Carbon Pricing versus Green Finance

Lasse Heje Pedersen*

This version: April 25, 2025

Abstract

Economics recommends combating climate change with carbon pricing, but green finance — such as ESG investing and sustainable finance regulations — is becoming widespread. In a unified model of carbon pricing and green finance, I show: (1) If carbon prices reflect their social cost, green finance should not be used. (2) With too low carbon prices, green finance can, under certain conditions, implement the social optimum by raising each firm's cost of capital by its "externality rate," a metric proportional to carbon emissions. (3) However, the calibrated costs of capital exceed feasible levels for carbon-intensive ("brown") firms and stranded assets.

Keywords: ESG investing, sustainable finance regulation, carbon allowances, carbon offsets, carbon credits
JEL Codes: G1, H23, E44, O44, Q5

*AQR Capital Management, Copenhagen Business School, and CEPR; e-mail: lhp001@gmail.com; www.lhpedersen.com. I am grateful for helpful comments from Darrell Duffie, Niels Gormsen, Theis I. Jensen, Marc Eskildsen, Nicolae Garleanu, William Hogan, Markus Ibert, Claus Thustrup Kreiner, David Lando, Maria Olesen, Lukasz Pomorski, Andrei Shleifer, Aleh Tsyvinski; seminar participants at Harvard University, Olin Business School at Washington University in Saint Louis, New York University, Yale University, University of Amsterdam, and Copenhagen Business School; and conference participants at the European Energy Policy Conference by MIT and Cambridge University, Cornell-OFR Conference on Global Climate Finance and Risks, 24th Macro Finance Workshop, Outstanding Paper Award Ceremony at the SFI Research Days; as well as to Clara Egehoved Tørsløv for excellent research assistance. AQR Capital Management is a global investment management firm, which may or may not apply similar investment techniques or methods of analysis as described herein. The views expressed here are those of the author and not necessarily those of AQR. I gratefully acknowledge support from the Center for Big Data in Finance (Grant no. DNRF167).

Climate change is the ultimate challenge for economics according to the Nobel prize lecture of Nordhaus (2019). Nordhaus recommends a Pigouvian carbon tax equal to the "social cost of carbon," which he estimates to be 43 dollars per metric ton of carbon emissions ($/tCO$_2$) in one of his models.

However, carbon pricing is growing slowly in the real world, while various forms of green finance are promoted by UN, EU, many central banks, creditors, pension funds, and other investors. Green finance includes investment policies based on "environmental, social, and governance" (ESG) concerns (particularly the environmental part) as well as different kinds of "sustainable finance regulation." While carbon pricing seeks to affect the *cost of emissions*, green finance seeks instead to affect the *cost of capital* of emitting firms.

Using a different mechanism to fight climate change raises the questions: Can green finance accomplish the same as a carbon tax? If so, how does 43$/tCO$_2$ translate into cost of capital (which is measured in different units)?

This paper answers these questions in a unified model of carbon pricing and green finance. Green finance can accomplish the same as a carbon tax when the cost of capital can be controlled, firms can commit to their future emissions, and stranded assets are not a severe problem. In that case, a carbon tax of 43$/tCO$_2$ translates into an increase in the cost of capital of 43 times each firm's emission-to-value ratio. For example, for the average brown electricity firm (one of the most polluting industries), the carbon tax translates to a 3.2 percentage points (pp) increase in the cost of capital.

The model includes green and brown electricity firms, goods producers, and consumers. The model is more general than integrated assessment models (IAMs) in terms of its multi-pronged carbon-mitigation mechanisms, but it takes the social cost of carbon as given. The goal is a tractable translation between the cost of pollution and the cost of capital.

The model delivers the following striking result. Even when the carbon price is too low, the social optimum can nevertheless be achieved by setting the cost of capital of any firm i to its *sustainable discount rate*, r^x_{it}, at time t. The sustainable discount rate (or *externality-adjusted cost of capital*) is:

$$r^x_{it} = r + x_{it} = r + (S_{t+1} - \tau_{it+1})\frac{X_{it+1}}{v_{it}} \tag{1}$$

The sustainable discount rate is the "normal" cost of capital, r, plus the *externality rate* (or *carbon burden rate*), x_{it}. The externality rate, $x_{it} = (S_{t+1} - \tau_{it+1})\frac{X_{it+1}}{v_{it}}$, stems from the firm's total externality, X_{it+1}, measured as the direct carbon emission in tCO$_2$. To convert this externality into dollars, it is multiplied by the social cost of carbon, S_{t+1}, net of any

paid carbon taxes, τ_{it+1}. Finally, to convert the dollar externality into a cost of capital, it is divided by firm value, v_{it}. The sustainable discount rate can be written more informally as

$$r_{it}^{x} = r + \textit{Social cost of carbon, net of carbon tax (\$/tCO}_2\textit{)} \times \frac{\textit{Emission (tCO}_2\textit{)}}{\textit{Value of firm (\$)}} \tag{2}$$

I show that this discount rate "works" because it implies that the firm experiences a capital cost corresponding to an implicit carbon tax rate of $S_{t+1} - \tau_{it+1}$. Adding the explicit carbon tax rate, τ_{it+1}, the firm faces a total carbon tax rate equal to the social cost of carbon, S_{t+1}, as desired.

The sustainable discount rate (1) has several natural properties. It increases linearly in the amount of pollution, X_{it+1}, and the cost of this pollution, S_{it+1}. It is scaled by the size of the firm, so merging identical firms does not change the rate.

Further, the sustainable discount rate is lower if the firm is already paying for its damages via an explicit tax, τ_{it+1}. In particular, if the firm is already paying the efficient tax, $\tau_{it+1} = S_{it+1}$, then the sustainable discount rate equals the normal rate, r. So, if some countries charge polluting firms a high carbon tax and other countries do not, then the sustainable discount rate should only be elevated for polluting firms in the countries without the carbon tax.

Note also that the sustainable discount rate only depends on the firm's direct, so-called scope-1, emissions. For example, if a polluting electricity firm has a high sustainable discount rate, electricity prices go up, so users of electricity need not be penalized as well.

The sustainable discount rate (1) motivates firms to care about pollution, but how can this cost of capital arise in equilibrium? I address this issue in two ways.

First, I show how sustainable discount rates can be implemented via carbon-based capital-income taxes. Naturally, when a firm's owners are taxed based on emissions, they increase their required returns accordingly.

Second, I consider a voluntary system in which each investor feels responsible for the firm's carbon emissions, in the proportion to their ownership. In other words, when the firm pollutes, investors experience a disutility proportional to the social cost of these carbon emissions, which is essentially a "negative dividend." While this assumption may be unrealistic, it is interesting because it turns out to be needed to implement (1). Investors convert the carbon-based negative dividend to dollars based on the social cost, $(S_{t+1} - \tau_{it+1})X_{it+1}$, and compute the effect on their return by dividing by firm value. So, if an investor expects a financial return of r_{it}, the return net of the emission disutility is $r_{it} - x_{it}$. In equilibrium, this "net return," $r_{it} - x_{it}$, must equal the normal rate, r, for all firms, so the equilibrium

required return is $r_{it}^x = r + x_{it}$.

This required return is exactly the same as (1). In other words, (1) has two related properties: (a) firms behave as if they pay a carbon tax; (b) investors require this rate if they internalize any untaxed externalities.

What are the conditions for this green finance solution to work and what are the challenges? One condition is that firms and investors must use (1) as a forward-looking incentive scheme to mimic a carbon tax. Note in this connection that the ex ante cost of capital depends on the firm's future emissions, so the firm must be able to commit to low emissions to secure a low cost of capital. Further, the firm value in the denominator is endogenous and depends on future emissions. Creating such a fine-tuned incentive scheme seems challenging given that a firm's cost of capital depends on the collective actions of a broad group of investors and creditors with a range of preferences and limited control by regulators.

Another challenge with implementing (1) is the sheer magnitude, both theoretically and empirically. Theoretically, the problem is that, to achieve the social optimum with green finance, firms' cost of capital must reflect a market in which the average investor fully internalizes the firms' externalities. For example, *all* investors must view the emissions as negative dividends priced based on the *full* social cost of carbon. In the real world, market prices are set based on a *subset* of investors that *partially* internalizes the externalities and another subset that only pursues financial objectives.

To evaluate the magnitude of (1) empirically, one simply multiplies the social cost of carbon, say 43\$/tCO_2, with each firm's emission-to-value ratio.[1] Using data on emissions and firm values, I find that a social cost of 43\$/tCO_2 corresponds to elevating the sustainable discount rate by 0.01pp for the median firm among all, and 0.19pp for the market-value-weighted average firm. These modest average adjustments reflect that the majority of firms have few emissions. However, a minority of firms account for most of the economy's emissions, and their sustainable discount rates must be elevated by much more. This result is seen from Figure 1, which plots the sustainable discount rates averaged across firms in each industry, over a realistic range of the social cost of carbon, S_t. Indeed, Nordhaus (2019) provides estimates of S_t from his baseline number of 43\$/tCO_2 to upwards of 300\$/tCO_2. The most polluting industry in the figure is brown electricity. For this industry, the discount rate must be elevated by 3.2% to mimic a carbon tax of 43\$/tCO_2. Furter, this number increases to 27% with a social cost of carbon of 300\$/tCO_2. These calibrated magnitudes are much larger than the current green-finance effects in real markets (Eskildsen et al., 2024) and surveys among

[1] The calibrations in this paragraph are explained in more detail and extended in Section 6.

investors reveal little willingness to accept significantly lower returns for green investments than brown (Edmans et al., 2024).

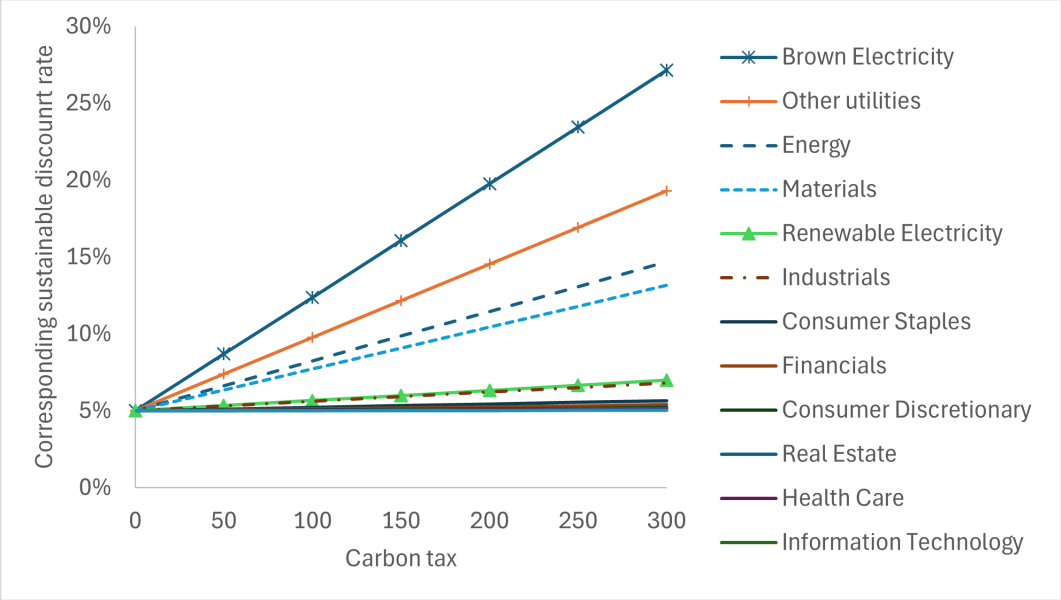


Figure 1: **Translating a Carbon Tax into Sustainable Discount Rates.** Each firm's sustainable discount rate, r_{it}^{x}, is calibrated based on equation (1) as a function of the social cost of carbon, S_{t+1}. The figure reports the value-weighted average within each industry. The data is from 2021, the competitive rate is set to $r = 5\%$, and the paid carbon tax is set to $\tau_{it+1} = 0$.

The challenge with the magnitude of (1) is most extreme if there are firms that are so polluting that their sustainable discount rates become infinite. This problem can arise when the corresponding firm value in the denominator becomes zero. In other words, a firm with a pollution worse than the value of its output should be shut down in the social optimum, so such a firm is considered worthless by sustainable investors (a so-called "stranded asset"). However, if the economy has a single investor willing to buy the stranded asset for a price of zero in order to make an infinite financial return, then the green-finance approach breaks down. Green finance cannot realistically be implemented when the discount rate in (1) becomes too large, much less infinite.

However, stranded assets can in some cases be handled by letting green finance depend on indirect emissions. In particular, I provide alternative green-finance implementations that can address stranded assets and have relevance for several countries' new focus on industrial policy. These alternative implementations involve the following elements:

- Sustainable discount rates that depend on both direct emissions and indirect emissions

due to electricity use, i.e., so-called scope-1 and scope-2 emissions.

- Providing cash subsidies to green electricity firms.
- Providing subsidized discount rates to green electricity firms.

In sum, the paper unifies two distinct literatures reviewed in Section 1: the economics literature on carbon pricing, and the finance literature on ESG investing and sustainable finance regulation. What new results arise from combining these literatures? I show when green finance can implement the social optimum, provide the implementation via sustainable discount rates, calibrate the magnitude, point out limitations with stranded assets, and present several extensions, including alternative implementations of the social optimum, long-term sustainable discount rates, and policy implications discussed in the conclusion.

1 Understanding Carbon Pricing and Green Finance

This section provides a brief overview of the literature and the real-world institutional arrangement for fighting climate change.

1.1 Carbon Pricing

IAMs that estimate the social cost of carbon are developed in Nordhaus (1994) and other papers by Nordhaus, Peck and Teisberg (1992), Golosov et al. (2014), Daniel et al. (2019), and other papers surveyed in Nobel Committee (2018). Also, there is public finance research on second-best outcomes with abstract forms of "imperfect instruments," which I complement by studying the empirically relevant alternative, green finance.[2] I discuss next the three main types of real-world carbon pricing mechanisms.

Carbon Taxes

While the use of carbon taxes has been growing, such taxes still only affects a minority of global emissions as seen in Figure 2. There is evidence that carbon taxes work, including Andersson (2019) who empirically finds "a significant causal effect of carbon taxes on emissions" in Sweden. The implementation of carbon taxes varies across regions in terms of the

[2]While a carbon tax is the standard Pigouvian solution to an externality, ESG investing and sustainable finance regulation can be viewed as examples of imperfect instruments. Imperfect instruments are discussed in Diamond (1973), Dávila and Walther (2022), and references therein, but, to my knowledge, the focus on instruments that work via the cost of capital is new.

price of emitting carbon, disclosures of future prices, the sectors covered, possible exceptions (often related to competitiveness), the point of regulation (i.e., who pays the tax), and the scope of regulation (what emissions, direct or indirect, do they pay for). For example, the point of regulation can be at the source (that is, the firms actually emitting carbon, e.g. by burning fossil fuels), upstream (producers, distributors, and importers of the fossil fuels), and downstream (e.g., firms using electricity produced with emissions).

Carbon Allowances

Another way to price carbon is via an emissions trading system (ETS). This form of carbon pricing has overtaken carbon taxes in terms of its global coverage, as seen in Figure 2. Introducing an ETS means that the government places a cap on total emissions in certain sectors of the economy, and then auctions or distributes tradable emission allowances to entities covered by the cap. This system is known as a "cap-and-trade," or "baseline-and-credit" if firms are allowed a baseline emission and can sell credits if they reduce emissions below the baseline. Each carbon allowance typically represents the permission to emit one metric ton of CO_2. Allowances can be traded until they are finally "surrendered" for the appropriate year to the regulator.

Carbon Offsets

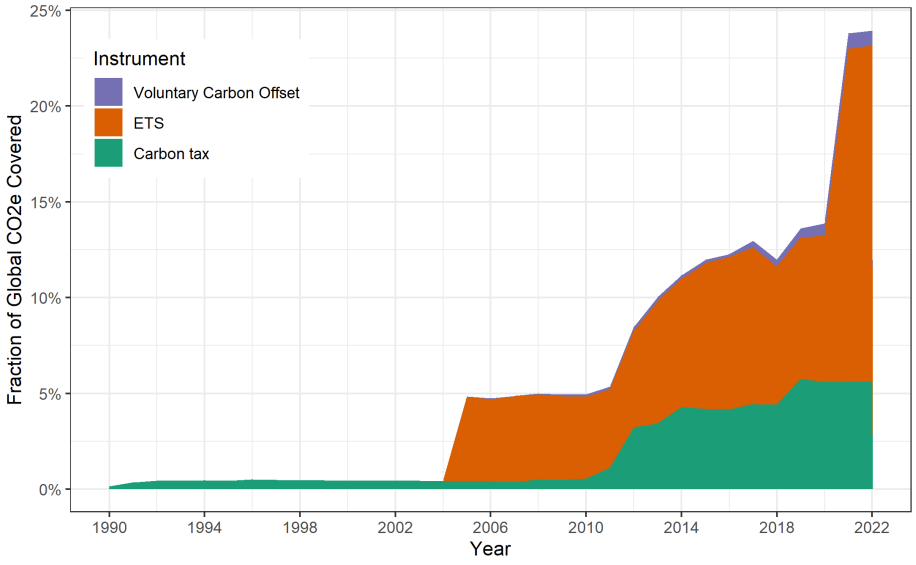
A carbon offset is an instrument that, in principle, represents one metric ton of carbon dioxide equivalent avoided or removed from the atmosphere. Some carbon offsets are part of a regulatory framework and there is also a market for voluntary carbon offsets, where firms buy offsets without a regulatory reason. Offsets should be real, additional, verifiable, enforceable, and permanent, but offsets often have little credibility.[3]

1.2 Green Finance

While carbon pricing is still not widespread, different forms of green finance are becoming prevalent, at least in terms of stated intentions, as seen in Figure 2.

[3] Calel et al. (2021) estimate for wind farms in India "that at least 52% of approved carbon offsets were allocated to projects that would very likely have been built anyway. In addition to wasting scarce resources, we estimate that the sale of these offsets to regulated polluters has substantially increased global carbon dioxide emissions." Kaplan et al. (2023) propose an auditable framework for offset accounting.

(a) Coverage of Carbon Taxes, Allowance Markets, and Voluntary Offsets



(b) Coverage of ESG Investing and Sustainable Finance Regulation

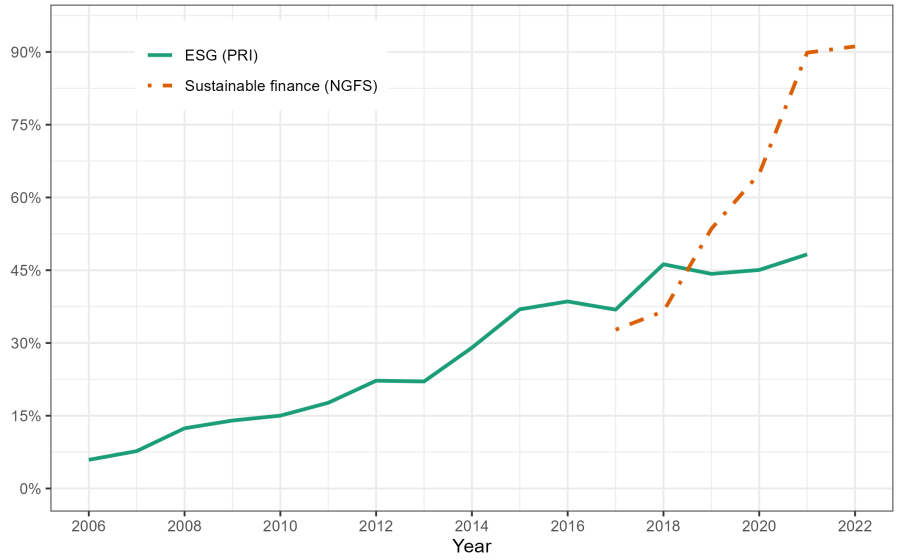


Figure 2: **Global Coverage of Carbon Pricing and Green Finance.** Panel (a) shows the fraction of global emissions covered by carbon taxes, carbon allowance markets (ETS), and voluntary carbon offsets. Panel (b) shows the collective assets under management (AUM) represented by all signatories of the Principles of Responsible Investment as a fraction of the combined market value of global equity markets and global fixed income markets (solid line); and the total GDP of all countries with a regulator that is a member of The Network of Central Banks and Supervisors for Greening the Financial System (NGFS), as a fraction of global GDP (dash-dotted line).

Sustainable Finance Regulation

Many investors and regulators are interested in complementing the regulation of carbon pricing with other tools. For example, the Network for Greening the Financial System is an extensive group of central banks and financial supervisors who aim to "help strengthening the global response required to meet the goals of the Paris agreement and to enhance the role of the financial system to manage risks and to mobilize capital for green and low-carbon investments in the broader context of environmentally sustainable development."[4] Also, the European Banking Authority is rolling out an effort "to integrate ESG risks considerations in the banking framework and support the EU's efforts to achieve the transition to a more sustainable economy."[5]

Regulators run climate stress tests (Acharya et al., 2023), impose wide-ranging sustainability disclosure requirements,[6] consider capital requirements for banks that differ for green and brown loans (Dafermos and Nikolaidi, 2021; Oehmke and Opp, 2023; Thakor and Song, 2023), and impose other types of green finance regulation (Huang and Kopytov, 2023). These regulations can affect the relative cost of capital of green and brown firms.

ESG Investing

At the same time, many investors are including environmental, social, and governance criteria in their investment decisions. For example, 4,902 institutional investors "with an estimated total of AUM of US$121.3 trillion" have signed the Principles for Responsible Investments, an organization furthering ESG investing supported by the United Nations (PRI Annual Report, 2021-2022). Likewise, a number of sustainable investment products for retail investors are gathering interest (and regulation). These investor preferences can raise the required returns of brown firms relative to the required returns of green firms (Pástor et al., 2021; Pedersen et al., 2021)), affect the choice of capital structure (Feldhütter and Pedersen, 2024), improve welfare (Hong et al., 2023), and interact with political decisions (Allen et al., 2023; Heeb et al., 2023). Empirically, ESG investing has been found to affect the cost of capital of stocks (Hong and Kacperczyk, 2009; Bolton and Kacperczyk, 2021; Eskildsen et al., 2024) and bonds (Flammer, 2021; Baker et al., 2022; Feldhütter and Pedersen, 2024), as well as corporate managers' perceived costs of capital (Gormsen et al., 2024). The present value of

[4] See https://www.ngfs.net/en.

[5] See, e.g., https://www.eba.europa.eu/eba-publishes-its-roadmap-sustainable-finance.

[6] E.g., EU has wide-raging new initiatives including a Sustainable Finance Disclosures Regulation, a Corporate Sustainability Reporting Directive, its double materiality assessment, and the underlying taxonomy for sustainable activities. Also, some banks already measure their "financed emissions" (PCAF, 2022).

the carbon burden is empirically large (Pastor et al., 2024). The beneficial effects of ESG have been questions by Berk and Van Binsbergen (2025), Arnold (2023), and Hartzmark and Shue (2023).

My model derives new results by unifying this literature with the carbon pricing literature, and it can also be applied to other elements of ESG investing. For example, investors concerned with the social element (the "S" in ESG) may disinvest from alcohol or tobacco firms to raise their cost of capital. As with carbon, it is interesting to contrast this ESG approach with a Pigouvian tax. Indeed, such firms with social issues can also be regulated more directly via alcohol and tobacco taxes.

Carbon Footprint or Financed Emissions

Some investors and creditors compute their "carbon footprint" (Frankel et al., 2015; Funk, 2020) or "financed emissions" (PCAF, 2022). An investor's carbon footprint is the amount of emissions that the investor is implicitly responsible for via the security holdings. If the investor owns ω_i dollars of each firm i with value v_i and total emissions X_i, then the carbon footprint is:

$$\textit{Carbon footprint} = \sum_i \frac{\omega_i}{v_i} X_i \tag{3}$$

In other words, if the investor owns 10% of a company, the investor inherits 10% of the firm's emissions, and the investor then sums these inherited emissions across investments i. If investors place a large value on having a low carbon footprint, then the resulting investment behavior can lead to sustainable discount rates (1), as shown later.

Green Capital Income Taxes

Since green finance seeks to affect the cost of capital of green versus brown firms, another potential avenue for this effect is via capital income taxes. In other words, suppose that capital income earned by investing in brown firms was subject to a higher tax than that of green capital income. If all firms must provide investors the same after-tax return despite this tax difference, brown firms would face a higher cost of capital. Or, similarly, investors could be taxed on wealth held in brown assets — the browner the asset, the higher the tax.

Such taxes could potentially enforce a large spread between green and brown costs of capital. In contrast, as discussed before, ESG investing relies on some investors' non-financial pursuit of a ethical motive and, when other investors go for the highest return, the return-

seeking investors limit how much the cost of capital can change in equilibrium. Further, with ESG, the green investors pay the "tax" in form of lower green returns while the return-seeking investors collect the "tax" in the form of higher brown returns, and the "tax" is partly lost to society due to imperfect diversification (rather than everyone pays the same tax and everyone is diversified).

That said, capital taxes come with other practical problems, which may be why this tool has not been proposed, to my knowledge. Also, such taxes would violate the Chamley-Judd no-capital-income-tax rule from public finance, although this rule has been largely overturned (Straub and Werning, 2020). In any case, while all taxes create distortions, here the tax would be designed to offset another distortion, namely the pollution externality.

Engagement, Impact Investing, and Other Forms of Green Finance

Rather than just focusing on the cost of capital, impact investors seek to directly engage with firms' management to reduce pollution and other externalities (Landier and Lovo, 2024). More generally, Hart and Zingales (2017) argue that firms should maximize shareholder welfare. Oehmke and Opp (2024) present a theory of a large impact investor, which trades off financial returns against reductions in pollution relative to the counterfactual scenario with no investment in a given firm. My notion of green finance with commitment is related to engagement. I show that, when a firm can commit to its active owners, sustainable discount rates can reverse engineer a carbon tax (in a way that does not depend on counterfactuals).

Focusing instead on an international agreement, Adrian et al. (2022) estimate the cost of paying firms to replace coal with renewables. More broadly, governments can subsidize green firms or green innovation in various ways, and the model also has results on efficient ways to subsidize green industries via transfers or lower costs of capital.

2 Stylized Model

This section derives the paper's main results in the simplest possible setting. The section shows how sustainable discount rates should depend on emission, and the challenges with relying on such discount rates.

A firm exists for two time periods, time 0 and time 1. At time 0, the firm chooses its capital, K, as described below. At time 1, the firm produces its output, Y. If the firm ignores its pollution, then it has a standard "AK-production-technology," meaning that its output is AK with productivity $A > 0$. To produce this output, the firm burns an amount,

$X = fK$, of fossil fuel at no cost, where $f > 0$ captures how dirty the firm is.

The firm can also choose to use a smaller amount of fuel, $X \in [0, fK]$, although this lowers the output. Specifically, output depends on fuel via

$$Y = AK - \frac{1}{fK}(fK - X)^2 \tag{4}$$

Hence, output is increasing in fuel over the relevant range $X \in [0, fK]$. For example, the firm can choose to be completely clean, $X = 0$, with an output of $AK - fK$. (Also, Appendix B contains an example with a Cobb-Douglas production function.)

Given a carbon tax rate of τ, the firm's profit is

$$\Pi = AK - \frac{1}{fK}(fK - X)^2 - \tau X \tag{5}$$

The profit-maximizing emission is $X = (1 - \frac{\tau}{2})fK$, which naturally decreases in the tax.

The capital K is measured in terms of the number of power plants operated by the firm. At time 0, the firm starts with one power plant, $K = 1$, which was acquired in the past. To capture investment and disinvestment in a simple way, the firm can only choose among the following options:

(i) buy another power plant, i.e., $K = 2$ with investment cost $I = 1$;

(ii) keep the existing plant, i.e., $K = 1$ with no investment cost, $I = 0$;

(iii) or shut down the firm, i.e., $K = 0$ with no resale value, $I = 0$.

To make this choice, the firm first computes the present value of each power plant, $v_1 = \frac{\Pi(K=1)}{1+r_i}$ using the discount rate r_i. Here, the subscript "1" refers to the value of one unit of capital, and the production function has the property that everything scales with capital.

If each plant generates a loss, $v_1 \leq 0$, the firm is shut down ($K = 0$). If each plant is profitable, but with a smaller present value than the investment cost, $v_1 \in (0, 1)$, then the firm keeps $K = 1$. Lastly, if the present value is above the investment cost, $v_1 \geq 1$, the firm invests ($K = 2$), yielding a total value of $2v_1 - 1$.

The firm is owned by a representative household with utility $\frac{Y - SX}{1+r} - I$, where S is the social cost of carbon and r is the opportunity cost of investing. Hence, when the carbon tax equals the social cost, $\tau = S$, and the discount rate is $r_i = r$, then the market value of the firm equals its social value. In this case, all the firm's choices are socially optimal, both its ex ante investment choice and its ex post pollution choice.

Suppose instead that $\tau = 0$, and the firm has a sustainable discount rate of $r_i > r$, which can depend on firm characteristics, (A, f). If the firm is not shut down, each plant will emit $X = fK = f$, given the zero carbon tax. Hence, the emission is above the socially optimal level. Each plant therefore has a profit of $AK = A$ and a value of $v_1 = \frac{A}{1+r_i}$.

To ensure efficient investment decisions, sustainable investors must set the discount rate such that the firm's market value equals its social value computed in light of its excessive emissions, $\frac{A-Sf}{1+r}$. In other words, investors must find r_i such that $\frac{A}{1+r_i} = \frac{A-Sf}{1+r}$. The solution is

$$r_i = r + \frac{Sf}{(A - Sf)/(1+r)} \tag{6}$$

where r_i is set to infinity if the denominator is zero or negative. Since the resulting market value equals the social value, the firm makes the socially optimal investment choice regarding (i), (ii), or (iii) when it computes the present value of its profit with this discount rate. However, two problems remain. First, the firm's pollution choice is suboptimal, as mentioned before. Second, if the discount rate is infinity, the corresponding market value is zero. But, if any investor exists who wants to buy the firm for zero and make a profit next period of $A > 0$, then this method breaks down.

To get green finance to implement the social optimum, the economy needs a more powerful form of sustainable discount rates. Suppose that the firm can commit to any level of future pollution, X, and that the discount rate can depend on this commitment, $r_i(X)$. To reach the optimum, the value of each power plant with no carbon tax and the discount rate of $r_i(X)$ must equal the social value for each X. That is, $\frac{A-\frac{1}{f}(f-X)^2}{1+r_i(X)} = v_1(X) = \frac{A-\frac{1}{f}(f-X)^2-SX}{1+r}$, so solving for $r_i(X)$ gives

$$\begin{aligned} r_i(X) &= \frac{(1+r)(A-\frac{1}{f}(f-X)^2)}{A-\frac{1}{f}(f-X)^2-SX} - 1 \\ &= \frac{r(A-\frac{1}{f}(f-X)^2)+SX}{A-\frac{1}{f}(f-X)^2-SX} \\ &= \frac{r(A-\frac{1}{f}(f-X)^2-SX)+(1+r)SX}{A-\frac{1}{f}(f-X)^2-SX} \\ &= r + \frac{SX}{(A-\frac{1}{f}(f-X)^2-SX)/(1+r)} = r + \frac{SX}{v_1(X)} \end{aligned} \tag{7}$$

where the denominator must be strictly positive, otherwise $r_i(X)$ is set to infinity. This

stylized model illustrates the main findings in this paper.

First, the sustainable discount rates can implement the social optimum. The mechanism behind this feat is that the firm gets cheap financing in exchange for low emission.

Second, the sustainable discount rate must be the normal rate, r, plus the externality rate, as seen in (7). This finding matches (1) from the introduction given that $\tau = 0$ here.

Third, the sustainable discount rate must depend on future emissions. If it only depends on the firm's exogenous characteristics (A, f) as in (6), then the firm's ex post pollution is distorted. So such a simple form of sustainable finance cannot achieve the social optimum.

Fourth, the firm faces a commitment problem: Once it has received cheap financing by promising a low emission and invested in a new power plant, then the firm has an incentive to choose a high emission of $X = fK$ to maximize profit in the future.

Fifth, the sustainable discount rate must be infinite for a firm with a zero or negative social value. In this case, the firm should be shut down, since it does more harm than good. However, if the economy has any investor who wants to profit from buying the power plant for zero in order to make a future profit of A, thus earning an infinite return, then the plant is not shut down. In this case, sustainable discount rates cannot handle such stranded assets.

The rest of the paper shows that these results hold in a general model. It also shows what type of investor preferences or taxes give rise to sustainable discount rates. Further, it derives additional results on how to handle stranded assets, supply-chain effects, and green subsidies.

3 General Model of Carbon Pricing and Green Finance

This section presents an economy consisting of firms producing consumption goods using capital, labor, and electricity; the electricity is supplied by green and brown electricity producers; and the goods are consumed by households who work, invest, and experience disutility from pollution. Firms take their carbon taxes and discount rates as given, and Section 5 discusses how discount rates are set to mimic the effect of a carbon tax.

Firms Producing Consumption Goods

The economy has N price-taking firms producing consumption goods, indexed $i = 1, ..., N$. At time $t = 1, 2, ...$, each firm i has a capital, K_{it}, which is determined by its prior investment, $I_{it-1} \in \mathbb{R}$, via $K_{it} = (1-\delta)K_{it-1}+I_{it-1} \geq 0$, where $\delta \in [0, 1]$ is the depreciation rate. The firm chooses its labor workforce, L_{it}, green electricity usage, G_{it}, and brown electricity usage, B_{it}.

The output also depends on how much the firm allows itself to directly emit, $X_{it} = X_{it}^{scope1}$, where I use the short-hand notation without "scope 1" in the superscript. These factors generate the firm's output via the production function, $Y_{it} = Y_{it}(z_{it})$, which increases in all its inputs, $z_{it} = (K_{it}, L_{it}, G_{it}, B_{it}, X_{it})$. Output only depends on the total amount of electricity used, i.e., green and brown electricity enter the production function via their sum, $G_{it} + B_{it}$.

The firm's time t profit, Π_{it}, is paid as a dividend and consists of its output less labor costs at wage w_t, electricity costs with green electricity price p_{gt} and brown electricity price p_{bt}, and carbon taxes:

$$\Pi_{it}(z_{it}) = Y_{it} - w_t L_{it} - p_{gt} G_{it} - p_{bt} B_{it} - \tau_{it} X_{it} - \tau_{it}^2 f_b B_{it} - \tau_{it}^2 f_g G_{it} \tag{8}$$

Here, τ_{it} is a scope-1 carbon tax rate proportional to its own direct emissions, X_{it}. Similarly, τ_{it}^2 is a potential scope-2 carbon tax rate based on indirect pollution. The scope-2 carbon tax on brown electricity, $f_b B_{it}$, assumes a so-called fossil intensity of f_b, which captures the pollution per unit of electricity, often measured in tons of carbon emissions per kilo Watt hour (tCO_2/kWh).

The fossil intensity multiplied by the brown electricity usage, $f_b B_{it}$, therefore equals the implicit emission, which is multiplied by τ_{it}^2 to get the tax. Likewise, the last term, $\tau_{it}^2 f_g G_{it}$, is a scope-2 carbon tax related to the use of green electricity, based on a fossil intensity, f_g, of green electricity.

The fossil intensities used for carbon tax purposes, f_b and f_g, need not equal the true fossil intensities of electricity firms, defined in the next section. The notation allows such a difference so that the model can capture the situation in which the source of electricity cannot be determined, so that green and brown electricity must face the same scope-2 tax, that is, $f_b = f_g$.

The firm chooses its actions, z_{t+1}, at time $t+1$ to maximize its endogenous value, V_t:

$$V_{it}(K_{it}) = \max_{z_{it+1}} \left(\frac{\Pi_{it+1}(z_{it+1}) + V_{it+1}(K_{it+1})}{1 + r_{it}} - I_{it} \right) \tag{9}$$

taking the discount rate, r_{it}, as given.

Finally, firm 1 is a particularly simple asset: It has no emissions and it provides a way to save consumption goods via $Y_{1t} = (1 + r + \delta) K_{1t}$. This asset implies that the competitive cost of capital is constant, r, which simplifies the notation.

Production of Electricity: Green and Brown Electricity Firms

The economy has two technologies for electricity, green and brown, where brown is more polluting. The representative green electricity producer uses its capital, $K_{gt} = (1-\delta)K_{gt-1} + I_{gt-1}$, and other inputs (say land), H_{gt}, to produce electricity, $G_t = G_t(z_{gt})$, where $z_{gt} = (K_{gt}, H_{gt})$ contains the endogenous variables. The production generates an emission of $X_{gt} = F_g G_t$ with fossil intensity F_g. The corresponding profit is

$$\Pi_{gt}(z_{gt}) = (p_{gt} + \nu_{gt} - \tau_{gt}F_g)G_t(z_{gt}) - \chi_{gt}(z_{gt}) \tag{10}$$

where ν_{gt} is a proportional government subsidy for green electricity, τ_{gt} is the direct carbon tax rate, and χ_{gt} captures production costs (measured in units of consumption goods).

Similarly, the brown electricity firm uses its capital, $K_{bt} = (1-\delta)K_{bt-1} + I_{bt-1}$, and other inputs (say fuel), H_{bt}, to produce electricity, $B_t = B_t(z_{bt})$ and emission, $X_{bt} = F_b B_t$, where $z_{bt} = (K_{bt}, H_{bt})$ and $F_b > F_g$ is the fossil intensity. The brown electricity producer has profit

$$\Pi_{bt}(z_{bt}) = (p_{bt} - \tau_{bt}F_b)B_t(z_{bt}) - \chi_{bt}(z_{bt}) \tag{11}$$

where τ_{bt} is the direct carbon tax and χ_{bt} is the production cost. The firm can essentially "shut down" by choosing $H_{bt} = 0$, which implies a zero production, $B_{it} = 0$, at zero cost, $\chi_{bt} = 0$.

Finally, the brown and green electricity firms have discount rates, r_{bt} and r_{gt}. Based on these discount rates, they maximize their respective values, V_{bt} and V_{gt}, as in (9).

Households

Each time period, the representative household works, invests, and consumes. The household earns $w_t L_t$ by supplying all its labor, L_t. The household chooses how many shares θ_{it} to own of each firm $i \in \mathcal{I}$, where $\mathcal{I} = \{1, ..., N, b, g\}$, and the number of shares outstanding is normalized to 1 for all firms. These choices determine consumption:

$$C_t = w_t L_t + \sum_{i\in\mathcal{I}} \theta_{it-1}(\Pi_{it} + V_{it}) - \sum_{i\in\mathcal{I}} \theta_{it}(V_{it} + I_{it}) + \mathcal{G}_t \tag{12}$$

where $\mathcal{G}_t$ comes from the government, which runs a balanced budget.[7] In other words, consumption comes from labor income, government transfer, the profit and value of the

[7] I.e., $\mathcal{G}$ is carbon taxes less green subsidies, $\mathcal{G}_t = \sum_i (\tau_{it}X_{it} + \tau_{it}^2 f_b B_{it} + \tau_{it}^2 f_g G_{it}) + \tau_{bt}F_b B_t + (\tau_{gt}F_g - \nu_{gt})G_t$.

incoming portfolio, θ_{it-1}, less what is saved via the new portfolio, θ_{it}.

The household's utility is

$$U = \sum_{t=1}^{\infty} \beta^t \left[u_t \left(C_t \right) - d_t \left(X_t \right) \right] \tag{13}$$

where $\beta < 1$ is the time-preference rate, u_t is the utility of consumption, and d_t is the damage of aggregate carbon emissions, $X_t = \sum_i X_{it} + X_{gt} + X_{bt}$.

Equilibrium and Planner Problem

The *social planner's problem* is to maximize utility (13) by choosing the actions of firms, z_{it}, and households, θ_{it}, subject to the following resource constraints:

$$\begin{aligned} \sum_i G_{it} &= G_t \\ \sum_i B_{it} &= B_t \\ \sum_i L_{it} &= L_t \\ \theta_{it} &= 1 \end{aligned} \tag{14}$$

A solution z_{it}^*, θ_{it}^* exists, which is ensured by all production functions and utility functions being differentiable and being sufficiently well behaved.[8] The goal of the rest of the paper is *not* to find this solution, which would require specifying parametric production and utility functions, whereas the model has been written abstractly. Rather, the rest of the paper takes the solution as given and then presents alternative market-based implementations of this socially optimal outcome.

A *competitive equilibrium* is a wage, w_t, prices of green and brown electricity, p_{gt}, p_{bt}, discount rates, r_{it}, and firm values, V_{it}, such that firms choose their actions, z_{it}, to maximize (9) with maximum given by V_{it}, households choose θ_{it} to maximize their utility taking prices and total emissions as given, and electricity, labor, and equity markets clear (14). As part of their competitive behavior, households assume that they cannot affect emissions, just like they cannot affect prices. In a competitive equilibrium, discount rates are pinned down at the competitive rate for all firms, $r_{it} = r$,

[8] The solution is an interior optimum, except that the brown electricity firm cannot disinvest, $I_{bt-1} \geq 0$, so its capital could get stranded in the sense that the carbon tax could get so high that the firm optimally chooses to shut down.

A *green-finance equilibrium* is the same as a competitive equilibrium, except that households choose $\theta_{it} = 1$ without regard to their own utility, which means that discount rates are unrestricted. Instead household somehow coordinate to "set" discount rates for the greater good.

To understand the similarities and differences between these definitions, note that, in both cases, the economic analysis centers around implementing the social optimum, but the tools are different. A competitive equilibrium takes discount rates as given, seeking to find carbon taxes that implement the social optimum. In contrast, a green-finance equilibrium takes carbon taxes as given, seeking to find discount rates that implement the social optimum.

Section 5.1 derives discount rates that sustain the socially optimal equilibrium by making firms take their emission into account. To understand the green-finance equilibrium at a deeper level, Section 5.2 derives conditions for these reverse-engineered rates to maximize a form of "sustainable utility," where households care about the emissions of firms in their portfolio choice.

Before presenting these green-finance results, the next section discusses optimal carbon pricing in a competitive equilibrium, as a well-known benchmark.

4 Carbon Pricing without Green Finance

Before turning to the main focus of the paper — sustainable discount rates — it is useful to establish benchmarks based on carbon pricing. In particular, this section presents three propositions that are mirrored in the next section. The first one contains the well-known result that carbon pricing can implement the social optimum. All proofs are in the appendix.

Proposition 1 ($*$-Equilbrium and Social Cost of Carbon) *The social optimum z_{it}^* is a competitive equilibrium when the wage is $w_t^* = \partial Y_{it}/\partial L_{it}(z_{it}^*)$, all firms face scope-1 carbon taxes of $\tau_{it} = S_t$, where $S_t = d_t'(X_t^*)/u_t'(C_t^*)$, electricity prices are $p_t^* = p_{gt} = p_{bt} = \partial Y_{it}/\partial G_{it}(z_{it}^*)$, the discount rate is r for all firms, firm values are $V_{it}^* = \sum_{s=1}^{\infty} \frac{\Pi_{it+s}^* - I_{it+s}^*}{(1+r)^s} - I_{it}^*$, and households invest without regard to firms' emissions.*

This equilibrium, henceforth denoted as the star-equilibrium, is the focal point of the rest of the paper, which shows other ways of implementing the same outcome. The *social cost of carbon* is defined as S_t, which is seen to equal the marginal utility cost of pollution, relative to the marginal utility of consumption.

When carbon emission is taxed at its social cost, the market equilibrium reduces global emissions in a socially optimal way, as is well-known in economics. The new part of the

proposition is the obvious addendum, that with such an optimal carbon tax, there is no need for green finance (i.e., $r_{it} = r$ for all i). In fact, green finance distorts the equilibrium if carbon is already being taxed. As the saying goes "too many cooks spoil the broth," so if the "cook" is the carbon tax, there should not also be green finance.

It is useful to also state the well-known result that the optimum can also be reached via a scope-2 implementation for comparison with a corresponding result for green finance, especially since the latter result has a novel element related to stranded assets, as discussed in the next section.

Proposition 2 (Scope-2 Carbon Pricing) *The social optimum is a competitive equilibrium in which each goods producer faces the scope-1 carbon tax $\tau_{it} = S_t$ as well as a scope-2 carbon tax on indirect emissions from electricity ($\tau^2_{it} = S_t$, $f_b = F_b$, $f_g = F_g$), while electricity producers face no carbon tax, electricity prices are $p_{gt} = p^*_t - (S_t - \tau_t)F_g$ and $p_{bt} = p^*_t - (S_t - \tau_t)F_b < p_{gt}$, and $r_{it} = r$, $w_t = w^*_t$, and $V_{it} = V^*_{it}$.*

If the regulator wishes to pursue scope-2 carbon pricing, but cannot distinguish whether a goods producer buys green or brown electricity, then the following implementation can be used.

Proposition 3 (Green Subsidies) *The social optimum is a competitive equilibrium in which each goods producer faces the scope-1 carbon tax $\tau_{it} = S_t$ as well as a scope-2 carbon tax on all electricity whether green or brown ($\tau^2_{it} = S_t$, $f_b = f_g = F_b$), electricity producers face no carbon tax, but green electricity producers receive a proportional subsidy, $\nu_{it} = S_t(F_b - F_g)$, and prices are $p_{gt} = p_{bt} = p^*_t - (S_t - \tau_t)F_b$, $r_{it} = r$, $w_t = w^*_t$, and $V_{it} = V^*_{it}$.*

Here, goods producers are over-taxed on their electricity usage since they even pay a scope-2 tax on their green electricity usage. Therefore, electricity prices are low in equilibrium, which hurts green electricity firms. The low electricity price means that the green electricity firms are essentially paying a carbon tax even though they are not polluting. Therefore, to make the equilibrium socially optimal, green electricity producers must have their "implicit tax" refunded, and this happens via a green subsidy.

Several countries are starting to subsidize green energy, and the proposition provides an argument for using such subsidies if green and brown electricity firms get the same price for their electricity and the brown electricity firms do not pay a carbon tax (or only pay a low tax). In this case, green electricity firms could receive a subsidy equal to the carbon taxes that brown competitors should be paying for the same amount of electricity. To reach the first-best outcome, Proposition 3 shows that users of electricity should simultaneously be taxed on their indirect emissions.

5 Green Finance and Sustainable Discount Rates

5.1 Can Green Finance Mimic Carbon Pricing?

This section considers the situation in which the scope-1 carbon tax is below the social cost of carbon, $\tau_{it} < S_t$, and, for notational simplicity, the scope-2 carbon tax is zero, $\tau_{it}^2 = 0$. The key question is whether green finance can make up for the too low carbon tax, and incentivize firms to care about pollution.

In other words, despite the low carbon tax, can a sustainable discount rate, r_{it}, be found such that each firm chooses its endogenous variables exactly as in the $*$-equilibrium from in Proposition 1, where the carbon tax equals S_t and the discount rate is r? That is, when a firm chooses its endogenous variables, z_{it+1}, its maximization problem must be the same when using the sustainable discount rate and low carbon tax (left-hand side) and in the $*$-equilibrium (right-hand side) in the following equation:

$$\max_{z_{it+1}} \frac{\Pi_{it+1} + V_{it+1}}{1 + r_{it}} - I_{it} = V_{it} = \max_{z_{it+1}} \frac{\Pi_{it+1} + V_{it+1} - (S_{t+1} - \tau_{it+1})X_{it+1}}{1 + r} - I_{it} \tag{15}$$

The problem on the right uses the competitive discount rate r and includes the "missing" carbon tax, $(S_{t+1} - \tau_{it+1})X_{it+1}$, such that the total carbon tax is $S_{t+1}X_{it+1}$. These problems coincide when, for each action z_{it+1}, it holds that

$$\begin{aligned}
r_{it} &= \frac{(1+r)(\Pi_{it+1} + V_{it+1})}{\Pi_{it+1} + V_{it+1} - (S_{t+1} - \tau_{it+1})X_{it+1}} - 1 \\
&= \frac{r(\Pi_{it+1} + V_{it+1}) + (S_{t+1} - \tau_{it+1})X_{it+1}}{\Pi_{it+1} + V_{it+1} - (S_{t+1} - \tau_{it+1})X_{it+1}} \\
&= \frac{r(\Pi_{it+1} + V_{it+1} - (S_{t+1} - \tau_{it+1})X_{it+1}) + (1+r)(S_{t+1} - \tau_{it+1})X_{it+1}}{\Pi_{it+1} + V_{it+1} - (S_{t+1} - \tau_{it+1})X_{it+1}} \\
&= r + \frac{(S_{t+1} - \tau_{it+1})X_{it+1}}{(\Pi_{it+1} + V_{it+1} - (S_{t+1} - \tau_{it+1})X_{it+1})/(1+r)} \\
&= r + \frac{(S_{t+1} - \tau_{it+1})X_{it+1}}{v_{it}}
\end{aligned} \tag{16}$$

where $v_{it}(z_{it+1})$ is defined by the last equation. As seen from (15), v_{it} is the time-t firm value after investment, $v_{it} = V_{it} + I_{it}$, in equilibrium.

In sum, the firm's optimization problem is literally the same with *(i)* the sustainable discount rate (16) and a too low carbon tax τ_i, and *(ii)* the normal discount rate r and the right carbon tax S. So green finance can, in principle, work! But note that the cost of capital at time t must depend on emissions at time $t+1$, so the firm must be able to

commit to its future emission. In other words, when the firm chooses among any level of pollution and other actions, the firm and the market must understand how the cost of capital (including the firm value in the denominator) reacts, and let the choices depend on this feedback. Therefore, a "simple" sustainable discount rate that only depends on ex ante characteristics cannot work in general (except in special cases, e.g., if pollution is a known function of capital).

The following proposition formally states that sustainable discount rates reverse engineer the socially optimal $*$-equilibrium from Proposition 1. Before stating the proposition, note that the denominator, v_{it}, in (16) must be strictly positive, otherwise the discount rate is set to infinity (which is problematic for reasons discussed below). To ensure that this positivity is not a problem, the proposition is stated under the condition that firm values are strictly positive in the $*$-equilibrium, $V_{it}^* + I_{it}^* > 0$. Said differently, there are no stranded assets, meaning firms that pollute so much that they have no social value.

Proposition 4 (Sustainable Discount Rates) *If firms can commit to their future actions, z_{it+1}, and $V_{it}^* + I_{it}^* > 0$ for all $i \in \mathcal{I}$, then the social optimum can be implemented by a green-finance equilibrium with the following sustainable discount rates*

$$r_{it}^x(z_{it+1}) = r + \frac{(S_{t+1} - \tau_{it+1})X_{it+1}}{v_{it}(z_{it+1})} \tag{17}$$

electricity prices, $p_{gt} = p_{bt} = p_t^*$, and firm values, $v_{it} = V_{it} + I_{it} = V_{it}^* + I_{it}^*$, equal to those in the socially optimal $*$-equilibrium.

To understand the issue with stranded assets, consider for example the brown electricity firm's profit, $\Pi_{bt} = (p_{bt} - \tau_{bt}F_b)B_t(z_{bt}) - \chi_{bt}(z_{bt})$. As regulation increases τ_{bt}, the profit decreases, especially if the electricity price, p_{bt}, does not increase much, for example because electricity can be alternatively supplied by a green electricity firm or goods producers can reduce their electricity demand. If the carbon tax and fossil intensity, F_b, are high enough, the brown electricity firm eventually chooses to shut down, reducing production to zero to avoid a negative profit. In this case, profit, investment, and the value of the brown electricity firm become zero.

What is the challenge with reaching the same outcome with green finance? At a superficial level, dividing by zero in (17) corresponds to an infinite discount rate. At a more economic level, as long as the carbon tax is low, the brown electricity has a positive profit, Π_{bt}. Hence, if any investor exists who wishes to buy the firm for zero in order to make an infinite return, then the firm will not be shut down. While sustainable investors can seek to raise the

discount rate to a degree, thus reducing the stock price, raising the discount rate to infinity can simply not be done if any purely-financial investor exists.

Interestingly, this shortcoming of green finance can be addressed via another form of green finance that targets goods producers' scope-2 emissions, $X_{it+1}^{scope2} = F_b B_{it+1} + F_g G_{it+1}$. Indeed, the next two results demonstrate how sustainable discount rates can mimic Propositions 2 and 3. These results are stated under the notational convenient assumption that all electricity firms face the same carbon tax, $\tau_{bt+1} = \tau_{gt+1} = \tau_{t+1}$. The propositions derive sustainable discount rates, where the denominator is $v_{it}(z_{it+1}) = (\Pi_{it+1}^*(z_{it+1}) + V_{it+1}^*(z_{it+1}))/(1+r)$, defined based on the $*$-equilibrium (a definition that matches the one above and also covers the more complex cases with different electricity prices and profits).

Proposition 5 (Handling Stranded Assets via Scope 2) *If firms can commit to future actions, $V_{it}^* + I_{it}^* > 0$ for $i = 1, .., N$, and goods producers can choose their electricity provider, then the social optimum can be implemented by discount rates for goods producers equal to*

$$r_{it}^{x(1+2)}(z_{it+1}) = r + \frac{(S_{t+1} - \tau_{it+1})X_{it+1} + (S_{t+1} - \tau_{t+1})X_{it+1}^{scope2}(z_{it+1})}{v_{it}(z_{it+1})} \tag{18}$$

discount rates for electricity producers equal to r, electricity prices $p_{gt} = p_t^* - (S_t - \tau_t)F_g$ and $p_{bt} = p_t^* - (S_t - \tau_t)F_b < p_{gt}$, and firm values $v_{it} = V_{it} + I_{it} = V_{it}^* + I_{it}^*$.

This implementation mirrors that of Proposition 2, but uses green finance instead of carbon taxes. The implicit scope-2 carbon taxes imply that goods producers are willing to pay less for brown electricity.

This implementation works even if brown electricity is a stranded asset. To understand why, note first that brown electricity has the normal discount rate r, so the issue of dividing by a zero value does not arise. Instead, brown electricity is penalized via a low electricity price, coming from goods producers' capital cost. Hence, in the case of stranded assets, no one will buy the brown electricity for a price that is worthwhile for the electricity company, so brown electricity firms collapse, as they would in the social optimum.

This effect can even be accomplished without distinguishing green and brown electricity, as seen from the next result, which mirrors Proposition 3.

Proposition 6 (Subsidizing Green Firms' Cost of Capital) *If firms can commit future actions and $V_{it}^* + I_{it}^* > 0$ for $i = 1, .., N$, then the social optimum can be implemented*

by discount rates for goods producers equal to

$$r_{it}^{x(1+2+)}(z_{it+1}) = r + \frac{(S_{t+1} - \tau_{it+1})X_{it+1} + (S_{t+1} - \tau_{t+1})F_b(G_{it+1} + B_{it+1})}{v_{it}(z_{it+1})} \tag{19}$$

depending on scope 1+2 carbon burden rates computed as if all electricity is brown, letting brown electricity producers' discount rate equal $r_{bt} = r$*, subsidizing green electricity producers' discount rate as*

$$r_{gt} = r - \frac{(S_{t+1} - \tau_{t+1})(F_b - F_g)G_{t+1}}{v_{gt}(z_{gt+1})} \tag{20}$$

electricity prices $p_{gt} = p_{bt} = p_t^* - (S_t - \tau_t)F_b$*, and firm values* $v_{it} = V_{it} + I_{it} = V_{it}^* + I_{it}^*$.

Here, green electricity faces a subsidized discount rate. It might be surprising that a discount rate of a non-polluting firm is changed relative to the competitive rate r, but this change is optimal because it reverses an implicit tax that goes in the opposite direction. In particular, since goods producers are effectively paying a high scope-2 tax on all their electricity consumption, including the green part, the equilibrium electricity price is low, which hurts green electricity firms. Green electricity firms' low discount rate compensates these firms for the low electricity price. More broadly, the proposition provides an argument for subsidizing green electricity as long as green and brown electricity firms sell their electricity at the same price and brown electricity is not taxed directly at its social cost.

5.2 What Preferences Generate Sustainable Discount Rates?

If the economy consists of many small investors, each of whom take total emissions as given, then the competitive equilibrium discount rate is r, as seen in Proposition 1. Nevertheless, green finance is modeled by letting investors require other costs of of capital, r_{it}. Such sustainable discount rates can informally be motivated by investors' environmental, social, and governance (ESG) considerations, sustainable finance regulation, or sustainable finance capital taxation, as explained in Section 1.2.

To be more precise on what is needed to generate sustainable discount rates, suppose that the household chooses its investments as follows. At time t, the household buys θ_{it} shares in each firm, corresponding to a dollar investment in firm i of $\omega_{it} = \theta_{it}(V_{it} + I_{it})$, and a total dollar investment of $W_t = \sum_i \theta_{it}(V_{it} + I_{it})$.

Recall that firm 1 is a "savings asset," which always yields a return of r without emissions. Therefore, it is helpful to consider the investment in firm 1 as the residual, $W_t - \sum_{i \neq 1} \omega_{it}$, of

the total investment, W_t, minus the investments in ω_{it} in the other firms, $2, ..., I, b, g$. Using this notation, the future value of the portfolio investments can be written as:

$$\sum_i \theta_{it}(\Pi_{it+1} + V_{it+1}) = \sum_i \omega_{it} \frac{\Pi_{it+1} + V_{it+1}}{V_{it} + I_{it}} = \sum_{i \neq 1} \omega_{it}(1 + r_{it}) + (W_t - \sum_{i \neq 1} \omega_{it})(1 + r)$$

For any portfolio ω_{it}, the investor owns the fraction $\frac{\omega_{it}}{v_{it}}$ of firm i. Suppose that the investor feels responsible for the same fraction of emissions, $\frac{\omega_{it}}{v_{it}} X_{it+1}$. In other words, the investor feels responsible for the portfolio's "carbon footprint," as defined in (3) in Section 1.2. In particular, suppose that the investor experiences a disutility equal to the carbon footprint multiplied by the social cost of carbon net of paid carbon taxes, $S - \tau$. In that case, the investor's "sustainable utility" over its investment is

$$\sum_{i \neq 1} \omega_{it}(1 + r_{it}) + (W_t - \sum_{i \neq 1} \omega_{it})(1 + r) - \sum_i (S_{t+1} - \tau_{it+1}) \frac{\omega_{it}}{v_{it}} X_{it+1} \tag{21}$$

With this objective function, the first-order condition with respect to ω_i, which makes the investor indifferent across the various investments i, is

$$r_{it} = r + \frac{(S_{t+1} - \tau_{it+1}) X_{it+1}}{v_{it}} \tag{22}$$

This equation shows that, if the investor fully internalizes the firm's externalities, then the financial return, r_{it}, must compensate the investor for the externality-to-value ratio.

Interestingly, equation (22) is exactly the same as (17), namely the condition that the sustainable discount rate is elevated by the externality rate. So green finance works when discount rates reflect that the *average* investor *fully* internalizes the externalities of their investments. So, if the economy has financial investors who disregard the firms' pollution in their portfolio choice, then sustainable investors must compensate by internalizing more than the full externalities.

Another way to derive the same condition is to assume that the investor faces a green capital income tax, as discussed in Section 1.2. In particular, if the investor faces a capital income tax equal to the carbon footprint as in (21) and maximizes after-tax performance, then required returns must satisfy (22).

In sum, sustainable preferences or capital income taxes can provide a foundation for the discount rates in Proposition 4. Further, similar preferences or taxes could provide foundations for the alternative sustainable discount rates in Propositions 5 and 6.

6 Calibrating Sustainable Discount Rates

To evaluate whether the different sustainable discount rates from Propositions 4–6 can be implemented in practice, this section calibrates the corresponding magnitudes of the required changes in the cost of capital.

Sustainable Discount Rates based on Scope-1 Emissions

Figure 1 in the introduction plots different industries' calibrated sustainable discount rates corresponding to Proposition 4. These discount rates are computed as follows. Data on estimated scope-1 emissions measured in tCO_2 per fiscal year come from Trucost for the fiscal year 2021 (the end of my sample). The market value of each firm is estimated as the market value of equity plus the book value of debt at the beginning of the calendar year 2021, based on data from CRSP and COMPUSTAT. For illustrative purposes, the competitive rate is set to $r = 5\%$, the paid carbon tax is set to $\tau_i = 0$ for all firms, and then the firm-value-weighted average sustainable discount rate is computed for each industry. Industries are based on 2-digit GICS industries, except that Utilities is split into Renewable Electricity, Brown Electricity, and Other Utilities.[9]

Beyond the insights from Figure 1 discussed in the introduction, it is also interesting to consider the full cross section of US stocks. Figure 4 shows the sustainable discount rate for all US stocks based on Nordhaus's arguably modest estimate of S =43\$/tCO_2. The figure shows that the pollution is concentrated among a small fraction of firms, while the majority of firms have sustainable discount rates close to r.

The median scope-1 burden rate is 0.01% among all firms for which the relevant data are available, and the market-value-weighted average among these firms (i.e, weighted by the sum of market equity and book debt) is 0.19%. Also, the market-weighted average of the scope 1+2 burden rate is 0.23% with a median of only 0.04%.

As another way to think of these numbers, suppose that all US firms actually paid a carbon tax of S =43\$/tCO_2. This tax would impose a cost on the firms corresponding to 0.19% of their value, based on a value-weighted average. This magnitude seems like a bearable cost, especially given that firms would adapt to become greener, and the additional government revenues could be used to reduce other taxes.

[9] Utilities is GICS 55, which is split as follows. Renewable Electricity is GICS 55105020; Brown Electricity is GICS 55101010, except permnos 44206, 15720, 15859, 24205 (which have low emissions, e.g. because they include a lot of renewables); and Other Utilities consists of GICS 55102010, 55103010, 55104010, and 55105010.

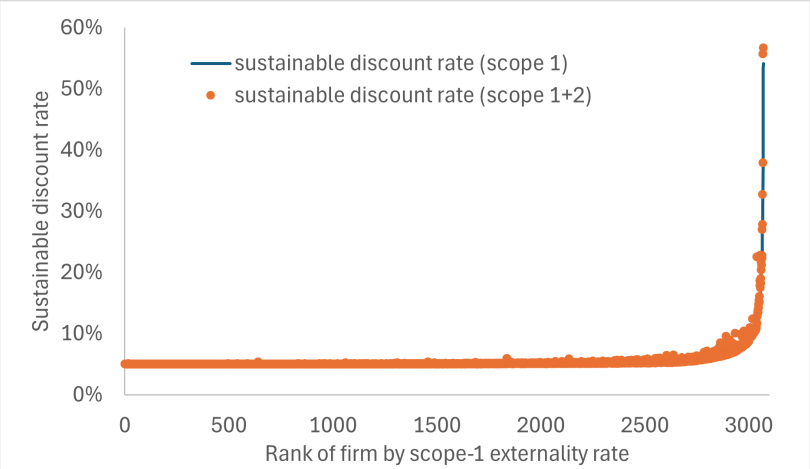


Figure 3: **Sustainable discount rates for all US firms if $S = 43\$/\text{tCO}_2$.** The figure plots each firm's scope-1 sustainable discount rate, r_{it}^{x}, and its scope-1 plus scope-2 discount rate, $r_{it}^{x(1+2)}$, using (17) and (18), respectively. The emission data is from Trucost and firm values are based on CRSP and COMPUSTAT, 2021.

Similarly, if investors could agree, they should be happy to collectively impose higher costs of capital on all firms, thus earning 0.19% more on the market portfolio. But, investors who only focus on their financial return would have an incentive to concentrate their investments in brown firms that offer much higher returns. Such a deviation would not be attractive for an environmental investor concerned with the carbon footprint of owning brown stocks. However, if some investors overweight brown stocks, environmental investors would need to overweight green stocks.

For example, suppose that financial investors buy the top third brownest firms and the environmental investors buy the rest. In this case, the financial investors own the firms responsible for more than 97% of all carbon emission, while the sustainable investors own firms responsible for less than 3% of all emissions. In this case, the financial investors would outperform by about 1% per year. Further, this outperformance reaches 5% using a five times higher social cost of carbon, e.g. based on EPA (2023), and higher yet if financial investors concentrate on a smaller subset of brown firms or shortsell green stocks. Such a return difference could make the financial investors more than 4 times richer than the environmental investors over a 30-year retirement saving period, $1.10^{30}/1.05^{30} = 4$.

Sustainable Discount Rates based on Scope-1+2 Emissions and Green Subsidies

Figure 4 shows calibrations of the alternative sustainable discount rates based on Propositions 5 and 6, respectively. Recall that the former is based on scope 1+2 emissions, and the

latter also relies on subsidized discount rates for green electricity.

To compute the calibration in Figure 4(a), I use the same method and data as in the previous section, plus data on scope-2 emissions, also from Trucost. For panel (b), two additional assumptions are needed. First, to estimate (19), that is, each firm's scope-2 electricity consumption as if all electricity is brown, each scope-2 estimate is multiplied by $1/0.6 = 1.67$ based on the idea that about 60% of US electricity comes from fossils while about 40% comes from renewables and nuclear.[10] Finally, to calibrate (20), the fossil intensity of brown electricity is set to $F_b = 820 \times 10^{-6}\text{tCO}_2/\text{kWh}$ based on the median estimate of coal's lifecycle emission from Schlömer et al. (2014). Further, G is approximated by converting the dollar value of revenues into kWh by dividing by the average price of electricity in all sectors in 2021, which is estimated as 0.11\$/kWh based on data from EIA[11] and then using the median value of G/v_g in the Renewable Electricity sector.

Figure 4(a) shows that non-electricity firms have higher scope-1+2 sustainable discount rates relative to the approach illustrated in Figure 1 in the introduction. However, brown electricity firms have lower costs of capital, which is helpful since this industry has the highest discount rate with the scope-1 approach in Figure 1.

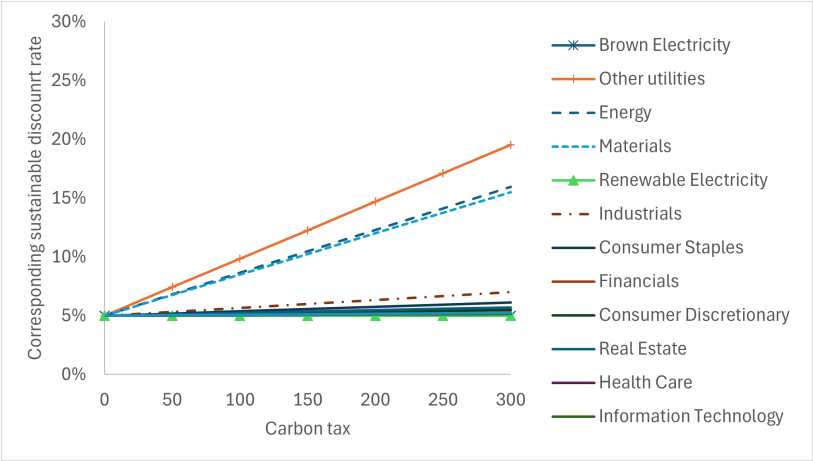
If investors only know a firm's total electricity consumption (not whether the electricity comes from solar or fossil-based energy, say), then the model suggests another scope-2 approach, illustrated in Figure 4(b). Electricity users should face a sustainable discount rate as in (19), where the scope-2 emission is computed as if all electricity is brown, thus raising their sustainable discount rates relative to panel (a). At the same time, green electricity producers receive the subsidized rate (20), as seen by the downward-sloping line in Figure 4(b).

Alternative Calibration

One issue with Propositions 4–6 is that the sustainable discount rates depend on future emissions and endogenous firm values, but these could change if firms transition to be greener to avoid punitive discount rates. Instead of relying on firm values, consider an alternative calibration based on a zero-profit condition for brown electricity, the most polluting sector. Suppose that brown electricity is produced with a constant-returns-to-scale technology with output $B_t = aK_{bt}$, cost $\chi_{bt} = \bar{\chi}aK_{bt}$, and the firm chooses capital K_{bt+1} to maximize

[10] See, e.g., https://www.epa.gov/green-power-markets/us-electricity-grid-markets.
[11] See The U.S. Energy Information Administration (EIA).

(a) Sustainable discount rate = 5%+carbon burden rate(scope 1+2)



(b) Subsidized cost of capital for green electricity firms

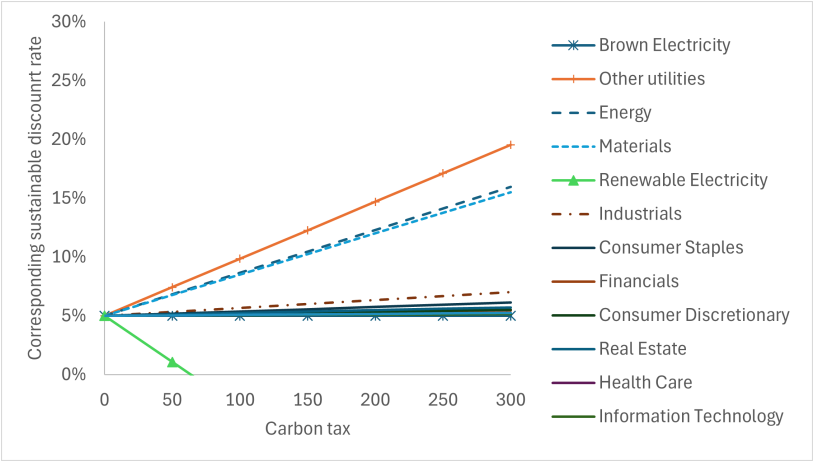


Figure 4: **Translating a Carbon Tax into Scope-1+2 Sustainable Discount Rates.** Panel (a) shows goods producers' calibrated sustainable discount rates, $r_{it}^{x(1+2)}$, based on scope-1+2 emissions via equation (18) as a function of the social cost of carbon, S_t, while electricity firms have normal discount rates. In panel (b), green electricity firms face the subsidized rate (20), and other firms penalized for all electricity consumption (19). The data is from 2021, the competitive rate is $r = 5\%$, and the paid carbon tax is set to $\tau_{it} = 0$.

$$\begin{aligned}\frac{\Pi_{bt+1} + V_{bt+1}}{1 + r_{bt}} - I_{bt} &= \frac{(p_{bt+1} - \tau_{bt+1}F_b - \bar{\chi})aK_{bt+1} + (1-\delta)K_{bt+1}}{1 + r_{bt}} - I_{bt} \\ &= \frac{(p_{bt+1} - \tau_{bt+1}F_b - \bar{\chi} - \frac{r_{bt}+\delta}{a})aK_{bt+1}}{1 + r_{bt}} + (1+\delta)K_{bt}\end{aligned} \tag{23}$$

using that $I_{bt} = K_{bt+1} - (1+\delta)K_{bt}$ and that the future value of capital is $(1-\delta)$ if the firm is investing. The brown electricity price, p_{bt+1}, that satisfies the first-order condition with

respect to K_{bt+1} is therefore

$$p_{bt+1} = \tau_{bt+1} F_b + \bar{\chi} + \frac{r_{bt} + \delta}{a} \tag{24}$$

Consider how we can choose the sustainable discount rate r_{bt} such that the price is the same in the following two cases: *(i)* a carbon tax of S_{t+1} and a discount rate of r, and *(ii)* a too low carbon tax of τ_{bt+1} and a discount rate of r_{bt}. The result is

$$r_{bt} = r + (S_{t+1} - \tau_{bt+1}) F_b a = r + \frac{(S_{t+1} - \tau_{bt+1}) F_b}{(r+\delta)/a}(r+\delta) \tag{25}$$

To calibrate this sustainable discount rate, set the competitive rate to $r = 5\%$, the depreciation rate to $\delta = 5\%$, the fossil intensity of brown electricity to $F_b = 820 \times 10^{-6} \text{tCO}_2/\text{kWh}$ as in the previous section, the social cost of carbon emissions to $S_{t+1} = 43\$/\text{tCO}_2$ based on the low estimate from Nordhaus (2019), the carbon tax to $\tau_{bt+1} = 0\,\$/\text{tCO}_2$, and the productivity parameter a and cost parameter $\bar{\chi}$ are chosen such that the competitive electricity cost is $(r+\delta)/a + \bar{\chi} = 0.11\$/\text{kWh}$ based on the number from the previous section, and the profit margin is 40%.[12] In other words, 60% of revenue is lost to cost of revenue, $\bar{\chi} = 0.60 \times 0.11$, while the rest goes to the capital, $(r+\delta)/a = 0.40 \times 0.11$. Based on these assumptions, the cost of capital for brown electricity is calibrated as

$$\begin{aligned} r_{bt} &= r + (S_{t+1} - \tau_{bt+1}) F_b a = r + \frac{(S_{t+1} - \tau_{bt+1}) F_b}{(r+\delta)/a}(r+\delta) \\ &= 5\% + \frac{43\$/tCO_2 \times 820 \times 10^{-6} tCO_2/\text{kWh}}{0.4 \times 0.11\$/\text{kWh}}(5\% + 5\%) \\ &= 5\% + \frac{0.035}{0.044} \times 10\% = 5\% + 8.0\% = 13\% \end{aligned} \tag{26}$$

In other words, a carbon tax equal to the social cost, $S_{t+1} = 43\$/\text{tCO}_2$, would raise the cost of electricity by $0.035\$/\text{kWh}$. To achieve the same effect via green finance, the cost of capital must increase from 5% to 13%.

It is interesting to compare this 8% increase in the cost of capital to the calibration in the previous section based on carbon burden rates, x_{it}. With $S_{t+1} = 43\$/\text{tCO}_2$, the average carbon burden rate for Brown Electricity firms is 3.2%, and some firms have carbon burden rates of 5.4% and 8.9%, for example, which is similar to the calibration in this section.[13]

[12] The mean EBITDA/Revenue for firms in the Brown Electricity sector was 40% in 2021.

[13] "O G E ENERGY CORP" (permno 24440) and "N R G ENERGY INC" (permno 90071), respectively.

7 Extensions

7.1 Long-Term Sustainable Discount Rates for a Green Transition

The previous section shows how 1-period sustainable discount rates are related to emissions over the coming year. To understand the case of long-term discount rates, consider the following version of Gordon's growth model for a firm with a constant long-term discount rate of r. The firm has free cash flows before carbon taxes, D_t, that grow at the rate $g_D < r$ such that $D_t = (1+g_D)^{t-1}D$, where $D > 0$ is the first free cash flow. Also, the firm has externalities that are known to grow at the rate $g_X < r$ such that $S_t X_t = (1+g_X)^{t-1}SX$. With a socially optimal carbon tax rate of S_t, the firm value at time 0 is

$$\begin{aligned} v &= \frac{D-SX}{1+r} + \frac{D(1+g_D)-SX(1+g_X)}{(1+r)^2} + \frac{D(1+g_D)^2-SX(1+g_X)^2}{(1+r)^3} + ... \\ &= \frac{D}{r-g_D} - \frac{SX}{r-g_X} \end{aligned} \tag{27}$$

Suppose instead that there is no carbon tax, but the firm has a higher long-term discount rate, $\bar{r}_i^x$, that applies in all time periods, yielding a value of

$$v = \frac{D}{\bar{r}_i^x - g_D} \tag{28}$$

If green finance mimics the effect of a carbon tax, then (27) and (28) must be equal. Solving for $\bar{r}_i^x$ yields[14]

$$\bar{r}_i^x = r + \frac{SX}{v}\frac{r-g_D}{r-g_X} = r + x\frac{r-g_D}{r-g_X} \tag{29}$$

This equation shows that the long-term sustainable discount rate is elevated by the current externality rate, $x = \frac{SX}{v}$, scaled by $\frac{r-g_D}{r-g_X}$, which naturally differs from the short-term sustainable discount rate, $r_i^x = r + x$. For most firms, the growth of emissions is hopefully below earnings growth, $g_X < g_D$ (even when taking into account that S_t also grows according to

[14] To calculate the long-term sustainable discount rate, consider

$$\begin{aligned} \bar{r}_i^x &= \frac{D}{\frac{D}{r-g_D} - \frac{SX}{r-g_X}} + g_D = \frac{D + g_D(\frac{D}{r-g_D} - \frac{SX}{r-g_X})}{v} \\ &= \frac{r\frac{D}{r-g_D} - g_D\frac{SX}{r-g_X}}{v} = \frac{r\left(\frac{D}{r-g_D} - \frac{SX}{r-g_X}\right) + r\frac{SX}{r-g_X} - g_D\frac{SX}{r-g_X}}{v} = r + \frac{SX}{v}\frac{r-g_D}{r-g_X} \end{aligned}$$

Nordhaus), which means that long-term sustainable discount rates are smaller than short-term rates, $\bar{r}_i^x < r_i^x$. Moreover, the more aggressively a firm commits to decarbonization (i.e., the more negative g_X becomes), the lower the corresponding long-term sustainable discount rate, $\bar{r}_i^x$, assuming all other parameters remain constant.

7.2 Uncertainty

The model can also be extended to handle uncertainty. For example, the social cost of carbon, S_t, could have random variation over time and each firm's production function could be affected by random productivity shocks. In this case, the equations capturing the firms' and planner's problems need an expectation. For green finance to work, then firms must be able to commit to a state-dependent plan for its future emissions and actions. The sustainable discount rate is then the expected value of (17) conditional on the firm's planned actions and on all time-t information. Further, the firm must be able to compute its sustainable discount rate for any potential plan of action, realizing that its discount rate will be higher if its plan contains many states with high emissions, thus mirroring the realized carbon tax in each state. Clearly, such commitments and foresight are even more challenging in practice.

7.3 Greenwashing: Externalities via Taxes or Voluntary Markets

Carbon pricing in a regulated system, via taxes or emission trading systems (see extension in Appendix C), is a powerful way to make firms care about their pollution via a simple tool. Every firm simply pays for their externality. Implementing green finance via a capital income tax could also work, as shown above, but is potentially more complex.

Carbon taxes and sustainable capital income taxes have the potential advantage that the government raises a revenue that can be used to reduce income taxes or other distortionary taxes. So these approaches can potentially lead to a so-called "double dividend," meaning that they both reduce pollution and lead to potential economic benefits associated with a lower income tax.

In contrast, relying on voluntary markets does not raise any revenues and comes with several challenges, often termed greenwashing. In fact, relying on voluntary markets to fix externalities is almost a contradiction in terms. This is true both for carbon pricing and green finance. Extending the model to include voluntary carbon offset markets (Appendix D) shows that the low and varying offset prices observed in the real world are telltale signs of greenwashing.

Likewise, relying on investors' and creditors' voluntary implementation of sustainable discount rates is also challenging. This approach requires that investors and creditors forego higher returns such that their collective actions result in the right total change in the cost of capital—a tall order. Further, the green finance approach is susceptible to various avoidance methods by brown firms. For example, firms who report many hundreds of sustainability measures can try to appear greener than they really are in order to lower their cost of capital, for example by gaming simple aggregate sustainability scores. Also, firms can use various corporate structures to lower their cost of capital, for instance a non-polluting firm can provide financing to a polluting firm (as the pollution of the subsidiary might not be captured by the parent, even via its scope-2 or scope-3 measures).

Green finance is also challenged by stranded assets, as discussed before. But, what if the stranded asset is bundled with a useful green asset? Said differently, can green finance better handle a situation in which a firm owns two assets, a green and a brown? Clearly, carbon pricing still works with such two bundled assets. At first sight, it may seem that green finance also works if the firm has an incentive to shut down the use of the stranded asset to lower the firm's overall cost of capital, thus increasing the value of the joint firm. The problem with this approach is that the firm can make more money by selling off the stranded asset for a positive price, as it happens in the real world, and then be left with a green asset, which typically ends up having a low cost of capital (despite the sale).

Carbon prices and green finance differ in how they can handle foreign emissions. Europe's carbon border adjustment mechanism seeks to put domestic and foreign firms in equal footing and simultaneously create an incentive for foreign governments to implement a carbon tax. Also, green finance has the advantage that investments flow across borders.

In sum, externalities can be handled by pricing their costs via carbon taxes or carbon capital income taxes, while voluntary systems are challenging. Perhaps a third possibility is to impose lots of rules on what firms are allowed to do, disclosure requirements, and other forms of central micro-management. This third approach likely suffers from greenwashing problems and significant productivity losses.

7.4 Green versus Brown Technology Choice

In the model, goods producers and electricity firms have different propensities to pollute, but each firm can only invest in its own technology (i.e., its production function is given). The model can also capture firms' technology choices as follows. To capture technology choice, interpret all (or a collection of) firms as a representative firm. When this representative firm

decides on all its investments (i.e., chooses I_{it} for each i), it makes a choice across green and brown technologies. Hence, the model can also capture how carbon pricing and green finance can push firms, industries, and the economy to invest in greener technologies.

8 Conclusion: Everything, Everywhere, All at Once?

The UN secretary general, António Guterres, stated that "our world needs climate action on all fronts — everything, everywhere, all at once"[15] and, indeed, global climate action has proliferated in various forms of carbon pricing and green finance. This proliferation stands in contrast to the recommendation from economics that climate change should be fought using a single tool everywhere at once—a carbon tax.

However, if carbon pricing is not politically feasible, then other approaches should be considered. To capture the UN's "everything" approach, this paper introduces a model that unifies carbon-pricing and green-finance approaches and yields several results.

First, the model shows how to "translate" a carbon tax rate to sustainable discount rates. Such sustainable discount rates can in principle address climate change if the cost of capital can be sufficiently controlled by the collective actions of investors, creditors, and regulators. The sustainable discount rate equals the normal cost of capital plus each firm's carbon burden rate. The carbon burden rate is the social cost of a firm's untaxed emissions, scaled by firm value.

Second, the sustainable discount rate has several natural properties. It increases with emissions and the social cost of carbon. Further, if a firm is already paying a high carbon tax, then it should not also face a high cost of capital. In contrast, in reality Europe has a high carbon tax and also a significant focus on sustainable finance regulation, while other regions have neither.

Third, the model is straightforward to calibrate using data on emissions and standard market data. For example, a carbon tax of 14\$/tCO_2 corresponds to an increase in the cost of capital of 1% for the average US brown electricity company. A more meaningful carbon tax of 190\$/tCO_2 corresponds to a cost-of-capital increase of 14%, which is hardly feasible in practice.

Looking at firms' actual cost of capital, Eskildsen et al. (2024) estimate that the cost of capital of the top third brown firms exceed that of the top third green firms by about 0.6%, with a difference of just over 1% when looking at more extreme polluters. The latter number

[15] https://www.un.org/sg/en/content/sg/statement/2023-03-20/secretary-generals-video-message-for-press-conference-launch-the-synthesis-report-of-the-intergovernmental-panel-climate-change

corresponds to a carbon tax of about 14\$/tCO_2. Based on these estimates, green finance is already making a meaningful difference, but scaling up the effect to accomplish a green transition seems challenging.

Scaling the effect is challenging because the real world has different types of investors: sustainable investors willing to give up returns for a greener investment and investors focused on their financial return who partly undo the green premium. Further, sustainable discount rates imply that firms essentially pay a carbon tax to the financial investors, while sustainable investors do not receive any of the tax—and a nontrivial part of the tax is actually lost to society because both types of investors suffer from imperfect diversification. This inefficient and unfair situation would likely be difficult for sustainable investors to accept if sustainable discount rates were to deviate an order of magnitude from competitive rates.

The model also shows how other tools can be applied, including green subsidies or low capital costs for green firms. In sum, the model indicates that a focus on green finance to answer UN's call for "everything, everywhere, all at once" can be a second-best attempt of taking many small steps when the optimal leap is unattainable. However, using sustainable discount rates is challenging because discount rates are difficult to affect and the approach is challenged by stranded assets. Moreover, greenwashing and regulatory burdens present further challenges in the real world. Hence, a country or region that is able to implement a carbon tax—such as Europe—has a better chance of a green transition coupled with sustainable economic growth by focusing on a carbon tax without the regulatory burden of sustainable finance.

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Appendix

Appendix A provides proofs of all propositions. Appendix B presents a stylized model with a Cobb-Douglas production function. Appendix C and Appendix D contain extensions with carbon allowance markets and carbon offset markets, respectively.

A Proofs

Proof of Propositions 1–3. To derive the first-order conditions of the planner's problem, note that when $\theta_{it} = 1$, consumption (12) can be written as final output minus electricity costs and investment:

$$C_t = \sum_i Y_{it} - \chi_{bt} - \chi_{gt} - \sum_i I_{it} - I_{bt} - I_{gt} \tag{A.1}$$

Hence, the planner's problem is to maximize

$$\begin{aligned}\sum_{t=1}^{T} \beta^t \Bigg[u_t\Bigg(\sum_i Y_{it} - \chi_{bt} - \chi_{gt} - \sum_i I_{it} - I_{bt} - I_{gt} \Bigg) - d_t\Bigg(\sum_i X_{it} + F_g G_t + F_b B_t \Bigg) \\ - \lambda_{Lt}\Bigg(\sum_i L_{it} - L_t \Bigg) - \lambda_{gt}\Bigg(\sum_i G_{it} - G_t \Bigg) - \lambda_{bt}\Bigg(\sum_i B_{it} - B_t \Bigg) \Bigg]\end{aligned} \tag{A.2}$$

with Lagrange multipliers $\lambda_{Lt}, \lambda_{gt}, \lambda_{bt}$ for the resource constraints (14). Recall that K_{it} is considered the choice variable rather than I_{it}, using the connection, $I_{it} = K_{it+1} - (1-\delta)K_{it}$. The first-order conditions (FOCs) with respect to each element of $z_{it} = (K_{it}, L_{it}, G_{it}, B_{it}, X_{it})$,

$z_{bt} = (K_{bt}, H_{bt})$, and $z_{gt} = (K_{gt}, H_{gt})$ are as follows:

$$\frac{\beta u'_t}{u'_{t-1}}\left(\frac{\partial Y_{it}}{\partial K_{it}} + 1 - \delta\right) = 1 \tag{A.3}$$

$$\frac{\partial Y_{it}}{\partial L_{it}} = \frac{\lambda_{Lt}}{u'_t} \tag{A.4}$$

$$\frac{\partial Y_{it}}{\partial G_{it}} = \frac{\lambda_{gt}}{u'_t} \tag{A.5}$$

$$\frac{\partial Y_{it}}{\partial B_{it}} = \frac{\partial Y_{it}}{\partial G_{it}} = \frac{\lambda_{bt}}{u'_t} \tag{A.6}$$

$$\frac{\partial Y_{it}}{\partial X_{it}} = \frac{d'_t}{u'_t} \tag{A.7}$$

$$\frac{\beta u'_t}{u'_{t-1}}\left(\left[\frac{\lambda_{bt}}{u'_t} - \frac{d'_t}{u'_t}F_b\right]\frac{\partial B_t}{\partial K_{bt}} - \frac{\partial \chi_{bt}}{\partial K_{bt}} + 1 - \delta\right) \leq 1 \text{ with equality unless } I_{bt-1} = 0 \tag{A.8}$$

$$\left(\frac{\lambda_{bt}}{u'_t} - \frac{d'_t}{u'_t}F_b\right)\frac{\partial B_t}{\partial H_{bt}} \leq \frac{\partial \chi_{bt}}{\partial H_{bt}} \text{ with equality unless } H_{bt} = 0 \tag{A.9}$$

$$\frac{\beta u'_t}{u'_{t-1}}\left(\left[\frac{\lambda_{gt}}{u'_t} - \frac{d'_t}{u'_t}F_g\right]\frac{\partial G_t}{\partial K_{gt}} - \frac{\partial \chi_{gt}}{\partial K_{gt}} + 1 - \delta\right) = 1 \tag{A.10}$$

$$\left(\frac{\lambda_{gt}}{u'_t} - \frac{d'_t}{u'_t}F_g\right)\frac{\partial G_t}{\partial H_{gt}} = \frac{\partial \chi_{gt}}{\partial H_{gt}} \tag{A.11}$$

where the fourth FOC uses that green and brown electricity only enter the production function via their sum, so all forms of electricity have the same marginal productivity. These FOCs are satisfied at the social optimum, denoted by $*$. Given the assumption that firm 1 has $Y_{it} = (1 + r + \delta)K_{it}$, the first FOC shows that

$$\frac{\beta u'_t}{u'_{t-1}} = \frac{1}{1+r} \tag{A.12}$$

The rest of the proof establishes that the competitive problem has the same first-order conditions under the stated assumptions. When $r_{it} = r$, the firm's problem (9) can be iterated forward to get

$$\begin{aligned} V_{i0} &= \frac{\Pi_{i1}(z_{i1}) + V_{i1}(K_{i1})}{1+r} - I_{i0} \\ &= \sum_{t=1}^{\infty} \frac{\Pi_{it}(z_{it}) - I_{it}}{(1+r)^t} - I_{i0} \\ &= \sum_{t=1}^{\infty} \frac{\Pi_{it}(z_{it}) - K_{it+1} + (1-\delta)K_{it}}{(1+r)^t} - I_{i0} \end{aligned} \tag{A.13}$$

using $I_{it} = K_{it+1} - (1-\delta)K_{it}$. Goods producer i has the following first order conditions with respect to each element of $z_{it} = (K_{it}, L_{it}, G_{it}, B_{it}, X_{it})$:

$$\frac{\frac{\partial Y_{it}}{\partial K_{it}} + 1 - \delta}{1+r} = 1 \tag{A.14}$$

$$\frac{\partial Y_{it}}{\partial L_{it}} = w_t \tag{A.15}$$

$$\frac{\partial Y_{it}}{\partial G_{it}} = p_{gt} + \tau_{it}^2 f_g \tag{A.16}$$

$$\frac{\partial Y_{it}}{\partial B_{it}} = \frac{\partial Y_{it}}{\partial G_{it}} = p_{bt} + \tau_{it}^2 f_b \tag{A.17}$$

$$\frac{\partial Y_{it}}{\partial X_{it}} = \tau_{it} \tag{A.18}$$

Similarly, the brown electricity firm has first-order conditions w.r.t. $z_{bt} = (K_{bt}, H_{bt})$,

$$\frac{(p_{bt} - \tau_{bt}F_b)\frac{\partial B_t}{\partial K_{bt}} - \frac{\partial \chi_{bt}}{\partial K_{bt}} + 1 - \delta}{1+r} \leq 1 \text{ with equality unless } I_{bt-1} = 0 \tag{A.19}$$

$$(p_{bt} - \tau_{bt}F_b)\frac{\partial B_t}{\partial H_{bt}} \leq \frac{\partial \chi_{bt}}{\partial H_{bt}} \text{ with equality unless } H_{bt} = 0 \tag{A.20}$$

and the green electricity firm has first-order conditions w.r.t. $z_{gt} = (K_{gt}, H_{gt})$,

$$\frac{(p_{gt} + \nu_{gt} - \tau_{gt}F_g)\frac{\partial G_t}{\partial K_{gt}} - \frac{\partial \chi_{gt}}{\partial K_{gt}} + 1 - \delta}{1+r} = 1 \tag{A.21}$$

$$(p_{gt} + \nu_{gt} - \tau_{gt}F_g)\frac{\partial G_t}{\partial H_{gt}} = \frac{\partial \chi_{gt}}{\partial H_{gt}} \tag{A.22}$$

These FOCs (A.14)–(A.22) as seen to be the same as the planner's first order conditions (A.3)–(A.11) under the assumptions of Proposition 1, namely $w_t = \frac{\partial Y_{it}}{\partial L_{it}}(z_{it}^*) = \frac{\lambda_{Lt}}{u_t'}$, $p_t^* = p_{gt} = p_{bt} = \frac{\partial Y_{it}}{\partial G_{it}}(z_{it}^*) = \frac{\partial Y_{it}}{\partial B_{it}}(z_{it}^*) = \frac{\lambda_{gt}}{u'(C_t^*)} = \frac{\lambda_{bt}}{u'(C_t^*)}$, $\tau_{it} = \tau_{bt} = \tau_{gt} = S_t = \frac{d_t'}{u_t'}$, $\tau_{it}^2 = 0$, $r_{it} = r_{bt} = r_{gt} = r$, and $\nu_{gt} = 0$.

Finally, households' FOC with respect to θ_{it} is

$$\frac{\beta u_t'}{u_{t-1}'}(\Pi_{it} + V_{it}) = V_{it-1} + I_{it-1} \tag{A.23}$$

which is satisfied because of (A.12) and because of the firm's value, $V_{it-1} = \frac{\Pi_{it}+V_{it}}{1+r} - I_{it-1}$.

Proposition 2 assumes instead that $\tau_{it} = S_t$, $\tau_{bt} = \tau_{gt} = 0$, $\tau_{it}^2 = S_t$, $f_b = F_b$, $f_g = F_g$, $r_{it} = r_{bt} = r_{gt} = r$, and $\nu_{gt} = 0$. In this case, all first order conditions (A.14)–(A.22)

continue to be satisfied at the social optimum when electricity prices are $p_{gt} = p_t^* - S_t F_g$ and $p_{bt} = p_t^* - S_t F_b$. Indeed, as in the $*$-equilibrium in Proposition 1, goods producers face net-of-taxes electricity prices equal to p_t^* and green and brown electricity producers face net-of-taxes prices of $p_t^* - S_t F_g$ and $p_t^* - S_t F_b$, respectively.

Proposition 3 assumes that $\tau_{it} = S_t$, $\tau_{bt} = \tau_{gt} = 0$, $\tau_{it}^2 = S_t$, $f_b = f_g = F_b$, $r_{it} = r_{bt} = r_{gt} = r$, and $\nu_{gt} = S_t(F_b - F_g)$. In this case, all first order conditions (A.14)–(A.22) continue to be satisfied at the social optimum when electricity prices are $p_{gt} = p_{bt} = p_t^* - S_t F_b$, again because all firms face the same prices net of taxes and subsidies. □

The following three proofs make use of the following lemma. To state the lemma, recall that the quantities from the equilibrium in Proposition 1 are denoted by $*$, i.e., energy prices are p_t^*, firm actions z_{it}^*, profits, Π_{it}^*, and values, V_{it}^*.

Lemma 1 (Taxes as Discount Rates) *For a given set of electricity prices (p_{gt}, p_{bt}), carbon taxes $(\tau_{it}, \tau_{it}^2, f_g, f_b, \tau_{gt}, \tau_{bt})$, and wage, w_t^*, suppose that there exists $(\pi_{it})_{i=1,...,N,g,b}$ such that each firm's profit is $\Pi_{it}(z_{it}) = \Pi_{it}^*(z_{it}) + \pi_{it}(z_{it})$ and, for all $i \in \mathcal{I}' = \{1, ..., N, g, b\} \setminus \{i \,|\, \pi_{it} \equiv 0\}$ it holds that $\Pi_{it}^*(z_{it}) + V_{it}^*(z_{it}) > 0$. Then these prices and taxes can be sustained as an equilibrium with actions z_{it}^* and firm values V_{it}^* when each firm $i \in \mathcal{I}'$ has the discount rate*

$$r_{it}(z_{it+1}) = r + \frac{\pi_{it+1}(z_{it+1})}{(\Pi_{it+1}^*(z_{it+1}) + V_{it+1}^*(z_{it+1}))/(1+r)} \tag{A.24}$$

and the remaining firms have discount rates r. Firm i experiences this discount rate as equally costly as a tax of $\pi_{it}(z_{it+1})$, which offsets its higher profit.

Proof of Lemma 1. Suppose that electricity prices are given by (p_{gt}, p_{bt}) and each firm's future value is $V_{it+1} = V_{it+1}^*$, for $i = 1, ..., N, g, b$. Then each firm chooses the same actions,

$z_{it+1} = z^*_{it+1}$, and achieves the time-t value:

$$\begin{aligned} V_{it} &= \max_{z_{it+1}} \left(\frac{\Pi_{it+1} + V^*_{it+1}}{1 + r_{it}} - I_{it} \right) \\ &= \max_{z_{it+1}} \left(\frac{\Pi^*_{it+1} + \pi_{it+1} + V^*_{it+1}}{1 + r + \frac{\pi_{it+1}}{(\Pi^*_{it+1} + V^*_{it+1})/(1+r)}} - I_{it} \right) \\ &= \max_{z_{it+1}} \left(\frac{\Pi^*_{it+1} + \pi_{it+1} + V^*_{it+1}}{(1+r)\frac{\Pi^*_{it+1} + V^*_{it+1} + \pi_{it+1}}{\Pi^*_{it+1} + V^*_{it+1}}} - I_{it} \right) \\ &= \max_{z_{it+1}} \left(\frac{\Pi^*_{it+1} + V^*_{it+1}}{1+r} - I_{it} \right) \\ &= V^*_{it} \end{aligned} \tag{A.25}$$

and these actions constitute an equilibrium, as in the $*$-equilibrium. □

Proof of Proposition 4. The proof can be seen an application of Lemma 1 with $p_{gt} = p_{bt} = p^*_t$ and $\pi_{it} = (S_{t+1} - \tau_{it+1})X_{it+1}$ for all i.

The fact the sustainable discount rate can implement the social optimum is also shown in Section 5.1 under the stated assumptions. In particular, this section shows that the firm's problem is the same in the green finance equilibrium and the $*$-equilibrium (i.e., the equilibrium with carbon tax rate S_t and discount rate r).

The sustainable discount rate, r_{it}, is defined in (17) as the ratio of future externalities to the current firm value, v_{it}. Note that the current firm value defined in (15) contains the future firm value, $V_{it+1}(K_{it+1})$. To be explicit about this term, note that, when firms use the discount rate (17) that depends on future actions, (15) holds in equilibrium so that

$$v_{it} = V_{it} + I_{it} = \frac{\Pi_{it+1} + V_{it+1}}{1 + r_{it}} = \frac{\Pi^*_{it+1} + V_{it+1}}{1 + r} \tag{A.26}$$

where $\Pi^*_{it+1+s} = \Pi_{it+1} - (S_{t+1} - \tau_{it+1})X_{it+1}$ is the profit when the carbon tax is $\tau_{it+1+s} = S_{t+1+s}$ as in the $*$-equilibrium. The firm value in the numerator of (17) is found by iterating (A.26) forward,

$$V_{it+1}(K_{it+1}) = \max \sum_{s=1}^{\infty} \frac{\Pi^*_{it+1+s} - I_{it+1+s}}{(1+r)^s} - I_{it+1} = V^*_{it+1} \tag{A.27}$$

This equation also proves the last statement in the proposition, namely that firm values are the same as in Proposition 1 where the discount rate is r and the carbon tax rate is S_t.

Finally, note that, while all terms in (A.26) are the same in equilibrium, the sustainable discount rate, r_{it}, must be defined based on the firm value, v, in (15), i.e., using the last term of (A.26). Indeed, the second to last expression in (A.26) cannot be used as it includes the object to be defined, r_{it}. The first expression does not work if the firm value, V_{it}, is interpreted as a constant that only depends on the lagged state variable, K_{it}, since the sustainable discount must take into account that the firm value depends on emissions next period. □

Proof of Proposition 5. Since a goods producer faces the discount rate (18), the proof seeks to establish that such a firm is implicitly paying scope-1 and scope-2 carbon taxes, corresponding to first-order conditions (A.14)–(A.22) with $\tau_{it} = S_t$, $\tau_{bt} = \tau_{gt} = \tau_t$, $\tau_{it}^2 = S_t - \tau_t$, $f_b = F_b$ and $f_g = F_g$, $r_{it} = r_{bt} = r_{gt} = r$, and $\nu_{gt} = 0$. Suppose therefore that electricity prices reflect these conditions, that is, $p_{gt} = p_t^* - (S_t - \tau_t)F_g$ and $p_{bt} = p_t^* - (S_t - \tau_t)F_b$, where $p_t^* = \frac{\partial Y_{it}}{\partial G_{it}} = \frac{\partial Y_{it}}{\partial B_{it}}$ is the electricity price in the social optimum as above.

The rest of the proof is an application of Lemma 1. That is, the proof checks that all firms have discount rates consistent with their "profit adjustments," π_i.

Given the low energy prices (along with the fact that the firm is not paying actual scope-2 taxes), goods producers have $\pi_{it} = (S_{t+1} - \tau_{it+1})X_{it+1} + (S_t - \tau_t)F_b B_{it+1} + (S_t - \tau_t)F_g G_{it+1} = (S_{t+1} - \tau_{it+1})X_{it+1} + (S_t - \tau_t)X_{it+1}^{scope2}$ for all i.

Further, electricity firms have $\pi_g \equiv \pi_b \equiv 0$, since they face low electricity prices and low carbon taxes, yielding the same net-of-tax electricity revenue as in the social optimum. For example for brown electricity, $p_{bt} - \tau_t F_b = p_t^* - (S_t - \tau_t)F_b - \tau_t F_b = p_t^* - S_t F_b$.

In sum, behaving as in social optimum is an equilibrium based on Lemma 1. □

Proof of Proposition 6. The proof seeks to establish that the assumed discount rates mean that goods producers face a scope-1 carbon tax of $(S_t - \tau_{it})X_{it}$ embedded in their discount rate plus an explicit scope-1 tax of $\tau_{it}X_{it}$, yielding a total scope-1 carbon tax of $S_t X_{it}$. In addition, they face a scope-2 tax of $(S_t - \tau_t)F_b(G_{it} + B_{it})$ embedded in the discount rate. At the same time, green electricity producers face an embedded subsidy corresponding to a cash subsidy of $(S_t - \tau_t)(F_b - F_g)G_t$.

If so, the competitive problem has first-order conditions (A.14)–(A.22) with $\tau_{it} = S_t$, $\tau_{bt} = \tau_{gt} = \tau_t$, $\tau_{it}^2 = S_t - \tau_t$, $f_b = f_g = F_b$, $\nu_{gt} = (S_t - \tau_t)(F_b - F_g)$, and $r_{it} = r_{bt} = r_{gt} = r$. Suppose that electricity prices are consistent with such an equilibrium, that is, electricity prices equal $p_{gt} = p_{bt} = p_t^* - (S_t - \tau_t)F_b$.

The rest of the proof is an application of Lemma 1. That is, the proof checks that all firms have discount rates consistent with their "profit adjustments," π_i.

Brown electricity firms have $\pi_g \equiv 0$, since they face the same net-of-tax price as in the social optimum, $p_{bt} - \tau_t F_b = p_t^* - (S_t - \tau_t)F_b - \tau_t F_b = p_t^* - S_t F_b$.

Green electricity firms face a net-of-tax price of $p_{gt} - \tau_t F_g = p_t^* - (S_t - \tau_t)F_b - \tau_t F_g = p_t^* - S_t F_g - (S_t - \tau_t)(F_b - F_g)$. Given that this net-of-tax price is too low, $\pi_{gt} = -(S_t - \tau_t)(F_b - F_g)G_t$.

Finally, goods producers benefit from their low scope-1 tax, the low energy prices, and no actual scope-2 tax, so $\pi_{it} = (S_t - \tau_t)X_{it} + (p_t^* - p_t)(G_{it} + B_{it}) = (S_t - \tau_t)X_{it} + (S_t - \tau_t)F_b(G_{it} + B_{it})$

In sum, behaving as in social optimum is an equilibrium based on Lemma 1. □

B Model with Cobb-Douglas Production Function

This section derives some of the paper's main results in another stylized model, thus complementing the one in Section 2. A firm exists for two time periods, time 0 and time 1. At time 0, the firm chooses its capital, K, as described below. At time 1, the firm produces its output, Y, using two inputs, capital, K, and fuel, X. The firm has a Cobb-Douglas production function:

$$Y(K, X) = AK^{\alpha}X^{\beta} - \phi X \tag{B.1}$$

where A is productivity, α, β capture the reliance on capital and fuel, and ϕ is the fuel cost.

Using fuel leads to carbon emission. With a carbon tax rate of τ (measured per unit of fuel), the firm's profit is

$$\Pi = Y - \tau X = AK^{\alpha}X^{\beta} - \phi X - \tau X = AK^{\alpha}X^{\beta} - \tilde{\tau}X \tag{B.2}$$

where $\tilde{\tau} = \phi + \tau$ is the total fuel and tax cost. The profit-maximizing emission is

$$X = \left(\frac{\beta A K^{\alpha}}{\tilde{\tau}}\right)^{\frac{1}{1-\beta}} \tag{B.3}$$

which naturally decreases in the tax. The corresponding profit is $A^{\frac{1}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}(1-\beta)\tilde{\tau}^{-\frac{\beta}{1-\beta}}K^{\frac{\alpha}{1-\beta}}$.

At time 0, the firm chooses its capital by maximizing its present value, less investment cost

$$\frac{\Pi + K}{1 + r_i} - K = \frac{A^{\frac{1}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}(1-\beta)\tilde{\tau}^{-\frac{\beta}{1-\beta}}K^{\frac{\alpha}{1-\beta}} - r_i K}{1 + r_i} \tag{B.4}$$

where the depreciation is set to zero for simplicity. The capital that maximizes this present

value can be computed to be

$$K = \left(\frac{A^{\frac{1}{1-\beta}} \beta^{\frac{\beta}{1-\beta}} \alpha}{r_i \tilde{\tau}^{\frac{\beta}{1-\beta}}} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \tag{B.5}$$

The firm is owned by a representative household with utility $\frac{Y-SX}{1+r} - I$, where S is the social cost of carbon and r is the opportunity cost of investing. Hence, when the carbon tax equals the social cost, $\tau = S$, and the discount rate is $r_i = r$, then the market value of the firm equals its social value. In this case, all the firm's choices are socially optimal, both its ex ante investment choice, K^*, and its ex post pollution choice, X^*.

Suppose instead that the carbon tax is $\tau = 0$, and the firm has a sustainable discount rate of $r_i > r$, which can depend on the firm's characteristics, (A, α, β, ϕ). Sustainable investors seek to ensure the first-best capital choice, K^*. To achieve this capital choice, the investors must ensure that (B.5) gives K^* when the discount rate is r_i and $\tilde{\tau} = \phi$. Therefore, $r_i \phi^{\frac{\beta}{1-\beta}} = r(\phi + S)^{\frac{\beta}{1-\beta}}$, that is,

$$r_i = r \left(1 + \frac{S}{\phi} \right)^{\frac{\beta}{1-\beta}} \tag{B.6}$$

This discount rate increases in the ratio of social cost, S, to private cost, ϕ. The sensitivity to this ratio increases in β, because a high β means that output is more fuel-sensitive.

While this discount rate ensures that the firm chooses the first-best capital, the outcome is nevertheless distorted. Indeed, once the firm has chosen this capital at time 0, then, at time 1, it will choose the fuel in (B.3) based on the zero carbon tax, $\tilde{\tau} = \phi$, thus choosing a higher level of pollution than in the social optimum, $X > X^*$.

To get green finance to implement the social optimum, the economy needs a sustainable discount rate that depends on the firm's actions, $r_i(K, X)$. In particular, the firm value with no carbon tax and the discount rate of $r_i(K, X)$ must equal the social value for each K and

X. That is, $\frac{Y(K,X)+K}{1+r_i(K,X)} = v(K,X) = \frac{Y(K,X)+K-SX}{1+r}$, so solving for $r_i(K,X)$ gives

$$\begin{aligned} r_i(K,X) &= \frac{(1+r)(Y(K,X)+K)}{Y(K,X)+K-SX} - 1 \\ &= \frac{r(Y(K,X)+K)+SX}{Y(K,X)+K-SX} \\ &= \frac{r(Y(K,X)+K-SX)+(1+r)SX}{Y(K,X)+K-SX} \\ &= r + \frac{SX}{(Y(K,X)+K-SX)/(1+r)} = r + \frac{SX}{v(K,X)} \end{aligned} \tag{B.7}$$

as before.

C Equilibrium Price of Carbon Allowances

Until now, the model was based on an exogenous tax on carbon emissions, τ_{it}. Suppose instead that all firms must buy carbon allowances in order to emit carbon at an endogenous price of τ. Specifically, any firm must buy a number of carbon allowances equal to its emissions. Hence, with a price of carbon allowances given by τ, the firm's problem becomes the same as before, except that the common price of carbon allowances, τ, is now determined in equilibrium such that the sum of all emissions equals a fixed cap. Naturally, for every tax rate, there exists an equivalent level of the total carbon cap, and vice versa.

D Carbon Offsets

To model carbon offsets, suppose that a firm can reduce its carbon tax by buying carbon offsets of various qualities, $(o_{i,q})_{q\in[0,1]}$, where $o_{i,q}$ is the number of offsets of quality q bought by firm i. The firm faces a net carbon tax that depends on its emission less its allowed offsets, $\tau_{it}(X_{it} - \sum_{q\geq\bar{q}_i} o_q)^+$, where $\bar{q}_i$ is the minimal allowed quality for this firm (and the superscript "+" indicates that the tax can never be negative).

The price of an offset of quality q is denoted ϕ_q, and offsets are supplied by a representative producer of offsets with a linear production technology. This firm can produce offsets of quality q for each quality $q \in [0,1]$. A quality of $q = 1$ means that the producer reduces carbon one-for-one with the number of offsets sold, e.g. via carbon capture. A lower quality means that the carbon reduction is expected to be less than the claimed amount. Specifically, the quality q indicates the ratio of the actual expected carbon reduction to the claimed

amount. In equilibrium, offsets of quality q must have a price that is linear in quality, $\phi_q = \bar{\phi} q$, where $\bar{\phi} = \phi(1)$ is the marginal cost of producing true offsets. Otherwise, if $\phi_q < \phi(1)$ then offset sellers profit from selling a package of $1/q$ offsets of quality q as a single offset of quality 1. Vice versa if $\phi_q > \phi(1)$, offsets of quality 1 can be profitably split into $1/q$ offsets of quality q.

If firm i buys any offsets, then the firm buys offsets of the minimum quality $\bar{q}_i$ allowed since the price of offsets is increasing in quality q. Buying these cheapest-allowed offsets costs $\phi_{\bar{q}_i}$, so this is economical if the cost is lower than the carbon tax that is saved, $\phi_{\bar{q}_i} \leq \tau_{it}$. If so, the firm buys enough of these offsets to cover its entire carbon emission and is then able to state that it is "carbon neutral." Otherwise, if $\phi_{\bar{q}_i} > \tau_{it}$, it is cheaper to pay the carbon tax, and the firm buys zero offsets. Therefore, each firm i faces an "effective carbon price" of $\tilde{\tau}_{it} = \min\{\tau_{it}, \phi_{\bar{q}_i}\}$. Similarly, green and brown electricity companies face effective carbon prices of $\tilde{\tau}_{gt}$ and $\tilde{\tau}_{bt}$, respectively. From the perspective of the planner's problem, if true offset costs are above the externality cost, $\phi_1 \geq S_t$, then offsets are not socially useful, and otherwise all emissions should be offset. As a result, the planner's effective social cost of carbon is $\tilde{S}_t = \min\{S_t, \phi_1\}$. Hence, all results go through with carbon taxes, τ_{it}, replaced by the corresponding effective carbon price, $\tilde{\tau}_{it}$, and the social cost of carbon, S_t, replaced by $\tilde{S}_t$.

Further, offsets lead to some additional results. First, to reach the social optimum, all firms must only be allowed to buy high-quality offsets, $\bar{q}_i = 1$. In such a socially optimal equilibrium, all offsets with positive trading volume have the same price—because a ton of carbon is a ton of carbon.

In other words, all the price differences seen in real-world offset markets must reflect poor quality (greenwashing). In fact, the model predicts that the price is proportional to the quality.

The low and variable offset prices seen in the real world create distortions. To see why, suppose for example that some carbon offsets only reduce emissions by 10% of their claimed effect. Why is this a problem if such offsets only trade at a "fair" price of 10% of the price of fully credible offsets? The problem is that, if firms can buy such offsets to reduce their carbon tax one-for-one, then the firm's carbon tax is essentially reduced to 10% of the intended level, thus distorting the firm's incentive to reduce emissions.