The Distributional Effects of Trade across Local Labor Markets and Heterogeneous Consumers*

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Abstract

I study how global trade and technology shocks propagate across local labor markets and affect the welfare of consumers with heterogeneous consumption baskets along the income distribution. I develop a tractable general equilibrium Ricardian-Roy framework that allows for non-homothetic expenditure patterns. Methodologically, it reconciles heterogeneous workers and non-homothetic demand with the "exact-hat algebra" approach for counterfactual analysis, which imposes minimal data requirements. I also derive a generalized welfare gains formula for the distributional impact of trade shocks that applies to any demand system. Apart from rich distributional implications, the framework gives rise to a novel – the Consumption Inequality – channel of the aggregate welfare gains. I apply the model to revisit the impact of the China shock for a sample of 40 countries and 722 US commuting zones. I find that the China shock led to overall gains for most countries. The aggregate US gains were 0.71%, about 1/3 of which is attributed to the Consumption Inequality channel. The shock disproportionately benefited low-income US consumers and commuting zones by reducing the relative prices of low-income elastic sectors. The distribution of welfare gains across income quantiles varied significantly between countries.

Keywords: international trade, welfare gains and losses, income inequality, non-homothetic demand, heterogeneous consumers, local labor markets.

JEL Codes: F1, F6, D11, D33, J2.

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1 Introduction

It is well known that global trade and technology shocks may generate winners and losers at the country level. While the aggregate welfare effects are often found to be modest, there is growing empirical evidence of sizable distributional impacts of trade and trade policies. Generally, once a country is exposed to a global shock, it may generate winners and losers along several dimensions. For instance, consider the impact of an arbitrary shock that has heterogeneous sectoral exposure. Based on differences in sectoral employment patterns, while some local labor markets may gain from trade openness and changes in a country's specialization, others might be adversely affected by the shock. At the same time, it may change relative prices across various sectors by affecting the wage distribution among workers. This implies winners and losers among heterogeneous consumers – the expenditure channel of welfare change. So, the resulting welfare impact of any shock for a particular group of individuals tends to be jointly determined by the interaction of earnings and expenditure channels in general equilibrium.

To provide a specific example, consider a low-income individual who lives in a region that specializes in import-competing manufacturing industries (e.g., the US Rust Belt). On the one hand, the rising Chinese import competition is likely to reduce the individual's nominal wage. On the other hand, low-income individuals may disproportionally benefit from the expenditure channel if the shock reduces the relative prices of low-income elastic goods/sectors. Depending on which of the two effects is dominating, it may result in increasing or lowering the country's welfare inequality in response to the shock.

In this paper, I aim to study how global trade and technology shocks propagate across local labor markets and affect the welfare of local consumers with heterogeneous consumption baskets along the income distribution. Such a goal essentially gives rise to several underlying questions: *i*) How do non-homothetic consumption patterns affect the equilibrium outcomes in a general equilibrium? *ii*) What are the distributional consequences across local consumers with heterogeneous incomes and the implications for the aggregate welfare gains? *iii*) What is the relative importance of the non-homothetic effects and each channel of welfare change?

To study these questions theoretically and quantitatively, I propose a tractable general equilibrium Ricardian-Roy framework that allows for non-homothetic expenditure patterns. In particular, I introduce the Almost Ideal Demand System (Deaton and Muellbauer 1980) – an arbitrary first-order approximation to any demand system – into a multi-sectoral extension of the Eaton and Kortum (2002) model that is enriched with Roy (1951)–type allocation of workers across sectors and many local markets as in Galle, Rodríguez-Clare, and Yi (2023). The model's flexible demand side implies that the aggregate expenditure shares for each sector may depend on other sectors' price indices and income distribution across individual con-

sumers. In particular, the Almost Ideal Demand System implies that the impact of income distribution on the aggregate expenditure shares can be summarized by sufficient statistics – mean income adjusted by the Theil index of income inequality. As a result, expenditure shares may change in response to shocks through both homothetic (through changes in sectoral price indices) forces and non-homothetic (through changes in mean income and Theil index of income inequality) forces. Apart from affecting the equilibrium outcomes of aggregate variables, non-homothetic effects also generate distributional welfare consequences of any shock along the income distribution. In this context, the Theil index governs the inequality margin of adjustment to trade and technology shocks across local labor markets and heterogeneous consumers.

Deviation from the standard Cobb-Douglas case of constant expenditure shares gives rise to two main methodological challenges. First, to deliver theory-consistent predictions of the impacts of any shock, it is imperative to reconcile heterogeneous consumers and non-homothetic demand with the counterfactual analysis based on relative changes – the *exact-hat algebra* approach (Dekle, Eaton, and Kortum 2008). By imposing mild data requirements, the exact-hat algebra approach is a cornerstone of the quantitative analysis in the corresponding literature. The second challenge is that the well-known (exact) welfare change formula by Arkolakis, Costinot, and Rodríguez-Clare (2012) inevitably ceases to hold in such a setup and requires a more complex welfare analysis of trade shocks. I address these methodological issues in Sections 2 and 3, respectively.

In particular, I combine the convenient market share aggregation properties of the Almost Ideal Demand System, which is based on sufficient statistics, with the results from the supply side of the model, which endogenously generates the Fréchet income distribution. The supply side of the model gives rise to a closed-form expression for the Theil index of income inequality at each local labor market. I also show that solving for changes in the aggregate expenditure shares for each country boils down to solving a system of linear equations for each country. It nests counterfactual changes in the aggregate consumption shares obtained under the homothetic Translog expenditure function and constant (Cobb-Douglas) sectoral expenditure shares as special cases. Exploiting the supply-side properties of the model, I also derive analytical expressions for counterfactual values of the country-level Theil indices. As a result, the model remains highly tractable, and the data requirements to solve for any counterfactual equilibrium based on the "exact-hat algebra" approach are similar to the standard case of constant expenditure shares across sectors. Notably, the spatial dimension represented by heterogeneous local labor markets differing in their patterns of comparative advantage gives rise to the "Average income" and "Inequality" margins of adjustment to trade shocks.

As for the second methodological challenge, given a non-homothetic demand side, the welfare analysis based on the standard tools of the exact price index cannot be applied in

general. So, I use the *Equivalent Variation* as a measure, to a first-order approximation, of welfare changes associated with changes in prices and income. I derive a parsimonious welfare change formula for the distributional impact of any foreign/non-local shock for consumers with a given income level that applies to any demand system across sectors. It generalizes the well-known ACR (Arkolakis, Costinot, and Rodríguez-Clare 2012) formula for the Ricardian model and the one derived by Galle, Rodríguez-Clare, and Yi (2023) that are based the Cobb-Douglas/CES frameworks. In particular, it shows that an income group's local welfare change in response to any trade shock can be decomposed into a country-level shift in trade openness, the local market's change in the degree of specialization, and the covariance of these terms with expenditure patterns of an income group. It also complements the formula for infinitesimal trade shocks derived by Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2019) and encompasses their flexible demand system.[1]

In addition to the traditional Ricardian and Roy channels of welfare gains, the framework also gives rise to *the Consumption Inequality* channel of the aggregate welfare effects of trade and technology shocks.[2] It suggests the importance of taking into account non-homothetic expenditure patterns for studying both the distributional and aggregate impact of shocks. Moreover, I analytically decompose the Consumption Inequality channel at the local labor market level into three different margins: *i*) the "*Pro-poor*" margin, *ii*) the "*Average income*" margin, and *iii*) the "*Inequality*" margin. I show that the "Pro-poor" margin amplifies the aggregate local welfare gains when the nature of a shock is "pro-poor" and, conversely, dampens the aggregate local welfare gains if the nature of a shock is "pro-rich". The "Average income" margin implies that if the nature of a shock is "pro-poor", local markets with relatively lower average income than the country-level one gain from this margin and lose otherwise. Similarly, the "Inequality" margin implies that local markets with a relatively lower Theil index of income inequality gain if the nature of a shock is "pro-poor". These results are driven by the demand side property that the aggregate expenditure shares across sectors depend on average income adjusted by (the exponent of) the Theil index of income inequality. So, local labor markets with relatively lower average income and lower income inequality tend to spend disproportionally more on low-income elastic than their relatively more affluent counterparts. As a result, for example, a "pro-poor" shock, by reducing relative prices of low-income elastic sectors, tends to benefit low-income and low-inequality local labor markets. In Section

[1] The relation between my formula and the one derived by Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2019) (ACDR 2019) can be summarized as follows. First, while the ACDR (2019) formula is derived from a flexible (non-parametric) demand system within a sector, I allows for an arbitrary (including the one used in ACDR 2019) demand system across sectors. Second, while the ACDR (2019) formula allows for a monopolistically competitive framework with variable markups, it does not take into account heterogeneous consumers as well as the Roy-type comparative advantages and the corresponding channel of welfare gains.

[2] I also analytically decompose the welfare impact of any shock into homothetic vs. non-homothetic channels and discuss the difference between the Consumption Inequality channel and the non-homothetic channel.

4, I quantify the contribution of the Consumption Inequality channel as well as each of its margins.

From a research and policymaker's perspective, the framework possesses several advantages. First, it allows for studying the heterogeneous impact of trade policy on individuals jointly via the earnings and expenditure channels in general equilibrium. Second, despite the very flexible supply and demand side, the model remains analytically tractable. In particular, the Almost Ideal Demand System – combined with the convenient supply-side properties – provides closed-form expressions for the aggregate expenditure shares and a local market's welfare response to any foreign/non-local shock. It also allows for the analytically tractable decomposition of the Consumption Inequality channel that isolates its different margins. Third, as has been noted, given the quantitative nature of the model in line with recent literature built on Eaton and Kortum (2002), it allows for counterfactual analysis in relative changes (the "exact–hat algebra" approach) that imposes very mild data requirements for counterfactual predictions and welfare analysis. In particular, one does not need to identify factual levels of "fundamentals" like trade costs or technological parameters. So, it conveniently allows for many potential applications studied in the recent literature, such as the US-China "Tariff War", the impact of AI/Automation, Climate Change, etc.

I apply the model to revisit the impact of the China shock on a sample of 40 countries and the universe of the US local labor markets represented by 722 US Commuting Zones. I find that while the China shock led to aggregate gains for most of the countries, some countries experienced net losses. As for the United States, the aggregate welfare impact of the shock is estimated to be 0.71%. It suggests larger country-level gains than most of the previous findings in the quantitative trade literature (see, e.g., Galle, Rodríguez-Clare, and Yi 2023), which mainly relies on the simplifying assumption of homothetic expenditure shares. In particular, a notable part of this increase (0.26%) is attributed to the Consumption Inequality channel driven by non-homothetic expenditure patterns across sectors. For the US, I also quantify the contribution of each margin of the Consumption Inequality channel. I find that each of these margins accounts for a sizable part of the distributional impact of the China shock across local labor markets and that Commuting Zones with lower mean income were more likely to gain from the shock. At the individual level, the China shock favored relatively poor consumers by reducing the relative prices of low-income elastic sectors. I also find that the distribution of welfare gains across income quantiles varied significantly between countries.

Related Literature This paper is related to several strands of the literature. First of all, it complements Ricardian-Roy models and models with trade and sectoral reallocation (see, e.g., Adão 2016, Costinot and Vogel 2015, Caliendo, Dvorkin, and Parro 2019, Galle, Rodríguez-

Clare, and Yi 2023, Kim and Vogel 2020, etc.).[3] While these papers focus on the distributional impact of trade through the earnings channel, I also study the expenditure effects in general equilibrium that occur under non-homothetic demand. In particular, I derive the Consumption Inequality channel and analytically decompose it into several margins. I also quantify the contribution of each of these margins as well as traditional Ricardian and Roy channels to the total welfare change.

Another related strand of the literature is trade models with non-homothetic demand (see, e.g., Arkolakis, Costinot, Donaldson, and Rodriguez-Clare 2019, Artuc, Porto, and Rijkers 2019, Borusyak and Jaravel 2021, Faber 2014, Fajgelbaum, Grossman, and Helpman 2011, Fajgelbaum and Khandelwal 2016, Fan, Li, Xu, and Yeaple 2020, Feenstra and Romalis 2014, Fieler 2011, Flam and Helpman 1987, Nigai 2016, Simonovska 2015, etc.).[4] In contrast to these papers, I reconcile the non-homothetic demand side represented by a first-order approximation to any demand system with a multi-country and multi-sector Ricardian model enriched with Roy-like heterogeneous workers and heterogeneous local markets differing in their patterns of comparative advantage. Within this flexible framework, I also provide a counterfactual analysis of any foreign/non-local shock based on *the exact-hat algebra* approach that imposes only mild restrictions on the data availability and does not require identifying factual levels of the model's fundamentals like trade costs and technology parameters. Thus, the paper bridges the literature on the distributional effects of trade through the expenditure channel and the quantitative trade and spatial models built on Eaton and Kortum (2002).

The rest of the paper is organized as follows. In Section 2.4, I propose a theoretical framework and derive the main theoretical results for counterfactual analysis. In Section 3, I provide welfare analysis to study the aggregate and distributional impact of an arbitrary foreign/non-local shock. In Section 4, I apply the model to revisit the distributional impact of the China shock. Section 5 provides concluding remarks. The Appendix A contains routine derivations and proofs. Extensions of the baseline model are discussed in the Appendix B. In the (Online) Appendix C, I discuss some supplementary results of the paper.

2 Theory

In this theory part, I derive and discuss the main theoretical results of the paper. In particular, in Section 2.1, I describe the baseline environment and the supply side of the baseline

[3] It also complements the recent literature on trade and inequality and models with heterogeneous workers (see, e.g., Antràs, De Gortari, and Itskhoki 2017, Dix-Carneiro and Traiberman 2023, Helpman, Itskhoki, and Redding 2010, Sampson 2014, Yeaple 2005, etc.).

[4] By allowing for the non-homothetic demand side, it is also related to recent papers on structural change (see, e.g., Comin, Lashkari, and Mestieri 2021, Eckert and Peters 2022), spatial models with non-homotheticity (see, e.g., Couture, Gaubert, Handbury, and Hurst 2024), and the literature on inflation inequality (see, e.g., Argente and Lee 2021, Atkin, Faber, Fally, and Gonzalez-Navarro 2024, Cravino and Levchenko 2017, Jaravel 2021, etc.).

Ricardian-Roy model. Section 2.2 describes equilibrium conditions and comparative statics equations for an unspecified demand side. I introduce a non-homothetic demand in Section 2.3. Derivations of counterfactual changes in the aggregate shares of consumption are presented in Section 2.4. Section 2.5 derives counterfactual changes for the average income and income inequality.

2.1 Environment and the Supply side

While the demand side and its implications are the main novel parts of the framework, it is instructive to start with the supply side of the model that closely follows a standard Ricardian-Roy multi-sector quantitative trade model as in Galle, Rodríguez-Clare, and Yi 2023 that also relies on Eaton and Kortum (2002) and Roy (1951)-like models (see, e.g., Burstein, Hanson, Tian, and Vogel 2020, Burstein, Morales, and Vogel 2019, Hsieh, Hurst, Jones, and Klenow 2019, Lagakos and Waugh 2013, Lee 2024, etc.). I assume that there are *i*) N countries indexed by $n, i \in \mathcal{N} \equiv \{1, 2, ..., N\}$, *ii*) J sectors indexed by $j, k \in \mathcal{J} \equiv \{1, 2, ..., J\}$, and *iii*) G_n country-specific local markets indexed by $g, r \in \mathcal{G}_n \equiv \{1, 2, ..., G_n\}$.

Consumers In each country $n \in \mathcal{N}$, consumers derive utility from consuming J sectoral (aggregate) goods. Each sector $j \in \mathcal{J}$ resembles Eaton and Kortum (2002), and each consumer derives utility from consumption of a continuum of varieties with (sector-specific) constant elasticity of substitution $\sigma_j > 1$. While preferences within sectors can be aggregated to the standard CES consumption index, preferences across sectors are allowed to be non-homothetic.[5] As has been mentioned, deviation from the standard Cobb-Douglas case gives rise to two main methodological challenges. First, one has to reconcile heterogeneous consumers with non-homothetic demand to the exact-hat algebra approach (Dekle, Eaton, and Kortum 2008) for counterfactual analysis. Second, the well-known exact welfare change formula by Arkolakis, Costinot, and Rodríguez-Clare (2012) ceases to hold in such a setup, thereby requiring a more complex welfare analysis of trade shocks. I will address both of these methodological issues in Sections 2 and 3, respectively.

Trade is subject to iceberg costs, implying that for one unit of a variety from sector j to arrive in country n, country i must ship $\tau_{inj} \geq 1$ units. Technologies to produce a variety ω in each country n and sector j exhibit constant returns to scale with productivity φ_{nj} drawn from the Fréchet distribution:

$$F_{nj}(\varphi) \equiv \Pr\left\{\varphi_{nj}(\omega) \leq \varphi\right\} = \exp\left(-T_{nj}\varphi^{-\theta_j}\right),$$

where $\theta_j > \sigma_j - 1$ and $T_{nj} > 0$ are the distribution's shape and scale parameters, respectively.

[5] The demand side will be discussed in detail in Section 2.3.

Note that T_{nj} denotes the level of aggregate productivity of country n in the production of goods in sector j. The parameter θ_j governs the sector-specific dispersion of productivities across goods within each country, where a higher value of θ_j implies a lower heterogeneity.

Workers and Labor supply Each country has its own set of labor markets $\mathcal{G}_n = \{1, 2, ..., G_n\}$. In the model's baseline version, I assume that labor markets are local, implying that workers are mobile across sectors but cannot migrate across labor markets. Each worker from labor market $g \in \mathcal{G}_n$ independently draws a number of efficiency units z_{ngj} (or, equivalently, productivity level) in each sector j from the following Fréchet distribution:

$$F_{ngj}(z) \equiv \Pr\{z_{ngj} \leq z\} = \exp\left(-A_{ngj} z^{-\alpha_{ng}}\right),$$

where $\alpha_{ng} > 1$ *and* $A_{ngj} > 0$ *are the shape and scale parameters of the corresponding distribution.*

The scale parameter A_{ngj} governs the level of aggregate productivity of workers in sector j at local market $g \in \mathcal{G}_n$. Note also that α_{ng} governs the dispersion of local productivities for $g \in \mathcal{G}_n$, where a higher value of α_{ng} corresponds to a smaller dispersion. In contrast to the baseline multi-sectoral extensions of Eaton and Kortum (2002), which feature a common wage per efficiency unit within a country (see, e.g., Caliendo and Parro 2015 and Costinot, Donaldson, and Komunjer 2012), the model implies that wages are country-sector specific.[6] In what follows, $\mathbf{w} \equiv \{w_{nj}\}_{n\in\mathcal{N}, j\in\mathcal{J}}$, where w_{nj} denotes the *wage per efficiency unit* in sector j of country n. Each worker from local market $g \in \mathcal{G}_n$ draws a vector of sectoral productivities $\boldsymbol{z} = (z_1, z_2, .., z_J)$ and makes a choice regarding which sector to apply for employment. For given wages, let $\Omega_{nj} \equiv \{\boldsymbol{z} \text{ s.t. } w_{nj} z_j \geq w_{nk} z_k \ \forall k \in \mathcal{J}\}$ denote the set of sectoral productivities, under which a worker ends up applying to sector j. One can readily verify that the share of workers in labor market g that are employed in sector j is given by

$$\pi_{ngj} \equiv \int_{\Omega_{nj}} \mathrm{d}F_{ng}(\boldsymbol{z}) = \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_{k=1}^{J} A_{ngk} w_{nk}^{\alpha_{ng}}}. \tag{2.1}$$

Notably, the Fréchet distribution of worker productivity also implies that the share of income earned by workers at local market $g \in \mathcal{G}_n$ in sector j is also given by π_{ngj}.

In what follows, Y_{ng} and L_{ng} denote the labor market's $g \in \mathcal{G}_n$ total income and total amount of workers, respectively. So, given 2.1, one can readily verify that the labor market's $g \in \mathcal{G}_n$ the total supply of efficiency units of labor in sector j is given by

$$Z_{ngj} \equiv L_{ng} \int_{\Omega_{nj}} z_j \mathrm{d}F_{ng}(\boldsymbol{z}) = \Gamma\left(1 - 1/\alpha_{ng}\right) L_{ng} A_{ngj} \left(\frac{w_{nj}}{\Upsilon_{ng}}\right)^{\alpha_{ng}-1},$$

[6] As noted by Galle, Rodríguez-Clare, and Yi (2023), sector-specific wage per efficiency unit is one of the implications of a Roy (1951)-like supply-side featuring an upward-sloping sectoral labor supply curve.

where $\Upsilon_{ng} \equiv \left(\sum_{k=1}^{J} A_{ngk} w_{nk}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$ *and* $\Gamma(\cdot)$ *is the Gamma function.*

Similarly, one can readily infer that the labor market's $g \in \mathcal{G}_n$ total income is given by

$$Y_{ng} \equiv \sum_{j \in \mathcal{J}} w_{nj} Z_{ngj} = \Gamma\left(1 - 1/\alpha_{ng}\right) L_{ng} \Upsilon_{ng}.$$

2.2 General equilibrium and Comparative statics

In this subsection, I derive the equilibrium and comparative statics equations. For expositional purposes, I leave the demand side unspecified and postpone the corresponding discussion until Section 2.3.

Note that the baseline version of the model imposes the assumption that there are no trade costs across local labor markets within a country. Hence, technologies and prices of goods are set at the country level. It also implies that while wages can differ across sectors, they are equal across local labor markets for a particular sector j.

One can readily infer that the excess labor demand for efficiency units of labor in sector j of country i is given by

$$\mathcal{D}_{ij} \equiv \underbrace{\frac{1}{w_{ij}} \sum_{n \in \mathcal{N}} \lambda_{inj} S_{nj} X_n}_{Demand} - \underbrace{\sum_{g \in \mathcal{G}_i} Z_{igj}}_{Supply}, \tag{2.2}$$

where:

i) $\lambda_{inj} = \frac{T_{ij}(\tau_{inj} w_{ij})^{-\theta_j}}{\sum_{n'} T_{n'j}\left(\tau_{n'nj} w_{n'j}\right)^{-\theta_j}}$ *is the share of country* i *in spending of country* n *in sector* j*;*

ii) $X_n = Y_n + D_n$ *is the total expenditures of country* n *with* $Y_n \equiv \sum_{g \in \mathcal{G}_n} Y_{ng}$ *and* D_n *denoting its total income and trade imbalance (via transfers), respectively;*

iii) S_{nj} *is the aggregate share of sector* j *in expenditures of country* n*.*[7]

Note that $\frac{1}{w_{ij}} \sum_{n \in \mathcal{N}} \lambda_{inj} S_{nj} X_n$ represents the demand for efficiency units of labor of sector j labor in country i. Similarly, $\sum_{g \in \mathcal{G}_i} Z_{igj}$ represents the supply of efficiency units of labor in sector j in country i. Note also that each $\mathcal{D}_{ij}$ from (2.2) is the function of the NxJ wage matrix $\mathbf{w} \equiv \{w_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{J}}$. So, there is a system of NxJ equations: $\{\mathcal{D}_{ij}(\mathbf{w}) = 0\}_{i \in \mathcal{N}, j \in \mathcal{J}}$.

2.2.1 Comparative statics

A counterfactual equilibrium analysis is of primary interest. For this purpose, one can solve the model in *relative changes* (the *exact-hat algebra* approach by Dekle, Eaton, and Kortum 2008). In what follows, $\hat{x} \equiv x'/x$ for any x, where x' is the counterfactual value and x is the current value. The salient advantage of this approach is that one does not need to identify current

[7] As discussed later, the aggregate expenditure shares $\{S_{nj}\}$ are allowed to be non-constant and dependent on sectoral prices and the whole distribution of income across individual consumers.

levels of the model's "fundamentals" like trade costs $\{\tau_{inj}\}$ and technology parameters $\{T_{nj}\}$, $\{A_{ngj}\}$.

One can readily verify that for each equilibrium condition, $\mathcal{D}_{ij}(\mathbf{w}) = 0$, its counterfactual counterpart, $\mathcal{D}'_{ij}(\mathbf{w}') = 0$, can be re-written in "relative changes" as follows:

$$\mathcal{D}'_{ij}(\hat{\mathbf{w}}) = 0 \Longleftrightarrow \sum_{n\in\mathcal{N}} \hat{\lambda}_{inj}\lambda_{inj}\hat{S}_{nj}S_{nj}\left(\sum_{g\in\mathcal{G}_n}\hat{Y}_{ng}Y_{ng} + \hat{D}_n D_n\right) = \sum_{g\in\mathcal{G}_i}\hat{\pi}_{igj}\pi_{igj}\hat{Y}_{ig}Y_{ig}, \tag{2.3}$$

where:

i) $\hat{\lambda}_{inj} \equiv \frac{\lambda'_{inj}}{\lambda_{inj}} = \frac{\hat{T}_{ij}(\hat{\tau}_{inj}\hat{w}_{ij})^{-\theta_j}}{\sum_{n'}\lambda_{n'nj}\hat{T}_{n'j}\left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$ *is the relative change in trade share of country* i *in import of country* n*;*

ii) $\hat{Y}_{ig} \equiv \frac{Y'_{ig}}{Y_{ig}} = \hat{y}_{ig} = \left(\sum_{j=1}^{J}\pi_{igj}\hat{A}_{igj}\hat{w}_{ij}^{\alpha_{ig}}\right)^{\frac{1}{\alpha_{ig}}}$ *is the relative change in total (and average) income of workers at local market* g *in country* i*;*

iii) $\hat{\pi}_{igj} \equiv \frac{\pi'_{igj}}{\pi_{igj}} = \frac{\hat{A}_{igj}\hat{w}_{ij}^{\alpha_{ig}}}{\sum_k \pi_{igk}\hat{A}_{igk}\hat{w}_{ik}^{\alpha_{ig}}}$ *is the relative change in sector* j*'s employment share of workers at local market* g *in country* i*.*

Note that (2.3) implies a system of NxJ equations, $\left\{\mathcal{D}'_{ij}(\hat{\mathbf{w}}) = 0\right\}$, that solves for an equilibrium matrix of relative changes in country-sector-specific wages, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}_{i\in\mathcal{N},j\in\mathcal{J}}$.

The demand side of the model will imply the aggregate consumption shares may change in response to shocks: $\hat{S}_{nj}S_{nj} \neq S_{nj}$. Note that in contrast to the standard Ricardian(–Roy) models (see, e.g., Galle, Rodríguez-Clare, and Yi 2023), individual expenditure shares of each consumer are now non-homothetic and allowed to depend on: i) sectoral price indices in country n, $\mathbf{P}_n \equiv (P_{n1}, P_{n2}, ..., P_{nJ})$, ii) individual expenditure/income level y of this consumer. As a result, the aggregate expenditure shares for each sector depend on sectoral prices and the whole distribution of income across individual consumers. Importantly, these non-homothetic expenditure patterns, apart from affecting equilibrium $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}_{i\in\mathcal{N},j\in\mathcal{J}}$ and the aggregate outcomes, also generate distributional consequences of any shock along the income distribution. The rest of this section derives the main results and discusses novel implications driven by the non-homothetic demand side.

2.3 Non-homothetic demand

In the previous sections, the demand side has been intentionally left unspecified to disentangle the supply and demand forces. In this part of the paper, I introduce a demand system into the framework and derive expressions crucial for comparative static and welfare analysis. Specifically, I use the Almost Ideal Demand System (Deaton and Muellbauer 1980) across sectors. Apart from allowing non-homothetic effects, one of the salient advantages of this

demand system is its flexibility.[8] In particular, as noted by Deaton and Muellbauer (1980), it gives an arbitrary first-order approximation to any demand system derived from utility-maximizing behavior. So, its implications allow us to examine the general role of homothetic and non-homothetic effects in Ricardian(-Roy) models. Moreover, it will be shown that it nests several popular demand specifications in the literature as special cases.

The indirect utility of a consumer with income y function is given by

$$\mathcal{V}(\mathbf{P}_n, y) = \ln\left(\left(\frac{y}{a(\mathbf{P}_n)}\right)^{\frac{1}{b(\mathbf{P}_n)}}\right), \tag{2.4}$$

where

i) $a(\mathbf{P}_n) \equiv \exp\left(\underline{a}_n + \sum_{j=1}^{J} a_{nj} \ln P_{nj} + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{J} \gamma_{njk} \ln P_{nj} \ln P_{nk}\right)$ *is the homothetic (Translog) price index;*

ii) $b(\mathbf{P}_n) \equiv \exp\left(\sum_{j=1}^{J} \beta_{nj} \ln P_{nj}\right)$ *is the non-homothetic component of preferences.*

For any $n \in \mathcal{N}$, restrictions on parameters are given by *i)* $\sum_{j=1}^{J} a_{nj} = 1$; *ii)* $\sum_{j=1}^{J} \beta_{nj} = \sum_{j=1}^{J} \gamma_{njk} = 0 \; \forall \, j \in \mathcal{J}$, and *iii)* $\gamma_{njk} = \gamma_{nkj} \; \forall j, k \in \mathcal{J}$.

Applying Roy's identity to the indirect utility function yields an individual–y's sectoral share of consumption as[9]

$$s_{nj}(\mathbf{P}_n, y) = a_{nj} + \sum_{k=1}^{J} \gamma_{njk} \ln P_{nk} + \beta_{nj} \ln\left(\frac{y}{a(\mathbf{P}_n)}\right). \tag{2.5}$$

Note that parameters β_{nj} are semi-elasticities of sectoral market shares to income. In particular, sectors with $\beta_{nj} > 0$ have positive income elasticities of individual market shares. Analogously, sectors with $\beta_{nj} < 0$ have negative income elasticities of individual market shares.

The supply side of the model implies the (endogenous) income distribution at local labor market g in country n obeys the Fréchet distribution and is given by $H_{ng}(y) = \exp\left\{-\left(y/\Upsilon_{ng}\right)^{-\alpha_{ng}}\right\}$, where $\Upsilon_{ng} \equiv \left(\sum_{j=1}^{J} A_{ngj} w_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$. As a result, one can analytically derive the mean local

[8] The Almost Ideal Demand System also permits the exact aggregation over consumers. In contrast to non-homothetic CES preferences (see, e.g., Fieler 2011, Comin, Lashkari, and Mestieri 2021), the presence of the representative consumer facilitates the exact-hat algebra approach for counterfactuals in a Ricardian(-Roy) model.

[9] Alternatively, one can apply Shephard's lemma to the expenditure function. The corresponding expenditure function is given by $\ln e(\mathbf{P}_n, u) = \ln a(\mathbf{P}_n) + u \cdot b(\mathbf{P}_n)$.

income, $y_{ng} \equiv Y_{ng}/L_{ng}$, and the Theil index, $\mathcal{T}_{ng}$, for a local market $g \in \mathcal{G}_n$:[10]

$$y_{ng} = \int_{\mathscr{Y}} y \, \mathrm{d}H_{ng}(y) = \Gamma\left(1 - 1/\alpha_{ng}\right) \left(\sum_{j=1}^{J} A_{ngj} w_{nj}^{\alpha_{ng}} \right)^{1/\alpha_{ng}}, \tag{2.6}$$

$$\mathcal{T}_{ng} = \int_{\mathscr{Y}} \frac{y}{y_{ng}} \ln\left(\frac{y}{y_{ng}}\right) \mathrm{d}H_{ng}(y) = \ln\left(\Gamma\left(1 - 1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi\left(1 - 1/\alpha_{ng}\right)}{\alpha_{ng}}, \tag{2.7}$$

where $\Gamma(\cdot)$ *is the Gamma function, and* $\psi(\cdot) \equiv \Gamma'(\cdot)/\Gamma(\cdot)$ *is the Digamma function.*

Now, let $H_n(\cdot)$ denote the c.d.f. of income in country n. This distribution is continuous and represents a (finite) mixture distribution. As a result, aggregating across heterogeneous consumers and exploiting the Theil index decomposition properties yield the aggregate expenditure shares of the following form:

Proposition 1. *The aggregate expenditure share of sector* j *in country* n *is given by:*

$$S_{nj} \equiv \int_{\mathscr{Y}} \frac{y}{y_n} s_{nj}(\mathbf{P}_n, y) \, \mathrm{d}H_n(y) = s_{nj}(\mathbf{P}_n, \, y_n \cdot \exp(\mathcal{T}_n)), \tag{2.8}$$

where country n*'s mean income,* y_n*, and the Theil index of the income distribution,* $\mathcal{T}_n$*, satisfy:*

$$y_n \equiv \int_{\mathscr{Y}} y \, \mathrm{d}H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} y_{ng},$$

$$\mathcal{T}_n \equiv \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \mathrm{d}H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng} + \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln\left(\frac{y_{ng}}{y_n}\right).$$

Proof. See the Appendix A.1. □

In what follows, $\tilde{y}_n \equiv y_n \cdot \exp(\mathcal{T}_n)$ is the inequality-adjusted mean income for country n. It implies that the aggregate shares of consumption are equal to expenditure shares of the "inequality-adjusted representative consumer" in the sense that this representative consumer possesses mean income yet adjusted to the (country-level) Theil index of income inequality. Proposition 1 states that if there is any level of income heterogeneity, then the aggregate expenditure shares of high-income elastic sectors are higher than in the case of no income inequality (i.e., $\mathcal{T}_n = 0$). Analogously, aggregate shares of low-income elastic sectors are lower than in the income homogeneity scenario.

As Proposition 1 also states, the country-level Theil index can be represented as the sum

[10] The Lorenz curve at a local market $g \in \mathcal{G}_n$ is given by $\mathscr{L}_{ng}(u) = \frac{\int_0^u (-\ln \rho)^{-1/\alpha_{ng}} \mathrm{d}\rho}{\Gamma(1-1/\alpha_{ng})}$, where u denotes a particular quantile.

of its *within* and *between* components:

$$\mathcal{T}_n = \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng}}_{\mathcal{T}_{n,\,within}} + \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln \frac{y_{ng}}{y_n}}_{\mathcal{T}_{n,\,between}}, \tag{2.9}$$

where $\ell_{ng} \equiv L_{ng}/L_n$ is the labor share, y_{ng} is the average income of a local labor market g.

This decomposition implies that the country-level Theil index is given by the (weighted) average inequality within each local labor market plus the inequality between local labor markets. Hence, given $\mathcal{T}_{ng}$ and data on y_{ng} and ℓ_{ng} for all $g \in \mathcal{G}_n$, one can readily infer the country-level Theil index, $\mathcal{T}_n$.

2.4 Counterfactual changes in the aggregate shares of consumption

In this part, I provide an analysis of counterfactual changes in the aggregate shares of consumptions. In particular, to solve the system (2.3) for the equilibrium wage matrix of relative changes, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}$, we need to derive counterfactual values of aggregate sectoral shares $S'_{nj} \equiv \hat{S}_{nj} S_{nj}$. To proceed with the analysis, it is convenient to represent each counterfactual aggregate expenditure share as the sum of its current sectoral share and an absolute change $S'_{nj} \equiv \Delta S_{nj} + S_{nj}$, where $\Delta S_{nj} \equiv S'_{nj} - S_{nj}$. Recall that the current levels of the aggregate consumption shares S_{nj} are available from the data. So, for counterfactual predictions, we need to derive $\Delta S_{nj} \equiv S'_{nj} - S_{nj}$ for any $n \in \mathcal{N}$ and $j \in \mathcal{J}$. One can readily verify that

$$\Delta S_{nj} = \underbrace{\sum_{k=1}^{J} \gamma_{njk} \ln \hat{P}_{nk}}_{homothetic} + \underbrace{\beta_{nj} \ln \left(\hat{\hat{y}}_n / \hat{a}_n \right)}_{non-homothetic}, \tag{2.10}$$

where $\hat{P}_{nk} = \hat{w}_{nk} \hat{\lambda}_{nnk}^{1/\theta_k} / \hat{T}_{nk}^{1/\theta_k}$ for any sector $k \in \{1, .., J\}$.

Note that change in each aggregate expenditure share, ΔS_{nj}, can be decomposed into homothetic and non-homothetic parts. In particular, $\beta_{nj} \ln \left(\hat{\hat{y}}_n / \hat{a}_n \right)$ represents a change in S_{nj} driven by non-homotheticity. One can readily verify that if $\beta_{nj} = 0$ for any $n \in \mathcal{N}$ and $j \in \mathcal{J}$, then changes in aggregate sectoral shares of consumption may still change due to homothetic forces captured by changes in relative sectoral prices. Note also that if we additionally impose that $\gamma_{njk} = 0$ for any $n \in \mathcal{N}$ and $j, k \in \mathcal{J}$, then the model reduces to the basic Cobb-Douglas case, implying that aggregate expenditure shares do not change in any counterfactual scenario (i.e., $\Delta S_{nj} = 0$ for any $n \in \mathcal{N}$ and $j \in \mathcal{J}$).

Now, the matter of primary interest for the exact-hat algebra counterfactual analysis is that it is possible to derive tractable expressions for relative changes in the inequality-adjusted

mean income, $\hat{\tilde{y}}_n$, and in the homothetic price index, $\hat{a}_n$. In particular, it can be shown that the relative change in the homothetic price index component, $\ln \hat{a}_n$, can be represented as:[11]

$$\ln \hat{a}_n \equiv \ln(a'_n/a_n) = \sum_{k=1}^{J} S_{nk} \ln \hat{P}_{nk} + \sum_{k=1}^{J} \frac{\Delta S_{nk}}{2} \ln \hat{P}_{nk}. \tag{2.11}$$

Combining (2.11) with (2.10), I derive counterfactual absolute changes in the aggregate expenditure shares in a matrix form.

Proposition 2. *Given the matrix of relative changes in wages, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}$, counterfactual changes in the aggregate sectoral expenditure shares, $\mathbf{\Delta S}_n \equiv (\Delta S_{n1}, \Delta S_{n2}, ..., \Delta S_{nJ})^T$, in country n are given by:*

$$\mathbf{\Delta S}_n = (\text{diag}(\mathbf{1}) + \mathbf{\Omega}_n)^{-1} \left(\tilde{\mathbf{P}}_n + \tilde{\mathbf{b}}_n \right), \tag{2.12}$$

where

i) $\tilde{\mathbf{P}}_n \equiv (\tilde{P}_{n1}, \tilde{P}_{n2}, ..., \tilde{P}_{nJ})^T$ *is a* $Jx1$ *vector with* j*'s element given by* $\tilde{P}_{nj} = \sum_{k=1}^{J} \gamma_{njk} \ln \hat{P}_{nk}$*;*

ii) $\tilde{\mathbf{b}}_n \equiv (\tilde{b}_{n1}, \tilde{b}_{n2}, ..., \tilde{b}_{nJ})^T$ *is a* $Jx1$ *vector with* j*'s element given by* $\tilde{b}_{nj} = \beta_{nj} \left(\ln \hat{\tilde{y}}_n - \sum_{k=1}^{J} S_{nk} \ln \hat{P}_{nk} \right)$*;*

iii) $\text{diag}(\mathbf{1})$ *is the identity matrix of size* J*;*

iv) $\mathbf{\Omega}_n \equiv \begin{pmatrix} \frac{\beta_{n1}}{2} \ln \hat{P}_{n1} & ... & \frac{\beta_{n1}}{2} \ln \hat{P}_{nJ} \\ \vdots & \ddots & \vdots \\ \frac{\beta_{nJ}}{2} \ln \hat{P}_{n1} & ... & \frac{\beta_{nJ}}{2} \ln \hat{P}_{nJ} \end{pmatrix}$ *is a country-specific* JxJ *matrix of cross-interactions between sectoral (semi-)elasticities and changes in sectoral prices.*

Proposition 2 demonstrates that solving for changes in the aggregate expenditure shares for each country n boils down to solving a country-specific linear system. It is also straightforward to see that (2.12) nests counterfactual changes in the aggregate consumption shares under the homothetic Translog expenditure function and fixed (Cobb-Douglas) sectoral expenditure shares. It is instructive to discuss both of these special cases separately.

Special Case 1 (Homothetic Translog expenditure function) The case of the Translog expenditure function arises when $\{\beta_{nj}\} = 0$ and implies no cross-interactions between sectoral (semi-)elasticities and changes in sectoral prices (i.e., $\mathbf{\Omega}_n = \mathbf{0}_{JxJ}$) and no interactions between sectoral (semi-)elasticities and the inequality-adjusted mean income (i.e., $\tilde{\mathbf{b}}_n = \mathbf{0}_{Jx1}$). As a result, the formula (2.12) reduces to $\mathbf{\Delta S}_n = \tilde{\mathbf{P}}_n$, implying that changes in the aggregate expenditure shares in any country n do not depend on the distribution of income across consumers

[11] Alternatively, one can apply Shephard's lemma to obtain: $\hat{a}_n \approx \sum_{j=1}^{J} \left(S_{nj} - \beta_{nj} \ln \left(\frac{\tilde{y}_n}{a(\mathbf{P}_n)} \right) \right) \hat{P}_{nj}$. Such an approach, while computationally more straightforward, is much less advantaged than (2.11) for a few reasons. Firstly, it relies on an approximation, while the formula (2.11) represents the exact change between a current value and a counterfactual value. Secondly, in this case, the counterfactual analysis would require information on the current values of homothetic price indices, $a(\mathbf{P}_n)$, for each country.

and are driven only by relative changes in sectoral homothetic price indices and substitution patterns across them captured by the matrix $\{\gamma_{njk}\}$.

Special Case 2 (Cobb-Douglas expenditure shares) Another special case arises when, in addition to the restrictions imposed by the Translog case, each element of the matrix of substitution patterns across sectoral price indices is zero (i.e., $\gamma_{njk} = 0$ for any $n \in \mathcal{N}$ and $j, k \in \mathcal{J}$). In this case $\tilde{\mathbf{P}}_n = \mathbf{0}_{J\text{x}1}$, implying also that $\mathbf{\Delta S}_n = \mathbf{0}_{J\text{x}1}$. It can readily be inferred that it corresponds to the standard case of constant aggregate expenditure shares, implying Cobb-Douglas preferences across sectors as in Galle, Rodríguez-Clare, and Yi (2023).

The following subsection provides details on counterfactual changes in the inequality-adjusted mean income, $\hat{\tilde{y}}_n$.

2.5 Counterfactual changes in income inequality and mean income

This is a direct corollary from Proposition 1 that the impact of any changes in the distribution of income within a country on the aggregate sectoral expenditure shares is leveraged by changes in the inequality-adjusted mean income. Recall that the inequality-adjusted mean income is given by $\tilde{y}_n \equiv y_n \cdot \exp(\mathcal{T}_n)$. So, one can readily verify that the relative change in $\tilde{y}_n$ is given by:

$$\hat{\tilde{y}}_n \equiv \tilde{y}'_n / \tilde{y}_n = \hat{y}_n \cdot \exp(\Delta \mathcal{T}_n), \tag{2.13}$$

where $\Delta \mathcal{T}_n \equiv \mathcal{T}'_n - \mathcal{T}_n$ is the difference between counterfactual and actual values of the country-level Theil indices.

The formula above suggests that any trade or technology shock may affect both mean income and inequality. It is straightforward to show that the relative change in the country-level average income, $\hat{y}_n$, is given by

$$\hat{y}_n \equiv \frac{y'_n}{y_n} = \sum_{g \in \mathcal{G}_n} \frac{y_{ng}}{y_n} \hat{Y}_{ng}, \tag{2.14}$$

where $\hat{Y}_{ng} = \hat{y}_{ng} = \left(\sum_{j=1}^{J} \pi_{ngj} \hat{A}_{ngj} \hat{w}_{nj}^{\alpha_{ng}} \right)^{\frac{1}{\alpha_{ng}}}$ is the relative change in total (and average) income of workers at local market g in country n.

Now, recall from (2.7) that the local Theil index depends exclusively on α_{ng} and can be analytically expressed via the Gamma and Digamma functions as $\mathcal{T}_{ng} = \ln\left(\Gamma\left(1 - 1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi(1-1/\alpha_{ng})}{\alpha_{ng}}$. Given that α_{ng} is constant, the local Theil index does not change in a counterfactual scenario, $\mathcal{T}'_{ng} = \mathcal{T}_{ng}$ for any $g \in \mathcal{G}_n$. Yet, in the case of many local labor markets, the country-level Theil index is subject to change in a counterfactual equilibrium due to changes in both

the *within* and *between* components. So, in general, $\mathcal{T}'_n \neq \mathcal{T}_n$, as the following proposition states.

Proposition 3. *A counterfactual value of the country-level Theil index is given by:*

$$\mathcal{T}'_n = \underbrace{\sum_{g \in \mathcal{G}_n} \frac{\hat{Y}_{ng} Y_{ng}}{\hat{Y}_n Y_n} \mathcal{T}_{ng}}_{\mathcal{T}'_{n,\,within}} + \underbrace{\sum_{g \in \mathcal{G}_n} \frac{\hat{Y}_{ng} Y_{ng}}{\hat{Y}_n Y_n} \ln\left(\frac{\hat{y}_{ng} y_{ng}}{\hat{y}_n y_n}\right)}_{\mathcal{T}'_{n,\,between}}. \tag{2.15}$$

where

i) $\mathcal{T}_{ng}$ *is a current value of the Theil index at local market* g *in country* n*;*
ii) $\mathcal{T}'_{n,\,within}$ *denotes a counterfactual "within" component of the Theil index in country* n*;*
iii) $\mathcal{T}'_{n,\,between}$ *denotes a counterfactual "between" component of the Theil index in country* n*;*
iv) $\hat{Y}_n \equiv Y'_n / Y_n = \sum_{g \in \mathcal{G}_n} \frac{Y_{ng}}{Y_n} \hat{Y}_{ng}$ *is the relative change in total income in country* n*.*

Proof. See the Appendix A.2. □

Note that the expression (2.15) makes it clear that if we consider only one labor market g for a country n, then the country-level Theil index is necessarily left unchanged in any counterfactual scenario, $\mathcal{T}'_n = \mathcal{T}_n$. Thus, apart from non-homotheticities, enriching a Ricardian-Roy model with the spatial dimension gives rise to the inequality margin of adjustment to trade shocks. This change in the Theil index, $\Delta\mathcal{T}_n$, translates to an additional source of Consumption Inequality between sectors through the non-homothetic mechanism.

Data requirements for counterfactual analysis based on the exact–hat algebra. To perform counterfactual analysis, one needs to solve for an equilibrium matrix of relative changes in country-sector-specific wages implicitly defined by (2.3). To solve for $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}_{n \in \mathcal{N}, j \in \mathcal{J}}$, one need structural parameters $\{\alpha_{ng}, \theta_j, \gamma_{njk}, \beta_{nj}\}$ estimated as well as data on:

i) sectoral expenditure, trade, and employment shares $\{S_{nj}, \lambda_{inj}, \pi_{ngj}\}$*;*
ii) total income and employment levels across local labor markets $\{Y_{ng}, L_{ng}\}$*;*
iii) current levels of trade imbalances and their relative changes $\{D_n, \hat{D}_n\}$*;*
iv) relative changes in (trade cost and technology) fundamentals $\{\hat{\tau}_{inj}, \hat{T}_{nj}, \hat{A}_{ngk}\}$*.*

Given an equilibrium matrix $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}$, one can restore the other endogenous variables of interest and make welfare predictions. Note that one does not need to estimate some structural demand parameters of the model: $\{\underline{a}_n\}$ and $\{a_{nj}\}$. Importantly, given structural parameters $\{\alpha_{ng}, \theta_j, \gamma_{njk}, \beta_{nj}\}$, the data requirements to solve for a counterfactual equilibrium are almost the same as in the standard case of constant (Cobb-Douglas) sectoral consumption shares. Specifically, the only additional data one needs is information on the local-market-specific total amount of labor, $\{L_{ng}\}$. This information is needed for calculations of local

market-specific mean incomes that, in turn, allow for calculations of counterfactual values of the country-level mean income and the Theil index.

3 Welfare Analysis

In this Section, I provide the main results of the welfare analysis. Given a non-homothetic demand side, the analysis based on the exact price index cannot generally be applied. So, I use the *Equivalent Variation* (EV) as a measure, to a first-order approximation, of welfare changes associated with changes in prices and income.[12] It represents the amount of additional income, given the current prices, that would have the same effect on consumer welfare as if the economic environment had changed. Formally, the Equivalent Variation is defined by the following condition:

$$\frac{\partial \ln \mathcal{V}_n}{\partial \ln y}\frac{EV(y)}{y} = \frac{\partial \ln \mathcal{V}_n}{\partial \ln y}\mathrm{d}\ln y + \sum_{j=1}^{J}\frac{\partial \ln \mathcal{V}_n}{\partial \ln P_{nj}}\mathrm{d}\ln P_{nj}, \tag{3.1}$$

where the right-hand side of the equation above is obtained by totally differentiating the indirect utility function $\mathcal{V}_n(\mathbf{P}_n, y)$ of a consumer with individual income y in country n.

In what follows, let $\mathrm{d}\ln \mathcal{W}_n(y)$ denote the corresponding change in the welfare measure for an individual with income level y. One can readily infer that rearranging terms in (3.1) and applying Roy's identity directly yields

$$\mathrm{d}\ln \mathcal{W}_n(y) \equiv \frac{EV(y)}{y} = \mathrm{d}\ln y - \sum_{j=1}^{J} s_{nj}(\mathbf{P}_n, y)\mathrm{d}\ln P_{nj}. \tag{3.2}$$

Note that (3.2) leaves the demand and supply features of the model unspecified. However, to make counterfactual predictions on the distributional impacts of trade or technology shocks, one needs to infer $\mathrm{d}\ln y$ and $\mathrm{d}\ln P_{nj}$ based on the model's structure. In the next subsection, I propose a generalized welfare change formula based on the supply-side properties of the Ricardian-Roy model.

3.1 The distributional welfare impact of shocks: a generalized formula

In this subsection, I derive a generalized welfare change formula for the distributional impact of trade that applies to any demand system across sectors. It combines the Ricardian- and

[12] It is common in the literature to exploit the Equivalent Variation as a measure of welfare changes under a non-homothetic demand (see, e.g., Fajgelbaum and Khandelwal 2016, Borusyak and Jaravel 2021, etc.).

Roy-like terms and also gives rise to a novel – *the Consumption Inequality* – channel of the aggregate impact of trade or technology shocks.

Note that the formula allows one to examine the distributional impact of any *foreign/non-local* shock. In the spirit of Arkolakis, Costinot, and Rodríguez-Clare (2012), I first introduce a formal definition of these shocks.

Definition 1. Let $\boldsymbol{L} \equiv \{L_{ng}\}$, $\boldsymbol{T} \equiv \{T_{nj}\}$, $\boldsymbol{A} \equiv \{A_{ngj}\}$, and $\boldsymbol{\tau} \equiv \{\tau_{nij}\}$. A foreign/non-local shock for a local labor market g in country n is any change from $(\boldsymbol{L}, \boldsymbol{T}, \boldsymbol{A}, \boldsymbol{\tau})$ to $(\boldsymbol{L'}, \boldsymbol{T'}, \boldsymbol{A'}, \boldsymbol{\tau'})$ such that $L_{ng} = L'_{ng}$ and $\{T_{nj}, A_{ngj}, \tau_{nnj}\}_{j\in\mathcal{J}} = \{T'_{nj}, A'_{ngj}, \tau'_{nnj}\}_{j\in\mathcal{J}}$.

The definition above imposes just mild restrictions on the nature of a shock. In particular, it implies that a shock is foreign/non-local in the sense that it does not affect *i*) labor endowments of the local labor market, *ii*) the domestic country's technological levels, *iii*) the local labor market's technological levels, and *iv*) domestic country's ability to serve its own market.[13] The proposition below states the welfare impact of such a shock on a particular group of individuals.

Proposition 4 (Generalized formula)**.** *Consider any foreign/non-local shock. A first-order approximation to the welfare change of income-y individuals at local market g in country n is given by*

$$\frac{\Delta \mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \underbrace{\sum_{j=1}^{J} \frac{s_{ngj}(y)}{\theta_j}\left(1-\hat{\lambda}_{nnj}\right)}_{consumer's\ Ricardian-type\ gains} + \underbrace{\frac{1-\left(\sum_{j=1}^{J} s_{ngj}(y)\hat{\pi}_{ngj}\right)}{\alpha_{ng}}}_{consumer's\ Roy-type\ gains}, \tag{3.3}$$

where $\hat{\lambda}_{nnj} = \frac{\hat{w}_{nj}^{-\theta_j}}{\sum_{n'} \lambda_{n'nj}\hat{T}_{n'j}\left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$, $\hat{\pi}_{ngj} = \frac{\hat{w}_{nj}^{\alpha_{ng}}}{\sum_k \pi_{ngk}\hat{w}_{nk}^{\alpha_{ng}}}$, *and* $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}_{n\in\mathcal{N},j\in\mathcal{J}}$ *is implicitly given by the solution of (2.3).*

Proof. See the Appendix A.3. □

As one can see, this parsimonious formula applies to any demand system. Depending on preferences across sectors, the schedule of $\{s_{ngj}(y)\}$ may take different forms. I provide an alternative representation of this Proposition to better grasp the intuition of distributional effects at play.

Proposition 5 (Alternative representation)**.** *Consider any foreign/non-local shock. A first-order approximation to the welfare change of income-y individuals at local market g in country n is given by*

[13] Note also that, to examine welfare change implications aggregated to the country level, one need additionally impose the assumption that $L_{ng} = L'_{ng}$ for any $g \in \mathcal{G}_n$.

$$\frac{\Delta \mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \underbrace{\left(\overline{\frac{1-\hat{\lambda}_{nn}}{\theta}}\right) + J \cdot cov\left(\frac{1-\hat{\lambda}_{nn}}{\theta}, s_{ng}(y)\right)}_{Ricardian-type\ gains} + \underbrace{\frac{1-\overline{\hat{\pi}_{ng}} - J \cdot cov\left(\hat{\pi}_{ng}, s_{ng}(y)\right)}{\alpha_{ng}}}_{Roy-type\ gains}, \tag{3.4}$$

where

i) $\left(\overline{\frac{1-\hat{\lambda}_{nn}}{\theta}}\right) \equiv \frac{1}{J}\sum_{j=1}^{J}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}\right)$ *denotes the (trade elasticity-adjusted) average change in trade openness of a country* $n \in \mathcal{N}$ *across sectors;*

ii) $cov\left(\frac{1-\hat{\lambda}_{nn}}{\theta}, s_{ng}(y)\right)$ *is the covariance between changes in trade openness of a country* $n \in \mathcal{N}$ *across sectors and income-specific sectoral expenditure shares;*

iii) $\overline{\hat{\pi}_{ng}} \equiv \frac{1}{J}\sum_{j=1}^{J}\hat{\pi}_{ngj}$ *denotes the average change in specialization of a local market* $g \in G_n$*;*

iv) $cov\left(\hat{\pi}_{ng}, s_{ng}(y)\right)$ *is the covariance between changes in the specialization pattern of a local market* $g \in G_n$ *across sectors and income-specific sectoral expenditure shares.*

The proof is straightforward given Proposition 4. It is instructive to discuss each of these terms separately. First, consider the Ricardian-type gains of the welfare decomposition above. The first term, $\left(\overline{\frac{1-\hat{\lambda}_{nn}}{\theta}}\right)$, determines a part of gains from an increase in a country's trade openness that is uncorrelated with sectoral consumption profile $\{s_{ngj}(y)\}$ of income$-y$ individuals. So, this term simply states that a country tends to benefit from increased specialization at the country level. As for the second term of the Ricardian-type gains, $J \cdot cov\left(\frac{1-\hat{\lambda}_{nn}}{\theta}, s_{ng}(y)\right)$, it states that a group of income-y individuals is better off from trade if their consumption pattern is correlated with of change in the country's specialization.

As for the Roy-type terms, they capture the welfare effects of changes in a particular local market's market specialization. Specifically, note that $\overline{\hat{\pi}_{ng}} < 1$ implies that $\frac{1}{J}\sum_{j=1}^{J}\frac{\Delta\pi_{ngj}}{\pi_{ngj}} < 0$. So, $\overline{\hat{\pi}_{ng}} < 1$ implies a higher degree of specialization in a counterfactual equilibrium. As a result, one can readily verify from Proposition 5, that regardless of the consumption/income profile, individuals within a given local market tend to benefit from increased specialization in this local market.

Propositions 4 and 5 also allow for a further decomposition of the Ricardian- and Roy-type gains into the individual, local, and aggregate components, respectively:

$$\frac{\Delta \mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \underbrace{\sum_{j=1}^{J}\frac{s_{ngj}(y) - \tilde{s}_{ngj}}{\theta_j}\left(1-\hat{\lambda}_{nnj}\right)}_{individual\ Ricardian\ gains} + \underbrace{\frac{1}{\alpha_{ng}}\sum_{j=1}^{J}\left(\tilde{s}_{ngj} - s_{ngj}(y)\right)\hat{\pi}_{ngj}}_{individual\ Roy\ gains}$$

$$+\underbrace{\sum_{j=1}^{J}\frac{\tilde{s}_{ngj}-S_{nj}}{\theta_j}\left(1-\hat{\lambda}_{nnj}\right)}_{\text{local Ricardian gains}}+\underbrace{\frac{1}{\alpha_{ng}}\sum_{j=1}^{J}\left(S_{nj}-\tilde{s}_{ngj}\right)\hat{\pi}_{ngj}}_{\text{local Roy gains}}+\underbrace{\sum_{j=1}^{J}\frac{S_{nj}}{\theta_j}\left(1-\hat{\lambda}_{nnj}\right)}_{\text{aggregate Ricardian gains}}+\underbrace{\frac{1}{\alpha_{ng}}\left(1-\sum_{j=1}^{J}S_{nj}\hat{\pi}_{ngj}\right)}_{\text{aggregate Roy gains}}, \tag{3.5}$$

where $\tilde{s}_{ngj}$ denotes a local aggregate expenditure share on sector $j \in \mathcal{J}$ for consumers at a labor market $g \in \mathcal{G}_n$.

3.2 A novel channel of the aggregate welfare change

Compared to the case of Cobb-Douglas or any other homothetic preferences across sectors, I show that, compared to standard Ricardian(-Roy) models, my framework gives rise to a novel – *the Consumption Inequality* – channel of the aggregate impact of trade or technology shocks. To demonstrate it straightforwardly, it is instructive to move from an unspecified demand system that is considered in Section 3.1 back to the Almost Ideal Demand System. I also start with discussing an individual's welfare gains formula. In particular, combining Propositions 4 and 5, the welfare impact of any foreign/non-local shock for income-y individuals at local market $g \in \mathcal{G}_n$ is now given by

$$\frac{\Delta\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \sum_{j=1}^{J} S_{nj}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}+\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right)+\ln\left(\frac{y}{\tilde{y}_n}\right)\sum_{j=1}^{J}\beta_{nj}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}+\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right). \tag{3.6}$$

First, I briefly discuss the traditional Ricardian and Roy channels of aggregate welfare gains. The former one is captured by the term $\sum_{j=1}^{J} S_{nj}\frac{1-\hat{\lambda}_{nnj}}{\theta_j}$. It is straightforward to see that, given the trade elasticity within sectors $\{\theta_j\}$, a country tends to gain from a shock if it increases the country's trade openness in, at least, those sectors that constitute a large share of the country's aggregate consumption.

The Roy channel is captured by the term $\sum_{j=1}^{J} S_{nj}\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}$. One can readily infer that it can be alternatively rewritten as

$$\frac{1}{\alpha_{ng}}\sum_{j=1}^{J} S_{nj}\left(1-\hat{\pi}_{ngj}\right)=\frac{1}{\alpha_{ng}}\left(1-\sum_{j=1}^{J}S_{nj}\hat{\pi}_{ngj}\right)=\frac{1}{\alpha_{ng}}\left(1-\frac{\sum_j S_{nj}\hat{A}_{ngj}\hat{w}_{nj}^{\alpha_{ng}}}{\sum_k \pi_{ngk}\hat{A}_{ngk}\hat{w}_{nk}^{\alpha_{ng}}}\right).$$

Recall also that $\hat{\pi}_{ngj}/\hat{\pi}_{ngk}=\left(\hat{A}_{ngj}/\hat{A}_{ngk}\right)\left(\hat{w}_{nj}/\hat{w}_{nk}\right)^{\alpha_{ng}}$ for any $j,k \in \mathcal{J}$. Moreover, the autarky scenario implies that the employment shares must be aligned with the aggregate consumption shares given by $\{S_{nj}\}$. So, the expression above makes clear that the Roy channel is positive if, in response to a shock, the local labor market's g specialization pattern $\{\pi_{ngj}\}$ across sectors shifts further away from what would have been observed under the autarky

scenario, $\{S_{nj}\}$. Similarly, if the local labor market's g specialization pattern shifts closer to the one that would have been observed under the autarky scenario, then the Roy channel of the aggregate welfare gains turns out to be negative. Note also that, regardless of the nature of the shock, the role channel is magnified by the local market's degree of heterogeneity of workers.[14]

Apart from traditional Ricardian and Roy channels, an individual's welfare change depends on the consumer's income level relative to the inequality-adjusted mean income in the economy. Whether high- or low-income consumers are those who benefit from a shock is determined by the sign of the term $\sum_{j=1}^{J}\beta_{nj}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}+\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right)$. One can readily infer that it is proportional to the covariance between sectoral income (semi-)elasticities, $\{\beta_{nj}\}$, and sector-specific Ricardian-type, $\left\{\frac{1-\hat{\lambda}_{nnj}}{\theta_j}\right\}$, and Roy-type, $\left\{\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right\}$, forces of welfare change, respectively.

To grasp the intuition of this covariance, I separately consider each of these terms. First, note that $\sum_{j=1}^{J}\beta_{nj}\frac{1-\hat{\lambda}_{nnj}}{\theta_j}$ captures the covariance between sectoral income elasticities and changes in trade openness across sectors. If this term is positive, it implies that the country's trade openness pattern has been shifted towards high-income elastic sectors (i.e., sectors with positive β's) in response to a shock. It also implies that ceteris paribus, relatively richer consumers tend to be better off from this shock. Analogously, if $\sum_{j=1}^{J}\beta_{nj}\frac{1-\hat{\lambda}_{nnj}}{\theta_j}<0$, then the country's trade openness pattern has been shifted towards low-income elastic sectors. Lastly, $\sum_{j=1}^{J}\beta_{nj}\frac{1-\hat{\lambda}_{nnj}}{\theta_j}=0$ implies that changes in the country-level specialization pattern has been uncorrelated with sectoral income elasticities of consumption and thereby has no effect for the aggregate level of gains from trade. Importantly, the aggregate impact of the term $\sum_{j=1}^{J}\beta_{nj}\frac{1-\hat{\lambda}_{nnj}}{\theta_j}$ can be either sign regardless of whether the "negative" or "positive" shock took place. For example, let's assume that $\theta_j=\theta$ for any sector $j\in\mathcal{J}$ and changes in the country's trade openness have been proportional across all sectors, $1-\hat{\lambda}_{nnj}=\tilde{\lambda}$. In this particular case, it does not matter whether the shock has led to an increase in the country's trade openness ($\tilde{\lambda}>0$) or the shock has moved the country closer to the autarky ($\tilde{\lambda}<0$) – in either of these cases the country-level *relative* sectoral specialization pattern remains unaffected and has no impact on the Consumption Inequality channel of welfare gains: $\sum_{j=1}^{J}\beta_{nj}\frac{1-\hat{\lambda}_{nnj}}{\theta_j}=\frac{\tilde{\lambda}}{\theta}\sum_{j=1}^{J}\beta_{nj}=0$.

Now, let $\left(\frac{\Delta\mathcal{W}_{ng}}{\mathcal{W}_{ng}}\right)^{agg}\equiv\int_{\mathscr{Y}}\frac{\Delta\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)}\mathrm{d}H_{ng}(y)$ denote the aggregate welfare change in country n at local market g. Integrating across the corresponding income distribution yields the following parsimonious formula of the aggregate welfare gains at the local market $g\in\mathcal{G}_n$.

[14]Recall that the higher the value of α_{ng}, the lower the heterogeneity of workers within the local labor market g in country n.

Proposition 6. *Consider any foreign/non-local shock. A first-order approximation to the aggregate welfare change at local market g in country n is given by*

$$\left(\frac{\Delta \mathcal{W}_{ng}}{\mathcal{W}_{ng}}\right)^{agg} \approx \underbrace{\sum_{j=1}^{J} S_{nj}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}+\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right)}_{\text{the Ricardian+Roy channels}} + \underbrace{\Theta_{ng}\sum_{j=1}^{J}\beta_{nj}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}+\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right)}_{\text{the Consumption Inequality channel}}, \quad (3.7)$$

where

i) $\Theta_{ng} \equiv \frac{\gamma+\psi(1-1/\alpha_{ng})}{\alpha_{ng}} + \ln\left(\frac{y_{ng}}{y_n}\right) + (\mathcal{T}_{ng}-\mathcal{T}_n)$ *and* γ *denoting the Euler–Mascheroni constant;*

ii) $\hat{\lambda}_{nnj} = \frac{\hat{w}_{nj}^{-\theta_j}}{\sum_{n'}\lambda_{n'nj}\hat{T}_{n'j}\left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$ *is the change in domestic share in sector j in country n;*

iii) $\hat{\pi}_{ngj} = \frac{\hat{w}_{nj}^{\alpha_{ng}}}{\sum_k \pi_{ngk}\hat{w}_{nk}^{\alpha_{ng}}}$ *is the change in employment share of sector j at local labor market $g \in \mathcal{G}_n$;*

iv) $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}_{n\in\mathcal{N},j\in\mathcal{J}}$ *is implicitly given by the solution of (2.3).*

It directly follows from (3.7) that, in addition to the standard Ricardian and Roy channels of welfare change, the model gives rise to a novel – *the Consumption Inequality* – channel at the aggregate (local) level.[15]

3.2.1 Decomposing the Consumption Inequality channel

One can take a closer look at this novel channel. In particular, note that the total local impact of the Consumption Inequality channel, $\Theta_{ng}\cdot\sum_{j=1}^{J}\beta_{nj}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}+\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right)$, can be decomposed into three different margins: **i**) the "Pro-poor" margin; **ii**) the "Average income" margin; and **iii**) the "Inequality" margin. To better grasp the mechanics of the Consumption Inequality channel, it is instructive to consider each of these margins separately.

The "Pro-poor" margin corresponds to the term $\frac{\gamma+\psi(1-1/\alpha_{ng})}{\alpha_{ng}}\left(\sum_{j=1}^{J}\beta_{nj}\left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j}+\frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}}\right)\right)$. It tends to amplify the aggregate local welfare gains when the nature of a shock is "pro-poor" and, conversely, dampens the aggregate local welfare gains if the nature of a shock is "pro-rich". Formally, note that the term $\frac{\gamma+\psi(1-1/\alpha_{ng})}{\alpha_{ng}}$ is always negative because of the Digamma function, $\psi(\cdot)$, being strictly negative along the relevant interval and $\gamma+\psi(x)<0$ for any $x\in(0,1)$. Moreover, it can be shown that $\lim_{\alpha_{ng}\to\infty}\psi\left(1-1/\alpha_{ng}\right) = -\gamma$, where γ is the Euler–Mascheroni constant. Hence, it is straightforward to see that the "Pro-poor" margin vanishes when $\alpha_{ng}\to\infty$: $\lim_{\alpha_{ng}\to\infty}\frac{\gamma+\psi(1-1/\alpha_{ng})}{\alpha_{ng}} = 0$. The intuition is that when α_{ng} increases, the distribution of skills within a local market g in country n is going to be more and more homogeneous. It directly implies that the equilibrium distribution of income is becoming more

[15] It is straightforward to aggregate this formula further to the country level.

and more homogeneous as well. As a result, as $\alpha_{ng} \to \infty$, the "Pro-poor" margin disappears because there is no variance in income distribution across consumers. Note that the margin is "pro-poor" in the sense that it amplifies the aggregate (local) welfare gains when the nature of a shock is "pro-poor" (i.e., $\sum_{j=1}^{J} \beta_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}} \right) < 0$). Similarly, it tends to dampen the aggregate (local) welfare gains when the nature of a shock is "pro-rich" (i.e., $\sum_{j=1}^{J} \beta_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}} \right) > 0$).

The "Average income" margin corresponds to the term $\ln\left(\frac{y_{ng}}{y_n}\right)\left(\sum_{j=1}^{J} \beta_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}} \right)\right)$. The sign of this effect is driven by the deviation of the local market's average income from the country-level average income. In particular, it implies that if the nature of a shock is "pro-poor", local markets with relatively lower average income than the country-level one gain from this margin and lose otherwise. Recall from Proposition 1 that the aggregate expenditure shares across sectors depend on average income adjusted by (the exponent of) the Theil index of income inequality. The same mechanism applies to the consumption pattern of local labor markets. So, conditional on the Theil indices, local labor markets with relatively lower average income spend disproportionally more on low-income elastic than their relatively richer counterparts. As a result, for example, a "pro-poor" shock, by reducing relative prices of low-income elastic sectors, tends to benefit low-income local labor markets.

The "Inequality" margin corresponds to the term $(\mathcal{T}_{ng} - \mathcal{T}_n)\left(\sum_{j=1}^{J} \beta_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{ngj}}{\alpha_{ng}} \right)\right)$. It is driven by the deviation of the local market's Theil index from the country-wide Theil index of income inequality. For instance, it implies that if the nature of a shock is "pro-poor", then local markets with relatively lower income inequality gain from this margin. To better grasp the intuition, again recall from Proposition 1 that the aggregate expenditure shares across sectors depend on average income adjusted by (the exponent of) the Theil index of income inequality. In other words, if there is any level of income heterogeneity, then the aggregate expenditure shares of high-income elastic sectors are higher than in the case of no income inequality (i.e., $\mathcal{T}_n = 0$). Analogously, aggregate shares of low-income elastic sectors are lower than in the income homogeneity scenario. Note that the same logic applies to the expenditure patterns at the level of local labor markets. So, conditional on the average income, local labor markets with lower inequality than the economy-wide Theil index tend to spend disproportionally less on high-income elastic sectors while having disproportionally larger expenditure shares of low-income elastic sectors. As a result, "pro-poor" shocks, by reducing relative prices of low-income elastic sectors, tend to benefit local labor markets with lower levels of income inequality. It is straightforward to see that the opposite mechanism applies to "pro-rich" shocks.

3.2.2 The role of geography and skill heterogeneity in welfare implications

The formula (3.7) and the Consumption Inequality channel decomposition shed light on the local market's incidence of any shock. For example, if a country-level shock is "pro-poor", then it tends to favor local labor markets with *i*) a higher level of workers' skill heterogeneity (i.e., when α_{ng} is low), *ii*) lower average income ($y_{ng} < y_n$) and *iii*) lower income inequality ($\mathcal{T}_{ng} < \mathcal{T}_n$).

Now, note that if one imposes the assumption that workers in each local labor market within a country draw their sectoral productivities from the distribution with common shape parameter (i.e., $\alpha_{ng} = \alpha_n$ for any $g \in \mathcal{G}_n$, while allowing for local markets' "absolute advantages" captured by differences in $\{A_{ngj}\}$), then the Theil index of income inequality is the same for all labor markets within a country. In this case, the "Inequality" margin of (3.7) that is driven by differences in Theil indices is equal to zero for all labor markets. If, in addition, workers are assumed to be homogeneous within each local labor market (i.e., $\alpha_{ng} \to \infty$), then differences in the welfare response to a shock across local labor markets are driven solely by differences in the average local incomes (i.e., the "Average income" margin) and the formula (3.7) reduces to:

$$\left(\frac{\Delta \mathcal{W}_{ng}}{\mathcal{W}_{ng}}\right)^{agg} \approx \sum_{j=1}^{J} S_{nj} \frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \ln\left(\frac{y_{ng}}{y_n}\right) \sum_{j=1}^{J} \beta_{nj} \frac{1-\hat{\lambda}_{nnj}}{\theta_j}.$$

Alternatively, if one abstracts away from the geographical distribution of economic activity and assumes only one labor market within country n (i.e., $G_n = 1$), then both the "Average income" and the "Inequality" margins disappear, implying that only the "Pro-poor" margin remains being operated. So, in this particular case, the country-level welfare response to a shock is given by

$$\left(\frac{\Delta \mathcal{W}_{n}}{\mathcal{W}_{n}}\right)^{agg} \approx \sum_{j=1}^{J} S_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{nj}}{\alpha_n}\right) + \frac{\gamma + \psi\left(1-1/\alpha_n\right)}{\alpha_n} \sum_{j=1}^{J} \beta_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{nj}}{\alpha_n}\right).$$

Note that the expression above implies that the Consumption Inequality channel is driven by the term $\frac{\gamma+\psi(1-1/\alpha_n)}{\alpha_n}$ that magnifies the aggregate welfare gains when the nature of a shock is "pro-poor" and, conversely, dampens the aggregate effects if the nature of a shock is "pro-rich".

In Section 4, I quantitatively study the contribution of each traditional channel and each margin of the Consumption Inequality channel to the total welfare change in response to the China shock.

3.2.3 Relation to other welfare gains formulas

As has been mentioned, the derived formulas (4) and (5) apply to any demand system across sectors. To highlight the difference from some existing well-known, it is enough to consider special cases of the aggregate (local) welfare gains formula for the Almost Ideal Demand System (3.7). Specifically, I will separately consider four different special cases.

Special Case 1 (Homothetic Translog expenditure function) It can be readily inferred that if one shuts down non-homotheticities (i.e., $\beta_{nj} = 0$ for any $j \in \mathcal{J}$ and $n \in \mathcal{N}$), then $\Theta_{ng} \sum_{j=1}^{J} \beta_{nj} \left((1 - \hat{\lambda}_{nnj})/\theta_j + (1 - \hat{\pi}_{ngj})/\alpha_{ng} \right) = 0$ for any $g \in \mathcal{G}_n$ and the Consumption Inequality channel disappears for any local labor market. As a result, (3.7) reduces to

$$\left(\frac{\Delta \mathcal{W}_{ng}}{\mathcal{W}_{ng}} \right)^{agg} \approx \sum_{j=1}^{J} S_{nj} \left(\frac{1 - \hat{\lambda}_{nnj}}{\theta_j} + \frac{1 - \hat{\pi}_{ngj}}{\alpha_{ng}} \right). \tag{3.8}$$

The expression above represents the welfare gains for the particular case of homothetic Translog expenditure function across sectors, implying variable elasticity of substitution across them. As has been discussed in Section 2.4, $\{\beta_{nj}\} = 0$ implies no cross-interactions between sectoral income (semi-)elasticities and changes in sectoral prices (i.e., $\mathbf{\Omega}_n = \mathbf{0}_{J \text{x} J}$) as well as no interactions between sectoral income elasticities and the inequality-adjusted mean income (i.e., $\tilde{\mathbf{b}}_n = \mathbf{0}_{J \text{x} 1}$). As a result, changes in the aggregate expenditure shares in any country n are driven only by homothetic forces through (relative) changes in sectoral price indices and substitution patterns across them captured by the matrix of $\{\gamma_{njk}\}$: $\mathbf{\Delta S}_n = \tilde{\mathbf{P}}_n$, where $\tilde{\mathbf{P}}_n$ is a Jx1 vector with j's element given by $\tilde{P}_{nj} = \sum_{k=1}^{J} \gamma_{njk} \ln \hat{P}_{nk}$.

Special Case 2 (Constant expenditure shares as in Galle, Rodríguez-Clare, and Yi 2023) If one additionally imposes that the matrix of substitution patterns across sectoral price indices is zero (i.e., $\{\gamma_{njk}\} = 0$), then the model's welfare change predictions reduce to the case of Cobb-Douglas preferences across sectors studied by Galle, Rodríguez-Clare, and Yi (2023). The corresponding first-order approximation of the aggregate welfare change in local labor market g in country n is given by

$$\left(\frac{\Delta \mathcal{W}_{ng}}{\mathcal{W}_{ng}} \right)^{agg} \approx \sum_{j=1}^{J} S_{nj} \left(\frac{1 - \hat{\lambda}_{nnj}}{\theta_j} + \frac{1 - \hat{\pi}_{ngj}}{\alpha_{ng}} \right). \tag{3.9}$$

Note that while this expression is qualitatively equivalent to the case of the Translog expenditure function (3.8), these two special cases deliver quantitatively different predictions, even conditional on factual trade shares, $\{S_{nj}\}$, and structural parameters, $\{\theta_j\}$ and $\{\alpha_{ng}\}$. The reason is that both counterfactual trade shares ($\hat{\lambda}_{nnj}$) and employment shares ($\hat{\pi}_{ngj}$) de-

pend on the matrix of counterfactual (relative) changes in wages, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}$. This matrix, in turn, is indirectly affected by model-implied changes in aggregate expenditure shares, $\mathbf{\Delta S}_n = (\text{diag}(\mathbf{1}) + \mathbf{\Omega}_n)^{-1}\left(\tilde{\mathbf{P}}_n + \tilde{\mathbf{b}}_n\right)$, via the system of labor market clearing conditions (2.3). In the case of fixed expenditure shares, apart from the absence of non-homothetic forces ($\mathbf{\Omega}_n = \mathbf{0}_{J\text{x}J}$ and $\tilde{\mathbf{b}}_n = \mathbf{0}_{J\text{x}1}$), there are also no any changes in the aggregate expenditure shares that are driven homothetic forces via changes in relative prices across sectors (i.e., $\tilde{\mathbf{P}}_n = \mathbf{0}_{J\text{x}1}$ in contrast to the Translog case). As a result, $\mathbf{\Delta S}_n = (\text{diag}(\mathbf{1}) + \mathbf{\Omega}_n)^{-1}\left(\tilde{\mathbf{P}}_n + \tilde{\mathbf{b}}_n\right)$ reduces to the case of constant aggregate expenditure shares: $\mathbf{\Delta S}_n = \mathbf{0}_{J\text{x}1}$.

Special Case 3 (Multi-sector extensions of Eaton and Kortum 2002) Note also that if, in addition to the assumption of constant (Cobb-Douglas) shares of consumption, one also dispenses with Roy-type worker heterogeneity, then the corresponding welfare term disappears (i.e., $\sum_{j=1}^{J} S_{nj}(1 - \hat{\pi}_{ngj})/\alpha_{ng} = 0$ for any $g \in \mathcal{G}_n$) and welfare gains are equal for each local labor market and given by

$$\frac{\Delta \mathcal{W}_n}{\mathcal{W}_n} \approx \sum_{j=1}^{J} S_{nj}\left(\frac{1 - \hat{\lambda}_{nnj}}{\theta_j}\right). \tag{3.10}$$

The formula above determines, to the first-order, welfare gains in multi-sector extensions of the standard Eaton and Kortum (2002) model (see, e.g., Costinot, Donaldson, and Komunjer 2012 or Caliendo and Parro 2015 under the case of no input-output linkages).

Special Case 4 (Eaton and Kortum 2002) Lastly, if one assumes a single-sector case (i.e., $J = 1$), then the formula (3.10) reduces to

$$\Delta \mathcal{W}_n / \mathcal{W}_n \approx \frac{1 - \hat{\lambda}_{nn}}{\theta}. \tag{3.11}$$

One can readily infer that it corresponds to the first-order approximation of welfare change in the baseline Eaton and Kortum (2002) model. As a result, the generalized formula presented in this Section nests the main existing cases in the corresponding literature that tend to rely heavily on the assumption of homothetic preferences. So, in addition to the general tractability of the model, the welfare analysis remains analytically tractable as well, and one can predict *ex-ante* the welfare consequences of any foreign/non-local shock. In Section 4, I examine the quantitative implications of the Consumption Inequality channel and other model features by revisiting the impact of the China shock.

4 Application: the China shock

4.1 Data

To quantify the aggregate and distributional impacts of the China shocks, data from several sources are collected. In particular, data on the aggregate expenditure shares $\{S_{nj}\}_{n\in\mathcal{N},j\in\mathcal{J}}$ and trade shares $\{\lambda_{inj}\}_{i,n\in\mathcal{N},j\in\mathcal{J}}$ are constructed from the World Input-Output Database (WIOD). Data on employment shares across different US commuting zones $\{\pi_{gj}\}_{g\in\mathcal{G}_{US},j\in\mathcal{J}}$ are constructed based on the County Business Patterns database (CBP) and the WIOD database. Country-level employment shares for other countries, $\{\pi_{nj}\}_{n\in\mathcal{N},j\in\mathcal{J}}$, are constructed using the WIOD database. As for the US CZ-level income $\{Y_g\}_{g\in\mathcal{G}_{US}}$ and labor force $\{L_g\}_{g\in\mathcal{G}_{US}}$ data, they are obtained from the American Community Survey (ACS). The corresponding country-level data on total income $\{Y_n\}_{n\in\mathcal{N}}$ and labor force $\{L_n\}_{n\in\mathcal{N}}$ for the sample of other countries are obtained from the World Bank Open Data. Based on the data collected, the following subsection describes the calibration strategy.

4.2 Calibration

4.2.1 Geographical distribution of skill heterogeneity

Recall that the supply side of the model implies that the Theil index of income inequality at the local market g in country n is given by $\mathcal{T}_{ng} = \ln\left(\Gamma\left(1-1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi(1-1/\alpha_{ng})}{\alpha_{ng}}$, where α_{ng} is shape parameter of the Fréchet distribution of income and also a Roy-Fréchet (shape) parameter of the local-level distribution of worker productivity. It suggests that one can calibrate parameters $\{\alpha_{ng}\}$ based on information about the distribution of Theil indices across the economy. In terms of data availability, however, Theil indices are rarely available, especially at the level of local markets. Yet, one can alternatively infer $\{\alpha_{ng}\}$ from data on the Gini indices of income inequality. Specifically, in our case of the Fréchet–distributed local incomes, the Gini index of income inequality at the local market g in country n is given by $2^{1/\alpha_{ng}} - 1$. So, given the Fréchet income distribution, there is a direct map between the Gini and Theil indices of income inequality.

Figure 1: Calibrated values of $\alpha_{US,g}$ for 722 CZs.

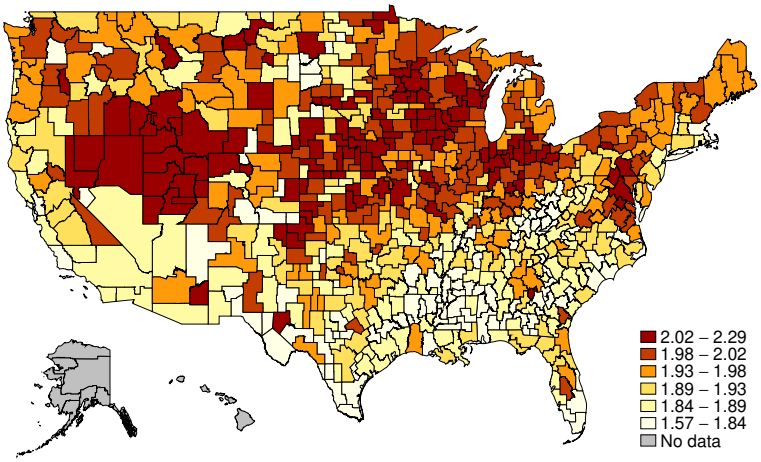


Figure 1 shows calibrated values of $\alpha_{US,g}$ based on information about Gini indices for each of the 722 Commuting Zones across the mainland US. Recall that the higher the value of α_{ng}, the lower the heterogeneity of workers within the local labor market $g \in \mathcal{G}_n$, also implying lower income inequality. So, Figure 1 suggests that Commuting Zones from the northern part of the US generally tend to have lower income inequality than their southern counterparts. However, as one can see from Figure 2, there is no clear relationship between the mean income of Commuting Zones and the income inequality captured by the inferred local Theil index, $\mathcal{T}_{ng} = \ln\left(\Gamma\left(1 - 1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi(1-1/\alpha_{ng})}{\alpha_{ng}}$.

Figure 2: Local income inequality vs. the local mean income across the US CZs.

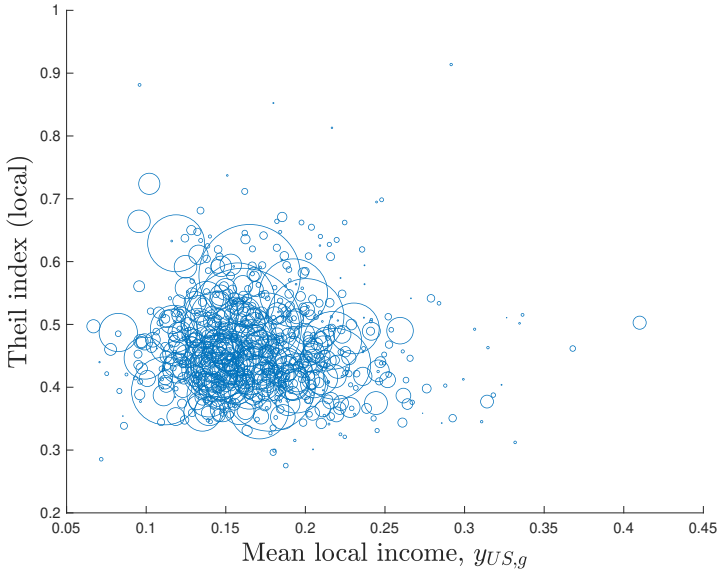


Figure 2 and the direct mapping between $\mathcal{T}_{ng}$ and α_{ng} suggest that, while both "Pro-poor" and "Inequality" margins may contribute a sizable part to the aggregate gains of a particular

local labor market, differences in magnitudes of the Consumption Inequality channel across local labor market are going to be mainly driven by differences in the "Average income" margin. I will discuss it in more detail in the corresponding subsection.

4.2.2 Calibration of the China shock

I calibrate the China shock in the spirit of Caliendo, Dvorkin, and Parro (2019) and Galle, Rodríguez-Clare, and Yi (2023). Specifically, to cope with potential endogeneity driven by the US demand shocks and isolate that part of the China shock that is driven by China's supply forces, I obtain predicted 2000-2007 relative changes in the China-US trade shares by running the ("first-stage") regression in the spirit of the IV-strategy by Autor, Dorn, and Hanson (2013):

$$\frac{\lambda^{2007}_{Ch,US,j}}{\lambda^{2000}_{Ch,US,j}} = \delta_0 + \delta_1 \cdot \frac{\sum_{n \in Other} \lambda^{2007}_{Ch,n,j}}{\sum_{n \in Other} \lambda^{2000}_{Ch,n,j}} + \varepsilon_j,$$

where $Other = \{\text{Australia, Denmark, Finland, Germany, Japan, Spain}\}$ is a set of developed countries similar to that used by Autor, Dorn, and Hanson (2013).

After obtaining the predicted (relative) changes in trade shares, $\hat{\lambda}^{data}_{Ch,US,j} \equiv \frac{\lambda^{2007}_{Ch,US,j}}{\lambda^{2000}_{Ch,US,j}}$, China's sector-specific changes in technology parameters, $\left\{\hat{T}_{China,j}\right\}_{j \in \mathcal{J}}$, are calibrated such that the model-predicted changes in trade shares, $\left\{\hat{\lambda}^{model}_{Ch,US,j}\right\}_{j \in \mathcal{J}}$, match those predicted by the regression analysis, $\left\{\hat{\lambda}^{data}_{Ch,US,j}\right\}_{j \in \mathcal{J}}$.[16]

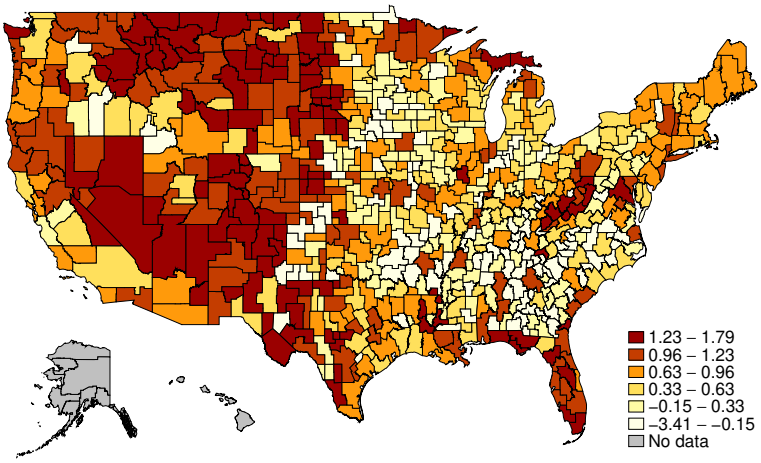
4.3 The aggregate and local market effects of the China shock

In this subsection, I discuss the results of the aggregate effects of the China shock as well as how shock is distributed across local labor markets (CZs). The aggregate welfare impact of the China shock across Commuting Zones averages 0.71%. It implies larger country-level gains than most of the previous findings in the quantitative trade literature based on the simplifying assumption of homothetic expenditure shares. The reason is that a significant part of this increase (0.26%) is attributed to the Consumption Inequality channel that operates via non-homothetic expenditure patterns across sectors. In the following subsection, I will discuss the Consumption Inequality channel in more detail and will quantify each margin of this channel. As for the traditional channels, the Ricardian channel operates at the country level and contributes 0.28% to the aggregate welfare gains of the US economy. The Roy channel, in contrast to the Ricardian one, is heterogeneous across local labor markets, and its aggregate

[16] Calibration of $\{\theta_j\}$ is based on Head and Mayer (2014), while demand parameters $\{\beta_{nj}\}$ are calibrated to match sectoral income elasticities in Fajgelbaum and Khandelwal (2016).

impact across Commuting Zones averaged 0.17%. Figure 3 shows the geographical distribution of welfare changes across different Commuting Zones.

Figure 3: The aggregate welfare change (in %) across CZs.



As one can note from Figure 3, the geographical distribution of the China shock across local labor markets tends to demonstrate spatially correlated patterns. The question is whether Commuting Zones, which, for instance, gained from the shock, tended to have some features in common. Figure 4 helps to shed light on this by plotting the aggregate (local) welfare response versus mean local income for each Commuting Zone.

Figure 4: The aggregate welfare change vs. the local mean income across the US CZs.

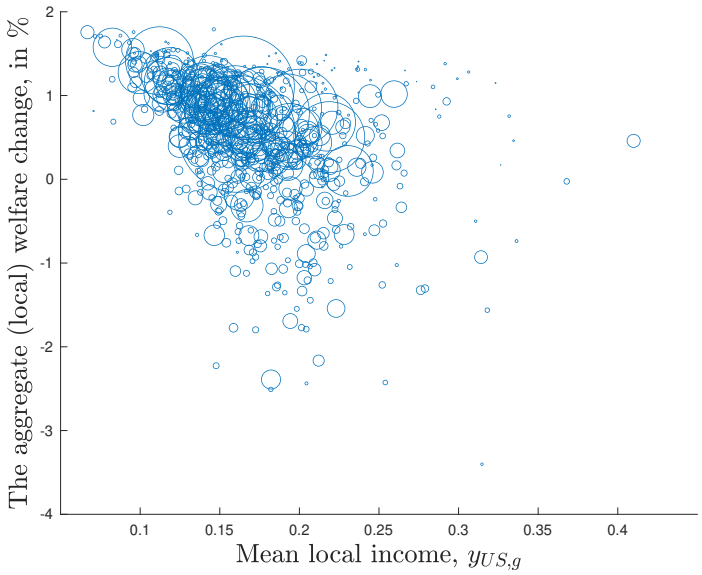


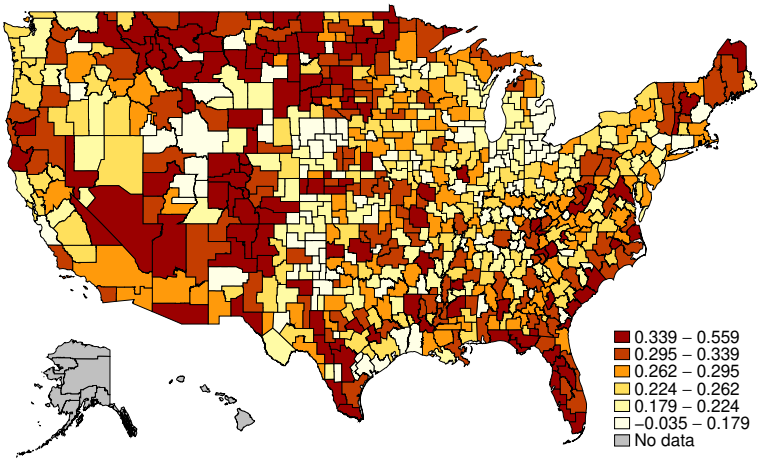
Figure 4 shows that Commuting Zones with lower mean income were more likely to gain from the shock. In contrast, relatively more affluent commuting Zones tended to bear wel-

fare losses from the China shock, also accompanied by large variance in the aggregate local impact. The following subsection uncovers the mechanism of these via the presence of the Consumption Inequality channel.

4.4 The Consumption Inequality channel

In this subsection, I quantitatively study the aggregate and distributional impacts of the Consumption Inequality channel. First, note that while its aggregate impact across Commuting Zones is mainly positive, averaging 0.26%, there is significant heterogeneity in its geographical incidence. As Figure 5 demonstrates, its aggregate local impact ranges from slightly negative (–0.04%) to strictly positive (0.56%).

Figure 5: The Consumption Inequality channel (in %) across CZs.



As before, the question is whether Commuting Zones, which gained relatively more from the shock, tended to have some features in common. Figure 6 shows that low-income Commuting Zones were more likely to gain from the Consumption Inequality channel, implying that the incidence of the shock was mainly "pro-poor".

Figure 6: The Consumption Inequality channel vs. the local mean income across the US CZs.

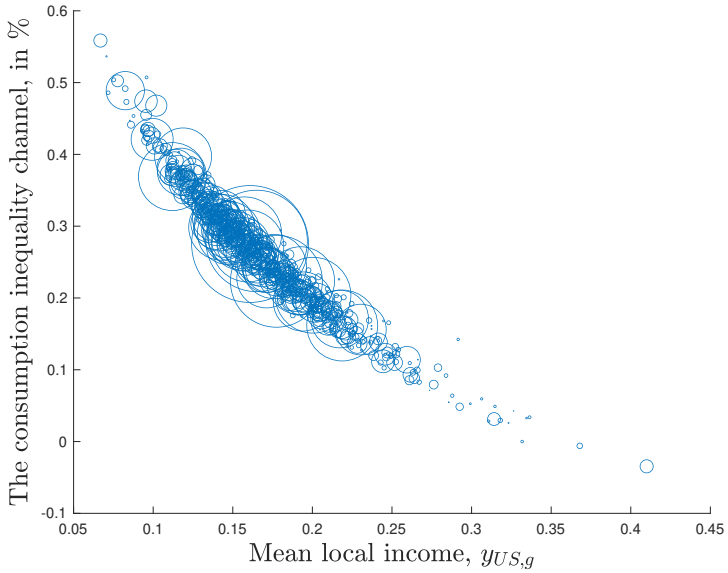


Figure 6 also clearly demonstrates that the Consumption Inequality channel was the main driving force behind the pattern that Commuting Zones with lower mean income were more likely to gain from the China shock (as was previously shown by Figure 4). To better understand the mechanism through which it operates, it is instructive to consider the effects of each of the three margins of the Consumption Inequality channel.

4.4.1 The "Pro-poor" margin of the Consumption Inequality channel

Recall that the "Pro-poor" margin amplifies the aggregate local welfare gains when the nature of a shock is "pro-poor" and, conversely, dampens the aggregate local welfare gains if the nature of a shock is "pro-rich" Figure 7 shows that the resulting effect of the "Pro-poor" margin is positive for each CZ in response to the China shock. It suggests that the China shock generally favored relatively low-income consumers via the reduction in relative prices of low-income elastic sectors.

Figure 7: The "Pro-poor" margin vs. the local mean income across the US CZs.

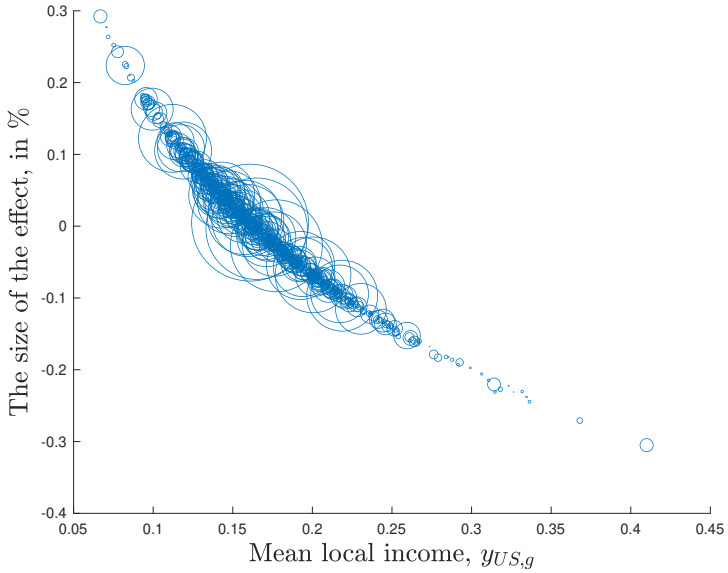


Figure 7 shows that there is no correlation between mean local income and the size of the "Pro-poor" margin effect. The reason is that this margin heavily hinges on the local market's level of skill heterogeneity captured by α_{ng}. However, recalling from Figure 2, there is no clear relationship between the mean local income and the Theil index of income inequality (as well as the level of skill heterogeneity). Despite this, the contribution of this margin to the aggregate local welfare gains is sizable and ranges from 0.15% to 0.48%.

4.4.2 The "Average income" margin of the Consumption Inequality channel

The "Average income" margin creates winners and losers from the Consumption Inequality channel across Commuting zones. In particular, as Figure 8 demonstrates, CZs with relatively lower average income gain from this margin (up to 0.29%), while others – lose (up to –0.30%).

Figure 8: The "Average income" margin vs. the local mean income across the US CZs.

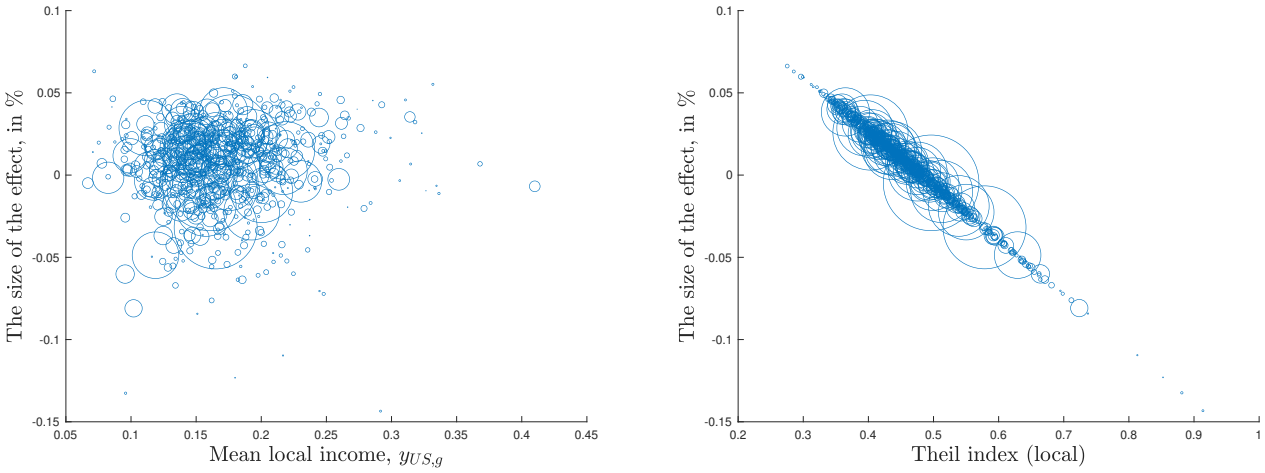


To better understand this pattern, recall from the theory part that the aggregate expenditure shares across sectors depend on average income adjusted by (the exponent of) the Theil index of income inequality. A similar mechanism applies to the consumption pattern of local labor markets. So, conditional on the Theil indices, local labor markets with relatively lower average income spend disproportionally more on low-income elastic than their relatively more affluent counterparts. As a result, the China shock, by reducing relative prices of low-income elastic sectors, benefited low-income Commuting Zones through this margin.

4.4.3 The "Inequality" margin of the Consumption Inequality channel

The "Inequality" margin also creates winners (up to 0.07%) and losers (up to –0.14%) across Commuting Zones. First, note that the left panel of Figure 9 shows that there is no correlation between mean local income and the size of the "Inequality" margin effect. The reason is similar to that described for the "pro-poor" margin. In particular, one can recall from Figure 2 that there is no clear relation between the mean local income and the Theil index of income inequality.

Figure 9: The "Inequality" margin vs. the local mean income and Theil index across the US CZs.



The right panel of Figure 9 shows that the size of the effect is strictly correlated with the value of the local Theil index. It is not surprising, given the definition of the "Inequality" margin. Recall that it is driven by a deviation of the local market's Theil index from the country-wide Theil index of income inequality. Given that the nature of the China shock is "pro-poor", local markets with relatively lower income inequality gain from this margin. To better grasp the intuition, again recall that the aggregate expenditure shares across sectors depend on average income adjusted by (the exponent of) the Theil index of income inequality. So, conditional on the average incomes, local labor markets with relatively lower income inequality spend disproportionally more on low-income elastic than their relatively wealthier counterparts. So, the China shock, by reducing relative prices of low-income elastic sectors, benefited low-income Commuting Zones through this margin.

4.5 The impact of the China shock on other countries

In this subsection, I apply the welfare change formula (3.7) to quantify the aggregate (country-level) impact of the China shock on other countries in the sample. Recall that the formula (3.7) applies to examine the welfare consequences of any foreign/non-local shock. So, it cannot be directly applied to quantify the impact of the China shock on China's economy. Table 1 summarizes the welfare changes for the sample of 38 other countries and the "Rest of the World" (RoW).

Table 1: The aggregate impact of the China shock for a sample of 39 countries.

Country	$\Delta W_n/W_n$	Country	$\Delta W_n/W_n$	Country	$\Delta W_n/W_n$
AUS	1.61%	GRC	–0.04%	NLD	0.65%
AUT	0.78%	HUN	0.04%	POL	2.23%
BEL	0.11%	IND	2.69%	PRT	0.57%
BRA	1.88%	IDN	2.06%	ROU	1.42%
BGR	0.95%	IRL	0.46%	RUS	1.37%
CAN	0.61%	ITA	0.78%	SVK	1.79%
CYP	0.43%	JPN	1.75%	SVN	1.45%
CZE	2.14%	KOR	0.05%	ESP	2.06%
DNK	0.71%	LVA	0.34%	SWE	–0.91%
EST	0.19%	LTU	2.15%	TUR	0.04%
FIN	1.26%	LUX	2.21%	UK	0.40%
FRA	1.77%	MLT	3.72%	TWN	2.62%
GER	0.45%	MEX	1.27%	RoW	2.20%

Note that while the China shock led to aggregate gains for most countries, a few countries experienced net losses. To grasp the intuition, recall that for countries other than the US, I follow the corresponding quantitative trade literature by imposing the simplifying assumption that there is only one labor market within each country. Recall also that if one abstracts away from the geographical distribution of economic activity, then both the "Average income" and the "Inequality" margins of the Consumption Inequality channel disappear for this country, and only the "Pro-poor" margin remains being operated. It implies that, in this particular case, the country-level welfare responses to the China shock are given by

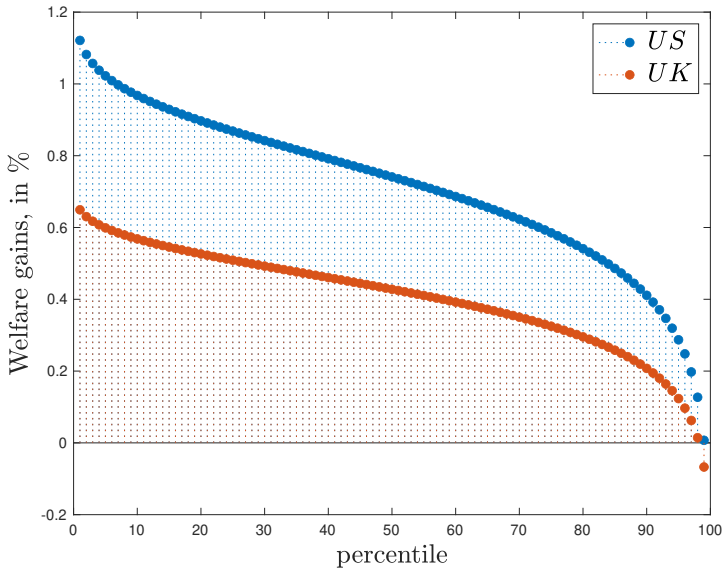
$$\left(\frac{\Delta \mathcal{W}_n}{\mathcal{W}_n}\right)^{agg} \approx \sum_{j=1}^{J} S_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{nj}}{\alpha_n}\right) + \frac{\gamma + \psi\left(1-1/\alpha_n\right)}{\alpha_n} \sum_{j=1}^{J} \beta_{nj} \left(\frac{1-\hat{\lambda}_{nnj}}{\theta_j} + \frac{1-\hat{\pi}_{nj}}{\alpha_n}\right).$$

The expression above implies that the Consumption Inequality channel is driven by the term $\frac{\gamma+\psi(1-1/\alpha_n)}{\alpha_n}$ that magnifies the aggregate welfare gains when the China shock was "pro-poor" for this country, and, conversely, dampens the aggregate welfare effects if the nature of a shock was "pro-rich". So, even if a country increases its foreign trade shares $(1-\hat{\lambda}_{nnj})$ for each sector j in response to the China shock, a country may still bear aggregate losses through, for example, *i*) the Roy channel via a decrease in the country's specialization, *ii*) the Consumption Inequality channel if, for example, the country's trade openness pattern has been shifted towards high-income elastic sectors, thereby benefiting relatively affluent consumers.

4.6 The distributional effects across income quantiles

In the previous sections, the distributional effects of the China shock were studied through the lens of heterogeneous exposure across local labor markets. However, given the presence of heterogeneous workers and non-homothetic expenditure patterns, the model also implies notable distributional consequences along the income distribution. In this subsection, I provide a comparison of the US welfare gains patterns with some other countries.

Figure 10: The distribution of welfare gains across income quantiles in the US vs. the UK.



As one can see from Figure 10, the China shock disproportionately benefited low-income US consumers. This aligns with the previous findings that low-income commuting zones gained from the shock via reductions in the relative prices of low-income elastic sectors. This Figure also compares the distribution of welfare gains across income quantiles in the US versus that in the UK. One can readily see that the lower the percentile, the larger the divergence in welfare gains between the two countries. In particular, while low-income individuals in the UK gain more than high-income individuals in the country, they benefit relatively less than their counterparts in the US.

Figure 11: The distribution of welfare gains across income quantiles in the US vs. Germany.

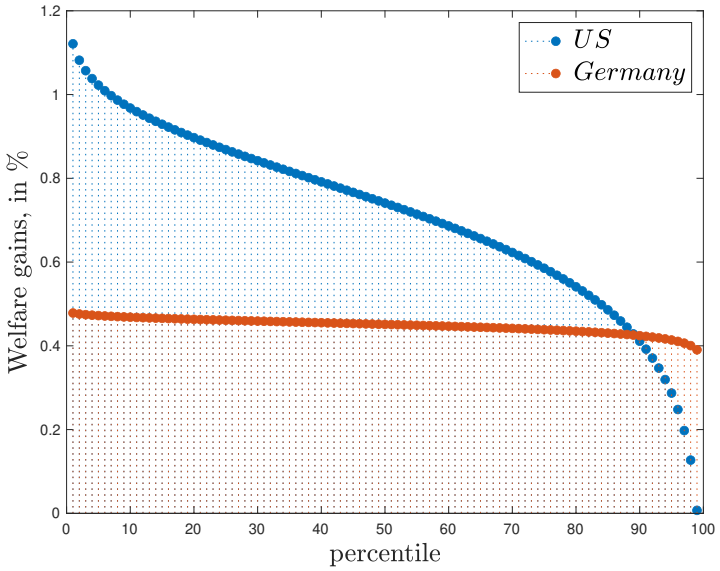
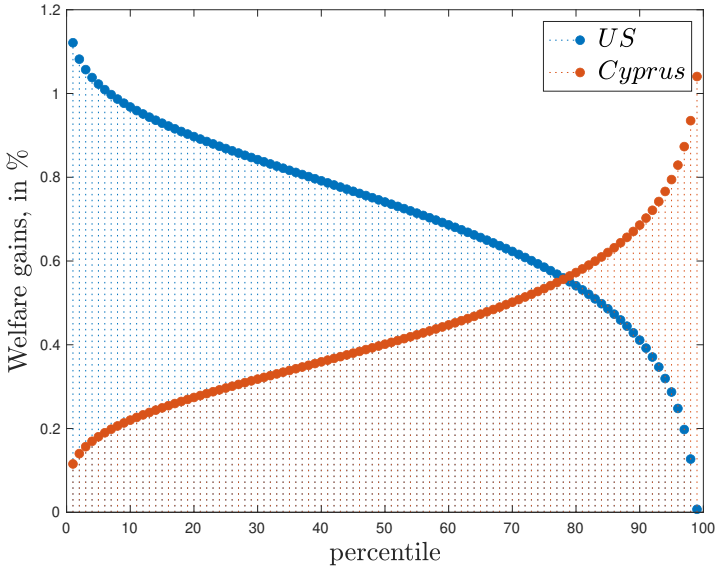


Figure 11 demonstrates that, compared to the US, the distributional effect of the China shock across income quantiles in Germany was quite neutral. It suggests that the gains through non-homothetic forces for low-income individuals were not as pronounced in Germany as in the US.

Figure 12: The distribution of welfare gains across income quantiles in the US vs. Cyprus.



As shown in Figure 12, the China shock may disproportionately benefit high-income US consumers. In particular, in Cyprus, the nature of the China shock was "pro-rich" in that it

reduced the relative prices of high-income elastic sectors. As a result, high-income consumers there disproportionately benefited from the shock.

5 Conclusion

In this paper, I study the aggregate and distributional consequences of trade shocks across different local labor markets and heterogeneous consumers in general equilibrium. I propose a tractable theoretical framework that combines Ricardian-Roy supply forces and comparative advantage with non-homothetic demand in a unified quantitative model. Methodologically, I show how to reconcile an arbitrary first-order approximation to any demand system and heterogeneous consumers with the counterfactual analysis based on relative changes that do not need to identify levels of fundamentals – the exact-hat algebra approach. I also derive a generalized welfare change formula that applies to any demand system. Apart from rich distributional implications, the framework gives rise to the Consumption Inequality channel of the aggregate welfare gains. From a research and policymaker's perspective, the framework's main advantages are that it *i*) allows for studying the heterogeneous impact of trade policy across consumers with different expenditure patterns, *ii*) remains analytically tractable, and *iii*) imposes only mild data requirements for counterfactual predictions and welfare analysis.

I apply the model to revisit the impact of the China shock on a sample of 40 countries and the universe of the US local labor markets represented by 722 US Commuting Zones. I find that while the China shock led to aggregate gains for most of the countries, some countries experienced net losses. The aggregate welfare impact of the shock on the US economy is estimated to be 0.71%, implying larger country-level gains than most of the previous findings in the quantitative trade literature based on the simplifying assumption of homothetic expenditure shares. A significant part of this increase (0.26%) is attributed to the Consumption Inequality channel that operates via non-homothetic expenditure patterns across sectors. For the US, I also quantify the contribution of each margin of the Consumption Inequality channel. I find that each of these margins accounts for a sizable part of the distributional impact of the China shock across local labor markets and that Commuting Zones with lower mean income were more likely to gain from the shock. At the individual level, the China shock favored relatively poor consumers by reducing the relative prices of low-income elastic sectors. Moreover, the distribution of welfare gains across income quantiles varied significantly between countries.

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Appendix

A Proofs

In Appendix A, I provide proofs of the main propositions as well as some auxiliary derivations.

A.1 Proof of Proposition 1

Proof. The proof consists of two parts. The first part applies the aggregation result from Deaton and Muellbauer (1980) to the case of continuous distribution. The second part exploits the decomposition properties of the Theil index.

Part i

By definition, the share of sector j in the aggregate expenditures of country n is the weighted integral of individual market shares:

$$S_{nj} \equiv \frac{\int_{\mathscr{Y}} y s_{nj}(\mathbf{P}_n, y)\, \mathrm{d}H_n(y)}{\int_{\mathscr{Y}} y\, \mathrm{d}H_n(y)} = \int_{\mathscr{Y}} \frac{y}{y_n} s_{nj}(\mathbf{P}_n, y)\, \mathrm{d}H_n(y).$$

So, if $s_{nj}\left(\mathbf{P}_n, y\right) = \alpha_{nj} + \sum_{k=1}^{J} \gamma_{njk} \ln P_{nk} + \beta_{nj} \ln\left(y/a(\mathbf{P}_n)\right)$, we obtain

$$S_{nj} = \int_{\mathscr{Y}} \frac{y}{y_n} s_j\left(\mathbf{P}_n, y\right) \mathrm{d}H_n(y) = \alpha_{nj} + \sum_{k=1}^{J} \gamma_{njk} \ln P_{nk} - \ln a(\mathbf{P}_n) + \beta_{nj} \int_{\mathscr{Y}} \frac{y}{y_n} \ln y\, \mathrm{d}H_n(y).$$

Now, consider the last term of this expression. One can readily verify that

$$\int_{\mathscr{Y}} \frac{y}{y_n} \ln y\, \mathrm{d}H_n(y) = \ln y_n + \int_{\mathscr{Y}} \frac{y}{y_n} \ln y\, \mathrm{d}H(y) - \int_{\mathscr{Y}} \frac{y}{y_n} \ln y_n\, \mathrm{d}H_n(y)$$

$$= \ln y_n + \underbrace{\int_{\mathscr{Y}} \ln\left(y/y_n\right)^{y/y_n}\, \mathrm{d}H_n(y)}_{=\mathcal{T}_n} = \ln y_n + \mathcal{T}_n = \ln\left(y_n \cdot \exp(\mathcal{T}_n)\right).$$

Part ii

Next, we derive the mean income in an arbitrary country n, y_n, and the corresponding country-level Theil index of income inequality.

First, recall that the income distribution at local market g in country n is given by

$$H_{ng}(y) = \exp\left\{-\left(y/\Upsilon_{ng}\right)^{-\alpha_{ng}}\right\},$$

where $\Upsilon_{ng} \equiv \sum_{j=1}^{J} A_{ngj} w_{nj}^{\alpha_{ng}}$.

Hence, the country-level distribution of income can be represented as a (finite) mixture distribution:

$$H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} H_{ng}(y),$$

where ℓ_{ng} is the labor share of the market g within country n (i.e., $\sum_{g \in \mathcal{G}_n} \ell_{ng} = 1$).

As a result, exploiting the properties of moments of the mixture distribution yields

$$y_n \equiv \int_{\mathscr{Y}} y \, \mathrm{d}H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} y \, \mathrm{d}H_{ng}(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \Upsilon_{ng} \Gamma\left(1 - 1/\alpha_{ng}\right)$$

$$= \sum_{g \in \mathcal{G}_n} \sum_{j=1}^{J} \ell_{ng} \Gamma\left(1 - 1/\alpha_{ng}\right) A_{ngj} w_{nj}^{\alpha_{ng}}.$$

Now, I derive the country-level Theil index. By definition of the Theil index we obtain

$$\mathcal{T}_n \equiv \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \mathrm{d}H_n(y) = \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \sum_{g \in \mathcal{G}_n} \ell_{ng} \mathrm{d}H_{ng}(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} \frac{y_{ng}}{y_n} \frac{y}{y_{ng}} \ln\left(\frac{y}{y_{ng}} \frac{y_{ng}}{y_n}\right) \mathrm{d}H_{ng}(y)$$

$$= \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} \frac{y_{ng}}{y_n} \frac{y}{y_{ng}} \ln\left(\frac{y}{y_{ng}}\right) \mathrm{d}H_{ng}(y) + \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} \frac{y_{ng}}{y_n} \frac{y}{y_{ng}} \ln\left(\frac{y_{ng}}{y_n}\right) \mathrm{d}H_{ng}(y)$$

$$= \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng} + \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln\left(\frac{y_{ng}}{y_n}\right) \underbrace{\int_{\mathscr{Y}} \frac{y}{y_{ng}} \mathrm{d}H_{ng}(y)}_{=1}.$$

Thus, we obtain

$$\mathcal{T}_n \equiv \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \mathrm{d}H_n(y) = \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng}}_{\text{within}_n} + \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln \frac{y_{ng}}{y_n}}_{\text{between}_n}.$$

Note also that the expression above reflects the decomposability property of the Theil index into the *within*– and *between*–group components. □

A.2 Proof of Proposition 3

Proof. As was previously derived, the Theil index is given by

$$\mathcal{T}_n = \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng} + \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln \frac{y_{ng}}{y_n}.$$

Recall that $\ell'_{ng} = \ell_{ng}$ and $\mathcal{T}'_{ng} = \mathcal{T}_{ng}$ for any $n \in \mathcal{N}$ and $g \in \mathcal{G}_n$. Hence, applying *the*

exact-hat algebra approach yields

$$\mathcal{T}_n' = \sum_{g\in\mathcal{G}_n} \frac{\ell_{ng}\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\mathcal{T}_{ng} + \sum_{g\in\mathcal{G}_n} \frac{\ell_{ng}\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\ln\left(\frac{\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\right)$$

$$= \sum_{g\in\mathcal{G}_n} \frac{\frac{L_{ng}}{L_n}\hat{y}_{ng}\frac{Y_{ng}}{L_{ng}}}{\hat{y}_n\frac{Y_n}{L_n}}\mathcal{T}_{ng} + \sum_{g\in\mathcal{G}_n} \frac{\frac{L_{ng}}{L_n}\hat{y}_{ng}\frac{Y_{ng}}{L_{ng}}}{\hat{y}_n\frac{Y_n}{L_n}}\left(\ln\frac{\hat{y}_{ng}\frac{Y_{ng}}{L_{ng}}}{\hat{y}_n\frac{Y_n}{L_n}}\right).$$

As a result, a counterfactual value of the Theil index is given by

$$\mathcal{T}_n' = \underbrace{\sum_{g\in\mathcal{G}_n} \frac{\hat{Y}_{ng}Y_{ng}}{\hat{Y}_n Y_n}\mathcal{T}_{ng}}_{\text{within}_n'} + \underbrace{\sum_{g\in\mathcal{G}_n} \frac{\hat{Y}_{ng}Y_{ng}}{\hat{Y}_n Y_n}\ln\left(\frac{\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\right)}_{\text{between}_n'}.$$

□

A.3 Proof of Proposition 4

Proof. Consider the equivalent variation formula that applies to any demand system,

$$\frac{\mathrm{d}\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \equiv \frac{EV}{y} = \underbrace{\frac{\mathrm{d}y}{y}}_{\substack{earnings\\channel}} \underbrace{-\sum_{j=1}^{J} S_{nj}\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{agg.\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}\left(\tilde{s}_{ngj}-S_{nj}\right)\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{local\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}\left(s_{ngj}(y)-\tilde{s}_{ngj}\right)\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{individual\ expenditure\ effect}.$$

In this model, the earnings channel is expressed as changes in the expected income of individuals with current income level y. Given that labor is perfectly substitutable in production, the earnings channel is equivalent to the (expected changes in the average income y_{ng}). So, we have

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \mathrm{d}\ln y_{ng} - \sum_{j=1}^{J} S_{nj}\mathrm{d}\ln P_{nj} - \sum_{j=1}^{J}\left(\tilde{s}_{ngj}-S_{nj}\right)\mathrm{d}\ln P_{nj} - \sum_{j=1}^{J}\left(s_{ngj}(y)-\tilde{s}_{ngj}\right)\mathrm{d}\ln P_{nj}.$$

Now, we want to expand the expression for $\mathrm{d}\ln P_{js}$. Recall that

$$P_{nj} = \zeta_j\,\Phi_{nj}^{-1/\theta_j} = \zeta_j\left(\sum_{i\in\mathcal{N}} T_{ij}\left(\tau_{inj}w_{ij}\right)^{-\theta_j}\right)^{-1/\theta_j}$$

where $\zeta_j = \left[\Gamma\left(1-\frac{\sigma_j-1}{\theta_j}\right)\right]^{1/(1-\sigma_j)}$ is some sector-specific constant.

Now, note that taking logs and totally differentiating this expression yields

$$\mathrm{d}\ln P_{nj} = -\left(1/\theta_j\right)\frac{\sum_{i\in\mathcal{N}}\mathrm{d}\left[T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\right]}{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}} \iff \mathrm{d}\ln P_{nj} = -\frac{1}{\theta_j}\frac{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\frac{\mathrm{d}\left[T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\right]}{T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}}{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}$$

$$\mathrm{d}\ln P_{nj} = -\left(1/\theta_j\right)\frac{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\mathrm{d}\ln\left(T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\right)}{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}$$

$$\mathrm{d}\ln P_{nj} = \sum_{i\in\mathcal{N}}\frac{T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}{\sum_{n'\in\mathcal{N}}T_{n'j}(\tau_{n'nj}w_{n'j})^{-\theta_j}}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij})$$

Recall that $\lambda_{inj} = \frac{T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}{\sum_{n'}T_{n'j}\left(\tau_{n'nj}w_{n'j}\right)^{-\theta_j}}$. Hence, the formula above is equivalent to

$$\mathrm{d}\ln P_{nj} = \sum_{i\in\mathcal{N}}\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}).$$

Hence, the formula for $\mathrm{d}\ln W_{ng}$ can be re-written as follows:

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \mathrm{d}\ln y_{ng} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}S_{nj}\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij})$$

$$-\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\tilde{s}_{ngj}-S_{nj}\right)\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(s_{ngj}(y)-\tilde{s}_{ngj}\right)\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}). \tag{A.1}$$

Now, we want to expand the first term of the expression above, $\mathrm{d}\ln y_{ng}$. First, note that $\mathrm{d}\ln y_{ng} = \mathrm{d}\ln Y_{ng}$. Recall that $y_{ng} = \frac{Y_{ng}}{L_{ng}} \propto \left(\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$. Local market g's productivity levels, $\{A_{ngj}\}_{j=1}^{J}$, are assumed to remain constant in a counterfactual scenario. As a result, we obtain

$$\ln Y_{ng} = const + \left(1/\alpha_{ng}\right)\ln\left(\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\right),$$

Taking the total differential yields

$$\mathrm{d}\ln Y_{ng} = \left(1/\alpha_{ng}\right)\mathrm{d}\ln\left(\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\right)$$

$$\mathrm{d}\ln Y_{ng} = \left(1/\alpha_{ng}\right)\frac{\sum_{j\in\mathcal{J}}A_{ngj}\mathrm{d}\left(w_{nj}^{\alpha_{ng}}\right)}{\sum_{k\in\mathcal{J}}A_{ngk}w_{nk}^{\alpha_{ng}}} \iff \mathrm{d}\ln Y_{ng} = \left(1/\alpha_{ng}\right)\frac{\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\frac{\mathrm{d}\left(w_{nj}^{\alpha_{ng}}\right)}{w_{nj}^{\alpha_{ng}}}}{\sum_{k\in\mathcal{J}}A_{ngk}w_{nk}^{\alpha_{ng}}}$$

$$\mathrm{d}\ln Y_{ng} = \frac{1}{\alpha_{ng}} \frac{\sum_{j\in\mathcal{J}} A_{ngj} w_{nj}^{\alpha_{ng}} \mathrm{d}\ln\left(w_{nj}^{\alpha_{ng}}\right)}{\sum_{k\in\mathcal{J}} A_{ngk} w_{nk}^{\alpha_{ng}}}$$

$$\iff \mathrm{d}\ln Y_{ng} = \sum_{j\in\mathcal{J}} \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_{k\in\mathcal{J}} A_{ngk} w_{nk}^{\alpha_{ng}}} \mathrm{d}\ln w_{nj}.$$

Recall that $\pi_{ngj} = \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_k A_{ngk} w_{nk}^{\alpha_{ng}}}$, so the expression above is equivalent to

$$\implies \mathrm{d}\ln Y_{ng} = \sum_{j\in\mathcal{J}} \pi_{ngj} \mathrm{d}\ln w_{nj}.$$

Hence, we can re-write the expression for $\mathrm{d}\ln W_{ng}$ as follows

$$\mathrm{d}\ln \mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}} \pi_{ngj} \mathrm{d}\ln w_{nj} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} S_{nj}\lambda_{inj} \mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij})$$

$$- \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} \left(\tilde{s}_{ngj} - S_{nj}\right)\lambda_{inj} \mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} \left(s_{ngj}(y) - \tilde{s}_{ngj}\right)\lambda_{inj} \mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}). \tag{A.2}$$

Now, note that the formula for trade share $\lambda_{inj} = \frac{T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}{\sum_{n'} T_{n'j}\left(\tau_{n'nj}w_{n'j}\right)^{-\theta_j}}$. Recall that domestic technology levels, $\{T_{nj}\}_{j=1}^{J}$, are assumed to remain constant. Hence,

$$\ln\left(\lambda_{inj}/\lambda_{nnj}\right) = \ln\left(\left(\frac{T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}}{T_{nj}^{-1/\theta_j}w_{nj}}\right)^{-\theta_j}\right) \implies \frac{\mathrm{d}\ln\left(\lambda_{inj}/\lambda_{nnj}\right)}{\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}/w_{nj}\right)} = -\theta_j,$$

Similarly, recall that local market g's productivity levels, $\{A_{ngj}\}_{j=1}^{J}$, are assumed to remain constant in a counterfactual scenario. So, one can readily infer that the formula for employment share $\pi_{ngj} = \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_k A_{ngk} w_{nk}^{\alpha_{ng}}}$ implies

$$\ln\left(\pi_{ngj}/\pi_{ngk}\right) = \ln\left(\frac{A_{ngj}}{A_{ngk}}\left(\frac{w_{nj}}{w_{nk}}\right)^{\alpha_{ng}}\right) \implies \frac{\mathrm{d}\ln\left(\pi_{ngj}/\pi_{ngk}\right)}{\mathrm{d}\ln\left(\frac{w_{nj}}{A_{ngk}^{1/\alpha_{ng}}w_{nk}}\right)} = \alpha_{ng}.$$

Now, solving for $\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right)$ and $\mathrm{d}\ln w_{nj}$, we obtain:

$$\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right) = \frac{\mathrm{d}\ln\left(\lambda_{inj}/\lambda_{nnj}\right)}{-\theta_j} + \mathrm{d}\ln w_{nj},$$

$$\mathrm{d}\ln w_{nj} = \frac{\mathrm{d}\ln\left(\pi_{ngj}/\pi_{ngk}\right)}{\alpha_{ng}} + \mathrm{d}\ln\left(A_{ngk}^{1/\alpha_{ng}}w_{nk}\right).$$

Combining these two equations yields

$$\Longrightarrow \mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right) = \frac{\mathrm{d}\ln\left(\lambda_{inj}/\lambda_{nnj}\right)}{-\theta_j} + \frac{\mathrm{d}\ln\left(\pi_{ngj}/\pi_{ngk}\right)}{\alpha_{ng}} + \mathrm{d}\ln\left(A_{ngk}^{1/\alpha_{ng}}w_{nk}\right)$$

For brevity, I temporarily re-write the expression for $\mathrm{d}\ln\mathcal{W}_{ng}(y)$ in an equivalent, yet a more compact way without decomposing the expenditure channel on different effects:

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}}\pi_{ngj}\mathrm{d}\ln w_{nj} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right).$$

Now, combining it with derived expressions for $\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right)$ and $\mathrm{d}\ln w_{nj}$ yields

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) =$$

$$\sum_{j\in\mathcal{J}}\pi_{ngj}\left(\frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}} + \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\right) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln\frac{\lambda_{inj}}{\lambda_{nnj}}}{-\theta_j} + \frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}} + \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\right).$$

One can readily verify that

$$\sum_{j\in\mathcal{J}}\pi_{ngj}\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk}) = \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\left(\sum_{j\in\mathcal{J}}\pi_{ngj}\right) = \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})$$

and

$$-\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk}) = -\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\left(\sum_{j\in\mathcal{J}}s_{ngj}(y)\sum_{i\in\mathcal{N}}\lambda_{inj}\right) = -\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk}).$$

So, we obtain

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}}\pi_{ngj}\left(\frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln\frac{\lambda_{inj}}{\lambda_{nnj}}}{-\theta_j} + \frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right).$$

Next, recalling that $\sum_{j=1}^{J}\pi_{ngj} = 1$ and $\sum_{j=1}^{J}\mathrm{d}\pi_{ngj} = 0$, one can readily verify that

$$\sum_{j\in\mathcal{J}}\pi_{ngj}\left(\frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right) = \frac{1}{\alpha_{ng}}\left[\sum_{j\in\mathcal{J}}\mathrm{d}\pi_{ngj} - \sum_{j\in\mathcal{J}}\pi_{ngj}\mathrm{d}\ln\pi_{ngk}\right] = -\frac{\mathrm{d}\ln\pi_{jgk}}{\alpha_{ng}}.$$

Hence, rearranging terms, formula for $\mathrm{d}\ln\mathcal{W}_{ng}(y)$ can be re-written as follows

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = -\frac{\mathrm{d}\ln\pi_{jgk}}{\alpha_{ng}} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln\frac{\lambda_{inj}}{\lambda_{nnj}}}{-\theta_j} + \frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right)$$

$$= -\frac{\mathrm{d}\ln \pi_{jgk}}{\alpha_{ng}} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{inj}}{-\theta_j} - \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\right)$$

$$= -\frac{\mathrm{d}\ln \pi_{jgk}}{\alpha_{ng}} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right) + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{inj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\right).$$

Now, note that the third term $\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{inj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\right)$ is equal to

$$\sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\sum_{i\in\mathcal{N}} \mathrm{d}\lambda_{inj} + \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj} = \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}.$$

As a result, we obtain the following expression for $\mathrm{d}\ln \mathcal{W}_{ng}(y)$:

$$\mathrm{d}\ln \mathcal{W}_{ng}(y) = -\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right).$$

Recalling that $\sum_{i=1}^{N}\lambda_{inj} = 1$ yields

$$\mathrm{d}\ln \mathcal{W}_{ng}(y) = -\sum_{j\in\mathcal{J}} s_{ngj}(y)\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right).$$

$$\iff \mathrm{d}\ln \mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}}\left(\mathrm{d}\ln \lambda_{nnj}^{-s_{ngj}(y)/\theta_j} + \mathrm{d}\ln \pi_{ngj}^{-s_{ngj}(y)/\alpha_{ng}}\right)$$

$$\iff \mathrm{d}\ln \mathcal{W}_{ng}(y) = -\sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\mathrm{d}\ln \lambda_{nnj} - \sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\alpha_{ng}}\mathrm{d}\ln \pi_{ngj}. \tag{A.3}$$

As a result, for large shocks we obtain the following approximated (up to the first-order) formula:

$$\hat{\mathcal{W}}_{ng}(y) - 1 \approx -\sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\left(\hat{\lambda}_{nnj} - 1\right) - \sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\alpha_{ng}}\left(\hat{\pi}_{ngj} - 1\right).$$

$$\iff \hat{\mathcal{W}}_{ng}(y) - 1 \approx \sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\left(1 - \hat{\lambda}_{nnj}\right) + \sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\alpha_{ng}}\left(1 - \hat{\pi}_{ngj}\right),$$

Recalling that $\sum_{j\in\mathcal{J}} s_{ngj}(y) = 1$ for any level of income y, we can simplify the expression above as

$$\hat{\mathcal{W}}_{ng}(y) - 1 \approx \underbrace{\sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\left(1 - \hat{\lambda}_{nnj}\right)}_{consumer's\ Ricardian-type\ gains} + \underbrace{\frac{1 - \sum_{j\in\mathcal{J}} s_{ngj}(y)\hat{\pi}_{ngj}}{\alpha_{ng}}}_{consumer's\ Roy-type\ gains}, \tag{A.4}$$

where $\hat{\lambda}_{nnj} = \frac{\hat{w}_{nj}^{-\theta_j}}{\sum_{n'}\lambda_{n'nj}\hat{T}_{n'j}\left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$ *and* $\hat{\pi}_{ngj} = \frac{\hat{w}_{nj}^{\alpha_{ng}}}{\sum_k \pi_{igk}\hat{w}_{ik}^{\alpha_{ig}}}$. □

A.4 Proof of Proposition 10

Proof. The proof is based on a general formula (C.1) for decomposing a welfare change as a response to small price changes. Here, I specify the implications of this formula under the model's supply and demand side.

First, note that $\sum_{j=1}^{J} S_{nj} = 1$ implies that $-\sum_{j=1}^{J} S_{nj} \frac{\mathrm{d}P_{nj}}{P_{nj}} = 1 - \sum_{j=1}^{J} S_{nj} \hat{P}_{nj}$. Note also that the Almost Ideal Demand System implies $(\tilde{s}_{ngj} - S_{nj}) = \left(\ln \frac{\tilde{y}_{ng}}{\tilde{y}_n}\right) \beta_{nj}$ and $(s_{ngj}(y) - \tilde{s}_{ngj}) = \left(\ln \frac{y}{\tilde{y}_{ng}}\right) \beta_{nj}$. Moreover, by definition of sectoral income (semi-)elasticities, $\sum_{j=1}^{J} \beta_{nj} = 0$. Employing these results yields

$$\sum_{j=1}^{J} (\tilde{s}_{ngj} - S_{nj}) \frac{\mathrm{d}P_{nj}}{P_{nj}} = -\left(\ln \frac{\tilde{y}_{ng}}{\tilde{y}_n}\right) \sum_{j=1}^{J} \beta_{nj} \hat{P}_{nj},$$

$$\sum_{j=1}^{J} (s_{ngj}(y) - \tilde{s}_{ngj}) \frac{\mathrm{d}P_{nj}}{P_{nj}} = -\left(\ln \frac{y}{\tilde{y}_{ng}}\right) \sum_{j=1}^{J} \beta_{nj} \hat{P}_{nj}.$$

Now, recalling that $\hat{P}_{nj} = \hat{w}_{nj} \left(\hat{\lambda}_{nnj} / \hat{T}_{nj}\right)^{1/\theta_j}$, yields the following expression:

$$\frac{\Delta \mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \underbrace{\hat{y}_{ng} - 1}_{\substack{\text{earnings} \\ \text{channel}}} + \underbrace{1 - \sum_{j=1}^{J} S_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{\text{agg. expenditure effect}} \underbrace{- \left(\ln \frac{\tilde{y}_{ng}}{\tilde{y}_n}\right) \sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{\text{local expenditure effect}} \underbrace{- \left(\ln \frac{y}{\tilde{y}_{ng}}\right) \sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{\text{individual expenditure effect}},$$

where $\hat{y}_{ng} = \left(\sum_{j=1}^{J} \pi_{ngj} \hat{A}_{ngj} \hat{w}_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$ *and* $\hat{\lambda}_{nnj} = \frac{\hat{T}_{nj} \hat{w}_{nj}^{-\theta_j}}{\sum_{n'}^{N} \lambda_{n'nj} \hat{T}_{n'j} \left(\hat{\tau}_{n'nj} \hat{w}_{n'j}\right)^{-\theta_j}}$. □

B Extensions

B.1 Heterogeneous occupations

Each local market (CZ) is now enriched with many occupations indexed $o \in \mathcal{O} \equiv \{1, 2, .., O\}$. While production technologies are national, CZs are heterogeneous in stock of workers across occupations. Workers are assumed to be mobile across sectors yet immobile across occupations and CZs. A firm with productivity $\tilde{z}$ has the following production function to produce a variety ω:

$$q(\omega) = \tilde{z}(\omega) \left(\sum_{o=1}^{O} \mu_o^{1/\varsigma} o^{\frac{\varsigma-1}{\varsigma}} \right)^{\frac{\varsigma}{\varsigma-1}}.$$

Cost minimization implies that the unit cost is $\frac{\left(\sum_{o=1}^{O} \mu_o w_o^{1-\varsigma}\right)^{\frac{1}{1-\varsigma}}}{\tilde{z}(\omega)}$ and the **cost share** of occupation o is given by:

$$\Xi_o \equiv \frac{w_o o}{\sum_{o'=1}^{O} w_{o'} o'} = \frac{\mu_o w_o^{1-\varsigma}}{\sum_{o'=1}^{O} \mu_{o'} w_{o'}^{1-\varsigma}}, \tag{B.1}$$

where Ξ_o is assumed to be country– and sector–specific (Ξ_{noj}).

B.1.1 Labor supply

Each worker from CZ $g \in \mathcal{G}_i$ and occupation $o \in \mathcal{O}$ draws productivity vector $\mathbf{z} = (z_1, z_2, .., z_J)$ and makes a choice on which sector to apply for employment:

$$F_{igoj}(z) \equiv \Pr\left\{z_{igo}(j) \leq z\right\} = \exp\left\{-A_{igoj} z^{-\alpha_{igo}}\right\}.$$

Let $\Omega_{ioj} \equiv \{\boldsymbol{z} \text{ s.t. } w_{ioj} z_j \geq w_{iok} z_k \; \forall k\}$, where w_{ioj} is the occupation o's **wage** (per efficiency unit) in sector j of country i. Similarly, π_{igoj} now denotes the country i's share of workers in CZ g that are employed in sector j and occupation o:

$$\pi_{igoj} = \int_{\Omega_{ioj}} dF_{igo}(\boldsymbol{z}) = \frac{A_{igoj} w_{ioj}^{\alpha_{igo}}}{\Upsilon_{igo}^{\alpha_{igo}}} = \frac{A_{igos} w_{ioj}^{\alpha_{igo}}}{\sum_k A_{igok} w_{iok}^{\alpha_{igo}}},$$

where $\Upsilon_{igo} \equiv \left(\sum_k^J A_{igok} w_{iok}^{\alpha_{igo}}\right)^{1/\alpha_{ig}}$ and $F_{ig}(\boldsymbol{z})$ is the joint probability distribution of $\boldsymbol{z}$.

Let Z_{igoj} to denote the sum of efficiency units supplied to a sector j among workers in a local labor market $g \in \mathcal{G}_i$ and occupation $o \in \mathcal{O}$:

$$Z_{igoj} \equiv L_{igo} \int_{\Omega_{ioj}} z_{igoj} dF_{igo}(\boldsymbol{z}) \propto \frac{\pi_{igoj} L_{igo}}{w_{ioj}} \Upsilon_{igo}.$$

The country i's average income of workers of a local labor market g in occupation o is given by:

$$y_{igo} \equiv \frac{Y_{igo}}{L_{igo}} = \frac{\sum_k w_{iok} Z_{igok}}{L_{igo}} \propto \Upsilon_{igo}.$$

Note also that the country i's share of income earned by workers of local labor market g in occupation o and sector j is given by π_{igoj}:

$$\frac{w_{ioj} Z_{igoj}}{\sum_k w_{iok} Z_{igok}} = \pi_{igoj}.$$

B.1.2 Comparative statics

A counterfactual eq-m system solves for $\hat{\mathbf{w}} \equiv \{\hat{w}_{ioj}\}$, where each of the NxOxJ equations ($\mathcal{D}'_{ioj}\{\hat{w}_{ioj}\} = 0$) can be written as:

$$\hat{\Xi}_{ioj}\Xi_{ioj} \sum_n \hat{\lambda}_{inj}\lambda_{inj}\hat{S}_{nj}S_{nj}\left(\sum_{g\in\mathcal{G}_i}\hat{Y}_{ng}Y_{ng} + \hat{D}_n D_n\right) = \sum_{g\in\mathcal{G}_i}\hat{\pi}_{igoj}\pi_{igoj}\hat{Y}_{igo}Y_{igo}, \tag{B.2}$$

where

i) $\hat{\Xi}_{ioj} = \frac{\hat{w}_{ioj}^{1-\varsigma_i}}{\sum_{o'=1}^{O}\Xi_{io'j}\left(\hat{w}_{io'j}\right)^{1-\varsigma_i}}$ is the relative change in the production cost share of occupation o in sector j of country i;

ii) $\hat{\lambda}_{inj} = \frac{\hat{T}_{ij}\hat{\tau}_{inj}^{-\theta_j}\left(\sum_{o'=1}^{O}\Xi_{io'j}\hat{w}_{io'j}^{1-\varsigma_i}\right)^{\frac{-\theta_j}{1-\varsigma_i}}}{\sum_{n'}\lambda_{n'nj}\hat{T}_{n'j}\hat{\tau}_{n'nj}^{-\theta_j}\left(\sum_{o'=1}^{O}\Xi_{n'o'j}\hat{w}_{n'o'j}^{1-\varsigma_{n'}}\right)^{\frac{-\theta_j}{1-\varsigma_{n'}}}}$ is the relative change in trade share of country i in spendings of country n within sector j;

iii) $\hat{Y}_{ngo} = \left(\sum_k \pi_{ngok}\hat{A}_{ngok}\hat{w}_{nok}^{\alpha_{ngo}}\right)^{1/\alpha_{ngo}}$ is the change in total income of occupation–o workers in CZ g of country n;

iv) $\hat{\pi}_{igos} = \frac{\hat{A}_{igos}\hat{w}_{ios}^{\kappa_{igo}}}{\sum_k \pi_{igok}\hat{A}_{igok}\hat{w}_{iok}^{\kappa_{igo}}}$ is the change in employment sectoral share of occupation–o workers in CZ g of country n.

B.1.3 Welfare Analysis. Heterogeneous Occupations

Proposition 7. *The welfare change of income-y individuals employed in occupation o at local market g in country n can be decomposed into five channels:*

$$\frac{\mathrm{d}\mathcal{W}_{ngo}(y)}{\mathcal{W}_{ngo}(y)} \equiv \frac{EV}{y}$$

$$= \underbrace{\frac{\mathrm{d}y_{ngo}}{y_{ngo}}}_{\substack{earnings\\channel}} \underbrace{-\sum_{j=1}^{J} S_{nj}\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{agg.\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}(\tilde{s}_{ngj}-S_{nj})\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{local\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}(\tilde{s}_{ngoj}-\tilde{s}_{ngj})\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{occupational\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}(s_{ngoj}(y)-\tilde{s}_{ngoj})\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{individual\ expenditure\ effect}, \tag{B.3}$$

where $\tilde{s}_{ngj}$ denotes an aggregate share of sector j at local market g in country n; $\tilde{s}_{ngoj}$ is an aggregate share of sector j for occupation o at local market g in country n.

In addition to the previous effect, we now also have the occupational expenditure effect that operates at the local occupation level.

Proposition 8. *Given a shock to trade costs and/or technology levels, a first-order approximation to the (expected) exact welfare change of income-y individuals employed in occupation o at local market g in country n is given by*

$$\frac{\Delta\mathcal{W}_{ngo}(y)}{\mathcal{W}_{ngo}(y)} \approx \underbrace{\hat{y}_{ngo}-1}_{\substack{earnings\\channel}} \underbrace{+1-\sum_{j=1}^{J} S_{nj}\frac{\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}\left(\sum_{o'=1}^{O}\Xi_{no'j}\hat{w}_{no'j}^{1-\varsigma_n}\right)^{\frac{1}{1-\varsigma_n}}}_{aggregate\ expenditure\ effect} \underbrace{-\left(\ln\frac{\tilde{y}_{ng}}{\tilde{y}_n}\right)\sum_{j=1}^{J}\beta_{nj}\frac{\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}\left(\sum_{o'=1}^{O}\Xi_{no'j}\hat{w}_{no'j}^{1-\varsigma_n}\right)^{\frac{1}{1-\varsigma_n}}}_{local\ expenditure\ effect}$$

$$\underbrace{-\left(\ln\frac{\tilde{y}_{ngo}}{\tilde{y}_{ng}}\right)\sum_{j=1}^{J}\beta_{nj}\frac{\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}\left(\sum_{o'=1}^{O}\Xi_{no'j}\hat{w}_{no'j}^{1-\varsigma_n}\right)^{\frac{1}{1-\varsigma_n}}}_{occupational\ expenditure\ effect} \underbrace{-\left(\ln\frac{y}{\tilde{y}_{ngo}}\right)\sum_{j=1}^{J}\beta_{nj}\frac{\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}\left(\sum_{o'=1}^{O}\Xi_{no'j}\hat{w}_{no'j}^{1-\varsigma_n}\right)^{\frac{1}{1-\varsigma_n}}}_{individual\ expenditure\ effect}, \tag{B.4}$$

where $\hat{y}_{ngo}=\left(\sum_{j=1}^{J}\pi_{ngoj}\hat{A}_{ngoj}\hat{w}_{noj}^{\alpha_{ngo}}\right)^{1/\alpha_{ngo}}$ *and* $\hat{\lambda}_{nnj}=\frac{\hat{T}_{nj}\hat{\tau}_{nnj}^{-\theta_j}\left(\sum_{o'=1}^{O}\Xi_{no'j}\hat{w}_{no'j}^{1-\varsigma_n}\right)^{\frac{-\theta_j}{1-\varsigma_n}}}{\sum_{n'}^{N}\lambda_{n'nj}\hat{T}_{n'j}\hat{\tau}_{n'nj}^{-\theta_j}\left(\sum_{o'=1}^{O}\Xi_{n'o'j}\hat{w}_{n'o'j}^{1-\varsigma_{n'}}\right)^{\frac{-\theta_j}{1-\varsigma_{n'}}}}$.

B.2 Input-output linkages

By introducing the Input-Output linkages, it is implied that labor is not the only factor of production and sectoral composite goods are used as intermediate goods. In particular, the unit cost of production is now given by

$$c_{nj}=\left(\sum_{o=1}^{O}\mu_{noj}w_{noj}^{1-\varsigma_n}\right)^{\frac{1-\chi_{nj}}{1-\varsigma_n}}\prod_{k=1}^{J}P_{nk}^{\chi_{nkj}}, \tag{B.5}$$

where $\sum_{k=1}^{J}\chi_{nkj}\equiv\chi_{nj}$ and $1-\chi_{nj}$ is the share of labor in the value added.

One can readily verify that if $\chi_{nj} = 0 \; \forall \; n \in \mathcal{N}$ and $j \in \mathcal{J}$, then the model reduces to the baseline setup with heterogeneous occupations.

Numerous intermediate derivations imply that each of the NxOxJ equations ($\mathcal{D}'_{ioj}\left\{\hat{w}_{ioj}\right\} = 0$) for counterfactuals can be written as

$$\hat{\Xi}_{ioj}\Xi_{ioj}\left(1-\chi_{ij}\right)\sum_{n}\hat{\lambda}_{inj}\lambda_{inj}\left(\hat{S}_{nj}S_{nj}\left(\sum_{g\in\mathcal{G}_i}\hat{Y}_{ng}Y_{ng}+\hat{D}_nD_n\right)+\sum_{k=1}^{J}\chi_{njk}\hat{\mathcal{Y}}_{nk}\mathcal{Y}_{nk}\right)=\sum_{g\in\mathcal{G}_i}\hat{\pi}_{igoj}\pi_{igoj}\hat{Y}_{igo}Y_{igo}, \tag{B.6}$$

where

i) $\hat{\lambda}_{inj}=\hat{T}_{ij}\left(\hat{\tau}_{inj}\left(\frac{\sum_{o'=1}^{O}\Xi_{io'j}\hat{w}_{io'j}^{1-\varsigma_i}}{\hat{P}_{nj}}\right)^{\frac{\left(1-\chi_{ij}\right)}{1-\varsigma_i}}\prod_{k=1}^{J}\left(\hat{P}_{ik}/\hat{P}_{nj}\right)^{\chi_{ikj}}\right)^{-\theta_j}$ is the change in trade share of country i in spendings of country n within sector j;

ii) $\hat{P}_{nj}=\prod_{k=1}^{J}\left(\frac{\hat{\lambda}_{nnk}}{\hat{T}_{nk}}\right)^{\gamma_{njk}/\theta_k}\left(\sum_{o'=1}^{O}\Xi_{no'k}\hat{w}_{no'k}^{1-\varsigma_n}\right)^{\frac{(1-\chi_{nk})\gamma_{njk}}{1-\varsigma_n}}$ is the change in the price index of sector j in country n;

iii) $\hat{\mathcal{Y}}_{nk}\mathcal{Y}_{nk}=\sum_{i=1}^{N}\hat{\lambda}_{nik}\lambda_{nik}\left(\hat{S}_{nk}S_{nk}\left(\sum_{g\in\mathcal{G}_i}\hat{Y}_{ng}Y_{ng}+\hat{D}_nD_n\right)+\sum_{\tilde{k}=1}^{J}\chi_{nj\tilde{k}}\hat{\mathcal{Y}}_{n\tilde{k}}\mathcal{Y}_{n\tilde{k}}\right)$.

B.2.1 Welfare analysis. Input-output linkages

Proposition 9. *Given a shock to trade costs and/or technology levels, a first-order approximation to the (expected) exact welfare change of income-y individuals employed in occupation o at local market g in country n is given by*

$$\frac{\Delta\mathcal{W}_{ngo}(y)}{\mathcal{W}_{ngo}(y)}\approx\underbrace{\hat{y}_{ngo}-1}_{\substack{earnings\\channel}}+\underbrace{1-\sum_{j=1}^{J}S_{nj}\hat{P}_{nj}}_{agg.\ expenditure\ effect}\underbrace{-\left(\ln\frac{\tilde{y}_{ng}}{\tilde{y}_n}\right)\sum_{j=1}^{J}\beta_{nj}\hat{P}_{nj}}_{local\ expenditure\ effect}\underbrace{-\left(\ln\frac{\tilde{y}_{ngo}}{\tilde{y}_{ng}}\right)\sum_{j=1}^{J}\beta_{nj}\hat{P}_{nj}}_{occupational\ expenditure\ effect}\underbrace{-\left(\ln\frac{y}{\tilde{y}_{ngo}}\right)\sum_{j=1}^{J}\beta_{nj}\hat{P}_{nj}}_{individual\ expenditure\ effect},$$

where $\hat{y}_{ngo}=\left(\sum_{j=1}^{J}\pi_{ngoj}\hat{A}_{ngoj}\hat{w}_{noj}^{\alpha_{ngo}}\right)^{1/\alpha_{ngo}}$, $\hat{P}_{nj}=\prod_{k=1}^{J}\left(\frac{\hat{\lambda}_{nnk}}{\hat{T}_{nk}}\right)^{\gamma_{njk}/\theta_k}\left(\sum_{o'=1}^{O}\Xi_{no'k}\hat{w}_{no'k}^{1-\varsigma_n}\right)^{\frac{(1-\chi_{nk})\gamma_{njk}}{1-\varsigma_n}}$

and $\hat{\lambda}_{nnk}=\hat{T}_{nk}\left(\frac{\hat{\tau}_{nnk}\left(\sum_{o'=1}^{O}\Xi_{no'k}\hat{w}_{no'k}^{1-\varsigma_n}\right)^{\frac{-\theta_k(1-\chi_{nk})}{1-\varsigma_n}}\prod_{k'=1}^{J}\hat{P}_{nk'}^{\chi_{nk'k}}}{\hat{P}_{nk}}\right)^{-\theta_k}$.

C Online Appendix

In this Appendix, I discuss some supplementary results of the main part of the paper.

C.1 Earnings vs. Expenditure effects of welfare change

In this subsection, I decompose the total welfare change on the *earnings/wage channel* and several distinct types of *expenditure effects*. In particular, one can readily verify that the welfare

change of income-y individuals at local market g in country n can be decomposed into four channels:

$$\frac{\mathrm{d}\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \equiv \frac{EV}{y} = \underbrace{\frac{\mathrm{d}y}{y}}_{\substack{earnings \\ channel}} \underbrace{-\sum_{j=1}^{J} S_{nj}\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{agg.\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}\left(\tilde{s}_{ngj}-S_{nj}\right)\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{local\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}\left(s_{ngj}(y)-\tilde{s}_{ngj}\right)\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{individual\ expenditure\ effect}, \tag{C.1}$$

where $\tilde{s}_{ngj}$ denotes an aggregate share of sector j at local market g in country n.

The proof is straightforward given the formula 3.2. In particular, it is just an alternative way of representing the expenditure effect as a sum of the aggregate, local, and individual expenditure effects, respectively.

Now, I revisit the implications of the formula above under the Almost Ideal Demand System. Recall that the model's Roy-like labor supply implies that workers self-select into different sectors based on their comparative advantage. As a result, when considering a particular worker, any trade or technology shock may force this worker to change in the employment sector. Thus, studying the welfare effects of a shock, I study the expected welfare change for individuals with income level y in country n at local market g. Specifically, the earnings/wage channel of welfare change represents the expected change in the average income of individuals with current income y, i.e., weighted across all sectors. The following proposition provides a formal statement.

Proposition 10. *Given a shock to trade costs and/or technology levels, a first-order approximation to the (expected) exact welfare change of income-y individuals at local market g in country n is given by*

$$\frac{\Delta\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \underbrace{\hat{y}_{ng}-1}_{\substack{earnings \\ channel}} + \underbrace{1-\sum_{j=1}^{J} S_{nj}\frac{\hat{w}_{nj}\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{agg.\ expenditure\ effect} \underbrace{-\left(\ln\frac{\tilde{y}_{ng}}{\tilde{y}_{n}}\right)\sum_{j=1}^{J}\beta_{nj}\frac{\hat{w}_{nj}\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{local\ expenditure\ effect} \underbrace{-\left(\ln\frac{y}{\tilde{y}_{ng}}\right)\sum_{j=1}^{J}\beta_{nj}\frac{\hat{w}_{nj}\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{individual\ expenditure\ effect}, \tag{C.2}$$

where $\hat{y}_{ng}=\left(\sum_{j=1}^{J}\pi_{ngj}\hat{A}_{ngj}\hat{w}_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$ *and* $\hat{\lambda}_{nnj}=\frac{\hat{T}_{nj}(\hat{\tau}_{nnj}\hat{w}_{nj})^{-\theta_j}}{\sum_{n'}^{N}\lambda_{n'nj}\hat{T}_{n'j}\left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$.

Proof. See the Appendix A.4. □

Proposition 10 exploits the result that changes in a sectoral price index are driven by changes in sectoral wage, trade share, and productivity: $\hat{P}_{nj}=\hat{w}_{nj}\left(\hat{\lambda}_{nnj}/\hat{T}_{nj}\right)^{1/\theta_j}$ for any $n\in N$ and $j\in J$. One can readily verify that i) *the earnings channel* (EC) operates at the local level, ii) *the aggregate expenditure effect* (AEE) operates at the country level, iii) *the local expenditure effect* (LEE) operates at the local level, and iv) *the individual expenditure effect* (IEE) operates at the local level along the income distribution.

Note that nothing requires any of these channels to be always positive or negative. In particular, the earnings channel is positive (negative) if $\hat{y}_{ng} > 1$ (< 1). The aggregate expenditure effect, in turn, is positive (negative) if $\sum_{j=1}^{J} S_{nj}\hat{w}_{nj}\hat{\lambda}_{nnj}^{1/\theta_j}/\hat{T}_{nj}^{1/\theta_j} < 1$ (> 1). Put it differently, it is positive if the "weighted" average of relative changes of sectoral prices is lower than 1.[17]

As for the local and individual expenditure effects, note that $\sum_{j=1}^{J}\beta_{nj} = 0$ directly implies that $\sum_{j=1}^{J}\beta_{nj}\frac{\hat{w}_{nj}\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}} = J \cdot cov\left(\beta_n, \frac{\hat{w}_n\hat{\lambda}_{nn}^{1/\theta}}{\hat{T}_n^{1/\theta}}\right)$, or, equivalently, $\sum_{j=1}^{J}\beta_{nj}\hat{P}_{nj} = J \cdot cov\left(\beta_n, \hat{P}_n\right)$. Hence, depending on a particular shock, the covariance between the vector of sectoral price changes and income semi-elasticities of market share, $cov\left(\beta_n, \hat{P}_n\right)$, can be either positive or negative. Moreover, the term $\ln\left(y/\tilde{y}_{ng}\right)$ in expression for the individual expenditure effect directly implies that the sign of the effect for relatively rich local consumers (i.e., those consumers who are richer than the local inequality-adjusted mean income, $\tilde{y}_{ng}$) are the opposite to the sign of the expenditure effect for relatively poor consumers. Similarly, the term $\ln\left(\tilde{y}_{ng}/\tilde{y}_n\right)$ in expression for the local expenditure effect states that the sign of the effect for relatively rich local labor markets (i.e., commuting zones with the local inequality-adjusted mean income higher than the country-level one, $\tilde{y}_n$) are the opposite to the sign of the local effect for relatively poor local labor markets.

C.1.1 Homothetic vs. Non-homothetic channels of welfare change

Apart from the decomposition of welfare changes on the earning and expenditure effects discussed earlier, the welfare change can also be alternatively decomposed into just two separate channels: the *homothetic* channel and the *non-homothetic* channel, respectively. Note that the non-homothetic channel does not entirely absorb all of the expenditure effects. In particular, a part of the aggregate expenditure effect (AEE) is driven by the homothetic channel. To isolate the non-homothetic part of the aggregate expenditure effect, I rewrite it as follows

$$\text{AEE}_n = 1 - \sum_{j=1}^{J} S_{nj}\hat{P}_{nj} = 1 - \sum_{j=1}^{J}\left(S_{nj} - \beta_{nj}\ln\frac{y}{a(\mathbf{P}_n)}\right)\hat{P}_{nj} - \ln\left(\frac{y}{a(\mathbf{P}_n)}\right)\sum_{j=1}^{J}\beta_{nj}\hat{P}_{nj}, \quad \text{(C.3)}$$

where $\hat{P}_{nj} = \hat{w}_{nj}\left(\hat{\lambda}_{nnj}/\hat{T}_{nj}\right)^{1/\theta_j}$.

One can readily verify that the local expenditure effect and individual expenditure effects are entirely absorbed by the non-homothetic channel. As a result, one can derive the following first-order welfare decomposition of a given shock into the homothetic and non-homothetic channels.

[17] Note also that $\sum_{j=1}^{J} S_{nj}\hat{w}_{nj}\left(\hat{\lambda}_{nnj}/\hat{T}_{nj}\right)^{1/\theta_j} = J \cdot cov(S_n, \hat{P}_n) + \bar{\hat{P}}_n$, where $\bar{\hat{P}}_n \equiv \frac{1}{J}\sum_{j=1}^{J}\hat{P}_{nj}$ is an average relative change of sectoral prices in country n.

$$\frac{\Delta \mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \underbrace{\hat{y}_{ng} - \sum_{j=1}^{J} \left(S_{nj} - \beta_{nj} \ln \left(\frac{y}{a(\mathbf{P}_n)} \right) \right) \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{homothetic\ channel} \underbrace{- \left(\ln \frac{y^2}{a(\mathbf{P}_n)\tilde{y}_n} \right) \sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{non-homothetic\ channel} \tag{C.4}$$

The proof is straightforward given Proposition 10 and formula (C.3).