{x}_h) + \varepsilon\Pi_{\widehat{\theta}_l}(\hat{x}_h) = 0.$$

However, they incur losses on the high types:

$$\Pi_{\theta_h}(\hat{x}_h) = -\frac{\varepsilon}{(1-\varepsilon)\,\mu}\Pi_{\widehat{\theta}_l}(\hat{x}_h) < 0,$$

and generate profits from the behavioral types, $\Pi_{\widehat{\theta}_l}(\hat{x}_h) > 0$, who are cross-subsidizing the high types. Interestingly, even if lenders were able to screen out the behavioral types, they would have no incentive to do so, as their presence sustains equilibrium through cross-subsidization.

The competitive equilibrium contract $\hat{x}_h$ is obtained from the unique solution of the maximization problem

$$\begin{array}{ll} \underset{x=(s,d,p)\in\mathcal{C}\cup\mathcal{L}}{\text{maximize}} & U_{\theta_h}(x) \\ \text{subject to} & U_{\theta_l}(x) \leq U_{\theta_l}(x_l^\circ)\ \text{(IC)} \\ & \Pi_{\theta_h}(x) \geq -\frac{\varepsilon}{(1-\varepsilon)\mu}\Pi_{\widehat{\theta}_l}(x)\ \text{(CC)} \end{array}$$

Because the binding competitive constraint (CC) is relaxed–since $\Pi_{\widehat{\theta}_l}(x) > 0$–the high type obtains higher utility. Whenever, $\Phi+\delta+\tau > 1$, we have $U_{\theta_h}(\hat{x}_h) > U_{\theta_h}(x_h^*) > U_{\hat{x}_h}(x_l^\circ)$. The behavioral types, who mistakenly underestimate their liquidity shocks to be θ_h, also select the SCC contract $\hat{x}_h$.

With SCC contracts, high-type borrowers attain strictly higher utility in equilibrium than in the equilibrium without behavioral types. In contrast, under loan contracts alone,

the presence of behavioral types reduces the utility of high-type borrowers when these types cannot be properly screened out, since low-type behavioral borrowers are unprofitable to lenders.

Our theoretical results in this subsection show that SCC contracts exhibit robust properties: they perform well even when agents make mistakes, due to the structure of penalties and the dynamic screening mechanism. Behavioral types are gradually screened out through early withdrawals, generating profits for the lender. In a competitive lending market, these profits are used to cross-subsidize high types, thereby improving their overall welfare. These features closely mirror the properties identified by Gottlieb and Smetters (2021) in the context of life insurance markets.

2.8 Moral Hazard During Saving and Credit Phases

We now extend the benchmark model with adverse selection by introducing a moral hazard component during both the saving and credit phases. At the beginning of period $t = 1$ (the saving period) and again at $t = 2$ (the credit period), the agent chooses an effort level $\varepsilon_1, \varepsilon_2 \in [1, \infty)$ that increases the probability of success from θ to $\theta\varepsilon_t$. The cost of effort is given by $c(\varepsilon_t) = \frac{\kappa}{2}(\varepsilon_t - 1)^2$.

This specification reflects the idea that higher-type agents incur lower marginal costs when increasing the probability of success through effort. We also consider an alternative specification in which the probability of success is independent of type and given by $\theta + \varepsilon_t$, with a cost of $c(\varepsilon_t) = \frac{\kappa}{2}\varepsilon_t^2$. This yields qualitatively similar results (see Appendix for details).

Additionally, we assume that during the credit phase the agent faces a default cost φ, which captures real-world penalties such as a damaged credit score, social stigma, or exclusion from future borrowing opportunities. This default cost is specific to the credit phase and does not apply during the saving period.

The utility of a type-θ agent who selects contract $x = (s, d, p)$ and chooses effort levels $(\varepsilon_1, \varepsilon_2)$ is given by:

$$\begin{aligned} U_\theta(s, d, p \mid \varepsilon_1, \varepsilon_2) = \beta^2\theta\varepsilon_1 \left[\theta\varepsilon_2(v - p) - (1 - \theta\varepsilon_2)\varphi - c(\varepsilon_2)\right] \\ + \beta \left[(1 - \theta\varepsilon_1)(1 - \tau)s - \theta\varepsilon_1 d - c(\varepsilon_1)\right] - s. \end{aligned}$$

We characterize the agent's optimal effort choices in the following proposition.

Proposition 7 *(Moral Hazard During Saving and Credit Phases) The optimal effort levels during the saving and credit phases are given by:*

$$\varepsilon_1^* = \frac{\theta}{\kappa}\left(\beta v_2^* - d - (1-\tau)s\right) \quad \textit{(saving phase)}$$

$$\varepsilon_2^* = \frac{\theta}{\kappa}\left(\varphi + v - p\right) \quad \textit{(credit phase)}$$

where the continuation value is $v_2^* = \theta\varepsilon_2^*(v-p) - (1-\theta\varepsilon_2^*)\varphi - c(\varepsilon_2^*)$.
Moreover, the optimal effort in the credit phase is strictly higher than in the saving phase, i.e., $\varepsilon_2^* > \varepsilon_1^*$. *As a result, the withdrawal rate exceeds the default rate under an SCC contract.*

A higher loan payment p increases moral hazard and lowers effort, providing a rationale for lenders to cap repayment obligations (see also Tirole (2010)).

There are two main reasons why agents exert strictly greater effort during the credit phase than during the saving phase. First, default during the credit phase carries an additional cost φ, reflecting real-world consequences such as a tarnished credit score or reduced access to future credit. No such penalty exists for early withdrawal during the saving phase—instead, the agent recovers $(1-\tau)s$, which further reduces the incentive to exert effort early on. Second, prior to receiving the loan, the agent is required to make a down payment d, increasing their financial stake in the contract and strengthening their incentive to avoid default. Together, these forces help explain why default is less likely than early withdrawal in SCCs.

3 Institutional Setting and Data

To provide empirical support for the model's predictions we explore a credit contract in Brazil – Consorcio – that exhibits features of an SCC.

3.1 Consumer Lending and Consorcios in Brazil

First, we provide a brief overview of consumer lending in Brazil and a more detailed description of consorcio contracts.

Consumer Lending in Brazil In 2019, 17 percent of the working age population (17-64) in Brazil had no checking account with a bank and 55 percent had no active credit report with a loan balance greater than 200 BRL (40 USD). The mean interest rate for car loans was around 24 percent. For marginal borrowers, the rate is substantially higher.

While there is no centralized credit scoring system such as FICO score in the U.S., banks have access to information about individuals' current and past credit information from the Central Bank, which they can use for internal credit scoring models. Due to features of the Brazilian economy and labor market, information about borrowers is often incomplete. A large fraction of Brazilian businesses operate in the informal sector and 28.5 percent of workers are employed informally (Doornik, Schoenherr, and Skrastins, 2023). This prevents individuals from providing hard information on their employment and income. As a result, information asymmetry is a major obstacle in consumer lending markets in Brazil.

Consorcios Consorcios are a financial product in which participants pool funds to save towards the purchase of durable goods. Groups are typically administered by the finance division of a manufacturer, who provides the good towards which individuals save, a bank, or a specialty finance company. The administrator is in charge of marketing the consorcio, selecting applicants, managing payments, organizing the allocation of credit, and enforcing contracts. The administrator is compensated for these services through an administrative fee levied on all participants. Active screening of applicants is virtually non-existent. In practice, anyone with a social security number in Brazil can sign up for a consorcio group.

Prospective participants are provided with several pieces of information when considering to sign up for a group. They know the identity of the administrator, the price of the good, the number of months for which the group is set to operate, and a target number of participants. Once the group is formed, all participants contribute identical pre-determined payments at regular intervals, typically monthly. The payments are adjusted for inflation and changes in the price of the underlying good. In addition, monthly payments cover the administrative fee and establish a guarantee fund that covers losses resulting from defaults of individual participants. In most groups a fraction of the payments is used to insure the good against damage to preserve its value as collateral. All participants are required to continue their monthly contributions even after receiving credit from the group. Eventually, every participant that does not default during the savings period receives credit. Due to the organization of the group through a central and independent administrator and enforcement

through physical collateral, personal relations between consorcio participants are uncommon and participants of the same group are not known to each other.

Credit Allocation All participants in a consorcio group start out as savers making equal monthly contributions to the group. Every month some participants receive credit from the group which they are required to repay through their monthly contributions. The decision which members receive credit in a given month is based on two mechanisms: lotteries and auctions. The relative number of allocations by lottery and auctions varies by group. By law, each period at least one good has to be allocated through a lottery.

Lotteries are based on the outcome of the national lottery (Loteria Federal), which is broadcast on TV. Each participant receives a ticket number at the start of the group. Based on an algorithm, which is known to all participants, the national lottery number is translated into a ticket number and the participant holding the respective ticket number is declared the winner of the lottery. Algorithms are designed such that at the beginning of the group each participant has the same unconditional probability of winning the lottery at any point in time over the duration of the group.

In credit auctions, participants bid a fraction of the total value of the good. Rather than constituting an additional payment, bids are equivalent to a higher down payment, and future contributions are adjusted accordingly. For example, if the value of a good is \$5,000 with monthly contributions of \$100 and a participant bids 40 percent, they would pay \$2,000 immediately and would cease monthly payments 20 months early. In some groups, auctions are designed differently. For example, some auctions are capped such that bids may not exceed a fraction x of the total payments, and if several participants bid the same value, the algorithm related to the lottery determines the auction winner among the participants that bid the capped value.

Default After an individual obtains credit, the good is purchased. The good serves as physical collateral and can be seized if payments are late.[6] Participants are not allowed to sell the good to somebody else without the administrator's approval, to ensure that the good is not transferred to somebody that is a high credit risk to the group.

[6] A supporting feature sustaining consorcio groups in Brazil is the ease and speed of recovery of physical collateral in Brazil that enables administrators to recover those quickly upon default.

If a participant defaults before receiving credit, her past payments are retained by the group until they win a lottery. When a previously defaulted participant is declared the winner of a lottery, her funds are released and paid to her instead of obtaining credit to purchase the good. However, the defaulted participant receives only a fraction of her previous payments since default carries a contractually specified penalty of on average about 25 percent of the payments. Before February 2009, participants who default before receiving credit had to wait until the end of the group to have their payments minus the penalty returned.

Due to this contractual design, defaults of participants before receiving credit do not affect the required payments of other participants. However, defaults after receiving credit impose costs to the group if the liquidation value of the good is not sufficient to recover the full amount of credit that is owed to the group. This is more likely if the outstanding credit is higher, which applies to participants that default soon after receiving credit and for participants that obtained credit early in the group. The resulting losses are first covered by the guarantee fund, which is designed to be sufficiently generous to prevent a collapse of the group. If losses exceed the capacity of the guarantee fund, participants absorb the losses through higher contributions. In practice, losses from defaults virtually never exceed the capacity of the guarantee fund. At the termination date of the group, any remaining funds in the reserve fund are split equally and repaid to the participants.

Aggregate Statistics Consorcios are common in Brazil. In 2015, consorcios had 9,908,527 participants, about a third of them in groups that finance cars, which we analyze in this paper. The 3,681,235 participants in those groups are equivalent to 2.4 percent of the working age population or 7.4 percent of formally employed individuals in Brazil. 370,095 individuals, or 0.24 percent of the working age population obtained a car through a consorcio group in 2015 alone. These sales amount to about a quarter of all car sales on credit in Brazil in 2015.

The average car value across all groups is BRL 29,414 (about USD 10,000). Average monthly payments amount to about 1.8 percent of the value of the car and also cover administrative fees (11 percent of the monthly payment), and a guarantee fund (0.03 percent of the monthly payment). The average duration of a consorcio group is 68.2 months and the mean number of participants is 340 (median is 245), where the average group comprises participants from 125 different municipalities in 17 different states (out of 27). On average, less than two participants in a group are from the same municipality. The share of cars allocated through lotteries is 40 percent with the rest allocated through auctions.

24.17 percent of participants default during the savings period. An additional 6.6 percent of participants default after receiving credit, in which case the car is seized by the administrator and liquidated. If the liquidation value of the car is higher than the outstanding payments, the defaulted participant keeps the difference.

3.2 Data

The data for this paper comes from three main sources. Data on consorcios is from the Sistema de Administracao de Grupos/Cotas de Consorcio (SAG) database, which is maintained by the Banco Central do Brasil (BCB). Information on bank loans is derived from SCR (Sistema de Informações de Crédito do Banco Central), a restricted-access credit registry managed by the Central Bank of Brazil (BCB). Finally, information on labor markets outcomes is available through RAIS (Relação Anual de Informações Sociais), an employer-employee matched database that includes employment information and wages for all formally employed workers in Brazil.

The database on consorcios provides information on the administrator, all participants, the good that is being allocated, and the dates when credit is awarded to participants. The bank loan credit registry (SCR) provides information on loan characteristics (e.g., outstanding value, interest rate, issue date, maturity), issuing bank, location of the loan among other things. Our primary focus is on loans issued and consorcio groups started between 2008-02 and 2010-02, centered two years around the reform that changed the cost of defaulting in the savings period for consorcios (see Section 5).

The consorcio and bank credit registry databases provide the social security number of all individuals. This allows us to match them to the RAIS database. The RAIS database records information on all formally employed workers in a given year and is maintained by the Ministry of Labor and Employment of Brazil. All formally-registered firms in Brazil are legally required to report annual information on each worker that the firm employs. RAIS includes detailed information on the employer (tax number, sector of activity, establishment size, geographical location), the employee (social security number, age, gender, education), and the employment relationship (wage, tenure, type of employment, hiring date, layoff date, reason for layoff, etc.). By the end of 2015, the database covers about 50 million formal employees.

3.3 Consorcios as Random Multi-period SCCs

Our baseline framework features a single savings period (in addition to the down payment), followed by one credit period. However, our empirical analysis focuses on SCCs that unfold over multiple periods.

SCCs can be naturally generalized to a multi-period setting. Define a multi-period SCC by $x = (T_s, p_s; T_c, p_c|\tau)$ where p_s is the per-period deposit during the savings phase, p_c is the per-period installment during the credit phase, T_s (T_c) is the number of savings (credit) periods, and τ is the forfeiture rate applied to accumulated deposits when a borrower withdraws early.

The borrower makes an initial deposit p_s at $t = 0$. Each subsequent period $t = 1, \ldots, T_s$ she either (i) remains active with probability θ and deposits another p_s, or (ii) experiences a liquidity shock with probability $1 - \theta$ and withdraws, receiving $(1 - \tau)tp_s$ —-her past deposits net of the penalty. After T_s successful deposits the lender advances a loan of size I to purchase the asset, to be repaid over T_c periods in equal installments p_c.

The expected utility of a type-θ borrower and the corresponding expected profit for the lender are given by expressions (7) and (8) below, which generalizes expression (1) and (2).[7]

Both the borrower's and the lender's problems satisfy the usual multidimensional marginal-rate-of-substitution (MRS) conditions. Hence, we conjecture that a competitive equilibrium exists in this multi-period environment. While the algebra becomes significantly more intricate, the key qualitative features established in the single-period model—namely ex-ante mitigation of adverse selection, ex-post dynamic screening, and the presence of moral hazard—are expected to continue to hold.

RoSCAs and consorcios, which we examine in our empirical analysis, are financial products that adopt the structure of SCCs. In these arrangements, participants agree to make

[7]Given a multi-period SCC $x = (T_s, p_s; T_c, p_c|\tau)$, the borrower's expected utility and the lender's expected profit for a type θ agent are given respectively by:

$$U_\theta(x) = U_\theta(p|\tau) := (\theta\delta)^{T_s}(v - p_c)\left(\sum_{t=1}^{T_c}\theta^t\beta^t\right) - \sum_{t=1}^{T_s}\theta^{t-1}\beta^t(\theta p_s - (1-\theta)(1-\tau)tp_s) - p_s, \tag{7}$$

and

$$\Pi_\theta(x) = \Pi_\theta(p|\tau) := \theta^{T_s}\left(p_c\left(\sum_{t=1}^{T_c}\theta^t\right) - I\right) + \sum_{t=1}^{T_s}\theta^{t-1}(\theta p_s - (1-\theta)(1-\tau)tp_s) + p_s \tag{8}$$

installment payments—typically on a monthly basis—for a fixed number of periods (on average, 38 months). Unlike the deterministic SCCs discussed so far, these products incorporate a random allocation mechanism: in each period, a lottery (or bidding process) determines the winner. The selected participant is awarded the loan and transitions from the savings phase to the credit phase. This process is repeated until all participants have received the loan.

RoSCAs and consorcio can be viewed as random SCCs, in contrast to the deterministic SCCs analyzed thus far.[8] To see this formally, consider a representative consorcio in which each participant makes installment payments of p for T periods. The expected award date is $T/2$. Let π_t denote the probability that an individual wins in period $t = 1, \ldots, T$ conditional on not having won earlier: $\pi_t = \frac{1}{T+1-t}$.

Define a deterministic SCC $x^{(t)} = (t, p; T + 1 - t, p \mid \tau)$ in which the borrower makes t saving-period deposits before switching to the credit phase and then repays the loan over the remaining $T - t$ periods. The consorcio contract can be represented as a probabilistic mixture of these deterministic contracts, with weights π_t.

Besley, Coate, and Loury (1993) analyze a bidding variant in which participants may bid for early awards. With homogeneous agents, they prove that individuals are indifferent between bidding and not bidding. Hence a bidding consorcio can be interpreted as a non-bidding scheme in which early winning probabilities are adjusted to reflect awards allocated to bidders. Consequently, bidding consorcios can also be modelled as probability-weighted combinations of the deterministic contracts $x^{(t)}$.

The standard multidimensional marginal-rate-of-substitution (MRS) conditions remain valid in the random SCC framework. We therefore conjecture that a competitive equilibrium exists, and that the qualitative properties derived for deterministic SCCs—such as ex-ante mitigation of adverse selection and ex-post dynamic screening—continue to hold.

[8] Lotteries embedded in savings and credit products are common in developing countries, and a growing empirical literature finds that prize-linked savings schemes substantially increase savings rates (see Cole, Iverson, and Tufano (2022); Gertler et al. (2023)), providing a strong rationale for incorporating lotteries into the design of consorcios as well to increase demand for the product.

4 Empirical Predictions

In this section, we describe the empirical predictions of the model that we can test in the data. Based on the strengths and limitations of our institutional setting and data, we differentiate between empirical predictions that we can carefully identify and test in the data, and empirical predictions for which we can provide suggestive supportive evidence.

Positive Selection The main focus of our theory is to illustrate the mechanisms that allow SCCs to thrive in the presence of bank loans for the same product. Specifically, we show that SCCs have an edge over standard bank loans under high information asymmetry as they are able to achieve positive selection of borrowers conditional on observable characteristics.

In our empirical setting, information asymmetry varies across borrowers. As shown in Lemma 1, standard loan contracts are optimal in the case of full information. They achieve a separating equilibrium without the need for an extra savings period. Not requiring an extra savings period benefits agents due to their impatience. In addition, as shown in Proposition 2, if information asymmetry is low, loan contracts may still dominate SCCs even for high quality borrowers (especially if discount rates are low). Thus, borrowers for whom information asymmetry is low choose standard loan contracts over consorcios, irrespective of their type.

However, for high type borrowers for whom information asymmetry is high, for example borrowers working in the informal sector or with undocumented wealth and education, it is optimal to choose a SCC (see Proposition 2). That is because the marginal rate of substitution between savings and down payments is higher for the high type making SCCs a stronger screening device allowing high types to signal their type with lower overall deposits (savings plus down payment). An intuitive way of understanding this theoretical insight is that SCCs expose borrowers to an additional period during which they can default - the mandatory savings period - which is costly for borrowers, since SCCs feature a high default penalty during the savings period. Since low type borrowers are more likely to default, the expected cost of default during the savings period is higher than for high type agents. As a result, SCCs achieve a positive selection on unobservables, especially when the penalty for default during the savings period is high (Corollary 2). This implies that conditional on observable characteristics, consorcios attract relatively higher quality borrowers.

We can summarize the empirical predictions from the positive selection properties of SCCs for our setting as follows:

1. The unconditional relative default rate between bank loan and SCC borrowers depends on the relative size of the set of borrowers for which information asymmetry is low, who choose bank loans, and for which information asymmetry is high, among whom only the high type chooses the SCC contract. As a result, the prediction for the difference between the unconditional default rates for bank loans and SCC contracts is ambiguous.

2. Conditional on observable characteristics, SCCs achieve better selection, and therefore conditional on observable information default rates are lower in SCCs.

3. The higher the penalty during the savings period in an SCC, the more likely it is for higher quality borrowers to select into the SCC contract. Thus, the average quality of borrowers in SCCs is a positive function of the size of the penalty and default rates are a negative function of the penalty.

Our empirical setting is well-suited to test these implications of the model, as we can estimates differences in default rates controlling for a wide range of personal and contract characteristics. In addition, a reform that only changed the severity of the penalty during the default period allows us to sharply estimate the effect of the penalty on selection into the contract (observable characteristics) and realized withdrawal and default rates while keeping all other features of the contract constant.

Dynamic Screening A second feature that improves borrower quality in SCCs is dynamic screening. The mechanism is intuitive. Conditional on selection into the contract, relatively lower quality borrowers are more likely to withdraw during the savings period. As a result, the pool of borrowers that ultimately obtains a loan in an SCC is of higher quality than the pool of borrowers that select into the SCC (Proposition 4).

It should be noted that this type of dynamic screening also exists for bank loans. For example, in our model some agents who choose the bank loan over the SCC experience a liquidity shock before making the down payment in period $t = 1$. Since such shocks are more likely for low quality borrowers, similar to SCCs, the pool of borrowers that ultimately

obtain a standard loan is of higher quality than the pool of borrowers that chose to pursue a standard loan in the first place.

There are two differences with respect to dynamic screening between SCCs and bank loans. First, for SCCs, we can observe the initial pool and the withdrawal rate during the savings period. For bank loans, this is not possible, since individuals that intend to make a down payment for a bank loan but experience a liquidity shock during their autarchic savings period cannot be differentiated in the data from the pool of individuals that never intended to obtain a bank loan in the first place. Second, SCCs can exploit the financial windfall from withdrawals during the savings period to subsidize interest rates for remaining borrowers, which in turn allows them to be even more attractive for high quality borrowers, who are more likely to obtain a loan in SCCs and benefit from the subsidized interest rate in SCCs as implied by Proposition 5.

We can summarize the empirical predictions from the dynamic screening property of SCCs for our setting as follows:

1. Average default rates in SCCs are higher during the savings period than during the lending period as lower quality borrowers are more likely to default before obtaining a loan.

2. Withdrawal rates decline over the course of the savings period as the lowest quality borrowers are more likely to withdraw early and the pool of remaining SCC participants improves over time.

3. Similarly, default rates are lower for individuals that obtain a loan at a later point during the lifetime of an SCC as the lowest quality borrowers are more likely to default early and the pool of remaining SCC participants improves over time.

Our institutional setting allows us to provide descriptive evidence consistent with these predictions of the model. The design on consorcios with multiple payments during the savings period allows us to assess the predictions about time-series variation in borrower quality in more detail by providing more granular time-series evolution in changes to the pool of participants.

Moral Hazard The dynamic nature of moral hazard in borrowing also has implications for the empirical predictions of the model. Specifically, in SCCs default costs other than the

penalty during the savings period are higher during the credit period after agents obtained a loan relative to the savings period before agents obtain a loan for two main reasons. First, default during the credit period negatively affect agents' ability to borrow in the future by attaching a default flag to their credit report, which does not apply to withdrawals during the savings period. Second, once individuals obtain credit, the default penalty is effectively zero, as they are no longer returned accumulated savings in case of default. Third, moral hazard decreases with the total amount of deposits made by the agent over time. Together, this implies a monotonic decrease in moral hazard over time, as well as a sharp decline in moral hazard when agents make the down payment to obtain the loan, which instantly increases their total deposits by a large amount.

Given that in our empirical setting with consorcios down payments are more continuous, the third channel is somewhat muted, especially when allocation of credit is based on lotteries which do not require a discontinuous payment upon receipt of credit. Nevertheless, the risk of obtaining a bankruptcy flag in credit record implies a discontinuous decline in moral hazard at the beginning of the credit period, and the higher accumulated savings during the credit relative to the savings period imply lower moral hazard-driven defaults during the credit relative to the savings period (Proposition 7).

We can summarize the empirical predictions from moral hazard implications for our setting as follows:

1. At a given point in time (relative to the start of the savings period) default rates are lower for individuals who are in the credit phase of the SCC than for individuals who are in the savings period of the SCC.

Our data allow us to provide descriptive evidence consistent with this prediction of the model by comparing default rates for credit in SCCs and withdrawal rates of individuals before obtaining credit.

5 Empirical Analysis

In this section, we describe the analysis to provide empirical support for the predictions of the model, outlined in Section 4.

Borrower Characterisitics and Realized Default Rates One of the key prediction of our model is that consorcios achieve a positive selection of borrowers conditional on observable information. Put differently, consorcio participants are positively selected in terms of unobservable characteristics. This implies that even if consorcio participants may look worse in terms of observable charactersitics, we may see better lower default rates for credit contracts allocated to consorcio participants than for standard bank loans on the same product (although the model's prediction is ambiguous depending on the severity of information asymmetry for the average borrower). *Conditional* on controlling for observable characteristics, our model implies that we should see lower default rates for consorcios relative to standard bank loans.

To assess whether the data support this prediction, we start by comparing observable characteristics for consorcio participants as well as bank borrowers that obtain an auto loan. Descriptive statistics on borrowers are reported in Panel A of Table 1. Overall, consorcio participants exhibit characteristics commonly associated with riskier borrowers relative to bank borrowers. Specifically, consorcio participants are 17 percentage points less likely to be formally employed than bank borrowers, are 2.8 percentage points more likely to be currently be in default on an existing bank loan, and 2.4 percentage points more likely to have defaulted on a bank loan in the past five years.

Panel B compares contract characteristics of consorcio participants and bank borrowers. Consorcio participants tend to finance somewhat more expensive cars than bank loans (BRL 29,414 vs. BRL 23,144, respectively), whereas the average credit amount extended to borrowers is smaller for consorcio participants (BRL 12,535 vs. BRL 17,658). Consequently, consorcios exhibit a lower LTV of about 0.4 compared with 0.8 for banks. The average maturity is shorter for consorcios with 38 months and compared with 43 months for bank loans. The implied interest rate of 16% in consorcios is lower than for bank loans, where the average rate is 24%.

About 24% of all consorcio participants withdraw during the savings period. It is not possible to compare this rate to individuals that saved for a down payment on a loan but then withdrew due to liquidity shocks or other reasons, which is akin to withdrawals during the savings period in consorcios. As a result, we have no way of interpreting the 24% withdrawal rate as high or low compared to standard bank loans. We can, however, compare realized default rates for credit allocated in consorcios and through bank loans. We observe that the realized default rate conditional on obtaining credit is lower for consorcios with 6.6% relative

to 9.0% for bank borrowers.

To make sure that the documented differences in borrower characteristics and realized default rates that we observe are not driven by differences in contract characteristics, we complement the descriptive evidence in Table 1 by estimating

$$y_i = \alpha_{mt} + \delta \cdot Consorcio_i + \epsilon_{it}, \tag{9}$$

where y_i is the outcome of interest for a contract i issued at time t. The dummy variable $Consorcio_i$ takes the value of one for credit issued by a consorcio and zero for bank loans. We compare borrowers of bank and consorcio contracts signed within the same municipality and the same month by adding municipality-time fixed effects (α_{mt}).

We start by confirming that the documented differences in borrower charateristics are not driven by differences in contract characteristics attracting different sets of borrowers. The results are reported in Table 2. In column I, we find that consorcio participants are 17 percentage points less likely to be formally employed. The magnitude of the difference in formal employment rates is similar after controlling for contract characteristics, such as total credit amount, maturity, LTV deciles, and comparing loans for cars of similar value in columns II to IV. Controlling for the same contract characteristics, we find that consorcio participants are 1.9 percentage points more likely to be currently in default on a bank loan and 1.3 percentage points more likely to have defaulted on a bank loan in the past five years in columns V and VI, respectively. In addition, we find that after controlling for differences in contract characteristics, consorcio participants have an about 12.4 percent worse credit score.

To aggregate the information on borrower characteristics, in Figure 2, we estimate a predicted probability of default based on observable borrower characteristics. We estimate our own risk model that predicts the probability of default over the next 12 months given that there is no standardized, readily available credit score system, like FICO scores in the U.S., for Brazil. We estimate the default risk model as

$$Default_{it,t+12} = \alpha + \beta \cdot X_{it} + \epsilon_{it}, \tag{10}$$

where $Default_{it,t+12}$ is a dummy variable that takes the value of 1 if individual i defaults on payments within the next twelve months, X_{it} is a vector of borrower characteristics: gender, age, employment characteristics (status, income, tenure), and credit market characteristics

(repayment history over the past five years, whether an individual is banked). Specifically, to predict default risk in month t, we estimate the model based on a random sample of 50,000 contracts from $t-12$ to $t-1$. This ensures that we use past information in predicting future performance to avoid a look-ahead bias. Finally, we split the predicted default risk measure into 100 equally spaced bins, akin to assigning credit scores based on the predicted default risk.

Consistent with the descriptive statistics on individual borrower characteristics, we find that the distribution of expected default rates for consorcio participants (orange solid line) is shifted to the right, i.e., predicted default rates are higher for consorcio participants than for bank borrowers (blue solid line). However, in contrast to the predicted patterns, we find that realized default rates on credit for consorcio participants are lower than for bank borrowers. Specifically, realized default rates of consorcio participants (green dashed line) are 2.4 percentage points lower than for bank borrowers (yellow dashed line). In particular, default rates for consorcio participants are lower for a given predicted default rate, except for the lowest predicted default rates. This is consistent with positive selection into consorcios based on unobservable characteristics. That is, for the same observable risk characteristics, consorcio participants are, in fact, safer borrowers. In addition, the flatter slope of realized default rates for consorcio participants relative to bank borrowers suggests that observable risk is less informative about realized default rates in consorcios, consistent with the SCC design in consorcios using additional information due to its unique screening properties that are not exploited in standard bank loans.

We also examine differences in realized default rates more formally, controlling for contract characteristics, in Table 3. Controlling for month-municipality fixed effects, we find that consorcio participants are 1.5 percentage points less likely to default relative to bank loans in column I. The difference in default rates becomes consistently stronger once we control for more contract characteristics, such as car values, total credit, maturity, and LTV in columns II to IV, and after controlling for observable borrower characteristics based on employment and credit risk in columns V to VII. In column VIII, we add the predicted default risk measure from our credit risk model as a control, which implies that we compare differences in realized default rates for consorcio participants and bank borrowers that are predicted to be equally likely to default based on their observable characteristics. We find that realized default rates are 4.8 percentage points lower for consorcio participants relative to bank borrowers after controlling for observable risk characteristics.

Altogether, the evidence from predicted and realized default rates supports the model's prediction that SCC design improves borrower quality conditional on observable characteristics by inducing positive selection.

Regulatory Change The second main testable prediction of our model is that positive selection into SCCs is a positive function of the expected cost of default during the savings period. To provide empirical support for this prediction of the model, we exploit a regulatory change in 2009 that altered the expected cost of default during the savings period for consorcios while keeping other features of the contract constant.

Prior to February 2009, when an individual defaults during the savings period, their savings minus a penalty are returned to them after all non-defaulted participants receive credit and the group is dissolved. This is costly for participants since they not only face a large penalty on their savings, but also have to wait for their savings to be returned, often for years. This feature was salient to participants, and consorcio participants sued administrators (mostly unsuccessfully) questioned whether withholding their savings until maturity violated existing laws (Superior Tribunal de Justiça, 2009; Júnior, 2014).

To eliminate legal uncertainty, on October 8, 2008, the federal government announced a reform that would require changes to the timing of savings being returned to defaulted consorcio participants.[9] The reform was widely discussed in the public domain.[10] As a result of the reform, groups that start after February 2009 may retain the savings of defaulted participants only until their ticket number is drawn in one of the lotteries held to determine the recipient of credit. In expectation, participants have to wait for their savings to be returned half as long after the reform compared to before the reform. This significantly reduced the expected costs of default during the savings period.[11]

To test whether the reduction in expected default costs during the savings period after the

[9]The specific law is 11,795/2008, available at `https://www.planalto.gov.br/ccivil_03/_ato2007-2010/2008/lei/l11795.htm`.

[10]For instance, Júnior (2014), Menezes (2009), Costa (2009), and dos Santos and Rebelatto (2009).

[11]A back-of-the-envelope calculation shows that the regulatory change led to a 24% decrease in the effective penalty rate. Consider a typical consorcio with a 38-month duration and a 25% withdrawal penalty. Assume a member discount rate of 2% per month, based on the average annual interest rate of 24% for car loans (see Table 1). Under the previous regulation, where the proceeds saved are returned at the end of the group, the effective penalty for a member dropping out after 12 months is calculated as $\tau = 1 - (0.75/(1.02)^{26}) = 55\%$ (i.e., after 26 months). After the regulatory change, the proceeds are returned when the member is randomly chosen (on average after 13 months), resulting in an effective penalty rate of $\tau = 1 - (0.75/(1.02)^{13}) = 42\%$. This represents a 24% decrease in the penalty.

reform affected selection into consorcios, we start by plotting withdrawal rates for consorcio participants during the savings period around the reform in Figure 4. Consistent with lower cost of default, we observe that withdrawals during the savings period sharply increase after the reform. While this evidence is consistent with worse selection after the reform, it is also conceivable that lower default costs lead to higher moral hazard during the savings period. Put differently, higher withdrawal rates after the reform may simply reflect lower expected costs of default during the savings period.

Next, we examine how the regulatory change affected observable borrower characteristics and realized default rates. In addition, we use bank loan borrowers as a control groups to account for potential unrelated changes in borrower characteristics and realized default rates in the time-series around the reform. Specifically, we estimate

$$y_i \quad = \quad \alpha_{cm} + \alpha_{mt} + \delta \cdot Consorcio_i \cdot Post_t + \epsilon_i, \tag{11}$$

where y_i is outcome of interest for contract i that is either issued by a bank or a consorcio in month t. $Consorcio_i$ is a dummy variable that takes the value one for consorcio contracts and zero for bank loans. $Post_t$ equals one for the post-reform period from February 2009 to February 2010 and zero for the pre-reform period from February 2008 to January 2009. The municipality by consorcio (α_{cm}) fixed effects absorb time invariant differences among consorcio and bank participants within a municipality, and municipality by time (α_{mt}) fixed effects ensure we compare outcomes for consorcio participants and bank borrowers for contracts issued in the same municipality and the same month. The coefficient of interest is δ, which measures the effect of reducing the cost of default during the savings period in consorcios.

The results are reported in Table 4. We find that observable characteristics of consorcio participants deteriorate relatively to bank borrowers after the reform. Specifically, consorcio participants experience a relative drop in formal employment by 3.7 percentage points (columns I), a 0.4 percentage point higher increase in the likelihood of being in default on a current bank loan (column II), a 0.4 percentage point increase of having been in default on a bank loan during the past five years (column III), and a 10.3 percent relative increase in their credit risk measure (column IV).

On examining changes in realized default rates around the reform in column V, we find that consorcio participants experience a 2.5 percentage point higher increase in default rates after the reform relative to bank borrowers. In Figure 5, we plot quarter-by-quarter estimates

of differences in realized default rates for consorcio participants and bank borrowers around the reform. While we observe parallel trends in default rates for contracts signed up to the reform, we observe a significant relative jump in realized default rates for consorcio contracts signed after the reform. This suggests that the increase in realized default rates is driven by the regulatory change rather than being a result of time-series trends in default rates for consorcio participants relative to bank borrowers.

Together, the evidence from the regulatory changes suggests that reducing the expected costs of default during the savings period leads to more adverse selection into SCCs, consistent with the predictions of the model. Higher realized default rates also suggest that the effects around the reform is not driven by changes in moral hazard during the savings period. If agents are more likely to withdraw during the savings period after the reform because default is less costly, we should not expect to see higher default rates *after* credit is allocated, since there is no variation in default cost and moral hazard after receiving credit around the reform.

Dynamic Screening and Moral Hazard Another implication of our model is that the pool of participants increases over time as lower quality borrowers are more likely to experience a liquidity shock in a given period and therefore exit the pool of prospective borrowers earlier at a higher rate. So examine this dynamic screening feature of SCCs, we plot the withdrawal rate (blue plot) and default rate (red plot) over the following 12 months conditional on the number of months that have passed since the consorcio started operating in Figure 3. For example, the values for $t = 12$ are the withdrawal and default rates over the next 12 months, respectively, for individuals that do not or do obtain credit through a credit lottery 12 months after the consorcio started operating.

We find that both withdrawal and default rates decline quite steeply in the beginning with the decline slowing down over time as predicted by the model. Intuitively, the dynamic screening effect weakens over time, since the lowest quality borrowers are more likely drop out early leading to lower heterogeneity in types of remaining participants over time reducing the strength of the dynamic screening effect.

Our model also predicts that moral hazard is higher during the savings period compared to the credit period, for example because default during the credit period leads to individuals obtaining a default flag on their credit history making access to future credit harder, whereas

this does not apply to withdrawals during the savings period. Consistent with this argument, we observe lower default rates for loans relative to withdrawal rates even though at a given point in time the lottery winner is drawn from the same pool of participants that remain savers.

6 Conclusion

In this paper, we explore a contract design – savings-and-credit contract (SCC) – that reduces information asymmetry in lending by providing the opportunity for high-quality borrowers to signal their type. The main feature generating a separating equilibrium is a mandatory savings period with a high penalty for default before providing credit. We show that an SCC design can expand access to credit and dominate classic loan contracts in the presence of information frictions. We also show that an SCC is a more efficient separating device than a single down payment in standard spot loans, since for bad borrowers committing to the savings period adds additional exposure to costly default risk.

We provide empirical support for the predictions of the model using data on consorcio participants and bank borrowers in Brazil. Comparing bank borrowers' and consorcio participants' observable characteristics in the cross section, we show that consorcio participants appear riskier than bank borrowers along a broad range of characteristics. Notwithstanding, default rates for consorcio participants are about 5 percentage points lower than for bank loans. Together, this suggests that consorcios manage to identify safer borrowers among the pool of observably riskier borrowers. To isolate the role of the penalty during the savings period, we exploit a regulatory reform that changes the expected default cost associated with the default penalty. We find that the relative quality of consorcio borrowers declines when the default penalty is lower both in terms of observable characteristics and realized default rates.

Signaling through SCCs is most valuable when borrowers face higher real interest rates or when there is more unobservable information, which often applies to financial markets in mid-income and developing countries with sparse information environments. Thus, innovation in contract design may help to allocate credit to individuals with profitable investment opportunities more effectively. Consistent with the view, features of SCC design exist in a wide range of mid-income and developing countries, such as Brazil, Cambodia, China,

Colombia, India, Mexico, Nigeria, South Africa, or Turkey. This suggests that SCCs are an effective tool to improve financial intermediation and expand credit to a broader set of otherwise underserved households.

While the literature on RoSCAs primarily emphasizes the role of social capital, we focus on the positive selection effect of a default penalty within a combined savings-and-credit contract. Our analysis suggests that the expected cost of default during the savings period contributes to positive selection into RoSCAs, helping to explain why they attract low-risk individuals (Levenson and Besley, 1996) and can offer lower interest rates (Kapoor et al., 2011), similar to our findings for Consorcios.

While we do not explore the role of SCC design as a commitment device to saving, in principle, the SCC design may generate savings incentives for individuals who struggle to commit to saving in the absence of strong incentives (Laibson, 1997; Karlan, Ratan, and Zinman, 2014; Ericson and Laibson, 2019). While in theory there should be strong demand for contracts that help solving commitment problems, little take-up has been documented in practice (Laibson, 2015) and evidence on whether such contracts can improve welfare is mixed (Allcott et al., 2021; Carrera et al., 2021). A design that provides better incentives to overcome present-bias problems, for instance, via long-term contracting (Gottlieb and Zhang, 2021), may improve take-up and welfare effects. Whether SCC design is successful at solving commitment problems and can enhance welfare for present-biased individuals is an interesting question for future research.

References

Adams, William, Liran Einav, and Jonathan Levin, 2009, Liquidity constraints and imperfect information in subprime lending, *American Economic Review* 99, 49–84.

Akerlof, George A., 1970, The market for lemons: Quality uncertainty and the market mechanism, *Quarterly Journal of Economics* 84, 488–500.

Allcott, Hunt, Joshua Kim, Dmitry Taubinsky, and Jonathan Zinman, 2021, Are High-Interest Loans Predatory? Theory and Evidence from Payday Lending, *The Review of Economic Studies* 89, 1041–1084.

Anderson, Siwan, Jean-Marie Baland, and Karl Ove Moene, 2009, Enforcement in informal saving groups, *Journal of Development Economics* 90, 14–23.

Attanasio, Orazio, Britta Augsburg, Ralph De Haas, Emla Fitzsimons, and Heike Harmgart, 2015, The impacts of microfinance: Evidence from joint-liability lending in Mongolia, *American Economic Journal: Applied Economics* 7, 90–122.

Augsburg, Britta, Ralph De Haas, Heike Harmgart, and Costas Meghir, 2015, The impacts of microcredit: Evidence from bosnia and herzegovina, *American Economic Journal: Applied Economics* 7, 183–203.

Aydin, Deniz, 2024, Forbearance vs. interest rates: Experimental tests of liquidity and strategic default triggers.

Banerjee, Abhijit, Emily Breza, Esther Duflo, and Cynthia Kinnan, 2021, Can microfinance unlock a poverty trap for some entrepreneurs?, Working Paper 26346, National Bureau of Economic Research.

Banerjee, Abhijit V., Timothy Besley, and Timothy W. Guinnane, 1994, Thy Neighbor's Keeper: The Design of a Credit Cooperative with Theory and a Test*, *The Quarterly Journal of Economics* 109, 491–515.

Banerjee, Abhijit V., Esther Duflo, Rachel Glennerster, and Cynthia Kinnan, 2015, The miracle of microfinance? Evidence from a randomized evaluation, *American Economic Journal: Applied Economics* 7, 22–53.

Bari, Faisal, Kashif Malik, Muhammad Meki, and Simon Quinn, 2024, Asset-based microfinance for microenterprises: Evidence from pakistan, *American Economic Review* 114, 534–74.

Berg, Tobias, Valentin Burg, Ana Gombović, and Manju Puri, 2019, On the Rise of FinTechs: Credit Scoring Using Digital Footprints, *The Review of Financial Studies* 33, 2845–2897.

Berg, Tobias, Andreas Fuster, and Manju Puri, 2022, Fintech lending, *Annual Review of Financial Economics* 14, 187–207.

Besanko, David, and Anjan V Thakor, 1987, Collateral and rationing: sorting equilibria in monopolistic and competitive credit markets, *International Economic Review* 671–689.

Besley, Timothy, and Stephen Coate, 1995, Group lending, repayment incentives and social collateral, *Journal of Development Economics* 46, 1 – 18.

Besley, Timothy, Stephen Coate, and Glenn Loury, 1993, The economics of rotating savings and credit associations, *American Economic Review* 83, 792–810.

Bester, Helmut, 1985, Screening vs. rationing in credit markets with imperfect information, *American Economic Review* 75, 850–855.

Brueckner, Jan K, 2000, Mortgage default with asymmetric information, *Journal of Real Estate Finance and Economics* 20, 251–274.

Burke, Jeremy, Julian Jamison, Dean Karlan, Kata Mihaly, and Jonathan Zinman, 2022, Credit Building or Credit Crumbling? A Credit Builder Loan's Effects on Consumer Behavior and Market Efficiency in the United States, *The Review of Financial Studies* 36, 1585–1620.

Cao, Jin, Emilia Garcia-Appendini, and Cedric Huylebroek, 2024, Banking on deposit relationships: Implications for hold-up problems in the loan market, Working paper.

Carrera, Mariana, Heather Royer, Mark Stehr, Justin Sydnor, and Dmitry Taubinsky, 2021, Who Chooses Commitment? Evidence and Welfare Implications, *The Review of Economic Studies* 89, 1205–1244.

Chan, Yuk-Shee, and Anjan V Thakor, 1987, Collateral and competitive equilibria with moral hazard and private information, *Journal of Finance* 42, 345–363.

Cole, Shawn, Benjamin Iverson, and Peter Tufano, 2022, Can gambling increase savings? empirical evidence on prize-linked savings accounts, *Management science* 68, 3282–3308.

Costa, Gilberto, 2009, Mudanças em consórcios começam a valer hoje, `https://noticias.uol.com.br/cotidiano/ultimas-noticias/2009/02/06/mudancas-em-consorcios-comecam-a-valer-hoje.htm` [Accessed: 2024-03-14].

Crepon, Bruno, Florencia Devoto, Esther Duflo, and William Pariente, 2015, Estimating the impact of microcredit on those who take it up: Evidence from a randomized experiement in morocco, *American Economic Journal: Applied Economics* 7, 123–150.

Deaton, Angus, 1992, *Understanding consumption* (Oxford University Press).

Doornik, Bernardus van, David Schoenherr, and Janis Skrastins, 2023, Strategic formal layoffs: Unemployment insurance and informal labor markets, *American Economic Journal: Applied Economics* 15, 292–318.

dos Santos, Rodrigo, and Merlim Rebelatto, 2009, Lei amplia consórcios mas complica desistências, `https://www.conjur.com.br/2009-fev-12/lei-consorcios-abre-possibilidades-complica-desistencia/` [Accessed: 2024-03-14].

Einav, Liran, Mark Jenkins, and Jonathan Levin, 2012, Contract pricing in consumer credit markets, *Econometrica* 80, 1387–1432.

El-Gamal, Mahmoud, Mohamed El-Komi, Dean Karlan, and Adam Osman, 2014, Bank-insured rosca for microfinance: Experimental evidence in poor egyptian villages, *Journal of Economic Behavior & Organization* 103, S56–S73.

Ericson, Keith Marzilli, and David Laibson, 2019, Chapter 1 - intertemporal choice, in B. Douglas Bernheim, Stefano DellaVigna, and David Laibson, eds., *Handbook of Behavioral Economics - Foundations and Applications 2*, volume 2 of *Handbook of Behavioral Economics: Applications and Foundations 1*, 1–67 (North-Holland).

Fuster, Andreas, Paul Goldsmith-Pinkham, Tarun Ramadorai, and Ansgar Walther, 2022, Predictably unequal? the effects of machine learning on credit markets, *The Journal of Finance* 77, 5–47.

Gertler, Paul, Brett Green, and Catherine Wolfram, 2024, Digital Collateral, *Quarterly Journal of Economics* .

Gertler, Paul, Sean Higgins, Aisling Scott, and Enrique Seira, 2023, Using lotteries to attract deposits.

Ghatak, Maitreesh, and Timothy W. Guinnane, 1999, The economics of lending with joint liability: theory and practice1this version: May 1999.1, *Journal of Development Economics* 60, 195–228.

Gottlieb, Daniel, and Kent Smetters, 2021, Lapse-based insurance, *American Economic Review* 111, 2377–2416.

Gottlieb, Daniel, and Xingtan Zhang, 2021, Long-term contracting with time-inconsistent agents, *Econometrica* 89, 793–824.

Gupta, Arpit, and Christopher Hansman, 2022, Selection, leverage, and default in the mortgage market, *Review of Financial Studies* 35, 720–770.

John, Kose, and Joseph Williams, 1985, Dividends, dilution, and taxes: A signalling equilibrium, *the Journal of Finance* 40, 1053–1070.

Júnior, Alberto B., 2014, Consórcio –as regras de devolução de valores e a posição dos tribunais, `https://abac.org.br/servicos/clipping/novembro2014/06_11_14/jus2.pdf` [Accessed: 2024-03-14].

Kapoor, Mudit, Antoinette Schoar, Preethi Rao, and Sharon Buteau, 2011, Chit funds as an innovative access to finance for low-income households, *Review of Market Integration* 3, 287–333.

Karlan, Dean, Aishwarya Lakshmi Ratan, and Jonathan Zinman, 2014, Savings by and for the poor: A research review and agenda, *Review of Income and Wealth* 60, 36–78.

Karlan, Dean, and Jonathan Zinman, 2011, Microcredit in theory and practice: Using randomized credit scoring for impact evaluation, *Science* 332, 1278–1284.

Klonner, Stefan, and Ashok S Rai, 2007, Adverse selection in credit markets: evidence from a policy experiment.

Laibson, David, 1997, Golden eggs and hyperbolic discounting, *The Quarterly Journal of Economics* 112, 443–477.

Laibson, David, 2015, Why don't present-biased agents make commitments?, *American Economic Review* 105, 267–72.

Leland, Hayne E, and David H Pyle, 1977, Informational asymmetries, financial structure, and financial intermediation, *The Journal of Finance* 32, 371–387.

Levenson, Alec R., and Timothy Besley, 1996, The anatomy of an informal financial market: Rosca participation in taiwan, *Journal of Development Economics* 51, 45–68, Interamerican Seminar on Macroeconomics.

Matthews, Steven, and John Moore, 1987, Monopoly provision of quality and warranties: An exploration in the theory of multidimensional screening, *Econometrica: Journal of the Econometric Society* 441–467.

Meager, Rachael, 2019, Understanding the average impact of microcredit expansions: A Bayesian hierarchical analysis of seven randomized experiments, *American Economic Journal: Applied Economics* 11, 57–91.

Meager, Rachael, 2022, Aggregating distributional treatment effects: A Bayesian hierarchical analysis of the microcredit literature, *American Economic Review* 112, 1818–1847.

Menezes, Fabiane Z., 2009, Nova lei de consórcios prevê mais segurança, `https://www.gazetadopovo.com.br/imoveis/nova-lei-de-consorcios-preve-mais-seguranca-becwz4d4c67b3unjjcmfdw0ge/` [Accessed: 2024-03-14].

Milde, Hellmuth, and John G Riley, 1988, Signaling in credit markets, *Quarterly Journal of Economics* 103, 101–129.

Morduch, Jonathan, 1999, The microfinance promise, *Journal of Economic Literature* 37, 1569–1614.

Riley, John G., 1979, Informational equilibrium, *Econometrica* 47, 331–359.

Riley, John G, 2001, Silver signals: Twenty-five years of screening and signaling.

Rochet, Jean-Charles, and Lars A Stole, 2003, The economics of multidimensional screening, *Econometric Society Monographs* 35, 150–197.

Rothschild, Michael, and Joseph Stiglitz, 1976, Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *The Quarterly Journal of Economics* 90, 629–649.

Scholten, Ulrich, 2000, Rotating savings and credit associations in developed countries: The german–austrian bausparkassen, *Journal of Comparative Economics* 28, 340–363.

Spence, Michael, 1973, Job Market Signaling, *The Quarterly Journal of Economics* 87, 355–374.

Stiglitz, Joseph E, and Andrew Weiss, 1981, Credit rationing in markets with imperfect information, *American Economic Review* 71, 393–410.

Stroebel, Johannes, 2016, Asymmetric information about collateral values, *The Journal of Finance* 71, 1071–1112.

Superior Tribunal de Justiça, 2009, Processos sobre consórcios no stj aumentaram 380% desde o ano 2000, `https://www.direitonet.com.br/noticias/exibir/11578/Processos-sobre-consorcios-no-STJ-aumentaram-380-desde-o-ano-2000` [Accessed: 2024-03-14].

Tarozzi, Alessandro, Jaikishan Desai, and Kristin Johnson, 2015, The impacts of microcredit: Evidence from ethiopia, *American Economic Journal: Applied Economics* 7, 54–89.

Tirole, Jean, 2010, *The theory of corporate finance* (Princeton university press).

van Doornik, Bernadus, Dimas Fazio, Tarun Ramadorai, and Janis Skrastins, 2024a, Housing and fertility, Working paper, Banco Central do Brasil.

van Doornik, Bernadus, David Schoenherr, and Janis Skrastins, 2025, Escaping death: Individual mobility and female mortality, Working paper, Banco Central do Brasil.

van Doornik, Bernardus, Armando Gomes, David Schoenherr, and Janis Skrastins, 2024b, Financial access and labor market outcomes: Evidence from credit lotteries, *American Economic Review* 114, 1854–81.

Zambrano, Andres Felipe, Luis Felipe Giraldo, Monica Tatiana Perdomo, Iván Darío Hernández, and Jesús María Godoy, 2023, Rotating savings and credit associations: A scoping review, *World Development Sustainability* 3, 100081.

Figure 1: **Model Timeline**

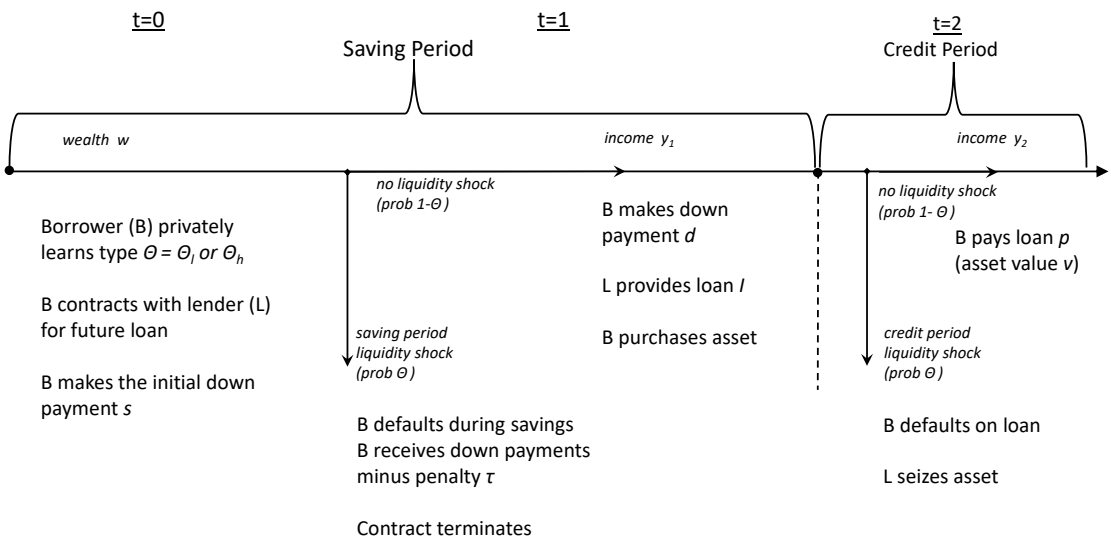
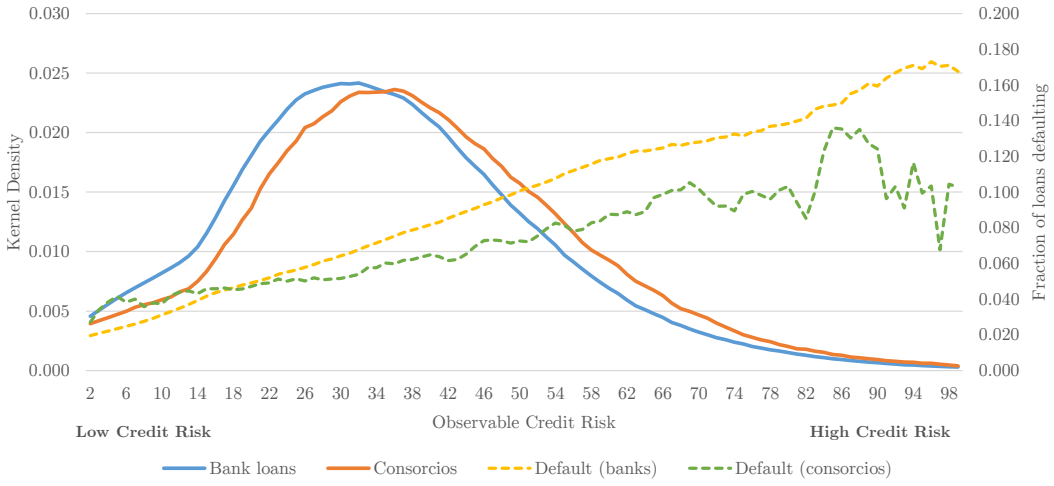
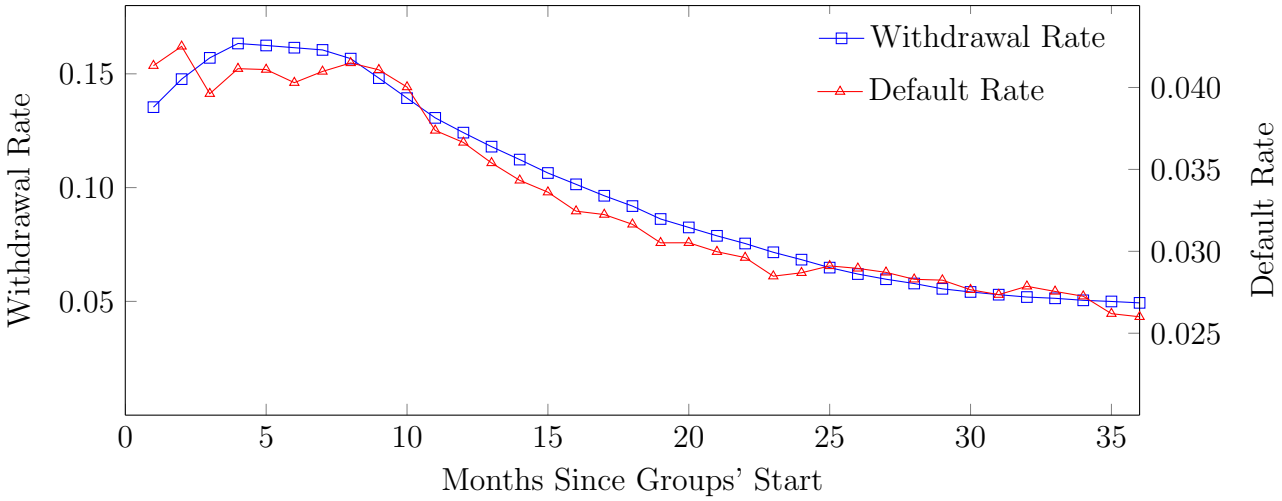


Figure 2: **Predicted and Realized Default Rates**



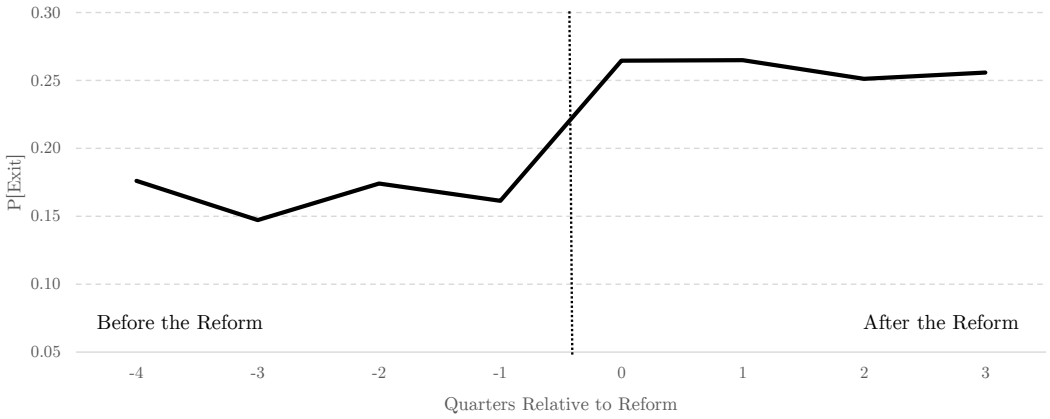
This figure depicts the kernel density plots of predicted default rate (left axis) and realized default rates (right axis) separately for bank loans (blue) and consorcio borrowers (orange) for each of the 100 equally-spaced default rate bins (x-axis). Default is predicted following the default model in equation (10). The dashed lines depict realized default rates for bank loans (yellow) and consorcio borrowers (green).

Figure 3: **Dynamic Screening**



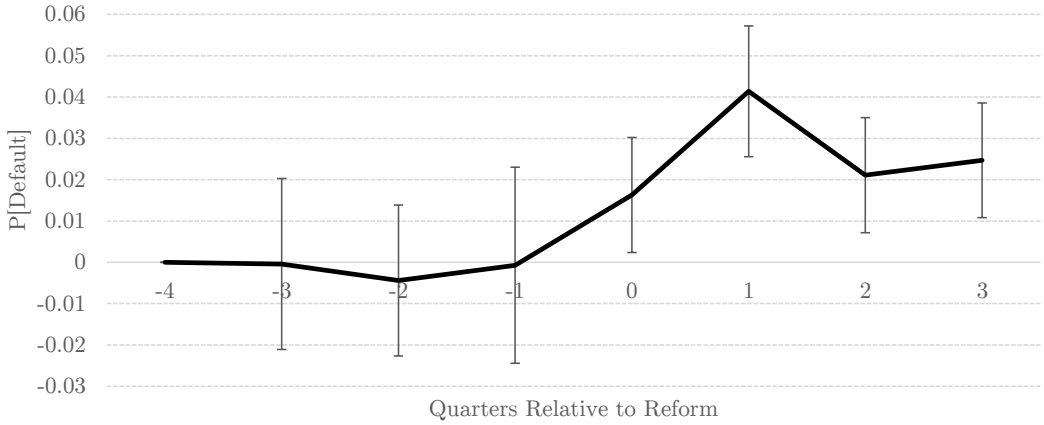
The figure plots the 12-month forward withdrawal rate (left axis) and default rate (right axis) against months since consorcio group inception. The withdrawal rate at month t is the fraction of members who are still in the savings period and have not yet received credit that withdraw over the subsequent 12 months. The default rate at month t is the fraction of members who have already received credit and are not in default that default over the subsequent 12 months.

Figure 4: **Withdrawal Rates During Savings Period Around the Reform**



The figure depicts the fraction of people who withdraw during the savings period in consorcios in a given quarter from February 2008 to February 2010. The x-axis shows the distance to the reform in quarters, where 0 is the quarter when the reform first applies (February to April 2009).

Figure 5: **Realized Default Rates Around the Reform**



This figure depicts the differences in realized default rates for consorcio borrowers relative to bank loans estimated in equation (11) with 95 percent confidence bounds and normalized to zero in the quarter before the reform. The x-axis depicts the distance to the reform in quarters, where 0 is the quarter when the reform first applies (February to April 2009).

Table 1: **Descriptive Statistics**

	Consorcios	Loans (Banks)	Difference
Panel A: Individual Characteristics			
Male	0.69	0.68	-0.01
Age	35.01	39.68	4.67
Formal employment share	0.42	0.59	0.17
Currently in default on a bank loan	0.067	0.039	-0.028
Default on a bank loan in past 5 years	0.049	0.025	-0.024
Panel B: Contractual Characteristics			
Value good (BRL)	29,414	23,144	-6,270
Loan (BRL)	12,535	17,658	5,123
LTV	0.395	0.791	0.396
Maturity	38	43	5
Interest rate (implied)	0.16	0.24	0.08
Withdrawal rate	0.24		
Default rate	0.066	0.090	0.024

This table provides descriptive statistics on individuals and credit contracts. Panels A provides descriptive statistics on individuals separately for consorcio participants and bank borrowers as well as the differences between them. Panel B provides contract characteristics and outcomes separately for consorcios participants and bank borrowers as well as the differences between them.

Table 2: **Observable Characteristics**

	I	II	III	IV	V	VI	VII
Dep. Var.:	$Formally\ Employed_i$				$Default\ Currently_i$	$Default\ Past\ 5\ yrs_i$	$Credit\ Risk_i$
$Consorcio_i$	-0.171***	-0.194***	-0.143***	-0.148***	0.019***	0.013***	0.124***
	(0.009)	(0.009)	(0.008)	(0.008)	(0.001)	(0.001)	(0.008)
Month-Municipality FE	yes	yes	yes	yes	yes	yes	yes
Month-Colat val. dec. FE				yes	yes	yes	yes
Contract-level controls							
Total credit		yes	yes	yes	yes	yes	yes
Maturity		yes	yes	yes	yes	yes	yes
Month-LTV dec			yes	yes	yes	yes	yes
Clustered SE	muni	muni	muni	muni	muni	muni	muni
Observations	3,998,993	3,956,788	3,956,788	3,194,796	3,194,796	3,194,796	2,976,424
R^2	0.049	0.096	0.133	0.106	0.034	0.030	0.083

This table shows the results from estimating equation (9) where the dependent variable is a dummy variable that equals one if individual i is formally employed and zero otherwise in columns I through IV, a dummy variable that equals one if individual i is currently in default on bank credit in column V, a dummy variable that equals one if individual i defaulted on bank credit contract within the last five years in column VI, and the credit score estimated in equation (10) in column VII. $Consorcio_i$ is a dummy variable that equals one for consorcio contracts and zero for bank loans. *Total credit* and *Maturity* are the credit volume and maturity, respectively. Standard errors are reported in parentheses. The bottom of the table provides information on fixed effects and the clustering of standard errors. *** denotes statistical significance at the 1% level.

Table 3: **Realized Default Rates**

	I	II	III	IV	V	VI	VII	VIII
Dep. Var.:	$P[Default_i]$							
$Consorcio_i$	-0.015***	-0.032***	-0.059***	-0.053***	-0.058***	-0.055***	-0.054***	-0.048***
	(0.003)	(0.004)	(0.004)	(0.004)	(0.002)	(0.004)	(0.004)	(0.002)
Month-Municipality FE	yes	yes	yes	yes	yes	yes	yes	yes
Month-Colat val. dec. FE				yes	yes	yes	yes	yes
Contract-level controls								
Total credit		yes	yes	yes	yes	yes	yes	yes
Maturity		yes	yes	yes	yes	yes	yes	yes
Month-LTV dec			yes	yes	yes	yes	yes	yes
Ex-ante controls					$Employed$	$DefCur$	$Def5yr$	$CrRisk$
Clustered SE	muni	muni	muni	muni	muni	muni	muni	muni
Observations	3,980,198	3,956,788	3,194,796	3,194,796	3,194,796	3,194,796	3,194,796	3,194,796
R^2	0.036	0.132	0.077	0.083	0.084	0.085	0.084	0.115

This table shows the results from estimating equation (9). $Consorcio_i$ is a dummy variable that equals one for consorcio contracts and zero for bank loans. *Total credit* and *Maturity* are the credit volume and maturity, respectively. *Employed* is a dummy variable that equals one if individual i is formally employed and zero otherwise, $DefCur$ is a dummy variable that equals one if individual i is currently in default on a bank credit contract, $Def5yr$ is a dummy variable that equals one if individual i defaulted on a bank credit contract in the last five years in, $CrRisk$ is the credit score estimated in equation (10). Standard errors are reported in parentheses. The bottom of the table provides information on fixed effects and the clustering of standard errors. *** denotes statistical significance at the 1% level.

Table 4: **Regulatory Constraints**

	I	II	III	IV	V
Dep. Var.:	$Formally\ Employed_i$	$Default\ Currently_i$	Default $Past\ 5\ yrs_i$	Credit $Risk_i$	$P[Default_i]$
$Consorcio_i * Post_t$	-0.037***	0.004**	0.004**	0.103***	0.025***
	(0.006)	(0.002)	(0.004)	(0.012)	(0.004)
Observations	4,018,126	4,018,126	4,018,126	3,778,051	3,999,403
R^2	0.056	0.031	0.029	0.061	0.041
Month-Municipality FE	yes	yes	yes	yes	yes
Consorcio-Muni FE	yes	yes	yes	yes	yes
Clustered SE	muni	muni	muni	muni	muni

This table shows the results from estimating equation (11) where the dependent variable is a dummy variable that equals one if individual i is formally employed and zero otherwise in columns I, a dummy variable that equals one if individual i is currently in default on bank credit in column II, a dummy variable that equals one if individual i defaulted on bank credit in the last five years in column III, the credit score estimated in equation (10) in column IV, and a dummy variable that equals one if the contract defaults and zero otherwise in column VI. $Consorcio_i$ is a dummy variable that equals one for consorcio contracts and zero for bank loans. $Post_t$ equals one for post-reform period after February 2009 and zero before. Standard errors are reported in parentheses. The bottom of the table provides information on fixed effects and the clustering of standard errors. *** and ** denote statistical significance at the 1% and 5% levels, respectively.

Appendix A. Proofs

Appendix A.1. Proof of Lemma 1

First consider the simplified problem of maximizing agents welfare, subject to the feasibility constraints and the lender's break-even constraint (CC):

$$\begin{array}{ll} \underset{s,d,p}{\text{maximize}} & U_\theta\left(s,d,p|\tau\right) \\ \text{subject to} & \Pi_\theta\left(s,d,p|\tau\right) \geq 0 \text{ (CC)} \\ & s \geq 0, s \leq w_0, p \geq 0, p \leq \bar{p} \end{array}$$

Our analysis demonstrates that the solution of this problem is the loan contract $d = \underline{d}_\theta$ and $p = \bar{p}$, and thus, expanding the contract space to include SCC does not enhance borrower utility under full information.

Consider the Lagrangian

$$\mathcal{L} = U_\theta\left(s,d,p\right) + \lambda_1 \Pi_\theta\left(s,d,p\right) - \lambda_2(p - \bar{p}) + \lambda_2 s.$$

By the Karush-Kuhn-Tucker (KKT) theorem, there exist Lagrange multipliers $\lambda_1, \lambda_2, \lambda_3 \geq 0$ for the three inequalities above so that the three FOCs hold:

$$\begin{aligned} \frac{\partial U_\theta\left(s,d,p\right)}{\partial s} + \lambda_1 \frac{\partial \Pi_\theta\left(s,d,p\right)}{\partial s} + \lambda_3 \frac{\partial s}{\partial s} &= 0 \\ \frac{\partial U_\theta\left(s,d,p\right)}{\partial d} + \lambda_1 \frac{\partial \Pi_\theta\left(s,d,p\right)}{\partial d} &= 0 \\ \frac{\partial U_\theta\left(s,d,p\right)}{\partial p} + \lambda_1 \frac{\partial \Pi_\theta\left(s,d,p\right)}{\partial p} + \lambda_2 \frac{\partial\left(-p\right)}{\partial p} &= 0 \end{aligned}$$

The complementary slackness conditions are:

$$\begin{aligned} \lambda_1 \Pi_\theta\left(s,d,p\right) &= 0 \\ \lambda_2\left(p - \bar{p}\right) &= 0 \\ \lambda_3 . s &= 0 \end{aligned}$$

The three FOCs simplify to

$$\begin{aligned}
\partial s &: (1-\theta)\beta(1-\tau) - 1 + \lambda_1(-(1-\theta)(1-\tau)+1) + \lambda_3 = 0 \\
\partial d &: -(\theta\beta) + \theta\lambda_1 = 0 \Rightarrow \lambda_1 = \beta > 0 \\
\partial p &: -(\theta\beta)^2 + \theta^2\lambda_1 - \lambda_2 = 0 \Rightarrow \lambda_2 = \theta^2\beta(1-\beta) > 0
\end{aligned}$$

Solving the first FOC yields

$$\lambda_3 = 1 - (1-\theta)\beta(1-\tau) - \beta(1-(1-\theta)(1-\tau)) = 1-\beta.$$

Thus $\lambda_1 = \beta > 0$, $\lambda_2 = \theta^2\beta(1-\beta) > 0$, and $\lambda_3 = 1-\beta > 0$, and from the three complementary slackness conditions, $s = 0$, $p = \bar{p}$, and $\Pi_\theta(s = 0, d, \bar{p}) = 0$, which implies $d = \underline{d}_\theta$. Therefore, the loan contract with $d = \underline{d}_\theta$ and $p = \bar{p}$ is the unique solution to the problem.

The maximum agent θ utility, is therefore

$$U_\theta(\underline{d}_\theta, \bar{p}) := \theta\beta(\theta\beta(v-\bar{p}) - \underline{d}_\theta)$$

Replacing the value of $\underline{d}_\theta := I - (\theta\bar{p} + (1-\theta)\vartheta)$ we get that

$$U_\theta(\underline{d}_\theta, \bar{p}) > 0 \text{ if and only } \theta > \widehat{\theta} = \frac{I-\vartheta}{(1-\beta)(\bar{p}-\vartheta) + \beta v}$$

which completes the proof.

Appendix A.2. Proof of Proposition 1

The Pareto-dominant separating equilibrium is obtained by maximizing the utility of the high type subject to the lender breaking even (CC) and the incentive compatibility condition (IC), that the low type does not prefer the high type loan compared to his full information loan, $p_l^\circ = \bar{p}$ and $d_l^\circ = I - (\theta_l\bar{p} + (1-\theta_l)\vartheta)$.

Consider the problem

$$\begin{aligned}
\underset{d,p}{\text{maximize}} \;& U_{\theta_h}(d,p) \\
\text{subject to } & \Pi_{\theta_h}(d,p) \geq 0 \text{ (CC)} \\
& U_{\theta_l}(d,p) \leq U_{\theta_l}(d_l^\circ, p_l^\circ) = \bar{U}_{\theta_l} \text{ (IC)} \\
& d \geq 0, d \leq y_1, p \geq 0, p \leq \bar{p}
\end{aligned}$$

Disregard the constraints on d and p momentarily, and consider the maximization just with constraints IC and CC. Both of these constraints must bind at the optimum. Indeed, by the KKT theorem, there exists Lagrange multipliers $\lambda_1, \lambda_2 \geq 0$ for the two inequalities above so that the two FOCs,

$$\begin{aligned}
\frac{\partial U_{\theta_h}(d,p)}{\partial d} + \lambda_1 \frac{\partial \Pi_{\theta_h}(d,p)}{\partial d} - \lambda_2 \frac{U_{\theta_l}(d,p)}{\partial d} &= 0 \\
\frac{\partial U_{\theta_h}(d,p)}{\partial p} + \lambda_1 \frac{\partial \Pi_{\theta_h}(d,p)}{\partial p} - \lambda_2 \frac{U_{\theta_l}(d,p)}{\partial p} &= 0,
\end{aligned}$$

and the two complementary slackness conditions hold:

$$\begin{aligned}
\lambda_1 . \Pi_{\theta_h}(d,p) &= 0 \\
\lambda_2 . \left(U_{\theta_l}(d,p) - \bar{U}_{\theta_l} \right) &= 0
\end{aligned}$$

The two FOCs simplify to

$$\begin{aligned}
\partial d &: -(\theta_h \beta) + \theta_h \lambda_1 + \lambda_2 (\theta_l \beta) = 0 \\
\partial p &: -(\theta_h \beta)^2 + \theta_h^2 \lambda_1 + \lambda_2 (\theta_l \beta)^2 = 0
\end{aligned}$$

which has the unique solution

$$\begin{aligned}
\lambda_1 &= \tfrac{\beta^2}{\theta_h - \beta\theta_l}(\theta_h - \theta_l) > 0 \\
\lambda_2 &= \tfrac{\theta_h^2}{\theta_l(\theta_h - \beta\theta_l)}(1-\beta) > 0
\end{aligned}$$

Thus, both inequalities bind at the optimum.

Therefore, the solution of the optimization is just obtained by solving the system of linear equations:

$$\begin{aligned}
&\Pi_{\theta_h}(d,p) = 0 \\
&U_{\theta_l}(d,p) = \bar{U}_{\theta_l} = U_{\theta_l}(d_l^\circ, p_l^\circ).
\end{aligned}$$

which yields explicitly:

$$d_h^\circ = d_l^\circ + \frac{\theta_l \beta (p_l^\circ - \vartheta)(\theta_h - \theta_l)}{\theta_h - \beta\theta_l} \text{ and } p_h^\circ = p_l^\circ - \frac{(p_l^\circ - \vartheta)(\theta_h - \theta_l)}{\theta_h - \beta\theta_l}.$$

Notice that the solution is such that $p_h^* < p_l^*$ and $d_h^* > d_l^*$, thus the constraint $p_h^* \leq \bar{p}$, also holds. Finally, the income in period $t = 1$ need to be high enough $y_1 \geq d_h^*$, for the high type to be able to separate.

Appendix A.3. Proof of Lemma 2

We first show that the single-crossing-property holds in the contracting space $x = (s, d, p)$.

The marginal rate of substitution between deposits s and down payments d or loan payments p, defined as

$$MRS_{s,d}^{\theta} = -\frac{\partial U_\theta(x)}{\partial s} / \frac{\partial U_\theta(x)}{\partial d} = -\frac{1}{\theta\beta}\left(1 - \beta(1-\theta)(1-\tau)\right),$$

and between deposits s and down payments d or loan payments p,

$$MRS_{s,p}^{\theta} = -\frac{\partial U_\theta(x)}{\partial s} / \frac{\partial U_\theta(x)}{\partial p} = -\frac{1}{\theta^2\beta^2}\left(1 - \beta(1-\theta)(1-\tau)\right),$$

because

$$\frac{\partial U_\theta(x)}{\partial s} = -\left(1 - (1-\theta)\beta(1-\tau)\right), \ \frac{\partial U_\theta(x)}{\partial d} = -\theta\beta \text{ and } \frac{\partial U_\theta(x)}{\partial p} = -\theta^2\beta^2.$$

Both MRS are increasing in the type θ because

$$\begin{aligned}
\frac{\partial MRS_{s,d}^{\theta}}{\partial \theta} &= \frac{1}{\theta^2\beta}(\tau\beta + 1 - \beta) > 0 \text{ and} \\
\frac{\partial MRS_{s,p}^{\theta}}{\partial \theta} &= \frac{1}{\theta^3\beta^2}\left(2 - \beta(2-\theta)(1-\tau)\right) > 0.
\end{aligned}$$

Let $x' = (s', d', p')$ and $x(t) = tx' + (1-t)x$, for $t \in [0, 1]$.

Suppose (i) holds: $s' > s$, $d' = d$, and $p' < p$, and that $U_\theta(x') = U_\theta(x)$. Then for all

$\theta' > \theta$ we have

$$\begin{aligned}
U_{\theta'}(x') - U_{\theta'}(x) &= \int_0^1 \frac{dU_{\theta'}(x(t))}{dt} dt = \int_0^1 \left(\frac{\partial U_{\theta'}(x(t))}{\partial s} \frac{ds}{dt} + \frac{\partial U_{\theta'}(x(t))}{\partial p} \frac{dp}{dt} \right) dt \\
&= \int_0^1 \left(\frac{\partial U_{\theta'}(x(t))}{\partial s} (s' - s) + \frac{\partial U_{\theta'}(x(t))}{\partial p} (p' - p) \right) dt \\
&= \int_0^1 \left(\left(\frac{\partial U_{\theta'}(x(t))}{\partial s} / \frac{\partial U_{\theta'}(x(t))}{\partial p} (s' - s) + (p' - p) \right) \frac{\partial U_{\theta'}(x(t))}{\partial p} \right) dt \\
&= \theta'^2 \beta^2 \int_0^1 \left(\left(MRS_{s,p}^{\theta'} (s' - s) - (p' - p) \right) \right) dt \\
&> \theta^2 \beta^2 \int_0^1 \left(\left(MRS_{s,p}^{\theta} (s' - s) - (p' - p) \right) \right) dt = \\
&= U_\theta(x') - U_\theta(x) = 0,
\end{aligned}$$

which implies $U_{\theta'}(x') > U_{\theta'}(x)$.

Suppose (ii) holds: $s' > s$, $d' < d$, and $p' = p$ and that $U_\theta(x') = U_\theta(x)$. Then for all $\theta' > \theta$ we have

$$\begin{aligned}
U_{\theta'}(x') - U_{\theta'}(x) &= \int_0^1 \frac{dU_{\theta'}(x(t))}{dt} dt = \int_0^1 \left(\frac{\partial U_{\theta'}(x(t))}{\partial s} \frac{ds}{dt} + \frac{\partial U_{\theta'}(x(t))}{\partial d} \frac{dd}{dt} \right) dt \\
&= \int_0^1 \left(\frac{\partial U_{\theta'}(x(t))}{\partial s} (s' - s) + \frac{\partial U_{\theta'}(x(t))}{\partial d} (d' - d) \right) dt \\
&= \theta' \beta \int_0^1 \left(\left(MRS_{s,d}^{\theta'} (s' - s) - (d' - p) \right) \right) dt \\
&> \theta \beta \int_0^1 \left(\left(MRS_{s,d}^{\theta} (s' - s) - (d' - p) \right) \right) dt = \\
&= U_\theta(x') - U_\theta(x) = 0,
\end{aligned}$$

which implies $U_{\theta'}(x') > U_{\theta'}(x)$ .The converse case $\theta' < \theta$ is similar.

Appendix A.4. Proof of Proposition 2

Our analysis demonstrates that the menu of contracts where the high type selects $x_h^* \in \mathcal{C} \cup \mathcal{L}$, solution of the maximization problem below, and the low type selects its full information optimum $x_l^* = x_l^\circ \in \mathcal{L}$ is the competitive equilibrium.

Let x_h^* be the solution of the maximization problem:

$$\begin{array}{ll} \underset{x=s,d,p}{\text{maximize}} & U_{\theta_h}(s,d,p) \\ \text{subject to} & \Pi_{\theta_h}(s,d,p) \geq 0 \text{ (CC)} \\ & U_{\theta_l}(s,d,p) \leq U_{\theta_l}(x_l^\circ) = \bar{U}_{\theta_l} \text{ (IC)} \\ & s \geq 0, s \leq w_0, d \geq 0, d \leq y_1, p \geq 0, p \leq \bar{p} \end{array}$$

The first constraint is the lender breakeven constraint (CC), the second is the incentive compatibility constraint (IC) to ensure the low-type does prefer the contract designated for him, and the last constraints ensure that the contract specified for the high type is feasible and belong to $\mathcal{C} \cup \mathcal{L}$.

Consider the Lagrangian $\mathcal{L}$ and disregard momentarily the constraints on d and p :

$$\mathcal{L} = U_{\theta_h}(s,d,p) + \lambda_1 \Pi_{\theta_h}(s,d,p) - \lambda_2 (U_{\theta_l}(s,d,p) - \bar{U}_{\theta_l}) + \lambda_3 s - \lambda_4 (s - w_0).$$

By KKT theorem there exist Lagrange multipliers $\lambda_i \geq 0$, $i = 1, 2, 3$, and 4, for the inequalities above so that the three FOCs hold:

$$\begin{aligned} \frac{\partial U_{\theta_h}(s,d,p)}{\partial s} + \lambda_1 \frac{\partial \Pi_{\theta_h}(s,d,p)}{\partial s} - \lambda_2 \frac{\partial U_{\theta_l}(s,d,p)}{\partial s} + \lambda_3 - \lambda_4 &= 0 \\ \frac{\partial U_{\theta_h}(s,d,p)}{\partial d} + \lambda_1 \frac{\partial \Pi_{\theta_h}(s,d,p)}{\partial d} - \lambda_2 \frac{\partial U_{\theta_l}(s,d,p)}{\partial d} &= 0 \\ \frac{\partial U_{\theta_h}(s,d,p)}{\partial p} + \lambda_1 \frac{\partial \Pi_{\theta_h}(s,d,p)}{\partial p} - \lambda_2 \frac{\partial U_{\theta_l}(s,d,p)}{\partial p} &= 0 \end{aligned}$$

and the four complementary slackness conditions hold:

$$\begin{aligned}
\lambda_1.\Pi_\theta(s,d,p) &= 0 \\
\lambda_2.\left(U_{\theta_l}(s,d,p) - U_{\theta_l}\right) &= 0 \\
\lambda_3.s &= 0 \\
\lambda_4.(s-w_0) &= 0
\end{aligned}$$

The FOCs simplify to

$$\begin{aligned}
\partial s &: (1-\theta_h)\beta(1-\tau) - 1 + \lambda_1\left(-(1-\theta_h)(1-\tau)+1\right) + \lambda_2\left(1-(1-\theta_l)\beta(1-\tau)\right) + \lambda_3 - \lambda_4 = 0 \\
\partial d &: -(\theta_h\beta) + \theta_h\lambda_1 + (\theta_l\beta)\lambda_2 = 0 \\
\partial p &: -(\theta_h\beta)^2 + \theta_h^2\lambda_1 + (\theta_l\beta)^2\lambda_2 = 0
\end{aligned}$$

Solving the equations yield

$$\lambda_1 = \tfrac{\beta^2}{\theta_h - \beta\theta_l}(\theta_h - \theta_l) > 0 \text{ and } \lambda_2 = \tfrac{\theta_h^2}{\theta_l(\theta_h - \beta\theta_l)}(1-\beta) > 0,$$

which shows that both the IC and CC constraints are binding.

Replacing the values of λ_1 and λ_2 yields

$$\lambda := \lambda_4 - \lambda_3 = \frac{1}{\theta_l(\theta_h - \beta\theta_l)}(1-\beta)(\theta_h - \theta_l)(\theta_h - \beta\theta_h - \beta\theta_l + \tau\beta\theta_h)$$

Taking into account that $\theta_l = (1-\Phi)\theta$ and $\theta_h = \theta$, and that $\beta = \frac{1}{1+\delta}$ we can simplify the value of λ :

$$\begin{aligned}
\lambda &= \frac{(1-\beta)}{(1-\Phi)\theta(\theta - \beta(1-\Phi)\theta)}(\theta - (1-\Phi)\theta)(\theta(1-\beta+\tau\beta) - \beta(1-\Phi)\theta) \\
&= \frac{\delta}{(\delta+1)}\frac{\Phi}{(1-\Phi)}\frac{(\Phi+\tau+\delta-1)}{(\Phi+\delta)}
\end{aligned}$$

(i) Consider the case where $\Phi + \tau + \delta > 1$:

Then the solution of the maximization problem has $s > 0$, and $\lambda_3 = 0$ and $\lambda_4 = \lambda > 0$.

For any given $s > 0$, let $d(s)$ and $p(s)$ be the unique solution of the system of two linear

equations $\Pi_\theta(s, d(s), p(s)) = 0$ (CC) and $U_{\theta_l}(s, d(s), p(s)) = U_{\theta_l}(x_l^\circ) = \bar{U}_{\theta_l}$ (IC):

$$\begin{aligned} p(s) &= p_h^\circ + s\left[\frac{\theta_h(1-\beta) + \tau\beta(\theta_h - \theta_l)}{\beta\theta_h\theta_l(\theta_h - \beta\theta_l)}\right] \text{ and} \\ d(s) &= d_h^\circ - s\left[\frac{\beta\theta_l\theta_h(\theta_h - \beta\theta_l) + \theta_h^2(1-\beta) + \tau(\beta\theta_h^2(1-\theta_l) - \beta^2\theta_l^2(1-\theta_h))}{\beta\theta_l\theta_h(\theta_h - \beta\theta_l)}\right]. \end{aligned}$$

where p_h° and d_h° are the solution of Proposition 1. It is easy to verify that $p(s)$ is increasing in s and $d(s)$ is decreasing in s, thus $d(s) \leq d_h^\circ \leq y_1$ and $p(s) \geq 0$.

So far we have ignored the remaining constraints $s \leq w_0$, $d \geq 0$,and $p \leq \bar{p}$ which we now take into account.

Let s_h^* is the largest value in $[0, w_0]$ such that $p(s_h^*) \leq \bar{p}$ and $d(s_h^*) \geq 0$. Note that there is a unique solution s_h^* because $p(s)$ is increasing in s and $d(s)$ is decreasing in s, and the solution of the problem is $x_h^* = (s_h^*, d_h^*, p_h^*) = (s_h^*, d(s_h^*), p(s_h^*))$.

(ii) Consider the case where $\Phi + \tau + \delta \leq 1$:

Then the solution of the maximization problem has $s = 0$, and thus $\lambda_4 = 0$ and $\lambda_3 = -\lambda \geq 0$. The solution of the maximization problem is the same as in Proposition 1 with $x_h^* = (0, d_h^\circ, p_h^\circ)$, the unique solution of the system of two linear equations $\Pi_\theta(0, d_h^\circ, p_h^\circ) = 0$ (CC) and $U_{\theta_l}(0, d_h^\circ, p_h^\circ) = U_{\theta_l}(x_l^\circ) = \bar{U}_{\theta_l}$ (IC).

Appendix A.5. Proof of Proposition 3

We showed in the previous proposition that when $\Phi + \tau + \delta > 1$ the solution is $x_h^* = (s_h^*, d_h^*, p_h^*)$.

Let $\Delta p = p_h^* - p_h^\circ$ and $\Delta d = d_h^* - d_h^\circ$, where

$$\begin{aligned} \Delta p &= s_h^*\left[\frac{\theta_h(1-\beta) + \tau\beta(\theta_h - \theta_l)}{\beta\theta_h\theta_l(\theta_h - \beta\theta_l)}\right] \text{ and} \\ \Delta d &= -s_h^*\left[\frac{\beta\theta_l\theta_h(\theta_h - \beta\theta_l) + \theta_h^2(1-\beta) + \tau(\beta\theta_h^2(1-\theta_l) - \beta^2\theta_l^2(1-\theta_h))}{\beta\theta_l\theta_h(\theta_h - \beta\theta_l)}\right]. \end{aligned}$$

The welfare gain is $\Delta U^*_{\theta_h} = U_{\theta_h}(x^*_h) - U_{\theta_h}(x^\circ_h)$, where

$$\begin{aligned} U_{\theta_h}(x^*_h) &= \theta_h\beta(\theta_h\beta(v-p^*_h)-d^*_h)+(1-\theta_h)\beta(1-\tau)s^*_h - s^*_h \text{ and} \\ U_{\theta_h}(x^*_h) &= \theta_h\beta(\theta_h\beta(v-p^\circ_h)-d^\circ_h). \end{aligned}$$

Thus

$$U_{\theta_h}(x^*_h) - U_{\theta_h}(x^\circ_h) = -\theta_h\beta(\theta_h\beta\Delta p + \Delta d) + (1-\theta_h)\beta(1-\tau)s^*_h - s^*_h$$

Subsitituting the expressions above for Δp and Δp, after simplifying, yields:

$$U_{\theta_h}(x^*_h) - U_{\theta_h}(x^\circ_h) = s^*_h \times \frac{(1-\beta)}{\theta_l(\theta_h-\beta\theta_l)}(\theta_h-\theta_l)(\theta_h-\beta\theta_h-\beta\theta_l+\tau\beta\theta_h)$$

Taking into account that $\theta_l = (1-\Phi)\theta$ and $\theta_h = \theta$, and that $\beta = \frac{1}{1+\delta}$ we can further simplify to get

$$U_{\theta_h}(x^*_h) - U_{\theta_h}(x^\circ_h) = s^*_h \times \frac{\delta}{1+\delta}\left(\frac{\Phi}{1-\Phi}\right)\left(\frac{\Phi+\delta+\tau-1}{\Phi+\delta}\right) > 0,$$

We note that an alternative shorter proof can be obtained from the fact that the lagrangian multiplier λ_4 of constraint $s \leq w_0$ is

$$\lambda_4 = \frac{\partial U_{\theta_h}}{\partial w_0} = \frac{\delta}{1+\delta}\left(\frac{\Phi}{1-\Phi}\right)\left(\frac{\Phi+\delta+\tau-1}{\Phi+\delta}\right)$$

To prove the monotonicity of the welfare gain, consider the function

$$H(\Phi,\delta) = \left(\frac{\delta}{1+\delta}\right)\left(\frac{\Phi}{1-\Phi}\right)\left(\frac{\Phi+\delta+\tau-1}{\Phi+\delta}\right)$$

Both derivatives $\frac{\partial H(\Phi,\delta)}{\partial \Phi} > 0$ and $\frac{\partial H(\Phi,\delta)}{\partial \delta} > 0$ are strictly positive. Indeed all the functions in parenthesis are increasing

$$\frac{\partial\left(\frac{\Phi}{1-\Phi}\right)}{\partial\Phi} = \frac{1}{(1-\Phi)^2} > 0 \text{ and } \frac{\partial\left(\frac{\delta}{1+\delta}\right)}{\partial\delta} = \frac{1}{(\delta+1)^2} > 0$$

and

$$\frac{\partial\left(\frac{\Phi+\delta+\tau-1}{\Phi+\delta}\right)}{\partial\delta}=\frac{\partial\left(\frac{\Phi+\delta+\tau-1}{\Phi+\delta}\right)}{\partial\Phi}=\frac{1-\tau}{(\Phi+\delta)^2}>0$$

Therefore, the welfare gain is stritcly increasing in both Φ and δ.

Appendix A.6. Proof of Proposition 4

The withdrawal rate among the fraction λ of high-type agents and the fraction $(1-\lambda)$ of low-type is given byThe implied withdrawal rate is:

$$h_w=\lambda\left(1-\theta_h\right)+(1-\lambda)\left(1-\theta_l\right).$$

After withdrawal the remaining population is $\lambda\theta_h+(1-\lambda)\,\theta_l$. Among the $\lambda\theta_h$ high types not withdrawing, $(1-\theta_h)\,\lambda\theta_h$ defaults, and amAmong low-type borrowers, a fraction $(1-\lambda)\theta_l$ continue in the contract without withdrawing. Of these, a share $(1-\theta_l)(1-\lambda)\theta_l$ ultimately default. Therefore the default rate is The implied default rate in the pooled population is:

$$h_d=\frac{\left(1-\theta_h\right)\lambda\theta_h+\left(1-\theta_l\right)(1-\lambda)\,\theta_l}{\lambda\theta_h+(1-\lambda)\,\theta_l}$$

The withdrawal rate is higher than the default rate

$$h_w>h_d$$

because

$$h_w-h_d=\lambda\left(1-\lambda\right)\frac{\left(\theta_h-\theta_l\right)^2}{\theta_l+\lambda\left(\theta_h-\theta_l\right)}>0.$$

After one period, the share of high types increases from λ to $\frac{\lambda\theta_h}{\lambda\theta_h+(1-\lambda)\theta_l}$, and as time progresses, most of the remaining population will consist of high types as this effect compounds. For example, during the n-th period the fraction of high types among those who remain is $\frac{\lambda\theta_h^n}{\lambda\theta_h^n+(1-\lambda)\theta_l^n}$, which converges to one as $n\to\infty$. Thus, over time, the population becomes dominated by high types, and the default rate converges to that of high-type borrowers, $1-\theta_h$.

Appendix A.7. Proof of Proposition 5

The lender has a U-shaped quadratic profit function

$$\Pi_\theta(x) = (p-\vartheta)\,\theta^2 - (I-\vartheta-d-s\,(1-\tau))\,\theta + s\tau$$

$p-\vartheta > 0$, and the product of the two roots is equal to $s\tau > 0$. The minimum of the profit function is

$$\theta_{\min}(x) = \frac{1}{2\,(p-\vartheta)}\,(I-\vartheta-d-s\,(1-\tau))$$

The quadratic profit function is decreasing below the minimum point $\theta_{\min}(x)$ and necessarily has a root $\theta_l(x) \in (0,1)$ below this minimum point

$$\theta_l(x) = \frac{(I-\vartheta-d-s\,(1-\tau)) - \sqrt{(I-\vartheta-d-s\,(1-\tau))^2 - 4\,(p-\vartheta)\,s\tau}}{2\,(p-\vartheta)}$$

and θ_h is the largest root with positive derivative.

Appendix A.8. Proof of Proposition 6

The behavioral types $\widehat{\theta_l}$ mistakenly make the same choices as the high type θ_h. Therefore, among the borrowers choosing a contract offered to the high type, there are $(1-\varepsilon)\mu$ strategic high types and ε behavioral types. The lender's profit in the presence of behavioral types is given by:

$$\Pi^{\varepsilon}_{\theta_h}(x) := (1-\varepsilon)\mu\Pi_{\theta_h}(x) + \varepsilon\Pi_{\widehat{\theta_l}}(x).$$

Lenders in a competitive market break even, and the Pareto-dominating competitive separating equilibrium maximizes utility for both low and high types, subject to the lender's break-even (CC) and incentive compatibility (IC) constraints. Let $\hat{x}_h$ be the solution to the following program, in which the only difference from the earlier case is that the lender's profit function is now $\Pi^{\varepsilon}_{\theta_h}(x)$:

$$\begin{array}{ll} \underset{x=(s,d,p)\in\mathcal{C}\cup\mathcal{L}}{\text{maximize}} & U_{\theta_h}(x) \\ \text{subject to} & U_{\theta_l}(x) \leq U_{\theta_l}(x_l^\circ)\ \text{(IC)} \\ & \Pi^{\varepsilon}_{\theta_h}(x) \geq 0\ \text{(CC)} \end{array}$$

The utility $U^{\varepsilon}_{\theta_h}$ attained by the high type in the presence of behavioral types is at least as high as the utility without behavioral types, i.e., $U_{\theta_h}(x^*_h)$, since the contract x^*_h from Proposition 2 satisfies both constraints. The lender's profit constraint with behavioral types is

$$(1-\varepsilon)\mu\Pi_{\theta_h}(x^*_h) + \varepsilon\Pi_{\widehat{\theta}_l}(x^*_h) = \varepsilon\Pi_{\widehat{\theta}_l}(x^*_h) > 0,$$

because $\Pi_{\widehat{\theta}_l}(x^*_h) > 0$ for all $\widehat{\theta}_l < \theta_l(x^*_h)$.

However, x^*_h is no longer an equilibrium contract because lenders make strictly positive profits:

$$\Pi^{\varepsilon}_{\theta_h}(x^*_h) = \varepsilon\Pi_{\widehat{\theta}_l}(x^*_h) > 0,$$

and can thus offer a more attractive contract $\hat{x}_h$ in equilibrium. As in Proposition 2, the equilibrium contract must satisfy the lender's break-even condition with equality, and the optimal contract delivers strictly higher utility to the high type.

Moreover, the optimal solution $\hat{x}_h$ must be an SCC contract rather than a loan contract. To see why, suppose we solve the same maximization problem above while restricting the contract space to loan contracts, i.e., $x \in \mathcal{L}$. In this case, the presence of behavioral types would reduce the high-type's utility, since:

$$\Pi^{\varepsilon}_{\theta_h}(x) = (1-\varepsilon)\mu\Pi_{\theta_h}(x) + \varepsilon\Pi_{\widehat{\theta}_l}(x) < (1-\varepsilon)\mu\Pi_{\theta_h}(x),$$

because $\Pi_{\widehat{\theta}_l}(x) < 0$ for all loan contracts given that $\widehat{\theta}_l < \theta_h$.

Given that when $\Phi + \delta + \tau > 1$, we have:

$$U_{\theta_h}(\hat{x}_h) > U_{\theta_h}(x^*_h) > U_{\theta_h}(x^\circ_h),$$

the resulting welfare gain from using SCC contracts rather than loans increases even further in the presence of behavioral types.

Appendix A.9. Proof of Proposition 7

We solve the agent's problem by backward induction.

Credit Phase ($t = 2$): At the credit stage, the agent chooses effort ε_2 to maximize:

$$\max_{\varepsilon_2} \ \theta\varepsilon_2(v-p) - (1-\theta\varepsilon_2)\varphi - c(\varepsilon_2),$$

where the cost of effort is $c(\varepsilon_2) = \frac{\kappa}{2}(\varepsilon_2 - 1)^2$. The first-order condition (FOC) is:

$$\theta(v - p + \varphi) - \kappa\varepsilon_2 = 0,$$

which yields the unique optimal solution:

$$\varepsilon_2^* = \frac{\theta}{\kappa}(v - p + \varphi).$$

Saving Phase ($t = 1$): Let v_2^* denote the continuation value from period 2:

$$v_2^* = \theta\varepsilon_2^*(v - p) - (1 - \theta\varepsilon_2^*)\varphi - c(\varepsilon_2^*).$$

The agent chooses ε_1 to maximize:

$$\max_{\varepsilon_1} \ \beta^2\theta\varepsilon_1 v_2^* + \beta\left[(1 - \theta\varepsilon_1)(1 - \tau)s - \theta\varepsilon_1 d - c(\varepsilon_1)\right] - s.$$

The corresponding FOC is:

$$\beta\theta v_2^* - \theta(1 - \tau)s - \theta d - \kappa\varepsilon_1 = 0,$$

which implies the optimal effort level:

$$\varepsilon_1^* = \frac{\theta}{\kappa}\left(\beta v_2^* - d - (1 - \tau)s\right).$$

Note that $\varepsilon_1^* > 0$, since otherwise the agent would prefer to withdraw and obtain $(1-\tau)s$ rather than continue and receive a net continuation value $\beta v_2^* - d$.

Comparing Efforts: The next result establishes that $\varepsilon_2^* > \varepsilon_1^*$. First, observe that:

$$v - p > v_2^* = \theta\varepsilon_2^*(v - p) - (1 - \theta\varepsilon_2^*)\varphi - c(\varepsilon_2^*),$$

so:

$$v - p > \beta v_2^*.$$

Since $\varphi > 0$ and $d + (1 - \tau)s > 0$, it follows that:

$$v - p + \varphi > \beta v_2^* - d - (1 - \tau)s,$$

which implies:

$$\varepsilon_2^* > \varepsilon_1^*.$$

Alternative Specification: Now consider the case in which success probability is type-independent and given by $\theta + \varepsilon_t$, with cost $c(\varepsilon_t) = \frac{\kappa}{2}\varepsilon_t^2$. The agent's utility is:

$$\begin{aligned} U_\theta(s, d, p \mid \varepsilon_1, \varepsilon_2) = &- s + \beta \left[(1 - (\theta + \varepsilon_1))(1 - \tau)s - (\theta + \varepsilon_1)d - \tfrac{1}{2}\kappa\varepsilon_1^2\right] \\ &+ \beta^2(\theta + \varepsilon_1)\left[(\theta + \varepsilon_2)(v - p) - (1 - (\theta + \varepsilon_2))\varphi - \tfrac{1}{2}\kappa\varepsilon_2^2\right]. \end{aligned}$$

A similar derivation yields:

$$\varepsilon_1^* = \frac{1}{\kappa}\left(\beta v_2^* - d - (1 - \tau)s\right), \quad \varepsilon_2^* = \frac{1}{\kappa}(v - p + \varphi),$$

where

$$v_2^* = (\theta + \varepsilon_2^*)(v - p) - (1 - (\theta + \varepsilon_2^*))\varphi - \tfrac{1}{2}\kappa(\varepsilon_2^*)^2.$$

By the same reasoning as above, it follows that $\varepsilon_2^* > \varepsilon_1^*$.