Persistent or Temporary? How Earnings Shock Misperception Leads to Household Financial Distress*

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march 2025

Abstract

This paper examines how earnings shock misperception fuels financial distress. Households must learn about the persistence of income shocks by observing noisy signals, leading to borrowing and default decisions based on imperfect information. A benchmark model with perfect information is compared to two learning-based models: one with Bayesian updating and another with diagnostic expectations, where agents overreact to recent shocks. The model is calibrated to match observed delinquency, bankruptcy, and debt accumulation rates. Learning amplifies financial distress by causing agents to misinterpret transitory shocks as persistent, leading to excessive borrowing and delayed repayment difficulties. Diagnostic expectations further increase default risk as households overreact to positive and negative income fluctuations. Simulations show how these misperceptions result in higher debt levels, increased interest rates, and more frequent delinquency and bankruptcy episodes. These findings highlight the importance of earnings misperception in driving financial distress.

JEL Classification: D14, D84, E21, G51
Keywords: Earnings Shocks, Learning, Diagnostic Expectations, Financial Distress, Bankruptcy, Delinquency.

*First version: February 2025. The views expressed herein are those of the authors and do not reflect the views of the FRB of St. Louis or the Federal Reserve System.

1 Introduction

Knowing whether a change in earnings is short-term or long-term can be the key to staying financially secure or falling into distress. By comparing predictions from three different models of household financial distress, this study uncovers the powerful role that earnings shock misperception plays in shaping financial outcomes.

To understand the impact of earnings shock misperception, this analysis compares a standard model that incorporates both formal and informal debt non-repayment with two novel extensions. The first introduces rational learning, where households update their beliefs about the persistent component of labor earnings based on observed realizations. The second also includes learning but departs from rational expectations, allowing for a cognitive bias (diagnostic expectations) where individuals overweight recent information when forming beliefs about the future.

The analysis follows a straightforward strategy: the model is first estimated under learning and then compared, using the same parameters, to the perfect information and diagnostic expectations frameworks. This comparison reveals how different learning mechanisms shape household financial behavior and, crucially, how misperceptions about income persistence fuel financial distress. The model with Bayesian learning–and even more so the model with diagnostic expectations–generates substantially higher rates of delinquency and bankruptcy.

Simulation results illustrate the underlying mechanism. When young households mistakenly attribute high income to the persistent component, they accumulate excessive debt, believing their earnings will remain elevated. However, when fortunes reverse–a common occurrence when income increases are largely transitory–households find themselves overleveraged, facing repayment difficulties, and ultimately defaulting. Under diagnostic expectations, these reversals are even more extreme, as households overreact to recent income fluctuations, amplifying financial distress. These findings highlight the power of expectation-driven household-level debt cycles, underscoring the role of misjudged optimism and sudden pessimism in shaping household financial stability.

The models used in this paper allow households for non-repayment of debt in the tradition of Athreya (2002), Livshits, MacGee and Tertilt (2007), and Chatterjee, Corbae, Nakajima and Ríos-Rull (2007). The first model, based on Athreya, Mustre-del Río and Sánchez (2019), follows a standard framework incorporating both formal bankruptcy and delinquency as forms of non-repayment of credit card debt. Two additional models extend this framework in ways not previously explored in the literature. One introduces learning about the persistent component of labor earnings, similar to Guvenen (2007) and Lee and Sæverud (2023). In that setup, households update their beliefs using Bayesian learning based on observed earnings realizations over time. The final model departs from rational expectations. We assume that behavioral households adopt diagnostic expectations, following Bordalo, Gennaioli, Porta and Shleifer (2019) and Bordalo, Gennaioli, Ma and Shleifer (2020).

2 Setup

The economy consists of a continuum of finitely lived, risk-averse households. Each individual enters the labor market at age $n = n_0$, retires at age $n = T$, and lives up to age $n = N$.

2.1 Income Process

Household income follows a restricted income profile (RIP), given by

$$\tilde{y}_n = \exp\left(\ell(n) + z_n + e_n\right),$$

where $f(n)$ represents the deterministic component that depends solely on age, z_n is the persistent stochastic component, and e_n is an i.i.d. transitory shock, distributed as $\mathcal{N}(0, \sigma_e^2)$.

The persistent component evolves as a random walk:

$$z_n = z_{n-1} + \epsilon_n,$$

where $\epsilon_n \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is an i.i.d. innovation.

We assume that $\ell(n)$ is common knowledge. Each period, households observe their realized income $\tilde{y}_n$. Additionally, we define the residual income realization y_n as

$$y_n \equiv z_n + e_n = \ln(\tilde{y}_n) - \ell(n),$$

which is also observable since both $\tilde{y}_n$ and $\ell(n)$ are known. However, households cannot distinguish between the stochastic components of income–persistent and transitory. That is, they do not directly observe either z_n or e_n.

2.2 Rational Belief Updating

Households enter the labor market with a prior belief about z_n and gradually update their beliefs through the realizations of residual income. Every household starts with the same prior,

$$z_{n_0} \sim \mathcal{N}(\mu_{z,0}, \sigma_{z,0}^2),$$

where the subscript n_0 indicates entry into the labor market. Additionally, we assume that this prior is aligned with the objective distribution of z_{n_0}. Thus, there is no initial misbelief about z_{n_0}, even under diagnostic expectations.

A rational household enters age n with prior beliefs about z_n, which we refer to as $z_{n|n-1}$, and has the probability density $f(z_n|Y_{n-1})$, where Y_{n-1} denotes the full history of residual income observed up to the previous period. At age n, the household and the lender observe a new signal—

its residual income realization y_n—and update its estimate of the persistent income component using Bayes' rule:

$$f(z_n|Y_n) = \frac{f(y_n|z_n)f(z_n|Y_{n-1})}{\int f(y_n|z)f(z|Y_{n-1})\,\mathrm{d}z}, \tag{1}$$

which iteratively defines the household's rational beliefs.

With normal priors and shocks, the rational household's and lenders' learning problem reduces to a textbook Kalman filter process. Specifically, at age n, the prior about z_n is summarized by

$$\mathcal{N}(z_{n|n-1}, \Sigma_{n|n-1}).$$

Crucially, in a Kalman filter framework, the variance depends only on the number of signals received rather than the specific history of realized signals. In our setting, the number of observed realizations maps one-to-one with age n, allowing us to express the variance solely as a function of age. This property is fundamental: to fully characterize the distribution of z_n, it suffices to know the mean and the age.

Given the prior $\mathcal{N}(z_{n|n-1}, \Sigma_{n|n-1})$, their belief updating follows:

$$\begin{aligned}
z_{n|n} &= z_{n|n-1} + K_n\left(y_n - z_{n|n-1}\right), \\
\Sigma_{n|n} &= \frac{\sigma_e^2}{\Sigma_{n|n-1} + \sigma_e^2}\Sigma_{n|n-1}, \\
\Sigma_{n|n-1} &= \Sigma_{n-1|n-1} + \sigma_\epsilon^2, \\
K_n &= \frac{\Sigma_{n|n-1}}{\Sigma_{n|n-1} + \sigma_e^2}, \\
z_{0|-1} &\equiv \mu_{z,0}, \quad \Sigma_{0|-1} \equiv \sigma_{z,0}^2.
\end{aligned}$$

Given the posterior, a household's belief about next period's residual income realization follows a normal distribution with mean equal to the updated belief about income, $z_{n|n}$, and variance incorporating the updated uncertainty $\Sigma_{n|n}$ along with the variances of the income shocks:

$$y_{n+1} \sim \mathcal{N}\left(z_{n|n}, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2\right).$$

Note that the mean of residual income in the next period equals the mean of the belief about z in the current period, as z follows a random walk and both e and ϵ have zero means. Similarly, since the shocks are independent, the variance is the sum of the individual variances.

2.3 Distorted Belief Updating

We consider the possibility that households form distorted beliefs based on the extent of diagnostic expectations, parameterized by $\theta \geq 0$, following Bordalo et al. (2020). Specifically, these distortions arise from Kahneman and Tversky's representativeness heuristic.

In line with Bordalo et al. (2019)'s proposal for a diagnostic Kalman filter, we define the representativeness of a state at age n as the likelihood ratio:

$$R_n(z) = \frac{f(z|Y_n)}{f\left(z|Y_{n-1} \cup z_{n|n-1}\right)}.$$

Here, $Y_n = Y_{n-1} \cup y_n$ represents the history up to today and $Y_{n-1} \cup z_{n|n-1}$ represents the history up to today under the assumption that the new residual income equals the ex-ante forecast, $y_n = z_{n|n-1}$. This implies that state z is more representative at age n if the new signal y_n increases its probability relative to the scenario where the residual income matches the forecast. Intuitively, the most representative states z are those whose likelihood has increased the most in response to the latest residual income realization.

Households then overweight representative states by using a distorted posterior, replacing the rational posterior (1) with

$$f^\theta(z_n|Y_n) = f(z_n|Y_n)\left[R_n(z_n)\right]^\theta \frac{1}{Z_n},$$

where Z_n is a normalization factor ensuring that $f^\theta(z_n|Y_n)$ integrates to one.

The parameter $\theta > 0$ measures the extent to which beliefs deviate from rational updating due to representativeness. When $\theta = 0$, beliefs remain rational and are given by the Bayesian conditional distribution $f(z_n|Y_n)$. For $\theta > 0$, the diagnostic density $f^\theta(z_n|Y_n)$ inflates the probability of representative states while deflating that of unrepresentative ones. Crucially, this distortion leads to systematic mistakes: states that have become relatively more likely are overweighted, even if they remain unlikely in absolute terms.

Note that after observing the current residual income realization y_n, the diagnostic posterior follows a normal distribution:

$$\mathcal{N}(z^\theta_{n|n}, \Sigma_{n|n}),$$

where the posterior mean $z^\theta_{n|n}$ is adjusted according to diagnostic expectations (for proof, see Proposition 1 of Bordalo et al. (2020)):

$$\begin{aligned}
z^\theta_{n|n} &= z_{n|n-1} + (1+\theta)K_n\left(y_n - z_{n|n-1}\right), \\
\Sigma_{n|n} &= \frac{\sigma_e^2}{\Sigma_{n|n-1} + \sigma_e^2}\Sigma_{n|n-1}, \\
K_n &= \frac{\Sigma_{n|n-1}}{\Sigma_{n|n-1} + \sigma_e^2}, \\
z_{0|-1} \equiv \mu_{z,0}, &\quad \Sigma_{0|-1} \equiv \sigma^2_{z,0}.
\end{aligned}$$

Thus, a household's distorted belief about next period's residual income realization follows:

$$y_{n+1} = \quad z^\theta_{n|n} + \epsilon_{n+1} + e_{n+1} \sim \mathcal{N}(z^\theta_{n|n}, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2).$$

Since z follows a random walk, the mean of residual income in the next period equals the mean of the belief about z in the current period. However, this belief is distorted due to households' diagnostic expectations.

A key feature of diagnostic expectations is what the literature calls *rational reversion*. The diagnostic distortion applies only to the posterior update within each period. However, at the end of the period, households reassess their beliefs rationally, as if re-evaluating the entire history of realizations. This reversion ensures that the prior for the next period remains rational. As a direct implication, diagnostic expectations always start with a rational prior and there is no need to carry from one period to the other the value of the distorted belief. The key distinction between the lender and the behavioral households lies in their *posterior mean*. While the lender maintains entirely rational beliefs, their variance updates remain identical to households holding rational expectations.

2.4 Timeline

The timeline within each period (year/age) unfolds as follows:

1. Households and lenders enter age n with a prior about the persistent income component: $z_{-1} \sim \mathcal{N}(z_{-1}, \Sigma_{n|n-1})$, and assets a.
2. At the beginning of the period, income is realized: y_n observed by households and lenders.
3. Both households and lenders update beliefs based on income.
4. Lender offers debt prices, given updated beliefs.
5. Households choose actions based on prices:
    - Default, delinquency, or repayment
    - Consumption and borrowing/saving
6. Households with distorted beliefs obtain rational beliefs (rational reversion).

2.5 Value Functions

We write the value functions in a way that allows for the possibility of households distorted beliefs arising from diagnostic expectations when $\theta > 0$. This formulation accommodates the presence of cognitive biases, where agents may hold imperfect or biased beliefs. The case of rational learning is nested within this framework and can be obtained by setting $\theta = 0$. In this case, agents' expectations are updated entirely rationally. Therefore, the model captures both scenarios: one with households distorted beliefs ($\theta > 0$) and the other with perfectly rational expectations ($\theta = 0$).

The household's decision at each age n is to either declare bankruptcy, choose delinquency, or repay debt. The value functions associated with these choices incorporate beliefs about future

income and the persistence of income shocks. In particular, given the current residual income realization y, the belief about the persistent component carried over from the previous period z_{-1}, and the asset or debt level a, households' lifetime utility at age n is given by:

$$G_n(y, z_{-1}, a) = \max\Big\{ \underbrace{V_n(y, z_{-1}, a)}_{\text{Repayment}}, \underbrace{B_n(y, z_{-1})}_{\text{Bankruptcy}}, \underbrace{D_n(y, z_{-1}, a)}_{\text{Delinquency}} \Big\}.$$

These value functions evolve with age n, reflecting changes in the income profile, the learning process, and the finite lifespan of households.

The lifetime utility from declaring bankruptcy at age n is:

$$B_n(y, z_{-1}) = u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \int G_{n+1}(y', z, 0)\, \mathrm{d}\Phi(y'; z^\theta, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2),$$

subject to:

$$\begin{aligned} z &= z_{-1} + K_n(y - z_{-1}), \\ z^\theta &= z_{-1} + (1+\theta) K_n(y - z_{-1}), \\ c &= \underbrace{\exp\left(\ell(n) + y\right)}_{\text{Income}} - \mathbf{c}_{BK}. \end{aligned}$$

Here, $\mathbf{c}_{BK}$ represents the cost of filing for bankruptcy, and debt is reset to zero. The function $\Phi(y; \mu, \sigma^2)$ denotes the cumulative distribution function (CDF) of a normal distribution with mean μ and variance σ^2. The term $h(n)$ accounts for household size in equivalent adults.

Compared to rational households ($\theta = 0$), those with distorted beliefs set the mean of the residual income distribution at z^θ instead of z. Thus, when $\theta > 0$, households *overreact* to the current income realization when assessing the persistent income component.

Similarly, the lifetime utility from delinquency is given by:

$$\begin{aligned} D_n(y, z_{-1}, a) = u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \int \Big[& \gamma G_{n+1}(y', z, 0) \\ & + (1-\gamma) G_{n+1}(y', z, a') \Big]\, \mathrm{d}\Phi(y'; z^\theta, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2), \end{aligned}$$

subject to:

$$\begin{aligned} a' &= (1+\eta) a, \\ z &= z_{-1} + K_n(y - z_{-1}), \\ z^\theta &= z_{-1} + (1+\theta) K_n(y - z_{-1}), \\ c &= \underbrace{\min\{\exp\left(\ell(n) + y\right), \tau\}}_{\text{Income net of garnishment}}. \end{aligned}$$

With probability γ, the debt is forgiven, while with probability $1 - \gamma$, the debt rolls over at a penalty interest rate η.

If the household chooses to repay its debt, it must settle the outstanding balance and may borrow anew using the lender's offered menu of borrowing costs, q. The lifetime utility from repayment is given by

$$V_n(y, z_{-1}, a) = \max_{a', c} u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \int G_{n+1}(y', z, a')\, \mathrm{d}\Phi(y'; z^\theta, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2),$$

subject to:

$$\begin{aligned} z &= z_{-1} + K_n(y - z_{-1}), \\ z^\theta &= z_{-1} + (1+\theta)K_n(y - z_{-1}), \\ c &= a - a' q_n(z, a') + exp(\ell(n) + y). \end{aligned}$$

The price of debt is determined by the lender's rational posterior on the household's persistent income component, the amount of debt taken, and age, as policy functions vary with age. Note that lenders consider rational beliefs about future residual income when pricing debt, while the repayment policy follows the household's distorted beliefs. The repayment decision function is:

$$R_n(y, z_{-1}, a) = \begin{cases} 1, & \text{if } V_n = \max\{V_n, D_n, B_n\}, \\ 2, & \text{if } D_n = \max\{V_n, D_n, B_n\}, \\ 0, & \text{otherwise.} \end{cases}$$

Since default and delinquency are household choices, a risk-neutral lender charges a premium to ensure zero expected profit. Debt prices depend on the household's probability of default or delinquency. Consequently, debt prices are set as

$$\begin{aligned} q_n(z, a') = \frac{1}{1+r} \int \Big[& 1(R_{n+1}(y', z, a') = 1) \\ & + 1(R_{n+1}(y', z, a') = 2)(1-\gamma)(1+\eta) q_{n+1}(z', a'') \Big] \mathrm{d}\Phi(y'; z, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2), \end{aligned}$$

subject to:

$$\begin{aligned} z' &= z + K_n(y' - z), \\ a'' &= (1+\eta)a' \end{aligned}$$

3 Calibration

The benchmark model for calibration is the rational learning model. The next section explores counterfactual scenarios to assess the impact of learning and diagnostic expectations on financial distress.

Table 6 includes parameters that are either well-established in the literature or based on empirical estimates. The risk-free rate ($r = 0.03$) reflects long-term averages. The risk aversion parameter

takes the standard value ($\sigma = 2$). Parameters governing income processes, such as the standard deviations of persistent and transitory shocks ($\sigma_\epsilon = 0.10$, $\sigma_e = 0.22$), follow from Kaplan and Violante (2010). The debt roll-over rate ($\eta = 0.20$) takes the same value that in Livshits, MacGee and Tertilt (2007). The bankruptcy cost ($\mathbf{c}_{BK} = 0.165$) and the family size are obtained directly from data.

Table 1: *Calibrated Parameters*

Parameter	Value	Meaning	Basis
T	65	Retirement age	Standard
σ	2.00	Risk aversion	Standard
r	0.03	Risk-free rate	Standard
η	0.20	Debt roll-over rate	Livshits, MacGee and Tertilt (2007)
σ_ϵ	0.10	Std of persistent shock	Kaplan and Violante (2010)
σ_e	0.22	Std of transitory shock	Kaplan and Violante (2010)
ϱ_n	-	Mortality rate	Kaplan and Violante (2010)
$\mu_{z,0}$	0	Initial mean of persistent shock	Kaplan and Violante (2010)
$\sigma_{z,0}$	0.39	Initial Std of persistent shock	Kaplan and Violante (2010)
$\mathbf{c}_{BK}$	0.165	Bankruptcy cost	BK cost to mean income
$h(n)$	-	Family size	Survey of Consumer Finances
μ	0.7	Family size scaling parameter	Buhmann et al. (1988)
θ	$\{0, 0.5, 1\}$	DE parameter	Bordalo et al. (2020)

The remaining three parameters are estimated to minimize the distance to key moments. The estimation targets five sets of moments, each including three age groups to capture the life-cycle profile of financial distress and household debt.

The first two sets contain information about the share of individuals in financial distress. The first three moments are the share of individuals in DQ, which declines with age, and the second set includes bankruptcy rates, which remain relatively low across age groups. These two moments are crucial for capturing how frequently individuals fail to repay their debt obligations.

The next three sets of moments provide information on borrowing behavior. The first measures debt participation, which increases slightly with age. The second captures the debt-to-income ratio, following a similar pattern, with younger individuals carrying slightly higher relative debt burdens. Lastly, the coefficient of variation (CV) of debt is considered, which is highest among the oldest group, reflecting greater heterogeneity in debt holdings over the life cycle.

Table 2 reports the estimated parameters that minimize the distance between the model and empirical targets, along with standard errors (in parentheses). While each parameter is jointly determined, it closely aligns with specific targets. The discount factor ($\beta = 0.814$) helps match moments related to debt. The discharge probability of delinquent debt ($\gamma = 0.179$) reduces the likelihood of bankruptcy for households remaining in delinquency, making it key to matching

the observed bankruptcy rate. Finally, the parameter governing the cost of delinquency ($\tau = 3.488$) is crucial for reproducing both the incidence and life-cycle profile of delinquency. Standard errors are relatively small, with noticeable uncertainty only in the estimated value of the discharge probability of delinquent debt.

Table 2: *Estimated Parameters*

Parameter	Value
Discount factor, β	0.814
	(0.008)
Discharge prob of DQ debt, γ	0.179
	(0.038)
DQ consumption cost, τ	3.488
	(0.040)

Note: Standard errors in parentheses.

With these parameters, the model successfully replicates key patterns of financial distress and debt participation across age groups, as shown in Table 3. DQ rates align well with the data, with small deviations across age groups. The model closely matches DQ rates for individuals aged 35-44 and 45-54 while slightly overpredicting DQ for younger households. Bankruptcy rates follow a similar pattern, with predictions tracking observed rates well for middle-aged and older households, despite a higher estimate for younger individuals.

Debt participation exhibits a strong match, particularly for those aged 35-44. The model slightly underestimates participation for individuals aged 45-54 but captures the overall life-cycle trend. The coefficient of variation (CV) of debt closely aligns with the data across all groups, reflecting a realistic dispersion in borrowing. The largest discrepancies appear in mean debt-to-income ratios, where the model underpredicts debt holdings across age groups. However, given that only three parameters were chosen to fit fifteen moments, the model's ability to match key financial outcomes with such precision underscores its robustness in capturing the core mechanisms driving default and borrowing behavior.

4 The Role of Shocks Misperception for Financial Distress

In this section, we analyze the role of shock misperception in financial distress, first in partial equilibrium and then in general equilibrium. To do so, we maintain the same parameters and compute two alternative economies: one with perfect information and the other with diagnostic expectations.

Table 3: *Fit of targeted moments*

Statistic	Data	Learning Model
% in DQ, age 25-34	11.16	15.20
% in DQ, age 35-44	10.81	9.63
% in DQ, age 45-54	8.91	9.07
% in BK, age 25-34	0.64	1.98
% in BK, age 35-44	0.78	0.75
% in BK, age 45-54	0.66	0.60
% in Debt, age 25-34	41.49	47.96
% in Debt, age 35-44	44.55	46.10
% in Debt, age 45-54	45.70	44.12
(debt/income)%, age 25-34	3.310	1.775
(debt/income)%, age 35-44	3.472	0.848
(debt/income)%, age 45-54	3.159	0.723
CV of debt, age 25-34	1.465	1.494
CV of debt, age 35-44	1.560	1.519
CV of debt, age 45-54	1.703	1.784

4.1 Partial Equilibrium Results

This section compares the alternative frameworks holding all parameters fixed and keeping the risk-free interest rate constant. The results reveal that incorporating learning and diagnostic expectations significantly increases the incidence of both informal (delinquency) and formal (bankruptcy) default compared to the perfect information benchmark.

Table 4 compares the learning model with the perfect information model. Under learning, delinquency increases significantly across all age groups. Among younger households (ages 25-34), delinquency rises by 21.9%. The increase is even more pronounced for middle-aged (ages 35-44) and older (ages 45-54) households, where delinquency rises by 40.5% and 31.6%, respectively. A similar pattern emerges for bankruptcy rates, though the overall incidence remains lower. These results suggest that earnings shock misperception under learning leads to greater difficulty in meeting debt obligations.

Table 4 also compares the learning model with the model incorporating diagnostic expectations. While diagnostic expectations further amplify financial distress, the overall impact remains relatively small. Among younger households (ages 25-34), delinquency increases by only 3.0%, while for middle-aged (ages 35-44) and older (ages 45-54) households, the increases are 11.1% and 6.5%, respectively. Bankruptcy rates show a similar pattern, with moderate increases despite the addition of diagnostic expectations. Notably, this limited effect arises even though we set $\theta = 1$,

Table 4: *Financial Distress under Perfect Information, Learning, and Diagnostic Expectations (PE)*

Statistic	(1) Perfect Information	(2) Learning (Benchmark)	% Change (2) vs. (1)	(3) Diagnostic Expectations	% Change (3) vs. (2)
% in DQ, age 25-34	12.47	15.20	21.9	15.66	3.0
% in DQ, age 35-44	6.85	9.63	40.5	10.70	11.1
% in DQ, age 45-55	6.89	9.07	31.6	9.66	6.5
% in BK, age 25-34	1.80	1.98	10.0	2.08	5.1
% in BK, age 35-44	0.53	0.75	41.5	0.88	17.3
% in BK, age 45-55	0.48	0.60	25.0	0.71	18.3

a value that is large relative to the literature. These findings suggest that while diagnostic expectations exacerbate financial distress, their influence is modest compared to the shift from perfect information to learning.

4.2 General Equilibrium Results

One possible reason for the increased financial distress under learning and diagnostic expectations is that they lead households to borrow more. To control for this possibility, this section presents results where the market adjusts the interest rate to ensure that the total level of debt remains the same across the three economies, following the approach of Huggett (1993). Since the learning economy serves as the benchmark, the level of debt is fixed at its value in that economy, keeping the interest rate in the learning economy at 3%.

Table 5 compares financial distress under perfect information and learning in general equilibrium. As in partial equilibrium, learning increases both delinquency and bankruptcy rates across all age groups. Among younger households (ages 25-34), delinquency rises by 21.2%, while middle-aged (ages 35-44) and older (ages 45-54) households experience even larger increases of 41.4% and 31.8%, respectively. Bankruptcy rates follow a similar pattern, with increases of 10.6% for younger households, 41.5% for middle-aged households, and 27.7% for older households.

Compared to the partial equilibrium results, the magnitude of the changes is slightly lower for younger households but remains similar for older groups. An important distinction in general equilibrium is that the interest rate declines from 3.375% under perfect information to 3% under learning, an 11.1% decrease. This suggests that borrowing demand was higher under perfect information, requiring a higher interest rate to restrain borrowing. Under learning, misperception of income shocks makes households more cautious, leading to lower equilibrium interest rates despite higher financial distress.

Table 5 compares financial distress under learning and diagnostic expectations in general equilibrium. As in partial equilibrium, introducing diagnostic expectations further increases delinquency and bankruptcy rates across all age groups, though the effects remain relatively small.

Table 5: *Financial Distress under Perfect Information, Learning, and Diagnostic Expectations (GE)*

Statistic	(1) Perfect Information	(2) Learning (Benchmark)	% Change (2) vs. (1)	(3) Diagnostic Expectations	% Change (3) vs. (2)
% in DQ, age 25-34	12.54	15.20	21.2	15.61	2.7
% in DQ, age 35-44	6.81	9.63	41.4	10.52	9.2
% in DQ, age 45-55	6.88	9.07	31.8	9.54	5.2
% in BK, age 25-34	1.79	1.98	10.6	2.06	4.0
% in BK, age 35-44	0.53	0.75	41.5	0.86	14.7
% in BK, age 45-55	0.47	0.60	27.7	0.69	15.0
Interest Rate (%)	3.375	3.000	-11.1%	4.242	41.4%

Delinquency rises by 2.7% for younger households (ages 25-34), 9.2% for middle-aged households (ages 35-44), and 5.2% for older households (ages 45-54). Bankruptcy rates also increase, with the largest effects observed for middle-aged (14.7%) and older households (15.0%).

Compared to the partial equilibrium results, the increases in financial distress under diagnostic expectations are slightly muted in general equilibrium. A key difference is the sharp rise in the equilibrium interest rate, which increases from 3% under learning to 4.242% under diagnostic expectations, a 41.4% jump. This suggests that diagnostic expectations initially increase borrowing demand, prompting the market to raise interest rates to restore equilibrium. The higher borrowing costs, in turn, reduce overall borrowing levels, mitigating the rise in financial distress.

4.3 How Earnings Shocks Trigger Financial Distress

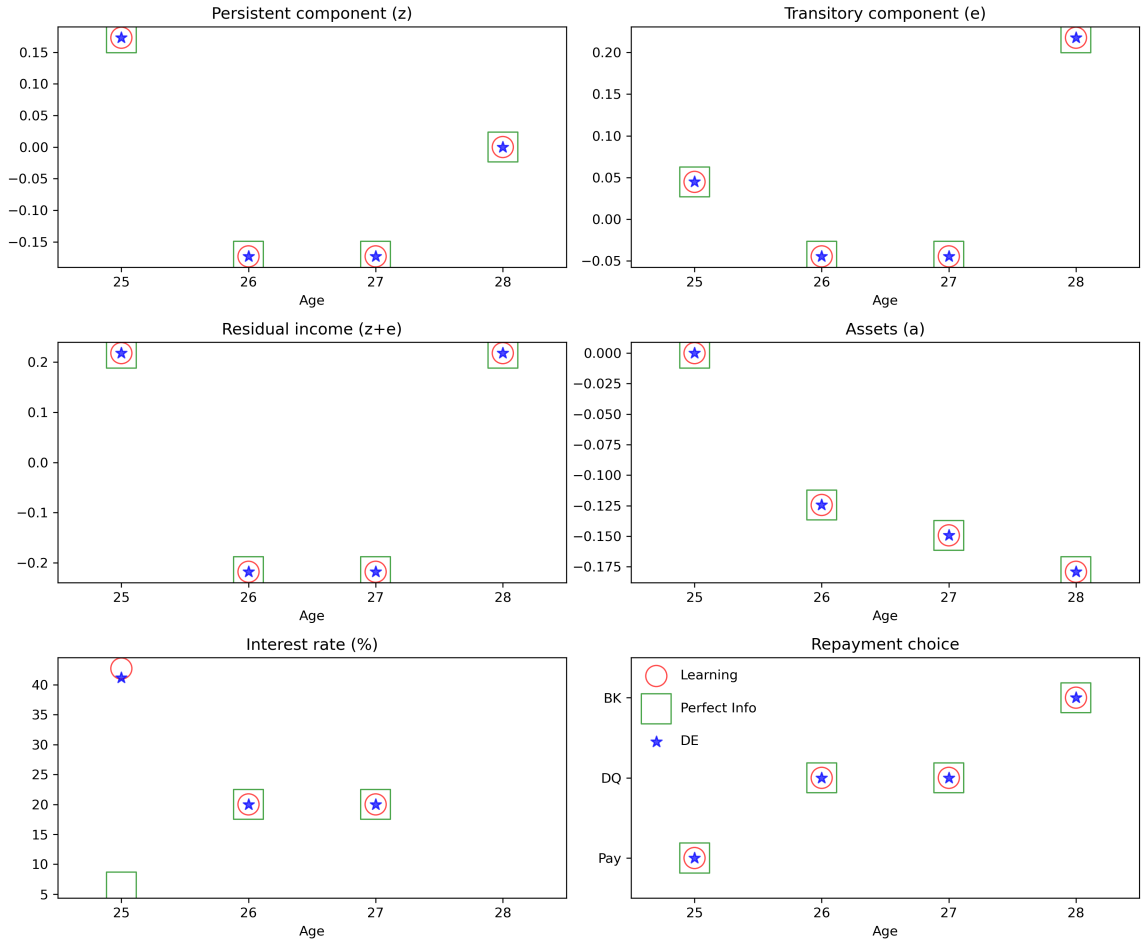
To illustrate how individuals slide into financial distress, this subsection and the next two present examples that highlight the model's dynamics. In each scenario, three types of agentsâperfect information, rational learning, and diagnostic expectationsâbegin with zero assets.

Figure 1 shows an initial case where individuals receive a substantial positive persistent income shock ($z_{25} = 0.16$) and a minor positive transitory income shock ($e_{25} = 0.04$). Motivated mainly by impatience and expectations of sustained high future income, all three agent types borrow ($a_{26} = -0.125$), boosting current consumption beyond their income.

However, in the subsequent year, the persistent income shock sharply reverses ($z_{26} = -0.16$), accompanied by a small negative transitory shock ($e_{26} = -0.04$). Faced with unexpectedly reduced income, all agents skip their debt payments and fall into delinquency. As a result, their debt grows significantly because lenders apply a punitive 20% interest rate, increasing debt to $a_{27} = -0.15$.

The unfavorable income shocks persist into the following year ($z_{27} = -0.16$, $e_{27} = -0.04$), prompting agents to remain delinquent. Consequently, their debt escalates further by another 20% ($a_{28} = -0.18$).

Figure 1: Labor Earnings Shocks and Financial Distress



Finally, at age 28, a positive transitory income shock provides temporary relief. The agents strategically use part of this unexpected income to file for bankruptcy, effectively eliminating their accumulated debt and beginning anew the next period.

In this example, all three agent types made identical decisions, experiencing financial distress similarly. The subsequent two examples will demonstrate how learning and diagnostic expectations can increase the likelihood of falling into financial distress.

4.4 How Misperceptions of Future Income Lead to Financial Distress

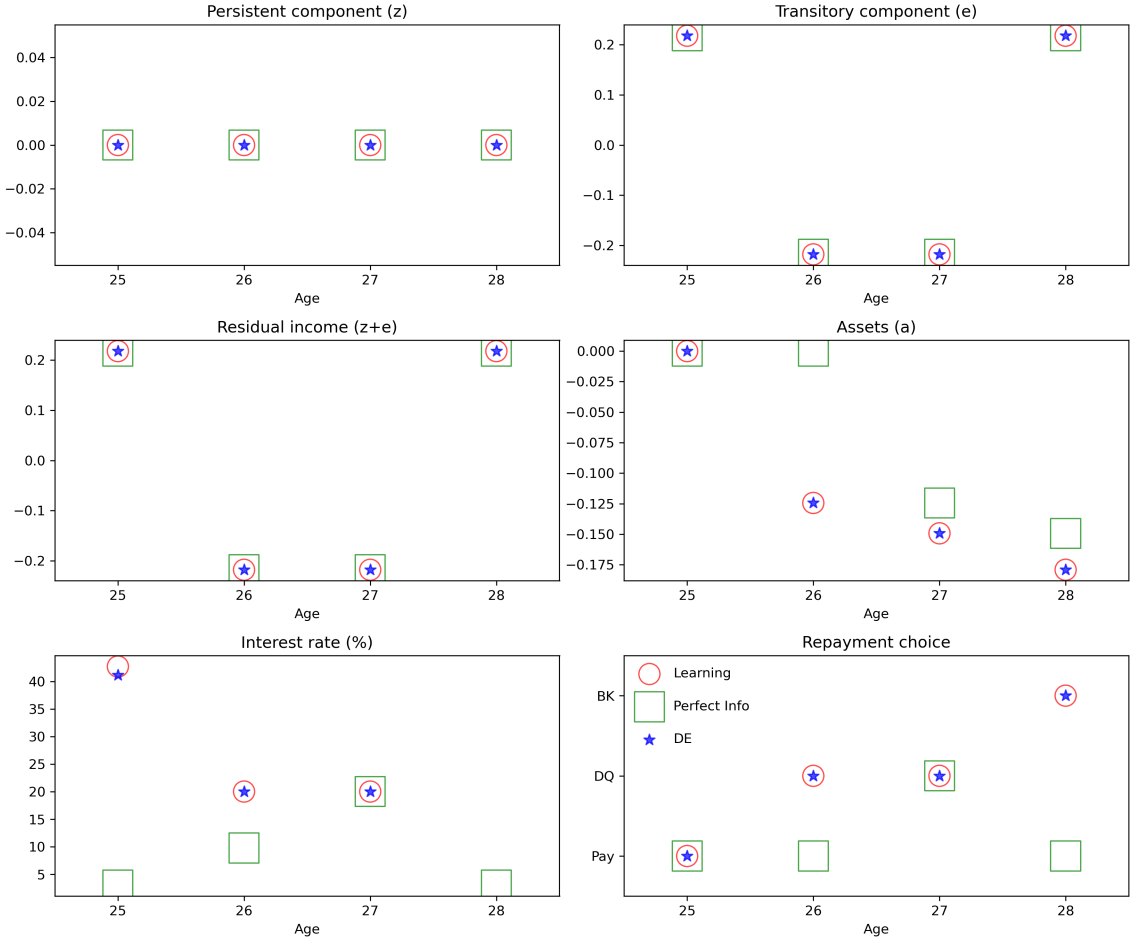
The example in Figure 2 traces an individual's financial decisions during the first four years of adulthood. Initially, this individual starts with zero assets and no persistent income shock ($z_{25} = 0$) but receives a positive transitory income shock ($e_{25} = 0.2$). Thus, residual income at age 25 is higher ($z_{25} + e_{25} = 0.2$) than initially anticipated.

As seen in the middle-right plot, asset choices at age 26 significantly differ between learning-based models and the perfect-information model. Under perfect information, assets remain un-

changed ($a_{26} = 0$). Conversely, the learning models generate greater borrowing ($a_{26} \approx -0.125$), as individuals overestimate future income prospects. This borrowing raises interest rates due to increased default risk, as depicted in the bottom-left panel.

The next year, the individual faces a negative transitory shock ($e_{26} = -0.2$) without a persistent shock ($z_{26} = 0$). Agents using learning models revise income expectations downward and, burdened by debt, enter delinquency. Meanwhile, the agent with perfect information, having avoided debt initially, chooses moderate borrowing ($a_{27} \approx -0.125$) to maintain stable consumption.

Figure 2: Example of the Role of Learning in Financial Distress



At age 27, experiencing identical shocks ($z_{27} = 0$, $e_{27} = -0.2$), the learning-model agent remains delinquent, increasing debt by 20% due to penalty rates. The perfect-information agent, now financially vulnerable, also enters delinquency to mitigate the consumption drop.

By age 28, with a positive transitory shock ($e_{28} = 0.2$) but no persistent improvement ($z_{28} = 0$), the perfect-information agent quickly repays the debt and exits distress. However, agents using rational learning or diagnostic expectations, encumbered by larger debts and pessimistic beliefs, ultimately resort to bankruptcy.

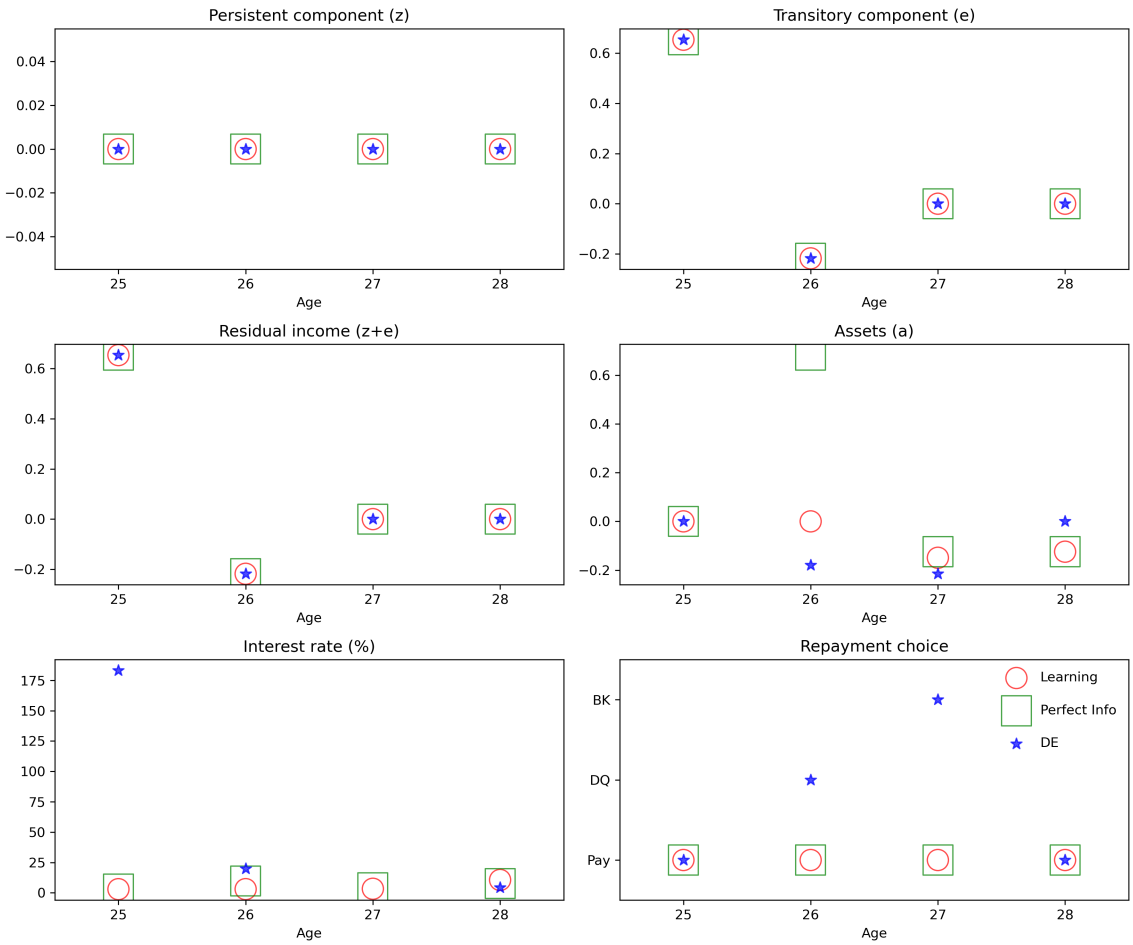
This example demonstrates how mistaken beliefs about income stability can push individuals

toward debt delinquency and eventually bankruptcy.

4.5 Exaggerated Optimism and the Path to Financial Distress

The second example, presented in Figure 3, illustrates how diagnostic expectations amplify financial distress compared to perfect information and rational learning. The example begins at age 25, with zero initial assets, no persistent income shock ($z_{25} = 0$), but a substantial positive transitory income shock ($e_{25} = 0.6$). As a result, residual income that year is notably high ($z_{25} + e_{25} = 0.6$), significantly exceeding prior expectations.

Figure 3: Example of Exaggerated Optimism and Financial Distress



The middle-right panel highlights substantial differences in asset decisions for age 26 across the three cases. With perfect information, awareness that the income boost is temporary motivates substantial saving ($a_{26} \approx 0.6$). Under rational learning, moderate optimism about future income leads to full consumption of current income, resulting in zero asset accumulation ($a_{26} = 0$). Under diagnostic expectations, exaggerated optimism fuels excessive consumption financed through borrowing, despite the already high income level, anticipating continued high income.

In the following year, the individual experiences no persistent income shock ($z_{26} = 0$) and a negative transitory income shock ($e_{26} = -0.2$). Agents with rational learning and perfect information, possessing no debt, respond by borrowing modestly to smooth their consumption. In contrast, the agent driven by diagnostic expectations sharply revises future income beliefs downward and, burdened by significant debt, chooses delinquency.

At age 27, with no income shocks occurring ($z_{27} = 0, e_{27} = 0$), agents with rational learning and perfect information rollover their manageable debt. However, the individual guided by diagnostic expectations, confronting substantial accumulated debt, opts for bankruptcy.

This example demonstrates clearly how diagnostic expectations, by magnifying misperceptions of long-term income prospects, lead to elevated financial distress.

5 Estimation of Alternative Models

We estimate each model by minimizing its distance to the same set of empirical moments. Our objective is to clarify the roles of learning and diagnostic expectations in driving financial outcomes.

Table 6 presents parameter estimates across models. Estimates from the perfect information model are statistically indistinguishable from those of the benchmark rational learning model, except for the discount factor β, which is significantly lower. This difference aligns with the intuition that the perfect information model, unable to generate sufficient financial distress on its own, compensates by requiring households to be more impatient to match observed data.

Table 6: *Model Parameters Estimated*

Parameter	Perfect Information	Learning (Benchmark)	Learning with DE ($\theta = 0.5$)	Learning with DE (θ flexible)
β	0.793	0.814	0.825	0.812
	(0.007)	(0.008)	(0.005)	(0.008)
γ	0.174	0.179	0.180	0.180
	(0.055)	(0.038)	(0.026)	(0.030)
τ	3.486	3.488	3.486	3.489
	(0.057)	(0.040)	(0.037)	(0.040)
θ	-	-	-	0.075
				(0.189)

The third estimated model incorporates agents with diagnostic expectations, setting the degree of it at $\theta = 0.5$ to match previous empirical estimates. For instance, Bordalo et al. (2020) find an average θ of approximately 0.5 when analyzing professional forecasters' expectations across various aggregate variables, such as nominal GDP. We adopt this value as a benchmark. In this model, the estimated discount factor β is significantly higher compared to both the perfect information and

rational learning models, implying that diagnostic expectations reduce the degree of household impatience required to explain the observed levels of financial distress.

The final column reports our attempt to estimate the degree of diagnostic expectations, θ. We obtain an estimate of $\theta = 0.075$, but it comes with large standard errors, indicating that the moments we match do not effectively identify the parameter θ. Consequently, for the analysis of distinguishing implications, we focus on the diagnostic expectations model with $\theta = 0.5$.

Table 7 compares how well the alternative estimated models match the data. All models successfully replicate the targeted moments. The learning model achieves the smallest minimized distance to targets, but the gap with diagnostic-expectations models is narrow. This suggests that both learning and diagnostic expectations may play a role in accounting for household's financial distress.

Table 7: *Fit of Targeted Moments in Alternative Economies*

Parameter	Perfect Information	Learning (Benchmark)	Diagnostic Exp (θ flexible)	Diagnostic Exp ($\theta = 0.5$)
% in DQ, age 25-34	12.88	15.20	15.36	15.46
% in DQ, age 35-44	7.33	9.63	9.71	9.89
% in DQ, age 45-55	7.61	9.07	9.14	9.12
% in BK, age 25-34	1.80	1.98	2.00	1.96
% in BK, age 35-44	0.60	0.75	0.76	0.78
% in BK, age 45-55	0.51	0.60	0.61	0.60
% in Debt, age 25-34	49.60	47.96	48.51	51.28
% in Debt, age 35-44	48.02	46.10	46.30	45.54
% in Debt, age 45-55	47.11	44.12	44.65	42.94
Debt/Income%, age 25-34	1.852	1.775	1.793	1.827
Debt/Income %, age 35-44	0.887	0.848	0.851	0.826
Debt/Income %, age 45-55	0.752	0.723	0.720	0.680
CV of Debt, age 25-34	1.465	1.494	1.485	1.504
CV of Debt, age 35-44	1.559	1.519	1.514	1.547
CV of Debt, age 45-54	1.762	1.784	1.774	1.836
Distance	50.428	47.360	47.658	47.967

6 The Welfare Cost of Misperceptions

In this section, we analyze the welfare effects associated with eliminating misperceptions about income shocks. Specifically, we define welfare in an economy as the expected discounted sum of utility derived from future consumption flows, $c_{n\,n=n_0}^{N}$:

$$E[V] \equiv E\left[\sum_{n=n_0}^{N} \beta^n u(c_n)\right].$$

We use Monte Carlo simulations to obtain the distribution of V, interpreting $E[V]$ as the welfare of an âaverageâ household born into the economy. We quantify the welfare gains from eliminating misperceptions as:

$$\text{Welfare Change (\%)} \equiv \left(\frac{E[V_{bench}]}{E[V_{new}]} - 1\right) \times 100.$$

Since we employ a constant relative risk aversion (CRRA) utility function with parameter $\sigma = 2$, these welfare changes can be interpreted directly in consumption-equivalent units.

First, we examine the welfare impact of reducing informational frictions, allowing households to better distinguish persistent from transitory income shocks. Although the model lacks a single parameter that adjusts informational friction independently of the underlying income process, we treat the perfect-information economy as an extreme scenario, where informational friction is entirely removed relative to the rational learning benchmark. Column 1 of Table 8 reports the welfare gain from fully eliminating informational friction. Compared with the rational learning benchmark, households experience a welfare increase of 0.46%.

Table 8: *Percentage of Welfare Change*

	Eliminating Information Friction	Dropping Diagnostic Expectation ($\theta = 0.5$)	Total
Welfare Gain(%)	0.46	0.11	0.57

Second, we analyze the welfare effect of eliminating diagnostic expectations. This scenario can be interpreted as the result of effective financial literacy education, as in Exler et al. (2024). Column 2 of Table 8 reports the welfare gain from shifting household expectations toward rational learning. Relative to the economy with diagnostic expectations, households experience an average welfare gain of 0.11%. Similar to the case of informational friction, households with diagnostic expectations borrow excessively, face less favorable price schedules, and incur higher default and delinquency costs, resulting in lower overall welfare.

7 Conclusion

These findings suggest that households may not default simply because of low income–they default because they misperceive their income prospects. When earnings shocks are misunderstood, debt dynamics spiral out of control because misplaced optimism and exaggerated pessimism drive borrowing and repayment decisions. The model shows that even small misjudgments about in-

come persistence–whether through Bayesian learning or diagnostic overreaction–translate into higher financial distress, riskier debt accumulation, and more frequent bankruptcies. In a world where access to credit is expanding and households rely increasingly on volatile income sources, these results raise fundamental questions: Is household financial distress amplified by poor forecasting? If earnings misperception is a core driver of financial instability, then interventions targeting expectation formation or simply limiting borrowing, rather than debt relief, may be key to preventing household financial distress.

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A Appendix: Value Functions in Perfect Information Economy

In perfect information economy, the value functions at age n are written as

$$G_n(y,z,a) = \max\{V_n(y,z,a), B_n(y,z), D_n(y,z,a)\}$$

where agents fully observe their persistent income component z.

The lifetime utility of bankruptcy then writes as

$$\begin{aligned} B_n(y,z) &= u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint G_{n+1}(z'+e',z',0)\mathrm{d}P(z'|z)\mathrm{d}\Phi(e';0,\sigma_e^2) \\ s.t.\ c &= \exp\left(l(n)+y\right) - c_{BK} \\ c &\geq 0 \end{aligned}$$

where f_b is bankruptcy filing cost and $\Phi(x;\mu,\sigma^2)$ is the cdf of a normal distribution with mean μ and variance σ^2 at x.

The lifetime value of delinquency from age n is

$$\begin{aligned} D_n(y,z,a) = u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint \big[&\gamma G_{n+1}(z'+e',z',0) \\ &+(1-\gamma)G_{n+1}(z'+e',z',a')\big]\,\mathrm{d}P(z'|z)\mathrm{d}\Phi(e';0,\sigma_e^2) \\ a' &= (1+\eta)a \end{aligned}$$

where common notations as in bankruptcy case are omitted here.

The value of repayment writes as

$$\begin{aligned} V_n(y,z,a) = \max_{a',c}\ & u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint G_{n+1}(z'+e',z',a')\mathrm{d}P(z'|z)\mathrm{d}\Phi(e';0,\sigma_e^2) \\ s.t.\ c &= a - a'q_n(z,a') + exp(l(n)+y) \end{aligned}$$

Debt Prices To simplify notation, we denote

$$R_n(y,z,a) = \begin{cases} 1\ if\ V_n = \max\{V_n, D_n, B_n\} \\ 2\ if\ D_n = \max\{V_n, D_n, B_n\} \\ 0\ if\ otherwise \end{cases}$$

Therefore, the risk-neutral lender sets up the debt prices as

$$q_n(z,a') = \frac{1}{1+r}\iint \Big[1_{R_{n+1}(z'+e',z',a')=1} + 1_{R_{n+1}(z'+e',z',a')=2}(1-\gamma)(1+\eta)q_{n+1}(z',a'')\Big]\, \mathrm{d}P(z'|z)\mathrm{d}\Phi(e';0,\sigma_e^2)$$
$$a'' = (1+\eta)a'$$

The prices are adjusted by the probability that households default or choose delinquency.

B Value Functions at Retirement and Pre-Retirement

We assume that households' uncertainty about their persistent income component is fully eliminated at $n = n_{ret}$. In addition, we assume that the retired households receives a fraction of their persistent income component realization at retirement age T, z_T, using the replacement formula as

$$f(z) \equiv A_0 \min\{\exp(z + l(T)), \exp(l(T))\} + A_2 \max\{\exp(z + l(T)) - \exp(l(T)), 0\} \tag{2}$$

Therefore, the replacement ratio will be A_0 if agents' income at retirement is lower than the median level, and will decrease gradually to A_2 if it exceeds. In addition, we assume that there is no over-reaction at and after age of retirement, in the case of diagnostic expectation. We also assume that the households can still choose default but not delinquency at age T, but will not be able to choose either after retirement.

Since we assume households are to die at the age N, the value function at death will be

$$V_N(z,a) = u\left(\frac{c}{h(n)}\right)$$
$$s.t.\, c = a + f(z)$$

where we use z to refer the observed persistent income realization at retirement.

The value functions for after-retirement households then writes as

$$V_n(z,a) = \max_{a',c}\ u\left(\frac{c}{h(n)}\right) + \varrho_n \beta V_{n+1}(z,a')$$
$$s.t.\ c = a + f(z) - \frac{a'}{1+r}$$
$$a' \geq 0,\ T < n < N$$

where the debt prices is $\frac{1}{1+r}$ since default/delinquency cannot be chosen.

At retirement age T, households are not able to choose delinquency. However, default is still a choice. Therefore, value functions at retirement age writes as

$$G_T(y,z,a) = \max\{V_T(y,z,a), B_T(y,z)\}$$

where value of repayment is similar to value function after retirement age except for income being

$exp(l(T)+y)$. The value function for bankruptcy writes as

$$B_T(y,z) = u\left(\frac{c}{h(n)}\right) + \varrho_T \beta V_T(z_T, 0)$$
$$s.t.\ c = exp(l(T)+y) - c_{BK}$$

Although knowing that uncertainty will be eliminated at retirement age, households are still subjective to over-reaction at $n = T-1$, if we are in a economy with diagnostic expectation. Specifically, we have

$$G_n(y, z_{-1}, a) = \max\{V_n(y,z,a), B_n(y,z_{-1}), D_n(y,z_{-1},a)\}$$

where we still use z_{-1} to refer the prior belief carried from last last period. The value functions then write as

$$V_n(y,z_{-1},a) = \max_{a',c}\ u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint G_{n+1}(z'+e', z', a')\mathrm{d}\Phi(e'; 0, \sigma_e^2)\mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2)$$
$$s.t.\ c = a - a'q(z,a') + exp(l(n)+y)$$
$$q_n(z,a') = \frac{1}{1+r}\iint \left[1_{R_{n+1}(z'+e',z',a')=1}\right]\mathrm{d}\Phi(e'; 0, \sigma_e^2)\mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2)$$

$$B_n(y,z_{-1}) = u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint G_{n+1}(z'+e', z', 0)\mathrm{d}\Phi(e'; 0, \sigma_e^2)\mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2)$$
$$s.t.\ c = exp(l(n)+y) - c_{BK},$$

$$D_n(y,z,a) = u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint \left[\gamma G_{n+1}(z'+e', z', 0) + (1-\gamma)G_{n+1}(z'+e', z', a')\right]\mathrm{d}\Phi(e'; 0, \sigma_e^2)\mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2)$$
$$s.t. a' = (1+\eta),\ n = T-1$$

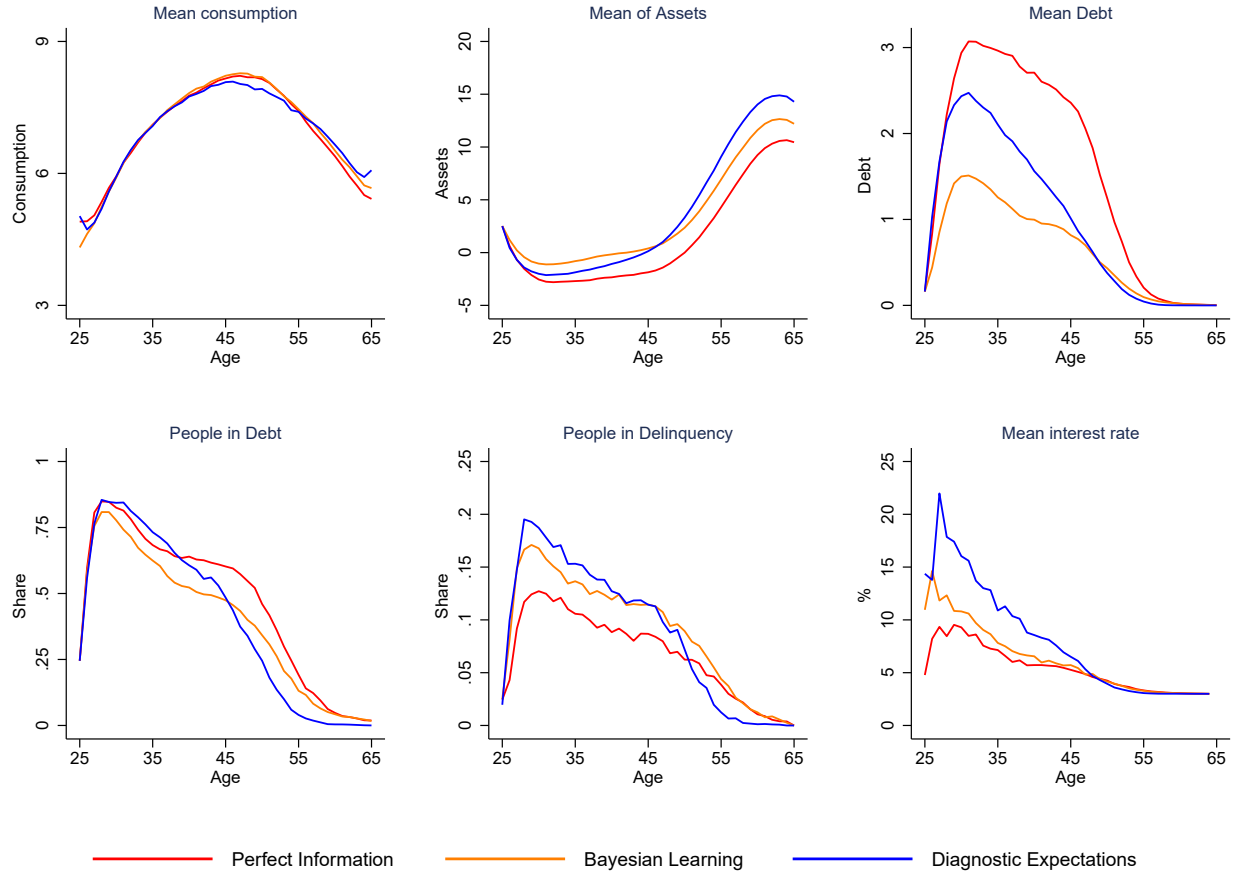
where z again refers to the posterior after updating.

C Life-cycle profiles

D Definition of Moments

This appendix provides a description of the moments definition and construction.

Figure 4: Lifecycle profiles



D.1 Unconditional Fraction of Individuals in DQ and BK

Define the unconditional probability of DQ share at age t is defined as follows

$$\text{Prob}(DQ|t) = \frac{\sum_i^{N_t} \mathbf{1}_{DQ_{i,t}}}{N_t}$$

Accordingly, the moments we used is computed as follows

$$\text{\% of Financial Distress, age T to T+9} = \frac{\sum_{t=T}^{T+9} N_t \text{Prob}(DQ|t)}{\sum_{t=T}^{T+9} N_t}, \ T = 25, 35, 45$$

where N_t is defined as the number of individuals at age t.

Similar, we could define the moments for BK as

$$\text{\% of Bankruptcy, age T to T+9} = \frac{\sum_{t=T}^{T+9} N_t \text{Prob}(BK|t)}{\sum_{t=T}^{T+9} N_t}, \ T = 25, 35, 45$$

D.2 Unconditional Mean of Debt to Income Ratio

We define the debt of individual i at age t as follows

$$debt_{i,t} = \begin{cases} 0, & \text{if no credit card debt} \\ \text{amount of credit card debt}, & \text{if has credit card debt} \end{cases}$$

The unconditional mean of debt to income ratio, by age bins, is defined as follows

$$\text{mean(debt)/mean(income), age T to T+9} = \frac{\frac{\sum_{t=T}^{T+9}\sum_i |debt_{i,j}|}{\sum_t N_t}}{\frac{\sum_{t=T}^{T+9}\sum_i income_{i,j}}{\sum_{t=T}^{T+9} N_t}},\ T = 25, 35, 45$$

D.3 Unconditional Fraction of Individuals in Debt

We define, in the data, individual i at age t as being in debt if this individual still has positive amount of credit card debt after most recent repayment. Therefore, we define share of people in debt, both in the data and model, as

$$\text{\% in Debt, age T to T+9} = \frac{\sum_{t=T}^{T+9}\sum_i^{N_t} \mathbf{1}_{|debt_{i,t}|>0}}{\sum_{t=T}^{T+9} N_t},\ T = 25, 35, 45$$

D.4 CV_{debt}

We simply define CV_{debt} both in the model and data as

$$CV_{debt}\text{, age T to T+9} = \frac{std(debt_{i,t}|T \leq t \leq T+9, |debt_{i,t}| > 0)}{mean(|debt_{i,t}||T \leq t \leq T+9, |debt_{i,t}| > 0)},\ T = 25, 35, 45$$

where $std(\cdot)$ and $mean(\cdot)$ refers to sample standard deviation and sample mean respectively. Notice that this measure is conditional on having a positive absolute value of debt.