Transition Risks

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Abstract

The ongoing transition to a low-carbon economy creates transition risks and opportunities for firms. We show that emission intensity is an insufficient predictor of a firm's transition risk; other factors are also critical, including market structure, where emissions are released in the value chain, and the drivers of decarbonization—whether driven by costly regulations or productivity-enhancing innovation. Modelling a supply chain with Melitz-type industries, we show that high emissions do not necessarily translate to high transition risks. Moreover, commonly used risk metrics can misclassify some firms as vulnerable when they might actually benefit from economy-wide decarbonization. We outline conditions under which a firm's Scope 1, 2, and 3 emissions accurately reflect its risk. We also identify scenarios in which aggregating these different emissions scopes enhances (or obscures) the true picture of a firm's transition risk. Our paper lays out a framework for evaluating and improving the wide variety of transition-risk metrics currently used to guide investor decisions, inform stakeholders, and comply with climate-risk disclosure mandates.

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1 Introduction

The ongoing shift to a low-carbon economy creates risks and opportunities for firms; the associated threats to firm profitability are often referred to as transition risks. With nearly all companies being potentially affected by a low carbon transition, transition risks are increasingly important to investors and asset managers, central banks, and government and public stakeholders.

Such risks and opportunities could be massive. Net-zero by 2050 is expected to require a tripling of renewable energy investments alone to ∼$5 trillion per year (**?**) and trillions more in other capital investments per year (**?**). Governments and central banks have concerns about financial stability risks arising from an unordered transition (**?**), and the risk of 'stranded assets' (**?**), while individual and institutional investors have an often strong interest in managing their own transition risks. Before the risks of a low-carbon transition can be managed, they must be understood, measured, and disclosed. Both industry-led and regulator-led approaches to this are widespread. Institutions with tens of trillions in assets under management have endorsed the Task Force on Climate-related Financial Disclosures recommendations on measuring and disclosing their transition risks (**?**), while the U.S., E.U., Japan, U.K., Australia, and many other countries require companies to disclose their climate risks and mitigation efforts (**??????**).

Such disclosures typically include a company's emissions, risk management strategies, and any metrics and targets used to manage risks. For example, the U.S. Securities and Exchange Commission has issued rules on how firms must disclose climate-related risks, including by disclosing their Scope 1 and 2—but not Scope 3, as initially proposed—emissions (**?**)[0]. A plethora of industry-led metrics are also in widespread, though not mandated, use. These include carbon intensity and emission metrics (**?**), ESG rating systems, (**?**), and ratings of firm-level emission trajectories (**???**), all of which can be used to assess transition risks.

Understanding and assessing transition risks is far from straightforward. Risks to firms, reflecting the complexity of any economy-wide transition, arise from the interaction of a number of causes and can affect firms through various channels. For example, which firms benefit and which lose out from declines in aggregate emissions can depend on whether emission declines result from regulations, technology improvements, or preference changes. How an individual firm is affected will depend on its own production structure and that of its competitors, and on the firm's upstream and downstream value chain linkages. Despite—or perhaps because of—this complexity, the disclosure and measurement of transition risks is commonly based on a firm's emissions and does not explicitly take into account the economic causes

[0] We define and discuss emission Scopes in Section 2.

or channels through which low-carbon transition risks are created. For example, in disclosing risks it is common practice to aggregate emissions released at different stages of a value chain—such as adding upstream Scope 2 emissions to direct Scope 1 emissions—into a single measure of a firm's emissions, and to average or compare such measures across diverse industries as in a portfolio's average carbon intensity. What is not clear, however, is whether all emissions in a value chain create equal economic risks to firms such that they can be simply added, or what conditions might affect how firms' emissions are comparable across different industries.

The scale of transition risks and the increasing importance of efforts to manage them raises a question that is the focus of this paper: what are the key economic features that create and mediate transition risks, and how should such risks be measured? In this paper we develop a structural Melitz-type model of transition risks to firms that incorporates different economic shocks that reduce emissions, and includes upstream and downstream value-chain linkages between firms (**?**). In our model, entry costs are endogenous—they vary depending on the mass of entrants. In this context, we show how shocks to productivity, cost or demand will drive adjustments along multiple margins: changes in the mass of entrants and varieties available to consumers, as well as the output (and profits) of operational firms. We use our model to lay out a microfounded economic framework of transition risks, including the causes of transition risks and the factors that mediate risks to firms, and how such factors can be incorporated into transition risk metrics by used industry, regulators, and investors.

Our model delivers several findings on how transition risks affect firms and which have significant implications for the trillions in investments informed by the disclosure of transition risks. First, different causes of emission reductions—such as policy induced cost increases versus technology improvements that raise the productivity of low-carbon technologies—create different types of transition risks. As a result, transition risks to a firm's profits can be both positively and negatively correlated with its own carbon emissions depending on the type of low-carbon transition that occurs.[1] This limits the usefulness of a firm's emissions as a stand-alone proxy for transition risks.

Second, the sign and magnitude of risks are also affected by where in its value chain a firm's emissions are affected. Emissions associated with the upstream production of inputs, directly released by a firm, or associated with the downstream use of its products interact with the type of transition in creating risks. The risks created by 1 tonne of CO_2eq depend on whether they are Scope 1, 2, or 3 emissions; thus adding different emission scopes together can lead to misleading conclusions about risks. Of particular note is that we find risks from downstream emissions do not need to have the same sign or magnitude

[1] Following current industry practice we use "carbon emissions" and "greenhouse gas emissions" interchangeably.

as direct or upstream emissions. This suggests Scope 3 emissions should be unbundled into upstream and downstream emissions, rather than the current practice of summing all (non-electricity) emissions that are external to the company, regardless of whether they occur upstream or downstream.

Third, market structure also mediates risks. This includes factors like market concentration, barriers to entry, demand elasticities, and substitutability between inputs in addition to whether a firm sells relatively 'brown' goods that are emission intensive, or relatively 'green' goods that are not. Intuitively, whether a sold product is a green substitute for downstream emissions or the brown emission-intensive good itself is critical to risk, even when comparing firm's with the same Scope 1, 2, and 3 emissions.

Combined, these findings show how higher emissions do not necessarily mean higher transition risks—even compared to similar companies in the same industry. These are potentially significant limitations to the widespread measurement of emissions in assessing transition risks, to aggregating firms' Scope 1, 2, and 3 emissions, and to reporting the average emission intensity of investment portfolios. Our results also provide guidance on how to improve the disclosure of transition risks. They support the use of taxonomy-based systems like **?** as an important factor in interpreting a firm's emissions and classifying risks. Our results also clarify the conditions under which aggregating emission Scopes, and averaging emissions across firms in a portfolio, can be done without substantial risk of creating misleading risk measures.

A broad literature going back to **?** studies how economic shocks and technological transitions affect firms. Two features distinguish a low-carbon transition in particular. The first feature is the incredible breadth of low-carbon transitions. Firms can be directly affected by shifts in demand, technological changes, and punitive and supportive policies. Importantly, each firm typically emits only a small share of the lifecycle emissions of the product or service it sells. As a result, how demand, technology, and policy changes indirectly affect firms by propagating impacts through value chains is critical to understanding the risks firms face. In considering how upstream and downstream shocks propagate through value chains to affect firms, our approach has similarities to work by **?** and **?** which study how economic shocks propagate through production networks. However, in order to study on how transition risks are created and how they related to emissions we do not incorporate a full multi-sector network approach. Our work also has similarities to **?** who qualitatively discuss some of the same transition risk channels that we model.

The second feature is that declines in carbon emissions are, of course, central to a low-carbon transition. Our model shows how carbon emissions are related to transition risks through the interaction of policy, technology, and demand shocks and firm's production structure and value-chain linkages. That the level or intensity of emissions is not a straight-forward proxy for transition risks is critical to a

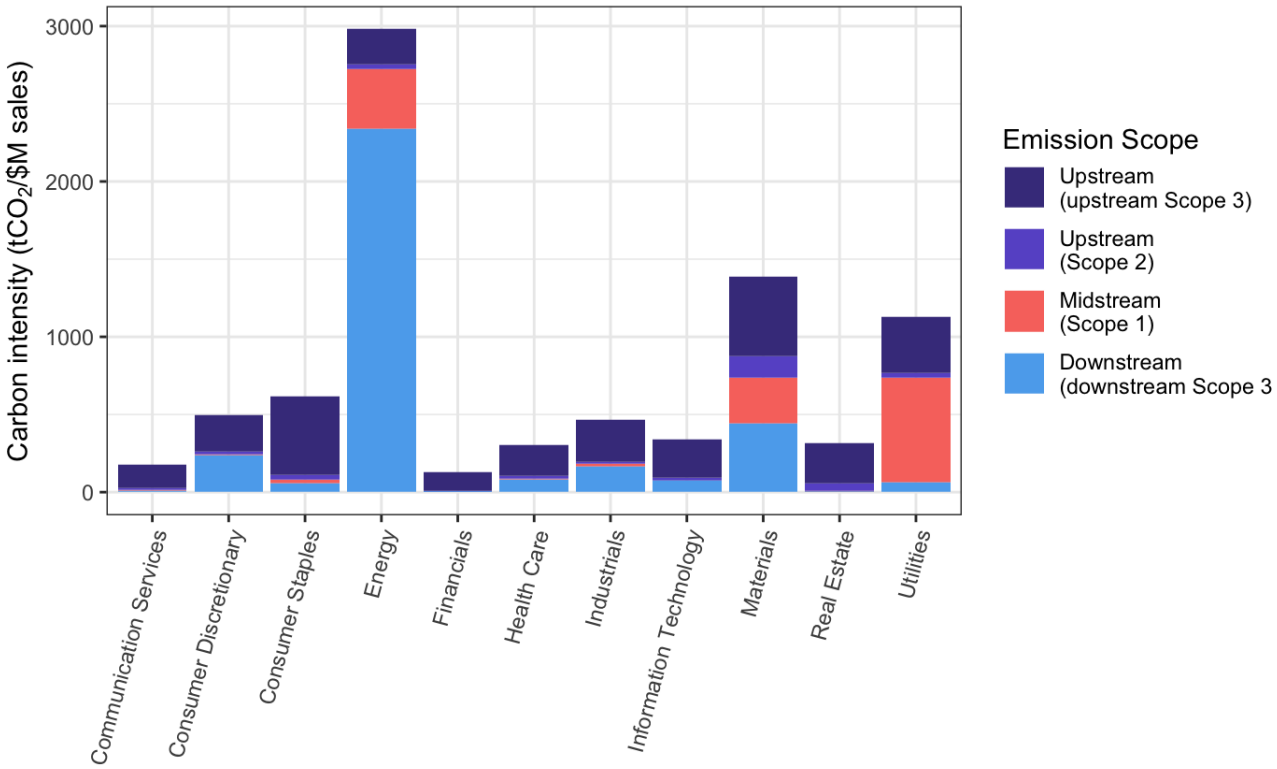
wide variety of academic and industry work on transition risks. This includes how climate policy risk can affect the financial system (**?**) and cause macroeconomic impacts (**?**), and the scale of stranded asset risk (**?**). Regulators are increasingly requiring firms to disclose their emissions (**?**), and firms representing two-thirds of global market capitalization already disclose emissions through CDP (**?**). Financial products frequently combine emissions across diverse firms and emission scopes into single summary 'financed emission' or 'carbon footprinting' statistics (**???**). Other metrics assess risk or climate mitigation progress by comparing a company's emissions against a carbon budget derived from scenarios of a future low-carbon economy (**???**). Hundreds of other transition risk metrics have been developed by practitioners (**??**) and our work may help explain previous findings that risk metrics can diverge substantially from each other (**???**). That we find different policy shocks can have a different sign and magnitude of correlations with a firm's emissions is also important to revealed-preference transition risk measures. For example, how asset prices are correlated with price movements of other brown or green firms (**???**) or through quantifying the frequency of climate discussions in earnings reports (**?**).

2 The scale and scope of firms' emissions

Carbon emissions are central to the challenge of climate change and transition risks. Three Scopes of emissions have been defined by the **?** to standardise the disclosure and comparison of emissions across firms. Scope 1 emissions are those released directly by a company from its own operations, Scope 2 are emissions released during the production of purchased or acquired electricity, steam, heat and cooling used by a company, and Scope 3 emissions "are all indirect emissions (not included in scope 2) that occur in the value chain of the reporting company, including both upstream and downstream" (**?**).

While Scope 1, 2, and 3 are relatively simple categories they are not based on obviously economically meaningful distinctions between emissions. In particular, Scope 2 and 3 both include upstream emissions, while Scope 3 combines both upstream and downstream emissions. In considering transition risks arising from value chains it will be important to differentiate between upstream and downstream emissions. To do so we define three mutually exclusive categories of emissions: Midstream emissions are those released directly by a firm and are equivalent to Scope 1 emissions. Upstream emissions are those occurring upstream in a value chain from the production of a firm's inputs; these consist of upstream Scope 3 emissions and Scope 2 emissions. Downstream emissions are those released downstream in a firm's value chain and are the downstream component of Scope 3 emissions. Downstream emissions are primarily from the transportation, use, and disposal of sold products (**?**).

Figure 1: Median Carbon Intensity



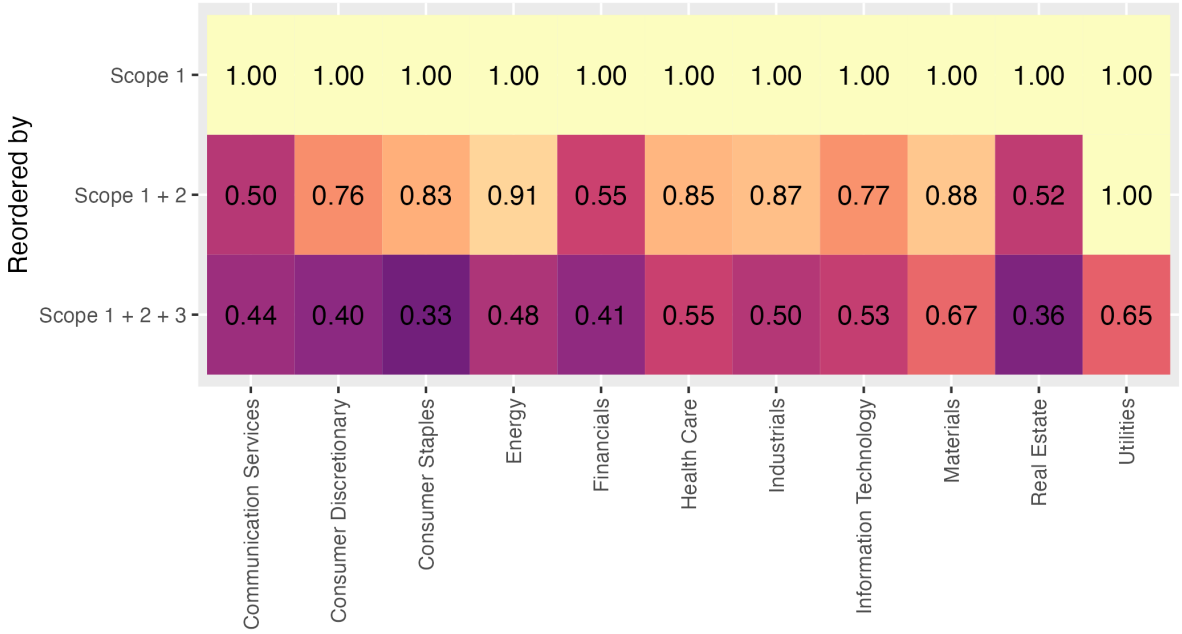
Median carbon intensity by emission Scope within each GICS sector. Data on firm's annual emissions is compiled by MSCI (**?**) with data on sales, revenue, and GICS sector from Refinitiv Eikon (**?**). We have Scope 1 and 2 data for 8,653 publicly traded companies, and complete Scope 3 data for a 2,225 firm subset of these. Scope 3 emissions data are a mix of emissions disclosed by firms and emissions estimated by MSCI. MSCI reports upstream and downstream emissions separately.

Figure 1 shows the median emissions intensity in tonnes per million USD sales across firms within GICS sectors for Upstream, Midstream, and Downstream emissions, as well as Scope 1, 2, and 3 categories[2]. Among these firms 87% of emissions are Scope 3, 3% are Scope 2, and only 11% come from Scope 1. Firms in the Energy, Materials, and Utilities sectors have an unsurprisingly substantially higher emissions intensity, and the importance of Upstream versus Downstream emissions varies notably across sectors; MSCI estimates that 29% of Scope 3 are upstream emissions and 71% are downstream emissions.

A key question in assessing transition risks is what Scopes should be included when measuring a firm's emissions? Different sources of risk may depend on different emission Scopes, and some emissions like Scope 2 emissions from electricity could have lower or higher abatement costs than other emission Scopes. Which emission Scopes to use would not be of particular importance if the choice did not

[2]note that "Energy" in the GICS definition is predominantly fossil fuel extraction and processing firms. Renewable energy firms are categorised to other sectors.

Figure 2: Probability of remaining in the top quartile



Fraction of firms in the top quartile of Scope 1 Carbon Intensity (Scope 1 emissions normalised by revenue) remaining in the top quartile when they are ordered by, respectively, Scope 1 + Scope 2 Carbon Intensity, and Scope 1 + 2 + 3 Carbon Intensity.

change the ordering of which firms were considered to have 'high' emissions; for example, if firms with high Scope 1 necessarily also have high Scope 2 and 3, there would be no need to include other Scopes in comparing firms. However, in Figure 2 we show that which Scope is included when measuring a firm's emissions has a large effect on which firms are considered to have 'high' emissions. Figure 2 shows the fraction of firms in the top quartile of Scope 1 carbon intensity within a GICS sector that remain in the top quartile after reordering firms by the indicated alternative emissions Scope. For example, only 44% of Communication Services firms in the top quartile of emissions measured using Scope 1 carbon intensity are still in the top quartile when their emissions also include Scope 2 and 3 emissions. Note this is a substantial change in which firms have high emissions as 25% of firms will remain in the top quartile even when firms are ordered completely at random. The substantial reordering of firms as additional emission Scopes are included is particularly notable given Scope 1 emissions are still included in all measures shown in Figure 2. This demonstrates that the Scope of emissions included has a very large effect on which companies are considered 'high' emissions, which will affect all risk metrics and conclusions derived from comparing firms' emissions.[3]

[3]We also find similar results showing the Scope of emissions has a large effect on which firms are considered high emissions when calculating Kendall's rank correlation coefficient for the above emission categories.

3 Model of a single industry

In order to illustrate the various channels through which carbon intensity correlates with profit risk from green transition shocks, we develop a model of a closed economy with multiple industries. We begin by examining a representative industry—industry k. In subsequent sections will expand the analysis to allow for multiple, interacting industries that are either part of the same value chain or act as suppliers of complementary and/or substitute products. We begin our analysis of the representative industry by looking at an operational firm in industry k.

3.0.1 Firms

There is a mass M_k of firms operating in industry k producing differentiated goods that contribute to a composite

$$Q_k = \left[\int_0^{M_k} {q_{ki}}^{\frac{\gamma_k - 1}{\gamma_k}} di \right]^{\frac{\gamma_k}{\gamma_k - 1}} \tag{1}$$

where q_{ki} is consumption of variety i produced by industry k and $\gamma_k > 1$ gives the elasticity of substitution between industry k's varieties.

Let R_k denote spending on the mass M_k varieties offered by industry k and let Ω_k be the set of varieties available from industry k. Maximizing Q_k subject to the budget constraint $\int_{i \in \Omega_k} p_{ik} q_{ik} di \leq R_k$ yields demand for variety ik as follows:

$$q_{ik} = \frac{R_k {p_{ik}}^{-\gamma_k}}{{P_k}^{1-\gamma_k}} \tag{2}$$

where

$$P_k \equiv \left[\int_{j \in \Omega_k} {p_{jk}}^{1-\gamma_k} dj \right]^{\frac{1}{1-\gamma_k}} \tag{3}$$

is the price index for composite k; the maximized value of Q_k thus solves $R_k = Q_k P_k$.

Let c_{ik} be the marginal cost facing firm i in industry k. Throughout this paper we assume there are a sufficiently large number of firms in each industry so that each individual firm takes their industry's price index, spending, and mass of varieties as outside their own influence.[4] This "small firm" assumption, combined with the CES nature of preferences in the industry, translates to individual firms perceiving

[4] This "small firm" assumption rules out intentional raising rivals' costs (RRC) strategies and a host of other strategic interactions.

their demand to be isoelastic, generating the standard markup rule

$$p_{ik} = \frac{\gamma_k}{\gamma_k - 1} c_{ik}. \tag{4}$$

Suppose that, as in **?**, the marginal cost c_{ik} facing firm i in industry k is given by c_k/θ_i where c_k is common to all firms in that industry and θ_i is a firm-specific productivity parameter. c_k measures the industry's "raw" marginal cost. In Melitz-type models, θ_i is typically a realization of a random variable. We assume θ is Pareto distributed with probability distribution function $g_k(\theta)$ and cumulative density function $G_k(\theta))$ given by

$$\begin{aligned} g_k(\theta) &= (1+\eta_k)(\gamma_k - 1)\theta^{-(\gamma_k + \eta_k(\gamma_k - 1))} \\ 1 - G_k(\theta) &= \theta^{-(1+\eta_k)(\gamma_k - 1)} \end{aligned} \tag{5}$$

for $\theta \geq 1$ where $\eta_k > 0$. As c_k is common to all firms in industry k, we can drop the i subscripts and characterize a firm by its productivity θ and industry. Recalling equation (4), a firm with productivity θ will charge

$$p(\theta) = \frac{\gamma_k}{\gamma_k - 1} \frac{c_k}{\theta}. \tag{6}$$

Let π_{ik} denote revenues-less-variable costs. If we assume constant marginal cost, then, by (2) and (4),

$$\pi_k(\theta) = \left[p_k(\theta) - \frac{c_k}{\theta}\right] q_k(\theta) = \frac{p_k(\theta) q_k(\theta)}{\gamma_k} = \frac{R_k [p_k(\theta)]^{1-\gamma_k}}{\gamma_k {P_k}^{1-\gamma_k}} \tag{7}$$

and so revenues-less-variable costs ends up being a simple scalar of the firm's revenues.

Also as in **?**, assume that operating in industry k involves a fixed cost F_k. Define θ_k^* as the lowest productivity level at which a firm can operate profitably in industry k. The variable θ_k^* will satisfy the Zero Cutoff Profit (ZCP) condition

$$\pi_k(\theta_k^*) = F_k \tag{8}$$

which, using (7), can be written as

$$\frac{R_k [p_k(\theta_k^*)]^{1-\gamma_k}}{\gamma_k {P_k}^{1-\gamma_k}} = F_k \tag{9}$$

where, from (3), the price index for industry k is[5]

$$P_k = \left[\int_{\theta_k^*}^{\infty} \frac{g_k(\theta_k)M_k}{1-G_k(\theta_k^*)} p_k(\theta)^{1-\gamma_k} d\theta\right]^{\frac{1}{1-\gamma_k}} \tag{10}$$

$$\equiv \mathcal{M}_k \frac{c_k}{\theta_k^a} \tag{11}$$

in which we make use of the following variable definitions:

$$\theta_k^a \equiv \left[\int_{\theta_k^*}^{\infty} \frac{g_k(\theta)}{1-G_k(\theta_k^*)} \theta^{\gamma_k-1} d\theta\right]^{\frac{1}{\gamma_k-1}} \tag{12}$$

is a weighted mean measuring average productivity in industry k;[6]

$$\mathcal{M}_k \equiv M_k^{\frac{1}{1-\gamma_k}} \frac{\gamma_k}{(\gamma_k-1)}$$

measures the extent to which direct markups and love-for-variety magnify (or shrink) the average costs $\frac{c_k}{\theta_k^a}$ to arrive at a price index for industry k.

3.0.2 Entry

Any firm aspiring to operate in industry k must pay an up-front cost F_k^E before it learns its θ, i.e. how productive it would be should it operate in that industry. In most Melitz-based models, the mass of entrants E into an industry is endogenous; the mass of aspiring entrepreneurs entering the industry depends on expected returns from entry. We parameterize entry as follows: let $F_k^E(E_k)$ denote the entry costs for industry k and define

$$\xi_k(E_k) \equiv \frac{dF_k^E}{dE_k}\frac{E_k}{F_k^E} \tag{13}$$

as the elasticity of the entry cost with respect to the mass of entrants. Classic Melitz-type models assume entry costs are unvarying and so ξ_k will be zero in these models. In contrast, in models following **?**, ξ_k is effectively infinite.[7] We assume ξ_k is non-negative. Aspiring firms enter the industry until the expected payoff from entry equals F_k^E. As we show in Appendix A.2, the Free Entry condition can be

[5] By equations (2) and (10) we can write the total amount of composite-k produced as

$$Q_k = M_k^{\frac{\gamma_k}{\gamma_k-1}} q_k(\theta_k^a).$$

[6] As $q_k(\theta)/q_k(\theta_k^a) = \theta/\theta_k^a$ then the weights are simply firms' relative output.

[7] The function $F_k^E(E_k)$ would be a step function with $F_k^E = 0$ when E_k is less than some exogenously determined value $\bar{E}_k$ and F_k^E is arbitrarily high whenever E_k exceeds $\bar{E}_k$.

written as the following function of the mass of entrants E_k, and the cutoff productivity θ_k^*:

$$F_k^E \left(\frac{M_k}{1 - G_k(\theta_k^*)} \right) = \frac{[1 - G_k(\theta_k^*)]F_k}{\eta_k} \tag{14}$$

Entrants to industry k with θ below θ_k^* will exit immediately so that the mass of firms operating, M_k, is

$$M_k = E_k[1 - G_k(\theta_k^*)]. \tag{15}$$

Using (5) and its CDF we have that

$$\theta_k^a = \theta_k^* \left[\frac{1 + \eta_k}{\eta_k} \right]^{\frac{1}{\gamma_k - 1}}. \tag{16}$$

Note that using (9) and (6) to replace terms in (7) shows how revenue-less-variable costs of a firm operating in the industry depends critically on the operating cost F_k and cutoff productivity θ_k^*:

$$\pi_k(\theta) = F_k \left[\frac{\theta}{\theta_k^*} \right]^{\gamma_k - 1}. \tag{17}$$

Given F_k is constant, θ_k^* is a sufficient statistic for the profits, $\pi_k(\theta)$, of a firm with productivity θ operating in industry k. As per (17), operating profits will fall as the industry gets tougher, i.e. as θ_k^* rises.

3.0.3 Costs

We close out our description of industry k by discussing costs. Generating a unit of raw output entails combining $j = 1, ..., J$ inputs in a CES fashion.[8] It is straightforward to show that a firm's raw marginal costs c_k will take the form

$$c_k = \left[\sum_{j=1}^{J} \left[\frac{w_{kj}}{\mu_{kj}} \right]^{1 - \delta_k} \right]^{\frac{1}{1 - \delta_k}} \tag{19}$$

in which $\delta_k \geq 0$ is the elasticity of substitution between inputs in industry k and the μ_{kj} are input-productivity weights. Correspondingly, the unit-input requirement—i.e. amount of input j needed in

[8] Specifically, assume the raw output of a firm in industry k using inputs $l_1, l_2, ..., l_J$ is

$$\left[\sum_{j=1}^{J} \left[\mu_{kj} l_{kj} \right]^{\frac{\delta_k - 1}{\delta_k}} \right]^{\frac{\delta_k}{\delta_k - 1}} \tag{18}$$

where the μ_{kj} are factor-productivity weights and δ_k is the elasticity of substitution between inputs in industry k.

order to produce one unit of raw output in industry k—is

$$a_{kj} = \frac{{w_{kj}}^{-\delta_k} {\mu_{kj}}^{\delta_k - 1}}{{c_k}^{-\delta_k}}. \tag{20}$$

Note for completeness that, for a firm with productivity θ operating in industry k, the minimized cost of producing one unit of output is c_k/θ.

Let w_{kj} measure net prices for factor j:

$$w_{kj} = e_{kj}\tau_{kj} + r_{kj} \tag{21}$$

where r_{kj} is the market price for input j in industry k, e_{kj} measures emissions arising due to use of one unit of input j in industry k and τ_{kj} is the effective tax rate on said emissions. Treating e_{kj}, r_{kj}, τ_{kj} as parameters, these equations define the unknowns—π, c_k, p_k, P_k, M_k, q_k, R_k, θ_k^*, θ_k^a, w_{kj}, and F_k^E.

In the next section we examine percentage changes rather than changes in levels.

3.1 Changes in firm profits

Our goal is to evaluate how the profits of an operational firm and the size of an industry are affected by the transition to a low-carbon economy. We use changes in emission prices and technological innovations as general proxies for the main ways a transition is likely to occur, which includes regulation-based cost increases on emission intensive activities and final consumption, carbon pricing, subsidies of green alternatives, and technological changes that lower the cost of 'green' products or reduce the emissions intensity of production and consumption.

To see these effects we start by log-linearizing the equilibrium conditions. Let the symbol $\widehat{\ }$ denote percentage change of a variable, e.g. $\widehat{M_k} \equiv \frac{dM_k}{M_k}$ measures the percentage change in the mass of varieties available (and firms operating) in industry k.

Define $-\nu_k$ as the price elasticity of demand for composite Q_k with respect to P_k, i.e.

$$\nu_k \equiv -\frac{\partial Q_k}{\partial P_k}\frac{P_k}{Q_k}. \tag{22}$$

Throughout this paper we maintain the assumption $\gamma_k > \nu_k$, which stipulates that demand for a product within industry k is more elastic than demand for the industry's composite as a whole. We also assume ν_k is non-negative and not equal to unity.

Log differentiating the equilibrium conditions (Appendix A.3) yields several equations describing

how shocks affect firms.

Define ι_{kj} as input j's share of a k-industry firm's costs

$$\iota_{kj} \equiv \frac{w_{kj} a_{kj}}{c_k} = \left[\frac{\frac{w_{kj}}{\mu_{kj}}}{c_k} \right]^{1-\delta_k} \tag{23}$$

and ψ_{kj} as the share of pollution fees in the net price of factor j in industry k

$$\psi_{kj} \equiv \frac{\tau_{kj} e_{kj}}{w_{kj}} \tag{24}$$

Log differentiating the cost function (19) yields

$$\widehat{c_k} = \frac{\sum_{j=1}^{J} \left[\frac{w_{kj}}{\mu_{kj}} \right]^{1-\delta_k} [\widehat{w_{kj}} - \widehat{\mu_{kj}}]}{\sum_{j=1}^{J} \left[\frac{w_{kj}}{\mu_{kj}} \right]^{1-\delta_k}} \tag{25}$$

$$= \sum_{j=1}^{J} \iota_{kj} [\widehat{w_{kj}} - \widehat{\mu_{kj}}] \tag{26}$$

and the net factor-price equation (21) yields

$$\widehat{w_{kj}} = \psi_{kj} [\widehat{\tau_{kj}} + \widehat{e_{kj}}] + [1 - \psi_{kj}] \widehat{r_{kj}} \tag{27}$$

Changes in costs affect firm entry and exit, and change the cutoff productivity for firms to remain profitable. Taking the derivative of the Free Entry condition (Appendix A.5) yields the change in mass of firms

$$\widehat{M_k} = -\mathcal{Z}_k (\gamma_k - 1) \widehat{\theta_k^*} \tag{28}$$

and how the cutoff productivity changes with $\widehat{c_k}$

$$\widehat{\theta_k^*} = \mathcal{H}_k (\nu_k - 1) \widehat{c_k}. \tag{29}$$

where $\mathcal{Z}_k$ and $\mathcal{H}_k$ are composite structural parameters given by[9]

[9] $\mathcal{H}_k$ is only a parameter in the case of isoelastic demand.

$$\mathcal{Z}_k \equiv \frac{1+\xi_k}{\xi_k}[1+\eta_k] > 1 \tag{30}$$

$$\mathcal{H}_k \equiv \frac{1}{(\nu_k - 1) + \mathcal{Z}_k(\gamma_k - \nu_k)} \tag{31}$$

where ξ_k is the elasticity of entry costs to the mass of firms, ν_k the price elasticity of demand for composite Q_k, γ_k the elasticity of substitution between industry k firms, and η_k governs the Pareto distribution firm productivity. $\mathcal{H}_k$ is positive so the effect of a cost rise on the cutoff productivity θ_k^* depends on the sign of $\nu_k - 1$: if demand for composite k is price elastic (inelastic), then higher production costs translate to an industry in which low-productivity firms find it harder (easier) to break even. It's worth noting, however, that regardless of whether demand for composite k is price-elastic or -inelastic, increased production costs necessarily raise the cost of composite k:

$$\widehat{P_k} = \frac{\mathcal{Z}_k[\gamma_k - 1]}{\nu_k - 1 + \mathcal{Z}_k[\gamma_k - \nu_k]}\widehat{c_k} \tag{32}$$

$$\widehat{P_k} \equiv \mathcal{A}_k \widehat{c_k} \tag{33}$$

$\mathcal{A}_k$ governs how much the cost of composite k rises; when demand for composite k is price-elastic, $\mathcal{A}_k > 1$, and when demand is price-inelastic $0 < \mathcal{A}_k < 1$.

Lastly, log differentiating the Zero Cutoff Profit condition 8 shows that profits earned by an operational firm with productivity θ are decreasing in the cutoff productivity level θ_k^*:

$$\widehat{\pi_k(\theta)} = -[\gamma_k - 1]\widehat{\theta_k^*}$$

Together, these equations yield how the profits earned by an operational firm change as a function of tax rates, productivity, emission intensities, and input prices

$$\widehat{\pi_k(\theta)} = -(\gamma_k - 1)\mathcal{H}_k(\nu_k - 1)\sum_{j=1}^{J} \iota_{kj}\Big[\psi_{kj}[\widehat{\tau_{kj}} + \widehat{e_{kj}}] + [1-\psi_{kj}]\widehat{r_{kj}} - \widehat{\mu_{kj}}\Big]. \tag{34}$$

How much a firm's profits change in response to shocks is mediated by $\mathcal{H}_k$. $\mathcal{H}_k$ depends on several parameters; these include the elasticity of demand for industry k's output ν_k, the elasticity of entry costs to the mass of firms ξ_k, and the elasticity of substitution across industry k varieties γ_k. Together these parameters govern how the much the cutoff productivity θ_k^* increases with an increase in raw

marginal costs c_k, and as a result, $\mathcal{H}_k$ scales how much firm profits fall as raw costs increase.

In summary, equations (28), (29) and (34) tell us how industry size, productivity, and profits respond to changes in taxes, emission intensity, input prices, and input productivity. In looking at each of these shocks in turn we will want to see how risks to a firm's profits are related to its carbon emissions.

3.2 A firm's carbon emissions

As discussed in Section 2 a firm's carbon intensity is commonly defined by practitioners as the firm's emissions normalised by the firm's value. To define carbon intensity in line with common practice we assume that a firm's value is proportional to the annuity value of its stream of profits. We abstract away from other factors that affect firm value. As the annuity value of a firm's stream of profits will be proportional to their annual profits, we drop the scaling factor between profits and firm value and define a firm's carbon intensity as its emissions normalized by its profits. We introduce a firm's midstream emissions and midstream carbon intensity here, and later discuss upstream and downstream emissions.

The emissions z_k released directly by a firm in industry k with productivity θ depend on the emissions-intensity of its inputs, its unit input requirements, and quantity of output produced, and are given by

$$z_k(\theta) = \sum_{j=1}^{J} e_{kj} a_{kj} \frac{q_k(\theta)}{\theta}$$

Expressing profits $\pi_k(\theta)$ in terms of output and markups on marginal costs using equations (6) and (7) an average industry k firm's midstream carbon intensity SM_{CI} is defined as[10]

$$SM_{CI} \equiv \frac{z_k(\theta)}{\pi_k(\theta)} = \frac{\sum_{j=1}^{J} e_{kj} a_{kj} \frac{q_k(\theta)}{\theta}}{\frac{c_k q_k(\theta)}{(\gamma_k - 1)\theta}} \tag{35}$$

$$= (\gamma_k - 1) \frac{\sum_{j=1}^{J} e_{kj} a_{kj}}{c_k} \tag{36}$$

SM_{CI} is equivalent to a firm's Scope 1 carbon intensity as a firm's direct midstream emissions are its Scope 1 emissions. A firm's upstream carbon intensity SU_{CI} and downstream carbon intensity SD_{CI} will be defined similarly. SU_{CI} will be the upstream emission attributed to a midstream firm normalised by the midstream firm's profits, while SD_{CI} will be the downstream emissions attributed to a firm normalized by the midstream firm's profits.

[10] see Appendix A.7 for details.

3.3 The direct effects of a transition

In this subsection we consider how a low-carbon transition that directly impacts a firm creates risks to firm profits through changes in technology and regulatory costs. To help compare results between upstream, midstream, and downstream shocks, we focus on a firm in industry $k = m$, where m denotes a midstream industry. We discuss how firm profits are affected by various midstream shocks and in Section 7 we compare results to upstream and downstream shocks and elaborate on the implications for measuring and disclosing transition risks.

We restrict our attention to cases where demand for the industry-m composite is elastic, $\nu_m > 1$.

How firm profits change due to various shocks is given by equation (34). We consider the effects of a change in emission prices $\widehat{\tau_{mj}}$, emissions intensity $\widehat{e_{mj}}$, and productivity $\widehat{\mu_{mj}}$ separately. We show the effects of an emissions price and productivity shock in equations (37)-(39), and leave the similar analysis of transition risks from changes in the input-emissions intensity e_{mj} to Appendix D.[11] We also express these changes in profits in terms of a firm's SM_{CI} using (36); in doing so we simplify by assuming that only a single input produces emissions. To clarify notation we denote brown inputs that produce emissions (and general inputs when the distinction is not necessary) with a regular subscript such as j, and denote green inputs that do not produce emissions with j', such that $e_{mj} \neq 0$ and $e_{mj'} = 0$. For some shocks we also express changes in profits both in terms of key model parameters and expressed in terms of $SMCI$, which are equivalent equations. $\oplus$ and $\ominus$ summarise the sign all of terms in the equation other than the listed shock; for example, $\ominus \cdot \widehat{\tau_{mj}}$ means an increase in τ_{mj} results in a decrease in π_m.

$$\text{Carbon price:} \quad \begin{cases} \widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m(\nu_m - 1)\sum_{j=1}^{J} \frac{e_{mj}a_{mj}}{c_m}\tau_{mj}\widehat{\tau_{mj}} & = \ominus\, \widehat{\tau_{mj}} \\ \widehat{\pi_m} = -\mathcal{H}_m(\nu_m - 1)SM_{CI}\tau_{mj}\widehat{\tau_{mj}} & = \ominus\, \widehat{\tau_{mj}} \end{cases} \tag{37}$$

$$\text{Green-input productivity:} \quad \begin{cases} \widehat{\pi_m} = (\gamma_m - 1)\mathcal{H}_m(\nu_m - 1)\sum_{j=1}^{J} \frac{w_{mj'}a_{mj'}}{c_m}\widehat{\mu_{mj'}} & = \oplus\, \widehat{\mu_{mj'}} \end{cases} \tag{38}$$

$$\text{Brown-input productivity:} \quad \begin{cases} \widehat{\pi_m} = (\gamma_m - 1)(\mathcal{H}_m(\nu_m - 1)\sum_{j=1}^{J} \frac{w_{mj}a_{mj}}{c_m}\widehat{\mu_{mj}} & = \oplus\, \widehat{\mu_{mj}} \\ \widehat{\pi_m} = \mathcal{H}_m(\nu_m - 1)SM_{CI}\frac{w_{mj}}{e_{mj}}\widehat{\mu_{mj}} & = \oplus\, \widehat{\mu_{mj}} \end{cases} \tag{39}$$

[11] We show the associated changes in firms' output-emissions intensity due to these shocks in Appendix A.8.

Increases in carbon prices

An increase in emission price τ_{mj} on a firm's directly released midstream emissions raises the productivity cutoff θ_m^* and shrinks the mass of firms operating and reduces profits for operational firms. The change in industry m firm profits in (37) show that the larger the share of input costs affected by the emission price $\frac{e_{mj}a_{mj}}{c_m}$, or the larger the emissions intensity SM_{CI}, the greater the reduction in profits from an increase in τ_{mj}. This is as expected; the more emission intensive a firm is—reflected in its SM_{CI}—the greater risk the firm faces from increases in the cost of its emissions.[12]

However, (37) also reflects a limitation in using a firm's carbon intensity SM_{CI} as a proxy for transition risks as the scale of risk is mediated by the composite parameter $\mathcal{H}_m$. As noted in (31), $\mathcal{H}_m$ depends on the price elasticity of demand, elasticity of entry costs to the mass of entrants, and the elasticity of substitution between inputs—all of which can vary across industries. Thus firms in different industries experiencing the same carbon price face different risks to their profits as mediated by $\mathcal{H}_m$—even if they have the same SM_{CI} carbon intensity.

Input productivity changes

We consider the effects of a change in input-productivity μ_{mj} on a brown emissions intensive input and in $\mu_{mj'}$ on an alternative green input. Innovations in input productivity, whether to a green or brown input, raise a firm's profits as per equations (38) and (39). While an increase in $\mu_{kj'}$ for a green input necessarily reduces a firm's output-emissions intensity, increase in μ_{kj} for an emissions intensive input can either raise or lower the output-emissions intensity depending on relative input prices and δ_k (Appendix A.8). Importantly, note that SM_{CI} has no direct relationship to the change in profits from a green innovation i.e. it does not appear in (38). The effect of green innovations on firm profit depends on the share of costs coming from the green input—not the emissions intensity or share of costs coming from a brown input. As a result, while a firm's carbon intensity SM_{CI} can be used as a proxy for risks from increases in regulatory costs, it is not a proxy for risks or benefits to firms from green innovations. We elaborate on this later in Section 7.

[12]Note that if an industry m firm is less emission intensive than a competitor its profits might rise due to a carbon price but this channel is reflected through downstream demand.

4 Upstream value chain transition risks

4.1 Modelling upstream transition risks

Transition risks are also created through shocks that affect the emissions released during the production of a midstream firm's inputs; that is, shocks that occur upstream in a firm's value chain. We study upstream shocks by assuming upstream firms produce output using the same general production structure as the midstream firm in Section 3. For this analysis we look at a single midstream industry m and let there be U upstream industries, indexed $u = 1, \ldots, U$.

A firm in industry m uses the upstream composites as its only inputs; the firm's cost of using one unit of composite u is simply P_u. Setting $r_{mj} = P_u$ and summing across all U composites in equation (34) gives the change in midstream firm profits due to upstream shocks:

$$\widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m(\nu_m - 1)\sum_{u=1}^{U} \iota_{mu}(1 - \psi_{mu})\widehat{P_u}. \tag{40}$$

To solve for $\widehat{P_u}$, let L_u denote the number of inputs used by upstream industry u; then equations (26) and (32) allow us to write $\widehat{P_u}$ as the weighted sum of shocks to upstream industry u:

$$\begin{aligned} \widehat{P_u} &= \mathcal{A}_u \sum_{l=1}^{L_u} \iota_{ul}(\widehat{w_{ul}} - \widehat{\mu_{ul}}) \\ &= \mathcal{A}_u \sum_{l=1}^{L_u} \iota_{ul}\Big[\psi_{ul}[\widehat{\tau_{ul}} + \widehat{e_{ul}}] + [1 - \psi_{ul}]\widehat{r_{ul}} - \widehat{\mu_{ul}}\Big]. \end{aligned} \tag{41}$$

4.2 Upstream emissions

The upstream emissions attributed to a midstream firm are those released during the production of the goods ultimately used as inputs by the midstream firm. Producing the midstream output quantity $\frac{q_m(\theta_m)}{\theta_m}$ requires a quantity of input j given by

$$l_{mj} = a_{mj}\frac{q_m(\theta_m)}{\theta_m}$$

with j purchased from an upstream firm in industry u. We assume that input j is purchased from an average upstream firm such that

$$l_{mj} = q_u(\theta_u^a).$$

Production of $q_u(\theta_u^a)$ releases upstream emissions of

$$z_u(\theta_u^a) = \sum_{l=1}^{L} e_{ul} a_{ul} \frac{q_u(\theta_u^a)}{\theta_u^a}.$$

Thus the upstream emissions attributed to the midstream firm from the production of a single input are

$$\begin{aligned} z_u^{\text{from upstream input j}}(\theta_m) &= \sum_{l=1}^{L} e_{ul} a_{ul} \frac{q_u(\theta_u^a)}{\theta_u^a} && (42)\\ &= \sum_{l=1}^{L} e_{ul} a_{ul} \frac{l_{mj}}{\theta_u^a} && (43)\\ &= \sum_{l=1}^{L} e_{ul} a_{ul} \frac{a_{mj} \frac{q_m(\theta_m)}{\theta_m}}{\theta_u^a} && (44)\\ &= \frac{a_{mj} q_m(\theta_m)}{\theta_u^a \theta_m} \sum_{l=1}^{L} e_{ul} a_{ul}. && (45) \end{aligned}$$

Summing over all inputs j with $j \equiv u$ and assuming as before that only a single upstream input l produces emissions in each upstream industry u, the upstream carbon intensity is

$$\begin{aligned} SU_{CI} &\equiv \frac{z_u}{\pi_m} && (46)\\ &= \frac{\sum_u^U e_{ul} a_{ul} a_{mu} \frac{q_m(\theta_m)}{\theta_u^a \theta_m}}{\frac{c_m}{(\gamma_m - 1)} \frac{q_m(\theta_m)}{\theta_m}} && (47)\\ &= \frac{(\gamma_m - 1)}{c_m} \sum_u^U \frac{e_{ul} a_{ul} a_{mu}}{\theta_u^a} && (48) \end{aligned}$$

4.3 Interpreting upstream transition risks

We consider each upstream shock from equation (40) separately. We simplify comparisons by assuming that the midstream use of input j produces no midstream emissions such that $\psi_{mj} = 0$.[13] This assumption also results in $w_{mu} = P_u$ such that $\frac{w_{mu}}{c_u} = \frac{P_u}{c_u} = \frac{\mathcal{M}_u}{\theta_u^a}$ as per (11). We also consider only the case of elastic demand ($\nu_m > 1$). Transition risks from each shock can then be expressed as[14]

[13] Assuming j also produces emissions when used midstream complicates the algebra and interpretation without changing the economics, as $\widehat{w_{mu}} = \widehat{P_u}$ no longer holds as we must account directly for the midstream emissions cost in percentage changes in w_{mu}.

[14] See Appendix B.1 for details.

$$\text{Carbon price: } \begin{cases} \widehat{\pi_m} = -\dfrac{(\gamma_m - 1)}{c_m}\mathcal{H}_m(\nu_m - 1)\displaystyle\sum_{j=u=1}^{U} \mathcal{A}_u\mathcal{M}_u\frac{e_{ul}a_{ul}a_{mu}}{\theta_u^a}\tau_{ul}\widehat{\tau_{ul}} & = \ominus\widehat{\tau_{ul}} \end{cases} \tag{49}$$

$$\text{Green-input productivity: } \begin{cases} \widehat{\pi_m} = \dfrac{(\gamma_m - 1)}{c_m}\mathcal{H}_m(\nu_m - 1)\displaystyle\sum_{j=u=1}^{U} \mathcal{A}_u\mathcal{M}_u\frac{w_{ul'}a_{ul'}a_{mu}}{\theta_u^a}\widehat{\mu_{ul'}} & = \oplus\widehat{\mu_{ul'}} \end{cases} \tag{50}$$

$$\text{Brown-input productivity: } \begin{cases} \widehat{\pi_m} = \dfrac{(\gamma_m - 1)}{c_m}\mathcal{H}_m(\nu_m - 1)\displaystyle\sum_{j=u=1}^{U} \mathcal{A}_u\mathcal{M}_u\frac{e_{ul}a_{ul}a_{mu}}{\theta_u^a}\frac{w_{ul}}{e_{ul}}\widehat{\mu_{ul}} & = \oplus\widehat{\mu_{ul}} \end{cases} \tag{51}$$

As expected a carbon price applied to upstream emissions (49) reduces the midstream firm's profits by increasing its input costs while an increase in productivity upstream—whether to a green input (50) or brown input (51)—increases midstream profits by reducing input costs. Thus both carbon price and green-input productivity changes have the same sign of effect on a firm's profits whether they occur upstream, or directly to a firm.

However, the magnitude of upstream transition risks are different from midstream risks. Recall how in Section 3.3 the risk to a firm's profits from a midstream carbon price was directly proportional to its SM_{CI}; this is no longer the case for an upstream carbon price. The differences can be seen by considering under what conditions upstream risks from carbon prices and green innovations in equations (49) to (50) can be expressed directly in terms of SU_{CI} from equation (48), as was done for midstream risks in equations (37) to (38).

The magnitude of upstream risks is not necessarily proportional to a firm's SU_{CI} for two main reasons. First, for risks to be proportional to SU_{CI} the ratio of input costs to input-emissions intensity $\frac{w_{ul}}{e_{ul}}$ must be the same across different midstream inputs $j \equiv u$. $\frac{w_{ul}}{e_{ul}}$ will generally differ substantially when upstream emissions across upstream industries u come from different sources of greenhouse gas emissions—which is of course common. In other words, how much a carbon price affects input costs depends in part on the magnitude of emissions, but also on how expensive those emissions are to begin with—the more expensive an emission-generating input is, the less the effect on input costs per dollar of increase in a carbon price.

Second, the terms $\mathcal{A}_u$ and $\mathcal{M}_u$ mediate the scale of changes in profits and vary across industries u. $\mathcal{M}_u$ (equation 11) expresses how an industry's price index—and price paid when using its output

as an input downstream—is related to its raw marginal costs c_u and average productivity, and reflects markups. $\mathcal{A}_u$ (equation 32) is the passthrough rate in percentage terms and expresses how much the price index changes due to an increase in raw costs c_u. Depending on the market concentration and demand curve faced by firms they may pass on greater than, less than, or the full 100% of their increases in costs.[15] Thus, the upstream market structure—reflected in $\mathcal{M}_u$ and $\mathcal{A}_u$—affects the degree a carbon price applied upstream is passed through to midstream firms, and this affects the magnitude of risk from upstream carbon prices vis-a-vis midstream carbon prices. Only in the special case of perfect competition and no love-of-variety effect are $\mathcal{M}_u = \mathcal{A}_u = 1$ and upstream carbon price risks are directly proportional SU_{CI}, just as midstream carbon price risks are proportional to SM_{CI}. Together, these two terms result in upstream transition risks being a weighed average—with weights largely unknown in practice—of a firm's upstream emissions.

In addition, upstream green productivity improvements have no definite relationship to the firm's SU_{CI} just like for midstream green productivity improvements. We return to the implications of these results for measuring risks in Section 7.

5 Downstream value chain transition risks

This section examines how a midstream firm is affected by changes downstream in its value chain. Both midstream and downstream industries are characterised by the Melitzian structure described in section 3—where firms have heterogeneous productivity and entry and operating involve fixed costs. We denote the downstream industry by d and midstream by m. To keep things simple we assume the downstream d-type firms use only two inputs: the composite produced by midstream industry m and a generic alternative input o with factor price w_{do} and productivity μ_{do}. We also only consider downstream shocks that directly affect downstream firms; downstream shocks thus affect midstream firms through changing demand for m. We do not model more complicated channels where downstream consumers might, for example, be concerned about the lifecycle emissions of a product and thus take a midstream firm's upstream emissions into account. To enhance clarity, in the discussion below we let θ refer to a downstream firm's productivity and use φ to denote the productivity of a midstream firm. Accordingly, φ_m^a and φ_m^* will denote average and cutoff productivity in the midstream industry.

Let Q_m^{demanded} denote aggregate demand for m from the d-industry. Aggregating across all opera-

[15] See e.g. **?**.

tional downstream firms, demand for the midstream composite is given by

$$Q_m^{\text{demanded}} = \int_{\theta_d^*}^{\infty} \frac{g_d(\theta) M_d}{1 - G_d(\theta_m^*)} \frac{q_d(\theta)}{\theta} a_{dm} d\theta$$

where $\frac{q_d(\theta)}{\theta} a_{dm}$ measures midstream composite required as input by a downstream firm with productivity θ. Substituting in for q_d using (2), making use of the definition of θ_d^a (12) and collecting terms yields

$$Q_m^{\text{demanded}} = \frac{M_d a_{dm} R_d p_d(\theta_d^a)^{-\gamma_d}}{P_d^{1-\gamma_d} \theta_d^a} \tag{52}$$

Collectively, equation (52) plus equations (6), (8), (10), (14), (16), (17), (19), (20), and (21)—each evaluated once for $k = d$ and then again for $k = m$ (and using φs in lieu of θs where appropriate)—define equilibrium for the model with two vertically-linked industries m and d.

5.1 Comparative Statics

Log differentiating (52) gives

$$\widehat{Q_m^{\text{demanded}}} = \widehat{M_d} + \widehat{a_{dm}} + \widehat{R_d} - \gamma_d \widehat{p_d(\theta_d^a)} - [1 - \gamma_d] \widehat{P_d} - \widehat{\theta_d^a}. \tag{53}$$

Substitute in for $\widehat{R_d}$ using (82) and for $\widehat{P_d} = \frac{\widehat{M_d}}{1-\gamma_d} + \widehat{p_d(\theta_d^a)}$ using the log differential of (10) and collect terms to obtain

$$\widehat{Q_m^{\text{demanded}}} = \left[\frac{1 - \nu_d}{1 - \gamma_d}\right] \widehat{M_d} + \widehat{a_{dm}} - \widehat{\theta_d^a} - \nu_d \widehat{p_d(\theta_d^a)}. \tag{54}$$

In this section, our primary interest concerns how factor demand in the downstream industry affects midstream profitability. So as to shine the spotlight on substitutability, we make the following assumptions for the remainder of this section.

A1: $\nu_d = 1$,

A2: $\gamma_m > \delta_d$.

A1 stipulates unit-elastic demand for the downstream composite while **A2** indicates there is greater substitutability between the varieties comprising composite Q_m than there is between that composite and the outside input.

Under **A1** and log differentiating (6) and (20), (54) becomes

$$\widehat{Q_m^{\text{demanded}}} = [\delta_d - 1]\iota_{do}\left[\widehat{\mu_{dm}} + \widehat{w_{do}} - \widehat{\mu_{do}}\right] - \left[\iota_{dm} + \iota_{do}\delta_d\right]\psi_{dm}\left[\widehat{e_{dm}} + \widehat{\tau_{dm}}\right] - \left[\iota_{dm} + \iota_{do}\delta_d\right][1 - \psi_{dm}]\widehat{P_m}. \quad (55)$$

Equation (55) indicates how demand from the downstream industry for the midstream composite varies with exogenous parameters. As with equation (22), $-\nu_m$ is the price-elasticity of demand for the midstream composite. Setting $\widehat{\mu_{dm}} = \widehat{w_{do}} = \widehat{\mu_{do}} = \widehat{e_{dm}} = \widehat{\tau_{dm}} = 0$ in (55) and rearranging shows

$$\nu_m \equiv -\frac{\widehat{Q_m^{\text{demanded}}}}{\widehat{P_m}} = \left[\iota_{dm} + \iota_{do}\delta_d\right][1 - \psi_{dm}]. \quad (56)$$

Note for later use that $\nu_m \in [0, \max\{\delta_d, 1\}]$; as we have assumed $\gamma_k > 1$ for all k then, by **A2**, it's also the case that $\gamma_m > \nu_m$.

We now turn our attention to examining variation in midstream supply. Use the feature that $R_m = P_m Q_m$ to substitute in for R_m in the zero-profit condition (9) to derive an equation expressing equilibrium supply of the midstream composite as a function of the price index P_m and other variables. Log differentiating and isolating $\widehat{Q_m}$ delivers

$$\widehat{Q_m} = -\gamma_m \widehat{P_m} + (\gamma_m - 1)[\widehat{c_m} - \widehat{\varphi_m^*}]. \quad (57)$$

Equate the right hand sides of (55) and (57) to get an expression in which the only endogeneous variables are $\widehat{P_m}$ and $\widehat{\varphi_m^*}$:

$$\left[\nu_m - \gamma_m\right]\widehat{P_m} - (\gamma_m - 1)\widehat{\varphi_m^*} = -(\gamma_m - 1)\widehat{c_m} + [\delta_d - 1]\iota_{do}[\widehat{\mu_{dm}} + \widehat{w_{do}} - \widehat{\mu_{do}}] - \frac{\nu_m \psi_{dm}}{1 - \psi_{dm}}[\widehat{\tau_{dm}} + \widehat{e_{dm}}]. \quad (58)$$

Using the (58) along with the log derivatives of the price index (11), free entry condition (101), and Zero Cutoff Profit condition (8) delivers the companion to (34), the change in profits for the midstream firm from a downstream shock—see Appendix C.1 for details.

$$\widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m(\nu_m - 1)\widehat{c_m} - (\gamma_m - 1)\mathcal{H}_m[1 - \delta_d]\iota_{do}[\widehat{\mu_{dm}} + \widehat{w_{do}} - \widehat{\mu_{do}}] - (\gamma_m - 1)\mathcal{H}_m\nu_m\frac{\psi_{dm}}{[1 - \psi_{dm}]}[\widehat{\tau_{dm}} + \widehat{e_{dm}}] \quad (59)$$

5.2 Downstream emissions

The Greenhouse Gas Protocol defines a number of categories of Scope 3 emissions; for downstream emissions these are primarily from the transportation, further processing, and use of sold products. For example, a midstream firm might sell plastic resin which is an intermediate input to a downstream firm that produces plastic handles; the downstream firm uses natural gas in processing the plastic resin into plastic handles and releases emissions from their onwards transportation, which can be attributed to the midstream firm.[16] We model such downstream emissions as follows.

A downstream firm with productivity θ_d purchasing quantities of inputs l_{dm} from m and l_{do} from o releases downstream emissions z_d and has output $\frac{q_d(\theta_d)}{\theta_d}$ according to

$$z_d(\theta_d) = e_{dm} l_{dm} + e_{do} l_{do} \tag{60}$$

$$= (e_{dm} a_{dm} + e_{do} a_{do}) \frac{q_d(\theta_d)}{\theta_d} \tag{61}$$

Integrating over all downstream firms across their θ_d the total downstream emissions Z_d are

$$Z_d = M_d (e_{dm} a_{dm} + e_{do} a_{do}) \frac{q_d(\theta_d^a)}{\theta_d^a} \tag{62}$$

$$\tag{63}$$

and downstream quantity $\frac{q_d(\theta_d^a)}{\theta_d^a}$ requires a quantity of the m composite Q_m according to (52) and (2) of

$$Q_m^{demanded} = M_d a_{dm} \frac{q_d(\theta_d^a)}{\theta_d^a} \tag{64}$$

$$\tag{65}$$

Each individual midstream firm with output $q_m(\varphi_m)$ is attributed their cost share of Z_d. That is, the midstream firm is attributed emissions based both on their input cost share ι_{dm} in downstream production, as well as their share $\frac{q_m(\varphi_m)}{Q_m^{demand}}$ of the total composite Q_m^{demand}. Emissions attributed to the midstream firm $z_d(\varphi_m)$ are then

[16] See example 10.1 in **?**

$$
\begin{aligned}
z_d(\varphi_m) &= \iota_{dm} \cdot \frac{q_m(\varphi_m)}{Q_m^{demand}} \cdot Z_d && (66)\\
&= \iota_{dm} \frac{q_m(\varphi_m)}{M_d a_{dm} \frac{q_d(\theta_d^a)}{\theta_d^a}} \cdot M_d(e_{dm}a_{dm} + e_{do}a_{do}) \frac{q_d(\theta_d^a)}{\theta_d^a} && (67)\\
&= \iota_{dm} \frac{q_m(\varphi_m)}{a_{dm}} \cdot (e_{dm}a_{dm} + e_{do}a_{do}) && (68)
\end{aligned}
$$

Note that as per the Greenhouse Gas Protocol **?** different attribution schemes are permitted, such as attributing all downstream Scope 3 emissions to a midstream firm regardless of their input cost shares (i.e. setting $\iota_{dm} = 1.$) We attribute emissions based on cost shares to avoid double-counting emissions and because the downstream input cost share ι_{dm} is an important determinant of risks.

As was done for SM_{CI} and SU_{CI} the downstream carbon intensity SD_{CI} is a midstream firm's downstream emissions $z_d(\varphi_m)$ divided by its profits π_m,

$$
\begin{aligned}
SD_{CI} &= \frac{z_d(\varphi_m)}{\pi_m(\varphi_m)} && (69)\\
&= \frac{\iota_{dm}(e_{dm} + e_{do}\frac{a_{do}}{a_{dm}})q_m(\varphi_m)}{\frac{c_m q_m(\varphi_m)}{(\gamma_m - 1)\varphi_m}} && (70)\\
&= (\gamma_m - 1)\iota_{dm} \frac{(e_{dm} + e_{do}\frac{a_{do}}{a_{dm}})}{c_m} \varphi_m && (71)\\
&= (\gamma_m - 1)\mathcal{M}_m \iota_{dm} \frac{(e_{dm} + e_{do}\frac{a_{do}}{a_{dm}})}{P_m} && (72)
\end{aligned}
$$

where the last line uses $\mathcal{M}_m$ as defined in equation (10).

5.3 Interpreting downstream transition risks

Equations (73) to (76) summarise how downstream shocks create transition risks to midstream firms. We also show carbon pricing risks expressed in an equivalent form using SD_{CI}.[17] Importantly, downstream emissions do not necessarily create the same sign or magnitude of transition risks as upstream or midstream emissions. We discuss this important point further in Section 6. A carbon price applied to downstream emissions reduces profits when the midstream firm sells a brown good or sells a complementary green good. Unsurprisingly, a downstream carbon price can also raise midstream firm profits if the firm sells a substitute green good used in downstream production. For example, a firm that sells energy efficient machinery that is a substitute for emissions-intensive machinery may

[17] See Appendix C.2 for the derivation of profit changes in terms of SD_{CI}.

benefit from a downstream carbon price that applies to both energy efficient and emissions-intensive machinery. Similarly, how a productivity improvement to downstream production affects midstream profits depends both on whether the improvement affects the midstream good m or alternative good o, but also whether m and o are complements ($\delta_d < 1$) or substitutes ($\delta_d > 1$). Downstream green productivity improvements do not necessarily raise midstream profits such as when the midstream firm sells a brown good m that is a substitute to the green good o, or when the midstream firm sells a green good m that is a complementary good to a brown good o. Note that green innovations—if defined as productivity improvements for goods that do not release emissions when used—do not necessarily reduce downstream emissions when green and brown goods are complements. We focus only on situations where green innovations do reduce emissions, and discuss cases where emissions rise in Appendix C.3.

Brown good; carbon price τ_{dm}
$e_{dm} \neq 0, e_{do} = 0$

$$\begin{cases} \widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m \nu_m \dfrac{e_{dm}}{P_m} \tau_{dm} \widehat{\tau_{dm}} & = \ominus \widehat{\tau_{dm}} \\ \widehat{\pi_m} = -\mathcal{H}_m \nu_m \dfrac{SD_{CI}}{\mathcal{M}_m \iota_{dm}} \tau_{dm} \widehat{\tau_{dm}} & = \ominus \widehat{\tau_{dm}} \end{cases} \tag{73}$$

Green good; carbon price τ_{do}
$e_{dm} = 0, e_{do} \neq 0$

$$\begin{cases} \widehat{\pi_m} = +(\gamma_m - 1)\mathcal{H}_m(\delta_d - 1)\iota_{do}\psi_{do}\widehat{\tau_{do}} & = \oplus(\delta_d - 1)\widehat{\tau_{do}} \\ \widehat{\pi_m} = +\mathcal{H}_m(\delta_d - 1)\dfrac{SD_{CI}}{\mathcal{M}_m}\tau_{do}\widehat{\tau_{do}} & = \oplus(\delta_d - 1)\widehat{\tau_{do}} \end{cases} \tag{74}$$

Green good m;
Green-input productivity on m:

$$\begin{cases} \widehat{\pi_m} = +(\gamma_m - 1)\mathcal{H}_m(\delta_d - 1)\iota_{do}\widehat{\mu_{dm}} & = \oplus(\delta_d - 1)\widehat{\mu_{dm}} \end{cases} \tag{75}$$

Brown good m;
Green-input productivity on o

$$\begin{cases} (\gamma_m - 1)\widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m(\delta_d - 1)\iota_{do}\widehat{\mu_{do}} & = \ominus(\delta_d - 1)\widehat{\mu_{do}} \end{cases} \tag{76}$$

6 Comparing upstream, midstream and downstream transition risks

In this section we compare upstream, midstream, and downstream transition risks from Sections 3.3, 4.3, and 5.3. This comparison highlights several important results for the measurement and disclosure of transition risks.

6.1 Types of transition risks

Comparing upstream, midstream, and downstream shocks shows there is no single 'type' of transition risk—the sign and magnitude of transition risks depends on how an economy transitions to a low-carbon

future, and the specific market and production structure of firms. That increases in regulatory costs like carbon pricing can decrease a firm's profits while green innovations may instead increase the same firm's profits—with both reducing emissions—seems almost trivially obvious. However, carbon pricing and green innovation are not simple opposites of each other where one makes firms better off and the other makes them worse off. Instead, carbon pricing and green innovation reflect fundamentally different paths towards a low-carbon future the world might go down, which create fundamentally different risks to firms that may need to be measured and disclosed in different ways. As can be seen from equations (37) and (38), a carbon price applied to a firm decreases its profits the higher the firm's midstream carbon intensity SM_{CI}, while a green innovation's effect on profits is not similarly proportional to the firm's SM_{CI}. This implies that while SM_{CI} can be used as a proxy for a firm's transition risks from a carbon price applied to its midstream emissions, it is not necessarily a good proxy for transition risks arising from improvements in green technologies.

It is intuitive that some paths towards a low-carbon future will require substantial substitution towards existing green alternatives, such as from ICE vehicles towards existing EVs. Along this path the only way to reduce emissions is to use fewer brown emission-intensive products, and use more low-emissions green products instead. And increasing the cost of emissions (including through implicit cost increases such as via increased emission regulations) or through changes in downstream demand is necessary to achieve such a shift from brown to green products. Along such a path a firm with high emissions is typically at high risk as the world must at some point use less of its products. In contrast, an alternative path is one where green innovation allows existing products to be produced with lower emissions. Such innovation does not require a shift from a currently brown to a currently green product—instead, it changes the emissions, and brown versus green status, of existing products. Along such a path a high emissions firm is not necessarily at high risk as green innovations can reduce its own emissions. In other words, a high emission firm relative to a similar low emissions firm may be at highest risk from a carbon price, yet also best positioned to benefit from a green technology that reduces its emissions. And the differences between costly-emissions driven transitions versus green-innovation driven transitions is even starker once the extended impacts on a firm's value chain are considered.

6.2 Upstream versus downstream emissions

Critically, how carbon pricing and green innovation affect a firm's profits depends whether the shock occurs upstream, directly to the firm, or downstream. This can be seen by comparing the effect of carbon pricing on upstream and midstream profits (equations 49 and 37) and downstream (equations

73 and 74), and similarly for green innovation in (50), (38), (75), and (76).

Upstream carbon pricing and direct midstream carbon pricing both reduce firm profits by raising the cost of inputs. Similarly, green innovations upstream or midstream both reduce costs and increase firm profits. Yet this pattern does not necessarily continue for downstream carbon pricing or green innovations. For example, if a midstream firm sells a product that is a green input in downstream production then a downstream carbon price could either increase or decrease the firm's profits depending on how demand for its output is affected. Similarly, a downstream green innovation could also increase—or decrease—the firm's profits, depending on whether the green product sold is a substitute or complement with downstream emission-intensive inputs. This highlights how downstream emissions can have the opposite sign of effect on transition risks as upstream or midstream emissions do.

6.3 Magnitude of risks

The magnitude of risks to a firm's profits from increases in the cost of emissions are not necessarily proportional to its carbon intensity; as a result, even firms with higher Scope 1 + 2 carbon intensity can face lower risks than firms with lower carbon intensity. This occurs because the market structure of upstream inputs and downstream demand both affect how risks are transmitted to a midstream firm and measures of upstream, midstream, and downstream emissions do not reflect market structure.

This can be seen by comparing the risks to a firm that has 1 tonne of upstream, midstream, or downstream emissions and that experiences a uniform carbon price applied to all of its emissions. Setting $SU_{CI} = SM_{CI} = SD_{CI} = 1$ tonne $CO_{2eq}/value$ and assuming the same increase in carbon price $d\tau_{ul} = d\tau_{mj} = d\tau_{dm}$, equations (49), (37), and (73)/(74) show the resulting changes in profits. As discussed in Section 4.3, the upstream carbon price risk is mediated by $\mathcal{M}_u$ and $\mathcal{A}_u$ which capture how the price paid for a midstream input is related to upstream industry costs. Together, these two terms result in upstream transition risks being a weighted average—with the weights largely unknown in practice—of a firm's upstream emissions; upstream transition risks are not directly proportional to SU_{CI}.

The magnitude of risk from 1 tonne of downstream emissions is different again. Importantly, how a midstream firm's profits are affected by downstream emissions depends on whether the firm sells a green (74) or emissions-intensive brown good (73); this distinction is not relevant for midstream and upstream risks. If a firm sells a brown good then its price elasticity of demand ν_m affects the scale of risks it faces; the more elastic its demand, the greater profits will fall from an increase in costs. In contrast, if the firm sells a green good then profits scale with the elasticity of substitution δ_d between

downstream brown and green inputs. The greater the substitution, the more a midstream firm will benefit as a carbon price induces greater substitution to purchases of its own output. The downstream input share ι_{dm} also affects the magnitude of risk. When the firm sells a brown good as per (73) the greater the input share of the brown good then the lower the emissions intensity e_{dm} must be for a given SD_{CI}; risks to profits are also correspondingly lower. Similarly, $\mathcal{M}_m$ appears in the denominator and has the opposite effect from $\mathcal{M}_u$ in upstream carbon price risks. The higher the markup of the midstream firm's output price P_m to its marginal costs, the lower a firm's SD_{CI} as its downstream emissions are divided by a larger firm value. Thus the greater the markup $\mathcal{M}_m$, the lower the e_{dm} must be for a given SD_{CI} and the lower the cost increase from a carbon price.

The above comparisons are for carbon prices. The risks from green innovations are also not directly comparable across 1 tonne of upstream, midstream, and downstream emissions. Comparing equations (50), (38), and (75)/(76) shows that not only do innovation shocks propagate differently depending on where they occur in a value chain, but that green innovation risks do not have a straightforward relation to a firm's carbon intensity.

6.4 Further implications: non-marginal changes and quantity targets

There are two further situations where a high emissions firm can be at lower risk than a similar low-emissions firm in the same industry.

First, while two firms with the same SM_{CI} may face the same risk from cost-based policies like carbon pricing, they may still face different risks from policies that use cost-based politics to achieve specified reductions in the quantity of emissions. This is analogous to the prices-versus-quantities tradeoff of carbon taxes and cap-and-trade policies. We saw from equation (37) that a high SM_{CI} firm is at higher risk from an emissions price $d\tau_{mj}$ than a similar low emissions firm. Such an emissions price policy leaves the quantity of emissions reduced up to the firm—Appendix A.8 equation (119) gives the associated reduction in emissions. Importantly—and in contrast to the change in profits—this change in output-emissions intensity due to $d\tau_{mj}$ depends directly on δ_m, e_{mj}, and w_{mj}, in addition to SM_{CI}. As a result, while the high SM_{CI} firm will face a higher risk to its profits than the low SM_{CI} firm, it may simultaneously reduce its emissions intensity by more. The converse is that a policy that specifies a reduction in z_m/q_m has different implicit emission costs to the firms, and under such a quantity policy the high SM_{CI} firm can be a lower risk than the low SM_{CI} firm.

Second, this is analysis is based on marginal changes in prices and productivities. When changes in price or productivity are non-marginal i.e. "large" changes the risks to a firm are even less related to

their emissions and equations (49), (37), and (73)/(74) do not hold. In this case, risks to firm profits are not proportional to their carbon intensity in even the simplest case of comparing two firms in the same industry with the same midstream SM_{CI}. This is a further limitation in using a firm's carbon intensity as a proxy for its transition risks.

7 Implications for risk metrics

Aggregating emission Scopes in risk metrics

It is common for Scope 1, 2, and at times Scope 3 emissions to be added together when firms, investors, and regulators compare emissions across companies and financial products. Our results reveal significant limitations in aggregating emissions which have important implications for the disclosure of emissions and transition risks. Fortunately, the sign of risk from carbon prices (and green innovations) is the same for upstream and midstream emissions. This means the ubiquitous practice of aggregating Scope 1 and 2 emissions is consistent with higher emissions being correlated with higher risk from a cost on emissions. However, as the magnitude of risk from Scope 1 and 2 emissions differs a firm with higher Scope 1 + 2 emissions does not necessarily mean it is at higher risk than an otherwise similar firm with lower Scope 1 + 2 emissions, if the two firms have different shares from Scope 1 and 2 emissions. Adding Scope 3 emissions to Scope 1 + 2 further complicates comparing company emissions; firms with higher total financed emissions (Scope 1 + 2 + 3) are not necessarily at higher risk from an emissions price. For example, a firm with high Scope 3 emissions that sells a green product might see its profits rise due to a carbon price upstream if demand for its output rises due to the carbon price, despite increases in costs from its Scope 1 + 2 emissions.

Average emission intensities

Portfolios of companies, such as widely available exchange-traded funds, commonly report their emission intensities such as the weighed-average carbon intensity (WACI). Average emission intensities do not necessarily reflect average or aggregate risks to a portfolio as they are implicitly comparing firms in different industries with different market structures and supply chains. As discussed in Section 3.3, risks differ across industries as an industry structure—including its price elasticity of demand, elasticity of entry costs to the mass of entrants, and the elasticity of substitution between varieties—all affect $\mathcal{H}_m$ and how much profits change during a transition. None of these are captured by a firm's emission intensity. So while the average emissions intensity of a portfolio is a weighted average of firms' SM_{CI},

the average risk to the portfolio is a weighted average including $\mathcal{H}_m$. This can make it challenging to use aggregated financed emission metrics like weighted average carbon intensity outside of narrowly defined industries.

8 Conclusion

This paper highlights how financial transition risks to firms arise from shifts to a low-carbon economy. We show this by developing a microfounded Melitz-type model that includes upstream and downstream value-chain linkages between firms. Using this model we examine productivity, cost, and demand shocks and identify factors causing and mediating transition risks. Our findings offer a basis for developing improved metrics for assessing transition risks and can guide risk and emissions disclosure standards for industry stakeholders, regulators, and investors.

We find that different causes of emission reductions—such as policy-induced cost increases versus technological improvements that enhance the productivity of 'green' low-carbon alternatives—can create different transition risks. Importantly, risks can be either positively or negatively correlated with a firm's emissions depending on the nature of the transition, and the upstream, direct, or downstream location of emissions within a firm's value chain. Downstream emissions in particular can have distinct impacts compared to direct or upstream emissions, both in magnitude and sign. Other important factors in mediating the sign and magnitude of transition risks include whether a firm produces low-carbon "green" or emission-intensive "brown" goods, and market structure factors such as market concentration, barriers to entry, price elasticity of demand, and substitutability between the inputs to production. For example, green innovations that reduce emissions are typically a benefit to firms when the innovation occurs upstream, while green innovations downstream can create risks or raise profits. In contrast, carbon pricing upstream creates risks while carbon pricing downstream can also raise or lower profits. Together, these factors create notable limitations in using a firm's Scope 1, 2, and 3 emissions in assessing transition risks and in comparing the carbon intensity of firms and investment portfolios.

The challenge of assessing transition risks can in part be addressed by complementing emissions-based metrics utilising a firm's Scope 1, 2, and 3 emissions with classifications of economic activities like the EU's Taxonomy Regulation, and disaggregating Scope 3 into upstream and downstream emissions. Taxonomy-based classifications can help specify whether downstream emissions are likely to be a source of risk—or potential benefit—to a firm; that is, do firm's sell 'green' or 'brown' products, and are such products substitutes or complements to downstream emission-generating activities.

Disaggregating Scope 3 emissions into upstream and downstream emissions would enable clearer

comparisons of the scale of firms' upstream and downstream emissions, and permit upstream Scope 3 to be combined with Scope 2. Indeed, Scope 2 emissions from purchased energy—many of which have readily available low-carbon alternatives—may turn out to be a lower risk category than other upstream emissions, which are currently excluded from most risk disclosure methods based on Scope 1 and 2 emissions. Perhaps more importantly, this disaggregation would clarify the scale of a firm's downstream Scope 3 emissions separately from its upstream emissions. Comparing firms within narrowly defined categories would also ameliorate some of the limitations of emissions-based metrics.

A Midstream model

A.1 Appendix: Zero Profit Condition

The total costs facing a firm operating in industry k with productivity θ is $\frac{c_k}{\theta} q_k(\theta) - F_k$. As firms will remain operating as long as they earn positive profits then, using (7), the Zero Profit Condition can be written

$$\frac{R_k[p_k(\theta_k^*)]^{1-\gamma_k}}{\gamma_k {P_k}^{1-\gamma_k}} = F_k \tag{77}$$

where

$$p_k(\theta_k^*) = \frac{\gamma_k}{\gamma_k - 1} \frac{c_k}{\theta_k^*} \tag{78}$$

is the price charged by a firm in industry k with cutoff-productivity θ_k^* and, from (3), the price index for industry k is as given in (10).[18]

A.2 Appendix: Entry

When entry is possible, equilibrium requires that aspiring firms enter the industry until the expected payoff from entry equals F_k^E as per the following Free Entry (FE) condition:

$$\int_{\theta_k^*}^{\infty} g_k(\theta) \left[\pi_k(\theta) - F_k\right] d\theta = F_k^E(E_k). \tag{79}$$

Or, using (17),

$$F_k^E(E_k) = [1 - G_k(\theta_k^*)] F_k \left[\left(\frac{\theta_k^a}{\theta_k^*} \right)^{\gamma_k - 1} - 1 \right]. \tag{80}$$

[18] By equations (10) and (2) we can write the total amount of composite-k produced as

$$Q_k = M_k^{\frac{\gamma_k}{\gamma_k - 1}} q_k(\theta_k^a).$$

Recalling that θ_k^a depends only on θ_k^* as per equation (12), the preceding equation suggests that the cutoff-productivity θ_k^* is fixed unless entry costs vary with the mass of entrants. Given (16) and (80), the free entry condition can be written as

$$F_k^E\left(\frac{M_k}{1-G_k(\theta_k^*)}\right) = \frac{[1-G_k(\theta_k^*)]F_k}{\eta_k} \tag{81}$$

A.3 Appendix: Changes in Costs

Log differentiating $R_k = Q_k P_k$ with (22) gives

$$\widehat{R_k} = [1-\nu_k]\widehat{P_k} \tag{82}$$

while log differenting the zero cutoff-profits condition ((9)) gives

$$[1-\nu_k]\left[\widehat{c_k} - \widehat{\theta_k^*}\right] = \frac{\gamma_k - \nu_k}{\gamma_k - 1}\widehat{M_k}. \tag{83}$$

Log differentiating the cost function (19) yields

$$\widehat{c_k} = \frac{\sum_{j=1}^{J}\left[\frac{w_{kj}}{\mu_{kj}}\right]^{1-\delta_k}\left[\widehat{w_{kj}} - \widehat{\mu_{kj}}\right]}{\sum_{j=1}^{J}\left[\frac{w_{kj}}{\mu_{kj}}\right]^{1-\delta_k}}. \tag{84}$$

We defined

$$\iota_{kj} \equiv \frac{w_{kj}a_{kj}}{c_k}$$

as input j's share of a k-industry firm's costs; by (19) and (20), j's share can also be written as

$$\iota_{kj} = \left[\frac{\frac{w_{kj}}{\mu_{kj}}}{c_k}\right]^{1-\delta_k}. \tag{85}$$

Two comments are in order. First is that ι_{kj} is in the unit interval even when $\delta_k > 1$. The second is that j's share in k's costs is decreasing (increasing) in $\frac{w_{kj}}{\mu_{kj}}$ if δ_k is larger (smaller) than 1. Specifically,

$$\frac{\partial ln\iota_{kj}}{\partial lnw_{kj}} = (1-\delta_k)(1-\iota_{kj}); \tag{86}$$

given $1-\iota_{kj}$ is non-negative, (86) tells us that whether an inputs share in unit costs rises or falls with the input's own factor price depends pivotally on the size of δ_k. This isn't surprising. When δ_k is high, firms can easily replace input j with close substitutes. But when δ_k is low, even though the firm's

unit-input requirement for the now-more-costly input j falls, the drop in a_{kj} won't be so large as to outweigh the increase in w_{kj} and the factor's share in k's costs rises.

Rewriting (84) in terms of cost shares gives

$$\widehat{c_k} = \sum_{j=1}^{J} \iota_{kj} \left[\widehat{w_{kj}} - \widehat{\mu_{kj}}\right]. \tag{87}$$

Meanwhile, from the net factor-price equation (21),

$$\widehat{w_{kj}} = \psi_{kj}[\widehat{\tau_{kj}} + \widehat{e_{kj}}] + [1 - \psi_{kj}]\widehat{r_{kj}} \tag{88}$$

where

$$\psi_{kj} \equiv \frac{\tau_{kj} e_{kj}}{w_{kj}} \tag{89}$$

measures the share of pollution fees in the net price for using factor j in industry k.

The change in unit-input requirements is given by

$$\widehat{a_{kj}} = -\delta_k \widehat{w_{kj}} + (\delta_k - 1)\widehat{\mu_{kj}} + \delta_k \sum_{j=1}^{J} \iota_{kj} \left[\widehat{w_{kj}} - \widehat{\mu_{kj}}\right] \tag{90}$$

A.4 Appendix: Changes in Price Index

Log differentiating ((78)) gives

$$\widehat{p_k(\theta_k^*)} = \widehat{c_k} - \widehat{\theta_k^*} \tag{91}$$

while log differentiate ((16)) yields

$$\widehat{\theta_k^*} = \widehat{\theta_k^a} \tag{92}$$

and so we can write the log differentiated version of ((10)) as

$$\widehat{P_k} = \widehat{c_k} - \widehat{\theta_k^*} - \frac{1}{\gamma_k f - 1} \widehat{M_k}. \tag{93}$$

Substituting in for $\widehat{M_k}$ and $\widehat{\theta_k^*}$ in (93) and collecting terms yields

$$\widehat{P_k} = \frac{\mathcal{Z}_k[\gamma_k - 1]}{\nu_k - 1 + \mathcal{Z}_k[\gamma_k - \nu_k]} \widehat{c_k} \tag{94}$$

$$\widehat{P_k} \equiv \mathcal{A}_k \widehat{c_k} \tag{95}$$

When $\gamma_k > 1$, $\mathcal{Z}_k > 1$, and $\gamma_k > \nu_k > 1$, we have $\mathcal{A}_k > 1$. When $\gamma_k > 1$, $\mathcal{Z}_k > 1$, $\gamma_k > \nu_k$ but $\nu_k < 1$, we have $\mathcal{A}_k < 1$.

Sign $\mathcal{A}_k$

Let $\gamma_k = \nu_k + \alpha$ and $\mathcal{Z}_k = 1 + \kappa$. From $\gamma_k > 1$, $\gamma_k > \nu_k$ and $\mathcal{Z}_k > 1$ we have $\alpha > 0$ and $\kappa > 0$. Then

$$\begin{aligned}
\mathcal{A}_k &\equiv \frac{\mathcal{Z}_k[\gamma_k - 1]}{\nu_k - 1 + \mathcal{Z}_k[\gamma_k - \nu_k]} && (96)\\
&= \frac{\mathcal{Z}_k[\nu_k + \alpha - 1]}{\nu_k - 1 + \mathcal{Z}_k[\nu_k + \alpha - \nu_k]} && (97)\\
&= \frac{\mathcal{Z}_k[\nu_k - 1] + \mathcal{Z}_k\alpha}{\nu_k - 1 + \mathcal{Z}_k\alpha} && (98)\\
&= \frac{(1+\kappa)[\nu_k - 1] + \mathcal{Z}_k\alpha}{\nu_k - 1 + \mathcal{Z}_k\alpha} && (99)\\
&= 1 + \frac{\kappa[\nu_k - 1]}{\nu_k - 1 + \mathcal{Z}_k\alpha} && (100)
\end{aligned}$$

which shows $\mathcal{A}_k > 1$ if $\nu_k > 1$ and $\mathcal{A}_k < 1$ if $\nu_k < 1$.

A.5 Appendix: Changes in FE and ZCP

Next examine the FE and ZCP conditions. Log differentiating ((14)) gives the log-linearized version of the Free Entry condition:

$$\xi_k(E_k)\widehat{M_k} = -[1 + \xi_k(E_k)][1 + \eta_k](\gamma_k - 1)\widehat{\theta_k^*} \tag{101}$$

where $\xi_k(E_k)$ is the entry cost elasticity as defined in (13). Notice that, if ξ is zero then the cutoff-productivity level θ_k^* is constant in equilibrium, implying in turn that the profits earned by an operational firm are also constant as per (17). Why would this be? When $\xi = 0$, all adjustment in the market occurs via entry: anything that makes the industry intrinsically more profitable attracts exactly enough entry that average prices are driven back to their original level and any given firm makes the same profits as before. Similarly, if there's a cost hike, there's exactly enough exit (or non-entry) that average prices return to their original level and remaining firms fare no worse than before the cost hike. Such extreme responsiveness of entry to changes in raw costs, e.g., seems implausible, which is why we allow for the possibility ξ is greater than zero.

To obtain closed-form expressions for $\widehat{\theta_k^*}$, recall we assumed $\xi_k \in (0, \infty)$, i.e. the mass of entrants is variable and affects entry costs, and that ν_k is non-negative and not equal to unity.

A.6 Sign $\mathcal{H}_k$

$\mathcal{H}_k$ was defined in (31) as

$$\mathcal{H}_k \equiv \frac{1}{(\nu_k - 1) + \mathcal{Z}_k(\gamma_k - \nu_k)} \tag{102}$$

To see the sign of $\mathcal{H}_k$ recall $\mathcal{Z}_k$ and γ_k are both greater than unity and $\gamma_k > \nu_k$. Let $\alpha = \gamma_k - \nu_k$, such that

$$\begin{aligned}
\mathcal{H}_k &\equiv \frac{1}{(\nu_k - 1) + \mathcal{Z}_k(\gamma_k - \nu_k)} && (103)\\
&= \frac{1}{(\gamma_k - \alpha - 1) + \mathcal{Z}_k(\alpha)} && (104)\\
&= \frac{1}{(\gamma_k - 1) + \alpha(\mathcal{Z}_k - 1)} && (105)
\end{aligned}$$

Which is $\mathcal{H}_k > 0$ as $\gamma_k - 1 > 0$ and $\mathcal{Z}_k - 1 > 0$.

A.7 Appendix—Midstream Emissions

A firm's midstream emissions depend on the emission intensity of its inputs, its unit input requirements, and quantity of output produced. For a firm in industry k with average productivity θ_k^a the emissions associated with its final output $q_k(\theta_k^a)$ are

$$z_k(\theta_k^a) = \sum_{j=1}^{J} e_{kj} a_{kj} \frac{q_k(\theta_k^a)}{\theta_k^a}$$

Using profits from equations (6) and (7) an average industry k firm's midstream carbon intensity (SM_{CI}, 'scope-mid carbon intensity') is defined as follows,

$$\begin{aligned}
SM_{CI} &\propto \frac{\text{firm's midstream emissions}}{\text{firm value}} && (106)\\
&= \frac{z_k(\theta_k^a)}{\pi_k(\theta_k^a)} && (107)\\
&= \frac{\sum_{j=1}^{J} e_{kj} a_{kj} \frac{q_k(\theta_k^a)}{\theta_k^a}}{\frac{c_k q_k(\theta_k^a)}{(\gamma_k - 1)\theta_k^a}} && (108)\\
&= (\gamma_k - 1)\frac{\sum_{j=1}^{J} e_{kj} a_{kj}}{c_k} && (109)
\end{aligned}$$

To simplify the analysis we later assume that only a single input j produces emissions. Using

equations (20), (85), and (89), the SM_{CI} can also be written

$$SM_{CI} \equiv (\gamma_k - 1)\frac{\sum_{j=1}^{J} e_{kj}a_{kj}}{c_k} \tag{110}$$

$$= (\gamma_k - 1)\frac{\sum_{j=1}^{J} e_{kj}\frac{{w_{kj}}^{-1}(w_{kj}/\mu_{kj})^{1-\delta_k}}{{c_k}^{-1}{c_k}^{1-\delta_k}}}{c_k} \tag{111}$$

$$= (\gamma_k - 1)\sum_{j=1}^{J}\frac{\iota_{kj}\psi_{kj}}{\tau_{kj}} \tag{112}$$

$$= (\gamma_k - 1)\sum_{j=1}^{J}\frac{\iota_{kj}e_{kj}}{w_{kj}} \tag{113}$$

A.8 Midstream output-emissions intensity

We define an average midstream firm's output-emissions intensity as its emissions (whether upstream, midstream, or downstream) normalized by its final physical output $q_k(\theta_k^a)$. For the midstream output-intensity this is

$$\frac{z_k(\theta_k^a)}{q_k(\theta_k^a)} = \frac{\sum_{j=1}^{J} e_{kj}a_{kj}\frac{q_k(\theta_k^a)}{\theta_k^a}}{q_k(\theta_k^a)} \tag{114}$$

$$= \frac{1}{\theta_k^a}\sum_{j=1}^{J} e_{kj}a_{kj} \tag{115}$$

Note θ_k^a is a productivity shock that has already been realized by operating firms. As a result, shocks that affect a firm do not affect its already-realised θ_k^a. Log-linearising and taking the derivative provides the changes in physical output-emissions intensity; for a midstream firm where only one input j produces emissions, this is

$$\widehat{\left(\frac{z_m(\theta_m^a)}{q_m(\theta_m^a)}\right)} = \widehat{e_{mj}} + \widehat{a_{mj}} \tag{116}$$

$$= \widehat{e_{mj}} - \delta_m\widehat{w_{mj}} + (\delta_m - 1)\widehat{\mu_{mj}} + \delta_m\widehat{c_m} \tag{117}$$

$$= \widehat{e_{mj}} - \delta_m\widehat{w_{mj}} + (\delta_m - 1)\widehat{\mu_{mj}} + \delta_m\sum_{j=1}^{J}\iota_{mj}\left[\widehat{w_{mj}} - \widehat{\mu_{mj}}\right] \tag{118}$$

where $\widehat{w_{mj}} = e_{mj}\widehat{\tau_{kj}} + \tau_{kj}\widehat{e_{mj}} + \widehat{r_{kj}}$

Considering each shock separately,

$$\text{Carbon price:} \quad \begin{cases} \widehat{\left(\frac{z_m}{q_m}\right)} = -\delta_m \psi_{mj}[1-\iota_{mj}]\widehat{\tau_{mj}} & = \ominus\widehat{\tau_{mj}} \\ \widehat{\left(\frac{z_m}{q_m}\right)} = -\delta_m \tau_{mj}[\frac{e_{mj}}{w_{mj}} - \frac{SM_{CI}}{\gamma_m - 1}]\widehat{\tau_{mj}} \end{cases} \tag{119}$$

$$\text{Brown-input productivity:} \quad \begin{cases} \widehat{\left(\frac{z_m}{q_m}\right)} = [\delta_m(1-\iota_{mj})-1]\widehat{\mu_{mj}} & = \pm\cdot\widehat{\mu_{mj}} \\ \widehat{\left(\frac{z_m}{q_m}\right)} = [\delta_m \frac{w_{mj}}{e_{mj}}(\frac{e_{mj}}{w_{mj}} - \frac{SM_{CI}}{(\gamma_m-1)}) - 1]\widehat{\mu_{mj}} \end{cases} \tag{120}$$

$$\text{Green-input productivity:} \quad \begin{cases} \widehat{\left(\frac{z_m}{q_m}\right)} = -\delta_m \iota_{mj'}\widehat{\mu_{mj'}} & = \ominus\cdot\widehat{\mu_{mj'}} \end{cases} \tag{121}$$

$$\text{Input-emissions intensity:} \quad \begin{cases} \widehat{\left(\frac{z_m}{q_m}\right)} = [1-\delta_m\psi_{mj}(1-\iota_{mj})]\widehat{e_{mj}} = \pm\cdot\widehat{e_{mj}} \\ \widehat{\left(\frac{z_m}{q_m}\right)} = [1-\delta_m\tau_{mj}(\frac{e_{mj}}{w_{mj}} - \frac{SM_{CI}}{\gamma_m-1})]\widehat{e_{mj}} \end{cases} \tag{122}$$

B Appendix: Upstream Model

B.1 Appendix: Risks to a midstream firm from upstream shocks

Midstream firms in industry m purchase their inputs j from upstream firms. Midstream production of $\frac{q_m(\theta)}{\theta}$ requires a quantity l_{mj} of input j according to

$$l_{mj} = a_{mj}\frac{q_m(\theta)}{\theta}$$

Each input j is purchased from a unique upstream industry u. A specific composite Q_u (such as $u=1$) supplies all inputs j (such as $j=1$) across all midstream firms. Thus the indexes u and j are the same.

Upstream production of Q_u releases emissions. By equations (2) and (10) we can write the total amount of composite-u produced as

$$Q_u = M_u^{\frac{\gamma_u}{\gamma_u-1}} q_u(\theta_u^a)$$

Production of $q_u(\theta_u^a)$ releases emissions according to

$$z_u(\theta) = \sum_{l=1}^{L} e_{ul}a_{ul}\frac{q_u(\theta)}{\theta}$$

Aggregating over all upstream firms, the aggregate emissions Z_u from producing composite Q_u are

$$Z_u = M_u z_u(\theta_u^a)$$

A firm in m purchasing l_{mj} of composite Q_u is attributed the upstream emissions required to produce l_{mj}.

Assume that a midstream firm purchases its input from an average upstream firm. Producing $\frac{q_m(\theta_m)}{\theta_m}$ requires a quantity of input j

$$l_{mj} = a_{mj}\frac{q_m(\theta_m)}{\theta_m}$$

which purchased from an upstream firm in industry m

$$l_{mj} = q_u(\theta_u^a)$$

with associated upstream emissions of

$$z_u(\theta_u^a) = \sum_{l=1}^{L} e_{ul}a_{ul}\frac{q_u(\theta_u^a)}{\theta_u^a}$$

Thus

$$\begin{aligned} z_m^{\text{from upstream input j}}(\theta_m) &= \sum_{l=1}^{L} e_{ul}a_{ul}\frac{q_u(\theta_u^a)}{\theta_u^a} && (123)\\ &= \sum_{l=1}^{L} e_{ul}a_{ul}\frac{l_{mj}}{\theta_u^a} && (124)\\ &= \sum_{l=1}^{L} e_{ul}a_{ul}\frac{a_{mj}\frac{q_m(\theta_m)}{\theta_m}}{\theta_u^a} && (125)\\ &= \frac{a_{mj}q_m(\theta_m)}{\theta_u^a\theta_m}\sum_{l=1}^{L} e_{ul}a_{ul} && (126) \end{aligned}$$

Summing over all inputs u the total upstream emissions attributed to a midstream firm with productivity θ_m are

$$z_m^{upstream}(\theta_m) = \sum_{u}^{U}\sum_{l}^{L^u} e_{ul}a_{ul}\frac{a_{mu}q_m(\theta_m)}{\theta_u^a\theta_m} \quad (127)$$

A firm's upstream emissions intensity is defined similarly to its midstream emissions intensity; as the ratio of a firm's upstream emissions normalised by its own (midstream) profit. Assuming as before that only a single upstream input l in each industry u produces emissions,

$$SU_{CI} \equiv \frac{\sum_{j\equiv u}^{U} e_{ul}a_{ul}a_{mu}\frac{q_m(\theta_m^a)}{\theta_u^a\theta_m^a}}{\pi_m} \tag{128}$$

$$= \frac{q_m(\theta_m^a)}{\theta_m^a}\frac{\sum_{j\equiv u}^{U}\frac{e_{ul}a_{ul}a_{mu}}{\theta_u^a}}{\pi_m} \tag{129}$$

$$= \frac{q_m(\theta_m^a)}{\theta_m^a}\frac{\sum_{j\equiv u}^{U}\frac{e_{ul}a_{ul}a_{mu}}{\theta_u^a}}{\frac{c_m q_m(\theta_m^a)}{(\gamma_m-1)\theta_m^a}} \tag{130}$$

$$= \frac{(\gamma_m-1)}{c_m}\sum_{j\equiv u}^{U}\frac{e_{ul}a_{ul}a_{mu}}{\theta_u^a} \tag{131}$$

We can now see how midstream profits are affected by an upstream carbon price that affects multiple inputs j via (40) and (41)

$$\widehat{\pi_m} = -(\gamma_m-1)\mathcal{H}_m(\nu_m-1)\sum_{j=1}^{J}\iota_{mj}(1-\psi_{mj})\widehat{P_u}$$

$$\widehat{P_u} = \mathcal{A}_u\sum_{l=1}^{L}\iota_{ul}\Big[\psi_{ul}[\widehat{\tau_{ul}}+\widehat{e_{ul}}]+[1-\psi_{ul}]\widehat{r_{ul}}-\widehat{\mu_{ul}}\Big]$$

Combining these delivers:

$$\text{Carbon price: } \widehat{\pi_m} = -(\gamma_m-1)\mathcal{H}_m(\nu_m-1)\sum_{j=1}^{J}\iota_{mj}(1-\psi_{mj})\mathcal{A}_u\sum_{l=1}^{L}\iota_{ul}\psi_{ul}\widehat{\tau_{ul}} \tag{132}$$

$$\text{Productivity: } \widehat{\pi_m} = -(\gamma_m-1)\mathcal{H}_m(\nu_m-1)\sum_{j=1}^{J}\iota_{mj}(1-\psi_{mj})\mathcal{A}_u\sum_{l=1}^{L}\iota_{ul}(-\widehat{\mu_{ul}}) \tag{133}$$

note $\iota_{ul}\psi_{ul} = \frac{\tau_{ul}e_{ul}a_{ul}}{c_u}$ and $\iota_{mu} = \frac{w_{mu}a_{mu}}{c_m}$, and assume $\psi_{mj} = 0$ and the initial carbon price is zero such that $\frac{w_{mu}}{c_u} = \frac{\mathcal{M}_u}{\theta_u^a}$.

B.2 Upstream output-emissions intensity

The emissions released by a firm (equation (3.2)) and the unit-input requirement (equation (20)) give the upstream emissions z_u associated with the midstream firm's production of q_m of raw output and with upstream input use $q_u(\theta_u^a) = q_u * \theta_u^a = a_{mu}q_m$. Upstream emissions associated with upstream use

of inputs l are

$$z_u = \sum_{l=1}^{L^u} e_{ul} a_{ul} q_u = \sum_{l=1}^{L^u} e_{ul} a_{ul} \frac{q_u(\theta_u^a)}{\theta_u^a} = \sum_{l=1}^{L^u} e_{ul} a_{ul} \frac{a_{mu} q_m(\theta_k^a)}{\theta_u^a \theta_k^a}$$

with output emissions intensity

$$\frac{z_u(\theta_m^a)}{q_m(\theta_m^a)} = \sum_{l=1}^{L^u} \frac{e_{ul} a_{ul} a_{mu}}{\theta_u^a \theta_k^a}$$

We maintain the assumption that emissions are released by a single input in each stage of production. The upstream emissions-output intensity depends on the input-emissions intensity, unit-input requirements, and the average upstream productivity. θ_m^a and θ_u^a are both already-realised productivities for an operating firm, and are unaffected by upstream shocks.

Using changes in a_{mu} and a_{ul} from equation (90) and noting $\widehat{a_{mu}}$ is only affected by the input price $r_{mu} = P_u$ (equation (41)) through the cost w_{mu},

$$\widehat{a_{mu}} = \delta_m(\iota_{mu} - 1)\widehat{w_{mu}} = \delta_m(\iota_{mu} - 1)(1 - \psi_{mu})\widehat{P_u} \tag{134}$$

delivers

$$\begin{aligned}
\widehat{\left(\frac{z_u(\theta_m^a)}{q_m(\theta_m^a)}\right)} &= \widehat{e_{ul}} + \widehat{a_{ul}} + \widehat{a_{mu}} && (135)\\
&= \widehat{e_{ul}} - \delta_u\widehat{w_{ul}} + (\delta_u - 1)\widehat{\mu_{ul}} + \delta_u\widehat{c_u} + \delta_m(\iota_{mu} - 1)(1 - \psi_{mu})\widehat{P_u} && (136)\\
&= \widehat{e_{ul}} - \delta_u\widehat{w_{ul}} + (\delta_u - 1)\widehat{\mu_{ul}} + \delta_u\widehat{c_u} + \delta_m(\iota_{mu} - 1)(1 - \psi_{mu})\mathcal{A}_u\widehat{c_u} && (137)\\
&= \widehat{e_{ul}} - \delta_u\widehat{w_{ul}} + (\delta_u - 1)\widehat{\mu_{ul}} + (\delta_u - \delta_m(1 - \iota_{mu})(1 - \psi_{mu})\mathcal{A}_u)\widehat{c_u} && (138)
\end{aligned}$$

Note we have to distinguish here between brown productivity shocks μ_{ul} that directly affect the brown input as well as affecting costs through c_u, from green productivity shocks $\mu_{ul'}$ that only affect c_u. Considering each upstream shock in turn,

$$\text{Carbon price:} \quad \begin{cases} \widehat{\left(\frac{z_u}{q_m}\right)} = \psi_{ul}\left[-\delta_u(1-\iota_{ul}) - \delta_m \mathcal{A}_u \iota_{ul}(1-\psi_{mu})(1-\iota_{mu})\right]\widehat{\tau_{ul}} \\ \qquad\qquad = \ominus\widehat{\tau_{ul}} \end{cases} \tag{139}$$

$$\text{Brown-input productivity:} \quad \begin{cases} \widehat{\left(\frac{z_u}{q_u}\right)} = \left[-1 + \delta_u - \iota_{ul}\left(\delta_u - \delta_m(1-\psi_{mu})(1-\iota_{mu})\mathcal{A}_u\right)\right]\widehat{\mu_{ul}} \\ \qquad\qquad = \mp\widehat{\mu_{ul}} \end{cases} \tag{140}$$

$$\text{Green-input productivity:} \quad \begin{cases} \widehat{\left(\frac{z_u}{q_u}\right)} = -\iota_{ul'}\left[\delta_u - \delta_m(1-\psi_{mu})(1-\iota_{mu})\mathcal{A}_u\right]\widehat{\mu_{ul'}} \\ \qquad\qquad = \mp\widehat{\mu_{ul'}} \end{cases} \tag{141}$$

$$\text{Input-emissions intensity:} \quad \begin{cases} \widehat{\left(\frac{z_u}{q_m}\right)} = \left[1 - \psi_{ul}\left(\delta_u(1-\iota_{ul}) + \delta_m \mathcal{A}_u \iota_{ul}(1-\psi_{mu})(1-\iota_{mu})\right)\right]\widehat{e_{ul}} \\ \qquad\qquad = \pm\widehat{e_{ul}} \end{cases} \tag{142}$$

Green productivity improvements change emissions in more than one way. First, a green productivity improvement lowers the cost of the upstream firm u using the green input l—this lowers emissions if the upstream firm substitutes more to a green input. However, the green productivity improvement also lowers the cost to the midstream firm m of using the upstream firm's good u. This increases demand for u, creating an offsetting increase in use of u, and a corresponding increase in upstream emissions.

Note that decreases in e_{ul} can potentially raise emissions. This occurs when upstream emissions are priced. A reduction in e_{ul} reduces costs from using the emissions-generating input l. This induces the upstream firm u to use more of l, and induces the midstream firm to use more of input u. This increase in use offsets the direct reduction in e_{ul}. Note that if emissions are not priced then $\psi_{ul} = 0$, and a decrease in e_{ul} reduces $\frac{z_u}{q_m}$.

C Appendix: Downstream Model

C.1 Downstream demand derivation

Under **A1**, (54) becomes

$$\widehat{Q_m^{\text{demanded}}} = \widehat{a_{dm}} - \widehat{\theta_d^a} - \widehat{p_d(\theta_d^a)}. \tag{143}$$

Log differentiating (6) gives

$$\widehat{p_d(\theta_d^a)} = \widehat{c_d} - \widehat{\theta_d^a} \tag{144}$$

while log differentiating (20) gives

$$\widehat{a_{dm}} = -\delta_d \widehat{w_{dm}} + [\delta_d - 1]\widehat{\mu_{dm}} + \delta_d \widehat{c_d} \tag{145}$$

and so (143) becomes

$$\widehat{Q_m^{\text{demanded}}} = -\delta_d \widehat{w_{dm}} + [\delta_d - 1]\widehat{\mu_{dm}} + [\delta_d - 1]\widehat{c_d}. \tag{146}$$

Substituting in for $\widehat{c_d}$ using (26)[19] and for $\widehat{w_{dm}}$ using (88)[20] and collecting terms gives

$$\widehat{Q_m^{\text{demanded}}} = [\delta_d - 1]\iota_{do}\left[\widehat{\mu_{dm}} + \widehat{w_{do}} - \widehat{\mu_{do}}\right] - \left[\iota_{dm} + \iota_{do}\delta_d\right]\psi_{dm}\left[\widehat{e_{dm}} + \widehat{\tau_{dm}}\right] - \left[\iota_{dm} + \iota_{do}\delta_d\right][1 - \psi_{dm}]\widehat{P_m}. \tag{149}$$

Equation (55) indicates how demand from the downstream industry for the midstream composite varies with exogenous parameters.

As with equation (22), $-\nu_m$ is the price-elasticity of demand for the midstream composite. Setting $\widehat{\mu_{dm}} = \widehat{w_{do}} = \widehat{\mu_{do}} = \widehat{e_{dm}} = \widehat{\tau_{dm}} = 0$ in (55) and rearranging shows

$$\nu_m \equiv -\frac{\widehat{Q_m^{\text{demanded}}}}{\widehat{P_m}} = \left[\iota_{dm} + \iota_{do}\delta_d\right][1 - \psi_{dm}].$$

Note for later use that $\nu_m \in [0, \max\{\delta_d, 1\}]$; as we have assumed $\gamma_k > 1$ for all k then, by **A2**, it's also the case that $\gamma_m > \nu_m$.

We now turn our attention to examining variation in midstream supply. Use the feature that $R_m = P_m Q_m$ to substitute in for R_m in the zero-profit condition (9) to derive an equation expressing equilibrium supply of the midstream composite as a function of the price index P_m and other variables. Log differentiating and isolating $\widehat{Q_m}$ delivers

$$\widehat{Q_m} = -\gamma_m \widehat{P_m} + (\gamma_m - 1)[\widehat{c_m} - \widehat{\varphi_m^*}]. \tag{150}$$

Equate the right hand sides of (57) and (55) to get an expression in which the only endogeneous variables

[19] Specifically,

$$\widehat{c_d} = \iota_{dm}[\widehat{w_{dm}} - \widehat{\mu_{dm}}] + \iota_{do}[\widehat{w_{do}} - \widehat{\mu_{do}}] \tag{147}$$

[20] Specifically,

$$\widehat{w_{dm}} = \psi_{dm}[\widehat{\tau_{dm}} + \widehat{e_{dm}}] + [1 - \psi_{dm}]\widehat{P_m} \tag{148}$$

are $\widehat{P_m}$ and $\widehat{\varphi_m^*}$:

$$\left[\nu_m-\gamma_m\right]\widehat{P_m}-(\gamma_m-1)\widehat{\varphi_m^*}=-(\gamma_m-1)\widehat{c_m}+[\delta_d-1]\iota_{do}[\widehat{\mu_{dm}}+\widehat{w_{do}}-\widehat{\mu_{do}}]-\frac{\nu_m\psi_{dm}}{1-\psi_{dm}}[\widehat{\tau_{dm}}+\widehat{e_{dm}}]. \quad (151)$$

To obtain an expression for $\widehat{P_m}$, substitute in for $\widehat{M_d}$ in (93) using (101) and collect terms :

$$\widehat{P_m}=\widehat{c_m}+[\mathcal{Z}_m-1]\widehat{\varphi_m^*}.$$

Use this to substitute in for $\widehat{P_m}$ in (151), then collect terms and rearrange to isolate $\widehat{\varphi_m^*}$

$$\widehat{\varphi_m^*}=\mathcal{H}_m(\nu_m-1)\widehat{c_m}+\mathcal{H}_m(1-\delta_d)\iota_{do}[\widehat{\mu_{dm}}+\widehat{w_{do}}-\widehat{\mu_{do}}]+\mathcal{H}_m\nu_m\frac{\psi_{dm}}{[1-\psi_{dm}]}[\widehat{\tau_{dm}}+\widehat{e_{dm}}]. \quad (152)$$

where $\mathcal{H}_m$ was defined in (31). Replacing the θ variables in (17) with φs and log differentiating delivers the companion to (34), the change in profits for the midstream firm from a downstream shock

$$\widehat{\pi_m}=-(\gamma_m-1)\mathcal{H}_m(\nu_m-1)\widehat{c_m}-(\gamma_m-1)\mathcal{H}_m[1-\delta_d]\iota_{do}[\widehat{\mu_{dm}}+\widehat{w_{do}}-\widehat{\mu_{do}}]-(\gamma_m-1)\mathcal{H}_m\nu_m\frac{\psi_{dm}}{[1-\psi_{dm}]}[\widehat{\tau_{dm}}+\widehat{e_{dm}}] \quad (153)$$

As noted earlier, **A2** ensures $\gamma_m > \nu_m$. $\mathcal{H}_m$ is positive and so signs for the various comparative statics are as follows:

$$\mathbf{sgn}\left[\frac{\widehat{\pi_m}}{\widehat{c_m}}\right]=\mathbf{sgn}\left[1-\nu_m\right], \quad \mathbf{sgn}\left[\frac{\widehat{\pi_m}}{\widehat{\mu_{dm}}}\right]=\mathbf{sgn}\left[\frac{\widehat{\pi_m}}{\widehat{w_{do}}}\right]=\mathbf{sgn}\left[-\frac{\widehat{\pi_m}}{\widehat{\mu_{do}}}\right]=\mathbf{sgn}\left[\delta_d-1\right]$$

and

$$\mathbf{sgn}\left[\frac{\widehat{\pi_m}}{\widehat{\tau_{dm}}}\right]=\mathbf{sgn}\left[\frac{\widehat{\pi_m}}{\widehat{e_{dm}}}\right]=(-) \quad (154)$$

C.2 Downstream profit changes in terms of SD_{CI}

Case 1: Emissions come from m, $e_{dm}\neq 0, e_{do}=0$. Changes in profits are given by (59).

$$\widehat{\pi_m} \quad =-(\gamma_m-1)\mathcal{H}_m\nu_m\tfrac{\psi_{dm}}{[1-\psi_{dm}]}\widehat{\tau_{dm}} \quad =\ominus\widehat{\tau_{dm}} \quad (155)$$

$$(156)$$

We use several relations. As $\psi_{dm} = \frac{\tau_{dm} e_{dm}}{w_{dm}}$ and $w_{dm} = \tau_{dm} e_{dm} + P_m$,

$$\frac{\psi_{dm}}{[1-\psi_{dm}]} = \frac{\frac{\tau_{dm} e_{dm}}{w_{dm}}}{1 - \frac{\tau_{dm} e_{dm}}{w_{dm}}} = \frac{\tau_{dm} e_{dm}}{w_{dm} - \tau_{dm} e_{dm}} = \frac{\tau_{dm} e_{dm}}{P_m}$$

For $e_{dm} \neq 0$,

$$SD_{CI} = (\gamma_m - 1)\mathcal{M}_m \iota_{dm} \frac{e_{dm}}{P_m}$$

These can be used to express changes in profit in terms of SD_{CI},

$$\widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m \nu_m \tfrac{\psi_{dm}}{[1-\psi_{dm}]} \widehat{\tau_{dm}} \tag{157}$$

$$= -(\gamma_m - 1)\mathcal{H}_m \nu_m \tfrac{\tau_{dm} e_{dm}}{P_m} \widehat{\tau_{dm}} \tag{158}$$

$$= -\mathcal{H}_m \nu_m \tfrac{SD_{CI}}{\mathcal{M}_m \iota_{dm}} \tau_{dm} \widehat{\tau_{dm}} \tag{159}$$

$$\tag{160}$$

Case 2: Emissions come from o, $e_{dm} = 0, e_{do} \neq 0$.

We use: $\iota_{do}\psi_{do} = \frac{\tau_{do} e_{do} a_{do}}{c_d}$, $w_{dm} = P_m$ because $e_{dm} = 0$, and note from definition of ι_{dm} that $\frac{a_{dm}}{\iota_{dm}} = \frac{c_d}{w_{dm}}$.

Recall,

$$SD_{CI} = (\gamma_m - 1)\mathcal{M}_m \iota_{dm} \frac{e_{do} \frac{a_{do}}{a_{dm}}}{P_m}$$

The change in profits is then

$$\widehat{\pi_m} = +(\gamma_m - 1)\mathcal{H}_m(\delta_d - 1)\iota_{do}\psi_{do}\widehat{\tau_{do}} \tag{161}$$

$$= +(\gamma_m - 1)\mathcal{H}_m(\delta_d - 1)\frac{\tau_{do} e_{do} a_{do}}{c_d}\widehat{\tau_{do}} \tag{162}$$

$$= +\mathcal{H}_m(\delta_d - 1)\frac{SD_{CI}}{\mathcal{M}_m \iota_{dm}}\frac{P_m a_{dm}}{c_d}\tau_{do}\widehat{\tau_{do}} \tag{163}$$

$$= +\mathcal{H}_m(\delta_d - 1)\frac{SD_{CI}}{\mathcal{M}_m \iota_{dm}}\frac{P_m a_{dm}}{c_d}\tau_{do}\widehat{\tau_{do}} \tag{164}$$

$$= +\mathcal{H}_m(\delta_d - 1)\frac{SD_{CI}}{\mathcal{M}_m}\tau_{do}\widehat{\tau_{do}} \tag{165}$$

$$\tag{166}$$

C.3 Downstream output-emissions intensity

Downstream emissions attributed to an average midstream firm are

$$z_d(\varphi_m^a) = \iota_{dm}(e_{dm}a_{dm} + e_{do}a_{do})\frac{q_m(\varphi_m^a)}{a_{dm}}$$

with downstream emissions output-intensity

$$\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)} = \iota_{dm}(e_{dm} + e_{do}\frac{a_{do}}{a_{dm}})$$

Consider two cases: the midstream firm sells a brown good and downstream emissions come from using the midstream firm's output ($e_{dm} \neq 0, e_{do} = 0$), or the midstream firm sells a green good and emissions come from the alternative input ($e_{dm} = 0, e_{do} \neq 0$).

How shocks affect the downstream output-emissions intensity depends on whether downstream inputs are complements or substitutes.

Case 1: Midstream sells a brown good, $e_{do} = 0$:

If $e_{do} = 0$, then changes in the output-intensity of downstream emissions are determined only by the input emissions intensity e_{dm} and changes in input shares ι_{dm}

$$\begin{aligned}
\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} &= \widehat{\iota_{dm}} + \widehat{e_{dm}} && (167)\\
&= \widehat{w_{dm}} + \widehat{a_{dm}} - \widehat{c_d} + \widehat{e_{dm}} && (168)\\
&= \widehat{w_{dm}} - \delta_d\widehat{w_{dm}} + (\delta_d - 1)\widehat{\mu_{dm}} + \widehat{e_{dm}} + \delta_d\widehat{c_d} - \widehat{c_d} && (169)\\
&= (1-\delta_d)\widehat{w_{dm}} + (\delta_d - 1)\widehat{\mu_{dm}} + \widehat{e_{dm}} + (\delta_d - 1)\widehat{c_d} && (170)\\
&= (1-\delta_d)\widehat{w_{dm}} + (\delta_d - 1)\widehat{\mu_{dm}} + \widehat{e_{dm}} + (\delta_d - 1)\sum_{j=m,o}\iota_{dj}\left[\widehat{w_{dj}} - \widehat{\mu_{dj}}\right] && (171)
\end{aligned}$$

Summarising shocks separately

$$\text{Carbon price } \tau_{dm}\left\{\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} = (1-\delta_d)\psi_{dm}(1-\iota_{dm})\widehat{\tau_{dm}} \quad = \oplus(1-\delta_d)\cdot\widehat{\tau_{dm}} \right. \tag{172}$$

$$\text{Green-input productivity: } \left\{\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} = (1-\delta_d)\iota_{do}\widehat{\mu_{do}} \quad = \oplus(1-\delta_d)\cdot\widehat{\mu_{do}} \right. \tag{173}$$

$$\text{Input-emissions } e_{dm}\left\{\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} = (1+\psi_{dm}(1-\delta_d)(1-\iota_{dm}))\,\widehat{e_{dm}} \quad = \pm\cdot\widehat{e_{dm}} \right. \tag{174}$$

Case 2: Midstream sells a green good, $e_{dm} = 0$:

$$\begin{aligned}
\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} &= \widehat{\iota_{dm}} + \widehat{e_{do}} + \widehat{a_{do}} - \widehat{a_{dm}} && (175)\\
&= \widehat{w_{dm}} + \widehat{a_{dm}} - \widehat{c_d} + \widehat{e_{do}} + \widehat{a_{do}} - \widehat{a_{dm}} && (176)\\
&= \widehat{w_{dm}} - \widehat{c_d} + \widehat{e_{do}} - \delta_d\widehat{w_{do}} + (\delta_d - 1)\widehat{\mu_{do}} + \delta_d\widehat{c_d} && (177)\\
&= \widehat{w_{dm}} - \delta_d\widehat{w_{do}} + (\delta_d - 1)\widehat{\mu_{do}} + \widehat{e_{do}} + (\delta_d - 1)\widehat{c_d} && (178)\\
&= \widehat{w_{dm}} - \delta_d\widehat{w_{do}} + (\delta_d - 1)\widehat{\mu_{do}} + \widehat{e_{do}} + (\delta_d - 1)\sum_{j=m,o}\iota_{dj}\left[\widehat{w_{dj}} - \widehat{\mu_{dj}}\right] && (179)
\end{aligned}$$

Summarising shocks separately

$$\text{Carbon price } \tau_{do}\begin{cases}\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} = -\psi_{do}[\delta_d + \iota_{do}(1-\delta_d)]\widehat{\tau_{do}} & \\ \qquad\qquad = \ominus\cdot\widehat{\tau_{do}} \text{ for } \delta_d > 1 & \\ \qquad\qquad = \pm\cdot\widehat{\tau_{do}} \text{ for } \delta_d < 1 & \end{cases} \quad (180)$$

$$\text{Green-input productivity: } \begin{cases}\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} = (1-\delta_d)\iota_{dm}\widehat{\mu_{dm}} & = \oplus(1-\delta_d)\cdot\widehat{\mu_{dm}}\end{cases} \quad (181)$$

$$\text{Input-emissions } e_{do}\begin{cases}\widehat{\left(\frac{z_d(\varphi_m^a)}{q_m(\varphi_m^a)}\right)} = [1-\psi_{do}(\iota_{do}+\delta_d(1-\iota_{do}))]\,\widehat{e_{do}} & = \pm\cdot\widehat{e_{do}}\end{cases} \quad (182)$$

These results show how a downstream green-input productivity improvement can in some cases raise a midstream firm's output-emissions intensity. This occurs when the green-input sold by a midstream firm is a complement for emissions-intensive production. When this occurs, a green-input productivity reduces the cost of using the green input, which then increases demand for the complementary emissions intensive good, causing the emissions-intensity to rise.

D Input-emissions intensity

This section shows transition risks from changes in $\widehat{e_{kj}}$ occuring upstream, midstream, and downstream. Many emissions sources, like fossil fuels, release an approximately fixed amount of emissions per amount used; for such sources the emissions intensity of an input e_{kj} is fixed. However, other inputs can be thought of as an aggregate bundle which do not have a fixed emissions intensity. For example, transportation or industrial heat is an input to production that technology can reduce the emissions

intensity of. In such situations, e_{kj} can be interpreted as a technology parameter that can vary. Changes in e_{kj} do not change the productivity of the input, unlike μ_{kj}, and only reduce costs to the extent that the input's emissions themselves are costly. For upstream, midstream, and downstream see Sections 4.3, 3.3 and 5.3. When expressing results in terms of a firm's carbon intensity we maintain the assumption that a single input releases emissions.

$$\text{Upstream } e_{ul}\text{:} \quad \begin{cases} \widehat{\pi_m} = -\dfrac{(\gamma_m - 1)}{c_m}\mathcal{H}_m(\nu_m - 1)\displaystyle\sum_{j=u=1}^{U} \mathcal{A}_u\mathcal{M}_u\frac{e_{ul}a_{ul}a_{mu}}{\theta_u^a}\tau_{ul}\widehat{e_{ul}} & = \ominus\widehat{e_{ul}} \end{cases} \tag{183}$$

$$\text{Midstream } e_{mj}\text{:} \quad \begin{cases} \widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m(\nu_m - 1)\displaystyle\sum_{j=1}^{J}\frac{\tau_{mj}a_{mj}}{c_m}e_{mj}\widehat{e_{mj}} & = \ominus\,\widehat{e_{mj}} \\ \widehat{\pi_m} = -\mathcal{H}_m(\nu_m - 1)SM_{CI}\tau_{mj}\widehat{e_{mj}} & = \ominus\,\widehat{e_{mj}} \end{cases} \tag{184}$$

$$\text{Downstream } e_{dm} \text{ for m} \begin{cases} \widehat{\pi_m} = -(\gamma_m - 1)\mathcal{H}_m\nu_m\dfrac{\psi_{dm}}{[1-\psi_{dm}]}\widehat{e_{dm}} & = \ominus\widehat{e_{dm}} \\ \widehat{\pi_m} = -\mathcal{H}_m\nu_m\dfrac{SD_{CI}}{\mathcal{M}_m}\tau_{dm}\widehat{e_{dm}} & = \ominus\widehat{e_{dm}} \end{cases} \tag{185}$$

$$\text{Downstream } e_{do} \text{ for o:} \quad \begin{cases} \widehat{\pi_m} = +(\gamma_m - 1)]\mathcal{H}_m(\delta_d - 1)\iota_{do}\psi_{do}\widehat{e_{do}} & = \oplus(\delta_d - 1)\widehat{e_{do}} \\ \widehat{\pi_m} = +\mathcal{H}_m(\delta_d - 1)\dfrac{SD_{CI}}{\mathcal{M}_m}\tau_{do}\widehat{e_{do}} & = \oplus(\delta_d - 1)\widehat{e_{do}} \end{cases} \tag{186}$$

Input-emissions intensity changes

As long as emissions are priced a reduction in e_{mj} or e_{ul} raises industry m profits as per equations (184) or (183), and in general raises profits more the larger a firm's SM_{CI}. Similarly, if a firm sells a brown good then a downstream reduction in e_{dm} also raises profits. In contrast, if a firm sells a downstream green substitute good then a reduction in e_{do} can lower profits.