Certain vs. Uncertain Timing: Financial Markets and Pricing Implications

Confidential and Not for Distribution

Indira Puri*
Stern School of Business
New York University

May 29, 2025

Abstract

While financial research has extensively explored risk over outcomes, it has focused less on risk over the timing of payment. I study such risk both theoretically and empirically. Theoretically, I prove that certain textbook stochastic discount factor frameworks can struggle to price uncertain timing, because the underlying space does not accommodate these assets. Expanding these frameworks results in a tradeoff between arguably desirable pricing properties and flexibility in pricing uncertain timing. Despite these theoretical challenges, however, uncertain timing does appear to be priced in financial markets. Comparing dividend-free American and European call options on the same underlying, offered by the same exchange, with identical contract specifications, the short side requires higher compensation for the American than the European, over and above pricing effects of an early exercise premium. These results are not fully explained by hedging considerations, re-investment risk, liquidity effects, informed trading, or contract specifications, but are consistent with markets pricing uncertain timing. Together, these theoretical and empirical results suggest that the dedicated study of certain vs. uncertain timing may deepen pricing models and yield insights into empirical prices.

*I thank Yakov Amihud, John Campbell, Ian Dew-Becker, Steve Figlewski, Nikunj Kapadia, Peter Klibanoff, Theresa Kuchler, Andrew Lo, Sydney Ludvigson, Lasse Pedersen, Matt Richardson, Tony Saunders, Alexi Savov, Marti Subrahmanyam, Johannes Stroebel, and Jeff Wurgler for valuable feedback.

1 Introduction

Although risk aversion is now taken as a foundational building block in financial models, the notion of risk aversion was not always well-accepted. The St. Petersburg Paradox, introduced by Nicholas Bernoulli in 1713, posited a thought experiment on a gamble with infinite expected value but very high variance. The fact that many people did not find the gamble appealing was evidence in favor of risk aversion, according to Bernoulli. But not all were satisfied. For example, his contemporary the Comte de Buffon argued that the response could instead be due to small probabilities being ignored; or to the improbability of such a gamble existing in reality due to capital constraints.

Today, however, risk aversion is taken as given, and implies a preference for certainty to uncertainty. To illustrate, suppose that all uncertainty is resolved immediately, and all payments made in 10 days. Which do you prefer: \$2 for sure, or the gamble {\$1 with 50% probability and \$3 with 50% probability}? Risk aversion would say the certain outcome, \$2.

Now consider a different setting, in which money and time are flipped. Suppose that the amount of money is fixed, but there is uncertainty over the timing of payment. All uncertainty is still resolved immediately. You know you will receive \$10, but you don't know when you will receive it. Would you prefer that \$10 in two months for sure, or the gamble {50%, 1 month, 50% 3 months}?

This question clearly reflects a different kind of uncertainty - uncertainty over the time of payment rather than over the outcome. However, I show in this paper that pricing models treat these two types of risk very differently. In particular, certain straightforward textbook pricing frameworks are not sufficiently rich to price uncertain timing. An expansion of these frameworks trades off flexibility with other desirable properties. However, an empirical exploration in options markets suggests that uncertain timing should be priced. My work does not take a strong stand on the direction of the pricing; instead, I show that it is difficult to price uncertain timing using canonical asset pricing approaches in a coherent and flexible way; but market prices suggest a non-zero pricing of timing risk.

I start with a theoretical exploration, which yields five theorems on how the stochastic discount factor and other pricing frameworks treat uncertain timing. Theorems 1 - 4 cumulatively make the following point: in the utility space, lotteries are

probability measures. In the stochastic discount factor framework, assets are random variables. In making the move from probability measures to random variables, the choice of state space in the SDF framework becomes critical. Certain standard state spaces, such as consumption-based state spaces or finite state spaces, are not sufficiently rich to allow for assets with uncertain timing. This does not mean that it cannot be done, but rather that the most straightforward textbook approaches have difficulty pricing uncertain timing. Theorem 5 considers models that go beyond existing SDF or utility approaches. It shows that any utility function that meets a number of arguably desirable properties cannot consistently price a preference for certain timing. In particular, this means that flexibility in pricing uncertain timing, even outside standard models, is limited: a collection of arguably desirable pricing properties will predict a preference for uncertain timing for at least some lotteries.

In detail, Theorem 1 considers a standard SDF framework, where the outcome of an asset is its payment, and the filtration consists of past realized consumption. The theorem shows that, under this framework - regardless of choice of the SDF - it is not possible to price uncertain timing separately from its marginals on the observable space. The intuition is that the space on which assets can be defined, within this canonical framework, is not sufficiently rich to allow for asset-specific timing risk.

A natural response is to expand the state space to allow for more than realized consumption. Theorem 2 considers arbitrary expansions of the state space. It shows that there is no parsimonious (finite-state) SDF framework that can coherently price uncertain timing.

The next possibility is to get rid of either parsimony or as outcomes as payoffs. Theorems 3 and 4 show that removing either assumption is sufficient to price assets with uncertain timing. However, the ability to price does not necessarily mean that other desirable pricing properties are met. Theorem 5 shows that any utility-based SDF that falls in a wide class with arguably desirable properties such as history independence, no probability distortion, and dynamic consistency – even thinking creatively beyond existing models – cannot consistently price a preference for certain timing.

Together, these theoretical results indicate that pricing assets with uncertain timing in a clean, straightforward, and fully flexible way is theoretically challenging.

I next turn to an empirical exploration of certain vs. uncertain timing. Options markets provide an ideal setting in which to study the question in a relatively clean

way. The reason is that American options can be exercised any time up to and including expiry, while European options can be exercised only at expiry. The short side of an American is therefore exposed to timing risk that the short side of an otherwise identical European is not.

The goal then becomes to identify the pricing due to this timing risk. To do so, I utilize results from prior literature showing that the long side empirically exercises early. Merton (1973), one of the classical theorems in derivatives, shows that American and European call options with no dividends in the lifetime of the option, should be priced identically. However, subsequent research finds an empirical tendency of the long side to exercise early, for example due to idiosyncracy, lack of sophistication, or capital constraints (ex. Finucane (1997); Poteshman and Serbin (2003); Bryzgalova et al. (2024)). I take these findings as given. I then ask - given that the long side empirically exercises early, how does the short side respond to that early exercise threat? Any pricing difference on the short side over and above pricing differences induced by the long side, would be consistent with a pricing of uncertain timing.

To test for the pricing of timing risk empirically, I therefore find otherwise identical American and European options, that is, offered by the same exchange, with the same underlying and contract specifications. This is hard in general because most underlyings typically have only one option type – American or European – associated with them. However, options on the S&P 100 index have both American and European types with identical contract specifications, and which are offered by the same exchange. I restrict attention to options that have no dividends in their lifetime. I then consider the difference in ask prices over and above the difference in bid prices, and find this difference to be positive and statistically significant, consistent with a pricing of uncertain timing. While the pricing turns out to be positive - consistent with a preference for certain timing - because uncertain timing has not historically been a focus of financial research, the interesting finding is that there is a non-zero difference in pricing on the short side.

A natural question is whether there are other channels which could also produce this pricing result. I test for eight alternative plausible explanations. First, the test above uses quote data, and executed prices in trades can sometimes differ. I therefore check whether trades in these otherwise identical American and European options are priced differently, and find that the American is more expensive, consistent with the quote-based results. A second alternative explanation is that the options have

liquidity differences. I therefore control for liquidity by restricting attention only to minutes and options which have similar bid-ask spreads (measured as %) and show that the results continue to hold. In addition, any symmetric liquidity premium gets differenced out in the specification, because the object of interest is the ask difference after controlling for the bid difference across the two contract types. A third possible explanation is that the difference in pricing represents an adversarial response to informed trading on the long side. Because past work shows that small trades, particularly in index options, are uninformed (Muravyev, 2016), I test for this by quantifying the proportion of small trades in both the American and European classes, and find these to be the majority and a similar proportion across the two classes. A fourth possible explanation is that the results are driven disproportionately by options with that are in- or out- of the money; but the results appear in options that are closer to at-the-money. A fifth possible explanation is that the difference reflects re-investment risk. However, the pricing difference appears even in options with only a few days to expiry, in whose lifetime there are no FOMC meetings. A sixth possible explanation is that microstructure differences between the two options may result in differential willingness to pay. But, by construction, the index options I consider have identical settlement time, contract size, strike prices, strike price intervals, position limits, margin rules, expiration date, and expiration months, and are offered by the same exchange – that is, they have identical contract specifications, except that one is American and the other European. A seventh possible explanation is that inventory pressures can cause bids to adjust different from asks - but a more flexible specification that allows for this, continues to find a pricing difference in asks over and above what can be explained by bids. An eighth possible explanation is that dynamic hedging of American options may make them more complex for the short side; to test for this, I consider options with gamma close to zero, and find that the pricing difference on the short side continues to hold. Cumulatively, these robustness checks, and the main specification, suggest an unwillingness to be short the American over and above pricing of an early exercise premium on the long side, providing evidence consistent with an aversion to holding uncertain timing.

Finally, as a supplement to these empirical tests, I elicit willingness to pay for uncertain timing in surveys. These find a pricing of timing risk; and incentivized pricing narratives indicate that the majority of respondents price uncertainty in the time and money domains in a conceptually similar way.

The focus of the paper is on whether uncertain timing is priced within theory and in markets: any non-zero result would be interesting. The direction, though secondary from the perspective of this paper, also bears brief discussion. Either direction - or even no pricing of uncertain timing - could plausibly appear in financial markets. On the one hand, individuals who have been trained in finance may think through the term structure and have a preference for uncertain timing. For example, my informal polls of finance professors find somewhere around two-thirds preferring uncertain timing; and some models developed by finance professors appear to have become adopted over time by markets, particularly for options (MacKenzie, 2023). On the other hand, laboratory experiments on human subjects tend to show timing-risk aversion (Chesson and Viscusi, 2003; DeJarnette et al., 2020), even though animal studies find the opposite pattern of timing-risk seeking (Kacelnik and Bateson, 1996). Which effect wins out is in financial markets is therefore in question, and our priors might suggest that financial market participants, being sophisticated and trained through financial models, would prefer uncertain timing. This makes it all the more surprising that the data seem to suggest the opposite.

The paper primarily contributes to a literature on theoretical properties of the stochastic discount factor, and to a debate over whether there is a right SDF to use (Hansen and Jagannathan (1991), Wachter (2006), Verdelhan (2010), van Binsbergen (2016), van Binsbergen and Koijen (2017), Campbell (2017), Verdelhan (2018), van Binsbergen (2020), among many others). It also contributes to a literature on empirical option pricing, particularly index options and empirical behavior surrounding American options (Muravyev, 2016; Bryzgalova et al., 2024). There are a number of other strands of literature the paper also speaks to, which are comprehensively reviewed in Section 6.

The paper proceeds as follows. Section 2 derives the theoretical results on asset pricing frameworks. Section 3 shows that options markets have prices that are consistent with a pricing of uncertain timing, and tests for robustness to various alternative explanations. Section 4 elicits willingness-to-pay and pricing narratives in surveys and shows that the results are consistent with the market findings. Section 5 discussions potential implications across finance. Section 7 concludes.

2 Stochastic Discount Factors and Uncertain Timing

I start by defining the standard framework for Arrow-Debreu securities and the SDF.

Definition 1 (Arrow-Debreu Securities)**.**
From Arrow and Debreu (1954), let $S = 1, 2, .., |S|$ be a finite set of states, $T = 1, 2, ...,$ a set of time periods, $S \times T$ with generic element (s, t) the set of state-time pairs, and $a_{s,t}$ an Arrow-Debreu security that pays 1 unit of consumption in state s at time t and 0 units of consumption in all other states and times.

Definition 2 (The Stochastic Discount Factor Framework)**.**
From Duffie (2001), there exists a probability space $(\Omega, \{\mathcal{F}_t\}_{t\in T}, P)$, where Ω is a set of states, $\{\mathcal{F}_t\}$ a filtration, T is a set of times (typically the natural numbers), and P a probability measure. In this framework, assets are adapted processes: $X = (X_1, X_2, ...)$ where each X_t is a random variable measurable with respect to $\mathcal{F}_t$. Assume at time zero the existence of a stochastic discount factor $M = \{M_t\}_{t\in T}$: the price of asset X is given by $Price(X) = E_P[\sum_t M_t X_t]$.

I then make explicit one assumption that tends to be common and implicit. A standard way of describing the stochastic discount factor is to say that there is substitution across time and over states. For example, the SDF is often used interchangeably with the phrase 'intertemporal marginal rate of substitution' (Hansen and Jagannathan, 1991; Cochrane and Hansen, 1992; Lustig and Verdelhan, 2012). Implicit in this phrase is the idea that something - typically consumption - is being substituted across states and time. This is formalized in the below statement of outcomes as payoffs.

Assumption (Outcomes as Payoffs)**.** *Fix any asset A. For any state and time (s, t), the value that A takes on in (s, t) is the amount of consumption it pays in that state.*

To isolate the impact of uncertain timing, "assets" here are thought of as probabilistic consumption processes (similar to Epstein et al. (2014)) representing the entirety of consumption. We then think of the price as the time-zero consumption equivalent for the entire probabilistic stream.

Definition 3 (Assets with Uncertain Timing)**.**
Let R^+ be the set of nonnegative consumption amounts, and let $\mathcal{T} = \{1, 2, \ldots, T\}$ be

a finite set of time periods. For any $x > 0$ and any $t \in T$, define the consumption stream

$$c^t(x) \;=\; (c_1, \ldots, c_T) \quad \textit{where} \quad c_s = \begin{cases} x, & s = t, \\ 0, & s \neq t. \end{cases}$$

$\delta_{c^t(x)}$ is the Dirac measure on the stream $c^t(x)$. An asset with uncertain timing *delivering payoff x is a probability measure $(p_t)_{t\in\mathcal{T}}(x)$ over $\{\delta_{c^t(x)}\}_{t\in\mathcal{T}}$.*

Denote the family of all assets with uncertain timing by

$$A_{\text{Unc. Tim.}} = \Big\{ (p_t)_{t\in\mathcal{T}}(x) :\; x > 0,\; p_t \geq 0,\; \sum_{t\in\mathcal{T}} p_t = 1 \Big\}.$$

2.1 Pricing Uncertain Timing with Consumption-Based States

A first approach towards pricing uncertain timing is to think of states as outcome-based, where here, outcomes are consumption. This might seem like a reasonable approach because several early models such as Lucas (1978) and Mehra and Prescott (1985) implicitly use consumption-based state spaces (Cochrane, 2005); as do later canonical models such as Campbell and Cochrane (1999).

As Theorem 1 shows, however, outcome-based states are not able to price assets with uncertain timing separately from their marginals.

Assumption (Outcome-Based States)**.** *Assume that $C = \mathbb{R}^+$ is the consumption space, then $\Omega = C^T$ and the filtration relies only on realizations of consumption up to and including time t: $\mathcal{F}_t = \{A \subseteq 2^{C^T} : \forall (c_0, .., c_T), (c'_0, ..., c'_T) \in A,\; c_i = c'_i \;\forall i \leq t\}$.*

Theorem 1. *Consider any stochastic discount factor framework under the assumptions of outcomes as payoffs, and outcome-based states. It is not possible to separately price assets A and A', where these assets are defined as follows:*

- *Asset A pays out 1 unit of consumption almost surely with respect to P, but the timing of payment is uncertain: it pays out 1 unit of consumption at period t_1 with probability p_A and at period $t_2 > t_1$ with probability $1 - p_A$.*
- *Asset A' is defined as a probability measure with respect to Ω, which pays out 1 unit of consumption at period t_1 with probability p_A, and independently pays out 1 unit of consumption at period t_2 with probability $1 - p_A$.*

The intuition is that both A and A' have the same observable consumption outcomes - therefore, they cannot be priced separately. A more general intuition is that the uncertain timing asset must be defined on a richer space than what is allowed by outcome-based states. It is not well-defined with respect to the filtration because it contains information beyond the history of consumption, via its perfect negative correlation across time. The uncertain timing asset therefore needs to be projected down into the filtration to be priced. Think of a three-dimensional object being projected into a two-dimensional space. In this case, what is lost by working on the less rich space where states depend only on the history of observed consumption and the SDF therefore prices the marginals, is the perfect negative correlation between time periods inherent in the uncertain timing asset.

A natural question is how then the SDF framework can capture, for example, autocorrelation in consumption. The answer is that there is a single probability measure P, and therefore when states are outcome-based, only one probabilistic dependency on states can be captured. In particular, it is not possible to simultaneously price two dependencies on past consumption - above, A_2 denotes independence while A_1 perfect dependence. More generally, while the states here are outcome-based, what is needed for the theorem is that the set of assets with uncertain timing is defined on a richer space than what the set of observable states can accommodate. This is proven formally in Theorem 1 in Appendix A.1.1. These theorems makes explicit the critical role of the choice of state space in the SDF framework. In particular, the most straightforward textbook approaches do not have state spaces sufficiently rich to price uncertain timing separately from its marginals.

A natural reaction is to keep the SDF framework but to allow the state space to expand arbitrarily. We turn to these expansions of the state space next.

2.2 Pricing Uncertain Timing: Parsimony

We will now keep outcomes-as-payoffs but remove outcome-based states, instead allowing states to be arbitrary.

One appealing feature of the Arrow-Debreu approach is that while the set of states is finite, the cardinality of the assets that can be priced is uncountably infinite. To see this, observe that the consumption payoff of an asset in a state can be any number in $\mathbb{R}^+$; therefore the set of payoffs with even two states is $\mathbb{R}^{+2}$, whose cardinality is

uncountably infinite. But the number of state prices needed with two states is only two.

I formalize this particular appealing feature as *parsimony*: the cardinality of the set of states is finite (as in textbook introductions to the SDF, such as Campbell (2017) and Cochrane (2005)). As a result, an uncountably infinite set of assets can be priced with a finite-dimensional SDF.

Definition 4 (Parsimony)**.**
Say that a pricing framework as defined in Definition 2 is ***parsimonious*** *if the cardinality of* Ω *is finite.*

Keeping the SDF framework of Definition 2, but allowing the state space to expand arbitrarily, Theorem 2 shows that there is no parsimonious stochastic discount factor framework that allows for the coherent pricing of all assets with timing uncertainty. Coherence is a weak requirement, formalized below. It says, take any asset with uncertain timing and hold fixed all but two of its probabilities of paying in different time periods. If you change those probabilities, the price of the asset should change. The condition takes no position on whether the price increases or decreases - only that it is not the same but is responsive to changes in probabilities over timing of payment.

Definition 5 (Coherence)**.**
Let $p + x$ *be an asset with uncertain timing, whose probability of paying out in period* t *is* $p_x(t)$*. Pick* $t_1 < t_2$ *arbitrarily, where* $p_x(t_1) > 0$ *and* $p_x(t_2) > 0$*. Create a new asset with uncertain timing* p'_x *such that* $p'_x(t) = p_x(t)$ *for all* $t \neq t_1, t_2$*, and* $p'x(t_1) = p_x(t_1) + \epsilon$ *,* $p'_x(t_2) = p_x(t_2) - \epsilon$ *for some* $\epsilon < p_x(t_2)$*. Then the price of asset* p_x *should not be the same as the price of asset* p'_x*.*

Theorem 2. *There is no parsimonious stochastic discount factor framework with outcomes as payoffs that can coherently price assets with uncertain timing.*

2.3 Pricing Uncertain Timing, Ignoring Parsimony or Removing Outcomes as Payoffs

A next step is to either expand the state space in a way that ignores parsimony, or to allow assets to take on values that deviate from the amount of consumption

they pay out in a given state and time. Theorems 3 and 4 show that, with either of these restrictions removed, there now exist (possibly trivial) SDF frameworks that can coherently price all assets with uncertain timing. However, these frameworks need not have other desirable pricing properties.

Theorem 3. *There exists a parsimonious SDF framework without outcomes as payoffs that can uniquely price all assets with uncertain*

Theorem 4. *There exists a non-parsimonious stochastic discount factor framework with outcomes as payoffs that can coherently price the set of assets with uncertain timing.*

The proof of Theorem 3 is one of existence. It chooses a trivial SDF framework, e.g. one in which assets take on values that are exactly their prices, and the probability measure and SDF are meaningless. As this proof sketch suggests, existence is not the same as an interpretable pricing framework, or one with desirable properties. The proof of Theorem 4 similarly constructs an arbitrary SDF framework to meet the sole goal of being able to price uncertain timing – but without considering whether desirable pricing properties hold within that framework. We turn to one set of desirable properties next.

2.4 Pricing Uncertain Timing: Flexibility and Tradeoffs

I now turn to working with utility frameworks to understand whether they allow full flexibility in the pricing of uncertain timing. It turns out not. A consistent preference for certain timing can interfere with arguably desirable properties, even if one thinks creatively beyond standard models. I start by precisely defining the space and properties. I then prove the statement. For completeness, I also derive results on one popular non-recursive utility model and for negative shocks.

2.4.1 Space and Definitions

Per-period consumption lies in $C \subseteq \mathbb{R}^+ \cup \{0\}$ and time takes on values $T = 1, 2, ...$ The infinite dimensional space C^∞ represents deterministic consumption streams. Vector $\vec{c} \in C^\infty$ refers to receiving consumption $\bar{c}$ at each period t. Pair $(c, t) \in C \times T$ denotes receiving consumption c in period t, and zero consumption in all other periods. Let Δ be the set of simple lotteries over $C \times T$ endowed with the topology of weak

convergence. A typical element of Δ is denoted m and a generic realization of m denoted $c(\vec{m})$. We study a complete and transitive ordering $\succeq$ over Δ.

Consumption lotteries, defined as lotteries that pay out uncertain consumption at a specified date $\bar{t}$ are denoted $q^{(\bar{t})} \in \Delta$, and time lotteries, defined as lotteries that pay out certain consumption $x \in C$ at an uncertain date, are denoted $p^{(x)} \in \Delta$. Notation $\vec{\bar{c}} + (x, t)$ denotes the deterministic consumption stream of $\bar{c}$ in each period, except in period t where consumption is $\bar{c} + x$. Notation $\vec{\bar{c}} + p^{(x)}$ denotes the lottery which, with probability $p^{(x)}(t)$, gives consumption vector $\vec{\bar{c}} + (x, t)$. Unless otherwise specified, for both time and consumption lotteries, all uncertainty is resolved immediately. This assumption is made to focus on the risk aspects of the models, rather than on resolution period effects.

Definition 6 (Prefer Certain to Uncertain Timing)**.**
Say that the preference $\succeq$ prefers certain to uncertain timing if, for any $\bar{c} \in C$, $x \in \mathbb{R}$, and time lottery $p^{(x)} \in \Delta$ with $\mathbb{E}\left[p^{(x)}\right] \in C \times T$, consumption stream $\vec{\bar{c}} + (x, \mathbb{E}\left[p^{(x)}\right]) \succ \vec{\bar{c}} + p^{(x)}$. Analogously, the preference prefers uncertain to certain timing or is indifferent over timing risk if the above holds with $\prec$ or $\sim$, respectively.

In words, given any constant consumption stream, the agent prefers certain to uncertain timing if, for time lottery $p^{(x)}$, they prefer to receive consumption at its deterministic expected time rather than play the time lottery.

Definition 6 imposes restrictions on behavior only at certainty; it says nothing about mean-preserving spreads.

The restriction to constant consumption is to focus on the pure timing uncertainty aspect of the preference, rather than on deterministic consumption variability. This definition and framework generalizes ones used in the decision theory literature (Ebert, 2020) to account for shocks of any sign and non-zero baseline consumption, as is typically the case in finance.

Definition 7 (Convex Time Discounting)**.**
Say that a representation U has convex discounting in the absence of risk if, for any deterministic $t, t', t'' \in T$ with $t'' = \frac{t+t'}{2}$, $\frac{1}{2}U((x,t)) + \frac{1}{2}U((x,t')) > U((x,t''))$.

The next set of definitions, regarding recursive preferences, are standard (Chew and Epstein (1990)).

Definition 8 (Utility Certainty Equivalent)**.**
Say that function $I : M([0,1]) \to [0,1]$ is a utility certainty equivalent operator if it is continuous, increasing with respect to first-order stochastic dominance, and has $I(x) = x$ for every $x \in \mathbb{R}_+$.

Definition 9 (Image Measure)**.**
Given a function $U : \Delta \to [0,1]$ and a probability measure $m \in \Delta$, define the image measure $m \cdot U^{-1} \in M([0,1])$ by letting $[m \cdot U^{-1}](B) \coloneqq m\left(\left\{c(\vec{m}) : U(c(\vec{m})) \in B\right\}\right)$ for all $B \in \mathcal{B}([0,1])$, where $\mathcal{B}$ is the Borel measure.

Definition 10 (Recursive Representation)**.**
Say that representation $U : \Delta \to [0,1]$ is recursive if it takes the form:

$$U(c, m) = W(c, I(m \cdot U^{-1})),$$

where $I : M([0,1]) \to [0,1]$ is a utility certainty equivalent operator, and $W : C \times [0,1] \to [0,1]$ is a continuous function, strictly increasing in its second argument.

Definition 11 (Kreps-Porteus Recursive Representation)**.**
Say that representation $U : \Delta \to [0,1]$ is a Kreps-Porteus recursive representation if it is a recursive representation and $I(m \cdot U^{-1}) = \phi^{-1}\mathbb{E}[\phi(m \cdot U^{-1})]$ for some increasing and continuous $\phi : \mathbb{R} \to \mathbb{R}$.

Assumption (Differentiability)**.** *Assume that the Kreps-Porteus function U is twice differentiable and that U' and U'' are well-defined at all but at most a finite number of consumption paths in C^∞.*

2.4.2 Theoretical Tradeoffs

The Kreps-Porteus class is characterized by dynamic consistency, history independence, and no probability distortion (Kreps and Porteus, 1978; Strzalecki, 2023). Dynamic consistency helps ensure that the willingness to pay for an asset remains consistent if nothing else changes. History independence is useful for tractability and to avoid having to impose a starting value for initial consumption. No probability distortion allows for standard expectation taking. As a result, all three properties are arguably desirable.

Theorem 5 shows that it is not possible to have these properties alongside convex discounting and a preference for certain timing. Whereas the fact that, for positive

shocks, certain standard utility functions predict a preference for uncertain timing is known (Chesson and Viscusi, 2003; DeJarnette et al., 2020), one might think that creatively going beyond these standard functions, it is possible to obtain a consistent preference for certain timing. My theorem below shows that this is not the case. Even thinking beyond any existing model in the literature, even including models that have never been written down, it is not possible to have full flexibility in pricing uncertain timing, if one wishes to do so in a debate-free way.

Theorem 5. *Suppose U is a differentiable Kreps-Porteus representation with convex time discounting. Then U does not satisfy a preference for certain to uncertain timing.*

2.4.3 Habit Formation

Theorem 6 shows that giving up history independence in one common way, e.g. via habit formation, also does not help accommodate a preference for certain timing.

Theorem 6. *The Campbell and Cochrane (1999) habit formation model does not satisfy a preference for certain timing.*

2.4.4 Positive vs. Negative Shocks

Corollaries 1 and 2 show that additive utility models (e.g., expected utility and hyperbolic discounting) predict a preference for uncertain timing for positive shocks and a preference for certain timing for negative shocks.[1] While the result for positive shocks is known and apparent from the convexity of the time discounting function, these results are nevertheless included here for completeness because decision theory derivations tend to use positive shocks and zero baseline consumption. Corollaries 1 and 2, in contrast, work with the full generality of Definition 6. The dichotomy between positive and negative shocks is one we will want to be aware of when testing for the pricing of uncertain timing in real-world markets, where shocks can take any sign.

Corollary 1. *Any expected utility model, $U(p) = \int_{\vec{c(p)} \in p} \sum \delta^t u(c(p)_t) dp(\vec{c(p)})$ prefers uncertain timing for all time lotteries $p^{(x)}$ when $x > 0$, and certain timing when $x < 0$.*

[1] I thank John Campbell for suggesting I include this result.

Corollary 2. *Any hyperbolic discounting model* $U(p) = \int_{\vec{c(p)} \in p} \sum \beta \delta^t u(c(p)_t) dp(\vec{c(p)})$ *prefers uncertain timing all time lotteries* $p^{(x)}$ *when* $x > 0$ *and* $p^{(x)}(t = 0) = 0$*, and certain timing when* $x < 0$ *and* $p^{(x)}(t = 0) = 0$*.*

3 Option Pricing and Uncertain Timing

3.1 Overview

Options constitute one of the world's largest and most actively traded asset classes. In the first quarter of 2024 alone, total notional value in options traded in the US was 40 trillion dollars (Office of the Comptroller of the Currency, 2024), larger than the US national debt (Department of the Treasury, 2024), and over 10 times larger than annual social security benefits (Social Security Administration, 2024).

There are two types of commonly traded options: American and European. European options may be exercised only on the designated expiration date. American options can be exercised any time up to and including expiry. One of the classical results in derivatives is Merton's theorem (Merton, 1973): absent dividend considerations, the value of an American call option is equal to the value of a European call option with the same strike price and expiration date.

Model-free tests of Merton's theorem have been limited because, in the real world, assets typically have only one option type associated with them, American or European. To the extent that the theorem has been challenged, it is in the idea that the long side of the option may exercise early due to irrationality, transaction costs, or short sale constraints (Finucane, 1997; Poteshman and Serbin, 2003; Jensen and Pedersen, 2016). The idea that early exercise may be valuable for the long side of the option is called the early exercise premium (Bryzgalova et al., 2024).

This paper contains an idea for a further channel that challenges Merton's theorem: that the short side of the American option requires compensation to hold timing risk. This unwillingness to hold timing risk is conceptually similar to standard risk aversion. Standard theory says the short side holds no compensatable risk because if an option is idiosyncratically exercised early, unlinked to market variables, there is no re-investment risk. However, the idea that timing uncertainty is inherently undesirable, whether for practical or psychological reasons, would predict that such risk is compensated.

I test for an aversion to timing risk over and above an early exercise premium by finding a setting with otherwise identical American and European options, and adopting a difference-in-difference approach.

Such a setting is hard to find in general, because most underlyings have only one option type (American or European) on them. However, there is (at least) one exception. CBOE offers both American and European options on the S&P 100, which have the same contract size, trading fees, settlement times, and other contract-specific details.[2] These otherwise identical American and European options will be the focus of analysis.

To isolate timing risk over and above an early exercise premium, I look at the following object:

$$(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}) - (\text{bid}_{\text{American}} - \text{bid}_{\text{European}}) \tag{1}$$

Using this differences in differences approach accomplishes several goals. First, it captures the timing risk premium over and above the early exercise premium. This is because the early exercise premium is thought to arise on the long side as a valuation of flexibility (whether rational (Jensen and Pedersen, 2016) or irrational (Poteshman and Serbin, 2003)). Therefore, the second difference (i.e., the bid difference) captures the value of flexibility to the buy side. And this second difference is differenced out.

The overall term can therefore be thought of as the premium in the ask of the American not attributable to higher pricing from the early exercise premium. This approach allows me to remain agnostic as to the source of the early exercise premium (an interesting but separate question), and focus only on the goal of this paper, which is an aversion to timing risk on the short side of an American, over and above the pricing in of desires for flexibility on the long side.

[2]There also exist both American and European options on the S&P 500; in particular, American-style contracts on SPDR and European-style contracts on the S&P 500 index. However, these differ in at least three ways that OEX and XEO do not. First, the most liquid of the S&P 500 options, SPY and SPX, have different contract sizes. In contrast, OEX and XEO have the same contract size. Second, all S&P 500 index options have different CBOE trading fees than SPY, meaning that even after matching on contract size, one would not be able to differentiate fees from pricing premia (CBOE, 2024*a*). This is because it is not possible to see participant identity in the data, and the fees differ by participant type. In contrast, OEX and XEO have the same CBOE trading fees for all participants. Third, SPY versus SPX (and other S&P 500 index options) settle 15 minutes apart; that difference in settlement time can contribute to what is known as a wildcard premium (Fleming and Whaley, 1994), further affecting pricing. In contrast, OEX and XEO have the same settlement time.

The differences-in-differences approach also allows me to control for unobservable differences between the American and European options. While all contract specifications are the same (by construction) and controls for liquidity are included (Section 3.3.1, below), it is possible that the option classes differ in unobservable ways. To the extent that these unobservable differences result in structurally higher prices in one options class or the other, those differences should show up in both the bid and the ask. Subtracting out the bid difference therefore also allows me to control for any unobservable factors that would symmetrically affect the spread.

I then perform a number of robustness checks. For example, I test whether the result appears in trades: that is, whether traded American options are more expensive than traded Europeans, and find that they are. I similarly test for a number of alternative explanations for the result, including moneyness explanations; wildcard premia; hedging considerations; adversarial thinking; inventory considerations; reinvestment risk; and further liquidity considerations.

This exercise contributes to a small literature on the pricing of American options relative to European ones. The canonical paper in this literature is Merton (1973), which argues that a dividend-free American call option should be priced equivalently to a dividend-free European call, a theoretical prediction that I show my findings violate. Prior literature challenging Merton's theorem focuses on the early exercise premium (Sung, 1995; Zivney, 1991; Harvey and Whaley, 1991; Li et al., 2024), typically using midpoints; I focus here on timing risk and separate out bid and ask quotes to identify an aversion to timing risk.[3]

3.2 Linkage to Theory

The motivating example in the introduction considered with certainty {\$10 in 2 months for sure} against receiving receiving that \$10 in a timing gamble, which was {50%, 1 month, 50% 3 months}.

The lottery corresponding to the options analysis is even more egregious. In

[3]To my knowledge, there is one prior paper with a reduced-form comparison of American to European options that considers bids versus asks; it finds that most of the premium shows up on the ask side (Dueker and Miller, 2002). The paper hypothesizes that it may be because the options in question were about to be discontinued leading to wider bid-ask spreads. Timing uncertainty provides a more general way to rationalize why differences in American versus European options may show up in ask quotes - the short side requires compensation to bear timing risk. Further, the options that I consider here are continuously traded and not discontinued at any point in the sample.

particular, from the perspective of the short side of an option, money can arrive or depart only at expiry for a European option but at any time up to and including expiry for an American option. As a result, an example that would conceptually correspond to the options comparison is $x in, say, 7 days for sure ('Sure Later Timing'), against $x in 1 day with some probability; 2 days with some probability; and so on, up to 7 days ('Uncertain Sooner Timing'). The second lottery time-dominates the first one, in that all payments are received or made weakly sooner.

Is $x positive or negative? When the short side owns the underlying (i.e., a covered call), $x is a positive cash flow akin to selling one's asset. When the short side does not own the underlying (i.e., a naked short call), $x will generally be negative. We are not able to observe either identity or asset ownership in the data, so we will not be able to say with certainty whether $x should be treated as positive or negative in this dataset.

The exercise is nevertheless informative because standard pricing models (e.g., Merton (1973)), do not account for a preference for certain timing. However, what I will show is that the market appears to prefer 'Sure Later Timing' to 'Uncertain Sooner Timing.'

As a follow-up question, one could ask which representative preference models the findings theoretically map to. When $x is positive, finding a preference for 'Sure Later Timing' relative to 'Uncertain Sooner Timing' goes against standard theories (see Theorem 5 and Corollary 1), which would predict a preference for uncertainty. In fact, it strongly goes against standard theory, because 'Uncertain Sooner Timing' strictly dominates 'Sure Later Timing.' When $x is negative, finding a preference for 'Sure Later Timing' would be in line with standard preference theories (Corollary 1), but not standard option pricing models which do not having a timing preference component.

3.3 Data

For the cleanest possible results, I use minute-by-minute quote data from CBOE, which offers both OEX and XEO.[4] I compare quotes within the same minute. CBOE

[4]Past research has shown that minute-by-minute data is superior to end-of-day quotes (such as those offered by OptionMetrics), which are subject to end-of-day volatility (Andersen et al., 2017) and end-of-day quotes occurring after the settlement value of the underlying is determined (van Binsbergen et al., 2012). For completeness, I re-do analysis in OptionMetrics data, in Online Appendix D.

treats both OEX and XEO identically for the purpose of fees, settlement dates and times, contract size, expiration dates, tick size, and other institutional features (CBOE, 2024*c*,*b*). The minute-by-minute quote data is available starting in 2012. Starting in 2020, the volume of contracts traded across these asset classes drops below 100,000; for example, in 2024 it was only on the order of 3,000 contracts. The sample I use is therefore Jan 2012 - Dec 2019.

Data on dividends paid by companies in the S&P 100 comes from Compustat, whose series starts in 2002; data on end-of-day levels of the S&P 100 also comes from Compustat. Data on Federal Open Market Committee meeting dates comes from the Federal Reserve Board. For robustness, Appendix B.2 also collects dividend data by hand and shows that it maps well to Compustat.

3.3.1 Cleaning

Following standard data cleaning procedure for options data (Muravyev and Pearson, 2020), I drop any quotes with a bid less than \$0.05 or where the bid-ask spread is negative.

Both OEX and XEO options settle to the closing price on the day of exercise, regardless of the time in the day when the option is exercised. Timing uncertainty is therefore not present within a given day, but rather over different days. To study timing uncertainty, I therefore drop quotes where the option expires on the same day as the quote, because such options are not subject to timing uncertainty - all parties know that they will be exercised that day and that the settlement value will be the day's closing value regardless of the time of exercise.

While most contract specifications are held fixed by CBOE, there are two features that differentiate the American and European options that we must control for. The first and most important is dividends. When a stock is known to start paying a dividend on a particular date, the stock price adjusts downwards by the dividend amount on that date (Hull, 2018). The S&P 100 is a weighted average of stock prices. Consequently, if for example an American and European option expire on a date with a dividend payment, the holder of the American option may be willing to pay a premium - say, up to the anticipated dividend amount - to exercise their option before the stock price update for the dividend occurs. We should therefore restrict our attention to options in whose lifetime, including at initiation and expiry, there are no dividend considerations for any of the stocks in the S&P 100. This restriction

substantially reduces our sample, but is also an important step towards identification, by removing the standard explanation for American option premia.[5]

I therefore focus attention on dates and options where, were a market participant to enter the option on that date, they would not face any dividend considerations for the lifetime of the option. To find such dates, I first find the set of dates on which S&P 100 dividends paid are zero.[6] I then fill in holidays and weekends (for which dividends paid are also zero, because these are not trading days).

The second feature we must rule out as a source of the premium is liquidity: in this case, the American option is the more traded product, perhaps because it was introduced before the European version (Harvey and Whaley, 1991). Following Amihud and Mendelson (1986) and Bryzgalova et al. (2022), I use the bid-ask spread, that is, (ask - bid)/midpoint, as the illiquidity measure. Restricting this is a little tricky because if, for example, the price premium for the American option arises from sellers demanding compensation for timing uncertainty, imposing a tight bid-ask spread means that it will look as though American bids are higher than European bids in the sample - although actually what is happening is that American asks are higher than European asks and the data restriction imposes tight bid-ask spreads. Thus, we want some liquidity threshold to compensate for liquidity premia, but not such a tight liquidity threshold that we are no longer able to differentiate between movement on the bid and on the ask side. I use a 5% threshold in the main text, e.g. restrict attention to minutes where the difference in bid-ask spreads between the OEX and XEO quotes is less than 5%. For robustness, Table 3 re-performs the analysis for other thresholds and also without liquidity corrections. As the liquidity correction becomes looser, the difference in asks further dominates the difference in bids, consistent with pricing premia for American options arising at least partially from compensation for timing uncertainty for the short side (rather than entirely from a willingness to pay for flexibility on the long side).

Finally, following Constantinides et al. (2013) and Ni et al. (2008), to focus on credible quotes, I drop options with a moneyness (defined as (underlying-strike)/strike)

[5]Li et al. (2024) also consider the midpoints of options I study here, and find that at longer horizons, OEX is more expensive than XEO 86% of the time, consistent with an aversion-to-timing-uncertainty interpretation. However, they (1) do not correct for dividend concerns and (2) do not adjust for liquidity differences, and (3) do not separate the early exercise premium on the long side from an aversion to holding timing risk on the short side.

[6]Compustat calculates dividends based on the ex-dividend date, which is also the relevant date for dividend-based American option strategies.

of below -0.2 or above 0.2. As is the case in other option markets, these tend to be less traded than those within the 0.2 moneyness band; here, after applying all filters above, options which are outside this moneyness band have only 5 contracts traded across the entire sample.

The notional value of all options in the final cleaned dataset is $3.65 billion.

3.4 Summary Statistics

Table 1 provides summary statistics for moneyness, days to expiry, and bid-ask spreads for all options in the analysis dataset post cleaning (Section 3.3.1). The bid-ask spreads are similar because of the imposed liquidity controls. Because companies within the S&P 100 pay out dividends at different times (Harvey and Whaley, 1991), in this data, the longest lived contracts with strictly zero dividend considerations have a lifetime from quote date to expiry of seven days. The median quote is in-the-money, but the moneyness distribution skews left; as a robustness check (Section 3.7), I restrict consideration to options that are in-the-money and show that the results continue to hold. Appendix B.1 provides further descriptive statistics for these options.

3.5 Baseline Results

Table 2, Column 1 provides the baseline results. The mean ask difference (OEX - XEO) is $0.33; midpoint $0.28; and bid $0.22; all are statistically significant at the 1% level. From the perspective of standard and parsimonious pricing theory, the interesting part of this empirical exercise is that timing risk is priced; whether to annualize and the precise method of annualizing is secondary.

These results show that (1) the market prices American call options without dividend considerations higher than otherwise identical European options, and (2) that this higher pricing comes both from the long and the short side, but more so from the short side.

Figure 1 plots the average ask, bid, and midpoint difference, aggregated by week, over time. This figure shows that the ask difference is usually higher than the bid difference across all time periods considered. This indicates that the results of Table 2 are largely consistent across time and not the result of a few outliers.

Timing uncertainty is the primary feature that distinguishes OEX from XEO in

TABLE 1: Summary Statistics for OEX and XEO Options

	Both	OEX	XEO
Bid-Ask Spread (%):			
Mean		6.27%	6.16%
Median		2.90%	2.69%
Standard Deviation		11.22%	11.30%
Moneyness:			
Mean	0.09		
Median	0.08		
Standard Deviation	0.06		
N. Contracts by Days to Expiry:			
1	743,713		
2	240,515		
3	112,668		
4	68,476		
5	11,036		
7	26,577		

Note: This table lists summary statistics for the main sample of OEX and XEO options, cleaned according to the procedure in Section 3.3. The bid-ask spread is calculated as $100 * (ask - bid)/midpoint$. Days to expiry is the number of days between the quote date and the expiration date of the option. Moneyness is calculated as $(underlying - strike)/strike$. The top line follows Equation 1, e.g. the ask difference between the American and European less the bid difference between the American and European.

this dataset because, by construction, we have controlled for other contract characteristics: quote time, tick size, liquidity, type of settlement, tick size, expiration date, etc. One supply side story consistent with this evidence is that sellers require compensation for bearing the timing risk inherent in American options. The timing risk here arises from the long side possibly exercising early. I take no position on why the long side might exercise early. To check for one approximate proxy for possibility of early exercise and whether it correlates with additional compensation required by the short side, I consider moneyness; the more in-the-money an option is, the more tempted the long side may be to exercise (this is a very crude proxy). Table 4 regresses the log of the ask difference less the bid difference on the log of moneyness. The results indicate that for in-the-money options, after controlling for bid-ask-spreads, a 1% increases in moneyness increases the ask difference less bid difference by 1.03% on average. While the R^2 for these regressions is low, the positive and statistically significant coefficient is consistent with uncertain timing being priced.

The robustness sections, next, shows that these baseline results continue to hold in various subsamples of the data.

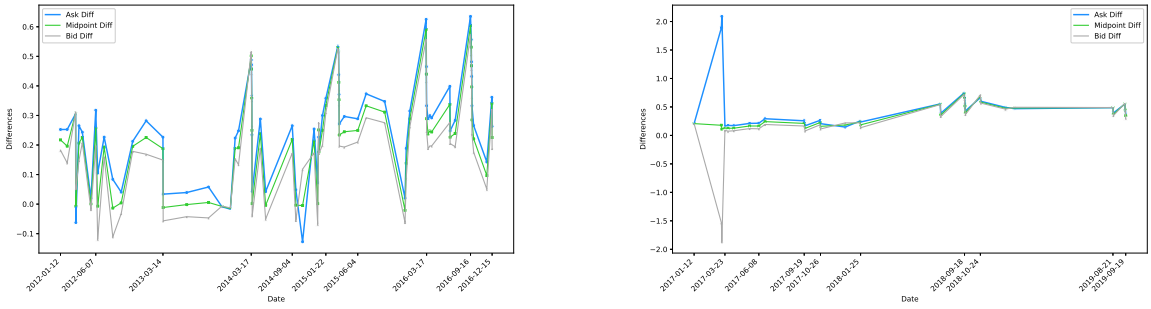
TABLE 2: Pricing Differences between Dividend-Free American (OEX) and European (XEO) Options

	Baseline	In the Money	No Fed Meetings	Positive Open Interest
Ask Difference - Bid Difference	\$0.1084***	\$0.1121***	\$0.1084***	\$0.0536***
	(0.0373)	(0.0384)	(0.0373)	(0.0189)
Ask Difference	\$0.3319***	\$0.3388***	\$0.3319***	\$0.2168***
	(0.0250)	(0.0259)	(0.0250)	(0.0236)
Midpoint Difference	\$0.2777***	\$0.2827***	\$0.2777***	\$0.1900***
	(0.0191)	(0.0198)	(0.0191)	(0.0240)
Bid Difference	\$0.2235***	\$0.2267***	\$0.2235***	\$0.1631***
	(0.0284)	(0.0292)	(0.0284)	(0.0278)
Ask Difference Quartiles:				
25%	\$0.20	\$0.20	\$0.20	\$0.00
50%	\$0.30	\$0.30	\$0.30	\$0.20
75%	\$0.50	\$0.50	\$0.50	\$0.40
Midpoint Difference Quartiles:				
25%	\$0.15	\$0.15	\$0.15	\$0.025
50%	\$0.25	\$0.25	\$0.25	\$0.15
75%	\$0.40	\$0.40	\$0.40	\$0.30
Bid Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.10	\$0.00
50%	\$0.20	\$0.20	\$0.20	\$0.10
75%	\$0.40	\$0.40	\$0.40	\$0.30
Number of Observations	1,202,985	1,169,096	1,202,985	205,223
Number of Unique Days	101	101	101	98

*Standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.*

Note: This table shows the difference in quoted prices (OEX (American) - XEO (European)) between otherwise identical OEX and XEO options in dollar terms. Standard errors are clustered by date and by contract, where a contract is a (strike price, expiration date) pair. The baseline column performs this analysis on the data cleaned as described in Section 3.3. The in-the-money column restricts the cleaned data to only in-the-money options. The no-fed-in-lifetime column restricts the cleaned data to only options in whose lifetime there is no Fed meeting. The positive open interest column restricts the cleaned data to only option quotes and minutes for which both OEX and XEO have strictly positive open interest.

FIGURE 1: Ask Difference - Bid Difference is Positive Across Sample



(A) 2012-2016 (B) 2017-2024

Note: This figure plots average price differences for asks, bids, and midpoints, aggregated daily. Days with data are shown as points on each line. Because some dates have large magnitude differences, for ease of visual processing, Figure 1a focuses on the period 2012-2016 and Figure 1b 2017-2020. The figure shows that ask differences (in blue) are generally positive and higher than bid differences (in gray) over time.

TABLE 3: OEX - XEO Pricing Differences Under Different Liquidity Thresholds

	Baseline (Liquidity Difference < 5%)	Liquidity Difference < 1%	Liquidity Difference < 10%	No Liquidity Controls
Ask Difference - Bid Difference	\$0.1084***	\$0.0743***	\$0.1253**	\$0.1075*
	(0.0373)	(0.0044)	(0.0560)	(0.0575)
Ask Difference	\$0.3319***	\$0.3225***	\$0.3356***	\$0.3179***
	(0.0250)	(0.0199)	(0.0312)	(0.0313)
Midpoint Difference	\$0.2777***	\$0.2853***	\$0.2730***	\$0.2641***
	(0.0191)	(0.0200)	(0.0187)	(0.0180)
Bid Difference	\$0.2235***	\$0.2482***	\$0.2104***	\$0.2103***
	(0.0284)	(0.0202)	(0.0359)	(0.0363)
Ask Difference Quartiles:				
25%	\$0.20	\$0.20	\$0.20	\$0.10
50%	\$0.30	\$0.30	\$0.30	\$0.30
75%	\$0.50	\$0.50	\$0.50	\$0.40
Midpoint Difference Quartiles:				
25%	\$0.15	\$0.15	\$0.10	\$0.10
50%	\$0.25	\$0.25	\$0.25	\$0.25
75%	\$0.40	\$0.40	\$0.40	\$0.40
Bid Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.10	\$0.10
50%	\$0.20	\$0.20	\$0.20	\$0.20
75%	\$0.40	\$0.40	\$0.40	\$0.40
Number of Observations	1,202,985	1,089,218	1,244,861	1,301,399
Number of Unique Days	101	101	101	101

*Standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.*

Note: This table shows the difference in quoted prices (OEX (American) - XEO (European)) between otherwise identical OEX and XEO options in dollar terms, with different liquidity controls. Standard errors are clustered by date and by contract, where a contract is a (strike price, expiration date) pair. The baseline column performs this analysis on the data cleaned as described in Section 3.3. The second column restricts to options and dates or minutes where the the difference in bid-ask spread (measured as a percentage) between XEO and OEX is less than 1%; the third column less than 10%; and the fourth column makes no liquidity restrictions.

TABLE 4: Ask Difference - Bid Difference and Moneyness

	Full Sample		In-the-Money	
	(1)	(2)	(3)	(4)
Log Moneyness	0.586***	1.028***	0.544***	1.033***
	(0.206)	(0.523)	(0.213)	(0.528)
Bid-Ask Spread Difference Control	N	Y	N	Y
R^2	0.018	0.094	0.014	0.110
Observations	1,202,985	1,202,985	1,169,096	1,169,096

*Standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.*

Note: This table shows the result of regressing the signed log of ask difference − bid difference (Equation 1) on the signed log of moneyness, measured as (underlying-strike)/strike). The signed log of x is defined as $\text{sign}(x)log(|x|)$, which allows x to take on both positive and negative values. Standard errors are clustered by date and by contract. Specifications (1) and (3) run this regression without any controls. Specifications (2) and (4) include a control for the difference in bid-ask spreads (measured as a %) between the two option classes. Specifications (1) and (2) use the full sample, while (3) and (4) use in-the-money options only.

3.6 Replicating Results in Trades

Here I test for whether the quote-based results - of an unwillingness to be short an American over and above an unwillingness to be short European - are consistent with trades as well. The separation between the short and long side will not be as clean as for the quotes, but the fact that similar results appear for trades provides additional evidence towards an aversion to holding uncertain timing.

First, as in Section 3.3.1, consider only options in whose lifetime there are no dividends. I then match each XEO trade with an OEX trade with the same strike price and expiry, where both trades occur within 10 minutes of each other. If there are multiple such OEX trades, I take the one closest in time to the XEO trade. I then compare the traded price for the OEX and XEO trades. This method of cleaning yields 1189 trades in the sample across 62 days.

Following the matching procedure above finds that American options trade on average 3.75 cents higher than European options ($p < 0.05$, with standard errors clustered by date); as a percentage of the option price, this difference is on average 5.5%.

3.7 Assessing Possible Explanations

3.7.1 Out-of-the-Money Anomalies.

The second column of Table 2 restricts analysis to the subsample of quotes that are in-the-money. The results continue to hold: on average, ask quotes for the American option are $0.34 higher than for the European, midpoints $0.28 higher, and bid quotes

$0.23 higher.

3.7.2 Re-Investment Risk.

In addition to an aversion to uncertain timing, another reason that the short side for the American option may require additional compensation is if there is a risk of interest rates changing during the lifetime of the option. To check for this, I use Federal Reserve meeting dates to determine whether there is a Fed meeting during the lifetime of any of the options in the sample. Column 3 of Table 2 shows that restricting to no-dividend days of the S&P 100 is a stronger restriction than restricting to options in whose lifetime there are no Fed meetings, so that the baseline sample already includes only options with no re-investment risk.

3.7.3 Strictly Positive Open Interest.

One concern may be that option quotes are not meaningful for options which do not have positive open interest. This is unlikely to explain the results because CBOE has designated market makers for option classes, who are required to uphold certain requirements, including minimum quote contract size requirements (CBOE, 2019). As a further explicit check, Column 4 of Table 2 restricts attention to options and minutes for which both OEX and XEO have strictly positive open interest. The result of timing uncertainty being priced continues to hold.

3.7.4 Executed Prices.

Similarly, one may argue that quotes and executed prices differ. Section 3.6 computes the difference in traded prices for OEX and XEO and finds that dividend-free OEX call options trade at higher prices than otherwise identical XEO options. These trading results are consistent with the results from quotes.

3.7.5 Wildcard Premium Differences.

Fleming and Whaley (1994) document that some options have wildcard premiums, meaning that the settlement price of the contract is established before the last time at which the option holder can decide to exercise. This would affect a comparison of options on the S&P 500, because PM-settled SPX (European) options are exercised at the close of the trading day, but holders of SPY (American) have the option to exercise

after the closing price of the underlying - so one is subject to a wildcard premium while the other is not. This is one of the reasons I chose not to use the S&P 500 for the reduced form comparison. A benefit of the options on the S&P 100 considered here is that both stop trading at the same time and have identical settlement specifications, and therefore there is no wildcard premium difference between them.

3.7.6 Informed Trading

The next set of explanations we will consider explores the idea that the short side of the market may be strategically reacting to actions from the long side of the market. We will consider these explanations one by one, but they can also be answered by showing that the options we consider are traded largely by uninformed investors. If the trading is uninformed, that alleviates strategic concerns that might affect pricing.

There are several reasons to believe that the trades corresponding to our main sample are uninformed. First, past research shows that trading in index options is generally uninformed, with informed investors typically locating in single-name stocks, or at best stock options, rather than index options (Pan and Poteshman, 2006; Chan et al., 2002). But second, we can perform an explicit check for uninformed trading by looking at trade size, because Muravyev (2016) shows that small option trades tend to be uninformed in that they do not predict market movements. In addition, small option trades have been shown to be one (noisy) proxy for retail trading in options, which past work finds to be idiosyncratic with suboptimal behavior(Poteshman and Serbin, 2003; Barber and Odean, 2013; Bauer et al., 2009; Bryzgalova et al., 2022). Appendix B.4 checks the proportion of small trades (few contracts traded, small monetary amount transacted, or both) and shows that these small trades constitute over over 50% of trades corresponding to the main sample. Within buys - which is what the short side of the market would be responding to - small trades account for approximately 57-83% of trades, depending on the measure used. From past research, we conclude that a large portion of trades corresponding to the main sample are uninformed, which means strategic responses to them are less likely to be driving the results.

3.7.7 Adversarial Thinking.

Because of the constraint that these options not have dividend considerations in their lifetime, these are generally short-dated options (Table 1). One might ask why the long side of an option would choose to enter an American rather than European; in particular, rationally a market participant should be willing to pay more for a short-dated American than European if either (1) the long side values the timing flexibility, or (2) the long side intends to exercise right away. If (2) holds, then the short side, anticipating this, may adjust their prices. However, standard theory predicts they would adjust prices in the opposite direction from what I find. This is because an American where the long side exercises immediately is equivalent to a European expiring today; but we know from both standard models and a large empirical literature that European options with more days to expiry are more expensive, because they have a higher implied volatility (Bakshi and Kapadia, 2003). This kind of adversarial thinking would therefore predict that American options trade at a discount relative to European options, the opposite of what I find here.

Another type of adversarial thinking might posit that options are exercised only on days disadvantageous to the short side of the option. Robustness check 'Re-Investment Risk,' above, shows that the result holds even when the most common market variables related to this, interest rate considerations, are controlled for. In addition, such considerations should be minimal one day to expiry, but the effect shows up for such options too. Finally, arguments of this form should be minimal if trading on the long side is uninformed, as discussed in Section 3.7.6, above.

3.7.8 Hedging Considerations.

One reason American options may be harder to hold for the short side than European ones is that, when there is uncertainty over the day of exercise, hedging becomes more complicated, and the short side of the American may have a practical or psychological aversion to dealing with this complexity (Puri, 2024), or may have budgeting needs that make changes in hedging difficult.

To show that there is a pricing of timing uncertainty over and above differential hedging considerations, Appendix B.3 re-performs the analysis for only options whose hedging requirements are not expected to change over their lifetime. In particular, it re-conducts the test for options whose gamma is close to zero. This means that the

amount of hedging needed for being short the option is not expected to change over the option's lifetime. Though the sample becomes an order of magnitude smaller, the qualitative result: a pricing premium on the short side, over and above that on the long side (Equation 1) continues to hold.

3.7.9 Asymmetric Changes in Bids and Asks - For Example Due to Inventory

Specification 1 treats asks and bids symmetrically. But, it is possible that the long side moves differently than the short side. For example, under some models (Ho and Stoll, 1981), if a market maker has accumulated excessive long positions due to rising bids, they might lower ask prices to encourage sales and rebalance inventory. The main specification does not allow for that flexible difference in changes in bids relative to asks. To allow for this flexibility, I run the regression:

$$\left(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}\right)_{m,o} = \beta_0 + \beta_1 \left(\text{bid}_{\text{American}} - \text{bid}_{\text{European}}\right)_{m,o} + \epsilon_{m,o},$$

where m denotes the minute and o the option. We are interested in whether $\beta_0 > 0$ even after allowing β_1 to be flexible (e.g., not imposing that it is one). The regression yields a statistically significant $\beta_0 = 0.37$ ($p < 0.01$), showing that there is an unwillingness to be short the American over and above flexible pricing incorporation of the long side.

3.7.10 Further Liquidity Considerations

Figlewski (2022) shows theoretically that when the liquidity of the options market is highly variable, an American option can be more valuable than a European one because the American allows the long side to exercise at times when the market is relatively illiquid. However, the baseline analysis controls for liquidity, by construction (Section 3.3.1). In addition, this is a reason why the long side may value the American, but not a reason for the short side to price the American at a premium; per the 'Adversarial Thinking' section above, standard option value arguments imply that the short side should be willing to offer options at a discount if they believe early exercise is likely. In addition, the 'Hedging' section above shows that the results hold even for options where the short side has no need to change their hedging requirements, e.g. their own budgeting considerations are not affected.

3.7.11 Rare Disasters

Seo and Wachter (2019) documents that index put options are routinely more expensive to purchase than predicted by Black-Scholes, and finds that a model including a small probability of a rare disaster provides a better approximation of prices. My findings do not contradict this, but neither can they be fully explained by rare disasters. First, the analysis is on call options which, all else equal, should be less appealing in the face of rare disasters; the analysis of Seo and Wachter (2019) focuses on put options, which provide insurance should a rare disaster occur. Second, the empirical test is reduced form and within-index options; it does not rely on Black-Scholes pricing in any way.

3.7.12 Early Resolution of Uncertainty

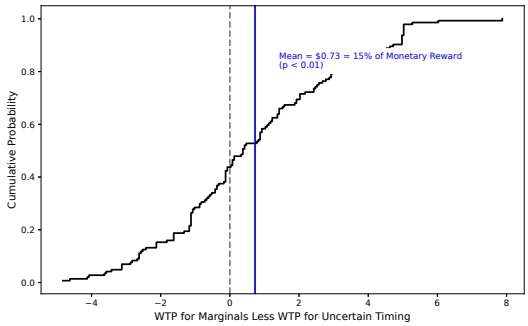
Ai et al. (2023) find evidence of a preference for early resolution of uncertainty using FOMC announcements. In the context of American options, uncertainty is resolved sooner if the long side chooses to exercise early. As a result, if the short side prefers early resolution, they should be more willing to be short the American than the European. But this is the opposite of what the data suggests (Table 2). In particular, here the short side requires additional compensation for the American than the European option.

4 Willingness-to-Pay and Pricing Narratives

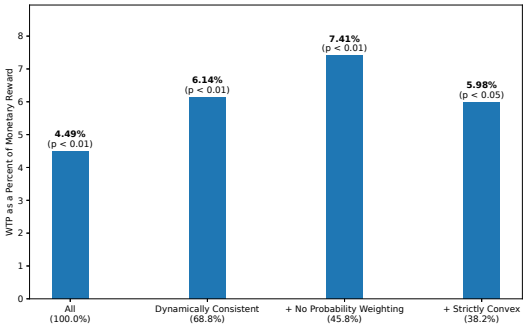
As a supplement to the market results, this section elicit willingness to pay for lotteries to further test the predictions of some of the above derived theorems. Consistent with the market-based results, uncertain timing is priced. Further, it is priced in a manner that differs from pricing of the marginals. Finally, the pricing of uncertain timing holds even amongst those individuals with with otherwise desirable properties. An incentivized elicitation of the approach used to price finds that many respondents self-describe as pricing of certainty in the time and money domains similarly.[7]

[7]Self-descriptions are often used in surveys (ex. Schotter (2003), Bailey et al. (2018)) – here I use them in the context of studying attitudes towards timing risk.

FIGURE 2: Willingness-to-Pay Measures



(A) WTP for Marginals − WTP for Uncertain Timing (Theorem 1)



(B) WTP for Certain − Uncertain Timing and Theoretical Properties (Theorem 5)

Note: Panel (A) shows the cumulative density function for difference in willingness to pay for the marginals of the uncertain timing lottery less the willingness to pay for the uncertain timing lottery itself. Panel (A) also provides a label for the mean difference and the p-value of that difference. Panel (B) shows, as a percentage of the money at stake ($5), the difference in willingness to pay for certain and uncertain timing. The first bar shows this overall; the second within the subset of respondents who are dynamically consistent; the third bar within the subset of respondents who are dynamically consistent and do not distort probabilities; and the fourth bar within the subset of respondents who are dynamically consistent, do not distort probabilities, and display locally convex deterministic discounting. The percentages on the bottom of the x-axis show the fraction of respondents within each group. For example, 68.8% of respondents are dynamically consistent, while 38.2% satisfy all three properties. The p value is shown underneath the mean value for each bar.

4.1 Data and Methodology

The survey was run on Prolific in March 2025; IRB approval was obtained from the New York University Institutional Review Board, under ID IRB-FY2024-8127. Respondents received an average payment of $4.23, and the median time taken was 21 minutes. Willingness to pay was measured via standard incentivized multiple price lists (Holt and Laury, 2002), that is, a question and row were selected at random for 10% of respondents, who then received a bonus payment according to their response in that row. A series of eleven comprehension questions ensured that participants understood the difference between sure money and sure timing; sure money and uncertain timing; and uncertain money and sure timing (Online Appendix C). There were 144 respondents total.

The main three questions to test for attitudes towards uncertain timing were elicitations of willingness to pay (WTP) today for the lotteries: (1) {50%, $5 in 2 days; 50%, $5 in 8 days}; (2) {50%, $0 in 2 days; 50%, $5 in 2 days}; (3) {50%, $0 in 8 days; 50%, $5 in 14 days}; and (4) $5 in 5 days. The prediction of the standard SDF framework is that, for any SDF, holding all else equal, WTP for (2) + WTP for (3) = WTP for (1). The prediction of Definition 6 is that the WTP for (1) should be

TABLE 5: Incentivized Narratives

Panel A: Sample Narratives	**Panel B: Classification**	
(1) I simply just went for less risks.	GENERAL PRICING RULE	51%
(2) I tried to play it relatively safe.	MONEY CERTAINTY PRICING, TIME UNCERTAINTY TOLERANT	17%
(3) I chose the safe option with no probability and was a guarantee up until $4 when I changed to the probability option.	MONEY CERTAINTY PRICING, TIME CERTAINTY PRICING	17%
(4) For lotteries that include a 50% chance of receiving nothing, I elected to receive everything over a minimal amount. For lotteries that paid out and varied only in time, I elected to receive the deferred payment for anything less than the immediate award.	MONEY UNCERTAINTY TOLERANT, TIME UNCERTAINTY TOLERANT	11%
	MONEY UNCERTAINTY TOLERANT, TIME CERTAINTY PRICING	3%
(5) I chose to not look at the timeframe of when I would get the bonus because I knew that I would get it eventually. Instead, I only focused on the dollar amount that would be given to me. In the case of a lottery, I calculate the dollar amount by taking the expected payoff from the lottery.		

weakly lower than the WTP for (4).

In addition, to measure correlations of the answers to these questions with (a) dynamic consistency, (b) no probability distortion, and (c) convex discounting, that is, some of the properties at odds with a preference for certain timing in Theorem 5, the survey included additional standard questions for deterministic time discounting (Sprenger, 2015) and probability distortion vs. complexity aversion (Puri, 2024); these questions are described in detail in Online Appendix C.

The order of the questions was randomized within the survey for each participant. Directly after the uncertain timing question, the participant was surprised with a question which asked them to describe how they made their prior choices. The question was made incentive compatible by having, for 5% of respondents, a new participant read the original participant's description, and having the new participant attempt to replicate the original participant's choices on the timing risk questions. If the new participant correctly replicated the original participant's choice on a randomly selected question, both earn a bonus.

4.2 Results

The spread between the willingness to pay for the marginals vs. the willingness to pay for uncertain timing with the same marginals, is depicted in Figure 2a. In contrast to the marginalization approach of certain canonical SDF frameworks (Theorem 1), these direct elicitations reveal an average difference of \$0.73 in willingness-to-pay, equivalent to 15% of the \$5 monetary award in the uncertain timing lottery.

Figure 2b shows that the statistically significant willingness to pay for certain timing remains even after applying properties that financial economists have historically found appealing; namely, dynamic consistency, no probability distortion, and locally convex discounting.[8]

Table 5, Panel A provides examples of incentivized narratives about how respondents approached uncertain timing and substitution across time and money. Table 5, Panel B classifies these into general choice rules (e.g., no differentiation between time and money in approach to choice); stated certainty preference in both domains; certainty preference in money but uncertainty tolerant over time; etc. The classification is done using the procedure described in Online Appendix C. Overall, 68% of respondents report pricing narratives of either a general pricing rule across domains or a higher WTP for certainty in both domains.

5 Potential Applications Across Finance

Future work can build on the idea that (1) markets appear to price uncertain timing but (2) standard pricing models may not have the ability or flexibility to capture this pricing, in a variety of ways.

Bond Spreads. Callable and convertible bonds are subject to timing uncertainty. Pricing models for these bonds typically conceptualize them as a standard bond plus an option (Brealey et al., 2020). Just as options prices differ from predictions of classical theorems, consistent with a pricing of uncertain timing (Section 3), corporate bond spreads may differ as well.

[8] Online Appendix C performs a robustness check in which the sample is restricted only to high financial literacy respondents who satisfy monotonicity conditions, and shows that the results continue to hold.

Corporate Decisions. Companies often have uncertainty not only over the magnitude of cash flows on an uncertain project, but also when they will occur. One puzzle in the corporate literature is that companies appear to use hurdle rates that are higher than their weighted average cost of capital (Graham, 2022). It may be the case that one reason for these high hurdle rates is that the cost of capital estimate does not account for the uncertain timing of return inherent in some projects.

Mortgage-Backed Securities. In mortgages, prepayment risk is at heart uncertainty over the time at which the mortgage will be paid back. In practice, prepayment is often predicted using interest rate factors, but research shows that much of the pricing of MBS reflects something beyond interest rates (Boyarchenko et al., 2019). It may be the case that, as with the options markets in this paper, MBS are pricing (possibly difficult to observe) uncertain timing in the time of mortgage prepayment.

Policy Investment Decisions. Several international institutions and nonprofits claim that both developing and developed nations under-invest in infrastructure relative to the nation's needs (on Foreign Relations, 2023; Bank, 2024, 2025). One part of a rationale could be that infrastructure projects can be subject to uncertain timing and unpredictable delays. A pricing of uncertain timing may therefore be relevant in policy contexts.

Convenience Yields. In times of fiscal shutdown or economic crisis, both the likelihood of default and, the probability that payments might be delayed, increase. It may be the case that convenience yields decrease in such instances due not only to outcome uncertainty but also due to uncertain timing.

6 Connections to Literature

The contribution of this paper spans financial subfields. In showing that the standard stochastic discount factor framework is unable to capture timing uncertainty, the paper contributes to a literature on the stochastic discount factor and its properties (Ross (1978), Harrison and Kreps (1979), Hansen and Singleton (1982), Hansen and Jagannathan (1991), Duffie (2001), Hansen and Scheinkman (2017)).

By showing that pricing frameworks do not accommodate timing uncertainty, but a preference for certain timing shows up empirically in real-world markets, the paper also contributes to literature that thinks about how to model the representative investor; a topic extensively thought about and discussed by, for example, Campbell and Cochrane (1999); Lettau and Ludvigson (2001); Daniel et al. (2002); Lewellen and Nagel (2006), and Verdelhan (2010).

In showing a possible friction between theory and empirical pricing, the paper complements work on the term structure of the equity premium (van Binsbergen et al., 2012; van Binsbergen and Koijen, 2017), which shows that standard theory does not match existing models on dividend strips. In this case, empirical pricing seems to suggest a pricing of uncertain timing, which can be difficult to model theoretically in a fully flexible way, especially within certain textbook SDF frameworks.

In studying empirical pricing of options, the paper contributes to a literature on actual prices of options and how these either adhere to, or deviate from, standard theory. Past work has studied a willingness to pay for flexibility on the long side (Hull, 2018), which my specification explicitly distinguishes from; as well as frictions across markets (Bryzgalova et al., 2024); and the identification and behavior of retail trades (Bryzgalova et al., 2022). Here, I study the short side of an index option market and find evidence that equilibrium pricing is consistent with an aversion to bearing timing risk.

Because pricing appears throughout finance, the paper also has implications for how to think about discounting risky cash flows (Graham, 2022), when the time of project completion or return may be uncertain; welfare implications associated with climate finance, where the timing of subsidy or disaster may be uncertain (Bolton and Kacperczyk, 2020; Stroebel and Wurgler, 2021; Pastor et al., 2024); valuation in venture capital and private equity, where the timing of return is uncertain (Da Rin et al., 2013; Metrick and Yasuda, 2021; Gourier et al., 2024); the pricing of corporate bonds, where the timing of conversation or call is uncertain (Brealey et al., 2020); and more generally, has implications for financial decisions in any setting with uncertain timing. In particular, one implication is that an approach towards evaluating uncertain timing with standard pricing models or frameworks - as typically done when needed, for example the pricing of callable bonds draws on standard option pricing theories - may or may not accurately reflect empirical market treatment.

The paper also complements a large literature on the elasticity of intertemporal

substitution, a component of preferences whose calibration is thought to be first order for financial decisions (Hansen and Singleton, 1982; Vissing-Jorgensen, 2002; Woodford, 2003). It also complements a smaller literature on the timing of uncertainty resolution (Epstein et al., 2014). Here, I show that no matter what the calibration of the EIS, and even when all uncertainty is resolved immediately, it is theoretically challenging to flexibly price uncertain timing.

The finance literature has not historically considered whether standard pricing frameworks treat uncertain timing accurately. This paper shows that such attitudes are relevant empirically and theoretically. In particular, standard ways of using the SDF have strong implications for the pricing of timing risk, while real-world markets with sophisticated participants feature pricing consistent with compensation for bearing uncertain timing. This is not, however, the first paper to think about these topics in the world; there is a small but rich interdisciplinary literature. In biology, Kacelnik and Bateson (1996) show that foraging animals like pigeons and rats are temporally risk seeking. In decision theory, Chesson and Viscusi (2003) show that expected utility is temporally risk-seeking over gains, but unincentivized experiments seem to show the opposite. Onay and Onculer (2007) generalize these results to time-separable preferences, and show that temporal risk aversion may be partially accommodated by a model that includes probability weighting. DeJarnette et al. (2020) leverage this past work to show that temporal risk aversion is axiomatically incompatible with a property they introduce called stochastic impatience, across a wide range of utility functions, and perform incentivized experiments supporting certain timing; they and Dillenberger et al. (2024) also derive results on both properties within Epstein-Zin. Ebert (2020) introduces terminology and discusses possible ways to formalize temporal attitudes.

In addition to contributing to the finance literature, the paper contributes to this interdisciplinary literature by bringing the topic of timing risk to applications and theoretical frameworks relevant for finance. First, on the empirical side, I provide some of the first real-world evidence of the pricing of certain timing in large financial markets with sophisticated participants. Second, on the theory side, this is a first paper across the interdisciplinary literature to study temporal risk within the stochastic discount factor framework central to asset pricing. In addition, I expand the theory to show that even moving beyond commonly-used models, it is not possible to accommodate empirical pricing without giving up at least one property financial economists

have historically found appealing. Third, I elicit willingness-to-pay measures which supplement the market results.

Finally, the paper also has policy implications for the recent discussion around discount rates, especially as regards pricing of longer-term government policies (Joskow et al., 2023). Whereas prior work has calibrated a long-run discount rate parameter (Lucas, 2023), the present work shows that in certain canonical SDF treatments, such calibration, and its application, simultaneously pins down not only deterministic discounting but also (perhaps inadvertently) prices uncertain timing.

7 Conclusion

This paper explored the pricing of certain vs. uncertain timing both in the stochastic discount factor framework and empirically in financial markets. While certain textbook SDF approaches are not rich enough to accommodate the pricing of uncertain timing, uncertain timing appears to be priced in options markets. Expanding beyond these straightforward textbook approaches allows for the pricing of uncertain timing, but full flexibility in the method of pricing conflicts with other desirable properties such as dynamic consistency, the future mattering less than the present, and history independence.

These findings suggest that better understanding both theoretical and empirical components of uncertain timing may deepen financial models and understanding of pricing in financial markets. For example, in many settings, uncertainty in outcomes and uncertainty over time are intertwined. Future work may wish to seriously examine the uncertain timing aspect of empirical settings such as corporate bond pricing, investment decisions, and policy decisions, among others.

References

Ai, Hengjie, Ravi Bansal, Hongye Guo, and Amir Yaron. 2023. "Identifying Preference for Early Resolution from Asset Prices." National Bureau of Economic Research w31087, Cambridge, MA.

Amihud, Yakov, and Haim Mendelson. 1986. "Asset pricing and the bid-ask spread." *Journal of Financial Economics*, 17: 223–249.

Andersen, Torben G., Nicola Fusari, and Viktor Todorov. 2017. "Short-Term Market Risks Implied by Weekly Options." *The Journal of Finance*, 72: 1335–1386.

Arrow, Kenneth J., and Gerard Debreu. 1954. "Existence of an Equilibrium for a Competitive Economy." *Econometrica*, 22: 265.

Bailey, Michael, Ruiqing Cao, Theresa Kuchler, and Johannes Stroebel. 2018. "The Economic Effects of Social Networks: Evidence from the Housing Market." *Journal of Political Economy*, 126: 2224–2276.

Bakshi, Gurdip, and Nikunj Kapadia. 2003. "Delta-Hedged Gains and the Negative Market Volatility Risk Premium." *Review of Financial Studies*, 16: 527–566.

Bank, European Central. 2024. "Mind the gap: Europe's strategic investment needs and how to support them."

Bank, World. 2025. "Infrastructure Finance."

Barber, Brad M., and Terrance Odean. 2013. "The Behavior of Individual Investors." In *Handbook of the Economics of Finance*. Vol. 2, 1533–1570. Elsevier.

Bauer, Rob, Mathijs Cosemans, and Piet Eichholtz. 2009. "Option trading and individual investor performance." *Journal of Banking & Finance*, 33: 731–746.

Bolton, Patrick, and Marcin T. Kacperczyk. 2020. "Carbon Premium around the World." *SSRN Electronic Journal*.

Boyarchenko, Nina, Andreas Fuster, and David O Lucca. 2019. "Understanding Mortgage Spreads." *The Review of Financial Studies*, 32: 3799–3850.

Brealey, Richard A., Stewart C. Myers, and Franklin Allen. 2020. *Principles of corporate finance. The McGraw-Hill/Irwin series in finance, insurance, and real estate.* Thirteenth edition, international student edition ed., New York, NY:McGraw-Hill Education.

Bryzgalova, Svetlana, Anna Pavlova, and Taisiya Sikorskaya. 2022. "Retail Trading in Options and the Rise of the Big Three Wholesalers." *SSRN Electronic Journal.*

Bryzgalova, Svetlana, Anna Pavlova, and Taisiya Sikorskaya. 2024. "Strategic Arbitrage in Segmented Markets." *Journal of Financial Economics, Forthcoming.*

Campbell, John, and John Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 107: 205–251.

Campbell, John Y. 2017. *Financial decisions and markets: a course in asset pricing.* Princeton ; Oxford:Princeton University Press.

CBOE. 2019. "Cboe Options Regulatory Circular 19-032."

CBOE. 2024*a*. "Cboe U.S. Options Fee Schedules."

CBOE. 2024*b*. "OEX Options Product Specifications."

CBOE. 2024*c*. "XEO Options Product Specifications."

Chan, Kalok, Y. Peter Chung, and Wai-Ming Fong. 2002. "The Informational Role of Stock and Option Volume." *Review of Financial Studies*, 15: 1049–1075.

Chesson, Harrell W., and W. Kip Viscusi. 2003. "Commonalities in Time and Ambiguity Aversion for Long-Term Risks." *Theory and Decision*, 54: 57–71.

Chew, Soo Hong, and Larry G Epstein. 1990. "Nonexpected utility preferences in a temporal framework with an application to consumption-savings behaviour." *Journal of Economic Theory*, 50: 54–81.

Cochrane, John H. 2005. *Asset Pricing: Revised Edition.* s.l:Princeton University Press.

Cochrane, John H., and Lars Peter Hansen. 1992. "Asset Pricing Explorations for Macroeconomics." *NBER Macroeconomics Annual*, 7: 115–165.

Constantinides, George M., Jens Carsten Jackwerth, and Alexi Savov. 2013. "The Puzzle of Index Option Returns." *Review of Asset Pricing Studies*, 3: 229–257.

Daniel, Kent, David Hirshleifer, and Siew Hong Teoh. 2002. "Investor psychology in capital markets: evidence and policy implications." *Journal of Monetary Economics*, 49: 139–209.

Da Rin, Marco, Thomas Hellmann, and Manju Puri. 2013. "A Survey of Venture Capital Research." In *Handbook of the Economics of Finance*. Vol. 2, 573–648. Elsevier.

DeJarnette, Patrick, David Dillenberger, Daniel Gottlieb, and Pietro Ortoleva. 2020. "Time Lotteries and Stochastic Impatience." *Econometrica*, 88: 619–656.

Department of the Treasury. 2024. "What is the national debt?" Treasury.gov. Department of the Treasury, United States Government.

Dillenberger, David, Daniel Gottlieb, and Pietro Ortoleva. 2024. "Stochastic impatience and the separation of time and risk preferences." *Theoretical Economics*, Forthcoming.

Dueker, Michael J., and Thomas W. (Jr) Miller. 2002. "Directly Measuring Early Exercise Premiums Using American and European S&P 500 Index Options."

Duffie, Darrell. 2001. *Dynamic asset pricing theory.* . 3rd ed ed., Princeton (N.J.):Princeton university press.

Ebert, Sebastian. 2020. "Decision Making When Things Are Only a Matter of Time." *Operations Research*, 68: 1564–1575.

Epstein, Larry G., Emmanuel Farhi, and Tomasz Strzalecki. 2014. "How Much Would You Pay to Resolve Long-Run Risk?" *American Economic Review*, 104: 2680–2697.

Figlewski, Stephen. 2022. "An American Call *Is* Worth More Than a European Call: The Value of American Exercise When the Market Is Not Perfectly Liquid." *Journal of Financial and Quantitative Analysis*, 57: 1023–1057.

Finucane, Thomas J. 1997. "An empirical analysis of common stock call exercise: A note." *Journal of Banking & Finance*, 21: 563–571.

Fleming, Jeff, and Robert E. Whaley. 1994. "The Value of Wildcard Options." *The Journal of Finance*, 49: 215–236.

Gourier, Elise, Ludovic Phalippou, and Mark M. Westerfield. 2024. "Capital Commitment." *The Journal of Finance*, 79: 3407–3457.

Graham, John R. 2022. "Presidential Address: Corporate Finance and Reality." *The Journal of Finance*, 77: 1975–2049.

Hansen, Lars Peter, and Jose A. Scheinkman. 2017. "Stochastic Compounding and Uncertain Valuation." In *After the Flood: How the Great Recession Changed Economic Thought*. 21–50. Chicago:University of Chicago Press.

Hansen, Lars Peter, and Kenneth J. Singleton. 1982. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica*, 50: 1269.

Hansen, Lars Peter, and Ravi Jagannathan. 1991. "Implications of Security Market Data for Models of Dynamic Economies." *Journal of Political Economy*, 99: 225–262.

Harrison, J.Michael, and David M Kreps. 1979. "Martingales and arbitrage in multiperiod securities markets." *Journal of Economic Theory*, 20: 381–408.

Harvey, Campbell R., and Robert E. Whaley. 1991. "S&P 100 Index Option Volatility." *The Journal of Finance*, 46: 1551–1561.

Holt, Charles A, and Susan K Laury. 2002. "Risk Aversion and Incentive Effects." *American Economic Review*, 92: 1644–1655.

Ho, Thomas, and Hans R. Stoll. 1981. "Optimal dealer pricing under transactions and return uncertainty." *Journal of Financial Economics*, 9: 47–73.

Hull, John. 2018. *Options, futures, and other derivatives.* . Tenth Edition ed., New York, NY:Pearson.

Jensen, Mads Vestergaard, and Lasse Heje Pedersen. 2016. "Early option exercise: Never say never." *Journal of Financial Economics*, 121: 278–299.

Joskow, Paul, Christopher Knittel, Leonid Kogan, Andrew W. Lo, Deborah Lucas, Robert C. Merton, Gilbert E. Metcalf, John Parsons, Robert Pindyck, Richard Schmalensee, and Antoinette Schoar. 2023. "Letter to the White House Re: Proposed Revisions to Circular A-94 on Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs."

Kacelnik, Alex, and Melissa Bateson. 1996. "Risky Theories-The Effects of Variance on Foraging Decisions." *American Zoologist*, 36: 402–434.

Kreps, David M., and Evan L. Porteus. 1978. "Temporal Resolution of Uncertainty and Dynamic Choice Theory." *Econometrica*, 46: 185.

Lettau, Martin, and Sydney Ludvigson. 2001. "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying." *Journal of Political Economy*, 109: 1238–1287.

Lewellen, J, and S Nagel. 2006. "The conditional CAPM does not explain asset-pricing anomalies." *Journal of Financial Economics*, 82: 289–314.

Li, Weihan, Jin E. Zhang, Xinfeng Ruan, and Pakorn Aschakulporn. 2024. "An empirical study on the early exercise premium of American options: Evidence from OEX and XEO options." *Journal of Futures Markets*, 44: 1117–1153.

Lucas, Deborah. 2023. "Reflections on What Financial Economics Can and Cannot Teach Us About the Social Discount Rate." *Annual Review of Financial Economics*, 15: 185–195.

Lucas, Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica*, 46: 1429.

Lustig, Hanno, and Adrien Verdelhan. 2012. "Exchange Rates in a Stochastic Discount Factor Framework." In *Handbook of Exchange Rates.* . 1 ed., ed. Jessica James, Ian W. Marsh and Lucio Sarno, 391–420. Wiley.

MacKenzie, David. 2023. "Black-Scholes at 50: how a pricing model for options changed finance." *Financial Times*.

Mehra, Rajnish, and Edward C. Prescott. 1985. "The equity premium: A puzzle." *Journal of Monetary Economics*, 15: 145–161.

Merton, Robert C. 1973. "An Intertemporal Capital Asset Pricing Model." *Econometrica*, 41: 867.

Metrick, Andrew, and Ayako Yasuda. 2021. *Venture capital & the finance of innovation.* . Third edition ed., Hoboken, NJ:Wiley.

Muravyev, Dmitriy. 2016. "Order Flow and Expected Option Returns." *The Journal of Finance*, 71: 673–708.

Muravyev, Dmitriy, and Neil D Pearson. 2020. "Options Trading Costs Are Lower than You Think." *The Review of Financial Studies*, 33: 4973–5014.

Ni, Sophie X., Jun Pan, and Allen M. Poteshman. 2008. "Volatility Information Trading in the Option Market." *The Journal of Finance*, 63: 1059–1091.

Office of the Comptroller of the Currency. 2024. "Quarterly Report on Bank Trading and Derivatives Activities: First Quarter 2024."

Onay, Selcuk, and Ayse Onculer. 2007. "Intertemporal choice under timing risk: An experimental approach." *Journal of Risk and Uncertainty*, 34: 99–121.

on Foreign Relations, Council. 2023. "The State of U.S. Infrastructure." *Council on Foreign Relations. Written by James McBridge, Noah Berman, and Anshu Siripurapu.*

Pan, Jun, and Allen M. Poteshman. 2006. "The Information in Option Volume for Future Stock Prices." *Review of Financial Studies*, 19: 871–908.

Pastor, Lubos, Robert F. Stambaugh, and Lucian A. Taylor. 2024. "Carbon Burden."

Poteshman, Allen M., and Vitaly Serbin. 2003. "Clearly Irrational Financial Market Behavior: Evidence from the Early Exercise of Exchange Traded Stock Options." *The Journal of Finance*, 58: 37–70.

Puri, Indira. 2024. "Simplicity and Risk." *Journal of Finance*, Forthcoming.

Ross, Stephen A. 1978. "A Simple Approach to the Valuation of Risky Streams." *The Journal of Business*, 51: 453.

Schotter, Andrew. 2003. "Decision Making with Naive Advice." *American Economic Review*, 93: 196–201.

Seo, Sang Byung, and Jessica A. Wachter. 2019. "Option Prices in a Model with Stochastic Disaster Risk." *Management Science*, 65: 3449–3469.

Social Security Administration. 2024. "Summary: Actuarial Status of the Social Security Trust Funds." ssa.gov. Social Security Administration, United States Government.

Sprenger, Charles. 2015. "Judging Experimental Evidence on Dynamic Inconsistency." *American Economic Review*, 105: 280–285.

Stroebel, Johannes, and Jeffrey Wurgler. 2021. "What do you think about climate finance?" *Journal of Financial Economics*, 142: 487–498.

Strzalecki, Tomasz. 2023. *Decision Theory.* Cambridge, MA.

Sung, Hyun Mo. 1995. "The early exercise premia of America put options on stocks." *Review of Quantitative Finance and Accounting*, 5: 365–373.

van Binsbergen, Jules, and Ralph S.J. Koijen. 2017. "The term structure of returns: Facts and theory." *Journal of Financial Economics*, 124: 1–21.

van Binsbergen, Jules H. 2016. "Good-Specific Habit Formation and the Cross-Section of Expected Returns." *The Journal of Finance*, 71: 1699–1732.

van Binsbergen, Jules H. 2020. "Duration-Based Stock Valuation." *SSRN Electronic Journal*.

van Binsbergen, Jules, Michael Brandt, and Ralph Koijen. 2012. "On the Timing and Pricing of Dividends." *American Economic Review*, 102: 1596–1618.

Verdelhan, Adrien. 2010. "A Habit-Based Explanation of the Exchange Rate Risk Premium." *The Journal of Finance*, 65: 123–146.

Verdelhan, Adrien. 2018. "The Share of Systematic Variation in Bilateral Exchange Rates." *The Journal of Finance*, 73: 375–418.

Vissing-Jorgensen, Annette. 2002. "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution." *Journal of Political Economy*, 110: 825–853.

Wachter, J. 2006. "A consumption-based model of the term structure of interest rates." *Journal of Financial Economics*, 79: 365–399.

Woodford, Michael. 2003. *Interest and prices: foundations of a theory of monetary policy.* Princeton, N.J. ; Woodstock, Oxfordshire [England]:Princeton University Press.

Zivney, Terry L. 1991. "The Value of Early Exercise in Option Prices: An Empirical Investigation." *The Journal of Financial and Quantitative Analysis*, 26: 129.