Leverage and Exchange Rate Cycles

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Abstract

I develop a dynamic general equilibrium model that unifies the analysis of leverage cycles and exchange rate dynamics in emerging markets, treating the corporate sector as de facto financial intermediaries. Firms' endogenous, time-varying risk-bearing capacity—shaped by their credit risk and expected exchange rate paths—plays a central role in determining currency market outcomes. A novel pecuniary externality arises: corporate foreign-currency borrowing drives aggregate leverage, which affects exchange rates under financial frictions and, in turn, feeds back into firms' cross-state financing terms. This externality, coupled with firms' inability to commit to future debt policies, gives rise to endogenous sudden stops. The model illuminates a fundamental tension between ex-post economic resilience and ex-ante moral hazard, demonstrating how debt maturity structures, convenience yields, and exchange rate volatility can influence financial stability through their impact on firms' state-contingent risk-bearing capacity. Consistent with key empirical patterns, the framework explains persistently positive currency risk premia and their negative correlation with capital inflows. By linking currency risk premia asymmetrically to crisis-state volatility, it offers a micro-founded alternative to existing rare-disaster models.

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> La pression exercée en tout point d'un fluide en équilibre se transmet intégralement dans toutes les directions.
>
> — *Le principe de Pascal*

Over the past half-century, financial globalization has transformed emerging market economies (EMEs), allowing unprecedented access to international capital. However, this path remains fraught with peril, as many lessons have been learned through the crucible of severe financial crises. From the Latin American debt crisis of the 1980s to the Mexican Tequila Crisis in 1994 and the Asian Financial Crisis in 1997, a common thread emerges: unsustainable credit booms fueled by foreign borrowing culminate in sudden stops and sharp currency depreciations, triggering widespread financial instability (Corsetti, Pesenti and Roubini 1999). These experiences underscore a crucial insight: to understand an economy's vulnerability to debt crises, we must analyze leverage cycles and exchange rate dynamics jointly, not in isolation.

This paper addresses several critical questions within a unified analytical framework. First, what drives the cyclical pattern of leverage—why do economies repeatedly transition from periods of low leverage and rising debt accumulation into episodes of high leverage followed by abrupt deleveraging? Adopting a portfolio-balancing perspective, I demonstrate how the currency market liquidity and time-varying risk-bearing capacity of the corporate sector provides a central mechanism explaining these leverage cycles, in particular when the slow moving crisis happens.

Second, the recent literature on the global financial cycle (Miranda-Agrippino and Rey 2021) highlights a robust correlation between international capital flows and exchange rate dynamics. Yet the theoretical foundations behind this empirical regularity remain incomplete. This paper develops a general equilibrium model in which firms strategically engage in external borrowing, continuously extracting endogenous gains from trade by exploiting deviations from uncovered interest parity (UIP) and doing risk shifting. Importantly, debt servicing obligations shape the path of international imbalances, influencing exchange rates in a feedback loop. In this framework, firms' endogenous risk-bearing capacity is explicitly forward-looking, coming from the price impact of debt issuance, and influenced by market expectations of future exchange rate movements, offering novel theoretical insights into several persistent puzzles.

Third, understanding the underlying mechanisms behind sudden stops—episodes of sharp reversals in capital flows—remains essential, particularly for EMEs. Departing from traditional models emphasizing behavioral biases or financial constraints, this paper highlights how the properties of debt contract and pecuniary externalities generate sudden-stop dynamics. Specifi-

cally, by embedding firms' strategic external financing decisions in a general equilibrium setting, I illustrate how seemingly stable leverage dynamics can abruptly unravel, exposing systemic vulnerabilities.

The theoretical model builds upon and extends recent dynamic corporate finance literature—particularly the influential contributions of DeMarzo and He (2021) (hereafter DH) and DeMarzo, He and Tourre (2023) (hereafter DHT)—which analyze firms' leverage dynamics in the absence of commitment.[1] A key innovation of my model is its adaptation to the characteristic financial structures of EMEs, achieved through two fundamental modifications.

First, I explicitly incorporate balance-sheet currency mismatches—often called the "original sin" of EMEs (Eichengreen and Hausmann 1999)—assuming firms issue foreign-currency (FC) debt while their equity values are denominated in local currency (LC). Second, I model a continuum of firms whose individual leverage decisions collectively shape aggregate currency risk premiums. This setup generates a novel pecuniary externality: each firm's borrowing and repayment decisions affect aggregate leverage, influencing exchange rates and thereby feeding back into other firms' financing conditions. Thus, an aggregation problem presents.

In equilibrium, risk-neutral firms strategically issue long-term FC debt to foreign creditors who face lower risk-free interest rates. Firms' incentives to borrow externally arise from two main channels: carry trade profits from exploiting UIP deviations, and risk-shifting opportunities arising from limited liability and currency mismatches. Crucially, the currency risk premium emerges endogenously—even in a risk-neutral environment—because firms internalize the adverse impact of their leverage on debt prices and financing terms. This endogenous channel fundamentally differs from DH and DHT, where gains from borrowing are exogenous, derived either from tax shields (DH) or creditor-borrower discount rate differentials (DHT).[2]

From a portfolio-balancing perspective, the model demonstrates that firms play a market-balancing role analogous to financial intermediaries with limited risk-bearing capacity, a view closely related to the approaches in Gabaix and Maggiori (2015) (GM) and Itskhoki and Mukhin (2021) (IM). However, my analysis diverges from theirs in two critical respects. First, whereas their frameworks rely on short-term debt instruments, my model explicitly incorporates long-term debt with exponential maturity, creating persistent, infinite-horizon currency market imbalances due to debt repayment flows. Second, the corporate sector's risk-bearing capacity in my setting is endogenous and forward-looking, influenced not only by current credit risk but also by market expectations of future exchange rate dynamics and associated debt servicing costs.[3]

[1] In emerging markets, weak regulatory or judicial institutions often make enforceable commitments to future borrowing paths unrealistic, showcasing no-commitment problems.

[2] In these earlier models, lack of commitment leads firms toward aggressive debt accumulation until marginal issuance benefits equal marginal equity dilution costs—reminiscent of the Coase conjecture. Unlike these settings, where equilibrium adjustment occurs primarily through debt prices, my model endogenizes the gains from trade themselves.

[3] Although firms are risk-neutral, their "effective" risk aversion arises because their financing terms (debt prices)

This mechanism clearly links leverage cycles to exchange rate dynamics. Firms' strategic borrowing decisions redistribute currency market imbalances across time, influencing the trajectory of exchange rates through endogenous variations in currency risk premia.

To understand why international capital flows themselves display cyclical behavior and their negative correlation with currency risk premiums (Kalemli-Özcan and Varela 2024, hereafter KV), I further extend the model to include two distinct sectors. While the benchmark model has only the firm sector balancing currency market imbalances from trade and financial repayments, the extended model introduces local investors as a second sector. Relative differences in risk-bearing capacities between sectors determine how market imbalances are distributed. When firms have high risk-bearing capacity, local investors "free-ride" by accumulating FC assets, leading to robust capital flows, both in and out. As firms' risk-bearing capacity deteriorates, investors provide necessary liquidity to firms, causing capital inflows and outflows to contract simultaneously.

The equilibrium dynamics in the currency market can be intuitively understood through an analogy with Pascal's Law.[4] In this analogy, deterioration in firms' balance sheets acts like a localized "pressure point." The market's invisible hand responds by equalizing firms' debt rollover costs and external intermediation margins, redistributing this pressure across the entire economic system. Just as fluid moves unevenly to different areas until uniform pressure is established, market equilibrium involves cross-sectoral redistribution of imbalances: when the corporate sector's risk-bearing capacity weakens, local investors change their asset portfolio weights to provide additional liquidity. This process systematically shifts how imbalances are absorbed by different sectors. Thus, this portfolio-balancing mechanism highlights how leverage decisions and financing terms naturally emerge from the interaction between aggregate imbalances and the cross-sectional distribution of risk-bearing capacities.

Leveraging continuous-time methods, the model delivers a fully global solution to the economy's dynamic equilibrium, reducing its characterization to ordinary differential equations (ODE). This analytical tractability enables precise investigation of the mechanisms underlying sudden stops.

In particular, the framework reproduces the sudden-stop phenomenon by illuminating how debt prices and capital inflows can exhibit highly nonlinear reactions to economic shocks. When overall leverage is moderate, small perturbations to earnings or exchange rates barely affect market conditions. But once the economy drifts into a near-default crisis state—characterized by elevated leverage—relatively modest shocks can trigger pronounced declines in debt prices and capital inflows, effectively constituting a sudden stop.

This sudden-stop dynamic emerges endogenously through pecuniary externalities that dis-

depend on market perceptions of default risk, linking current decisions to future conditions. It is the role of price impact that creates risk premium.

[4]Pascal's Law states that pressure applied to an enclosed fluid is transmitted uniformly throughout the fluid.

courage firms from retiring debt in a timely manner. As aggregate leverage rises, attempts to deleverage provoke significant demand for FC, pushing up the LC risk premium. A higher LC risk premium creates profitable carry-trade opportunities, dulling firms' motivation to reduce debt. At the same time, the elevated risk premium hints at future currency stabilization or appreciation, lowering expected future effective debt burdens. These forces contain perceived default risk and keep debt prices from tumbling until leverage nears a critical boundary, where abrupt price drops ensue.

The model thus highlights a central tension in financial stability analysis: aggressive deleveraging in a near-default crisis boosts ex-post resilience and firms' current risk-bearing capacity, yet paradoxically encourages firms to delay initial debt reductions—an echo of moral hazard. Several factors shape this balancing act, including debt maturity structures and market liquidity conditions, and no single configuration provides a one-size-fits-all solution. Short-term debt maturities impose rapid deleveraging during crises and uphold debt prices but amplify the likelihood of sudden stops and defaults. Longer maturities, by contrast, smooth out deleveraging pressures over time but naturally harder to be reduced frequently, complicating recovery. While ample pre-crisis liquidity—in the form of lower convenience yields or subdued exchange rate volatility—fosters resilience by enabling debt repayment, plentiful liquidity during crises can erode ex-ante risk management discipline and encourage risk-shifting, ultimately undermining financial stability despite its immediate benefits. Somewhat counterintuitively, higher volatility during crises can promote beneficial ex-ante deleveraging by elevating conditional default probabilities, thereby delaying the onset of a credit bust.

Finally, this model contributes a novel, endogenous approach to rare-disaster modeling. Standard rare-disaster frameworks introduce disaster events exogenously to explain elevated risk premia (Rietz 1988, Barro 2006, Gabaix 2012, Farhi and Gabaix 2015). By contrast, this setting ties catastrophic outcomes directly to the economy wide default event. Since debt values largely hinge on extreme downside scenarios, a firm's effective risk-bearing capacity becomes disproportionately sensitive to near default period volatility rather than fluctuations in normal times. Consequently, the currency risk premium also reflects near default crisis-period volatility, offering a potential resolution to the puzzle raised by Lustig and Verdelhan (2019) regarding a not high enough current-period volatility but observed high risk premiums in the currency market.

Related Literature This paper connects several distinct strands of literature.

At the firm level, the paper is closely related to dynamic capital structure models, particularly recent developments in leverage dynamics without commitment. For the individual firm's problem, I directly adopt the equilibrium solving methodology developed in DH and DHT.

Another strand of literature focuses on leverage dynamics in a macroeconomic setting, particularly in explaining the credit spread puzzle. Chen (2010) analyzes how business cycle risks

shape firms' leverage decisions, while Chen, Xu and Yang (2021) investigates maturity structure over the business cycle.

My paper builds on the literature examining exchange rate dynamics through the lens of imperfect financial markets, following GM and IM. The key innovation lies in incorporating an infinite-horizon sector with endogenous time-varying risk-bearing capacity. In addition, the multiple-sector model highlights how cross-sectional differences in risk-bearing capacity can lead to capital flows that equate margins and produce a multi-layered currency risk premium structure. Aside from the required premium for imbalances-bearing, the wedge also appears in the currency risk premium—notably, the firm sector's wedge arising from risk-shifting incentives. This finding connects to a growing literature on the role of wedges in international risk sharing and exchange rate determination, particularly studies examining convenience yields (Jiang, Krishnamurthy and Lustig 2021, Jiang, Krishnamurthy, Lustig and Sun 2024).

This paper also contributes to studies on financial frictions in macroeconomics. The analysis of financial stability heavily relies on financial constraints. The seminal papers by Kiyotaki and Moore (1997) and Bernanke and Gertler (1989) show that financial friction can amplify economic shocks through the net worth channel. He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014) study financial constraints with a global solution and emphasize the nonlinearity of shock responses. Another strand of leverage cycle models emphasizes the marginal buyer's valuation (Geanakoplos 1996, 2003, 2010). Leverage starts when the collateral constraint is loose and busts when asset valuations drop and lending withdraws. In an international context, Mendoza (2010) incorporates a collateral constraint into a business cycle model, generating an amplification effect that explains sudden stops. My model, however, abstains from imposing any financial constraints; instead, it utilizes a Leland-type framework with pecuniary externality to shed light on financial stability. While Brunnermeier and Sannikov (2015) also use pecuniary externalities to explain sudden stops, their mechanism differs: firms' failure to internalize production decisions' price impact leads to overproduction, with sudden stop fears and illiquid capital fire sales amplifying market illiquidity.

The paper proceeds as follows: Section 1 introduces the benchmark one-sector model. Section 2 incorporates the "intermediation" function of debt issuance in a two-sector model. Section 3 examines financial stability. Section 4 extends the analysis to stochastic volatility, and Section 5 concludes.

I. Benchmark: a Model with Leveraging Firms

This section introduces a baseline model that isolates the role of the firm sector in determining leverage and exchange rate dynamics (Figure 1). The firm sector accesses international credit

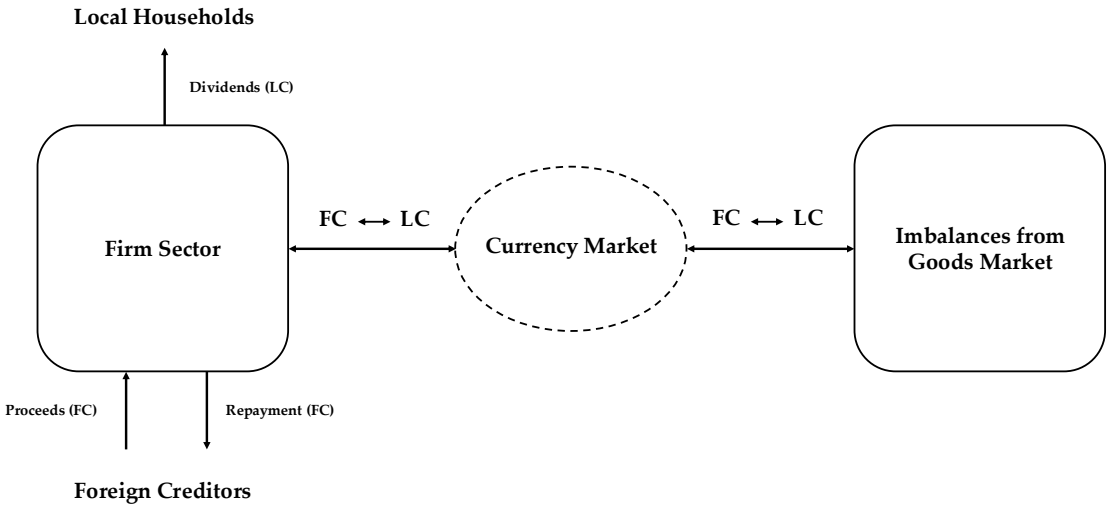


FIGURE 1. Benchmark Model

markets by issuing long-term defaultable bonds denominated in FC to foreign creditors, who benefit from lower FC risk-free rates.[5] Firms' participation in the currency market directly results from their financing activities: they convert FC debt proceeds into LC for dividend payments and subsequently exchange LC back into FC to make coupon and principal payments. For simplicity, firms are assumed neither to hold FC risk-free assets nor to maintain FC reserves.

To clearly highlight the leverage decisions of firms and ensure analytical tractability, I deliberately adopt the simplifications introduced by GM. Specifically, these simplifications neutralize the influence of household marginal utility of consumption on exchange rate dynamics. This approach allows a sharper focus on firms' leverage behavior while preserving the key features of a general equilibrium framework.

I.1. Model Setup

Consider a small open economy.

Households Households derive their utility from consuming a bundle consisting of tradable and nontradable goods:

$$C_t = \left(C_{NT,t}^{\chi_t} C_{T,t}^{\iota_t}\right)^{\frac{1}{\theta_t}}$$

[5]Throughout the paper, "bond" and "debt" are used interchangeably.

where $\theta_t = \chi_t + \iota_t$. Here, $C_{NT,t}$ represents consumption of nontradable goods, and $C_{T,t}$ denotes consumption of tradable goods. For simplicity and clarity, I choose nontradable goods as the local numeraire, giving them a price of one—let's call this LC also. Analogously, assume "foreign households" price tradable goods at unity in FC. Thus, the exchange rate, s_t, is simply the price of tradable goods expressed in LC terms.

Each period, local households receive endowments: Y_t units of nontradable goods and $Y_{T,t}$ units of tradable goods. Households face a restricted investment space—they can hold only LC-denominated, risk-free bonds paying an interest rate r_t^{LC}. Their optimization problem then becomes:

$$\begin{aligned} \max_{C_{NT,t}, C_{T,t}} \quad & \mathbb{E}\left[\int_0^\infty e^{-\rho t} \ln C_t \, dt\right] \\ \text{s.t.} \quad & dW_t^h = \left(r_t^{LC} W_t^h + Y_t + s_t Y_{T,t} - C_{NT,t} - s_t C_{T,t} + \text{Div}_t\right) dt, \end{aligned}$$

where Div_t denotes dividends paid in LC by domestic firms—I'll discuss this in detail shortly.

Intratemporal utility maximization implies simple first-order conditions (with multiplier ν_t):

$$\frac{\chi_t}{C_{NT,t}} = \nu_t, \quad \frac{\iota_t}{C_{F,t}} = \nu_t s_t$$

To simplify our story without losing the essence, I set $\chi_t = 2Y_t$. Given market clearing—which I'll verify later—we obtain $C_{NT,t} = 2Y_t$ and thus $\nu_t = 1$. Consequently, tradable goods consumption measured in LC is exactly ι_t. I further simplify by assuming $\iota_t = s_t Y_{T,t} - \overline{\mu} Y_t$ for some constant $\overline{\mu}$, leading to net exports proportional to Y_t: $NX_t = s_t(Y_{T,t} - C_{T,t}) = \overline{\mu} Y_t$. This assumption conveniently maintains the homogeneity of equilibrium in Y_t, helping us neatly pin down firms' leverage decisions in a Leland-type setting.

Households' Euler equation ensures that discounted LC bond returns, given by $M_t R_t^{LC}$ (with state-price density $M_t = e^{-\rho t} \frac{\chi_t}{C_{NT,t}}$ and $\frac{dR_t^{LC}}{R_t^{LC}} = r_t^{LC} dt$), form a local martingale. This immediately pins down the equilibrium risk-free interest rate at the subjective discount rate: $r_t^{LC} = \rho$.

Lastly, I assume the aggregate state Y_t evolves according to a geometric Brownian motion (GBM):

$$\frac{dY_t}{Y_t} = \mu \, dt + \boldsymbol{\sigma} \cdot d\boldsymbol{B}_t,$$

where $\{\boldsymbol{B}_t\}_{t \geq 0}$ is an n-dimensional Brownian motion on a standard probability space $(\Omega, \mathcal{F}, \mathbb{P})$, capturing both local and global aggregate risks. Its variance-covariance matrix is denoted $\boldsymbol{\Sigma}$. I adopt the standard regularity assumption:

ASSUMPTION 1. *Parameters* $(\rho, \mu, \boldsymbol{\sigma})$ *satisfy*

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} Y_t \, dt \mid Y_0 = Y\right] < \infty.$$

Firms The economy is populated by a continuum of identical firms, indexed by $j \in [0, 1]$.[6] For simplicity, I abstract from idiosyncratic risks and firm heterogeneity, assuming each firm identically receives an instantaneous endowment of nontradable goods Y_t.

The central mechanism of this model revolves around the firms' leverage decisions. Firms issue external debt to foreign creditors without committing to future issuance paths.

Firm's Debt Structure Firms leverage themselves by issuing FC denominated bonds exclusively to foreign creditors. [7] Let $\tilde{F}_{j,t}$ represent the total face value of each firm's outstanding FC bonds. Regardless of the issuance time, bonds carry a constant coupon rate $c > 0$ and exhibit exponential maturity, with a constant amortization rate $\xi > 0$. Thus, the expected debt maturity equals $1/\xi$.

Between $[t, t + dt]$, firms must service FC coupon payments $c\tilde{F}_{j,t}\, dt$ and principal repayments $\xi\tilde{F}_{j,t}\, dt$ to avoid default. Firms' default policies follow a Leland-type framework and are fully endogenous. A zero recovery rate is assumed upon default. Because debt contracts are homogeneous across firms (identical c, ξ), identical firms' leverage and default decisions become perfectly correlated in equilibrium, resulting in economy-wide credit bust given default. Given symmetry, the firm index j is dropped in subsequent notation.

Each firm's outstanding debt evolves according to their cumulative debt issuance process, $\{\tilde{\Gamma}_t\}_t$:

$$d\tilde{F}_t = -\xi\tilde{F}_t \, dt + d\tilde{\Gamma}_t.$$

For smooth issuance without jumps, we have $d\tilde{\Gamma}_t = \tilde{G}_t dt$, with $\tilde{G}_t$ as the instantaneous rate of debt issuance. Smooth issuance equilibrium arises naturally when the equity value function is strictly convex in individual debt outstanding, which I will show later.

Firm Managers Firm managers make debt issuance and default decisions. Managers exchange FC net debt proceeds into LC, combine them with nontradable endowments, and distribute divi-

[6] This atomistic setup serves two key purposes: it ensures firms remain price-takers in the exchange rate market while granting them monopoly power over their own debt issuance, consistent with realistic market practices and generating pecuniary externalities.

[7] While currency denomination choices are beyond the scope of this paper, empirical evidence strongly supports this setup: between 2003 and 2017, the local-currency share of external corporate debt increased modestly, from just 8.5% to 13% (Du and Schreger 2022). Domestic local-currency debt issuance would introduce only second-order effects on international balance-of-payments dynamics. Additionally, debt-related tax shield benefits associated with local-currency issuance, as noted by DH, are not considered here.

dends to households. Over interval $[t, t+dt]$, dividends paid are:

$$d\tilde{C}_t = \left[Y_t - (c+\xi)\tilde{F}_t s_t\right] dt + \tilde{p}_t s_t \, d\tilde{\Gamma}_t,$$

where $\tilde{p}_t$ is the equilibrium FC price of newly issued debt, and s_t is the exchange rate, both determined endogenously.

Managers are assumed risk-neutral and motivated solely by maximizing dividend payouts, reflecting their reputation or incentive structures. Let τ_b denote the endogenous default time. The firm's managerial objective is:

$$V_t = \sup_{\tilde{\Gamma}_t, \tau_b} \mathbb{E}_t \left[\int_t^{\tau_b} e^{-\rho(\tau - t)} \left(\left[Y_\tau - (c+\xi)\tilde{F}_\tau s_\tau\right] d\tau + \tilde{p}_\tau s_\tau \, d\tilde{\Gamma}_\tau \right) \right]. \tag{1}$$

The firm's main currency risk exposure arises from the mismatch between FC debt obligations and LC equity dividends: LC depreciation increases leverage burdens, whereas appreciation eases them. This exchange-rate risk is central to firms' ability to service their debts and the equilibrium leverage dynamics.

Foreign Creditors Foreign creditors, operating in FC with a constant risk-free rate $r < \rho$ (recall ρ is the LC risk-free rate), hold firms' debt. The interest rate differential provides fundamental incentives for firms' debt issuance, analogous to carry trade strategies, though similarly exposed to exchange-rate fluctuations.

It is assumed that these creditors are risk-neutral and discount future payments at the FC risk-free rate. The FC-denominated debt price per unit face value is thus:

$$\tilde{p}_t = \mathbb{E}_t \left[\int_t^{\tau_b} e^{-(r+\xi)(\tau - t)} (c+\xi) d\tau \right], \tag{2}$$

representing the expected present value of future coupon and principal payments up until either the firm defaults or the debt matures.

In a symmetric equilibrium, where all firms are identical, corporate debts are priced uniformly across firms, resulting in a common debt price. However, deviations in an individual firm's leverage choices affect its default probability and thus its debt price, granting firms monopoly power over their own debt issuance. Importantly, while each firm internalizes the impact of issuance decisions on its debt pricing, the atomistic structure ensures firms remain price-takers in exchange-rate markets, where their individual actions are negligible.

To differentiate economy-wide quantities from individual variables, I denote aggregate variables without tildes. Aggregating across firms $j \in [0,1]$ defines aggregate debt outstanding $F_t = \int_j \tilde{F}_{j,t} \, dj = \tilde{F}_t$, cumulative issuance $\Gamma_t = \int_j \tilde{\Gamma}_{j,t} \, dj = \tilde{\Gamma}_t$, and instantaneous aggregate issuance

$G_t = \int_j \tilde{G}_{j,t}\, dj = \tilde{G}_t$ under continuous issuance. Aggregate external debt evolves according to $dF_t = -\xi F_t\, dt + d\Gamma_t$. In a symmetric equilibrium, all debts are identical, including their pricing. Consequently, I denote the common debt price as $p_t = \tilde{p}_t$. The capital inflow—new bond issuance minus repayment—can then be expressed as:

$$dL_t = p_t d\Gamma_t - (c + \xi)F_t dt \tag{3}$$

$$= p_t G_t dt - (c + \xi)F_t dt \tag{4}$$

The second equality holds in a smooth issuance equilibrium.

The flow-based currency market clearing condition, also known as the Balance of Payments (BoP) condition, is given by:

$$s_t\, dL_t + Y_t \cdot \overline{\mu}\, dt = 0. \tag{5}$$

This equation pins down the exchange rate dynamics within the model.

Incidentally, this condition implies a straightforward expression for dividend payments:

$$\text{Div}_t = (1 - \overline{\mu})Y_t.$$

Consequently, households' consumption of nontradable goods becomes $((1+\overline{\mu})+(1-\overline{\mu}))Y_t = 2Y_t$.

Though stark, this baseline model structure is sufficient to illustrate key insights clearly, setting the stage for more detailed analysis and additional complexities introduced in subsequent sections.

I.2. Equilibrium Characterization

A key methodological contribution of this paper is solving a leverage dynamics model in a macroeconomic setting. The equilibrium characterization follows a three-step procedure:

Step 1: Characterize individual firms' optimal issuance policy as a function of relevant state variables, addressing the firm-level optimization problem.

Step 2: Aggregate individual issuance decisions while simultaneously determining the exchange rate process and equilibrium aggregate capital inflows.

Step 3: Finally, the optimal issuance policies and exchange rate dynamics derived in the earlier steps are linked to the individual firm manager's value function. The last step involves solving this function using differential equations. Once the manager's value function is solved, the equilibrium characterization is complete.

I focus on Markov Perfect Equilibria (MPE), where the payoff-relevant states involve exoge-

nous earnings (Y_t), the individual firm's debt level $\left(\tilde{F}_t\right)$, the aggregate economy's debt level (F_t), and the exchange rate (s_t). Following standard Leland-type modeling approaches, I scale the system by earnings and adjust for exchange rates to ensure consistent denomination: Y_t/s_t serves as the FC-denominated scaling factor. In this system, the exchange rate level appears only as a scaling factor, making the focus on its changes rather than its level. The state space reduces to $\left(\tilde{f}_t, f_t\right) = \left(\frac{s\tilde{F}_t}{Y_t}, \frac{sF_t}{Y_t}\right)$, representing individual and aggregate debt-to-earnings ratios. I will later verify that these two state variables adequately capture the model's dynamics.

The default strategy is then defined within the state space $\{(\tilde{f}_t, f_t) : 0 \leq \tilde{f}_t, f_t \leq +\infty\}$. An admissible default policy is determined by a stopping time τ_b with respect to the filtration $\mathcal{F}_t$. Let $\mathcal{O}$ denote the endogenous default region. The optimal stopping time τ_b is given by:

$$\tau_b = \inf\left\{t \geq 0 : (\tilde{f}_t, f_t) \in \mathcal{O}\right\}.$$

The model's homogeneity properties yield:

$$V(\tilde{F}_t, F_t, Y_t, s_t) = v(\tilde{f}_t, f_t) \cdot Y_t, \qquad p(\tilde{F}_t, F_t, Y_t, s_t) = p(\tilde{f}_t, f_t),$$

where $v(\tilde{f}_t, f_t)$ represents the earnings-scaled manager's value. [8]

I first analyze some general properties of the equilibrium manager's value and debt prices. Many of these results are drawn from DH and serve as the foundation for subsequent steps.

Lemma 1. *In any MPE, the manager's value function $V\left(\tilde{F}_t, F_t, Y_t, s_t\right)$ is convex and decreasing in $\tilde{F}_t$, with the negative debt price in LC acting as a subgradient and also decreasing in $\tilde{F}_t$:*

$$s_t \cdot p(\tilde{F}_t, F_t, Y_t, s_t) \in -\partial_{\tilde{F}} V(\tilde{F}_t, F_t, Y_t, s_t).$$

If $V\left(\tilde{F}_t, F_t, Y_t, s_t\right)$ is strictly convex in $\tilde{F}_t$, and therefore $v\left(\tilde{f}_t, f_t\right)$ is also strictly convex in $\tilde{f}_t$, then the debt price p is strictly decreasing in $\tilde{F}_t$ (or equivalently, v is strictly decreasing in $\tilde{f}_t$), and the optimal cumulative issuance path $\tilde{\Gamma}_t$ is continuous in t.

The micro-macro separation—where individual deviating firms cannot influence aggregate outcomes—preserves DH's core insight: only equilibrium debt prices rather than exchange rates discipline firms by supporting manager's value and deterring excessive issuance. Manager value's strict convexity necessitates smooth issuance, as lumpy adjustments would be suboptimal.

Following Lemma 1, I focus on characterizing the smooth MPE, where all firms follow a continuous cumulative issuance path. The analysis proceeds by conjecturing such equilibrium's existence and characterizing scaled manager's value, then verifying strict convexity in $\tilde{F}_t$ (or $\tilde{f}_t$).

[8] With slightly abuse of notation, I still use p for the debt price function.

I postulate an Ito process for the exchange rate evolution:

$$\frac{ds_t}{s_t} = \mu_s dt + \boldsymbol{\sigma_s} \cdot d\boldsymbol{B_t}, \tag{6}$$

where μ_s and $\boldsymbol{\sigma_s}$ represent the endogenously determined drift and volatility vector, respectively.

In a smooth issuance equilibrium, the manager's value satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}\rho V(\tilde{F}_t, F_t, Y_t, s_t) = \sup_{\tilde{G}_t} \Big\{ & Y_t - (c+\xi)\tilde{F}_t s_t + p_t \tilde{G}_t s_t + (\tilde{G}_t - \xi \tilde{F}_t) V_{\tilde{F}} + (G_t - \xi F_t) V_F \\ & + s_t \mu_s V_s + \mu Y_t V_Y + \frac{1}{2}\|\boldsymbol{\sigma}_s\|^2 s_t^2 V_{ss} + \frac{1}{2}\|\boldsymbol{\sigma}\|^2 Y^2 V_{YY} + \boldsymbol{\sigma}' \boldsymbol{\Sigma} \boldsymbol{\sigma_s} s_t Y_t V_{sY} \Big\},\end{aligned} \tag{7}$$

Let $V^0(\tilde{F}_t, F_t, Y_t, s_t)$ denote the maximum attainable manager's value when the firm refrains from future debt issuance but still cannot commit to this strategy. Similarly, let $v^0(\tilde{f}_t, f_t)$ represent the earnings-scaled manager's value of such a firm. We have the following result.

Lemma 2. *In any smooth MPE, debt price and manager's value should satisfy*

$$s_t \cdot P(\tilde{F}_t, F_t, Y_t, s_t) = -V_{\tilde{F}}(\tilde{F}_t, F_t, Y_t, s_t), \quad \textit{or equivalently } p(\tilde{f}_t, f_t) = -v_{\tilde{f}}(\tilde{f}_t, f_t). \tag{8}$$

And the value of the (earnings-scaled) manager's value equals the no-future-issuance (earnings-scaled) manager's value:

$$V(\tilde{F}_t, F_t, Y_t, s_t) = V^0(\tilde{F}_t, F_t, Y_t, s_t), \quad \textit{or equivalently } v(\tilde{f}_t, f_t) = v^0(\tilde{f}_t, f_t).$$

The intuition behind this result lies in the fact that, despite the presence of multiple firms in the economy, each firm acts as a monopolist to its own debt. Following the Coase conjecture logic, firms issue debt aggressively until reaching a point where the marginal cost of increased default probability precisely offsets the marginal benefit of debt proceeds. This dynamic eliminates any potential net benefit from debt issuance in the absence of commitment power.

While each firm manager's valuation can be analyzed as though the firm abstains from future debt issuance, the aggregate debt dynamics continue evolving along the equilibrium issuance path, ensuring the presence of G_t in the individual firm's HJB equations. This aspect differentiates this model from the setup considered in DH. Specifically, each firm's issuance decision generates a "pecuniary externality" on the broader economy: individual decisions collectively shape aggregate capital inflows, influencing exchange rate dynamics and consequently affecting all firms' valuations. Importantly, individual firms do not internalize this externality, and it remains unaddressed by adjustments in debt pricing. Thus, while the Coase conjecture neutralizes

direct issuance benefits, it leaves the externality channel intact. Due to this complexity, individual managerial values cannot simply be analyzed as if all other firms also refrained from issuing debt. And we need to solve a PDE in the end.

Step 1: Individual Firms Policy

To further characterize firms' optimal issuance policy, we need to determine the level at which firms have no further incentive to issue more external debt, occurring when $s_t \cdot p(\tilde{F}_t, F_t, Y_t, s_t) = -V_{\tilde{F}}(\tilde{F}_t, F_t, Y_t, s_t)$.

LEMMA 3. *In a smooth MPE, a firm's equilibrium issuance policy, i.e., the optimal issuance rate, is given by:*

$$\frac{\tilde{G}^*}{\tilde{F}} = \frac{\rho - r - \mu_s + \varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}}}{\varepsilon_{p\tilde{F}}}, \tag{9}$$

where the elasticities are defined as follows: the debt price-exchange rate elasticity is: $\varepsilon_{ps} = -\frac{s_t}{p}\frac{\partial p}{\partial s_t}$, the debt price-earnings elasticity is: $\varepsilon_{pY} = \frac{Y_t}{p}\frac{\partial p}{\partial Y_t}$, and the debt price-outstanding elasticity is: $\varepsilon_{p\tilde{F}} = -\frac{\tilde{F}}{p}\frac{\partial p}{\partial \tilde{F}}$. [9] *All of them are functions of the state variables.*

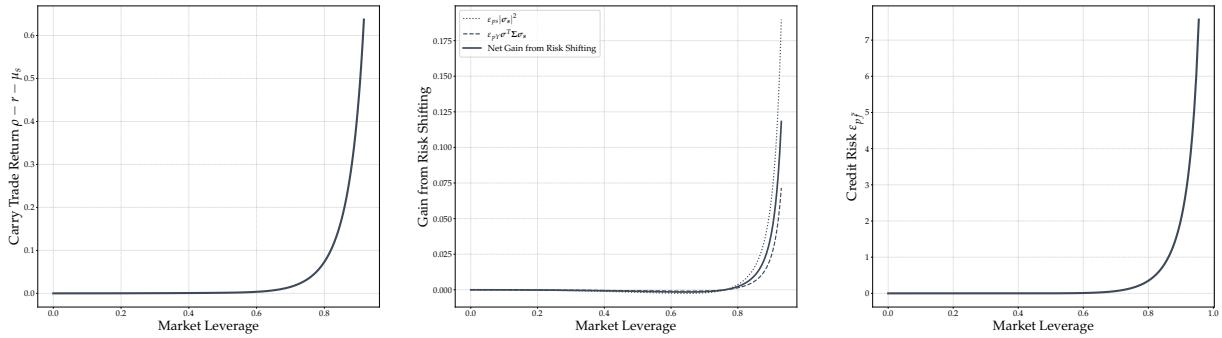
The optimal debt issuance strategy balances the benefits and costs associated with issuing debt. The numerator in the equation represents the benefits, which are derived from two primary sources: the *carry trade return* and the *gain from risk shifting*. Figure 2 illustrates each components, and it can be obtained after solving the model.

The carry trade return, $\rho - r - \mu_s$, is straightforward: firms profit when the local-foreign interest rate differential exceeds expected exchange rate depreciation.[10]

The risk-shifting mechanism, captured by $\varepsilon_{ps}|\boldsymbol{\sigma}s|^2 - \varepsilon pY\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}}$, operates through more subtle channels. Default options and limited liability generate convex manager's value in equilibrium, creating asymmetric payoff distributions between manager's holders and creditors: if the exchange rate moves favorably, the firm benefits while its debt repayment remains unchanged, whereas adverse movements' impact is bounded by limited liability. This asymmetry enables firms to gamble exchange rate shocks and transfer risk to creditors through increased exchange rate exposure. Unlike DH's framework where liability-side risks are absent, this setup allows firms to enhance manager's value by enlarging effective LC liability volatility through external debt issuance.

[9] I omit the superscript $*$ in subsequent discussions for simplicity.

[10] The prevalence of carry trade incentives is well-documented in empirical studies. For instance, Bruno and Shin (2017) shows that high cash-holding non-financial firms are more likely to issue dollar-denominated bonds when dollar carry trades are favorable. Similarly, Acharya and Vij (2022) documents increased foreign currency debt issuance among Indian firms when US-India interest rate differentials widen.



A. Carry Trade Return B. Gain From Risk Shifting C. Credit Risk

FIGURE 2. Optimal Issuance Policy: Gain and Loss

More specifically, the price-exchange rate elasticity and price-earnings elasticity are the same, and can be written as:

$$\varepsilon_{ps} = \varepsilon_{pY} = -\frac{v_{\tilde{f}\tilde{f}}\tilde{f} + v_{\tilde{f}f}f}{v_{\tilde{f}}}.$$

This equation stems from debt prices' dependence on earnings-scaled effective debt ($\tilde{f}$ and f), where proportional changes in exchange rates and earnings leave prices unchanged given the $\frac{s}{Y}$ scaling factor. The term $-v_{\tilde{f}\tilde{f}}\tilde{f}/v_{\tilde{f}}$ measures the convexity of the earnings-scaled manager's value function, with greater convexity amplifying manager's-creditor incentive misalignment and risk-shifting behavior. The $v_{\tilde{f}f}f$ term arises as exchange rate movements affect scaled aggregate debt, introducing interaction effects through $v_{\tilde{f}f}$ when measuring convexity. When exchange rate risks and earnings risks are driven by independent shocks, the second term, $\varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}$, vanishes. In this case, any additional exchange rate risk exposure brought on by new debt issuance will be fully added to the firm's liability side. However, positive correlation between these risks provides natural hedging, diminishing the marginal impact of debt issuance on liability volatility and moderating risk-shifting incentives.

The price-debt outstanding elasticity, $\varepsilon_{p\tilde{F}}$, represents the marginal cost of issuing new debt. As firms increase their outstanding debt, elevated default probability leads to declining debt prices, with this price sensitivity intensifying at higher leverage levels. Given this relationship, $\varepsilon_{p\tilde{F}}$ serves as my measure of firm-level credit risk throughout the paper.

To facilitate further analysis, I introduce an additional scaling for external debt issuance:

$$\tilde{G}(\tilde{F}_t, F_t, s_t, Y_t)/\tilde{F}_t = \tilde{g}\left(\frac{s_t\tilde{F}_t}{Y_t}, \frac{s_t F_t}{Y_t}\right) \cdot \frac{Y_t}{s_t\tilde{F}_t} = \tilde{g}(\tilde{f}_t, f_t)/\tilde{f}_t.$$

Applying equations (8) and (9) confirms that $\tilde{g}(\tilde{f}, f)$ depends only on the variables $(\tilde{f}, f)$. In equilibrium, where $\tilde{f} = f$, the aggregate counterpart of this scaled debt issuance is denoted by

$g = g(f) = \tilde{g}(f, f)$.

Thus, the dynamics for the individual state variable, $\tilde{f}_t$, and the aggregate state variable, f_t, follows:

$$\frac{d\tilde{f}_t}{\tilde{f}_t} = \left(\mu_s - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} + \frac{\tilde{g}}{\tilde{f}} - \xi\right) dt + (\boldsymbol{\sigma}_{\boldsymbol{s}} - \boldsymbol{\sigma}) \cdot d\boldsymbol{B}_{\boldsymbol{t}}, \tag{10}$$

$$\frac{df_t}{f_t} = \left(\mu_s - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} + \frac{g}{f} - \xi\right) dt + (\boldsymbol{\sigma}_{\boldsymbol{s}} - \boldsymbol{\sigma}) \cdot d\boldsymbol{B}_{\boldsymbol{t}}. \tag{11}$$

Step 2: Exchange Rate and Capital Flows

In this baseline model, the BoP condition emerges solely at the deterministic (dt) level, as I intentionally avoid specifying explicit sources of randomness. Thus, only a single restriction is imposed on the exchange rate dynamics. To maintain simplicity without losing essential insights, I assume a constant and exogenous volatility ($\boldsymbol{\sigma}_s$), deferring a detailed specification to subsequent sections. Later, after explicitly modeling the sources of currency market risk, I will provide microfoundations to complete the model.

The equilibrium values for the LC risk premium, defined as $RP_t = \rho - r - \mu_s$, and the capital inflow, $p_t g_t - (c + \xi) f_t$, must now be established. In a symmetric equilibrium ($g_t = \tilde{g}_t$), optimal debt issuance policy implies an aggregate issuance rate:

$$\frac{g_t}{f_t} = \left.\frac{RP_t + \varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{\varepsilon_{p\tilde{f}}}\right|_{\tilde{f}_t = f_t} = \left.\frac{RP_t + \varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{-v_{\tilde{f}\tilde{f}} \cdot f_t / v_{\tilde{f}}}\right|_{\tilde{f}_t = f_t}. \tag{12}$$

Combining this with the BoP condition leads directly to the following result:

LEMMA 4. *In a smooth MPE, aggregate bond issuance rate and currency risk premiums are:*

(a) *The currency risk premium is:*

$$RP_t = \frac{\varepsilon_{p\tilde{f}}}{p \cdot f} \cdot \left[(c + \xi) f - \overline{\mu}\right] - \underbrace{\left(\varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}}\right)}_{wedge}.$$

(b) *The aggregate new debt issuance rate is:*

$$\frac{g_t}{f_t} = \frac{1}{p_t f_t}\left((c + \xi) f - \overline{\mu}\right)$$

This result extends the exchange rate financial determination framework of GM and IM by incorporating time-varying risk-bearing capacity. In this simplified framework, the BoP condition

mandates that all debt repayment flows and exogenous flows must be balanced by new debt issuance. Thus, aggregate issuance flow's level $(c + \xi)f - \overline{\mu}$ is predetermined regardless of debt prices, exchange rates, or their expected movements.

The first part of the currency risk premium is the imbalances' price impact. The ratio $\frac{\varepsilon_{p\tilde{f}}}{p \cdot f}$ represents the required premium per unit of currency flow. An increase in firms' credit risk ($\varepsilon_{p\tilde{f}}$) naturally reduces this capacity, whereas an increase in the market value of debt ($p \cdot f$) enhances it. Consequently, firms require a smaller premium as their total market value of outstanding debt rises. Provided debt prices remain stable, a larger outstanding debt can partially mitigate the negative effects of heightened credit risk.

At first glance, this relationship might appear counterintuitive—why would firms become seemingly more willing to bear greater currency market imbalances as their debt burdens grow? The explanation lies in the nature of a pecuniary externality. Since the market value of debt ($p \cdot f$) captures the discounted value of future repayment flows, larger future obligations imply greater future demand for foreign currency, driving up the LC risk premium. This heightened premium stabilizes the exchange rate, thereby effectively reducing individual firms' future debt burdens and making them less sensitive—or in effect, less "risk-averse"—to current imbalances.[11] In other words, current credit risk immediately depresses debt prices, while the pecuniary externality acts through future expectations. As illustrated numerically in the appendix (Figure D.4C), the immediate impact of credit risk dominates, ensuring that firms' effective risk-bearing capacity generally declines with rising leverage.

The second component of the currency risk premium arises directly from firms' risk-shifting behavior. By engaging in risk shifting, firms realize certain private gains that effectively lower the premium they require to absorb additional currency imbalances.

It is instructive to consider why a positive currency risk premium emerges even in a risk-neutral environment. Suppose that at the current debt price without additional issuance, p^0, firms must issue debt at rate $g^0 = \frac{1}{p^0}\left[(c + \xi)f - \overline{\mu}\right]$ to clear the market imbalance. However, debt issuance itself negatively impacts prices, implying that after price adjustments, issuing at rate g^0 would no longer suffice to fully offset the imbalance. Consequently, firms must increase their issuance above g^0 to meet repayment obligations, but doing so requires additional incentives—specifically, higher carry-trade returns combined with gains from risk shifting. Thus, even under risk neutrality, a positive currency risk premium arises naturally from the endogenous interaction between debt issuance, price impacts, and market-clearing constraints. The sensitivity of debt prices to issuance, closely related to default risk, determines the magnitude of the risk premium. Hence, the term $\frac{\varepsilon_{p\tilde{f}}}{p \cdot f}$ effectively encapsulates firms' aggregate risk-bearing capacity.

[11] When debt is homogeneous, the rate at which existing debt decays is identical to the decay rate of newly issued debt. Thus, the debt price is also discounted at the rate of ξ, creating a straightforward market value form to capture such an effect.

In sum, long-term debt issuance affects future repayment flows, which contribute to currency market imbalances. Both individual and aggregate firm debt levels endogenously impact the corporate sector's risk-bearing capacity, thereby affecting both the pricing of these imbalances and the magnitude of risk-shifting wedge.

Step 3: Manager's Value Function

We observe that both the currency risk premium and issuance policy are fundamentally tied to the manager's value function. Once we solve for this function, we can establish the conjectured smooth equilibrium and back out every prices and quantities.

To begin, let's consider a scenario where the firm never defaults. In this case, the manager's value is expressed as the discounted cash flows from earnings minus the value of the debt. From Lemma 2, we can calculate the manager's value under the assumption of no future debt issuance. Under these conditions, the LC value of outstanding bonds, given current debt level $\tilde{F}_t$, is:

$$\mathbb{E}\left[\int_t^\infty e^{-(\rho+\xi)(\tau-t)}(c+\xi)\tilde{F}_t s_\tau d\tau\right].$$

The corresponding earnings-scaled no-default manager's value can be expressed as:

$$\overline{v}(\tilde{f},f) = \frac{1}{\rho-\mu} - \underbrace{(c+\xi)\tilde{f}\cdot\mathbb{E}\left[\int_t^\infty e^{-(\rho+\xi)(\tau-t)}\frac{s_\tau}{s_t}d\tau\right]}_{\text{No-default Bonds Value}}. \tag{13}$$

The term $\mathbb{E}\left[\int_t^\infty e^{-(\rho+\xi)(\tau-t)}\frac{s_\tau}{s_t}d\tau\right]$ captures how bond values in LC depend on future exchange rate dynamics. This expectation accounts for the the exchange rate shocks over time. Crucially, it depends only on the aggregate state variable f, not individual firm strategies, since exchange rate drift is determined by aggregate debt flows that individual firms cannot influence. Defining:

$$E(f) = \mathbb{E}\left[\int_t^\infty e^{-(\rho+\xi)(\tau-t)}\frac{s_\tau}{s_t}d\tau\right],$$

which satisfies the following ODE:

$$(\rho+\xi)E = 1 + \mu_f(f)\cdot fE_f + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2}\cdot f^2E_{ff}. \tag{14}$$

Here, $\mu_f(f) = \mu_s - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s + \frac{g}{f} - \xi$ includes the endogenous exchange rate drift, μ_s, and the issuance rate, $\frac{g}{f}$, as detailed in Lemma 4. All of these are functions of f.

To further compute the manager's value function, we need to incorporate an additional de-

fault option value into the no-default manager's value, denoted by D:

$$D(\tilde{f}, f) = \mathbb{E}^0 \left[e^{-(\rho+\xi)\tau_b} \left(0 - \overline{v}(\tilde{f}_B, f_B) \right) \right], \tag{15}$$

where the expectation assumes no future issuance, τ_b represents endogenous default timing, and $(\tilde{f}_B, f_B)$ denotes the default boundary in state space. This default option value reflects that, once the firm defaults, its value is preserved by the zero lower bound, eliminating any negative non-default manager's value. Unlike the E function, the default option value D depends on both the individual state variable $\tilde{f}$ and the aggregate state variable f.[12]

Under the no-issuance condition, $D(\tilde{f}, f)$ satisfies:

$$\begin{aligned}(\rho + \xi)D =& \tilde{f}(\mu_s - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} - \xi) \cdot D_{\tilde{f}} + f\left(\mu_s - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} + \frac{g}{f} - \xi\right) \cdot D_f \\ &+ \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}}}{2}\left(D_{ff}f^2 + 2D_{\tilde{f}f}\tilde{f}f + D_{\tilde{f}\tilde{f}}\tilde{f}^2\right).\end{aligned} \tag{16}$$

As shown in Appendix B, this PDE admits a separable solution:

$$D(\tilde{f}, f) = \left(\frac{\tilde{f}}{f}\right)^{\eta} \cdot N(f),$$

where η is a constant determined by the boundary conditions, and $N(f)$ is an unknown function satisfying the following ODE:

$$(\rho + \xi)N = -\frac{g}{f}\eta + \mu_f(f) \cdot fN'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}}}{2} \cdot f^2 N''(f). \tag{17}$$

Finally, the manager's value under no debt issuance is characterized as:

$$v(\tilde{f}, f) = \frac{1}{\rho - \mu} - (c + \xi)\tilde{f} \cdot E(f) + \left(\frac{\tilde{f}}{f}\right)^{\eta} \cdot N(f), \tag{18}$$

with corresponding debt price:

$$p(\tilde{f}, f) = (c + \xi) \cdot E(f) - \eta\left(\frac{\tilde{f}}{f}\right)^{\eta-1} \frac{1}{f} \cdot N(f). \tag{19}$$

PROPOSITION 1. *The manager's value function for individual firms can be characterized through a*

[12] On one hand, both the endogenous default time and position are affected by future exchange rate dynamics and the evolution of aggregate debt, which are determined by the aggregate state variable. On the other hand, the individual firm's current debt matures exponentially, setting a deterministic trajectory for future individual debt levels until default. Consequently, $D(\tilde{f}, f)$ is a function of both $\tilde{f}$ and f.

two-dimensional state space comprising individual and aggregate debt-to-earnings ratios. Both state variables follow a strong Markov process, with dynamics described by equations (10) *and* (11).

The manager's value can be solved as if the individual firm makes no issuance, $\frac{\tilde{g}}{\tilde{f}} = 0$ *and the earning scaled manager's value function is*

$$v(\tilde{f}, f) = \frac{1}{\rho - \mu} - (c + \xi)\tilde{f} \cdot E(f) + \left(\frac{\tilde{f}}{f}\right)^{\eta} \cdot N(f),$$

where η *is a constant determined as a solution to the characteristic equation. The corresponding debt price is:*

$$p(\tilde{f}, f) = (c + \xi) \cdot E(f) - \eta \left(\frac{\tilde{f}}{f}\right)^{\eta - 1} \frac{1}{f} \cdot N(f).$$

When evaluated at $\tilde{f} = f$*, the equilibrium manager's value simplifies to:*

$$\hat{v}(f) = v(f, f) = \frac{1}{\rho - \mu} - (c + \xi) f \cdot E(f) + N(f).$$

The equilibrium is characterized by two coupled second-order ODEs:

$$(\rho + \xi)E(f) = 1 + \mu_f(f) \cdot fE'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 E''(f)$$
$$(\rho + \xi)N(f) = -\frac{g}{f}\eta + \mu_f(f) \cdot fN'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 N''(f).$$

where

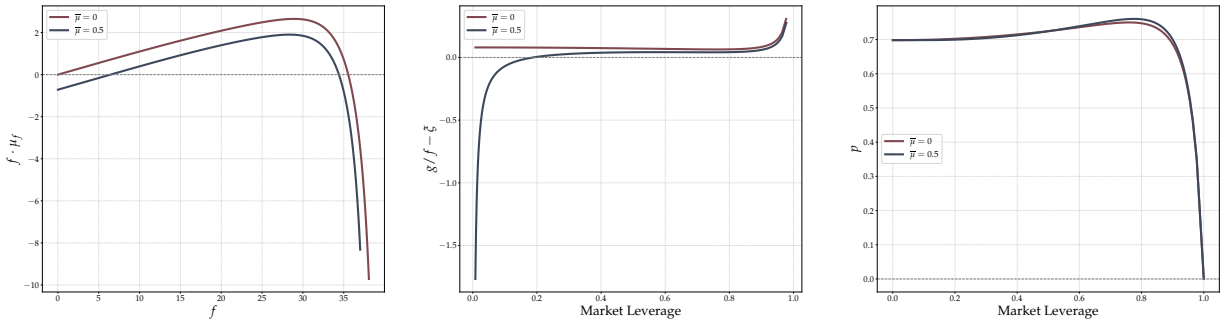
$$\mu_f(f) = \rho - r - RP_t - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s + \frac{g}{f} - \xi.$$

Here, RP_t *and* $\frac{g}{f}$ *are functions of* f *as specified in Lemma 4, with boundary conditions detailed in Appendix B.*

I.3. Equilibrium Properties

Appendix D.3 provides detailed equilibrium quantities and prices.

This simplified, one-sector model offers valuable insights into the private sector's leverage dynamics, particularly regarding external debt sustainability. International imbalances here stem from two main flows: the trade surplus or deficit ($\overline{\mu}$), and debt repayment obligations ($(c + \xi)f$). Initially, debt issuance responds mainly to trade flows when debt obligations remain modest. If



A. State Variable Drift B. Net Issuance Rate C. Debt Prices

FIGURE 3. Equilibrium Dynamics: Net Exporter and Non Exporter

the economy runs a trade surplus ($\overline{\mu} > 0$), the model exhibits two stable stochastic steady states. Under moderate fundamental shocks, firms steadily accumulate debt, sustainably supported by net exports generating sufficient FC liquidity. Conversely, weaker fundamental shocks induce deleveraging, guiding firms back to a stable zero-debt state.

However, the story changes dramatically if the economy experiences exceptionally strong fundamental shocks, coming from either fundamental (Y_t) or exchange rate (s_t). Firms' effective leverage rises, eventually surpassing an unstable intermediate debt threshold. Beyond this point, rising repayment needs force firms to continually issue more debt, initiating a slow-moving but ultimately accelerating debt crisis.

The private sector thus enters a spiral of increasing leverage: initially gradual but rapidly accelerating. Early on, falling debt prices—dropping below par together with increasing repayment burdens—force firms to issue more debt just to sustain stable capital inflows. As firms transition toward very high leverage states, debt prices plunge toward zero, further accelerating issuance in a self-reinforcing loop (see Figure D.5). This dynamic parallels the sovereign debt spiral documented by Lorenzoni and Werning (2019) (LW), where rising default expectations lower debt prices and thus force governments to issue even more debt, validating market pessimism.[13]

Central to this spiral is firms' equilibrium response to deteriorating credit conditions. LW addresses this issue via an equilibrium selection that favors increased debt issuance.[14] In my model, the absence of alternative sources of currency market liquidity ensures firms similarly increase issuance.[15]

In this one sector model, the absence of alternative liquidity doesn't immediately threaten stability, since exchange rate adjustments (high LC risk premium implies a reverting phenomenon)

[13]Unlike LW, which admits multiple equilibria, my framework produces a unique smooth equilibrium.

[14]In LW, the context is sovereign debt. The authors argue that if a government needs to finance a deficit of 100 million euros and plans to sell 10 million bonds at 10 euros each, but the bond price falls to 9 euros, resulting in a 10 million euro shortfall, it is more likely to draw on liquid reserves and increase bond issuances in the coming weeks rather than immediately raising taxes or cutting spending.

[15]The flexibility increases once a second liquidity-providing sector is introduced, explored later in the paper.

naturally reduce future debt burdens. This mechanism establishes a second, high-debt-level stable steady state. But we should also keep in mind, there is no debt retirement in FC, since the net issuance rate is still positive. Yet, as subsequent sections with multi-sector highlight, such stability hinges critically on this extreme exchange rate movements—a fragile buffer.

So far, the discussion reflects a macroeconomic portfolio-balance perspective. At the micro-level, prices must also support this leverage spiral:

(1) The currency risk premium starts at zero when firms' risk-bearing capacity is practically unlimited (see Figure D.4).[16] The numerical result also shows that the premium rises steadily with outstanding debt levels.[17] Importantly, this positive premium exists regardless of whether the economy runs a trade surplus or deficit. This rising profitable carry trade opportunities thus incentivizes firms' continued debt issuance.

(2) More strikingly, debt prices initially rise with increasing debt levels, only declining sharply once leverage reaches extreme levels.[18] This seemingly paradoxical pattern arises from pecuniary externalities: high aggregate debt levels imply substantial future repayment flows, creating pronounced deviations from UIP in the future. These deviations predict future currency appreciation, reducing the expected future debt burden and momentarily signaling lower default risk despite higher current leverage.[19]

Nevertheless, this one-sector setup has limitations. Its simplified structure fixes capital inflows, which is defined as $p_t g_t - (c + \xi) f_t$, at $\overline{\mu}$, thereby obscuring interactions between capital flow dynamics and currency risk premia. Regarding financial stability, although growing debt pushes the economy closer to the default boundary, the rising currency risk premium and anticipated currency appreciation provide a natural cushion against immediate defaults. The substantial negative drift near default implies only severe shocks trigger systemic defaults. While this exchange rate-driven stabilization offers novel insights—achieving stability through price rather than quantity adjustments—the model's predicted explosive currency risk premium may exceed empirically observed magnitudes.

[16] Formally, as $f \to 0$, if $\frac{\partial p}{\partial \tilde{f}} \to 0$, and $\varepsilon_{ps}, \varepsilon_{pY} \to 0$, then $RP_t \to 0$.

[17] If the gain from risk shifting is less sensitive to the outstanding debt level f than RP_t, we can theoretically show that the local currency risk premium is increasing in f.

[18] This finding does not contradict the result that debt prices are non-increasing with a firm's own debt level, $\tilde{f}$, because we are analyzing these properties along the equilibrium line. As market leverage $(\frac{p_t f_t}{p_t f_t + v_t})$rises, both individual and aggregate debt levels increase.

[19] I thank Stefan Nagel for insights sparking this discussion. Empirically, one may indeed observe a positive correlation between a firm's debt price and its debt level due to this exchange rate mechanism.

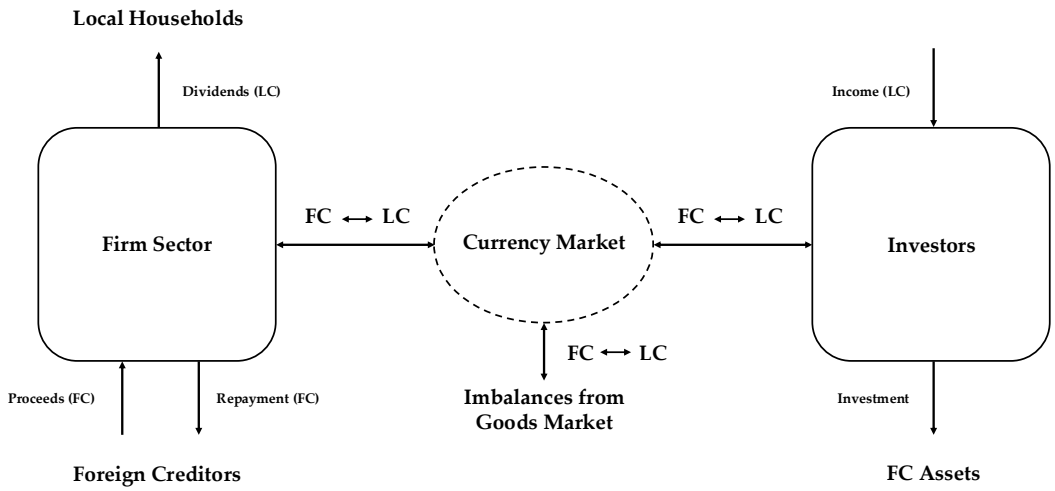


FIGURE 4. Model Sketch: Two Sectors

II. Two-sector Model and Capital Flows

The simplified one-sector model has clarified how firm-level leverage cycles influence exchange rate dynamics. However, it leaves unanswered a key question: why do capital flows themselves exhibit pronounced cyclical patterns, forming what we observe as a global financial cycle?

To answer this question, I now extend the framework by incorporating another sector that generates meaningful currency flows and explicitly including their activities in the aggregate BoP condition. This expansion fundamentally alters the economic function of firms' debt issuance: rather than merely rolling over existing obligations (aside from balancing trade flows), firms now also intermediate currency demands across sectors, balancing external and internal financial needs.

Specifically, I introduce a "local investor" sector composed of sophisticated domestic funds with direct access to global asset markets. With this addition, the model captures two distinct capital flows: inflows, occurring when foreign creditors purchase domestic corporate bonds, and outflows, emerging when local investors deploy capital abroad. This richer structure offers a more comprehensive view of the interactions underpinning global financial cycles.

II.1. Additional Setup

Local Investors

Local investors receive an instantaneous endowment of ζY_t units of nontradable goods. Unlike households or firms, these investors do not immediately consume or distribute dividends. In-

stead, they temporarily invest their endowments over each interval $[t, t+\Delta t]$ and pay out the accumulated gross returns—both in nontradable and tradable goods—to households at time $t+\Delta t$. The analysis naturally transitions into continuous time as $\Delta t \to dt$.

Local investors have access to two distinct risk-free investment opportunities. They can invest in FC assets yielding interest rate r, with the asset supply matching local investor demand, or LC assets yielding interest rate ρ. [20]

At any given time t, investors allocate a fraction a_t of their income into FC assets, acquiring $\frac{a_t \zeta Y_t}{s_t}$ units, and invest the remainder, $(1-a_t)\zeta Y_t$, into LC assets. Local investors select their portfolio weights to maximize their risk-adjusted short-term returns measured in LC. Additionally, investors have a non-pecuniary preference for holding foreign assets, captured through a convenience yield parameter $\lambda > 0$. Given their Constant Relative Risk Aversion (CRRA) preferences with risk aversion parameter γ and the assumed exchange rate dynamics described in equation (6), the investors' optimization problem is: [21]

$$\max_{a} \qquad \mathbb{E}_t \left[\frac{R_t^{1-\gamma}}{1-\gamma} \right], \tag{20}$$

$$\text{s. t.} \quad R_t = \left[(1-a_t)\, e^{\rho \Delta t} + a_t e^{\left(r+\lambda+\mu_s - \frac{1}{2}\|\boldsymbol{\sigma}_s\|^2\right)\Delta t + \boldsymbol{\sigma_s} \cdot \Delta \boldsymbol{B_t}} \right]. \tag{21}$$

exploiting the log-normality of $s_{t+\Delta t}/s_t$.

Lemma 5. *The optimal fraction of the portfolio allocated to foreign assets, a_t^*, is given by:*

$$a_t^* = \frac{\mu_s + r + \lambda - \rho}{\gamma \|\boldsymbol{\sigma}_s\|^2}. \tag{22}$$

This result follows directly from the classic mean-variance portfolio framework under CRRA utility, assuming log-normal returns for the risky asset.[22]The resulting capital outflows during the interval $[t, t+dt]$ are expressed simply as:

$$dA_t = a_t^* \zeta Y_t dt. \tag{23}$$

And the expanded BoP can be written as,

$$s_t\, dL_t - dA_t = -Y_t \cdot \overline{\mu} dt. \tag{24}$$

[20]To maintain a constant LC interest rate, household preference parameters are adjusted accordingly: $\chi_t = (2+\zeta)Y_t$ and $\iota_t = s_t \left(Y_t + \frac{a_{t-}}{s_{t-}} \zeta Y_{T,t-} \right) - \overline{\mu} Y_t$.

[21]This setup is stylized since the resulting aggregate returns distributed to households are of order $O((dt)^2)$. However, it conveniently avoids additional state variables for investors' wealth, effectively assuming a mean-variance portfolio choice.

[22]The optimal portfolio share a_t^* may become negative, indicating short positions. In this simplified model, negative values represent immediate short positions because long-term investment strategies are not modeled. In a more realistic scenario, negative flows might correspond to temporary divestments rather than outright short-selling.

II.2. Equilibrium Characterization

The solution method follows the same approach as in the one-sector model. The first step—deriving an individual firm's optimal issuance policy—remains unchanged, with Lemma 3 still applicable. The only difference emerges in the second step, where the BoP condition is modified.

LEMMA 6. *In a smooth MPE, the aggregate bond issuance rate and exchange rate dynamics are characterized by:*

(a) *The currency risk premium, with* $\omega(f) = \frac{\varepsilon_{p\tilde{f}}}{\alpha p_t f_t + \varepsilon_{p\tilde{f}}}$, *and* $\alpha = \hat{\gamma}\|\boldsymbol{\sigma}_s\|^2$ *(here* $\hat{\gamma} = \gamma/\zeta$*):*

$$RP_t = \omega(f) \cdot \left[\lambda + \alpha\Big((c+\xi)f_t - \overline{\mu}\Big)\right] - (1-\omega(f)) \cdot \left(\varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s\right). \tag{25}$$

(b) *The issuance rate:*

$$\frac{g_t}{f_t} = \Big(\alpha p_t f_t + \varepsilon_{p\tilde{f}}\Big)^{-1} \left\{\left[\lambda + \alpha\Big((c+\xi)f_t - \overline{\mu}\Big)\right] + \varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s\right\}. \tag{26}$$

Intuitively, the currency risk premium exhibits a two-layered structure, reflecting interactions among distinct market participants. The first layer captures the scenario in which the investor sector alone bears market imbalances, echoing the framework of IM: $\lambda + \alpha\left((c+\xi)f_t - \overline{\mu}\right)$. Here, λ denotes the convenience yield wedge, while α reflects the pricing of market imbalances $(c + \xi)f_t - \overline{\mu}$.

The second layer highlights the influence of the firm sector through the scaling factor $\omega(f)$, incorporating α, debt price impact, and externalities. This factor scales both the baseline risk premium and the risk-shifting wedge, with the latter remains $(1-\omega)$. Unlike the simplified structure in Lemma 4, here an additional coefficient α precedes the term $p \cdot f$. This coefficient quantifies how other sectors moderate the future repayment flow's impact on the currency risk premium, thereby diminishing the pecuniary externality on the firm sector's risk-bearing capacity. The simpler one-sector model emerges as a special case: as $\alpha \to +\infty$, we get $\omega(f) \to 0$, resulting in the entire risk-shifting wedge being fully reflected in RP_t.

This layered structure can also be interpreted from various institutional perspectives. For local investors, the risk premium expression becomes:

$$RP_t = \underbrace{\hat{\gamma} \cdot \left((c+\xi)f_t - p_t g_t - \overline{\mu}\right)\|\boldsymbol{\sigma}_s\|^2}_{\text{Required premium for remaining imbalances}} + \underbrace{\lambda}_{\text{Convenience yield wedge}},$$

where $\hat{\gamma} = \gamma/\zeta$ quantifies local investors' required compensation for bearing one unit of risk.

Firm issuance behavior reflects distinct incentive channels. Without firm issuance, the cur-

rency risk premium simplifies to $\lambda + \alpha\left((c+\xi)f_t - \overline{\mu}\right)$; a higher premium signals more lucrative potential carry trades, incentivizing further debt issuance. Additional incentives come from risk-shifting behavior, which, although indirectly reducing the currency risk premium, primarily provides direct encouragement to issue debt—this second-order indirect effect remains smaller relative to the direct incentive.

These issuance incentives are moderated by $\left(\alpha p_t f_t + \varepsilon_{p\tilde{f}}\right)^{-1}$, capturing two key effects. Rising credit risk ($\varepsilon_{p\tilde{f}}$) suppresses issuance incentives, whereas increases in $\alpha p_t f_t$ lower today's price of imbalances, consequently reducing the LC risk premium and issuance.

This integrated framework, blending "rollover" and "intermediation," contrasts sharply with simpler one-sector models by achieving equilibrium through joint adjustments in prices and quantities. The corporate debt issuance must simultaneously balance two margins: for a given market imbalance $(c+\xi)f_t - \overline{\mu}$, the equilibrium debt price and issuance volume align the premiums required across sectors. Insufficient issuance from firms shifts the imbalance onto investors with lower risk-bearing capacity, causing the LC risk premium to rise. This rise, in turn, motivates firms—who have greater risk-bearing capacity—to step up issuance until equilibrium is restored. Conversely, excessive issuance prompts escalating credit risk, rapidly eroding profitability and moderating firms' incentives. Thus, the market's "invisible hand" operates naturally to equate risk-bearing capacities across participants, forging a fundamental connection between leverage and exchange rate dynamics.[23]

The final step in solving the manager's value function follows the one-sector model's approach, differing only in the coefficients of the system of ODEs. I restate it here.

PROPOSITION 2. *In an economy with two sectors, the earning scaled manager's value function is still*

$$v(\tilde{f}, f) = \frac{1}{\rho - \mu} - (c+\xi)\tilde{f} \cdot E(f) + \left(\frac{\tilde{f}}{f}\right)^{\eta} \cdot N(f),$$

where η is a constant and it is a solution to the characteristic function.

The equilibrium is still characterized by two coupled second order ODEs:

$$(\rho+\xi)E = 1 + \mu_f \cdot fE'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}}}{2} \cdot f^2 E''(f)$$
$$(\rho+\xi)N = -\frac{g}{f}\eta + \mu_f \cdot fN'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}}}{2} \cdot f^2 N''(f).$$

[23] As mentioned in the introduction, this equilibrating mechanism parallels Pascal's Law.

where

$$\mu_f = \rho - r - RP_t - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} + \frac{g}{f} - \xi$$

and RP_t, and $\frac{g}{f}$ are now given in Lemma 6. The modified boundary conditions are detailed in Appendix B.

II.3. Equilibrium Properties

The numerical solutions for all major equilibrium quantities and prices are presented in Appendix D.4. Below, I analyze the key equilibrium properties.

Capital Flows Earlier, we raised the question of how equilibrium capital inflows, defined as $p_t g_t - (c + \xi) f_t$, vary across states of the economy.

For market leverage below roughly 0.6, the extended model closely mirrors one-sector dynamics, characterized by stable capital inflows sustained by firms' debt issuance. The only difference is the additional amount of intermediation needs. In this phase, local investors maintain long FC positions, incentivized by a positive convenience yield and moderate LC risk premiums.[24] As leverage continues rising, escalating debt-service obligations generate higher FC demand, compelling firms to issue more debt and initiating a leveraging spiral.

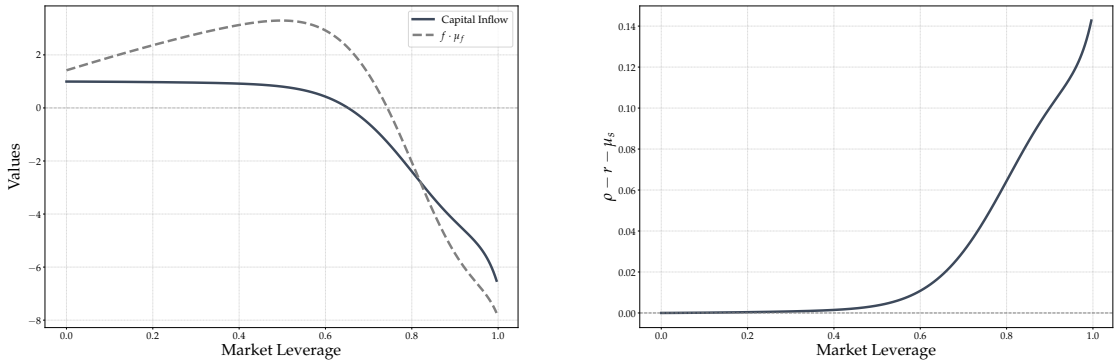
The dynamics notably diverge from the one-sector model once market leverage surpasses the critical threshold of approximately 0.6. Beyond this point, firms' debt issuance no longer fully offsets currency market imbalances, causing issuance rates to decline. Investors are thus forced to unwind their long FC positions, reversing capital flow direction and transferring liquidity provision responsibilities to local investors.[25] This nonlinearity sharply contrasts the monotonic issuance acceleration and constant capital inflow predicted by the one-sector model. Now capital flows exhibit state-contingent shock sensitivity: low leverage regions comfortably absorb moderate shocks, whereas higher leverage states amplify even small perturbations.

Thus, to the same question in the one-sector model: How do firms respond when rising credit risk reduces issuance proceeds—maintain issuance or adjust issuance to meet obligations, the expanded model demonstrates a state contingent response. Initially, firms increase issuance rates to maintain stable inflows, but eventually scale back issuance as high leverage leads to declining debt prices and greater default risks. Naturally, the precise leverage level where this adjustment occurs significantly influences the economy's resilience—a critical stability issue explored .

LC Risk Premium The two-sector model preserves essential elements of the currency risk

[24]The benchmark convenience yield is calibrated at $\lambda = 145$ bps, consistent with dollar convenience yield estimates for advanced economies from Koijen and Yogo (2019). The convenience yield for EMEs likely exceeds this benchmark, suggesting even stronger capital inflows in low-leverage environments.

[25]This reversal aligns closely with foreign asset patterns observed in EMEs, where local investors accumulate foreign assets during stable periods but liquidate these positions during crises to supply needed FC liquidity.



A. Equilibrium Dynamics

B. LC Risk Premium

FIGURE 5. Two-sector Model

premium dynamics introduced earlier while achieving greater empirical realism. Consistent with observed patterns in EMEs, the LC risk premium remains positive and monotonically rises with leverage. The dominant driver remains the flow-price interaction: higher market leverage reduces aggregate risk-bearing capacity, necessitating higher premiums to absorb imbalances (see Figure D.9C). Simultaneously, increased leverage raises these imbalances themselves.

The model also realistically generates persistent LC depreciation episodes, aligning with widely documented empirical findings in EMEs. During normal leverage states—far from the default boundary—the currency risk premium fails to fully offset the interest rate differential effects on exchange rates and predicts depreciation.[26] Unlike the one-sector model's excessively high UIP deviations, the expanded framework yields more plausible values. Additionally, exchange rate drift near the stochastic steady state approaches zero, reflecting empirical findings of exchange rates' near-random-walk behavior. In other words, the model predicts leverage-dependent exchange rate predictability: heightened predictability emerges at both low and high leverage extremes, whereas moderate leverage states—near the stochastic steady state—display relatively low predictability.

Explicitly modeling firms' intermediation role endogenously generates a negative equilibrium relationship between market leverage and capital inflows. Since higher leverage increases the LC risk premium, the model naturally reproduces the negative correlation between LC risk premiums and capital inflows documented empirically. However, consistent with recent evidence, this relationship may weaken or vanish once leverage and other local risk factors are controlled for.[27]

LC Risk Premium v.s. Interest Rate Differentials I further examine the cross-sectional relationship between currency risk premia and interest rate differentials through comparative statics

[26]The benchmark interest rate differential is set at $\rho - r = 0.07$.

[27]For instance, Kalemli-Özcan and Varela (2024) demonstrate that incorporating broader local risk indicators, such as economic policy uncertainty, eliminates the correlation between capital inflows and the LC risk premium.

with respect to $\rho - r$ in Appendix F. Unconditionally, the positive relationship between LC risk premiums and interest rate differentials arises primarily from their shared association with market leverage: higher interest rate differentials generate larger effective leverage for given debt levels f, magnifying the risk premia. When conditioning on market leverage, the firms' credit risk ($\varepsilon_{p\tilde{f}}$) is largely accounted for, variations in the market price of imbalances primarily reflect pecuniary externality effects. During normal periods, the conditional LC risk premium maintains a positive relationship with interest rate differentials: higher differentials predict lower market value of debt at fixed leverage levels, attenuating pecuniary externalities and reducing firms' risk-bearing capacity.

III. Implications for Financial Stability

I now utilize the integrated framework developed above to explore financial stability concerns surrounding external financing in EMEs. Specifically, I conduct comparative statics on key model parameters to shed light on two critical questions: what triggers sudden stops in capital inflows, and what drives economies toward excessive external borrowing? The global equilibrium solution of the model makes addressing these issues possible.

To quantitatively assess financial stability, I conduct model simulations and follow Calvo, Izquierdo and Mejía (2008)'s empirical definition of sudden stops (detailed definition in Appendix D.2).

III.1. Debt Maturity

To examine the effect of debt maturity on equilibrium properties, I conduct comparative statics with respect to the parameter ξ. In the baseline model, the expected debt maturity is set to $1/\xi = 6.58$ years.[28]

Short-term FC debt is often seen as a primary source of vulnerability in EMEs' financial systems and a leading cause of crises. The conventional wisdom emphasizes that shorter maturities necessitate more frequent refinancing, potentially triggering liquidity shortages when FC prices remain elevated. However, this perspective often conditions on ex-post financial crises, after substantial debt has already been issued. With rational expectations, firms would anticipate the vulnerabilities associated with short-term debt and adjust their leverage decisions proactively. Likewise, creditors anticipating potential default risks among EM firms would also incorporate this into a higher premium, which could correct market behavior before crises unfold. This paper seeks to explore, from the standpoint of a rational firm, whether there are inherent properties of

[28]This value matches the average maturity of EMEs' foreign-targeted bonds, as calculated using the SDC Platinum dataset.

TABLE 1. Financial Stability Measurements

	Financial Distress			Sudden Stop			Lifetime	
	% of Time	*#N*	*Recovered*	*% of Time*	*#N*	*Recovered*	Δ_0	Δ_{sss}
A. Maturity								
3 years	14.79%	0.25	68.21%	5.95%	0.35	70.78%	15.95	5.96
6.58 years	20.31%	0.24	60.82%	5.61%	0.19	65.80%	17.09	7.26
10 years	22.83%	0.23	56.90%	5.36%	0.16	65.05%	17.98	7.88
B. Convenience Yield								
145 bps	20.31%	0.24	60.82%	5.61%	0.19	65.80%	17.09	7.26
[145, 400] bps	26.01%	0.24	68.33%	5.80%	0.16	73.35%	20.39	10.33
[50, 400] bps	26.44%	0.22	68.75%	6.28%	0.16	74.87%	27.07	11.98
C. Stochastic Volatility								
Lower Normal Vol	31.49%	0.22	66.20%	5.31%	0.12	79.45%	16.26	11.90
Benchmark	24.43%	0.21	59.90%	5.54%	0.14	65.58%	15.66	9.29
Lower Crisis Vol	20.39%	0.22	56.50%	5.30%	0.16	61.29%	13.41	7.28

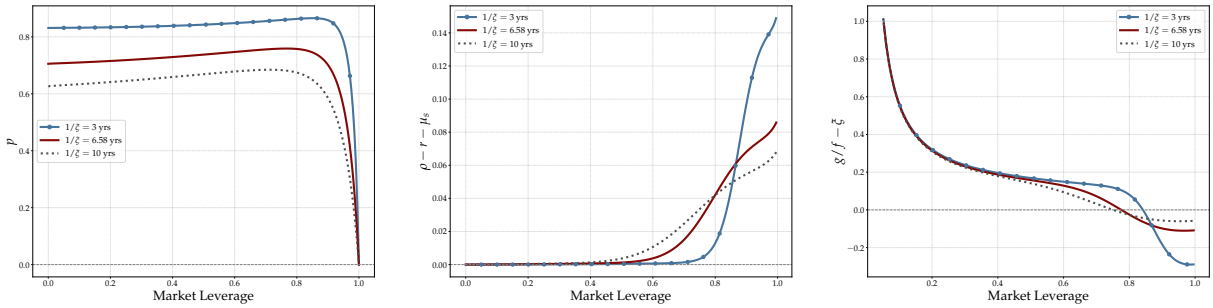
Notes: The "% of Time" measures the length of distress or sudden stop periods relative to the entire lifetime of the economy. The number of distress periods and sudden stops is converted to an annual frequency. The "Recovered" variable measures the percentage of distress periods or sudden stops that do not directly lead to default. The lifetime measures the number of years it takes to progress from $f \approx 0$ and from a stable stochastic steady state to default. If there are multiple stable stochastic steady states, the one with the smallest f is selected. Financial distress and sudden stop measurements are all simulate starting from such stable stochastic steady state.

short-term debt that might lead firms and creditors to "overlook" potential instability, ultimately generating unintended outcomes such as sudden stops.

Maturity and Financial Stability The model predicts more frequent episodes of financial distress and sudden stops when the external debt maturity of the firm sector is shorter. Table 1 Panel A presents these findings. Simulation results, whether starting from the stochastic steady state (SSS) or from a low debt level, indicate that shorter-term debt is associated with a reduced expected time to default. These results underscore that the probability of distress periods, sudden stops, or even an economy-wide credit bust increases as debt maturity shortens.

However, shorter maturities generate a nuanced stability trade-off: while they increase sudden stop frequency, they do not necessarily precipitate immediate systemic defaults. Most distress episodes resolve through recovery. Moreover, shorter-term debt implies that, on average, the economy spends less time in financial distress. This reveals a fundamental tension: shorter-term debt leads to more frequent financial instability but enhances conditional crisis resilience.

The Debt Ratchet Effect These properties arise from the debt ratchet phenomenon, initially characterized by Admati, DeMarzo, Hellwig and Pfleiderer (2018) and further explored by DH, whereby debt reduction is achieved only through maturing debt rather than buybacks through-



A. Debt Price B. LC Risk Premium C. Net Issuance Rate

FIGURE 6. Comparative Statics: Debt Maturity

out the leverage dynamics.[29] Longer maturities reduce natural debt amortization, maintaining less negative net issuance at higher leverage (see Figure 6C). Conversely, shorter maturities provide natural commitment to debt reduction when creditworthiness deteriorates through more frequent repayment obligations.

Simultaneously, the higher repayment obligations associated with short-term debt increase the demand for FC. Firms rely more heavily on external liquidity provision in the currency market, which drives up the LC risk premium and even leads to expectations of LC appreciation (see Figure 6B). This effect also reduces firms' effective liabilities in LC. As a result, shorter maturities allow firms to lower their effective debt burden more readily than longer maturities, which explains the faster recovery from financial instability observed with shorter-term debt. [30]

However, short-term debt's natural commitment mechanism generates countervailing ex-ante effects. Improved credit conditions keep debt prices high, even at elevated leverage, reducing the market price of imbalances and enabling less costly intermediation and rollovers (See Figure D.13). This allows investors to maintain high demand for FC assets, delaying debt reduction until the economy reaches very high leverage levels. Consequently, the economy edges closer to its default boundary, becoming more susceptible to default as even small adverse shocks could push it over the boundary. When debt prices and capital inflows eventually begin to decline, the reversal is more sudden, resembling a "sudden stop" event—a rapid and pronounced retraction of capital inflows. Thus, while shorter maturities improve recovery potential, they also create con-

[29] In this model, the debt ratchet effect is sensitive to parameter choices, particularly the convenience yield. Appendix E demonstrates that debt buybacks can occur when market leverage is low. But under this section's parameter setting, debt ratchet effect remains.

[30] Traditionally, short-term debt is viewed as safer due to considerations related to debt overhang and risk shifting. In the classic paper Myers (1977), the value insensitivity of short-term debt to immediate investments mitigates debt overhang. This investment-related mechanism is absent here. But, this model instead confirms that longer-term debt heightens risk-shifting incentives, as smaller repayment obligations reduce the probability of endogenous default, which is the part where issuer and creditors' incentives are aligned (see Figure D.14E). In traditional frameworks, risk-shifting typically involves taking on risky projects, whereas in this model, it manifests through issuing additional debt. Moreover, pecuniary externalities introduce an exchange rate channel that further reinforces the safety of short-term debt.

ditions conducive to sudden stops, in contrast to longer maturities, where capital inflows decrease more gradually (See Figure D.14C). Thus, shorter external debt maturities increase vulnerability to excessive borrowing driven by debt price insensitivity to leverage. [31]

Rather than characterizing this as tension between commitment and price insensitivity, the fundamental trade-off emerges between ex-post recovery capacity and ex-ante moral hazard—a classic financial stability policy challenge.[32] Enhanced ex-post economic resilience paradoxically strengthens ex-ante risk-bearing capacity, encouraging risk-taking behavior that pushes the economy closer to the default boundary. In this case, moral hazard emerges as investors exploit the firm sector's cheap liquidity provision to satisfy FC preferences while firms delay debt retirement. In the next example, we will see how this moral hazard manifests through firms' risk-shifting behavior.

III.2. Convenience Yield

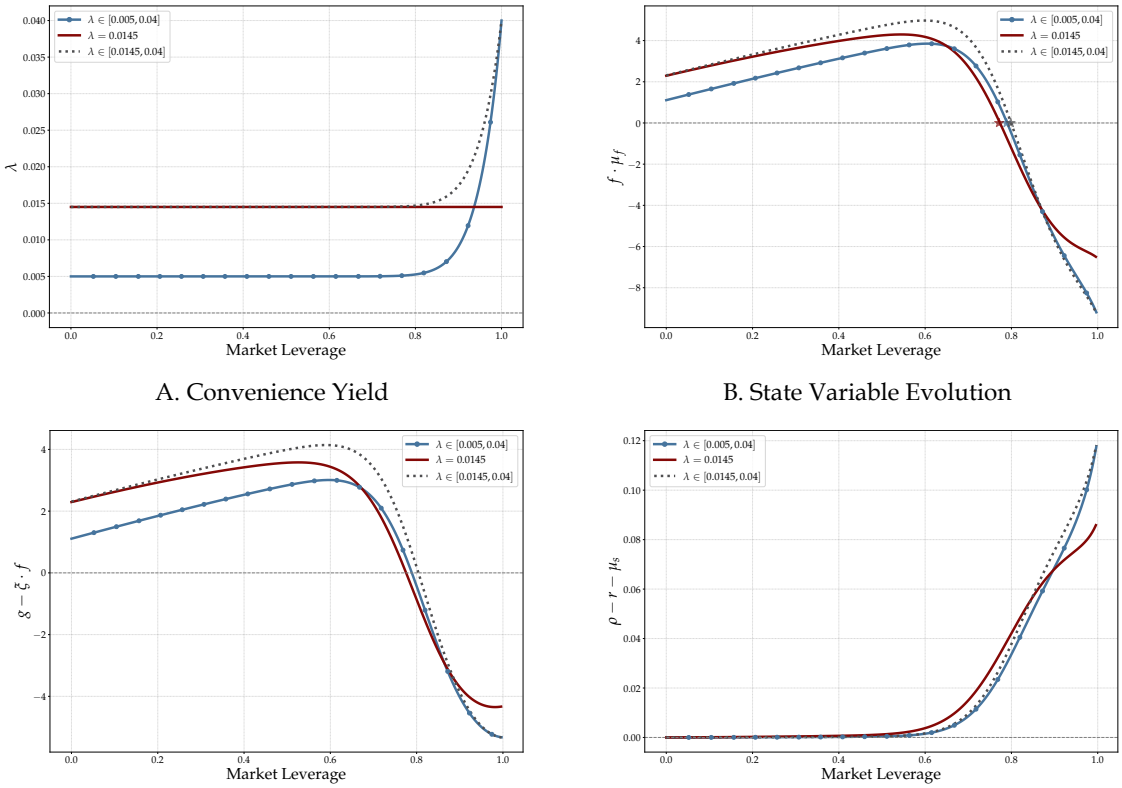
In this paper, the portfolio balancing perspective links an economy's leverage cycle to its foreign asset demand. I examine this connection's implications for financial stability by analyzing how convenience yield—representing local investors' FC asset preference—affects leverage dynamics. Detailed numerical results appear in Appendix D.6.

I first analyze convenience yield's role during crises (market leverage near 1) by introducing a reduced-form state-dependent convenience yield $\lambda(f)$ that rises with market leverage (see Figure 7A). This adjustment aligns with empirical findings, such as those by Jiang, Krishnamurthy and Lustig (2021), who document countercyclical dollar convenience yields, as measured by the Treasury basis, rising during global crises.

Higher crisis-period convenience yields generate enhanced economic resilience, evidenced by extended lifetimes and improved recovery rates (Table 1). The state variable evolution (Figure 7B) shows that high crisis-period convenience yields trigger reversion to safer leverage levels when market leverage peaks. The stabilization mechanism operates through two channels (Figures 7C and 7D): First, rising convenience yields induce active debt retirement by reducing risk-shifting incentives relative to low convenience yield scenarios (See Figure D.18E). Higher intermediation costs force a greater fraction of future debt to be self-rolled rather than intermediated through investors, increasing default probability and diminishing risk-shifting gains. Conversely, lower

[31] A paper that explores short-term debt vulnerabilities due to its high market value is Gertner and Scharfstein (1991). In their model, a higher debt price leads to increased market leverage, resulting in greater ex-post debt overhang in times of financial distress. This overhang channel differs from the ex-ante price effects influencing leverage dynamics in this paper. Diamond and He (2014) also observes that shorter-term debt's insensitivity to firm value changes leads to more volatile future overhang, triggering earlier default for short-term debt. A similar finding here is that long-term debt supports a higher debt capacity, with the default boundary f_b being larger for long-term debt.

[32] This tension echoes Larry Summers' concerns during the Bear Stearns rescue in the global financial crisis, as quoted by Bernanke: "The Fed may well be on a path that: (1) will fail to contain systemic risk, (2) will invite legitimately all kinds of charges of 'helping friends,' (3) be negative for moral hazard" (Bernanke 2015)



A. Convenience Yield

B. State Variable Evolution

C. Net Issuance Rate

D. LC Risk Premium

FIGURE 7. Comparative Statics: Convenience Yield

convenience yields encourage exploitation of cheap currency market liquidity for risk-shifting.[33]

Beyond direct deleveraging effects, exchange rate dynamics provide an indirect stabilization mechanism: debt repayment increases FC demand, raising the currency risk premium and thereby reducing expected LC depreciation or inducing appreciation. This exchange rate adjustment alleviates firms' effective debt burden—an indirect benefit of higher convenience yield.

While reduced crisis-period liquidity mitigates moral hazard during crises, the improved ex-ante risk-bearing capacity—paralleling the short-term debt case—increases normal-period moral hazard, evidenced by delayed debt retirement in Figure 7B. Though this generates an ex-post recovery versus ex-ante moral hazard trade-off, recovery benefits dominate in this specification.

To further enhance stability while minimizing these adverse effects, improving pre-crisis (market leverage between 0.5 and 0.8) currency market liquidity proves effective. I demonstrate this by reducing normal-period convenience yield to 50 basis points (Figure 7A). This lower

[33]A common concern of moral hazard problem during crises is government provision of liquidity, which can crowd out private incentives for liquidity provision. Here, the investor sector parallels the government, and firms the private sector. By expecting public liquidity support, private firms may forego self-provision during crisis. They have a greater capacity to do their own good before that. The government mitigates this by offering crisis liquidity at penalty rates, following Bagehot's principle, analogous to elevated convenience yields.

convenience yield enables firms to access more affordable liquidity through reduced investor-demanded premiums on LC assets. Consequently, firms require a smaller fraction of rollovers when capital inflows become unsustainable, lowering pre-crisis issuance rates and slowing progression toward high leverage (Figure 7C).

We should also note that pre-crisis liquidity provision offers an additional advantage: its enhancement of firms' risk-bearing capacity creates smaller ex-ante moral hazard concerns compared with crisis-period restrictions on risk-shifting. This advantage stems from the nonlinear response of optimal issuance policies to risk-bearing capacity across leverage levels. When market leverage is low (below 0.5), firms' inherent safety means debt prices and issuance policies show limited sensitivity to default risk reductions. Consequently, pre-crisis liquidity provision generates less tension between debt reduction and ex-ante moral hazard than crisis-period risk-shifting restrictions, suggesting the efficacy of early policy action over crisis-time responses.

IV. Model Extension: Stochastic Volatility and Rare Disasters

Up to this point, exchange rate volatility has not been explicitly considered. The flows analyzed thus far operate at $O(dt)$ without diffusion terms, resulted from the deterministic leverage dynamics over infinitesimal intervals and the simplified modeling of investor flows.[34] This section explores some implications of the model in an environment with stochastic exchange rate volatility. Before doing so, I offer simple modifications that endogenize exchange rate volatility.[35]

First, suppose there is a noise flow in the currency market, $Y_t \cdot \overline{\boldsymbol{\sigma}}\, d\boldsymbol{B}_t$. One interpretation is the presence of noise traders who take a long-short zero capital position in FC and LC risk-free bonds. If they short LC bonds and go long FC bonds, then $Y_t \cdot \overline{\boldsymbol{\sigma}}\, d\boldsymbol{B}_t < 0$, indicating increased demand for FC in the spot market.

To balance these noise-trader flows, I introduce central bank's intervention and movements on households' equity holdings. These flows respond to exchange rate shocks, thereby offsetting the noise traders' actions.

Central Bank Assume the central bank holds substantial FC reserves and conducts sterilized exchange rate interventions. After exchanging currencies, it sterilizes the intervention through open market operations. For example, if the central bank exchanges LC for FC, it immediately shorts LC bonds to sterilize the operation, and vice versa. Under this sterilized policy, the amount of nontradable goods (i.e., LC) in the market remains unchanged.

[34] If local investors take long-term positions in currencies and their positions depend on the aggregate state, changes in those positions should include $O(d\boldsymbol{B}_t)$ terms.

[35] Although the model can accommodate exogenous exchange rate volatility, explicitly modeling sources of volatility adds theoretical completeness without materially affecting the core implications. The properties discussed here are robust to both exogenous and endogenous treatments of volatility.

Suppose the central bank follows the policy rule

$$dZ_t^{CB} = \alpha_c Y_t \cdot \frac{ds_t}{s_t} = \alpha_c Y_t \cdot (\mu_s dt + \boldsymbol{\sigma_s} \cdot d\boldsymbol{B_t}) \tag{27}$$

where dZ_t^{CB} represents the central bank's intervention flow over $[t, t + \Delta t]$, measured as LC risk-free assets purchased against an equivalent short position in FC, adjusted for the exchange rate s_t. The parameter α_c is a positive constant quantifying the central bank's responsiveness to exchange rate fluctuations; it captures how aggressively the bank reacts to exchange rate movements. Thus, when the LC is depreciating $\left(\frac{ds_t}{s_t} > 0\right)$, the central bank purchases LC and sells FC to mitigate future depreciation.

Including the term Y_t preserves system homogeneity and maintains analytical tractability in a Leland-type framework. Economically, it can be interpreted as the central bank's capacity to act: a larger Y_t implies a more robust local economy and a greater ability to intervene.

With this intervention, the exchange rate's volatility remains constant at $\frac{\overline{\sigma}}{\alpha_c}$, consistent with the benchmark model's assumption of constant exchange rate volatility. To extend this framework to stochastic volatility, I further introduce a equity demand structure.

Households Equity Holdings In the benchmark model, firm managers simply rebate all cash flows to households. Here, consider a small modification: firm managers are also equity holders in their own firms and consume the dividend payouts rather than remitting them to households. Their objective function remains formally the same but now reflects utility from consumption rather than from reputation.

Households can also purchase equity shares from firm managers by exchanging tradable goods for nontradable goods and then using the latter to acquire levered equity. A reduced-form specification of households' levered equity holdings is assumed, consistent with the idea that when tradable goods become more expensive (i.e., a higher s_t), households choose to consume fewer tradable goods and instead purchase firm equity, which pays dividends in nontradable goods.[36]

Households' equity shares are assumed given by

$$Z_{et} = \alpha_e \log(s_t) + \eta_e' \log(\boldsymbol{k}_t) + \kappa_{et}$$

where α_e is a positive constant. The model remains flexible enough to incorporate other aggregate factors affecting households' demand, collected in the vector $\boldsymbol{k}_t$, and a latent demand term κ_{et}.

[36] In essence, firm managers sell equity to households in exchange for FC to repay obligations. To maintain a constant interest rate as before, the following adjustments are made to preference parameters: $\chi_t = (1 + \overline{\mu} + \zeta)Y_t + Z_{et}(1 - \overline{\mu})Y_t$ and $\iota_t = s_t(Y_t + \frac{a_{t-}}{s_{t-}}\zeta Y_{T,t-}) - \overline{\mu}Y_t + \alpha_c \mu_s Y_t - \tilde{V}_t \cdot K$, where K is the drift term of dZ_{et}.

These exogenous variables evolve according to

$$d\boldsymbol{k}_t \;=\; \boldsymbol{k}_t\left(\boldsymbol{\mu}_k\, dt + \boldsymbol{\sigma}_k \cdot d\boldsymbol{B}_t\right), \quad d\kappa_e \;=\; \mu_{\kappa e}\, dt + \boldsymbol{\sigma}_{\kappa e} \cdot d\boldsymbol{B}_t.$$

Firms' levered equity trades at $\tilde{V}$, the minimum price acceptable to local equity holders, and the decision to sell equity is determined by households' demand. This setup does not affect firms' leverage decisions but does alter the exchange rate dynamics. The currency market clearing condition now becomes

$$s_t\, dL_t - dA_t + dZ_t^{CB} + \tilde{V}_t dZ_{et} = -Y_t \cdot \overline{\mu} dt - Y_t \cdot \overline{\sigma} d\boldsymbol{B_t}.$$

Lemma 7 details the necessary modifications to the baseline model:

LEMMA 7. *In an equilibrium with equity flows, aggregate bond issuance rate and exchange rate dynamics are:*

(a) *The currency risk premium, with* $\omega(f) = \frac{\varepsilon_{p\tilde{f}}}{\alpha\nu(f)p_t f_t + \varepsilon_{p\tilde{f}}}$, $\nu(f) = \frac{1}{1-\alpha(\alpha_c+\alpha_e v)} > 0$, *and* $\alpha(f) = \hat{\gamma}\|\boldsymbol{\sigma}_s\|^2$, *can be written as:*[37]

$$RP_t = \omega(f)\cdot\left\{\nu(f)\cdot\left[\lambda + \alpha(f)\cdot Flow\right] + (1-\nu(f))\cdot(\rho - r)\right\} - (1-\omega(f))\cdot\left(\varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}\right). \tag{28}$$

where

$$Flow \;= (c+\xi)f_t - \bar{\mu} - v\eta_e'\left(\boldsymbol{\mu}_k - \frac{1}{2}\operatorname{diag}\left(\boldsymbol{\sigma}_k\boldsymbol{\Sigma}\boldsymbol{\sigma}_k'\right)\right) - \mu_{\kappa e}v + \frac{\alpha_e v}{2}\|\boldsymbol{\sigma}_s\|^2$$

(b) *The exchange rate volatility is:*

$$\boldsymbol{\sigma_s} = -\frac{1}{\alpha_c + \alpha_e v}\left(\overline{\boldsymbol{\sigma}} + (\eta_e'\boldsymbol{\sigma}_k + \boldsymbol{\sigma_{\kappa e}})v\right) \tag{29}$$

(c) *The issuance rate,* g/f, *is:*

$$\frac{g_t}{f_t} = \left(\alpha\nu(f)p_t f_t + \varepsilon_{p\tilde{f}}\right)^{-1}\cdot\left\{\nu(f)\cdot\left[\lambda + \alpha(f)\cdot Flow\right] + (1-\nu(f))\cdot(\rho - r) + \varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}\right\}. \tag{30}$$

The logic behind this stochastic volatility is straightforward. Equity prices affect the proceeds generated from equity holdings' movements. During periods of low equity prices, each unit of equity flow generates smaller proceeds, making noise flows more influencial—reflected in

[37] Note that the involvement of central bank creates an additional layer for the currency risk premium.

the volatility function's denominator. The numerator introduces additional randomness through characteristic-based and latent demand shocks. Implementing this extension requires only updating the ODE coefficients as outlined in Proposition 2.

IV.1. Rare Disasters

Having established the microfoundations for stochastic volatility, I examine how the model endogenously generates "rare disaster" features in exchange rate dynamics. This feature emerges naturally from the nonlinearity generated by the debt contract and remains robust to alternative specifications of endogenous volatility.

I analyze the relationship between exchange rate volatility and the currency risk premium by comparing two scenarios: constant volatility versus volatility that increases with market leverage.[38] The stochastic volatility specification matches empirical patterns from major EMEs, where volatility typically hovers around 0.1 in normal times but spikes to 0.25 during near-default crises. The constant volatility case is calibrated higher than stochastic volatility during normal periods conditioning on the market leverage, but stochastic volatility exceeds it near the default boundary (Figure 8A).

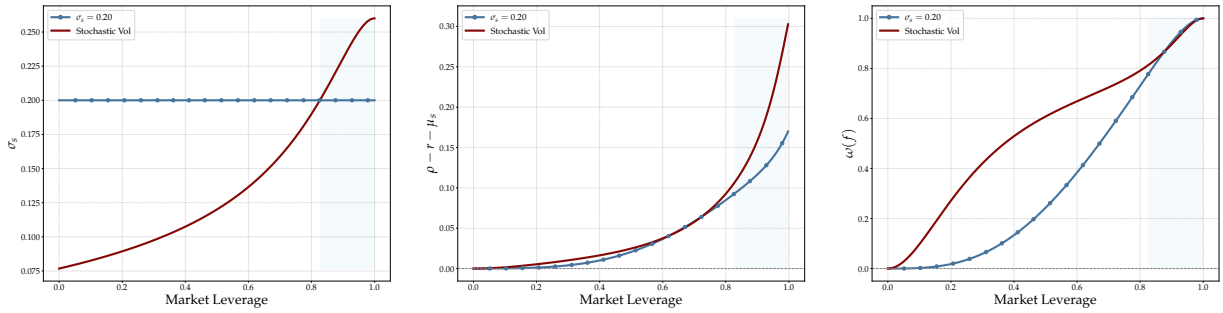
I define the crisis regime as beginning when stochastic volatility surpasses constant volatility (82.8% market leverage). Simulations from the SSS indicate the economy spends merely 2.5% of time above this threshold, confirming these crises as genuinely "rare" events.

Currency risk premium patterns exhibit striking properties (Figure 8B). During normal periods, conditioning on market leverage, the model generates lower currency risk premiums when crisis-period volatility is lower, despite higher current volatility. This demonstrates the premium's heightened sensitivity to crisis-state volatility—a fundamental feature of rare disaster models.

The currency risk premium's heightened sensitivity to crisis periods reflects fundamental properties of the firm sector's risk-bearing capacity. The asymmetric nature of debt contracts—where payoffs are capped on the upside but fully exposed to downside risks—means that high volatility during crises, even if rare, has an outsized impact on firms' default probability and debt prices. This deterioration in debt prices impairs firms' ability to reduce imbalance-driven premiums. Figure 8C captures this asymmetric response, showing that the firm sector's ability to mitigate the currency risk premium, drops significantly (higher $\omega(f)$) when crisis-period volatility is high. This finding further suggests that market leverage alone insufficiently characterizes risk-bearing capacity, since it also depends on cross-state conditions.

Previous literature, such as Farhi and Gabaix (2015), has employed rare disaster models to address exchange rate puzzles. My model, however, introduces two key distinctions. First, rather

[38] These volatility specifications are implemented through calibration of α_c and α_e, with $\boldsymbol{\eta_e} = \mathbf{0}$ and suppressed latent demand.



A. Exchange Rate Volatility B. LC Risk Premium C. Reduction Rate

FIGURE 8. Rare Disasters: Illustration

than relying on an SDF dependent on exogenous crises, it links disasters to endogenous default events through debt contracts. Second, instead of assuming exogenous processes for crisis probability and severity, it endogenizes these features through firms' leverage dynamics. The model demonstrates that the firm sector's risk-bearing capacity is disproportionately impacted by crisis-state volatility and varies with the leverage cycle, thereby generating endogenous 'rare disaster' behavior in exchange rates.

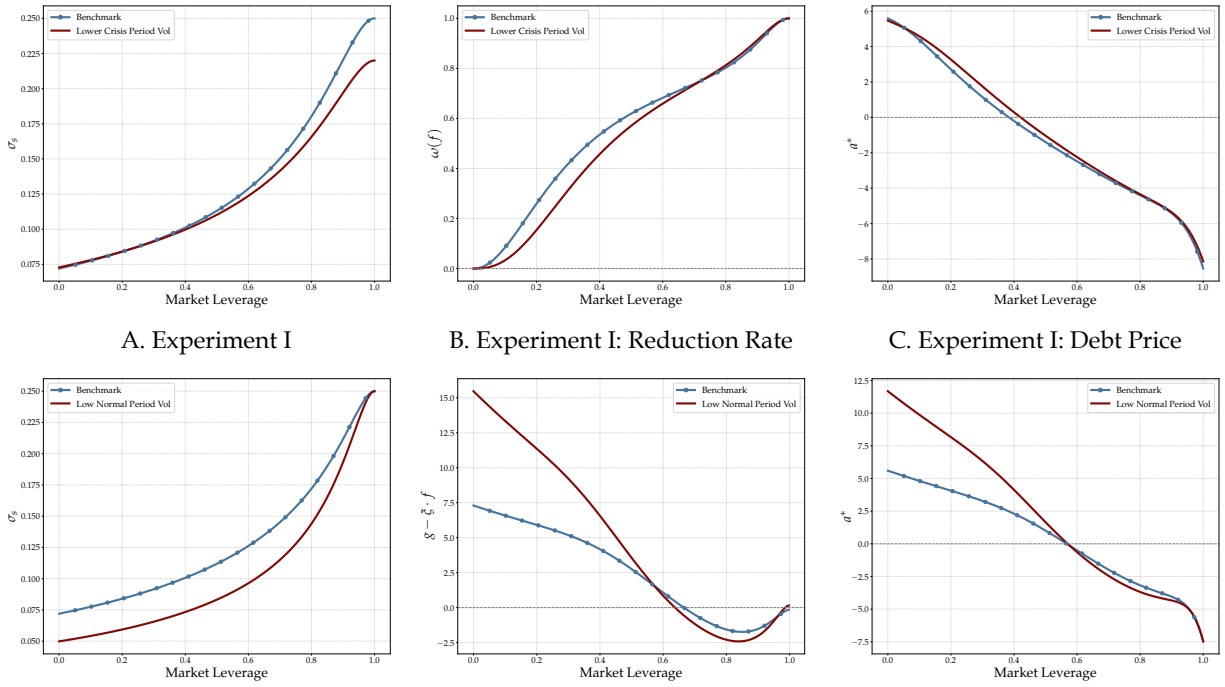
One application of this rare disaster framework is to address the trade-off between exchange rate volatility and the currency risk premium, as identified by Lustig and Verdelhan (2019). By making the premium sensitive to crisis-state volatility, the model can generate substantial premiums even with moderate current volatility, provided crisis-state volatility is sufficiently high. The model's calibrations align well with empirical evidence from KV, generating premiums below 0.05 in normal times that surge to around 0.15 during crises with realistic volatility schemes.

IV.2. Crisis and Normal Period Volatilities

Market volatility is often viewed as a source of financial instability. However, volatility itself is neutral—it doesn't inherently trigger market crashes. Similar to the convenience yield, volatility is simply a condition to which rational market participants respond, adjusting their behavior optimally in anticipation of future market dynamics.

This model suggests that crisis-period volatility can enhance economic resilience from an ex-ante perspective, while pre-crisis volatility poses the greater threat of pushing the economy toward default. This result emerges from the fundamental trade-off between ex-ante and ex-post effects.

I examine this through two comparative static exercises. The first compares economies with similar normal-period volatilities but divergent crisis-period volatilities (Figure 9A); the second contrasts economies with different normal-period volatilities but similar crisis-period levels (Figure 9D).



A. Experiment I | B. Experiment I: Reduction Rate | C. Experiment I: Debt Price

D. Experiment II | E. Experiment II: Net Issuance | F. Experiment II: Investors Position

FIGURE 9. Two Experiments for Exchange Rate Volatility

Panel C of Table 1 presents the financial stability metrics. The simulations reveal that higher crisis-period volatility extends the expected time to economy-wide default and increases recovery probability from distress or sudden stop events. This counterintuitive finding suggests that economies become more resilient under high crisis-period volatility.

The stability enhancement mechanism operates through heightened ex-post default probability under elevated exchange rate volatility, inducing preemptive reductions in net debt issuance at moderate leverage levels. Increased volatility raises firms' inherent default risk, triggering earlier debt price deterioration and deleveraging relative to low-volatility scenarios (see Figures D.19C and D.19D). This preventive behavior promotes market stability. [39]

From a portfolio balancing perspective, high crisis-period volatility raises firms' conditional default probability in crises, diminishing their ex-ante risk-bearing capacity before reaching crisis regions—evidenced by reduced ability to lower currency risk premia (Figure 9B). Consequently, the investor sector must bear more imbalances ex-ante compared to cases where firms require

[39]While expected effective debt reduction rates near market leverage of 1 are slightly higher under high crisis volatility (See Figure D.19E) due to elevated LC risk premia, the higher default probability stems from the volatility term itself—high exchange rate volatility increases the likelihood of extreme adverse shocks.

Unlike the convenience yield case where crisis-period increases reduce risk-shifting through higher self-rollover costs, high crisis-period volatility actually encourages risk-shifting, elevating debt issuance at certain points (Figure D.19B). However, the net stabilizing effect of higher crisis-period volatility prevails by keeping firms below leverage levels where risk-shifting becomes severe.

lower premium for bearing imbalances (Figure 9C), leading to earlier debt retirement. This parallels the reduced ex-ante moral hazard observed in the convenience yield analysis.

Normal-period volatility generates opposing effects: higher volatility during pre-crisis periods (market leverage 0.5-0.8) reduces investor sector liquidity provision, forcing greater reliance on self-rollover and delayed debt retirement (Figures 9E and 9F)—analogous to how high pre-crisis convenience yields accelerate progression toward high leverage states.

These findings yield nuanced policy implications for balancing ex-ante and ex-post stability concerns. While policymakers should reduce pre-crisis volatility to maintain market liquidity, eliminating crisis-period volatility may be undesirable. Some crisis volatility beneficially induces deleveraging, though elevated exchange rate volatility and associated risk-shifting indeed reduce ex-post resilience conditional on crises. Unlike the convenience yield case where ex-post costs (risk-shifting) dominate ex-ante benefits (early deleveraging), here ex-ante benefits outweigh ex-post costs of volatility-induced default risk. This contrast demonstrates that no universal principle resolves the ex-ante/ex-post trade-off—policymakers must carefully calibrate policy targets to specific circumstances.

V. Conclusion

This paper develops a dynamic model that illuminates how leverage and exchange rates coevolve in EMEs, offering a unified framework to tackle key questions about the drivers of external debt accumulation, the coexistence of aggressive borrowing and rational expectations, and the sources of vulnerability to debt crises.

Central to the analysis is the role of the corporate (firm) sector in performing a market-balancing function akin to financial intermediaries with limited risk-bearing capacity. The model highlights how firms' time-varying risk-bearing capacity—shaped by both individual credit risk and expected exchange rate paths—underpins the joint evolution of leverage and exchange rates. Two important insights emerge.

First, interpreting non-financial firms in this way underscores that the traditional notion of "carry trade," usually understood as risk-free arbitrage across currencies, may be too narrow. Borrowing and lending across borders in pursuit of cheaper financing also qualifies as a form of carry trade, driven by the same fundamental incentives.

Second, given that the direction of intermediation is interchangeable and the model makes no assumptions about firms' real activities, we can alternatively interpret the firm sector as financial intermediaries providing currency exchange services to other economic sectors. This interpretation particularly illuminates intermediaries' leverage dynamics in environments where LC risk-

free rates exceed FC rates and these intermediaries actively channel foreign credit into the local economy.

Beyond its immediate contributions, the tractable, globally solved framework in this paper opens several avenues for future inquiry. One fruitful direction involves embedding the model in a broader macroeconomic setting to study how financial and real sectors interact more comprehensively. Another is to incorporate explicit time-series interest rate dynamics—currently absent from the model's focus on currency risk premia—to better understand how fluctuating interest rate differentials shape firms' risk-bearing capacities. Finally, the model's emphasis on liquidity conditions and cross-sectional variations in risk-bearing capacity offers insights for policymakers seeking to bolster financial stability in EMEs. As the global financial landscape continues to evolve, understanding the complex dynamics of leverage and exchange rates will remain crucial for policymakers, investors, and academics alike.

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Online Appendices *for* Leverage and Exchange Rate Cycles

Lewei He

March 2025

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Appendix A. Proofs for Lemmas in Sections I, II and IV

A.1. *Proof for Lemma 1*

Proof. Similar to the approach in DeMarzo and He (2021), the key observation is that an individual firm's decision has no price impact on exchange rate dynamics but will take into account debt issuance's impact on the debt price. With this in mind, the rest of the reasoning follows similarly.

Given any initial debt level $\tilde{F}'$, equity holders can adjust the debt level to $\tilde{F}$ by issuing $\tilde{F} - \tilde{F}'$ at the FC price $p(\tilde{F}, F, Y, s)$. Therefore, the equity value must satisfy:

$$V\left(\tilde{F}', F, Y, s\right) \geq V(\tilde{F}, F, Y, s) + s \cdot p(\tilde{F}, F, Y, s) \cdot \left(\tilde{F} - \tilde{F}'\right)$$

Thus, the negative debt price, $-s \cdot p$, is a subgradient of V, implying that V is convex. Additionally, since s is invariant with respect to $\tilde{F}$, the debt price p must be decreasing in $\tilde{F}$. Given that $s \cdot p > 0$, we can also conclude that V is decreasing in $\tilde{F}$.

When V is strictly convex, the inequality above becomes strict, meaning any discrete issuance becomes suboptimal. In this case, the subgradient becomes the derivative at that point. Using the fact that $V\left(\tilde{F}_t, F_t, Y_t, s_t\right) = v\left(\tilde{f}_t, f_t\right) \cdot Y_t$, we can express the second derivative as:

$$V_{\tilde{F}\tilde{F}} = \frac{s^2}{Y} \cdot v_{\tilde{f}\tilde{f}}$$

Therefore, $v_{\tilde{f}\tilde{f}} > 0$, which confirms that v is also strictly convex. □

A.2. *Proof for Lemma 2*

Proof. The same as DeMarzo and He (2021). The objective function in equation (7) is linear in $\tilde{G}_t$, with the coefficient $s_t \cdot p_t + V_{\tilde{F}}$. This coefficient represents the net marginal gain from issuing debt, accounting for the revenue from the debt sale minus the incremental cost of future debt obligations for equity holders (note that $V_{\tilde{F}} < 0$). In a smooth MPE, where equity holders optimally issue debt continuously, the following condition must hold:

$$s_t \cdot P(\tilde{F}_t, F_t, Y_t, s_t) = -V_{\tilde{F}}(\tilde{F}_t, F_t, Y_t, s_t), \tag{A.1}$$

or equivalently:

$$p(\tilde{f}_t, f_t) = -v_{\tilde{f}}(\tilde{f}_t, f_t). \tag{A.2}$$

Substituting equation (A.1) into the HJB equation for the equity value gives:

$$\begin{aligned}\rho V(\tilde{F}_t, F_t, Y_t, s_t) =& Y_t - (c+\xi)\tilde{F}_t s_t - \xi \tilde{F}_t V_{\tilde{F}} + (G_t - \xi F_t) V_F \\ &+ s_t \mu_s V_s + \mu Y_t V_Y + \frac{1}{2}\|\boldsymbol{\sigma}_s\|^2 s_t^2 V_{ss} + \frac{1}{2}\|\boldsymbol{\sigma}\|^2 Y_t^2 V_{YY} + \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} s_t Y_t V_{sY},\end{aligned} \tag{A.3}$$

which is independent of the individual issuance, $\tilde{G}_t$ now.

Given an equilibrium smooth issuance strategy and default strategy, modifying the issuance strategy to $\tilde{G}_t = 0$ makes no changes to the equity value since setting $\tilde{G}_t = 0$ in equation (7) directly implies equation (A.3). Hence, the equity value $V(\tilde{F}_t, F_t, Y_t, s_t)$ can be obtained under no future issuance. Thus, $V \leq V^0$ since V^0 is the maximum achievable equity value under no issuance given any default strategies.

But at the same time, $V \geq V^0$ by definition. We must have $V = V^0$. Scaling by Y_t gives the second equality. □

A.3. Proof for Lemma 3

Proof. Denoting the optimal issuance strategy of an individual firm by $\tilde{G}_t^*$, we can write the HJB equation for the debt price as follows:

$$\begin{aligned}r p(\tilde{F}_t, F_t, Y_t, s_t) =& c + \xi - \xi p + \left(\tilde{G}_t^* - \xi \tilde{F}_t\right) p_{\tilde{F}} + (G - \xi F) p_F + s_t \mu_s p_s + \mu Y_t p_Y \\ &+ \frac{1}{2}\|\boldsymbol{\sigma}_s\|^2 s_t^2 p_{ss} + \frac{1}{2}\|\boldsymbol{\sigma}\|^2 Y^2 p_{YY} + \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} s_t Y_t p_{sY}.\end{aligned} \tag{A.4}$$

On the other hand, differentiating equation (A.3) with respect to $\tilde{F}$, and substituting equation (A.1) into it, gives:

$$\begin{aligned}-\rho p(\tilde{F}_t, F_t, s_t, Y_t) s_t =& -(c+\xi) s_t + \xi p s_t + \xi \tilde{F}_t p_{\tilde{F}} s_t - (G - \xi F) p_F s_t + s_t \mu_s (-p - p_s s_t) - \mu Y_t p_Y s_t \\ &+ \frac{1}{2} s_t^2 \|\boldsymbol{\sigma}_s\|^2 (-p_{ss} s_t - 2 p_s) + \frac{1}{2} Y_t^2 \|\boldsymbol{\sigma}\|^2 (-p_{YY} s_t) + \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_{\boldsymbol{s}} s_t Y_t \left(-p_{sY} s_t - p_Y\right).\end{aligned} \tag{A.5}$$

By comparing these two equations, we can derive the optimal issuance level. □

A.4. Proof for Lemma 4

Proof. Aggregate issuance implied by the BoP condition. Then LC risk premium implied by equation (12). □

A.5. Proof for Lemma 5

The proof closely follows Campbell (2017), Chapter 2.

Proof. The investors' problem is:

$$\max_{a} \qquad \mathbb{E}_t\left[\frac{\hat{C}_t^{1-\gamma}}{1-\gamma}\right], \tag{A.6}$$

$$\text{s.t.} \quad \hat{C}_t = \zeta Y_t \cdot \left[(1-a)e^{\rho\Delta t} + ae^{(r+\lambda+\mu_s-\frac{\|\boldsymbol{\sigma_s}\|^2}{2})\Delta t+\boldsymbol{\sigma_s}\cdot\Delta\boldsymbol{B_t}}\right]. \tag{A.7}$$

Following the approximation to the objective function in Campbell (2017), equation (A.6) is equivalent to:

$$\max\left[\log\left(\mathbb{E}_t\hat{C}_t^{1-\gamma}\right)\right] = \max\left[(1-\gamma)\mathbb{E}_t\hat{c}_t + \frac{1}{2}(1-\gamma)^2\sigma^2_{\hat{c}_t}\right],$$

where $\hat{c}_t = \log\hat{C}_t$, and $\mathbb{E}_t\hat{c}_t$ and $\sigma^2_{\hat{c}_t}$ are its conditional mean and variance.[1]

Let $e^{c_{t-}} = \zeta Y_t$, the budget constraint is:

$$\hat{c}_t = r_{p,t} + c_{t-},$$

where $r_{p,t} = \log\left((1-a)e^{\rho\Delta t} + ae^{(r+\lambda+\mu_s-\frac{\|\boldsymbol{\sigma_s}\|^2}{2})\Delta t+\sigma_s\cdot\Delta\boldsymbol{B_t}}\right)$ is the log return on the portfolio. Substituting this into the objective function, we can restate the problem as:

$$\max_{a_t} \left[\mathbb{E}_t r_{p,t+1} + \frac{1}{2}(1-\gamma)\sigma^2_{pt}\right], \tag{A.8}$$

We know the excess return of the portfolio is:

$$\begin{aligned} r_{p,t} - \rho\Delta t &= \log\left(1 + a_t\left(e^{(\mu_s+r+\lambda-\rho-\frac{\|\boldsymbol{\sigma_s}\|^2}{2})\Delta t+\boldsymbol{\sigma_s}\cdot\Delta\boldsymbol{B_t}} - 1\right)\right) \\ &\approx \left[a_t(\mu_s + r + \lambda - \rho) - \frac{1}{2}a_t^2\|\boldsymbol{\sigma}_s\|^2\right]\Delta t + a_t\boldsymbol{\sigma_s}\cdot\Delta\boldsymbol{B_t}, \end{aligned}$$

where the approximation is through a second-order Taylor expansion of $\log(1 + a(e^x - 1))$ around $x = 0$. Then the problem (A.8) is equivalent to:

$$\max_{a_t} \quad a_t(\mu_s + r + \lambda - \rho) - \frac{1}{2}a_t\|\boldsymbol{\sigma}_s\|^2 + \frac{1}{2}(1-\gamma)a_t^2\|\boldsymbol{\sigma}_s\|^2. \tag{A.9}$$

[1] When $1-\gamma < 0$, it becomes a minimization problem, but the solution remains the same.

The solution is:

$$a_t = \frac{\mu_s + r + \lambda - \rho}{\gamma \|\boldsymbol{\sigma}_s\|^2}, \tag{A.10}$$

which is the standard mean-variance form. □

A.6. Proof for Lemma 6

Proof. The BoP condition requires

$$\overline{\boldsymbol{\sigma}} - \alpha_c \boldsymbol{\sigma_s} = 0 \Rightarrow \sigma_s = \frac{\overline{\sigma}}{\alpha_c},$$

and

$$p_t g_t - (c + \xi) f_t + \overline{\mu} - \alpha_c \mu_s = a_t^* \equiv \frac{\mu_s + r + \lambda - \rho}{\gamma \|\boldsymbol{\sigma}_s\|^2}.$$

Using the optimal issuance policy evaluated along the equilibrium line $\tilde{f} = f$:

$$g_t / f_t = \frac{\rho - r - \mu_s + \varepsilon_{pY} \phi \sigma_s \sigma + \varepsilon_{ps} \|\boldsymbol{\sigma}_s\|^2}{-f_t v_{\tilde{f}\tilde{f}} / v_{\tilde{f}}}.$$

we can express the aggreagte issuance rate and the currency risk premium in terms of the equity value function as in the lemma. □

A.7. Proof for Lemma 7

Proof. Using Ito's lemma, we can express

$$dZ_{et} = \left(\alpha_e \mu_s + \eta_e' \boldsymbol{\mu}_k + \mu_{\kappa e} + \frac{1}{2} \left(-\alpha_e \sigma_s \sigma_s' - \eta_e' \text{diag} \left(\boldsymbol{\sigma}_k \boldsymbol{\Sigma} \boldsymbol{\sigma}_k' \right) \right) \right) dt + \left(\alpha_e \boldsymbol{\sigma_s} + \eta_e' \boldsymbol{\sigma}_k + \boldsymbol{\sigma_{\kappa e}} \right) \cdot d\boldsymbol{B_t}.$$

The BoP condition now requires

$$\begin{aligned} (O(dt)) \qquad \frac{\mu_s + r + \lambda - \rho}{\gamma \|\boldsymbol{\sigma}_s\|^2} =& p_t g_t - (c + \xi) f_t + \overline{\mu} + (\alpha_c + \alpha_e v) \mu_s \\ & - \alpha_e v \frac{1}{2} \|\boldsymbol{\sigma}_s\|^2 + v \eta_e' \left(\boldsymbol{\mu}_k - \frac{1}{2} \text{diag} \left(\boldsymbol{\sigma}_k \boldsymbol{\Sigma} \boldsymbol{\sigma}_k' \right) \right) + \mu_{\kappa e} v \end{aligned}$$

$$(O(d\boldsymbol{B_t})) \qquad \overline{\boldsymbol{\sigma}} + \alpha_c \boldsymbol{\sigma_s} + \left(\alpha_e \boldsymbol{\sigma_s} + \eta_e' \boldsymbol{\sigma}_k + \boldsymbol{\sigma_{\kappa e}} \right) v = 0$$

which implies that

$$\boldsymbol{\sigma_s} = -\frac{1}{\alpha_c + \alpha_e v}\left(\overline{\sigma} + (\eta_e' \boldsymbol{\sigma}_k + \boldsymbol{\sigma_{\kappa e}})v\right)$$

and by defining $\nu(f) = \frac{1}{1+\alpha(\alpha_c+\alpha_e v)}$

$$\mu_s = \nu(f)\left[-r - \lambda + \rho + \alpha\left(p_t g_t - (c+\xi)f_t + \bar{\mu} + v\eta_e'\left(\boldsymbol{\mu}_k - \frac{1}{2}\operatorname{diag}\left(\boldsymbol{\sigma}_k \boldsymbol{\Sigma} \boldsymbol{\sigma}_k'\right)\right) + \mu_{\kappa e} v - \frac{\alpha_e v}{2}\|\boldsymbol{\sigma}_s\|^2\right)\right].$$

It implies

$$\begin{aligned} RP_t = \rho - r - \mu_s =& \nu(f)\left[\lambda - \alpha\left(p_t g_t - (c+\xi)f_t + \bar{\mu} + v\eta_e'\left(\boldsymbol{\mu}_k - \frac{1}{2}\operatorname{diag}\left(\boldsymbol{\sigma}_k \boldsymbol{\Sigma} \boldsymbol{\sigma}_k'\right)\right) + \mu_{\kappa e} v - \frac{\alpha_e v}{2}\|\boldsymbol{\sigma}_s\|^2\right)\right] \\ &+ (1 - \nu(f))(\rho - r) \end{aligned}$$

Define

$$\text{Flow } = (c+\xi)f_t - \bar{\mu} - v\eta_e'\left(\boldsymbol{\mu}_k - \frac{1}{2}\operatorname{diag}\left(\boldsymbol{\sigma}_k \boldsymbol{\Sigma} \boldsymbol{\sigma}_k'\right)\right) - \mu_{\kappa e} v + \frac{\alpha_e v}{2}\|\boldsymbol{\sigma}_s\|^2$$

we can write:

(A.11)

$$RP_t = \omega(f) \cdot \left\{\nu(f) \cdot \left[\lambda + \alpha \cdot \text{Flow}\right] + (1 - \nu(f)) \cdot (\rho - r)\right\} - (1 - \omega(f)) \cdot \left(\varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}\right).$$

The issuance-to-outstanding ratio, g/f, is:

(A.12)

$$\frac{g_t}{f_t} = \left(\alpha\nu(f)p_t f_t + \varepsilon_{p\tilde{f}}\right)^{-1}\left\{\nu(f) \cdot \left[\lambda + \alpha \cdot \text{Flow}\right] + (1 - \nu(f)) \cdot (\rho - r) + \varepsilon_{ps}\|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY}\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}\right\}.$$

□

Appendix B. Proof for Propositions 1 and 2

In this section, I primarily present the proof for Proposition 2 with the involvement of equity flows as in Section IV, since it is the most comprehensive model. However, I will also discuss the modifications required for the proof of Proposition 1 for The One-sector Model, and Proposition 2 for the model without equity flows.

In the main text setup, I use $(\tilde{f}, f)$ as the state variables representing the scaled equity value. While writing the HJB equation, it is advantageous to change the variables to $a = \frac{\tilde{f}}{f} = \frac{\tilde{F}}{F}$ and f as the state variables, to facilitate variable separation.

I am slightly abusing notation by still using v as the scaled equity value function, i.e. $v(a, f)$. One can verify the following transformations from the original system, using $(\tilde{f}, f)$ as state variables, to the transformed system with (a, f) as state variables. The left-hand side (LHS) represents the original system, while the right-hand side (RHS) represents the transformed system:

$$
\begin{aligned}
&\tilde{f} v_{\tilde{f}} = a v_a \\
&f v_f = -a v_a + f v_f \\
&f^2 v_{ff} = 2a v_a + a^2 v_{aa} + f^2 v_{ff} - 2\tilde{f} v_{a\tilde{f}} \\
&\tilde{f} f v_{\tilde{f}f} = -a v_a - a^2 v_{aa} + \tilde{f} v_{af} \\
&\tilde{f}^2 v_{\tilde{f}\tilde{f}} = a^2 v_{aa}
\end{aligned}
$$

Next I summarize key equilibrium variables and formulas under the new system of state variables, which will be used for characterizing equity value.

1. **Price**:

$$p = -v_a(a, f) \cdot \frac{1}{f}$$

2. **Price Elasticities**:

$$\varepsilon_{ps} = 1 - \frac{v_{af}}{v_a} f, \quad \varepsilon_{pY} = 1 - \frac{v_{af}}{v_a} f, \quad \varepsilon_{p\tilde{f}} = -a \frac{v_{aa}}{v_a}.$$

3. **Issuance Policy**:

$$\frac{\tilde{g}_t}{\tilde{f}_t} = \frac{\rho - r - \mu_s + \left(1 - \frac{v_{af}}{v_a} f\right)\left(-\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s + \|\boldsymbol{\sigma}_s\|^2\right)}{-a \cdot \frac{v_{aa}}{v_a}}$$

4. **Aggregate Issuance Rate**:

$$\frac{g_t}{f_t} = -\left(\alpha \nu(f) v_a + \frac{v_{aa}}{v_a}\right)^{-1} \left[\nu(f) \cdot \left[\lambda + \alpha \cdot \text{Flow}\right] + (1 - \nu(f)) \cdot (\rho - r - \frac{1}{2}\|\boldsymbol{\sigma}_s\|^2)\right.$$

$$+\left(1-\frac{v_{af}}{v_a}f\right)\left(-\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}+\|\boldsymbol{\sigma}_s\|^2\right)\Bigg]$$

where

$$\text{Flow} = (c+\xi)f_t - \overline{\mu} + v\eta_e'\left(\boldsymbol{\mu}_k - \frac{1}{2}\text{diag}\left(\boldsymbol{\sigma}_k\boldsymbol{\Sigma}\boldsymbol{\sigma}_k'\right)\right) + \mu_{\kappa e},$$
$$\nu(f) = \frac{1}{1+\alpha(\alpha_c+\alpha_e v)}.$$

In Section II:

$$\text{Flow} = (c+\xi)f_t - \overline{\mu}, \quad \nu = \frac{1}{1+\alpha\alpha_c},$$

and there is no Itô term, $-\frac{1}{2}\|\boldsymbol{\sigma}_s\|^2$ in the expression g_t/f_t.
In The One-sector Model of :

$$\frac{g_t}{f_t} = \frac{1}{p_t}\cdot((c+\xi)f_t - \overline{\mu})$$

5. **Currency Risk Premium**:

$$\begin{aligned} RP_t =& \omega(f)\cdot\left\{\nu(f)\cdot\left[\lambda+\alpha\cdot\text{Flow}\right] + (1-\nu(f))\cdot\left(\rho - r - \frac{1}{2}\|\boldsymbol{\sigma}_s\|^2\right)\right\} \\ &- (1-\omega(f))\cdot\left(1-\frac{v_{af}}{v_a}f\right)\left(-\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}+\|\boldsymbol{\sigma}_s\|^2\right). \end{aligned}$$

where $\omega(f) = \frac{A(f)}{A(f)+B(f)}$, and

$$A(f) = -\frac{v_{aa}}{v_a}\Big|_{a=1}, \quad B(f) = -\alpha\nu(f)v_a\Big|_{a=1}.$$

In Section II, $\nu(f) = \nu = \frac{1}{1+\alpha\alpha_c}$. In The One-sector Model, set $\alpha = \nu = 1$:

$$RP_t = \frac{A(f)}{B(f)}\left((c+\xi)f_t - \overline{\mu}\right) - \left(1-\frac{v_{af}}{v_a}f\right)\left(-\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}+\|\boldsymbol{\sigma}_s\|^2\right).$$

6. **Dynamics of** f:

$$\frac{df_t}{f_t} = \left(\mu_s - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s} + \frac{g}{f} - \xi\right)dt + (\boldsymbol{\sigma_s}-\boldsymbol{\sigma})\cdot d\boldsymbol{B_t}$$

Substituting $\mu_s = \rho - r - RP_t$ and $\frac{g_t}{f_t}$:

$$\mu_s + \frac{g_t}{f_t} = \rho - r + \frac{-A(f)+1}{A(f)+B(f)} \left\{ \nu(f) \cdot \left[\lambda + \alpha \cdot \text{Flow} \right] + (1-\nu(f)) \cdot \left(\rho - r - \frac{1}{2} \|\boldsymbol{\sigma}_s\|^2 \right) \right\} + \frac{B(f)+1}{A(f)+B(f)} \left(1 - \frac{v_{af}}{v_a} f \right) \left(-\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s + \|\boldsymbol{\sigma}_s\|^2 \right).$$

And in Section I,

$$\mu_s + \frac{g}{f} = \rho - r + \frac{-A(f)+1}{B(f)} \left((c+\xi) f_t - \overline{\mu} \right) + \left(1 - \frac{v_{af}}{v_a} f \right) \left(-\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s + \|\boldsymbol{\sigma}_s\|^2 \right).$$

7. **Dynamics of** a:

$$\frac{da}{a} = -\frac{g}{f} dt.$$

under the deviating strategy of no future issuance. In equilibrium, $a = 1$ always.

B.1. Derivation of Equity Value

We now characterize the equity value function. In the main text, I decompose the equity value as follows:

$$v(a, f) = \overline{v}(a, f) + D(a, f)$$

which is the non-default equity value plus the default option value.

We only need to rewrite it using state variables (a, f) rather than $(\tilde{f}, f)$.

(a) **No Default Equity Value:** The value of equity assuming no default, denoted by $\overline{v}(a, f)$, is given by:

$$\overline{v}(a, f) = \frac{1}{\rho - \mu} - (c+\xi) a f \cdot \mathbb{E} \left[\int_t^\infty e^{-(\rho+\xi)(\tau - t)} \frac{s_\tau}{s_t} d\tau \right]$$

Here, $E(f) = \mathbb{E} \left[\int_t^\infty e^{-(\rho+\xi)(\tau-t)} \frac{s_\tau}{s_t} d\tau \right]$ is fully determined by f, and characterizes the impact of future exchange rate dynamics. $E(f)$ satisfies the following ODE:

$$(\rho + \xi) E = 1 + \mu_f(f) \cdot f E_f + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 E_{ff} \tag{B.1}$$

(b) **Default Option Value:** The default option value, $D(a, f)$, is expressed as:

$$D(a, f) = \mathbb{E}^0 \left[e^{-(\rho+\xi)\tau_b} (0 - \overline{v}(a_b, f_b)) \right]$$

Here, $(a_b, f_b) \in \partial O$, where O denotes the default region. The expectation is taken over the defaulting time τ_b and the defaulting individual and aggregate state variables' position (a_b, f_b). $D(a, f)$ satisfies the following PDE:

$$(\rho + \xi)D = -\frac{g}{f} \cdot aD_a + \mu_f(f) \cdot fD_f + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 D_{ff} \tag{B.2}$$

Next, we transform the PDE presented in equation (B.2) into ODEs, leveraging its separable form.

Conjecture that D adopts a multiplicative form, i.e., $D(a, f) = M(a)N(f)$. By substituting and rearranging, we obtain:

$$\eta = -\frac{aM'(a)}{M(a)} = \left(\frac{g}{f}\right)^{-1} \left[(\rho + \xi)N - \mu_f(f) \cdot fN'(f) - \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 N''(f)\right]$$

Here, η serves as the separation constant.[2] $M(a)$ solves

$$\eta = -\frac{aM'(a)}{M(a)}$$

Without loss of generality, I normalize $M(a)$ such that the constant multiplier is 1, resulting in: [3]

$$M(a) = a^\eta$$

and $N(f)$ should solve the following ODE:

$$(\rho + \xi)N = -\frac{g}{f}\eta + \mu_f(f) \cdot fN'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 N''(f)$$

Thus far, we have identified a class of solutions parameterized by the separation constant η, which satisfy the PDE Equation (B.2): $D \in span(\mathcal{W})$, where

$$\mathcal{W} = \left\{\left(\frac{\tilde{f}}{f}\right)^\eta \cdot N(f) : \eta \in \mathbb{R}\right\}, \tag{B.3}$$

since any linear combination of these solutions also constitutes a solution to the PDE. To determine the specific solutions to our problem, we must apply the boundary conditions and solve for the appropriate values of η (or a specific linear combination).

[2] See Evans (2010) Chapter 4.

[3] We can always scale the $N(f)$ function to absorb the multiplier.

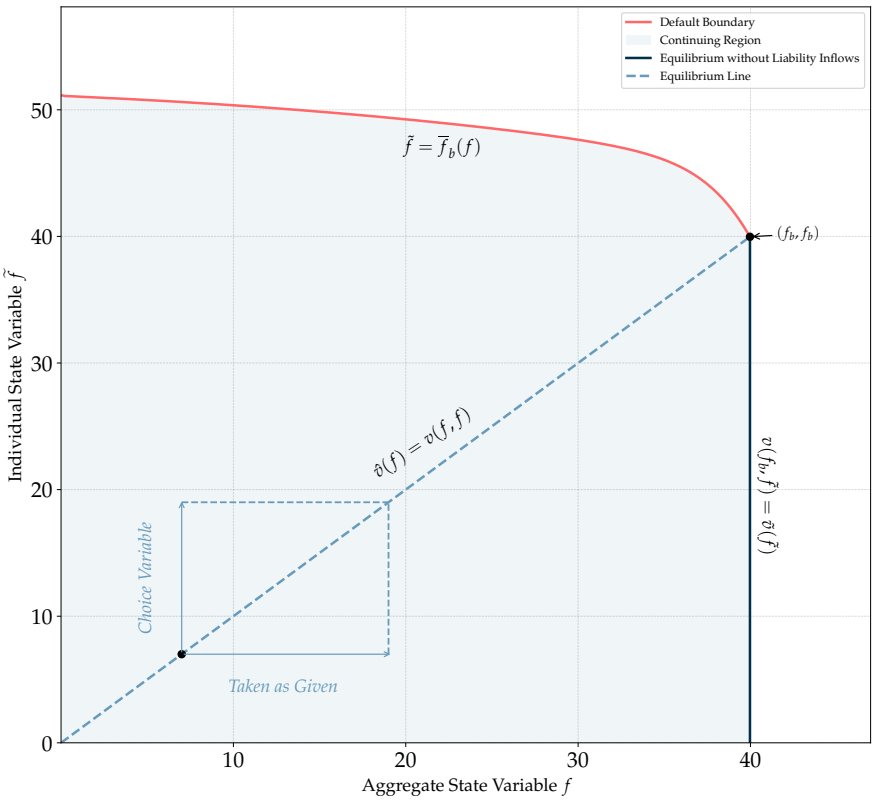


FIGURE B.1. State Space Illustration

B.2. Boundary Conditions

Having completed the variable separation task, we can now return to the $(\tilde{f}, f)$ system to clearly delineate the boundaries, as this system offers a more intuitive understanding. But keep in mind, it is equivalent to do it in the (a, f) system.

First, we focus on the equilibrium line where $\tilde{f} = f$. Drawing from standard Leland-type models, I conjecture a threshold equilibrium. Consider a threshold equilibrium characterized by a boundary $f_b > 0$. Define the continuation region along $\tilde{f} = f$ as $\mathcal{C}^e = \{(\tilde{f}, f) : 0 \leq \tilde{f} = f < f_b\}$. In this equilibrium, a firm continues operations if and only if $(\tilde{f}, f) \in \mathcal{C}^e$.

The derivation of the default boundary requires examining the properties of the equity value function at $f = f_b$, represented by the vertical line in Figure B.1. At this boundary, the economy experiences an economy-wide default as f reaches f_b. However, a deviating firm with positive equity value will continue operating, corresponding to the value $\tilde{f} < f_b$.[4]

[4]The equity value function is decreasing in $\tilde{f}$, implying that the region below the default boundary constitutes a continuation region.

ASSUMPTION B.1. *After an economy-wide default (excluding a measure-zero set of firms), the following balance of payments condition holds:*

$$-dA_t + dZ_t^{CB} = -Y_t(\overline{\mu}dt + \overline{\boldsymbol{\sigma}} \cdot d\boldsymbol{B_t}), \tag{B.4}$$

that is, there are no future capital inflows through debt or equity, and the asset outflows balance the exogenous flows.

This assumption specifies one boundary condition for our analysis. When other firms default, each individual firm anticipates that the exchange rate will adjust to maintain the balance of payments condition in an equilibrium devoid of capital inflows. In this scenario, the exchange rate drift and volatility become constants:

$$\frac{ds_t}{s_t} = \hat{\mu}dt + \hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} \cdot d\boldsymbol{B_t}, \tag{B.5}$$

where

$$\hat{\mu} = (1 + \alpha\alpha_c)^{-1}\left[\rho - r - \lambda + \alpha\overline{\mu}\right],$$
$$\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} = \frac{\overline{\boldsymbol{\sigma}}}{\alpha_c},$$

The equity value in this equilibrium, denoted by $\tilde{v}(\tilde{f})$, is derived in Appendix C.[5]

The implications for our analysis are as follows. The value matching condition requires that along the line $f = f_b$, we must have $v(\tilde{f}_t, f_b) = \tilde{v}(\tilde{f}_t)$. Since this condition holds for any $\tilde{f}_t$, it follows that $v_{\tilde{f}}(\tilde{f}_t, f_b) = \tilde{v}'(\tilde{f}_t)$, given that both of them are assumed to be differentiable. At the default point (f_b, f_b), the equity value must be zero, and the smooth pasting condition requires continuity in the first order derivative at this boundary:

$$v(f_b, f_b) = 0, \quad v_{\tilde{f}}(f_b, f_b) = 0.$$

These conditions imply $\tilde{v}(f_b) = v(f_b, f_b) = 0$ and $\tilde{v}'(f_b) = v_{\tilde{f}}(f_b, f_b) = 0$. These conditions are satisfied when $f_b = \hat{f}_b$, where $\hat{f}_b$ represents the default boundary in an equilibrium without capital inflow, derived in Appendix C:

$$f_b = \hat{f}_b = \frac{\tilde{\eta}}{\tilde{\eta} - 1}\frac{1}{\rho - \mu}\frac{\rho + \xi - \hat{\mu}}{c + \xi} > 0 \tag{B.6}$$

[5] In The One-sector Model without investors and central bank, we assume firms hold a common belief about exchange rate dynamics after an economy-wide default, following the GBM in equation (B.5). For calibration purposes, we set this exchange rate process identical to the model in Section II for consistent comparison.

where $\tilde{\eta} > 0$ solves the quadratic equation:

$$\frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma_s}\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}}{2}\tilde{\eta}^2 + \left(\hat{\mu} - \mu + \|\boldsymbol{\sigma}\|^2 - \xi - \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma_s}\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma_s}}{2}\right)\tilde{\eta} - (\rho + \xi) = 0 \tag{B.7}$$

Under Assumption C.1, we have $\tilde{\eta} > 1$. I summarize these results in the following lemma:

LEMMA B.1. *The following results hold:*

(a) *The value matching condition:* $v(f_b, f_b) = \tilde{v}(f_b) = 0$
(b) *The smooth pasting condition:* $\tilde{v}'(f_b) = v_{\tilde{f}}(f_b, f_b) = 0$
(c) *The default boundaries coincide:* $f_b = \hat{f}_b$

Furthermore, the value matching condition, $v(\tilde{f}_t, f_b) = \tilde{v}(\tilde{f}_t)$, constrains the value of η and consequently determines the form of the equity value function.

LEMMA B.2. *The equity value function takes the form:*

$$v(\tilde{f}, f) = \frac{1}{\rho - \mu} - (c + \xi)\tilde{f} \cdot E(f) + \left(\frac{\tilde{f}}{f}\right)^{\eta} \cdot N(f) \tag{B.8}$$

where $\eta = \tilde{\eta}$. *Moreover, the functions* E *and* N *satisfy the boundary conditions:*

$$E(f_b) = \frac{1}{\rho + \xi - \hat{\mu}}, \quad N(f_b) = \frac{1}{\rho - \mu}\frac{1}{\eta - 1} \tag{B.9}$$

Proof. The value matching condition requires:

$$v(\tilde{f}, f_b) = \tilde{v}(\tilde{f}_t) = \frac{1}{\rho - \mu} - \frac{(c + \xi)\tilde{f}}{\rho + \xi - \hat{\mu}} + \left(\frac{\tilde{f}}{\hat{f}_b}\right)^{\tilde{\eta}} \frac{1}{\rho - \mu}\frac{1}{\eta - 1} \tag{B.10}$$

From previous characterizations, we know:

$$v(\tilde{f}, f) = \frac{1}{\rho - \mu} - (c + \xi)\tilde{f} \cdot E(f) + D(\tilde{f}, f) \tag{B.11}$$

where

$$D(\tilde{f}, f) \in \text{span}\,(\mathcal{W}) \tag{B.12}$$

Evaluating at $f = f_b$, to satisfy equation (B.10) for all $\tilde{f} \in [0, f_b]$, we must have:

$$E(f_b) = \frac{1}{\rho + \xi - \hat{\mu}} \tag{B.13}$$

$$N(f_b) = \frac{1}{\rho - \mu}\frac{1}{\eta - 1} \tag{B.14}$$

$$D(\tilde{f}, f_b) = \left(\frac{\tilde{f}}{f_b}\right)^{\tilde{\eta}} \cdot N(f_b) \tag{B.15}$$

which necessarily implies $\eta = \tilde{\eta}$. □

Having established the functional form of the equity value function and the default point along the equilibrium line, we must extend our analysis to the broader state space. The default boundary within the equilibrium line, (f_b, f_b), represents just one point on the default boundary. When considering deviations, a firm must account for all potential issuance policies and all resulted future default points that may differ from the equilibrium default point (f_b, f_b). Thus, while we know one point (f_b, f_b) on this free boundary, we must derive its complete form, represented by the red curve in Figure B.1.

Lemma B.3 (Default Boundary Properties). *The default boundary in a smooth issuance equilibrium can be expressed as a function of f, denoted as $\overline{f}_b(f)$ for $f \in (0, f_b]$.*

Proof. The existence and uniqueness of this boundary function follows from two key properties of the equity value function $v(\tilde{f}, f)$:

(a) The boundary values satisfy:

$$\lim_{\tilde{f}\to+\infty} v(\tilde{f}, f) = 0 \quad \text{and} \quad \lim_{\tilde{f}\to 0} v(\tilde{f}, f) > 0 \tag{B.16}$$

since firms has positive equity value with zero debt outstanding and default under infinite debt burden.

(b) $v(\tilde{f}, f)$ is strictly decreasing in the firm's debt-to-earnings ratio, $\tilde{f}$, in a smooth equilibrium

These properties ensure the existence of a well-defined default boundary, $\overline{f}_b(f)$.[6] □

To characterize this boundary, we employ value matching and smooth pasting conditions. The value matching condition requires:

$$v(\overline{f}_b(f), f) = \frac{1}{\rho - \mu} - (c + \xi)\overline{f}_b(f) \cdot E(f) + \left(\frac{\overline{f}_b(f)}{f}\right)^{\eta} \cdot N(f) = 0 \tag{B.17}$$

[6] We allow $\overline{f}_b(f)$ to take the value of $+\infty$.

The smooth pasting conditions require:

$$v_{\tilde{f}}(\overline{f}_b(f), f) = -(c+\xi)\cdot E(f) + \eta\left(\frac{\overline{f}_b(f)}{f}\right)^{\eta-1}\frac{1}{f}\cdot N(f) = 0 \tag{B.18}$$

$$v_f(\overline{f}_b(f), f) = -(c+\xi)\overline{f}_b(f)\cdot E'(f) + \left(\frac{\overline{f}_b(f)}{f}\right)^{\eta} N'(f) + \frac{-\eta}{f}\left(\frac{\overline{f}_b(f)}{f}\right)^{\eta}\cdot N(f) = 0 \tag{B.19}$$

Combining equations (B.17) and (B.18), we find

$$\frac{1}{\rho-\mu} + (1-\eta)\left(\frac{\overline{f}_b(f)}{f}\right)^{\eta}\cdot N(f) = 0 \tag{B.20}$$

$$\Rightarrow \quad \overline{f}_b(f) = \left(\frac{1}{\rho-\mu}\frac{1}{\eta-1}\frac{1}{N(f)}\right)^{1/\eta}\cdot f, \tag{B.21}$$

and

$$N(f) = \frac{1}{\rho-\mu}\frac{1}{\eta-1}\left((\rho-\mu)\frac{\eta-1}{\eta}(c+\xi)fE(f)\right)^{\eta} \tag{B.22}$$

Thus, we have obtained a characterization of the default boundary dependent on either function $N(f)$ or function $E(f)$ that we are going to solve.

We can futher verify that equation (B.19) is satisfied by direct substitution:

$$\begin{aligned}
v_f(\overline{f}_b, f) &= -(c+\xi)\overline{f}_b\cdot E'(f) + \left(\frac{\overline{f}_b}{f}\right)^{\eta} N'(f) + \frac{-\eta}{f}\left(\frac{\overline{f}_b}{f}\right)^{\eta}\cdot N(f) \\
&= \frac{\eta\overline{f}_b}{f^2}\Big((\rho-\mu)(\eta-1)N(f)\Big)^{(1-\eta)/\eta} N(f) - \frac{\overline{f}_b}{f}\Big((\rho-\mu)(\eta-1)N(f)\Big)^{(1-\eta)/\eta} N'(f) \\
&\quad + \frac{1}{\rho-\mu}\frac{1}{\eta-1}\frac{N'(f)}{N(f)} + \frac{-\eta}{f}\frac{1}{\rho-\mu}\frac{1}{\eta-1} \\
&= \frac{\eta}{f}\Big((\rho-\mu)(\eta-1)N(f)\Big)^{-1} N(f) - \Big((\rho-\mu)(\eta-1)N(f)\Big)^{-1} N'(f) \\
&\quad + \frac{1}{\rho-\mu}\frac{1}{\eta-1}\frac{N'(f)}{N(f)} + \frac{-\eta}{f}\frac{1}{\rho-\mu}\frac{1}{\eta-1} \equiv 0,
\end{aligned}$$

where we substitute

$$\begin{aligned}
E'(f) = &-\frac{1}{\rho-\mu}\frac{\eta}{\eta-1}\frac{1}{(c+\xi)f^2}\Big((\rho-\mu)(\eta-1)N(f)\Big)^{1/\eta} \\
&+ \frac{1}{\rho-\mu}\frac{1}{\eta-1}\frac{1}{(c+\xi)f}\Big((\rho-\mu)(\eta-1)N(f)\Big)^{(1-\eta)/\eta}(\rho-\mu)(\eta-1)N'(f)
\end{aligned}$$

$$= \frac{1}{(c+\xi)f}\left((\rho-\mu)(\eta-1)N(f)\right)^{(1-\eta)/\eta}\left(-\frac{\eta}{f}N(f)+N'(f)\right).$$

This derivation significantly simplifies our problem by reducing it to a single ODE rather than a system involving two unknown functions, $E(f)$ and $N(f)$. Equation (B.22) provides a direct relationship between these functions, enabling us to solve one ODE for either $E(f)$ or $N(f)$ and thereby determine the equity value function. I provide such details in Appendix B.3.

By evaluating equations (B.17) and (B.18) at $f = f_b$, together with the boundary conditions in Lemma B.2, we obtain $\overline{f}_b(f_b) = f_b$, confirming that (f_b, f_b) is indeed a crossing point of the default boundary and the 45-degree line.[7]

To establish the existence of a threshold equilibrium, we must verify the single crossing property between $\overline{f}_b(f)$ and the 45-degree line. This property ensures that firms do not default before reaching (f_b, f_b) in equilibrium. The monotonicity of $N(f)$ provides a sufficient condition for this property.

Lemma B.4 (Single Crossing Property). *If $N(f)$ is strictly increasing in f, then $\overline{f}_b(f)$ intersects the line $\tilde{f} = f$ exactly once at (f_b, f_b).*

Proof. Given that

$$\overline{f}_b(f) = \left(\frac{1}{\rho-\mu}\frac{1}{\eta-1}\frac{1}{N(f)}\right)^{1/\eta}\cdot f, \tag{B.23}$$

we need to show that

$$\left(\frac{1}{\rho-\mu}\frac{1}{\eta-1}\frac{1}{N(f)}\right)^{1/\eta} > 1 \quad \text{for } 0 < f < f_b \tag{B.24}$$

[7] Equation (B.19) is also satisfied at this point:

$$\begin{aligned}
0 = v_f(f_b, f_b) &= -(c+\xi)f_b \cdot E'(f_b) + N'(f_b) + \frac{-\eta}{f_b}\cdot N(f_b) \\
&= \frac{\eta}{f_b}\left((\rho-\mu)(\eta-1)N(f_b)\right)^{(1-\eta)/\eta} N(f_b) \\
&\quad - \left((\rho-\mu)(\eta-1)N(f_b)\right)^{(1-\eta)/\eta} N'(f_b) \\
&\quad + N'(f_b) + \frac{-\eta}{f_b}\cdot N(f_b) \\
&= \underbrace{\left[-\left((\rho-\mu)(\eta-1)N(f_b)\right)^{(1-\eta)/\eta} + 1\right]}_{=0} \cdot \left[N'(f_b) - \frac{\eta}{f_b}\cdot N(f_b)\right] \equiv 0
\end{aligned}$$

The last equality follows from the boundary condition $N(f_b) = \frac{1}{\rho-\mu}\frac{1}{\eta-1}$.

If $N(f)$ is strictly increasing in f, the left-hand side of inequality (B.24) is strictly decreasing in f. Since

$$\left(\frac{1}{\rho-\mu}\frac{1}{\eta-1}\frac{1}{N(f_b)}\right)^{1/\eta} = 1, \tag{B.25}$$

the result follows. □

While direct verification of $N(f)$ being strictly increasing requires solving the underlying ODE, and I do not find an explicit way to derive this result analytically, extensive numerical analysis under various parameter configurations consistently confirms both the monotonicity of $N(f)$ and the satisfaction of the single crossing property.[8]

After solving for $N(f)$, we obtain the default boundary through equation (B.21). Given this boundary and the monotonicity of equity value with respect to the individual firm's debt-to-earnings ratio $\tilde{f}$, we can characterize the continuation region, which lies below the default boundary:

$$\mathcal{C} = \{(\tilde{f}, f) \mid 0 \leq \tilde{f} < \overline{f}_b(f), 0 < f \leq f_b\} \tag{B.26}$$

We now examine the boundary behavior as $\tilde{f} \to 0$. For a firm with negligible debt, its equity value converges to that of an unlevered firm:

$$v(0, f) = \frac{1}{\rho-\mu}, \quad \forall\, 0 < f \leq f_b \tag{B.27}$$

This property is readily verified using the equity value function given in equation (B.8).

Then I consider the boundary behavior as $f \to 0$. We focus on nontrivial equilibria where $f_b > 0$. The single crossing property implies $\inf_f \overline{f}_b(f) > 0$. The following lemma establishes the necessary boundary conditions for $N(f)$ and $E(f)$.

Lemma B.5. *The following conditions must be satisfied at the boundary $f \to 0$:*

$$\lim_{f\to 0} N(f) = 0, \quad \lim_{f\to 0} fE(f) = 0 \tag{B.28}$$

Proof. We know that $v(0, f) = \frac{1}{\rho-\mu}$ for any $0 < f \leq f_b$, and $v(\tilde{f}, f)$ is decreasing in $\tilde{f}$. Let $\inf \overline{f}_b = \delta > 0$. Then we must have $0 < v(\hat{\delta}, f) < \frac{1}{\rho-\mu}$ and thus finite for any $f \in (0, f_b)$ and $\hat{\delta} \in (0, \delta)$. This

[8] The numerical verification is performed across a wide range of economically relevant parameter values, with particular attention to the stability of these properties under parameter perturbations.

implies:

$$-(c+\xi)\hat{\delta}\cdot\lim_{f\to 0}E(f)+\left(\hat{\delta}\right)^{\eta}\cdot\lim_{f\to 0}\frac{N(f)}{f^{\eta}}=\text{finite} \tag{B.29}$$

This finiteness condition requires that both $\lim_{f\to 0}E(f)$ and $\lim_{f\to 0}\frac{N(f)}{f^{\eta}}$ are finite. Since $\eta>1$, we have:

$$\lim_{f\to 0}N(f)=0. \tag{B.30}$$

Equation (B.22) then implies $\lim_{f\to 0}fE(f)=0$. □

These boundary conditions are sufficient to determine a unique solution to the ODE system for $E(f)$ and $N(f)$.[9]

Lemma B.6 (Smooth Issuance Equilibrium). *The equity value function given by equation* (B.8) *characterizes a smooth issuance equilibrium.*

Proof. This follows from its strict convexity in $\tilde{f}$, which is ensured by $\eta>1$ and $N(f)>0$. □

B.3. Solution Approach

The analytical characterization provided earlier offers multiple approaches for solving the problem.

B.3.1. Approach Using an ODE System

The most direct method involves solving the coupled ODE system for $E(f)$ and $N(f)$:

$$\begin{aligned}(\rho+\xi)E&=1+\mu_f(f)\cdot fE'(f)+\frac{\|\boldsymbol{\sigma}\|^2+\|\boldsymbol{\sigma}_s\|^2-2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2}\cdot f^2E''(f),\\(\rho+\xi)N&=-\frac{g}{f}\eta+\mu_f(f)\cdot fN'(f)+\frac{\|\boldsymbol{\sigma}\|^2+\|\boldsymbol{\sigma}_s\|^2-2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2}\cdot f^2N''(f).\end{aligned}$$

[9] For $E(f)$, we obtain a terminal value problem that can be recast as an initial value problem (IVP), while for $N(f)$ we have a boundary value problem (BVP). The uniqueness can first be established for the ODE governing $E(f)$ and then extended to $N(f)$ through equation (B.22). Under the condition that $E(f)$ is continuously differentiable, and given that the ODE coefficients of $E'(f)$ and $E''(f)$ as functions of (f,E,E') are continuously differentiable with respect to each component and thus Lipschitz, we obtain existence and local uniqueness. If we further assume E and E' remain bounded (which is verified numerically), the uniqueness extends globally by the continuation theorem.

This system requires four boundary conditions:

$$E(f_b) = \frac{1}{\rho + \xi - \hat{\mu}}, \quad N(f_b) = \frac{1}{\rho - \mu}\frac{1}{\eta - 1},$$

and

$$\lim_{f \to 0} N(f) = 0.$$

plus the smooth pasting condition at (f_b, f_b):

$$0 = v_f(f_b, f_b) = -(c + \xi) f_b \cdot E'(f_b) + N'(f_b) + \frac{-\eta}{f_b} \cdot N(f_b) \tag{B.31}$$

which serves as a Cauchy-type boundary condition.

B.3.2. ODE Approach with $N(f)$

Exploiting the one-to-one relationship between $E(f)$ and $N(f)$, we can reduce the system to a single ODE:

$$(\rho + \xi)N = -\frac{g}{f}\eta + \mu_f(f) \cdot fN'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 N''(f),$$

with boundary conditions:

$$\lim_{f \to 0} N(f) = 0, \quad N(f_b) = \frac{1}{\rho - \mu}\frac{1}{\eta - 1}.$$

This approach requires the following substitutions:

$$E(f) = \frac{1}{\rho - \mu}\frac{\eta}{\eta - 1}\frac{1}{(c + \xi)f}\left((\rho - \mu)(\eta - 1)N(f)\right)^{1/\eta},$$
$$E'(f) = \frac{1}{(c + \xi)f}\left((\rho - \mu)(\eta - 1)N(f)\right)^{(1-\eta)/\eta}\left(-\frac{\eta}{f}N(f) + N'(f)\right),$$

for the coefficients.

B.3.3. ODE Approach with $E(f)$

Alternatively, we can formulate a single ODE for $E(f)$:

$$(\rho + \xi)E = 1 + \mu_f(f) \cdot fE'(f) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\boldsymbol{\sigma}_s}{2} \cdot f^2 E''(f),$$

with boundary conditions:

$$E(f_b) = \frac{1}{\rho + \xi - \hat{\mu}},$$

The smooth pasting condition (B.31) is automatically satisfied when we substitute:

$$N(f) = \frac{1}{\rho - \mu}\frac{1}{\eta - 1}\left((\rho - \mu)\frac{\eta - 1}{\eta}(c + \xi)fE(f)\right)^{\eta},$$
$$N'(f) = \frac{1}{\rho - \mu}\frac{\eta}{\eta - 1}\left((\rho - \mu)\frac{\eta - 1}{\eta}(c + \xi)fE(f)\right)^{\eta}\left(\frac{1}{f} + \frac{E'(f)}{E(f)}\right)$$

We can verify that $\lim_{f \to 0} fE(f) = 0$ serving as an additional check.

Appendix C. Equilibrium without Capital Inflows

This section analyzes the model in the absence of capital inflows. The balance of payment condition (B.4) implies:

$$\frac{ds_t}{s_t} = \hat{\mu}dt + \hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} \cdot d\boldsymbol{B_t}$$

where

$$\hat{\mu} = (1 + \alpha\alpha_c)^{-1} [\rho - r - \lambda + \alpha\overline{\mu}]$$
$$\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} = \frac{\overline{\boldsymbol{\sigma}}}{\alpha_c}$$

Following similar steps as in the main text, the equity value is equivalent to the no-issuance equity value. The Hamilton-Jacobi-Bellman (HJB) equation is:

$$\begin{aligned}\rho V(\tilde{F}_t, F_t, s_t, Y_t) =& Y_t - (c + \xi)\tilde{F}_t s_t - \xi\tilde{F}_t V_{\tilde{F}} + (G_t - \xi F_t)V_F \\ &+ s_t\hat{\mu}V_s + \mu Y V_Y + \frac{1}{2}|\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}}|^2 s_t^2 V_{ss} + \frac{1}{2}\|\boldsymbol{\sigma}\|^2 Y^2 V_{YY} + \boldsymbol{\sigma'\Sigma\hat{\sigma}}_{\boldsymbol{s}} s_t Y_t V_{sY}.\end{aligned}$$

And the HJB equation for debt price is:

$$\begin{aligned}rp(\tilde{F}_t, F_t, s_t, Y_t) =& c + \xi - \xi p + (\tilde{G}_t - \xi\tilde{F}_t)p_{\tilde{F}} + (G_t - \xi F_t)p_F \\ &+ s_t\hat{\mu}p_s + \mu Y p_Y + \frac{1}{2}|\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}}|^2 s_t^2 p_{ss} + \frac{1}{2}\|\boldsymbol{\sigma}\|^2 Y^2 p_{YY} + \boldsymbol{\sigma'\Sigma\hat{\sigma}}_{\boldsymbol{s}} s_t Y_t p_{sY}.\end{aligned}$$

Comparing the HJB equations for debt price and equity value yields the optimal issuance level:

$$\tilde{G}^* = \frac{\rho - r - \hat{\mu} + Y_t\frac{p_Y}{p}\boldsymbol{\sigma'\Sigma\hat{\sigma}}_{\boldsymbol{s}} - s_t\frac{p_s}{p}|\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}}|^2}{-p_{\tilde{F}}/p}.$$

The system can be scaled using the homogeneity of the value function:

$$\begin{aligned}V(\tilde{F}_t, s_t, Y_t) &= \tilde{v}\left(\frac{s_t\tilde{F}_t}{Y_t}\right)\cdot Y_t = \tilde{v}(\tilde{f})\cdot Y_t, \\ \tilde{G}(\tilde{F}_t, s_t, Y_t) &= \tilde{g}\left(\frac{s_t\tilde{F}_t}{Y_t}\right)\cdot\frac{Y_t}{s_t} = \tilde{g}(\tilde{f})\cdot\frac{Y_t}{s_t}, \\ p(\tilde{F}_t, s_t, Y_t) &= p\left(\frac{s_t\tilde{F}_t}{Y_t}\right) = p(\tilde{f}).\end{aligned}$$

The firm's value can be expressed as the sum of the no-default value and the default option

value. The scaled firm value with no default and no future issuance is:

$$\begin{aligned}\overline{\tilde{v}}(\tilde{f}) &= \frac{1}{\rho-\mu} - \tilde{f}\cdot\mathbb{E}\left[\int_t^\infty e^{-(\rho+\xi)(\tau-t)}(c+\xi)\frac{s_\tau}{s_t}d\tau\right] \\ &= \frac{1}{\rho-\mu} - \frac{(c+\xi)\tilde{f}}{\rho+\xi-\hat{\mu}},\end{aligned}$$

where the second equality follows from the log-normal distribution of $\frac{s_\tau}{s_t}$.

The equity value is given by:

$$\tilde{v}(\tilde{f}) = \overline{\tilde{v}}(\tilde{f}) + \mathbb{E}^0\left[e^{-(\rho+\xi)\tau_b}\right]\left(0 - \overline{\tilde{v}}(\hat{f}_b)\right).$$

To characterize $h(\tilde{f}) = \mathbb{E}^0\left[e^{-(\rho+\xi)\tau_b}\right]$, we first derive the dynamics for $\tilde{f}$ using Itô's Lemma:

$$\frac{d\tilde{f}_t}{\tilde{f}_t} = \left(\hat{\mu} - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} - \xi\right)dt + (\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} - \boldsymbol{\sigma})\cdot d\boldsymbol{B}_{\boldsymbol{t}}$$

The function $h(\tilde{f})$ must satisfy:

$$(\rho+\xi)h(\tilde{f}) = \left(\hat{\mu} - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} - \xi\right)\tilde{f}h'(\tilde{f}) + \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}}}{2}\tilde{f}^2h''(\tilde{f})$$

with boundary conditions:

$$h(\hat{f}_b) = 1, \quad h(0) = 0.$$

The solution to this ODE is

$$h(\tilde{f}) = \left(\frac{\tilde{f}}{\hat{f}_b}\right)^{\tilde{\eta}},$$

where $\tilde{\eta} > 0$ solves:

$$\frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}}}{2}\tilde{\eta}^2 + \left(\hat{\mu} - \mu + \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}} - \xi - \frac{\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}_s\|^2 - 2\boldsymbol{\sigma}'\boldsymbol{\Sigma}\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}}}{2}\right)\tilde{\eta} - (\rho+\xi) = 0.$$

ASSUMPTION C.1. *Assume $\rho + \xi > \hat{\mu}$ and $\mu > \|\boldsymbol{\sigma}\|^2 - \boldsymbol{\sigma}'\boldsymbol{\Sigma}\hat{\boldsymbol{\sigma}}_{\boldsymbol{s}}$.*

Under Assumption C.1, it follows that $\tilde{\eta} > 1$.

The equity value function can then be expressed as:

$$\tilde{v}(\tilde{f}) = \frac{1}{\rho-\mu} - \frac{(c+\xi)\tilde{f}}{\rho+\xi-\hat{\mu}} + \left(\frac{(c+\xi)\hat{f}_b}{\rho+\xi-\hat{\mu}} - \frac{1}{\rho-\mu}\right)\left(\frac{\tilde{f}}{\hat{f}_b}\right)^{\tilde{\eta}}. \tag{C.1}$$

The default boundary is determined by the smooth pasting condition:

$$\tilde{v}'(\hat{f}_b) = -\frac{c+\xi}{\rho+\xi-\hat{\mu}} + \tilde{\eta}\frac{1}{\hat{f}_b}\left(\frac{(c+\xi)\hat{f}_b}{\rho+\xi-\hat{\mu}} - \frac{1}{\rho-\mu}\right) = 0$$

$$\Rightarrow \quad \hat{f}_b = \frac{\tilde{\eta}}{\tilde{\eta}-1}\frac{1}{\rho-\mu}\frac{\rho+\xi-\hat{\mu}}{c+\xi} > 0.$$

Appendix D. Numerical Solutions

D.1. Parameter Values

TABLE D.1. Baseline Parameters

Interest Rate	
LC Risk-free Rate	$\rho = 8\%$
FC Risk-free Rate	$r = 1\%$
Corporate Bonds	
Corporate Bonds Coupon Rate	$c = 1\%$
Maturity	$\frac{1}{\xi} = 6.58$
Economic Fundamentals	
Earning Drift	$\mu = 0.05$
Earning Volatility	$\sigma = \sqrt{0.02}$
Exgoenous Flow to Earning	$\overline{\mu} = 0.00$
Earning Exchange Rate Correlation	$\phi = 0.40$
Exchange Rate	
Exchange Rate Volatility	$\sigma_s = 0.15$
FC Convenience Yield	$\lambda = 145$ bps
Investors Sector	
Effective Risk Aversion	$\hat{\gamma} = 2/4$

For the numerical solution, I assume a two-dimensional Brownian motion

$$\boldsymbol{B_t} = (B_{1t}, B_{2t}),$$

where the two components have a correlation ϕ. I assume the earning process load on B_{1t}, while the exchange rate process (or exogenous flow) loads on B_{2t}, making σ_s one-dimensional. Therefore,

$$\frac{dY_t}{Y_t} = \mu dt + \sigma \cdot dB_{1t}, \quad \text{and } \frac{ds_t}{s_t} = \mu_s dt + \sigma_s \cdot dB_{2t}.$$

The coupon rate c equals to the FC risk-free rate, ensuring that risk-free corporate debt has a unit price. Exchange rate volatility is set at 0.15 in The One-sector Model, while in the intermediation model, α_c s calibrated to maintain the same 0.15 volatility level.

I intentionally set $\overline{\mu} = 0$ to ensure that, in The One-sector Model, there is no capital inflow, and firms issue debt exclusively for rollover purposes. Debt maturity is set at 6.58 , aligning with the average maturity of major EMEs' international-targeting bonds, calculated using new debt issuance data from the SDC Platinum dataset.

The convenience yield is set at 145 basis points, in line with the estimate by Koijen and Yogo (2019). The investor sector's risk aversion coefficient is 2, and $\zeta = 4$ chosen to represent an economy with a saving-to-GDP ratio of 0.4, and a total GDP of $10Y_t$. The model solutions give a default boundary around $f_b = 40$ and a stochastic steady-state range of $f \in [25, 35]$. These values correspond to approximately 400% of GDP for the default boundary and 250% to 350% of GDP for the steady-state range. This roughly aligns with recent global debt-to-GDP ratios, which are around 250% globally (approximately 300% for advanced economies and 200% for EMEs, with China reaching 260%-270% in the 2020s, according to the IMF's Global Debt Monitor).

Using the ODE solutions, I numerically simulate the model to obtain the stationary distribution and other financial stability indicators. I start the economy from either a low debt level, $f = 0.2$, or the lowest stable stochastic steady state. The simulation progresses daily until an economy-wide default or a 100-year time horizon, whichever occurs first. I perform 10,000 simulations and take the average across all f_t paths.

D.2. Measurements of Financial Stability

The financial stability measurement is based on Calvo et al. (2008).

The sudden stop measure is based on year-over-year changes. I simulate the model on a daily basis and compute monthly cumulative capital flow. The Sudden Stop phase meets the following criteria: (1) the year-over-year fall of the monthly cumulative capital inflow is at least two standard deviations below the overall mean of simulations, (2) the onset of a Sudden Stop phase is identified by the first instance when the year-over-year change in capital inflow drops to one standard deviation below the mean, and (3) the Sudden Stop phase concludes when the year-over-year change in capital inflow surpasses one standard deviation below the sample mean.

Additionally, I include another level-based measurement. I call it financial distress. The distress period phase meets the following criteria: (1) the annually cumulative capital inflow is at least two standard deviations below the overall mean of simulations, (2) the onset of a distress phase is identified by the first instance when the annually cumulative capital inflow drops to one standard deviation below the mean, and (3) the distress phase concludes when the annually cumulative capital inflow surpasses one standard deviation below the relative sample mean.

D.3. Numerical Solution for the One-sector Model

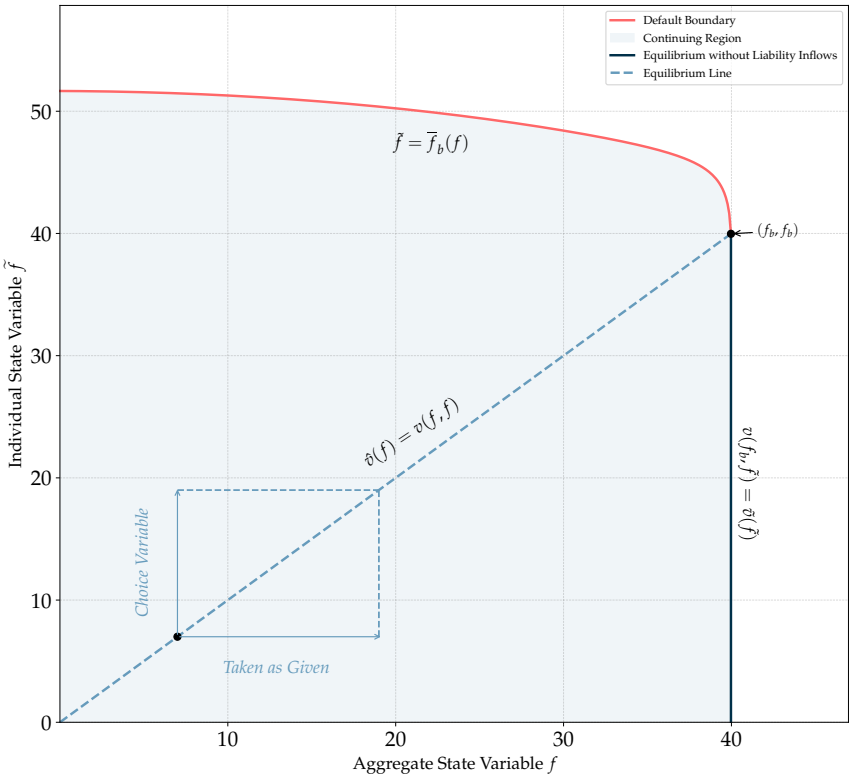
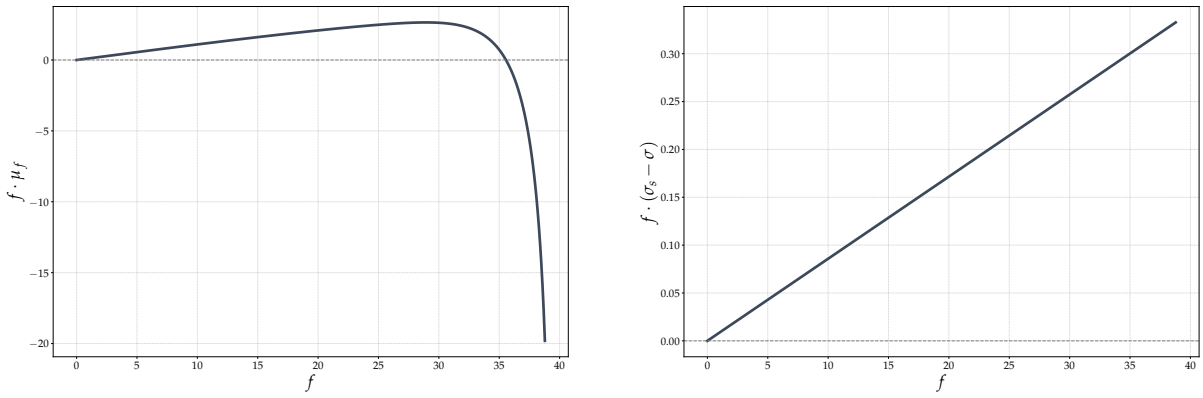


FIGURE D.1. The One-sector Model: State Space



A. State Variable Drift B. State Variable Volatility

FIGURE D.2. The One-sector Model: State Variable Dynamics

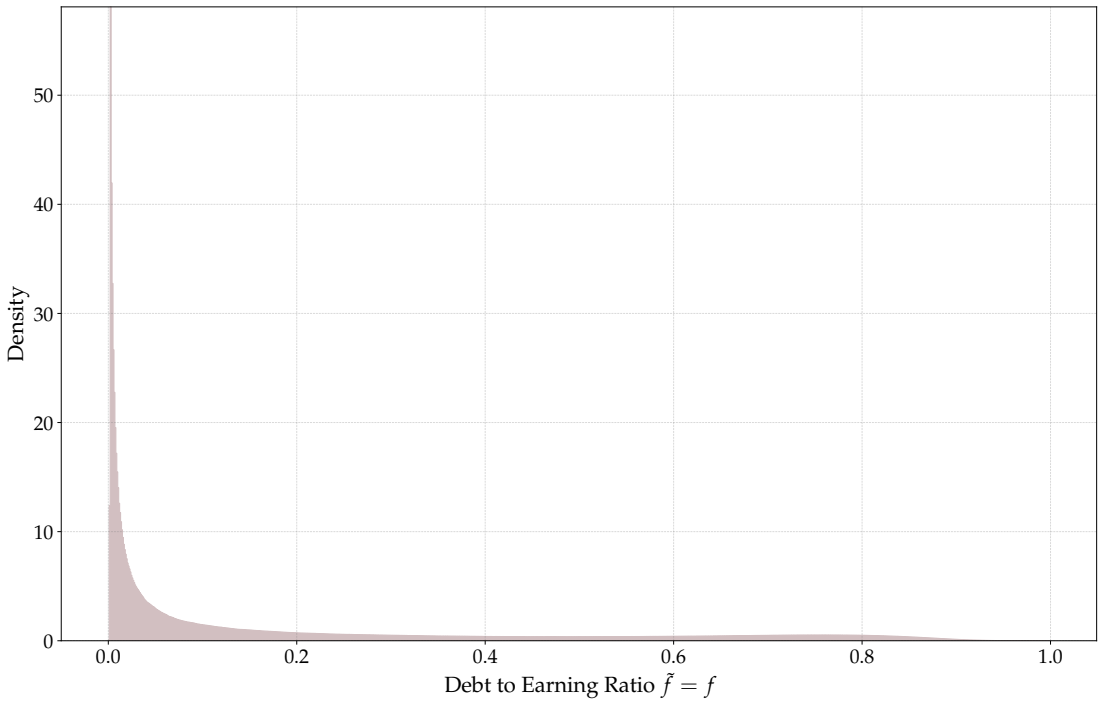
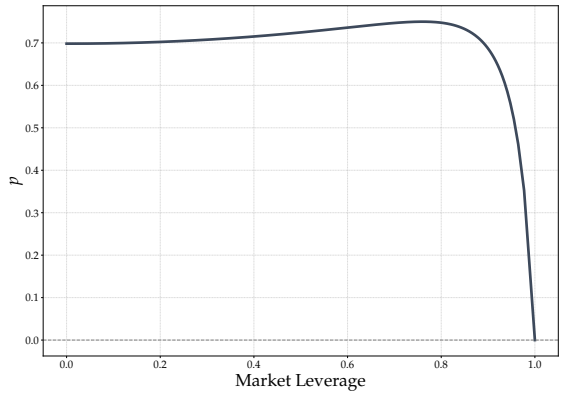
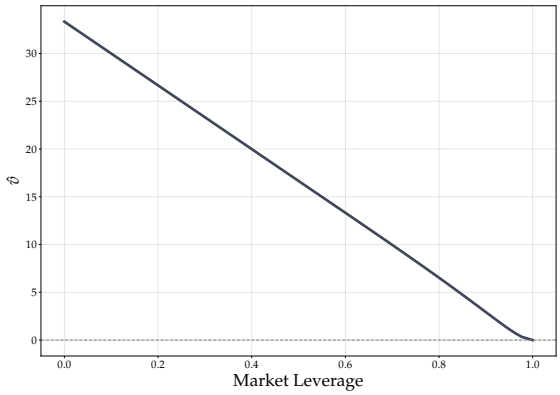


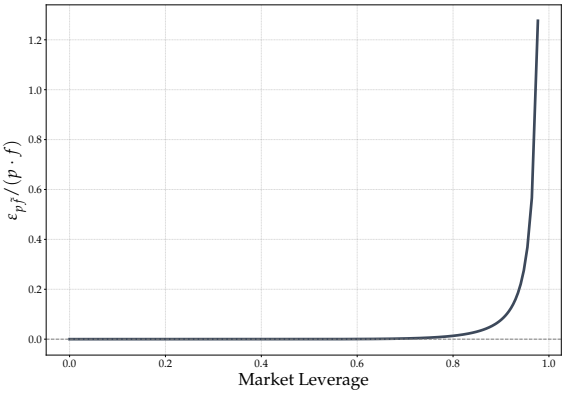
FIGURE D.3. The One-sector Model: Stationary Distribution



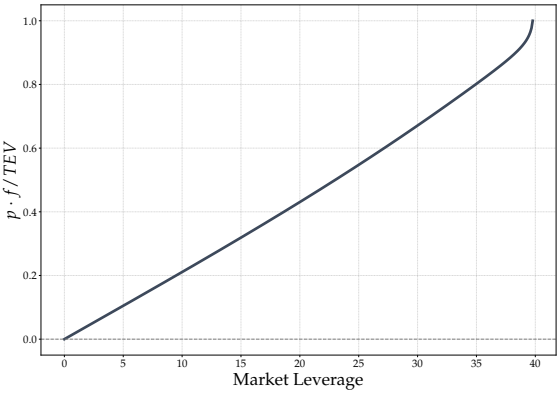
A. Debt Price



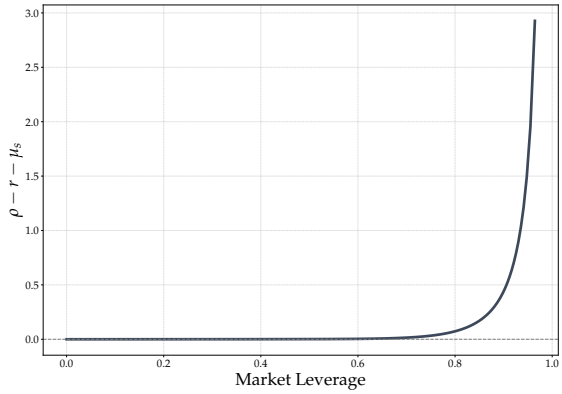
B. Equity Price



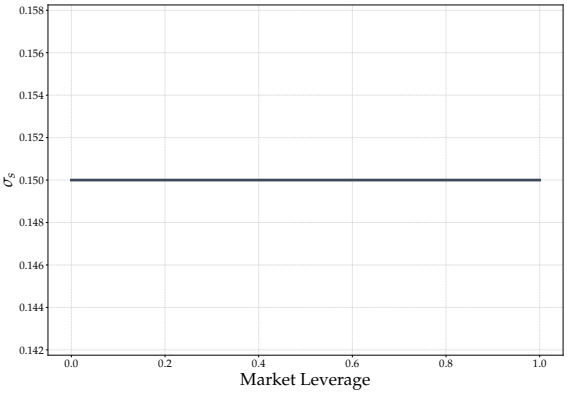
C. Price of Imbalances



D. Market Leverage

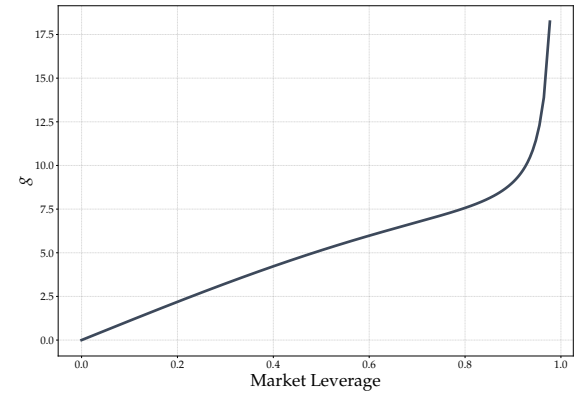


E. LC Risk Premium

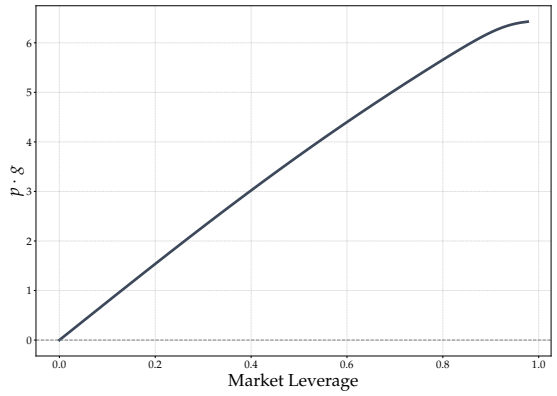


F. Exchange Rate Volatility

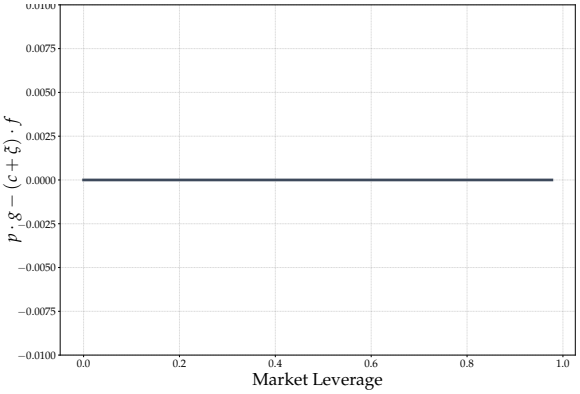
FIGURE D.4. The One-sector Model: Prices and Values



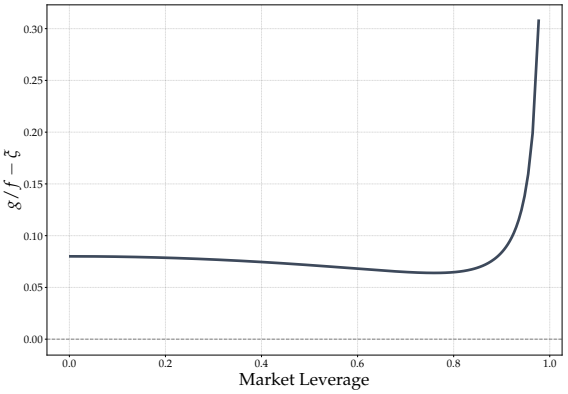
A. Optimal Issuance



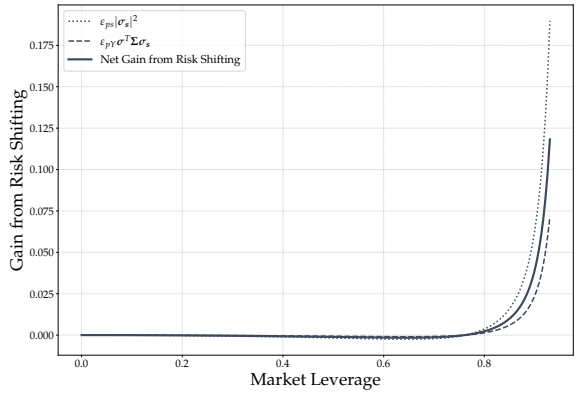
B. Proceeds



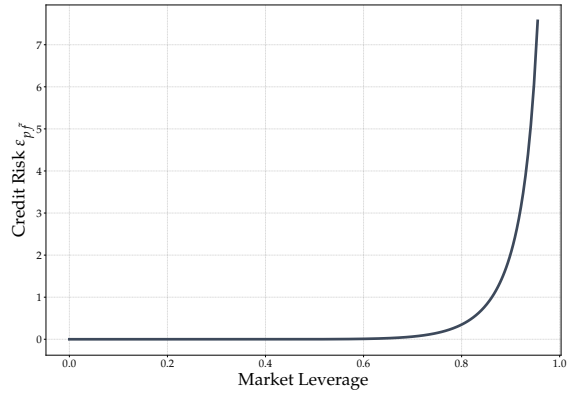
C. Capital Inflow



D. Net Issuance Rate



E. Gain from Risk-Shifting



F. Credit Risk

FIGURE D.5. The One-sector Model: Issuances and Flows

D.4. Numerical Solution for the Two-sector Model

For the model with intermediation, I adjust the parameters for exogenous flows as follows:

TABLE D.2. Adjusted Parameters

Economic Fundamentals	
Exgoenous Flow to Earning (Drift)	$\overline{\mu} = -0.09$
Exgoenous Flow to Earning (Volatility)	$\overline{\sigma} = 0.52$

I estimate these drift and volatility parameters by calculating the drift and volatility of agrgeagted flows not directly modeled in this paper: net portfolio equity flows, net FDI flows, net financial derivatives flows, and trade flows, based on data from The External Wealth of Nations database. Since my model's BoP condition focuses on currency market balance, I exclude valuation effects. Calculating valuation changes is achieved by taking the difference between the current account balance and the change in the net International Investment Position. I then subtract these valuation changes from the aggregated financial flows, obtaining the exogenous financial flow needed for the model.

Once I have these aggregate flows for each country, I scale them by the country's GDP for that year and run a regression for each country c:

$$\text{GDP Scaled Flow}_c = \mu_{flow,c} + \varepsilon_{c,t}$$

where μ_{flow} and $\text{Var}(\varepsilon_t)$ represent the drift and volatility terms needed for the model. For most countries, the drift (or the intercept) is statistically insignificant, so setting it to zero as a baseline aligns reasonably with reality. The average drift across countries is 0.01, and the average volatility is 0.052.

As discussed in the main text, this exogenous flow is assumed to be inelastic with respect to exchange rate dynamics, so it is beneficial to control for exchange rate effects. Additionally, valuation changes may involve assets beyond those considered here, so I include these changes as controls. For robustness, I run the following regression:

$$\text{GDP Scaled Flow}_c = \mu_{flow,c} + \beta_{1,c} s_{c,t} + \beta_{2,c} s_{t,c,end} + \beta_{3,c} \text{Valuation Change} + \varepsilon_t$$

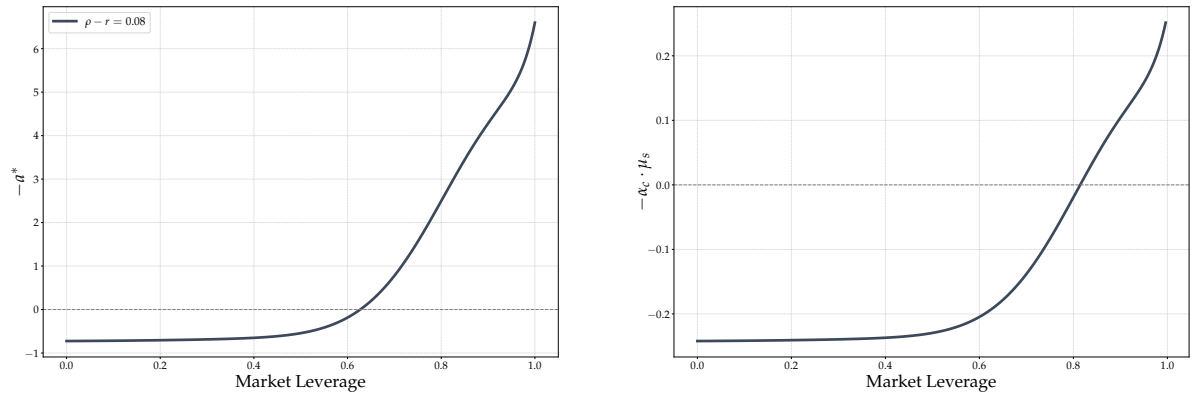
to control for any dependency on exchange rate fluctuations and potential mismeasurement of valuation changes.[10] With this approach, the mean volatility estimate remains at 0.052, and the mean intercept is -0.009 and most of them are still statistically insignificant, indicating that ex-

[10] $s_{c,t}$ is the average exchange rate over the period, and $s_{t,c,end}$ is the period-end exchange rate for country c.

change rate and valuation changes explain only minimal variation in the flows. This suggests that our aggregate measure of unmodeled flows serves its purpose effectively. I use these values as parameters for the exogenous flow. Given a GDP of $10Y_t$, I scale by 10 to set $\overline{\mu}$ and $\overline{\sigma}$.

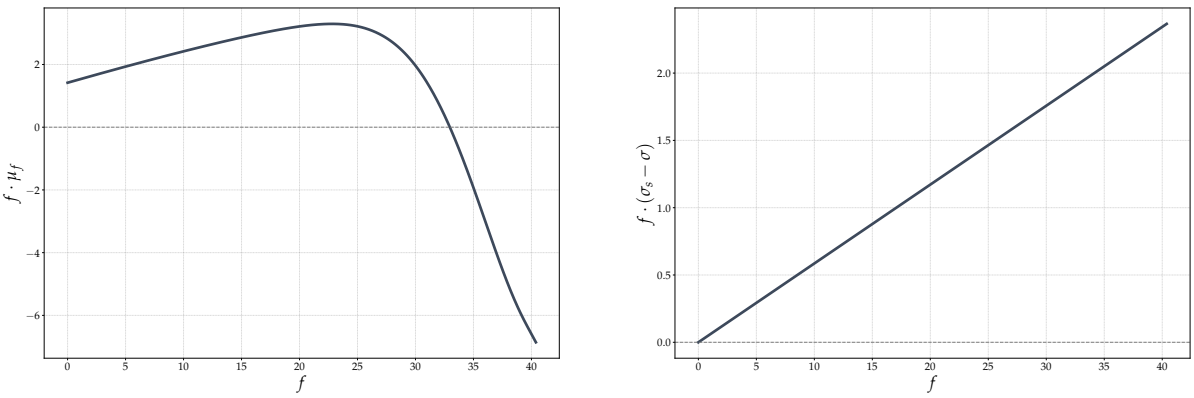
The efforts above are intended to establish reasonable values for these parameters, but it is important to note that the model's implications remain largely unaffected by variations in these values.

The model's state space is shown in Figure B.1, along with additional plots for The One-sector Model and sectoral flows in Figure D.6. From these plots, we observe that the central bank's intervention flow is relatively minor compared to asset outflows. Additionally, the directions of these two flows are largely aligned across the state space, suggesting that the central bank's primary function is to provide a microfoundation for exchange rate volatility, exerting only a small influence on total flows. The key interactions in the market are still driven by firm-initiated capital inflows and investor-driven asset outflows.



A. Assets Outflow B. Intervention Flow

FIGURE D.6. The Two-sector Model: Other Sectors



A. State Variable Drift B. State Variable Volatility

FIGURE D.7. The Two-sector Model: State Variable Dynamics

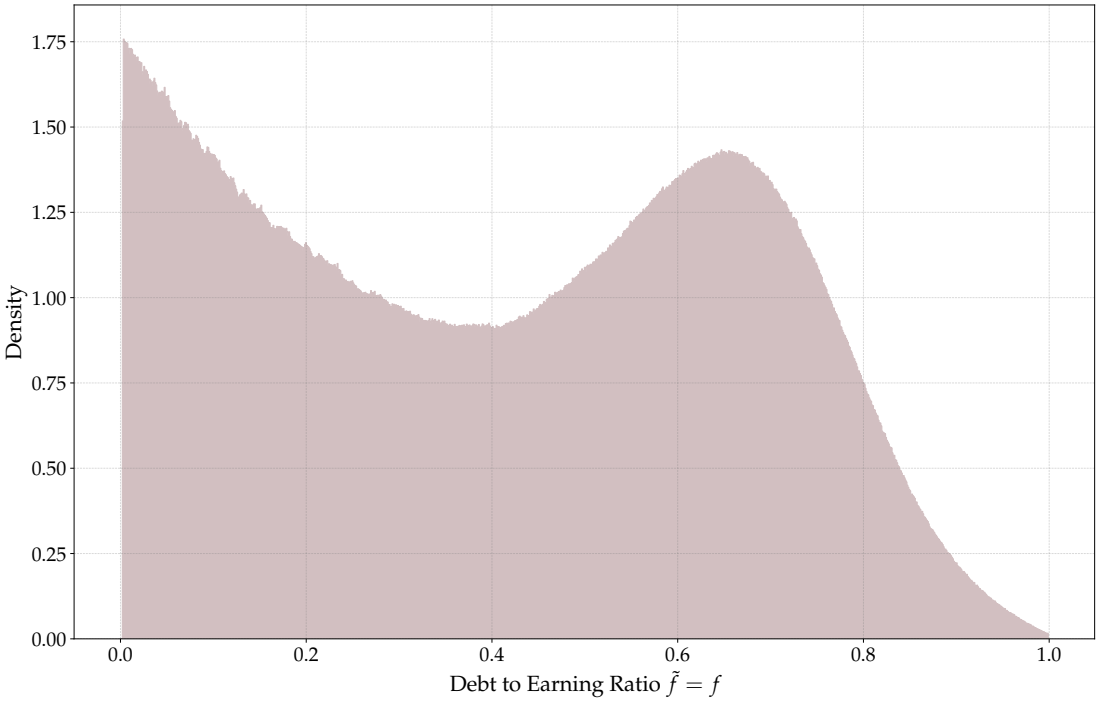
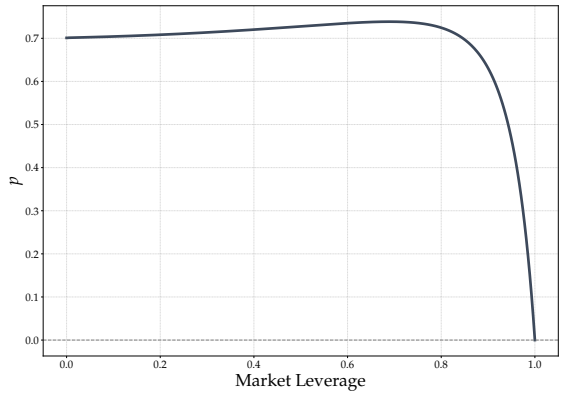
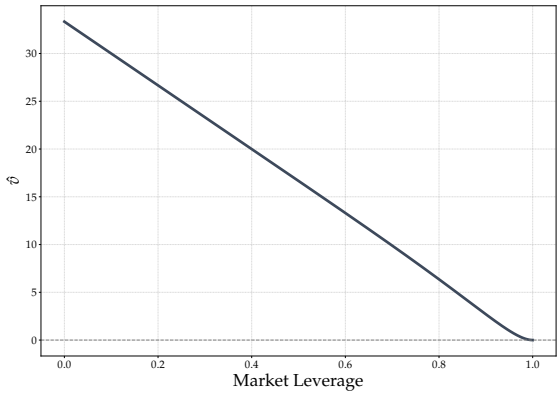


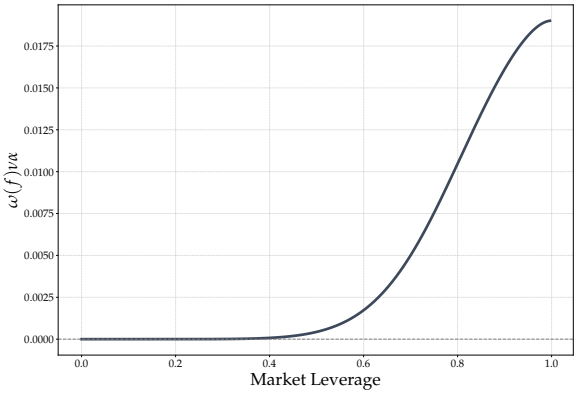
FIGURE D.8. The Two-sector Model: Stationary Distribution



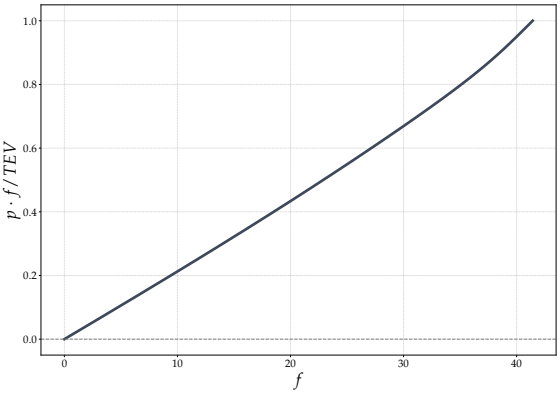
A. Debt Price



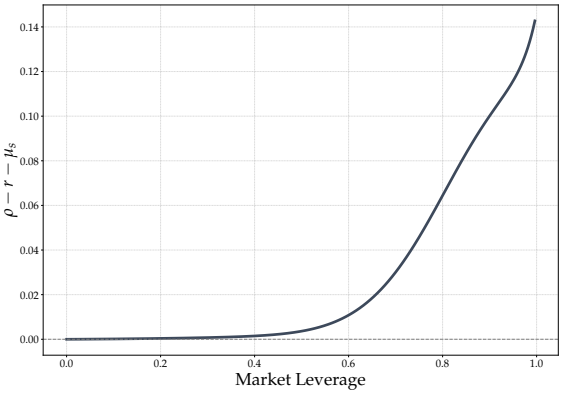
B. Equity Price



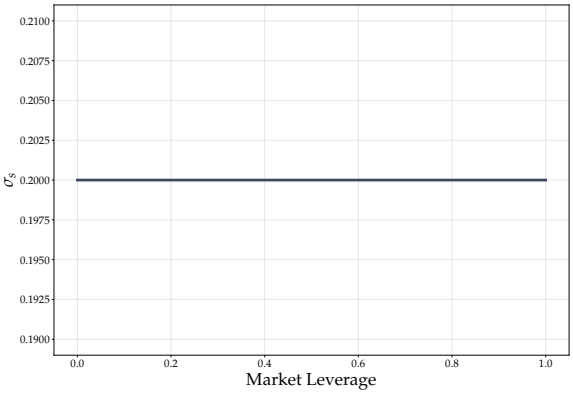
C. Price of Imbalances



D. Market Leverage

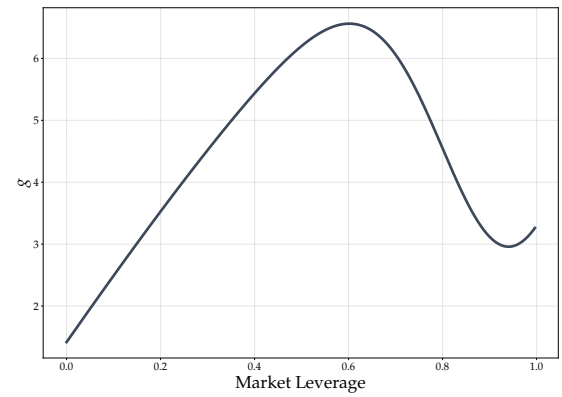


E. LC Risk Premium

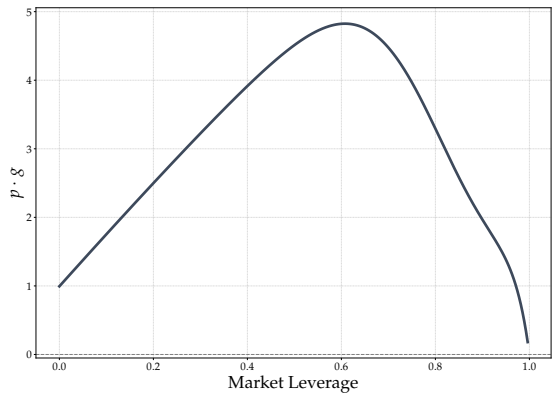


F. Exchange Rate Volatility

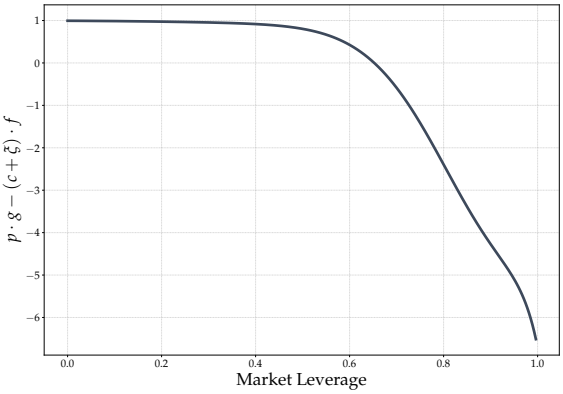
FIGURE D.9. The Two-sector Model: Prices and Values



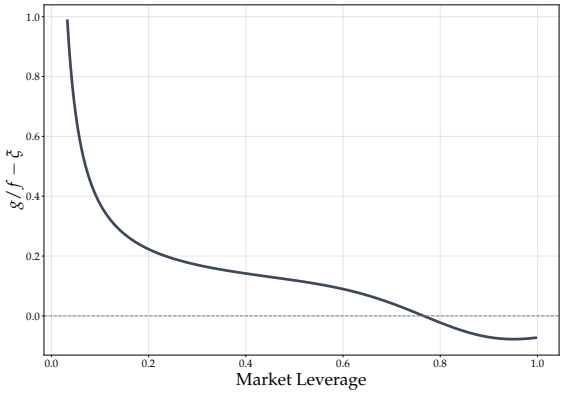
A. Optimal Issuance



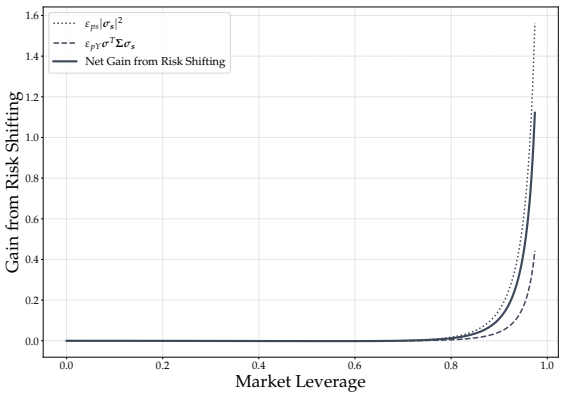
B. Proceeds



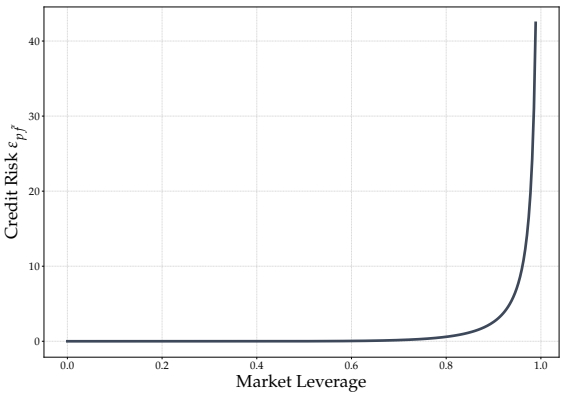
C. Capital Inflow



D. Net Issuance Rate



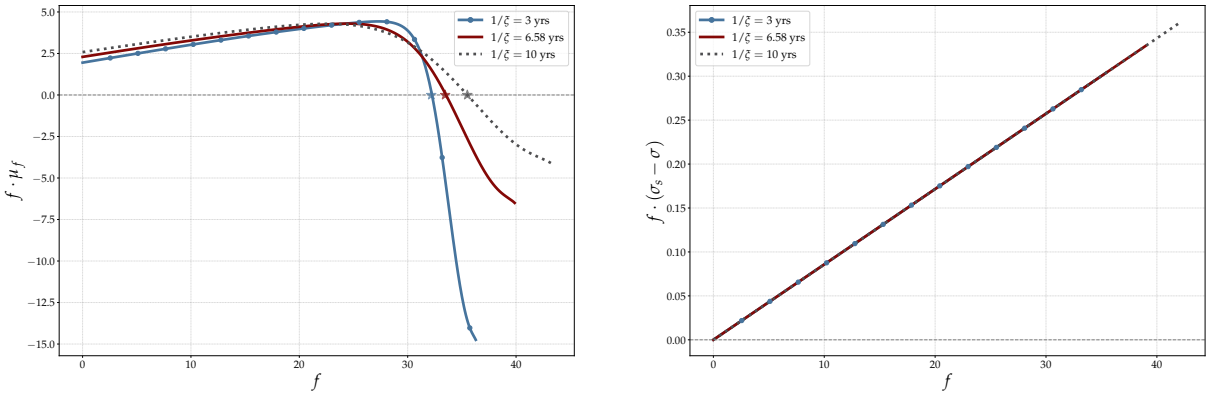
E. Gain from Risk-Shifting



F. Credit Risk

FIGURE D.10. The Two-sector Model: Issuances and Flows

D.5. Numerical Solution for Maturity Comparative Statics



A. State Variable Drift

B. State Variable Volatility

FIGURE D.11. Maturity Comparative Statics: State Variable Dynamics

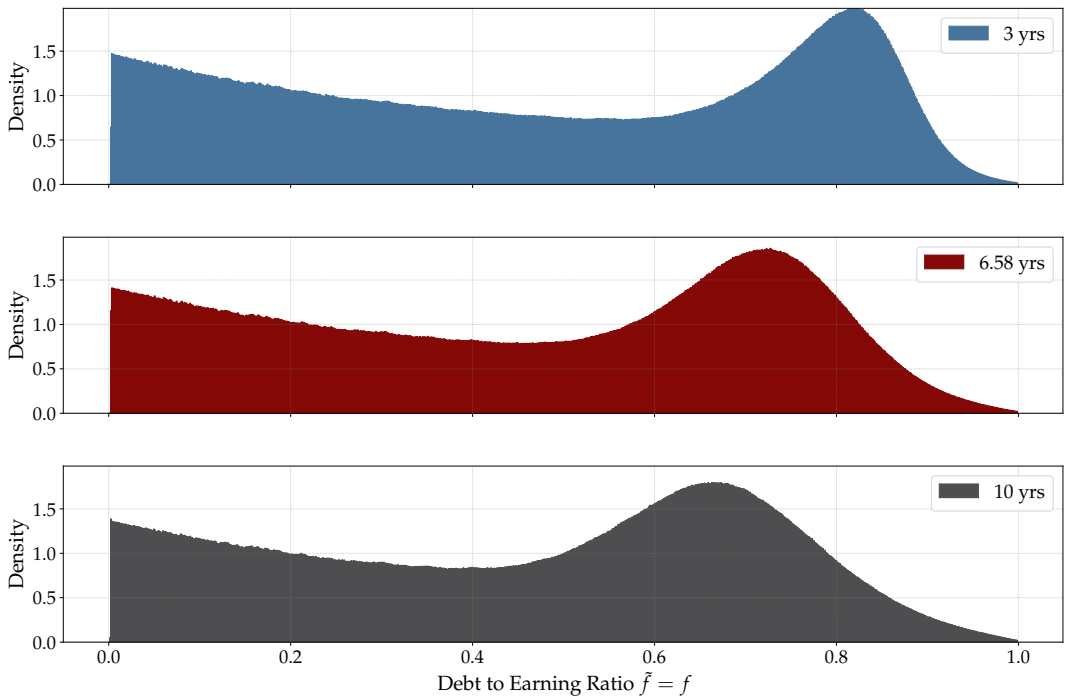
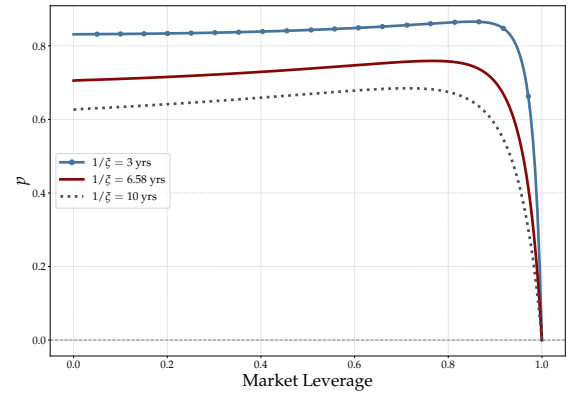
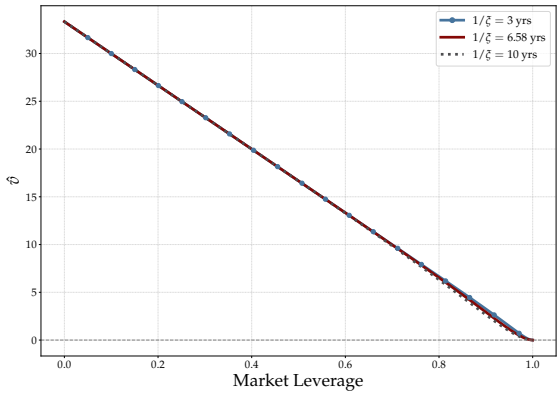


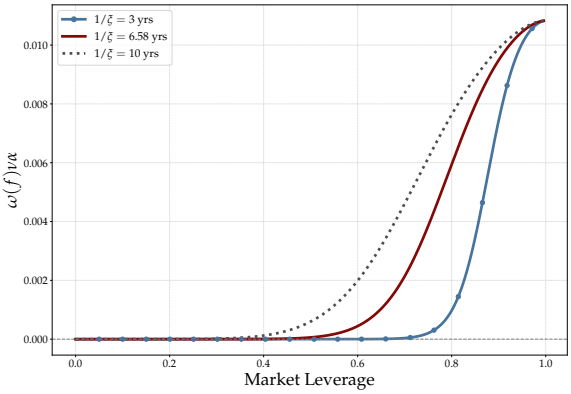
FIGURE D.12. Maturity Comparative Statics: Stationary Distribution



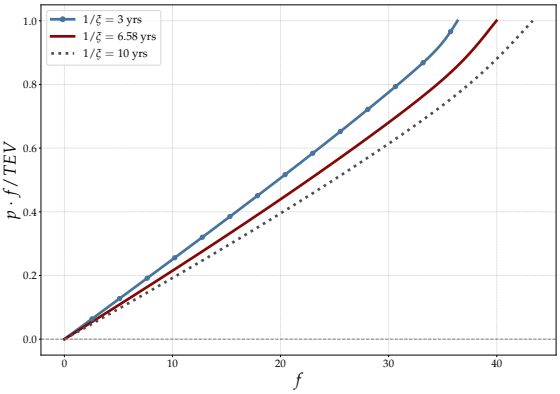
A. Debt Price



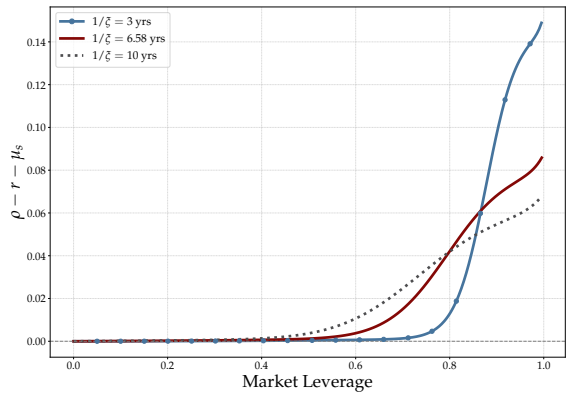
B. Equity Price



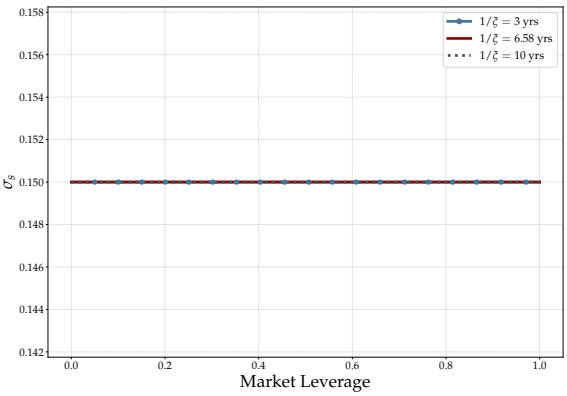
C. Price of Imbalances



D. Market Leverage

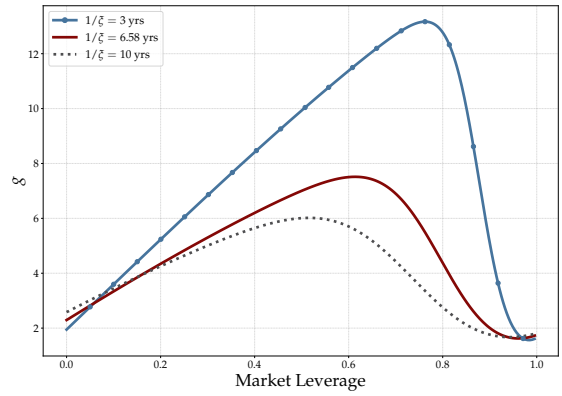


E. LC Risk Premium

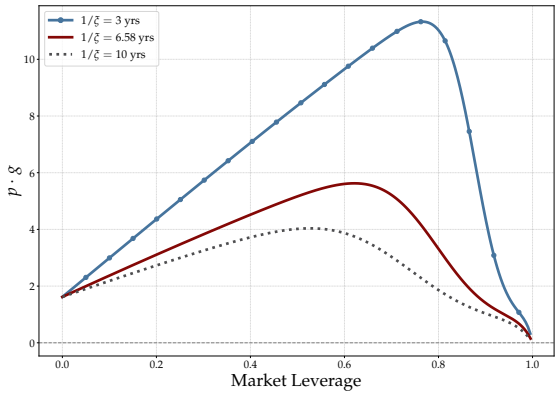


F. Exchange Rate Volatility

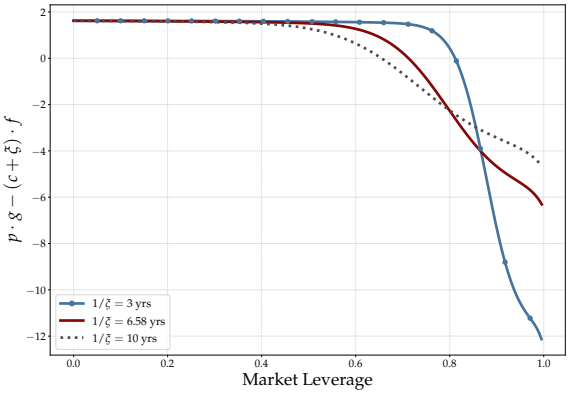
FIGURE D.13. Maturity Comparative Statics: Prices and Values



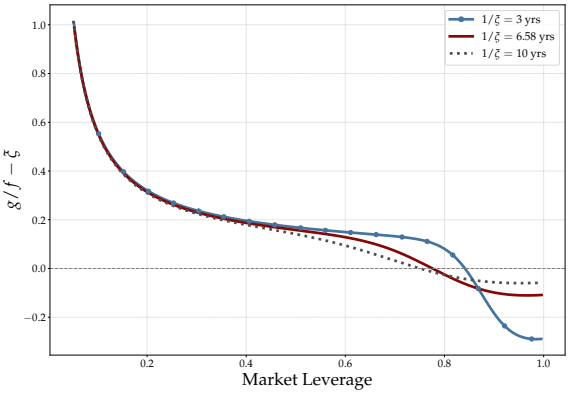
A. Optimal Issuance



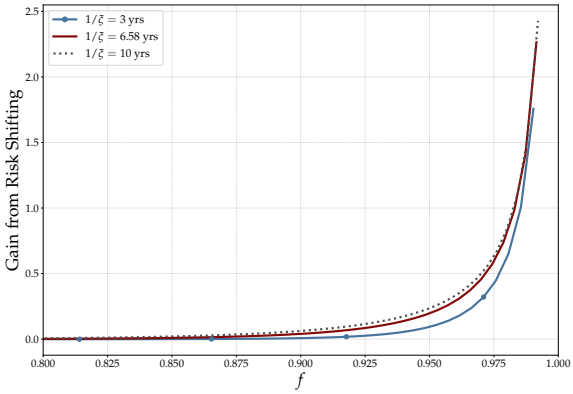
B. Proceeds



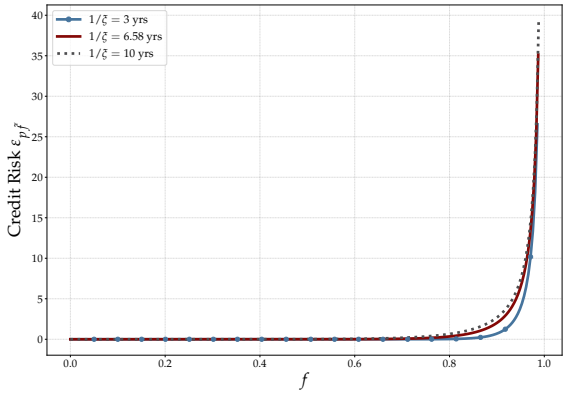
C. Capital Inflow



D. Net Issuance Rate



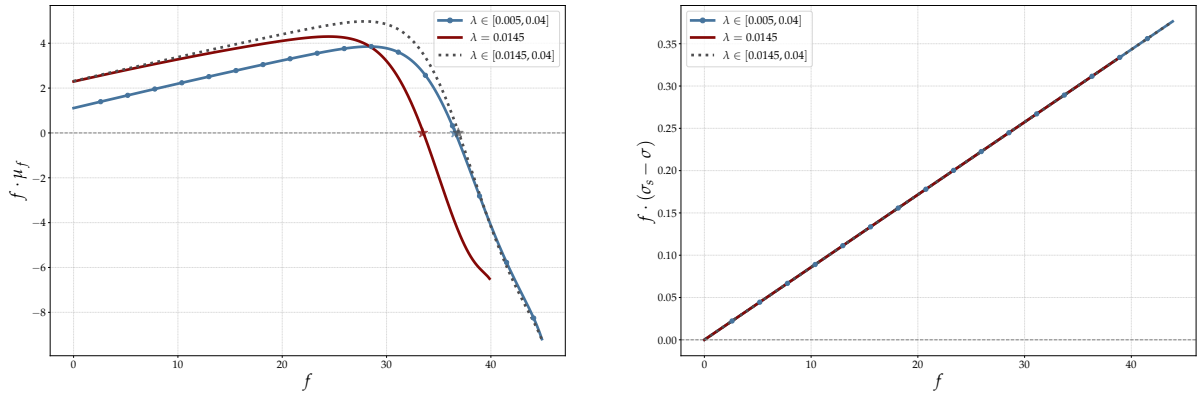
E. Gain from Risk-Shifting



F. Credit Risk

FIGURE D.14. Maturity Comparative Statics: Issuances and Flows

D.6. Numerical Solution for Convenience Yield Comparative Statics



A. State Variable Drift

B. State Variable Volatility

FIGURE D.15. State Variable Dynamics

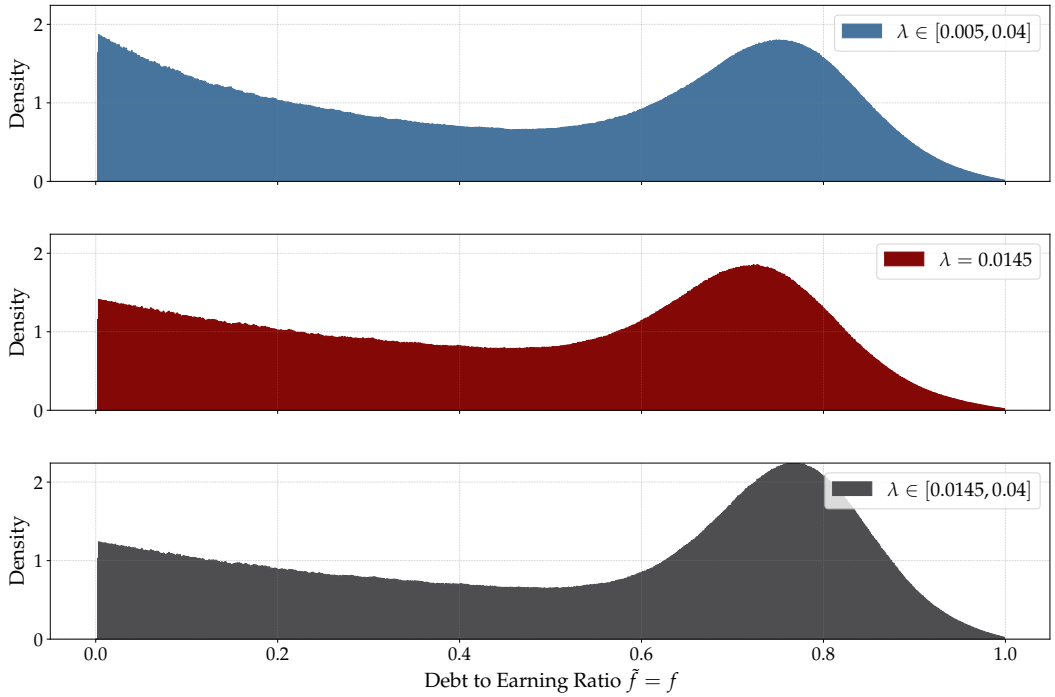
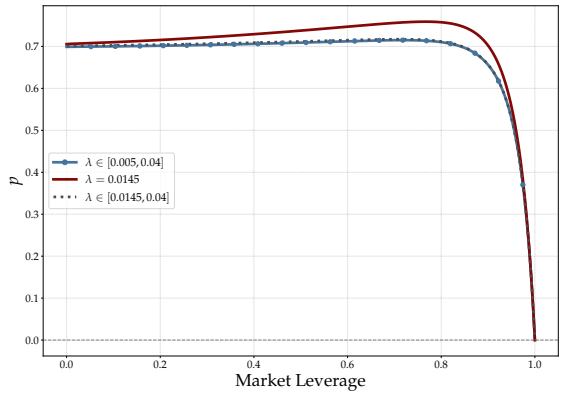
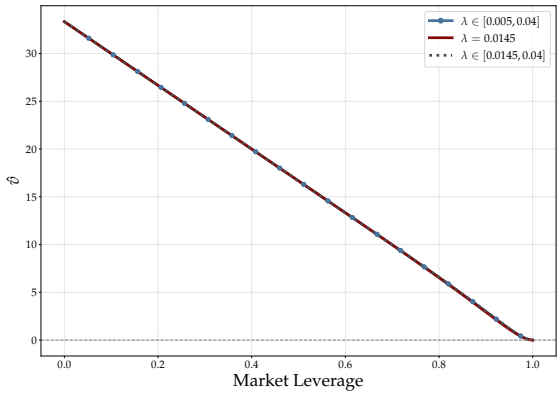


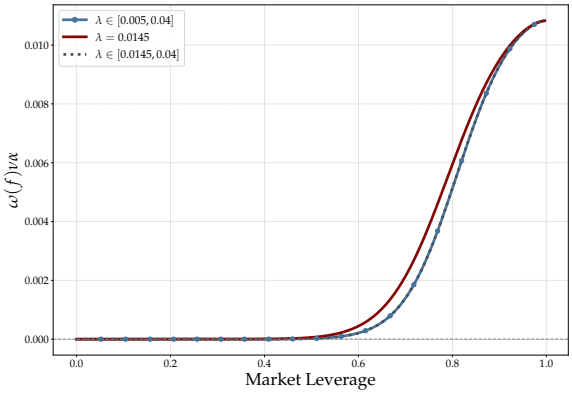
FIGURE D.16. Convenience Yield Comparative Statics: Stationary Distribution



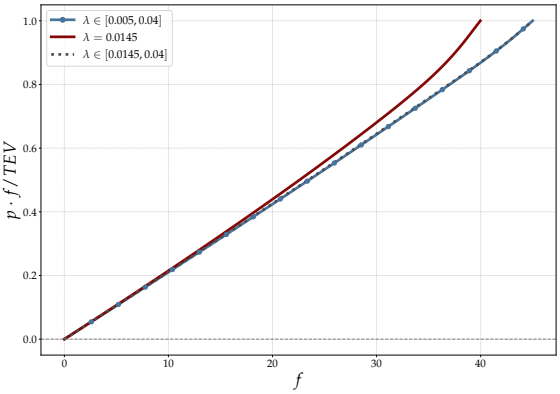
A. Debt Price



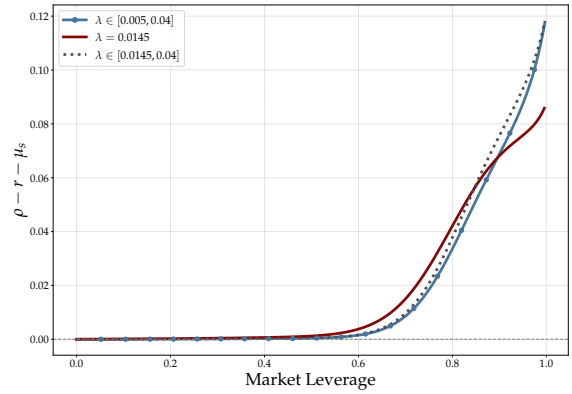
B. Equity Price



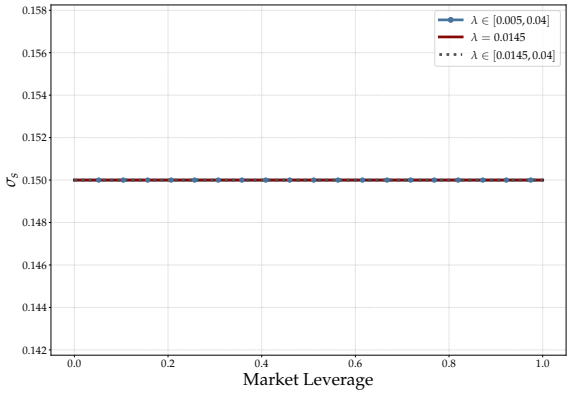
C. Price of Imbalances



D. Market Leverage

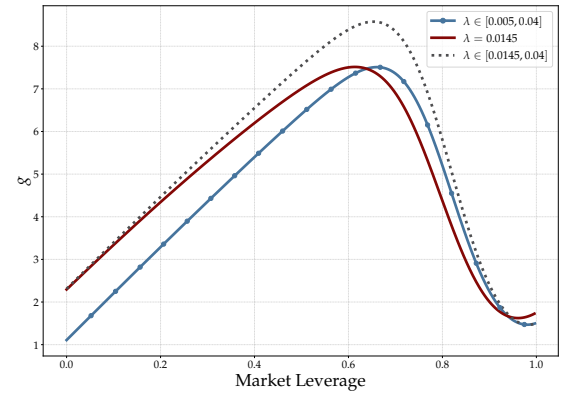


E. LC Risk Premium

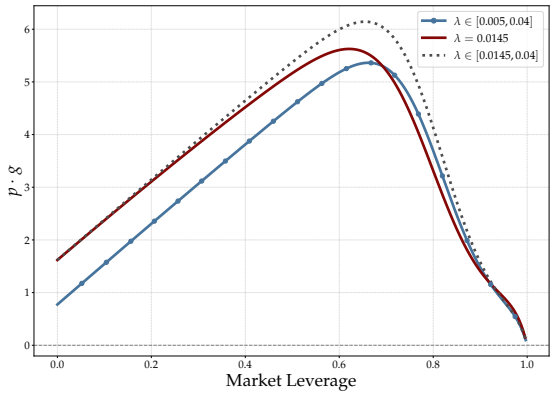


F. Exchange Rate Volatility

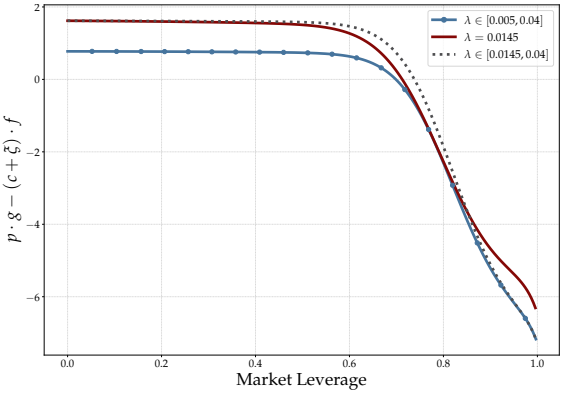
FIGURE D.17. Convenience Yield Comparative Statics: Prices and Values



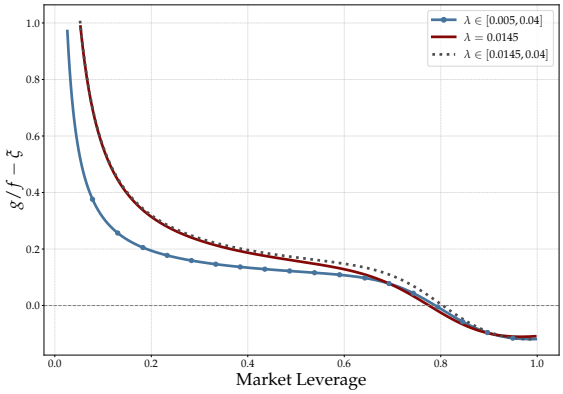
A. Optimal Issuance



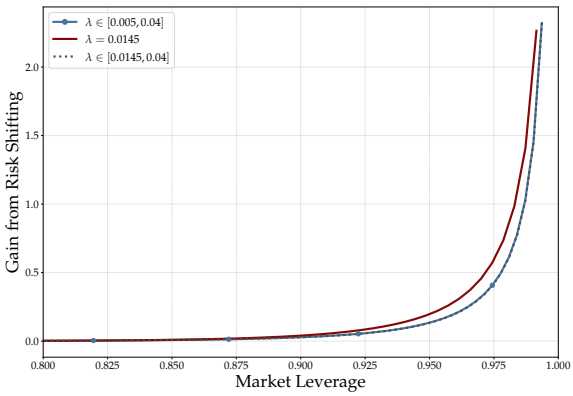
B. Proceeds



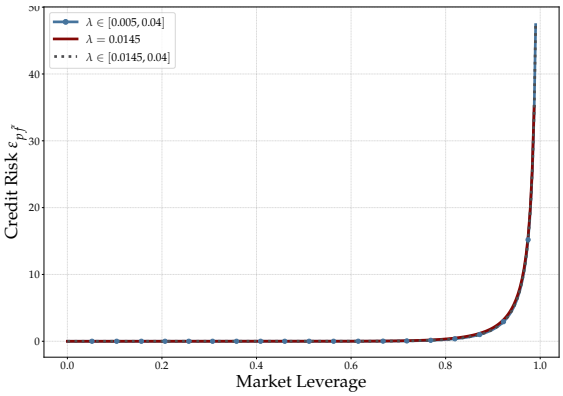
C. Capital Inflow



D. Net Issuance Rate



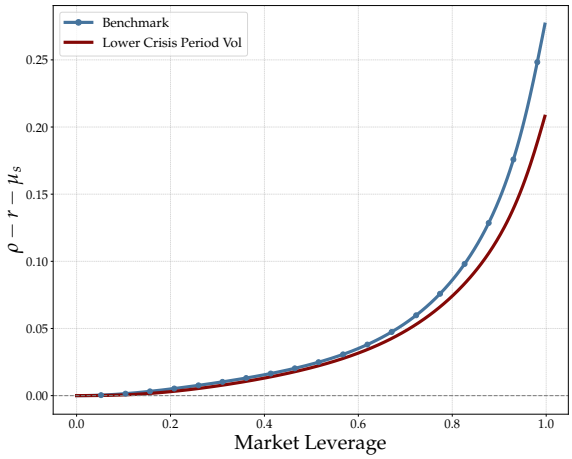
E. Gain from Risk-Shifting



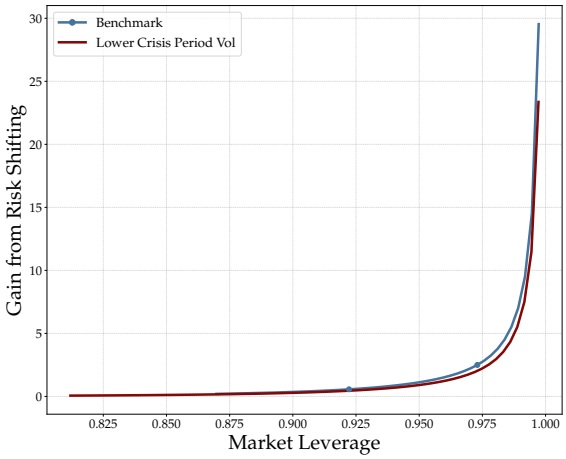
F. Credit Risk

FIGURE D.18. Convenience Yield Comparative Statics: Issuances and Flows

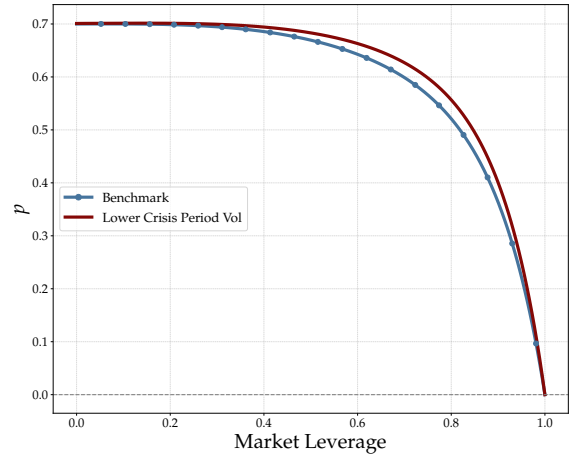
D.7. Additional Figures for Section IV.2



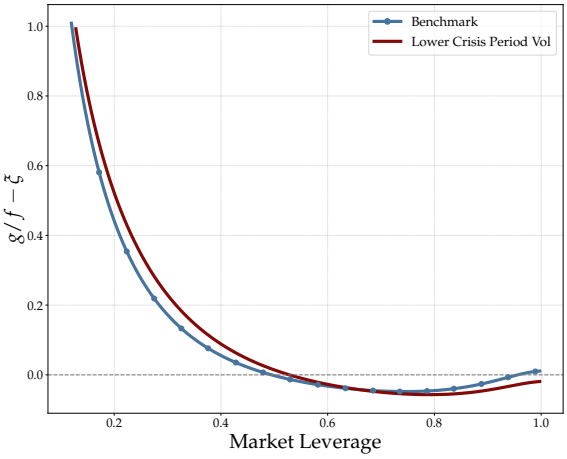
A. LC Risk Premium



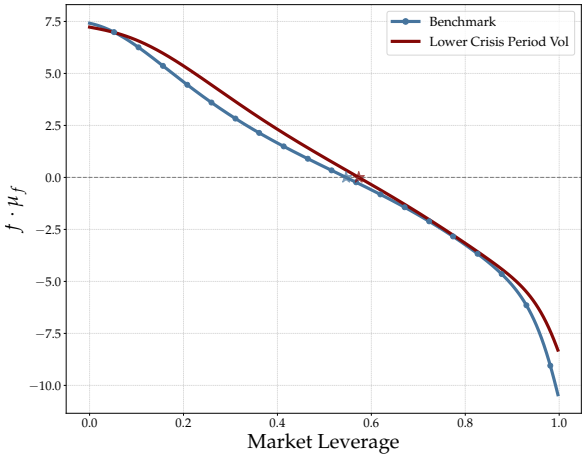
B. Gain From Risk-Shifting



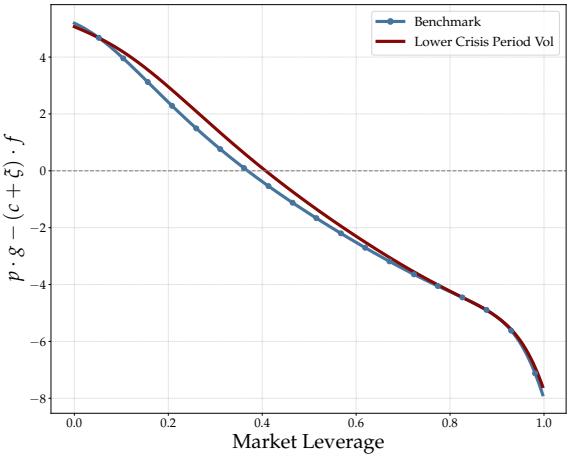
C. Debt Prices



D. Net Issuance Rate



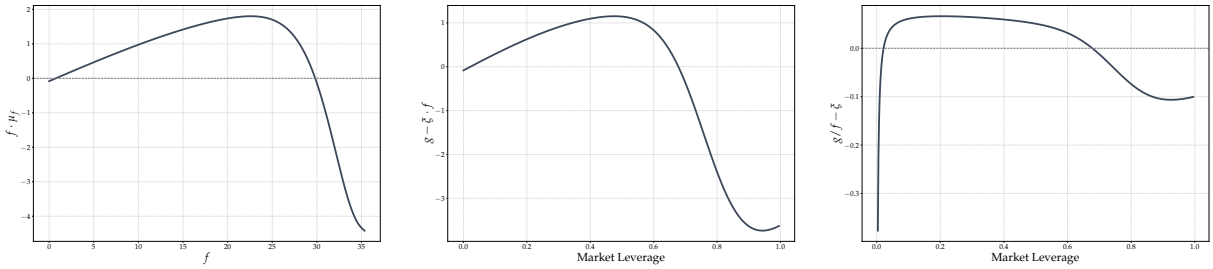
E. State Variable Drift



F. Capital Inflow

FIGURE D.19. Additional Figures for Experiment I

Appendix E. Zero Leverage: Comparison with DeMarzo and He (2021)



A. State Variable Evolution B. Net Issuance C. Net Issuance Rate

FIGURE E.1. Zero-Leverage Equilibrium

One notable feature of debt issuance, presented in DeMarzo and He (2021), is that when a firm's market leverage approaches zero, the model predicts an explosion in debt issuance. This feature, however, does not manifest in this paper. The key reason lies in the BoP condition embedded within this model. If a firm were to issue an infinite amount of foreign debt, the market would require a counterpart willing to take an equally vast currency position in the opposite direction. Yet, in my model, the dynamics of the exchange rate cannot sustain such extreme positions. Consequently, debt issuance, g, remains finite, even as market leverage dwindles toward zero.

Furthermore, this model does not inherently guarantee the debt ratchet effect—where firms persistently refrain from repurchasing outstanding debt. Indeed, under specific parameter settings, firms may opt for debt buybacks when their outstanding liabilities are relatively small, and market leverage is low. This results in zero leverage emerging as a stable stochastic steady state, diverging from the result in DeMarzo and He (2021). In their model, zero leverage constitutes an equilibrium, but it is unstable. Without a commitment mechanism, firms cannot capitalize on the profits from debt issuance, therefore, a zero-leverage equilibrium exists. But this equilibrium quickly disappears once debt is issued.

To illustrate this stable zero-leverage equilibrium, I adjusted the parameters by removing any convenience yield from holding FC assets, setting $\lambda = 0$. Under this configuration, investors maintain zero positions when the UIP holds. This corresponds to a scenario where the firm sector exhibits zero market leverage. LC depreciates in expectation, prompting the central bank to hold a long position in FC and a short position in LC. If exogenous flows create sufficient demand for LC, offsetting the CB's position, the residual market position forces the firm sector into a negative LC stance—effectively engaging in debt buybacks. Figure E.1 illustrates this scenario, with $\overline{\mu} = 0.3$, where firms reduce debt when market leverage is low. As market leverage rises, rollover requirements increase, debt issuance intensifies and turns positive.

The primary insight from these findings is that credit supply conditions influence the debt

ratchet effect. In DeMarzo and He (2021), credit supply is assumed to be perfectly elastic, meaning there are no constraints on the quantity of credit at a given debt price. However, this assumption is violated in my model due to the BoP condition. Specifically, the limited capacity of other market sectors to offset infinite debt issuance imposes a natural constraint.

Despite this, the debt ratchet effect persists once firms reach a higher level of market leverage: they do not repurchase debt when market leverage is elevated. This phenomenon hinges on the imbalances in the currency market that occur at high leverage levels. For equilibrium to be maintained, either the firm sector or other market participants must increase their holdings of LC to counterbalance the demand for FC triggered by debt repayment. If firms attempted to buy back debt, this would exacerbate the existing market imbalances. As a result, market forces—via adjustments in debt prices or exchange rate dynamics—effectively prevent such buybacks from occurring.

Appendix F. The Cross Section of Currency Risk Premia and Interest Rate Differentials

A key property of the currency risk premium is its positive loading on interest rate differentials, a relationship observed both in time series and cross-sectional data. The well-known Forward Premium Puzzle, identified by Fama (1984), reveals this pattern in time series data. Similarly, Lustig and Verdelhan (2007) find evidence in cross-sectional data, showing that sorting currencies into portfolios based on their interest rates and taking long positions on high-interest currencies while shorting low-interest ones yields a positive currency risk premium. This means that if we run a regression across a cross-section of currencies:

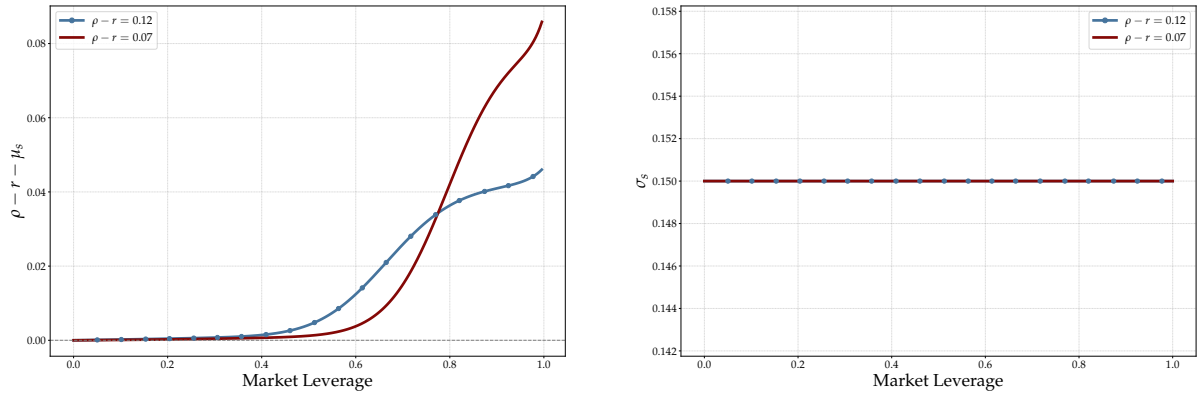
$$rx_{t+1}^i = \alpha_t^i + \beta_t \left(r_t^i - r_t^{\$} \right) + \varepsilon_{t+1}^i$$

β tends to be positive. Since this paper assumes a fixed constant interest rate, the model is better suited to explore this cross-sectional property.

To do so, I perform comparative statics with respect to $\rho - r$ using the model with both rollover and intermediation. This involves comparing two economies that are identical except for a slightly higher LC interest rate in one. As shown in Section IV, exchange rate volatility also significantly impacts the currency risk premium, so I use a constant volatility model where both economies share the same constant exchange rate volatility of 0.15.

The currency risk premium results are displayed in Figure F.1. The plot is shown against market leverage, but for now, let us first consider an unconditional perspective. Imagine two countries that both start with zero debt and begin issuing debt, but with different LC interest rates: one has an interest rate differential to the FC risk-free rate of 0.07, and the other, 0.12. Observing these countries over a long period, we should see that the high interest rate differential country tends to exhibit a higher currency risk premium, especially before the low differential country reaches a high leverage region.

Figure F.2A shows that with a larger interest rate differential, a country's debt capacity is lower, causing the market leverage to rise sooner at smaller debt levels, f. As a result, a small debt level can still lead to high market leverage in such an economy. Consequently, with similar drifts in state variables, the economy with $\rho - r = 0.12$ reaches a high leverage region near the stochastic steady state in approximately 4–5 years, while the economy with $\rho - r = 0.07$ takes around 10 years. The stationary distribution also indicates that the high-differential economy spends more time in high leverage regions than the low-differential one, leading to more frequent observations of elevated currency risk premiums. This pattern is confirmed by Figures F.2C and F.2D. Particularly within the first several years (the figure simulates 15 years), the low $\rho - r$ remains in a low-leverage



A. LC Risk Premium

B. Exchange Rate Volatility

FIGURE F.1. Exchange Rate Dynamics

state with a lower RP_t, while the high $\rho - r$ economy has already moved into a high-risk premium region. Thus, in this unconditional model, the positive factor loading of interest rate differentials arises from its positive correlation with market leverage.

We now turn to a conditional model: what if we control for market leverage? Can we still observe a positive coefficient for the interest rate differential? The answer is yes, as long as both economies are in a normal period with market leverage below approximately 75%, as shown in Figure F.1A. When leverage is below 75%, the risk premium rises earlier for economies with larger interest rate differentials. However, once market leverage becomes too high, this relationship reverses, predicting a smaller LC risk premium for high interest rate differentials.

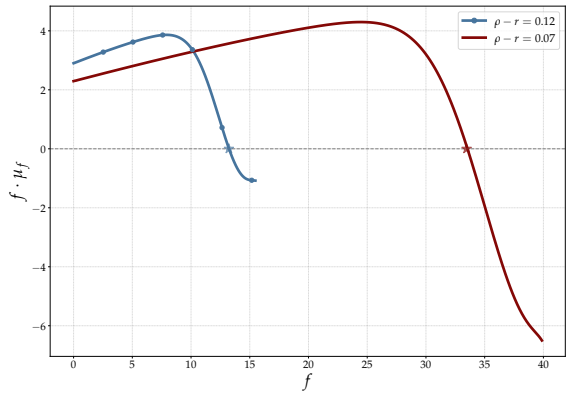
To explore the source of this result, I decompose the currency risk premium into its main components:

$$RP_t = \omega(f) \cdot \left\{ \nu \cdot \left[\lambda + \alpha \Big((c + \xi) f_t - \overline{\mu} \Big) \right] + (1 - \nu) \cdot (\rho - r) \right\} - (1 - \omega(f)) \cdot \left(\varepsilon_{ps} \|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY} \boldsymbol{\sigma}' \boldsymbol{\Sigma} \boldsymbol{\sigma}_s \right).$$

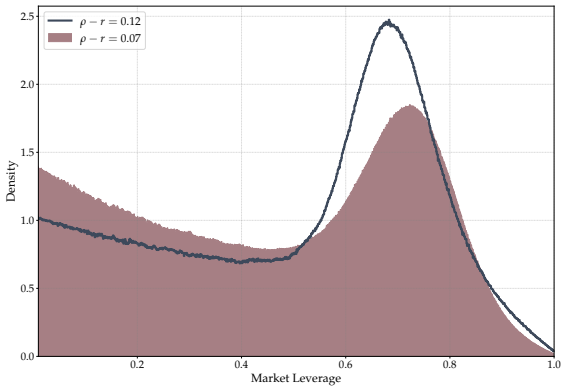
The components are defined as follows:

$$\begin{aligned}
&\text{Flow Component} = \omega(f) \cdot \nu \cdot \alpha \Big((c + \xi) f_t - \overline{\mu} \Big), \\
&\text{Interest Rate Differentials Component} = \omega(f) \cdot (1 - \nu) \cdot (\rho - r), \\
&\text{Risk-shifting Component} = -(1 - \omega(f)) \cdot \left(\varepsilon_{ps} \|\boldsymbol{\sigma}_s\|^2 - \varepsilon_{pY} \boldsymbol{\sigma}' \boldsymbol{\Sigma} \boldsymbol{\sigma}_s \right).
\end{aligned}$$

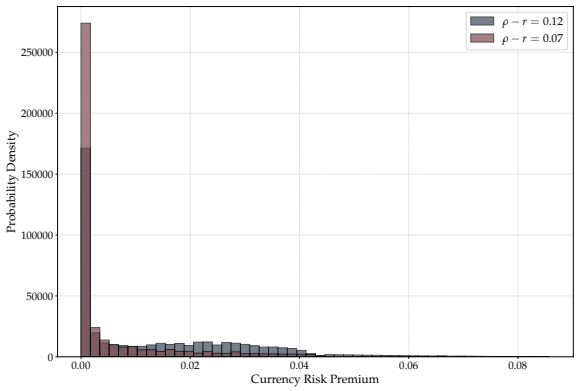
Figure F.3A shows each component's contribution to the currency risk premium. The conditional risk premium aligns closely with the Flow Component, which consists of two parts: the price of imbalances and the quantity of imbalances. The initial increase in the currency risk premium is driven by price, while the later reversal in the relationship is due to quantity effects.



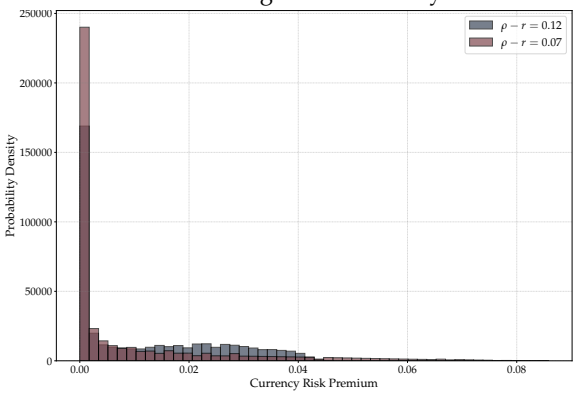
A. LC Risk Premium



B. Exchange Rate Volatility

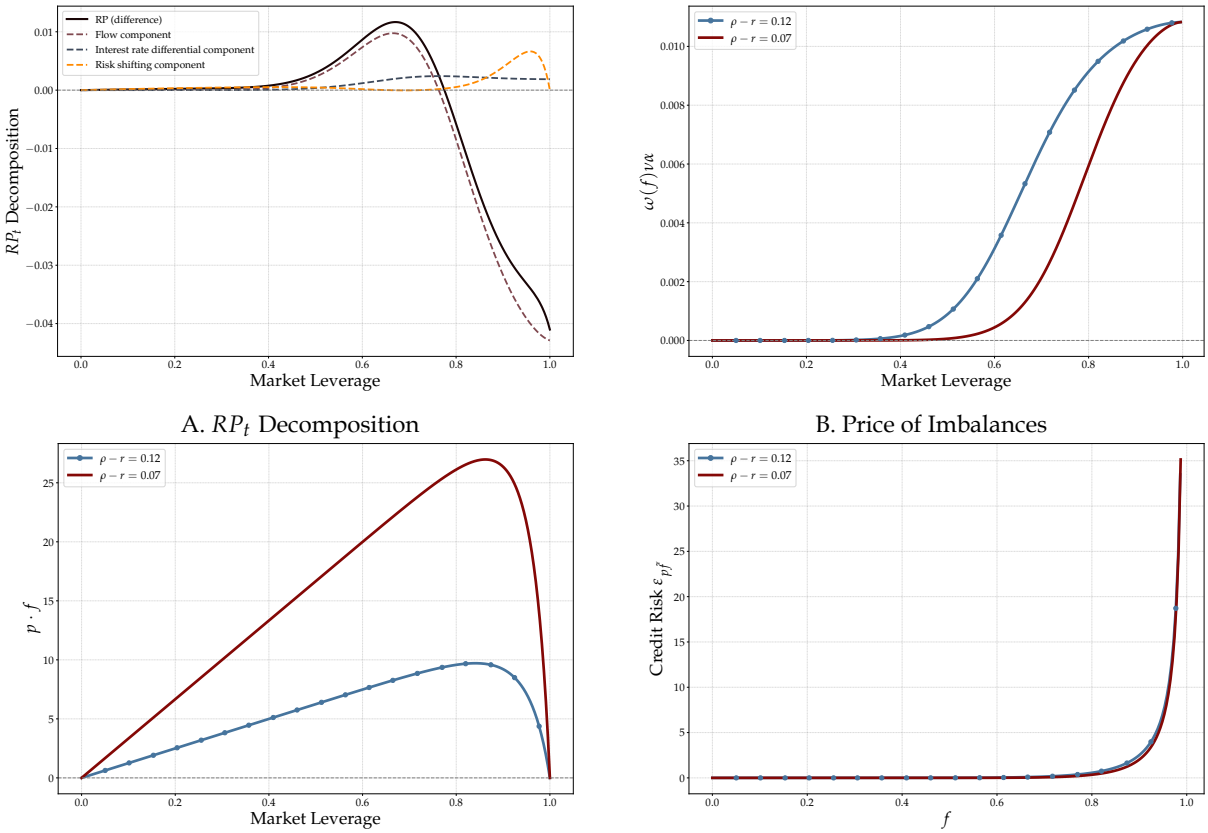


C. RP_t Distribution: 15 years



D. RP_t Distribution

FIGURE F.2. Unconditional Model



A. RP_t Decomposition

B. Price of Imbalances

C. Market Value of Debt

D. Credit Risk

FIGURE F.3. Conditional Model

Interestingly both price and quantity components are driven by debt capacity. As observed, a high interest rate differential limits debt capacity, with a smaller default boundary than for a low differential economy. Consequently, when market leverage approaches 1, and the price of imbalances converges, the higher debt capacity economy requires larger repayment flows, resulting in a higher risk premium.

For the price of imbalances, both economies share the same $\nu \cdot \alpha$ value, but differ in the firm sector's ability to reduce the risk premium, represented by $\omega(f) = \frac{\varepsilon_{p\tilde{f}}}{\alpha \nu p_t f_t + \varepsilon_{p\tilde{f}}}$. Controlling for market leverage helps measure credit risk, $\varepsilon_{p\tilde{f}}$, effectively; however, differences in $\omega(f)$ arise due to the aggregate debt quantity. With the same market leverage, a higher interest rate differential correlates with a smaller market value of total debt outstanding, diminishing firms' capacity to reduce imbalances. As discussed in the main text, market value of debt indicates a forward-looking channel: with smaller aggregate debt, firms anticipate lower future repayment flows and therefore a smaller future LC risk premium. This expectation increases their effective debt burden, making them less inclined to issue debt and demanding a higher premium today. Therefore, during normal periods, RP_t is higher for high interest rate differential currencies, but this relationship reverses in crisis periods.

Finally, for the modelling choice, as in KV, one key fact about the LC risk premium is that the average correlation between the LC risk premium and the interest rate differential in EMEs is 70%, which is a high number. At first glance, this poses a question to my modeling choice of a partial equilibrium model with fixed interest rate differentials. However, it is the high correlation between the interest rate differential and the LC risk premium that implies a small influence of the interest rate differential on the predictable component of exchange rate movement, μ_s:

$$\mu_s = \rho_t - r_t - RP_t$$

The components that affect the interest rate differential but are orthogonal to the currency risk premium, or the components that affect the currency risk premium but are orthogonal to the interest rate differential, are the factors that determine the predictable component of the exchange rate. By setting a fixed interest rate differential, $\rho - r$, this paper focuses on the latter and links it to the firm sector and, consequently, the market's risk-bearing capacity and the imbalances driven by the leverage cycle.