Are Brazilian Investors in Two New Goals-based Bonds for Retirement and Education Rational, Behavioral or Speculators? Evidence from Live Data (2023-2024)

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Abstract

In January 2023, Brazil issued a new instrument, Renda+ (or "Retirement Income Extra"), and in August 2023, it issued another, Educa+ ("Education Financing Extra"), to help retail investors better save for retirement security and a child's education, respectively. These bonds were designed with very specific cashflows for the respective goals, and which combined accumulation, decumulation, inflation and compounding in a single instrument to overcome the typical financial literacy challenges faced by average investors globally. These bonds, purchased voluntarily, on a simple two-question app, have been successful and are being considered by other countries in the developed and developing world as they address key social challenges. The only two questions to be answered are: (a) start date of targeted cashflows (e.g., one's retirement date or when a child would enter college); and (b) target real amount desired (e.g., R$ 5,000 per month) from such investments, with the idea being that ALL investors, of all levels of financial literacy and income levels can answer such questions. These instruments were issued with various start dates of payments to cater to different age cohorts to target their respective goal-cash flow needs. Interestingly, the retirement bond has been proposed for India and the US.

Brazil's Tesouro Nacional (NT) has kept very detailed records of the age of investors in the respective instruments and distributional data (along with gender, region of purchase etc.). One of the intriguing elements from an analysis of the data is that there is a mismatch between what specific instrument the app on NT's website would recommend to investors based on their target retirement date or child's date of college entrance (call it "guided choice" or "nudge"), and the actual age/ date distribution of investors based on data over the last 1.5-2 years. The unanswered research questions we are pursuing is whether the app is not entirely effective in educating/ guiding/ nudging investors, or whether investors are displaying rational behavior, exhibiting some form of behavioral bias, or are purely speculating (given that these are very long-duration bonds and hence offer oversized returns if say the investor goes long duration of 50 years and interest rates drop 1% in a short span of time), or some combination of the above. There are six puzzles that we examine within this data set, including why young people are buying bonds targeted to older investors? why older investors appear to be more risk averse and select bonds that are the guided choice? why we get unusual distributions by age in the education bond etc? We will examine the data for all start dates of each of Renda+ (8 instruments with start dates of 2030, 2035….2065) and Educa+ (17 instruments with start dates of 2030, 2031….2046), and try to discern if the portfolio choices reflect "rational", "behavioral" or "speculative" decisions.

Keywords: Retirement, Education Financing, Renda+, Educa+, Household Decision-making, Goals-based Investing, Behavioral Biases

JEL Codes: D14, D15, D53, D81, D91, G11, G41

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3. There were no non-disclosure arrangements that have any relevance to this paper.

Background.

Nobel Laureate William Sharpe is often quoted as saying that "decumulation (in DC pension funds) is the hardest problem in finance", though he himself states that this is hyperbole.[1] Yet, this quote is often used on LinkedIn by vendors to argue for annuities without having been made aware of innovations in financial markets in Brazil that greatly simplify this problem to be solved by those with NO financial literacy.

In 2023, Brazil issued two new instruments, Renda+ ("retirement income extra")[i], and Educa+ ("education financing extra")[ii], that change the paradigm of finance practice and theory, and retail sovereign debt issuance. These instruments reflect the new trend of Goals-based Investing (GBI) and correspond to the Cochrane (2022) observation that, "Switching to a payoff perspective may change lots of procedures in finance". Muralidhar (2025), focusing on Cochrane (2022)'s challenge to derive a general equilibrium (GE) in this framework, is able to derive a normative, (Janus) GE to price all assets, including such GBI bonds and the absolute risk-free asset, determine optimal asset allocations to these bonds and risky assets, while also being able to establish equilibrium wages paid to workers to be able to finance such portfolios. Given the success of this instrument, it has been proposed for countries as diverse as the US (Merton and Muralidhar 2020, called "SeLFIES") and India (Muralidhar 2023b, called "InDIAS" or Inflation Deferred Income Annuity-like Securities).

Brazil's Tesouro Nacional (NT) has kept detailed records of the portfolio choices made by investors, not only by security, but also by age of the investor, and this rich data set provides us with an opportunity to examine investor behavior in Brazil. We are able to ask difficult questions about whether individuals, with ostensibly low levels of financial literacy and financial resources, for whom these bonds were originally designed, have made rational, behavioral or speculative investment choices (defined a bit flexibly relative to an Expected Utility Theory

[1] https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4321188

(EUT)/non-GBI perspective). Not all questions are easily answered as we do not know the motivations of the investment decisions, just the revealed preferences in their choices, but we make an attempt nonetheless, and open the door to future research on investor behavior in a GBI world, with these new instruments or potentially to help NT and other governments to continue the process of innovation to improve such financial instruments.

The paper is structured as follows: Section I provides greater detail on Renda+ and Educa+, and the app to help financially unsophisticated investors make investment decisions for these two goals, either for themselves (retirement) or for their children (education). It demonstrates how EUT, by ignoring multiple stochastic goals, might not be the best model to apply in examining investor behavior in a GBI world. Section II provides high level detail on which Brazilians have been purchasing these instruments, by gender, region, age etc. as background to the puzzles that ensue. Section III raises the many questions that arise from the actual investment decisions of Brazilians, and Section IV attempts to answer them, while also defining the terms "rational", "behavioral" and "speculative", as these terms are relative to whether investors are wealth or income focused (Merton 2014). Section V addresses issues for future/continued research, and Section VI concludes.

Section I: Why Renda+ and Educa+?

EUT, with its primary assumption that "investors maximize the expected utility of wealth", ignores four realities of investing: (i) that investors have stochastic goals (and not a deterministic goal as assumed in Sharpe 1964); (ii) that investors have multiple stochastic goals (retirement, education, health etc. unlike single goal models in the literature); (iii) that they delegate to (hopefully) skillful agents (EUT ignores skill considerations even though some papers address agency and delegation); and (iv) they specify their objectives, especially their risk preferences, in a very focused/transparent manner, with clear absolute and relative risk goals (EUT has a non-observable risk aversion parameter). Muralidhar, Ohashi and

Shin (2014a) and (2014b), in a model they call the "Relative Asset Pricing Model" or RAPM, using a KUJ utility function with the stochastic liability as the denominator and the asset portfolio or wealth in the numerator, derive asset allocation and asset pricing recommendations to accommodate the first objection to using EUT in a GBI world. RAPM's objective is in keeping with the objective function of Sharpe-Tint (1990)[iii], and derives an asset pricing model in the presence of a single stochastic liability, common to all individuals. Merton (2014) argues that investors in defined contribution (DC) plans should focus on retirement income and not retirement wealth of EUT, implicitly endorsing the RAPM perspective. Muralidhar (2021, 2023a, and 2024a, 2024b, 2025) then extends this model to multiple stochastic goals (like retirement and education etc.) and calls it the Goals and Risk-based Asset Pricing Model (GRAPM). In short, in these "relative" models, there are three classes of assets: (i) the absolute risk-free asset; (ii) the relative risk-free asset or stochastic goal replicating asset (also termed Goals-based Arrow Debreu Securities or GADs in Muralidhar 2024b); and (iii) all other risky assets. Recall in EUT there is only the absolute risk-free asset (which is the relative risk-free asset as well in a deterministic world), and risky assets, aggregated into a non-observable "market portfolio". GRAPM has elements of Merton's (1973) Intertemporal Capital Asset Pricing Model, except that in reality, the stochastic driver of decisions is the liability (or *raison d'etre* for investing), and not a "stochastic opportunity set". As a result, asset allocation should be split across these three asset groupings, and not the Two Fund Separation of Tobin (1958), or the three fund separation of Merton (1973), which are not observed in practice.

Individuals are deemed "financially unsophisticated", in the terminology of Klapper et al (2015), if they could not answer basic questions about simple or compound interest, and inflation. Multi-period investing is all about these calculations, including accumulation and decumulation and hence the opening quote from Nobel Laureate William Sharpe. Hence, Muralidhar (2019), following Thaler (2013) and Merton (2012), argues that innovative governments can create

unique instruments to overcome financial literacy challenges (as opposed to wasting valuable resources trying to educate individuals on these concepts, when such literacy programs have been found to have limited and short-term impact). These instruments should embed accumulation, decumulation, compounding and inflation, thereby making a simple choice of these new instruments a way to overcome financial literacy challenges in goal achievement.

Muralidhar (2015), Muralidhar, Ohashi and Shin (2016) and Muralidhar (2016 a and b) advocate for the creation of a retirement income bond (originally called Bonds for Financial Security, BFFS, but then renamed SeLFIES in Merton and Muralidhar (2016)), and education bonds (called Bonds for Education and Student Tuition or BEST), respectively. Brazil issued two such GADs in 2023 – Renda+ (or "retirement income bond")[iv] in January 2023, and Educa+ ("education bond")[v], in August 2023.

For retirement, Renda+ pays R$1,000 real/year, monthly, for 20 years (tied to average life expectancy), starting at the retirement age, and indexed to the IPCA, the inflation index, most commonly used in Brazil. Renda+ was issued with 8 different start dates: 2030, 2035….2065. Educa+ offers R$1,000 real/year, paid monthly too, for 5 years, starting at age 18 (when a child goes to college because college in Brazil typically takes 5 years), and also indexed to IPCA (though a more ideal indexation would be to tuition inflation as suggested in the original BEST design). Educa+, similarly, had 17 different cash flow start dates: 2030, 2031, 2032…2046. The cash flows of generic Brazilian retirement and education bonds are provided in Figure 1, but there is no requirement that these GADs be solely debt instruments, as certain goals have equity-like components and those could be contemplated in future GADs as in Muralidhar (2024a).

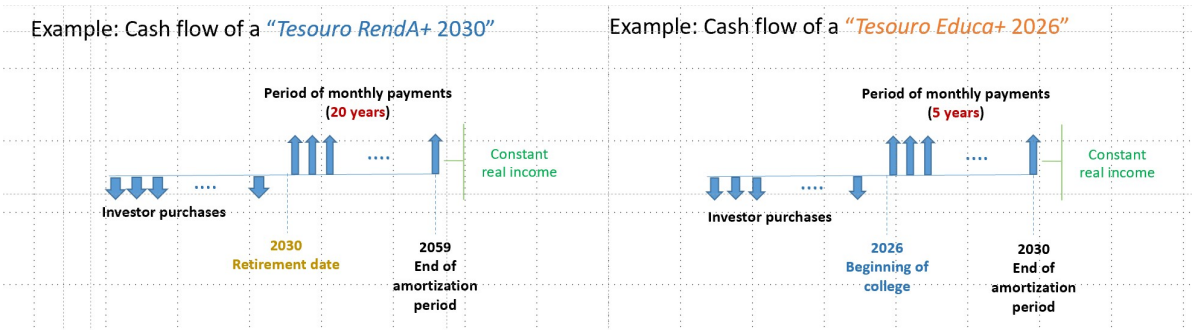
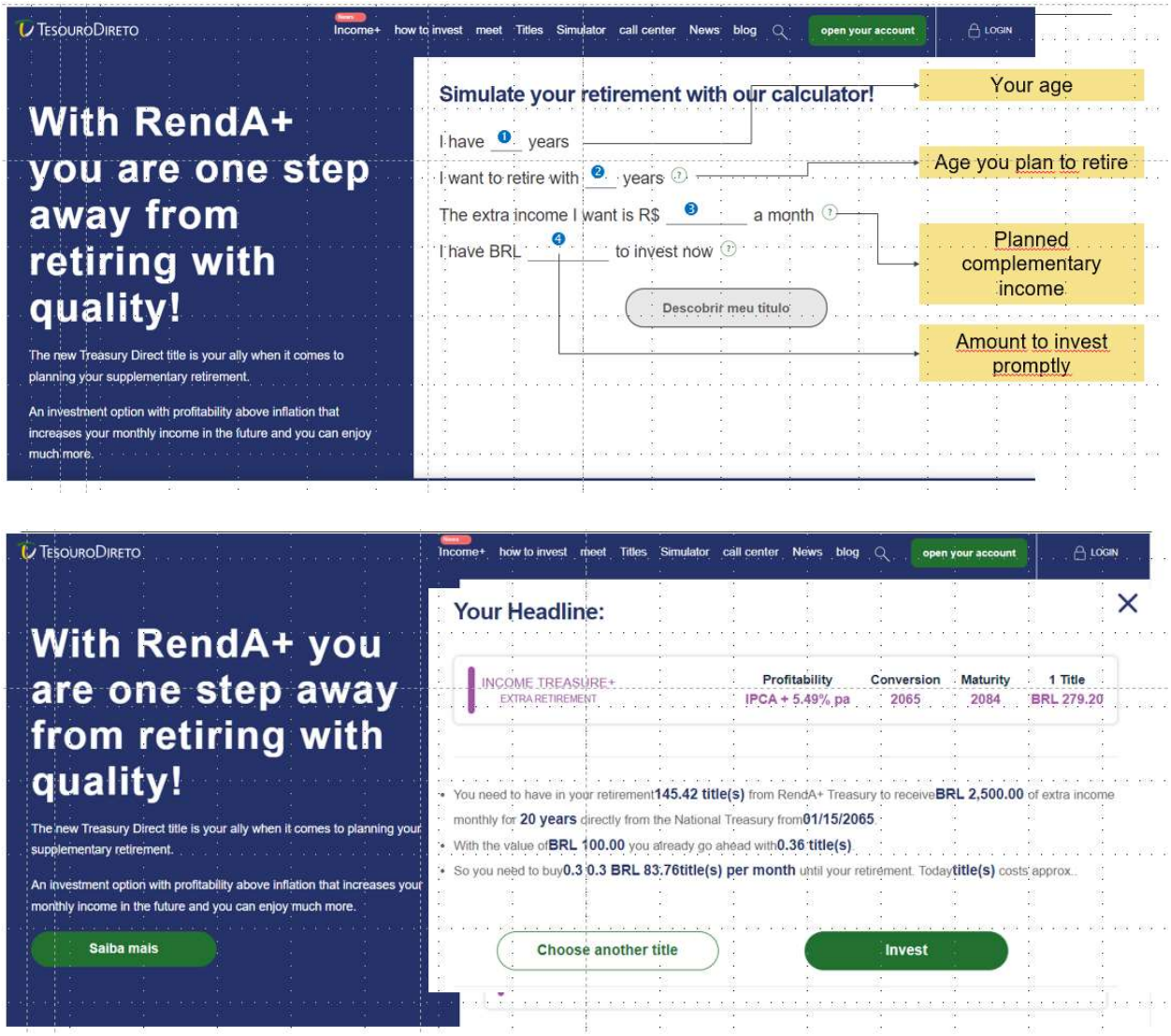


Figure 1: Cash flows of retirement (20 years starting in 2030), and education (5 years starting in 2026) bonds issued in Brazil (*Source: Tesouro National presentation to International Retail Debt Management Symposium, Washington DC, November 2024*).

The design was very specific to overcome the quote of Prof Sharpe: if an individual needed $50,000 real/year in retirement or for a child's college education, they just need to buy 50 of the respective bonds prior to the trigger event (with the flexibility to change their date as these are liquid and tradeable on a simple app offered by Brazilian Treasury), requiring nothing more than basic division (but see app later). *In short, to achieve one's goal requires buying just multiples of a single, relative risk-free security (Merton and Muralidhar 2020), as opposed to current practice of buying multiple securities to match goal cash flows (Coqueret et al 2017).* This is how these instruments can greatly improve goal-achievement and risk management in a goals-based world as the "guided choice" can be super simple and, ostensibly, an easy "nudge".

Brazil's NT went further and even provided a simple app, that required investors to answer two simple questions (their date of start of goals-based cash flows and target real income), and they would be prompted with the ideal instrument to meet their objectives. Figure 2 demonstrated how one can see this as "guided choice" for financially unsophisticated investors. Not only is the choice guided, but the execution of the investment decision is also trivial; in fact, buying securities other than the guided choice involves effort (and therein lie some puzzles).



Figures 2A (prompts) and 2B (results): NT App to help investors make "ideal" Renda+ and Educa+ investment choices (Source: National Treasury. Available at: https://www.tesourodireto.com.br/Rendamais/, in 20/06/23.)

We will focus most of our attention on Renda+ because Educa+ could have been purchased for a child by parents and grandparents. Hence, there is not always a one-to-one correspondence between the age of the recipient and the age of the purchaser.[vi] Whereas for Renda+, ostensibly, investors are purchasing these securities for themselves.

Section II – Who is Buying?

Brazil scores relatively poorly on the financial literacy assessments, assuming that these naïve questions are useful in the context of gauging financial literacy; Klapper et al (2015) claim that 28% of Brazilian adults are financially literate using their measure of five questions that test for compounding and inflation awareness. The demographics of the country are provided in Appendix I as background. Also, from NT we know that traditionally, 60-70% of investors in government bonds favor instruments with a maturity of 5 years or less.[2] Tesouro Direto is a National Treasury Program developed to facilitate the sale of federal government bonds to individuals. Launched in 2002, the Program was created with the aim of democratizing access to government bonds, encouraging savings and serving as a financial education tool. The program as of the end of 2024 had around three million active investors and a total stock of over R$150 billion. The issuance of bonds is 100% guaranteed by the National Treasury. Trading is carried out online with no minimum investment amount.

As of November 2023, 122,102 investors had purchased Renda+, with a median age of approximately 37-38 years, and with a breakdown by start/end date as in Table 1. Because Renda+ start dates are spaced out in 5-year intervals, there could be some clustering of allocations across different start dates (e.g., a 57-year-old in 2025, may allocate to both the 2030 and 2035 bonds, as there is no specific bond for their cash flows to start when they turn 65 in 2033). However, as we demonstrate later, less than 20% of investors are buying multiple securities across Renda+ issues. Moreover, while the dataset we have is as of November 2024 (so a static data set), NT informs us that when the price of long dated bonds declined dramatically, investors actually stepped in to buy these

[2] I thank Lucas Gurgel Leite for this useful data point.

long duration instruments. As a friend with experience as a trader in the Brazilian markets noted in private correspondence in February 2025, "

"Lots of friends pilling..into RendA+ these days… not even with the intent of using it for retirement but because it offers the biggest duration one can ever get down there in Brazil, and, because of the expectations the market will improve"

Start	End payments	TARGET AGE	NUMBER OF INVESTORS	PERCENT
2030	2049	60	26,801	21.9%
2035	2054	55	18,924	15.5%
2040	2059	50	15,645	12.8%
2045	2064	45	21,983	18.0%
2050	2069	40	11,155	9.1%
2055	2074	35	10,809	8.9%
2060	2079	30	6,299	5.2%
2065	2084	25	10,486	8.6%
			122,102	100%

Table 1: Distribution of Investors in Renda+ as of November 2024

With respect to Educa+, 125,670, with a median investor age of 38 years, had purchased Educa+ (even though Educa+ had been launched 7 months later), and the distribution of investors is provided in Table 2. In other words, once Educa+ was created, the demand growth for Renda+ tapered off and the growth in Educa+ investors has been much more rapid.

However, relative to the total activity in Tesouro Direto (3 million), there is still room for further penetration and ideas like offering these facilities to recipients of Bolsa Familia (50 million recipients) could transform the demand for these assets. But this initial phase of the program, given how innovative and transformational it is, allows NT to try different ideas to improve the product before attempting to make it more widespread.

What is interesting is that the skew in Educa+ is towards near term payments (not dissimilar to investor behavior across all NT products), and one part of that could be because the first payment only starts in 2030, so there is no ideal

instrument for say a 15-year-old today. The average age statistic and the fact that more investors have purchased Educa+ than Renda+ in a shorter span of time, in and of itself suggests that individuals will prioritize their children's education over their own retirement; either because of the nearness and smaller cost of education or because individuals prioritize their children over their own financial security (behaviorally "altruistic"?). For both these GADs, we do not have corresponding statistics on how much each age cohort bought of each security at the current time, so our analysis is limited by the current data set.

Start Payments	End Payments	TARGET AGE	NUMBER / AVERAGE AGE
2030	2035	13 (OR HIGHER)	61,367
			38.90
2031	2036	12	12,380
			38.96
2032	2037	11	7,204
			37.77
2033	2038	10	5,997
			40.18
2034	2039	9	4,976
			38.65
2035	2040	8	3,699
			38.64
2036	2041	7	3,309
			37.60
2037	2042	6	2,508
			37.47
2038	2043	5	2,557
			37.36
2039	2044	4	4,818
			38.11
2040	2045	3	2,182
			36.17
2041	2046	2	1,853
			37.25
2042	2047	1	1,811
			35.93
2043	2048	0	2,160
			35.51
2044	2049	-1	2,933
			36.09
2045	2050	-2	3,725
			36.40
2046	2051	-3	2,191
			35.85
Total			125,670

Table 2: Distribution of Investors in Educa+ as of November 2024

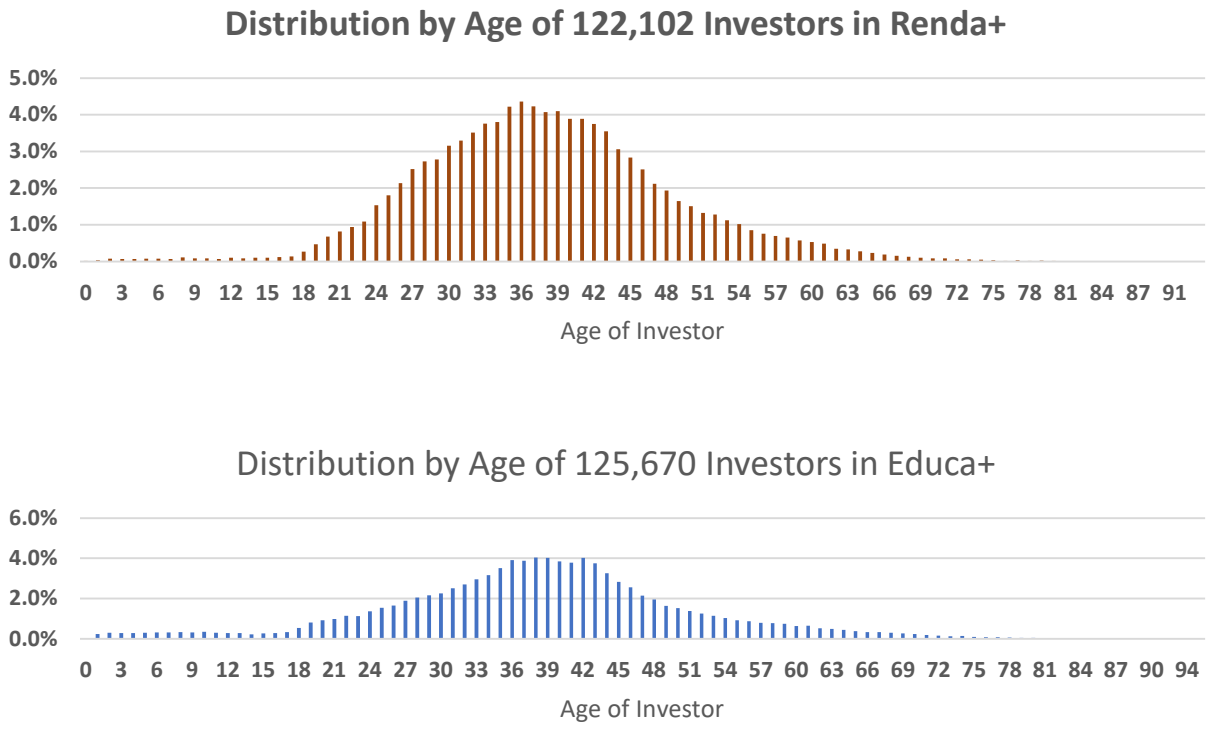


Figure 3A (Renda+) and 3B (Educa+): Distribution of Investors in the Two Instruments

From an earlier update from NT (August 1, 2024), the gender and marital status of Renda+ investors are provided in Figure 4A and 4B and similarly for Educa+ in Figures 4C and 4D.

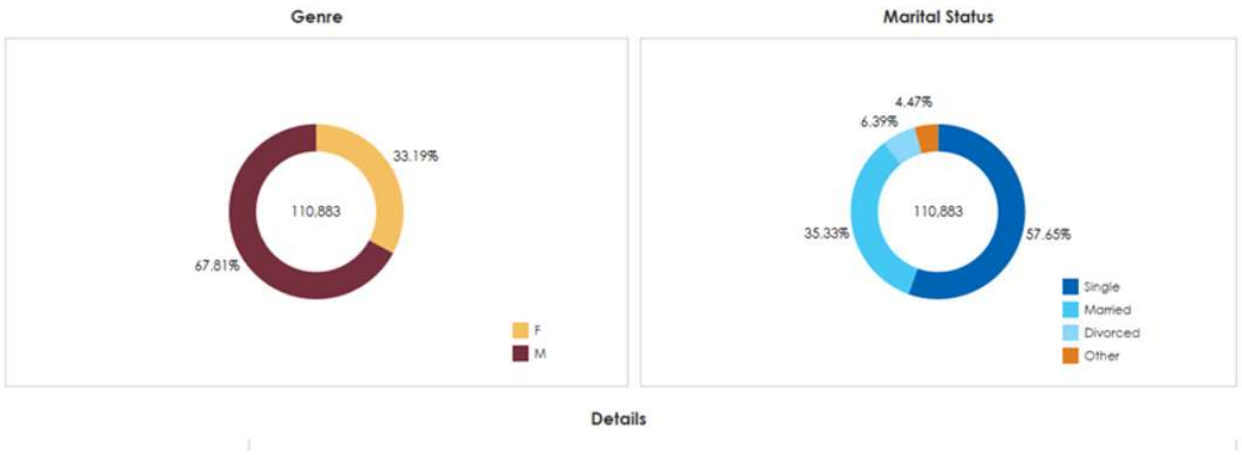


Figure 4A (Gender) and 4B (Marital Status): Renda+ Investors



Figure 4C (Gender) and 4D (Marital Status): Educa+ Investors

NT also reports that in Renda+, 53% of total investors have already made more than one contribution over time, and the more contributions the investor makes, the higher the median purchase ticket. In Educa+, the highest concentration of investors is that of a single contribution (70.6%). The demographic profile of the public that invests repeatedly is like that observed in Renda+, with a large proportion (59.8%) from the southeast region, and 64.8% made up of men. Despite this, the percentage of married and divorced people is higher than the percentage of singles in Educa+, which may indicate that the title is being used for their children's education. The median ticket per contribution is similar among the groups in terms of number of contributions.

Figure 5A provides a distribution of the investors by Cash Flow Start Date for Renda+ and 5B provides the same for Educa+. The Renda+ data suggests individuals closer to retirement, assuming these are pure retirement investors and not speculators, are the larger proportion of investors (i.e., especially because speculators might prefer the longer duration bonds given the bigger impact on price from small changes in lower interest rates since these are all long positions only) as noted in the quote from the Brazilian trader.

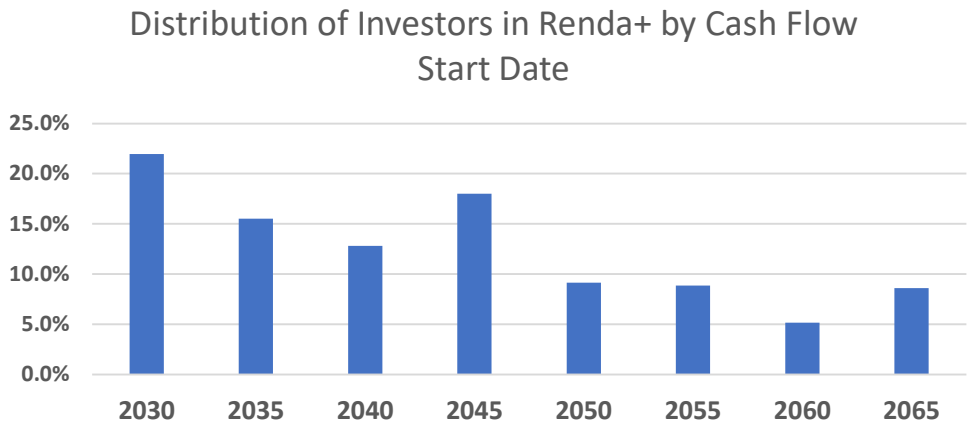


Figure 5A: Distribution of Investors in Renda+ by Cash Flow Start Date

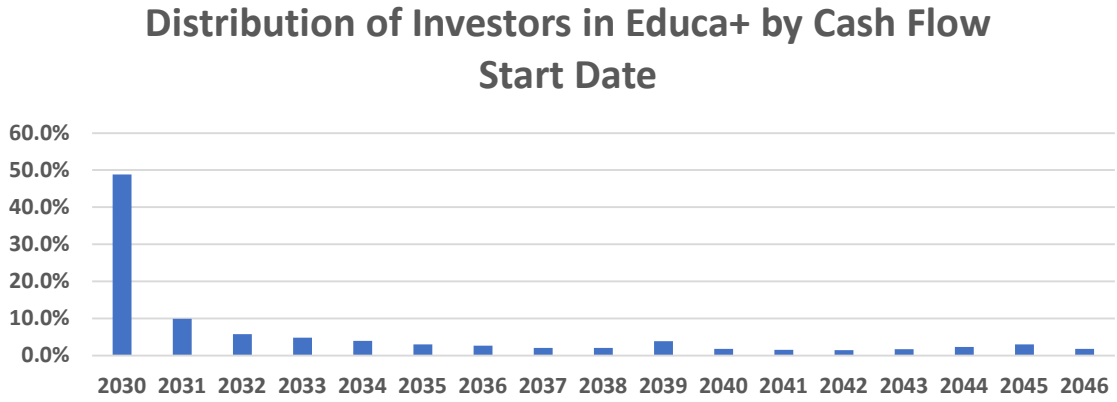


Figure 5B: Distribution of Investors in Educa+ by Cash Flow Start Date

Similarly, the data for Educa+ might suggest that individuals are targeting the instruments that are tailored for their children potentially 13 years and older, since the earliest cash flow start date in 2030 (and would be suited to a 13-year-old today). The skew is clearly more dramatic in Educa+ than in Renda+, which has a more substantial distribution for future payment dates, but this is similar to overall investor behavior to favor maturities that pay out in the next 5 years or less. In the case of Educa+, only 2031 touches 10%; all other start date allocation percentages are in single digits.

Section III – At Least 6 Puzzles

We do not have the complete portfolio data of these investors; namely, whether they are holding positions in what might be considered "risky" assets such as the Bovespa stock market. Kobor and Muralidhar (2018 and 2019) demonstrate how a dynamic mix of SeLFIES and say an equity index, with a funded status-based rebalancing, could help investors achieve their retirement income goal on a better risk-adjusted basis than traditional life-cycle products or a passive 60% Equities/40% Bonds portfolio. This lack of information notwithstanding, this dataset poses more questions than it answers. Basically, if the guided choice approach is working, and individuals treat Renda+ and Educa+ as the relative risk-free asset, then the distributions of investors should be very concentrated on the approximate date of retirement (in the case of Renda+), and a possibly bi-polar distribution for Educa+ - one specific bar for the age of child for which each Educa+ is tailored (e.g., age 13 for Educa+ 2030), and maybe another 25-30 years later, to represent the age of the parent who may be holding the security for their child. This is not always what the data shows.

1. *Puzzle 1 – Young People Buying Renda+ 2030*. The first major question that arises from this dataset, as Figure 6 demonstrates, is that a lot of young people are buying the 2030 Renda+ bond, which starts paying in 5 years; they ostensibly should not need these cash flows if they have (labor) income. As Figure 6 demonstrates, the target age is noted by the red bar (give or take a few years), but instead the bulk of the purchases have been made by individuals substantially younger than this target age, suggesting that individuals are largely ignoring the recommendations from the app, which was meant to greatly simplify goals-based investing, or are making the same behavioral bias to favor short maturity instruments that they exhibit in all NT bonds. Out of 26,801 investors in this series, just 364 are for the specific target retirement date, and even a count of +/-5 years around 60 years, yields a total investor count of approximately 3,000

investors! That is approximately 11% of the total number of investors in this series!

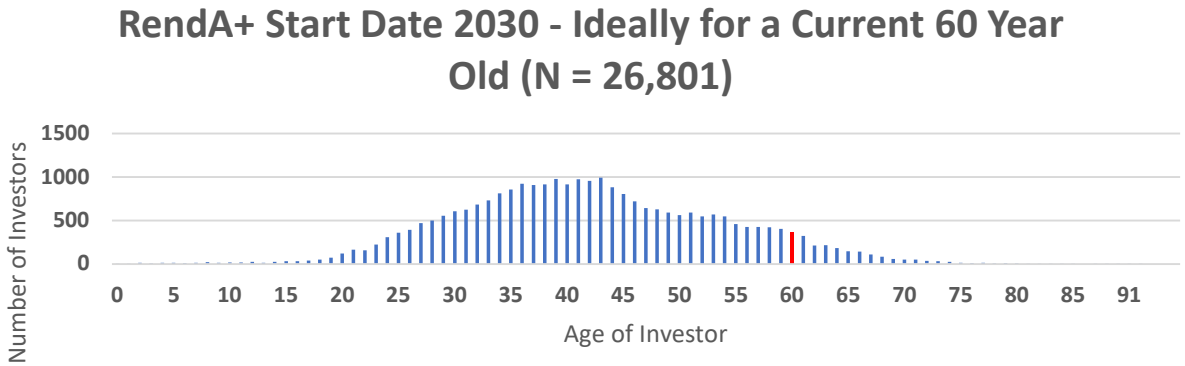


Figure 6 and Puzzle 1: Lots of Young People buying Bond that Pays Quickly

2. *Puzzle 2 – Older Investors Buying Long Duration Renda+*: Similarly, at the other end of the Renda+ series spectrum, old and young people are buying the 2065 Renda+ which is targeted to 25-years-olds - this is shown in Figure 7. Once again, the red bar highlights the "guided choice" for the age of the investor and there is a significant percentage of the distribution to the right of the red bar, suggesting older investors buying approximately 50-year duration instruments (often far exceeding their potential life expectancy). Ostensibly, older investors may have more money to invest, but this chart is tracking number of investors, and not ability to invest.

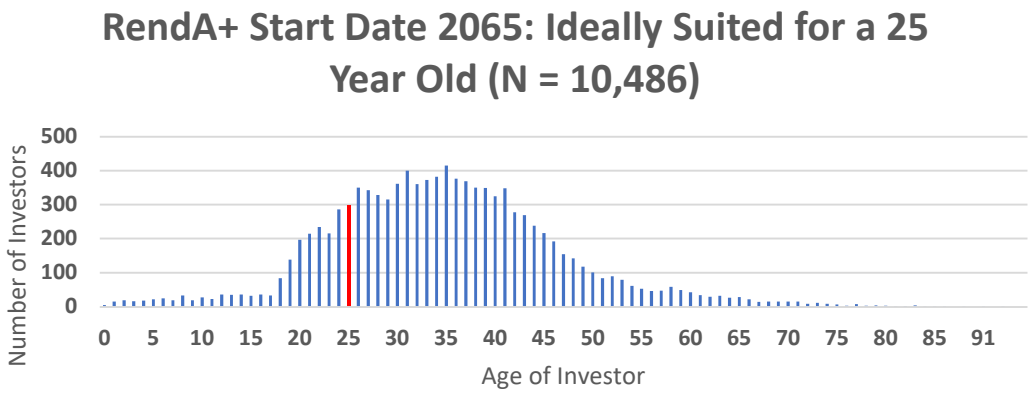


Figure 7 and Puzzle 2: Lots of "Old" People Buying Bond Targeted for 25 Year Olds

3. *Puzzle 3*: When one looks at the portfolio choices of say the 25-year-olds and the distribution of bonds to which they have allocated, one gets a very interesting picture as in Figure 8. Why is it that this age group is investing in such a wide spectrum of assets in nearly equal proportions and not the guided choice date highlighted by the green bar?

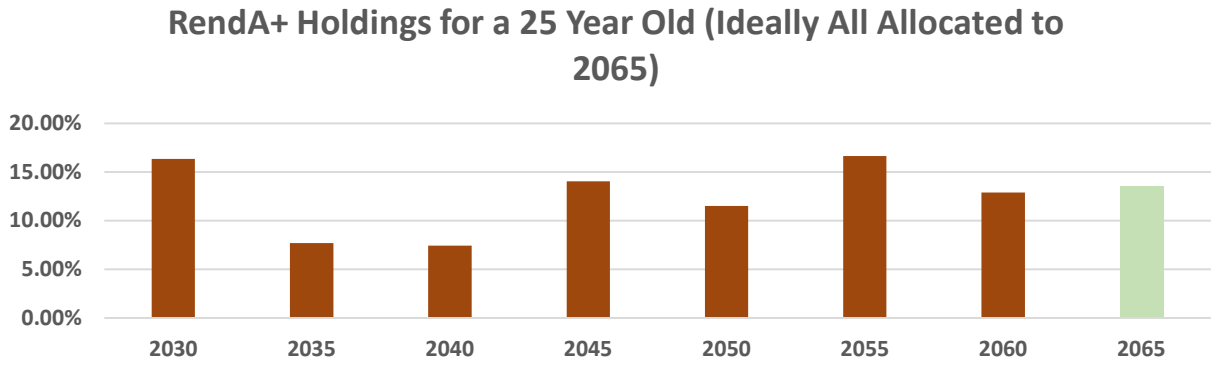


Figure 8 and Puzzle 3: Why are 25 Year Olds Diversifying Away from Relative Safe Asset?

4. *Puzzle 4*: When one looks at the portfolio choices of say the 60-year-olds (Renda+ 2030), and the distribution of bonds to which they have allocated, one gets a very interesting picture (Figure 9). Why is the allocation of this group much more concentrated in a few securities unlike the 25-year-olds?

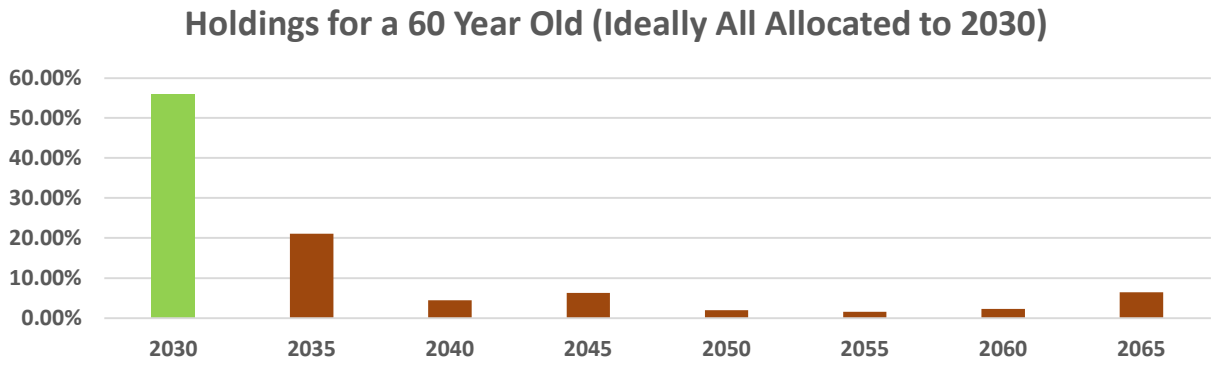


Figure 9 and Puzzle 4: Why are 55-60 Year Largely in the Relative Safe Asset?

5. *Puzzle 5*: We took another stab at the data and aggregated data based on age for the various Renda+ start dates (e.g., we added up all investors from 56 years onward for the 2030; then 51-55 onward for 2035 and so on). We were trying to see whether there would be an aggregation of investors to dates close to their approximate return date, but instead the percentages are clearly less than 50% of the investors. However, why do we see this hump shaped pattern, where 30 to 40-year-olds seem more inclined to invest in series that start paying close to/near the guided choice date?

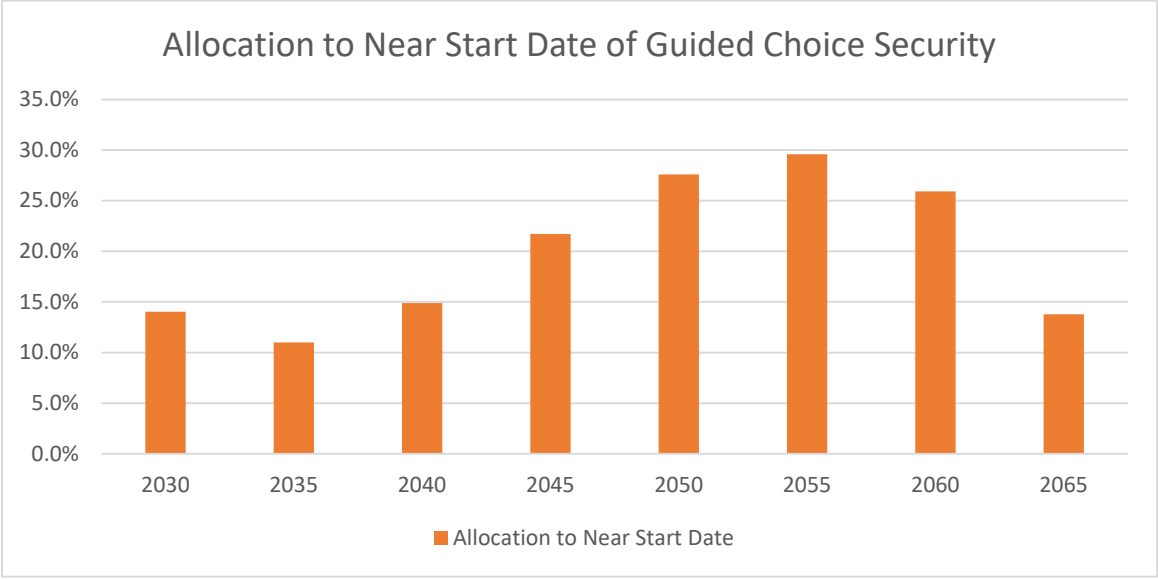


Figure 10 and Puzzle 5: Why Do 30-40 Years Olds Invest Closer to their Date of Retirement?

6. *Puzzle 6*: The Educa+ data is a bit harder to interpret as these allocations could have been made by parents and grandparents on behalf of children in their family. But as Table 2 shows, there is a clear clustering of allocations to the near payment dates with over 50% allocated to the nearest cash flow start date (and recall this instrument has a shorter duration than Renda+). In examining the 2030 allocation data, as Figure 11 shows, with a red bar indicating the target age for this issue, it is clear that parents and grandparents are holding these securities on behalf of their family members – some form of altruism. However, as we get into the later Educa+ payment start dates we start to see a more bipolar distribution (starting with Educa+ 2035), as shown in Figure 12 specifically for Educa+ 2040, but with a much smaller sample size of just 2,182. Even then, the spike in the youth holdings is not on the age we would expect (colored in red) for the cash flow start date. Why do we see differences in distribution shape and again, why is there less clustering than we would expect if the app is useful?

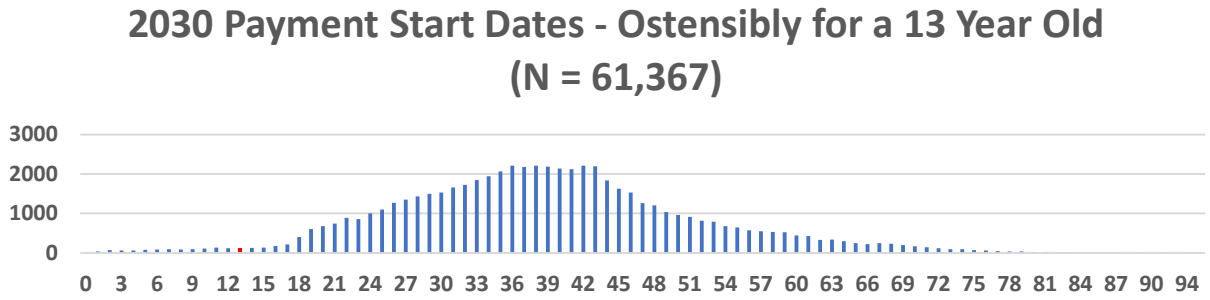


Figure 11 and Puzzle 6: Allocation by Age to Educa+ 2030

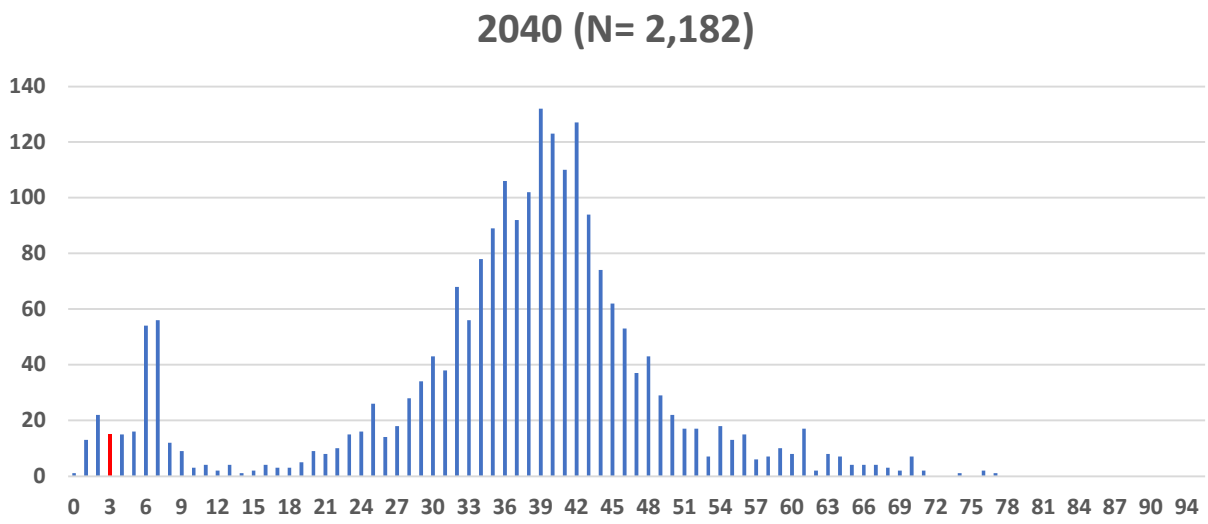


Figure 12 and Puzzle 6: Allocation by Age to Educa+ 2040

Section IV – Possible Explanations

Recall that we have no knowledge of why investors made these decisions and we are working with revealed preference in this data set. So, before we ascribe objectives to the decisions, we have to define the terms "rational", "behavioral" and "speculation"

Rational – if we follow EUT, this would indicate that individuals are cash-flow focused and trying to achieve some form of Tobin's Two Fund Separation; namely, holding the security closest to their target date of usage, plus some risky security. Remember, holding a security with a start date different from the security ideal for one's age is a risky choice. The allocation between these two options indicates their degree of risk-aversion. Higher the risk aversion the more likely they are to cash flow match or to be funded status oriented (than worry about a change in wealth).

Behavioral – while Kahneman-Tversky (1979) suggest loss-aversion occurs around some "critical value", but Muralidhar (2014), quoting the original KT text (see footnote)[vii] note that when individuals are just below the threshold funded

status level of 100%, then they must punt to achieve a binary goal like buying a house. While retirement and education are not such binary events as one can lower one's standard of living in retirement or potentially go to a lower cost college (as opposed to affording a house or not). If one has insufficient savings, the likelihood of gambling when they are close to the date of when cash flows are needed, and may not have saved sufficiently, can be seen as a form loss aversion. Behaviors that are also included are habit – previous retail debt behavior being repeated despite the fact that these instruments are different.

Speculation – could be revealed if investors appear to be taking a view on the direction of short-term or long-term interest rates as suggested by the casual comment by the trader friend noted earlier (and in some ways a rational decision, but geared more for principal appreciation than cash flow matching). Typically, here the goal is wealth maximization in our definition (or targeting a positive return on investment, and not cash flow matching).

1. *Puzzle 1 – Young People Buying Renda+2030*. One explanation for this result is that young individuals correctly anticipated that Brazil's central bank would be raising short term interest rates to combat high inflation and hence held shorter duration debt than for their age group to preserve capital.[viii] The price of this bond rose from R$ 1483.57 on Jan 30, 2023 to R$1720.7 on October 29, 2024 or a 15.98% total return over this period. Again, we do not know the entry date of their trade and it might help to have a snapshot of positions say at 6-month intervals to see if there are marked shifts in the holding patterns. More likely, this is behavioral/habit given the investing pattern across all other bonds.
2. *Puzzle 2 – Older Investors Buying Renda+ 2065*: One explanation for this result, assuming that investors are rational (and speculative), is that older investors are expecting long rates will decline and are taking a speculative bet on the price appreciation. This has been a pretty decent bet as the price of say RendA+ (bid) rose from R$ 160.32 to R$184.18, a 14.88% total return over nearly 2 years. Recall, we do not know the entry date of their

transaction, but this is the best explanation we can offer. An alternative explanation is that they are buying long duration instruments as a way to hedge longevity risk, as Renda+ does not directly hedge longevity risk (and why Merton and Muralidhar 2020 and Muralidhar 2018 propose different mechanisms to address this issue).

3. *Puzzle 3 – Allocation Distribution of 25 Year Olds to Renda+ Securities*: The interesting clarification from NT is that while "some customers have multiple holdings, it is not meaningful." The net total investors after accounting for multiple holding was approximately 104,000 (so at least 80% of the universe). Clearly, these investors are diversifying across the entire spectrum of available Renda+ bonds, but all of them are clearly lowering the average duration below the duration of their retirement cash flows (which would allocate in large part to the 2065 bond). Recall, that investing in any other Renda+ bond, is in effect a risky bet (and exploited by Muralidhar 2024a and b to derive GRAPM).
4. *Puzzle 4 – Portfolio Choice of 60 Year Olds*: Clearly, as one approaches one's retirement date, they tend to be less risk averse and there is a clustering in the nearest two cash flow start dates (and the allocation to the 2035 could either be based on start date of retirement or a hedge against longevity). The stark difference between Figure 8 and 9 demonstrates that as individuals age, the risk aversion to allocate to risky assets increases (ignoring different funded status situations which could affect risk aversion), and hence they favor the "safe" relative risk-free asset.
5. *Puzzle 5 – Why Mid-Life Individuals Invest Closer to their Target Age for Renda+*: Why this behavioral bias seems to creep in is not clear. Maybe these investors are more financially sophisticated? It would be an interesting topic for future research.
6. *Puzzle 6 – Why a Concentration of Investments in Educa+ 2030 and why bi-polar distributions for Educa+ 2035 onwards?* Again, in general, it appears that there is a habit element to favoring the earliest paying bond (i.e.,

within 5 years), but also the later distributions, being bi-polar, suggest a rational decision to favor the bond best suited for the child.

We try to summarize the guesses we make in Table 3.

Puzzle	Rational	Behavioral	Speculation
Puzzle 1 – Young buying Renda+ 2030	Likely: Liquidity	Most Likely: Behavioral Bias/Habit	Possible: Bet on Short Rates
Puzzle 2 – Old Buying Renda+ 2065	Unlikely: Longevity Hedging	Loss aversion?	Most Likely: Duration Speculation
Puzzle 3 – Young Spreading Allocation	Diversifying Risk	Possible: Confused by Choice?	
Puzzle 4 – 55-60 Year Olds = Concentrated	Cash flow match – risk aversion	Risk aversion	
Puzzle 5 – 30-40 Year Olds Likely to Invest Close to Renda+ Start Date		Guided Choice most effective for this group (relatively)?	
Puzzle 6 – Educa+ Distribution Smooth for 2030; Bipolar for 2035 onwards	Cash flow matching for younger kids. Desire to save something for older kids explains the 50% weight in 2030 bond?	Guided choice works for 2035 onwards. Possible bias because no way to hedge 15 year olds' cash flows? Habit for earliest paying bond?	Less speculation in Educa+ (possibly because of lower duration and availability of RendA+ for long duration bets?)

Table 3: Best Guess as to Behavior of Investors vis-à-vis the 6 Puzzles

Section V - Future/continued research

This data set and questions they raise offer a rich set of issues to be tackled in future research, including:

1. Have size of investment also calibrated to age of investors, so we are looking at allocation amounts and not just number of investors
2. Be able to get these same tables possibly at 6-month intervals of say June 30, 2023, December 31, 2023, June 30, 2024 and December 31, 2024, as that will capture the volatility of interest rates and portfolio positioning (and changes if any). "Cash flow investors" will not change their allocations because of different rates as their investments are programmatic; speculative or informed investors might accelerate or decelerate purchases (and sales) based on the interest rate cycle as suggested in Mantilla et al (2020).
3. For those 18,000 odd Renda+ investors holding multiple securities, it might be useful to see their portfolio choices and how they are diversifying – whether between close-dated Renda+, or whether they are splitting their portfolio across wide maturities, or even between Renda+ and Educa+ to create a "relative safe/risky" portfolio allocation as in Kobor and Muralidhar (2019).

Section V - Conclusion.

In January 2023, Brazil issued a new instrument, Renda+ (or "Retirement Income Extra"), and in August 2023, it issued another, Educa+ ("Education Financing Extra"), to help retail investors better save for retirement security and a child's education, respectively. These bonds were designed with very specific cashflows for the respective goals, and which combined accumulation, decumulation, inflation and compounding in a single instrument to overcome the typical financial literacy challenges faced by average investors globally. These bonds, purchased voluntarily, on a simple two-question app, have been successful and are being considered by other countries in the developed and

developing world as they address key social challenges. The only two questions to be answered are: (a) start date of targeted cashflows (e.g., one's retirement date or when a child would enter college); and (b) target real amount desired (e.g., R$ 5,000 per month) from such investments, with the idea being that ALL investors, of all levels of financial literacy and income levels can answer such questions. These instruments were issued with various start dates of payments to cater to different age cohorts to target their respective goal-cash flow needs and has clearly led to financial inclusion given that allocations can be made in slices as small as $1/time.

Brazil's NT data suggests that there is a major mismatch between what specific instrument the app on NT's website would recommend to investors based on their target retirement date or child's date of college entrance, and the actual age/date distribution of investors based on data over the last 1.5-2 years. The unanswered research questions we are pursuing is whether the app is useful in educating/guiding/nudging investors, or whether investors are displaying rational behavior, exhibiting some form of behavioral bias/habit (which is hard to break even with the app), or are purely speculating (given that these are very long-duration bonds), or some combination of the above. We examined six puzzles within this data set, including why young people are buying bonds targeted to older investors? why older investors appear to be more risk averse and select bonds that are the guided choice? why we get unusual distributions by age in the education bond etc? We tried to provide some initial answers and clearly the habit of buying near term paying bonds appears to manifest itself in both these bonds, even though they offer much greater duration than traditional 5-year bonds. Causal observations from trader friends seem to reinforce the idea that some investors are speculating on duration as with high real rates, a reduction in interest rates can lead to massive short-term windfall gains.

The data for all start dates of Renda+ (8 instruments with start dates of 2030, 2035….2065) and Educa+ (17 instruments with start dates of 2030, 2031….2046), suggest non-obvious portfolio choices and appear to reflect a

combination of "rational", "behavioral" or "speculative" decisions. More importantly, these questions and possible solutions offer NT (and other countries considering adopting/adapting these innovations), some issues to consider to improve the effective utilization of these new instruments to better serve citizens. The benefit is that the take-up of these bonds, while impressive, is still low relative to all investments in Tesouro Direto bonds, and hence gives NT a chance to continuously improve the design of the instruments and shape a research agenda to solicit investor feedback.

Appendix. Figure 1. Brazilian Demographics

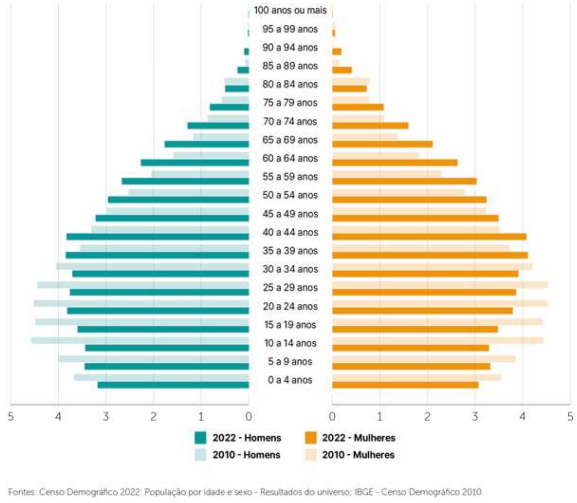
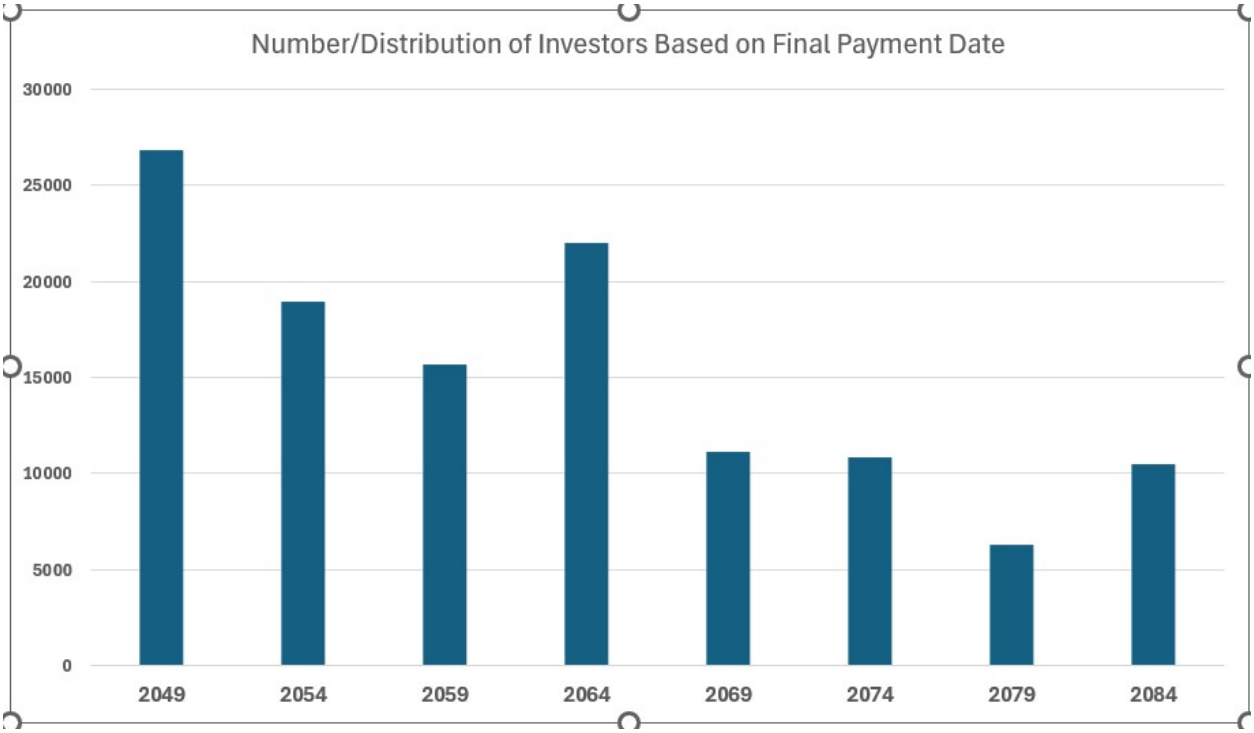


Figure 2: Distribution/Number of Investors in Brazil's Retirement Bond



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[ii] https://www.cnnbrasil.com.br/economia/tesouro-educa-novo-titulo-para-financiar-estudos-comeca-a-ser-negociado-na-b3-veja-como-funciona/.
[iii] That focuses only on asset allocation decisions of investors with stochastic liabilities and ignores asset pricing considerations.
[iv] https://borainvestir.b3.com.br/tipos-de-investimentos/Renda-fixa/tesouro-direto/tesouro-Renda-brasil-pode-ter-um-sistema-previdenciario-mais-sofisticado-que-o-de-outros-paises/
[v] https://www.cnnbrasil.com.br/economia/tesouro-educa-novo-titulo-para-financiar-estudos-comeca-a-ser-negociado-na-b3-veja-como-funciona/.
[vi] And Brazil has innovative technology to facilitate such purchases including a QR code for a child that can be used to designate the recipient of Educa+ purchases.
[vii] "[F]or example, the utility function of an individual who needs $60,000 to purchase a house may reveal an exceptionally steep rise near the critical value."
[viii] https://finance.yahoo.com/news/brazil-central-bank-pledges-hike-215048569.html