Resolving New Keynesian Puzzles*

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Abstract

New Keynesian models generate puzzles when confronted with the zero lower bound (ZLB) on nominal rates, such as the forward guidance puzzle and the paradox of flexibility. We show that these puzzles are resolved when monetary policy maintains a consistent history-dependence before, during, and after a ZLB episode. History dependent policy is both welfare-maximizing in theoretical models and an empirically supported description of real-world monetary policy. We show how to write monetary policy rules that preserve the same history dependence and policy objectives with and without the ZLB to resolve the puzzles in any New Keynesian model.

JEL Classifications: E31; E43; E58; D83; D84.

Key Words: New Keynesian puzzles, monetary policy, zero lower bound

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1 Introduction

The New Keynesian (NK) model offers an elegant framework for understanding monetary policy and has been widely adopted by policy institutions for both forecasting and scenario analysis. However, it encounters significant challenges in producing sensible predictions when nominal interest rates are constrained by the zero lower bound (ZLB), leading to several puzzles and paradoxes.

These puzzles have two distinct dimensions. The first is a set of *quantitative puzzles* with practical implications for policymakers, in which forecasts of real-world monetary policy announcements at the ZLB produce implausible and counterfactual results. The second is a set of *limit puzzles* that call into question the underlying assumptions of NK theory. These puzzles are typified by thought experiments involving anticipated shocks when interest rates are temporarily pegged. Such experiments predict the forward guidance puzzle, exhibiting what Farhi and Werning (2019) call "anti-horizon effects," where the power of a one-off shock grows without bound as its implementation date is pushed further into the future. Anticipated shocks in these settings may also have the wrong sign – as in the case of productivity and labor supply shocks in the paradox of toil (e.g., Eggertsson, 2010) – or become more powerful when prices are assumed to be more flexible, rather than moderating, as in the paradox of flexibility (e.g., Eggertsson and Krugman, 2012; Kiley, 2016).

In this paper, we show that the NK puzzles are resolved when monetary policy is history-dependent. The standard modeling convention for both quantitative applications and theoretical thought experiments assumes either a credible commitment to no history dependence in policy or commitment to the *wrong* history-dependent policy from the perspective of a welfare maximizing central bank. It is this policy mix that generates the NK puzzles.

We show that for quantitative models a history-dependent policy is supported by data and is the default modeling assumption outside of ZLB episodes. Specifically, away from the ZLB, quantitative NK models are closed with inertial Taylor (1993)-type rules that include a lagged interest rate term. This term is often justified on the grounds of "interest-rate smoothing," where a policymaker slowly implements policy changes due to concerns about financial market stability. However, such concerns are almost universally absent within complete market NK models.[1] Instead, from the perspective of the agents that inhabit the modeled economy, it has long been known, since at least Woodford (2003b), that lagged interest rates in the policy rule are a form of history-dependent policy. In fact, we show that it is equivalent to following an interest rate rule that responds to weighted averages of past inflation and output.

The NK puzzles arise in quantitative models because history dependence in policy ends at the ZLB. Interest rates pegged at zero percent no longer encode information about past output and inflation. The ZLB, when combined with a standard inertial policy rule, does not just impose a constraint on the central bank's choice of the interest rate, it changes the implicit policy regime.[2] Specifically, policy shifts from a history-dependent one that is rationalized by

[1] See the discussion in Woodford (2003a), chap. 6.

[2] The policy regime shift interpretation is also supported by Eggertsson and Schüle (2024) who argue that the

a central bank seeking to minimize deviations in inflation and output from some targets, to a discretionary policy that we show cannot be rationalized at all under those same objectives.

In contrast, we show that if the policy rule is rewritten such that it tracks weighted averages of past inflation rather than past interest rates, then no such implicit change in policy occurs at the ZLB and no quantitative puzzles occur. When interest rates are constrained, agents are perfectly able to track and forecast how monetary policy will respond to current outcomes in the future when it is unconstrained. Maintaining the continuity of history-dependent policy before, during, and after a ZLB episode resolves the quantitative puzzles.

The intuition for why history dependence resolves these puzzles is straightforward: firms and households anticipate that if inflation or output rise while interest rates are constrained, the central bank will respond to those outcomes once policy is no longer constrained. If inflation and inflation expectations rise today because of forward guidance, fiscal policy, or a negative productivity shock, then the central bank will pursue higher nominal interest rates in the future, all else being equal. Expectations of future monetary policy to today's outcomes restrain the general equilibrium effects of monetary policy and resolve the puzzles. The NK puzzles emerge specifically because standard modeling practices implicitly assume credible commitments by policymakers to *never* respond to economic conditions when policy is constrained. It is this policy scenario that gives rise to the NK puzzles and not the anticipation of shocks (monetary policy shocks included) while at the ZLB.

We propose a novel reformulation of the monetary policy rule that tracks weighted averages of past inflation and output rather than interest rates and which implements the same equilibrium dynamics as an inertial Taylor rule when there is no ZLB constraint. In other words, from the perspective of the agents in the model, including the central bank, there is no difference in the policy objectives between our formulations of the rule and the original rule when the ZLB does not bind. In contrast, when pairing our rule with the ZLB, the history-dependence is maintained and we successfully resolve the quantitative puzzles in estimated medium-scale models of the U.S. economy.

However, the degree of history dependence estimated for U.S. monetary policy is insufficient to eliminate limit puzzles. It instead has the effect of greatly dampening these puzzles such that they do not have a quantitatively meaningful effect at business cycle horizons. To investigate the required history dependence to resolve the limit puzzles, we return to the simple New Keynesian model and to generalizations of the limit puzzle definitions proposed by Diba and Loisel (2021). We study the limit puzzle under an interest rate rule that tracks a weighted average of past inflation, where the weights govern the degree of history dependence. We show that if policy responds aggressively enough to past inflation, i.e., it more than makes up for past misses to inflation from target as prescribed by optimal commitment policy in Eggertsson and Woodford (2003), then the limit puzzles are resolved as well.

The insight that history-dependent policy resolves the NK puzzles raises important questions about how the ZLB should be modeled in quantitative applications. We argue that continuity

standard ZLB modeling actually captures well what occurred in the United States during the Great Depression.

in history-dependent policy is appropriate, since the ZLB is, by definition, a constraint on the policy instrument; not a change in policy objectives. In fact, a key device that researchers often use to avoid modeling the ZLB is a shadow rate, which seeks to trace out how the inertial policy rule would behave if policy were unconstrained.[3] We provide a novel and simple way to write standard policy rules such that the ZLB is only a constraint allowing agents within the model to endogenously form views about the path of policy. Consequently, existing models and inference over their deep parameters can capture temporary interest-rate pegs, liquidity traps, and forward guidance much more plausibly than is currently appreciated.

Another contribution of our approach is that it provides a micro-founded resolution to the NK puzzles, grounded in agent optimization within the standard model. In particular, in the same spirit as Svensson (2003), it is by taking the optimization of the monetary policymaker seriously that plausibility is restored to the model's equilibrium predictions. Of course, researchers may wish to model non-optimizing behavior or policy mistakes in some instances. However, our reading of the literature suggests that this is not the intention of most researchers studying the ZLB constraint, and it directly contradicts what most central banks communicate they are attempting to do. For example, during the ZLB episode in the U.S. between 2009 and 2015, Bordo, Istrefi and Martínez (2024) show that language used by the Federal Reserve shifted away from discretion-based language and toward commitment-based, i.e., history dependent language. Likewise, discussions of the merits of history-dependent policy featured prominently in the Federal Reserve's review that recommended average inflation targeting (AIT) as a framework better suited to communicating policy at the ZLB (see, for example, Altig, Fuhrer, Giannoni and Laubach, 2020).

Lastly, unlike many studies that attribute the NK puzzles to fundamental flaws in rational expectations or market structures, our puzzle resolution preserves these assumptions and does not require alteration to the micro-foundations of the model. Instead, we provide insight into why other alternative policy regimes resolve the puzzles without resorting to these changes such as in Cochrane (2017) or McClung (2021), Diba and Loisel (2021), and Gibbs and McClung (2023) under the fiscal theory of the price level, interest on reserve rules, or indeterminate monetary policy coordinated on a sunspot, respectively. Our insight is that they all imply a continuity of history dependence to policy before, during, and after the ZLB.

This insight then begs the question of what do we learn from the NK puzzles? We argue that they reveal that general equilibrium effects writ large in NK models are the relevant findings. Indeed, Farhi and Werning (2019) show that even in the absence of the NK puzzles, standard models continue to predict large general equilibrium effects from zero interest rate policies. Consequently, the NK puzzles do not justify abandoning rational expectations in favor of bounded rationality or imperfect knowledge approaches such as those proposed by Angeletos and Lian (2018), Farhi and Werning (2019), Gabaix (2020), and Eusepi, Gibbs and Preston (2024), or market structures as in McKay, Nakamura and Steinsson (2017). Instead, these studies should be viewed as addressing a more fundamental reconciliation between the

[3]See Wu and Xia (2016) for a description.

model and empirical data.

Related Literature. The NK puzzles literature broadly splits into two main branches, with the forward guidance puzzle being the most significant from a quantitative and policy perspective. Seminal contributions by Del Negro, Giannoni and Patterson (2012) and Carlstrom, Fuerst and Paustian (2015) illustrate the puzzle and propose resolutions. Carlstrom et al. (2015) argue that introducing sticky information can reduce or eliminate puzzles, whereas Del Negro et al. (2012) question the plausibility of policy credibility at longer horizons. Like Del Negro et al. (2012), we highlight that standard forward guidance experiments yield implausible future interest rate paths. However, our explanation differs: we attribute these implausibilities to an unintended break in the modeled policy regime.

Many papers attempt to resolve the forward guidance puzzle by modifying fundamental assumptions, such as agents' expectations, demographics, or perceptions of policy credibility. For example, Del Negro, Giannoni and Patterson (2023) introduce a perpetual youth model, which shortens the horizon over which agents form expectations, producing effects similar to assuming imperfect credibility of forward guidance. Other authors explicitly leverage variations in policy credibility. Haberis, Harrison and Waldron (2019) and Gibbs and McClung (2023) demonstrate that partial credibility of the central bank's commitment to maintaining zero interest rates significantly reduces or eliminates the puzzle. Andrade, Gaballo, Mengus and Mojon (2019) posit that some agents interpret forward guidance as worse economic conditions rather than a credible promise of stimulus, which ameliorates the puzzle.

Numerous studies resolve the forward guidance puzzle by introducing information frictions or bounded rationality, thus weakening the general equilibrium impact of expectations. One set of approaches relies on agents having limited or sticky information (Kiley, 2016). Another set employs level-k reasoning, where agents' expectations differ in sophistication (Angeletos and Lian, 2018; Farhi and Werning, 2019; Evans, Gibbs and McGough, 2025). Gabaix (2020) assumes agents exhibit myopia, shortening their effective planning horizons. Finally, Eusepi et al. (2024) assume a mix of level-1 and adaptive learning beliefs. Each of these bounded-rationality frameworks reduces the strength of expectation-driven feedback loops, thereby mitigating or eliminating the puzzle.

A further approach in the literature uses incomplete markets to address the forward guidance puzzle. McKay, Nakamura and Steinsson (2016) and Hagedorn, Luo, Manovskii and Mitman (2019) demonstrate that introducing incomplete markets into heterogeneous-agent New Keynesian models significantly mitigates the puzzle. Similarly, Eggertsson, Mehrotra and Robbins (2019) find no puzzle in overlapping generations models with debt constraints. However, the robustness of incomplete-markets approaches is questioned by Farhi and Werning (2019) and Bilbiie (2020), who argue these models resolve the puzzle only under specific assumptions or parameterizations, limiting their general applicability.

A second, smaller branch of the literature addresses other NK puzzles, specifically those concerning fiscal multipliers, toil, and flexibility. The fiscal multiplier puzzle closely resembles the forward guidance puzzle, arising because anticipated fiscal expansions have disproportion-

ately large effects at the zero lower bound. Hills and Nakata (2018) propose resolving this puzzle by employing a shadow interest rate that encodes the appropriate history dependence of policy.[4] Bonciani and Oh (2023) show that using a shadow rate can similarly mitigate the paradox of flexibility. Notably, these other puzzles are generally subordinate to the forward guidance puzzle: resolving forward guidance issues typically addresses the other puzzles as well, but not necessarily the other way around. Empirically, Wieland (2019) finds no support for the paradox of toil, aligning with our broader argument that standard NK adaptations at the zero lower bound inadequately capture real-world monetary policy.

Finally, recent literature has questioned the very existence of the forward guidance puzzle. Eggertsson and Schüle (2024) argue, as we do, that the standard forward guidance experiment effectively represents a monetary policy regime change. They provide empirical evidence suggesting such regime changes resemble conditions seen during the Great Depression. However, their analysis does not quantitatively explain why the puzzle is notably absent in recent ZLB episodes. Our paper complements and extends their analysis by explicitly demonstrating that the puzzle disappears without regime changes and addresses related puzzles not covered in their work. Alternatively, Bianchi, Melosi and Nicoló (2024), like us, note that if expectations of future monetary policy actions are significantly reactive to current outcomes at the ZLB, then the forward guidance puzzle is resolved. However, they take a Markov-switching approach where reactions by the central bank occur probabilistically and exogenously rather than dependent on history. In contrast, we show how to make the anticipated policy response endogenous and how to implement it in standard modeling environments.

Outline. Section 2 shows how reformulating the standard monetary policy rule in an NK model can resolve the quantitative puzzles. Section 3 revisits standard optimal policy results from the NK literature to establish the disconnect between conventional monetary policy rules and optimal policy at the ZLB, which leads to the puzzles. Section 4 explores how much history dependence is required to eliminate the limit puzzles. Section 5 discusses the broader implications of our results.

2 Resolving New Keynesian Quantitative Puzzles

Quantitative NK models are typically closed with a rule of the following form:

$$i_t = (1-\rho_i)\bar{r} + \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t), \tag{1}$$

where i_t is the policy rate, π_t is inflation (assuming a zero percent inflation target), y_t is a measure of real activity (usually the output gap), and $\bar{r}$ is the steady state real interest. The parameters ϕ_π and ϕ_y capture the responsiveness of the central bank to current inflation and real activity, respectively, while ρ_i captures a preference for gradual policy adjustment.[5]

[4] However, Krippner (2020) highlights other limitations of using shadow rates to capture zero lower bound dynamics fully.

[5] The discussion here is based on the study of structural monetary models that are solved by standard first-order approximations using piece-wise solutions to capture the non-linearity at the ZLB. This assumption

History dependence is communicated here through $0 < \rho_i < 1$, which encodes past movements in inflation and output shaping expectations of future policy. Hence, standard Taylor-type rule (1) captures a form of history dependence. However, this is not the case when the rule is adapted to the ZLB.

To incorporate the ZLB, most researchers retain rule (1) and simply consider the following modification:

$$i_t = \max\left\{0, (1-\rho_i)\bar{r} + \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t)\right\}. \tag{2}$$

We argue that this formulation is inconsistent with the macroeconomic environment that most quantitative models are attempting to capture. This rule removes any history dependence to policy because lagged interest rates no longer capture past economic conditions once the ZLB is binding. Regardless of what occurs at the ZLB under rule (2), the monetary policymaker credibly commits to not respond to it. Bygones are bygones in the most strict sense when a model is closed with an interest rate rule like (2).

To understand why rule (2) represents policy regime change, consider a (seemingly) equivalent representation of rule (2):

$$i_t = \max\left\{0, \bar{r} + \phi_\pi \omega_t^\pi + \phi_y \omega_t^y\right\}, \tag{3}$$

where

$$\begin{aligned} \omega_t^\pi &= \omega_{t-1}^\pi + (1-\rho_i)(\pi_t - \omega_{t-1}^\pi) \\ \omega_t^y &= \omega_{t-1}^y + (1-\rho_i)(y_t - \omega_{t-1}^y) \end{aligned}$$

are weighted averages of past inflation and output (henceforth, we refer to rule (3) as a *weighted average rule*). Models with either rule (2) or rule (3) implement the same equilibrium outcomes in standard linearized economies when the ZLB never binds.[6] However, the weighted average rule (3) makes clear the degree of history dependence that is implicitly assumed in the inertial rule (2). The central bank responds to geometric averages of past inflation and output. High weight is placed on observations that occurred far in the past when ρ_i is large. Quite naturally, the history that policymakers are responding to is that of past inflation and output, consistent with real world statutory mandates.

Rule (2) and rule (3) make very different predictions for policy at the ZLB. Under the inertial rule (2), setting $i_t = 0$ deletes the central bank's objectives from the model at the ZLB. In addition, upon lift off of the interest rate, rule (2) does not return to approximating an

captures nearly the universe of models that are used in central banks and the simple environments in which the forward guidance policy is usually studied. Eggertsson and Singh (2019) show considering the non-linear environments does not eliminate the puzzles. In other words, the NK puzzles are not an artifact of approximation method.

[6]The equivalence holds for the rational solution of a first-order approximation of the economy in log-deviation from steady state form when initial conditions are the same, $\bar{r}$ is constant, and there is no monetary policy shock. Monetary policy shocks obviously propagate differently in equilibrium between these two rules. Likewise, if $\bar{r}$ tracks the natural rate shock, then the rules implement different policies, which is discussed in detail in Section 3.

optimizing central bank. Instead, the central bank down-weights its response to current and lagged output and inflation. Under the weighted average rule (3), however, exogenously setting $i_t = 0$ does not change what the central bank fundamentally cares about in the model. It simply implements exactly what the ZLB represents: a constraint on the choice of the instrument of policy. The central bank and the private sector do not lose their ability to track the evolution of output and inflation during the period interest rates are constrained; nor does the private sector lose its ability to formulate expectations about how policymakers will respond to current misses of their targets in the future.

2.1 Quantitative Implications

We now demonstrate the effect of rewriting the standard inertial policy rule as a weighted average rule using the standard model of Smets and Wouters (2007) by providing real-time out-of-sample forecasts of the effect of the following real-world Federal Reserve policy announcement:

> *The Committee currently anticipates that economic conditions ... are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013.*
>
> - FOMC Statement August, 9^{th} 2011

We interpret this policy as a credible commitment to hold interest rates at the ZLB for seven quarters and thereafter we enforce the ZLB only if shocks justify it. We take the posterior distribution from the original Smets and Wouters (2007) paper estimated from data through 2004 as given. No revised data from beyond 2004 is used in inference. We hold everything fixed apart from the form of the policy rule. We convert the monetary policy rule in the model given by

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_x x_t\right) + \phi_{dx}(x_t - x_{t-1}) + \epsilon_{r,t} \tag{4}$$

to a weighted average form given by

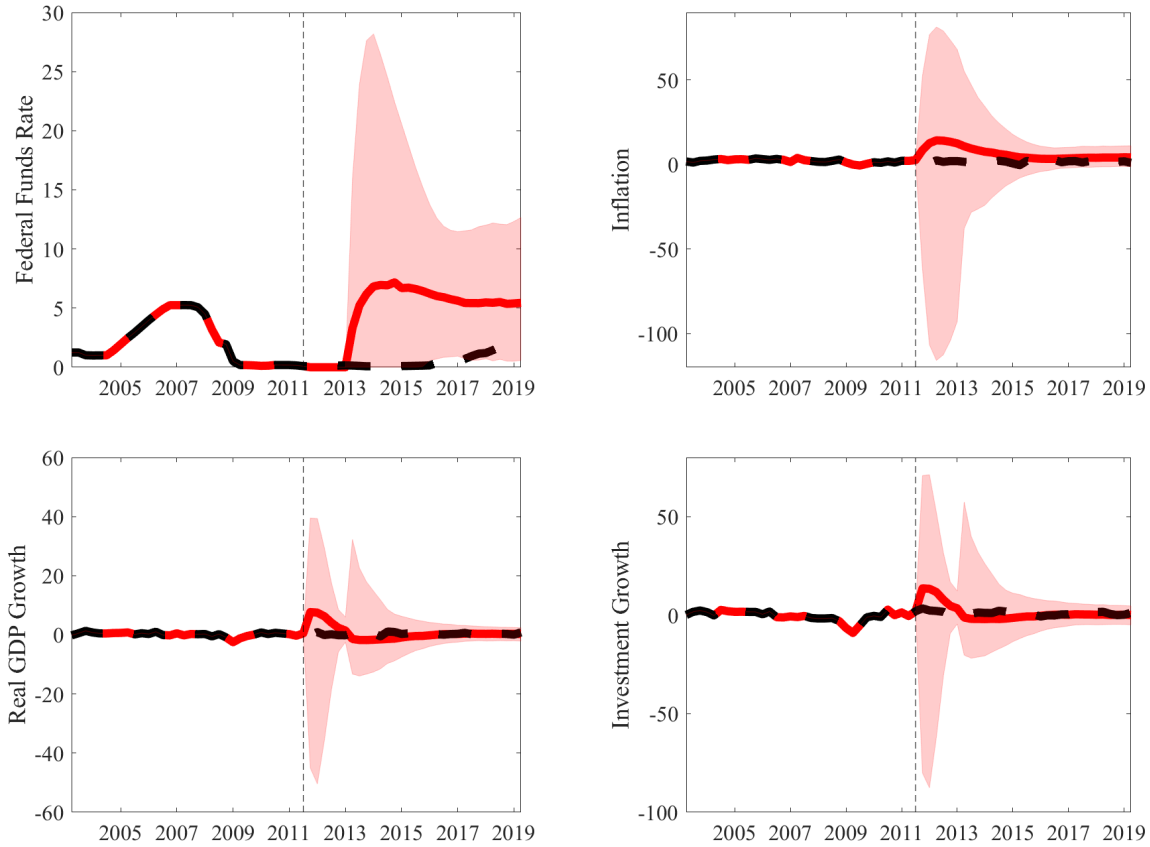
$$\begin{aligned} i_t &= \phi_\pi \omega_t^\pi + \phi_x \omega_t^x + \phi_{dx}\omega_t^{dx} + \omega_t^{\epsilon_r} \\ \omega_t^\pi &= \omega_{t-1}^\pi + (1-\rho_i)(\pi_t - \omega_{t-1}^\pi) \\ \omega_t^x &= \omega_{t-1}^x + (1-\rho_i)(x_t - \omega_{t-1}^x) \\ \omega_t^{dx} &= \rho_i \omega_{t-1}^{dx} + (x_t - x_{t-1}) \\ \omega_t^{\epsilon_r} &= \rho_i \omega_{t-1}^{\epsilon_r} + \epsilon_{r,t} \end{aligned} \tag{5}$$

where x_t is the output gap.

Figure 1 shows the forecasts under the inertial rule of the calendar-based forward guidance.[7]

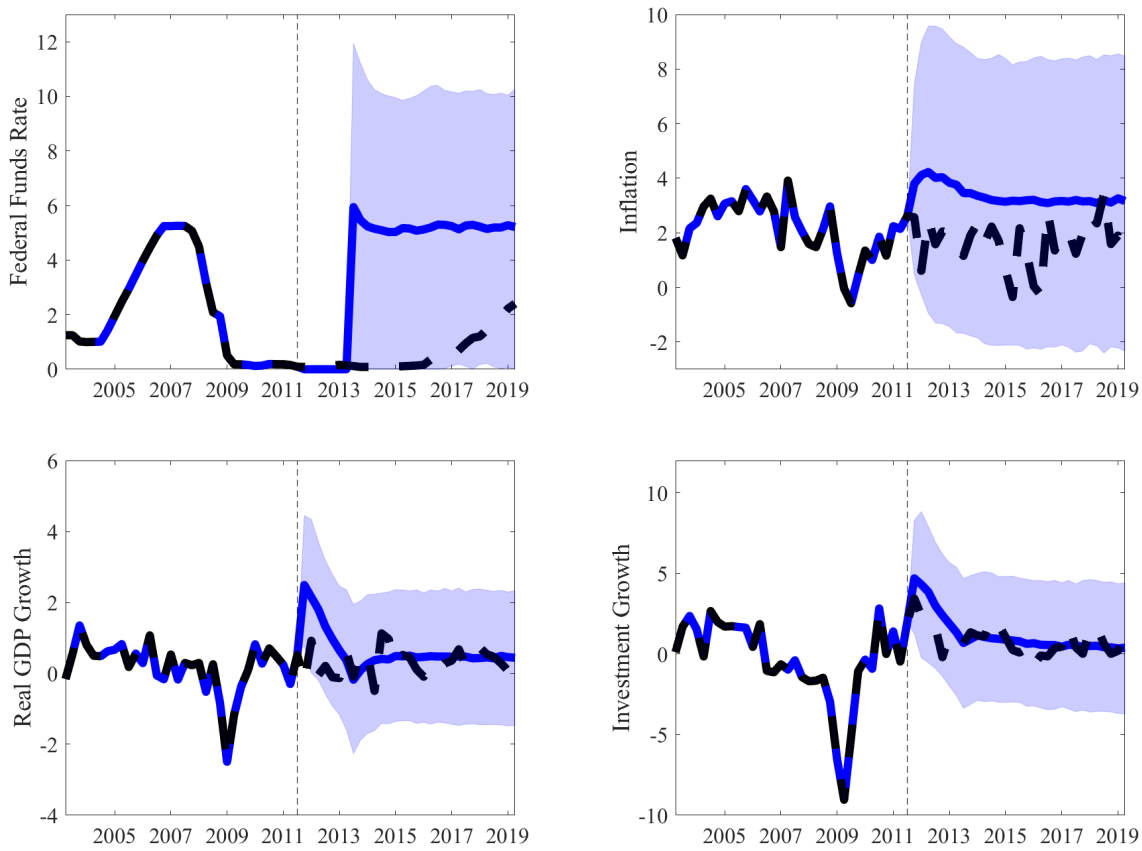
[7]The forecasts are constructed using Smets and Wouter's original replication files and data (which ends in 2004) to estimate the posterior distribution of the structural parameters. We then follow Cagliarini and Kulish (2013), Jones (2017), Kulish, Morley and Robinson (2017), and Kulish and Pagan (2017) to generate forecasts by sampling from the estimated posterior distribution of the parameters while enforcing a fully credible and known policy of zero interest rates for seven quarters. After seven quarters, monetary policy is governed by an occasionally binding constraints algorithm, where the policy rate is the maximum of zero or the interest rate implied by the model's interest rate rule.

Figure 1: Calendar-based forward guidance and the forward guidance puzzle



Notes: Out-of-sample forecasts of the August, 9^{th} 2011 Federal Reserve calendar-based forward guidance promise using the model of Smets and Wouters (2007). Inflation and the federal funds rates are presented in annual terms. The dashed line is the actual data. The solid line is the median forecast. The shaded region represents the 84th and 16th percentiles of forecasts. Inflation and the federal funds rates are annualized.

Figure 2: Calendar-based forward guidance without the forward guidance puzzle



Notes: Out-of-sample forecasts of the August, 9^{th} 2011 Federal Reserve calendar-based forward guidance promise using the model of Smets and Wouters (2007) with the interest rate rule written in the form of (3). Inflation and the federal funds rates are presented in annual terms. The dashed line is the actual data. The solid line is the median forecast. The shaded region represents the 84th and 16th percentiles of forecasts. Inflation and the federal funds rates are annualized.

The median forecasts for inflation, output growth, and investment each increase by more than 10% within a few quarters of the announcement with significant probability placed on the impacts ranging from +50% to -50%. Given these forecasts, it is not surprising that some of the first papers documenting the forward guidance puzzle came from within the Federal Reserve System. For example, the work by Del Negro et al. (2012) or Carlstrom et al. (2015), which to our knowledge are among the first papers to study the issues in Figure 1.

Figure 2 shows how the out-of-sample forecasts from Figure 1 change when we transform the interest rate rule of the Smets and Wouters model to the weighted average form. The forecasts no longer explode.[8] The responses are large, but quite sensible. Forward guidance here provides a significant boost to the economy, larger than what occurred on average, but not unreasonable given that the policy is assumed perfectly credible in the forecasts. For empirically relevant exercises like this one, the quantitative dimension of the forward guidance puzzle is absent at the posterior estimates of the model parameters when the weighted average rule is used to capture policy.

[8]Note that the forecasts also no longer exhibit the reversal puzzle (Carlstrom et al., 2015), where different parameterizations of the model can cause forecasts to flips signs.

The reason forecasts now look sensible is that the transmission of anticipated shocks is completely changed with history dependent policy ameliorating all of the NK puzzles. Figure 3 illustrates. It shows the impulse responses for anticipated monetary policy and productivity shocks. The first row of figures shows the impulse responses for output to a one-off 100-basis point monetary policy shock anticipated to occur either seven quarters (left) or forty quarters (right) in the future, where in the interim period the policy rate is pegged to steady state. The bottom row of figures shows the impulse responses for one-off one standard deviation productivity shock at the same horizons and with the same interim peg of the policy rate.[9]

The inertial rule (4) impulse responses are shown in red and display the forward guidance puzzle, the paradox of toil, and the paradox of flexibility. Evidence of the paradox of flexibility is the significant variance in the magnitude of the responses for different draws from the posterior, which we demonstrate further in Figure 4. The other endogenous variables have similar responses and are provided in the appendix.

The weighted average rule responses shown in blue behave very differently compared to the inertial rule responses. The quantitative aspects of all of the puzzles are resolved for the seven quarter scenario. However, the limit puzzles start to appear for the 40 quarter scenario. The response of output to the anticipated shocks increases in both cases when the shock is expected to occur farther into the future. However, the level of history dependence estimated in the data significantly dampens the effects of the puzzles such that they are much less relevant for policy at business cycle frequencies.

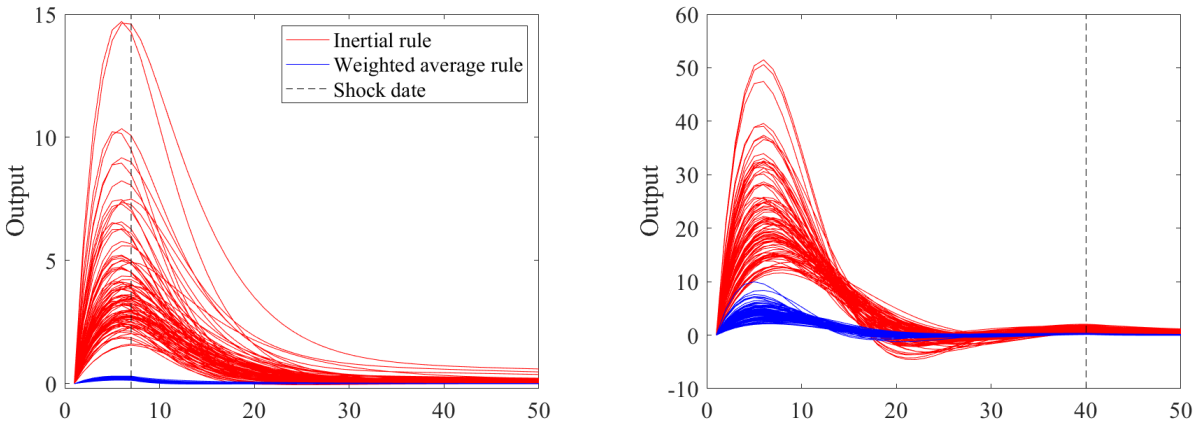
Figure 4 illustrates the fiscal multiplier puzzle and the effects of price flexibility on the outcomes. The figures plot the impulse response for output to an anticipated government spending shock seven quarters (left) and forty quarters (right) in the future. Responses for the remaining variables are shown in the appendix. The impulse responses are constructed using the median values of the posterior distributions of the deep parameters with the exception of ρ_i and ξ_p (the Calvo price reset probability). We set ξ_p to 0.1 (solid), 0.6 (dashed), or 0.9 (dotted) to demonstrate how price flexibility impacts the outcomes where lower values indicate more frequent price adjustment. In addition, we set $\rho_i = 0.99$ to better draw out the differences in outcomes due to history dependence, which are less pronounced for government spending shocks relative to the shocks shown in Figure 3. We again see that history dependence resolves the quantitative puzzles at business cycle frequencies with lower impacts of the anticipated shock under the weighted average rule and a lessening of the impact as prices become more flexible in the seven quarter case. However, the limit puzzles remain in the forty quarter impulse responses even for the very large value of ρ_i that we have chosen for this example.

Figures 2, 3, and 4 demonstrate that the standard model with policy modeled as a weighted average rule no longer exhibits the quantitative puzzles. In the next section, we show that this is not a fluke, it is consequence of the micro-foundations of the model, and illuminates how to eliminate the limit puzzles as well.

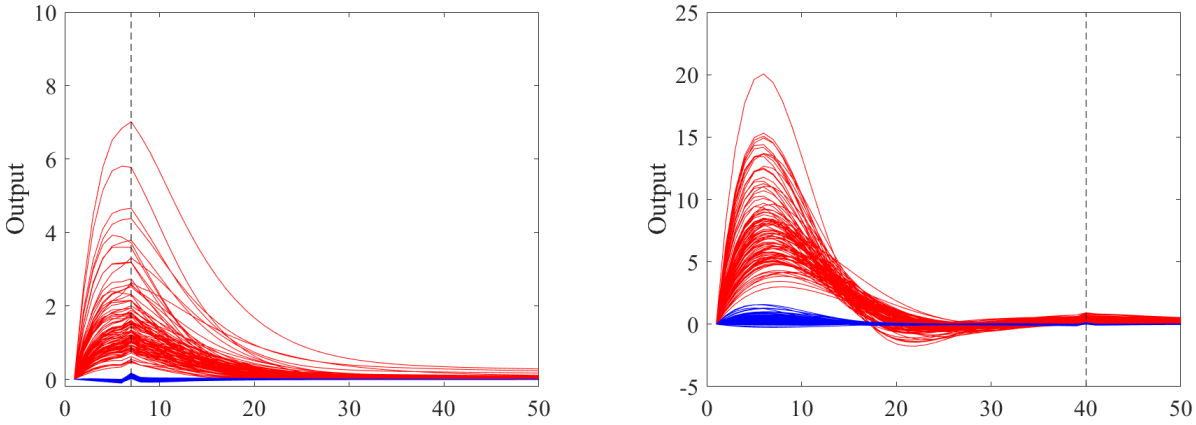
[9] We use the same size productivity shock for each impulse response (median estimate) shown in the bottom row.

Figure 3: The inertial rule vs the weighted average rule

Anticipated monetary policy shock



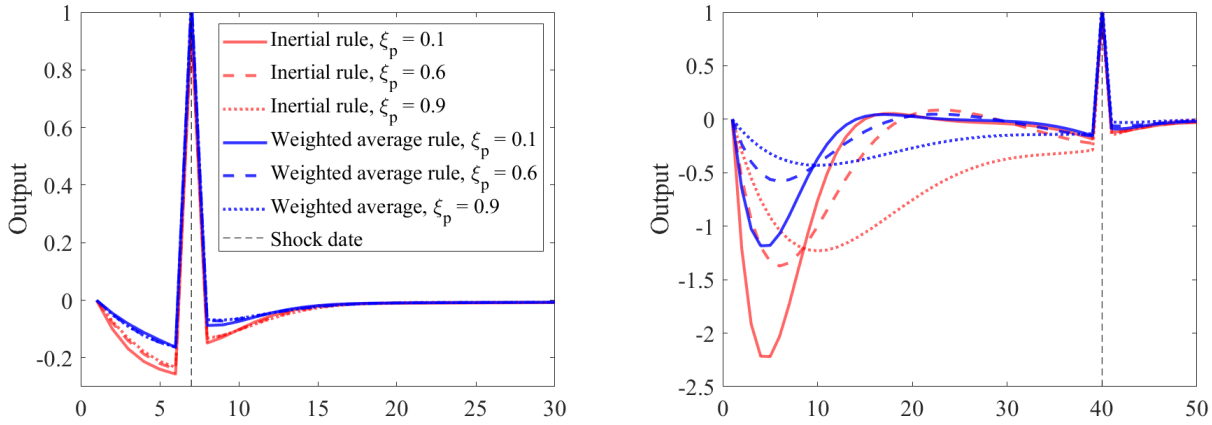
Anticipated negative productivity shock



Notes: The black dashed line indicates when the shock occurs. Each solid line corresponds to the impulse response implied by one draw from the posterior distribution of parameters for a one-time 100-basis point monetary policy shock anticipated to occur seven or forty quarters in the future under an interest rate peg (top) or a one-time one standard deviation negative productivity shock anticipated to occur seven or forty quarters in the future under an interest rate peg (bottom). In period 8 (41) policy returns to either the inertial rule or the weighted average rule. Neither shock has persistence.

Figure 4: An inertial rule vs a weighted average rule

Anticipated goverment spending policy shock



Notes: The black dashed line indicates when the shock occurs.

3 History dependence and optimal policy

This section establishes that history dependence to policy is not an ad hoc resolution. It is the micro-founded solution.

3.1 Classic optimal policy in the NK model and inertial rules

Consider the standard optimal monetary policy problem with commitment from the timeless perspective described by Clarida, Gali and Gertler (1999) or Woodford (2003a). The central bank seeks to maximize household welfare by committing to a policy to offset shocks to the real interest rate and to markups. The central bank seeks to minimize

$$\min_{\pi_t, y_t, i_t} \left\{ \frac{1}{2} E_t \sum_{T=t}^{\infty} \beta^{T-t} (\pi_t^2 + \alpha y_t^2) \right\}, \tag{6}$$

which is a quadratic approximation to household utility where α is the weight given to variation in output relative to inflation. Alternatively, we may view the loss function as that of an inflation targeting central bank with a dual mandate. The central bank seeks to minimize deviation of inflation from a target ($\bar{\pi} = 0$ in this case) while considering the output costs when implementing policy. We assert that most modelers roughly have in mind an objective of the form of (6) whenever they write down a reduced form monetary policy rule. The tacit assumption in the literature is that Taylor-type monetary policy rules approximately implement policy that minimizes (6).

That assumption is true when the ZLB is not present. Woodford (2001) shows this for non-inertial interest rate rules. Woodford (2003b) and Giannoni and Woodford (2005) show it is true of inertial interest rate rules that respond to lagged interest rates. It is not true, however, when the ZLB is considered. To understand why, consider first the optimal policy problem of a central bank that seeks to minimize (6) taking as given the first order conditions for household's and firm's decisions in the absence of the ZLB:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^n) \tag{7}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \mu_t \tag{8}$$

where Equations (7) and (8) are the standard NK IS and Phillips curves log-linearized around a zero inflation steady state, i_t is the policy rate, σ^{-1} is the intertemporal elasticity of substitution, r_t^n is a potentially autoregressive real interest rate shock with mean $\bar{r}$, β measures the rate of time preference, κ is composite parameter capturing the degree of price rigidity, and μ_t is a potentially autoregressive cost push shock.

The optimal target criterion that minimizes the central bank's loss function (6) from the timeless perspective is

$$y_t - y_{t-1} = -\frac{\kappa}{\alpha} \pi_t. \tag{9}$$

However, it is illustrative to study a refinement to the target criteria: the *unconditional* opti-

mal target criteria proposed by Jensen and McCallum (2002) and Blake (2001). The former shows numerically and the latter analytically that welfare on average is improved when time discounting by the central bank is ignored resulting in the targeting criteria:

$$y_t - \beta y_{t-1} = -\frac{\kappa}{\alpha}\pi_t. \tag{10}$$

This criterion is convenient because we can write this as

$$(1 - \beta L)y_t = -\frac{\kappa}{\alpha}\pi_t$$

where L is the lag operator. The inverse of $(1-\beta L)$ exists provided $|\beta| < 1$. Therefore, we can write

$$y_t = -\frac{\kappa}{\alpha}\frac{\pi_t}{1-\beta L}.$$

Proposition 1 (Two ways to implement optimal policy)**.** *The unconditional optimal target criterion (10) may be implemented by either of the following interest rate rules*

$$\textit{Optimal Rule 1} \quad \begin{cases} i_t &= \frac{\sigma\kappa}{\alpha(1-\beta)}\omega_t^\pi + \sigma E_t y_{t+1} + E_t\pi_{t+1} + r_t^n \\ \omega_t^\pi &= \omega_{t-1}^\pi + (1-\beta)(\pi_t - \omega_{t-1}^\pi) \end{cases} \tag{11}$$

$$\textit{Optimal Rule 2} \quad i_t = \beta i_{t-1} + \frac{\sigma\kappa}{\alpha}\pi_t + (1-\beta L)\left(\sigma E_t y_{t+1} + E_t\pi_{t+1} + r_t^n\right) \tag{12}$$

From Proposition 1, there are two things to note. First, optimal policy may be implemented with either a rule that includes lagged interest rates like an inertial rule (1), or one that includes a weighted average of past inflation like our weighted average rule (3). Either formulation implements the same policy when the ZLB is not present. Second, history dependence is more complicated in the second formulation (12) than in the first (11). To properly implement optimal policy, the lagged interest rate is not a sufficient statistic for history. Past forecasts and shocks are also required. In contrast, past inflation is all that is required to implement the rule (11).

Consider what this means for approximating optimal policy with a rule like (1):

$$i_t = (1-\rho_i)\bar{r} + \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi\pi_t + \phi_y y_t),$$

versus a rule like (3):

$$\begin{aligned} i_t &= \bar{r} + \phi_\pi\omega_t^\pi + \phi_y\omega_t^y \\ \omega_t^z &= \omega_{t-1}^z + (1-\rho_i)(\pi_t - \omega_{t-1}^z) \text{ for } z = \pi \text{ or } y \end{aligned}$$

when the central bank is faced only with demand shocks. Both rules fail to deliver the Blanchard and Galí (2007) Divine Coincidence, i.e., perfectly stabilizing output and inflation in response to r_t^n. Each rule only responds to r_t^n indirectly. However, only (3) offers a simple modification that engineers this result. Replace $\bar{r}$ with r_t^n in the weighted average inflation rule and it more closely approximates (11). However, do this substitution in the second rule (1) and it actually

worsens the approximation to optimal policy.

Why does the approximation to optimal policy worsen for Rule (1) when $\bar{r}$ is replaced with r_t^n? Because the lagged interest rate encodes the wrong history dependence when policy responds to r_t^n. The additional lag of the shock and expectations in (12) are there to undo the wrong history dependence that is encoded in lagged interest rates when responding to demand shocks.

The natural question here is if lagged interest rates are an issue, then why do the puzzles occur even when the policy rule includes no history dependence? Such as when a rule like

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t \tag{13}$$

is considered. The answer is that history dependence is an integral part of any sensible policy at the ZLB. The wrong history dependence or the omission of history dependence to policy is the problem.

Eggertsson and Woodford (2003) show that the optimal target criterion when $i_t \geq 0$ must be satisfied is a time-varying price level target:

$$\begin{aligned} \tilde{p}_t &= p_t^* \\ \tilde{p}_t &= p_t + \frac{\alpha}{\kappa} x_t \\ p_{t+1}^* &= p_t^* + \beta^{-1} \left((1 + \kappa\sigma^{-1}) - L \right) (p_t^* - \tilde{p}_t), \end{aligned}$$

where $\tilde{p}_t$ adjusts according to past misses in that target $(p_t^* - \tilde{p}_t)$ and L is again the lag operator. This criterion requires the central bank to be extremely history dependent. The central bank should not just be a price level targeter – perfectly make-up for past misses. It should promise to permanently overshoot on the price level in response to shocks that cause the ZLB to bind – more than make-up for past misses.

Implementing or approximating the optimal target criterion using an interest rate rule implies forward guidance. A central bank that follows an inertial rule (2) or a weighted average rule (3) may announce how long they intend to hold interest rates at zero in response to a shock. But that promise is explicitly paired with a promise to return to an interest rule that includes how policy will adjust to what occurred during the ZLB episode. Inertial policy rules (2) or the simple rule (13) promise to not respond at all to what has occurred during the ZLB episode. The absence of any promise is what generates the NK puzzles.

3.2 History dependence and forward guidance policy: an illustrative example

To illustrate the impact of either the wrong or an omission of history dependence to policy, we proceed in two steps. First, we replicate the ZLB thought experiment explored by Eggertsson and Woodford (2003) and Eggertsson, Egiev, Lin, Platzer and Riva (2021) of optimally responding to a real interest rate shock of uncertain duration that causes the ZLB to bind. The optimal policy to this shock implies a state-contingent forward guidance promise, where the number of quarters of zero interest policy is indexed by the expected duration of the shock.

Second, we use the optimal state-contingent forward guidance promise to simulate outcomes to the same shock under a promise that policy returns to either the inertial rule (2) or the weighted average rule (3), rather than optimal policy. We then compare the equilibrium outcomes.

The real interest rate shock follows a two-state reducible Markov process. In period one, the shock occurs and $r_t^n = r_S < 0$. We call this state S. The shock remains in effect in each period with probability $(1 - \delta)$. With complementary probability δ, the real interest rate returns to steady state, $r_t^n = r_N = \bar{r} > 0$. We call this state N. The central bank and the private sector understand the shock process. The central bank responds in the period the shock occurs with a state-contingent forward guidance promise, where for every possible duration of the shock, $\tau = 1, 2, 3, ...$, the central bank provides a promised duration of additional periods of zero interest rate policy in state N, e.g., $k_\tau = \{1, 2, 2, 3, 3, 4, ...\}$, which generates the appropriate initial conditions for optimal policy forever after.

Figure 5 shows the equilibrium outcomes under optimal policy compared to the approximations using either the inertial rule (2) or the weighted average rule (3) for realizations of the shocks lasting one through ten quarters. For comparison purposes, we have highlighted the paths associated with a shock lasting four quarters. We use the same calibration as Eggertsson and Woodford (2003) with $\beta = 0.99$, $\sigma = 2$, $\kappa = 0.02$, $r_N = 0.1$, $r_S = -0.005$ and $\delta = 0.1$.[10] For the policy rules, we assume $\rho_i = 0.8$, $\phi_\pi = 1.5$, and $\phi_y = 0.5$. Lastly, we set $\alpha = \kappa\sigma/\phi_\pi$ so that the weight placed on stabilizing inflation relative to output is comparable to the standard interest rate rule coefficients used in empirical Taylor-rules.[11] By construction, the forward guidance policy is the same under all three specifications. The only difference is the policy pursued after the interest rate lift off from zero.

The equivalence of the inertial rule (2) and the weighted average rule (3) is broken. The two rules generate extremely different equilibrium dynamics and welfare outcomes in response to the same shock and forward guidance policy. The weighted average rule continues to approximate optimal policy, while the inertial rule exhibits the forward guidance puzzle.

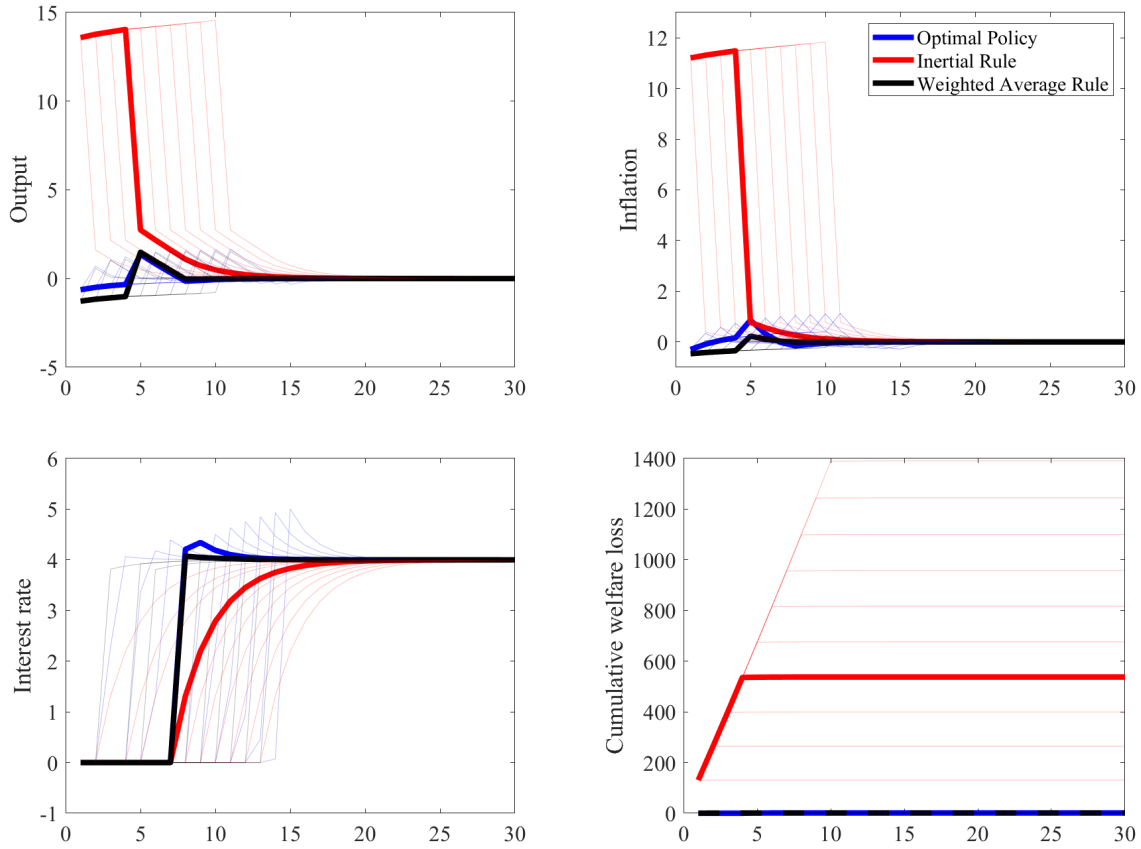
To understand the economics of the different outcomes, compare the highlighted paths of the interest rate for a shock that lasts four quarters in the bottom left panel of Figure 5. Optimal policy calls for a return to the neutral rate two quarters after liftoff. The weighted average rule returns policy to neutral in about eight quarters after liftoff. The inertial rule, however, does not return policy to neutral for more than 24 quarters (six years) after liftoff. Moreover, this policy is pursued despite the fact it is known with certainty that no further shocks will occur. Policymakers are promising to systematically make errors for years because policy is dependent on past interest rate realizations rather than past inflation and output realizations. Policymakers here have explicitly abandoned their dual mandate.

Figure 6 further illustrates the role of that history dependence plays. Here we plot the time zero expected paths for output under the inertial and weighted average rules for different

[10] The method is based on the original solution algorithm proposed by Eggertsson and Woodford (2003) and described more recently in Eggertsson et al. (2021).

[11] The same conclusions hold between the standard rules and optimal policy if we set $\phi_\pi = \kappa\sigma/\alpha$ and α at its welfare theoretic value. The welfare theoretic value of α is $\alpha = \kappa/\theta$ with $\theta = 7.87$.

Figure 5: Standard rules versus optimal rules without the ZLB



Notes: Optimal state-contingent forward guidance paired with different policy rules upon liftoff from the ZLB. Each line corresponds to a different realization of the Markov shock process. The outcomes for a shock lasting four quarters are highlighted to make comparisons across specifications easier. Inflation and interest rates are expressed in annual terms.

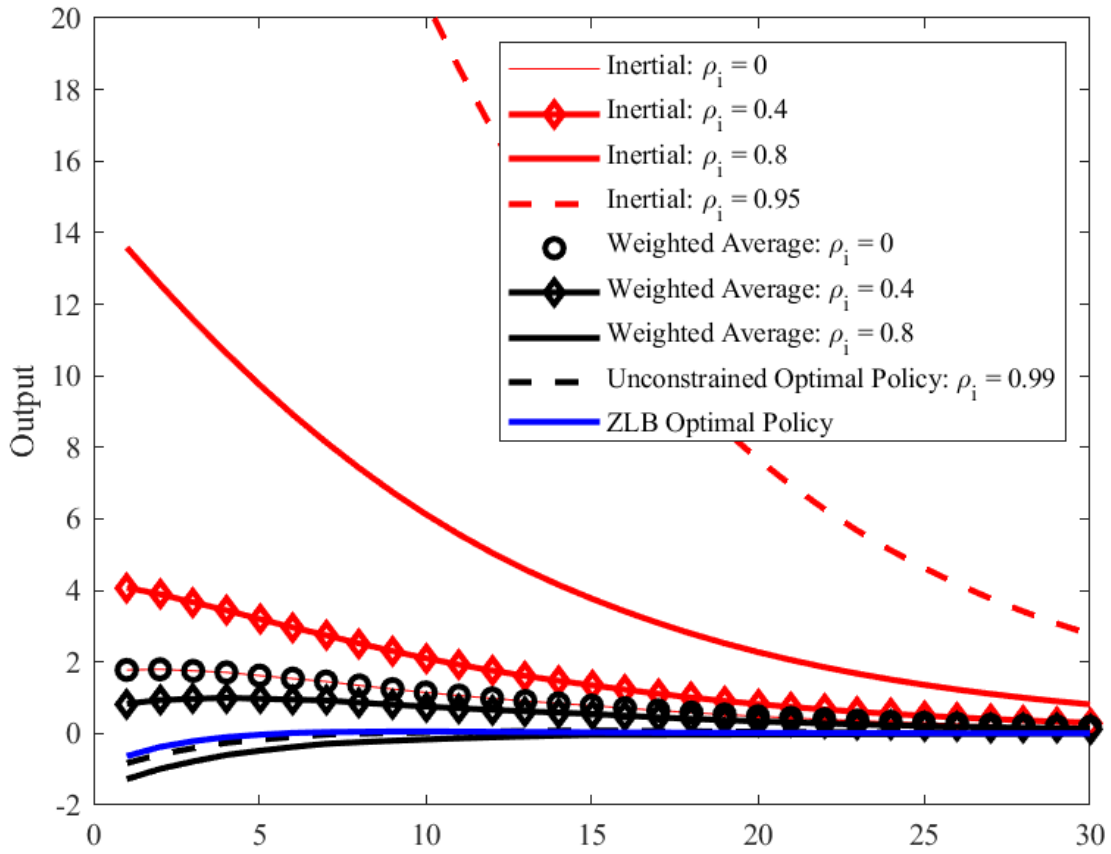
values of ρ_i. That is we simulate each possible realization of the shock and then weight each realization by the probability it occurs to form the expected path of output at the time the shock and the policy are known. We focus on output because it shows how much the constraint on the central bank's choice is relaxed in each scenario. Inflation dynamics behave similarly in each case.

The solid blue line shows the outcomes under optimal policy. The dashed black line that is closest to it corresponds to the weighted average rule with $\rho_i = 0.99$, which matches the history dependence in the unconditional target criterion and closely replicates a price level target. As we decrease ρ_i, the approximation of the policy rule to optimal policy is worse and the expected outcomes for output diverge.

The opposite relationship between ρ_i and approximating optimal policy occurs for the inertial rule. Less history dependence (ρ_i closer to zero) generates outcomes closer to optimal policy. When $\rho_i = 0$, the weighted average rule and the inertial rule are the same and we have the simple rule (13). Inertial rules encode the wrong history dependence at the ZLB delivering better outcomes with *less* history dependence.

A further implication of Figure 6 is that the stabilization outcomes possible under optimal policy are not driven by the forward guidance puzzle. Gibbs and McClung (2023) provide a

Figure 6: Approximating optimal policy as a function of history dependence



Notes: Each line corresponds to the time zero expectation of the path of output under the optimal state-contingent forward guidance promise paired with the indicated policy rule in the legend.

sufficient condition to rule out the forward guidance puzzle in equilibrium, and it is straightforward to numerically verify that the optimal commitment policy satisfies this condition under standard calibrations. In other words, the prediction under optimal commitment policy that the economy can be nearly perfectly stabilized in response to an arbitrarily large demand shock using forward guidance does not rely on the presence of the forward guidance puzzle.

4 Resolving New Keynesian Limit Puzzles

We now turn to quantifying how much of the *right* history dependence is required to resolve the limit puzzles. We do so by adopting the definitions of the NK puzzles put forward by Diba and Loisel (2021) and by analyzing the consequences of following a weighted average rule with different values for ρ_i under the limit puzzle thought experiment. We endeavor to follow Diba and Loisel (2021)'s analysis closely, as we believe they provide the most concrete treatment of the puzzles in the literature. We depart from their approach to introduce one aspect of generality. They focus solely on anticipated shocks when interest rates are pegged. We show that, in fact, all that is required is that monetary policy is passive. The standard thought experiment of a promised temporary interest rate peg is just an extreme form of passive policy.

To accommodate Diba and Loisel (2021)'s definitions, we modify the NK model defined by equations (7) and (8) in several ways. First, we remove the cost-push shock from the model and replace it with a supply shock that captures variation in marginal cost due to changes in labor supply (a_t). This shock allows us to explore the paradox of toil. Second, we introduce a government spending shock that may affect both supply and demand in the economy (g_t). This shock allows us to examine the fiscal multiplier puzzle. Finally, we assume that all shocks are i.i.d. and lack exogenous persistence. The NK model is now

$$y_t = E_t y_{t+1} - \sigma^{-1}\left(i_t - E_t\pi_{t+1} - r_t^n\right) + g_t - E_t g_{t+1} \tag{14}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\left(y_t - \delta_g g_t - a_t\right). \tag{15}$$

The thought experiment is as follows. Consider the effect of an anticipated shock that is known at time $t = T$ but occurs at time $t = T^* > T$. During the period $t = T, \ldots, T^*$, monetary policy is assumed to be passive. For $t > T^*$, monetary policy returns to an active regime. What is the effect of the shock at time T as $\Delta_p := T^* - T \to \infty$? In other words, what is the following limit?

$$\lim_{\Delta_p \to +\infty} \partial z_T / \partial X_{m,T+\Delta_p} \text{ where } z \in \{\pi, y\} \text{ and } X_m \in \{i^*, g\}.$$

where X_m is an anticipated monetary or fiscal shock.

Definition 1 (forward guidance puzzle) *When the policy rate is expected to be set passively during the next $\Delta_p > 0$ periods, the response of current inflation and output to an expected*

policy-rate shock Δ_p periods ahead, $\bar{i}_{t+\Delta_p}$, goes to plus or minus infinity with Δ_p, i.e.,

$$\lim_{\Delta_p \to +\infty} |\partial z_T / \partial \bar{i}_{T+\Delta_p}| = \infty \text{ where } z \in \{\pi, y\}.$$

Definition 2 (fiscal multiplier puzzle) *When the policy rate is expected to be set passively during the next $\Delta_p > 0$ periods, the response of current inflation and output to an expected expansionary government spending shock Δ_p periods ahead, $g_{T+\Delta_p} > 0$, goes to plus or minus infinity with Δ_p, i.e.,*

$$\lim_{\Delta_p \to +\infty} |\partial z_T / \partial g_{T+\Delta_p}| = \infty \text{ where } z \in \{\pi, y\}.$$

The paradox of toil and the flexibility are not strictly limit puzzles. These puzzles occur for any Δ_p. However, as these puzzles are framed using the same thought experiment, it is natural to consider them here.

Definition 3 (paradox of toil) *When the policy rate is expected to be set passively during the next $\Delta_p > 0$ periods, the response of current output to a positive supply shock Δ_p periods ahead, $a_{T+\Delta_p} > 0$, is weakly contractionary with Δ_p, i.e.,*

$$\partial y_T / \partial a_{T+\Delta_p} \leq 0.$$

Definition 4 (paradox of flexibility) *When the policy rate is expected to be set passively during the next $\Delta_p > 0$ periods, the response of current inflation and output to an expected shock Δ_p periods ahead goes to plus or minus infinity as κ goes to infinity, i.e.,*

$$\lim_{\kappa \to +\infty} |\partial z_T / \partial v_{T+\Delta_p}| = \infty \text{ where } z \in \{\pi, y\} \text{ and } v = \{i^*, g, a\}.$$

Definition 4 is likely more strict than some would like. For example, it allows the impact of shocks to increase as flexibility increases so long as it does so in bounded way, which is puzzling. We proceed, however, with the the definition put forward by Diba and Loisel (2021) and we discuss relaxations further in Section 4.2.

4.1 The Puzzles under a Standard Taylor Rule

To provide intuition for why history dependent policy resolves the puzzles, it is useful to first show why the puzzles emerge under standard policy.[12] Combining the transitory passive interest

[12] To simplify the analysis, we set $\rho_i = 0$ and $\phi_y = 0$. This simplification does not affect the conclusions.

rate regime with a policy of active interest rate setting for $t > T^*$, we have:

$$i_t = \begin{cases} \bar{i}_t + \phi\pi_t & \text{for } t = T, T+1, ..., T^* \\ \bar{i}_t + \phi^*\pi_t & \text{for } t > T^*, \end{cases} \tag{16}$$

where $0 \leq \phi < 1$, $\phi^* > 1$, and $\bar{i}_t$ is known at time $t = T$. The proposed policy nests the ZLB experiment when $\bar{i} = -\bar{r}$ and $\phi = 0$ for $t = T, T+1, \ldots, T^*$.

We can derive a closed-form solution for the equilibrium effects of the shocks described immediately above by using the Phillips curve to eliminate y_t in the IS curve. When $t < T^*$, the model may be expressed as

$$\left(L^{-2} - (1 + \beta^{-1} + \frac{\kappa}{\beta\sigma})L^{-1} + \left(\beta^{-1} + \frac{\kappa\phi}{\beta\sigma}\right)\right)\pi_t = X_t \tag{17}$$

$$X_t \equiv -\frac{\kappa}{\beta\sigma}(\bar{i}_t - r_t^n) - \frac{\kappa}{\beta}(a_t - E_t a_{t+1}) + \frac{\kappa}{\beta}(1 - \delta_g)\left(g_t - E_t g_{t+1}\right), \tag{18}$$

where L is the lag operator. Factoring the lag polynomial we have

$$(L^{-1} - \lambda_1)(L^{-1} - \lambda_2)\pi_t = X_t$$

where the eigenvalues for the economically relevant parameters satisfy $0 \leq \lambda_1 < 1 < \lambda_2$. Using the method of partial fractions, we can write this as

$$\pi_t = \frac{1}{\lambda_2 - \lambda_1} E_t \left[\frac{\lambda_1^{-1}}{1 - (\lambda_1 L)^{-1}} - \frac{\lambda_2^{-1}}{1 - (\lambda_2 L)^{-1}}\right] X_t.$$

Finally, under the assumed monetary policy and shocks processes, it follows that $E_t\pi_{T^*+j} = 0$ for all $j > 0$, which provides the necessary limit conditions to construct an unique rational expectation equilibrium by solving the model forward in time:

$$\begin{aligned} \pi_t &= \frac{1}{\lambda_2 - \lambda_1} E_t \left[\sum_{j=t}^{T^*} \left(\left(\frac{1}{\lambda_1}\right)^{j-t+1} - \left(\frac{1}{\lambda_2}\right)^{j-t+1}\right) X_j\right] \\ y_t &= \frac{1}{\kappa(\lambda_2 - \lambda_1)} E_t \left[\sum_{j=t+1}^{T^*} \left((1 - \beta\lambda_1)\left(\frac{1}{\lambda_1}\right)^{j-t+1} - (1 - \beta\lambda_2)\left(\frac{1}{\lambda_2}\right)^{j-t+1}\right) X_j\right] \\ &+ \frac{\lambda_1^{-1} - \lambda_2^{-1}}{\kappa\left(\lambda_2 - \lambda_1\right)} X_t + \delta_g g_t + a_t \end{aligned}$$

The origins of the puzzles are transparent when the solution is written in this form and $t = T$. The forward guidance puzzle and the fiscal multiplier puzzle are due to the fact that $\lambda_1 < 1$ when $0 \leq \phi < 1$, which makes $\lambda_1^{-\Delta_p}$ grow without bound as Δ_p increases. The paradox of flexibility occurs because $\lambda_1 \to 0$ as $\kappa \to \infty$, which implies that the current effect of any anticipated shock is larger when κ is larger, i.e., prices are more flexible, all else being equal.

The economics of these outcomes is also straightforward. Consider a promise to deliver a one-time reduction in the policy rate below the neutral rate in the future. By construction,

the central bank promises not to make up for or respond to any economic consequences of this policy once it returns to active management of the economy. This abrogation of stabilization policy causes inflation to surge, real interest rates to fall, and demand to rise, giving rise to the forward guidance puzzle.

The only link between this period of policy amnesia and the resumption of active policy is through chance. Firms that set prices under a Calvo pricing mechanism must assign some probability to the possibility that prices set during the passive regime may still be in effect when active policy resumes. Reducing the probability of firms being stuck with a price that is optimal under the passive regime but suboptimal under the active one naturally induces larger adjustments in prices and quantities, giving rise to the paradox of flexibility.

Finally, the paradox of toil arises due to the properties of X_t combined with the expected policy response. A positive supply shock reduces expected inflation, which, when combined with passive monetary policy, increases the expected real interest rate and reduces consumption and output.

4.2 The Puzzles under weighted average policy rule

We now consider the same thought experiment with a history dependent policy rule *after* the transitory passive interest rate regime ends ($t > T^*$). The policy we analyze captures both a change in the reaction of monetary policy to inflation and a change in degree of history dependence during the passive regime:

$$i_t = \begin{cases} \bar{i}_t + \phi \pi_t & \text{for } t = T, T+1, ..., T^* \\ \bar{i}_t + \phi^* \omega_t & \text{for } t > T^*, \end{cases} \tag{19}$$

$$\omega_t = \begin{cases} \rho \omega_{t-1} + \pi_t & \text{for } t = T, T+1, ..., T^* \\ \rho^* \omega_{t-1} + \pi_t & \text{for } t > T^*. \end{cases} \tag{20}$$

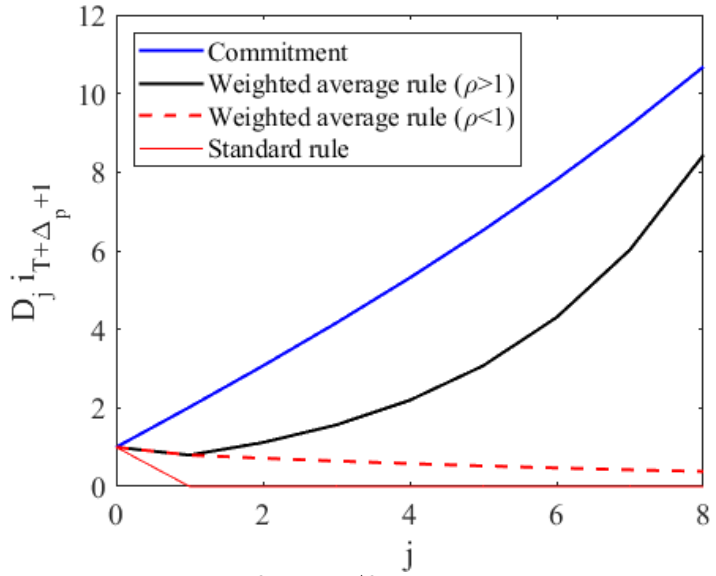
The idea behind this formulation of policy is that agents understand that past inflation outcomes matter for policy in the future even if these outcomes do not currently matter for the setting of the policy rate. The parameter ρ captures the degree to which policy responds to the outcomes in the first regime while in the second regime.

Figure 7 illustrates the history dependence imparted by ρ to the policy. The figure shows how strongly interest rates react in time $t = T^*+1$ to past inflation. Under the weighted average rule (19)-(20) with $\rho < 1$ there is history-dependence ($D_j i_{T+\Delta_p+1} := \partial i_{T+\Delta_p+1} / \partial \pi_{T+\Delta_p+1-j} > 0$) with declining weights over time. When $\rho > 1$, we observe the overshooting character of optimal policy with the response to distant misses larger than nearer term misses in inflation from target. When $\rho = 0$, we have the standard rule of the first regime where there is no history dependence to policy.

In what follows, we assume the economy is in steady state at $t = T - 1$ ($\omega_{T-1} = 0$).

Proposition 2 (Forward Guidance Puzzle)**.** *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, and $0 < \rho^* < 1$ exhibits the*

Figure 7: History dependence following lift off



Notes: Each line depicts $D_j i_{T+\Delta_p+1} := \partial i_{T+\Delta_p+1}/\partial \pi_{T+\Delta_p+1-j}$ for the specified policy rule. We normalize $D_0 i_{T+\Delta_p+1} = 1$ for each rule. We set $\rho = 0.9$ and $\rho = 1.4$, respectively, for weighted average rules shown.

forward guidance puzzle if $\rho < 1$. *The forward guidance puzzle is resolved if* $\rho > 1$.

The proof of the proposition is in the Appendix.[13] However, the calculations to arrive at the result are tedious. To sketch the intuition here, we consider a special case that is inspired by Cochrane (2017), Diba and Loisel (2021), and Gibbs and McClung (2023). We assume that $\rho = \rho^*$ and $\phi^* = \phi = 0$, while all else remains the same. We then assume that agents coordinate their expectations around the history that the central bank is tracking, ω_t.

To construct the equilibrium, start with the model expressed in terms of only inflation

$$\left(L^{-2} - (1 + \beta^{-1} + \frac{\kappa}{\beta\sigma})L^{-1} + \beta^{-1}\right)\pi_t = X_t$$

and use the equation for ω_t,

$$\pi_t = (1 - \rho L)\omega_t,$$

to eliminate π_t from (17). This results in a representation of the economy in terms of ω_t, the only endogenous state variable:

$$\left(L^{-2} - (1 + \beta^{-1} + \frac{\kappa}{\beta\sigma})L^{-1} + \beta^{-1}\right)(1 - \rho L)\omega_t = X_t.$$

Factoring the lag polynomial, we can write the model as

$$(L^{-1} - \lambda_1)(L^{-1} - \lambda_2)(1 - \rho L)\omega_t = X_t. \tag{21}$$

[13]The proof of Proposition 2 also addresses the price-level targeting case ($\rho = 1$), which only partially resolves the forward guidance puzzle: a monetary policy shock expected to occur in the infinite future has non-zero effects on inflation and output at $t = T$.

The expression above has three roots: $0 < \lambda_1 < 1 < \lambda_2$ and ρ. The last root, ρ, is a free parameter. If we set it above one, then we have two roots outside the unit circle and one root inside the unit circle, which satisfies the Blanchard-Kahn conditions without any further assumptions required to construct a unique rational expectations equilibrium.[14] When $\rho > 1$, then the forward guidance puzzle is resolved. If we set $\rho < 1$, we again would have too few roots larger than one to construct a stable equilibrium and the forward guidance puzzle is present. The equilibrium we construct here leverages the same reliance on a state-variable as Cochrane (2017) in constructing a puzzle-free Fiscal Theory of the Price Level equilibrium and Diba and Loisel (2021) in constructing a puzzle-free equilibrium with a money growth-type rule.

A corollary to the forward guidance puzzle result is that the impact of any anticipated monetary policy shock for a finite Δ_p is inversely related to the magnitude of ρ when it is less than one. This is because, all else equal, if $\rho' < \rho'' < 1$, then $(\rho')^{-1} > (\rho'')^{-1} > 1$ with corresponding effects on the dynamic multipliers.

Corollary 1 (Resolution of the Quantitative Puzzles)**.** *When $0 < \rho < 1$ and $\Delta_p > 0$,*

$$\frac{\partial}{\partial \rho}\left(\left|\frac{\partial z_T}{\partial \bar{i}_{T+\Delta_p}}\right|\right) < 0.$$

This result explains why the quantitative puzzles are resolved when $\rho < 1$. For ρ large enough, the promised duration of a passive regime required to induce significant current impacts becomes implausibly long for real-world applications.

The same arguments for the forward guidance puzzle hold for the fiscal multiplier puzzle.

Proposition 3 (Fiscal Multiplier Puzzle)**.** *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, $0 < \rho^* < 1$, and $\rho \neq \bar{\rho} < 1$ exhibits the fiscal multiplier puzzle if $\rho < 1$. The fiscal multiplier puzzle is resolved if $\rho > 1$.*

The only difference in this case is that there exists a $0 < \bar{\rho} < 1$ such that anticipated fiscal policy has no effect. This occurs because the fiscal policy shock appears in both the IS (14) and Phillips curves (15) and may perfectly offset in equilibrium.

Proposition 4 (The Paradox of Toil)**.** *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, and $0 < \rho^* < 1$ has the property that there exists a $\tilde{\rho}$ such that if $\rho < \tilde{\rho}$, then equilibrium exhibits the paradox of toil, and if $\rho > \tilde{\rho}$, the paradox of toil is resolved.*

For the paradox of toil, we have the following from our special case

$$\frac{\partial y_T}{\partial a_{T+\Delta_p}} = \frac{(1+\beta\rho-\beta(\lambda_1+\lambda_2))(1-\lambda_2^{-1})\lambda_2^{-\Delta_p} - (1-\beta\lambda_1)(1-\rho^{-1})\rho^{-\Delta_p}}{\beta(\rho-\lambda_2)}.$$

It is in fact possible to prove that if ρ is sufficiently large then $\frac{\partial y_T}{\partial a_{T+\Delta_p}} > 0$. For intuition, consider that the response of output to a supply shock is positive when ρ is near 1 and is

[14] This equilibrium is similar to those constructed using the approach of Bianchi and Nicolò (2021).

positive as ρ grows arbitrarily large:

$$\lim_{\rho \to \infty} \frac{\partial y_T}{\partial a_{T+\Delta_p}} = (\lambda_2 - 1)\lambda_2^{-\Delta_p - 1} > 0$$

and if $\rho = 1$ then $\frac{\partial y_T}{\partial a_{T+\Delta_p}} = \frac{\kappa \lambda_2^{-\Delta_p - 1}}{\beta \sigma} > 0$. In contrast, positive supply shocks can be contractionary for small ρ in this special equilibrium. For instance, consider the case when $\rho \to 0$:

$$\lim_{\rho \to 0} \frac{\partial y_T}{\partial a_{T+\Delta_p}} = -\infty.$$

Therefore, by continuity, the paradox of toil is resolved in our special equilibrium as long as ρ is sufficiently large. The economic intuition of the result is that if policy is sufficiently history dependent, then falling inflation expectations are paired with falling nominal interest rate expectations in the future, which leads to a fall in the expected real rate of interest rather than an increase as when ρ is small or zero.

Proposition 5 (Paradox of Flexibility)**.** *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, and $0 < \rho^* < 1$ does not exhibit the paradox of flexibility if $\rho > 0$.*

Lastly, the paradox of flexibility is resolved in our special case when there is history dependence because the relevant eigenvalue in the equilibrium that depends on κ is λ_2, which is equal to

$$\lambda_2 = \frac{\sqrt{\left(-\beta - \frac{\kappa}{\sigma} - 1\right)^2 - 4\beta\left(\frac{\kappa\phi}{\sigma} + 1\right)} + \beta + \frac{\kappa}{\sigma} + 1}{2\beta}$$

and

$$\lim_{\kappa \to \infty} \lambda_2^{-1} \kappa = \beta \sigma.$$

Therefore, the paradox of flexibility is resolved for all $\rho > 0$.

If one considers a weaker form of the paradox of flexibility, where responses to anticipated shocks are increasing in magnitude when κ increases but in a bounded way, then this weaker paradox still remains. However, it is clear why history dependence ameliorates the effect of increasing price flexibility on the endogenous response by firms to anticipated shocks. History dependence provides a link between current price setting under the temporary passive policy regime, to price setting under the active policy regime. The more responsive to history the policymaker is, the more current price setting must factor in the return to active policy for any price reset probabilities and hence any value of κ.

5 Plausible Predictions vs. Plausible Policy

The New Keynesian puzzles are sometimes cited as evidence of a fundamental failure of the model to sensibly capture the transmission of monetary policy. For example, Cochrane (2023) argues that the NK model's failure to generate intuitive predictions under interest rate pegs

supports the case for a fiscal-led determination of inflation as a more plausible theory. However, our results show that these critiques are misplaced. Rather than signaling a fundamental flaw in the model, these issues highlight the importance of specifying a plausible representation of monetary policy within the New Keynesian framework. In many ways our argument echoes the one made by Svensson (2003), where he writes on pages 429 and 430:

> "Monetary policy by the world's more advanced central banks these days is at least as optimizing and forward-looking as the behavior of the most rational private agents. I find it strange that a large part of the literature on monetary policy still prefers to represent central bank behavior with the help of mechanical instrument rules."

We argue that history dependence is an essential feature of any realistic description of monetary policy that seeks to make credible promises about the future. A central bank's optimal decision-making in this scenario naturally leads to policies that depend on past inflation and output. Ignoring history dependence at best amounts to assuming a purely discretionary monetary policy – one that, by design, does not attempt to shape expectations about the future, thereby ruling out forward guidance and promised interest rate pegs as policy tools. At worst, neglecting history dependence implies a central bank with undefined preferences over economic outcomes, which, when juxtaposed against optimizing firms and households, generates puzzling predictions that fail to align with real-world behavior, which encapsulates Cochrane's critique.

When monetary policy is viewed through the lens of the optimizing central bank, the qualitative puzzles are less puzzling. For example, the standard forecasts obtained modeling forward guidance policies like that of the FOMC statement of August 9^{th}, 2011 in Figure 1 are less puzzling if the thought experiment is reframed as follows: *What is the predicted response of the economy if the Federal Reserve credibly commits to abandon all statutory mandates for a set period of time, while fixing the interest rate below the neutral rate?* We argue a sensible prediction to this thought experiment is a sharp increase in inflation in the short run. This is not a puzzle; it is a plausible quantitative prediction of an extreme policy.

However, even this extreme framing does not resolve the strangeness of the limit puzzles. It still seems implausible that promises made progressively father in the future should have an increasing effect even if monetary policy is behaving in ways that we would never observe in reality. We argue that this dimension of the rational expectations NK model,i.e., the anti-horizon effect, is where the remaining puzzles in the model lie. We argue that the extensive literature aimed at resolving NK puzzles by dampening general equilibrium effects through mechanisms such as bounded rationality or various forms of "discounting" (e.g., Angeletos and Lian, 2018, Farhi and Werning, 2019, Gabaix, 2020, Eusepi et al., 2024, Evans et al., 2025) still plays a crucial role. These approaches remain vital for understanding economic dynamics and designing effective monetary policy even if they are no longer needed to resolve NK puzzles.

6 Conclusion

New Keynesian models of monetary policy are known to generate puzzling predictions for output and inflation in response to anticipated shocks when nominal interest rates are pegged or constrained by the zero lower bound. We show that all New Keynesian puzzles and paradoxes are a result of assuming that monetary policy has either the wrong or no history dependence. Adding an empirically plausible amount of history dependence to monetary policy over inflation and output during ZLB episodes resolves the quantitative puzzles. Considering policy that approximates an optimizing central bank resolves the limit puzzles.

Appendix For Online Publication

A1 Weighted average representations

We use lag operators to convert rules with interest rate smoothing to rules that are weighted averages of past output and inflation, where $LX_t = X_{t-1}$ and $L^n X_t = X_{t-n}$ for $n = ..., -2, -1, 0, 1, 2, ...$ Following Sargent (2009), we consider polynomials in the lag operator

$$A(L) = a_0 + a_1 L + a_2 L^2 + ... = \sum_{j=0}^{\infty} a_j L^j$$

where the a_j's are constants and

$$A(L)X_t = (a_0 + a_1 L + a_2 L^2 + ...)X_t = \sum_{j=0}^{\infty} a_j X_{t-j}.$$

We exploit the following relationship

$$A(L) = \frac{1}{1-\lambda L} = (1 + \lambda L + \lambda^2 L^2 + ...) = \sum_{j=0}^{\infty} \lambda^j,$$

where if $|\lambda| < 1$

$$\sum_{j=0}^{\infty} \lambda^j = \frac{1}{1-\lambda}.$$

Applying these operations to a Taylor-type rule with interest rate smoothing we can derive equation (1):

$$\begin{aligned}
i_t - \rho_i i_{t-1} &= (1-\rho_i)\bar{r} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t) \\
(1-\rho_i L) i_t &= (1-\rho_i)\bar{r} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t) \\
i_t &= \frac{1}{1-\rho_i L}\left((1-\rho_i)\bar{r} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t)\right) \\
&= (1-\rho_i)\sum_{j=0}^{\infty} \rho_i^j \bar{r} + (1-\rho_i)\left(\sum_{j=0}^{\infty} \rho_i^j \left(\phi_\pi \pi_{t-j} + \phi_y y_{t-j}\right)\right) \\
&= (1-\rho_i)\frac{1}{1-\rho_i}\bar{r} + (1-\rho_i)\left(\sum_{j=0}^{\infty} \rho_i^j \left(\phi_\pi \pi_{t-j} + \phi_y y_{t-j}\right)\right) \\
&= \bar{r} + (1-\rho_i)\sum_{j=0}^{\infty} \rho_i^j \left(\phi_\pi \pi_{t-j} + \phi_y y_{t-j}\right).
\end{aligned}$$

We arrive at the representation of rule with auxiliary variables ω_t^π and ω_t^y by way of the

following calculations:

$$\begin{aligned}
\omega_t^z &= \omega_{t-1}^z + (1-\rho_i)(z_t - \omega_{t-1}^z) \\
\omega_t^z &= \rho_i \omega_{t-1}^z + (1-\rho_i) z_t \\
\omega_t^z - \rho_i \omega_{t-1}^z &= (1-\rho_i) z_t \\
(1-\rho_i L)\omega_t^z &= (1-\rho_i) z_t \\
\omega_t^z &= \frac{(1-\rho_i)}{1-\rho_i L} z_t = (1-\rho_i) \sum_{j=0}^{\infty} \rho_i^j z_{t-j}
\end{aligned} \tag{A1}$$

Applying this representation for $z = y, \pi$ yields equation (3).

A2 Proofs of propositions

Proposition 1: The unconditional optimal targeting criteria is

$$y_t = -\frac{\kappa}{\alpha} \frac{\pi_t}{1-\beta L}.$$

Using the IS equation, we can write this as

$$E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) = -\frac{\kappa}{\alpha} \frac{\pi_t}{1-\beta L}.$$

Solving for i_t we have

$$i_t = \frac{\sigma\kappa}{\alpha} \frac{\pi_t}{1-\beta L} + \sigma E_t y_{t+1} + E_t \pi_{t+1} + r_t^n.$$

The first representation of the optimal rule comes from using (A1) where

$$\begin{aligned}
(1-\beta)\frac{\pi_t}{1-\beta L} &= \omega_t^\pi \\
\omega_t^\pi &= \omega_{t-1}^\pi + (1-\beta)(\pi_t - \omega_{t-1}^\pi).
\end{aligned}$$

The second representation of the optimal rule comes from multiplying both sides of the equation by $(1-\beta L)$:

$$\begin{aligned}
(1-\beta L) i_t &= (1-\beta L)\left(\frac{\sigma\kappa}{\alpha} \frac{\pi_t}{1-\beta L} + \sigma E_t y_{t+1} + E_t \pi_{t+1} + r_t^n\right) \\
i_t - \beta i_{t-1} &= \frac{\sigma\kappa}{\alpha}\pi_t + (1-\beta L)\left(\sigma E_t y_{t+1} + E_t \pi_{t+1} + r_t^n\right).
\end{aligned}$$

The proofs of propositions 2 - 5 require several Lemmas.

Lemma 1 (Terminal Solution)**.** *In the absense of shocks, the terminal regime ($t > T^*$) equilibrium law of motion for the economy is given by*

$$z_t = b_z^* \omega_{t-1}$$

where $z = \pi, y, \omega$, *and*

$$b_y^* = (b_y^* + \sigma^{-1} b_\pi^*) b_\omega^* - \sigma^{-1}\phi^* b_\omega^* \tag{A2}$$
$$b_\pi^* = \kappa b_y^* + \beta b_\pi^* b_\omega^* \tag{A3}$$
$$b_\omega^* = \rho^* + b_\pi^* \tag{A4}$$

and $0 < b_\omega^* < \rho^*$, $-\rho^* < b_\pi^* < 0$.

Proof of Lemma 1: b_ω^* is the only real root in $(-1, 1)$ of the third-order polymonial:[15]

$$P(b) = b^3 - \frac{\beta\rho^* + \beta + 1 + \kappa\sigma^{-1}}{\beta} b^2 + \frac{\beta\rho^* + \rho^* + 1 + \kappa\rho^*\sigma^{-1} + \kappa\sigma^{-1}\phi^*}{\beta} b - \frac{\rho^*}{\beta}$$

where $P(b)$ is obtained by substituting (A3) and (A4) into (A2). Because $P(0) < 0 < P(\rho^*) = \frac{\kappa\rho^*\phi^*}{\beta\sigma}$, $b_\omega^* \in (0, \rho^*)$ which implies that $-\rho^* < b_\pi^* < 0$. □

Lemma 2 (Recursion 1)**.** *Consider the following difference equation*

$$\lambda_j = \frac{\gamma_0}{\gamma_1 - \lambda_{j-1}}$$

where $\gamma_0 = \beta^{-1}(1 + \kappa\sigma^{-1}\phi) > 1$, $\gamma_1 = \beta^{-1} + 1 + \beta^{-1}\kappa\sigma^{-1} > 1$, *and* $\lambda_{-1} = (1 + \kappa\sigma^{-1}\phi)(1 + \kappa\sigma^{-1} + \beta(1 - b_\omega^*))^{-1} \in (0, 1)$. *The sequence* (λ_j) *has the following properties*

1. (λ_j) *is a convergent sequence:* $\lim_{j\to\infty} \lambda_j = \lambda^Z$ *and* $0 \leq \lambda^Z < 1$.
2. (λ_j) *is a monotone sequence: If* $\lambda_{-1} < (>)\lambda^Z$ *then* $\lambda_j > \lambda_{j-1}$ $(\lambda_j < \lambda_{j-1})$ *for* $j \geq 0$. *If* $\lambda_{-1} = \lambda^Z$ *then* $\lambda_j = \lambda^Z$ *for all* $j \geq 0$.
3. (λ_j) *is a bounded sequence:* $\lambda_j \in [\min\{\lambda_{-1}, \lambda^Z\}, \max\{\lambda_{-1}, \lambda^Z\}] \subseteq [0, 1)$ *for all* j.
4. $\lim_{\kappa\to\infty} \lambda_j = \phi$.

Proof of Lemma 2: Define $\lambda = h(\lambda)$, $h(\lambda) := \frac{\gamma_0}{\gamma_1 - \lambda}$. $h(\lambda)$ has the following properties:

1. h is continuous on $[0, 1]$.
2. $h(\lambda) - \lambda = 0$ has roots

$$\lambda^Z \to \frac{1}{2}\left(\gamma_1 - \sqrt{\gamma_1^2 - 4\gamma_0}\right), \lambda^A \to \frac{1}{2}\left(\sqrt{\gamma_1^2 - 4\gamma_0} + \gamma_1\right)$$

   Given γ_0 and γ_1 are composed of $0 < \beta < 1$, $\kappa > 0$, $\sigma > 0$, and $0 \leq \phi < 1$, then $0 \leq \lambda^Z < 1 < \lambda^A$.

3. $h'(\lambda) = \frac{\gamma_0}{(\gamma_1 - \lambda)^2} > 0$ when $\lambda \in [0, 1]$.

[15] A straightforward application of Blanchard-Kahn to the terminal regime model proves that there can only be one real root of P inside the unit circle.

4. $h''(\lambda) = \frac{2\gamma_0}{(\gamma_1-\lambda)^3} > 0$ when $\lambda \in [0,1]$.

5. $0 < h(0) = \gamma_0/\gamma_1 < h(1) = (1+\kappa\sigma^{-1}\phi)(1+\kappa\sigma^{-1})^{-1} < 1$.

6. Given any finite λ, $\lim_{\kappa\to\infty} h(\lambda) = \phi$.

Properties (1)-(5) and $\lambda_{-1} \in (0,1)$ imply assertions 1-3 of Lemma 2. Property 6 computes the limit for assertion 4. □

Lemma 3 (Recursion 2)**.** *Consider the following difference equations*

$$\begin{aligned} b_j &= \frac{\lambda_{j-1}}{\rho} b_{j-1} - \rho^{-1} \\ \lambda_j &= \frac{\gamma_0}{\gamma_1 - \lambda_{j-1}} \end{aligned}$$

where $b_{-1} < 0$, $\lambda_{-1} \in (0,1)$, *and* $\rho > 0$.

1. *If* $\rho > \lambda^Z$, *then* $\lim_{j\to\infty} b_j = (\lambda^Z - \rho)^{-1}$

2. *If* $\rho \leq \lambda^Z$, *then* $\lim_{j\to\infty} b_j = -\infty$

Proof of Lemma 3: We rely on assertions 1-3 of Lemma 2 to prove convergence of (b_j). Define $\Lambda := \sup\{\lambda_j : j = -1, 0, 1, 2, \ldots\} = \max\{\lambda_{-1}, \lambda^Z\}$. There are three cases to consider:

Case 1. $\rho \leq \lambda^Z$. It is straightforward to show that $\lim_{j\to\infty} b_j = -\infty$ (recall that $\rho > 0$, which implies that (b_j) is *eventually* monotonic and strictly decreasing.).

Case 2. $\rho > \Lambda$. We prove that $\lim_{j\to\infty} b_j = (\lambda^Z - \rho)^{-1}$ by showing that $\lim_{j\to\infty} |\hat{b}_j| = \lim_{j\to\infty} \hat{b}_j = 0$ where $\hat{b}_j = b_j - (\lambda^Z - \rho)^{-1}$. Define $\epsilon_j = (\rho(\lambda^Z - \rho))^{-1}(\lambda_j - \lambda^Z)$. Then we have:

$$\hat{b}_j = \frac{\lambda_{j-1}}{\rho}\hat{b}_{j-1} + \epsilon_{j-1}$$

given $\hat{b}_{-1} = b_{-1} - (\lambda^Z - \rho)^{-1}$. We first prove that $\lim_{j\to\infty} |\hat{b}_j| = 0$ using the squeeze theorem. We define a sequence, (b_j^*), such that $0 \leq |\hat{b}_j| \leq b_j^*$ for all j and show that $\lim_{j\to\infty} b_j^* = 0$. The sequence, (b_j^*), is defined recursively:

$$b_j^* = \frac{\Lambda}{\rho} b_{j-1}^* + |\epsilon_{j-1}|$$

given $b_{-1}^* = |\hat{b}_{-1}|$, where $\Lambda = \max\{\lambda_{-1}, \lambda^Z\} \geq \lambda_j$ for all $j \geq -1$. We now show that $|\hat{b}_j| \leq b_j^*$ for all j. Starting with $j = 0$:

$$\begin{aligned} |\hat{b}_0| &= |\frac{\lambda_{-1}}{\rho}\hat{b}_{-1} + \epsilon_{-1}| \leq |\frac{\lambda_{-1}}{\rho}\hat{b}_{-1}| + |\epsilon_{-1}| \\ &= \frac{\lambda_{-1}}{\rho}|\hat{b}_{-1}| + |\epsilon_{-1}| \leq \frac{\Lambda}{\rho}|\hat{b}_{-1}| + |\epsilon_{-1}| = b_0^*. \end{aligned}$$

For $j = 1$:

$$\begin{aligned} |\hat{b}_1| &= |\frac{\lambda_0}{\rho}\hat{b}_0 + \epsilon_0| \leq |\frac{\lambda_0}{\rho}\hat{b}_0| + |\epsilon_0| \\ &= \frac{\lambda_0}{\rho}|\hat{b}_0| + |\epsilon_0| \leq \frac{\Lambda}{\rho}|\hat{b}_0| + |\epsilon_0| \\ &\leq \frac{\Lambda}{\rho}b_0^* + |\epsilon_0| = b_1^*. \end{aligned}$$

The proof proceeds by induction, if $|\hat{b}_{j-1}| \leq b_{j-1}^*$ then

$$\begin{aligned} |\hat{b}_j| &= |\frac{\lambda_{j-1}}{\rho}\hat{b}_{j-1} + \epsilon_{j-1}| \leq |\frac{\lambda_{j-1}}{\rho}\hat{b}_{j-1}| + |\epsilon_{j-1}| \\ &= \frac{\lambda_{j-1}}{\rho}|\hat{b}_{j-1}| + |\epsilon_{j-1}| \leq \frac{\Lambda}{\rho}|\hat{b}_{j-1}| + |\epsilon_{j-1}| \\ &\leq \frac{\Lambda}{\rho}b_{j-1}^* + |\epsilon_{j-1}| = b_j^*. \end{aligned}$$

Because (i) $|\epsilon_j|$ is a strictly decreasing sequence and $\lim_{j\to\infty}\epsilon_j = 0$ and (ii) $\Lambda < \rho$, we have: $\lim_{j\to\infty} b_j^* = 0$. Finally, because $|\hat{b}_j| \leq b_j^*$ for all $j \geq 0$ and $\lim_{j\to\infty} b_j^* = 0$, we have: $\lim_{j\to\infty}|\hat{b}_j| = \lim_{j\to\infty}\hat{b}_j = 0$, which implies that $\lim_{j\to\infty} b_j = (\lambda^Z - \rho)^{-1}$.

Case 3. $\lambda^Z < \rho \leq \Lambda$. In this case, (λ_j) is strictly decreasing and $\lambda_{-1} \geq \rho$ (see Lemma 2). Hence, there exists K such that $\lambda_j < \rho$ for all $j \geq K$. Next, define $\hat{b}_j$, ϵ_j, as in case 2, and consider the following sequence:

$$b_j^* = \begin{cases} |\hat{b}_j| \text{ for } j \leq K \\ \frac{\lambda_K}{\rho}b_{j-1}^* + |\epsilon_{j-1}| \text{ for } j > K \end{cases}$$

Following the argument in case 2, it can be shown that $|\hat{b}_j| \leq b_j^*$ for all j and $\lim_{j\to\infty} b_j^* = 0$ which implies $\lim_{j\to\infty} b_j = (\lambda^Z - \rho)^{-1}$. □

Derivation of equilibrium output and inflation: For $t = T, \ldots, T^*$ expected inflation satisfies the following second-order difference equation in expected inflation:

$$E_T\pi_{t+2} - \gamma_1 E_T\pi_{t+1} + \gamma_0 E_T\pi_t = X_t \tag{A5}$$
$$X_t := E_T\left\{\frac{\kappa}{\beta}\left((1-\delta_g)(g_t - g_{t+1}) + a_{t+1} - a_t - \sigma^{-1}\bar{i}_t\right)\right\}$$

where $E_T\bar{i}_{T^*} = \bar{i}$, $E_T\bar{i}_t = 0$ if $t = T, \ldots, T^* - 1$, $\gamma_0 = \beta^{-1}(1 + \kappa\sigma^{-1}\phi) > 1$, and $\gamma_1 = \beta^{-1} + 1 + \beta^{-1}\kappa\sigma^{-1} > 1$. Equation (A5) is obtained by combining the Euler equation and Phillips curve after setting $E_T r_t^n = 0$ for all t. We now solve this expected difference equation backwards from $t = T^*$ to derive a first-order difference equation in expected inflation, which is used to derive the solution paths for π_t, y_t, ω_t.

Starting at $t = T^*$:

$$E_T\pi_{T^*+2} - \gamma_1 E_T\pi_{T^*+1} + \gamma_0 E_T\pi_{T^*} = (b^*_\omega - \gamma_1)E_T\pi_{T^*+1} + \gamma_0 E_T\pi_{T^*} = X_{T^*}$$

where we make use of

$$E_T\pi_{T^*+2} = b^*_\pi \omega_{T^*+1} = b_{\pi^*}(\rho + b_{\pi^*})\omega_{T^*} = b^*_\omega E_T\pi_{T^*+1}.$$

Then, we define the following

$$\begin{aligned} E_T\pi_{T^*+1} &= \lambda_{T^*+1}E_T\pi_{T^*} - (\gamma_1 - b^*_\omega)^{-1}X_{T^*} = \lambda_{T^*+1}(E_T\pi_{T^*} - \gamma_0^{-1}X_{T^*}) \\ \lambda_{T^*+1} &:= \frac{\gamma_0}{\gamma_1 - b^*_\omega}. \end{aligned}$$

Advancing to $t = T^* - 1$, we have

$$\begin{aligned} E_T\pi_{T^*+1} - \gamma_1 E_T\pi_{T^*} + \gamma_0 E_T\pi_{T^*-1} &= X_{T^*-1} \\ (\lambda_{T^*+1} - \gamma_1)E_T\pi_{T^*} + \gamma_0 E_T\pi_{T^*-1} - \gamma_0^{-1}\lambda_{T^*+1}X_{T^*} &= X_{T^*-1} \end{aligned}$$

which implies

$$\begin{aligned} E_T\pi_{T^*} &= \lambda_{T^*}E_T\pi_{T^*-1} - \gamma_0^{-1}\lambda_{T^*}\left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right) \\ \lambda_{T^*} &= \frac{\gamma_0}{\gamma_1 - \lambda_{T^*+1}}. \end{aligned}$$

Proceeding recursively backward in time for $t = T^* - 1, \ldots, T$, imposing $X_t = 0$ for $t < T^* - 1$, we have:

$$E_T\pi_{t+1} = \lambda_{t+1}E_T\pi_t - \left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right)\gamma_0^{-(T^*-t)}\Pi_{k=0}^{T^*-t-1}\lambda_{T^*-k}$$

where $\lambda_{t+1} = \frac{\gamma_0}{\gamma_1 - \lambda_{t+2}}$.

Following Gibbs and McClung (2023), the RE solution across both regimes in policy has a time-varying reduced form

$$z_t = b^z_t \omega_{t-1} + d^z_t.$$

where $\{b^z_t\}_{t=T}^{T^*}$ and $\{d^z_t\}_{t=T}^{T^*}$ for $z = \pi, y, \omega$ trace out the solution. The NK puzzles may be reduced to statements about the values b^z_T and d^z_T.

In periods $t > T^*$, by Lemma 1 the economy evolves according to

$$z_t = b^*_z \omega_{t-1},$$

where $z = \pi, y, \omega$. For ease of notation we note that these calculations assume perfect foresight

so that we can dispense with the expectations operator. Therefore, in period T^* we have,

$$\begin{aligned}
\pi_{T^*+1} &= \lambda_{T^*+1}(\pi_{T^*} - \gamma_0^{-1} X_{T^*}) \\
b_\pi^* \omega_{T^*} &= \lambda_{T^*+1}(\pi_{T^*} - \gamma_0^{-1} X_{T^*}) \\
b_\pi^*(\rho\omega_{T^*-1} + \pi_{T^*}) &= \lambda_{T^*+1}(\pi_{T^*} - \gamma_0^{-1} X_{T^*}) \\
&\dots \\
\pi_{T^*} &= \frac{b_\pi^* \rho}{\lambda_{T^*+1} - b_\pi^*}\omega_{T^*-1} + \frac{\gamma_0^{-1}\lambda_{T^*+1}}{\lambda_{T^*+1} - b_\pi^*} X_{T^*}
\end{aligned}$$

Then, employing the method of undetermined coefficients

$$b_{T^*}^\pi \omega_{T^*-1} + d_{T^*}^\pi = \frac{b_\pi^* \rho}{\lambda_{T^*+1} - b_\pi^*}\omega_{T^*-1} + \frac{\gamma_0^{-1}\lambda_{T^*+1}}{\lambda_{T^*+1} - b_\pi^*} X_{T^*}$$

$$b_{T^*}^\pi = \frac{b_\pi^* \rho}{\lambda_{T^*+1} - b_\pi^*},\ d_{T^*}^\pi = \frac{\gamma_0^{-1}\lambda_{T^*+1}}{\lambda_{T^*+1} - b_\pi^*} X_{T^*}$$

In period $T^* - 1$ we have

$$\begin{aligned}
\pi_{T^*} &= \lambda_{T^*}\pi_{T^*-1} - \gamma_0^{-1}\lambda_{T^*}\left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right) \\
b_{T^*}^\pi \omega_{T^*-1} + d_{T^*}^\pi &= \lambda_{T^*}\pi_{T^*-1} - \gamma_0^{-1}\lambda_{T^*}\left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right) \\
b_{T^*}^\pi(\rho\omega_{T^*-2} + \pi_{T^*-1}) + d_{T^*}^\pi &= \lambda_{T^*}\pi_{T^*-1} - \gamma_0^{-1}\lambda_{T^*}\left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right) \\
&\dots \\
\pi_{T^*-1} &= \frac{b_{T^*}^\pi \rho}{\lambda_{T^*} - b_{T^*}^\pi}\omega_{T^*-2} + \frac{1}{\lambda_{T^*} - b_{T^*}^\pi}\left(d_{T^*}^\pi + \gamma_0^{-1}\lambda_{T^*}\left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right)\right)
\end{aligned}$$

Employing the method of undetermined coefficients again, we have

$$b_{T^*-1}^\pi = \frac{\rho b_{T^*}^\pi}{\lambda_{T^*} - b_{T^*}^\pi},\ d_{T^*-1}^\pi = \frac{d_{T^*}^\pi + \gamma_0^{-1}\lambda_{T^*}(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1})}{\lambda_{T^*} - b_{T^*}^\pi}.$$

Proceeding recursively in this way, we can write coefficients for any $T \leq t \leq T^* - 1$ as

$$b_t^\pi = \frac{\rho b_{t+1}^\pi}{\lambda_{t+1} - b_{t+1}^\pi},\ d_t^\pi = \frac{d_{t+1}^\pi + \left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right)\gamma_0^{-(T^*-t)}\Pi_{k=0}^{T^*-t-1}\lambda_{T^*-k}}{\lambda_{t+1} - b_{t+1}^\pi}.$$

We can use the same method to recover the other elements of b_t^z and d_T^z:

$$\begin{aligned}
b_t^\omega &= \rho + b_t^\pi \\
b_t^y &= \kappa^{-1}(1 - \beta\lambda_{t+1})b_t^\pi \\
d_t^\omega &= d_t^\pi \\
d_{T^*}^y &= \frac{1 - \beta\lambda_{T^*+1}}{\kappa} d_{T^*}^\pi + \beta\lambda_{T^*+1}(\beta\gamma_0)^{-1}X_{T^*} + a_{T^*} + \delta_g g_{T^*} = \kappa^{-1}(1 - \beta b_\pi^*)d_{T^*}^\pi + a_{T^*} + \delta_g g_{T^*} \\
d_t^y &= \frac{1 - \beta\lambda_{t+1}}{\kappa} d_t^\pi + \frac{\beta}{\kappa}\left(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1}\right)\gamma_0^{-(T^*-t)}\Pi_{k=0}^{T^*-t-1}\lambda_{T^*-k}
\end{aligned}$$

To simplify the notation, let $j = 0$ correspond to T^* such that $j = 0, 1, ..., \Delta_p = T^* - T$. Then,

we can re-index the b_t^π, d_t^π, and λ_t recursions as

$$b_j^\pi = \frac{\rho b_{j-1}^\pi}{\lambda_{j-1} - b_{j-1}^\pi},\ d_j^\pi = \frac{d_{j-1}^\pi + \left(\gamma_0^{-1}\lambda_{-1}X_{T^*} + X_{T^*-1}\right)\gamma_0^{-j}\Pi_{k=0}^{j-1}\lambda_k}{\lambda_{j-1} - b_{j-1}^\pi}, \lambda_j = \frac{\gamma_0}{\gamma_1 - \lambda_{j-1}}.$$

Note that if we define $b_j = (b_j^\pi)^{-1}$,[16] then by Lemma 3 we have that

- If $\rho > \lambda^Z$, then $\lim_{j\to\infty} b_j^\pi = \lambda^Z - \rho$
- If $\rho \leq \lambda^Z$, then $\lim_{j\to\infty} b_j^\pi = 0$.

Therefore, as $j = \Delta_p \to \infty$, the dynamics of d_j^π may be approximated using limit laws when $\rho > \lambda^Z$ by

$$d_j^\pi = \frac{1}{\rho}d_{j-1}^\pi + \frac{\left(\gamma_0^{-1}\lambda_{-1}X_{T^*} + X_{T^*-1}\right)\gamma_0^{-j}\Pi_{k=0}^{j-1}\lambda_k}{\rho}.$$

and when $\rho \leq \lambda_Z$ by

$$d_j^\pi = \frac{1}{\lambda^Z}d_{j-1}^\pi + \frac{\left(\gamma_0^{-1}\lambda_{-1}X_{T^*} + X_{T^*-1}\right)\gamma_0^{-j}\Pi_{k=0}^{j-1}\lambda_k}{\lambda^Z}.$$

In addition, since $\gamma_0 > 1$ and by Lemma 2 $\lambda_j \in [0, 1)$ for all j, stability of the above recursions depends on $|\lambda^Z|$ and $|\rho|$.

We now analyse the impact of each shock individually to establish the puzzle properties.

Proposition 2 (Forward Guidance Puzzle). *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, and $0 < \rho^* < 1$ exhibits the forward guidance puzzle if $\rho < 1$. The forward guidance puzzle is resolved if $\rho > 1$.*

Proof of Proposition 2: Consider the dynamic multiplier on inflation to an interest rate shock Δ_p periods in the future:

$$\frac{\partial \pi_T}{\partial \bar{i}_{T+\Delta_p}} = \frac{\partial(b_T^\pi \omega_{T-1} + d_T^\pi)}{\partial \bar{i}_{T+\Delta_p}} = \frac{\partial d_T^\pi}{\partial \bar{i}_{T+\Delta_p}}$$

where

$$\partial d_T^\pi / \partial \bar{i}_{T+\Delta_p} = \frac{\partial d_{T+1}^\pi / \partial \bar{i}_{T+\Delta_p} - \kappa(\beta\gamma_0\sigma)^{-1}\gamma_0^{-\Delta_p}\Pi_{k=-1}^{\Delta_p-1}\lambda_{T+\Delta_p-k}}{\lambda_{T+1} - b_{T+1}^\pi}.$$

given $\partial d_{T+\Delta_p}^\pi / \partial \bar{i}_{T+\Delta_p} = -\kappa(\sigma\gamma_0\beta)^{-1}\lambda_{T+\Delta_p+1}(\lambda_{T+\Delta_p+1} - b_\pi^*)^{-1} < 0$. It immediately follows that $\partial d_{T+k}^\pi / \partial \bar{i}_{T+\Delta_p} < 0$ for $k = 0, \ldots, \Delta_p$. As $\Delta_p \to \infty$ we can approximate this as

$$\frac{\partial d_T^\pi}{\partial \bar{i}_{T+\Delta_p}} \approx \frac{\partial d_j^\pi}{\partial \bar{i}_0} = \begin{cases} \frac{1}{\rho}\frac{\partial d_{j-1}^\pi}{\partial \bar{i}_0} - \frac{\kappa(\beta\gamma_0\sigma)^{-1}\gamma_0^{-j}\Pi_{k=-1}^{j-1}\lambda_k}{\rho} & \text{if } \rho > \lambda^Z \\ \frac{1}{\lambda^Z}\frac{\partial d_{j-1}^\pi}{\partial \bar{i}_0} - \frac{\kappa(\beta\gamma_0\sigma)^{-1}\gamma_0^{-j}\Pi_{k=-1}^{j-1}\lambda_k}{\lambda^Z} & \text{if } \rho \leq \lambda^Z \end{cases}$$

Therefore, it follows that if $\rho > 1$, then

[16] Note that $b_t^\pi < 0$ for all t.

$$\lim_{j\to\infty}\frac{\partial d_j^{\pi}}{\partial \bar{i}_0}=0$$

because the last term goes to zero and $\rho^{-1}<1$. If $0<\rho<1$, then

$$\lim_{j\to\infty}\frac{\partial d_j^{\pi}}{\partial \bar{i}_0}=-\infty$$

because either $\rho^{-1}>1$ or $(\lambda^Z)^{-1}>1$. If $\rho=1$, then it follows that

$$\lim_{j\to\infty}\frac{\partial d_j^{\pi}}{\partial \bar{i}_0}=\bar{d}\neq 0,$$

because

$$\lim_{j\to\infty}\frac{\kappa(\beta\gamma_0\sigma)^{-1}\gamma_0^{-j}\Pi_{k=-1}^{j-1}\lambda_k}{\rho}=0.$$

The output response to the forward guidance shock depends completely on the inflation response

$$\partial y_T/\partial \bar{i}_{T+\Delta_p}=\partial d_T^y/\partial \bar{i}_{T+\Delta_p} \;=\; \kappa^{-1}(1-\beta\lambda_{T+1})(\partial d_T^{\pi}/\partial \bar{i}_{T+\Delta_p})-\sigma^{-1}\gamma_0^{-(\Delta_p+1)}\Pi_{k=-1}^{\Delta_p-1}\lambda_{T+\Delta_p-k}<0.$$

if $\Delta_p>0$. If $\Delta_p=0$:

$$\partial y_{T+\Delta_p}/\partial \bar{i}_{T+\Delta_p} \;=\; \partial d_{T+\Delta_p}^y/\partial \bar{i}_{T+\Delta_p}=\kappa^{-1}(1-\beta b_{\pi}^*)(\partial d_{T+\Delta_p}^{\pi}/\partial \bar{i}_{T+\Delta_p})<0.$$

Therefore, the results for inflation determine the forward guidance results for output. □

Corollary 1 (Resolution of the Quantitative Puzzles). *When* $0<\rho<1$ *and* $\Delta_p>0$,

$$\frac{\partial}{\partial\rho}\left(\left|\frac{\partial z_T}{\partial \bar{i}_{T+\Delta_p}}\right|\right)<0$$

where $z\in\{\pi,y\}$.

Proof of Corollary: If

$$\frac{\partial \pi_T}{\partial \bar{i}_{T+\Delta_p}}<0,$$

then

$$\frac{\partial}{\partial\rho}\left(\frac{\partial d_T^{\pi}}{\partial \bar{i}_{T+\Delta_p}}\right)>0$$

for $\Delta_p>0$ implies that the magnitude of the effect is decreasing as ρ increases. To show that this holds, recall that $\partial d_{T+k}^{\pi}/\partial \bar{i}_{T+\Delta_p}<0$ for $k=0,...,\Delta_p$ and consider the following

$$\frac{\partial}{\partial\rho}\left(\frac{\partial d_{T^*}^{\pi}}{\partial \bar{i}_{T^*}}\right)=0$$

$$\frac{\partial}{\partial\rho}\left(\frac{\partial d_{T^*-1}^{\pi}}{\partial \bar{i}_{T^*}}\right)=\frac{\partial d_{T^*-1}^{\pi}}{\partial \bar{i}_{T^*}}(\lambda_{T^*}-b_{T^*}^{\pi})^{-1}\frac{\partial b_{T^*}^{\pi}}{\partial\rho}$$

and

$$\frac{\partial b^{\pi}_{T^*}}{\partial \rho} = \frac{b^{\pi}_{T^*+1}}{\lambda_{T^*+1} - b^{\pi}_{T^*+1}} = \frac{b^*_{\pi}}{\gamma_0(\gamma_1 - b^*_{\omega})^{-1} - b^*_{\pi}} < 0,$$

which implies

$$\frac{\partial}{\partial \rho}\left(\frac{\partial d^{\pi}_{T^*-1}}{\partial \bar{i}_{T^*}}\right) > 0.$$

Then, by induction

$$\frac{\partial b^{\pi}_{k}}{\partial \rho} = \rho^{-1} b^{\pi}_{k} + (b^{\pi}_{k+1})^{-1} b^{\pi}_{k} \lambda_{k+1} (\lambda_{k+1} - b^{\pi}_{k+1})^{-1} \frac{\partial b^{\pi}_{k+1}}{\partial \rho} < 0$$

for all $k = 0, 1, 2, ...$ because $b^*_{\pi} < 0$ by Lemma 1, $\lambda_k \in [0, 1)$ by Lemma 2, and hence $b^{\pi}_{k} < 0$ for all k. Then, by induction:

$$\frac{\partial}{\partial \rho}\left(\frac{\partial d^{\pi}_{T+k}}{\partial \bar{i}_{T+\Delta_p}}\right) = (\lambda_{T+k+1} - b^{\pi}_{T+k+1})^{-1}\left(\frac{\partial d^{\pi}_{T+k}}{\partial \bar{i}_{T+\Delta_p}} \frac{\partial b^{\pi}_{T+k+1}}{\partial \rho} + \frac{\partial}{\partial \rho}\left(\frac{\partial d^{\pi}_{T+k+1}}{\partial \bar{i}_{T+\Delta_p}}\right)\right) > 0$$

for $k = 0, \ldots, \Delta_p - 2$. Therefore, the impact of the forward guidance shock is increasing as ρ decreases.

Turning to output, we have

$$\frac{\partial y_T}{\partial \bar{i}_{T+\Delta_p}} < 0$$

and

$$\begin{aligned}\frac{\partial}{\partial \rho}\left(\frac{\partial d^{y}_{T}}{\partial \bar{i}_{T+\Delta_p}}\right) &= \frac{\partial}{\partial \rho}\left(\kappa^{-1}(1 - \beta\lambda_{T+1})(\partial d^{\pi}_{T}/\partial \bar{i}_{T+\Delta_p}) - \sigma^{-1}\gamma_0^{-(\Delta_p+1)}\Pi_{k=-1}^{\Delta_p-1}\lambda_{T+\Delta_p-k}\right) \\ &= \kappa^{-1}(1 - \beta\lambda_{T+1})\frac{\partial}{\partial \rho}\left(\partial d^{\pi}_{T}/\partial \bar{i}_{T+\Delta_p}\right) > 0.\end{aligned}$$

□.

Proposition 3 (Fiscal Multiplier Puzzle). *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, $0 < \rho^* < 1$, and $\rho \neq \bar{\rho} < 1$ exhibits the fiscal multiplier puzzle if $\rho < 1$. The fiscal multiplier puzzle is resolved if $\rho > 1$.*

Proof of proposition 3: Consider the dynamic multiplier on inflation for a government spending shock Δ_p periods in the future:

$$\frac{\partial \pi_T}{\partial g_{T+\Delta_p}} = \frac{\partial (b^{\pi}_{T}\omega_{T-1} + d^{\pi}_{T})}{\partial g_{T+\Delta_p}} = \frac{\partial d^{\pi}_{T}}{\partial g_{T+\Delta_p}}$$

where

$$\partial d^{\pi}_{T}/\partial g_{T+\Delta_p} = \frac{d^{\pi}_{T+1}/\partial g_{T+\Delta_p} - \kappa(1 - \delta_g)(1 - \gamma_0^{-1}\lambda_{T+\Delta_p+1})\beta^{-1}\gamma_0^{-\Delta_p}\Pi_{k=0}^{\Delta_p-1}\lambda_{T+\Delta_p-k}}{\lambda_{T+1} - b^{\pi}_{T+1}}.$$

given $\partial d^{\pi}_{T+\Delta_p}/\partial g_{T+\Delta_p} = \kappa(\gamma_0\beta)^{-1}(1-\delta_g)\lambda_{T+\Delta_p+1}(\lambda_{T+\Delta_p} - b^*_{\pi})^{-1} > 0$.

As $j = \Delta_p \to \infty$, we can approximate this as

$$\frac{\partial d^{\pi}_{T}}{\partial g_{T+\Delta_p}} \approx \frac{\partial d^{\pi}_{j}}{\partial g_0} = \begin{cases} \frac{1}{\rho}\frac{\partial d^{\pi}_{j-1}}{\partial g_0} - \frac{\kappa(1-\delta_g)(1-\gamma_0^{-1}\lambda_{-1})\beta^{-1}\gamma_0^{-j}\Pi_{k=0}^{j-1}\lambda_k}{\rho} & \text{if } \rho > \lambda^Z \\ \frac{1}{\lambda^Z}\frac{\partial d^{\pi}_{j-1}}{\partial g_0} - \frac{\kappa(1-\delta_g)(1-\gamma_0^{-1}\lambda_{-1})\beta^{-1}\gamma_0^{-j}\Pi_{k=0}^{j-1}\lambda_k}{\lambda^Z} & \text{if } \rho \leq \lambda^Z \end{cases}$$

Note that $\partial d^{\pi}_{T+\Delta_P}/\partial g_{T+\Delta_p} = \kappa(\gamma_0\beta)^{-1}(1-\delta_g)\lambda_{T+\Delta_p+1}(\lambda_{T+\Delta_p} - b^*_{\pi})^{-1} > 0$. Therefore, it follows that if $\rho > 1$, then

$$\lim_{\Delta_p\to\infty} \frac{\partial d^{\pi}_{T}}{\partial g_{T+\Delta_p}} = 0$$

because the last term goes to zero and $\rho^{-1} < 1$.

For the case $\rho < 1$, we note that if $\partial d^{\pi}_{T+j}/\partial g_{T+\Delta_p} \leq 0$ for some $j \in [1,\ldots,\Delta_p]$ then $\partial d^{\pi}_{T+k}/\partial g_{T+\Delta_p} < 0$ for all $0 \leq k < j$. Hence, if $\partial d^{\pi}_{T+j}/\partial g_{T+\Delta_p} > 0$ for some $j \in [1,\ldots,\Delta_p]$, then $\partial d^{\pi}_{T+k}/\partial g_{T+\Delta_p} > 0$ for all $j \leq k \leq \Delta_p$. Thus, if $\rho < 1$ and $\partial d^{\pi}_{T+j}/\partial g_{T+\Delta_p} \leq 0$ for some $j \in [1,\ldots,\Delta_p]$ then

$$\lim_{\Delta_p\to\infty} \partial \pi_T/\partial g_{T+\Delta_p} = -\infty.$$

If, instead, $\partial d^{\pi}_{T+j}/\partial g_{T+\Delta_p} > 0$ for all $j \in [1,\ldots,\Delta_p]$ then

$$\lim_{\Delta_p\to\infty} \frac{\partial}{\partial \rho}\left(\frac{\partial d^{\pi}_{T}}{\partial g_{T+\Delta_p}}\right) = \sum_{k=0}^{\infty} \frac{\partial d^{\pi}_{T+k}}{\partial g_{T+\Delta_p}} \Pi_{i=1}^{k+1}(\lambda_{T+i} - b^{\pi}_{T+i})^{-1}\frac{\partial b^{\pi}_{T+k+1}}{\partial \rho} < 0$$

which implies existence of $\bar{\rho}$ such that $\lim_{\Delta_p\to\infty} \partial \pi_T/\partial g_{T+\Delta_p} = 0$ if $\rho = \bar{\rho}$. Therefore,

$$\lim_{\Delta_p\to\infty} \partial \pi_T/\partial g_{T+\Delta_p} = \pm\infty$$

if $\rho < 1$ and $\rho \neq \bar{\rho}$.[17] If $\rho = 1$, then it follows that

$$\lim_{j\to\infty} \frac{\partial d^{\pi}_{j}}{\partial g_0} = \bar{d}_g \neq 0,$$

because

$$\lim_{j\to\infty} -\frac{\kappa(1-\delta_g)(1-\gamma_0^{-1}\lambda_{-1})\beta^{-1}\gamma_0^{-j}\Pi_{k=0}^{j-1}\lambda_k}{\rho} = 0.$$

If $\Delta_p > 0$ then the output response is given by:

$$\partial d^{y}_{T}/\partial g_{T+\Delta_p} = \kappa^{-1}(1-\beta\lambda_{T+1})\frac{\partial d^{\pi}_{T}}{\partial g_{T+\Delta_p}} - \gamma_0^{-\Delta_p}(1-\delta_g)(1-\gamma_0^{-1}\lambda_{T+\Delta_p+1})\Pi_{k=0}^{\Delta_p-1}\lambda_{T+\Delta_p-k}.$$

If $\Delta_p = 0$ the output response is given by:

$$\partial y_{T+\Delta_p}/\partial g_{T+\Delta_p} \;=\; \partial d^{y}_{T+\Delta_p}/\partial g_{T+\Delta_p} = \kappa^{-1}(1-\beta b^*_{\pi})(\partial d^{\pi}_{T+\Delta_p}/\partial g_{T+\Delta_p}) + \delta_g.$$

[17] It is only necessary to assume $\rho \neq \bar{\rho}$ if $\bar{\rho} \in [0,1)$.

Hence, $\lim_{\Delta_p\to\infty} \partial y_T/\partial g_{T+\Delta_p}$ exists finitely if and only if $\lim_{\Delta_p\to\infty} \partial \pi_T/\partial g_{T+\Delta_p}$ exists finitely. If $\rho > 1$, then $\lim_{\Delta_p\to\infty} \partial y_T/\partial g_{T+\Delta_p} = 0$. This proves that the fiscal multiplier puzzle is resolved if $\rho > 1$. If $0 \leq \rho < 1$ and $\rho \neq \bar{\rho}$ then the fiscal multiplier puzzle is not resolved.

Proposition 4 (The Paradox of Toil). *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, and $0 < \rho^* < 1$ has the property that there exists a $\tilde{\rho}$ such that if $\rho < \tilde{\rho}$, then equilibrium exhibits the paradox of toil, and if $\rho > \tilde{\rho}$, the paradox of toil is resolved.*

The dynamic multiplier for a supply shock is:

$$\partial \pi_T/\partial a_{T+\Delta_p} = \partial d^{\pi}_T/\partial a_{T+\Delta_p} = \frac{\partial d^{\pi}_{T+1}/\partial a_{T+\Delta_p} + \kappa(1-\gamma_0^{-1}\lambda_{T+\Delta_p+1})\beta^{-1}\gamma_0^{-\Delta_p}\Pi_{k=0}^{\Delta_p-1}\lambda_{T+\Delta_p-k}}{\lambda_{T+1} - b^{\pi}_{T+1}}$$

given $\partial d^{\pi}_{T+\Delta_p}/\partial a_{T+\Delta_p} = -\kappa(\beta\gamma_0)^{-1}\lambda_{T+\Delta_p+1}(\lambda_{T+\Delta_p+1} - b^*_{\pi})^{-1} < 0$. The output response for $\Delta_p = 0$ is:

$$\frac{\partial y_{T+\Delta_p}}{\partial a_{T+\Delta_p}} = \frac{\partial d^{y}_{T+\Delta_p}}{\partial a_{T+\Delta_p}} = \kappa^{-1}(1-\beta b^*_{\pi})\frac{\partial d^{\pi}_{T+\Delta_p}}{\partial a_{T+\Delta_p}} + 1 = \frac{\beta b^*_{\pi}(\lambda_{T+\Delta_p+1} - \gamma_0) + (\beta\gamma_0 - 1)\lambda_{T+\Delta_p+1}}{\beta\gamma_0(\lambda_{T+\Delta_p+1} - b^*_{\pi})} > 0$$

and for $\Delta_p > 0$:

$$\partial y_T/\partial a_{T+\Delta_p} \;=\; \partial d^{y}_t/\partial a_{T+\Delta_p} = \kappa^{-1}(1-\beta\lambda_{T+1})\frac{\partial d^{\pi}_T}{\partial a_{T+\Delta_p}} + \gamma_0^{-\Delta_p}(1-\gamma_0^{-1}\lambda_{T+\Delta_p+1})\Pi_{k=0}^{\Delta_p-1}\lambda_{T+\Delta_p-k}.$$

It is straightforward to show that (a) if $\partial d^{\pi}_{T+j}/\partial a_{T+\Delta_p} < 0$ for some $j \in [0,\ldots,\Delta_p]$ then $\partial d^{\pi}_{T+k}/\partial a_{T+\Delta_p} < 0$ for all $j \leq k \leq \Delta_p$; (b) if $\partial d^{\pi}_{T+j}/\partial a_{T+\Delta_p} \geq 0$ or $\partial d^{\pi}_{T+j}/\partial a_{T+\Delta_p} < 0$ but sufficiently small in magnitude then $\partial d^{y}_{T+j}/\partial a_{T+\Delta_p} > 0$, for $j \in [0,\ldots,\Delta_p - 1]$; (c):

$$\frac{\partial}{\partial\rho}\left(\frac{\partial d^{\pi}_{T+\Delta_p}}{\partial a_{T+\Delta_p}}\right) = 0$$

$$\frac{\partial}{\partial\rho}\left(\frac{\partial d^{\pi}_{T+\Delta_p-1}}{\partial a_{T+\Delta_p}}\right) = \frac{\partial d^{\pi}_{T+\Delta_p-1}}{\partial a_{T+\Delta_p}}(\lambda_{T+\Delta_p} - b^{\pi}_{T+\Delta_p})^{-1}\frac{\partial b^{\pi}_{T+\Delta_p}}{\partial\rho} > 0$$

if $\frac{\partial d^{\pi}_{T+\Delta_p-1}}{\partial a_{T+\Delta_p}} < 0$, and hence, by induction:

$$\frac{\partial}{\partial\rho}\left(\frac{\partial d^{\pi}_{T+k}}{\partial a_{T+\Delta_p}}\right) = (\lambda_{T+k+1} - b^{\pi}_{T+k+1})^{-1}\left(\frac{\partial d^{\pi}_{T+k}}{\partial a_{T+\Delta_p}}\frac{\partial b^{\pi}_{T+k+1}}{\partial\rho} + \frac{\partial}{\partial\rho}\left(\frac{\partial d^{\pi}_{T+k+1}}{\partial a_{T+\Delta_p}}\right)\right) > 0$$

when $\frac{\partial d^{\pi}_{T+k}}{\partial a_{T+\Delta_p}} < 0$. By assertions (a)-(c), if $\rho > \tilde{\rho}$ for some $\tilde{\rho} \geq 0$, then $\partial y_T/\partial a_{T+\Delta_p} > 0$. This proves that the paradox of toil is resolved if ρ is sufficiently large. □

Proposition 5 (Paradox of Flexibility). *The NK model with monetary policy given by equations (14), (15), (19) , and (20) with $\phi^* > 1$, $0 \leq \phi < 1$, and $0 < \rho^* < 1$ does not exhibit the paradox of flexibility if $\rho > 0$.*

Define $R_t := \kappa^{-1} X_t$, $Q_t := \gamma_0^{-(T^*-t)}(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1})\Pi_{k=0}^{T^*-t-1}\lambda_{T^*-k} = \gamma_0^{-1}\lambda_{t+1}Q_{t+1}$, and $Q_{T^*-1} := \gamma_0^{-1}\lambda_{T^*}(\gamma_0^{-1}\lambda_{T^*+1}X_{T^*} + X_{T^*-1})$. We have

$$\lim_{\kappa\to\infty} Q_{T^*-1} = \sigma\beta R_{T^*-1}$$

which implies

$$\lim_{\kappa\to\infty} Q_t = \left(\lim_{\kappa\to\infty} \gamma_0^{-1}\lambda_{t+1}\right)\left(\lim_{\kappa\to\infty} Q_{t+1}\right) = 0$$

for $t = T, \ldots, T^* - 2$. For $t = T^*$

$$\lim_{\kappa\to\infty} d^\pi_{T^*} \quad = \quad \lim_{\kappa\to\infty} \frac{\gamma_0^{-1}\lambda_{T^*+1}\kappa}{\lambda_{T^*+1} - b^*_\pi} R_{T^*} = (\rho^* + \phi)^{-1}\sigma\beta R_{T^*}$$

Then for $t = T^* - 1$, if $\rho > 0$ or $\phi > 0$, then

$$\lim_{\kappa\to\infty} d^\pi_{T^*-1} = \left((\rho^* + \phi)^{-1}\sigma\beta R_{T^*} + \sigma\beta R_{T^*-1}\right)\left(\lim_{\kappa\to\infty}(\lambda_{T^*} - b^\pi_{T^*})^{-1}\right) = \frac{(\rho^* + \phi)^{-1}\sigma\beta R_{T^*} + \sigma\beta R_{T^*-1}}{\phi + \rho\rho^*(\phi + \rho^*)^{-1}}$$

and if $\rho = 0$ and $\phi = 0$, then

$$\lim_{\kappa\to\infty} d^\pi_{T^*-1} = \pm\infty.$$

Proceeding recursively for $t = T^* - 2, \ldots, T$:

$$\lim_{\kappa\to\infty} d^\pi_t \quad = \quad \lim_{\kappa\to\infty}\left(\frac{d^\pi_{t+1} + Q_t}{\lambda_{t+1} - b^\pi_{t+1}}\right)$$

If $\rho > 0$, then $\lim_{\kappa\to\infty} d^\pi_t$ exists for $t = T^* - 2, \ldots, T$ and through application of limit laws

$$\lim_{\kappa\to\infty} d^\pi_t \quad = \quad \frac{(\rho^* + \phi)^{-1}\sigma\beta R_{T^*} + \sigma\beta R_{T^*-1}}{\phi + \rho\rho^*(\phi + \rho^*)^{-1}}\left(\Pi_{i=1}^{\Delta_p - 1}\left(\phi - \lim_{\kappa\to\infty} b^\pi_{t+i}\right)\right)^{-1}.$$

Therefore, if $\rho = 0$ and $\phi = 0$, then $\lim_{\kappa\to\infty} d^\pi_t = \pm\infty$.

Now, we consider the output response:

$$\lim_{\kappa\to\infty} d^y_t = \lim_{\kappa\to\infty}\left((1 - \beta\lambda_{t+1})\kappa^{-1} d^\pi_t + \beta\kappa^{-1} Q_t\right) = \lim_{\kappa\to\infty}(1 - \beta\lambda_{t+1})\kappa^{-1} d^\pi_t$$

Hence, $\lim_{\kappa\to\infty} d^y_t = 0$ if $\rho > 0$. Therefore, the paradox of flexibility is resolved if $\rho > 0$. □

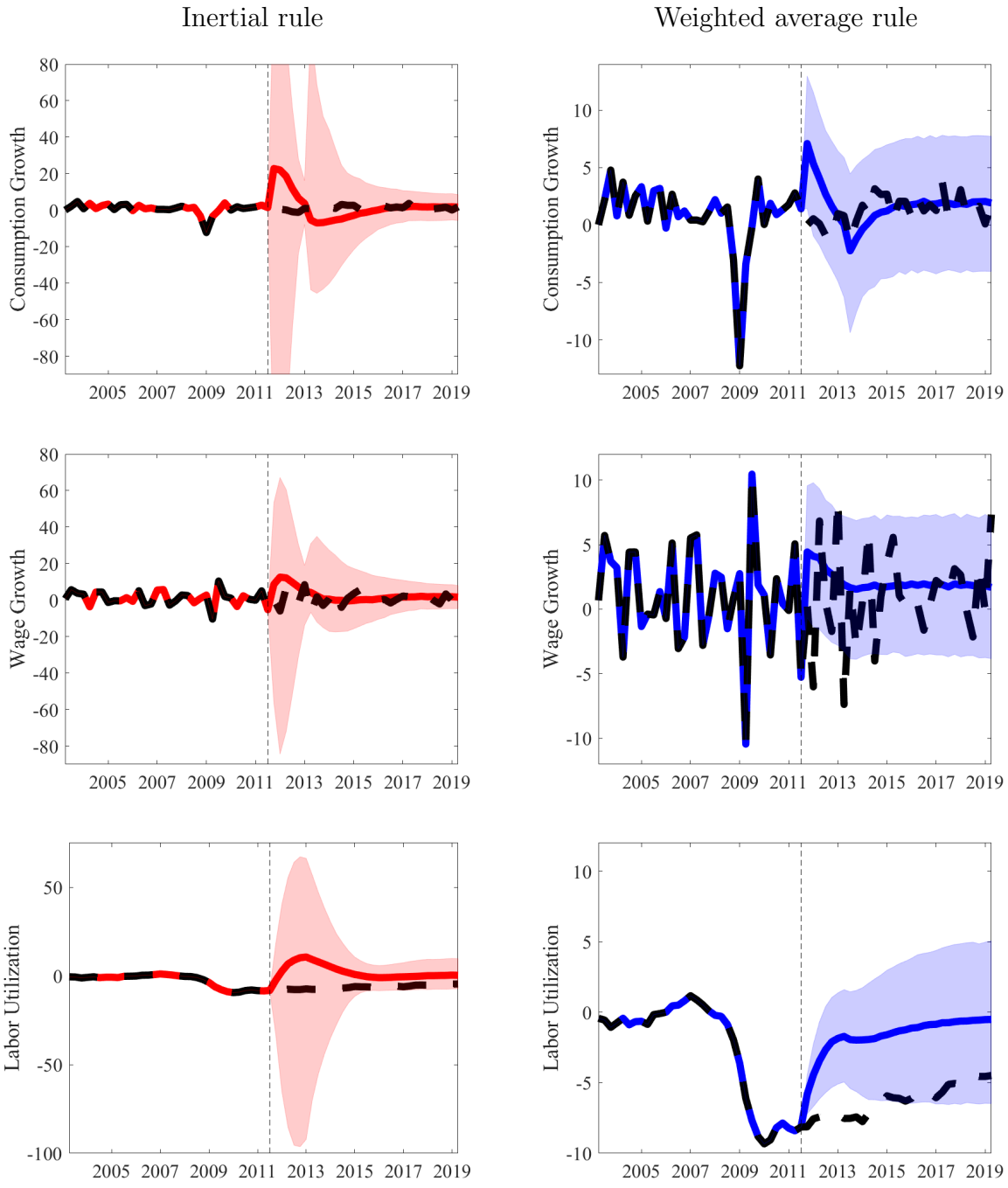
B3 Additional Figures

This appendix contains additional figures showing the effect on other variables of the Smets and Wouters model not shown in the main text.

Figure B1 shows real-time out-of-sample forecasts for three observables used in estimation of the Smets and Wouters model that are not shown in Figures 1 and 2.

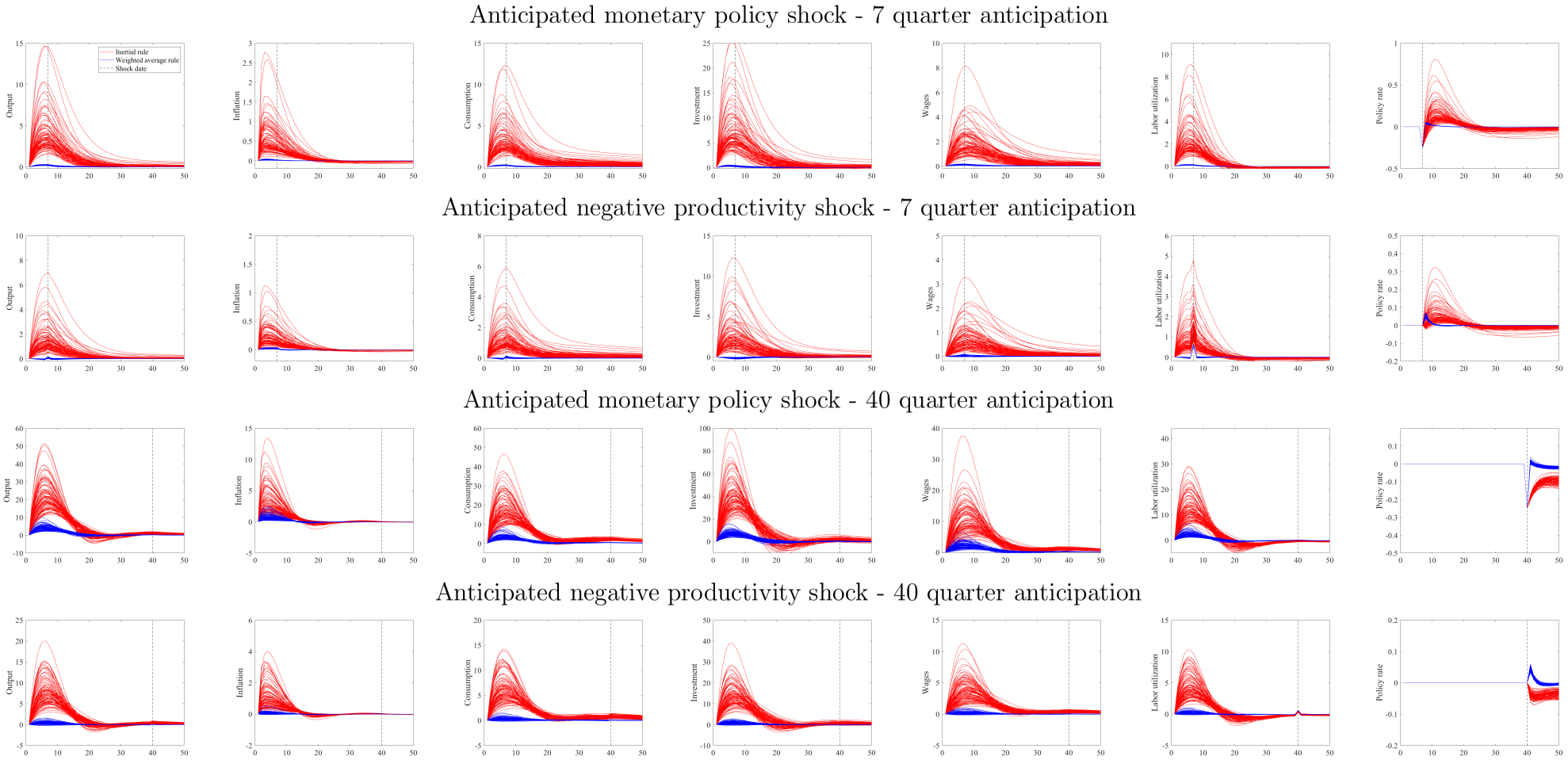
Figure B2 shows the impulse repsonses for inflation, consumption, interest rates, investment, wages and labor utilization.

Figure B1: Calendar-based forward guidance comparison



Out-of-sample forecasts of the August, 9^{th} 2011 Federal Reserve calendar-based forward guidance promise using the model of Smets and Wouters (2007) under different forms of the interest rate rule. The dashed line is the actual data. The solid line is the median forecast. The shaded region represents the 84th and 16th percentiles of forecasts.

Figure B2: The inertial rule vs the weighted average rule



Notes: The black dashed line indicates when the shock occurs. Each solid line corresponds to the impulse response implied by one draw from the posterior distribution of parameters for a one-time 100-basis point monetary policy shock anticipated to occur seven or forty quarters in the future under an interest rate peg (left) or a one-time one standard deviation negative productivity shock anticipated to occur seven or forty quarters in the future under an interest rate peg (right). In period 8 (41) policy returns to either the inertial rule or the weighted average rule. Neither shock has persistence.

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