Impact Investing with Shareholder Engagement*

PRELIMINARY DRAFT

Jean-François Chassagneux Roxana Dumitrescu Olivier David Zerbib

April 30, 2025

Abstract

In this paper, we study the impact of shareholder engagement on asset prices and corporate practices. We develop a dynamic equilibrium model where heterogeneous green investors choose their optimal asset allocation and costly engagement efforts, and a representative firm sets its greenhouse gas (GHG) emissions to minimize both its cost of capital and GHG abatement costs. Closed-form solutions deliver three key findings. First, the investors overweight the firm if it has high emissions, especially when engagement cost is low or green preferences are strong. Second, engagement-driven capital reallocations lower brown firms' cost of capital. Third, firms face dual mitigation pressure: they are incentivized to choose lower gross emissions and incur further cuts via investor engagement. These insights overturn standard sustainable asset-pricing predictions and explain why shareholder engagement is an effective vector of impact for sustainable investors.

Keywords: Shareholder engagement, Impact investing, Asset pricing.
JEL codes: C61, G12.

*Jean-François Chassagneux (jean-francois.chassagneux@ensae.fr), Roxana Dumitrescu (roxana.dumitrescu@ensae.fr), and Olivier David Zerbib (olivier-david.zerbib@ensae.fr) are all at CREST Laboratory, CNRS, Groupe des Écoles Nationales d'Économie et Statistique, Ecole Polytechnique, Institut Polytechnique de Paris, 5, avenue Henry Le Chatelier 91120 Palaiseau, France.

1 Introduction

Sustainable investing has developed in response to two distinct types of investor motivations: deontological motivations, which consist in aligning investments with ethical convictions (Riedl and Smeets, 2017), and consequentialist motivations, which seek to align the outcomes of investments with ethical convictions (Green and Roth, 2024). In practice, when it comes to pro-environmental (or "green") investing, the first motivation translates into maximizing the greenness of one's portfolio, while the second one aims to maximize the greenness of the economy as a whole, a strategy also referred to as "impact investing." These two motivations mirror Weber's (1921) distinction between the ethics of conviction and the ethics of responsibility.

Impact investors typically use two main approaches: investment screening and shareholder engagement. Investment screening excludes or underweights the assets of companies with high environmental footprints ("brown" companies) while overweighting those with low footprints ("green" companies). By shifting portfolio allocations, investor screening affects equilibrium asset prices, thereby modifying firms' expected returns (i.e., cost of capital). Specifically, it raises the cost of capital for brown companies and lowers it for green ones. However, De Angelis, Tankov, and Zerbib (2023) show that these shifts provide only weak incentives for brown firms to adopt greener practices.

Contrary to divestment strategies, shareholder engagement seeks to influence brown firms by maintaining or increasing investment and actively encouraging them to adopt greener practices. Engagement can occur through public mechanisms (e.g., voting at general meetings, tabling and withdrawing shareholder resolutions) or through private initiatives (e.g., meetings, calls, and letters addressed to company directors requesting changes to specific practices). Recent empirical studies highlight the effectiveness of shareholder engagement in promoting improved environmental practices (Dimson, Karakaş, and Li, 2015; Heeb and Kölbel, 2024; van der Kroft, Palacios, Rigobon, and Zheng, 2025). Moreover, Broccardo, Hart, and Zingales (2022) suggest that engagement may be more effective than environmental investment screens in influencing corporate behavior. However, to the best of our knowledge, no paper has yet modeled the impact of shareholder engagement on asset prices (or firms' cost of capital) or on companies' dynamic environmental responses.

To fill this gap, we develop a dynamic equilibrium model featuring a representative firm, which

produces output by emitting greenhouse gases (GHG), and two heterogeneous green investors. Each investor, in addition to having mean-variance preferences, has a different pro-environmental preference and wishes to reduce the firm's emissions through shareholder engagement, incurring a quadratic cost of effort. Stronger engagement efforts lead to greater reductions in the firm's emissions, all the more so when the investor's ownership of the firm's assets is high. As for the company, it chooses the optimum level of *gross* emissions to minimize the sum of its cost of capital and its abatement cost, as in Cartellier, Tankov, and Zerbib (2024). Thus, investors and the company interact as part of a Stackelberg game in which the company, acting as leader, chooses an optimal level of emissions, and investors, acting as followers, set their optimal asset allocation and engagement effort. *Net* emissions correspond to the firm's gross emissions, adjusted downward by the impact of investor engagement.

We derive closed-form expressions for investors' optimal allocations and engagement efforts, equilibrium expected returns, and the firm's optimal emissions. First, relative to a pure mean-variance benchmark, investors over-allocate their capital to the assets of a high-emitting firm, especially as engagement costs are low or pro-environmental preferences are strong. Similarly, optimal engagement effort increases with the investors' portfolio allocation to the firm, the firm's emissions, and the investors' pro-environmental preferences, but decreases with engagement costs.

Second, contrary to what is documented in the green finance literature, which focuses on investment screening (Pástor, Stambaugh, and Taylor, 2021; Pedersen, Fitzgibbons, and Pomorski, 2021; Zerbib, 2022), we show that expected returns fall for high-emitting firms when investors engage with them. Specifically, investors over-invest in firms where engagement is most effective in reducing emissions, thereby accepting lower expected returns. Conversely, they require higher expected returns to allocate their capital to firms where engagement yields limited emissions reductions.

Third, in response to the pressure exerted by investors—which grows with their pro-environmental preferences and decreases with the cost of engagement—firms are driven to reduce their emissions through two distinct channels: they are incentivized to select a lower optimal level of gross emissions, and they are compelled to further reduce emissions after investors' engagement takes place. This equilibrium framework enables us to illustrate reductions in corporate emissions under two different deterministic scenarios: when investors' pro-environmental preferences increase, and when engagement practices become more efficient, for example, thanks to improved proxy voting, which

significantly decreases engagement costs.

This paper contributes to two main areas of the quantitative sustainable finance literature. First, we add to the sustainable asset pricing literature. Several theoretical studies (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022) and empirical works (Bolton and Kacperczyk, 2021; Hsu, Li, and Tsou, 2023) document that investors' pro-environmental preferences (Riedl and Smeets, 2017) increase the cost of capital for brown firms. Indeed, green investors are willing to accept lower expected returns to hold the assets of green firms and require higher expected returns to hold the assets of brown firms. Recently, Favilukis, Garlappi, and Uppal (2024) demonstrated that, within a general equilibrium framework, substantial capital reallocation toward green firms can occur with only a minimal increase in brown firms' cost of capital. In this paper, we show that when investors practice shareholder engagement, pro-environmental preferences have an opposite effect on asset allocations and firms' cost of capital. This result is important because, given the prevalence and development of shareholder engagement, it highlights that sustainable investing does not trivially and homogeneously modify asset prices.

Second, our results contribute to the growing literature on impact investing. Although the increase in brown firms' cost of capital associated with green investing can incentivize firms to become greener, this effect remains limited (De Angelis et al., 2023). Moreover, pressure on the cost of capital may even have adverse effects, encouraging brown firms to pursue short-term, high-emission projects (Hartzmark and Shue, 2023). In fact, several recent studies emphasize that maximizing an investor's impact requires internalizing aggregate externalities across the economy, including those stemming from firms outside the investor's portfolio (Green and Roth, 2024; Oehmke and Opp, 2025). Strategies such as financing firms with high greening potential overlooked by other investors (Green and Roth, 2024) or targeting firms facing significant capital search frictions (Landier and Lovo, 2023) have been recommended. Our results show that shareholder engagement creates a double downward pressure on firms' GHG emissions: it incentivizes the firm to lower its optimal emissions, and it further reduces them through the engagement effect itself. Importantly, this mechanism operates while the cost of capital for brown firms decreases, as engagement becomes more effective when investors overweight the targeted firms.

The remainder of the paper is organized as follows. Section 2 introduces the model, detailing the market environment, the agents' objective functions, and their strategic interactions. Section 3 de-

rives the investors' optimal allocations, equilibrium expected returns, and firms' optimal emissions, and analyzes their behavior under deterministic economic scenarios. Section 4 concludes.

2 An equilibrium model with shareholder engagement

Market setting. Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space supporting a one dimensional Brownian Motion $(W_t)_{t\geq 0}$. We denote by $\mathbb{F} := (\mathcal{F}_t)_{t\geq 0}$ the filtration generated by W (augmented). On the associated filtered probability space with an infinite time horizon, we consider a market with two assets. The first asset is risk-free, in perfect elastic supply, and is assumed to have a zero interest rate. The other asset is risky and is issued at time 0 by a representative firm. Given a *gross emission* rate schedule, $(e_t)_{t\geq 0}$, which is a continuous $\mathbb{F}$-adapted process chosen by the company's manager, we assume that, in equilibrium, the stock price follows the dynamics:

$$dS_t = \mu_t(e.)dt + \sigma dW_t, \tag{1}$$

where the $\mathbb{R}$-valued $\mathbb{F}$-progressively measurable process $\mu_t(e.)$ is the expected returns of the asset, which is determined in equilibrium, and $\sigma \in \mathbb{R}_+^*$ is the constant and positive exogenous volatility. We normalize the quantity of stocks of the firm to one. Throughout the paper, we make the following assumption on the gross emissions rate that can be chosen by the company:

$$0 \leq e_t \leq \widehat{\Psi}, \tag{2}$$

where $\widehat{\Psi}$ is a positive constant representing the maximum rate that can be realized by the firm. The set of such gross emissions rate schedule is denoted by $\mathcal{E}$.

Firm emissions and shareholder engagement. By producing, the firm emits greenhouse gases, which we refer to as *net emissions*. Their rate of emission $(\psi_t)_{t\geq 0}$ is a continuous $\mathbb{F}$-adapted process. In addition to the firm, this market is also made up of two investors, indexed by $i \in \{1, 2\}$, with different pro-environmental preferences (formally defined a little further down). These two investors can push companies to reduce their emissions by engaging with them as shareholders. In practice, although sometimes public, shareholder engagement is mainly driven by investors via the

private channel,[1] through a range of pressures, from a simple phone call to requiring change in a private meeting with management. We denote by $\pi_t^i \in \mathbb{R}_+$ the shareholder engagement *effort* (or pressure) exerted on the company by investor i at time t. Similarly, we denote by $q_t^i \in [0,1]$ the weights of the company's assets held by investor i at time t. The processes $(q_t^i, \pi_t^i)_{t\geq 0}$ for $i \in \{1,2\}$ are assumed to be continuous $\mathbb{F}$-adapted processes. Thus, as the effect of shareholder pressure increases with asset ownership, when the firm sets gross emissions rate at time t, e_t, its emissions rate net of investor engagement at time t, ψ_t, are equal to :

$$\psi_t = (1 - f(q_t^1)\pi_t^1 - f(q_t^2)\pi_t^2)e_t, \tag{3}$$

where $f : \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing and concave function. We choose a concave function because the marginal impact of engagement decreases with the ownership size: a small ownership already allows investors to exert their influence on a company, for example, by raising their voice or building engagement coalitions.

Program of the investors. The optimization program for investors consists of two parts: a standard mean-variance criterion and additional factors related to pro-environmental preferences, inspired by Bouchard, Fukasawa, Herdegen, and Muhle-Karbe (2018) and Cartellier et al. (2024), among others. Specifically, pro-environmental preferences reflect investors' appetite to reduce the company's emissions by engaging with it at an instantaneous marginal engagement cost $(\alpha_t)_{t\geq 0}$, which is a continuous $\mathbb{F}$-adapted process. Moreover, throughout the paper, we assume that

$$\alpha_t \geq \widehat{\alpha}\,, \quad \text{for all } t \geq 0, \tag{4}$$

where $\widehat{\alpha} > 0$ is a positive constant. This engagement cost corresponds to all the expenses involved in screening and conducting the engagement, often through the work of dedicated teams (2 Degrees Investing, 2025). Thus, for a fix strategy (q^j, π^j) of the other investor $j \in \{1,2\}$, each investor $i \in \{1,2\} \setminus j$ determines, at each time t, her optimal allocation, q_t^i, and optimal engagement effort,

[1] See "Shareholder Engagement Inside and Outside the Shareholder Meeting" by Bowley, Hill, and Kourabas on the University of Oxford Faculty of Law blog: `https://blogs.law.ox.ac.uk/oblb/blog-post/2023/06/shareholder-engagement-inside-and-outside-shareholder-meeting`

π_t^i, by maximizing the following program:

$$\sup_{(q^i,\pi^i)\in\mathbb{A}} \mathbb{E}\left[\int_0^\infty e^{-\delta t}\left\{q_t^i dS_t - \frac{\gamma_s}{2}\langle q_t^i dS_t\rangle - \gamma_s\alpha_t\frac{(\pi_t^i)^2}{2} + \gamma_{\mathrm{e}}^i\left(f([q_t^1]_+)\pi_t^1 + f([q_t^2]_+)\pi_t^2\right)e_t dt\right\}\right], \quad (5)$$

where the space of controls $\mathbb{A}$ is defined as the set of continuous $\mathbb{F}$-adapted processes $(q_t,\pi_t)_{t\geq 0}$ satisfying $\pi \geq 0$ and $\mathbb{E}\Big[\int_0^{+\infty} e^{-\delta t}\left(|\pi_t|^2 + |q_t|^2\right)\mathrm{d}t\Big] < +\infty$. In the equation above, the notation $[x]_+$, for x a real number, stands for the positive part of x.

In the program, $q_t^i dS_t - \frac{\gamma_s}{2}\langle q_t^i dS_t\rangle$ is the mean-variance part, $\left(f([q_t^1]_+)\pi_t^1 + f([q_t^2]_+)\pi_t^2\right)e_t$ is the reduction of the firm's emissions imposed by both investors thanks to their engagement, α_t is the marginal unit cost of engagement and $\alpha_t\frac{(\pi_t^i)^2}{2}$ is the engagement cost for effort π_t^i, γ_e^i is the non-pecuniary pro-environmental preference parameter, γ_s is the financial risk aversion, which, for the sake of simplicity, is the same for both investors, and δ is the discount rate. Throughout the paper, we assume that $\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2 > 0$, so that there is always one investor at least prone to engage with the firm. Thus, the engagement effort, π_t^i, scales with the investor's allocation to the firms, $f([q_t^i]_+)$, since the effectiveness of her activism rises with the size of her capital stake. Expected returns on the firm's assets are determined in equilibrium based on investors' optimal allocations and such that the market clears. We denote $(\bar{q}^i(e.),\bar{\pi}^i(e.))$, for $i\in\{1,2\}$, the optimal allocation (if any) given the gross emissions rate schedule $(e_t)_{t\geq 0}$ of the firm management. These optimal allocations allow to determine an equilibrium asset return in (1) denoted $(\bar{\mu}(e.))_{t\geq 0}$ and it will be an *admissible solution* if, for all $t \geq 0$,

$$(\bar{q}_t^1,\bar{q}_t^2)\in[0,1]^2,\ (\bar{q}_t^i,\bar{\pi}_t^i)\neq(0,0) \text{ for } i\in\{1,2\}, \text{ and } f(\bar{q}_t^1)\bar{\pi}_t^1 + f(\bar{q}_t^2)\bar{\pi}_t^2 \leq 1\,. \quad (6)$$

It is worth noting that, deliberately, we do not specify a problem where the greening of companies modifies their financial risk, for three reasons. First, because the financial impact of greening a company is not consensual: while the abatement cost may lead to a drop in valuation in the short term given the investments mobilized, the reduction in financial transition risks[2] may work in the opposite direction, increasing the valuation of assets. Therefore, irrespective of investors'

[2]According to the United States Environmental Protection Agency, "transition risks are those associated with the pace and extent at which an organization manages and adapts to the internal and external pace of change to reduce greenhouse gas emissions and transition to renewable energy."

non-pecuniary preferences, the net effect of an abatement on contemporary asset returns and risks is not necessarily significant. Second, we intentionally focus this paper on preferences for environmental impact via shareholder engagement. Finally, this setup allows us to obtain simple and explicit expressions in closed-form formulas.

Program of the firms. The firm has linear utility and risk-neutral preferences (Lambrecht and Myers, 2017; De Angelis et al., 2023; Cartellier et al., 2024). It looks for the gross emissions schedule, $(e_t)_{t\geq 0}$, that minimizes the sum of its cost of capital and abatement cost over the entire period:

$$\inf_{e_\cdot \in \mathcal{E}} \mathbb{E}\left[\int_0^\infty e^{-\delta t}\left(\mu_t((e_\cdot)) - \kappa\left(1 - f(\bar{q}_t^1(e_\cdot))\bar{\pi}_t^1(e_\cdot) - f(\bar{q}_t^2(e_\cdot))\bar{\pi}_t^2(e_\cdot)\right)e_t\right)dt\right], \tag{7}$$

where κ is the marginal abatement rolling cost (De Angelis et al., 2023), $\bar{q}_t^1(e_\cdot)$ and $\bar{q}_t^2(e_\cdot)$ are the (admissible) optimal asset allocations of investors 1 and 2, respectively, $\bar{\pi}_t^1(e_\cdot)$ and $\bar{\pi}_t^2(e_\cdot)$ are the (admissible) optimal engagement efforts of investors 1 and 2, respectively, and $\mathcal{E}$ is the set of strategies for the firm management (have in mind (2)). Consistent with Cartellier et al. (2024), we specify the firms' program through the minimization of their cost of capital rather than the maximization of their asset price for several reasons: (i) the cost of capital is a fundamental variable in terms of corporate solvency and profitability, and is affected by capital flows from sustainable investors (Pástor et al., 2021; Zerbib, 2022; De Angelis et al., 2023; Cartellier et al., 2024); (ii) minimizing the cost of capital is a notion equivalent to maximizing the market value of assets at the date under consideration (McConnell and Sandberg, 1975; Nantell and Carlson, 1975); (iii) finally, the use of the cost of capital, through the expected return, yields closed-form formulas which make it possible to explain the transmission channels of the engagement effects at play.

In this framework, investors and the company interact as part of a Stackelberg game in which the company, acting as leader, chooses an optimal level of gross emissions rate, denoted $(e_t^\star)_{t\geq 0}$, and the investors, acting as followers, set their optimal asset allocation and engagement effort, which subsequently determines the asset equilibrium return, denoted $(\mu_t^\star)_{t\geq 0}$. The net emissions rate of the firm, $(\psi_t^\star)_{t\geq 0}$, correspond to the firm's gross emissions rate, adjusted downward by the impact of investor engagement.

3 Optimal shareholder engagement and impact on the firm

3.1 Results

In this section, we solve the above optimization problems and the associated Stackelberg game. We set, from now on,

$$\mathbb{R}_+ \ni q \mapsto f(q) = \sqrt{2q} \in \mathbb{R}_+. \tag{8}$$

This choice is purely technical as it allows to obtain closed-form formulae for the optimal allocations and engagements of the investors. Obviously, other specifications of f would also have led to closed form expression (e.g., $q \mapsto f(q) = q$). However, the above choice also allows to solve in closed form the firm management program (see Section 3.1.2 below), and facilitates the discussions.

3.1.1 Optimal allocation, optimal engagement, and equilibrium expected returns

We first focus on the investors' programs and the subsequent market equilibrium, which are the optimal responses to the gross emissions rate, $(e_t)_{t\geq 0}$, chosen by the firm management. In turn, the optimal choice of $e.$ is discussed in the next section and will allow us to solve completely the Stackelberg game under study.

Proposition 1. *Let $(e_t)_{t\geq 0} \in \mathcal{E}$ be a gross emissions rate schedule that satisfies*

$$e_t > 0\,, \quad \textit{for all } t \geq 0.$$

Assume that

$$\left| \frac{|\gamma_{\mathrm{e}}^1|^2 - |\gamma_{\mathrm{e}}^2|^2}{2\gamma_{\mathrm{s}}^2} \right| < \frac{\widehat{\alpha}\sigma^2}{\widehat{\Psi}^2} \quad \textit{and} \quad \frac{\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2}{\gamma_{\mathrm{s}}\widehat{\alpha}}\widehat{\Psi} + \frac{(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}{2\sigma^2\widehat{\alpha}\gamma_{\mathrm{s}}^3}\widehat{\Psi}^3 \leq 1, \tag{9}$$

then there exists a unique admissible solution according to (6)*. The asset's equilibrium return is*

given by

$$\bar{\mu}_t(e.) = \frac{\gamma_s \sigma^2}{2} - \frac{|\gamma_e^1|^2 + |\gamma_e^2|^2}{2\gamma_s \alpha_t} e_t^2. \tag{10}$$

The optimal allocations read

$$\bar{q}_t^1 = \frac{1}{2}\left(1 + \frac{|\gamma_e^1|^2 - |\gamma_e^2|^2}{2\sigma^2 \gamma_s^2 \alpha_t} e_t^2\right) \in (0,1) \ \textit{and} \ \bar{q}_t^2 = 1 - \bar{q}_t^1. \tag{11}$$

For $i \in \{1,2\}$, the optimal engagements read

$$\bar{\pi}_t^i = \frac{\gamma_e^i}{\gamma_s \alpha_t} f(\bar{q}_t^i) e_t, \tag{12}$$

where f is given in (8).

Proof. **Step 1)** For each investor, it is readily seen that the optimization problem is solved pointwisely. For $i \in \{1,2\}$, $t \geq 0$, we get from (5) that the following functions are to be maximized:

$$P_t^i(q,\pi) = q\mu_t - \frac{\gamma_s}{2}\left(\sigma^2 q^2 + \alpha_t \pi^2\right) + \gamma_e^i f([q]_+)\pi e_t \tag{13}$$

for $(q,\pi) \in \mathbb{R} \times \mathbb{R}_+$. Above $(\mu_t)_{t\geq 0}$ stands for any $\mathbb{F}$-progressively measurable process that will be determined through the market clearing condition below. We first study the critical points of P_t^i and will discuss the admissibility of the solutions later on in Step 2 below.

- Let $\mathcal{D} := \mathbb{R}^* \times \mathbb{R}_+^*$. The first order conditions on $\mathcal{D}$ lead to, denoting $(\bar{q}^i, \bar{\pi}^i)$ a critical point of P^i,

$$\bar{q}_t^i \sigma^2 = \frac{\mu_t}{\gamma_s} + \frac{\gamma_e^i}{\gamma_s} f'(\bar{q}_t^i) \mathbf{1}_{\{\bar{q}_t^i > 0\}} \bar{\pi}_t^i e_t \tag{14}$$

and

$$\bar{\pi}_t^i = \frac{\gamma_e^i}{\alpha_t \gamma_s} f(\bar{q}_t^i) \mathbf{1}_{\{\bar{q}_t^i > 0\}} e_t, \tag{15}$$

for $t \geq 0$. Equation (15) indicates that on $\mathcal{D}$, a critical point must satisfy $\bar{q}_t^i > 0$. From (8), we get to

$$\bar{q}_t^i \sigma^2 = \frac{\mu_t}{\gamma_{\rm s}} + \left(\frac{\gamma_{\rm e}^i}{\gamma_{\rm s}}\right)^2 \frac{e_t^2}{\alpha_t}\ ,\ t \geq 0, \tag{16}$$

and $\bar{\pi}^i$ follows from (15) (having in mind that $\bar{q}_t^i > 0$).

For $q > 0$ and $\pi > 0$, we compute the hessian of P^i:

$$\partial^2 P(q,\pi) = \begin{pmatrix} -\sigma^2 - \frac{\gamma_{\rm e}^i \pi}{2\sqrt{2} q^{\frac{3}{2}}} e_t & \frac{\gamma_{\rm e}^i}{\sqrt{2q}} e_t \\ \frac{\gamma_{\rm e}^i}{\sqrt{2q}} e_t & -\alpha \end{pmatrix} \tag{17}$$

At a critical point $(\bar{q}_t^i, \bar{\pi}_t^i)$, we have that the sum of the eigenvalues of $\partial^2 P(q,\pi)$ is negative. We also compute $\det(\partial^2 P(\bar{q}_t^i, \bar{\pi}_t^i)) = \alpha\sigma^2$. The critical point is thus a strict local maximum.

- On $\{0\} \times \mathbb{R}_+$, we straightforwardly obtain the necessary condition for optimality

$$(\bar{q}_t^i, \bar{\pi}_t^i) = (0,0). \tag{18}$$

- On $\mathbb{R}^* \times \{0\}$, one must have $\bar{q}_t^i < 0$: If this is not the case, it is always possible to find π' such that $P_t^i(\bar{q}_t^i, \pi') > P_t^i(\bar{q}_t^i, 0)$ as then $f(\bar{q}_t^i) > 0$ and $e_t > 0$ (by assumption).

Step 2) We now look for an admissible solution. From the discussion in the previous step and our definition of admissibility, see (6), we are only interested in a critical point satisfying (16) and such that $\bar{\pi}_t^i > 0$. So, to check that such a critical point exists, we must determine the equilibrium asset return that the allocations would imply. This equilibrium return is obtained thanks to the market-clearing condition $\bar{q}_t^1 + \bar{q}_t^2 = 1$. We compute:

$$\mu_t = \frac{\gamma_{\rm s}\sigma^2}{2} - \frac{|\gamma_{\rm e}^1|^2 + |\gamma_{\rm e}^2|^2}{2\gamma_{\rm s}\alpha_t} e_t^2.$$

This leads to the realized allocations

$$\begin{aligned} \bar{q}_t^1 &= \frac{\mu_t}{\sigma^2\gamma_{\rm s}} + \left(\frac{\gamma_{\rm e}^1}{\gamma_{\rm s}}\right)^2 \frac{e_t^2}{\sigma^2\alpha} \\ &= \frac{1}{2}\left(1 + \frac{|\gamma_{\rm e}^1|^2 - |\gamma_{\rm e}^2|^2}{2\sigma^2\gamma_{\rm s}^2\alpha_t} e_t^2\right), \end{aligned} \tag{19}$$

and the symetric formula holds for $\bar{q}_t^2$. From (9), we observe that $(\bar{q}_t^1, \bar{q}_t^2) \in (0,1)^2$ so that $(\bar{\pi}_t^1, \bar{\pi}_t^2) \in (0,1)^2$. We do have that $(\bar{q}_t^i, \pi_t^i)$ is maximum for the program of investor i and that it is unique due to the market clearing condition which sets the asset equilibrium return.

It remains to check the last condition of admissibility in (6). One then computes

$$\begin{aligned} f(\bar{q}^1)\bar{\pi}^1 + f(\bar{q}^2)\bar{\pi}^2 &= 2\frac{\gamma_{\mathrm{e}}^1}{\gamma_{\mathrm{s}}\alpha_t} e_t \bar{q}_t^1 + 2\frac{\gamma_{\mathrm{e}}^2}{\gamma_{\mathrm{s}}\alpha_t} e_t \bar{q}_t^2 \\ &= \frac{2e_t}{\gamma_{\mathrm{s}}\alpha_t}\left((\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)\bar{q}_t^1 + \gamma_{\mathrm{e}}^2\right) \\ &= \frac{e_t}{\alpha_t\gamma_{\mathrm{s}}}\left((\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)\left(1 + \frac{|\gamma_{\mathrm{e}}^1|^2 - |\gamma_{\mathrm{e}}^2|^2}{2\sigma^2\alpha_t\gamma_{\mathrm{s}}^2}e_t^2\right) + 2\gamma_{\mathrm{e}}^2\right) \\ &= \frac{\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2}{\gamma_{\mathrm{s}}\alpha_t}e_t + \frac{(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}{2\sigma^2\alpha_t\gamma_{\mathrm{s}}^3}e_t^3 \\ &\leq 1, \end{aligned}$$

from condition (9). This concludes the proof of the proposition. □

Investors' optimal quantity of assets, which are admissible solutions, deviates from that of the Merton portfolio (given in this setting by $\frac{\mu_t}{\sigma^2\gamma_s}$) via the term $\frac{\gamma_{\mathrm{e}}^i}{\gamma_{\mathrm{s}}} f'(\bar{q}_t^i)\mathbf{1}_{\{\bar{q}_t^i > 0\}}\bar{\pi}_t^i e_t$ in Equation (14). Compared to the Merton portfolio, since f' is positive, investors practicing shareholder engagement overweight the company's assets if it has high gross emissions, e_t, and if their engagement effort, $\bar{\pi}_t^i$, is high. That is, investors overweight a company's assets if their ability to reduce emissions through shareholder engagement is high. The higher their pro-environmental preferences, γ_e^i, the greater this overweight. Conversely, investors underweight a company when its greening potential through shareholder engagement is low.

In absolute terms (see Equation (11)), in equilibrium, the investor with the strongest pro-environmental preference has the strongest demand for the firm's assets and overweights her allocation relative to the weight without pro-environmental preferences ($\bar{q}_t^1 = \bar{q}_t^2 = 0.5$). As a result, the other investor underweights her allocation to provide liquidity. This extra demand increases as alpha decreases, the gap between investor pro-environmental preferences increases, and the firm's gross emissions grow. This optimal weighting contrasts sharply with that of impact investors practicing environmental investment screening (Pástor et al., 2021; De Angelis et al., 2023): they underweight the companies they want to push to transform to exert pressure on them through their

cost of capital. Similarly, and a fortiori, this optimal weight differs from that of deontological green investors who overweight green assets to align their portfolios with their ethical preferences (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022).

The optimal investor engagement effort is proportional to its impact on firm greening, $f(\bar{q}_t^i)e_t$ (see Equation (12)). Thus, the two optimal investor controls, $(\bar{q}_t^i, \bar{\pi}_t^i)$, are closely linked and positively correlated: holding more assets enables the investor to exert greater pressure on the company for the same engagement effort, just as increasing her engagement effort has a greater impact the more assets she holds. In addition, as with the optimal allocation, the optimal engagement effort scales with the pro-environmental preferences, γ_e^i, and decreases with the cost of engagement, α_t.

As a result of this optimal asset allocation, expected returns in equilibrium deviate from the CAPM via the term $\frac{|\gamma_e^1|^2+|\gamma_e^2|^2}{2\gamma_s\alpha_t}e_t^2$ in Equation (10). In practice, when a company has high gross emissions, its cost of capital falls, all the more so when the cost of engagement, α_t, is low. This effect increases with investors' pro-environmental preferences via $|\gamma_e^1|^2+|\gamma_e^2|^2$. This result naturally reflects the fact that high-emitting companies are potential targets for asset purchases—thereby increasing prices and decreasing expected returns—ahead of shareholder engagement. Indeed, investors accept a lower expected return when they expect to significantly reduce the firm's emissions, whereas they require a higher expected return when the potential for greening is limited. This finding is also in complete opposition to what has been highlighted in the literature on investors' green ethical preferences and environmental impact screening (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022; De Angelis et al., 2023): by greening their portfolios, they increase the cost of capital of brown companies relative to that of green companies, as they require a higher expected return to include them in their portfolios (ethical investors) or to force them to go green (impact investors). This clearly shows how environmental impact investing has a nuanced and non-uniform effect on asset returns when considering the heterogeneity of investor practices.

3.1.2 Shareholder engagement impact on the company's emissions

In this Section, we assume that the conditions in (9) hold true so that we can apply the results of Proposition 1. As stated in (7), the quantity to minimize for the firm's management is

$$\mathbb{E}\left[\int_0^\infty e^{-\delta t}\left(\mu_t((e.)) - \kappa\left(1 - f(\bar{q}_t^1(e.))\bar{\pi}_t^1(e.) - f(\bar{q}_t^2(e.))\bar{\pi}_t^2(e.)\right)e_t\right)dt\right],$$

and it is readily seen that this optimization can be solved pointwisely. This leads us to introduce

$$\begin{aligned} H_t(e_t) &:= \mu_t((e.)) - \kappa\left(1 - f(\bar{q}_t^1(e.))\bar{\pi}_t^1(e.) - f(\bar{q}_t^2(e.))\bar{\pi}_t^2(e.)\right)e_t \\ &= \frac{\gamma_{\mathrm{s}}\sigma^2}{2} - \kappa e_t + \frac{1}{\alpha_t\gamma_{\mathrm{s}}}\left(\kappa(\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2) - \frac{|\gamma_{\mathrm{e}}^1|^2 + |\gamma_{\mathrm{e}}^2|^2}{2}\right)e_t^2 + \frac{\kappa(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}{2\gamma_{\mathrm{s}}^2\sigma^2\alpha_t}e_t^4, \end{aligned} \tag{20}$$

where we used the formula from Proposition 1. This is the quantity that we will minimize with respect to e_t, and we will look for a positive solution, as required by Proposition 1. For later use, let us introduce

$$\bar{\gamma}_{\mathrm{e}} := \gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2 - \frac{2\gamma_{\mathrm{e}}^1\gamma_{\mathrm{e}}^2}{\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2}. \tag{21}$$

Notice that the coefficient of the second-degree monomial in the expression of H, whose sign will be of some importance in the discussion below, can be rewritten using $\bar{\gamma}_{\mathrm{e}}$. Then H rewrites as

$$H_t(e_t) = \frac{\gamma_{\mathrm{s}}\sigma^2}{2} - \kappa e_t + \frac{\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2}{2\alpha_t\gamma_{\mathrm{s}}}\left(2\kappa - \bar{\gamma}_{\mathrm{e}}\right)e_t^2 + \frac{\kappa(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}{2\gamma_{\mathrm{s}}^2\sigma^2\alpha_t}e_t^4. \tag{22}$$

It is also important to observe that when $\gamma_{\mathrm{e}}^1 = \gamma_{\mathrm{e}}^2$, namely the two investors have the same environmental preferences and are thus identical, the problem simplifies greatly as we just have to minimize a second-degree polynomial. We shall first discuss this case quickly and then move on to the general case.

Proposition 2. *Assume that $\gamma_{\mathrm{e}}^1 = \gamma_{\mathrm{e}}^2$ so that $\bar{\gamma}_{\mathrm{e}} = \gamma_{\mathrm{e}}^1$ and assume that $\frac{2\bar{\gamma}_{\mathrm{e}}}{\gamma_{\mathrm{s}}\widehat{\alpha}}\widehat{\Psi} \leq 1$ so that conditions (9) are fullfilled.*

In the case $2\kappa - \bar{\gamma}_{\mathrm{e}} > 0$*, the optimal gross emissions rate schedule* $e^\star$ *satisfies*

$$e_t^\star = \frac{\alpha_t \kappa \gamma_{\mathrm{s}}}{2\bar{\gamma}_{\mathrm{e}}(2\kappa - \bar{\gamma}_{\mathrm{e}})} \wedge \widehat{\Psi}, \quad \textit{for all } t \geq 0. \tag{23}$$

In the case $2\kappa - \bar{\gamma}_{\mathrm{e}} \leq 0$*,* $e_t^\star = \widehat{\Psi}$*, for all* $t \geq 0$*.*
Moreover, assuming $\alpha_t = \widehat{\alpha}$ *for all* $t \geq 0$*,* $\frac{2\bar{\gamma}_{\mathrm{e}}}{\gamma_{\mathrm{s}}\widehat{\alpha}}\widehat{\Psi} = 1$ *(the limit case of condition* (9) *in this setting), and* $\kappa > \gamma_{\mathrm{e}}$*, we have* $e_t^\star < \widehat{\Psi}$ *and*

$$\psi_t^\star = e^\star \frac{\kappa - \bar{\gamma}_{\mathrm{e}}}{2\kappa - \bar{\gamma}_{\mathrm{e}}} = \alpha_t \gamma_{\mathrm{s}} \frac{\kappa(\kappa - \bar{\gamma}_{\mathrm{e}})}{2\bar{\gamma}_{\mathrm{e}}(2\kappa - \bar{\gamma}_{\mathrm{e}})^2}, \quad \textit{for all } t \geq 0. \tag{24}$$

Proof. In this simple case, H reads

$$H_t(e_t) = \frac{\gamma_{\mathrm{s}}\sigma^2}{2} - \kappa e_t + \frac{\bar{\gamma}_{\mathrm{e}}}{\alpha_t \gamma_{\mathrm{s}}}\left(2\kappa - \bar{\gamma}_{\mathrm{e}}\right) e_t^2. \tag{25}$$

If $2\kappa - \bar{\gamma}_{\mathrm{e}} > 0$, then H_t is strictly convex and attains a minimum at

$$\mathfrak{r}_t = \frac{\alpha_t \kappa \gamma_{\mathrm{s}}}{2\bar{\gamma}_{\mathrm{e}}(2\kappa - \bar{\gamma}_{\mathrm{e}})},$$

which leads to $e_t^\star = \mathfrak{r}_t \wedge \widehat{\Psi}$, indeed on $[0, \mathfrak{r}_t]$, H_t is decreasing.
If $2\kappa - \bar{\gamma}_{\mathrm{e}} \leq 0$, then the minimization problem is concave and $e_t^\star = \widehat{\Psi}$. □

In this particular case featuring one generic investor with pro-environmental preferences who practices shareholder engagement ($\gamma_{\mathrm{e}}^1 = \gamma_{\mathrm{e}}^2 = \bar{\gamma}_{\mathrm{e}}$), the expression of gross and net emissions are given in Equations (23) and (24), respectively. In the case where $2\kappa > \bar{\gamma}_{\mathrm{e}}$, which yields a realistic calibration in Section 3.2, gross and net emissions fall as investors' pro-environmental preferences, $\bar{\gamma}_{\mathrm{e}}$, increase; conversely, they are maximal when both investors have no pro-environmental preferences. In addition, gross and net emissions fall as the engagement cost, α_t, falls; when engaging is free, emissions are zero (a limit case in our theoretical study), while they are maximal when emissions have a very high cost.

We now study the case of two genuinely different investors.

Theorem 3. *Assume that* $\gamma_{\mathrm{e}}^1 \neq \gamma_{\mathrm{e}}^2$ *and that conditions* (9) *hold true. Then, for all* $t \geq 0$*,* H_t *given in* (22) *has a unique minimum on* $\mathbb{R}_+$*, which is realized at* $\mathfrak{r}_t$*, the unique positive root of* $H_t'(x) = 0$*.*

The optimal gross emissions rate is thus set to

$$e_t^\star = \mathfrak{r}_t \wedge \widehat{\Psi}, \quad \textit{for all } t \geq 0. \tag{26}$$

Proof. The first order condition for the optimization problem is just $H'(e_t) = 0$. So one must solve, differentiating (22),

$$0 = -\kappa + \frac{\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2}{\alpha_t \gamma_{\mathrm{s}}} \left(2\kappa - \bar{\gamma}_{\mathrm{e}}\right) e_t + \frac{2\kappa(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}{\gamma_{\mathrm{s}}^2 \sigma^2 \alpha_t} e_t^3. \tag{27}$$

and we observe that $H'(0) = -\kappa < 0$.

Differentiating once more, we get

$$H''(e_t) = \frac{\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2}{\alpha_t \gamma_{\mathrm{s}}} \left(2\kappa - \bar{\gamma}_{\mathrm{e}}\right) + \frac{6\kappa(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}{\gamma_{\mathrm{s}}^2 \sigma^2 \alpha_t} e_t^2. \tag{28}$$

1. If $2\kappa \geq \bar{\gamma}_{\mathrm{e}}$ then $H'' > 0$. Equation (27) has only one real solution $\mathfrak{r}_t$, the minimization problem for H is convex. Moreover, $\mathfrak{r}_t > 0$ as $H'(0) < 0$ and it can be computed in closed form using Cardan's formula. Then, one sets

$$e_t^\star = \mathfrak{r}_t \wedge \widehat{\Psi}. \tag{29}$$

   In particular, if $2\kappa = \bar{\gamma}_{\mathrm{e}}$. Then one has $\mathfrak{r}_t = \left(\frac{\gamma_{\mathrm{s}}^2 \sigma^2 \alpha_t}{(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}\right)^{\frac{1}{3}}$.

2. $2\kappa < \bar{\gamma}_{\mathrm{e}}$ then H'' has two roots $\pm r$ with

$$r = \frac{\sigma}{|\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2|} \sqrt{\theta} \quad \text{with} \quad \theta := \gamma_{\mathrm{s}}(\gamma_{\mathrm{e}}^1 + \gamma_{\mathrm{e}}^2) \frac{\bar{\gamma}_{\mathrm{e}} - 2\kappa}{6\kappa}. \tag{30}$$

   One computes with the above notation

$$\begin{aligned} H'(r) &= -\kappa - \theta \frac{6\kappa}{\alpha_t \gamma_{\mathrm{s}}^2} r + \frac{2\kappa(\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2)^2}{\gamma_{\mathrm{s}}^2 \sigma^2 \alpha_t} r^3 \\ &= -\kappa - \theta \frac{6\kappa}{\alpha_t \gamma_{\mathrm{s}}^2} r + \frac{2\kappa\sigma}{\gamma_{\mathrm{s}}^2 \alpha_t |\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2|} \theta^{\frac{3}{2}} \\ &= -\kappa - \frac{4\kappa\sigma}{\gamma_{\mathrm{s}}^2 \alpha_t |\gamma_{\mathrm{e}}^1 - \gamma_{\mathrm{e}}^2|} \theta^{\frac{3}{2}} < 0. \end{aligned}$$

Since $H'(0) < 0$ and $\lim H'(x) = +\infty$ as $x \to +\infty$, there exists $\mathfrak{r}_t > r > 0$ such that $H'(\mathfrak{r}_t) = 0$ and which realizes indeed the unique minimum of H on $\mathbb{R}_+$. Then, one sets $e_t^\star = \mathfrak{r}_t \wedge \widehat{\Psi}$ to conclude the proof. □

The optimal gross emission rate, $(e_t^\star)_{t\geq 0}$, can be derived in closed form since it is built using a root of a polynomial of degree 3 (or one in the case of two identical investors). However, even if it exists, the closed-form expression of $e_t^\star$ is very convoluted. Below, we analyze numerically a realistic case featuring two heterogeneous investors: one green investor ($\gamma_{\mathrm{e}}^1 \neq 0$) and one non-green investor ($\gamma_{\mathrm{e}}^2 = 0$).

3.2 Calibration and scenarios

3.2.1 Calibration

We illustrate the effect of shareholder engagement on the emissions of a generic company in a configuration featuring a group of green investors and a group of non-green investors. To do so, we set financial risk aversion to $\gamma_{\mathrm{s}} = 1$, the non-pecuniary pro-environmental preference parameter of investor 1 to a significantly lower level, $\gamma_{\mathrm{e}}^1 = 0.2$, and that of investor 2 with no green preferences to $\gamma_{\mathrm{e}}^2 = 0$. We consider an asset with a volatility of 35%, and we define a company whose emissions are capped at $\widehat{\Psi} = 1$ thousand tons of CO2 per year.

The abatement cost, κ, and the engagement cost, $\alpha_t = \widehat{\alpha}$ for all $t \geq 0$, are hard to specify. Therefore, we calibrate κ and $\widehat{\alpha}$ such that: (i) the abatement cost and the engagement cost are positive, $\kappa > 0$, $\widehat{\alpha} > 0$, respetively; (ii) the gross and net emissions are positive, $e_t^\star > 0$ and $\psi_t^\star > 0$, respectively; (iii) the expected return is equal to the risk premium in the U.S. in 2024:[3] $\mu_t = 0.04$; and (iv) conditions (9) are satisfied. A couple $(\kappa, \widehat{\alpha})$ satisfying these conditions is $\kappa = 0.77$ and $\widehat{\alpha} = 0.5$. Below is a table summarizing the variable values.

In the Stackelberg game equilibrium, the results are as follows. Naturally, investor 2 exerts no engagement effort, $\bar{\pi}_t^2 = 0$, while investor 1 exerts an effort of $\bar{\pi}_t^1 = 0.3356$. To lend weight to her engagement effort, investor 1 over-allocates her capital to the firm relative to the optimum without green preferences ($q_t^1 = 0.5$) and purchases $q_t^1 = 0.6712$ shares, while investor 2 buys the remaining

[3] `https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histimpl.html`

Variable	Value	Description
γ_s	1	Financial risk aversion
γ_e^1	0.2	Investor 1 pro-environmental preference
γ_e^2	0	Investor 2 pro-environmental preference
κ	0.77	Abatement cost
$\widehat{\alpha}$	0.5	Engagement cost
μ	0.04	Expected return (2024 U.S. risk premium)
σ	0.35	Asset volatility
$\widehat{\Psi}$	1	Emission cap (10^3 tons of CO_2/year)

Table 1 Summary of the calibrated variables.

shares, $q_t^2 = 0.3288$, so that the market clears. Based on investors' expected optimal allocation, the firm chooses gross emissions of $e_t^\star = 0.7241$ tons of CO_2 per year (tCO_2/year) and, following investor 2's engagement, its net emissions are $\psi_t^\star = 0.5610$ tCO_2/year.

It is worth noting that the very existence of an investor practicing shareholder engagement encourages the company to lower its gross emissions and then forces it to reduce its net emissions. When the pro-environmental preferences of investor 1 are zero, $\bar{\gamma}_{\mathrm{e}}^1 = 0$, gross and net emissions are maximal, $e_t = \psi_t = 1$.

3.2.2 Economic scenarios

We study two economic scenarios as part of the calibration presented above. The first corresponds to a world in which the share of investors practicing shareholder engagement increases or, equivalently, the requirement of the existing group increases. In practice, we vary γ_{e}^1 from 0 (situation where there are no impact investors) to 0.4. Figure 1 shows that the green investor increases her asset allocation from 50% to over 70%, and her engagement effort from 0 to over 40%. As a result, the company's cost of capital is halved from 6% to 3%, and its gross and net emissions are significantly reduced from 1 tCO2/year to 0.45 tCO2/year and 0.35 tCO2/year, respectively. This scenario highlights the importance of shareholder engagement in reducing corporate emissions and encourages sustainable investors to integrate this approach into their practices. It also shows policymakers the crucial need to create conditions that facilitate shareholder engagement.

The second scenario reflects the reduction in engagement costs that public policies or private agreements could induce. To illustrate this, we vary $\widehat{\alpha}$ from 2 to 0.5 in Figure 2. As the cost

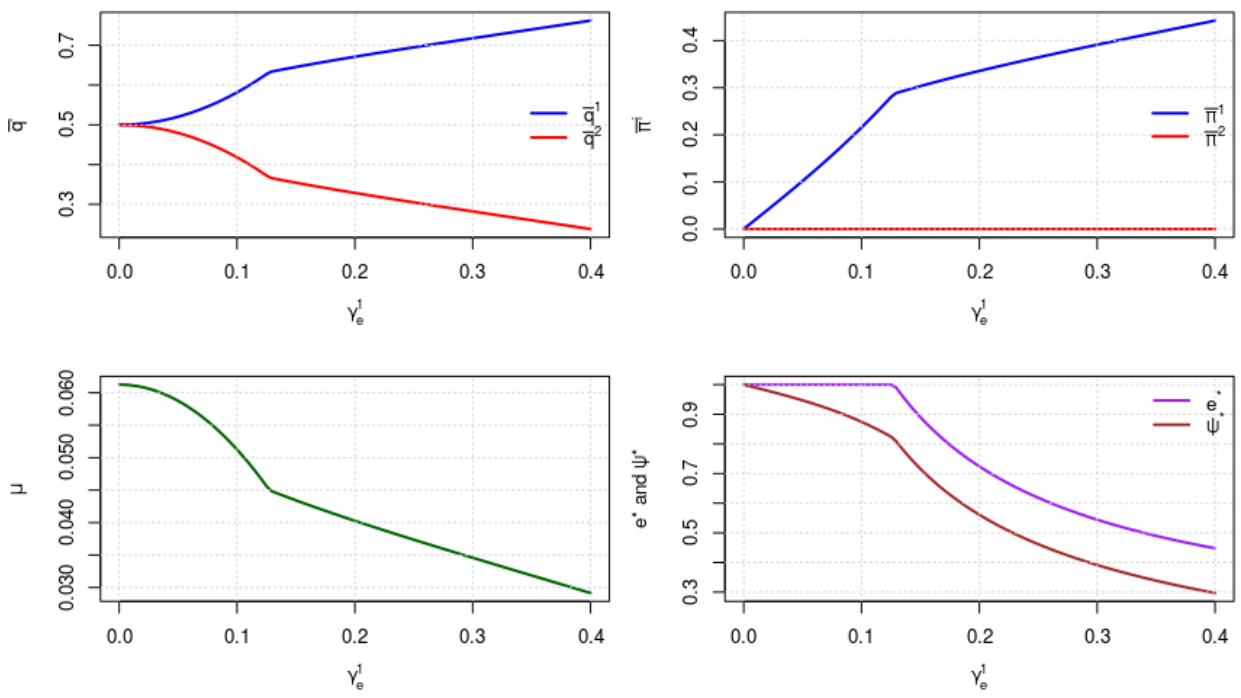


Figure 1. Evolution of the variables of interest as a function of γ_e.

of engagement falls, the green investor's engagement effort rises from 0.10 to over 0.30, and her asset allocation increases from 0.58 to 0.67, providing greater impact when engaging with the firm. Under these conditions, the company's cost of capital decreases from 5% to 4%. More importantly, its gross and net emissions fall from 1 tCO2/year and 0.94 tCO2/year to 0.72 tCO2/year and 0.57 tCO2/year, respectively. These results emphasize the extent to which lowering the cost of shareholder engagement, and more generally the barriers to engagement, is a factor enabling investors to push companies to significantly improve their environmental practices.

Several public policies and private initiatives could be implemented to facilitate shareholder engagement and reduce its cost: establishing a legal framework for proxy access allowing shareholder owning a modest stake to submit and have proposals included on the annual general meeting (AGM) ballot without board approval, inspired by the EU's Shareholder Rights Directive II and U.S. proxy-access rules; enhancing ESG disclosure requirements by mandating standardized, comparable non-financial reporting (climate, biodiversity, human rights) via frameworks like the EU CSRD or IFRS/SASB; establishing or strengthening a national Stewardship Code (as in the UK or Japan) setting best practices for institutional investors: pre-AGM dialogue, active voting, public vote

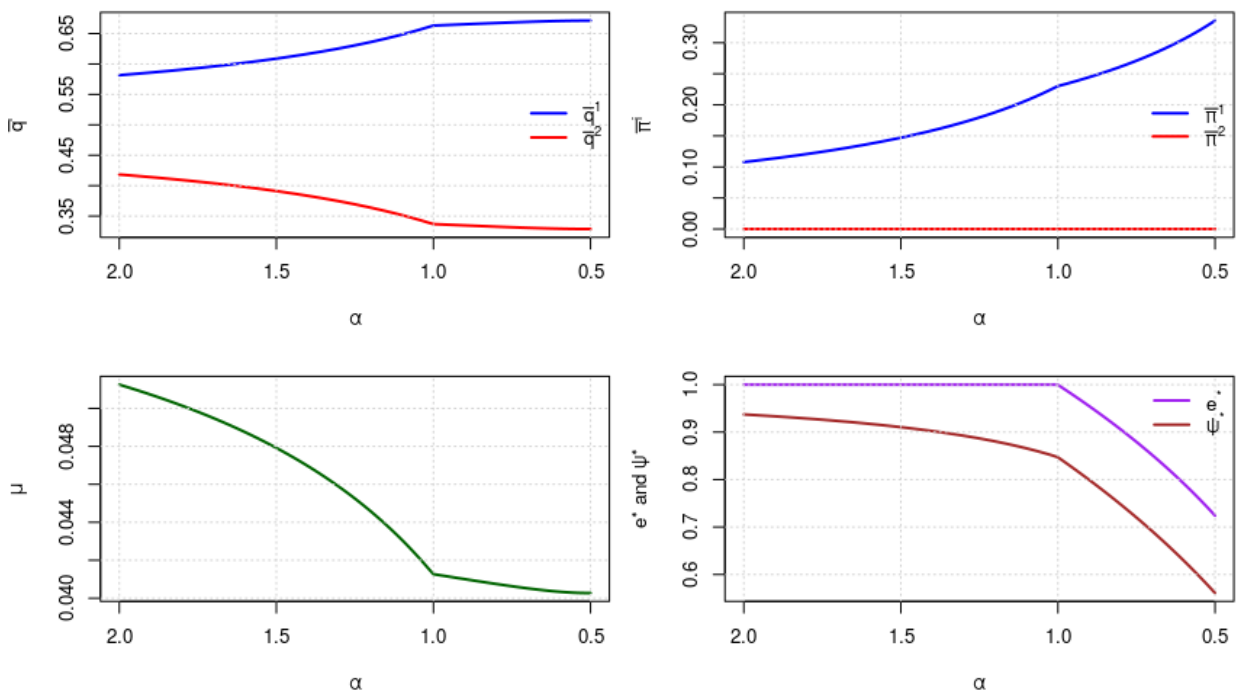


Figure 2. Evolution of the variables of interest as a function of α.

policies, and a watchdog to monitor compliance and deter greenwashing; offering a tax credit or deductibility for actual expenses incurred by funds during proxy votes, which would encourage retail investors (through retirement plans, life-insurance wrappers) to pool resources and amplify their voice; building a secure, regulator-run portal where investors and management can engage outside formal AGMs (Q&A forums, dedicated webinars, commitment tracking), ensuring transparency and archiving; launching campaigns (via securities regulators or public development banks) to teach retail shareholders about their rights, voting mechanics, and how to read ESG reports, with free e-learning modules on governance and proxy voting; simplifying and securing electronic voting (e-signatures, digital mandates) to reduce reliance on bank intermediaries, remove friction, and boost turnout in virtual or hybrid meetings; requiring asset managers to detail how they handle conflicts when voting (fees, issuer ties) and publish an annual engagement report showing which proposals they supported or opposed; introducing a non-binding shareholder vote on the company's climate strategy—akin to "say on pay"—to maintain continuous pressure on emissions targets and net-zero commitments; and promoting or subsidizing the creation of retail shareholder clubs or cooperatives with consolidated speaking rights, increasing their leverage without disrupting the

institutional framework.

4 Conclusion

In this paper, we propose a continuous-time equilibrium model in which heterogeneous investors exert costly shareholder engagement to decrease a representative firm's greenhouse-gas emissions. In contrast to standard screening strategies that raise brown firms' financing costs, our framework shows that shareholder engagement induces investors to overweight high-emitting firms, accepting lower expected returns, to secure deeper emissions cuts. More precisely, we establish three main results: (i) stronger pro-environmental preferences or lower engagement costs amplify capital allocations and engagement toward brown firms; (ii) such engagement mechanically reduces brown firms' equilibrium expected returns; and (iii) firms respond with a two-fold mitigation: they are incentivized to cut their gross emissions and subsequently are compelled to cut them further due to shareholder engagement.

References

2 Degrees Investing, 2025. Collective impact through collaborative engagement. Technical Report .

Bolton, P., Kacperczyk, M. T., 2021. Do investors care about carbon risk? Journal of Financial Economics 142, 517–549.

Bouchard, B., Fukasawa, M., Herdegen, M., Muhle-Karbe, J., 2018. Equilibrium returns with transaction costs. Finance and Stochastics 22, 569–601.

Broccardo, E., Hart, O., Zingales, L., 2022. Exit versus Voice. Journal of Political Economy 130, 3101–3145.

Cartellier, F., Tankov, P., Zerbib, O. D., 2024. Can investors curb greenwashing? Working Paper.

De Angelis, T., Tankov, P., Zerbib, O. D., 2023. Climate impact investing. Management Science 69, 7669–7692.

Dimson, E., Karakaş, O., Li, X., 2015. Active ownership. Review of Financial Studies 28, 3225–3268.

Favilukis, J., Garlappi, L., Uppal, R., 2024. Evaluating the impact of portfolio mandates. Working Paper.

Green, D., Roth, B., 2024. The allocation of socially responsible capital. Journal of Finance, Forthcoming Paper.

Hartzmark, S., Shue, K., 2023. Counterproductive sustainable investing: The impact elasticity of brown and green firms. Working Paper.

Heeb, F., Kölbel, J., 2024. The impact of climate engagement: A field experiment. Working Paper.

Hsu, P., Li, K., Tsou, C., 2023. The Pollution Premium. Journal of Finance 78, 1343–1392.

Lambrecht, B. M., Myers, S. C., 2017. The Dynamics of Investment, Payout and Debt. Review of Financial Studies 30, 3759–3800.

Landier, A., Lovo, S., 2023. ESG investing: How to optimize impact? Working Paper.

McConnell, J. T., Sandberg, C. M., 1975. The weighted average cost of capital: Some questions on its definition, interpretation, and use: Comment. Journal of Finance 30, 883–886.

Nantell, T. J., Carlson, C. R., 1975. The cost of capital as a weighted average. Journal of Finance 30, 1343–1355.

Oehmke, M., Opp, M. M., 2025. A Theory of Socially Responsible Investment. The Review of Economic Studies 92, 1193–1225.

Pedersen, L. H., Fitzgibbons, S., Pomorski, L., 2021. Responsible investing: The ESG-efficient frontier. Journal of Financial Economics 142, 572–597.

Pástor, L., Stambaugh, R. F., Taylor, L. A., 2021. Sustainable investing in equilibrium. Journal of Financial Economics 142, 550–571.

Riedl, A., Smeets, P., 2017. Why do investors hold socially responsible mutual funds? Journal of Finance 72, 2505–2550.

van der Kroft, B., Palacios, J., Rigobon, R., Zheng, S., 2025. Timing sustainable shareholder proposals in real asset investments. Working Paper.

Weber, M., 1921. In: Politik als Beruf, Mohr Siebeck, Tübingen.

Zerbib, O. D., 2022. A Sustainable Capital Asset Pricing Model (S-CAPM): Evidence from Environmental Integration and Sin Stock Exclusion. Review of Finance 26, 1345–1388.