The Local Root of Wage Inequality

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Preliminary draft. Do not circulate.

Abstract

Wage inequality varies substantially across space. I document two novel facts that highlight the role of employers —and the reallocation of workers between them— in shaping local wage inequality. First, the spatial concentration of high-paying employers explains one-third of the variation in wage inequality across cities. Second, between-city wage gaps widen over workers' careers as the wage gains from job switches are larger in bigger cities. I propose a spatial framework that rationalizes these facts through two ingredients: heterogeneous employers and on-the-job search. Productive employers agglomerate in large cities to hire more workers. Fiercer competition for workers endogenously arises and pushes productive employers to offer high wage to maintain their target size. Greater wage inequality follows. I provide a proof of identification that does not target within-city wage inequality. I show that the model endogenously give rise to the two novel facts documented in this paper. Finally, I quantify that the concentration of productive employers in large cities boosts aggregate wage by reducing employers' market power but generates large spatial welfare disparities.

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1 Introduction

Inequality varies substantially across space. In urban hubs, high-income earners live alongside low-wage workers who face high costs of living. In Paris for instance, workers at the 90th percentile of the local wage distribution earn hourly wages four times larger than workers at the 10th percentile. Meanwhile, after accounting for housing prices, low-wage workers earn real wages 15% lower than their counterparts in Lens, a typical mid-size French city with only a twofold pay gap between high- and low-wage workers. These large disparities have raised concerns among pundits and policy circles that modern metropolises are at the root of growing national inequality (e.g., United Nations, 2020; Goldin, 2023). Despite the extensive research on the benefits of agglomeration, little is known about what causes within-city wage inequality. Why do some cities exhibit higher wage inequality than others? Why are workers willing to accept lower real wages in large cities? And what does local wage inequality reveal about the functioning of local labor markets?

This paper offers answers to these questions in three parts. First, I document that employers ——and the reallocation of workers between them—— are instrumental in shaping local wage inequality. Second, I provide a framework that rationalizes between-city differences in wage inequality through the sorting of employers across frictional local labor markets. Third, I estimate the model, validate its quantitative relevance, and show that workers accept low real wages in large cities in anticipation of higher future income.

Specifically, in the first part of the paper, I leverage French matched employer-employee data to document two novel facts about the role of employers in shaping local wages. First, I show that the concentration of high-paying jobs in a few large locations accounts for one-third of the variation in wage inequality across cities. To quantify the relative importance of employers, I estimate a two-way fixed effects model à la Abowd et al. (1999) and decompose local wage variance into components attributable to worker and employer heterogeneity. I find that the dispersion in employer fixed effects accounts for one-third of the variation in wage inequality across commuting zones (CZs). Larger cities feature greater dispersion in employer premia: in the cross-section, doubling city size is associated with a 0.025 increase in the variance of employer fixed effects (relative to a mean of 0.052). This greater dispersion arises from the spatial concentration of high-paying jobs: whereas the 90th percentile of the employer fixed effects in Paris is 15% higher than in Lens, the 10th percentile is only 2% greater.

Second, I document that the reallocation of workers across employers generates substantial spatial gaps in lifetime incomes. I estimate that doubling city size increases the wage gains upon a job switch by 0.3 percentage points (relative to a mean of 1.3%). This pattern holds after accounting for the sorting of heterogeneous workers across space and the impact of locations on learning. Meanwhile, the average number of job switches is stable across locations. As a result, substantial differences in lifetime income arise. By the end of their careers, I estimate that workers in Paris earn wages 12% higher than those in Lens, despite an initial wage gap of only 0.1% prior to any

job switch. The estimated income profiles, net of worker heterogeneity, thus resemble qualitatively and quantitatively the local distributions of employer premia: workers start their career at jobs offering similar wages regardless of their locations, but transitions between employers generate spatial differences in the upper tail of wage distributions.

Altogether, these two facts underscore the key role of employers —and how workers reallocate between them— in shaping local wage inequality. In the second part of the paper, I build a theoretical framework that ties these two facts together through the sorting of employers across frictional labor markets.

Workers are ex-ante homogeneous. They choose where to reside to maximize their lifetime utility, which depends on current real income, expected future real income, and local amenities. Workers continuously search for better-paying opportunities: they start their careers unemployed and progressively experience wage growth by climbing the local job ladder. Employers differ in productivity and decide where to produce and hire workers. Search frictions constrain the effective size of employers. As in Burdett and Mortensen (1998), wages are an effective hiring tool: offering higher wages allows employers to attract and retain more workers from the local competitors.

In equilibrium, productive employers agglomerate in large cities. Their location decision depends on three considerations: how many workers live there, how intense is the local competition as captured by the local wage offer distribution, and how expensive is commercial housing. Complementarity in production between productivity and size implies that productive employers have a stronger willingness to produce in larger locations. Sorting occurs despite the absence of agglomeration forces (e.g., Gaubert, 2018) or TFP differentials (e.g., Bilal, 2023) as large cities allow employers to partially sidestep search frictions. Endogenous local competition dampens the size advantage of large cities: as productive employers agglomerate in large cities, the poaching competition intensifies, and employers are required to offer higher wages to reach their target size. This offsetting competition channel implies that a range of mid-productivity employers are indifferent between producing in large cities with fierce competition or in small locations where it is milder.

The endogenous differences in local competition rationalizes qualitatively the two novel document in this paper. The fierce competition in large cities requires productive employers, for which a large share of their workforce comes from poaching competitors, to offer high wages. In contrast, the wage offered by relatively unproductive employers are less sensitive to the poaching contest as most of their workforce are hired from unemployment. As a result, there are greater spatial variations in the right tail of wages than in the left and the wage gains from job switches are larger. The steep job ladder in large cities provides a rationale for why workers accept low real-paid jobs: because they anticipate higher future real earnings.

In the third part of the paper, I assess whether a quantitative version of the model can generate the two novel facts documented in this paper. I then use the estimated model to quantify the impact of employer sorting on local competition, aggregate wages, and spatial welfare disparities.

I extend the model in two significant ways. First, I allow cities to differ in Total Factor

Productivity (TFP) in addition to amenities. Second, employers face flexible curvature in job creation cost functions that let them grow without raising wages too rapidly. They also face idiosyncratic commercial housing costs to capture unobserved heterogeneity in location choice. The model is estimated on the employer fixed effects from the first part, which allows me to abstract from worker heterogeneity.

I provide a proof of identification for 15 of the 19 parameters by combining the strategy used in quantitative spatial models (Redding and Rossi-Hansberg, 2017) and wage-posting models (Bontemps et al., 2000). The remaining two parameters —the discount factor and the dispersion in workers' idiosyncratic preferences— are externally calibrated.

The key part of the estimation consists of separately identifying the three channels shaping employer sorting: city size, TFP, and the dispersion in idiosyncratic housing costs. City sizes are recovered from cities' employment share. Local profit opportunities are computed from firm-level data on location, size, and wages, together with the structure of the model. The dispersion in idiosyncratic costs is then estimated from the sensitivity of employers' location choice to the local profit opportunities. Given city sizes and idiosyncratic cost dispersion, TFPs are estimated as residuals to match each location's average wage net of the employer sorting predicted by the model.

I estimate that TFP gaps across space are minimal once employer sorting is accounted for. Given my estimates of city size, idiosyncratic cost dispersion, and search frictions, productive employers have strong incentives to locate in large cities to maximize their size. This productivity selection suffices to explain the average wage gaps across CZs. As a result, local TFPs only account for 0.70% of the between-city wage variance.

The estimation procedure explicitly targets the aggregate wage dispersion and city-specific average wages. It targets neither how wage inequality varies across space nor how steep are the local job ladders. I therefore assess the empirical relevance of my framework by testing whether the spatial sorting of employers endogenously generates these facts.

The model accounts quantitatively for the differences in inequality across cities. As in the data, it generates greater spatial variation in the right tail of the wage distribution than in the left. The targeted cross-sectional city size elasticity of average wages is 0.031. The size elasticity of the 10^{th} and 90^{th} percentiles are 0.002 and 0.054 in the model, compared to 0.005 and 0.053 in the data.

Endogenous differences in local competition shapes the local wage distribution. As productive employers concentrate in large cities, it intensifies the local competition, reduces employers' market power, and pushes wages up. For instance, employers charge a markdown 2 p.p. higher in Paris than they would if they were to produce elsewhere. The agglomeration of productive employers in large cities also intensifies the relative competition in small locations: unproductive employers now compete relatively more between each other, which increases the local markdown by 2 p.p. Aggregating across space, I find that the spatial sorting of employers increases the aggregate markdown — and therefore the aggregate wage — by 1%.

Turning to the second over-identification exercise, the model also matches the between-city

differences in job ladder steepness. The model predicts an average wage gain of 8% upon job switching, slightly above the 7% in the data when restricting the estimation to job switches associated with wage gains. These wage gains are 1.1 p.p. higher in cities 10% larger in the model, close to the empirical estimate of 0.97 p.p. Together, these two over-identification exercises validate the model's ability to account for the emergence of local wage inequality.

The steep job ladder in large cities translates into faster wage growth and higher lifetime consumption. The better expected future job opportunities imply that unemployed workers — and those on low rungs of the ladder — are indifferent between Paris and Lens despite despite earning 6% less in real terms. Meanwhile, they also imply that the average employed workers in Paris is willing to forego 3% of their lifetime consumption to avoid moving to Lens. I provide a first order welfare decomposition into three channels: amenities, contemporaneous real wages, and expected future real income. I show that, absent future job opportunities, workers are nearly indifferent as to where to live. I conclude that local job ladders — and how employer sorting across locations shapes them — are crucial to understand spatial welfare disparities.

Related literature This paper relates to several strands of literature. The first is the literature that studies the drivers of local wage inequality (Glaeser et al., 2009; Baum-Snow and Pavan, 2013; Eeckhout et al., 2014; Baum-Snow et al., 2018; Papageorgiou, 2022). This literature has primarily focused on the role played by worker heterogeneity. I emphasize that employers — and how workers reallocate across them — is a key driving force.

To separate the contribution of employer and worker heterogeneity, I estimate a two-way fixed effect model as in the original Abowd et al. (1999) and the long list of papers that followed (e.g. Card et al., 2013; Bonhomme et al., 2019; Song et al., 2019). Applying this method to local labor markets, Dauth et al. (2022) show that the high average wage of large location is partially explained by a better match between worker and firm, and Card et al. (2025) estimate the average wage impact of cities net of worker heterogeneity. While both papers focus on the differences in average wage across locations, I use this method to study within-city wage inequality.

I estimate that the reallocation of workers across employer spurs substantial wage growth, with larger gains in more unequal, bigger cities. This finding contributes to the literature on the dynamic impact of cities on wages (Baum-Snow and Pavan, 2012; Roca and Puga, 2017; Porcher et al., 2021; Eckert et al., 2022; Lhuillier, 2023). This literature has mostly investigated how cities shape learning. I find that the steeper job ladder in large cities — as measured by higher wages gains from job switching — explains 60% of the faster wage growth.

I microfound the existence of local job ladders by incorporating search frictions and on-the-job search into a quantitative spatial model, as in Schmutz and Sidibé (2019), Martellini (2020), and Heise and Porzio (2022). I contribute to these papers by allowing heterogeneous employers to sort across local labor markets, which I show is key to rationalize the steeper job ladder in large cities.[1]

[1]The role played by firm sorting on spatial inequality has also been documented by Combes et al. (2012), Behrens et al. (2014), Gaubert (2018), Bilal (2023), Oh (2023), and Franco (2024).

Most closely related to this paper, Lindenlaub et al. (2022) develop a framework where employers sort across frictional labor markets in which workers search on the job. Our papers complement each other: they study the impact sorting has on local labor shares whereas I analyze its impact for local wage inequality.

Finally, this paper connects to the literature on search frictions and monopsony power (Burdett and Mortensen, 1998; Bontemps et al., 2000; Engbom and Moser, 2021; Bilal and Lhuillier, 2021; Jarosch et al., 2021; Berger et al., 2022; Lamadon et al., 2022; Gouin-Bonenfant, 2022; Morchio and Moser, 2024). I contribute to it by showing that the sorting of employers across labor market shapes the rent-sharing process, reduces employers' market power, and increases aggregate wages.

The rest of this paper is organized as follows. Section 2 presents the two novel facts. Second 3 develops the theoretical framework that jointly accounts for them. Second 4 lays out the quantitative extensions of the model and its structural estimation. Second 5 concludes by assessing the quantitative relevance of the model.

2 Two facts about local wage inequality

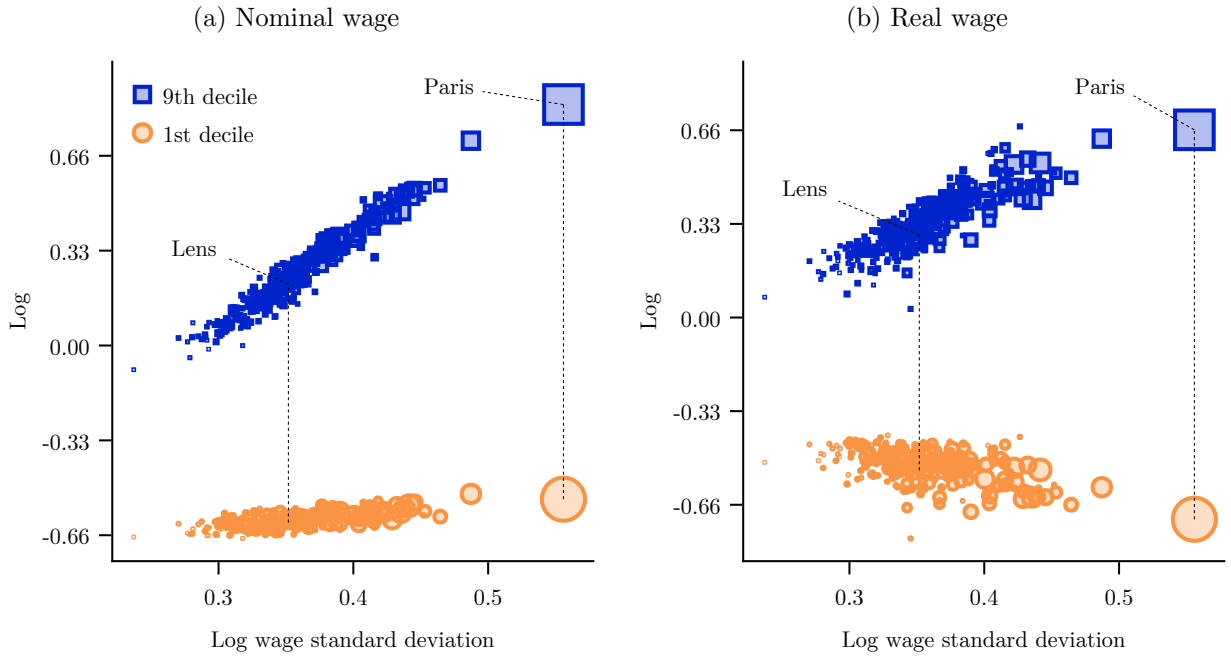
2.1 Data

I use employer tax records from France (*Déclaration Annuelle de Données Sociales*, DADS) between 2008 and 2019. This dataset comes in two formats. The first is a 4% representative panel that tracks the entire history of individuals in the labor market (*DADS panel*). The second format is a repeated cross-section covering the universe of employed workers (*DADS salariés*). Both datasets provide information on workers' earnings, the number of hours worked, the establishment where they are employed and their occupation, the location where they work and live, along with demographic information.

This dataset has two advantages. First, its panel structure, combined with the information of workers' employers, allow to separate the relative contribution of workers and employers to wage dispersion. Second, its large sample provides sufficient statistical power to estimate precisely statistics at relatively granular geographical units.

I apply the same sample restrictions on both datasets. I focus my analysis on full-time employed workers between 25 and 55 years old. Workers employed in the public sector in France have their wage determined nationally by their tenure rather than based on their productivity or the local competitiveness of the labor market. I therefore keep in the sample workers employed in the private sector, and I exclude the education and health industries due to their large fraction of public servants. I abstract from the labor supply decision and use gross hourly wage as my measure of labor income. I deflate wages by the aggregate consumption price index and express them in 2018 euros. Appendix A.1 provides more details on the construction of the sample and Table A.1 some aggregate summary statistics. Between 2008 and 2019, aggregate wage inequality is constant in France.

Figure 1: Hourly wage distributions by commuting zone



In both panels, each circle and rectangle is a commuting zone. Panel (a) displays the 1st (orange circle) and 9th (blue rectangle) deciles of the city nominal wage distribution. Panel (b) displays the 1st (orange circle) and 9th (blue rectangle) deciles of the city real wage distribution. Real wages are computed as nominal wages deflated by a citywide Cobb-Douglas price index with a housing expenditure share of 0.3. Nominal and real wages are normalized by their respective national average.

I define a city as a commuting zone (CZ). A commuting zone is a statistical area defined by the French statistical agency (INSEE). It consists of a collection of contingent municipalities clustered together to reduce the commuting flows across them. Commuting zones are thus the natural geographical unit when thinking of local labor markets —and the competition that takes place in them. There are 297 such areas in metropolitan France, which span the whole territory, with an average size of 31,823 employed workers. I use the terms commuting zones, cities, and locations interchangeably for the rest of the paper.

For computational feasibility and statistical accuracy, I sometimes group cities together according to their wage inequality. Specifically, I construct ten population-weighted deciles of within-city wage variance. The first decile contains 100 cities, Lens is in third group, and the tenth decile is composed of Paris and Saclay, the South-West suburb of Paris. Table A.2 provides summary statistics on each of the city cluster. This clustering accounts for 77% of the between-city variation in within-city wage variance.

2.2 Local wage inequality in France

Figure 1 displays the extent to which wage inequality varies across cities. Panel (a) plots the 10th (orange circles) and 90th (blue rectangles) percentiles of the city wage distribution against the local

standard deviation of log wages.

Two patterns strike out. First, there are substantial differences in wage inequality across CZs. For instance, the standard deviation of log wages is 0.56 in Paris, 58% larger than the standard deviation in Lens (0.35) — the average French city when it comes to wage inequality. For comparison, the standard deviation of log wages is 0.63 in the United States and 0.44 in France, countries often associated with high and low inequality. More unequal cities tend to be larger and have higher average wage and housing prices (Table A.2)

Second, large spatial variations in the right tail of the wage distributions cause the differences in wage inequality. By contrast, low-wage workers earn relatively similar wages everywhere. Going back to the earlier example, workers at the 90th percentile earn wages 87% higher in Paris than in Lens; this between-city wage gap is 8% for workers at the 10th percentile. Figure A.1a in Appendix A.4 shows that this pattern holds throughout the wage distribution: higher wage deciles are systematically more correlated with local wage inequality.

Figure 1b repeats the same exercise with real wages. Real wages are computed as nominal wages deflated by a citywide Cobb-Douglas price index with a housing expenditure share of 0.2.[2] Since the price index is constant within city, the standard deviation of log real wages is identical to that of nominal wages.

Figure 1b reveals that low-wage workers earn lower real earnings in more unequal cities. The higher housing prices indeed more than offset the moderately higher nominal wages. For instance, workers at the 10th percentile of the wage distribution in Paris earn real wages 15% lower than their counterpart in Lens. Figure A.1b extends the analysis to all the deciles of the wage distribution; on average, workers in the bottom 30% of the city wage distribution earn lower real wages in more unequal places. A similar pattern holds when the housing price index takes into account the sorting of workers across neighborhoods and when the analysis focuses on (domestic) migrants (Figure A.1b).

I now study the role of employers in shaping local wage inequalities.

2.3 Fact #1: high-paying jobs are spatially concentrated

I estimate the role of employers on local wage inequality via a two-way fixed effect model à la Abowd et al. (1999) —AKM henceforth. Specifically, I estimate

$$\log w_{it} = \alpha_i + \beta_{\ell(i,t)} x_{it} + \gamma_{j(i,t)} + \varepsilon_{it}, \tag{1}$$

where i is a worker, t is quarter-year, x_{it} denotes experience, and $\ell(i,t)$ and $j(i,t)$ are indices for the city group and job where i is employed at time t. A job is defined as an establishment $\times$ 4-digit occupation. The parameters α_i and γ_j are worker and job fixed effects. In addition, I allow cities to

[2]There does not exists publicly available price index at the commuting zone level in France. See Section 4.3 for details on the housing data.

offer differential returns to workers' experience as captured by β_ℓ. I define workers' total effect as the sum of the static and dynamic components: $\psi_{it} \equiv \alpha_i + \beta_{\ell(i,t)} x_{it}$.

Workers who move between jobs identify separately worker and job fixed effects if worker mobility is conditionally random (Card et al., 2013). In this case, the job fixed effects capture wage premia —the additional wage a worker earns when working at a particular job. To limit well-known econometric difficulties linked to limited mobility bias, I follow Bonhomme et al. (2019) and group workers and jobs each in 100 equally populated groups based on their unconditional mean wage. I then estimate equation (1) with OLS at the group level.[3]

The AKM model (1) implies two useful decomposition to understand what drives spatial wage inequality. The first decomposition breaks down the bewtween-city wage variance into differences in the average worker and average wage premia:

$$\mathbb{V}\text{ar}[\mathbb{E}_\ell(\log w_{it})] = \underbrace{\mathbb{V}\text{ar}[\mathbb{E}_\ell(\psi_{it})] + \mathbb{C}\text{ov}[\mathbb{E}_\ell(\psi_{it}), \mathbb{E}_\ell(\gamma_{j_{it}})]}_{\text{Worker}} + \underbrace{\mathbb{V}\text{ar}[\mathbb{E}_\ell(\gamma_{j_{it}})] + \mathbb{C}\text{ov}[\mathbb{E}_\ell(\psi_{it}), \mathbb{E}_\ell(\gamma_{j_{it}})]}_{\text{Wage premia}}. \quad (2)$$

The literature studying spatial wage inequality has thus far focused on this decomposition to understand the importance of employers (e.g., Combes et al., 2008; Dauth et al., 2022; Card et al., 2025). However, the between-city wage variance accounts for 12% of the aggregate variance in France. Furthermore, as documented in Section 2.2, there exists large differences in within-city wage inequality across space.

The second decomposition sheds light on why some cities are more unequal than others. Given (1), Paris is more unequal than Lens for three potential reasons. Workers may be more diverse. The pay premia may be more dispersed. Or the residuals may be more important. Formally, the wage variance of location ℓ reads:

$$\mathbb{V}\text{ar}_\ell[\log w_{it}] = \underbrace{\mathbb{V}\text{ar}_\ell[\psi_{it}] + \mathbb{C}\text{ov}_\ell[\psi_{it}, \gamma_{j(i,t)}]}_{\text{Worker}} + \underbrace{\mathbb{V}\text{ar}_\ell[\gamma_{it}] + \mathbb{C}\text{ov}_\ell[\psi_{it}, \gamma_{j(i,t)}]}_{\text{Wage premia}} + \underbrace{\mathbb{V}\text{ar}_\ell[\varepsilon_{it}]}_{\text{Residual}}. \quad (3)$$

Projecting each term onto the wage variance in a cross-city regression then quantifies the relative importance of worker, wage premia, and residual heterogeneity. Altogether, decompositions (2) and (3) provide a complete picture of the role of employers on spatial wage inequality.

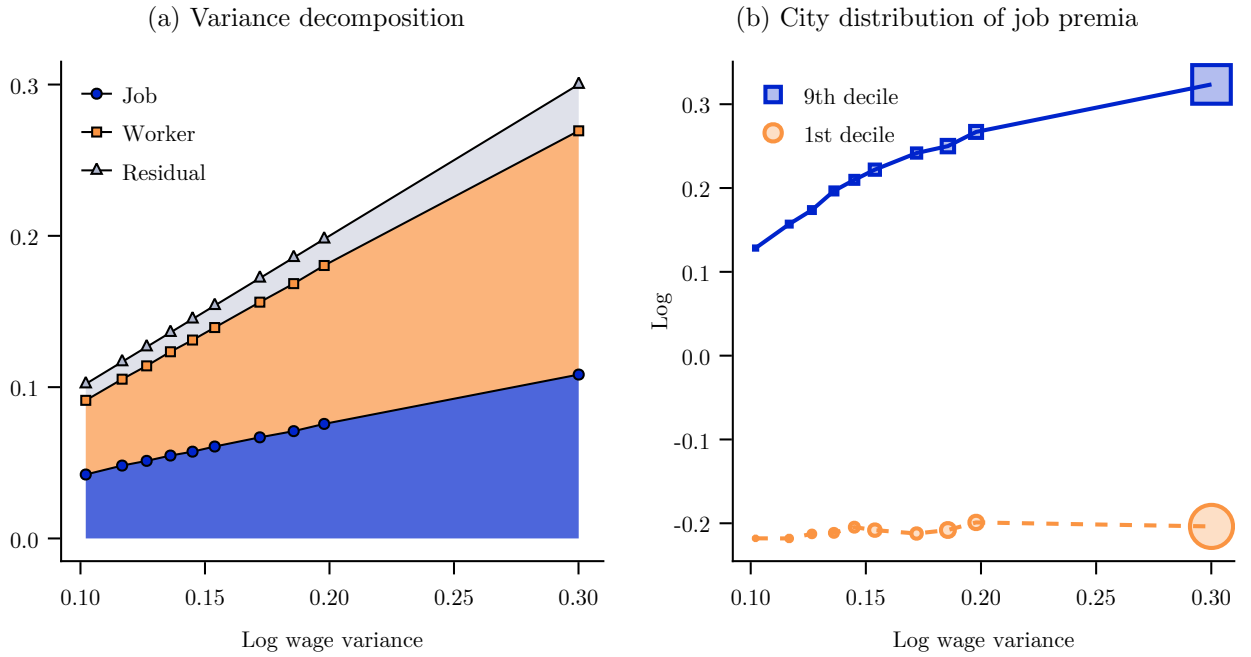
Table A.3 in Appendix A.3 presents the results from the first, between-city decomposition (2). I find that local wage premia accounts for a significant fraction — 36% — of the average wage gaps across CZs. This estimate is consistent with the previous results from the literature.

Figure 2a displays the novel, within-city variance decomposition (3). Each marker represents a city group. The blue, orange, and grey area depicts the location-specific dispersion in job fixed effects, workers fixed effects, and residuals, projected against the city wage variance.

Figure 2a shows that accounting for wage premia is key to understand local wage inequality.

[3]The results are virtually identical when varying the number of groups between 10 and 200.

Figure 2: The drivers of local wage inequality



Panel (a) displays the variance decomposition (3). Each marker is a city group. The blue area represents the dispersion in wage premia, $\mathbb{V}\text{ar}[\gamma_{it}] + \mathbb{C}\text{ov}[\psi_{it}, \gamma_{j(i,t)}]$. The orange area represents worker heterogeneity, $\mathbb{V}\text{ar}[\psi_{it}] + \mathbb{C}\text{ov}[\psi_{it}, \gamma_{j(i,t)}]$. The grey area represents the residuals, $\mathbb{V}\text{ar}[\varepsilon_{it}]$. Each measure is plotted against the location-specific wage variance, $\mathbb{V}\text{ar}_\ell[\log w_{it}]$. Panel (b) presents the local distribution of job fixed effects. The orange circles and blue rectangles are the 1st and 9th deciles of the local job fixed effect distribution. Both are plotted against the location-specific wage variance. Job fixed effects are normalized by their national average.

The dispersion in job fixed effects indeed explain 33% of the spatial variation in wage inequality. For instance, the dispersion in wage premia in Paris is 0.06 larger than in Lens, or 35% of the difference in wage variance (0.17). Meanwhile, worker heterogeneity and the residuals explain 57% and 10%.

Figure 2b unpacks why are wage premia more dispersed in unequal cities. Specifically, in the same spirit as Figure 1a, the figure depicts the 10th (orange circles) and 90th (blue rectangles) percentiles of the city-group job fixed effect distribution.

Figure 2b reveals that high-paying jobs are concentrated in a few locations whereas low-paying jobs are present everywhere. Qualitatively, the distributions of pay premia are indeed very similar to the wage distributions presented in Figure 1a. The bottoms are constant across space: workers employed at low-paying jobs in Paris earn a wage premia 0.01 log points higher than those in Lens. By contrast, there are large swings in the right-tail of distributions: workers employed at jobs in the 90th percentile of the job fixed effect distribution in Paris earn a premia 0.15 log points higher than those in Lens. Said differently, 23% of the jobs in Paris offer a premia in the top 10% of the aggregate distribution; this is 6% of the jobs in Lens (Figure A.2b).[4]

Figure A.3 in Appendix A.4 implements two robustness exercises that explore further why there

[4]Figure A.2a in Appendix A.4 repeats this exercise for every decile and shows that the right tail of the local employer fixed effect distribution is systemically fatter in more unequal cities.

is a greater variance of pay premia in unequal cities. First, Figure A.3a separates the role of sorting —as measured by the correlation between the job and worker fixed effects— from the variance in job fixed effects. The within-city variance decomposition (3) indeed bundles the two together. I do so because, everything else equal, the covariance is an increasing function of job heterogeneity. When splitting the two, I find that the correlation between jobs and workers fixed effect is stable across CZs. As a result, the greater variance of wage premia in unequal cities still explains 30% of the variation in local wage inequality.[5]

Second, the job fixed effects are estimated at the establishment $\times$ occupation. Given that I control for time-invariant unobserved worker heterogeneity and location-specific returns to experience, I indeed interpret pay differences between occupations for seemingly identical workers as pay premia. However, these may also capture time-varying unobserved skills. Furthermore, pay gaps may reflect compensating differentials rather than wage premia, in particular across occupations or industries. Figure A.3b quantifies the relative importance of occupation, industry, and firm heterogeneity on local wage inequality. The industry component captures 6% of the between-city differences in inequality, suggesting that compensating differentials are not driving the result. The occupation and firm components are equally important: they both explain 47% of the spatial variations in wage variance. As such, the dispersion in pay premia still accounts for a sizeable portion of local wage inequality even when abstracting from occupation heterogeneity.

The concentration of high-paying employers in a few locations is thus key to understand the spatial differences in wage inequality. I now investigate how wage premia shapes workers' career as they reallocate across employers.

2.4 Fact #2: the job ladder is steeper in unequal cities

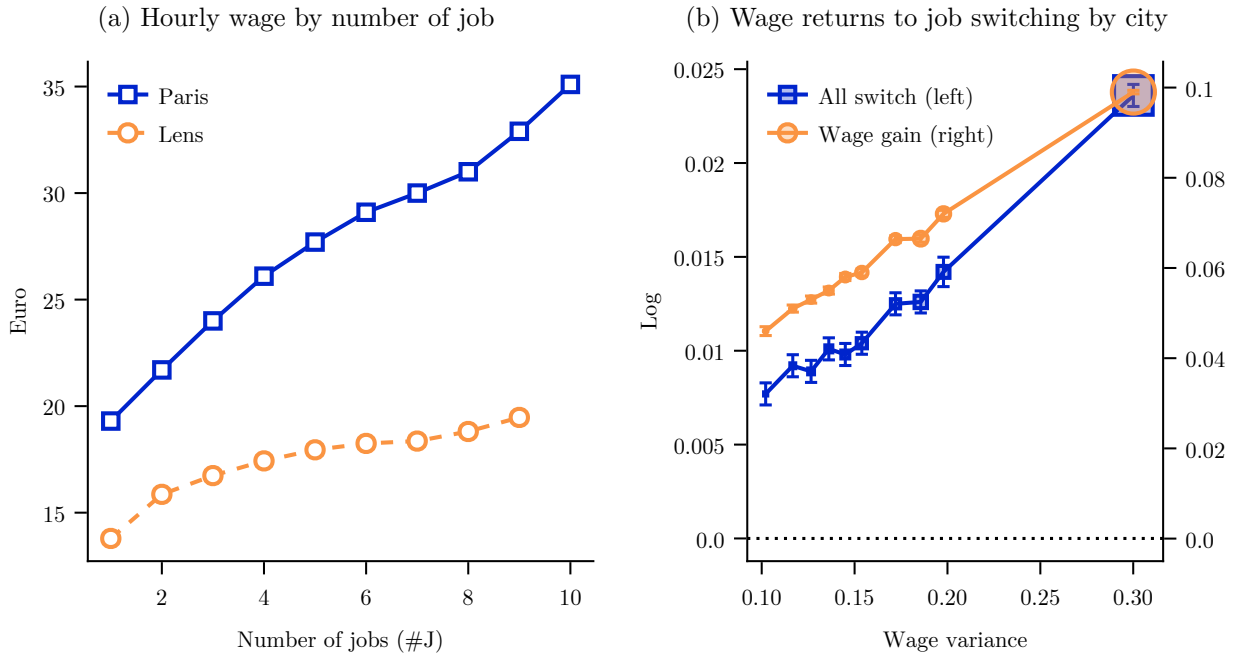
In this section, I document that, while the propensity of workers to switch jobs is similar across space, the wage returns to job switching are higher in more unequal cities.

Figure A.4 in Appendix A.4 depicts the average job switching rate by location (blue rectangles). An employed worker is said to switch jobs ($\text{J2J}_{it} = 1$) if they transition between pairs of establishment $\times$ occupation within two consecutive quarters. To avoid misclassifying transitions through unemployment as job switches, the right panel restricts the definition to job transitions associated with wage gains ($\text{J2J}^{+}_{it} = 1$).

Reallocation across employers are frequent. 7% of employed workers switch jobs on average per quarter. Unconditionally, job switches are more frequent in unequal cities. For instance, the job switching rate is 2.2 p.p. higher in Paris than in Lens. However, large — and unequal — cities attract young and skilled workers which are more incline to switch jobs. To compute the local job

[5]There does not exist an exact decomposition that separates the role of sorting from that of worker and job heterogeneity. Instead, section A.2 derives a first-order approximation that separates the different channels. Figure A.3a implements this approximation.

Figure 3: Local job ladders



Left panel plots the average hourly wage by the cumulative number of job held as defined by (5). The orange circles are in Lens, the blue rectangles in Paris. Right panel plots the estimates $\{\gamma_\ell\}_{\ell=1}^{L}$ in (6) against cities' wage variance. The vertical bars represent 95th confidence intervals.

switching rate net of worker sorting, I estimate a two-way fixed effect specification similar to (1):

$$z_{it} = \alpha_i + \beta x_{it} + \gamma_{\ell(i,t)} + u_{it}, \tag{4}$$

where $z_{it} = \text{J2J}_{it}$ or $z_{it} = \text{J2J}^{+}_{it}$. The location fixed effects $\{\gamma_\ell\}_{\ell=1}^{L}$ are depicted in Figure A.4 by the orange circles. I find that job switching rate are relatively constant across space once controlling for worker heterogeneity. If anything, I estimate them to be relatively lower in unequal places. For instance, the residualized rate is 0.3 p.p. lower in Paris than in Lens. A similar pattern holds for the job switches for which workers experience wage gains. I therefore conclude that variation in job-switching rates across cities does little to explain differences in local wage inequality.

By contrast, the wage gains accrued from job switching are substantially higher in more unequal locations. Figure 3a presents cross-sectional evidence on how the reallocation of workers across jobs interact with their wage profile. Specifically, it depicts for two cities, Paris and Lens, the average wage as a function of the number of job an individual has worked at. The number of job held by worker i at time t is defined by the cumulative sum of job switches between age 25 and t:

$$\#\text{J}_{it} = \sum_{\tau=\underline{t}_i}^{t} \text{J2J}_{i\tau}. \tag{5}$$

In (5), $\underline{t}_i$ denotes the year in which worker i was 25. For workers that I observe for the first time

after age 25, I infer their prior number of switches based on their age.[6] I also use equation (5) to compute the cumulative number of job switches that result in wage gains, denoted $\#\text{J}^+_{it}$.

Figure 3a highlights that the wage gap between workers in Paris and Lens grows as they move throughout the job ladder. While this gap is 40% for workers who enter the labor force, it rises to 80% after 9 job switches.

Three distinct forces may shape the wage profiles shown in Figure 3a. First, heterogeneous workers may sort across locations, and may do so differently across their lifecycle. Second, older workers, which have occupied more jobs, tend to be more experienced, and experience may be remunerated differentially across CZs. Third, the gains from switching jobs may be higher in Paris.

I estimate the local impact of job switching on wages net of sorting and learning through the reduced form model

$$\log w_{it} = \text{FE}_t + \text{FE}_i + \alpha_{\ell(i,t)} + \beta_{\ell(i,t)} x_{it} + \gamma_{\ell(i,t)} z_{it} + \varepsilon_{it}, \tag{6}$$

where $z_{it} = \#\text{J}_{it}$ or $z_{it} = \#\text{J}^+_{it}$. In (6), FE_i are worker fixed effects which control for the sorting of workers across space. The variable x_{it} is the experience of i at time t, and β_ℓ captures that particular cities may favor human capital accumulation. Finally, FE_t and α_ℓ are year and location fixed effects.

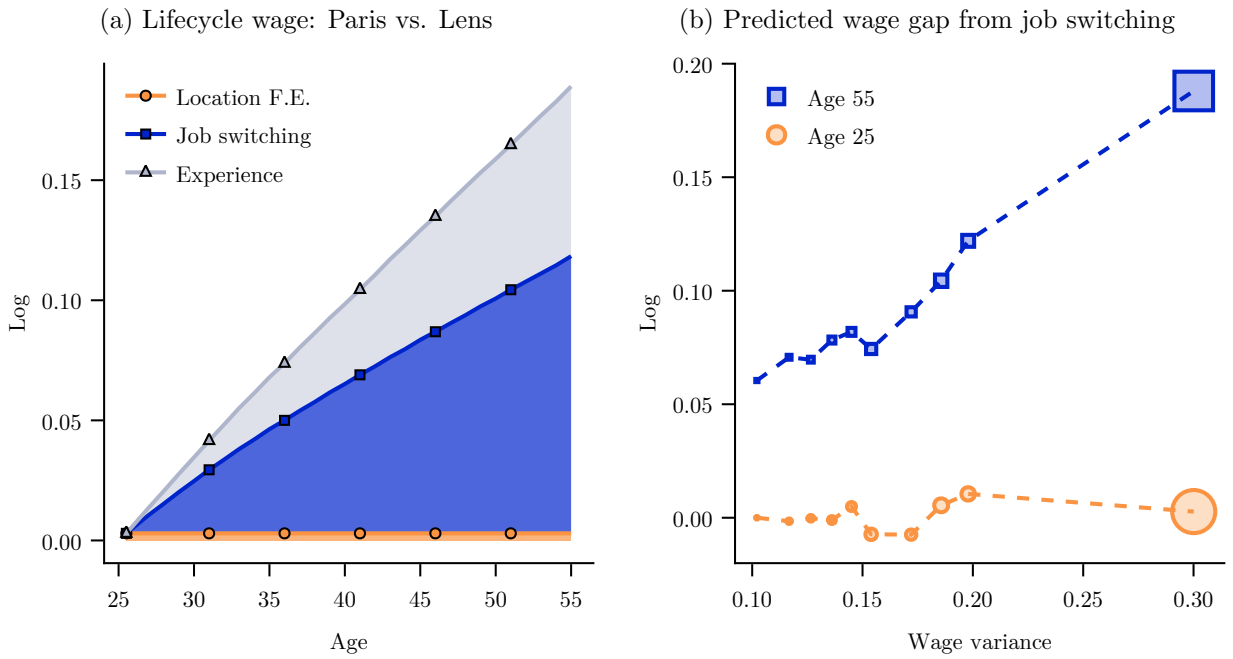
The main coefficients of interest are γ_ℓ: the local returns to job switching. Given the worker fixed effects and the city-specific returns to experience, γ_ℓ is identified from the wage growth of job switchers relative to the wage growth of job stayers. Identification then relies on random worker mobility across employers conditional on workers' fixed effect and experience. The local returns to experience are identified from the wage growth of job stayers. Finally, the location fixed effects α_ℓ measure the average wage in location ℓ, net of worker heterogeneity and before any job switch or experience. The difference $\alpha_{\ell'} - \alpha_\ell$ is therefore the wage gap between location ℓ' and ℓ for seemingly identical workers who have just entered the labor force.

The local returns to job switching are related to the pay premia estimated in Section 2.3. Indeed, both are identified from job switchers. However, whereas the job fixed effects estimate the wage differentials between any two employers, γ_ℓ measures the average wage change upon a job switch. It therefore quantifies the extent to which workers reallocate from low- to high-paying employers over time. If $\gamma_\ell > 0$, a job switch leads on average to a wage increase, consistent with the existence of a local job ladder. If $\gamma_{\ell'} > \gamma_\ell$, job switches yield larger wage gains in location ℓ' than ℓ, suggesting that the local job ladder is steeper.

Figure 3b plots the estimated returns to job switching against cities' wage variance. The blue rectangles are the estimates considering all switch, whereas the orange circles restrict the attention to switch associated to wage gains. The location fixed effects and returns to experience are reported separately in Figure A.5, Appendix A.4.

[6] I estimate $\text{J2J}_{it} = \alpha_t + \beta_{a_i} + u_{it}$, where a_i is the age of i. I then use the estimated age profile to infer workers' past number of switches: $\#\text{J}_{it_i} = \sum_{\tau=\underline{t}_i}^{t_i} \text{J2J}_{i\tau} = \sum_{a \leq a_{it_i}} \beta_a$, where t_i is the first year in which I observe i.

Figure 4: Lifetime consequences of job switching



Left panel plots the difference in the predicted average wage by age (7) between Paris and Lens. The orange circles display the difference in location fixed effects, $\alpha_{\ell'} - \alpha_{\ell}$. The blue rectangles display the impact of job switching assuming an identical job switching distribution across locations, $(\gamma_{\ell} - \gamma_{\ell'})\mathbb{E}\left[\#\mathrm{J}_{it} \mid a\right]$. The grey triangles display the impact of experience assuming an identical experience distribution across locations, $(\beta_{\ell} - \beta_{\ell'})\mathbb{E}\left[x_{it} \mid a\right]$. Right panel plots the predicted wage by location accruing from job reallocation. The orange circles are the predicted wage before any switching at age 25, i.e., the location fixed effects α_{ℓ}. The blue rectangles are the predicted wage after the average number of switches by age 55, $\alpha_{\ell} + \gamma_{\ell}\mathbb{E}\left[\#\mathrm{J}_{it} \mid a = 55\right]$. Both are plotted against cities' wage variance.

I find that job switching leads to wage growth in every location, but that the job ladder is steeper in cities with greater wage inequality. On average, a job switch leads to a 1.3% wage increase. In Paris, these wage gains are 2.4%, whereas they are 0.9% in Lens. Generalizing to all cities, workers gain an additional 0.2 p.p. upon switching jobs in cities where the wage variance is one standard deviation higher. This between-city relationship is even steeper when the independent variable is $\#\mathrm{J}_{it}^{+}$ (0.8 p.p.).

At the same time, I find that wage gaps across CZs are small for workers who enter the labor force once controlling for worker heterogeneity and the dynamic effects of cities. For instance, I estimate that entry-level wages in Paris exceed those in Lens by 0.3%, compared to an unconditional wage gap of 36%. Finally, consistent with the prior literature, I find that returns to experience are higher in more unequal and larger cities.

Table A.4 in Appendix A.3 provides several robustness exercises. For expositional clarity, I replace the city-specific returns to job switches in (6) with an interaction between job switching and city wage inequality. Column (1) reports the baseline estimates from this modified specification for all job switches and Column (2) for job switches associated to wage gains. Column (3) includes year-by-location fixed effects to control for location-specific wage trends. Human capital accumulation

may not be fully captured by experience; then, the wage growth of movers may confound the gains from reallocation with learning. I control more flexibly for human capital accumulation in two ways. Column (4) includes occupation fixed effects, thereby identifying returns to job switching from moves between establishments within occupations. Column (5) introduces worker-specific returns to experience to account for heterogeneity in learning abilities. The estimates are very stable throughout.

The steeper job ladder of unequal cities constitutes a strong source of wage growth. According to equation (6), holding constant the distribution of worker fixed effects across locations, the average wage of a worker with age a in location ℓ is

$$\mathbb{E}_\ell[\log w_{it} \mid a] = \alpha_\ell + \beta_\ell \mathbb{E}_\ell[x_{it} \mid a] + \gamma_\ell \mathbb{E}_\ell \left[\#\mathrm{J}_{it} \mid a\right], \tag{7}$$

Figure 4a uses (7) to plot the predicted wage difference between Paris and Lens along the lifecycle. There is virtually no wage discrepancy at entry in the labor market. The wage gap widens as workers reallocate between employers throughout their career. By age 55, the steeper job ladder in Paris implies that workers earn wages 12% greater than in Lens. The higher returns to experience further boosts the wage gap by 7 p.p. Said differently, the steeper job ladder explains 62% of the faster wage growth in Paris. Generalizing to all cities, I estimate that between-city differences in returns to job switching accounts for 59% of the between-city wage growth variance.

A steeper job ladder generates greater within-city wage inequality between workers who switch jobs and those who do not. Figure 4b concludes this section by plotting the wage inequality implied by the local returns to job switching. The orange circles correspond to the average wage upon entry, i.e., the location fixed effect α_ℓ. The blue rectangles show the wage predicted by the average number of switches at age 55, $\alpha_\ell + \gamma_\ell \mathbb{E}_\ell \left[\#\mathrm{J}_{it} \mid a\right]$. Qualitatively, Figure 4b closely tracks the local distribution of employer fixed effects plotted in Figure 2b: there are little variations in the left-tail of the wage distributions but there are large swings of the right-tail. Within cities, the inequality predicted by the reallocation of workers across employers is lower than the dispersion in pay premia, although this may be due to the fact that workers start on different rungs of the ladder and some experience more switches than others.

Altogether, Section 2.3 and 2.4 document that employers shape local wage inequality and workers' wage growth. In the next section, I build a framework that jointly account for these two novel facts through the sorting of employers across frictional labor markets.

3 A spatial theory of wage premia

In this section, I develop a framework that combines search frictions as in Burdett and Mortensen (1998) with an otherwise standard quantitative spatial model (Redding and Rossi-Hansberg, 2017). I concentrate here on the model's core feature. I characterize the equilibrium analytically, and in particular, show that it qualitatively replicates the two facts documented in Section 2.

3.1 Setup

The economy is comprised of two types of agents. There is a unit mass of *ex-ante* homogeneous workers and a mass $M \leq 1$ of heterogeneous employers. Workers and employers meet within L locations. Time is continuous. There is no aggregate shock, and I focus on the stationary equilibrium.

Cities Cities are indexed by ℓ with $\ell \in \{1, 2, \ldots, L\}$. They offer amenities A_ℓ to the workers that reside there. Cities differ in no other way *ex-ante*. I therefore order cities by their amenities: $A_1 < A_2 < \cdots < A_L$. I later allow for TFP differentials. However, focusing initially on amenities highlights how employer sorting endogenously creates wage differences across cities.

Workers Workers are risk neutral and discount the future at rate ρ. They exit the labor force at Poisson rate ξ, upon which they are replaced by new workers. They consume a freely traded good taken as the numéraire and local housing according to Cobb-Douglas preferences. Their lifetime utility is

$$\mathbb{E}_{t_0}\left[\int_0^\infty \mathrm{e}^{-\beta t}\left(\frac{h_t}{\alpha}\right)^\alpha\left(\frac{c_t}{1-\alpha}\right)^{1-\alpha} dt\right],$$

where I have already subsumed the stochastic exit probability into the effective discount rate, $\beta \equiv \rho + \xi$. The parameter α is the expenditure share on housing. The period t_0 denotes the time of entry into the labor force. The expectation is taken over future labor income. Workers do not have access to a savings devise: if they reside in location ℓ and earn labor income y_t, they face the per-period budget constraint $p_\ell h_t + c_t \leq y_t$.

Workers start their career as unemployed. Unemployed workers earn unemployment insurance b, which can alternatively be interpreted as revenues generated from home production. They receive job offers at Poisson rate λ^u. When employed, workers supply inelastically one unit of labor, so their labor income equals their wage. They also receive job offers at Poisson rate λ^e and fall back into unemployment at rate δ. I assume for now that the contact rates are exogenous and constant across space, with $\delta < \lambda^e \leq \lambda^u$.

Job offers are drawn from the local job offer distribution F_ℓ. That is, workers can only search for jobs in the city where they live.[7] Workers are free to migrate across cities, but jobs are tied to a location. Accordingly, if a worker wants to switch locations, they must quit their job. In the data, most job switches occur within cities, and workers who move between cities experience on average longer non-employment spells. The allocation of workers across space is characterized by the triplet $\{m_\ell, e_\ell, u_\ell\}_{\ell=1}^L$, which denote the total, employed, and unemployed measure of workers in each location. The allocation is feasible if $\sum_{\ell=1}^L m_\ell = 1$.

Employers Employers are infinitely lived and discount the future at rate r. They produce the numéraire of the economy. They are *ex-ante* heterogeneous, indexed by their time-invariant

[7] For a quantitative spatial model with wage posting and between-city search, see Heise and Porzio (2022).

productivity z. The aggregate distribution of productivity is Γ with support $[\underline{z}, \bar{z}]$, $\bar{z} \leq \infty$. I assume that Γ admits a finite and continuous density. The production — and revenues — generated by an employer with productivity z when they hire n workers is $R(z, n) = zn$. Given constant returns to scale, z is also the marginal product of labor (MPL) of a job at this employer. As such, I use employers and jobs interchangeably.[8] I assume that $\underline{z}$ is high enough relative to b so that any jobs is profitable in at least one city.

Employers hire workers in local frictional labor markets. The hiring process follows Burdett and Mortensen (1998). Employers have monopsony power over workers and are assumed to be atomistic. They post a single wage offer. They commit to a fixed and non-state-contingent wage that cannot be renegotiated throughout the employment spell. In particular, employers cannot make counteroffers when their workers receive alternative job opportunities. Rather, given their target size, they optimally set their wage offers *ex ante* to maximize hiring and minimize poaching.

Employers freely choose the city where they want to produce and hire workers.[9] They pay a unit housing cost r_ℓ upon producing in city ℓ. The endogenous distribution of jobs across space is summarized by two objects: the mass of employers in the city, M_ℓ, and the distribution of productivity, $d\Gamma_\ell$. The allocation $\{v_\ell, \Gamma_\ell\}_{\ell=1}^L$ is feasible if the number of jobs allocated across cities is not greater than the total number of jobs available:

$$\sum_{\ell=1}^{L} M_\ell d\Gamma_\ell(z) \leq M d\Gamma(z). \tag{8}$$

Housing markets The residential and commercial housing markets are segmented. In each of them, absentee land owners supply the local housing stocks. The residential housing supply is given by $L_\ell = \bar{L} p_\ell^\theta$, where θ is the residential housing supply elasticity. Likewise, the commercial housing supply is given by $H_\ell = \bar{H} r_\ell^\phi$ for ϕ the commercial housing supply elasticity.

I characterize the steady state equilibrium in three steps. First, I derive the spatial allocation of workers and the local labor supply curves that result from it. I then solve for the local wage distributions given the spatial allocation of employers. Finally, I characterize the spatial allocation of employers.

3.2 Local labor supplies

The local labor supply curves follow from the job-switching behavior of workers together with their choice of location. Let U_ℓ denote the lifetime utility of an unemployed worker in location ℓ.

[8] The framework does not model explicitly occupation as in the data, but the productivity z can be seen as capturing the total productivity of the pair occupation $\times$ establishment.

[9] Abstracting from mixed strategy is without loss of generality due to the CRS production technology.

This value satisfies the HJB equation

$$\beta U_\ell = \frac{A_\ell b}{P_\ell} + \lambda^u \int \max\{V_\ell(w) - U_\ell, 0\} dF_\ell(w), \tag{9}$$

where $P_\ell \equiv p_\ell^\alpha$ is the price index in location ℓ. The lifetime utility of unemployed workers in city ℓ consists of their contemporaneous real earnings adjusted for local amenities, and the expected value from future job opportunities. Unemployed workers choose their location to maximize their lifetime utility:

$$\bar{U} = \max_\ell U_\ell. \tag{10}$$

In equilibrium, unemployed workers are indifferent as to where to live.

By contrast, employed workers cannot move freely across locations. The lifetime utility of a worker employed at wage w in location ℓ satisfies

$$\beta V_\ell(w) = \frac{A_\ell w}{P_\ell} + \lambda^e \int \max\{V_\ell(w') - V_\ell(w), 0\} dF_\ell(w') + \delta[U - V_\ell(w)], \tag{11}$$

which also accounts for the utility loss associated with falling back into unemployment.

Within a city, employed workers behave as income maximizers. Their utility is increasing with their wage; as a result, they climb the local job ladder by continuously accepting better-paying job offers. The flow of workers up the job ladder determines the number of employees at each wage. In particular, the distribution of wages amongst employed workers is related to the wage offer distribution according to[10]

$$G_\ell(w) = \frac{F_\ell(w)}{1 + k(1 - F_\ell(w))}. \tag{12}$$

The parameter $k \equiv \lambda^e/\delta$ summarizes the speed at which workers climb the job ladder relative to the rate at which they fall back to unemployment. Given the wage offer distribution F_ℓ and the employment distribution G_ℓ, the number of employed workers per wage offer w in location ℓ is

$$n_\ell(w) = \frac{1}{\theta_\ell} \frac{1 + k}{\left[1 + k(1 - F_\ell(w))\right]^2}, \tag{13}$$

where $\theta_\ell \equiv M_\ell/e_\ell$ is the labor market tightness in city ℓ. Equation (13) is the labor supply curve in city ℓ. The supply curves slope upward: workers employed at high-wage jobs keep the same job for relatively longer since there are relatively fewer better-paying opportunities.

The labor supply curves differ across cities in two ways. First, the number of workers per job offer is lower in tighter labor markets. Second, the competition for workers — as represented by the job offer distribution — may be fiercer. In particular, in locations with a higher job offer distribution

[10]The derivation of the labor supply curve is similar to Burdett and Mortensen (1998) and detailed in Appendix B.1.

(in the first order stochastic dominance sense), the labor supply curve is shifted downward as workers transition more often to better paying opportunities. Search frictions, as summarized by k, determines the sensitivity of the supply curve to local competition. If frictions are high, workers transition infrequently across jobs, thus flattening the labor supply curve. In the limit of infinite frictions ($k \to 0$), the number of workers per job offer is entirely given by the market tightness.

Unemployed workers tradeoff a higher search efficiency against lower present earnings. They accept any wage offer greater than the city-specific reservation wage $\underline{w}_\ell$ given by

$$\underline{w}_\ell = b + (\lambda^u - \lambda^e) \int_{\underline{w}_\ell}^{\infty} \frac{1 - F_\ell(w)}{\beta + \delta + \lambda^e(1 - F_\ell(w))} dw. \tag{14}$$

In locations with a better job offer distribution (in the first order stochastic dominance sense), the option value of searching is higher, and therefore so is the reservation wage. The gap between the search efficiency of unemployed and employed workers, $\lambda^u - \lambda^e$, controls the sensitivity of the reservation wage to the job offer distribution. By contrast, the reservation wage is independent of local amenities and housing prices since these affect all workers equally.

Unemployed workers choose where to live considering the future stream of job opportunities together with housing prices and amenities. In equilibrium, housing prices adjust to make unemployed workers indifferent across locations. In cities with a better job offer distribution, expected future incomes are higher. As a result, the real reservation wage must be lower for unemployed workers to be willing to live elsewhere. Said differently, workers on low rungs of the ladder accept jobs with relatively lower real earnings in anticipation of high future wage growth.

Proposition 1 summarizes the results on the reservation wage.

Proposition 1 (Reservation wage)**.**
Consider two locations, ℓ and ℓ', and ℓ has a higher wage offer distribution in the FOSD sense. Then the nominal reservation wage is higher in ℓ, $\underline{w}_\ell \geq \underline{w}_{\ell'}$, strictly if $\lambda^e < \lambda^u$. Meanwhile, the real reservation wage is strictly lower, $A_\ell \underline{w}_\ell / P_\ell \leq A_{\ell'} \underline{w}_{\ell'} / P_{\ell'}$.

The wage offer distributions are equilibrium objects determined by employers' optimal wage posting. I now turn to solve for these wages.

3.3 Local wage distributions

I focus on the employer problem in steady state assuming that their discount rate is low. Employers choose where to produce, and which wage to offer, to maximize their flow profits:

$$\pi(z) = \max_\ell \pi_\ell(z) = \max_\ell \left\{ \max_{w,n} R(z,n) - wn - r_\ell \quad \text{s.t.} \quad n \leq n_\ell(w) \right\}. \tag{15}$$

Employers' hiring decision is constrained by the local labor supply curves. Wages then become an effective hiring tool: a higher wage allows employers to be on higher rungs of the ladder and thus to

hire and retain more workers.

The solution to (15) is given by two joint fixed points. First, given a spatial allocation of employers, the wage posted by an employer depends on the local wage offer distribution, which itself is a function of the wage-setting strategies of other firms. Second, employers factor in local prices when choosing their production location, themselves a function of other employers' location choice. I thus solve (15) in two stages. First, I derive the optimal wages for a given spatial allocation of employers. I then characterize employers' location decision.

Within cities, the results from Burdett and Mortensen (1998) apply for any spatial allocation of employers, $\{M_\ell, \Gamma_\ell\}_{\ell=1}^L$. Employers are on their labor supply curve, $n = n_\ell(w)$. The wage offer distributions are continuous over $[\underline{w}_\ell, \bar{w}_\ell]$.[11] The complementarity in the revenue function between employer productivity and their size implies that wages are strictly increasing in z within cities: more productive employers have a larger target size, and for that, offer higher wages. As a result, the rank of a job in the local wage offer distribution must correspond to the rank of the employer in the local employer distribution. That is, the wage offer distributions satisfy $F_\ell[w_\ell(z)] = \Gamma_\ell(z)$, where $w_\ell(z)$ is the optimal wage posted by z in city ℓ.

The wage distribution of city ℓ is determined by two functions. First, the number of workers employed at a job with productivity z is

$$n_\ell(z) \equiv n_\ell[w_\ell(z)] = \frac{1}{\theta_\ell} \frac{1+k}{\left[1 + k(1 - \Gamma_\ell(z))\right]^2}, \quad \forall z \in \text{ supp}\Gamma_\ell. \tag{16}$$

Employer size directly follows from the labor supply curves (13) and the rank-preserving condition $F_\ell[w_\ell(z)] = \Gamma_\ell(z)$. Second, the wage offered by an employer with productivity z is

$$w_\ell(z) = \underline{w}_\ell \left(\frac{n_\ell(\underline{z}_\ell)}{n_\ell(z)} \right) + \int_{\underline{z}_\ell}^{z} \zeta \left(\frac{n'_\ell(\zeta)}{n_\ell(z)} \right) \mathrm{d}\zeta, \quad \forall z \in \text{supp } \Gamma_\ell, \tag{17}$$

where $\underline{z}_\ell \equiv \min \Gamma_\ell$ is the least productive employer in ℓ. Given a spatial allocation of employers, the local wage distributions are explicitly determined by (16) and (17).

Optimal wages (17) result from the competition for workers along the local job ladder. At the bottom of the ladder, the least productive employers offer the reservation wage and only hire workers from unemployment. In the limit where $\lambda^e \to \lambda^u$, they offer the unemployment insurance b and their wage strategy is fully independent of the local competition. More productive employers offer higher wages to poach workers from their lower-paying competitors. This poaching game builds up throughout the job ladder, and the wage paid by an employer depends on the productivity of the other employers on the lower rungs. The higher the relatively productivity of an employer, the more competition it faces.

The competition for workers transforms spatial differences in employer productivity into spatial

[11] Continuity holds independently of the local employer distribution. For instance, if some employer distribution has a hole, employers with productivity to the right of it have no incentives to offer wages strictly greater than the wage offered by employers to the left of the hole as this would yield the same size.

differences in wages. Proposition 2 characterizes, for a particular spatial allocation of employers, the shape of the local wage distributions.

Proposition 2 (Local wage distribution)**.**
Consider two locations, ℓ and ℓ', such that the employer distribution in ℓ is higher in the FOSD sense. Then,

1. *The wage distribution in ℓ first order stochastically dominate that in ℓ', $G_\ell \succ_{FOSD} G_{\ell'}$;*
2. *The top-to-bottom wage gap is larger in ℓ, $\bar{w}_\ell - \underline{w}_\ell > \bar{w}_{\ell'} - \underline{w}_{\ell'}$;*
3. *The wage gains upon a job switch are larger in ℓ for workers on low rungs of the ladder: $\mathbb{E}_\ell[W/\underline{w}_\ell \mid switch] > \mathbb{E}_{\ell'}[W/\underline{w}_{\ell'} \mid switch]$.*

When all employers are relatively more productive (in the FOSD sense), the poaching competition is fiercer. As a result, the wage distribution shifts up, and every wage quantile is higher (Proposition 2.1). However, competition impacts employers differently depending on their position in the local productivity distribution. Relatively unproductive employers, who primarily hire unemployed workers, are insulated from local competition. There is therefore little spatial variation in the left-tail of the wage distributions. By contrast, there are larger variations in the right-tail as relatively productive employers face off all the local competition. Greater within-city wage inequality follows, together with faster wage growth as workers reallocate from low- to high-paying jobs (Proposition 2.2 and 2.3). Altogether, the endogenous local wage distributions resemble those documented in Section 2.

The location decision of employers spillovers onto the wage distribution. In cities where jobs are relatively productive, wages are relatively high not because they are offered by productive employers, but rather, because productive employers intensify the local competition. Endogenous wage gaps across space arise without the need for TFP differentials. This is in stark contrast with standard, competitive spatial model where TFP gaps — exogenous or endogenous — drive spatial wage inequality.

Proposition 2 holds to the extent that the local productivity distributions are ordered in term of first order stochastic dominance. I now show this is the case in equilibrium.

3.4 The spatial allocation of employers

The profits of an employer at its optimal wage offer and size is

$$\pi_\ell(z) = [z - w(z, \Gamma_\ell)]\, n[\Gamma_\ell(z), \theta_\ell] - r_\ell. \tag{18}$$

where, with a slight abuse of notation, I have made explicit the dependence of the optimal wage and size on the local distribution of employers. Employers produce in the location that maximize

their profits:

$$\pi_\ell(z) \geq \pi_{\ell'}(z) \text{ for all } \ell' \neq \ell \iff z \in \text{supp } \Gamma_\ell. \tag{19}$$

An equilibrium spatial allocation of employers is a tuple $\{M_\ell, \Gamma_\ell\}_{\ell=1}^L$ that satisfies feasibility (8) and profit maximization (19).

The complementarity between productivity and size drives employers' willingness to produce in particular locations. However, employer size is an equilibrium object. Labor market tightness — which depends on the local measure of employers — determines the level of the supply curve. Local competition — which depends on the within-city productivity distribution — shapes employers' relative size. The profitability of each location thus depends itself on the spatial allocation of employers, and standard optimal transport techniques cannot be used to solve the assignment problem (19) (Chade and Eeckhout, 2020). Nevertheless, a sharp characterization of employers' location choice exists.

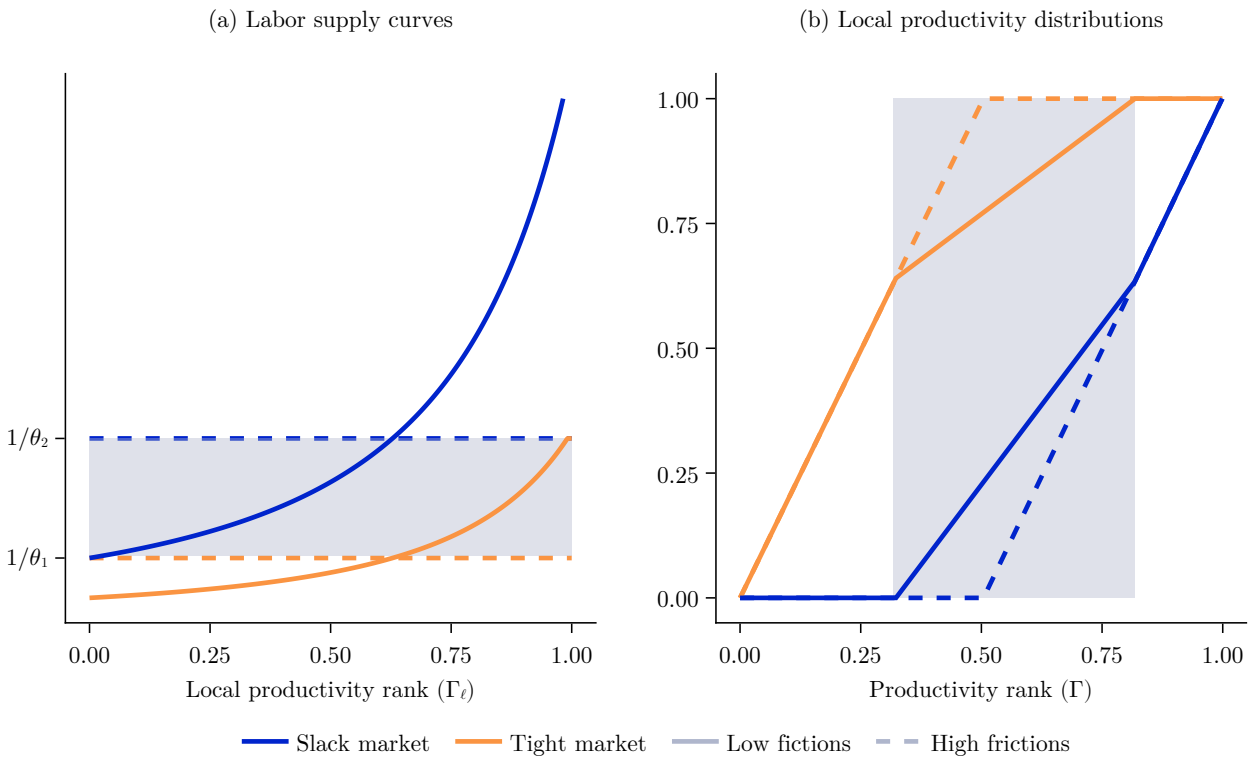
First, hold fix the spatial distribution of labor market tightness. All else equal, productive employers have a stronger willingness to pay to produce in slacker labor markets. In equilibrium, a tension however exists between tightness and competition: if all the productive employers produce in the slackest labor market, the local competition intensifies and most employers eventually hire only a few workers. The poaching competition thus introduces substitutability across locations. Weak search frictions accelerate the reallocation of workers to high-paying employers and intensify the tradeoff between labor market slackness and local competition. Proposition 3 formalizes this intuition.

Proposition 3 (Local productivity distributions)**.**
Consider two locations, ℓ and ℓ'. The location with the slacker labor market attracts relatively more productive employers: $\theta_\ell < \theta_{\ell'}$ if and only if $\Gamma_\ell \succ \Gamma_{\ell'}$. If the tightness are not too dissimilar, $\theta_{\ell'} < \theta_\ell(1+k)^2$, the productivity distributions overlap, $d\Gamma(supp\,\Gamma_\ell \cap supp\,\Gamma_{\ell'}) > 0$.

Figure 5(a) provides a visual representation of Proposition 3. The figure depicts the local labor supply curves, n_ℓ, against employers' rank in the local productivity distribution. The blue and orange curves refer to a location with a slack and a tight labor market. The solid lines refer to an allocation with weak search frictions ($k > 0$) whereas the dashed lines depict an allocation with infinite frictions ($k \to 0$).

When search frictions are infinite, employers only hire workers from unemployment and the local labor supply curves are flat. Employers in slacker labor markets are uniformly larger. The complementarity between productivity and size then guarantees that if an unproductive employer find it profitable to produce there, then so must a productive employer. As a result, the productivity distribution must be higher (in the FOSD sense) than in a tighter labor market. In addition, as in standard matching model, the uniformly greater size ensures that no positive measure of employers

Figure 5: The spatial distribution of employers



is indifferent between a slack and a tight labor market.[12]

When workers reallocate from low- to high-paying employers, the labor supply curves instead slope upwards and employers' relative productivity determines how many workers they can poach from the competition. Sorting must still prevail. Within any city, productive employers have a larger size than their unproductive competitors. Therefore, they still find it relatively more profitable to locate in slacker markets. However, the concentration of productive employers in slack labor markets intensify the local competition. Lower size follows for employers at the bottom of the distribution, which can now attain the same size in tighter labor markets with milder competition. As a result, a range of employers are indifferent, and the sorting of employers across space is no longer perfect.

Either way, employers always sort across space and the local productivity distribution can be ordered in the first order stochastic dominance. The equilibrium local wage distributions are described by Propositions 1 and 2 and resemble the ones observed in the data.

Proposition 3 characterizes the local productivity distribution holding fixed the labor market tightness and housing prices. In equilibrium, the labor market tightness grow as the number of employers in the city increase, and so do housing prices. Both forces act as sources of congestion and prevent all employers to produce in the largest city. In particular, when the housing congestion

[12]Whether employers prefer a slack or a tight labor market depends on the local housing prices, which are held fixed in Proposition 3. Accordingly, it can well be that all employers strictly prefer to locate in the tightest labor market. Then, Proposition 3 trivially holds as $\operatorname{supp} \Gamma_{\ell'} = \varnothing$ for all but one ℓ.

force is strong enough, large cities attract more employers but the labor market remains relatively slacker and thus attract more productive employers.[13]

Proposition 4 (Existence)**.**
Suppose that $\lambda^e \approx \lambda^u$ *and housing expenditures are large relative to unemployment insurance,* $b\bar{H}^{1/\phi} \approx 0$*. Then, there exists an equilibrium spatial allocation of jobs. In equilibrium, large cities attract more employers but have a slacker labor market:* $m_\ell > m_{\ell'} \iff M_\ell > M_{\ell'} \iff \theta_\ell < \theta_{\ell'}$*.*

Figure 5(b) depicts the equilibrium spatial allocation of employers that correspond to the labor supply curves shown on the left panel. The high housing prices in the large, slack labor market prevent low productivity employers from producing there. When search frictions are low, the labor supply curve jump up from a tight to a slack labor market, and all employers above a productivity threshold produce in the slacker location. In the presence of weak search frictions, mid-productivity employers are indifferent between the two cities. Finally, productive employers necessarily locate in the slackest location to maximize their size at the expense of larger wage bills.

Propositions 1 to 4 shows how the combination of employer sorting and on-the-job search leads to empirically relevant local wage distributions. They also imply agglomeration effects. Large cities attract relatively productive employers aiming to maximize their size. In response, the competition for workers intensifies, the wage distribution shifts up, and a greater number of workers choose to live there. These agglomeration effects occur despite the absence of increasing returns to scale or productivity spillovers.

I now estimate the model to assess whether the model quantitatively accounts for the emergence of local wage inequality. I then use the estimated framework to quantify the impact of employer sorting on local competition, aggregate wages, and spatial welfare disparities.

4 Extended model and estimation

4.1 Quantitative model

Extensions To structurally assess the empirical relevance of the model, I relax several of its strong assumptions. Across space, I allow cities to differ in terms of (exogenous) TFP T_ℓ.

On the employer side, I made two extensions. First, I let employers grow without increasing wages too rapidly by posting several vacancies — or more generally, exerting endogenous hiring effort. Vacancies are costly and come at a convex cost

$$c(v) = \frac{v^{1+\gamma}}{1+\gamma},$$

where $\gamma > 0$ denotes the vacancy cost elasticity. Employer size reflects their position in the local job

[13] Proving uniqueness is complicated as the underlying fixed point is non-linear and of infinite dimension. However, Proposition 4 holds in any equilibrium.

ladder together with their vacancy share:

$$n_\ell(w,v) = \frac{(1+k_\ell)e_\ell}{[1+k_\ell(1-F_\ell(w))]^2}\frac{v}{V_\ell}, \tag{20}$$

where V_ℓ is the aggregate number of vacancies posted in location ℓ,

$$V_\ell = M_\ell \int v_\ell(z) d\Gamma_\ell(z). \tag{21}$$

Search frictions are now city-specific and are determined by the matching function $\lambda_\ell^u = \mathcal{M}(u_\ell + \zeta e_\ell, V_\ell)/(u_\ell + \zeta e_\ell)$, where ζ is the relative search intensity of employed workers, $\lambda_\ell^e = \zeta\lambda_\ell^u$.

Second, employers face idiosyncratic housing or entry costs, $\{\varepsilon_\ell\}_{\ell=1}^L$. These costs smooth employers location choice and allow for an efficient numerical competition. Combined, an employer with productivity z and entry costs $\{\varepsilon_\ell\}_{\ell=1}^L$ solves

$$\pi(z, \{\varepsilon_\ell\}_{\ell=1}^L) = \max_{\ell,w,n,v} R(zT_\ell, n) - wn - c(v) - r_\ell - \varepsilon_\ell \quad \text{s.t.} \quad n \leq n_\ell(w,v), \tag{22}$$

where the total productivity of its jobs in location ℓ is zT_ℓ.

On the worker side, individuals have idiosyncratic preferences for each location. These preferences, which ensure uniqueness of the equilibrium when they are sufficiently dispersed across workers, are drawn upon entry in the labor force and remain constant thereafter. The flow utility of a worker with preferences $\{\omega_{\ell'}\}_{\ell'=1}^L$ in location ℓ is

$$u_\ell(c,h,\{\omega_{\ell'}\}_{\ell'=1}^L) = A_\ell\omega_\ell \left(\frac{h}{\alpha}\right)^\alpha \left(\frac{c}{1-\alpha}\right)^{1-\alpha}.$$

A full description of the model, together with a definition of the equilibrium, is presented in Appendix C.1.

Parametric assumptions I impose the following parametric assumptions. The aggregate productivity distribution is Pareto with shape σ. Its scale is normalized to ensure a mean productivity of one. The idiosyncratic entry costs of employers are i.i.d. across space. They are drawn from Gumbel distribution with inverse dispersion ϑ. Workers' idiosyncratic preferences are i.i.d. Frechet distributed with shape χ. Finally, the matching function is Cobb-Douglas with matching efficiency μ and elasticity ψ: $\mathcal{M}_\ell = \mu V_\ell^\psi (u_\ell + \zeta e_\ell)^{1-\psi}$.

4.2 Identification

The estimation of the model is organized in two blocks. The core of the estimation is contained in the first block which leverages the structure of the model to estimate parameters without the need for simulation. Proposition 5 proves identification of 15 of the 19 parameters.

Proposition 5 (Identification)**.**
Given aggregate data on flows in- and out- of employment, location-specific data on job switching rates, commercial and residential housing prices, and firm-level employment and wage data, the parameters δ, ζ, b, M, γ, μ, ψ, $\bar{L}$, θ, $\bar{H}$, ϕ, α and ξ are identified. The dispersion in employers' idiosyncratic entry cost ϑ is identified conditional on small TFP differentials. Local amenities $\{A_\ell\}_{\ell=1}^{L}$ are identified given workers' discount factor ρ and idiosyncratic location preferences χ.

I highlight here the intuition behind the proof and refer the reader to Appendix C.2 for the details. Worker flows — across employment status and employers — identify the search frictions. The aggregate employment-to-unemployment rate is equal to δ. The location-specific job switching rates J2J_ℓ identify the contact rates of employed workers λ_ℓ^e after accounting for rejected offers along the job ladder: $\text{J2J}_\ell = \delta((1+\delta/\lambda_\ell^e)\log(1+\lambda_\ell^e/\delta)-1)$. Importantly, the contact rates are identified from the flows rather than the wage returns to job switching. The aggregate unemployment-to-employment rate relative to the J2J rate pins down the search efficiency of employed workers ζ. Finally, the labor force exit probability ξ is identified from the average career duration: $\xi = 1/\mathbb{E}[\text{length career}]$.

The spatial allocation of unemployed workers identifies local amenities. Local amenities, housing prices, and the net present values of future job opportunities determine the number of unemployed workers in each location together with the taste shock dispersion:

$$\frac{u_\ell}{\sum_{\ell'} u_{\ell'}} = \frac{(A_\ell \mathcal{U}_\ell / P_\ell)^\chi}{\sum_{\ell'} (A_{\ell'} \mathcal{U}_{\ell'} / P_{\ell'})^\chi}. \tag{23}$$

In the above, $\mathcal{U}_\ell$ denote the lifetime utility of unemployed workers in ℓ net of amenities and housing prices, which depends on the (known) local wage offer distribution, search frictions, and the discount rate ρ. Inverting (23) identifies local amenities given an estimate for χ. Any spatial allocation of workers can be rationalized by the appropriate vector of amenities, and we can therefore treat it as given for the rest of the estimation.[14]

The parameters shaping employers optimal wage are identified from the firm-level wage and size data by extending the insights from Bontemps et al. (2000) to a setting with local labor markets. Specifically, the wage optimality condition behind (17) demands

$$z_j T_{\ell_j} \equiv \zeta_j = w_j + \left(\frac{1 + k_{\ell_j}(1-F_j)}{2k_{\ell_j}}\right) \frac{\partial w_{\ell_j}(F_j)}{\partial F}, \tag{24}$$

where j is an employer, ℓ_j denote their production location, F_j their rank in the local wage offer distribution, and $w_\ell(F)$ is the inverse function of $F_\ell(w)$. ζ_j refers to the total productivity — or marginal products of labor — of employer j gross of the local TFP. Equation (24) thus identifies employer productivity from wages net of markdowns.

Vacancy costs are intuitively identified from employer size net of their position in the job ladder. Employers' rank, together with the search frictions they face, determines the number of workers they

[14] This also implies that the parameters (ρ, χ) do not affect any of the estimates besides amenities.

hire per vacancy through the labor supply curves. Their vacancy share v_j/V_{ℓ_j} are then recovered from their size using (20). The vacancy optimality condition $\log v_j/V_{\ell_j} = \frac{1}{1+\gamma}\log[(\zeta_j - w_j)n_j] - \log V_{\ell_j}$ identifies the vacancy cost elasticity γ. It also pins down the number of vacancies in each location which, together with the spatial allocation of workers, identifies the matching function from spatial correlation between contact rates and labor market tightness: $\log \lambda^u_\ell = \log \mu + \psi \log(V_\ell/(u_\ell + \zeta e_\ell))$.

Finally, the dispersion in entry costs is identified from the sensitivity of employers' location decision to the local profit opportunities. The Gumbel entry cost indeed relates the spatial allocation of employers to local profits through

$$\log \Omega_\ell(z_j) = H(z_j) + \vartheta \pi_\ell(z_j), \tag{25}$$

where $\Omega_\ell(z)$ is the probability that z produces in ℓ and $H(z) = -\log \sum_{\ell'} \mathrm{e}^{\vartheta \pi_{\ell'}(z)}$ is the outside option. Realized profits depend on revenues per workers, employer size, vacancy cost and housing prices — all of which are known: $\pi_\ell(\zeta_j) = (\zeta_j - w_j)n_j - c(v_j) - r_{\ell_j}$. Meanwhile, the spatial allocation of employers with total productivity ζ is given by employers' location choice. When spatial TFP differentials are small, employers' productivity closely track their MPL, $z_j \approx \zeta_j$, and I can use (25) to identify ϑ.[15]

The second block of the estimation calibrates 2 of the remaining 4 parameters by indirect inference. Given the parameters identified by Proposition 5, I simulate the model and minimize the distance between a vector of empirical statistics and the same statistics in the model. First, I calibrate local TFPs to match average city wages. City wage can indeed be high due to the concentration of high-productivity employers (Proposition 2) or high TFP. I therefore set TFPs to match the average city wage net of the sorting predicted by the parameters already estimated. Second, I calibrate the productivity dispersion σ to match the aggregate wage variance. Finally, the estimate of ϑ may be biased if between-city TFP gaps are large. I therefore also calibrate ϑ by indirect inference. Specifically, I estimate (25) in the data and in the model, and set ϑ to match the empirical conditional correlation.

4.3 Data

I briefly describe the data used for the estimation before turning to the estimation results. Further details are included in Appendix C.3.

I set a quarterly frequency. To abstract from worker heterogeneity, I measure employers' wages via their employed fixed effects estimated in Section 2.3.[16] The model therefore accounts for 21% of

[15]The source of identification for the remaining parameters are standard. The aggregate housing expenditure share identifies the Cobb-Douglas preferences. The housing supply parameters are given by the housing market clearing conditions. Finally, the aggregate replacement rate determines the unemployment insurance.

[16]The conditional random mobility assumption behind the AKM specification is valid in random-search wage-posting models (Card et al., 2013). The log-linear specification is well-specified if workers' skills affect their wages multiplicatively and search frictions are constant across skills. When these assumptions fail, Bilal and Lhuillier (2021) shows that the AKM specification still fits well the data.

the total wage variance. Consistent with the sample constructed in Section 2.1, the average career lasts 30 years, or 120 quarters.

I solve the model for 10 city groups as defined in Section 2.1. Importantly, there remains 297 local labor markets in the model, but each local labor market within a group is homogeneous. While computationally feasible, including more city groups increases the measurement error in the within-city wage (i.e., employer fixed effect) distributions.

I obtain residential housing prices from the *Carte des Loyers* (Rental Map), which estimates the average monthly rent per meter square at the municipal level. I do not have access to city-level commercial rental rates. Instead, I define commercial housing prices as residential housing prices adjusted for the aggregate relative price of commercial to residential housing. I residualize residential housing prices by the mean worker fixed effect of each city to account for the sorting of workers across space.

The aggregate flows in and out of employment are obtained from the *Enquête emploi en continu* (Labor Force Survey). The local job switching rates are computed from the panel matched employer-employee data. Consistent with the model, I define a job switch as a transition between jobs associated with a wage increase taking place within a location and within two consecutive quarters. To account for the sorting of workers across space, I use the residualized switching rates estimated in (4).

I read the housing expenditure share from the aggregate national accounts (INSEE, 2020) and set the replacement rate to 0.6 (OECD, 2025). I normalize all nominal variables by the aggregate average wage.

Finally, I externally calibrate the discount rate and the dispersion in workers' idiosyncratic location preferences. I set $\rho = 0.013$ to match an annual real interest rate of 5%. I follow the literature and set the taste shock dispersion to 2 (e.g., Fajgelbaum et al., 2019). As explained in Section 4.2, these two parameters are relevant only for the estimation of amenities.

4.4 Results

Table 1 presents the parameter estimates. Search frictions are relatively strong and constant across space. The aggregate contact rate for unemployed workers is 0.18. The job switching rate is substantially lower (3.7%); yet, accounting for the fact that employed workers do not accept every offer they receive, I find that the search efficiency of employed workers is only 11% lower than that of the unemployed. I obtain a matching function elasticity of 0.17, towards the low end of the estimates found in the literature. However, this elasticity implies that the model fits well the between-city differences in contact rates with an R^2 of 0.84 (Figure C.7b). In equilibrium, the contact rates are relatively uniform across space: cities 10% larger have a contact rate 0.1 p.p. smaller.

The average labor supply elasticity in the model ($\mathbb{E}[\partial \log n_\ell(w)/\partial \log w]$) aligns with benchmark estimates. Unlike models that microfound upward-slopping labor supply curves via taste shocks, the

Table 1: Parameter estimates

	Parameter	Target	Empirical moment	Simulated moment	Parameter estimate
		A. External calibration			
ρ	Discount factor	Annual interest rate 5%			0.01
χ	Taste shock dispersion	Fajgelbaum et al. (2019)			2.00
		B. Model inversion			
α	Cobb-Douglas preference	Housing exp. share	0.20		0.20
$\boldsymbol{A}$	Amenities	Unemployment distribution	Fig. C.6a		Fig. C.6a
b	Unemployment insurance	Replacement rate	0.60		0.60
δ	Job destruction rate	EU rate	0.03		0.03
ζ	Rel. search efficiency	EE / UE rate	0.19		0.89
ξ	Exit rate	Average career length	120.0		0.01
μ	Matching efficiency	Avg. EE rate	0.04		0.29
ψ	Matching function elasticity	Correlation EE rate - tightness	0.17		0.17
M	Mass employers	Average size	7.59		0.11
γ	Vacancy cost elasticity	Vacancy optimality (48)	2.05		2.05
$\bar{L}$	Residential housing supply level	Avg. residential price	0.49[a]		180.4
θ	Residential housing supply elasticity	Correlation prices - expenditures	0.09		9.66
$\bar{H}$	Commercial housing supply level	Avg. commercial price	1.24		0.18
ϕ	Commercial housing supply elasticity	Corellation prices - employer demand	0.19		5.21
		C. Indirect inference			
σ	Productivity dispersion	Wage variance	0.15	0.15	13.17
ϑ	Entry cost dispersion	Equation (25)	2.52	2.49	4.96
$\boldsymbol{T}$	TFP	Average wage	Fig. C.7a	Fig. C.7a	Fig. C.6b

[a] Average residential price expressed in hundredths.

labor supply elasticity in my framework is not a structural parameter. Instead, it depends jointly on search frictions and the vacancy cost elasticity. I estimate a vacancy cost elasticity of 2.05, well within the range found in the literature. Together with the estimated search frictions, this yields an average labor supply elasticity of 5.77, close to the estimates of Lamadon et al. (2022) for the United States. The fit with this non-targeted moment validates the estimation of the key forces that shape employer competition and wage inequality in the model.

My estimate of the entry cost dispersion is within the range of the recent estimates. I find a dispersion of $\vartheta = 4.96$; converting it to a Fréchet elasticity, it reads 4.23, above the estimate of Giroud and Rauh (2019) but below those of Suárez Serrato and Zidar (2016).

Lastly, I estimate close to no spatial variation in TFP (Figure C.6b). The relatively low dispersion in idiosyncratic entry costs implies that the location choice of employers are sensitive to the local profit opportunities. Meanwhile, there are large disparities in city size, and as such, productive employers have strong incentives to concentrate in large cities to sidestep search frictions and

maximize their size. For instance, the mean employment-weighted employer productivity in Paris is 21% larger than in Lens. By contrast, the average wage gap is 13%. The spatial sorting of employers suffice to rationalize the between-city wage gap, and I infer that TFPs are very similar across locations: the between-city TFP variance explains 0.64% of the total between-city productivity variance.

The selection of employers across locations is thus sufficient to explain between-city wage inequality once their incentives to locate in large cities to sidestep search frictions are taken into account. I now turn to the last part of the paper which assesses whether this selection can also generate within-city wage distribution that match the facts documented in Section 2.

5 Employer sorting, search frictions, and local wage inequality

The estimation strategy proposed in Section 4.2 does not target the two facts documented in Section 2. While it is designed to fit the aggregate wage variance and the average wage of every location, it does not rely on any statistics related to the within-city wage distributions. Likewise, it targets the job switching rates but not the wage returns of those switches. In this section, I compare the model's predictions with the facts in Section 4.2 to evaluate how well employer sorting and search frictions explain local wage inequality.

5.1 Within-city wage inequality and local competition

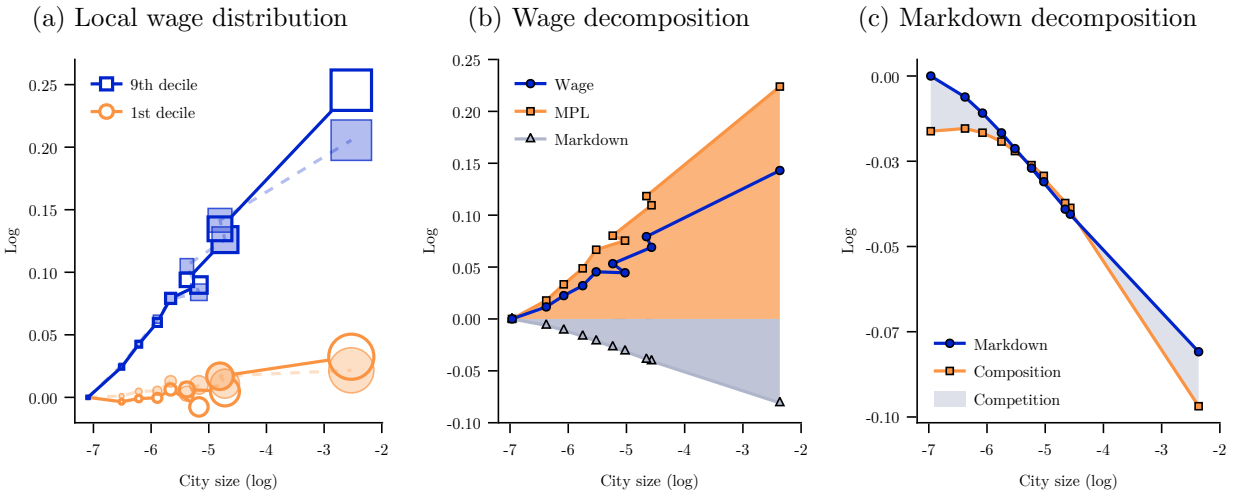
The model endogenously generates within-city wage inequality that closely resemble the local employer fixed distributions of Section 2.3. Figure 6a displays the equilibrium local wage distributions. The orange circles and blue rectangles depict the 1st and 9th deciles. The empty markers refer to the deciles in the model while the lightly shaded markers are the employer fixed effect deciles plotted in Figure 2b. Both sets of deciles are normalized to zero in the smallest city to facilitate the comparison.

As in the data, the bottom of the local wage distributions are relatively constant across space. Workers at the 10th percentile earn wages 2.7% higher in Paris than in Lens in the model, compared to 1.4% in the data. Equation (14) indeed shows that how sensitive are wages on low rungs of the ladder depends on the relative search efficiency of employed workers. The frequency at which employed workers switch jobs imply that their search efficiency is close to that of unemployed workers. As a result, the local reservation wages track closely workers' outside option, which is constant across space.[17]

Meanwhile, the right tail of the local wage distribution varies significantly across cities. The extra wage premia offered by high-paying employers in large cities are close to those estimated in the data. For instance, the wage gap between Paris and Lens for workers at the 9th decile is

[17] Adjusting the unemployment insurance by local TFP $b_\ell = bT_\ell$ to capture differences in home production productivity would not affect the results since $\mathbb{V}\text{ar}[\log T_\ell] \approx 0$.

Figure 6: Local wage distribution, employer selection and competition



25% in the model and 21% in the data. Combined, greater wage inequality arises in larger cities — consistent with Proposition 2.

Figure D.9 in Appendix D.2 conducts a more systematic comparison of within-city wage inequality in the model and in the data. Figure D.9a plots the within-city wage variance in the model and the data. The correlation between the two is very high (98%) although the model slightly over-predicts the wage inequality of large cities. Figure D.9b compares the size elasticity of local wage deciles in the model and the data. The two align closely.

Due to the absence of TFP differentials, the wage differences across locations entirely results from the sorting of employers.[18] The average employer productivity is 4% higher in cities twice as large. (Figure D.10a), consistent with estimates in the literature.[19] 7% of employers in the top productivity decile produce in Paris compared to 2% for the average employer (Figure D.10b).

The agglomeration of productive employers spillovers to local wages through the local competition. Figure 6b displays how rent-sharing varies across cities. I decompose average wages into markdowns and marginal products of labor according to the identity: $\mathbb{E}_\ell[\log w_\ell(z)] = \mathbb{E}_\ell[\log zT_\ell] + \mathbb{E}_\ell[\log \mu_\ell(z)]$. With this convention, a higher markdown $\mu_\ell(z)$ means less market power for employers. Figure 6b plots this decomposition relative to the least unequal city. The orange and grey areas represent the relative MPL and markdowns. The blue line plots the resulting wage gap.

I find that the average markdown is lower in large and unequal cities. For instance, it is 7 p.p. lower in Paris than in Lens (compared to a national average markdown of 82%). As a result, the between-city wage gaps are smaller than the productivity gaps.

[18] In the baseline model of Section 3, there is no scale effect and the measure of employers in every city has no impact on the local wage distribution. With vacancy posting, the number of employers in a city affects the local search frictions via the matching function, which may then feed to the local wage distribution. In practice, I find this effect to be small.

[19] However, the between-city differences in productivity are entirely driven by employer sorting: I estimate the size elasticity of city TFP to be 0.003.

Figure 6b therefore suggests that competition is milder in large cities, contradicting the intuition provided in Section 3. However, average markdowns reflect two forces. The first is a competition effect: a productive employer that reallocates its production from a low- to a high-competition labor market reduces its markdown. The second force is a composition channel. All else equal, productive employers charge higher markdowns because their overall position in the job ladder shields them away from the competition (Gouin-Bonenfant, 2022).

Figure 6c decomposes local markdowns into the competition and composition effects. The blue circles plot the average city markdown relative to the least unequal location — i.e., the mirror image of the grey triangles in Figure 6b. The orange rectangles represent the composition effect, defined as $\mathbb{E}_\ell[\sum_{\ell'} \mu_{\ell'}(z)]$. The grey area represents the competition effect.

Once accounting for the composition channel, I find that local competition boosts wages throughout the economy, and particularly so in small and large locations. In Paris, employers charge an average markdown 2 p.p. higher than what they would use elsewhere (relative to a mean of 78%). There, productive employers compete between each others, forcing them to pay larger premia. Meanwhile, local competition also pushes up the wages of small cities: the mean markdown of the smallest location is 2 p.p. higher than what it would be given its employer composition (relative to a mean of 85%). Absent the spatial sorting of employers, the relatively unproductive employers that populate small cities would hire most of their workers from unemployment, thus paying them at their outside option. The concentration of productive employers away from small cities means that unproductive employers now compete relatively more between each other which leads to higher markdowns.

The sorting of employers across space thus intensifies the competition in every labor market rather than redistributing it away from small places to large cities. As a result, the average aggregate markdown is 0.4 p.p. higher, which triggers an increase in the average national wage of 1%. Given this large aggregate effect, Figure 6c cautions against using average markdowns as a measure of local competition in the presence of employer heterogeneity.

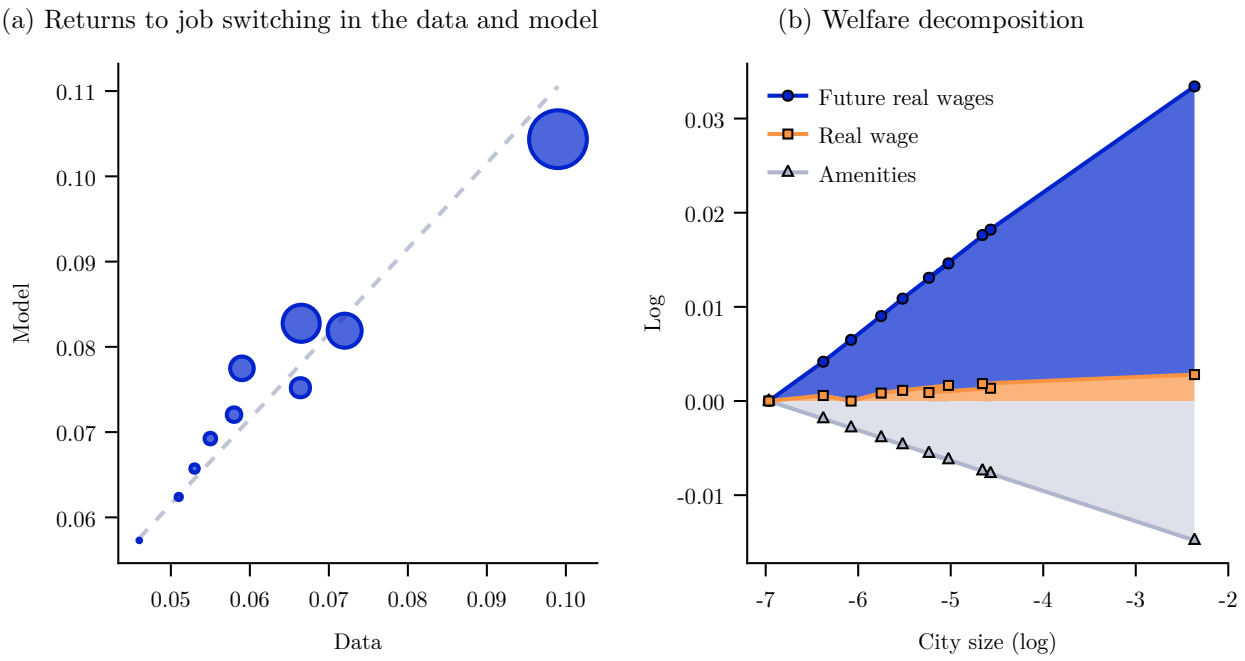
Altogether, Figure 6 shows that the selection of employers across frictional labor markets, and its consequences on local competition, explains the local wage inequality observed in the data. I now study what are the implications on workers' wage growth and welfare.

5.2 Local job ladder and welfare

In the model, within-city wage dispersion is intrinsically related to how much workers gain from moving across employers. In this section, I therefore assess whether search frictions are a good microfoundation for wage dispersion by testing if the model replicates the returns to job switching estimated in Section 3.

Figure 7a plots the estimated impact of job switching on wages in the model and in the data. In

Figure 7: Welfare implications of the local job ladders



the model, the local returns are computed as the average wage growth conditional on a job switch:

$$\mathbb{E}_\ell\left[\frac{W'}{W} \mid \text{switch}\right] = \int \frac{1}{1-F_\ell(w)} \int_w \frac{w'}{w} dF_\ell(w') dG_\ell(w)$$

In the data, I focus on job switches that result in wage gains to ensure consistency with the model's definition.

The returns to job switching in the model closely align with the empirical estimates: it leads to an average wage growth of 8%, slightly above the 7% observed in the data. Across space, I find that the wage gains from moving across employers are larger in big cities, as already suggested by Proposition 2. The correlation between the model's estimates and those in the data is 94%. I therefore conclude that local search frictions — and the employer reallocation they generate — are a valid microfoundation for local wage inequality. This microfoundation has strong welfare implication.

The higher expected wage gains are taken into account when unemployed workers decide where to live. As a result, they are willing to live in expensive cities despite initially lower real earnings. For instance, I find that the real earnings of unemployed workers, b/P_ℓ, are 6% lower in Paris than in Lens. When local amenities and idiosyncratic preferences are introduced, $A_\ell \mathbb{E}_\ell[\boldsymbol{\omega}] b/P_\ell$, the gap widens to 8%. However, the expected future wage gains are larger, and unemployed workers are indifferent between the two locations in equilibrium.[20]

[20] Introducing migration costs would break this indifference. However, given that the average welfare of employed workers is higher in larger cities, unemployed workers born there would have *higher* lifetime utility despite lower initial real earnings.

The expected wage gains from the local job ladder also generate non-trivial welfare gap across space for employed workers. The welfare of employed workers indeed need not be equalized across locations as workers cannot migrate without quitting their job. I find that, on average, employed workers are better off in larger cities. In the cross-section, doubling the size of a city leads to an increase in the average welfare of employed workers — expressed in lifetime consumption equivalence — of 1%. For instance, the average worker in Lens would be willing to give up 3% of its lifetime consumption to be employed at the average job in Paris.

What causes the welfare disparities across cities? In Appendix D.1, I show that, to a first order, the average welfare of employed workers in a given location can be decomposed into contributions from local amenities, average real earnings, and the expected real value of future job opportunities:

$$\log \kappa_\ell \approx \log A_\ell \mathbb{E}_\ell[\omega] + \eta \log \frac{\mathbb{E}_\ell[w]}{P_\ell} + (1-\eta) \log \frac{\mathbb{E}[O_\ell(w)]}{P_\ell}. \tag{26}$$

Here, κ_ℓ denotes by how much the lifetime consumption of workers in ℓ would have to be increased for them to be employed in the average location. $O_\ell(w)$ is the net present value of future expected income, and $\eta = \mathbb{E}[w]/(\mathbb{E}[w] + \mathbb{E}[O(w)]) \in [0,1]$ is the share of contemporaneous wage in the net present value of lifetime income. Figure 7b applies (26) to decompose between-city welfare disparities. The grey, orange, and blue areas represent the contributions of amenities, contemporaneous real wage, and expected future real income on local average welfare relative to the smallest location.

I find that employed workers are on average better off in larger cities almost entirely because the expected future job opportunities are better there. Amenities, gross of workers' idiosyncratic location preferences, are relatively worse in larger locations. Once taking into account the higher average real wage, welfare are nearly equalized across space. For instance, I find that the average employed workers in Paris is only 0.3% better off than in Lens if only amenities and contemporaneous wages are taken into account. The higher future expected wage opportunities turn the lifetime consumption gap to 3%.

Overall, Figure 6 and 7 confirms that employer sorting and local search frictions account for the between-city differences in wage inequality. Together, they generate substantial differences in lifetime consumption across locations that are absent from static spatial and search models.

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A Three facts about wage inequality

A.1 Data

Sample restrictions The employer tax records comes in two formats. The first format is a long-panel that tracks the labor market history of 4% of the French workforce. The second format is a repeated short-panel (two years) that provides information on the universe of jobs held by workers —where a job is defined a pair establishment × occupation. I apply the same restrictions on both datasets:

1. Exclude workers younger than 25 and older than 55 year old;
2. Exclude workers employed in the public sector;
3. Exclude the agriculture, education and health industries;
4. Keep only workers employed full-time;
5. Exclude workers that are non-employed for more than three years;
6. Exclude employment spell that lasts less than 30 days;
7. Exclude employment spell with no labor income or hours worked;

Construction of the panel Once a worker enters the long-panel, each of their employment spell are recorded. An employment spell is defined as a pair establishment × occupation. The dataset provides the start and end days of each employment spell. Workers can be observed multiple times within a given period if they work for multiple employers or if they switch employers. By contrast, a worker is observed only once per year if they work for a unique employer during that entire year.

I aggregate the data at the quarterly level. If workers hold multiple jobs within a quarter, I keep the job that provides the highest total labor income. I only keep the information (e.g., employer's ID, occupation, etc.) associated to that employment spell.

A.2 Accounting for sorting

The variance of log wages in any location reads

$$\mathbb{V}\text{ar}_\ell[\log w_{it}] = \mathbb{V}\text{ar}_\ell[\psi_{it}] + \mathbb{V}\text{ar}_\ell[\gamma_{it}] + 2\rho_\ell(\psi_{it}, \gamma_{it})\mathbb{S}\text{d}_\ell[\psi_{it}]\mathbb{S}\text{d}_\ell[\gamma_{it}] + \mathbb{V}\text{ar}_\ell[\varepsilon_{it}],$$

where $\rho(x, y)$ is the correlation between (x, y). The third term can thus be large because the correlation between the employer and fixed effects is larger, or either are very dispersed. No exact decomposition exists to separate those three effects. Instead, consider a first order approximation of the wage variance around the point where spatial inequality is the same everywhere; i.e., $(\mathbb{V}\text{ar}_\ell[\psi_{it}], \mathbb{V}\text{ar}_\ell[\gamma_{it}], \rho_\ell(\psi_{it}, \gamma_{it}), \mathbb{V}\text{ar}_\ell[\varepsilon_{it}]) \approx (\mathbb{V}\text{ar}[\psi_{it}], \mathbb{V}\text{ar}[\gamma_{it}], \rho(\psi_{it}, \gamma_{it}), \mathbb{V}\text{ar}[\varepsilon_{it}])$. To a first order,

we have

$$\begin{aligned}\mathbb{V}\mathrm{ar}_\ell[\log w_{it}] \approx \mathbb{V}\mathrm{ar}_\ell[\psi_{it}] + \rho(\psi_{it},\gamma_{it})(\mathbb{V}\mathrm{ar}_\ell[\psi_{it}] - \mathbb{V}\mathrm{ar}[\psi_{it}])\left(\frac{\mathbb{S}\mathrm{d}[\gamma_{it}]}{\mathbb{S}\mathrm{d}[\psi_{it}]}\right) + \\ \mathbb{V}\mathrm{ar}_\ell[\gamma_{it}] + \rho(\psi_{it},\gamma_{it})\left(\mathbb{V}\mathrm{ar}_\ell[\gamma_{it}] - \mathbb{V}\mathrm{ar}[\gamma_{it}]\right)\left(\frac{\mathbb{S}\mathrm{d}[\psi_{it}]}{\mathbb{S}\mathrm{d}[\gamma_{it}]}\right) + \\ 2\rho_\ell(\psi_{it},\gamma_{it})\mathbb{S}\mathrm{d}[\psi_{it}]\mathbb{S}\mathrm{d}[\gamma_{it}] + \mathbb{V}\mathrm{ar}_\ell[\varepsilon_{it}] + o_\ell,\end{aligned} \tag{27}$$

where o_ℓ is the approximation error. The first line is the impact of worker heterogeneity on local wage inequality. The second line is the impact of employer heterogeneity on local wage inequality. Finally, the third line captures the impact of sorting and the residuals.

A.3 Tables

Table A.1: Worker-level aggregate summary statistics

Table A.2: City cluster-level summary statistics

Cluster	# CZs	Size	Avg. wage	St. dev.	P10	P90	Rent	Smallest CZ	Largest CZ
1	100	8,518	16.61	0.31	11.33	23.48	8.41	Le Blanc	La Roche-sur-Yon
2	53	15,924	17.34	0.34	11.35	25.38	9.58	Châteaudun	Vannes
3	41	21,214	18.04	0.35	11.54	26.73	9.62	Commercy	Metz
4	29	28,976	18.54	0.37	11.55	28.15	10.38	Tergnier	Tours
5	24	36,931	19.44	0.38	11.79	29.88	10.48	Houdan	Rouen
6	13	62,581	19.37	0.39	11.64	30.16	11.34	Wissembourg	Bordeaux
7	19	50,164	20.33	0.41	11.75	32.48	11.62	Ambert	Toulouse
8	8	101,150	21.43	0.43	11.92	34.61	13.96	Chinon	Roissy
9	8	91,755	22.39	0.45	12.00	36.28	14.54	Étampes	Lyon
10	2	924,781	27.63	0.52	12.53	46.09	22.25	Saclay	Paris

The columns are: the cluster ID, the number of commuting zones in the cluster, the average number of employed worker, the average wage, the standard deviation of log wages, the 10^{th} percentile of the wage distribution, the 90^{th} percentile of the wage distribution, the rent per meter square, the smallest commuting zone in the cluster, and the largest commuting zone in the cluster. All statistics are computed at the CZ level and then averaged at the cluster.

Table A.3: Between-city wage variance decomposition

Table A.4: The local returns to job switching

A.4 Figures

Figure A.1: City wage deciles and local inequality

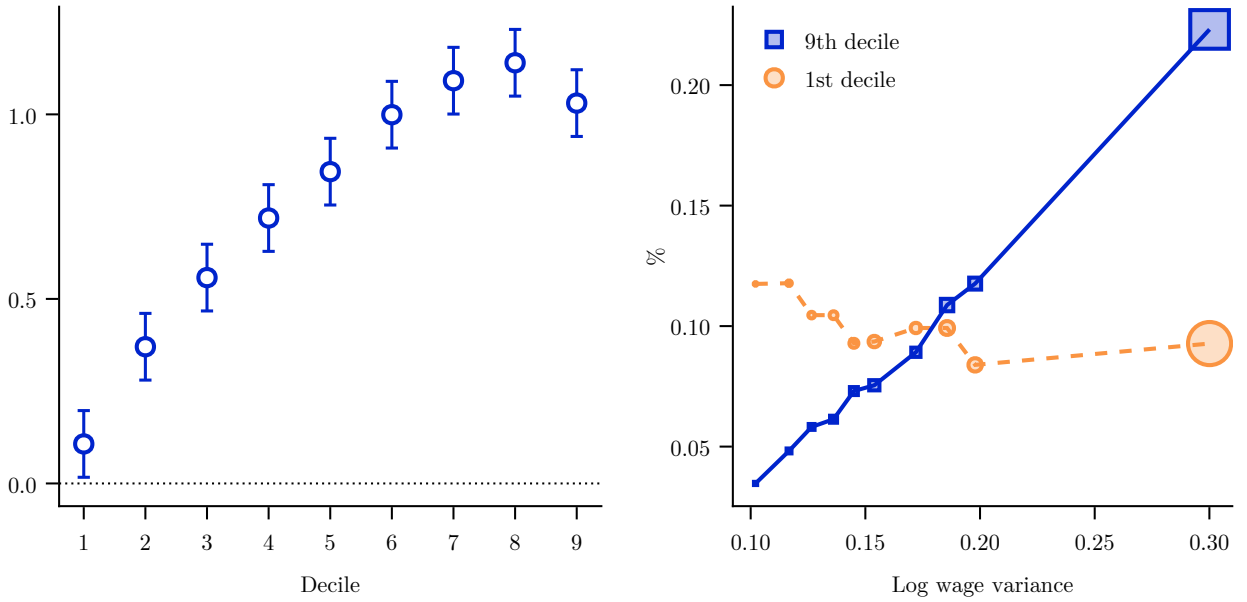


Both panels display the estimates of projections of city wage deciles onto local wage inequality. Let w_ℓ^d denotes the dth wage decile in location ℓ and V_ℓ location ℓ's variance of log wages. Each circle represents $\hat{\beta}^d$ estimated via $\log w_\ell^d = \alpha^d + \beta^d V_\ell + u_\ell^d$. The vertical bars are 95th confidence intervals. Panel (a) plots the $\hat{\beta}^d$ estimated on nominal wages. Panel (b) plots the $\hat{\beta}^d$ estimated on real wages. Real wages are computed as nominal wages deflated by a citywide Cobb-Douglas price index with a housing expenditure share of 0.3.

Figure A.2: Local distribution of employer fixed effects

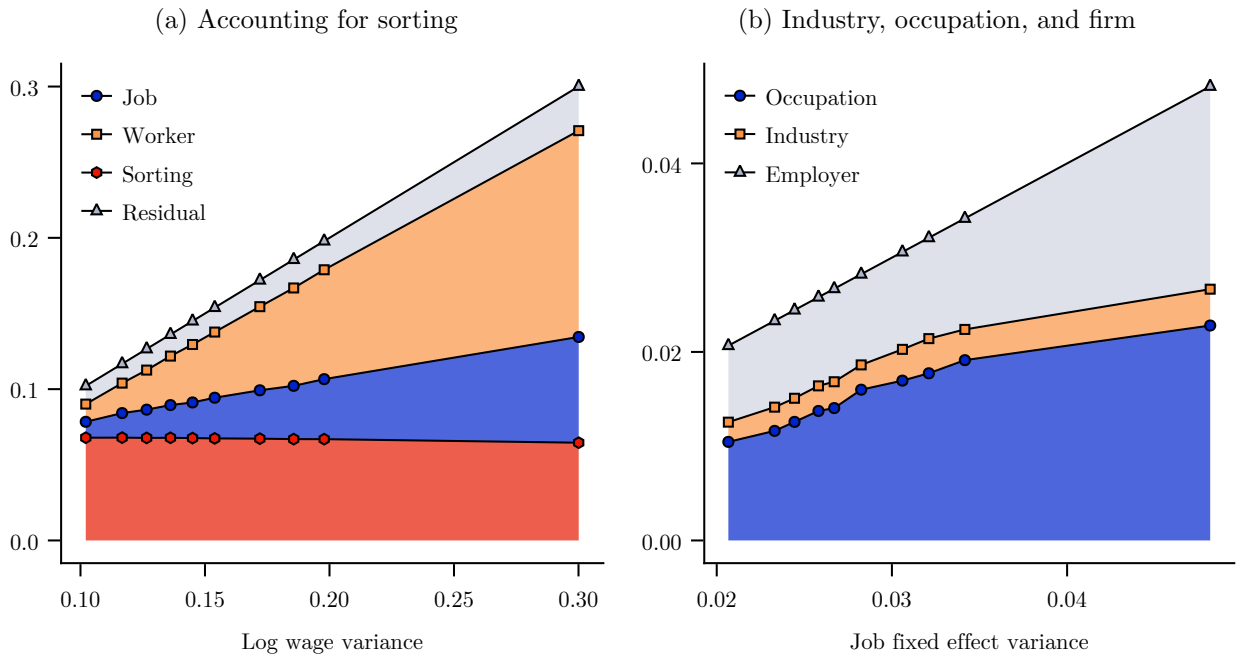
(a) Sensitivity of employer FE deciles to inequality

(b) Fraction of low- and high-paying jobs



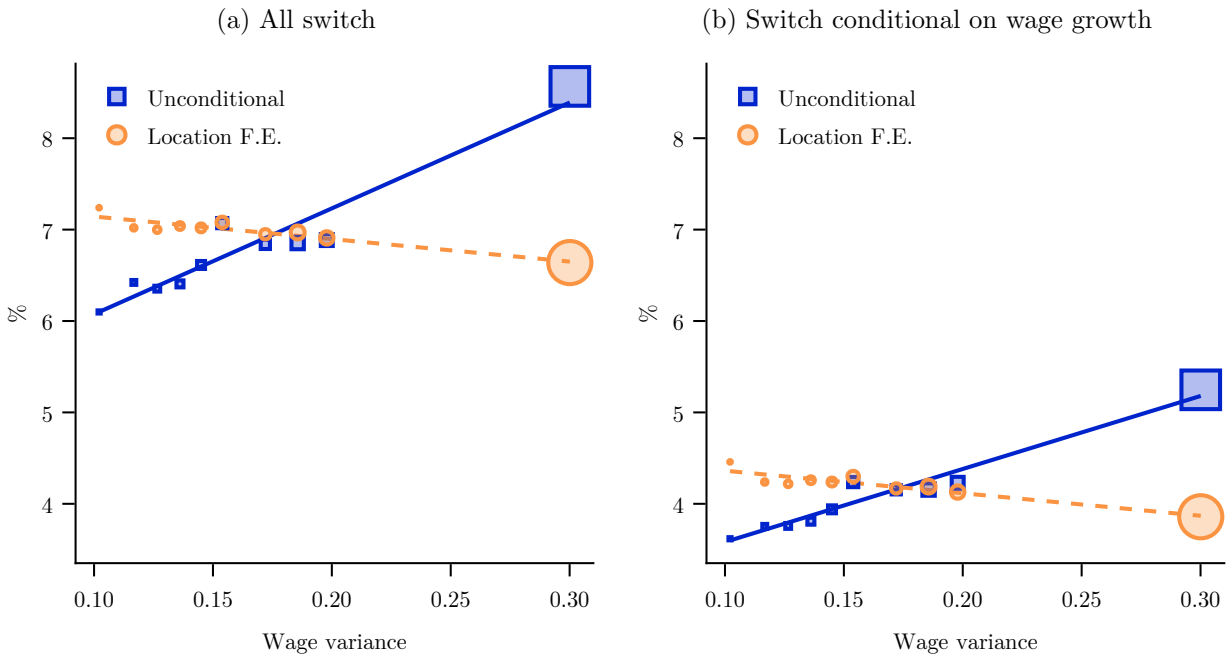
Panel (a) display the estimates of projections of city employer fixed effects deciles onto local wage inequality. Let ψ_ℓ^d denotes the dth employer fixed effects decile in location ℓ and V_ℓ location ℓ's variance of log wages. Each circle represents $\hat{\beta}^d$ estimated via $\psi_\ell^d = \alpha^d + \beta^d V_\ell + u_\ell^d$. The vertical bars are 95th confidence intervals. Panel (b) plots the city share of low- (orange circles) and high-paying (blue rectangles) jobs. Low and high-paying job defined as the 1st and 9th deciles of the national employer fixed effect distribution.

Figure A.3: Within-city wage variance decomposition — robustness



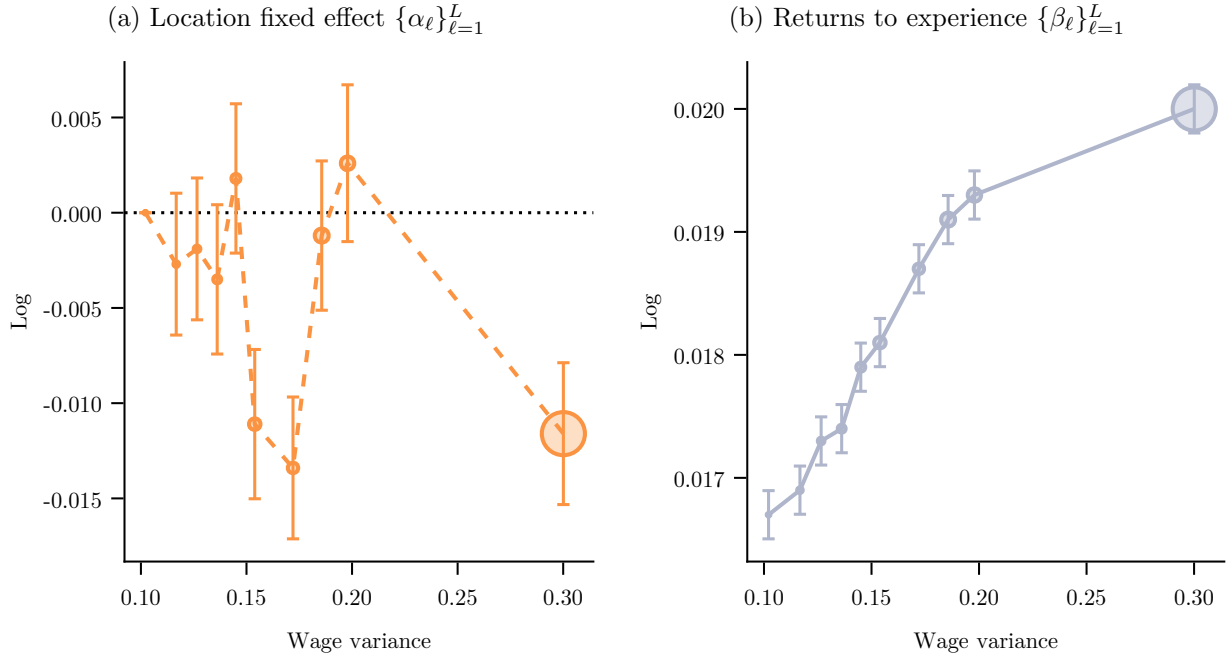
Panel (a) displays the variance decomposition (27). Each marker is a city group. The orange area represents the worker component (first line). The blue area represents the employer contribution (second line). The red area represents the sorting component (third line, first term). The grey area represents the residuals (third line, last two terms). Panel (b) decomposes the variance of the employer fixed effects into an occupation, industry, and establishment component by estimating $\gamma_j = \psi_{o(j)} + \zeta_{s(j)} + u_j$ for $o(j)$ and $s(j)$ the occupation and sector of job j. The blue area represents $\mathbb{V}\text{ar}[\psi_o] + \mathbb{C}\text{ov}[\psi_{o(j)}, \zeta_{s(j)}]$. The orange area represents $\mathbb{V}\text{ar}[\zeta_{s(j)}] + \mathbb{C}\text{ov}[\psi_{o(j)}, \zeta_{s(j)}]$ The grey area represents $\mathbb{V}\text{ar}[u_j]$.

Figure A.4: Local job switching rates



Left panel plots the average job switching rate by location. Job switch are defined as transition between pairs of establishment × occupation within two consecutive quarters. The blue rectangles are the unconditional job switch and the orange circles the average job switch net of worker heterogeneity as captured by the location fixed effects in (4). Right panel restricts the definition of a job switch to a transition between jobs associated with a wage gain.

Figure A.5: Determinants of lifetime wages – other parameters



Left panel plots the city fixed effect $\{\alpha_\ell\}_{\ell=1}^L$ in (6). Right panel plots the returns to experience parameters $\{\beta_\ell\}_{\ell=1}^L$ in (6). In both panels, the parameters are plotted against the cities' wage variance. The vertical bars represent 95th confidence intervals.

B A spatial theory of wage premia

B.1 Local supply curves

In location ℓ, the relative measure of unemployed and employed workers are such that the flow out of unemployment equates the flow into unemployment: $u_\ell/e_\ell = \delta/\lambda^u$. Consistency requires $u_\ell + e_\ell = m_\ell$, and therefore $u_\ell = \delta/(\lambda^u + \delta)m_\ell$ and $e_\ell = \lambda^u/(\lambda^u + \delta)m_\ell$.

The employment distribution, G_ℓ, is characterized by the within-city worker flows across jobs. In steady state, the flow of workers into the interval $[\underline{w}_\ell, w)$ has to be equal to the flow of workers out of the same interval, or $\lambda^u F_\ell(w)u_\ell = [\delta_\ell + \lambda^e \bar{F}_\ell(w)]e_\ell G_\ell(w)$. Solving for $G_\ell(w)$ and using $u_\ell/e_\ell = \delta/\lambda^u$ yields (12).

The labor supply curve is defined as the number of employed worker at wage w per wage offer, or $\lim_{\varepsilon\to 0^-}[G_\ell(w) - G_\ell(w-\varepsilon)]/[F_\ell(w) - F_\ell(w-\varepsilon)]e_\ell/v_\ell$. Taking the limit returns

$$n_\ell(w) = \frac{e_\ell}{v_\ell}\frac{1+k}{\left[1+k\bar{F}_\ell(w)\right]\left[1+k\bar{F}_\ell(w^-)\right]},$$

where $F_\ell(w^-) = \lim_{\varepsilon\to 0^-} F(w-\varepsilon)$. This is equivalent to equation (13) under a continuous wage offer distribution.

B.2 Reservation wage

Suppose for simplicity that F_ℓ admits a density. Differentiate the HJB of employed workers (11) with respect to w, integrate it back to w, and use $V_\ell(\underline{w}_\ell) = U$ to obtain

$$V_\ell(w) = U + \frac{A_\ell}{P_\ell}\int_{\underline{w}_\ell}^{w} \frac{1}{\beta + \delta + \lambda^e \bar{F}_\ell(w')}dw'.$$

Combine this expression into (9) and (11) to get

$$\beta U = \frac{A_\ell}{P_\ell}\left(b + \lambda^u \int_{\underline{w}_\ell} \frac{\bar{F}_\ell(w)}{\beta + \delta + \lambda^e \bar{F}_\ell(w)}dw\right), \tag{28}$$

$$\beta V_\ell(\underline{w}_\ell) = \frac{A_\ell}{P_\ell}\left(\underline{w}_\ell + \lambda^e \int_{\underline{w}_\ell} \frac{\bar{F}_\ell(w)}{\beta + \delta + \lambda^e \bar{F}_\ell(w)}dw\right) = \beta U. \tag{29}$$

Equating both equations and solving for $\underline{w}_\ell$ yield (14).

B.3 Proof of Lemma 1

Nominal Let $H_F(x)$ be the operator defined by

$$H_F(x) = \int_x \frac{\bar{F}(w)}{\beta + \delta + \lambda^e \bar{F}(w)}dw,$$

and $J_F(x)$ be defined by $J_F(x) = x - b - (\lambda^u - \lambda^e)H_F(x)$. The reservation wage solves $J_{F_\ell}(\underline{w}_\ell) = 0$. H satisfies two properties. First, $x \to H_F(x)$ is decreasing. Second, $F_1 \succ_{\text{FOSD}} F_2$ implies $H_{F_1}(x) > H_{F_2}(x)$ for any x. These properties carry through to J_F: $x \to J_F(x)$ is increasing and $F \to J_F(x)$ is weakly decreasing, strictly decreasing if $\lambda^u > \lambda^e$. Therefore, if $F_1 \succ_{\text{FOSD}} F_2$, $H_{F_1}(\underline{w}_2) \leq H_{F_2}(\underline{w}_2) = 0$, and it must be that $\underline{w}_1 \geq \underline{w}_2$, strictly if $\lambda^u > \lambda^e$.

Real From (29), the lifetime utility of a worker employed at the reservation wage is

$$\beta V_\ell(\underline{w}_\ell) = \frac{\underline{w}_\ell A_\ell}{P_\ell}\left(1 + \frac{\lambda^e H_F(\underline{w}_\ell)}{b + (\lambda^u - \lambda^e)\, H_F(\underline{w}_\ell)}\right),$$

where I have also used (14). We know that H_F is (strictly) increasing in F in the FOSD sense, and therefore so must $\lambda^e H_F/(b+[\lambda^u-\lambda^e]H_F)$. However, indifference (10) requires $V_\ell(\underline{w}_\ell) = \bar{U} = V_{\ell'}(\underline{w}_{\ell'})$. Therefore, $\underline{w}_\ell A_\ell/P_\ell$ is decreasing in F_ℓ in the FOSD sense.

B.4 Local wage offer distributions

I provide here the intuition for why there cannot be neither holes nor mass points in the wage offer distribution regardless of the shape of Γ_ℓ. For the full derivation, see Burdett and Mortensen (1998). To start, suppose there is a hole in F_ℓ between $\underline{w}$ and $\bar{w}$. We have $F_\ell(\underline{w}) \leq F_\ell(\bar{w})$, where the inequality is strict if there is a mass point at either $\underline{w}$ or $\bar{w}$. Therefore $n_\ell(\underline{w}) \leq n_\ell(\bar{w})$. However, by offering any wage in $(\underline{w}, \bar{w})$, an employer that used to post wage $\bar{w}$ would keep the same size while lowering its wage bill. This constitutes a profitable deviation, and therefore there cannot be holes in F_ℓ. Suppose now that there is a mass point at w. Take an employer with productivity z that offers wage w, and consider the deviation $w+\varepsilon$ for $\varepsilon > 0$ but small. For $\varepsilon \to 0^+$, we have $n(w+\varepsilon) > n(w)$ since there is a mass point at w, but $w+\varepsilon \to w$. Hence, the profit under wage offer w and $w+\varepsilon$ are respectively $(z-w)n(w) < (z-w-\varepsilon)n(w+\varepsilon)$, and offering wage $w+\varepsilon$ is a profitable deviation. This rules out any mass point in F_ℓ. Since neither arguments referred to Γ_ℓ, they hold for any Γ_ℓ.

I now show that $F_\ell[w_\ell(z)] = \Gamma_\ell(z)$. Since n_ℓ is strictly increasing in w, π_ℓ is continuously differentiable and strictly supermodular in (z, w). Directly applying Theorem 2.8.5. in Topkis (1998), it follows that w is strictly increasing in z. Given the ordering of wages, it must be that $F[w_\ell(z)] = \Gamma_\ell(z)$.

Finally, I derive the wage equation (17). Since there are no mass point in the wage offer distribution, we can take the first-order conditions of (19) with respect to w for almost every job productivity z,

$$\left(\frac{\partial R[z, n_\ell(w)]}{\partial n} - w\right)\frac{n'_\ell(w)}{n_\ell(w)} = 1.$$

Evaluating this equation at $w_\ell(z)$ and using the change of variable $n_\ell(z) = n_\ell[w_\ell(z)]$ yields

$$w'_\ell(z) = \frac{n'_\ell(z)}{n_\ell(z)}\left(z - w_\ell(z)\right), \tag{30}$$

after using $R(z,n) = z \cdot n$. Integrating this ODE with respect to w returns (17).

B.5 Proof of Proposition 2

Define $\eta_\ell(z) \equiv \theta_\ell n_\ell(z)$ as the tightness-independent size. Note that $\int \eta_\ell(z)\mathrm{d}\Gamma_\ell(z) = \Gamma_\ell(z)/(1 + k\bar{\Gamma}_\ell(z))$ is a proper cdf. Define $\eta^Q(q) \equiv \eta_\ell[\Gamma_\ell^{-1}(q)]$ as the size evaluated on rung q of the job ladder. Importantly, η^Q is identical across cities, which follows from search frictions being identical across space. Finally, let $\bar{w}_\ell$ denote the highest wage offered in city j. Throughout, I omit the city subscript whenever not necessary.

FOSD ordering (2.1) The wage paid by the employers at the q-th quantile of the wage distribution in city ℓ reads

$$\begin{aligned} w_\ell[\Gamma_\ell^{-1}(q)] &= \underline{w}_\ell\left(\frac{\eta^Q(0)}{\eta^Q(q)}\right) + \int_{\underline{z}_\ell}^{\Gamma_\ell^{-1}(q)} \zeta\left(\frac{\eta'_\ell(\zeta)}{\eta^Q(q)}\right)\mathrm{d}\zeta \\ &= \underline{w}_\ell\left(\frac{\eta^Q(0)}{\eta^Q(q)}\right) + \int_0^q \Gamma_\ell^{-1}(x)\left(\frac{\partial_x\eta^Q(x)}{\eta^Q(q)}\right)\mathrm{d}x, \end{aligned} \tag{31}$$

where the second equality follows from the change of variable $\Gamma_\ell(\zeta) \to x$. We guess and verify that $\Gamma_\ell \succ \Gamma_{\ell'}$ implies $F_\ell \succ F_{\ell'}$. So guess that the second ordering holds. From Proposition 1, we know that $\underline{w}_\ell \geq \underline{w}_{\ell'}$. Hence, for all $w \in [\underline{w}_{\ell'}, \underline{w}_\ell]$, we have $F_{\ell'}(w) \geq F_\ell(w) = 0$. From (31), $w_{\ell'}[\Gamma_\ell^{-1}(q)] < w_\ell[\Gamma_\ell^{-1}(q)]$ for all $q \in (0,1]$. In particular, $\bar{w}_{\ell'} < \bar{w}_\ell$, so that $F_{\ell'}(w) = 1 > F_\ell(w)$ for all $w \in [\bar{w}_{\ell'}, \bar{w}_\ell)$. Finally, for any $w \in (\underline{w}_\ell, \bar{w}_{\ell'})$, (31) implies that $w_\ell[\Gamma_\ell^{-1}(q_\ell)] = w = w_{\ell'}[\Gamma_{\ell'}^{-1}(q_{\ell'})]$ if and only if $q_\ell > q_{\ell'}$. Hence, $F_{\ell'}(w) = F_{\ell'}\{w_{\ell'}[\Gamma_{\ell'}^{-1}(q_l)]\} = \Gamma_{\ell'}[\Gamma_{\ell'}^{-1}(q_{\ell'})] = q_{\ell'} > q_\ell = \Gamma_\ell[\Gamma_\ell^{-1}(q_\ell)] = F_\ell\{w_\ell[\Gamma_\ell^{-1}(q_\ell)]\} = F_\ell(w)$, where the second equality follows from the rank-preserving property of the job ladder. Concluding, we have $F_{\ell'}(w) \geq F_\ell(w)$ for all w and $F_{\ell'}(w) > F_\ell(w)$ on the joint support of F_ℓ and $F_{\ell'}$, so that $F_\ell \succ F_{\ell'}$. From equation (12), it automatically follows that $G_\ell \succ G_{\ell'}$.

Inequality (2.2) Let $\Delta_{\ell\ell'} \equiv \bar{w}_{\ell'} - \underline{w}'_\ell - (\bar{w}_{\ell'} - \underline{w}_{\ell'})$ be the difference in top-to-bottom wage gap between city ℓ and ℓ'. Using (17), this difference writes

$$\Delta_{\ell\ell'} = (\underline{w}_\ell - \underline{w}_{\ell'})\left(1 - \frac{1}{(1+k)^2}\right) + \int_0^1 \left(\Gamma_\ell^{-1}(q) - \Gamma_{\ell'}^{-1}(q)\right)\left(\frac{\partial_x\eta^Q(q)}{\eta^Q(1)}\right)\mathrm{d}q.$$

If $\Gamma_\ell \succ \Gamma_{\ell'}$, this implies $\underline{w}_\ell \geq \underline{w}_{\ell'}$ and $\Gamma_\ell^{-1}(q) > \Gamma_{\ell'}^{-1}(q)$ for all $q \in (0,1)$, from which it follows that $\Delta_{\ell\ell'} > 0$. Note that this between-city difference in inequality is in level. Alternatively, the

top-to-bottom wage gap in ratio in city ℓ is

$$\frac{\bar{w}_\ell}{\underline{w}_\ell} = \frac{1}{1+k^2} + \frac{1}{\underline{w}_\ell}\int_0^1 \Gamma_\ell^{-1}(q)\left(\frac{\partial_x \eta^Q(q)}{\eta^Q(1)}\right)\mathrm{d}x.$$

When $\lambda^e \to \lambda^u$, we have $\underline{w}_\ell \to b$, so that $\Gamma_\ell \succ \Gamma_{\ell'}$ also implies $\bar{w}_\ell/\underline{w}_\ell > \bar{w}_{\ell'}/\underline{w}_{\ell'}$ for $\lambda^e \approx \lambda^u$.

E2E gains (2.3) The ℓ are dropped as long as comparisons across cities are absent. The expected wage growth upon a job-to-job transition in a given city is

$$\Delta^{\mathrm{EE}}(w) \equiv \mathbb{E}\left[\frac{w'}{w} \mid \text{switch from } w\right] = \frac{1}{\bar{F}(w)}\int_w \frac{w'}{w}\mathrm{d}F(w').$$

Consider this statistic for a worker employed on some rung q of the ladder,

$$\Delta^{\mathrm{EE}}[F^{-1}(q)] = \frac{1}{1-q}\int_{F^{-1}(q)} \frac{w'}{F^{-1}(q)}\mathrm{d}F(w'). \tag{32}$$

For $q \to 0$, we know that $F^{-1}(q) \to \underline{w} \approx b$, where the second approximation comes from $\lambda^e \approx \lambda^u$. Hence, for $q \to 0$, we have

$$\Delta^{\mathrm{EE}}[F^{-1}(q)] \to \int_b \frac{w'}{b}\mathrm{d}F(w')$$

If $\Gamma_\ell \succ G_{\ell'}$, then $F_\ell \succ F_{\ell'}$, and therefore $\Delta^{\mathrm{EE}}_\ell(\underline{w}_\ell) < \Delta^{\mathrm{EE}}_{\ell'}(\underline{w}_{\ell'})$. Since the inequality is strict and (32) is continuous in q, this holds for workers on low rungs of the ladder.

B.6 A competitive spatial matching model

Consider the simplest competitive spatial matching model. There are L cities that differ in size, m_ℓ, with $m_1 < m_2 < \cdots < m_L$. Cities are purposely homogeneous in TFP to study the implication for spatial wage inequality. There are heterogeneous firms indexed by their productivity z distributed according to Γ. Firms have a decreasing returns to scale technology: $R(z,n) = zn^\rho$.[21] Labor markets are competitive and segmented by locations. Employers pay housing cost r_ℓ to produce in location ℓ. The housing supply is $L_\ell = \bar{L} r_\ell^\chi$.

Employers solve

$$\pi(z) = \max_{\ell,n}\ zn^\rho - w_\ell n - r_\ell.$$

Their labor demand and optimal profits are

$$n_\ell(z) = \left(\frac{z\rho}{w}\right)^{\frac{1}{1-\rho}} \quad \text{and} \quad \pi(z) = \max_\ell\ (1-\rho)\rho^{\frac{\rho}{1-\rho}}\frac{z^{\frac{1}{1-\rho}}}{w^{\frac{\rho}{1-\rho}}} - r_\ell.$$

[21] Firm size is not well-defined absent DRS.

The spatial allocation of firms is described by $\{M_\ell, \Gamma_\ell\}_{\ell=1}^L$ for M_ℓ the measure of employers and Γ_ℓ the local productivity distribution.

The next proposition shows that wages are necessarily lower in larger cities.

Proposition B.1 (Wage distribution)**.**
In any equilibrium, wages are decreasing in city size: $m_\ell > m_{\ell'} \Rightarrow w_\ell < w_{\ell'}$.

Proof. Using the equilibrium condition for wages, the firm problem becomes

$$\max_\ell \ (1-\rho)\, \psi_\ell z^{\frac{1}{1-\rho}} - r_\ell,$$

where ψ_ℓ fully summarizes the profitability of ℓ:

$$\psi_\ell \equiv \left(\frac{m_\ell}{M_\ell \mathbb{E}_\ell \left[z^{\frac{1}{1-\rho}} \right]} \right)^\rho = \left(\frac{\rho}{w_\ell} \right)^{\frac{\rho}{1-\rho}}.$$

The constant ψ_ℓ is an equilibrium object that depends on the allocation of employers across space. However, given a distribution of $\{\psi_\ell\}_{\ell=1}^L$, the complementarity between (ψ_ℓ, z) implies that there is pure positive assortative matching between $\boldsymbol{\psi}$ and $\boldsymbol{z}$ — or pure negative assortative matching between $\boldsymbol{w}$ and $\boldsymbol{z}$ (Topkis, 1998).

We can now prove the proposition. From the expression for wages, wages are increasing in city size if

$$\frac{M_{\ell+1}}{M_\ell} \frac{\mathbb{E}_{\ell+1}\left[z^{\frac{1}{1-\rho}} \right]}{\mathbb{E}_\ell \left[z^{\frac{1}{1-\rho}} \right]} > \frac{m_{\ell+1}}{m_\ell} > 1.$$

That is, $M_\ell \mathbb{E}_\ell[z]$ must be increasing in m_ℓ. We also know that firms sort in the opposite direction of w_ℓ. Hence, $\mathbb{E}_\ell[z^{\frac{1}{1-\rho}}]$ is decreasing in m_ℓ. A necessary condition is therefore that M_ℓ is increasing in m_ℓ.

In any equilibrium with pure sorting, marginal firms must be indifferent between two locations. If w_ℓ is increasing in m_ℓ, this indifference is $\pi_\ell(\underline{z}_\ell) = \pi_{\ell+1}(\bar{z}_{\ell+1})$, or using the expression for profits and housing prices:

$$(1-\rho)\left[\psi_\ell - \psi_{\ell+1}\right] \underline{z}_\ell^{\frac{1}{1-\rho}} = \left(\frac{M_\ell}{\bar{L}} \right)^{1/\chi} - \left(\frac{M_{\ell+1}}{\bar{L}} \right)^{1/\chi} > 0.$$

Therefore, M_ℓ is decreasing in ℓ, and wages cannot be increasing in m_ℓ. □

B.7 Proof of Proposition 3

This section proves Proposition 3, and by doing so, uniquely characterizes the job distributions $\{\Gamma_\ell\}_{\ell=1}^L$ as a function of the vector of market tightness $\{\theta_\ell\}_{\ell=1}^L$. To do, I proceed in five parts. First,

I show that the support of the distribution is convex in each city (Lemma B.2). I then prove that local job distributions are necessarily ranked in terms of FOSD (Lemma B.3). Lemma B.4 continues by deriving the pdf of the job allocation in each city, while Lemma B.5 proves the condition for the overlapping support in the job distribution. Finally, Algorithm 1 concludes by uniquely solving for the cutoffs of the job productivity in each city.

Lemma B.1.

The profit function $\pi_\ell(z)$ is continuously differentiable.

Proof. From (**??**), $\pi'_\ell(z)$ is differentiable. Differentiating with respect to z returns $\pi'_\ell(z) = n_\ell(z)$. Since F_ℓ has no mass point, $n_\ell(z)$ is continuous in z. □

Lemma B.2 (Convex support)**.**

The support of the job distribution in each city is an interval.

Proof. Suppose not. That is, for some city ℓ, there exists at least one hole in the support of Γ_ℓ. Wlog, suppose there is a unique hole, such that $\operatorname{supp}\Gamma_\ell = [\underline{z}_\ell, z] \cup [z+\varepsilon, \bar{z}_\ell]$ for some $\varepsilon > 0$. The profit maximization condition (19) then requires

1. $\pi_\ell(z') \geq \pi_{\ell'}(z')$ for all $z' \in [\underline{z}_\ell, z]$ and all cities ℓ',
2. $\pi_\ell(z') < \pi_{\ell^\star}(z')$ for all $z' \in (z, z+\varepsilon)$ and at least one city $\ell^\star$,
3. $\pi_\ell(z') \geq \pi_{\ell'}(z')$ for all $z' \in [z+\varepsilon, \bar{z}_\ell]$ and all cities ℓ'.

The first two conditions occur only if $\pi'_\ell(z') < \pi'_{\ell^\star}(z')$ for $z' \in B^+(z)$.[22] Using the Envelope condition, this rewrites $n_\ell(z') = n_\ell(z) = \pi'_\ell(z) < \pi'_{\ell^\star}(z') = n_{\ell^\star}(z')$, where the first equality follows from n_ℓ being constant outside of the support of Γ_ℓ. It follows that $\pi'_\ell(z') < \pi'_{\ell^\star}(z')$ for all $z' \in (z, z+\varepsilon)$, and therefore $\pi_\ell(z+\varepsilon) < \pi_{\ell^\star}(z+\varepsilon)$, a contradiction with the third condition. □

Lemma B.3 (First order stochastic dominance ordering)**.**

For two cities ℓ and ℓ' and a vector of tightness that is equilibrium compatible, $\theta_\ell < \theta_{\ell'}$ if and only if $\Gamma_\ell \succ \Gamma_{\ell'}$.[23]

Proof. I start by showing that if $\theta_\ell < \theta_{\ell'}$, then $\Gamma_\ell \succ \Gamma_{\ell'}$. For the sake of contradiction, suppose that $\bar{\Gamma}_{\ell'}(z) \geq \bar{\Gamma}_\ell(z)$ for some z. Then,

$$\pi'_\ell(z) = \frac{1}{\theta_\ell}\frac{1+k}{(1+k\bar{\Gamma}_\ell(z))^2} \geq \frac{1}{\theta_\ell}\frac{1+k}{(1+k\bar{\Gamma}_{\ell'}(z))^2} > \frac{1}{\theta_{\ell'}}\frac{1+k}{(1+k\bar{\Gamma}_{\ell'}(z))^2} = \pi'_{\ell'}(z). \tag{33}$$

If $\Gamma_\ell(z), \Gamma_{\ell'}(z) \in (0,1)$, Lemma B.2 implies that $\pi_\ell(z) = \pi_{\ell'}(z)$. However, from equation (33), $\pi'_\ell(z') > \pi'_{\ell'}(z')$ for $z' \in B^+(z)$, and therefore $\pi_\ell(z') > \pi_{\ell'}(z')$. Lemma B.2 then implies $z = \bar{z}_{\ell'}$ and $z < \bar{z}_\ell$, or $\bar{\Gamma}_{\ell'}(z) = 0 < \bar{\Gamma}_\ell(z)$, a contradiction.

If $\Gamma_{\ell'}(z) = 0$ and $\Gamma_\ell(z) \in (0,1)$, then it must from Lemma B.2 that $\pi_{\ell'}(z) \leq \pi_\ell(z)$. Furthermore, the positive measure of jobs locating in city ℓ' requires the existence of $\underline{z}_{\ell'} \geq z$ such that $\pi_{\ell'}(\underline{z}_{\ell'}) \geq$

[22] Throughout the appendix, $B(x)$ refers to an open ball around x, $B^+(x)$ an open ball to the right of x, and similarly $B^-(x)$ to an open ball to the left of x.

[23] An equilibrium compatible vector of market tightness is such that $\theta_\ell < \infty$ for all ℓ.

$\pi_\ell(\underline{z}_{\ell'})$. If $\underline{z}_{\ell'} = z$, then it must be that $\pi_{\ell'}(\underline{z}_{\ell'}) = \pi_\ell(\underline{z}_\ell)$. However, (33) implies that $\pi'_{\ell'}(z') < \pi'_\ell(z')$ for all $z' \in B^+(\underline{z}_{\ell'})$, and therefore $\pi_{\ell'}(z') < \pi_\ell(z')$ for $z' \in B^+(\underline{z}_{\ell'})$, which contradicts Lemma B.2. Hence, it must be that $\underline{z}_{\ell'} > z$. But (33) implies $\pi'_{\ell'}(z') < \pi'_\ell(z')$ for all $z' \in (z, \underline{z}_{\ell'})$, and therefore $\pi_{\ell'}(\underline{z}_{\ell'}) < \pi_\ell(\underline{z}_{\ell'})$, a contradiction with the definition of $\underline{z}_{\ell'}$. It follows that $\underline{z}_{\ell'}$ does not exist, a contradiction.

Finally, suppose $\Gamma_\ell(z) = 1$, so that $z \geq \bar{z}_\ell$. If $\Gamma_{\ell'}(z) \in (0,1)$, then $\bar{z}_{\ell'} > \bar{z}_\ell$. Meanwhile, it must also be that $\pi_{\ell'}(\bar{z}_\ell) \leq \pi_\ell(\bar{z}_\ell)$. Finally, for all $z' \in [\bar{z}_\ell, \bar{z}_{\ell'})$, we have $\Gamma_{\ell'}(z') < \Gamma_\ell(z') = 1$, and therefore $\pi'_{\ell'}(z') < \pi'_\ell(z')$ from (33). However, this implies $\pi_{\ell'}(z') < \pi_\ell(z')$ for all $z' \in [\bar{z}_\ell, \bar{z}_{\ell'})$, which contradicts $\bar{z}_{\ell'} > \bar{z}_\ell$ from Lemma B.2. Hence, suppose that $\Gamma_{\ell'}(z) = 0$. However, in this case, a similar argument as in the previous paragraph shows that there does not exist a $\underline{z}_{\ell'}$, a contradiction. Hence, the only possibility is $\Gamma_{\ell'}(z) = 1$, a contradiction of $\Gamma_{\ell'} \succ \Gamma_\ell$.

The other direction automatically follows. Suppose not $\theta_\ell < \theta_{\ell'}$, such that $\theta_\ell \geq \theta_{\ell'}$. If $\theta_\ell > \theta_{\ell'}$, then we have already shown that $\Gamma_{\ell'} \succ \Gamma_\ell$, or not $\Gamma_\ell \succ \Gamma_{\ell'}$. If $\theta_{\ell'} = \theta_\ell$, it must be that $\Gamma_{\ell'}(z) = \Gamma_\ell(z)$ for all z. To see this, suppose for the sake of contradiction that $\bar{\Gamma}_{\ell'}(z) > \bar{\Gamma}_\ell(z)$ for some z. Note that, as in (33), this implies $\pi'_\ell(z) > \pi'_{\ell'}(z)$, and it is possible to re-use the same arguments as above to create a contradiction. By symmetry, $\bar{\Gamma}_\ell(z) > \bar{\Gamma}_{\ell'}(z)$ for some z also creates a contradiction. □

Lemma B.4 (Local job density)**.**
Define the functions $\mu^\ell : \mathbb{R}_+ \mapsto \mathbb{R}_+$ as the (employment-weighted) relative market tightness that employers z faces in city ℓ,

$$\mu^\ell(z) \equiv \frac{e_\ell \sqrt{\theta_\ell}}{\sum_{\ell' \in \mathcal{L}(z)} e_{\ell'} \sqrt{\theta_{\ell'}}} \quad \textit{for} \quad \mathcal{L}(z) \equiv \{\ell' \in \mathcal{J} : z \in [\underline{z}_{\ell'}, \bar{z}_{\ell'}]\}.$$

Then, if $z \in$ supp Γ_ℓ, the relative measure of z-jobs in ℓ is

$$\mathrm{d}\Gamma_\ell(z) = \left(\frac{M}{v_\ell}\right) \mu^\ell(z) \mathrm{d}\Gamma(z).$$

Before the proof, note that the function μ has three important properties. If all z-jobs locate in city ℓ, then $\mu^\ell(z) = 1$, so that $\mathrm{d}\Gamma_\ell(z) = M\mathrm{d}\Gamma(z)/v_\ell$. Second, $\mu^\ell(z)$ is decreasing in the number of overlapping cities, $\mathcal{L}(z)$. Finally, $\mu^\ell(z) \in [e_\ell\sqrt{\theta_\ell}/(\sum_{\ell' \in \{1,2,\ldots,L\}} e_{\ell'}\sqrt{\theta_{\ell'}}), 1]$, it is locally constant, and takes at most L values.

Proof. Take any $z \in$ supp Γ. If there exists a city ℓ so that $\pi_\ell(z) > \pi_{\ell'}(z)$ for all other ℓ', then all z employers locate in ℓ. The feasibility condition (8) then requires $v_\ell \mathrm{d}\Gamma_\ell(z) = M\mathrm{d}\Gamma(z)$. Otherwise, let $\mathcal{L}$ be the set of city for which $\pi_\ell(z) = \pi(z)$ for $\ell \in \mathcal{L}$. Unless $z \in \{\underline{z}_\ell, \bar{z}_\ell\}$ – a measure zero event, then this condition must also hold for a neighborhood around z. Hence, $n_\ell(z) = \pi'_\ell(z) = \pi'_{\ell'}(z) = n_{\ell'}(z)$

for any $(\ell, \ell') \in \mathcal{L}$. Differentiating $n_\ell(z) = n_{\ell'}(z)$ with respect to z implies

$$\frac{\mathrm{d}\Gamma_{\ell'}(z)}{\mathrm{d}\Gamma_\ell(z)} = \sqrt{\frac{\theta_\ell}{\theta_{\ell'}}},$$

for any $(\ell, \ell') \in \mathcal{L}$. But feasibility requires $\sum_{\ell' \in \mathcal{L}} v_{\ell'} \mathrm{d}\Gamma_{\ell'}(z) = M \mathrm{d}\Gamma(z)$, and therefore

$$\mathrm{d}\Gamma_\ell(z) = \frac{1}{\sqrt{\theta_\ell}} \frac{M \mathrm{d}\Gamma(z)}{\sum_{\ell' \in \mathcal{L}} e_{\ell'} \sqrt{\theta_{\ell'}}}.$$

Re-arranging and using the two functions $z \to \mu^\ell(z)$ and $z \to \mathcal{L}(z)$ yield the desired equation. □

An important aspect of Lemma B.4 is that, given $\mathcal{L}(z)$ and $\boldsymbol{\theta}$, the weighting function μ^ℓ is known. Hence, Lemma B.4 pins down the job distributions up to the thresholds $\{\underline{z}_\ell, \bar{z}_\ell\}_{\ell=1}^L$. Under perfect sorting, solving for these thresholds would be trivial. However, in this economy of matching with externality, perfect sorting does not always prevail.

Lemma B.5 (Overlapping supports)**.**
Take two cities ℓ and ℓ' so that $\theta_\ell < \theta_{\ell'}$. Then, supp $\Gamma_\ell \cap$ supp $\Gamma_{\ell'}$ has positive measure if and only if $\theta_{\ell'} < (1+k)^2 \theta_\ell$.

Proof. I first prove $\Leftarrow$. Suppose $\theta_{\ell'} < (1+k)^2 \theta_\ell$. For the sake of contradiction, suppose also that there is no overlap. Since $\theta_\ell < \theta_{\ell'}$, Lemma B.3 implies $\bar{z}_{\ell'} \leq \underline{z}_\ell$. Furthermore, $\pi_{\ell'}(\underline{z}_\ell) \leq \pi_\ell(\underline{z}_\ell)$ and $\pi_{\ell'}(z) > \pi_\ell(z)$ for $z \in B^-(\bar{z}_{\ell'})$. Hence, π_ℓ and $\pi_{\ell'}$ must cross at least once in $[\bar{z}_{\ell'}, \underline{z}_\ell]$ and π_ℓ crosses $\pi_{\ell'}$ from below; that is, there exists a $z^\star \in [\bar{z}_{\ell'}, \underline{z}_\ell]$ so that $\pi_\ell(z^\star) = \pi_{\ell'}(z^\star)$ and $\pi'_\ell(z) > \pi'_{\ell'}(z)$ for $z \in B^-(z^\star)$. By continuity, we therefore have $\pi'_\ell(z^\star) \geq \pi'_{\ell'}(z^\star)$. Furthermore, it must be that $\Gamma_{\ell'}(z^\star) = 1$. Combining these elements and using the Envelope theorem, this inequality writes

$$\frac{1}{\theta_\ell} \frac{1}{1+k} = n_\ell(\underline{z}_\ell) = \pi'_\ell(z^\star) \geq \pi'_{\ell'}(z^\star) = n_{\ell'}(\bar{z}_{\ell'}) = \frac{1+k}{\theta_{\ell'}},$$

where the first and last equality follows from n_ℓ and $n_{\ell'}$ being constant outside of their respective support. However, $\theta_{\ell'} < (1+k)^2 \theta_\ell$, a contradiction.

We now prove the other direction. For this, suppose not $\theta_{\ell'} < (1+k)^2 \theta_\ell$, or $\theta_{\ell'} \geq (1+k)^2 \theta_\ell$. We want to show that this implies no overlap. Hence, for the sake of contradiction, suppose that there is overlap in the job distribution. Lemma B.2 then implies supp $\Gamma_\ell \cap$ supp $\Gamma_{\ell'} = [\underline{z}_\ell, \bar{z}_{\ell'}]$ with $\underline{z}_\ell < \bar{z}_{\ell'}$. Employers in $[\underline{z}_\ell, \bar{z}_{\ell'}]$ must be indifferent between the two cities, $\pi_\ell(z) = \pi_{\ell'}(z)$ for all $z \in [\underline{z}_\ell, \bar{z}_{\ell'}]$, and therefore $\pi'_\ell(z) = \pi'_{\ell'}(z)$. By continuity, this must hold at $\underline{z}_\ell$, or using the Envelope theorem,

$$\frac{1}{\theta_\ell} \frac{1}{1+k} = n_\ell(\underline{z}_\ell) = \pi'_\ell(\underline{z}_\ell) = \pi'_{\ell'}(\underline{z}_\ell) = n_{\ell'}(\underline{z}_\ell) = \frac{1}{\theta_{\ell'}} \frac{1}{(1 + k\bar{\Gamma}_{\ell'}(\underline{z}_\ell))^2}.$$

However, $\theta_{\ell'} \geq (1+k)^2 \theta_\ell$, and this equality cannot hold for any $\Gamma_{\ell'}(\underline{z}_\ell) \in (0,1)$, a contradiction. □

For a given $\boldsymbol{\theta}$, Lemma B.4 and B.5, together with the boundary conditions boundary conditions $\underline{z}_1 = \underline{z}$, $\bar{z}_L = \bar{z}$, $\Gamma_\ell(\underline{z}_\ell) = 0$ and $\Gamma_\ell(\bar{z}_\ell) = 1$, pins down uniquely the set of cutoffs $\{\underline{z}_\ell, \bar{z}_\ell\}_{\ell=1}^L$. These cutoffs are derived in Algorithm 1.

Order cities by $1/\theta_\ell$. Normalize $\underline{z}_0 = \underline{z}$.
for $\ell \in \{1, 2, \ldots, L\}$ **do**
Either $\ell = 1$ and $\underline{z}_\ell = \underline{z}$, or $\ell > 1$ and the lower bound has been found in the previous iteration (step 2 or 3).
1. **if** $\theta_{\ell-1} < \theta_\ell(1+k)^2$ **then** From Lemma B.5, $\underline{z}_\ell \in (\underline{z}_{\ell-1}, \bar{z}_{\ell-1})$. Furthermore, $\pi'_\ell(z) = \pi'_{\ell-1}(z)$ for $z \in (\underline{z}_\ell, \bar{z}_{\ell-1})$ implies

$$\frac{1}{\theta_\ell}\left(\frac{1+k}{\left(1+k\bar{\Gamma}_\ell(\bar{z}_{\ell-1})\right)^2}\right) = \frac{1+k}{\theta_{j-1}},$$

which solves for $\Gamma_\ell(\bar{z}_{\ell-1})$.
else From Lemma B.5 $\underline{z}_\ell = \bar{z}_{\ell-1}$ and $\Gamma_\ell(\bar{z}_{\ell-1}) = 0$.
2. **foreach** $\ell' \in \{\ell+1, \ldots, L\}$ **do**
if $\theta_\ell < \theta_{\ell'}(1+k)^2$ *and* $\theta_{\ell-1} > \theta_{\ell'}(1+k)^2$ **then**
From Lemma B.5 $\bar{z}_{\ell-1} < \underline{z}_{\ell'} < \bar{z}_\ell$. For $z \in (\underline{z}_{\ell'}, \bar{z}_\ell)$, indifference requires $\pi'_\ell(z) = \pi'_{\ell'}(z)$, or

$$\frac{1}{\theta_\ell}\left(\frac{1+k}{1+k\bar{\Gamma}_\ell(\underline{z}_{\ell'})^2}\right) = \frac{1}{\theta_{\ell'}}\left(\frac{1}{1+k}\right). \tag{34}$$

Furthermore, since $\underline{z}_{\ell'} > \underline{z}_{\ell-1}$, jobs in $(\max\{\underline{z}_{\ell'-1}, \bar{z}_{\ell-1}\}, \underline{z}_{\ell'})$ are located in cities $\{\ell, \ldots, \ell'-1\}$. Lemma B.4 then implies $\mathrm{d}\Gamma_\ell(z) = \mu^\ell_{\ell,\ell'-1} M \mathrm{d}\Gamma(z)/v_\ell$, so that

$$\Gamma_\ell(\underline{z}_{\ell'}) = \Gamma_\ell\left(\max\{\bar{z}_{\ell-1}, \underline{z}_{\ell'-1}\}\right) + \left(\frac{M}{v_\ell}\right)\mu^\ell_{\ell,\ell'-1}\left(\Gamma(\underline{z}_{\ell'}) - \Gamma\left(\max\{\bar{z}_{\ell-1}, \underline{z}_{\ell'-1}\}\right)\right).$$

Combined with equation (34), these two equations returns $\underline{z}_{\ell'}$.
end
end
3. The cdf must integrate to one. Letting $\ell^\star \equiv \max\{\ell, \sup\{\ell' : \theta_{\ell'} > (1+k)^2\theta_\ell\}\}$, this requires

$$1 = \Gamma_\ell(\max\{\bar{z}_{\ell-1}, \underline{z}_{\ell^\star}\}) + \left(\frac{M}{v_\ell}\right)\mu^\ell_{\ell,\ell^\star}\left(\Gamma(\bar{z}_\ell) - \Gamma\left(\max\{\bar{z}_{\ell-1}, \underline{z}_{\ell^\star}\}\right)\right),$$

which solves for $\bar{z}_\ell$.
end

Algorithm 1: Boundaries solver

B.8 Proof of Proposition 4

The proof of Proposition 4 is broken up in several lemmas. Lemma B.6 proves that, in equilibrium, cities must necessarily have different market tightness. Lemma B.7 then derives the expression for the local prices that sustain the spatial job allocation. Finally, Lemma B.8 then orders cities based on their fundamentals.

Lemma B.6 (No symmetric equilibria)**.**
In equilibrium, no two cities can have the same labor market tightness.

Proof. Take two cities, ℓ and ℓ' with $e_\ell > e_{\ell'}$ (wlog). Suppose for the sake of contradiction that $\theta_\ell = \theta_{\ell'}$. Then, it must be that $v_{\ell'}/v_\ell = e_{\ell'}/e_\ell$, or $v_{\ell'} < v_\ell$. It follows that $r_{\ell'} < r_\ell$. At the same time, Lemma B.3 implies that $\Gamma_{\ell'} = \Gamma_\ell$, and therefore $\underline{z}_{\ell'} = \underline{z}_\ell = \underline{z}$. Furthermore, from equation (14), $\underline{w}_{\ell'} = \underline{w}_\ell \equiv \underline{w}$. Finally, employers must be indifferent between both locations. In particular, $\pi_{\ell'}(\underline{z}_{\ell'}) = (\underline{z} - \underline{w})n(\underline{z}) - r_{\ell'} = (\underline{z} - \underline{w})n(\underline{z}) - r_\ell = \pi_\ell(\underline{z}_\ell)$ must hold. However, this requires $r_{\ell'} = r_\ell$, a contradiction. □

Lemma B.7 (Local prices)**.**
Fix $\boldsymbol{\theta}$ and re-arrange cities so that θ_ℓ is decreasing in ℓ. Then, the spatial job allocation is sustained by a vector of local prices that satisfies the difference equation

$$\kappa_{\ell+1} - \kappa_\ell = \underline{z}_{\ell+1} n_{\ell+1}(\underline{z}_{\ell+1}) - \underline{z}_\ell n_\ell(\underline{z}_\ell) - \int_{\underline{z}_\ell}^{\underline{z}_{\ell+1}} n_\ell(\zeta)\mathrm{d}\zeta, \tag{35}$$

for κ_1 given. Furthermore, for two cities ℓ and ℓ', $\theta_{\ell'} < \theta_\ell \iff \kappa_\ell > \kappa_{\ell'}$.

Proof. For any city ℓ, take the city ℓ' such that $\theta_{\ell'} > \theta_\ell$ and there is no other third city l so that $\theta_{\ell'} > \theta_l > \theta_\ell$. If $z \in \text{supp } \Gamma_\ell$ and $z \in \text{supp } \Gamma_{\ell'}$, then it must be that $\pi_\ell(z) = \pi_{\ell'}(z)$ by profit maximization (19). By definition, $\underline{z}_\ell \in \text{supp } \Gamma_\ell$. Hence, if $\underline{z}_\ell \in \text{supp } \Gamma_{\ell'}$, then $\pi_\ell(\underline{z}_\ell) = \pi_{\ell'}(\underline{z}_\ell)$. For the sake of contradiction, suppose $\underline{z}_\ell \notin \text{supp } \Gamma_{\ell'}$; that is, $\bar{z}_{\ell'} < \underline{z}_\ell$ from Lemma B.2. Feasibility (8) then requires that there exists at least one city l for which $\pi_{\ell'}(z) < \pi_l(z)$ for $z \in B^+(\bar{z}_{\ell'})$, and thus $1 = \Gamma_{\ell'}(z) > \Gamma_l(z) > \Gamma_\ell(z) = 0$. However, from Lemma B.3, this is possible in equilibrium only if city l satisfies $\theta_{\ell'} > \theta_l > \theta_\ell$, a contradiction. We therefore conclude that $\pi_\ell(\underline{z}_\ell) = \pi_{\ell'}(\underline{z}_\ell)$.

To prove the second statement, I start by showing that $\theta_\ell < \theta_{\ell'}$ implies $\kappa_\ell > \kappa_{\ell'}$. For that, take two cities ℓ and ℓ' that are adjacent in the $\boldsymbol{\theta}$-space with $\theta_\ell < \theta_{\ell'}$. Since n_ℓ is a strictly increasing function, equation (35) implies

$$\begin{aligned}\kappa_\ell - \kappa_{\ell'} &\geq \underline{z}_\ell \left(n_\ell(\underline{z}_\ell) - n_{\ell'}(\underline{z}_\ell)\right) + \underline{z}_{\ell'} \left(n_{\ell'}(\underline{z}_\ell) - n_{\ell'}(\underline{z}_{\ell'})\right) \\ &= \underline{z}_\ell \left(n_\ell(\underline{z}_\ell) - n_{\ell'}(\underline{z}_\ell)\right) + \underline{z}_{\ell'} \left(n_{\ell'}(\underline{z}_\ell) - n_{\ell'}(\underline{z}_{\ell'})\right).\end{aligned}$$

The second term is positive since $\underline{z}_\ell > \underline{z}_{\ell'}$. Regarding the first term, Lemma B.2 requires that $\pi'_{\ell'}(z) = n_{\ell'}(z) \leq n_\ell(z) = \pi'_\ell(z)$ for $z \in B^+(\underline{z}_\ell)$, which must also hold at $\underline{z}_\ell$. Hence $\kappa_\ell > \kappa_{\ell'}$. To prove the other direction, suppose that $\kappa_\ell > \kappa_{\ell'}$. In equilibrium, it must either be that $\Gamma_\ell \succ \Gamma_{\ell'}$ or $\Gamma_\ell \prec \Gamma_{\ell'}$. For the sake of contradiction, suppose the latter. From Lemma B.3, it must then be that $\theta_\ell > \theta_{\ell'}$. Using the same derivation as above, we then conclude that $\kappa_\ell < \kappa_\ell$, a contradiction. Hence, $\Gamma_\ell \succ \Gamma_{\ell'}$ and $\theta_\ell < \theta_{\ell'}$. □

Lemma B.8 (City ordering)**.**
Suppose that $\lambda^e \approx \lambda^u$ and $c \gg b$. Then, in equilibrium, $e_\ell > e_{\ell'} \iff v_\ell > v_{\ell'} \iff \theta_\ell < \theta_{\ell'}$.
Proof. From Lemma B.7, $\theta_\ell < \theta_{\ell'}$ if and only if $\kappa_\ell > \kappa_{\ell'}$. Using $\kappa_\ell = \underline{w}_\ell n_\ell(\underline{z}_\ell) + r_\ell$, it follows that

$\theta_\ell < \theta_{\ell'}$ if and only if

$$\frac{b}{1+k}\left(\frac{1}{\theta_\ell}-\frac{1}{\theta_{\ell'}}\right)+c\left(v_\ell^\chi - v_{\ell'}^\chi\right)>0,$$

where $\lambda^e \approx \lambda^u$ implies $\underline{w}_\ell \approx b$ in both cities. If $c \gg b$, then $\kappa_\ell - \kappa_{\ell'} \approx c\left(v_\ell^\chi - v_{\ell'}^\chi\right) > 0 \iff v_\ell > v_{\ell'}$. Hence, $\theta_\ell < \theta_{\ell'}$ if and only if $v_\ell > v_{\ell'}$. Finally, for the sake of contradiction, suppose that $\theta_\ell < \theta_{\ell'}$ but $e_\ell < e_{\ell'}$. However, $\theta_\ell < \theta_{\ell'}$ holds only if $v_\ell > v_{\ell'}$. Together, $\theta_\ell = v_\ell/e_\ell > v_{\ell'}/e_{\ell'} = \theta_{\ell'}$, a contradiction. □

C Extended model and estimation

C.1 Quantitative model

Let κ denote the scale of the model. The model is scale invariant but this will be needed to map the model in the data since the empirical scale is unknown.

Workers We start with the worker problem. Their expected lifetime utility upon entry in the labor force is

$$U(\{\omega_\ell\}_{\ell=1}^L) = \mathbb{E}\left[\int \mathrm{e}^{-\beta}\omega_{\ell_t} A_{\ell_t} u(c_t, h_t)dt\right].$$

Workers are free to migrate as long as they are unemployed. Employed workers need to quit their jobs to migrate. Together with the facts that the economy is in steady state and that idiosyncratic preferences are time invariant, unemployed workers settle once in a location and then never choose to migrate again.

The problem of workers can therefore be broken down in two stages. In the later stage, workers are settled in location ℓ. They decide which jobs to accept and climb the local job ladder. The HJB equations describing the discounted lifetime utility of an unemployed and employed worker in location ℓ are

$$\beta U_\ell = \kappa b + \lambda_\ell^u \int \max\{V_\ell(w) - U_\ell, 0\} dF_\ell(w),$$
$$\beta V_\ell(w) = w + \lambda_\ell^e \int \max\{V_\ell(w) - U_\ell, 0\} dF_\ell(w) + \delta[U_\ell - V_\ell(w)].$$

These HJB equations are independent of workers' taste shocks, and therefore, need to be equalized across space. The job switching behaviors are identical to Section 3 and give rise to the same labor supply curves.

In the earlier stage, unemployed workers decide where to settle given the expected lifetime utility

of unemployed workers in each location. This location choice solves:

$$U = \max_{\ell} \frac{\omega_\ell A_\ell U_\ell}{P_\ell}.$$

Given the Frechet shocks, the fraction of unemployed workers who decide to locate in city ℓ is

$$\frac{u_\ell}{\sum_{\ell'} u_{\ell'}} = \frac{[A_\ell U_\ell / P_\ell]^\chi}{\sum_{\ell'} [A_{\ell'} U_{\ell'} / P_{\ell'}]^\chi}, \tag{36}$$

where $\bar{u} \equiv \sum_\ell u_\ell$. The flows in and out of unemployment imply

$$e_\ell = k_\ell^u u_\ell \quad \text{and} \quad m_\ell = u_\ell + e_\ell = u_\ell \left(1 + k_\ell^u\right). \tag{37}$$

Employers Employers solve (22) where $c(v) = \kappa v^{1+\gamma}/(1+\gamma)$. The vacancy optimality condition demands

$$(zT_\ell - w_\ell(z))\, n_\ell[w_\ell(z)] = \kappa v_\ell(z)^\gamma. \tag{38}$$

Potential profits in location ℓ after maximizing out vacancies are

$$\pi_\ell(zT_\ell) = \max_{w} \; c\kappa^{-\frac{1}{\gamma}} \left[(zT_\ell - w)\, n_\ell(w)\right]^{\frac{1}{c}} - r_\ell,$$

for $c \equiv \gamma/(1+\gamma)$ a constant. Given the spatial allocation of employers $\{\Gamma_\ell\}_\ell$, the solution to the employer's problem can be recasted as a system of two differential equations. For employers with $z \in \text{supp}\, \Gamma_\ell$, the wage optimality condition is

$$\frac{n_\ell'[w_\ell(z)]}{n_\ell[w_\ell(z)]}(zT_\ell - w_\ell(z)) = \frac{2k_\ell \mathrm{d}F_\ell[w_\ell(z)]}{1 + k_\ell \bar{F}_\ell[w_\ell(z)]}(zT_\ell - w_\ell(z)) = 1.$$

Within a city, wages are increasing in productivity. Accordingly, letting $\Upsilon_\ell(z) \equiv F_\ell[w_\ell(z)]$ denote the rank of an employer in the local wage offer distribution, we obtain

$$\Upsilon_\ell(z) = \frac{M_\ell}{V_\ell} \int_{\underline{z}_\ell}^{z} v_\ell(x) \mathrm{d}\Gamma_\ell(x).$$

The number of workers hired by employer z in ℓ is then

$$n_\ell(z) = n_\ell[w_\ell(z), v_\ell(z)] = \frac{(1+k_\ell)e_\ell}{[1 + k_\ell(1 - \Upsilon_\ell(z))]^2} \frac{v_\ell(z)}{V_\ell}. \tag{39}$$

With endogenous vacancies, $\Upsilon_\ell(z) \neq \Gamma_\ell(z)$. Using $\Upsilon_\ell(z)' = dF_\ell[w_\ell(z)] w_\ell'(z)$, the wage optimality rewrites

$$w_\ell'(z) = \frac{2k_\ell \mathrm{d}\Upsilon_\ell(z)}{1 + k_\ell \bar{\Upsilon}_\ell(z)}[zT_\ell - w_\ell(z)]. \tag{40}$$

Given Υ_ℓ and the boundary condition $w_\ell(\underline{z}_\ell) = \underline{w}_\ell$, (40) is an ODE that yields $w_\ell(z)$. Meanwhile, the expression for $\Upsilon_\ell(z)$ implies $d\Upsilon_\ell(z) = M_\ell v_\ell(z) d\Gamma_\ell(z)/V_\ell$. Used in the vacancy optimality condition, we obtain a second differential equation:

$$\Upsilon'_\ell(z) = \frac{M_\ell}{V_\ell}\left[\left(\frac{zT_\ell - w_\ell(z)}{\kappa}\right) n_\ell(z)\right]^{\frac{1}{\gamma}} \Gamma'_\ell(z). \tag{41}$$

Given the condition $\Upsilon_\ell(\underline{z}_\ell) = 0$ and $w_\ell(z)$, (41) yields $\Upsilon_\ell(z)$.

The local productivity distributions are given by the local profit opportunities, π_ℓ, together with the employers' idiosyncratic costs. The inverse dispersion in the entry cost is ϑ/κ. Given their Gumbel distribution, the probability that a firm with productivity z produces in city ℓ is

$$\Omega_\ell(z) = \frac{\mathrm{e}^{\vartheta \pi_\ell(zT_\ell)/\kappa}}{\sum_{\ell'} g_{\ell'} \mathrm{e}^{\vartheta \pi_{\ell'}(zT_{\ell'})/\kappa}}. \tag{42}$$

The local productivity distribution in city ℓ is then

$$\Gamma_\ell(z) = \frac{M}{M_\ell}\int^z \Omega_\ell(x)\mathrm{d}\Gamma(x) \quad \Longleftrightarrow \quad \mathrm{d}\Gamma_\ell(z) = \frac{M}{M_\ell}\Omega_\ell(z)\mathrm{d}\Gamma(z), \tag{43}$$

and the mass of firm in city ℓ is

$$M_\ell = M\int \Omega_\ell(x)\mathrm{d}\Gamma(x). \tag{44}$$

The idiosyncratic entry costs ensure full support of the local productivity distribution, so that the boundary condition to (42) is $\Gamma_\ell(\underline{z}) = 0$.

Frictions The contact rates are given by the local matching function:

$$\lambda^u_\ell = \mu\left(\frac{V_\ell}{u_\ell + \zeta e_\ell}\right)^\xi. \tag{45}$$

Aggregate prices There are three sets of aggregate quantities. The reservation wages $\{\underline{w}_\ell\}_{\ell=1}^L$ are given by (14). Commercial housing prices clear the commercial housing market:

$$\frac{\alpha}{p_\ell}\left(u_\ell b\kappa + e_\ell \mathbb{E}_\ell[w]\right) = \bar{L}p_\ell^\theta. \tag{46}$$

The residential housing prices clear the residential housing market:

$$M_\ell = \bar{H}(r_\ell/\kappa)^\phi, \tag{47}$$

where the housing supply has been scaled by κ, $H_\ell = \bar{H}(r_\ell/\kappa)^\phi$.

Definition C.1 (Equilibrium)**.**

An equilibrium is a collection of size policy functions, $\boldsymbol{n}(z)$, wage policy functions, $\boldsymbol{w}(z)$, vacancy policy functions, $\boldsymbol{v}(z)$, employer-rank distributions, $\boldsymbol{\Upsilon}(z)$, productivity distribution, $\boldsymbol{\Gamma}(z)$, residential and commercial housing prices, $\boldsymbol{p}$ and $\boldsymbol{r}$, reservation wages, $\boldsymbol{w}$, contact rates, $\boldsymbol{\lambda^u}$ and $\boldsymbol{\lambda^e}$, vacancy and employer measures, $\boldsymbol{V}$ and $\boldsymbol{M}$, spatial distribution of workers, $\boldsymbol{u}$ and $\boldsymbol{e}$, so that:

1. *The wage, employer-rank and productivity distributions satisfy the differential equations* (40), (41) *and* (43)*, subject to the appropriate boundary conditions and the spatial allocation of employers $\boldsymbol{\Omega}(z)$ given by* (42)*;*
2. *The vacancy and size policy functions satisfy* (38) *and* (39)*;*
3. *The mass of vacancy and employers are given by* (21) *and* (44)*;*
4. *The mass of unemployed and employed workers are given by* (36) *and* (37)*;*
5. *The contact rates are given by* (45)*;*
6. *The reservation wages are given by* (14)*;*
7. *The residential housing prices follows* (46) *and* (47)*.*

C.2 Proof of Proposition 5

Frictions The average UE rate, average EU rate, and location-specific average EE rate identifies the search frictions. First, the EU rate equates δ. Second, the location-specific average EE rates are

$$\mathrm{EE}_\ell = \lambda_\ell^e \int \bar{F}_\ell(w) dG_\ell(w) = \delta \left[\left(1 + \frac{1}{k_\ell} \right) \log\left(1 + k_\ell\right) - 1 \right].$$

The right-hand side is monotonically increasing in k_ℓ. Therefore, $k_\ell = h(\widehat{\mathrm{EE}}_\ell)$, for h the inverse of the above mapping. But $k_\ell = \zeta \lambda_\ell^u / \delta$, and $\zeta \lambda_\ell^u = \delta h(\widehat{\mathrm{EE}}_\ell)$. Third and last, the location-specific UE rate is λ_ℓ^u. Accordingly, the average UE rate is

$$\mathrm{UE} = \frac{\sum_\ell u_\ell \lambda_\ell^u}{\sum_\ell u_\ell} = \frac{1}{\zeta} \frac{1}{\sum_\ell \left(\frac{1}{\zeta \lambda_\ell^u} \right) \frac{e_\ell}{E}},$$

which follows from $u_\ell = \delta / (\delta + \lambda_\ell^u) m_\ell$ and $m_\ell = u_\ell + e_\ell = (\lambda_\ell^u + \delta) e_\ell / \delta$. In the denominator of the second term, e_ℓ / E refers to the employment share of location ℓ with $E = \sum_\ell e_\ell$. The denominator is therefore the employment-weighted mean of $1/\zeta\lambda_\ell^u$, which we know. The above expression thus identifies ζ. Given ζ, we separately identify $\{\lambda_\ell^u\}_\ell$.

Worker allocation The total mass of workers in ℓ reads $m_\ell = (\lambda_\ell^u + \delta) e_\ell / \lambda_\ell^u$. Consistency imposes $\sum_\ell m_\ell = 1$. Together:

$$\frac{1}{E} = \sum_\ell \left(\frac{\lambda_\ell^u + \delta}{\lambda_\ell^u} \right) \frac{e_\ell}{E}.$$

This expression identifies the aggregate mass of employed workers. We can then recover the location-specific measure of employed and unemployed workers via

$$e_\ell = \left(\frac{e_\ell}{E}\right) E \quad ; \quad u_\ell = \left(\frac{\delta}{\lambda_\ell^u}\right) e_\ell \quad ; \quad m_\ell = u_\ell + e_\ell.$$

We treat the spatial allocation of workers as given for now, and later show that any allocation can be rationalized by local amenities.

Unemployment insurance The UI rate is identified from the aggregate replacement rate. The location-specific replacement rate is $\text{RE}_\ell = \kappa b / \mathbb{E}[w_\ell]$. The aggregate replacement rate is therefore

$$\text{RE} = \frac{\sum_\ell \left(\frac{\kappa b}{\mathbb{E}[w_\ell]}\right) e_\ell}{\sum_\ell e_\ell}.$$

Inverting this expression identifies κb.

Employers Let j denote an employer in the sample. For each employer, we observe the wage it offers, w_j, its size, N_j, and its location, ℓ_j. From employers' location, we can compute the measure of firms in every location. Consistency indeed requires that the number of workers hired in each location equates the number of employed workers, $M_\ell \mathbb{E}_\ell[n(z)] = e_\ell$ holds. The average size of employers relative to the measure of employed workers thus identifies the number of employers: $m_\ell = e_\ell / \mathbb{E}_\ell[N(z)]$. The total measure of employers follows $M = \sum_\ell g_\ell m_\ell$.

Then, we can recover the productivity and vacancy of each employer from its size and wage:

1. Compute employers' rank in the local wage distribution, G_j.
2. Invert (12) to back out employers' rank in the wage offer distribution;

$$F_j = \frac{(1 + k_\ell) G_j}{1 + k_\ell G_j}.$$

3. Compute the number of workers per vacancy from employers' rank:

$$\eta_j = \frac{(1 + k_\ell) e_\ell}{[1 + k_\ell (1 - F_j)]^2}.$$

4. Compute employers' vacancy share from their size: $v_j / V_{\ell_j} = n_j / \eta_j$.
5. Invert the wage optimality condition to identifies employers' productivity:

$$z_j T_{\ell_j} \equiv \zeta_j = w_j + \left(\frac{1 + k_\ell (1 - F_j)}{2 k_\ell}\right) \left.\frac{\partial w}{\partial F}\right|_{w = w_j}.$$

In practice, I approximate $w \to F_\ell$ with a spline and use the approximation to compute $\partial w / \partial F$.

Given data on vacancy shares, the vacancy cost elasticity is identified the vacancy optimality

condition. Equation (38) indeed implies

$$\gamma = \sqrt{\frac{\sum_\ell e_\ell \mathbb{V}\text{ar}_\ell[\log((\zeta_j - w_j)\eta_j)]}{\sum_\ell e_\ell \mathbb{V}\text{ar}_\ell[\log \frac{v_j}{V_\ell}]}}. \tag{48}$$

Given an estimate for γ, the measure of vacancy in each location is then recovered as a location-specific residual from the optimality condition:

$$(1+\gamma)\log V_\ell + \log\kappa = \mathbb{E}_\ell[\log(\zeta_j - w_j)\eta_j] - \gamma\mathbb{E}_\ell\left[\log\frac{v_j}{V_\ell}\right].$$

From (42), the log probability that an employer with total productivity ζ produces in ℓ is

$$\log\Omega_\ell^\zeta(\zeta) = \vartheta\pi_\ell(\zeta) - \log\sum_{\ell'} \mathrm{e}^{\vartheta\pi_{\ell'}(\zeta T_{\ell'}/T_\ell)} \approx \vartheta\pi_\ell(\zeta) + H(\zeta), \tag{49}$$

where $H(\zeta)$ is some location-independent function and the approximation is exact when there are no dispersion in TFP, $T_{\ell'} = T_\ell$ for all ℓ. According to (49), projecting employers' location probabilities onto the local profit opportunities while flexibly controlling for employers' total productivity identifies ϑ. The left-hand side is known from employers' productivity and their location choice. Given the parameters already identifies, realized profits can be computed

$$\pi_j = \pi_{\ell_j}(\zeta_j) = (\zeta_j - w_j)n_j - c(v_j) - r_{\ell_j}.$$

In practice, I instrument local profit opportunities with employers' wage to reduce measurement error in employers' total productivity. I take a discrete approximation of ζ and replace $H(\zeta)$ by employer-group fixed effect. To account for the measurement error introduced by this discretization, together with the fact that TFP differentials may not be small, I replicate the same estimation technique in the model and set ϑ to match the 2SLS coefficient obtained in the data.

Matching function The matching function (45) implies

$$\log\lambda_\ell^u = \log\mu + \psi\log\left(\frac{V_\ell}{u_\ell + \zeta e_\ell}\right) + u_\ell, \tag{50}$$

for ϵ_ℓ some measurement error. Estimating (50) by OLS given our estimates of $\{\lambda_\ell^u, V_\ell, u_\ell, e_\ell\}_{\ell=1}^L$ and ζ identifies the matching function parameters (μ, ψ).

Housing supply The residential housing market clearing condition reads $\bar{L}p_\ell^\theta = \alpha I_\ell/p_\ell$, or

$$\log p_\ell = \left(\frac{1}{1+\theta}\right)\log\left(\frac{\alpha}{\bar{L}}\right) + \left(\frac{1}{1+\theta}\right)\log I_\ell, \tag{51}$$

where $I_\ell = u_\ell \kappa b + e_\ell \mathbb{E}[w_\ell]$ are total expenditures in ℓ. Accordingly, given data on residential housing prices, estimating the above relationship by OLS identifies $\{\bar{L}, \theta\}$.

Turning to the commercial housing supply parameters, commercial housing market clearing demands $\bar{H}(r_\ell/\kappa)^\phi = m_\ell$, or

$$\log r_\ell = \log \kappa - \frac{1}{\phi} \log \bar{H} + \frac{1}{\phi} \log M_\ell. \tag{52}$$

Estimating (52) by OLS identifies ϕ and a bundle of $(\kappa, \bar{H})$.

Worker preferences We conclude with the identification of worker preferences, $(\alpha, \boldsymbol{A})$. The Cobb-Douglas parameter α equates the aggregate housing expenditure share.

Amenities are identified from the location choice of unemployed workers. Specifically, given the dispersion in preferences χ, the spatial allocation of unemployed workers $\boldsymbol{u}$, housing prices $\boldsymbol{P}$, and the lifetime utility of unemployed workers in each location $\boldsymbol{U}$, (36) can be inverted to obtain local amenities. The first two vetors are known. The latter can be computed given data on the wage offer distribution and the discount factor β. Indeed, following similar steps as in Section B.2, I obtain

$$\beta U_\ell = \kappa b + \lambda_\ell^u \int_{\underline{w}_\ell} \frac{1 - F_\ell(w)}{\beta + \delta + \lambda^e (1 - F_\ell(w))} dw.$$

This conclude the proof.

C.3 Data

To be written.

Figure C.6: City characteristics

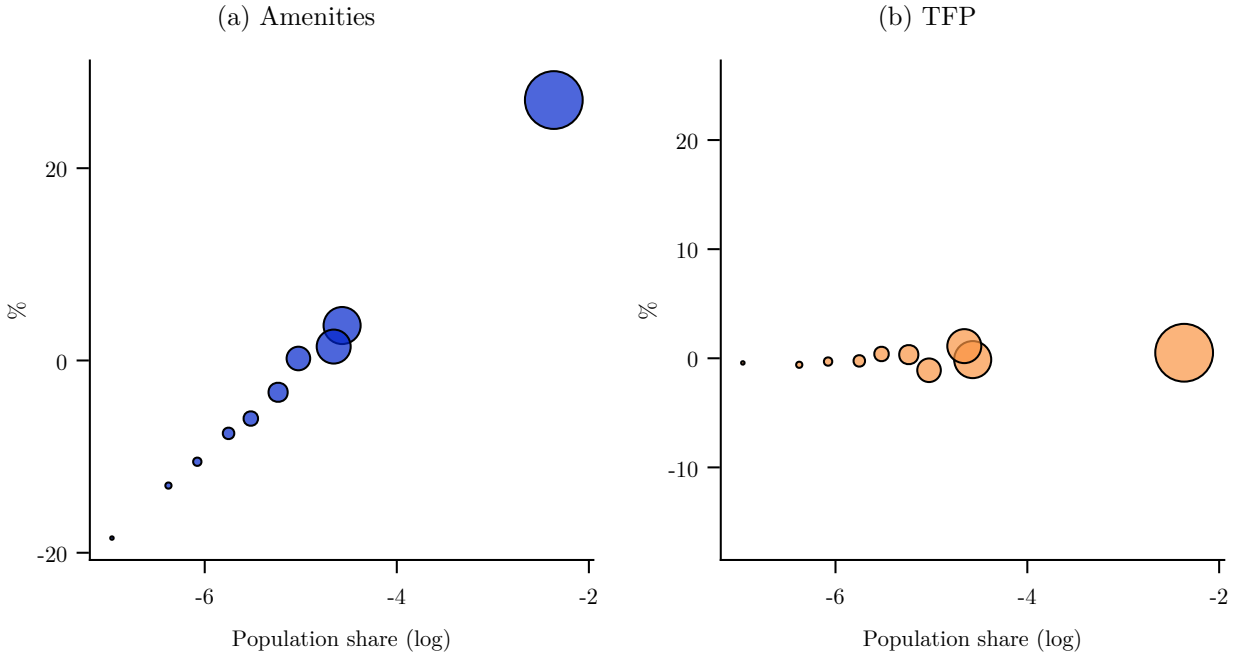


Figure C.7: Model fit

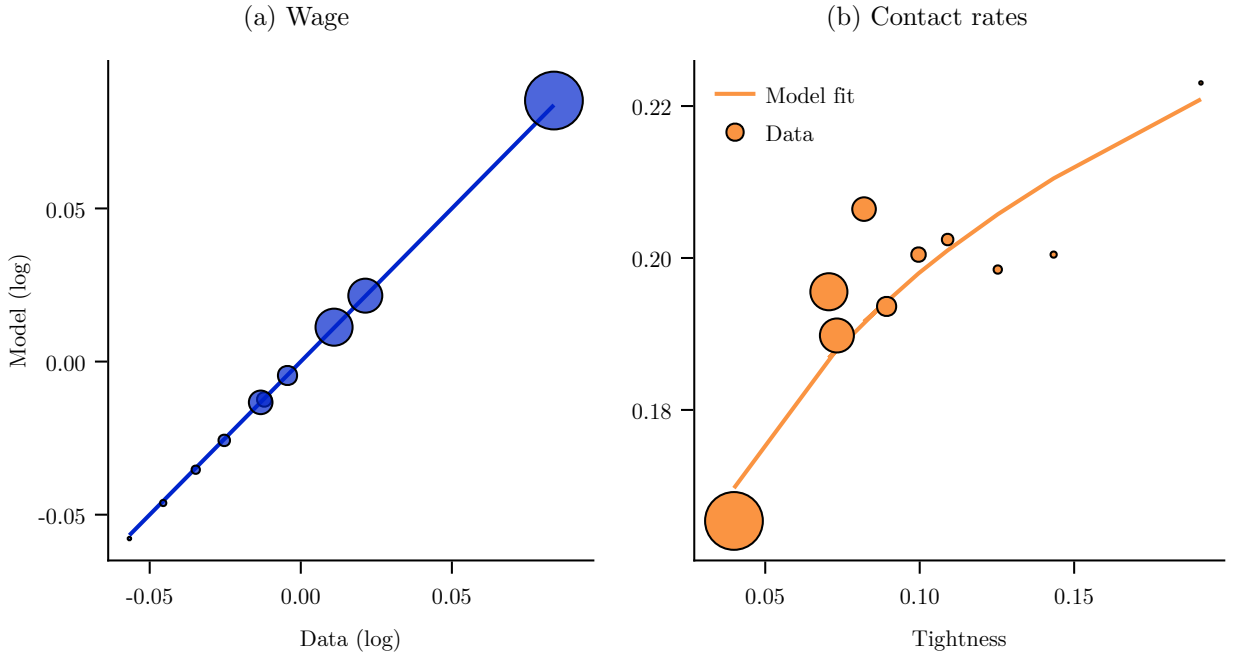
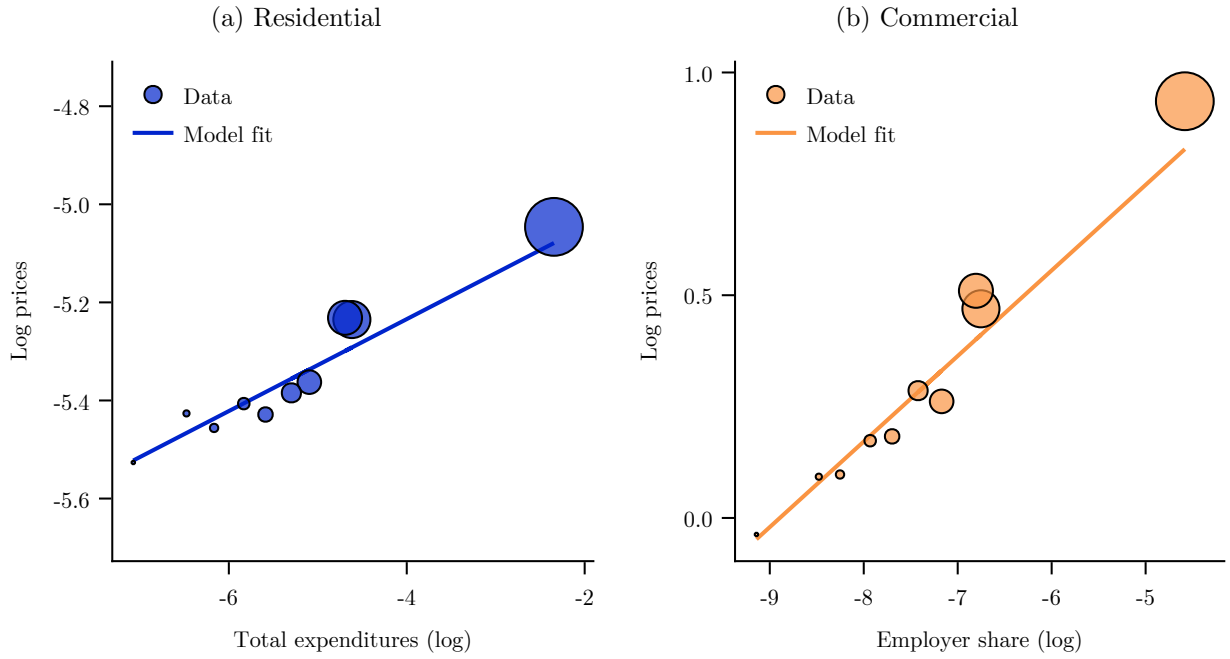


Figure C.8: Housing prices



D Employer sorting, search frictions, and local wage inequality

D.1 Welfare decomposition

To be written.

Figure D.9: Within-city wage inequality in the data and the model

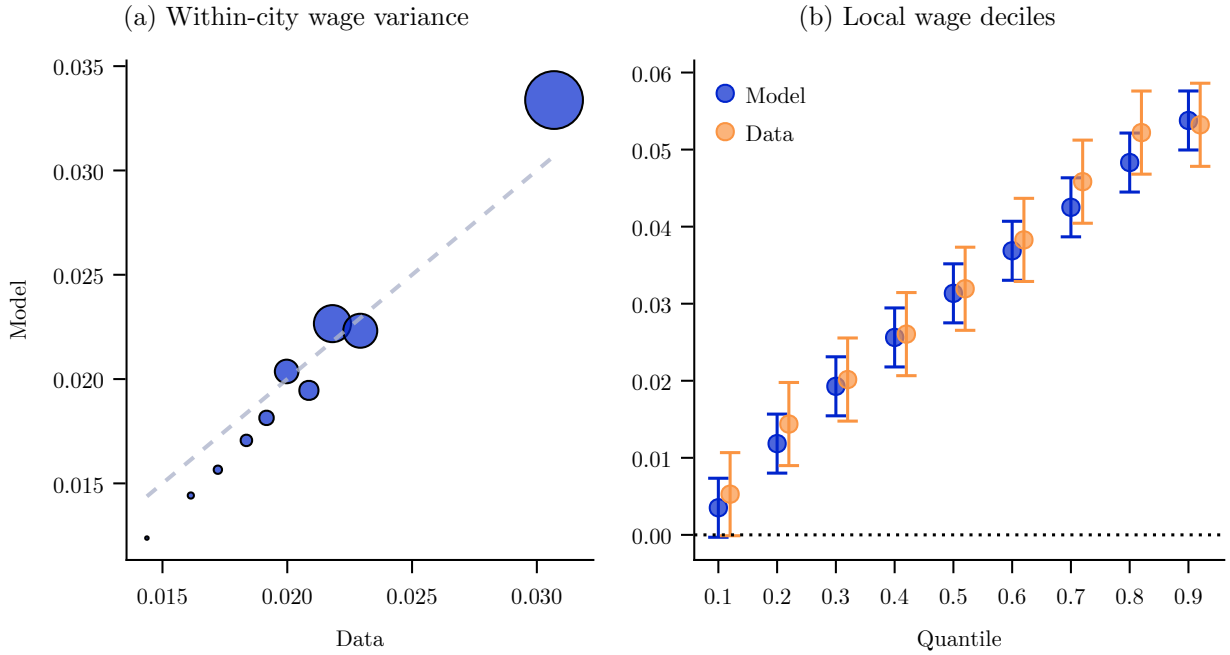


Figure D.10: Spatial allocation of employers

