Persistent or Temporary? How Earnings Shock Misperception Leads to Household Financial Distress[*]

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Abstract

This paper explores how misperceptions of earnings shocks amplify financial distress. Households infer shock persistence from observed income. A rational learning model replicates observed delinquency, bankruptcy, and debt accumulation. Perfect information significantly reduces distress compared to rational learning, while non-rational learning (diagnostic expectations) intensifies it. Matching observed distress rates requires more impatient agents under perfect information. Data on income and interest rates favors learning-based models. Misperceptions generate substantial welfare costs, particularly when delinquency and bankruptcy are possible. The findings underscore the key role of earnings shock misperceptions in driving financial distress and highlight the significance of learning mechanisms.

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1 Introduction

Knowing whether a change in earnings is short-term or long-term can be the key to staying financially secure or falling into distress. By comparing predictions from three different models of household financial distress, this study uncovers the powerful role that earnings shock misperception plays in shaping financial outcomes.

To understand the impact of earnings shock misperception, this analysis compares a standard model that incorporates both formal and informal debt non-repayment with two novel extensions. The first introduces rational learning, where households update their beliefs about the persistent component of labor earnings based on observed realizations. The second also includes learning but departs from rational expectations, allowing for a cognitive bias (diagnostic expectations) where individuals overweight recent information when forming beliefs about the future.

The analysis follows a straightforward strategy: the model is first estimated under rational learning and then compared, using the same parameters, to the perfect information and diagnostic expectations frameworks. This comparison reveals how different learning mechanisms shape household financial behavior and, crucially, how misperceptions about income persistence fuel financial distress. The model with rational learning–and even more so the model with diagnostic expectations–generates substantially higher rates of delinquency and bankruptcy. For example, going from perfect information to rational learning increases the incidence of delinquency and bankruptcy among individuals aged 35 to 44 by 40.5% and 41.5%, respectively. Moving from rational learning to diagnostic expectations further increases delinquency and bankruptcy by 11.1% and 17.3%, respectively, for the same age group.

If these results stem from additional borrowing, they may not hold in general equilibrium. To test this hypothesis, we allow the interest rate to adjust when going from economy A to B to keep the aggregate level of assets constant. The results remain consistent across partial and general equilibrium, although the interest rate must adjust significantly. For example, moving from rational learning to diagnostic expectations raises the risk-free interest rate from 3% to 4.24%, a 41.4% increase.

Simulated examples illustrate the underlying mechanism. In learning models, when young households receive high transitory income shocks, they associate a part of that with an increase in the persistent component. Consequently, they accumulate debt, believing their earnings will remain elevated. However, when fortunes reverse–a common occurrence when income increases are largely transitory–households find themselves overleveraged, facing repayment difficulties, and ultimately defaulting. Under

diagnostic expectations, these reversals are even more extreme, as households overreact to recent income fluctuations, amplifying financial distress. These findings highlight the power of expectation-driven household-level debt cycles, underscoring the role of misjudged optimism and sudden pessimism in shaping household financial stability.

Each of the three models–perfect information, rational learning, and diagnostic expectations–is estimated separately to match the same empirical facts regarding household borrowing and financial distress. All three models perform well, though the perfect information model achieves a slightly worse fit and requires more impatient households to replicate observed distress rates. Using these estimated models, the analysis examines the relationship between interest rates and income. The perfect information model generates a weak relationship because interest rates respond solely to the persistent component of income. In contrast, learning-based models produce a strong negative relationship, consistent with evidence from the Survey of Consumer Finances (SCF).

The final analysis quantifies the welfare impact of departures from perfect information. First, we compute the share of consumption that agents under perfect information are willing to forgo in each period to remain indifferent to the learning economy, where households cannot distinguish between transitory and persistent income components. We find that the inability to separate permanent from temporary income shocks imposes a substantial cost: households would sacrifice 0.46% of consumption each period. Additionally, rational learning households would forgo 0.11% of consumption per period to avoid diagnostic expectations.

The analysis also compares the welfare costs of departures from perfect information in settings without financial distress but with borrowing constraints. Allowing agents to distinguish between transitory and persistent shocks generates similar welfare gains across these environments. In contrast, explicitly incorporating financial distress nearly doubles the welfare gains from eliminating diagnostic expectations relative to an economy with a borrowing constraint calibrated to match aggregate borrowing.

There is extensive literature on heterogeneous-agent models of default. Early contributions, including Athreya (2002); Livshits, MacGee and Tertilt (2007); Chatterjee, Corbae, Nakajima and Ríos-Rull (2007), explicitly model bankruptcy following the legal framework and economic consequences of Chapter 7. Subsequent studies by Athreya, Sánchez, Tam and Young (2015, 2017) expand the analysis by introducing credit card delinquency, capturing a significantly higher incidence of default. The first model presented here, referred to as *perfect information*, follows closely Athreya, Mustre-del Río and Sánchez (2019) and incorporates both formal bankruptcy and delinquency as distinct forms of non-repayment of credit card debt.

To analyze the role of income-shock misperception, this paper introduces two additional models extending the existing framework in novel directions. The first extension, referred to as *rational learning*, features households that learn about the persistent component of labor earnings through Bayesian updating based on observed income realizations. Incorporating learning into heterogeneous-agent models significantly improves the ability to replicate consumption data, as demonstrated previously by Guvenen (2007), Guvenen and Smith (2014) and recently by Lee and Sæverud (2023). A crucial difference relative to these earlier studies involves explicitly modeling non-repayment through delinquency and bankruptcy.

The final model considered, referred to as *diagnostic expectations*, departs from rational expectations by assuming households form beliefs following diagnostic expectations as introduced in Bordalo, Gennaioli, Porta and Shleifer (2019) and Bordalo, Gennaioli, Ma and Shleifer (2020). These studies demonstrate that diagnostic expectations help match data on forecasts by analysts in financial markets and individual and consensus forecasts of macroeconomic variables. More recently, Bianchi, Ilut and Saijo (2023) develops a quantitative New Keynesian model with diagnostic expectations, emphasizing its role in explaining business cycle fluctuations. And Bordalo, Gennaioli, Shleifer and Terry (2021) study a model of real credit cycles in which lenders also have diagnostic expectations. The key innovation here is applying diagnostic expectations to households within a heterogeneous-agent framework to analyze and account for the incidence of financial distress.

Our analysis of diagnostic expectations relates to Exler, Livshits, MacGee and Tertilt (2024), as both consider non-rational agents within a heterogeneous agent model with default. However, they focus on over-optimism as a cognitive bias. Their key mechanism involves pooling behavioral and rational agents, demonstrating that policies like financial literacy improve welfare. In addition to considering a different cognitive bias, our work differs in that it does not involve pooling agents within the same economy. We focus instead on how earnings shock misperception leads to financial distress.

2 Setup

In this section, we will set up the model for an economy with information frictions and diagnostic expectations. The formulation of the perfect information economy, along with the value functions at and after retirement, will be detailed in Appendix A.

The economy consists of a continuum of finitely lived, risk-averse households. Each

individual enters the labor market at age $n = n_0$, retires at age $n = T$, and lives up to age $n = N$.

2.1 Income Process

Household income follows a restricted income profile (RIP) process proposed by Blundell and Preston (1998) and more recently used by Kaplan and Violante (2010),

$$\tilde{y}_n = \exp\left(\ell(n) + z_n + e_n\right),$$

where $\ell(n)$ represents the deterministic component that depends solely on age, z_n is the persistent stochastic component, and e_n is an i.i.d. transitory shock, distributed as $\mathcal{N}(0, \sigma_e^2)$.

The persistent component evolves as a random walk:

$$z_n = z_{n-1} + \epsilon_n,$$

where $\epsilon_n \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is an i.i.d. innovation.

We assume that $\ell(n)$ is common knowledge. Each period, households observe their realized income $\tilde{y}_n$. Additionally, we define the residual income realization y_n as

$$y_n \equiv z_n + e_n = \ln(\tilde{y}_n) - \ell(n),$$

which is also observable since both $\tilde{y}_n$ and $\ell(n)$ are known. However, households cannot distinguish between the stochastic components of income–persistent and transitory. That is, they do not directly observe either z_n or e_n.

2.2 Rational Belief Updating

Households enter the labor market with a prior belief about z_n and gradually update their beliefs through the realizations of residual income. Every household starts with the same prior,

$$z_{n_0} \sim \mathcal{N}(\mu_{z,0}, \sigma_{z,0}^2),$$

where the subscript n_0 indicates entry into the labor market. Additionally, we assume that this prior is aligned with the objective distribution of z_{n_0}. Thus, there is no initial misbelief about z_{n_0}, even under diagnostic expectations.

A rational household enters age n with prior beliefs about z_n, which we refer to as $z_{n|n-1}$, and has the probability density $f(z_n|Y_{n-1})$, where Y_{n-1} denotes the full history of residual income observed up to the previous period. At age n, the household and the lender observe a new signal—its residual income realization y_n—and update its estimate of the persistent income component using Bayes' rule:

$$f(z_n|Y_n) = \frac{f(y_n|z_n)f(z_n|Y_{n-1})}{\int f(y_n|z)f(z|Y_{n-1})\,\mathrm{d}z}, \tag{1}$$

which iteratively defines the household's rational beliefs.

With normal priors and shocks, the rational household's and lenders' learning problem reduces to a textbook Kalman filter process. Specifically, at age n, the prior about z_n is summarized by

$$\mathcal{N}(z_{n|n-1}, \Sigma_{n|n-1}).$$

Crucially, in a Kalman filter framework, the variance depends only on the number of signals received rather than the specific history of realized signals. In our setting, the number of observed realizations maps one-to-one with age n, allowing us to express the variance solely as a function of age. This property is fundamental: to fully characterize the distribution of z_n, it suffices to know the mean and the age.

Given the prior $\mathcal{N}(z_{n|n-1}, \Sigma_{n|n-1})$, their belief updating follows:

$$\begin{aligned}
z_{n|n} &= z_{n|n-1} + K_n\left(y_n - z_{n|n-1}\right), \\
\Sigma_{n|n} &= \frac{\sigma_e^2}{\Sigma_{n|n-1} + \sigma_e^2}\Sigma_{n|n-1}, \\
\Sigma_{n|n-1} &= \Sigma_{n-1|n-1} + \sigma_\epsilon^2, \\
K_n &= \frac{\Sigma_{n|n-1}}{\Sigma_{n|n-1} + \sigma_e^2}, \\
z_{0|-1} &\equiv \mu_{z,0}, \quad \Sigma_{0|-1} \equiv \sigma_{z,0}^2.
\end{aligned}$$

Given the posterior, a household's belief about next period's residual income realization follows a normal distribution with mean equal to the updated belief about income, $z_{n|n}$, and variance incorporating the updated uncertainty $\Sigma_{n|n}$ along with the variances of the income shocks:

$$y_{n+1} \sim \mathcal{N}\left(z_{n|n}, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2\right).$$

Note that the mean of residual income in the next period equals the mean of the be-

lief about z in the current period, as z follows a random walk and both e and ϵ have zero means. Similarly, since the shocks are independent, the variance is the sum of the individual variances.

2.3 Distorted Belief Updating

We consider the possibility that households form distorted beliefs based on the extent of diagnostic expectations, parameterized by $\theta \geq 0$, following Bordalo et al. (2020). Specifically, these distortions arise from Kahneman and Tversky's representativeness heuristic.

In line with Bordalo et al. (2019)'s proposal for a diagnostic Kalman filter, we define the representativeness of a state at age n as the likelihood ratio:

$$R_n(z) = \frac{f(z|Y_n)}{f\left(z|Y_{n-1} \cup \left\{z_{n|n-1}\right\}\right)}.$$

Here, $Y_n = Y_{n-1} \cup \{y_n\}$ represents the history up to today and $Y_{n-1} \cup \left\{z_{n|n-1}\right\}$ represents the history up to today under the assumption that the new residual income equals the ex-ante forecast, $y_n = z_{n|n-1}$. This implies that state z is more representative at age n if the new signal y_n increases its probability relative to the scenario where the residual income matches the forecast. Intuitively, the most representative states z are those whose likelihood has increased the most in response to the latest residual income realization.

Households then overweight representative states by using a distorted posterior, replacing the rational posterior (1) with

$$f^\theta(z_n|Y_n) = f(z_n|Y_n)\left[R_n(z_n)\right]^\theta \frac{1}{Z_n},$$

where Z_n is a normalization factor ensuring that $f^\theta(z_n|Y_n)$ integrates to one.

The parameter $\theta \geq 0$ measures the extent to which beliefs deviate from rational updating due to representativeness. When $\theta = 0$, beliefs remain rational and are given by the Bayesian conditional distribution $f(z_n|Y_n)$. For $\theta > 0$, the diagnostic density $f^\theta(z_n|Y_n)$ inflates the probability of representative states while deflating that of unrepresentative ones. Crucially, this distortion leads to systematic mistakes: states that have become relatively more likely are overweighted, even if they remain unlikely in absolute terms.

Note that after observing the current residual income realization y_n, the diagnostic

posterior follows a normal distribution:

$$\mathcal{N}(z^{\theta}_{n|n}, \Sigma_{n|n}),$$

where the posterior mean $z^{\theta}_{n|n}$ is adjusted according to diagnostic expectations [1]:

$$\begin{aligned}
z^{\theta}_{n|n} &= z_{n|n-1} + (1+\theta)K_n \left(y_n - z_{n|n-1}\right), \\
\Sigma_{n|n} &= \frac{\sigma_e^2}{\Sigma_{n|n-1} + \sigma_e^2}\Sigma_{n|n-1}, \\
K_n &= \frac{\Sigma_{n|n-1}}{\Sigma_{n|n-1} + \sigma_e^2}, \\
z_{0|-1} &\equiv \mu_{z,0}, \quad \Sigma_{0|-1} \equiv \sigma^2_{z,0}.
\end{aligned}$$

Thus, a household's distorted belief about next period's residual income realization follows:

$$y^{\theta}_{n+1} = \quad z^{\theta}_{n|n} + \epsilon_{n+1} + e_{n+1} \sim \mathcal{N}(z^{\theta}_{n|n}, \Sigma_{n|n} + \sigma^2_{\epsilon} + \sigma^2_e).$$

Since z follows a random walk, the mean of residual income in the next period equals the mean of the belief about z in the current period. However, this belief is distorted due to households' diagnostic expectations.

A key feature of diagnostic expectations is what the literature calls *rational reversion*. The diagnostic distortion applies only to the posterior update within each period. However, at the end of the period, households reassess their beliefs rationally, as if re-evaluating the entire history of realizations. This reversion ensures that the prior for the next period remains rational. As a direct implication, diagnostic expectations always start with a rational prior and there is no need to carry from one period to the other the value of the distorted belief. The key distinction between the lender and the behavioral households lies in their *posterior mean*. While the lender maintains entirely rational beliefs, their variance updates remain identical to households holding rational expectations.[2]

2.4 Timeline

The timeline within each period (year/age) unfolds as follows:

[1] For proof, see Proposition 1 of Bordalo et al. (2020)

[2] In Bordalo, Gennaioli, Shleifer and Terry (2021), firms and lenders form expectations diagnostically. We conjecture that the same assumption in our setup would exacerbate the role of diagnostic expectations in generating financial distress.

1. Households and lenders enter age n with a prior about the persistent income component: $z_{-1} \sim \mathcal{N}(z_{-1}, \Sigma_{n|n-1})$, and assets a.
2. At the beginning of the period, income is realized: y_n observed by households and lenders.
3. Both households and lenders update beliefs based on income.
4. Lender offers debt prices, given updated beliefs.
5. Households choose actions based on prices:
    - Default, delinquency, or repayment
    - Consumption and borrowing/saving
6. Households with distorted beliefs revert to rational beliefs (rational reversion).

2.5 Value Functions

We write the value functions in a way that allows for the possibility of households distorted beliefs arising from diagnostic expectations when $\theta > 0$. This formulation accommodates the presence of cognitive biases, where agents may hold imperfect or biased beliefs. The case of rational learning is nested within this framework and can be obtained by setting $\theta = 0$. In this case, agents' expectations are updated entirely rationally. Therefore, the model captures both scenarios: one with households distorted beliefs ($\theta > 0$) and the other with rational expectations ($\theta = 0$).

The household's decision at each age n is to either declare bankruptcy, choose delinquency, or repay debt. The value functions associated with these choices incorporate beliefs about future income and the persistence of income shocks. In particular, given the current residual income realization y, the belief about the persistent component carried over from the previous period z_{-1}, and the asset or debt level a, households' lifetime utility at age n is given by:

$$G_n(y, z_{-1}, a) = \max \Big\{ \underbrace{V_n(y, z_{-1}, a)}_{\text{Repayment}}, \underbrace{B_n(y, z_{-1})}_{\text{Bankruptcy}}, \underbrace{D_n(y, z_{-1}, a)}_{\text{Delinquency}} \Big\}.$$

These value functions evolve with age n, reflecting changes in the income profile, the learning process, and the finite lifespan of households.

The lifetime utility from declaring bankruptcy at age n is:

$$B_n(y, z_{-1}) = u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \int G_{n+1}(y', z, 0)\, \mathrm{d}\Phi(y'; z^\theta, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2),$$

subject to:

$$\begin{aligned} z &= z_{-1} + K_n(y - z_{-1}), \\ z^\theta &= z_{-1} + (1+\theta)K_n(y - z_{-1}), \\ c &= \underbrace{\exp\left(\ell(n) + y\right)}_{\text{Income}} - \mathbf{c}_{BK}. \end{aligned}$$

Here, $\mathbf{c}_{BK}$ represents the cost of filing for bankruptcy, and debt is reset to zero. The function $\Phi(y; \mu, \sigma^2)$ denotes the cumulative distribution function (CDF) of a normal distribution with mean μ and variance σ^2. The term $h(n)$ accounts for household size in equivalent adults.[3]

Compared to rational households ($\theta = 0$), those with distorted beliefs set the mean of the residual income distribution at z^θ instead of z. Thus, when $\theta > 0$, households *overreact* to the current income realization when assessing the persistent income component.

Similarly, the lifetime utility from delinquency is given by:

$$\begin{aligned} D_n(y, z_{-1}, a) = u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \int \Big[& \gamma G_{n+1}(y', z, 0) \\ & + (1-\gamma) G_{n+1}(y', z, a') \Big]\, \mathrm{d}\Phi(y'; z^\theta, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2), \end{aligned}$$

subject to:

$$\begin{aligned} a' &= (1+\eta)a, \\ z &= z_{-1} + K_n(y - z_{-1}), \\ z^\theta &= z_{-1} + (1+\theta)K_n(y - z_{-1}), \\ c &= \underbrace{\min\{\exp\left(\ell(n) + y\right), \tau\}}_{\text{Income net of garnishment}}. \end{aligned}$$

With probability γ, the debt is forgiven, while with probability $1 - \gamma$, the debt rolls over at a penalty interest rate η.

If the household chooses to repay its debt, it must settle the outstanding balance and may borrow anew using the lender's offered menu of borrowing costs, q. The lifetime

[3] Drozd and Serrano-Padial (2017) also adjusts consumption to reflect the variations in household size over the life cycle.

utility from repayment is given by

$$V_n(y, z_{-1}, a) = \max_{a', c} \; u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \int G_{n+1}(y', z, a') \, \mathrm{d}\Phi(y'; z^\theta, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2),$$

subject to:

$$\begin{aligned} z &= z_{-1} + K_n(y - z_{-1}), \\ z^\theta &= z_{-1} + (1+\theta) K_n(y - z_{-1}), \\ c &= a - a' q_n(z, a') + exp(\ell(n) + y). \end{aligned}$$

The price of debt is determined by the lender's rational posterior on the household's persistent income component, the amount of debt taken, and age, as policy functions vary with age. Note that lenders consider rational beliefs about future residual income when pricing debt, while the repayment policy follows the household's distorted beliefs. The repayment decision function is:

$$R_n(y, z_{-1}, a) = \begin{cases} 1, & \text{if } V_n = \max\{V_n, D_n, B_n\}, \\ 2, & \text{if } D_n = \max\{V_n, D_n, B_n\}, \\ 0, & \text{otherwise.} \end{cases}$$

Since default and delinquency are household choices, a risk-neutral lender charges a premium to ensure zero expected profit. Debt prices depend on the household's probability of default or delinquency. Consequently, debt prices are set as

$$\begin{aligned} q_n(z, a') = \frac{1}{1+r} \int \Big[& 1(R_{n+1}(y', z, a') = 1) \\ & + 1(R_{n+1}(y', z, a') = 2)(1-\gamma)(1+\eta) q_{n+1}(z', a'') \Big] \mathrm{d}\Phi(y'; z, \Sigma_{n|n} + \sigma_\epsilon^2 + \sigma_e^2), \end{aligned}$$

subject to:

$$\begin{aligned} z' &= z + K_{n+1}(y' - z), \\ a'' &= (1+\eta) a' \end{aligned}$$

3 Calibration

The benchmark model for calibration is the rational learning model. The next section explores counterfactual scenarios to assess the impact of learning and diagnostic expectations on financial distress.

Table 6 includes parameters that are either well-established in the literature or based on empirical estimates. The risk-free rate ($r = 0.03$) reflects long-term averages. The risk aversion parameter takes the standard value ($\sigma = 2$). Parameters governing income processes, such as the standard deviations of persistent and transitory shocks ($\sigma_\epsilon = 0.10$, $\sigma_e = 0.22$), follow from Kaplan and Violante (2010). The initial distribution of z is also taken from Kaplan and Violante (2010) ($\mu_{z,0} = 0, \sigma_{z,0} = 0.39$) and it coincides with the prior. The debt roll-over rate ($\eta = 0.20$) takes the same value as in Livshits, MacGee and Tertilt (2007). The bankruptcy cost ($\mathbf{c}_{BK} = 0.165$) and the family size are obtained directly from the data.

Table 1: *Calibrated Parameters*

Parameter	Value	Meaning	Basis
T	65	Retirement age	Standard
σ	2.00	Risk aversion	Standard
r	0.03	Risk-free rate	Standard
η	0.20	Debt roll-over rate	Livshits, MacGee and Tertilt (2007)
σ_ϵ	0.10	Std of persistent shock	Kaplan and Violante (2010)
σ_e	0.22	Std of transitory shock	Kaplan and Violante (2010)
ϱ_n	-	Mortality rate	Kaplan and Violante (2010)
$\mu_{z,0}$	0	Initial mean of persistent shock	Kaplan and Violante (2010)
$\sigma_{z,0}$	0.39	Initial Std of persistent shock	Kaplan and Violante (2010)
$\mathbf{c}_{BK}$	0.165	Bankruptcy cost	BK cost to mean income
$h(n)$	-	Family size	Survey of Consumer Finances
μ	0.7	Family size scaling parameter	Buhmann et al. (1988)
θ	$\{0, 0.5, 1\}$	DE parameter	Bordalo et al. (2020)

The remaining three parameters are estimated to minimize the distance to key moments. The estimation targets five sets of moments, each including three age groups to capture the life-cycle profile of financial distress and household debt.

The first two sets contain information about the share of individuals in financial distress. The first three moments are the share of individuals in DQ, which declines with age, and the second set includes bankruptcy rates, which remain relatively low across age groups. These two moments are crucial for capturing how frequently individuals fail to repay their debt obligations.

The next three sets of moments provide information on borrowing behavior. The first measures debt participation, which increases slightly with age. The second captures the debt-to-income ratio, following a similar pattern, with younger individuals carrying

slightly higher relative debt burdens. Lastly, the coefficient of variation (CV) of debt is considered, which is highest among the oldest group, reflecting greater heterogeneity in debt holdings over the life cycle. The detailed definition of moments is documented in Appendix B.

Table 2 reports the estimated parameters that minimize the distance between the model and empirical targets, along with standard errors (in parentheses). While each parameter is jointly determined, it closely aligns with specific targets. The discount factor ($\beta = 0.814$) helps match moments related to debt. The discharge probability of delinquent debt ($\gamma = 0.179$) reduces the likelihood of bankruptcy for households remaining in delinquency, making it key to matching the observed bankruptcy rate. Finally, the parameter governing the cost of delinquency ($\tau = 3.488$) is crucial for reproducing both the incidence and life-cycle profile of delinquency. Standard errors are relatively small, with noticeable uncertainty only in the estimated value of the discharge probability of delinquent debt.

Table 2: *Estimated Parameters*

Parameter	Value
Discount factor, β	0.814
	(0.008)
Discharge prob of DQ debt, γ	0.179
	(0.038)
DQ consumption cost, τ	3.488
	(0.040)

Note: Standard errors in parentheses.

With these parameters, the model successfully replicates key patterns of financial distress and debt participation across age groups, as shown in Table 3. DQ rates align well with the data, with small deviations across age groups. The model closely matches DQ rates for individuals aged 35-44 and 45-54 while slightly overpredicting DQ for younger households. Bankruptcy rates follow a similar pattern, with predictions tracking observed rates well for middle-aged and older households, despite a higher estimate for younger individuals.

Debt participation exhibits a strong match, particularly for those aged 35-44. The model slightly underestimates participation for individuals aged 45-54 but captures the overall life-cycle trend. The coefficient of variation (CV) of debt closely aligns with the data across all groups, reflecting a realistic dispersion in borrowing. The largest discrep-

Table 3: *Fit of targeted moments*

Statistic	Data	Learning Model
% in DQ, age 25-34	11.16	15.20
% in DQ, age 35-44	10.81	9.63
% in DQ, age 45-54	8.91	9.07
% in BK, age 25-34	0.64	1.98
% in BK, age 35-44	0.78	0.75
% in BK, age 45-54	0.66	0.60
% in Debt, age 25-34	41.49	47.96
% in Debt, age 35-44	44.55	46.10
% in Debt, age 45-54	45.70	44.12
(debt/income)%, age 25-34	3.310	1.775
(debt/income)%, age 35-44	3.472	0.848
(debt/income)%, age 45-54	3.159	0.723
CV of debt, age 25-34	1.465	1.494
CV of debt, age 35-44	1.560	1.519
CV of debt, age 45-54	1.703	1.784

ancies appear in mean debt-to-income ratios, where the model underpredicts debt holdings across age groups. However, given that only three parameters were chosen to fit fifteen moments, the model's ability to match key financial outcomes with such precision underscores its robustness in capturing the core mechanisms driving default and borrowing behavior.

4 The Role of Shocks Misperception for Financial Distress

In this section, we analyze the role of shock misperception in financial distress, first in partial equilibrium and then in general equilibrium. To do so, we maintain the same parameters and compute two alternative economies: one with perfect information and the other with diagnostic expectations.

4.1 Partial Equilibrium Results

This section compares the alternative frameworks holding all parameters fixed and keeping the risk-free interest rate constant. The results reveal that incorporating learning and diagnostic expectations significantly increases the incidence of both informal (delinquency) and formal (bankruptcy) default compared to the perfect information benchmark.

Table 4 compares the learning model with the perfect information model. Under learning, delinquency increases significantly across all age groups. Among younger households (ages 25-34), delinquency rises by 21.9%. The increase is even more pronounced for middle-aged (ages 35-44) and older (ages 45-54) households, where delinquency rises by 40.5% and 31.6%, respectively. A similar pattern emerges for bankruptcy rates, though the overall incidence remains lower. These results suggest that earnings shock misperception under learning leads to greater difficulty in meeting debt obligations.

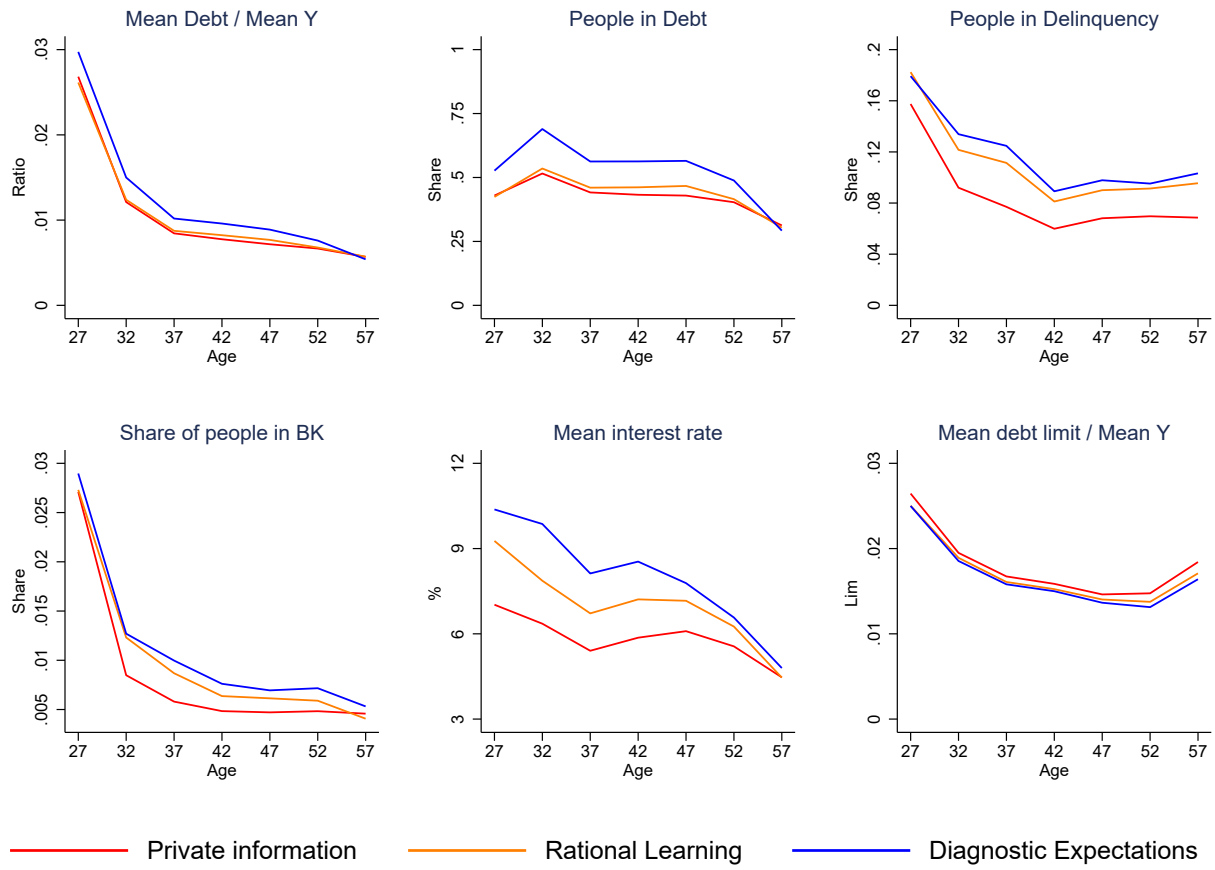
Table 4: *Financial Distress with Perfect Information, Learning, and Diagnostic Expectations (PE)*

Statistic	(1) Perfect Information	(2) Learning (Benchmark)	% Change (2) vs. (1)	(3) Diagnostic Expectations	% Change (3) vs. (2)
% in DQ, age 25-34	12.47	15.20	21.9	15.66	3.0
% in DQ, age 35-44	6.85	9.63	40.5	10.70	11.1
% in DQ, age 45-55	6.89	9.07	31.6	9.66	6.5
% in BK, age 25-34	1.80	1.98	10.0	2.08	5.1
% in BK, age 35-44	0.53	0.75	41.5	0.88	17.3
% in BK, age 45-55	0.48	0.60	25.0	0.71	18.3

Table 4 also compares the learning model with the model incorporating diagnostic expectations. While diagnostic expectations further amplify financial distress, the overall impact remains relatively small. Among younger households (ages 25-34), delinquency increases by only 3.0%, while for middle-aged (ages 35-44) and older (ages 45-54) households, the increases are 11.1% and 6.5%, respectively. Bankruptcy rates show a similar pattern, with moderate increases despite the addition of diagnostic expectations. Notably, this limited effect arises even though we set $\theta = 1$, a value that is large relative to the literature. These findings suggest that while diagnostic expectations exacerbate financial distress, their influence is modest compared to the shift from perfect information to learning.

Figure 1 illustrates the evolution of key variables over the life-cycle. In general, the values under perfect information are lower than those under rational learning, which are

Figure 1: Life-cycle Profiles under Perfect Information, Learning, and Diagnostic Expectations (PE)



in turn lower than those under diagnostic expectations for five out of six variables, including debt, delinquency, bankruptcy, and interest rates. However, this pattern does not hold for the final plot, which represents the maximum amount households can borrow, referred to as the "debt limit" in the plots. This suggests that household behavior (credit demand), rather than lender behavior (credit supply), drives this result.

4.2 General Equilibrium Results

One possible reason for the increased financial distress under learning and diagnostic expectations is that they lead households to borrow more. To control for this possibility, this section presents results where the market adjusts the interest rate to ensure that the total level of debt remains the same across the three economies, following the approach of Huggett (1993). Since the learning economy serves as the benchmark, the level of debt

is fixed at its value in that economy.

Table 5 compares financial distress under perfect information and learning in general equilibrium. As in partial equilibrium, learning increases both delinquency and bankruptcy rates across all age groups. Among younger households (ages 25-34), delinquency rises by 21.2%, while middle-aged (ages 35-44) and older (ages 45-54) households experience even larger increases of 41.4% and 31.8%, respectively. Bankruptcy rates follow a similar pattern, with increases of 10.6% for younger households, 41.5% for middle-aged households, and 27.7% for older households.

Compared to the partial equilibrium results, the magnitude of the changes is slightly lower for younger households but remains similar for older groups. An important distinction in general equilibrium is that the interest rate declines from 3.375% under perfect information to 3% under learning, an 11.1% decrease. This suggests that borrowing demand was higher under perfect information, requiring a higher interest rate to restrain borrowing. Under learning, misperception of income shocks makes households more cautious, leading to more precautionary savings and lower equilibrium interest rates despite higher financial distress.

Table 5: *Financial Distress with Perfect Information, Learning and Diagnostic Expectations (GE)*

Statistic	(1) Perfect Information	(2) Learning (Benchmark)	% Change (2) vs. (1)	(3) Diagnostic Expectations	% Change (3) vs. (2)
% in DQ, age 25-34	12.54	15.20	21.2	15.61	2.7
% in DQ, age 35-44	6.81	9.63	41.4	10.52	9.2
% in DQ, age 45-55	6.88	9.07	31.8	9.54	5.2
% in BK, age 25-34	1.79	1.98	10.6	2.06	4.0
% in BK, age 35-44	0.53	0.75	41.5	0.86	14.7
% in BK, age 45-55	0.47	0.60	27.7	0.69	15.0
Interest Rate (%)	3.37	3.00	-11.1%	4.24	41.4%

Table 5 compares financial distress under learning and diagnostic expectations in general equilibrium. As in partial equilibrium, introducing diagnostic expectations further increases delinquency and bankruptcy rates across all age groups, though the effects remain relatively small. Delinquency rises by 2.7% for younger households (ages 25-34), 9.2% for middle-aged households (ages 35-44), and 5.2% for older households (ages 45-54). Bankruptcy rates also increase, with the largest effects observed for middle-aged (14.7%) and older households (15.0%).

Compared to the partial equilibrium results, the increases in financial distress under

diagnostic expectations are slightly muted in general equilibrium. A key difference is the sharp rise in the equilibrium interest rate, which increases from 3% under learning to 4.242% under diagnostic expectations, a 41.4% jump. This suggests that diagnostic expectations initially increase borrowing demand, prompting the market to raise interest rates to restore equilibrium. The higher borrowing costs, in turn, reduce overall borrowing levels, mitigating the rise in financial distress.

4.3 How Earnings Shocks Trigger Financial Distress

To illustrate how individuals slide into financial distress, this subsection and the next two present examples that highlight the model's dynamics. In each scenario, three types of agents-perfect information, rational learning, and diagnostic expectations-begin with zero assets. **red**For the graphs, is it more good-looking if we link the points together?

Figure 2 shows an initial case where individuals receive a substantial positive persistent income shock ($z_{25} = 0.16$) and a minor positive transitory income shock ($e_{25} = 0.04$). Motivated mainly by impatience and expectations of sustained high future income, all three agent types borrow ($a_{26} = -0.125$), boosting current consumption beyond their income.

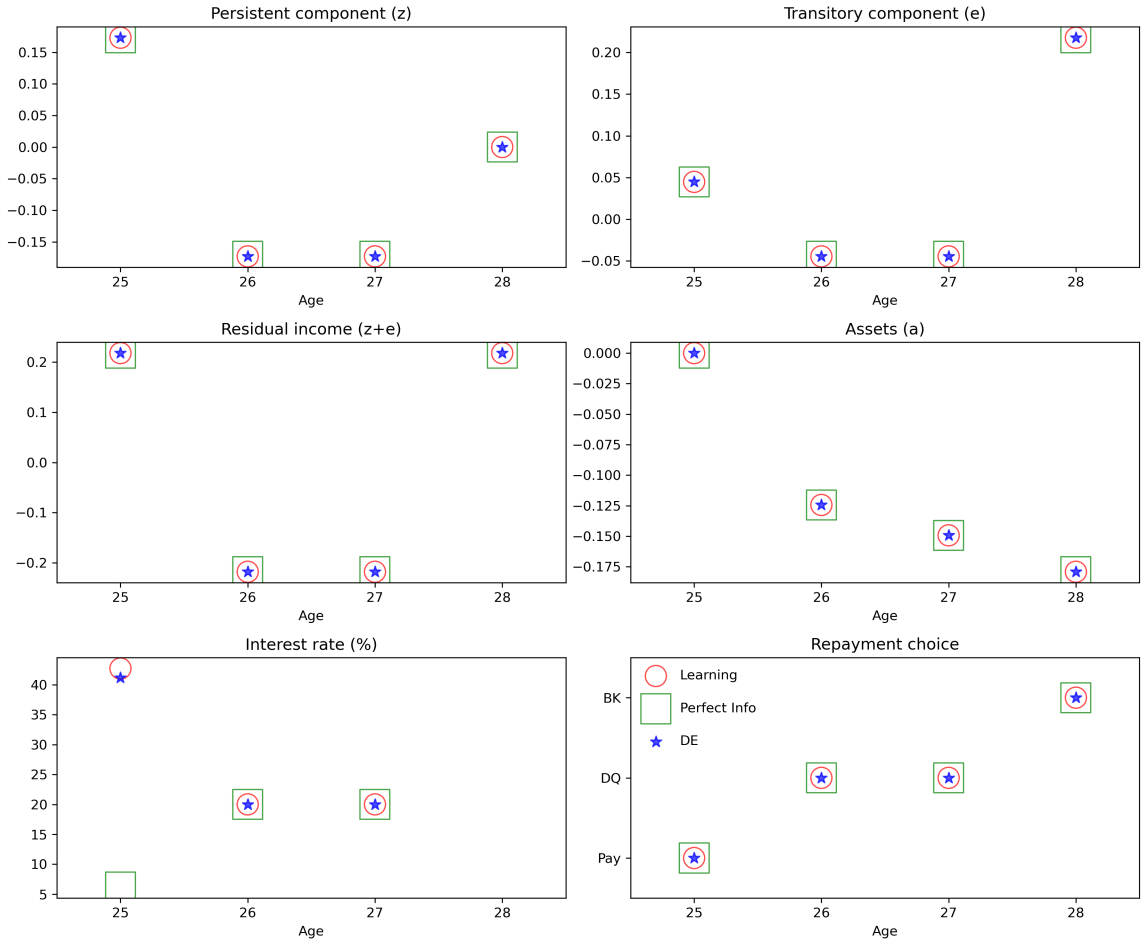
However, in the subsequent year, the persistent income shock sharply reverses ($z_{26} = -0.16$), accompanied by a small negative transitory shock ($e_{26} = -0.04$). Faced with unexpectedly reduced income, all agents skip their debt payments and fall into delinquency. As a result, their debt grows significantly because lenders apply a punitive 20% interest rate, increasing debt to $a_{27} = -0.15$.

The unfavorable income shocks persist into the following year ($z_{27} = -0.16$, $e_{27} = -0.04$), prompting agents to remain delinquent. Consequently, their debt escalates further by another 20% ($a_{28} = -0.18$).

Finally, at age 28, a positive transitory income shock provides temporary relief. The agents strategically use part of this unexpected income to file for bankruptcy, effectively eliminating their accumulated debt and beginning anew the next period.

In this example, all three agent types made identical decisions, experiencing financial distress similarly. The subsequent two examples will demonstrate how learning and diagnostic expectations can increase the likelihood of falling into financial distress.

Figure 2: Labor Earnings Shocks and Financial Distress



4.4 How Misperceptions of Future Income Lead to Financial Distress

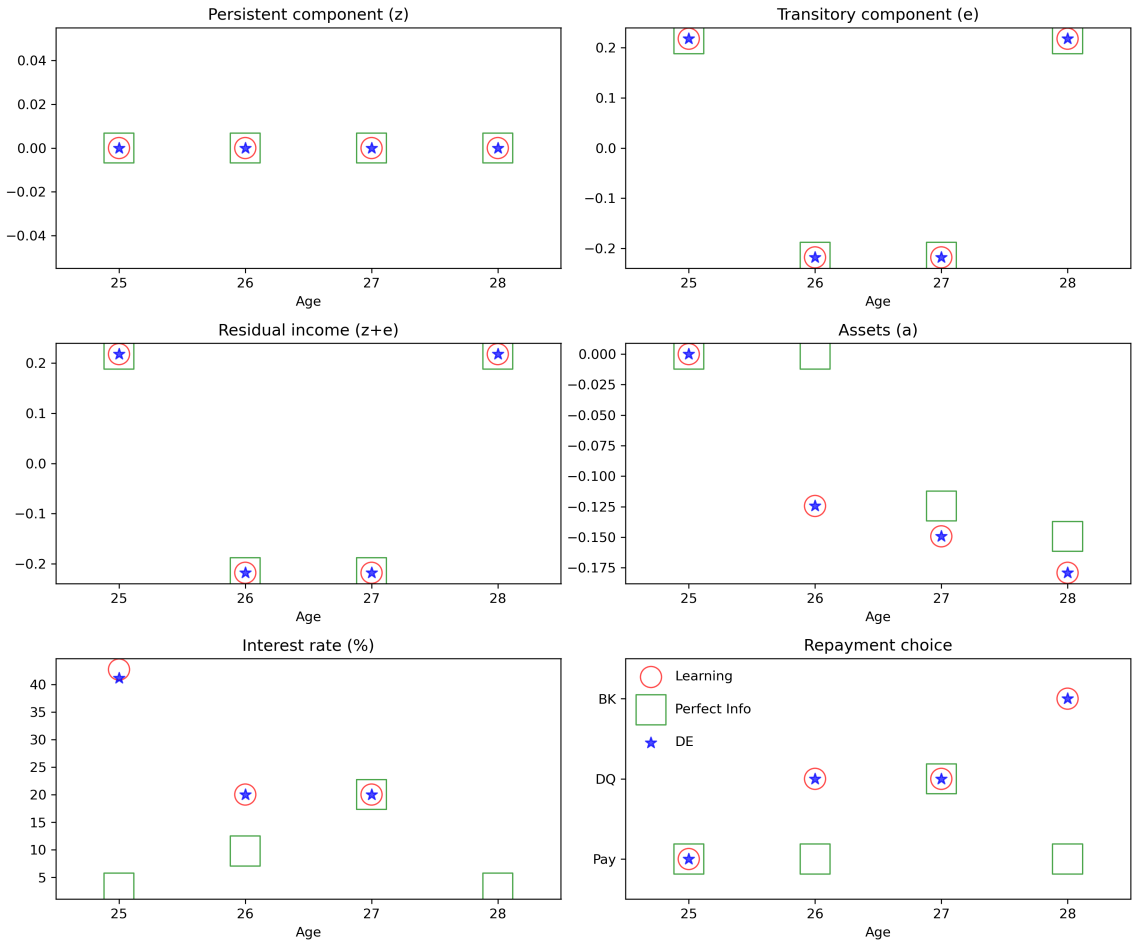
The example in Figure 3 traces an individual's financial decisions during the first four years of adulthood. Initially, this individual starts with zero assets and no persistent income shock ($z_{25} = 0$) but receives a positive transitory income shock ($e_{25} = 0.2$). Thus, residual income at age 25 is higher ($z_{25} + e_{25} = 0.2$) than initially anticipated.

As seen in the middle-right plot, asset choices at age 26 significantly differ between learning-based models and the perfect-information model. Under perfect information, assets remain unchanged ($a_{26} = 0$). Conversely, the learning models generate greater borrowing ($a_{26} \approx -0.125$), as individuals overestimate future income prospects. This borrowing raises interest rates due to increased default risk, as depicted in the bottom-left panel.

The next year, the individual faces a negative transitory shock ($e_{26} = -0.2$) without a persistent shock ($z_{26} = 0$). Agents using learning models revise income expectations

downward and, burdened by debt, enter delinquency. Meanwhile, the agent with perfect information, having avoided debt initially, chooses moderate borrowing ($a_{27} \approx -0.125$) to maintain stable consumption.

Figure 3: Example of the Role of Learning in Financial Distress



At age 27, experiencing identical shocks ($z_{27} = 0$, $e_{27} = -0.2$), the learning-model agent remains delinquent, increasing debt by 20% due to penalty rates. The perfect-information agent, now financially vulnerable, also enters delinquency to mitigate the consumption drop.

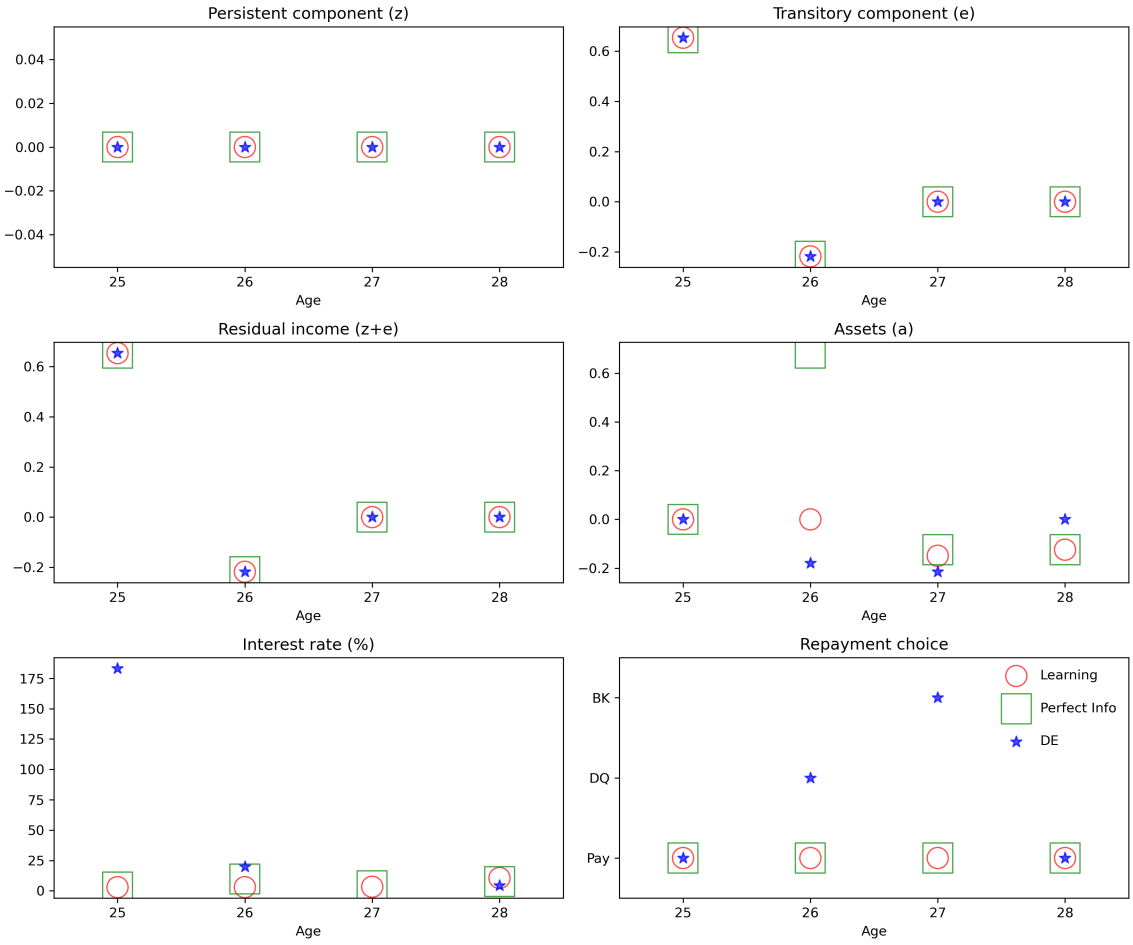
By age 28, with a positive transitory shock ($e_{28} = 0.2$) but no persistent improvement ($z_{28} = 0$), the perfect-information agent quickly repays the debt and exits distress. However, agents using rational learning or diagnostic expectations, encumbered by larger debts and pessimistic beliefs, ultimately resort to bankruptcy.

This example demonstrates how changes in beliefs about the permanent component of income can push individuals toward debt delinquency and eventually bankruptcy.

4.5 Exaggerated Optimism and the Path to Financial Distress

The second example, presented in Figure 4, illustrates how diagnostic expectations amplify financial distress compared to perfect information and rational learning. The example begins at age 25, with zero initial assets, no persistent income shock ($z_{25} = 0$), but a substantial positive transitory income shock ($e_{25} = 0.6$). As a result, residual income that year is notably high ($z_{25} + e_{25} = 0.6$), significantly exceeding prior expectations.

Figure 4: Example of Exaggerated Optimism and Financial Distress



The middle-right panel highlights substantial differences in asset decisions for age 26 across the three cases. With perfect information, awareness that the income boost is temporary motivates substantial saving ($a_{26} \approx 0.6$). Under rational learning, moderate optimism about future income leads to full consumption of current income, resulting in zero asset accumulation ($a_{26} = 0$). Under diagnostic expectations, exaggerated optimism fuels excessive consumption financed through borrowing, despite the already high income level, anticipating continued high income.

In the following year, the individual experiences no persistent income shock ($z_{26} = 0$) and a negative transitory income shock ($e_{26} = -0.2$). Agents with rational learning and perfect information, possessing no debt, respond by borrowing modestly to smooth their consumption. In contrast, the agent-driven by diagnostic expectations sharply revises future income beliefs downward and, burdened by significant debt, chooses delinquency.

At age 27, with no income shocks occurring ($z_{27} = 0$, $e_{27} = 0$), agents with rational learning and perfect information roll over their manageable debt. However, the individual guided by diagnostic expectations, confronting substantial accumulated debt, opts for bankruptcy.

This example demonstrates clearly how diagnostic expectations, by magnifying misperceptions of long-term income prospects, lead to elevated financial distress.

5 Distinguishing Model Implications

This section highlights key distinguishing implications across the three models and evaluates them against the data to isolate the role of earnings-shock misperceptions in driving household financial distress. To ensure a fair comparison, we first estimate each model by minimizing its distance to the identical set of empirical moments which we used for estimation in previous sections.

5.1 Estimation of Alternative Models

We estimate each model by minimizing its distance to the same set of empirical moments. Our objective is to clarify the roles of learning and diagnostic expectations in driving financial outcomes.

Table 6 presents parameter estimates across models. Estimates from the perfect information model are statistically indistinguishable from those of the benchmark rational learning model, except for the discount factor β, which is significantly lower. This difference aligns with the intuition that the perfect information model, unable to generate sufficient financial distress on its own, compensates by requiring households to be more impatient to match observed data.

The third estimated model incorporates agents with diagnostic expectations, setting the degree of it at $\theta = 0.5$ to match previous empirical estimates. For instance, Bordalo et al. (2020) find an average θ of approximately 0.5 when analyzing professional forecasters' expectations across various aggregate variables, such as nominal GDP. We adopt

Table 6: *Model Parameters Estimated*

Parameter	Perfect Information	Learning (Benchmark)	Learning with DE ($\theta = 0.5$)	Learning with DE (θ flexible)
β	0.793	0.814	0.825	0.812
	(0.007)	(0.008)	(0.005)	(0.008)
γ	0.174	0.179	0.180	0.180
	(0.055)	(0.038)	(0.026)	(0.030)
τ	3.486	3.488	3.486	3.489
	(0.057)	(0.040)	(0.037)	(0.040)
θ	-	-	-	0.075
				(0.189)

this value as a benchmark. In this model, the estimated discount factor β is significantly higher compared to both the perfect information and rational learning models, implying that diagnostic expectations reduce the degree of household impatience required to explain the observed levels of financial distress.

The final column reports our attempt to estimate the degree of diagnostic expectations, θ. We obtain an estimate of $\theta = 0.075$, but it comes with large standard errors, indicating that the moments we match do not effectively identify the parameter θ and we cannot reject the null hypothesis that $\theta = 0$. Consequently, for the analysis of distinguishing implications, we focus on the diagnostic expectations model with $\theta = 0.5$.

Table 7 compares how well the alternative estimated models match the data. All models successfully replicate the targeted moments. The rational learning model achieves the smallest minimized distance to targets, but the gap with the diagnostic-expectations model is narrow. This suggests that learning may play a role in accounting for households' financial distress.

5.2 Interest Rates and Income

Data from the Survey of Consumer Finances (SCF) can be useful to establish the relationship between credit card interest rates and household income. Establishing this empirical relationship matters because the three alternative models above yield distinct predictions regarding this connection. Hence, this empirical finding helps in distinguishing among competing models.

As the measure of income, we consider the answer to the question: *"How much was*

Table 7: *Fit of Targeted Moments in Alternative Economies*

Parameter	Perfect Information	Learning (Benchmark)	Diagnostic Exp ($\theta = 0.5$)	Diagnostic Exp (θ flexible)
% in DQ, age 25-34	12.9	15.2	15.5	15.4
% in DQ, age 35-44	7.33	9.63	9.89	9.71
% in DQ, age 45-55	7.61	9.07	9.12	9.14
% in BK, age 25-34	1.80	1.98	1.96	2.00
% in BK, age 35-44	0.60	0.75	0.78	0.76
% in BK, age 45-55	0.51	0.60	0.60	0.61
% in Debt, age 25-34	49.6	48.0	51.3	48.5
% in Debt, age 35-44	48.0	46.1	45.5	46.3
% in Debt, age 45-55	47.1	44.1	42.9	44.7
Debt/Income%, age 25-34	1.85	1.78	1.83	1.79
Debt/Income %, age 35-44	0.89	0.85	0.83	0.85
Debt/Income %, age 45-55	0.75	0.72	0.68	0.72
CV of Debt, age 25-34	1.47	1.49	1.50	1.49
CV of Debt, age 35-44	1.56	1.52	1.55	1.51
CV of Debt, age 45-54	1.76	1.78	1.84	1.77
Distance	50.4	47.4	48.0	47.7

the total income you (and your family living here) received from all sources before taxes and other deductions were made?" For interest rates, the analysis uses responses to the SCF question: *"What interest rate do you pay on the card where you have the largest balance?"*.

The analysis begins by examining the relationship between credit card interest rates and household income using binned scatterplots. Figure 6 highlights the central finding of this section: in the SCF data, there exists a tight and negative relationship between interest rates and income. This pattern remains robust across survey waves. The analysis controls for age, age squared, and the amount of credit card debt owed.

What does this relationship look like in the model? A clear difference across models is evident, as shown in Figure 6. This divergence arises primarily because, in learning-based models, income is observable and directly influences perceived credit risk. For a given amount of borrowed funds (controlled in the plots), higher income suggests higher future income through beliefs about the persistent income component. This reduces the expected probability of default, thereby lowering the interest rate lenders are willing to offer. In contrast, in the perfect information model, interest rates are determined by the persistent component z, while income also contains a transitory component e. The transitory component affects income but not the interest rate, thereby weakening the relation-

Figure 5: Interest rate and Income across years

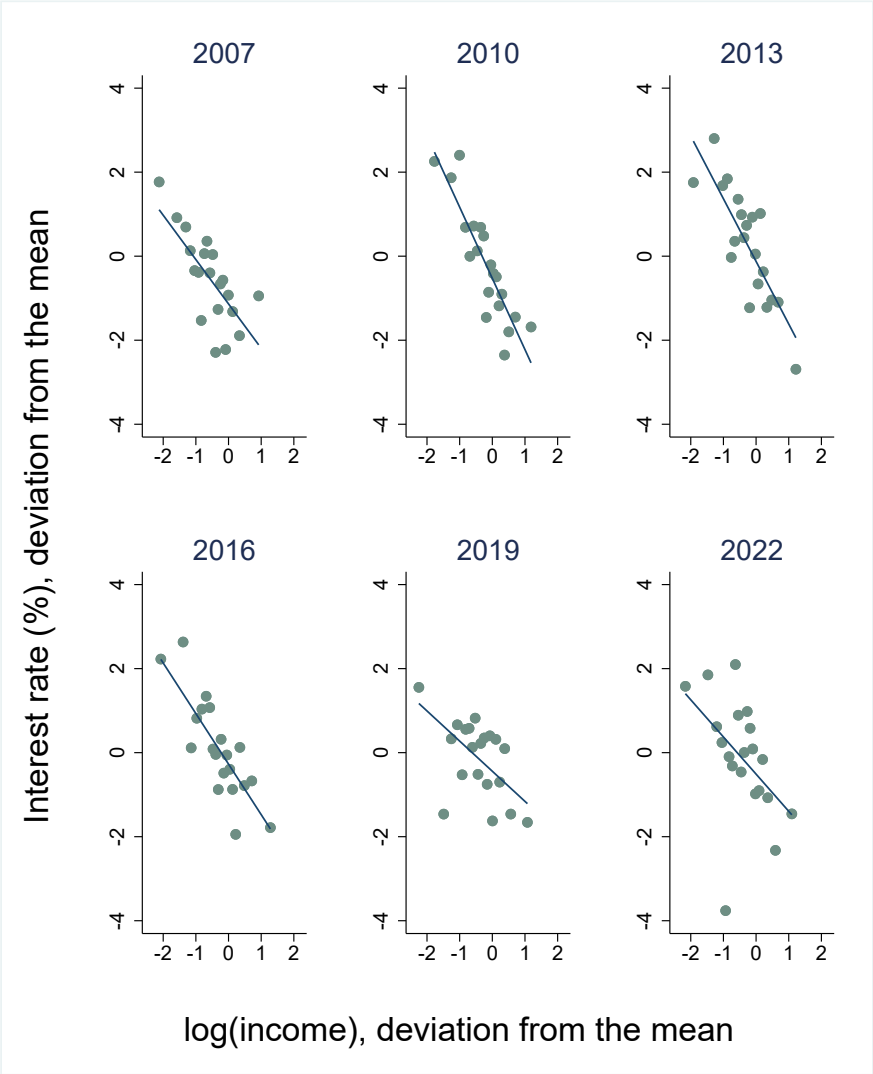
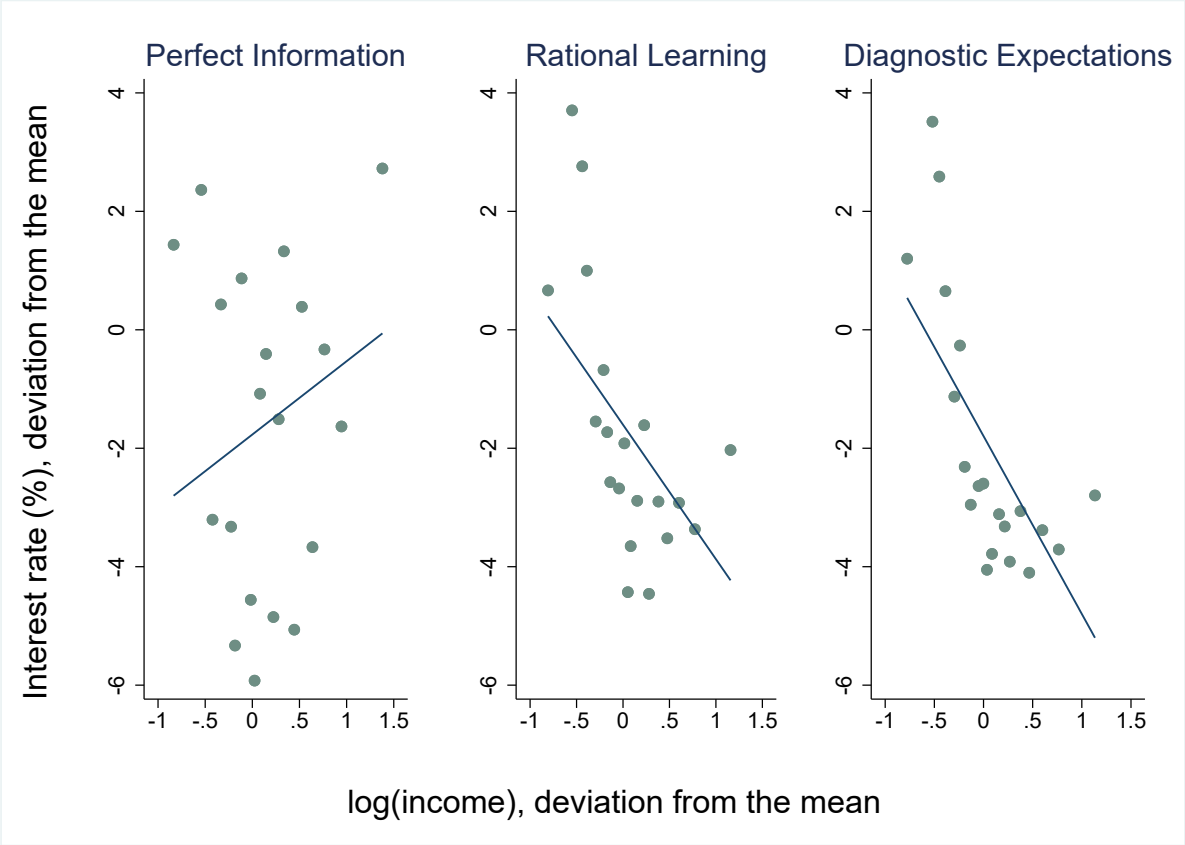


Figure 6: Relationship Between Interest Rates and Income in the Models



Note: The figure shows a binned scatterplot of demeaned interest rates against demeaned log total income, controlling for age, age squared, and credit card debt. The sample includes households aged 25-65 with positive credit card debt. Observations are weighted using SCF-provided weights.

ship between rates and income.

We also present evidence of this relationship with a formal regression analysis. The regression controls for age, age squared, and credit card debt levels. Credit card debt is measured by responses to the question: *"After the last payment was made, what was the total balance still owed?"* The regression incorporates survey weights and includes only households with positive credit card balances.

In addition, we consider an alternative question about income: *"In total, what was your (family's) annual income from wages and salaries in 2015, before deductions for taxes and anything else?"* Because both measures of income may reflect both predetermined factors (fixed effects not included in the model) and household decisions such as labor supply (also not included in the model), an instrumental variable is also considered. The instrument derives from responses to the SCF question: *"Is this income unusually high or low compared to what you would expect in a 'normal' year, or is it normal?"* This instrument captures variations

in income akin to income shocks modeled theoretically.

The results in Table 8 confirm the patterns illustrated by the scatterplots. Conditional on debt, higher household income correlates negatively with credit card interest rates. The standardized coefficients, reported in brackets, provide the clearest interpretation: a one-standard-deviation increase in income corresponds to a reduction in interest rates ranging from 0.13 to 0.29 standard deviations. Alternatively, the semi-elasticities indicate that doubling income leads to a decrease in interest rates between 1 and 2.22 percentage points.

Table 8: *Regression Results: Interest Rates and Income (OLS and IV)*

	OLS	IV	OLS	IV
log(Labor Earnings)	-0.99^{***}	-2.03^{***}	–	–
	(0.09)	(0.41)		
	[−0.13]	[−0.27]		
log(Total Income)	–	–	-1.20^{***}	-2.22^{***}
			(0.09)	(0.34)
			[−0.16]	[−0.29]
Observations	6,946	6,946	7,784	7,784
Weights	Yes	Yes	Yes	Yes
Sample	Debt>0	Debt>0	Debt>0	Debt>0
	Age 25-65	Age 25-65	Age 25-65	Age 25-65
First-stage F-stat	–	170.10	–	256.89
Instruments	–	unusually	–	unusually
	–	high or low	–	high or low

Note: Robust standard errors in parentheses; beta coefficients (normalized by standard errors) in brackets. Controls included in all regressions: credit card debt, age, and age squared. Instrumental variables regressions report first-stage F-statistics indicating instrument relevance. Significance levels: *** p<0.01, ** p<0.05, * p<0.1.

Table 9 reports the results from running equivalent regressions on data generated by the alternative models. These findings align closely with those from the scatterplots. Specifically, the standardized coefficients indicate that a one-standard-deviation increase in residual income reduces interest rates by 0.19 standard deviations under the learning model, and by 0.27 standard deviations under diagnostic expectations. In contrast, the same coefficient is small and positive in the data produced by the perfect information model.

Table 9: *Regression Results: Interest Rates and Residual Income (Alternative Models)*

	Perfect Info	Learning	Diagnostic Exp.
Residual Income (y)	1.24^{***}	-2.27^{***}	-3.00^{***}
	(0.02)	(0.01)	(0.01)
	[0.06]	[−0.19]	[−0.27]
Observations	679,796	581,924	567,987
Controls	Yes	Yes	Yes
Sample	Debt>0	Debt>0	Debt>0
	Age 25-65	Age 25-65	Age 25-65

Note: Robust standard errors in parentheses; beta coefficients (normalized by standard errors) in brackets. Controls included in all regressions: amount borrowed, age, and age squared. Significance levels: *** p<0.01, ** p<0.05, * p<0.1.

6 Welfare Gains of Eliminating Misperceptions

This section quantifies the welfare gains from eliminating misperceptions regarding income shocks.[4] As discussed below, some of these gains could be associated with policies in consumer credit markets.

First, we examine the welfare impact of eliminating informational frictions, allowing households to distinguish between persistent and transitory income shocks. Part of the gains obtained in this exercise can be associated with policies about financial literacy, in which households learn about the duration of transitions in labor markets. Column 1 of Table 10 reports the welfare gain from fully eliminating informational friction. Compared with the rational learning benchmark, households experience a very significant welfare increase of 0.61%. These gains are slightly higher (0.63%) for households that start with a positive transitory income shock, which is partially misperceived as a permanent shock.

Second, we analyze the welfare effect of eliminating diagnostic expectations. This scenario can be interpreted as the result of policies about disclosure of default risk, in which lenders inform households with diagnostic expectations and help them form a more accurate expectation of future events. Column 2 of Table 10 reports the welfare gain from shifting household expectations toward rational learning. Relative to the economy with diagnostic expectations, households experience an average welfare gain of 0.12%. Again, households that start with positive transitory income shocks have larger gains (0.135), as the misperception of that shock would be higher in the case of diagnostic expectations.

[4]The details of the computation of welfare gains are documented in Appendix C.

Table 10: *Welfare Gains, Consumption Equivalent (%)*

	Model with DQ and BK		Borrowing Constraint		No Borrowing	
	Eliminating Info. Friction	Dropping Diag. Exp.	Eliminating Info. Friction	Dropping Diag. Exp.	Eliminating Info. Friction	Dropping Diag. Exp.
Gains, CE %	0.613	0.120	0.607	0.079	0.538	0.054
$e_{25} \geq 0$	0.629	0.135	0.631	0.118	0.604	0.036
$e_{25} < 0$	0.597	0.106	0.582	0.040	0.471	0.071

We then implement two exercises to further validate our results. Both exercises remove the options for households to choose delinquency or bankruptcy. In the first exercise, we calibrate an external ad-hoc borrowing constraint such that the total assets in the economy remain the same as in the rational learning benchmark. In the second exercise, we do not allow households to borrow at all. Then, we compute the welfare gains of removing information friction and diagnostic expectations in these two economies. Together, these exercises help us isolate how delinquency and bankruptcy amplify the gains of removing information friction and diagnostic expectations.

The welfare impact of allowing households to distinguish between persistent and transitory income shocks is similar across the three setups. In the setup with no borrowing, the gains amount to 88% of those in the model with delinquency and bankruptcy. This indicates that delinquency and bankruptcy are not quantitatively significant in determining these gains. In contrast, the gains from eliminating diagnostic expectations are highly dependent on the presence of delinquency and bankruptcy. In the model with a borrowing constraint that allows for the same borrowing level as in the benchmark with rational learning, the gains are only 66% of the original value. In the model with no borrowing, the gains are reduced to 45% of the original value.

7 Conclusion

These findings suggest that households do not default simply because of low income–they default because they misperceive their income prospects. Misunderstood earnings shocks lead to borrowing driven by misplaced optimism. As beliefs adjust downward in response to negative income shocks, delinquency becomes a frequent response. Households remain in delinquency while negative shocks persist and debt accumulates. Positive shocks can trigger bankruptcy, offering a "fresh start" to escape elevated debt levels. The model demonstrates that even modest errors in perceiving income persistence–

whether through rational learning or diagnostic overreaction–result in heightened financial distress, riskier debt accumulation, and more frequent bankruptcy. If earnings misperception drives financial instability, as these results suggest, then policies that target expectation formation may be more effective in preventing household distress than traditional debt relief interventions.

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A Appendix: Additional Model Details

A.1 Perfect Information economy

In perfect information economy, the value functions at age n are written as

$$G_n(y,z,a) = \max\{V_n(y,z,a), B_n(y,z), D_n(y,z,a)\}$$

where agents fully observe their persistent income component z.

The lifetime utility of bankruptcy then writes as

$$\begin{aligned} B_n(y,z) &= u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint G_{n+1}(z'+e',z',0)\mathrm{d}P(z'|z)\mathrm{d}\Phi(e';0,\sigma_e^2) \\ s.t.\ c &= \exp\left(l(n)+y\right) - c_{BK} \\ c &\geq 0 \end{aligned}$$

where f_b is bankruptcy filing cost and $\Phi(x;\mu,\sigma^2)$ is the cdf of a normal distribution with mean μ and variance σ^2 at x.

The lifetime value of delinquency from age n is

$$\begin{aligned} D_n(y,z,a) = u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint \big[&\gamma G_{n+1}(z'+e',z',0) \\ &+(1-\gamma)G_{n+1}(z'+e',z',a')\big]\,\mathrm{d}P(z'|z)\mathrm{d}\Phi(e';0,\sigma_e^2) \\ a' &= (1+\eta)a \end{aligned}$$

where common notations as in bankruptcy case are omitted here.

The value of repayment writes as

$$\begin{aligned} V_n(y,z,a) = \max_{a',c}\ & u\left(\frac{c}{h(n)}\right) + \varrho_n\beta \iint G_{n+1}(z'+e',z',a')\mathrm{d}P(z'|z)\mathrm{d}\Phi(e';0,\sigma_e^2) \\ s.t.\ & c = a - a'q_n(z,a') + exp(l(n)+y) \end{aligned}$$

Debt Prices To simplify notation, we denote

$$R_n(y,z,a) = \begin{cases} 1 \; if \; V_n = \max\{V_n, D_n, B_n\} \\ 2 \; if \; D_n = \max\{V_n, D_n, B_n\} \\ 0 \; if \; otherwise \end{cases}$$

Therefore, the risk-neutral lender sets up the debt prices as

$$\begin{aligned} q_n(z,a') = \frac{1}{1+r} \iint \Big[& 1_{R_{n+1}(z'+e',z',a')=1} \\ & + 1_{R_{n+1}(z'+e',z',a')=2}(1-\gamma)(1+\eta) q_{n+1}(z', a'') \Big] \, \mathrm{d}P(z'|z) \mathrm{d}\Phi(e'; 0, \sigma_e^2) \\ a'' = (1+\eta)a' \end{aligned}$$

A.2 Value Functions at/pre Retirement

We assume that households' uncertainty about their persistent income component is fully eliminated at $n = n_{ret}$. In addition, we assume that the retired households receives a fraction of their persistent income component realization at retirement age T, z_T, using the replacement formula as

$$f(z) \equiv A_0 \min\{\exp(z + l(T)), \exp(l(T))\} + A_2 \max\{\exp(z + l(T)) - \exp(l(T)), 0\} \quad (2)$$

Therefore, the replacement ratio will be A_0 if agents' income at retirement is lower than the median level, and will decrease gradually to A_2 if it exceeds. In addition, we assume that there is no over-reaction at and after age of retirement, if in the case of diagnostic expectation. Households can still choose default but not delinquency at age T, but will not be able to choose either after retirement.

Since we assume households are to die at the age N, the value function at death will be

$$\begin{aligned} V_N(z,a) &= u\left(\frac{c}{h(n)}\right) \\ s.t. \; c &= a + f(z) \end{aligned}$$

where we use z to refer the observed persistent income realization at retirement.

The value functions for after-retirement households then writes as

$$V_n(z,a) = \max_{a',c} \; u\left(\frac{c}{h(n)}\right) + \varrho_n \beta V_{n+1}(z,a')$$
$$s.t.\ c = a + f(z) - \frac{a'}{1+r}$$
$$a' \geq 0,\ T < n < N$$

where the debt prices is $\frac{1}{1+r}$ since default/delinquency cannot be chosen.

At retirement age T, households are not able to choose delinquency. However, default is still a choice. Therefore, value functions at retirement age writes as

$$G_T(y,z,a) = \max\{V_T(y,z,a), B_T(y,z)\}$$

where value of repayment is similar to value function after retirement age except for income being $exp(l(T)+y)$. The value function for bankruptcy writes as

$$B_T(y,z) = u\left(\frac{c}{h(n)}\right) + \varrho_T \beta V_T(z_T, 0)$$
$$s.t.\ c = exp(l(T)+y) - c_{BK}$$

Although knowing that uncertainty will be eliminated at retirement age, households are still subjective to over-reaction at $n = T-1$, if in the case of diagnostic expectation. Specifically, we have

$$G_n(y,z_{-1},a) = \max\{V_n(y,z_{-1},a), B_n(y,z_{-1}), D_n(y,z_{-1},a)\}$$

where we still use z_{-1} to refer the prior belief carried from last last period. The value functions then write as

$$V_n(y,z_{-1},a) = \max_{a',c} \; u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \iint G_{n+1}(z'+e', z', a') \mathrm{d}\Phi(e'; 0, \sigma_e^2) \mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2)$$
$$s.t.\ c = a - a' q(z,a') + exp(l(n)+y)$$
$$q_n(z,a') = \frac{1}{1+r} \iint \left[1_{R_{n+1}(z'+e',z',a')=1}\right] \mathrm{d}\Phi(e'; 0, \sigma_e^2) \mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2)$$

$$B_n(y,z_{-1}) = u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \iint G_{n+1}(z'+e', z', 0) \mathrm{d}\Phi(e'; 0, \sigma_e^2) \mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2)$$
$$s.t.\ c = exp(l(n)+y) - c_{BK},$$

$$
\begin{aligned}
D_n(y, z_{-1}, a) = u\left(\frac{c}{h(n)}\right) + \varrho_n \beta \iint \big[\gamma G_{n+1}(z' + e', z', 0) \\
+ (1-\gamma) G_{n+1}(z' + e', z', a')\big] \, \mathrm{d}\Phi(e'; 0, \sigma_e^2) \mathrm{d}\Phi(z'; z, \Sigma_{n|n} + \sigma_\epsilon^2) \\
s.t. a' = (1+\eta)a, \; n = T-1
\end{aligned}
$$

where z again refers to the posterior after updating.

B Definition of Moments

This appendix provides a description of definition of the moments that we used for estimation.

B.1 Unconditional Fraction of Individuals in DQ and BK

Define the unconditional probability of DQ share at age t is defined as follows

$$
\text{Prob}(DQ|t) \equiv \frac{\sum_i^{N_t} \mathbf{1}_{DQ_{i,t}}}{N_t}
$$

Accordingly, the moments we used for measuring financial distress is computed as follows

$$
\text{\% of Financial Distress, age T to T+9} = \frac{\sum_{t=T}^{T+9} N_t \text{Prob}(DQ|t)}{\sum_{t=T}^{T+9} N_t}, \; T = 25, 35, 45
$$

where N_t is the number of individuals at age t.

Similar, we could define the moments for BK as

$$
\text{\% of Bankruptcy, age T to T+9} = \frac{\sum_{t=T}^{T+9} N_t \text{Prob}(BK|t)}{\sum_{t=T}^{T+9} N_t}, \; T = 25, 35, 45
$$

where $\text{Prob}(BK|t)$ is defined similarly as that for DQ.

B.2 Unconditional Mean of Debt to Income Ratio

We define the debt of individual i at age t as follows

$$debt_{i,t} \equiv \begin{cases} 0, & \text{if no credit card debt} \\ \text{amount of credit card debt}, & \text{if has credit card debt} \end{cases}$$

The unconditional mean of debt to income ratio, by age bins, is defined as follows

$$\text{mean(debt)/mean(income), age T to T+9} \equiv \frac{\frac{\sum_{t=T}^{T+9}\sum_i |debt_{i,j}|}{\sum_t N_t}}{\frac{\sum_{t=T}^{T+9}\sum_i income_{i,j}}{\sum_{t=T}^{T+9} N_t}},\ T = 25, 35, 45$$

B.3 Unconditional Fraction of Individuals in Debt

We define individual i at age t as being in debt if this individual still has positive amount of credit card debt after most recent repayment. Therefore, we define share of people in debt, both in the data and model, as

$$\text{\% in Debt, age T to T+9} = \frac{\sum_{t=T}^{T+9}\sum_i^{N_t} \mathbf{1}_{|debt_{i,t}|>0}}{\sum_{t=T}^{T+9} N_t},\ T = 25, 35, 45$$

B.4 CV_{debt}

We simply define CV_{debt} both in the model and data as

$$CV_{debt}\text{, age T to T+9} \equiv \frac{std(debt_{i,t}|T \leq t \leq T+9, |debt_{i,t}| > 0)}{mean(|debt_{i,t}||T \leq t \leq T+9, |debt_{i,t}| > 0)},\ T = 25, 35, 45$$

where $std(\cdot)$ and $mean(\cdot)$ refers to sample standard deviation and sample mean respectively. Notice that this measure is conditional on having active debt.

C Welfare Gains Computation

Welfare in the economy is defined as the expected discounted sum of utilities derived from consumption streams. The initial asset distribution is discretized with probabilities $\Pr_0(a_i)$. At each initial state (a_i), 10,000 sequences of income shocks $\{z_n, e_n\}_{n=n_0}^{T}$ are

simulated. These sequences determine consumption paths $\{c_n^B\}_{n=n_0}^T$ in the benchmark economy (B). Ex-ante welfare in economy B is thus calculated as:

$$EV^B \equiv \sum_i \frac{\Pr(a_i)}{10,000} \sum_{j=1}^{10,000} \sum_{n=n_0}^{T} \beta^n \varrho_n \frac{(c_n^B)^{1-\sigma}}{1-\sigma}.$$

Analogously, ex-ante welfare in an alternative economy (A) is computed as:

$$EV^A \equiv \sum_i \frac{\Pr(a_i)}{10,000} \sum_{j=1}^{10,000} \sum_{n=n_0}^{T} \beta^n \varrho_n \frac{(c_n^A)^{1-\sigma}}{1-\sigma}.$$

Welfare gain from economy A to economy B is defined as the percentage increase in consumption, Δ, required to equalize welfare across both economies. Thus, Δ satisfies:

$$EV^A(1+\Delta)^{1-\sigma} = EV^B.$$

Given the preference parameter $\sigma = 2$, this simplifies directly to:

$$\Delta = \frac{EV^A}{EV^B} - 1.$$