# Stochastic Social Preferences *

Thomas Dangl† Michael Halling‡ Jin Yu§ Josef Zechner¶

May 1, 2025

**ABSTRACT**

We develop a dynamic general equilibrium model with stochastic social preferences and endogenous corporate investment in the green transition. Firms' responses to changing investor tastes mitigate valuation effects of preference shocks occurring when investment is fixed. Thus, small changes in social preferences can have negligible cost-of-capital consequences, but large impact on corporate decisions. Our analysis shows that stochastic social preferences delay the transition, especially when preference shocks correlate positively with aggregate cash flows, and that they generate time-varying correlations between green and brown firms' returns. While risk aversion initially accelerates the transition it impedes it later due to risk-sharing considerations.

**JEL Classifications:** D62, D64, G11, G12, G31, G41.

**Keywords:** Social Preferences, Portfolio Choice, Corporate Investment in Social Responsibility.

*Acknowledgments: We thank Robert McDonald (discussant), Lubos Pastor (discussant), Carlo Sala (discussant), Hongyu Shan (discussant), and seminar and conference participants of the Cambridge Corporate Finance Theory Symposium 2022, the Irish Academy of Finance 4th Annual Conference 2023, the UBC Summer Finance Conference 2023, the Sydney Banking and Financial Stability Conference 2023, the Australasian Finance and Banking Conference 2023, the Western Finance Association Conference 2024, Curtin University, Hebrew University of Jerusalem, Tel Aviv University, TBEAR workshop KIT 2024, University of Mannheim, University of Melbourne, and the University of Western Australia for their comments. All errors are our responsibility.

†Vienna University of Technology, Theresianumgasse 27, A-1040 Vienna, Austria. Email: thomas.dangl@tuwien.ac.at

‡University of Luxembourg, 6, rue Richard Coudenhove-Kalergi, L-1359 Luxembourg. Email: michael.halling@uni.lu

§Department of Banking and Finance, Monash University, 900 Dandenong Road, Caulfield East, VIC 3145, Australia. Email: jin.yu@monash.edu

¶Vienna University of Economics and Business (CEPR and ECGI), Welthandelsplatz 1, 1020 Vienna, Austria. Email: josef.zechner@wu.ac.at

# 1 Introduction

Models of optimal portfolio choice in financial economics have evolved beyond their traditional exclusive focus on risk-return trade-offs to increasingly account for investors' non-pecuniary considerations. These considerations, often linked to investors' identities and ethical considerations, include responsibilities toward the environment, human rights, their religious beliefs, and political preferences. Among these, environmental concerns are particularly prominent, with a significant share of assets now owned or managed by investors who take into account firms' environmental standards.[1] Accordingly, this paper primarily interprets investors' *social preferences* through the lens of their environmental concerns, while recognizing that the insights extend to other forms of non-pecuniary considerations.

Social preferences are not constant over time and appear to change stochastically. In fact, Pastor et al. (2022) find that shocks to investor tastes were the main driver of return differences in the cross section of U.S. stocks with different ESG performances. The stochastic nature of the evolution of social preferences is also evidenced by recent pushbacks from some investors and law makers against the use of Socially Responsible Investment (SRI) filters in the context of delegated portfolio management. Also, inflows to ESG investment vehicles seem to have slowed or even reversed.[2] Furthermore, there exists academic evidence that social preferences change in response to past economic performance, consistent with a stochastic evolution of social preferences.[3]

The effects of the stochastic nature of social preferences on corporate investment strategies, on firms' cost of capital, and on the transition towards a green economy more generally remain theoretically largely unexplored. The only exception hereby is the seminal paper by Pastor et al. (2021) who, in an extension of their base model, allow for a one-time shock to investor tastes and for an additional round of trading once the preference shock has been

[1] As of the end of 2024, assets of signatories of the Principles of Responsible Investment amount to more than $128 trillion. Investors' social preferences are also revealed via yield spreads between green bonds and essentially identical non-green twins, see Pastor et al. (2022).

[2] For example, according to a Reuters article from November 21, 2024, "Global climate funds set for first annual outflows, Morningstar says", net withdrawals from global climate funds totaled $24 billion during the first nine months of 2024. Morningstar Sustainalytics attributed these outflows to factors including growing anti-ESG sentiment, see https://www.reuters.com/sustainability/sustainable-finance-reporting/global-climate-funds-set-first-annual-outflows-morningstar-says-2024-11-21/

[3] See, e.g., Exley et al. (2023).

realized.[4] Our paper provides a first fully dynamic stochastic general equilibrium model where investors make portfolio decisions and firms make investment choices at each point in time, anticipating the stochastic evolution of a representative investor's taste for social responsibility.

This framework generates several new insights that are unlikely to emerge in partial equilibrium models with fixed technology supply and deterministic social preferences. First, we show that firms' real options to switch to a green technology crucially affect price dynamics in response to preference shocks. When social preferences are sufficiently strong so that additional firms find it optimal to switch to green technologies, rationally anticipated supply effects undo much of the impact of shocks to social preferences on stock prices. Preference shocks only change share valuations significantly when they are far below the threshold where additional firms would find it optimal to switch to green technologies, or when most firms have already switched to a green technology, as supply is inelastic in these cases.

Second, we show that uncertainty about future social preferences delays the move to a greener economy. In the presence of stochastic social preferences, firms optimally delay the exercise of their real option to switch to a green technology. This delay effect becomes even stronger when preference shocks are positively correlated with aggregate cash flow shocks, as empirical evidence suggests. Such a correlation makes brown technologies more attractive, since it makes them less risky: when brown technologies experience low cash flows due to a negative common cash flow shock, they simultaneously tend to experience a positive valuation shock, due to weakening social preferences.

Third, we show that higher risk aversion initially helps the transition to greener technologies, but delays a complete transition of the economy away from polluting technologies. This finding highlights an important interaction between risk-sharing and corporate social responsibility. Investors demand both green and brown technologies to diversify risks. At the beginning of the transition, green technologies are in limited supply. This lowers the equilibrium cost of capital for green shares, thereby accelerating the transition. However, when green shares become predominant towards the end of the transition, scarcity in the supply of brown shares lowers their cost of capital, thereby slowing the transition towards a

[4] Gupta et al. (2022) focus on a different economic channel – search frictions in the sale of firms – and show that these frictions can generate significant social costs by delaying corporate green reforms. In an extension of their base model, they consider the case in which investors might become more socially responsible in the future, which results in a further delay to trade in their setup.

fully green economy.

Fourth, stochastic social preferences induce time-varying correlations between the returns of green and brown technologies. The intuition behind these correlation dynamics can be explained as follows. In the absence of supply responses, social preferences only affect brown firms' values, since the representative investor in our model perceives non-pecuniary costs only when holding brown shares. Consequently, when the cash flows of the two technologies are positively correlated, stochastic preferences actually lower the return correlation between the two stocks. This occurs because preference shocks do not affect the covariance, but they increase the standard deviation of brown firms' returns. This intuition holds when social preferences are weak or substantially below past highs, so that supply effects are irrelevant for equilibrium prices. However, as social preferences become stronger, it becomes more likely that the number of green firms will increase as a result, which affects green share prices negatively. Share prices of brown firms also drop, since the negative impact of stronger social preferences on investors' demand for brown firms always (weakly) exceeds the positive effect on the option value to switch. Thus, as supply effects start to matter, they introduce a positive covariance between the two stock returns, leading to an increase in their correlation. The resulting empirical prediction is that one should generally observe an increase in the correlation between brown and green firms as social preferences become stronger and some brown firms switching to the green technology becomes more likely.

Finally, we show that a small observed difference between the cost of capital of brown and green firms due to social preferences, which we refer to as the cost of capital gap, does not imply a small effect of social preferences on technology supply. Specifically, the sensitivity of the supply of green firms to a small increase in social preferences is not monotonically related to the cost of capital gap. Furthermore, the observed cost of capital gap can vary substantially when corporate technology supply is inelastic, i.e., when social preferences are well below past highs. Altogether, we find that the cost of capital gap is not a reliable indicator of the influence of social preferences on corporate behavior.

The analysis in this paper focuses on the effects of social preferences on corporate behavior through investors' portfolio decisions, intentionally excluding other channels that might influence firms' cash flows rather than their discount rates. For instance, social preferences could imply a consumer preference for goods and services from green firms over brown firms. Furthermore, agents with social preferences may directly influence managers' deci-

sions through voting at shareholder meetings or other informal channels, thereby impacting firms' cash flows. To isolate the effects of portfolio decisions, the model presented in this paper abstracts from such additional potential effects of social preferences.

Our paper is related to several strands of literature. First, there is a growing literature that analyzes the effect of social preferences on financial market equilibrium and corporate investment in a one-period, static framework. This literature includes Heinkel et al. (2001), Gollier and Pouget (2014), Hart and Zingales (2017), Pastor et al. (2021), Pedersen et al. (2021), Broccardo et al. (2022), Edmans et al. (2023), Landier and Lovo (2022), Oehmke and Opp (2024), Goldstein et al. (2022) and Berk and van Binsbergen (2024). These models differ in how they model social preferences. Some assume that social investors simply find it wrong to own shares of firms which do not accord with their ethical considerations, while others assume that social investors perceive non-pecuniary dividends from their portfolio firms, depending on how well they accord with their ethical standards. Yet another group of authors considers that social investors only care about the consequences that their portfolio decisions have on firms' negative externalities on society.[5]

All of the above papers use a static, essentially one-period framework. Only few papers analyze the role of social preferences for asset pricing and corporate investment in a dynamic framework. Such exceptions are Hong et al. (2023), Jagannathan et al. (2023), and Bustamante and Zucchi (2024). We differ from the above papers by providing a continuous-time framework that captures stochastic shocks to investors' tastes and allows both investors and firms to take this into account when making portfolio and investment decisions, respectively.

Finally, our paper is also related to a growing literature that provides experimental and empirical evidence on social preferences. Overall, this literature provides convincing support for the existence of social preferences that are reflected in agents' investment decisions. Studies that present such evidence include Riedl and Smeets (2017), Bauer et al. (2021), Krueger et al. (2020), Dyck et al. (2019), Bolton et al. (2020), Hartzmark and Sussman (2019), and Barber et al. (2021). Several papers within this literature provide evidence on the specific type of social preferences that investors have. Overall, these papers find that investors with social preferences do not seem to be motivated by the perceived consequences of their investment decisions, but rather by intrinsic ethical considerations (see, e.g., Ottoni-

[5]See Dangl et al. (2023) for a comprehensive discussion of different modeling approaches to social preferences.

Wilhelm et al., 2017; Hart et al., 2023; Heeb et al., 2022; Bonnefon et al., 2022; Cole et al., 2023; Humphrey et al., 2022).

Another strand of the literature suggests that social preferences might vary over time, for example in response to personal experiences (e.g., Choi et al., 2020, for exposure to heat and Andersen et al., 2024, for investors' children suffering from respiratory disease) or increased transparency (e.g., Fisman et al., 2023, find investor portfolio reallocations due to information from air monitoring stations installed in India). Our modeling approach accords with these experimental and empirical findings and features stochastic non-pecuniary payoffs that social investors receive from their portfolio holdings, depending on firms' social responsibility.

The rest of the paper is organized as follows. Section 2 develops the model. Investors' portfolio decisions are analyzed in Section 3.2 and Section 3.3 derives corporate equilibrium decisions. Numerical results are presented in Section 4 and Section 5 concludes.

# 2 The model

We consider a model where time is continuous and the horizon is infinite. The financial market consists of one riskless asset and two risky assets (shares). The riskless asset is in perfectly elastic supply and yields a constant return of $rdt$ over a time period $[t, t + dt)$. Equity shares are issued by firms that produce with one of two technologies, which we refer to as *green* and *brown*.

We normalize the total mass of all (green plus brown) firms and the number of shares to one and denote firm type (technology) by $f \in \{G, B\}$. The supply of firms with the green technology (the supply of green shares) is denoted by $S_G$, so that the supply of firms with the brown technology (the supply of brown shares), $S_B$, is $1 - S_G$.

**Investors and preferences.** There is a continuum of investors with heterogeneous social preferences. Investors are competitive, i.e., they do not act strategically and take prices as given. In aggregate, social preferences of the entire investor base are characterized by a

stochastic parameter, $g_t$, driven by persistent (random) shocks:[6]

$$dg_t = \mu_g dt + \sigma_g dz_{g,t}. \tag{1}$$

The shocks to social preferences, $dz_{t,g}$, may be correlated with shocks to firms' cash flows, as explained below.

We assume a representative investor who maximizes expected utility choosing consumption $C_t$ and the holdings of green and brown shares, $X_{G,t}$ and $X_{B,t}$

$$U_t = \max_{(X_{G,s},X_{B,s},C_s),s\geq t} E_t\left[\int_t^\infty e^{-\delta s}u(C_s)ds\right]$$

where $\delta > 0$ is the representative investor's discount rate and $u(C_t) = -\frac{e^{-\gamma C_t}}{\gamma}$ is an *instantaneous* CARA utility function on consumption. While the investor derives positive utility from consuming the physical consumption good, $C_{\$,t}$, there is a negative non-pecuniary dividend ("cold prickle") associated with holding brown stocks whenever $g_t$ is positive, i.e., her total time-$t$ consumption is therefore decomposed into

$$C_t dt = C_{\$,t}dt - g_t^+ X_{B,t}dt, \tag{2}$$

where $g_t^+ = \max\{g_t, 0\}$ prevents brown stocks from delivering positive non-pecuniary benefits when social preferences are weak such that $g_t$ is negative.[7]

**Firms, production technologies, and cash flows.** At time 0, all firms are endowed with the brown technology and each marginal firm has an option to switch to the green technology. The full transition to the green technology, i.e., the exercise of all individual firms' options requires total investment $I$. Brown firms' value-maximizing managers choose when to optimally exercise their option, rationally anticipating other brown firms' optimal investment policies and the effects of investors' stochastic social preferences. The investment

[6] We do not distinguish between social investors who care about environmental issues and pure financial investors. Consequently, we are agnostic about whether aggregate preferences increase (decrease) as a reaction to an increase (decrease) in the fraction of social investors or because existing social investors show stronger (weaker) preferences for green investments.

[7] The cutoff level at $g_t = 0$ is an arbitrary choice with no qualitative influence on the results.

is irreversible.[8] We hereby assume that each firm behaves competitively, taking prices as given.

Firms' cash flows are subject to aggregate shocks ($dz_{A,t}$) and technology-specific shocks ($dz_{f,t}$). Over a period $[t, t+dt)$, firms' cash flows are:[9]

$$dy_{B,t} = \mu dt + \sigma dz_{B,t} + \sigma_A dz_{A,t}, \tag{3}$$

$$dy_{G,t} = \mu dt + \sigma dz_{G,t} + \sigma_A dz_{A,t}. \tag{4}$$

Technology-specific and aggregate shocks to cash flows are, by definition, pairwise uncorrelated, i.e.,

$$cov_t[dz_{G,t}, dz_{B,t}] = cov_t[dz_{G,t}, dz_{A,t}] = cov_t[dz_{B,t}, dz_{A,t}] = 0.$$

However, green and brown firms' cash flows are correlated through aggregate shocks, i.e.,

$$\begin{aligned} cov_t[dy_{G,t}, dy_{B,t}] &= cov_t[\sigma dz_{G,t} + \sigma_A dz_{A,t}, \sigma dz_{B,t} + \sigma_A dz_{A,t}] \\ &= \sigma_A^2 dt = \rho(\sigma^2 + \sigma_A^2) dt \end{aligned}$$

where $\rho \equiv \frac{\sigma_A^2}{\sigma^2+\sigma_A^2}$ is defined as the correlation coefficient of the two cash flows.

Since there is evidence that shocks to financial returns are related to shocks in social preferences,[10] we also allow aggregate cash flow shocks and innovations to social preferences to be correlated, i.e.,

$$cov_t[dz_{A,t}, dz_{g,t}] = \rho_{Ag} dt.$$

# 3 Competitive market equilibrium

This section first derives firms' technology decisions (Subsection 3.1) and then the representative investor's portfolio decisions (Subsection 3.2). Subsection 3.3 combines supply and demand side analyses and provides equilibrium risk premia and share prices as solutions to

[8] While this assumption keeps the model tractable, it can be relaxed within our modeling framework. Appendix C provides a detailed discussion including selected results for the case where investment is fully reversible.

[9] We assume that expected cash flows of both firms equal $\mu dt$. If differences arise, e.g., caused by costly pollution abatement expenses incurred by green firms, the present value of future costs can, without loss of generality, be modelled as part the adjustment costs $I$, introduced above.

[10] See, e.g., Exley et al. (2023).

a fixed-point problem, i.e., the market-clearing prices and risk premia that are consistent with those used by firms and investors when determining optimal portfolio and investment decisions.

## 3.1 Firm valuation and optimal technology choice

As will be shown below, ex-dividend share prices depend on both social preferences, $g$, and the endogenously determined supply of green firms, $S_G$. In contrast, firms' cash flows, $dy_G$ and $dy_B$, follow stationary distributions, i.e., have no persistent component. While they impact investors' portfolio choice, they do not affect ex-dividend share prices as state variables. In this subsection, we take risk premia for green and brown shares, $\tilde{\pi}_G(g; S_G)$ and $\tilde{\pi}_B(g; S_G)$, as given and derive share prices $\tilde{P}_G(g; S_G)$ and $\tilde{P}_G(g; S_G)$ contingent on preferences $g$ and the supply of green shares $S_G$. Using this notation, we can derive the valuation equations for brown and green firms' stocks.

### 3.1.1 Brown firm valuation

To derive the share price of brown firms, we conjecture that their optimal technology choice is a free boundary problem. That is, for a given $S_G$, brown firms have no incentive to switch to the green technology by incurring the cost $I$ per unit of reformed firm as long as social preferences $g$ are below a critical threshold $\bar{g}(S_G)$, which depends on optimal choices by other brown firms in equilibrium. In this section, we take the optimal investment strategy of brown firms as given and derive valuation equations in the range of preferences where the supply of the green technology, $S_G$, is constant, i.e., $g < \bar{g}(S_G)$. The investment threshold will be optimally determined and the conjectured optimality of the free boundary will be verified in Section 3.3.

The share price of a brown firm can then be expressed as the present value of expected future dividends plus the value of a real option to switch to the green technology:

$$\tilde{P}_B(g_t; S_{G,t}) = \max_{T_t^* \geq t} E_t \left[ \int_t^{T_t^*} e^{-\int_t^\tau (r+\tilde{\pi}_B(g_s;S_{G,s}))ds} dy_{B,\tau} + e^{-\int_t^{T_t^*} (r+\tilde{\pi}_B(g_s;S_{G,s}))ds} \left( \tilde{P}_G(g_{T_t^*}, S_{G,T_t^*}) - I \right) \right]$$

where $T_t^*$ is the optimal time of the investment required to switch to the green technology,

defined as

$$T_t^* \equiv \inf\big(s \geq t \big| g_s = \bar{g}(S_{G,t})\big).$$

Applying the Feynman-Kac Theorem to the share price function, we obtain the Hamilton-Jacobi-Bellman (HJB) equation for the region $g < \bar{g}(S_G)$ where $S_G$ is fixed:[11]

$$(r + \tilde{\pi}_B(g; S_G))\tilde{P}_B(g; S_G) = \max_{T^*} \left[\mu + \mu_g \tilde{P}'_B(g; S_G) + \frac{1}{2}\sigma_g^2 \tilde{P}''_B(g; S_G)\right]. \tag{5}$$

### 3.1.2 Green firm valuation

Since green firms do not have a real option to switch technology, their share price is given by

$$\tilde{P}_G(g_t; S_{G,t}) = E_t\left[\int_t^\infty e^{-\int_t^\tau (r+\tilde{\pi}_G(g_s;S_{G,s}))ds} dy_{G,\tau}\right].$$

Applying the Feynman-Kac Theorem again, we obtain the following HJB equation for $\tilde{P}_G$:[12]

$$(r + \tilde{\pi}_G(g; S_G))\tilde{P}_G(g; S_G) = \mu + \mu_g \tilde{P}'_G(g; S_G) + \frac{1}{2}\sigma_g^2 \tilde{P}''_G(g; S_G). \tag{6}$$

Thus, Equations (5) and (6) state that the total return, i.e., the risk premium plus the riskless rate, required by investors (the left-hand sides of the equations), must equal the expected return provided by the green and brown stocks (the right-hand sides of the equations), respectively, where the expected dividends are

$$\mu dt = E_t[dy_G] = E_t[dy_B]$$

and expected capital gains or losses are

$$\left(\mu_g \tilde{P}'_f(g; S_G) + \frac{1}{2}\sigma_g^2 \tilde{P}''_f(g; S_G)\right) dt = E_t[d\tilde{P}_f(g; S_G)], \quad f \in \{B, G\}.$$

Note that non-pecuniary dividends, $-g_t^+$, do not explicitly enter the expressions for share

[11] See, e.g., Øksendal (2003).

[12] Since green firms do not face an optimization problem, their HJB equation is reduced to an ordinary differential equation (ODE).

prices. However, as will be shown, they affect share prices through their impact on the equilibrium risk premium.

## 3.2 Investors' portfolio decisions

We now turn to the demand side of the financial market. We hereby take the share price functions, $\hat{P}_G(g; S_G)$ and $\hat{P}_B(g; S_G)$, as given and first derive the conditional distribution of the excess financial returns of the two risky assets in the following Lemma. We then analyze the representative investor's demand for green and brown shares, $X_G$ and $X_B$, and solve for risk premia by inverting the demand.

Letting $W_t$ denote financial wealth at time $t$ and assuming that the supply of green assets does not change over the interval $[t, t+dt]$, the investor faces the following budget constraint[13]

$$\begin{aligned} W_{t+dt} &= (1+rdt)W_t - C_{\$,t}dt + (dy_{G,t} + d\hat{P}_G(g_t) - r\hat{P}_G(g_t)dt)X_{G,t} \\ &\quad +(dy_{B,t} + d\hat{P}_B(g_t) - r\hat{P}_B(g_t)dt)X_{B,t}. \end{aligned}$$

Denoting the wealth differential by $dW_t = W_{t+dt} - W_t$, the resulting wealth dynamics are

$$\begin{aligned} dW_t &= rW_t dt - C_{\$,t}dt + \left(dy_{G,t} + d\hat{P}_G(g_t) - r\hat{P}_G(g_t)dt\right) X_{G,t} \\ &\quad + \left(dy_B + d\hat{P}_B(g_t) - r\hat{P}_B(g_t)dt\right) X_{B,t} \\ &= rW_t dt - C_t dt + \left(dy_G + d\hat{P}_G(g_t) - r\hat{P}_G(g_t)dt\right) X_{G,t} \\ &\quad + \left(dy_B + d\hat{P}_B(g_t) - r\hat{P}_B(g_t)dt - g^+ dt\right) X_{B,t} \end{aligned} \tag{7}$$

where the second equality follows from Equation (2) stating that the investor derives utility from consuming the physical consumption good reduced by the non-pecuniary losses. Inspecting these wealth dynamics reveals that the wealth differential contains a linear combination of the excess financial returns of the two risky assets with their weights given by $X_{G,t}$ and $X_{B,t}$, respectively.

[13] For ease of exposition, we suppress the argument $S_G$ when the supply of technology is kept constant and write prices and risk premia as $\hat{P}_G(g)$, $\hat{P}_B(g)$, and $\hat{\pi}_G(g)$, $\hat{\pi}_B(g)$ where no ambiguity arises.

Lemma 1 characterizes the conditional distribution of the excess financial returns:[14]

**Lemma 1** *Over a period* $[t, t+dt)$ *and conditional on* $g_t$*, define the stock price differentials (i.e., capital gains/losses) as*

$$\begin{aligned} d\hat{P}_G(g_t) &\equiv \hat{P}_G(g_{t+dt}) - \hat{P}_G(g_t), \\ d\hat{P}_B(g_t) &\equiv \hat{P}_B(g_{t+dt}) - \hat{P}_B(g_t). \end{aligned}$$

*The excess financial returns are bivariate-normally distributed, i.e.,*

$$\begin{bmatrix} dy_{G,t} + d\hat{P}_G(g_t) - r\hat{P}_G(g_t)dt \\ dy_{B,t} + d\hat{P}_B(g_t) - r\hat{P}_B(g_t)dt \end{bmatrix} \sim \mathcal{N}(M_\Pi dt, M_\Sigma dt),$$

*where* $M_\Pi \equiv \begin{bmatrix} \hat{\pi}_G(g_t)\hat{P}_G(g_t) \\ \hat{\pi}_B(g_t)\hat{P}_B(g_t) \end{bmatrix}$ *with*

$$\begin{aligned} \hat{\pi}_G\hat{P}_G &= \mu + \mu_g\hat{P}'_G + \frac{1}{2}\sigma_g^2\hat{P}''_G - r\hat{P}_G, \\ \hat{\pi}_B\hat{P}_B &= \mu + \mu_g\hat{P}'_B + \frac{1}{2}\sigma_g^2\hat{P}''_B - r\hat{P}_B, \end{aligned}$$

*and* $M_\Sigma \equiv \begin{bmatrix} \Sigma_G(g_t) & \Sigma_{GB}(g_t) \\ \Sigma_{GB}(g_t) & \Sigma_B(g_t) \end{bmatrix}$ *with*

$$\begin{aligned} \Sigma_G &= \sigma_g^2(\hat{P}'_G)^2 + \sigma^2 + \sigma_A^2 + 2\rho_{Ag}\sigma_g\sigma_A\hat{P}'_G, \\ \Sigma_B &= \sigma_g^2(\hat{P}'_B)^2 + \sigma^2 + \sigma_A^2 + 2\rho_{Ag}\sigma_g\sigma_A\hat{P}'_B, \\ \Sigma_{GB} &= \sigma_g^2\hat{P}'_G\hat{P}'_B + \sigma_A^2 + \rho_{Ag}\sigma_A\sigma_g(\hat{P}'_G + \hat{P}'_B). \end{aligned}$$

We can now solve the investor's optimal portfolio decision. Let $U(W, g)$ denote the investor's value function. Using the results stated in Lemma 1 and applying the theorem of

[14] See Appendix A for a proof.

Feynman-Kac, we obtain the HJB equation of the investor:

$$\begin{aligned}\delta U(W,g;S_G) &= \max_{C,X_G,X_B} u(C)+(rW-C)U_W+\begin{bmatrix}X_G\\X_B\end{bmatrix}^T\left(M_\Pi-\begin{bmatrix}0\\g^+\end{bmatrix}\right)U_W \\ &\quad +\frac{1}{2}\begin{bmatrix}X_G\\X_B\end{bmatrix}^T M_\Sigma\begin{bmatrix}X_G\\X_B\end{bmatrix}U_{WW}+\begin{bmatrix}X_G\\X_B\end{bmatrix}^T M_H U_{Wg}+\mu_g U_g+\frac{1}{2}\sigma_g^2 U_{gg}. \quad (8)\end{aligned}$$

where $M_H \equiv \begin{bmatrix}\sigma_g^2\hat{P}_G'(g)+\rho_{Ag}\sigma_A\sigma_g\\ \sigma_g^2\hat{P}_B'(g)+\rho_{Ag}\sigma_A\sigma_g\end{bmatrix}$ and its first element reflects the co-movement of $dg$ with $dy_G+d\hat{P}_G$ and its second element reflects the co-movement of $dg$ with $dy_B+d\hat{P}_B$.

Following a standard method in the literature (see, e.g., Miao and Wang, 2007), we conjecture that the investor's value function takes the form

$$U(W,g;S_G)=-\frac{1}{\gamma r}e^{-\gamma r(W+H(g;S_G))}. \quad (9)$$

The following proposition summarizes the investor's optimal consumption and demand for risky assets. Appendix A provides the derivations.

**Proposition 1** *The representative investor's optimal consumption is given by*

$$C^*=r\left(W+H(g;S_G)\right), \quad (10)$$

*and the optimal demand for the risky assets is independent of investors' wealth, $W$, and given by*

$$\begin{bmatrix}X_G^*\\X_B^*\end{bmatrix}=\frac{1}{\gamma r}M_\Sigma^{-1}\left(M_\Pi-\begin{bmatrix}0\\g^+\end{bmatrix}\right)-H'(g)M_\Sigma^{-1}M_H, \quad (11)$$

*where $H(g;S_G)$ satisfies the differential equation*

$$\delta=r+\gamma r^2 H-\frac{(\gamma r)^2}{2}\begin{bmatrix}X_G^*\\X_B^*\end{bmatrix}^T M_\Sigma\begin{bmatrix}X_G^*\\X_B^*\end{bmatrix}-\mu_g\gamma r H'-\frac{1}{2}\sigma_g^2\gamma r\left(H''-\gamma r\left(H'\right)^2\right). \quad (12)$$

In models with CARA utility and exogenously fixed riskless interest rate, optimal consumption depends on investors' wealth, but demand for risky assets and, thus, prices of these

assets do not. To provide further intuition for the above results, we note that the first component of optimal demand, as given in Equation (11), is proportional to the investor's risk tolerance $1/\gamma$ and captures her demand for risky assets as a *myopic* mean-variance optimizer. The term $M_\Pi$ represents the investor's expected pecuniary returns, while $\begin{bmatrix} 0 \\ g^+ \end{bmatrix}$ captures the non-pecuniary losses associated with social preferences.

To interpret the second component of optimal demand, we first note that $H(g; S_G)$ captures the investor's certainty equivalent arising from the opportunity to invest in risky assets and to adjust consumption to fit her time preference $\delta$, in addition to consuming the constant interest from investing in the riskless asset (see Equation (10)). Thus, this part represents the investor's *intertemporal* demand for risky assets as a hedge against shocks to the investment opportunity set. Indeed, Lemma 1 implies that the investment opportunity set for risky assets varies with $g$ and is therefore subject to preference shocks.

Finally, solving Equation (11) for the required risk premia yields the investor's inverse demand function

$$\begin{bmatrix} \hat{\pi}_G \\ \hat{\pi}_B \end{bmatrix} = \begin{bmatrix} \hat{P}_G & 0 \\ 0 & \hat{P}_B \end{bmatrix}^{-1} \left( \gamma r \left( \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} + H' \begin{bmatrix} \sigma_g^2 \hat{P}_G' + \rho_{Ag}\sigma_g\sigma_A \\ \sigma_g^2 \hat{P}_B' + \rho_{Ag}\sigma_g\sigma_A \end{bmatrix} \right) + \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right). \quad (13)$$

## 3.3 Equilibrium asset prices and technology supply

In this subsection, we derive the competitive market equilibrium. That is, we solve for the equilibrium share prices, $P_G(g)$ and $P_B(g)$, and risk premia, $\pi_G(g)$ and $\pi_B(g)$, that (i) clear the financial market for the risky assets

$$X_G^* = S_G, \quad (14)$$

$$X_B^* = S_B = 1 - S_G, \quad (15)$$

and (ii) are consistent with the values conjectured by firms and the representative investor when solving their respective optimization problems

$$\begin{aligned} P_G(g;S_G) &:= \tilde{P}_G(g;S_G) = \hat{P}_G(g;S_G), \\ P_B(g;S_G) &:= \tilde{P}_B(g;S_G) = \hat{P}_B(g;S_G), \\ \pi_G(g;S_G) &:= \tilde{\pi}_G(g;S_G) = \hat{\pi}_G(g;S_G), \\ \pi_B(g;S_G) &:= \tilde{\pi}_B(g;S_G) = \hat{\pi}_B(g;S_G). \end{aligned}$$

Hence, evaluating inverse demand functions (13) at the equilibrium prices yields the following proposition:

**Proposition 2** *The equilibrium risk premia of green and brown shares are given by*

$$\begin{bmatrix} \pi_G \\ \pi_B \end{bmatrix} = \begin{bmatrix} P_G & 0 \\ 0 & P_B \end{bmatrix}^{-1} \left( \gamma r \left( \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} S_G \\ 1 - S_G \end{bmatrix} + H' \begin{bmatrix} \sigma_g^2 P_G' + \rho_{Ag}\sigma_g\sigma_A \\ \sigma_g^2 P_B' + \rho_{Ag}\sigma_g\sigma_A \end{bmatrix} \right) + \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right).$$

Proposition 2 implies that risk premia depend on social preferences, share prices, and technology supply.

Note that if the representative investor optimizes myopically and disregards social preferences, the equilibrium risk premia would follow a standard single-factor pricing model, where each stock's risk premium is simply the product of the market risk premium and the stock's market beta. However, as demonstrated by Pastor et al. (2021), market betas alone cannot fully account for risk premia in a world with social preferences. When these social preferences become stochastic, even a two-factor model is not sufficient to explain equilibrium risk premia, as investors also require compensation for risk arising from shocks to the investment opportunity set in response to changing social preferences, as Proposition 2 above implies. Thus, risk premia are determined by a three-factor model in our setup.

These factors correspond to three distinct portfolios: the market portfolio, a "social" portfolio capturing the effects of non-pecuniary payoffs arising from social preferences, and a hedging portfolio, reflecting the impact of stochastic changes in social preferences. In Appendix A we provide the proof of the following Corollary, which establishes that these three portfolios indeed span the equilibrium dollar risk premia of individual stocks in our model:

**Corollary 1** *The equilibrium dollar risk premia are determined by a three-factor asset pricing model*

$$\begin{bmatrix}\pi_G P_G\\ \pi_B P_B\end{bmatrix} = \pi_m P_m \begin{bmatrix}\beta_{G,m}\\ \beta_{B,m}\end{bmatrix} + \pi_h P_h \begin{bmatrix}\beta_{G,h}\\ \beta_{B,h}\end{bmatrix} + \pi_s P_s \begin{bmatrix}\beta_{G,s}\\ \beta_{B,s}\end{bmatrix} \tag{16}$$

*where $\pi_m P_m, \pi_h P_h$, and $\pi_s P_s$ are the dollar risk premium of the market, hedging and social portfolio, respectively; and $\beta_{f,m}, \beta_{f,h}$, and $\beta_{f,s}$, $f \in \{G, B\}$, are the market, hedging, and social beta of stocks of green and brown firms, respectively.*

Finally, substituting the equilibrium risk premia in the valuation equations (5) and (6), we characterize equilibrium share prices and firms' technology choices, as summarized in the proposition below:

**Proposition 3** *For $g < \bar{g}(S_G)$ the following system of ordinary differential equations (ODEs) holds:*

$$\begin{aligned}&\begin{bmatrix}rP_G\\ rP_B\end{bmatrix} + \left(\gamma r \left(\begin{bmatrix}\Sigma_G & \Sigma_{GB}\\ \Sigma_{GB} & \Sigma_B\end{bmatrix}\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix} + H' \begin{bmatrix}\sigma_g^2 P_G' + \rho_{Ag}\sigma_g\sigma_A\\ \sigma_g^2 P_B' + \rho_{Ag}\sigma_g\sigma_A\end{bmatrix}\right) + \begin{bmatrix}0\\ g^+\end{bmatrix}\right)\\ &= \begin{bmatrix}\mu\\ \mu\end{bmatrix} + \mu_g \begin{bmatrix}P_G'\\ P_B'\end{bmatrix} + \frac{1}{2}\sigma_g^2 \begin{bmatrix}P_G''\\ P_B''\end{bmatrix}.\end{aligned} \tag{17}$$

Furthermore, after imposing market-clearing, Equation (12) becomes[15]

$$\delta = r + \gamma r^2 H - \frac{(\gamma r)^2}{2}\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix}^T M_\Sigma \begin{bmatrix}S_G\\ 1-S_G\end{bmatrix} - \mu_g \gamma r H' - \frac{1}{2}\sigma_g^2 \gamma r \left(H'' - \gamma r \left(H'\right)^2\right). \tag{18}$$

In summary, for each $S_G \in [0,1]$, the system of differential equations (17) and (18) has to be jointly satisfied for $g \in (-\infty, \bar{g}(S_G)]$.

**Optimal investment and equilibrium technology supply.** At the free investment boundary, $\bar{g}(S_G)$, prices must satisfy the value-matching condition

$$P_B(g; S_G)\big|_{g=\bar{g}(S_G)} = P_G(g; S_G)\big|_{g=\bar{g}(S_G)} - I. \tag{19}$$

[15] In the competitive equilibrium, individual investors (and therefore the representative investor) do not internalize the impact of their own investment decisions on share prices. This is consistent with imposing the market-clearing conditions before solving for $H(g)$.

The optimal choice of $\bar{g}$ is determined by the smooth-pasting condition

$$\frac{\partial P_G(g; S_G)}{\partial g}\Big|_{g=\bar{g}(S_G)} = \frac{\partial P_B(g; S_G)}{\partial g}\Big|_{g=\bar{g}(S_G)}. \tag{20}$$

The value-matching condition implies that, for a given value of $S_G$, brown firms start switching to the green technology when social preferences $g$ reach the investment threshold $\bar{g}(S_G)$. The smooth-pasting condition is the first-order condition for optimal investment at the threshold $\bar{g}(S_G)$, applied by each single firm. Since firms behave competitively, brown firms switch until they are indifferent between switching to the green technology and staying brown, which determines the supply of the green technology in the competitive equilibrium[16]

$$\frac{\partial P_B}{\partial S_G}(\bar{g}; S_G) = 0, \tag{21}$$

which implies the boundary condition

$$\frac{dP_B}{dg}\Big|_{g=\bar{g}(S_G)} = \frac{\partial P_B}{dg}(\bar{g}; S_G) = \frac{\partial P_G}{dg}(\bar{g}; S_G) = \frac{dP_G}{dg}\Big|_{g=\bar{g}(S_G)}. \tag{22}$$

Any further positive shock $dg$ beyond $\bar{g}(S_G)$ motivates more brown firms to switch, thereby increasing the supply of green firms by $dS_G$ and reducing the remaining supply of brown firms by $-dS_G$ until the new supply of green firms satisfies $\frac{\partial P_B}{\partial S_G}(\bar{g}; S_G) = 0$, which implies $g + dg = \bar{g}(S_G + dS_G)$. At that point, brown firms are again indifferent between switching and remaining brown. Prices of both firms adjust to $P_G(g + dg; S_G + dS_G)$ and $P_B(g + dg; S_G + dS_G)$. Subsequent negative shocks to $g$ push $g$ into the interior region $g < \bar{g}(S_G + dS_G)$, where the solution is determined by the HJB equation (5). Consequently, the trailing maximum of the social preferences parameter $g$, defined as $g_{\max,t} = \sup_{0 \le s \le t} g_s$,

[16]The optimal decision to switch to the green technology in a competitive equilibrium also implies

$$P_G(g; S_G)\Big|_{g=\bar{g}(S_G)} = P_G(g; S_G + dS_G)\Big|_{g=\bar{g}(S_G)}$$

where $dS_G$ is the infinitesimal change in supply brought by competitors also switching to the green technology. Thus,

$$\frac{\partial P_G}{\partial S_G}(\bar{g}; S_G) = 0.$$

determines the equilibrium supply, denoted by $S_G(g_{\max,t})$.[17]

Firms' optimal investment decisions also imply boundary conditions for $H$, i.e., the term that captures the representative investor's investment opportunity set. As discussed, when social preferences increase and reach $\bar{g}$, then some brown firms switch to the green technology. This can be interpreted as some brown firms repurchasing their brown shares and financing this repurchase by issuing new green shares. These firms then use parts of the proceeds from the issued shares to fund the investment to adopt the green technology (recall that $P_G$ exceeds $P_B$ by exactly $I$ when social preferences reach $\bar{g}$). In a competitive equilibrium, the resulting supply change, $dS_G$, does not affect the representative investor's utility, since she is indifferent to small changes in her portfolio, i.e., $\partial U/\partial S_G = 0$ at the investment threshold (see Appendix B for a discussion of why the investors' utility is insensitive to changes in the supply of green shares at the boundary $\bar{g}$.). From the investor's value function (9), it follows that this indifference implies $\partial(W + H)/\partial S_G = 0$. Since prices of green and brown shares do not change with $S_G$ at the boundary, financial wealth $W$ does not change with $S_G$ either. This implies the boundary conditions that must be satisfied by $H$ at $\bar{g}$:

$$\frac{\partial H}{\partial S_G}(\bar{g}; S_G) = 0, \tag{23}$$

$$\frac{dH(g; S_G)}{dg}\Big|_{g=\bar{g}(S_G)} = \frac{\partial H}{\partial g}(\bar{g}; S_G). \tag{24}$$

We discuss conditions on $P_G$, $P_B$, and $H$ for $g \to \bar{g}_{S_G=100\%}$, at $g = 0$, and their asymptotic behavior for $g \to -\infty$ in Appendix B.

# 4 Numerical Results

This section provides a numerical analysis of the effects of stochastic social preferences on share prices and corporate technology choices. It thereby provides insights into the determinants of an economy's transition path towards green technologies and shows how the riskiness of green and brown stocks and their resulting risk premia evolve dynamically. We solve the system of ODEs (17) numerically with a Chebychev collocation approach, as described in

[17] To ease exposition, we suppress the argument $g_{\max,t}$ and simply denote the equilibrium supply by $S_G$.

Judd (1998) and Dangl and Wirl (2004).[18] Table 1 summarizes the parameter values for the base case of the numerical solutions. The parameters are chosen so that their values appear reasonable in absolute and in relative terms, but are not calibrated to match specific empirical observations.

## 4.1 Share Price Dynamics and Social Preferences

We start with the analysis of the relationship between share prices and social investor preferences. Figure 1 presents the case where 25% of all firms are initially endowed with the green technology or have already switched to this technology. Panel (A) illustrates the prices of green firms (dark green line) and brown firms (dark brown line) as functions of the social preference parameter $g$. Prices in dark green and dark brown are plotted up to the critical threshold $\overline{g}_{S_G=25\%}$, indicated by the vertical dashed line, at which social preferences are high enough to incentivize additional brown firms to switch to the green technology and both price functions adjust, as we discuss later.[19]

Panel (A) of Figure 1 shows the stock prices for different social preferences, starting with negative values of $g$. Recall that for $g \leq 0$, social investors do not perceive a negative non-pecuniary dividend from holding brown firms. However, Figure 1 shows that, even for negative values of $g$, the share price of green firms is above that of brown firms. Since the evolution of $g$ is stochastic, the price of brown firms already reflects that, in the future, (some) investors may internalize the negative externalities that these firms generate. As $g$ increases, the price wedge between green and brown firms increases, since it becomes more likely that investors will perceive negative non-pecuniary payoffs associated with their holdings of brown shares.

We also observe that an increase in $g$ affects both share prices negatively, although green shares less so than brown shares. The negative effect on green shares is a consequence of the market's rational anticipation of future supply effects. Investors understand that if $g$ increases by a sufficient amount, then some brown firms will switch and adopt the green technology, thereby reducing the market clearing prices of green shares.

[18] The Internet Appendix I provides further details on the numerical procedure applied. An application of this numerical solution method to environmental economics can be found in Dangl and Wirl (2007).

[19] Internet Appendix II compares the implications of the dynamic model for share prices and risk premia to a simple, static model in which $g$ is kept constant. It shows that accounting for stochastic preferences has important implications for asset prices, as we discuss in more detail there.

As $g$ approaches the critical threshold $\bar{g}_{S_G=25\%}$, the slope of the dark green line becomes more negative, and that of the dark brown line less negative. Both phenomena reflect the increasing (anticipated) supply effects on share prices, as discussed above. In other words, this makes the value of brown firms drop less as $g$ increases, and it makes the value of green firms decrease more, since investors rationally anticipate that it becomes increasingly likely that the supply of firms with the green technology increases. At the critical value of $\bar{g}_{S_G=25\%}$, additional brown firms start to switch, thereby increasing the total supply of green firms beyond 25%. This happens at $g = 1.49$. At this critical value of $g$, the slopes of the dark green and the dark brown lines are identical and the distance between the two lines equals $I$, the cost of switching to the green technology, as is evident from the value-matching and smooth-pasting conditions (19) and (20).

The fraction of green firms, $S_G$, is therefore determined by the historical maximum of $g$ and switching takes place gradually when $g$ reaches new maxima. Brown firms act competitively at the investment boundary, thus, at the optimal level of $S_G$, brown firms are indifferent between investing and staying brown, $\partial P_B/\partial S_G = 0$. To shed additional light on the switching behavior of brown firms and its pricing implications, Panel (A) of Figure 1 also illustrates the case in which all firms start out with the brown technology ($S_G = 0$) and shows the share prices in light brown and light green for values of $g$ that represent historical maxima. For low values of $g$ such that $g < \bar{g}_{S_G=0\%}$, no green firms exist, since no switching takes place and only the light brown line is shown. This line is below the dark brown line, since there are now only brown firms in this range, and there is less risk sharing in the economy, so that the market-clearing prices for brown firms are lower than in the case where 25% of firms are green (see the dark brown line). As before, the slope of the light brown line eventually becomes less negative, reflecting the increased supply effect on the share price of brown firms, but now this effect starts much earlier, i.e., for lower values of $g$. When $g$ reaches new maxima to the right of $\bar{g}_{S_G=0\%}$, brown firms gradually switch to the green technology. Thus, for $g \geq \bar{g}_{S_G=0\%} = 0.79$ a light green line is also shown in the figure, reflecting the value of green shares. For increasing values of $g$, more brown firms switch until eventually all have switched to the green technology when $g$ reaches $\bar{g}_{S_G=100\%} = 3.31$ for the first time, indicated by the right dotted vertical line.

Along the light brown and light green lines, brown firms are indifferent between staying brown and switching to green, and, consequently, the prices always differ exactly by the

investment cost $I$. Furthermore, both price functions become quite flat, which implies that at the investment threshold, any positive shock to $g$, which represents an increase of internalized social costs associated with holding brown firms, is largely offset by the decisions of some brown firms to adopt the green technology, i.e., by the associated increase in the supply of green firms and the decrease in the supply of brown firms. When $g$ surpasses $\bar{g}_{S_G=100\%}$ for the first time, the last brown firms have switched to the green technology and the light brown line disappears. The light green line becomes a horizontal line at this point, as the value of the green firms no longer depends on $g$.

Figure 2 also illustrates the relationship between share prices and social preferences, but does so for the case where there already exists a large number of green firms. Specifically, Figure 2 assumes that 75% of all firms are initially endowed with the green technology or have already switched to this technology in the past. In this case, if social preferences are sufficiently weak such that $g < -2.3$, the share prices of green firms in Panel (A) of Figure 2 are actually lower than those of brown firms. This is so since there is a larger supply of firms with this technology and, therefore, their contribution to systematic risk in the representative investor's portfolio is larger, which makes them less attractive. The opposite is true for firms with the brown technology, which offer attractive diversification benefits to the representative investor.

As social preferences become stronger, i.e., $g$ increases, the share price of brown firms decreases much faster than that of green firms, such that brown firms' stock prices eventually end up below those of green firms. Over this range of $g$, the impact of potential future technology switches is small, since there is already a large supply of green firms. The impact becomes visible and the slope of the brown curve becomes less negative, as soon as $g$ reaches a value of approximately 2.0. Finally, the switching boundary, i.e., the second dashed vertical line denoted by $\bar{g}_{S_G=75\%}$, is reached when $g$ equals the value of 2.68. We note that, as $g$ reaches that boundary, both stock price paths become less sensitive to $g$, due to the impact of supply on share prices in competitive equilibrium.

Panels (B) of Figures 1 and 2 illustrate the dynamics of the risk premia of green and brown firms as social preferences change. We start again with Figure 1 and focus on the dark brown and dark green curves, which reflect the case where 25% of firms are green. We observe that, even for negative values of $g$, brown firms exhibit higher risk premia. For this range of $g$, this is largely driven by the fact that only 25% of firms are green, whereas the

large supply of brown firms of 75% requires a higher risk premium to allow markets to clear. Once we get into the range where social preferences are positive, i.e., polluting technologies come with a negative non-pecuniary dividend, the required risk premium of brown firms rises sharply, as the stochastic $g$ now directly affects brown firms' non-pecuniary payoffs. However, the rate at which the required risk premium increases slows down as $g$ approaches the threshold $\bar{g}_{S_G=25\%}$, since at this point the supply impact, caused by switching from brown to green technology, becomes more significant and makes the brown firms less exposed to further increases in $g$.

Inspecting the dark green line, i.e. the required risk premium of the green firms when 25% of firms use this technology, we observe that the required risk premium is always below that of brown firms, and this has two reasons. First, only 25% of firms use this technology, and consequently they are in demand for risk sharing purposes. Second, in contrast to brown firms, green firms are not directly affected by the stochastically evolving social preferences, which are internalized by the representative investor. As a consequence, the required risk premium of green firms rises only very moderately as social preferences become stronger. Only when $g$ increases sufficiently such that it becomes more likely that the supply of green firms will increase due to switching firms, green firms' become more exposed to $g$, which requires a slightly increasing risk premium.

The light brown and light green curves represent the equilibrium risk premia when all firms are initially brown and, as we move to the right, each value of $g$ represents a new historic maximum. For low values of $g$, it is not optimal for brown firms to switch. The required risk premia are above the dark brown line, as there are more brown firms in this case. Once $g$ is positive, brown firms become substantially riskier, and this triggers a sharp increase in their required risk premium, similar to the one shown by the dark brown line. However, as firms start to switch, the supply of brown firms decreases, which slows down the increase in their risk premium associated with an increase in $g$. At $\bar{g}_{S_G=25\%}$ the light brown and the dark brown line touch, since both lines represent the required risk premium of brown firms when 25% of firms choose the green technology. Beyond this point, the light brown curve keeps going up since the higher perceived social loss associated with the brown stocks must be compensated by higher financial risk premia to ensure market clearing.

The light green line only starts at $g = 0.79$, since this is the level of social preferences that triggers the first brown firm to switch to the green technology. As $g$ increases further,

the representative investor requires an increasing risk premium, since the supply of green stocks increases and they contribute more to the overall risk of the market. Once all firms have switched to the green technology at $g = 3.31$, the required risk premium becomes independent of $g$ and remains constant.

Panels (A) of Figures 1 and 2 also suggest that the correlation between cum-dividend price changes of brown and green firms vary with social preferences since the stock price sensitivities with respect to changes in social preferences depend on $g$. Figure 3 illustrates the dynamics of the correlation between the stock returns of brown and green firms for changing $g$ and the base-case parameters underlying Figure 1.

To interpret Figure 3 we recall that the correlation of the cum-dividend price changes is defined by

$$\begin{aligned} \rho_{G,B} &= \frac{\Sigma_{GB}}{\sqrt{\Sigma_G}\sqrt{\Sigma_B}} \\ &= \frac{\sigma_A^2 + \sigma_g^2 P'_G P'_B + \rho_{Ag}\sigma_A\sigma_g(P'_G + P'_B)}{\sqrt{\sigma^2 + \sigma_A^2 + \sigma_g^2(P'_G)^2 + 2\rho_{Ag}\sigma_A\sigma_g P'_G}\sqrt{\sigma^2 + \sigma_A^2 + \sigma_g^2(P'_B)^2 + 2\rho_{Ag}\sigma_A\sigma_g P'_B}}. \end{aligned} \quad (25)$$

According to the numerator of the above expression, the correlation between the two stock returns has three components. The first represents the variance of the common cash flow shocks, $\sigma_A^2$, while the second captures the effect of stochastic social preferences, i.e., $\sigma_g^2$, multiplied by the product of the two stock price sensitivities, $P'_G P'_B$. The third component, finally, reflects that preference shocks and aggregate cash flows may be correlated, i.e., that $\rho_{Ag}$ may be different from zero. Since in the base case $\rho_{Ag} = 0$ (see Table 1), only the first two terms in the numerator are relevant when interpreting Figure 3.

The denominator of Equation (25) is equal to the product of the standard deviations of green and brown stocks. Note that, if prices are not exposed to shocks in social preferences, i.e., if $P'_G = P'_B = 0$, then the correlation coefficient between the two stocks is $\rho_{G,B} = 0.5$, since in the base case $\sigma = \sigma_A$ (see Table 1).

Figure 3 shows that, for very low values of $g$, the correlation coefficient converges to 0.5, since neither $P_G$ nor $P_B$ depend on $g$, i.e., $P'_G = P'_B = 0$ as $g \to -\infty$. This happens because the probability that brown shares will ever be perceived as generating negative non-pecuniary payoffs is effectively zero in this case. As $g$ increases, both brown and green share prices are affected by preference shocks, i.e., $P'_B$ and $P'_G$ are negative, increasing the covariance

between the two returns and pushing the correlation in Figure 3 above 50%. Only when $g$ approaches the threshold $\bar{g}_{s_G=25\%}$, we see again a small drop in the correlation due to the increasing supply effect, which reduces the sensitivity of stock prices to $g$, as it makes it more likely that the supply of green firms will go up.

Thus, the model implies that the correlation between the returns of green and brown firms increases as social preferences become stronger and move from past troughs towards their historical peaks. In contrast, when social preferences fall substantially below historical highs, one should observe a decrease in the correlation between brown and green firms.[20]

## 4.2 Time-Series Dynamics: An Example

To shed more light on the joint evolution of social preferences, supply of technologies, risk premia and stock return correlations, we next analyze the equilibrium along a specific path of the stochastic $g$-process. We hereby consider again the base-case parameters summarized in Table 1.

Panel (A) of Figure 4 shows the drawn path for $g$ (left axis) and the corresponding evolution of the fraction of green firms, $S_G$ (right axis). The initial value of $g$ at $t = 0$ is 0, a level for which brown firms do not have an incentive to switch to the green technology so that $S_G = 0$. At $t = 0.50$, $g$ reaches 0.79, which represents the initial threshold at which some brown firms decide to transition to the green technology, leading to an increase in $S_G$. $S_G$ continues to grow whenever $g$ reaches new maxima whereas it does not change in periods during which social preferences are below previous highs. At $t = 20$ more than 80% of firms have become green.

Panel (B) of Figure 4 shows the corresponding stock prices $P_B$ and $P_G$. For easier comparison, the green firms' stock price is adjusted for the cost of switching, i.e., the figure shows $P_G - I$, which represents the value that a brown firm could achieve by switching technology. Value maximizing corporate technology decisions imply that the price of green stocks adjusted for $I$ is always lower or equal to the share price of brown firms. The latter is the case in periods where $g$ reaches new peaks and some brown firms switch to the green technology, i.e., brown firms are indifferent between investing $I$ and becoming green and staying with the brown technology. In contrast, (cost-adjusted) share prices diverge when

[20] This relation is more pronounced when $S_G$ is less than 50% where brown firms' share price responds less to social preferences and thus, has a smaller standard deviation.

social preferences drop below previous levels. This valuation gap is particularly large between $t = 11.7$ and $t = 17.6$, where $g$ drops substantially below its previous maximum and stays below the maximum for an extended period. We also see that brown firms outperform green firms when social preferences drop below historic highs. In contrast, as social preferences recover from past troughs and move towards historical highs, we observe an outperformance of green firms, consistent with the empirical evidence presented in Pastor et al. (2022).

Panel (C) of Figure 4 shows the evolution of risk premia over time. Most of the time, brown firms have a higher cost of capital than green firms because social preferences make the representative investor less willing to hold them. However, this is not always the case. When social preferences fall significantly below previous highs, brown stocks become less costly to hold, as the representative investor internalizes to a lesser degree their externalities. Along the $g$-path considered in Figure 4 this happens, for example, between $t = 15$ and $t = 17$. During this period, social preferences are weak compared to previous highs and there is already a large supply of green firms, since $g$ had reached high values in the past. In this situation, brown firms actually experience a lower cost of capital than green firms. Once $g$ increases again, so does the equilibrium risk premium for brown shares. Note that, as $g$ continues to rise, the risk premium gap increases substantially to offset the large non-pecuniary disutility that the representative investor incurs from holding brown shares. Despite this, the supply of green firms remains constant at approximately 79% until time $t = 19.7$.

To induce further growth of the supply of green firms towards the end of the time period shown in Figure 4 requires a larger risk premium gap, as Panel (C) shows. This is needed because most firms have already switched to the green technology by this time, reflecting the high past maximum $g$. Therefore, brown firms become very valuable "diversifiers." As a consequence, it requires a large difference in the cost of capital between the two types of firms to induce additional brown firms to switch. This manifests itself in Panel (C) as we move toward time $t = 20$.

Finally, Panel (D) of Figure 4 illustrates the correlation between the two technologies' cum-dividend price changes. These correlation dynamics resemble the dynamics of the gap between the price of the green firms (adjusted for the cost of switching technology) and the brown firms presented in Panel (B). When the price gap widens, as is the case, for example, after time $t = 11.7$, the correlation between green and brown stocks decreases. In contrast,

when the price gap is small or zero (i.e., when we are at or near a new peak of the social preferences, which induces some additional brown firms to switch), the correlation tends to be high. We observe such situations, for example, at the beginning of the path of $g$, when $t = 3$, or again later, when $t$ is around 10.

## 4.3 Impact of Model Parameters on the Green Transition

In this subsection, we analyze how key model parameters influence the economy's transition toward the green technology. We begin by analyzing the effect of uncertainty of future preferences, $\sigma_g$, as illustrated in Figure 5. Panel (A) reveals that an increase in the uncertainty about future preference shocks delays corporate decisions to become green. For low preference uncertainty, i.e., $\sigma_g = 0.5$, the first brown firms already switch to the green technology when $g$ is slightly above 0.55. For this case, we also observe a fast rate at which the transition to the green technology occurs, i.e., a steep slope of the dark green line in Panel (A). Once $g$ has increased to approximately 2.5, the transition is complete.

In contrast, for a high value of preference uncertainty, i.e., $\sigma_g = 2$, the transition process starts much later, i.e., at a substantially higher value of $g$. Specifically, $g$ must reach a value of approximately 1.1 before the first brown firm switches to the green technology. Additionally, the transition also occurs more slowly, as the complete transition is only reached at a value of $g$ that is approximately 4.5. Thus, Panel (A) of Figure 5 shows that preference uncertainty slows an economy's transition toward the green technology.

Our finding that uncertainty delays the green transition aligns with insights from Bernanke (1983) and McDonald and Siegel (1986). However, unlike in their frameworks, brown firms in our model cannot strategically delay the transition to maximize the values of their real options, due to the competitive behavior among firms. In other words, if a given brown firm postpones its transition to the green technology, it will be preempted by other brown firms making the switch. The number of brown firms that need to transition to ensure that competitive firms are indifferent between switching and not switching depends on the supply effects on share prices. As the volatility of preference shocks increases, these supply effects become stronger, meaning that fewer brown firms need to change to counteract a given positive shock in social preferences. Thus, greater uncertainty about future social preferences delays the green transition in our framework.

Panel (B) of Figure 5 shows the impact of risk aversion on the supply of green firms. The higher the risk aversion of investors, the earlier firms start to switch to the green technology. However, the transition happens at a slower rate, i.e., the $S_G$-line becomes flatter as investors' risk aversion increases. Thus, for high degrees of risk aversion, even moderate social preferences induce some firms to switch to the green technology. This is so, since the green technology is initially a good diversifier for the representative investor's portfolio, since she only holds brown shares. This encourages some firms to switch even when social preferences are weak. However, diversification benefits decrease as the number of green companies increases. Eventually, when many brown firms have already transitioned, brown firms become more valuable diversifiers explaining the lower adoption rate for the green technology for a given rise in $g$.

It is interesting to observe that, for low values of risk aversion, the full transition happens over a narrow range of $g$ values. In this case, risk sharing is not very important and, as soon as preference levels are high enough to induce switching from brown to green technologies, the transition progresses very quickly.

Finally, we analyze the impact of investment costs on the transition toward a green economy. If switching from brown to green is costless, i.e., $I = 0$, then the base-case parameterization implies that, in the absence of social preferences, i.e., $g < 0$, 50% of firms choose green and 50% brown technologies. This maximizes the diversification benefits for the representative investor. As soon as social preferences become relevant, i.e., $g > 0$, some additional brown firms transition and become green. In comparison, when investment costs are positive, as captured by the other two lines, we do not observe any green firms in the absence of social preferences, as we consider an economy where all firms are initially endowed with the brown technology. Thus, the pure diversification benefits generated by having some green firms are insufficient to induce brown firms to switch. Furthermore, positive switching costs imply that the transition is delayed. It only starts when preference levels exceed specific thresholds, at which price differences equal the investment costs.

## 4.4 Interactions between Aggregate Cash Flows and Social Preferences

As discussed above, there is evidence that social preferences are affected by the state of the economy. In this subsection, we therefore allow shocks to social preferences, $dz_g$, and common cash flow shocks, $dz_A$, to be correlated and analyze the consequences. To this end, we compare an economy where the two shocks are negatively correlated ($\rho_{Ag} = -0.9$) to an economy where common cash flow shocks and social preference shocks are positively correlated ($\rho_{Ag} = 0.9$). The two cases are illustrated by Panels (A) and (B) of Figure 6.

Comparing Panel (A) to (B) of Figure 6, one first observes that a positive correlation between social preferences and the aggregate cash flow component makes brown firms relatively more, and green firms relatively less valuable. This is so since a positive correlation effectively makes green firms riskier compared to brown firms.

To derive the underlying intuition, consider a negative shock to the common cash flow component, $dz_A$, which coincides with a negative shock to social preferences, $dz_g$. Since both technologies are exposed to this negative common cash flow shock, both firms' dividends tend to be below expectations in such a state. However, holders of brown shares are partly hedged against such a negative cash flow shock, due to the weakening of social preferences that tends to occur at the same time. Such a preference shock makes social investors more willing to hold brown shares, and this extra demand for brown shares supports their share price.

Green shares, in contrast, do not enjoy the same natural hedge, as a weakening of social preferences in a state with negative common cash flow shocks does not lead to additional demand for green shares. Thus, a positive correlation between social preferences and common cash flow shocks makes stocks of brown firms more attractive compared to those of green firms.[21]

At $g = -3$, for example, Panel (B) shows that brown firms' share prices are approximately 89.5, which is higher than the share prices of green firms, which are approximately equal to

[21] Note that green share prices may still increase in response to a drop in social preferences, since such a preference change makes brown firms' real options to switch technology more out-of-the-money, i.e. it becomes less likely that the supply of green firms will increase. However, as can be seen, for example, in Figure 1, the increase in the price of green firms in response to a drop in $g$ is always less than that of brown firms. Thus, the hedging effect from preference shocks is more pronounced for brown firms.

86.3. This contrasts with the case of a negative correlation depicted in Panel (A). Here, for $g = -3$, both share prices are lower than those in Panel (B), but the prices of brown firms (approximately 82.8) are now below those of green firms (approximately 84.6). The fact that share prices are higher for positive $\rho_{Ag}$ is intuitive, as social costs due to holding brown firms tend to be lower (higher) when common shocks to cash flows are negative (positive) and this provides essentially a hedge to the representative investor and reduces the risk of her portfolio. The opposite is true for negative $\rho_{Ag}$.

Panels (A) and (B) of Figure 6 also show that the critical value of social preferences, $\bar{g}_{S_G=75\%}$, at which additional brown firms wish to switch to the green technology is higher when $\rho = 0.9$ compared to the case of a negative correlation. Also $\bar{g}(S_G = 100\%)$, i.e., the critical value of social preferences at which all firms have switched to the green technology, is substantially higher when the correlation is positive.

In summary, Panels (A) and (B) of Figure 6 imply that social preferences have weaker effects on corporate behavior if they evolve procyclically, i.e., if $g$ tends to decrease in bad and increase in good economic times, since this makes green firms riskier relative to brown firms,[22] and value-maximizing brown firms have less incentive to switch to a green technology.

## 4.5 Social preferences, firms' cost of capital and technology supply

We end Section 4 with an analysis of the effect of social preferences on the difference between green and brown firms' cost of capital and discuss whether this difference is a relevant measure of the importance of social preferences for the supply of brown versus green firms. To do this, we calculate the difference between the cost of capital of brown and green firms that is obtained in the absence of social preferences and compare it to the one that is obtained when the representative investor exhibits social preferences. We refer to this difference-in-differences as the cost of capital gap, in the spirit of Berk and van Binsbergen (2024).

Panel (A) of Figure 7 illustrates this cost of capital gap measure, the solid upwards sloping line, at the investment boundary, $\bar{g}(S_G)$, for different levels of $S_G$. It also plots the total derivative of the supply of green firms with respect to $g$ represented by the curved dashed line. Thus, we calculate the cost of capital gap and the derivative of $S_G$ along the envelope where $g = \bar{g}$, i.e., firms are at the threshold where the marginal brown firm is

[22] See Bansal et al. (2022) for empirical evidence.

indifferent between switching and not switching to the green technology.

The figure shows that, when $S_G = 0$ and the first brown firms switch to green, the presence of social preferences induces a cost of capital gap of slightly more than 1%. That is, the difference between the cost of capital of brown and green firms is slightly more than 1% larger if the representative investor exhibits social preferences than if she has no social preferences. However, even at this low implied value of social preferences, the supply of green firms is already sensitive to a small change in $g$, namely $dS_G/dg \approx 30\%$. At that point, the implied value of social preferences is $g = 0.79$, which means that non-pecuniary dividends from holding one share of brown firms are $-0.79$, compared to the expected monetary dividend of $\mu = 10$. Thus, a unit increase in $g$ reduces the expected total dividend of the representative investor by approximately $1/(10 - 0.79) \times 100\% = 11\%$ but increases $S_G$ by 30%. This sensitivity increases sharply with $S_G$ until it reaches a maximum of approximately 42%, which corresponds to a cost of capital gap of approximately 2.8% and a total supply of green firms of 0.45. As we consider stronger social preferences, which induce higher supplies of green firms, the sensitivity of the supply of green firms to increasing social preferences starts to decline. As $S_G$ approaches one, the sensitivity of the supply of green firms to $g$ drops again to approximately 0.38.

Panel (B) complements this analysis by providing an example of the cost of capital gap over time assuming the same stochastic evolution of $g$ that is considered in Figure 4. Thus, in this panel, the cost of capital gap is not only shown at the investment boundaries, $\bar{g}(S_G)$, but instead along a particular $g$-path. It reveals considerable variation in the cost of capital gap along the trajectory of $g$. It increases and becomes closely correlated with preference dynamics when $g$ moves close to the investment boundary, at which the supply of green firms reacts instantaneously to positive shocks in $g$. Conversely, when preferences are below their past peaks, the cost of capital gap remains close to zero for extended periods of time, such as between times 2.5 and 7.5 and again between times 15 and 17. However, it would be misleading to infer from such periods of low cost of capital gaps that social preferences had little impact on corporate technology choices. The low values in these intervals are actually a consequence of earlier shifts in social preferences, which had already led to a significant increase in the supply of green firms.

Overall, Figure 7 highlights that the cost of capital gap is not a useful metric to evaluate the impact of social preferences on corporate decisions. To address this question, it is

essential to consider the cumulative supply effects caused by the stochastic evolution of social preferences over time. Even when analyzing the effects of small changes in social preferences at a given point in time, Panel (A) of Figure 7 illustrates that the focus should be on the sensitivity of the supply of green firms to preference shifts, rather than the cost of capital gap. This corresponds to the derivative of $S_G$ with respect to $g$, which is not monotonically related to the cost of capital gap, as Panel (A) of Figure 7 clearly shows.

# 5 Conclusion

This paper analyzes an economy where social preferences evolve stochastically, and firms possess real options to adapt to investor preferences by transitioning from brown to green technologies. We find that in such a dynamic world, firms' responses to changing investor tastes mitigate or even reverse many of the effects that preference shocks would have in a world where corporate investment decisions are fixed exogenously. For example, stronger social preferences generally hurt not only brown firms' stock prices but also the ones of green firms, since the market rationally anticipates future corporate supply responses to such a preference shock.

We find that uncertainty about the future evolution of investor tastes delays the transition to a greener economy. When preference shocks are positively correlated with the state of the economy, i.e., social preferences are likely to become stronger in good economic states, as indicated by empirical evidence, then this delays the switch to green technologies even further. This is so, since a positive correlation makes brown firms less risky, as low cash dividends are likely to be partially offset by investors' preference shifts from green to brown technologies. The analysis also documents the effect of risk aversion and firms' risk characteristics on the transition to a greener economy. We find that higher risk aversion of investors induces some firms to switch earlier, but delays the complete transition.

The analysis also demonstrates that in a world with stochastic investor tastes and endogenous corporate decisions, the effect of green investors on the difference between brown and green firms' cost of capital is not a good measure of the impact that social preferences have on corporate decisions. We find that significant technology supply shifts can occur in response to a shock to investor tastes, even though the resulting effect on the cost of capital gap is small.

Overall, our findings suggest that understanding the interplay between investors' social preferences and corporate decisions requires acknowledging that preferences may change over time. In such a setting, firms' decisions are the solution to a dynamic problem. Such a new paradigm offers important insights, but also represents challenges for empirical analyses. Estimating cost of capital effects of social preferences is, at least conceptually, straightforward, but our analysis implies that this is not an appropriate empirical strategy to analyze the relevance of social preferences for corporate decisions. Instead, the analysis of stochastic preferences and endogenous corporate decisions suggests that empiricists should focus on novel research strategies. Important building blocks for such new empirical work include identifying proxies for the dynamics of social preferences, estimating the associated effects on corporate decisions, and analyzing the implied dynamics of stock return characteristics, such as the joint evolution of social preferences and stock returns of firms using green and brown technologies. While such novel empirical strategies may not be straightforward to execute, their advancement appears to be a promising and important direction for future research in finance.

# References


Andersen, S., Chebotarev, D., Filali-Adib, F. Z., Nielsen, K. M., 2024. A breath of change: Can personal exposures drive green preferences?, working paper.

Bansal, R., Wu, D. A., Yaron, A., 2022. Socially Responsible Investing in Good and Bad Times. The Review of Financial Studies 35, 2067–2099.

Barber, B. M., Morse, A., Yasuda, A., 2021. Impact investing. Journal of Financial Economics 139, 162–185.

Bauer, R., Ruof, T., Smeets, P., 2021. Get Real! Individuals Prefer More Sustainable Investments. The Review of Financial Studies 34, 3976–4043.

Berk, J. B., van Binsbergen, J. H., 2024. The impact of impact investing, journal of Financial Economics, forthcoming.

Bernanke, B. S., 1983. Irreversibility, Uncertainty, and Cyclical Investment. The Quarterly Journal of Economics 98, 85–106.

Bolton, P., Li, T., Ravina, E., Rosenthal, H., 2020. Investor ideology. Journal of Financial Economics 137, 320–352.

Bonnefon, J.-F., Landier, A., Sastry, P., Thesmar, D., 2022. The moral preferences of investors experimental evidence, working Paper.

Broccardo, E., Hart, O. D., Zingales, L., 2022. Exit vs. Voice. Journal of Political Economy 130, 3101–3145.

Bustamante, M. C., Zucchi, F., 2024. Dynamic carbon emission management, working paper.

Choi, D., Gao, Z., Jiang, W., 2020. Attention to Global Warming. The Review of Financial Studies 33, 1112–1145.

Cole, S., Jeng, L., Lerner, J., Rigol, N., Roth, B. N., 2023. What do impact investors do differently?, working Paper.

Dangl, T., Halling, M., Yu, J., Zechner, J., 2023. Social preferences and corporate investment, working paper.

Dangl, T., Wirl, F., 2004. Investment under uncertainty: calculating the value function when the bellman equation cannot be solved analytically. Journal of Economic Dynamics and Control 28, 1437–1460.

Dangl, T., Wirl, F., 2007. The consequences of irreversibility on optimal intertemporal emission policies under uncertainty. Central European Journal of Operations Research 15, 143–166.

Dyck, A., Lins, K. V., Roth, L., Wagner, H. F., 2019. Do institutional investors drive corporate social responsibility? international evidence. Journal of Financial Economics 131, 693–714.

Edmans, A., Levit, D., Schneemeier, J., 2023. Socially responsible divestment, working paper.

Exley, C. L., Lehr, N. H., Terry, S. J., 2023. Nonprofits in good times and bad times. Journal of Political Economy Microeconomics 1, 42–79.

Fisman, R., Ghosh, P., Sarkar, A., Zhang, J., 2023. Dirty air and green investments: The impact of pollution information on portfolio allocations, technical report National Bureau of Economic Research.

Goldstein, I., Kipytov, A., Shen, L., Xiang, H., 2022. On ESG investing: Heterogeneous preferences, information, and asset prices, working paper.

Gollier, C., Pouget, S., 2014. The "washing machine": Investment strategies and corporate behavior with socially responsible investors, working paper.

Gupta, D., Kopytov, A., Starmans, J., 2022. The pace of change: Socially responsible investing in private markets, working paper.

Hart, O., Thesmar, D., Zingales, L., 2023. Private sanctions. Economic Policy 39, 203–268.

Hart, O., Zingales, L., 2017. Companies should maximize shareholder welfare not market value. Journal of Law, Finance, and Accounting 2, 247–274.

Hartzmark, S. M., Sussman, A., 2019. Do investors value sustainability? a natural experiment examining ranking and fund flows. The Journal of Finance 74, 2789–2837.

Heeb, F., Kölbel, J. F., Paetzold, F., Zeisberger, S., 2022. Do Investors Care about Impact? The Review of Financial Studies 36, 1737–1787.

Heinkel, R., Kraus, A., Zechner, J., 2001. The effect of green investment on corporate behavior. The Journal of Financial and Quantitative Analysis 36, 431–449.

Hong, H., Wang, N., Yang, J., 2023. Welfare Consequences of Sustainable Finance. The Review of Financial Studies 36, 4864–4918.

Humphrey, J., Kogan, s., Sagi, J., Starks, L., 2022. The asymmetry in responsible investing preferences, working paper.

Jagannathan, R., Kim, S., McDonald, R., Xia, S., 2023. Environmental activism: Endogenous risk and asset prices, working paper.

Judd, K. L., 1998. Numerical Methods in Economics. MIT Pressså.

Krueger, P., Sautner, Z., Starks, L., 2020. The Importance of Climate Risks for Institutional Investors. The Review of Financial Studies 33, 1067–1111.

Landier, A., Lovo, S., 2022. Socially responsible finance: How to optimize impact, forthcoming, The Review of Financial Studies.

McDonald, R., Siegel, D., 1986. The value of waiting to invest. The Quarterly Journal of Economics 101, 707–727.

Miao, J., Wang, N., 2007. Investment, consumption, and hedging under incomplete markets. Journal of Financial Economics 86, 608–642.

Oehmke, M., Opp, M., 2024. A theory of socially responsbile investment, forthcoming in The Review of Economic Studies.

Øksendal, B., 2003. Stochastic Differential Equations. Springer.

Ottoni-Wilhelm, M., Vesterlund, L., Xie, H., 2017. Why do people give? testing pure and impure altruism. The American Economic Review 107, 3617–3633.

Pastor, L., Stambaugh, R., Taylor, L., 2021. Sustainable investing in equilibrium. Journal of Financial Economics 142, 550–571.

Pastor, L., Stambaugh, R., Taylor, L., 2022. Dissecting green returns. Journal of Financial Economics 146, 403–424.

Pedersen, L. H., Fitzgibbons, S., Pomorski, L., 2021. Responsible investing: The ESG-efficient frontier. Journal of Financial Economics 142, 572–597.

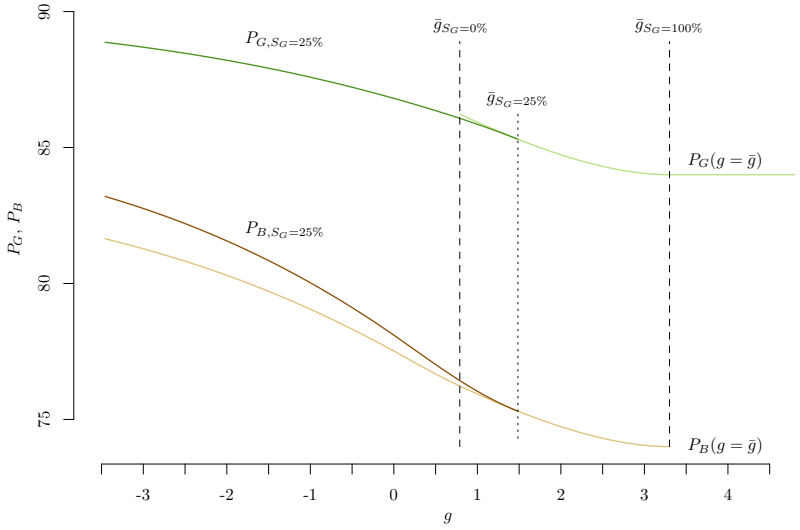
Riedl, A., Smeets, P., 2017. Why do investors hold socially responsible mutual funds? The Journal of Finance 72, 2505–2549.

**Table 1** Parameter Values

| | |
|---|---|
| Aggregate absolute risk aversion of all investors | $\gamma = 1.0$ |
| Expected cash-flows | $\mu = 10.0$ |
| Volatility of technology-specific cash-flow shocks | $\sigma = 2\sqrt{2}$ |
| Volatility of aggregate cash-flow shocks | $\sigma_A = 2\sqrt{2}$ |
| Risk-free rate of return | $r = 0.1$ |
| Expected non-pecuniary dividends | $\mu_g = 0.1$ |
| Volatility of non-pecuniary dividend shocks | $\sigma_g = 1.0$ |
| Correlation between preference shocks and aggregate cash-flow shocks | $\rho_{Ag} = 0.0$ |
| Costs to switch technology | $I = 10$ |

Figure 1: **Share prices and risk premia with dynamic technology choices (low $S_G$)** The figure shows prices (Panel (A)) as well as risk premia (Panel (B)) of brown and green firms for different levels of social investor preferences. In this case, 25% of all firms are initially endowed with the green technology or have already switched to this technology (i.e., $S_G = 0.25$).

(A) Share prices




(B) Risk premia

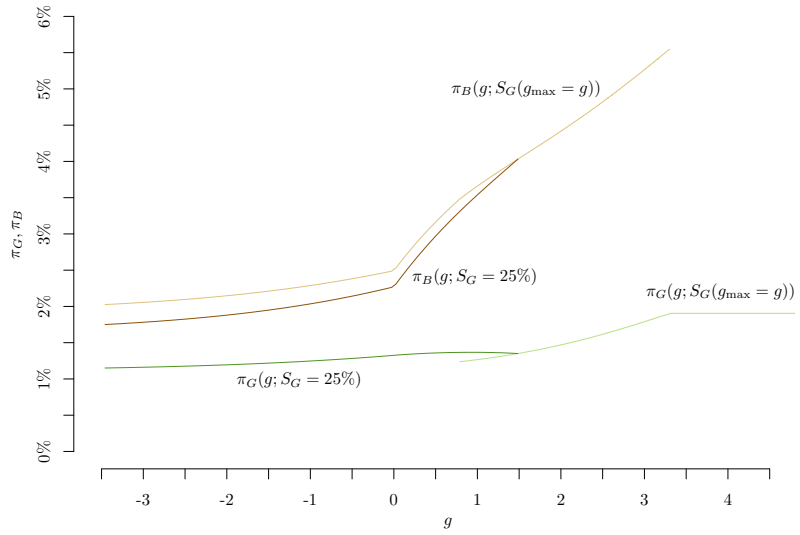
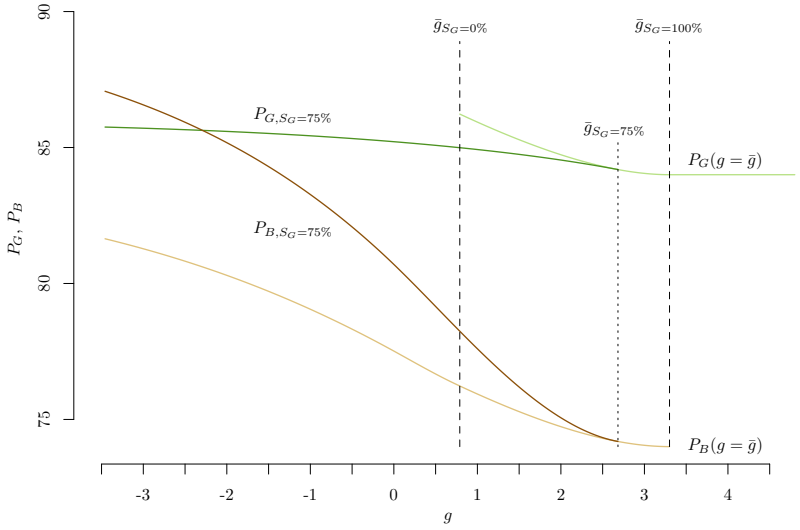


Figure 2: **Share prices and risk premia with dynamic technology choices (high $S_G$)** The figure shows prices (Panel (A)) as well as risk premia (Panel (B)) of brown and green firms for different levels of social investor preferences. In this case, 75% of all firms are initially endowed with the green technology or have already switched to this technology (i.e., $S_G = 0.75$).

(A) Share prices




(B) Risk premia

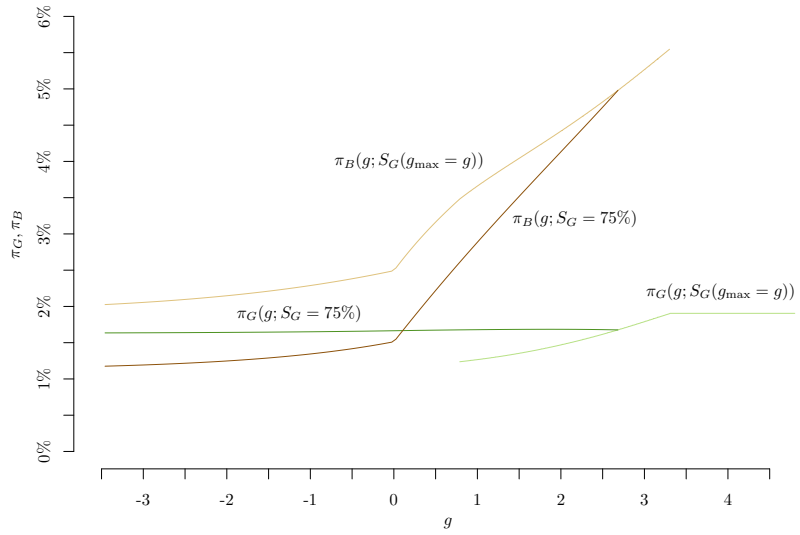


Figure 3: **Social preferences and correlations between green and brown firms**
The figure shows correlations between price changes of brown and green firms for different levels of social investor preferences.

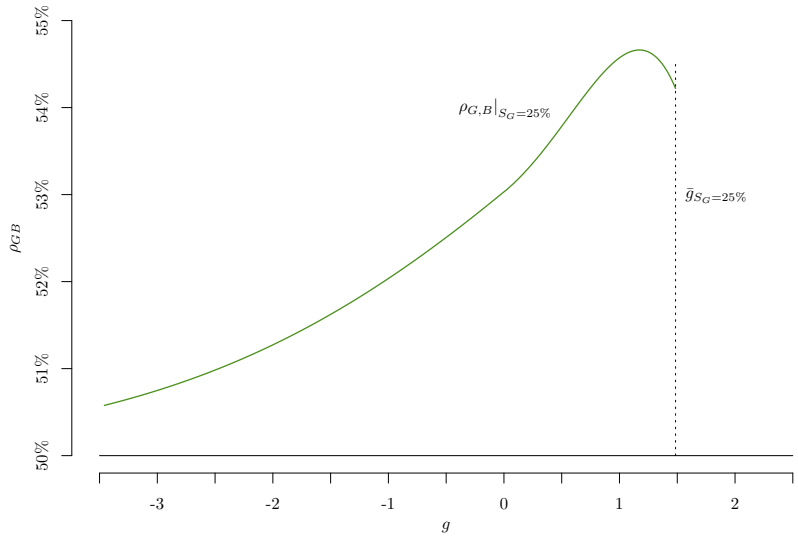
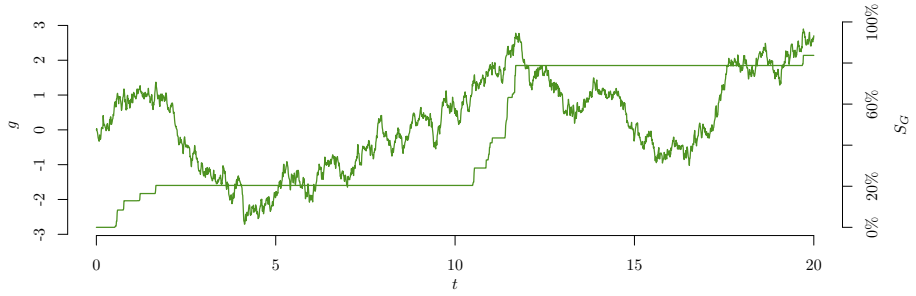
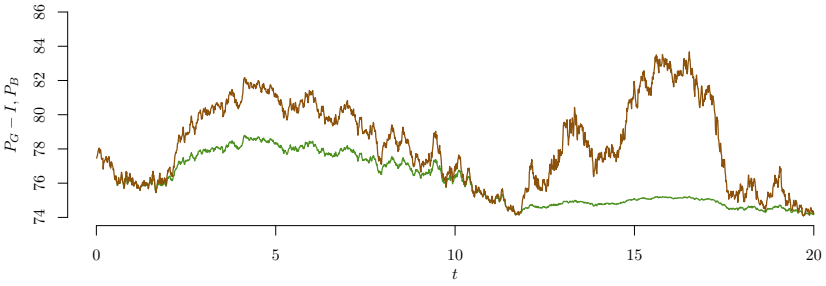


Figure 4: **A given path of social preferences.**
Panel (A) shows the sample path together with the supply of green firms; Panel (B) shows the share prices of green and brown firms for this path; Panel (C) plots risk premia of green and brown firms over the sample path; Panel (D) shows the correlation of cash flows and share price changes of green and brown firms.

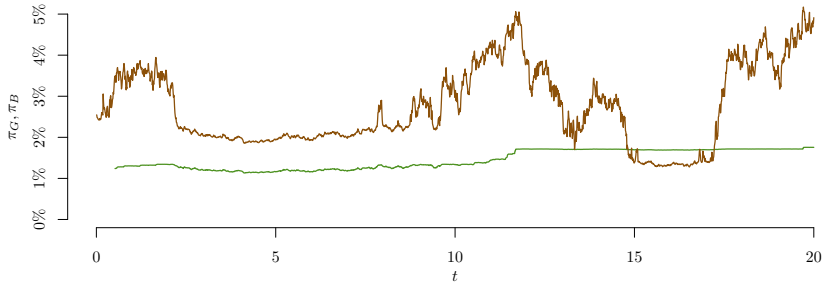
(A) Supply of green firms.




(B) Price of green and brown firms.




(C) Risk premia of green and brown firms.




(D) Correlation of cash flows and share price changes.

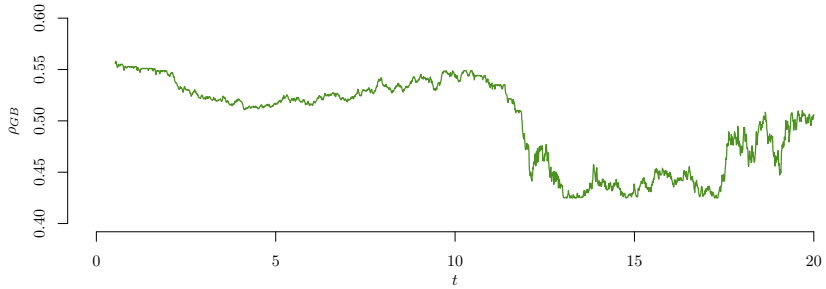
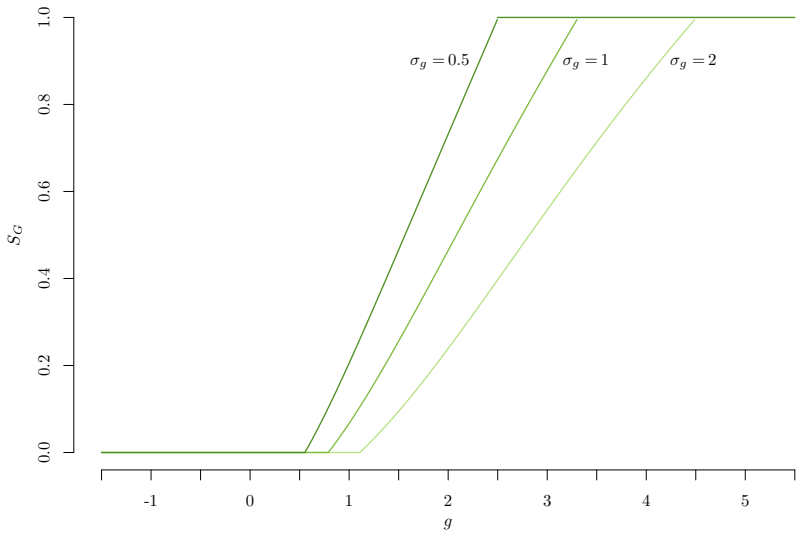
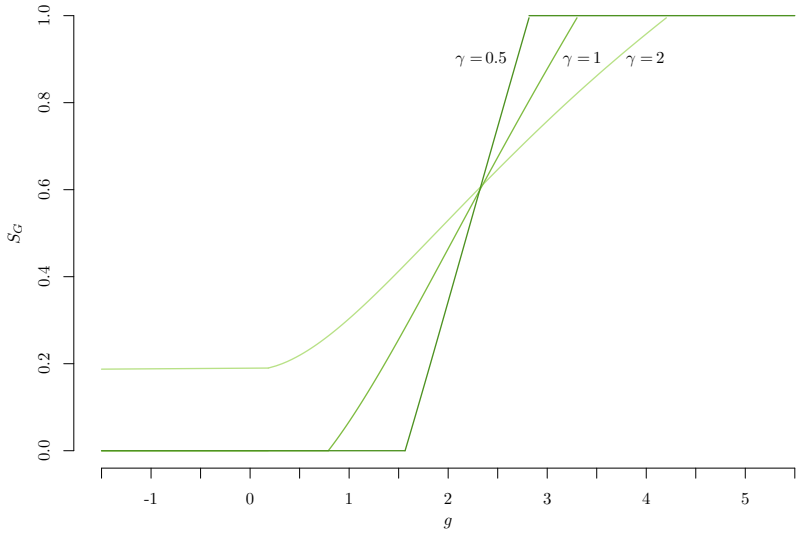


Figure 5: **Supply of green firms and comparative statics**
The figure illustrates how the supply of green firms depends on uncertainty about future social preferences (Panel (A)), risk aversion (Panel (B)), and investment costs (Panel(C)).

(A) Uncertainty about future social preferences




(B) Risk aversion



(C) Investment costs

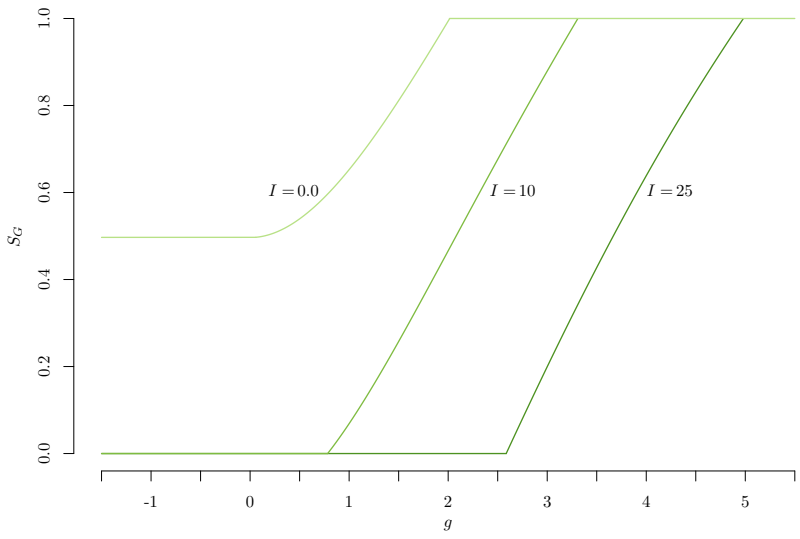
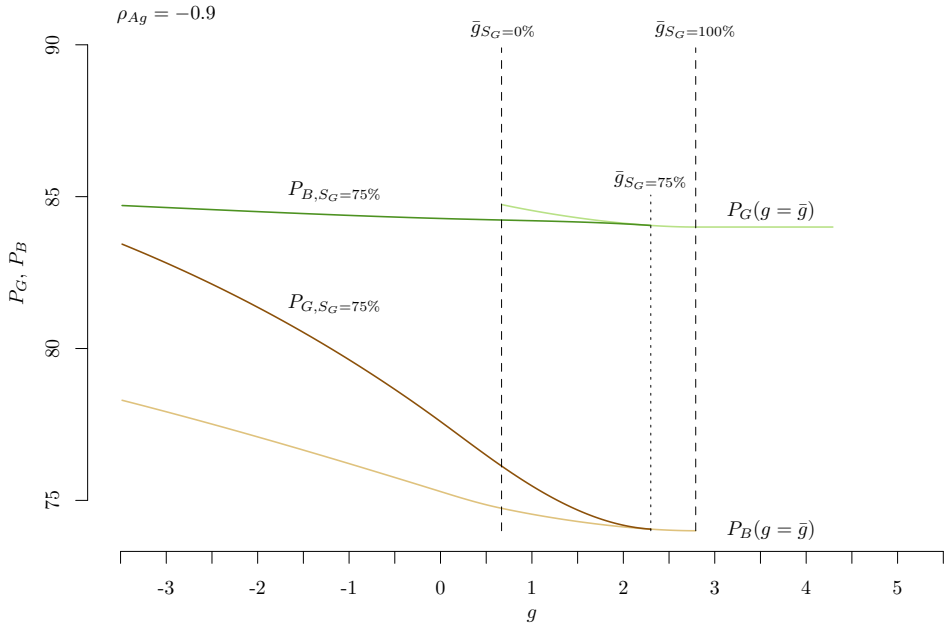


Figure 6: **Share prices with dynamic technology choices for different correlations between preference and cash-flow shocks**
The figure shows prices of brown and green firms across different levels of social investor preferences. Panel (A) illustrates results for the case of a negative correlation between preference shocks and common cash-flow shocks (i.e., $\rho_{Ag} = -0.9$) while Panel (B) focuses on a positive correlation (i.e., $\rho_{Ag} = 0.9$).

(A) Negative correlation $\rho_{Ag} = -0.9$



(B) Positive correlation $\rho_{Ag} = 0.9$

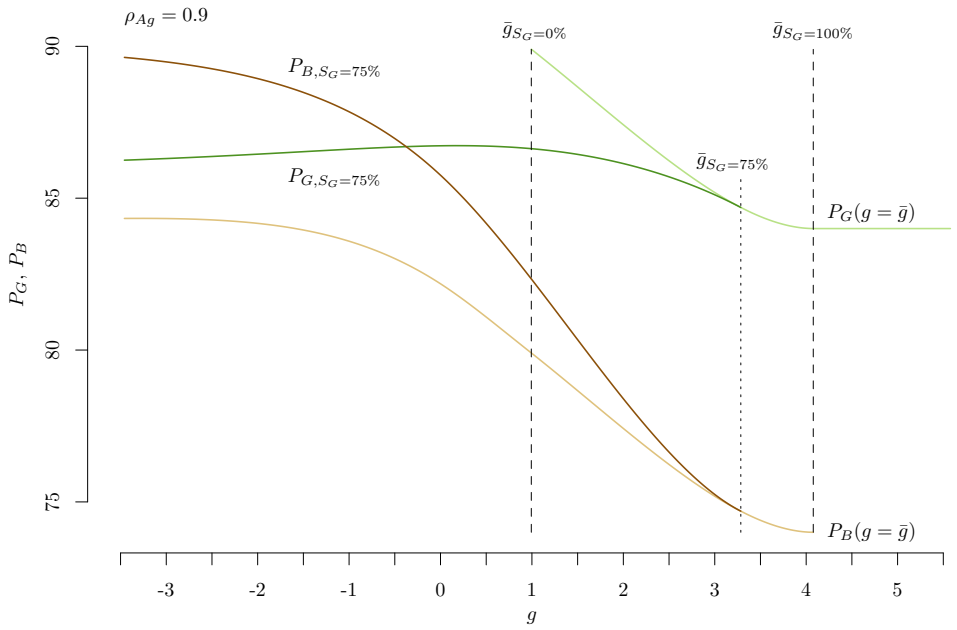
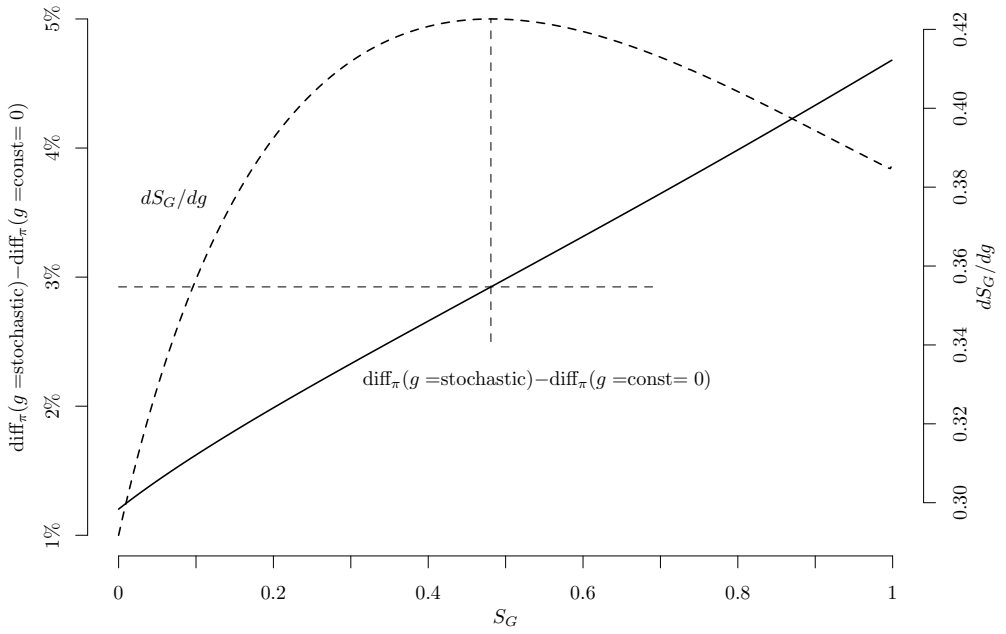
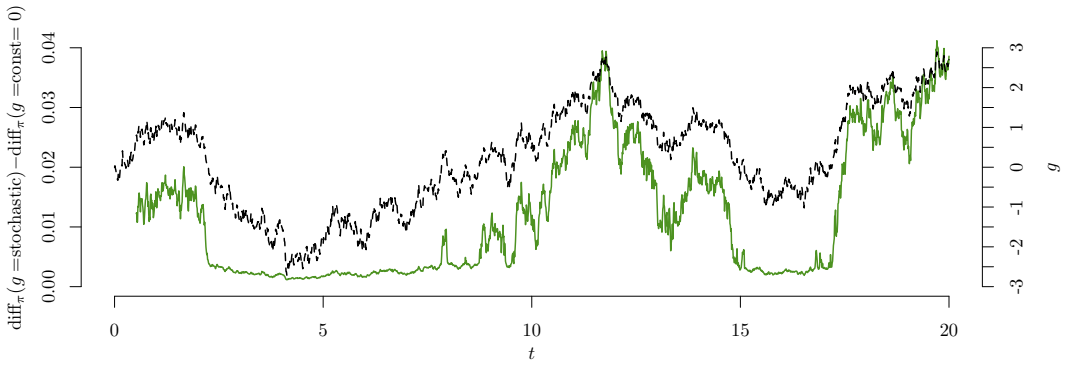


Figure 7: **Difference-in-differences of risk premia (cost of capital gap) and the sensitivity of $S_G$ to shocks in $g$**
In Panel (A), the solid line shows the diff-in-diff of risk premia (costs of capital) of green versus brown firms between our model with stochastic social preferences and a model without social preferences. The dashed line shows the sensitivity of the fraction of green firms to shifts in preferences $dS_G/dg$ at the investment threshold. Panel (B) shows the diff-in-diff of risk premia (solid line, left scale) over time, based on the realized path of $g$ (dashed line, right scale), which is identical to the one considered in Figure 4.

(A) Diff-in-diff and the sensitivity of $S_G$ to shocks in $g$



(B) Diff-in-diff of risk premia across time



# Appendix

## A Proofs

**Proof of Lemma 1.** Applying Itô's Lemma to $\hat{P}_G(g)$ and $\hat{P}_B(g)$, we can show that the vector of excess financial returns from holding the two risky assets is given by

$$\begin{bmatrix} dy_{G,t} + d\hat{P}_G(g_t) - r\hat{P}_G(g_t)dt \\ dy_{B,t} + d\hat{P}_B(g_t) - r\hat{P}_B(g_t)dt \end{bmatrix} = \underbrace{\begin{bmatrix} \mu + \mu_g \hat{P}'_G + \frac{1}{2}\sigma_g^2 \hat{P}''_G - r\hat{P}_G \\ \mu + \mu_g \hat{P}'_B + \frac{1}{2}\sigma_g^2 \hat{P}''_B - r\hat{P}_B \end{bmatrix} dt}_{\text{Expected excess returns = Risk premia}}$$
$$+ \underbrace{\begin{bmatrix} \hat{P}'_G(g_t) \\ \hat{P}'_B(g_t) \end{bmatrix}}_{\text{Price-preference sensitivity}} \underbrace{\sigma_g dz_{g,t}}_{\text{Preference shocks}} + \underbrace{\begin{bmatrix} \sigma dz_{G,t} + \sigma_A dz_{A,t} \\ \sigma dz_{B,t} + \sigma_A dz_{A,t} \end{bmatrix}}_{\text{Cash-flow shocks}}.$$

The rest follows from Brownian motion's basic properties and the definition of $M_\Pi$ and $M_\Sigma$.

**Proof of Proposition 1.** The investor makes optimal consumption and portfolio decisions to maximizes the right-hand side of Equation (8). First-order conditions imply that the representative investor's optimal consumption satisfies

$$u'(C^*) = U_W \tag{A.1}$$

and optimal demand for the risky assets is given by

$$\begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} = -\frac{U_W}{U_{WW}} M_\Sigma^{-1} \left( M_\Pi - \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right) - \frac{U_{Wg}}{U_{WW}} M_\Sigma^{-1} M_H. \tag{A.2}$$

Substituting the conjecture (9) into the condition for the optimal consumption (A.1) yields

$$C^*(W, g) = r(W + H(g)).$$

Moreover, the conjectured solution implies

$$U_W = (-\gamma r)U;\ U_{WW} = (\gamma r)^2 U;$$
$$U_{Wg} = (\gamma r)^2 U H';$$
$$U_g = (-\gamma r)UH';\ U_{gg} = (-\gamma r)U\left(H'' - \gamma r (H')^2\right).$$

Using the above partial derivatives to simplify Equation (A.2) yields Equation (11) stated in the proposition.

In the final step, we derive the ordinary differential equation for $H(g)$. Evaluating HJB equation (8) at the optimum yields

$$\begin{aligned}
\delta U(W,g) &= u(C^*) + (rW - C^*)U_W + \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix}^T \left[ \left( M_\Pi - \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right) U_W + M_H U_{Wg} \right] \\
&\quad + \frac{1}{2} \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix}^T M_\Sigma \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} U_{WW} + \mu_g U_g + \frac{1}{2}\sigma_g^2 U_{gg} \\
&= u(C^*) + (rW - C^*)U_W - \frac{1}{2} \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix}^T M_\Sigma \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} U_{WW} + \mu_g U_g + \frac{1}{2}\sigma_g^2 U_{gg}
\end{aligned}$$

where the second equality follows from rearranging Equation (A.2), i.e.,

$$\left( M_\Pi - \begin{bmatrix} 0 \\ g \end{bmatrix} \right) U_W + M_H U_{Wg} = -M_\Sigma \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} U_{WW}$$

and using it to simplify the HJB equation.

Using the conjectured solution (9) and its partial derivatives, we derive an ordinary differential equation for $H(g)$, which is stated in Proposition 1 as Equation (12)

$$\delta = r + \gamma r^2 H - \frac{(\gamma r)^2}{2} \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix}^T M_\Sigma \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} - \mu_g \gamma r H' - \frac{1}{2}\sigma_g^2 \gamma r \left( H'' - \gamma r \left(H'\right)^2 \right).$$

**Proof of Corollary 1.** In market equilibrium, the representative investor holds the market

and her optimal demand is given by:

$$\underbrace{\begin{bmatrix} S_G \\ 1-S_G \end{bmatrix}}_{\text{market portfolio}} = \underbrace{\frac{1}{\gamma r} M_\Sigma^{-1} \begin{bmatrix} \pi_G P_G \\ \pi_B P_B \end{bmatrix}}_{\text{MV optimal portfolio}} + \underbrace{\left( -\frac{1}{\gamma r} M_\Sigma^{-1} \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right)}_{\text{social portfolio}} + \underbrace{\left( -H' M_\Sigma^{-1} \begin{bmatrix} \sigma_g^2 P_G'(g) + \rho_{Ag}\sigma_A\sigma_g \\ \sigma_g^2 P_B'(g) + \rho_{Ag}\sigma_A\sigma_g \end{bmatrix} \right)}_{\text{hedging portfolio}}$$

where

(i) the MV optimal portfolio of the representative investor in the absence of non-pecuniary payoffs due to social preferences

(ii) the social portfolio reflects the demand resulting from *current* social preferences, i.e. non-pecuniary payoffs $g^+$;

(iii) the hedging portfolio captures the representative investor's hedging demand against shifts in *future* social preferences $dg$.

Denote the total, dollar return of green stocks by $dR_G \equiv dy_G + dP_G$ and that of brown stocks by $dR_B \equiv dy_B + dP_B$. In this case we have

$$\begin{bmatrix} \sigma_g^2 P_G'(g) + \rho_{Ag}\sigma_A\sigma_g \\ \sigma_g^2 P_B'(g) + \rho_{Ag}\sigma_A\sigma_g \end{bmatrix} = \frac{1}{dt} \begin{bmatrix} \sigma_{G,dg} \\ \sigma_{B,dg} \end{bmatrix},$$

where $\sigma_{G,dg} = \text{cov}(dR_G, dg)$ is the covariance of $dR_G$ and $dg$; and $\sigma_{B,dg} = \text{cov}(dR_B, dg)$ is the covariance of $dR_G$ and $dg$. We can therefore rewrite the expression for the hedging portfolio as

$$-\frac{H'}{dt} M_\Sigma^{-1} \begin{bmatrix} \sigma_{G,dg} \\ \sigma_{B,dg} \end{bmatrix}.$$

The market portfolio's total return is $dR_m = \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix}^T \begin{bmatrix} dR_G \\ dR_B \end{bmatrix}$, the social portfolio's total return is $dR_s = -\frac{1}{\gamma r} \begin{bmatrix} 0 \\ g^+ \end{bmatrix}^T M_\Sigma^{-1} \begin{bmatrix} dR_G \\ dR_B \end{bmatrix}$, and the hedging portfolio's total return is $dR_h = -\frac{H'}{dt} \begin{bmatrix} \sigma_{G,dg} \\ \sigma_{B,dg} \end{bmatrix}^T M_\Sigma^{-1} \begin{bmatrix} dR_G \\ dR_B \end{bmatrix}$.

To proceed, we note that the covariances of individual stock returns and portfolio returns are given by:

$$\begin{bmatrix}\sigma_{G,m}\\ \sigma_{B,m}\end{bmatrix} = M_\Sigma \left( \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix} \right) dt, \tag{A.3}$$

$$\begin{bmatrix}\sigma_{G,s}\\ \sigma_{B,s}\end{bmatrix} = M_\Sigma dt \left( -\frac{1}{\gamma r} M_\Sigma^{-1} \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right) = -\frac{1}{\gamma r} \begin{bmatrix} 0 \\ g^+ \end{bmatrix} dt, \tag{A.4}$$

$$\begin{bmatrix}\sigma_{G,h}\\ \sigma_{B,h}\end{bmatrix} = M_\Sigma dt \left( -\frac{H'}{dt} M_\Sigma^{-1} \begin{bmatrix} \sigma_{G,dg} \\ \sigma_{B,dg} \end{bmatrix} \right) = -H' \begin{bmatrix} \sigma_{G,dg} \\ \sigma_{B,dg} \end{bmatrix}. \tag{A.5}$$

Thus, we can rewrite the equilibrium risk premia defined in Proposition 2 as

$$\begin{aligned} \begin{bmatrix}\pi_G P_G\\ \pi_B P_B\end{bmatrix} dt &= \gamma r \left( M_\Sigma \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix} dt + H' \begin{bmatrix} \sigma_{G,dg} \\ \sigma_{B,dg} \end{bmatrix} \right) + \begin{bmatrix} 0 \\ g^+ \end{bmatrix} dt \\ &= \gamma r \begin{bmatrix}\sigma_{G,m}\\ \sigma_{B,m}\end{bmatrix} - \gamma r \begin{bmatrix}\sigma_{G,h}\\ \sigma_{B,h}\end{bmatrix} - \gamma r \begin{bmatrix}\sigma_{G,s}\\ \sigma_{B,s}\end{bmatrix} \\ &= \gamma r \begin{bmatrix} \sigma_{G,m} & \sigma_{G,h} & \sigma_{G,s} \\ \sigma_{B,m} & \sigma_{B,h} & \sigma_{B,s} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \end{aligned} \tag{A.6}$$

where the second equality follows from Equations (A.3), (A.4), and (A.5).

Next, the variances and covariances of the three portfolios are given by:

$$\sigma_m^2 = \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix}^T M_\Sigma \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix} dt, \tag{A.7}$$

$$\sigma_s^2 = \left( -\frac{1}{\gamma r} \begin{bmatrix} 0 \\ g^+ \end{bmatrix}^T M_\Sigma^{-1} \right) M_\Sigma dt \left( -\frac{1}{\gamma r} M_\Sigma^{-1} \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right) = \frac{1}{(\gamma r)^2} \begin{bmatrix} 0 \\ g^+ \end{bmatrix}^T M_\Sigma^{-1} \begin{bmatrix} 0 \\ g^+ \end{bmatrix} dt, \tag{A.8}$$

$$\begin{aligned}\sigma_h^2 &= \left(-\frac{H'}{dt}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}^T M_\Sigma^{-1}\right) M_\Sigma dt \left(-\frac{H'}{dt} M_\Sigma^{-1}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}\right)\\ &= \frac{(H')^2}{dt}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}^T M_\Sigma^{-1}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}, \end{aligned} \tag{A.9}$$

$$\sigma_{m,s} = \left(\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix}^T\right) M_\Sigma dt \left(-\frac{1}{\gamma r} M_\Sigma^{-1}\begin{bmatrix}0\\ g^+\end{bmatrix}\right) = -\frac{1}{\gamma r}\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix}^T\begin{bmatrix}0\\ g^+\end{bmatrix} dt, \tag{A.10}$$

$$\sigma_{m,h} = \left(\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix}^T\right) M_\Sigma dt \left(-\frac{H'}{dt} M_\Sigma^{-1}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}\right) = -H'\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix}^T\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}, \tag{A.11}$$

$$\sigma_{s,h} = \left(-\frac{1}{\gamma r}\begin{bmatrix}0\\ g^+\end{bmatrix}^T M_\Sigma^{-1}\right) M_\Sigma dt \left(\frac{-H'}{dt} M_\Sigma^{-1}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}\right) = \frac{1}{\gamma r} H'\begin{bmatrix}0\\ g^+\end{bmatrix}^T M_\Sigma^{-1}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}. \tag{A.12}$$

Therefore, the risk premium of the market portfolio is

$$\begin{aligned}\pi_m P_m dt &= \begin{bmatrix}S_G\\ 1-S_G\end{bmatrix}^T\left[\gamma r\left(M_\Sigma\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix} dt + H'\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}\right) + \begin{bmatrix}0\\ g^+\end{bmatrix} dt\right]\\ &= \gamma r\sigma_m^2 - \gamma r\sigma_{m,h} - \gamma r\sigma_{m,s} \end{aligned} \tag{A.13}$$

where the second line follows from Equations (A.7), (A.11), and (A.10). The risk premium of the social portfolio is

$$\begin{aligned}\pi_s P_s dt &= \left(-\frac{1}{\gamma r}\begin{bmatrix}0\\ g^+\end{bmatrix}^T M_\Sigma^{-1}\right)\left[\gamma r\left(M_\Sigma\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix} dt + H'\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}\right) + \begin{bmatrix}0\\ g^+\end{bmatrix} dt\right]\\ &= \gamma r\sigma_{m,s} - \gamma r\sigma_{s,h} - \gamma r\sigma_s^2 \end{aligned} \tag{A.14}$$

where the second line follows from Equations (A.10), (A.12), and (A.8). The risk premium

of the hedging portfolio is

$$\begin{aligned}\pi_h P_h dt &= \left(-\frac{H'}{dt}\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}^T M_\Sigma^{-1}\right)\left[\gamma r\left(M_\Sigma\begin{bmatrix}S_G\\ 1-S_G\end{bmatrix}dt + H'\begin{bmatrix}\sigma_{G,dg}\\ \sigma_{B,dg}\end{bmatrix}\right)+\begin{bmatrix}0\\ g^+\end{bmatrix}dt\right]\\ &= \gamma r\sigma_{m,h} - \gamma r\sigma_h^2 - \gamma r\sigma_{s,h} \qquad \text{(A.15)}\end{aligned}$$

where the second line follows from Equations (A.11), (A.9), and (A.12).

Summarizing Equations (A.13), (A.14), and (A.15) in compact, matrix form yields

$$\begin{bmatrix}\pi_m P_m\\ \pi_h P_h\\ \pi_s P_s\end{bmatrix}dt = \gamma r\begin{bmatrix}\sigma_m^2 & \sigma_{m,h} & \sigma_{m,s}\\ \sigma_{m,h} & \sigma_h^2 & \sigma_{s,h}\\ \sigma_{m,s} & \sigma_{s,h} & \sigma_s^2\end{bmatrix}\begin{bmatrix}1\\ -1\\ -1\end{bmatrix}.$$

Pre-multiplying both sides of the equation by the inverse of the variance matrix, we have

$$\begin{bmatrix}\sigma_m^2 & \sigma_{m,h} & \sigma_{m,s}\\ \sigma_{m,h} & \sigma_h^2 & \sigma_{s,h}\\ \sigma_{m,s} & \sigma_{s,h} & \sigma_s^2\end{bmatrix}^{-1}\begin{bmatrix}\pi_m P_m\\ \pi_h P_h\\ \pi_s P_s\end{bmatrix}dt = \gamma r\begin{bmatrix}1\\ -1\\ -1\end{bmatrix} \qquad \text{(A.16)}$$

Using Equations (A.16) and (A.6) to eliminate $\gamma r\begin{bmatrix}1\\ -1\\ -1\end{bmatrix}$, it is straightforward to show that

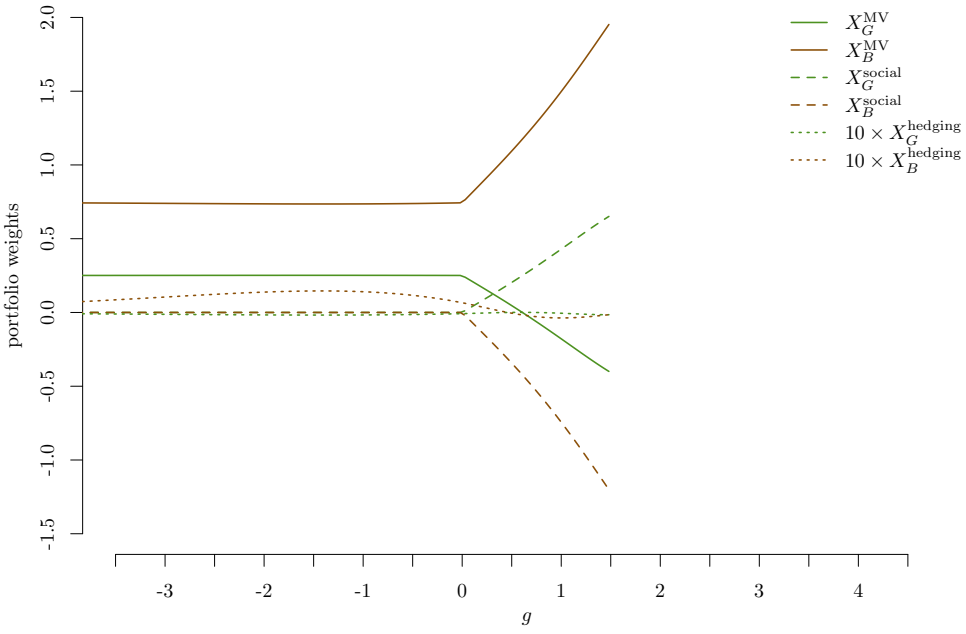
$$\begin{bmatrix}\pi_G P_G\\ \pi_B P_B\end{bmatrix} = \begin{bmatrix}\sigma_{G,m} & \sigma_{G,h} & \sigma_{G,s}\\ \sigma_{B,m} & \sigma_{B,h} & \sigma_{B,s}\end{bmatrix}\begin{bmatrix}\sigma_m^2 & \sigma_{m,h} & \sigma_{m,s}\\ \sigma_{m,h} & \sigma_h^2 & \sigma_{s,h}\\ \sigma_{m,s} & \sigma_{s,h} & \sigma_s^2\end{bmatrix}^{-1}\begin{bmatrix}\pi_m P_m\\ \pi_h P_h\\ \pi_s P_s\end{bmatrix} \qquad \text{(A.17)}$$

where

$$\begin{bmatrix}\sigma_{G,m} & \sigma_{G,h} & \sigma_{G,s}\\ \sigma_{B,m} & \sigma_{B,h} & \sigma_{B,s}\end{bmatrix}\begin{bmatrix}\sigma_m^2 & \sigma_{m,h} & \sigma_{m,s}\\ \sigma_{m,h} & \sigma_h^2 & \sigma_{s,h}\\ \sigma_{m,s} & \sigma_{s,h} & \sigma_s^2\end{bmatrix}^{-1} \equiv \begin{bmatrix}\beta_{G,m} & \beta_{G,h} & \beta_{G,s}\\ \beta_{B,m} & \beta_{B,h} & \beta_{B,s}\end{bmatrix}$$

is a 2-by-3 matrix of market, social, and hedging betas obtained in a multivariate regression of the individual stocks returns - $dR_G$ and $dR_B$ - on the three portfolio returns - $dR_m, dR_h$, and $dR_s$.

Figure A.1: **Portfolio decomposition** Portfolio holdings are decomposed into the mean-variance efficient portfolio, the social portfolio, and the hedging portfolio for base-case parameters and $S_G = 25\%$. The weights of the hedging portfolio are multiplied by a factor 10.



# B Boundary conditions for $P_G$, $P_B$, and $H$

**Boundary conditions at $\mathbf{g = \bar{g}_{S_G=100\%}}$.** When $g > \bar{g}_{S_G=100\%}$, social preferences become irrelevant and the entire stock market consists of green shares. Consequently, the price of green shares becomes constant, $P_G' = 0$, there is no intertemporal demand to hedge against shocks to social preferences, $H' = 0$, and the variance of the market portfolio becomes $\sigma^2 + \sigma_A^2$ and the ex-dividend share price becomes constant.

Substituting these conditions into Equations (17) and (12) and requiring $P_G(g; S_G)$ (in a trivial way also $P_B$) and $H(g; S_G)$ to be continuous and smooth as $g$ approaches $\bar{g}_{S_G=100\%}$ to avoid arbitrage, we get

$$\begin{aligned}
\lim_{(g,S_G)\to(\bar{g}_{S_G=100\%},100\%)} P_G(g; S_G) &= \frac{\mu}{r} - \gamma\left(\sigma^2 + \sigma_A^2\right), \\
\lim_{(g,S_G)\to(\bar{g}_{S_G=100\%},100\%)} \frac{\partial P_G(g; S_G)}{\partial g} &= 0, \\
\lim_{(g,S_G)\to(\bar{g}_{S_G=100\%},100\%)} P_B(g; S_G) &= \frac{\mu}{r} - \gamma\left(\sigma^2 + \sigma_A^2\right) - I, \\
\lim_{(g,S_G)\to(\bar{g}_{S_G=100\%},100\%)} \frac{\partial P_B(g; S_G)}{\partial g} &= 0, \\
\lim_{(g,S_G)\to(\bar{g}_{S_G=100\%},100\%)} H(g; S_G) &= \frac{\delta - r}{\gamma r^2} + \frac{\gamma}{2}\left(\sigma^2 + \sigma_A^2\right), \\
\lim_{(g,S_G)\to(\bar{g}_{S_G=100\%},100\%)} \frac{\partial H(g; S_G)}{\partial g} &= 0.
\end{aligned}$$

**Boundary conditions at the investment threshold $\mathbf{g = \bar{g}(S_G)}$** Value-matching and smooth-pasting conditions for share prices are stated and discussed in the main text in Equations (19) to (22). Boundary conditions that must be satisfied by $H$ at the boundary $\bar{g}(S_G)$ can be derived from utility maximization of competitive investors. If, after a positive shock to $g$ at the boundary $\bar{g}(S_G)$, a marginal firm of mass $dS_G$ considers to transform to the green technology, competitive investors must be willing to absorb exactly the change in asset supply by selling a marginal brown share at $P_B(g, S_G)$ and buying the newly created marginal green share at $P_G(g, S_G + dS_G) = P_B(g, S_G) + I$. If such a transaction increased investors' utility, investors would be willing to sell the brown shares at a slight discount, thereby

triggering further firms to change to the green technology (see the optimality condition for optimal corporate investment). If, otherwise, this transaction reduced utility, investors would buy the newly issued green shares only at a price below $P_B(g, S_G) + I$, which deters firms from transitioning to the green technology. Thus, in equilibrium, optimal investment implies $\partial U/\partial S_G = 0$. According to (21), asset prices do not change with $S_G$ at the boundary $\bar{g}$. Thus, at the boundary $\bar{g}$, wealth $W$ is not affected by the change in $S_G$, as can also be seen from the budget constraint (7). Consequently, from (9) it follows that $\partial H/\partial S_G = 0$.

**Boundary conditions at $\mathbf{g = 0}$.** Social preferences $g^+ = 0$ whenever $g < 0$; otherwise $g^+ = g$. No arbitrage requires share price functions be continuous and smooth at $g = 0$, i.e.,

$$\begin{aligned} P_G(g)\Big|_{g=0_-} &= P_G(g)\Big|_{g=0_+}, \qquad P_B(g)\Big|_{g=0_-} = P_B(g)\Big|_{g=0_+}, \\ \frac{\partial P_G(g)}{\partial g}\Big|_{g=0_-} &= \frac{\partial P_G(g)}{\partial g}\Big|_{g=0_+}, \qquad \frac{\partial P_B(g)}{\partial g}\Big|_{g=0_-} = \frac{\partial P_B(g)}{\partial g}\Big|_{g=0_+}. \end{aligned}$$

The boundary conditions for $H(g)$ are

$$\begin{aligned} H(g)\Big|g = 0_- &= H(g)\Big|g = 0_+, \\ \frac{\partial H(g)}{\partial g}\Big|_{g=0_-} &= \frac{\partial H(g)}{\partial g}\Big|_{g=0_+}. \end{aligned}$$

**Asymptotic behaviors of $\mathbf{P_G}, \mathbf{P_B}$, and $\mathbf{H}$ for $\mathbf{g \to -\infty}$.** For decreasing values of $g$, the value of the real option that brown firms can switch to the green technology vanishes and the supply of green shares $S_G$ becomes constant. Moreover, the investor's non-pecuniary dividends per share also vanish

$$g^+ = 0 \text{ as } g \to -\infty.$$

As a result, share prices are constant, i.e., $P_G' = P_B' = 0$, and therefore the investor does not have any hedging demand against preferences shocks, i.e., $H' = 0$. Furthermore, $M_\Sigma$ simplifies to

$$M_{\underline{\Sigma}} = \begin{bmatrix} \underline{\Sigma}_G, & \underline{\Sigma}_{GB} \\ \underline{\Sigma}_{GB}, & \underline{\Sigma}_B \end{bmatrix}$$

where

$$\begin{aligned}\underline{\Sigma}_G &= \sigma^2 + \sigma_A^2,\\ \underline{\Sigma}_B &= \sigma^2 + \sigma_A^2,\\ \underline{\Sigma}_{GB} &= \sigma_A^2.\end{aligned}$$

Using $P_G' = P_B' = 0$ to simplify Equations (17), the share prices are

$$\begin{bmatrix} P_G \\ P_B \end{bmatrix} = \frac{\mu}{r}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \gamma \begin{bmatrix} S_G\sigma^2 + \sigma_A^2 \\ (1-S_G)\sigma^2 + \sigma_A^2 \end{bmatrix}. \tag{B.1}$$

Next, using $H' = 0$ to simplify Equation (12) and solving for $H$ yields

$$H = \frac{\delta - r}{\gamma r^2} + \frac{\gamma}{2}\begin{bmatrix} S_G \\ 1-S_G \end{bmatrix}^T M_{\underline{\Sigma}} \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix}. \tag{B.2}$$

No arbitrage requires the price functions to be continuous and smooth as $g \to -\infty$,

$$\lim_{g\to-\infty} P_G(g) = \frac{\mu}{r} - \gamma(S_G\sigma^2 + \sigma_A^2), \qquad \lim_{g\to-\infty} P_B(g) = \frac{\mu}{r} - \gamma((1-S_G)\sigma^2 + \sigma_A^2),$$
$$\lim_{g\to-\infty} \frac{\partial P_G(g)}{\partial g} = 0, \qquad \lim_{g\to-\infty} \frac{\partial P_B(g)}{\partial g} = 0.$$

Finally, $H(g)$ is continuous and smooth as $g \to -\infty$, i.e.,

$$\begin{aligned}\lim_{g\to-\infty} H(g) &= \frac{\delta - r}{\gamma r^2} + \frac{\gamma}{2}\begin{bmatrix} S_G \\ 1-S_G \end{bmatrix}^T M_{\underline{\Sigma}} \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix},\\ \lim_{g\to-\infty} \frac{\partial H(g)}{\partial g} &= 0.\end{aligned}$$

# C Reversible transition

In the main text, the transition of firms to the green technology is assumed to be irreversible. While this assumption makes the analysis more tractable, there are good reasons why the results should qualitatively carry over to a framework where transition is reversible. First,

there is an asymmetry in how changes in social preferences affect incentives to switch to green versus reverting back to brown. Non-pecuniary payoffs, i.e. the cold prickle from holding brown firms, $g_t^+$, is naturally asymmetric. I.e. it is implausible that many investors would indeed feel positive non-pecuniary payoffs from holding polluting stocks, This is why we only consider non-pecuniary payoffs when $g_t$ is positive. Thus, $g_t^+$ increases, non-pecuniary payoffs induce more brown firms to switch to green. The opposite, however, is not the case. As $g_t$ becomes negative, a further "weakening" of social preferences does not make brown firms more attractive relative to green firms, i.e., $g_t^+$ simply remains zero. The only possible incentive for some firms to switch back from green to brown technology when $g_t^+$ drops is due to diversification. However, this benefit has a limited effect on prices, and for many plausible parameterizations, green firms would never find it optimal to transition back to brown.[23]

Second, the effect of irreversibility in standard real options theory stems from a positive value of waiting to invest which usually leads to delayed investment (the investment option must be sufficiently in the money to justify its exercise). In our model, however, competition among firms wipes out any value of waiting. If a firm waits with investment until the underlying project has positive net present value, competing firms would preempt it and invest early. Hence, the effect of irreversiblilty on the investment threshold is limited in a competitive setup.

To integrate (partial) reversibility into the existing model would (i) make the model more complex, (ii) add degrees of freedom regarding the conditions under which firms can switch back to brown (i.e., the expenses / proceeds associated), and (iii) increase the computational demand when solving the model numerically. However, the polar case, where a switch to the green technology is fully reversible, i .e. the investment cost $I$ is recovered when switching back to brown, can be solved analytically and can serve as an informative benchmark. This Appendix therefore provides the solution to this case.

When technology choices are fully reversible, $S_G$ is no longer path-dependent, i.e., $S_G = S_G(g)$. Furthermore $P_G = P_B + I = P$ for $S_G$ in the open interval $(0, 1)$ and $P = P(g)$, i.e.,

[23]The maximum price difference between shares of brown and green firms when $g_t \to -\infty$ and $S_G \to 1$ is given by $P_B - P_G \to \gamma\sigma^2$. Hence, if the costs for switching back to the brown technology exceed this level, there will be no reversion of the green transition at all. Assuming that switching back to brown costs the same amount $I$ as switching to the green technology, our base-case parameterization is such that firms would never transition back to being brown, even if possible.

it no longer depends explicitly on $S_G$.

According to Proposition 3, $P$ must simultaneously satisfy the system of PDEs

$$\begin{aligned}&\begin{bmatrix} rP \\ r(P-I) \end{bmatrix} + \left( \gamma r \left( M_\Sigma \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix} + H' \begin{bmatrix} \sigma_g^2 P' + \rho_{Ag}\sigma_g\sigma_A \\ \sigma_g^2 P' + \rho_{Ag}\sigma_g\sigma_A \end{bmatrix} \right) + \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right) \\ &= \begin{bmatrix} \mu \\ \mu \end{bmatrix} + \mu_g \begin{bmatrix} P' \\ P' \end{bmatrix} + \frac{1}{2}\sigma_g^2 \begin{bmatrix} P'' \\ P'' \end{bmatrix}, \end{aligned} \tag{C.1}$$

where $M_\Sigma$ simplifies to

$$M_\Sigma = \begin{bmatrix} \Sigma + \sigma^2 & \Sigma \\ \Sigma & \Sigma + \sigma^2 \end{bmatrix},$$

with $\Sigma = \sigma_g^2 (P')^2 + 2\rho_{Ag}\sigma_A\sigma_g P' + \sigma_A^2$.

$P$ simultaneously solves both PDEs of the system (C.1) only if these equations are identical. Large parts of the PDEs are equal anyhow, so the differences is

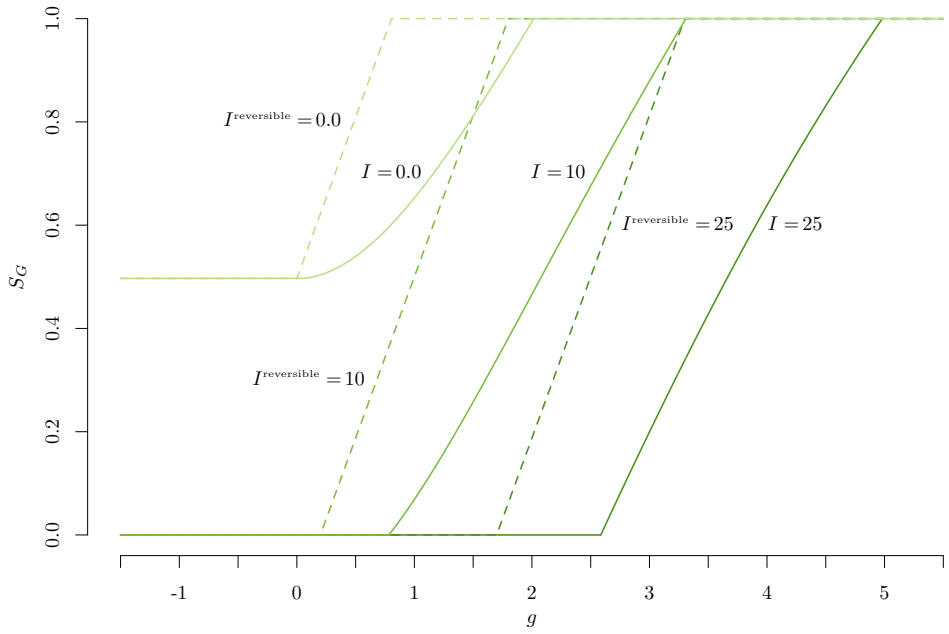
$$0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}^T \left( \begin{bmatrix} 0 \\ -rI \end{bmatrix} + \gamma r M_\Sigma \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix} + \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right),$$

which simplifies to

$$S_G = \frac{1}{2} + \frac{g^+ - rI}{2\gamma r \sigma^2}.$$

Figure C.1 shows the location of transition thresholds when investment in the green technology is fully reversible (dashed lines) together with thresholds presented in the main text (irreversible transition, solid lines) for different levels of investment costs $I$. Reversibility implies that $S_G = S_G(g_t)$ and that the green transition is completely reversed if social preferences deteriorate. Compared to the reversible case, irreversible transition takes place later (at higher levels of $g_t$) but is never reversed. Competition among firms prevents extreme delays of the investment which are observed in optimal investment decisions under uncertainty when firms have exclusive claims on the investment project.

Figure C.1: **Irreversible versus fully reversible investment**



# Internet Appendix

## I Numerical method for solving the system of Hamilton-Jacobi-Bellman equations

We numerically solve the system of differential equations (17) and (18) in Proposition 3 for given $S_G$ separately in the regions $g <= 0$ and $g > 0$. This appendix describes the employed numerical methodology. Boundary and optimality conditions are applied to the determined functions as described in the main text. We start with candidate solutions $\mathcal{P}_G$, $\mathcal{P}_B$, and $\mathcal{H}$ of the system of differential equations (17) and (18) in Proposition 3 which are elements of the polynomial space spanned by the first $n$ Chebyshev polynomials $T_0, \ldots, T_{n-1}$,

$$T_i(g) = \cos(i \cos^{-1}(g)), \quad i = 0, \ldots n-1.$$

The $3n$ coefficients $c_{G,i}$, $c_{B,i}$, and $c_{H,i}$ determine the functions $\mathcal{P}_G$, $\mathcal{P}_B$, and $\mathcal{H}$

$$\begin{aligned}
\mathcal{P}_G(g) &= \frac{1}{2}c_{G,0} + \sum_{i=1}^{n-1} c_{G,i} T_i(g), \\
\mathcal{P}_B(g) &= \frac{1}{2}c_{B,0} + \sum_{i=1}^{n-1} c_{B,i} T_i(g), \\
\mathcal{H}(g) &= \frac{1}{2}c_{H,0} + \sum_{i=1}^{n-1} c_{H,i} T_i(g).
\end{aligned}$$

Referring to the system of HJB equations (17) and (18), define the differential operators

$$
\begin{aligned}
\begin{bmatrix} \mathfrak{D}_G(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) \\ \mathfrak{D}_B(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) \end{bmatrix} &= \begin{bmatrix} \mu \\ \mu \end{bmatrix} + \mu_g \begin{bmatrix} \mathcal{P}'_G \\ \mathcal{P}'_B \end{bmatrix} + \frac{1}{2}\sigma_g^2 \begin{bmatrix} \mathcal{P}''_G \\ \mathcal{P}''_B \end{bmatrix} - \begin{bmatrix} r\mathcal{P}_G \\ r\mathcal{P}_B \end{bmatrix} \\
&\quad - \left( \gamma r \left( \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} S_G \\ 1 - S_G \end{bmatrix} + \mathcal{H}' \begin{bmatrix} \sigma_g^2 \mathcal{P}'_G + \rho_{Ag}\sigma_g\sigma_A \\ \sigma_g^2 \mathcal{P}'_B + \rho_{Ag}\sigma_g\sigma_A \end{bmatrix} \right) + \begin{bmatrix} 0 \\ g^+ \end{bmatrix} \right), \\
\mathfrak{D}_H(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) &= \delta - r - \gamma r^2 \mathcal{H} + \frac{(\gamma r)^2}{2} \begin{bmatrix} S_G \\ 1 - S_G \end{bmatrix}^T \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} S_G \\ 1 - S_G \end{bmatrix} \\
&\quad + \mu_g \gamma r \mathcal{H}' + \frac{1}{2}\sigma_g^2 \gamma r \left( \mathcal{H}'' - \gamma r \left( \mathcal{H}' \right)^2 \right).
\end{aligned}
$$

where $\Sigma_S$, $\Sigma_B$ and $\Sigma_{SB}$ are also evaluated at $\mathcal{P}_.$.

If $\mathcal{P}_G$, $\mathcal{P}_B$, $\mathcal{H}$ are solutions of (17) and (18), then

$$
\begin{bmatrix} \mathfrak{D}_G(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) \\ \mathfrak{D}_B(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) \\ \mathfrak{D}_H(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{I.1}
$$

The collocation method relaxes condition (I.1) such that it requires $\mathfrak{D}_G(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})$, $\mathfrak{D}_B(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})$, $\mathfrak{D}_H(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})$ to vanish only on the space spanned by the first $n$ Chebychev polynomials (and not everywhere).

This means, that after projecting $\mathfrak{D}_G(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})$, $\mathfrak{D}_B(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})$, $\mathfrak{D}_H(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})$ onto

the Chebychev basis, all projection coefficients $k_{.,i}$ must vanish,

$$\begin{aligned}
k_{G,i} &= \langle \mathfrak{D}_G(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) | T_i \rangle \\
&= \frac{2}{n} \sum_{j=0}^{n-1} \mathfrak{D}_G(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})(z_j^n) T_i(z_j^n) \\
&= 0, \\
k_{B,i} &= \langle \mathfrak{D}_B(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) | T_i \rangle \\
&= \frac{2}{n} \sum_{j=0}^{n-1} \mathfrak{D}_B(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})(z_j^n) T_i(z_j^n) \\
&= 0, \\
k_{H,i} &= \langle \mathfrak{D}_H(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H}) | T_i \rangle \\
&= \frac{2}{n} \sum_{j=0}^{n-1} \mathfrak{D}_H(\mathcal{P}_G, \mathcal{P}_B, \mathcal{H})(z_j^n) T_i(z_j^n) \\
&= 0 \\
i &= 0, \ldots, n-1,
\end{aligned} \tag{I.2}$$

and $z_j^n$, $j = 0, \ldots, n-1$ are the $n$ roots of $T_n$.

The relaxation translates the systems of HJB equations (17) and (18) into a system of $3n$ nonlinear equations (I.2), i.e., we have to find the set of $3n$ coefficients $c$ of $\mathcal{P}_G$, $\mathcal{P}_B$, and $\mathcal{H}$ that make the $3n$ coefficients $k$ of $\mathfrak{D}_.$ vanish.

For the treatment of value-matching conditions, we treat $c_{.,0}$, $c_{.,1}$ as a functions of the boundary conditions and drop two projections restrictions $k_{.,n-2} = 0$ and $k_{.,n-1} = 0$ from the set (I.2). Smooth-pasting conditions at free boundaries are met by variation of the free boundaries.

# II Comparison to a model with static preferences

In this appendix, we first solve for share prices and risk premia in a model with static preferences, i.e., social preferences $g$ and the supply of green firms $S_G$ enter the model as parameters. We then compare share prices and risk premia derived in the main text to those from the static model.

Since $g$ and $S_G$ are treated as model parameters, we note that the static model is just a special case of the main model in that $P'_G(g) = P''_G(g) = P'_B(g) = P''_B(g) = H'(g) = 0$. As a result, the system of ODEs (17) in Proposition 3 is reduced to

$$\begin{bmatrix} rP_G \\ rP_B \end{bmatrix} + \gamma r \begin{bmatrix} \sigma^2 + \sigma_A^2 & \sigma_A^2 \\ \sigma_A^2 & \sigma^2 + \sigma_A^2 \end{bmatrix} \begin{bmatrix} S_G \\ 1 - S_G \end{bmatrix} + \begin{bmatrix} 0 \\ g^+ \end{bmatrix} = \begin{bmatrix} \mu \\ \mu \end{bmatrix}.$$

Solving for the share prices yields

$$\begin{aligned} P_G &= \frac{\mu}{r} - \gamma(\sigma_A^2 + \sigma^2 S_G), \\ P_B &= \frac{\mu}{r} - \gamma(\sigma_A^2 + \sigma^2(1 - S_G)) - \frac{g^+}{r}. \end{aligned}$$

Using the prices of green and brown shares, we calculate their risk premia as follows:

$$\begin{aligned} \pi_G &= \frac{\mu}{P_G} - r, \\ \pi_B &= \frac{\mu}{P_B} - r. \end{aligned}$$

Figure II.1 compares the share prices and risk premia implied by the model with static preferences to the ones arising in our main model with dynamic preferences when the supply of green firms is low, i.e., 25%. The vertical, dotted line at which all the other lines end indicates the investment threshold, i.e., the level of social preferences, at which more brown firms would transition to the green technology in a world with dynamic preferences.

The solid lines represent the model with dynamic preferences and are identical to those shown in Figure 1, while the dashed lines capture the static case. As shown in the equation for $P_G$ derived just above, the price of green stocks does not depend on $g$ in the static model, as green stocks are not affected by the extent of "cold prickle"that social investors exhibit. On the other hand, prices of brown stocks depend linearly on $g$, as long as $g$ is positive and are also unaffected once $g$ becomes negative. We also see that $P_G > P_B$, as $S_G < S_B$ and, thus, green stocks are less affected by systematic risk.

Another interesting observation is that prices in a static model exceed prices in a dynamic one, almost everywhere in the graph. This is the case for green stocks, as their prices are

depressed in a dynamic model by the risk of more brown firms switching to become green in response to a sufficiently large positive shock in preferences (see also Panel (B) that shows that the risk premium of green stocks is larger in the dynamic model and increases towards the investment threshold).

In the case of brown stocks, the picture is more nuanced. Relatively far away from the investment threshold, $P_B$ is lower in a dynamic model compared to the static case, as investors rationally anticipate that preferences might increase in the future, making brown stocks less attractive and riskier (see also Panel (B)). However, once we approach the investment threshold, there is a countervailing positive effect arising from the endogenous decision of brown stocks to become green and, thus, $P_B$ in a dynamic model exceeds $P_B$ in a static model when we are close to the investment threshold.

Figure II.2 shows the same comparison in a world in which 75% of all firms are green. Qualitatively, the patterns look very similar but quantitatively we observe some changes. For example, the difference in $P_G$ between the static and the dynamic model becomes smaller compared to Figure II.1, as most of the firms are already green. We also observe that $P_G$ and $P_B$ intersect in the case of the dynamic model, as discussed before, as well as in the case of the static model. In the later case, however, that point is reached much faster, moving from right to left, reflecting that the in the dynamic model $P_B$ is depressed by the risk of increasing social preferences in the future.

Figure II.3 focuses on the time-series dimension, assuming a realized sample path of $g$, and highlights the importance of accounting for the stochastic nature of the preferences in the model, compared to a world in which investors assume $g$ to be constant. For this purpose, we assume that investors in a static model take the observed $g$ and $S_G$, which corresponds to the equilibrium $S_G$ according to the dynamic model, as given. Thus, investors in the static model experience the same dynamics of $g$ and $S_G$ but do not explicitly account for them. Figure II.3 uses the same sample path as Figure 4.

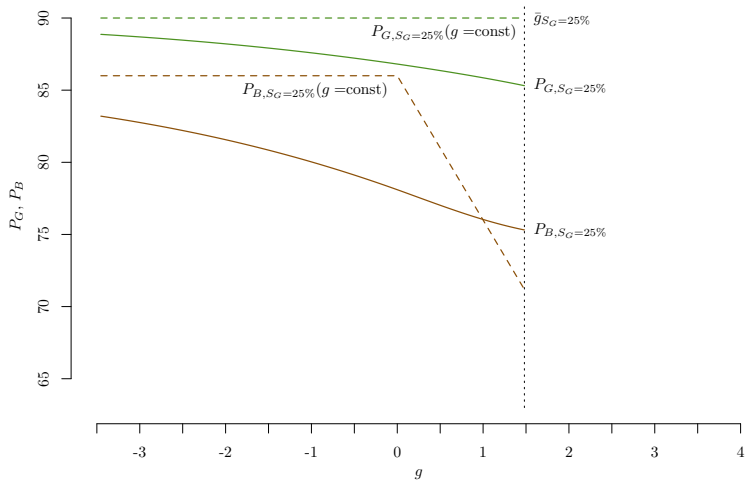
Specifically, Panel (B) shows the evolution of prices of green and brown firms over time. We observe that the price of green firms is basically flat and unaffected by variation in $g$, consistent with the derivations summarized at the beginning of this appendix. Prices of green stocks only change in response to $S_G$, as green firms become more systematically risky when $S_G$ increases.

Prices of brown firms vary inversely with $g$, as long as $g$ is positive, but are flat as soon

as $g$ becomes negative. It is also worth noting that price levels in Panel (B) of this figure are always higher than in Panel (B) of Figure 4 for green firms, as investors in a static world ignore the risk arising from endogenous technology transitions of brown firms. Similarly, prices of brown firms tend to be higher in the static case, as investors ignore uncertainty arising from stochastic preferences. However, when $g$ reaches very high levels, prices of brown stocks in the dynamic world exceed prices in the static world, as they are pushed upwards by the real option to switch to the green technology. Accounting for stochastic preferences and endogenous technology choice, thus, turn out to be important drivers of asset prices in our setup.

Figure II.1: **Share prices and risk premia in a model with stochastic preferences compared to a model with static preferences ($S_G = 0.25$).**
The figure compares prices (Panel (A)) as well as risk premia (Panel (B)) of brown and green firms for different levels of social investor preferences in a model with dynamic preferences (solid lines) to a model with static preferences (dashed lines). In this case, 25% of all firms are initially endowed with the green technology or have already switched to this technology (i.e., $S_G = 0.25$).

(A) Share prices




(B) Risk premia

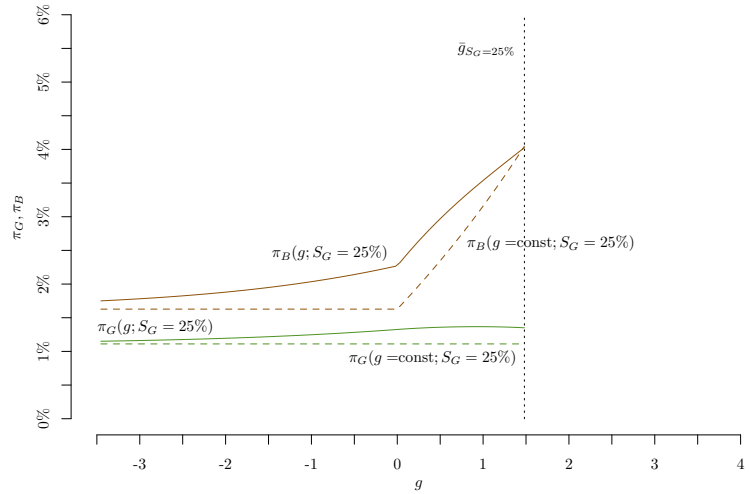
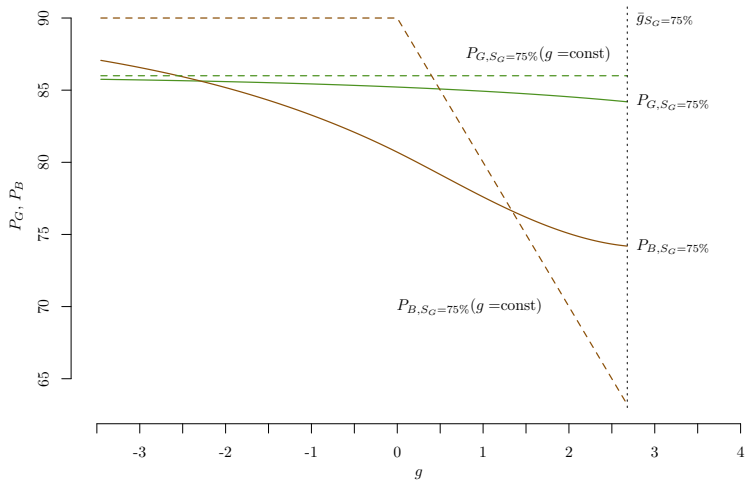


Figure II.2: **Share prices and risk premia in a model with stochastic preferences compared to a model with static preferences ($S_G = 0.75$).**
The figure compares prices (Panel (A)) as well as risk premia (Panel (B)) of brown and green firms for different levels of social investor preferences in a model with dynamic preferences (solid lines) to a model with static preferences (dashed lines). In this case, 25% of all firms are initially endowed with the green technology or have already switched to this technology (i.e., $S_G = 0.75$).

(A) Share prices




(B) Risk premia

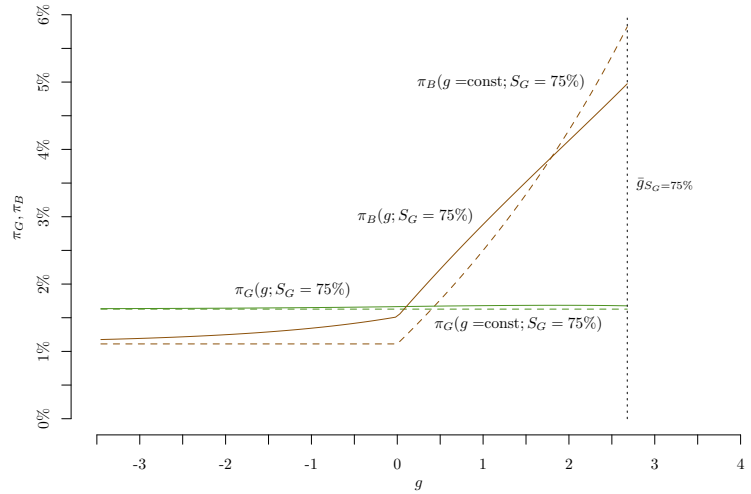
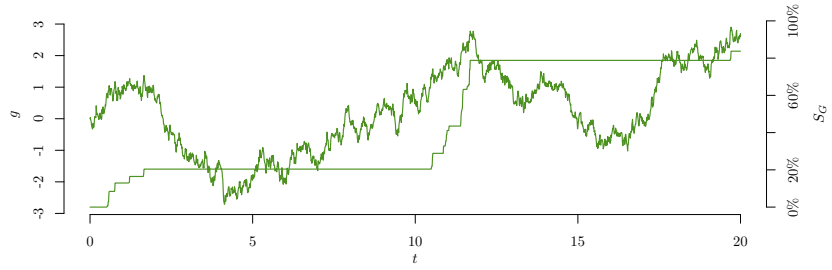


Figure II.3: **Share prices in a model with static preferences for a given path of social preferences.**
Panel (A) shows the same sample path of $g$ together with the supply of green firms according to the full dynamic model as those presented in Figure 4. Panel (B) shows prices of green and brown firms.

(A) Supply of green firms




(B) Prices in a "static" world

