The Churn Ladder

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March 21, 2025

Abstract

Firms vary considerably in their worker churn, the excess turnover of their labor force. Simple job ladder models predict that as workers climb the ladder to higher-wage employers, they also descend to lower-churn employers because high-wage firms poach from lower-wage firms. This paper validates the "churn ladder" empirically. However, job-to-job transitions are insufficient for all of the cross-sectional dispersion in churn and therefore the empirical ladder is considerably steeper. Separations to non-employment also correlate with churn and wages. We introduce firm-level and worker-level heterogeneity in separations, which results in endogenous sorting across firms and allows the job ladder model to match the empirical patterns of churn and wages. Worker heterogeneity in separation rates accounts for half of the cross-sectional dispersion in firm-level churn and we introduce an empirical test that rejects only firm-level heterogeneity in employment tenure.

*U.S. Census Bureau, CES; Federal Reserve Bank of Atlanta and IZA. We would like to thank Aniket Baksy, Nik Engbom, Chris Moser, Ioannis Kospentaris, Serdar Birinci, and Ryan Michaels for helpful comments. This research uses data from the Census Bureau's Longitudinal Employer Household Dynamics Program, which was partially supported by the following National Science Foundation Grants SES-9978093, SES-0339191 and ITR-0427889; National Institute on Aging Grant AG018854; and grants from the Alfred P. Sloan Foundation. Any opinions and conclusions expressed herein are those of the authors and do not represent the views of the U.S. Census Bureau, the Federal Reserve Bank of Atlanta or the Federal Reserve System. The Census Bureau has ensured appropriate access and use of confidential data and has reviewed these results for disclosure avoidance protection (Federal Statistical Research Data Center Project Number 1819: CBDRB-FY23-P1819-R10342, CBDRB-FY23-P1819-R10598, CBDRB- FY24-P1819-R11361). All errors are our own.

1 Introduction

Most firms experience no change in net growth rate, yet their hiring rate is about 7% per quarter, reflecting the replacement of lost workers. This "churn" is a critical aspect of worker reallocation across firms and hiring in general. Churn is not random: its heterogeneity at the firm level is large in the cross-section and highly correlated with net growth, wages, and other firm characteristics as noted by Tanaka et al. (2023) and Bachmann et al. (2017). We examine this variation in churn rates across the firm distribution and show that churn is negatively correlated with average wages. Equally important is the worker side, from which we observe that the employer's churn rate tends to decline as a worker moves from job to job throughout a continuous employment spell. Just as, on average, they make job-to-job transitions to higher-paying firms, these job switches are towards firms with ever-lower rates of turnover.

Our observations about worker churn are consistent with models of on-the-job search featuring a job ladder. These models of worker transitions would predict heterogeneity in churn: job ladder models imply both wage and churn ladders. Beginning with Burdett and Mortensen (1998), job ladder models generate heterogeneity in turnover as firms trade off a lower flow surplus of a match for an increase in the expected duration of the match. Thus, wages play an important retention role whereby offering higher wages, firms lose fewer workers from poaching and thereby have less turnover. This heterogeneity can exist even within otherwise identical firms, i.e. with equivalent productivity, as it arises from frictions in the labor market. The wage-churn tradeoff is an important implication of every job ladder model, and guides the strategy for structurally identifying features of firms (Bagger and Lentz (2018)) and jobs (Sorkin (2018)) in labor market data.

In this paper, we combine new evidence on the empirical shape of the churn ladder from both worker- and firm-side observations with theoretical insights into their relationship. We find significant dispersion in churn over the cross-section of firms and a negative relationship between churn and earnings that is qualitatively consistent with a broad class of job ladder models. We also follow this wage and turnover ladder at the worker level, across their

consecutive job transitions.[1] This controls for worker heterogeneity and potential selection, but the churn ladder is still apparent. As workers move from job to job, they move to higher-wage and lower-churn firms, climbing the wage ladder and descending the churn ladder. This is a data-intensive exercise that can only be conducted using a sufficiently long matched employer-employee panel of workers and firms. In our case, the Census Bureau's Longitudinal Employer-Household Dynamics (LEHD) dataset satisfies these requirements, covering nearly all workers and firms in the US.

Estimating the slope of the wage and churn ladder gives a quantitative relationship that can be directly compared to calibrated models of on-the-job search. We compute and estimate a parsimonious equilibrium model of on-the-job search. It incorporates the mechanisms that generate the wage and churn ladders and firm-level dispersion in turnover. Crucially, on-the-job search itself does not generate enough dispersion in churn to match the data. Rather, to generate high turnover rates at the bottom rungs, there must be high separations to nonemployment in addition to high poaching rates implied by on-the-job search.

Our model incorporates two possible explanations for the dispersion in churn and the relationship between poaching and separations into nonemployment. That is, it distinguishes two mechanisms to make the churn ladder steeper than induced by poaching alone. First, we allow firms to set policies that make all workers more or less likely to separate into nonemployment, dependent on the wage. Second, we introduce latent worker heterogeneity in the probability of separation to nonemployment. This means workers have different employment spell lengths and thus different likelihoods to climb up to the higher rungs of the job ladder. This is endogenous sorting: workers with low nonemployment separation probabilities are concentrated at high-wage, low-poaching firms, further lowering these firms' churn relative to the lower rungs. In our calibrated model, about half of the churn ladder's steepness over the wage distribution comes from worker heterogeneity, about one-fifth from firms, and the remainder, less than one-third, from the poaching-based on the job search mechanism.

The two explanations, firm policies and worker heterogeneity, also have implications that

[1] This continuous employment spell with potentially multiple jobs was coined an "employment cycle" in Wolpin (1992). In this paper, we refer to these "employment cycles" simply as "employment spells" and "continuous employment spells," interchangeably.

we can test empirically. We devise two test statistics using the means and variances of tenure conditional on firm-level wages. The intuition is that if firm policies alone drove the churn ladder, separation rates will be constant conditional on wage, and therefore tenure will be exponentially distributed. Worker heterogeneity increases average tenure and variance. We test against the null hypothesis that there is no worker-level heterogeneity in separation probabilities and reject it with both tests. This aligns with the findings in our calibrated model, which gives a role to both worker heterogeneity and firm policies in generating the cross-sectional dispersion in churn.

Our work is related to established models of the job ladder such as Burdett and Mortensen (1998) and Moscarini and Postel-Vinay (2017) which introduce the firm's retention motive into wage determination. In these models, a job ladder can exist both within ex-ante homogeneous firms and across heterogeneous levels of productivity.[2] Theory establishes a clear link between wage rungs and churn rungs of a job ladder, and we explore the empirical magnitude of this effect and how a model of the job ladder can deliver it.

In terms of measuring the job ladder in labor market data, seminal work by Topel and Ward (1992) highlighted the importance of job-to-job transitions to workers' wage gains. Moscarini and Postel-Vinay (2008); **?** first documented a link between job transitions and firm characteristics in the existence of a job ladder in firm size. We extend this measurement of the ladder to firm dynamics, via the churn rate, closely related to Bagger and Lentz (2018).

We then combine the structure of a partial equilibrium model of on-the-job search along with detailed microdata to back out the distribution of wage offers and resulting worker flows and turnover. Baksy et al. (2024) use a similar approach with wages from the CPS to measure the long-run decline of job transition rates and their cause. Gottfries and Teulings (2023) use match duration and employment histories in NLSY data to measure the contribution of on-the-job search in wage growth and dispersion throughout an "employment cycle."[3] With matched employer-employee data, we can study the interaction of the firm distribution of

[2]The latter somewhat weakens the strict one-to-one relationship between wage and turnover but maintains the general correlation because wages are still correlated with productivity, which itself has a one-to-one relationship with turnover.

[3]Rather than use the wage distribution, they infer the offer distribution from order statistics and the sequence of wage records, expanding on Barlevy (2008).

wages and churn and the worker job ladder.

Also quite related in incorporating both worker and firm heterogeneity, recent work by Karahan et al. (2019), Lentz et al. (2023), and Lamadon et al. (2024) identify latent heterogeneity in firms and workers, and the sorting between them. They use mobility for identification but focus primarily on the importance of this heterogeneity to wages, which is complementary to our focus on the distribution of turnover patterns across firms.

Understanding the source of separation heterogeneity has important implications for the role of the job ladder. We find evidence for the firm-specific differences in separation rates that are crucial to the premise of a "slippery" job ladder. This mechanism has been proposed to explain persistent earnings losses after job displacement via repeated unemployment risk in Pinheiro and Visschers (2015) and Jarosch (2023). However, we also reject that it is the only source of heterogeneity in separation rates. Worker-level latent heterogeneity and endogenous sorting produces a meaningful distinction between wage and earnings inequality and their sources. It is also consistent with the findings in Borovičková and Macaluso (2023) that workers with high and low end-of-career earnings experience similar numbers of job transitions, but of differing quality in terms of earnings gain. This paper and our findings also align with a growing literature on latent heterogeneity across workers in their flow patterns. For example, Hall and Kudlyak (2019); Gregory et al. (2021), and Ahn et al. (2023). We find these insights about heterogeneity in workers' separation rates to be critical in replicating the distribution of their employers, both in their wages and their gross flows.

2 Empirical estimates of wage and job ladders

2.1 Data

This paper's data comes from the LEHD, a matched employer-employee panel consisting of administrative data collected from quarterly state unemployment insurance files. These files cover most of wage and salary employment in the United States with the exception of federal employees, the self-employed, gig workers, and others not typically covered by the

unemployment insurance system. Our sample consists of a random 15% sample of workers from a 17 state sample over the time period spanning 1998-2013.[4]

At the employer, we observe firm identifiers at the State - Employer Identification Number (SEIN) level, through which we can link and observe employment and earnings of workers at the firm in that state.[5] We use several measures of establishment characteristics and dynamics from the Quarterly Workforce Indicators (QWI) files. We use employment weights to aggregate establishments within the state to the SEIN level. For each firm, we compute hires as employer-level "accessions," which includes all new employees and does not exclude recalls. We exclude firms that start up or shut down in that quarter as well as those with more than 200% turnover within a quarter.

From our 15% sample of workers, we use the job history file (JHF), which lists earnings at each job spell during a worker's lifetime. For each period in which the worker has earnings, we follow Hahn et al. (2017) by selecting a "dominant" employer as the job with maximum earnings over two quarters. A job-to-job transition is measured as a change in dominant job from one period to the next when the new dominant job had positive earnings in the same quarter as the old dominant job. This measure will potentially misclassify a worker as having made a job transition when they have worked continually at two jobs where the dominant job fluctuates between the two employers. Similarly, this measure also misses some true job-to-job transitions that occur over a weekend at the seam of two quarters since we require overlapping earnings. Time aggregation also implies that some job-to-job transitions we measure are transitions into non-employment for up to 11 weeks before transitioning to a new job.

We restrict our job transition sample in two principal ways. On the worker side, we restrict to workers who are between 25-65 years old when their job begins. This truncation by age is consistent with much of the earnings risk literature to reduce the prevalence of very short job spells. On the firm side, we remove transitions involving firms with fewer than

[4]Our sample includes data from California, Colorado, Hawaii, Idaho, Illinois, Indiana, Kansas, Maine, Maryland, Missouri, Montana, Nevada, North Dakota, Tennessee, Texas, Virginia and Washington. These states represent 42% of national employment.

[5]Multi-state firms will be counted as separate firms in each state. The firm-level data covers the firms' entire workforce, not just our 15% sample.

10 employees to eliminate large percentage changes from discrete employment changes. We accumulate worker earnings and firm growth and turnover into annual measures before and after a transition. Our sample restrictions result in 1.2 million job-to-job transitions. [6] We deflate earnings in all quarters to 2009 dollars.

2.2 Definitions

Our primary focus is on measuring a firm's "churn," the gross flows above and beyond net employment growth. To measure churn, we measure excess hires, $F_{i,t}$ as the number of hires beyond the ones needed for firm i's net employment growth in that period. This is equivalent to the churn measure in Elsby et al. (2017):

$$\begin{aligned} F_{i,t} =& H_{i,t} - \max(\Delta L_{i,t}, 0) = \min(H_{i,t}, S_{i,t}) \\ =& \frac{1}{2}\left(H_{i,t} - \max(\Delta L_{i,t}, 0) + S_{i,t} - \min(\Delta L_{i,t}, 0)\right) \ . \end{aligned} \tag{2.1}$$

where $H_{\ell,t} = \frac{\sum_{k=1}^{4} \mathbb{H}_{\ell,t-k}}{\frac{1}{2}(L_{\ell,t}+L_{\ell,t-4})}$, $H_{d,t} = \frac{\sum_{k=1}^{4} \mathbb{H}_{d,t+k}}{\frac{1}{2}(L_{d,t}+L_{d,t+4})}$ are the hiring rates and $S_{\ell,t} = \frac{\sum_{k=1}^{4} \mathbb{S}_{\ell,t-k}}{\frac{1}{2}(L_{\ell,t}+L_{\ell,t-4})}$ and $S_{d,t} = \frac{\sum_{k=1}^{4} \mathbb{S}_{d,t+k}}{\frac{1}{2}(L_{d,t}+L_{d,t+4})}$ are the separation rates.

Note that ℓ indexes the origin firm and d the destination firm associated with a transition.[7] For a transition in period t, we look 4 quarters prior to the transition and 4 quarters after the transition. So that our measure is not mechanically affected by the transition of the worker we observe, firm characteristics indexed t skip the period of transition, i.e. $t+1$ to $t+4$ and $t-1$ to $t-4$ just like we did with earnings growth.

Note that net job flows, or net employment growth at the firm can also be represented

[6] In order to measure annual earnings growth and firm dynamics, we require the origin and destination match to last 6 quarters. This restriction drops a substantial number of observations which is consistent with Hyatt and Spletzer (2017), who report that about $\frac{1}{3}$ of all job matches last less than 1 quarter.

[7] Since these annual measures are chosen to go with a transition at time t, we suppress the t notation for the firm and just denote them with d and ℓ.

as a function of employment, hires and separations:

$$\Delta L_{\ell,t} = \frac{\sum_{k=1}^{4} \mathbb{H}_{\ell,t-k} - \sum_{k=1}^{4} \mathbb{S}_{\ell,t-k}}{\frac{1}{2}(L_{\ell,t} + L_{\ell,t-4})}, \quad \Delta L_{d,t} = \frac{\sum_{k=1}^{4} \mathbb{H}_{d,t+k} - \sum_{k=1}^{4} \mathbb{S}_{d,t+k}}{\frac{1}{2}(L_{d,t} + L_{d,t+4})}. \tag{2.2}$$

We summarize average earnings of workers at the firm as the log of the average real earnings a firm pays its employees.

3 The firm distribution of churn and wages

We first want to consider the relationship between earnings and churn at the firm level. Figure 1 plots the quarterly excess hire rate, or churn rate, of firms over their percentile rank in terms of average wages. We can see that there is a strong monotonic decline in churn over the distribution of firms' average wages. Churn rates are very high at low-wage firms and the top of the firm wage distribution, the churn rate is nearly zero. This lines up well with the intuition behind most job ladder models, that firms face a tradeoff between their wage policy and worker retention. It is also consistent with the idea that high-wage firms poach from firms with lower wages.

3.1 Worker-level estimation: Churn

While this strong negative relationship between churn rates and firm wages is suggestive of a churn ladder, this correlation can reflect differences in worker composition across firms without necessarily being related to the job ladder. To address this, we document that looking within continuous employment spells, workers move up a wage ladder and down a churn ladder, from low-wage and high-turnover firms into higher-wage and low-turnover firms with each job transition. By looking within a worker, this movement doesn't reflect sorting on the characteristics of the worker. Further, we look within a continuous employment spell, which controls for the initial starting point out of unemployment. Rather, the pattern we find on the change in earnings and firm turnover with each job transition reflects the worker's

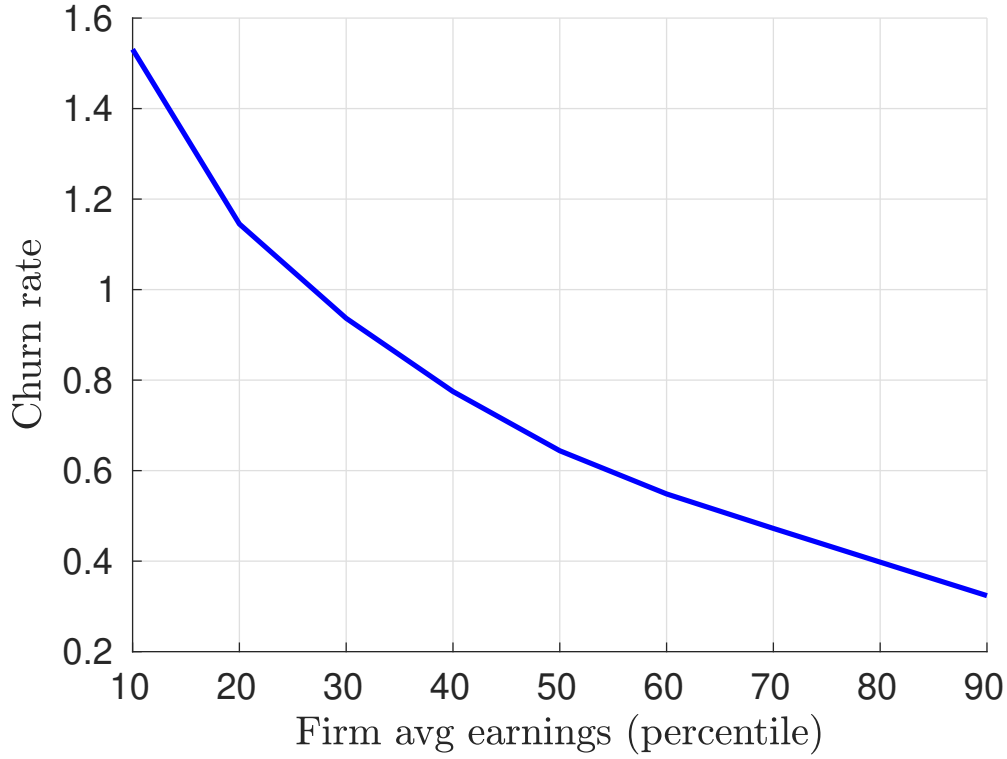


Figure 1: Nonparametric plot for the excess accession or "churn" rate (annual) over the percentiles of the average firm-level wage distribution.

marginal gain in wage from searching on the job and making a transition, and the firm's implied tradeoff between match duration and its share of match surplus for the same worker.

We run the following regression on our sample of job transition observations:

$$EAR_{ij} = \beta_0 + \sum_{i=1}^{N_j} \beta_i \mathbb{I}_{job_i} + \gamma_j + \varepsilon \tag{3.1}$$

Where the EAR is the excess accession rate of the firm in the year before the worker's separation for job i in continuous employment spell j. The index $\mathbb{I}_{job_i}$ is equal to one if the job is the ith job within a spell of employment. Including employment spell fixed effect γ means these coefficients are relative to the average for the employment spell. If a worker's first employer out of non-employment has low churn, then we want to look at that as the

base from which we look for changes.

The coefficients β_i reflect the change in churn from the transition to the i^{th} job in a spell from the baseline level of churn in the first job (which is normalized to 0). The inclusion of the employment spell fixed effect is particularly important because there is substantial heterogeneity not just across individuals, but across employment spells. By controlling for that heterogeneity, we isolate the effect of moving up the job ladder in a strict sense and quantify the worker's change in firm wage and turnover with each move. It is important to note that the employment spell fixed effect does not control for heterogeneity as it shows up in employment duration. That is, there is still a potential composition effect due to the survivorship of employment spells necessary to reach higher consecutive job transitions.

We can do the same exercise but looking instead at firms' average earnings. We run a similar regression with employment spell fixed effects except with the firm's average quarterly earnings as the dependent variable. This should correspond to the classical wage ladder that is often invoked when looking at earnings changes among job-to-job switchers.

Figure 2 plots the coefficients from the above regression, giving us a visual representation of the churn ladder and earnings ladder. Both begin at 0, which is a normalization because we are running this with fixed effects. For graphical ease, we look at log earnings and changes in the churn rate. We can see that within a job spell, workers move to lower-churn firms with each transition. The difference in magnitude of the decline in churn across consecutive job spells is substantial: after seven transitions we expect churn to decline by about 30 percentage points, which is about a quarter of the decline transiting across the wage distribution.

3.2 Worker-level estimation: Separations to Nonemployment

When we look at firm-level churn, these worker flows may be to other firms or to non-employment. Indeed, this observation is quite important to the story behind our first set of facts. When low-wage firms lose workers, are they losing them to other firms or to non-employment?

We find a very interesting pattern in Figure 3, which is that separations to non-employment

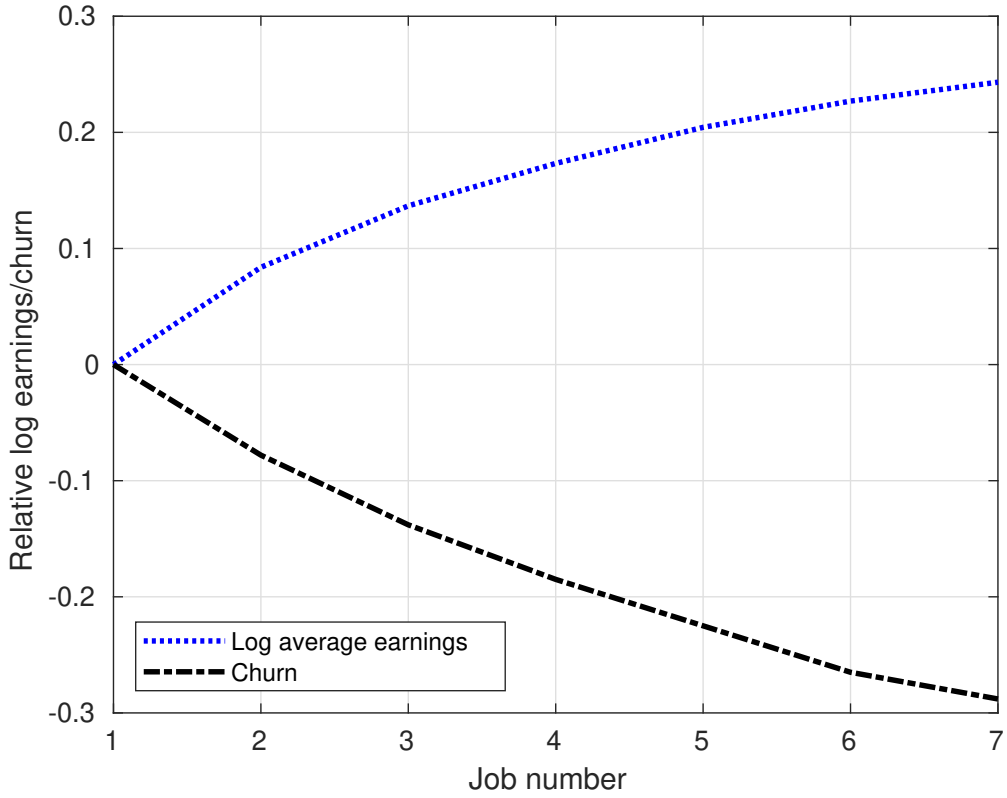


Figure 2: The firm's churn rate and log average earnings over the index of job transitions within a worker's continuous employment spell, normalized to 0 for the initial job in the spell.

and to other firms scale proportionally. So as churn rises, the fraction of workers that leave to other firms stays nearly constant. About one-quarter of separations are to other firms at every level of turnover. In this figure, we sort firms by their churn rate and then compute the fraction of these separations to job-to-job transitions. If we sort firms by their total separation rate or by their average wage rather than churn rates, the figure stays roughly constant.

Even as firms in the top decile of the churn distribution separate nearly ten times as many workers as those in the bottom decile, the fraction lost to poaching stays roughly constant. This proportionality is an important observation for theories of churn. In a simple job ladder model, high churn firms are such because they lose workers to higher wage poaching firms. On the other hand, the data shows that the majority of the separations are to non-

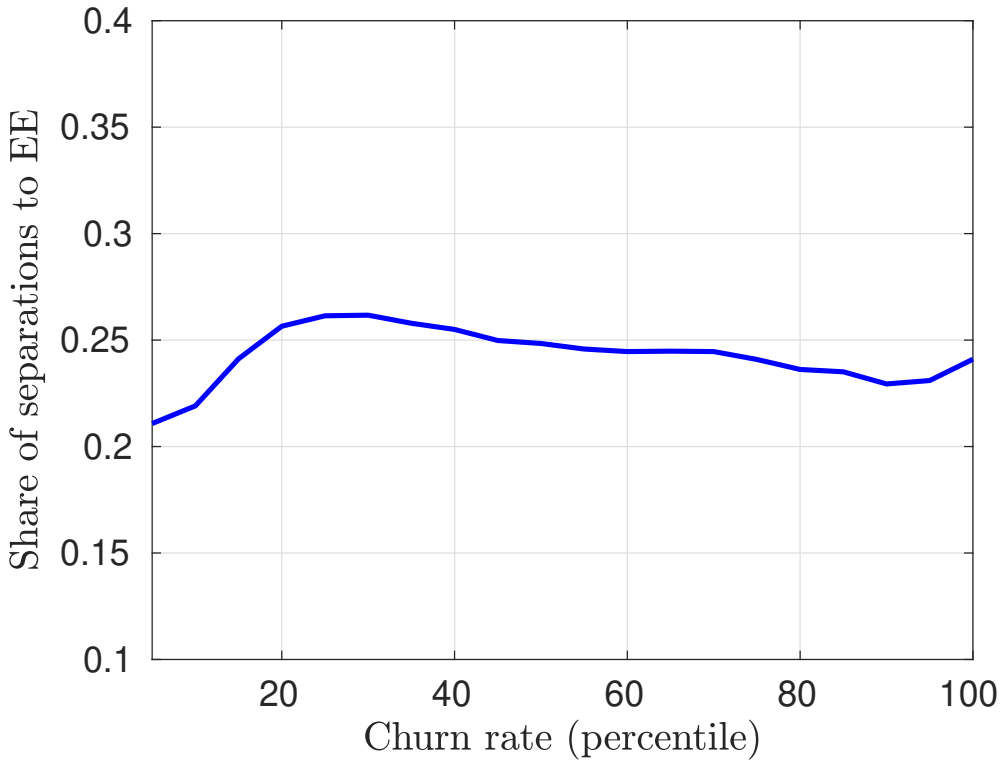


Figure 3: The share of separations from poaching by other firms as the churn rate increases. This is roughly constant around a quarter.

employment. Intuitively, that might not be surprising given how many separations overall are to non-employment. What is most striking, however, is that among low-separating firms, they also see the same proportion of workers lost to non-employment, even though far fewer workers leave.

4 How much churn in a job ladder

We start with a framework based on Burdett and Mortensen (1998). This job ladder model has a simple flow-balance condition that characterizes the distribution of firm size via hires and separations so that for a firm offering wage w to the worker, its size L is characterized by:

$$(\delta + \lambda_e(1 - F(w))L(w) = \lambda_u u + \lambda_e G(w)(1 - u) \tag{4.1}$$

where δ is the separation rate, λ_e is the rate of contact of employed workers, F is the distribution of offers from other firms, G the distribution of values among currently matched workers. We take as u the mass of unemployed workers, λ_u is their rate of contacting firms, and the overall measure of workers and firms as 1. The left-hand side is the number of workers exiting from a match with wage w and the right-hand side is the number of workers entering it. In this simple version, wage w summarizes the match value. In equilibrium the size of the firm, $L(w)$, is characterized by the firm indifference condition $L(w)(p - w) = \bar{\pi}$ where $\bar{\pi}$ is some constant level of profit and p is the productivity level of all firms. While we will not impose this equilibrium condition, it provides some useful intuition as to why the job ladder's wage-churn relationship is so general. Here, wages influence worker retention and thereby affect firm size and firm churn rates. A firm with lower turnover rates maintains a larger firm size by keeping its workers longer, resulting in a higher expected surplus from the same per-worker profit flow, $p - w$. Although higher-wage firms earn less flow profit on each worker, the decrease in turnover compensates for this.

Given the implied assumption that workers transit to higher-paying firms, $1 - F(w)$ defines the dominating offers, and known distributions and parameters, Equation 4.1 defines a relationship between wages and flows. This implies that one can take the empirical distribution of size and excess hires and infer the distribution of offered values or vice-versa.

Specifically, we can define the per-worker excess hires rate, $c(w)$, the hires above and beyond the growth rate of the firm, as

$$c(w) = \delta + \lambda_e(1 - F(w)) = \frac{\lambda_u u + \lambda_e G(w)(1 - u)}{L(w)} \ . \tag{4.2}$$

Because we are considering firms in steady-state, every hire is an "excess" hire and all gross worker flows are churn. Thus, $c(w)$ equals both the hire rate and the separation rate, whereas more generally it would be the minimum of the two. To obtain an empirical relationship

from this model, the first equation, $c(w) = \delta + \lambda_e(1 - F(w))$, is most useful.

Specifically, it is easy to see the monotonicity of churn on wages. Taking the derivative of $c(w)$ yields

$$c'(w) = -\lambda_e f(w)$$

which is weakly negative everywhere because by being a density, $f(w) \geq 0$. Both empirically and in one of the most famous results of Burdett and Mortensen (1998), there is positive mass on $f(w)$ over the entire domain of wages. This means that $c(w)$ strictly declines with wages.

Furthermore, we can define the flows in and out of the distribution below $F(w)$, given an earnings distribution $G(w)$. This flows equation relates the distributions $F(w)$ and $G(w)$ by:

$$F(w) = \frac{G(w)(\delta + \lambda_e)(1 - u)}{\lambda_u u + \lambda_e(1 - u)G(w)} \ . \tag{4.3}$$

This holds by equating the mass of workers falling down the job ladder to those who are climbing the job ladder, the same logic as presented in Jolivet et al. (2006). This last relationship is particularly useful empirically because $G(w)$ can be directly observed in the data, as can δ.

4.1 Heterogeneous separations and the job ladder

Based on our insights from our baseline model that delta is a key component of churn, we introduce heterogeneity at the worker and firm levels in δ. Thinking first about the former, the intuition is that different workers will fall off the job ladder by moving to nonemployment at different rates. This implies a pattern of sorting in which higher-wage firms at the top of the ladder accumulate low separation rate types. Low-wage firms that hire more directly from unemployment have more separations as the unemployed consist of many high-separation types. Going up the wage ladder, firms consist of more low-separation types who were able to remain employed for consecutive job-to-job transitions as high-separation types move back

to unemployment.

The intuition for firm-level differences in δ is that firms may bundle the wage and the stability of the job they offer. There are several reasons this may happen, for instance, if there are diminishing returns from monetary remuneration, longer-lived jobs might be more valuable. Mechanically, then we can build in a relationship whereby high-wage employers also offer lower δ values.

In addition to these two dimensions of heterogeneity, to empirically match the distribution of worker turnover and the relationship between wages and churn in job transitions, we must account for the possibility of workers moving to jobs that are dominated by the previous employer's wage level. We introduce a reallocation shock with an arrival rate of λ_r, in which a worker moves to a new job drawn from the entire offer distribution F regardless of the wage[8].

To add these three concepts, suppose the distribution of workers' separation rates is $D(\delta)$ and each worker gets a draw from this distribution, $\delta_i \in [0, \bar{\delta}]$, that is fixed forever. The firm side heterogeneity is defined by an elasticity ϵ_δ, such that it scales the separation rate by $e^{\epsilon_\delta w}$. This implies that conditional on δ_i, one could derive the same job-churn relationship from Equation 4.2 but at the firm level, however, a particular wage firm will have a $\delta(w)$ that depends on its accumulation of δ_i types and its own separation policy defined by ϵ_δ. The separation rate at wage w is therefore $\delta(w) = e^{\epsilon_\delta w} \int_0^{\bar{\delta}} s dD(s|w)$.

The reservation wage of every worker at a firm paying w is still w, although there is the possibility of a reallocation shock with probability λ_r. This means the mass of potential poaching offers is still $(1 - F(w))$ for on-the-job search. This comes because we forced all firm-side heterogeneity to be perfectly correlated with the wage, so in moving up in wages, the firm component of the separation risk also gets more favorable.

This means that our new expression for churn, $c(w)$, is now

$$c(w) = e^{\epsilon_\delta w} \int_0^{\bar{\delta}} s dD(s|w) + \lambda_r + \lambda_e(1 - F(w)) \tag{4.4}$$

[8]This reallocation or "Godfather" shock is an offer that the worker cannot refuse and induces a move to anywhere in the offer distribution regardless of the worker's current wage.

We now present a modified version of Equation 4.3, including both heterogeneity in workers and firms and our new reallocation shock, λ_r.

$$\int_{\underline{w}}^{w}\int_{0}^{\bar{\delta}}(se^{\epsilon_\delta z}+\lambda_r(1-F(w))+\lambda_e(1-F(w)))(1-u)h(z,s)dsdz=$$
$$\lambda_u(s)uF(w)+F(w)\int_{w}^{\bar{w}}\int_{0}^{\bar{\delta}}\lambda_r(1-u)h(z,s)dsdz \tag{4.5}$$

To see how this compares to our initial setup, just set $\delta_i = \delta, \epsilon_\delta = 0$ and we recover G as the marginal distribution, integrating over δ_i and from $z = 0$ to w: $G(w) = \int_{\underline{w}}^{w}\int_{0}^{\bar{\delta}} h(z,s)dsdz$.[9] Note that we are integrating over wages, using z to index them. Hence, the joint relationship between δ and w appears on the left-hand side of Equation 4.5 as the accumulation of firms' separation policy below wage w.

4.2 Mapping model to data

Suppose wages are defined on a discrete interval, $\{w_1,\ldots,w_W\}$ and there are a discrete set of types and $\{\delta_1,\ldots,\delta_\Delta\}$, which defines discrete densities h.[10] Then for wage value w_k we have the following:

$$\sum_{i=1}^{k}\sum_{j=1}^{\Delta}(\delta_j e^{\varepsilon_\delta w_i}+(\lambda_r+\lambda_e)(1-F(w_k)))(1-u)h(w_i,\delta_j)=$$
$$\lambda_u\sum_{j=1}^{\Delta}u_j p_j F(w_k)+F(w_k)\sum_{i=k+1}^{W}\sum_{j=1}^{\Delta}\lambda_r(1-u)h(w_i,\delta_j) \tag{4.6}$$

[9] Using this style of notation, we did not need to use the joint density h, instead we could use conditional distributions, $G(w|\delta)$ and we could have written equation 4.5 as

$$\int_{0}^{\bar{\delta}}(se^{\epsilon_\delta w}+\lambda_r(1-F(w))+\lambda_e(1-F(w)))(1-uG(w|s)ds=\lambda_u u(s)F(w)+F(w)\int_{0}^{\bar{\delta}}\lambda_r(1-u)G(w|s)ds$$

[10] We will abuse notation somewhat, continuing to use notation F, h to describe the distributions and densities associated with w although they are now discrete instead of continuous above.

Where the wage-specific separation rate is $\delta(w_k)$. That is, the effective separation rate for a firm at wage w_k is the worker-weighted average of separation rates δ_j for each j type of worker at wage w_k.

We can use Equation 4.6 to solve for $F(w_k)$

$$F(w_k) = \frac{\sum_{i=1}^{k}\sum_{j=1}^{\Delta}(\delta_j e^{\epsilon_\delta w_i} + \lambda_r + \lambda_e)(1-u)h(w_i,\delta_j)}{(\lambda_r+\lambda_e)\sum_{i=1}^{k}\sum_{j=1}^{\Delta}(1-u)h(w_i,\delta_j) + \lambda_u\sum_{j=1}^{\Delta}u_j p_j + \sum_{i=k+1}^{W}\sum_{j=1}^{\Delta}\lambda_r(1-u)h(w_i,\delta_j)} \tag{4.7}$$

And then our densities $h(w_i,\delta_j)$, $W\times\Delta$ values, are defined as a system of $W\times 2$ equations:

$$\delta(w_i) = \left(\sum_{j=1}^{\Delta}\delta_j\frac{h(w_i,\delta_j)}{g(w_i)}\right)e^{\varepsilon_\delta w_i} \tag{4.8}$$

$$g(w_i) = \sum_{j=1}^{\Delta}h(w_i,\delta_j) \tag{4.9}$$

$$\text{for } i \in \{1,...,W\}$$

and we can solve for the probability of each type by using the steady-state unemployment rate:

$$u = \sum_{j=1}^{\Delta}u_j p_j$$

Where u_j is the steady state for type j workers, which involves integrating over employment. Define the conditional density $g(w_i|\delta_j) = \frac{h(w_i,\delta_j)}{\sum_{s=1}^{W}h(w_s,\delta_j)}$. Then for type j, their unemployment rate is

$$u_j = \frac{\delta_j\sum_{i=1}^{W}e^{\epsilon_\delta w_i}g(w_i|\delta_j)}{\lambda_u + \delta_j\sum_{i=1}^{W}e^{\epsilon_\delta w_i}g(w_i|\delta_j)} \tag{4.10}$$

Of course, with $\Delta = 2$ types, we can use Equations 4.8, 4.9 and 4.10 to solve for this entire system, as given in Appendix C.

Using Bayes' rule:

$$\Pr[w = w_i|\delta = \delta_j] = \Pr[\delta = \delta_j|w = w_i]\frac{\Pr[w = w_i]}{\Pr[\delta = \delta_j]} \tag{4.11}$$

$$\frac{h(w_i, \delta_j)}{\sum_s h(w_s, \delta_j)} = \frac{h(w_i, \delta_j)}{\sum_j h(w_i, \delta_k)}\frac{g(w_i)}{(1 - u_j)} \tag{4.12}$$

and then

$$\sum_s h(w_s, \delta_j) = \sum_j h(w_i, \delta_k)\frac{(1 - u_j)}{g(w_i)} \tag{4.13}$$

5 A Calibrated job ladder model

In this section, we calibrate our model and compare the model-implied relationship between churn and wages with our empirical measures. To get a sense of the calibration, we first review how one calibrates a version of our model ignoring heterogeneity in worker and firm separation rates and setting our reallocation shock, λ_r, to 0. Doing so allows us to see how much the wage/retention tradeoff of on-the-job search can generate the observed distribution of turnover and wages.

We set our model to be quarterly to match the frequency of our data. We set the parameter values for δ, λ_u, and λ_e to match the separation rate, job-finding rate, and job transition rate, respectively, in the data. We normalize the outside option b to 0 and the productivity of a match, p, to 1. Rather than solve for equilibrium to back out the distribution of offers and wages, we use the normalized empirical CDF of firm wages as a direct input for the distribution G. This last part is possible because we have a relationship between the unobservable $F(w)$ and $G(w)$, which we do observe, in Equation 4.3. From the empirical measure of $G(w)$, we solve for $F(w)$ and then define churn $c(w) = \delta + \lambda_e(1 - F(w))$, invoking the steady-state assumption.

Adding in our extension involving heterogeneous separations for workers and firms, the

mapping of our model to the data is very similar except for the added heterogeneity in $\delta(w)$. While we still use the empirical wage distribution for G, we must now pin down the joint distribution of wages and separation types in the employment distribution where we only have the marginal distribution G as a function of the wage from our data. That is, rather than solving for $F(w)$ using 4.3, we solve the whole system in Equations 4.7-4.10. Holding fixed the proportion of workers of each type, $p1$ and $p2$, these identify the worker separation parameters δ_j, their distribution over wages in employment $H(w_i, \delta_j)$, and the reallocation rate, λ_r. The remaining variation in observed $c(w)$ is captured by ϵ_δ. With these values, we can solve for the offer distribution and our steady-state equilibrium that is consistent with our observed (normalized) empirical wage distribution.

Table 1: Calibrated Model parameters

Parameter	Value	Target
p_1	0.8	Ahn et al. (2023)
p_2	0.2	
λ_e	0.022	Quarterly EE rate 0.035 (J2J)
λ_u	0.084	Quarterly NE rate 0.084
λ_r	0.014	est. - slope of worker ladder
$\varepsilon_\delta w$	-0.529	est. - firm churn/wage dist.
s_1	0.028	est. - min $\delta(w)$ & $\delta(w)/J2J(w)$
s_2	0.37	

5.1 Results: The Burdett-Mortensen job ladder

First we look at how the job ladder with only on-the-job search generates the joint distribution of churn and wages. Qualitatively, poaching will deliver a churn ladder, but to what extent? Figure 4 plots the average churn rate of firms in each decile of firm average earnings in the data and our calibrated model. While our model can generate a negative relationship between earnings and churn, the elasticity is much higher in the data. This is to say, moving up the firm wage distribution, up the wage ladder, is associated with a much larger decline in churn rates in the data than in the model, indicating a much steeper empirical churn ladder.

From the shape of the wage distribution and transition rates we measure in our data, we can back out the steps of the job ladder in our model. Starting with the intuition that

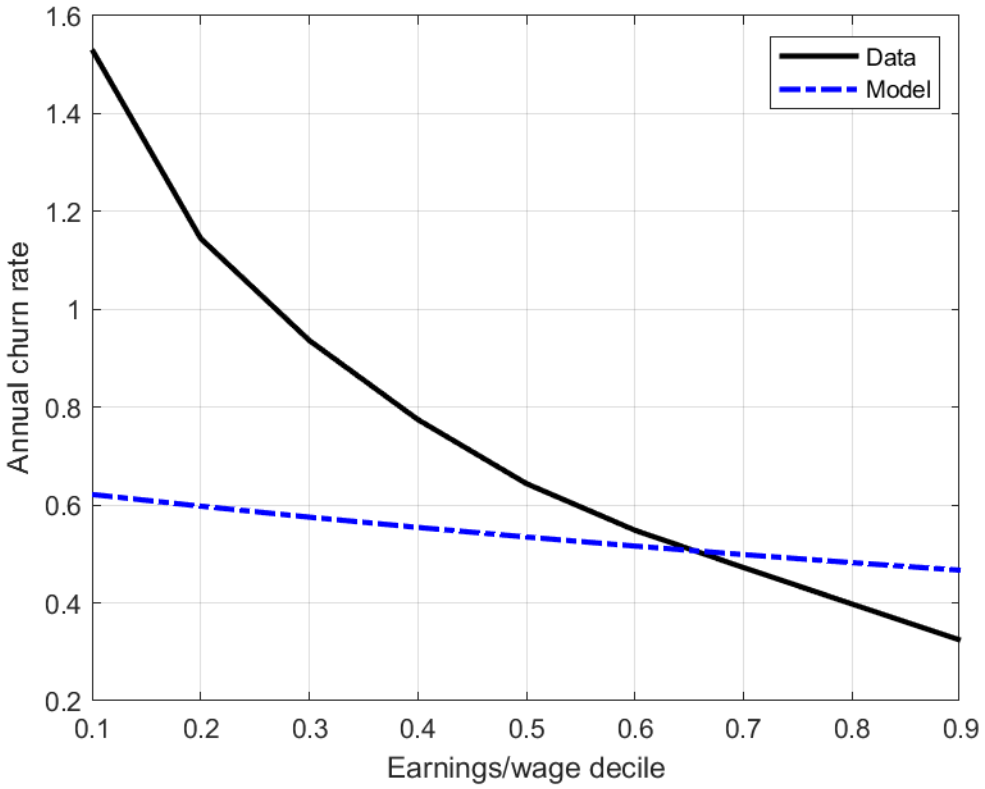


Figure 4: The relationship between annual churn rate and the earnings decile of the firm in the data and the calibrated model without separations heterogeneity or reallocation shocks.

the lowest rung of the ladder is populated by transitions from unemployment, we can trace the expected wage at each subsequent job transition based on the offer distribution and get the expected churn rate of firms at that wage. Doing so gives us a figure comparable to our empirical specification where we measured the decline in churn rate at each job index within continuous employment spells. Figure 5 compares the slope of the worker churn ladder in our model relative to our estimates in the data. The decline in churn with each job transition is again much smaller in the model.

To understand the mechanics of the simple model and why it fails to generate sufficient variation in churn, it is helpful to look at equation (4.2): $c(w) = \delta + \lambda_e(1 - F(w))$. It is clear that churn is increasing in the contact rate λ_e of employed workers searching on the job,

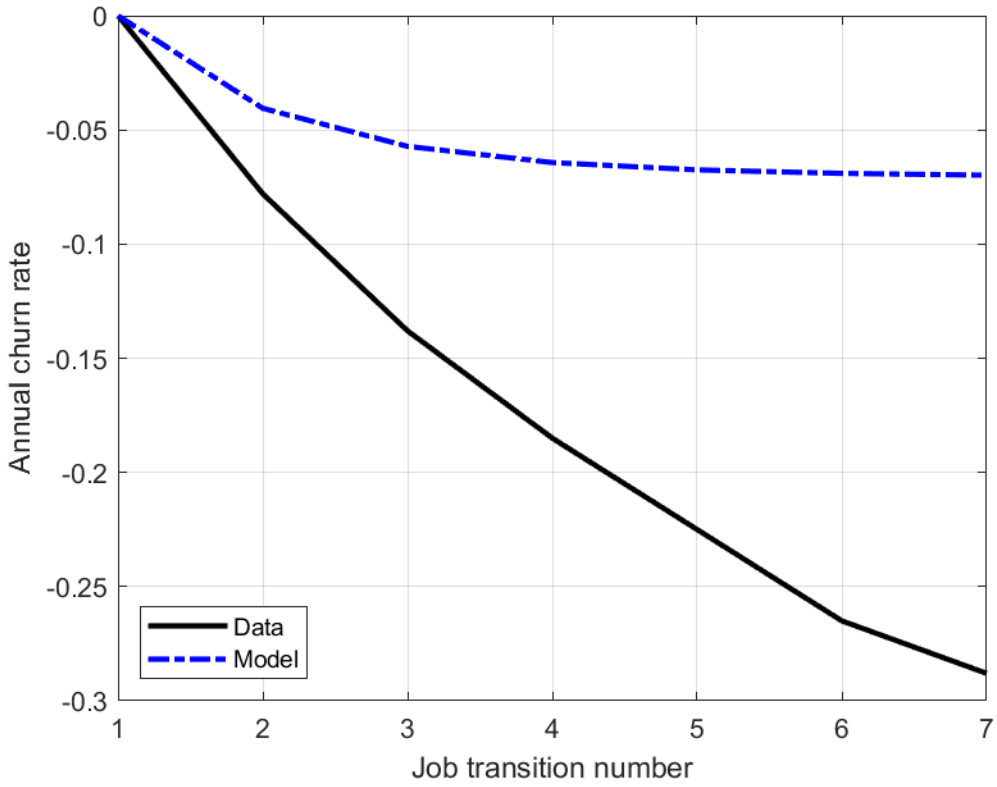


Figure 5: The relationship between the annual churn rate of the employer and the job ladder index in the data and the calibrated model without separations heterogeneity or reallocation shocks. The average churn rate of the initial job is normalized to 0.

but this increase depends on the offer distribution. As λ_e increases, the initial churn rate of employers hiring low-wage workers at the bottom of the job ladder increases, reflecting a high rate of poaching. But at the top of the job ladder, the churn rate of firms doesn't move much with λ_e since $(1 - F(w))$, the probability of the offer being higher than the worker's current wage, is small. The reason for this high baseline value of churn is evident in the appearance of a constant δ in equation (4.2). Separation rates to nonemployment are another determinant of churn and in our baseline model, this rate is identical for all workers. As long as δ is a constant, it imposes this floor on churn even at the highest wage rates.

To achieve enough variation in churn across the job ladder and the firm distribution, separation rates to nonemployment must also be correlated with wages. Indeed, when we

decompose worker separation rates at the firm into separations to other employers (EE) vs separations to nonemployment (EN), we found that EN rates are also negatively correlated with wage rank and positively correlated with churn.

5.2 Results: The job ladder with heterogeneous separations

We now look at the results of our calibrated model with our extension involving heterogeneous separations for workers and firms. Mechanically, worker heterogeneity gives the model the potential for lower baseline churn rates when firms mostly employ low-separation rate workers. Firms' wage-specific separation policies can further drive down all workers' separation probabilities as well. By lowering the total separation rate at high-wage firms, the lower bound on the churn distribution is now significantly lower. The heterogeneity in permanent worker separation types introduces variation in the composition of types along the job ladder through selection via longevity of employment spells. Low separation types have longer continuous employment spells and more opportunities to climb the job ladder without falling off to unemployment. Despite search being random, this selection results in low separation rate workers accumulating at high wage rank firms at the top of the wage ladder. Similarly, high separation rate workers disproportionately are present in low-wage firms and unemployment as they frequently fall off the job ladder. This heterogeneity in composition mirrors the exact type of heterogeneity that is not controlled for in our worker-level empirical specifications using employment spell fixed effects.

Underlying the endogenous relationships in our model, we backed out $F(w)$ via a system of Equations 4.7-4.10. As expected, $G(w)$ first order stochastic dominates $F(w)$, which must happen in any job ladder model. The sorting across types and wage-specific firm policies make this a bit more extreme: to get mass at the top of the earnings distribution requires relatively few offers because these jobs are mostly filled by low separation type workers and there are very few outflows to nonemployment or other jobs. Hence, $F(w)$ approaches 1 quite quickly.

To see the endogenous sorting, the left panel of Figure 6 plots the composition of worker

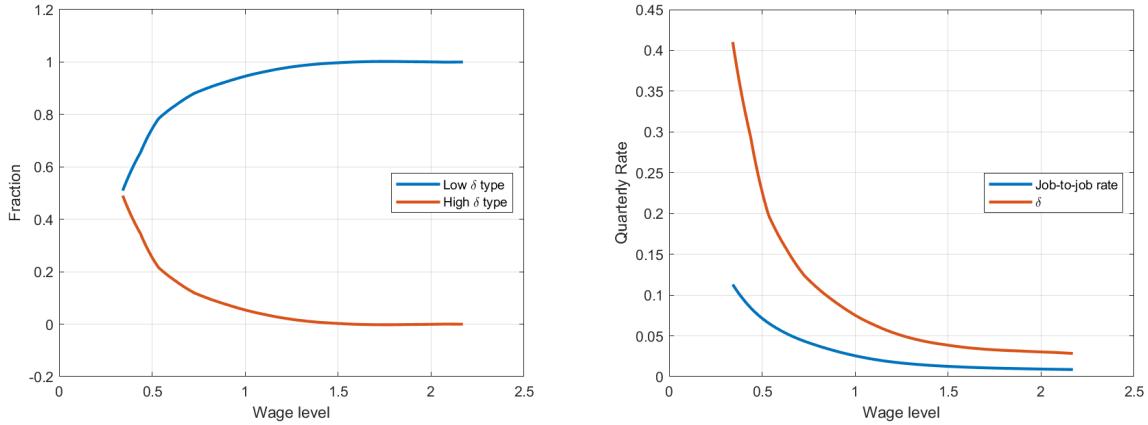


Figure 6: The types composition across wages (LEFT) and the rate of job-to-job transitions and the average separation rate at the firm (RIGHT). Job-to-job transitions include both on-the-job search and reallocation and the $\delta(w)$ integrates over all worker types over the firm wage distribution in our calibrated model.

types and separation probabilities over the firms' average wage distribution. At the lowest wage firms, their workforce is mostly composed of high separation rate workers: they are small and mostly recruit from unemployment, which low δ-workers rarely experience. At the top of the wage distribution are mostly low δ-workers, but who are joined to some extent by some lucky high δ types.

The result of this sorting, along with the firms' elasticity ϵ_δ creates our differences in $\delta(w)$ shown in the right panel of Figure 6 with the endogenous rate of job-to-job transitions. The job-to-job rate reflects the poaching hierarchy, and is quite steep in large part because of the steepness of $F(w)$. There are a lot of job transitions out of very low wage firms as there is a lot of churn and a high density of offers. $\delta(w)$ is higher in level but remains proportionally constant with job-to-job transition rates, calibrated to our empirical findings on the share of separations. Its decline over w reflects the sorting of lower δ types to higher wage levels as well as the firm-level separation policy declining in wage.

Returning to Figure 3, the model also targets this near-constant fraction of separations into non-employment. This relationship between separations to nonemployment vs. job-to-job is key to identifying the magnitudes of firms' elasticity ϵ_δ and the magnitude of the selection effect for workers. If separations to non-employment are primarily generated by

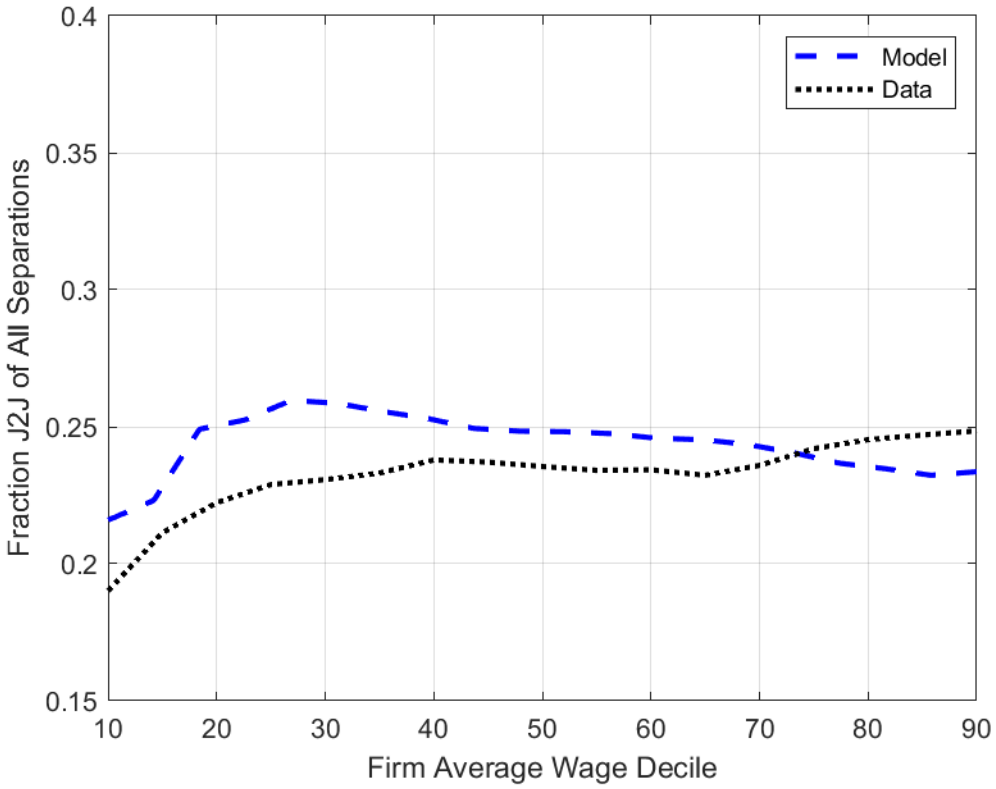


Figure 7: The composition of worker types (high and low separation probability) over the wage distribution.

the firm elasticity instead of worker composition, then job-to-job transitions contribute too much to turnover at the top of the wage distribution. Figure 7 shows how the calibrated model matches this fraction quite well.

Figure 9 compares the slope of the worker churn ladder in our model relative to our estimates in the data. The decline in churn with each job transition is now similar in magnitude in our model once we incorporate heterogeneity in separation rates. This is a natural result of matching the distribution of $\delta(w)$ that came from the model's endogenous sorting of types.

At the worker level, the composition of workers who remain employed through several job transitions consists mostly of those with a low separation probability. This variation in

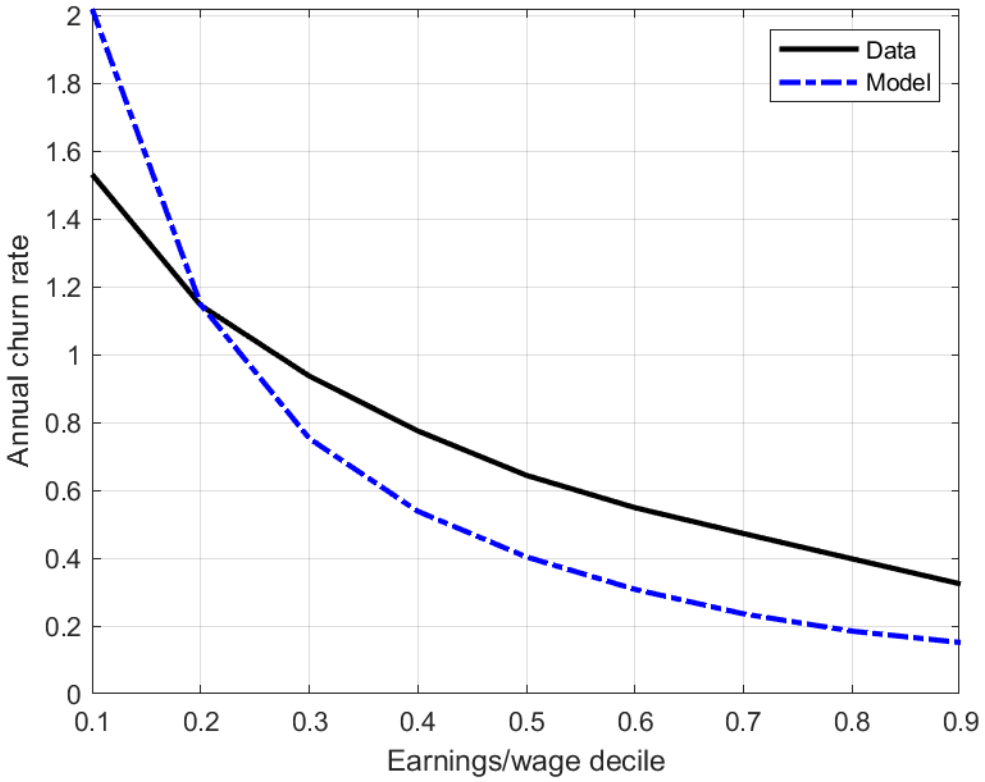


Figure 8: The relationship between annual churn rate and the earnings decile of the firm in the data and the calibrated model with worker heterogeneity.

composition mirrors the type of selection present in our regressions with employment-spell fixed effects. This sorting through the longevity of employment is present despite the fixed-effects specification controlling for heterogeneity across jobs within the employment spell and across workers.

In the model, we can split the separation rate into its components: those that are due to the relative composition of worker types, the firms' elasticity ϵ_δ, and job-to-job transitions. Figure 10 shows this breakdown across the wage distribution. We see that each source contributes to the overall separation rate. Throughout much of the wage distribution, worker composition and firms' δ policy contribute roughly equally in terms of level and have similar slope, except at the bottom of the wage distribution where composition accounts for nearly

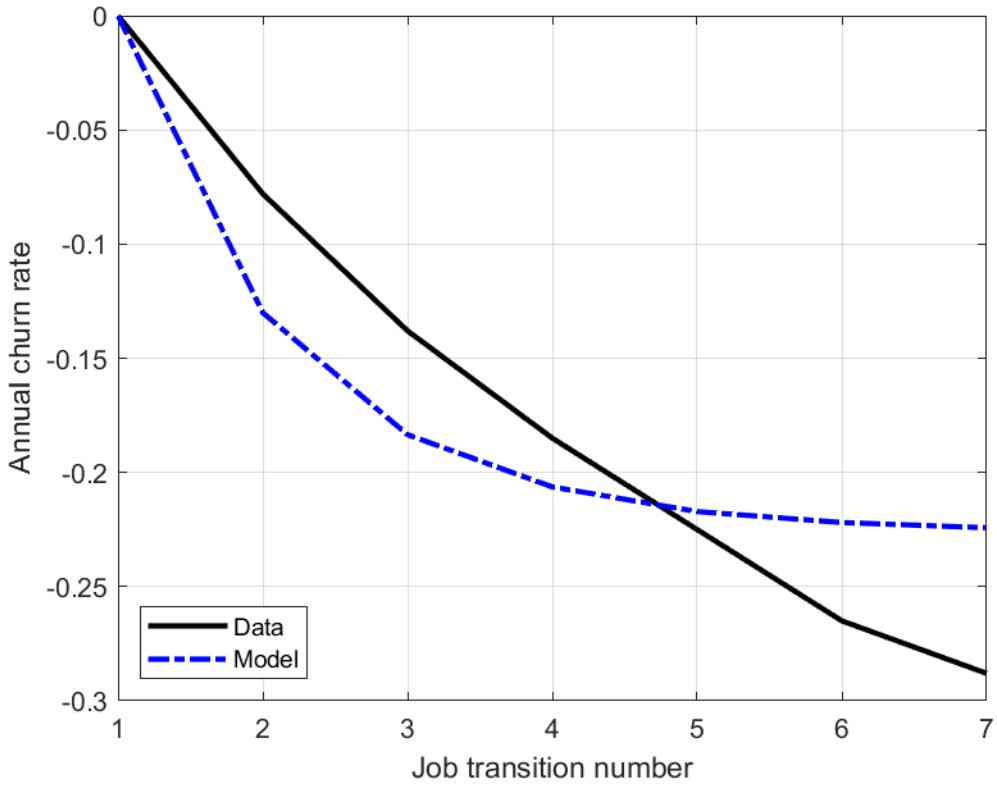


Figure 9: The relationship between the annual churn rate of the employer and the job ladder index in the data and the calibrated model. The average churn rate of the initial job is normalized to 0.

half of all separations. Reflecting our targeted constant fraction of total separations in Figure 7, about a quarter of overall turnover is generated by job-to-job transitions and the rest is due to separations to non-employment.

To summarize this decomposition presented in Figure 10, the total decline in churn is about 34% from the lowest wage level to the highest. The largest contributor to that is worker composition which creates about 17 percentage points of the decline, or half of it. The firm's δ policy, firm heterogeneity, generates another 7 percentage points. The classic explanation, differences in poaching and job-to-job rates, gives the other 10 percentage points.

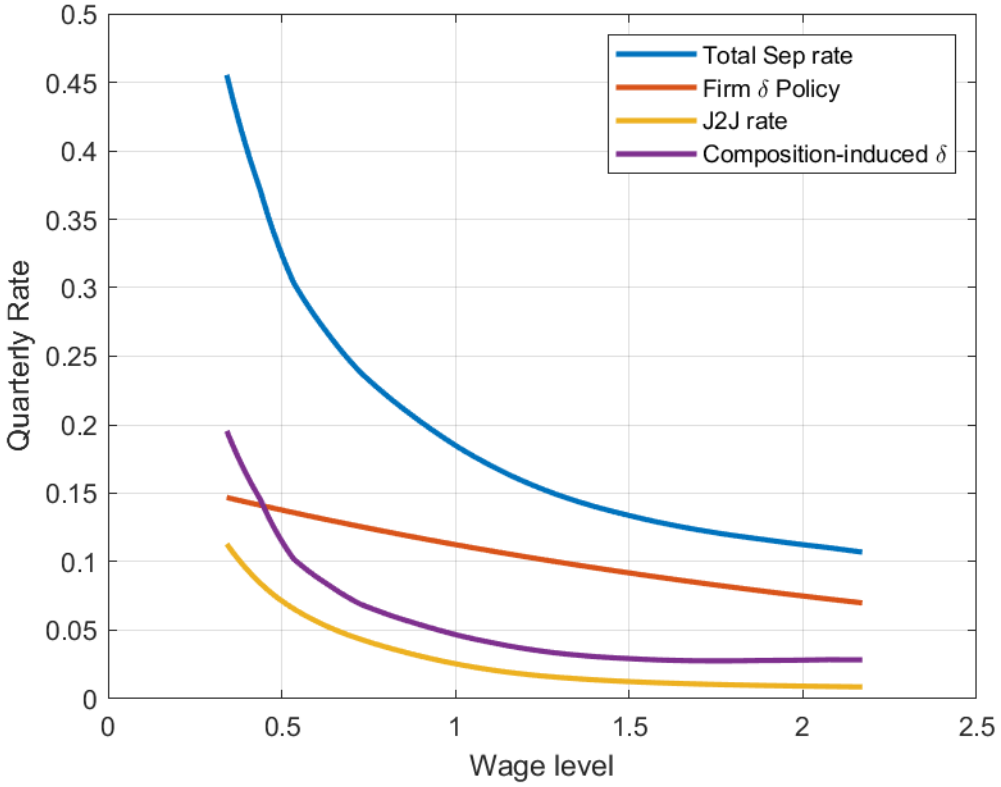


Figure 10: The contribution of worker types, firms, and job-to-job transitions to total turnover over the wage distribution.

5.3 Empirical tests of worker or firm heterogeneity

In our calibrated model, both firm policies and worker types could drive $\delta(w)$ heterogeneity. To match $\delta(w)$, either may be sufficient. However, whether $\delta(w)$ heterogeneity is driven by firm policies or worker types has very different implications for how we think about the job ladder and its role in the labor market, which we discuss in the next section. Fortunately, our model provides two simple tests of whether $\delta(w)$ is driven only by firm policies. We present both tests here, the first using the first moment of job tenure conditional on wage and the second using the second conditional moment.

To set up both tests, first, imagine there is no worker-level heterogeneity in δ_j. Then, there should be no within-wage variation in separation probabilities. For notational simplicity,

suppose the only reason for separation is into non-employment, $\delta(w)$.[11]

So in this case, without worker heterogeneity, the expected tenure at a given wage level should be $\tau(w) = \frac{1}{\delta(w)} = \frac{1}{\delta e^{-\epsilon w}}$. Now, suppose that the true data-generating process involves heterogeneous workers with two separation types, δ_1 and δ_2. Let $\mu_j(w) = \frac{h(w,\delta_j)}{g(w)}$, then the separation to non-employment can be written as

$$\begin{aligned}\delta(w) &= e^{-\epsilon w}\left[\mu(w)\delta_1 + (1-\mu(w))\delta_2\right] \\ &= \mu(w)\delta_1 e^{-\epsilon w} + (1-\mu(w))\delta_2 e^{-\epsilon w}\end{aligned}$$

This implies the expectation of tenure at a wage w,

$$\tau(w) = \mu(w)\frac{1}{\delta_1 e^{-\epsilon w}} + (1-\mu(w))\frac{1}{\delta_2 e^{-\epsilon w}} \ .$$

First-moment test: Now we can compare our null hypothesis, $\mu(w) = 1 \ \forall w$, to a case where δ incorporates heterogeneity $\delta(w) = \mu(w)\delta_1 e^{-\epsilon w} + (1-\mu(w))\delta_2 e^{-\epsilon w}$. Since $\frac{1}{x}$ is a strictly convex function, by Jensen's inequality the expectation of tenure in the no-heterogeneity case is strictly smaller than the expectation of tenure in the case with heterogeneity. That is,

$$\frac{1}{\delta_0 e^{-\epsilon_0 w}} < \mu(w)\frac{1}{\delta_1 e^{-\epsilon w}} + (1-\mu(w))\frac{1}{\delta_2 e^{-\epsilon w}}.$$

Where δ_0, ϵ_0 are the average separation and firm effects in which all heterogeneity in separation is driven by firm heterogeneity and there is no variance within a firm wage level. Our test is then against the null hypothesis that the expectation of tenure is consistent with the no-heterogeneity case. We can reject the null if the empirical mean of $\frac{1}{\delta(w)}$ is greater than $\frac{1}{\delta e^{-\epsilon w}}$ from the model calibration at each w.

This first-moment test can be formulated as $H_0 : E[\tau|w] = \frac{1}{\hat{\delta}(w_i)} \ \forall w_i \in \{w_1, \ldots, w_N\}$.

[11] Note, of course, that without on-the-job search there's no mechanism for dispersion in the wage offer distribution in our model anymore. So there would only be a single offered wage and a single variance of tenure exponentially distributed. For purposes of exposition, we are temporarily suspending our understanding of the Diamond Paradox.

We test this on a finite set of w_i, where we have estimated the empirical $\hat{\delta}(w_i)$s.

At each w_i, $\frac{1}{\hat{\delta}_i} - \hat{\tau}_i$, where we use $\hat{\delta}_i, \hat{\tau}_i$ to mean the separation rate and tenure at each wage level w_i. Under the null hypothesis, the match tenure is driven by a homogeneous Poisson process, so it is distributed exponentially with rate parameter δ_i, $\tau \sim exp(\delta_i)$. This means $2\delta_i\tau \sim \chi^2_2$ and hence our test statistic at each w_i is

$$2\hat{\delta}_i \left(\hat{\tau}_i - \frac{1}{\hat{\delta}_i} \right) \sim \chi^2_{2n_i - 1} \ .$$

To make this into a joint hypothesis, notice that $\delta(w_i)$ is independent across each w_i. Thus, the joint hypothesis test is distributed $\chi^2_{2\sum_{i=1}^N n_i - N}$

$$\sum_{i=1}^{N} 2\hat{\delta}_i \left(\hat{\tau}_i - \frac{1}{\hat{\delta}_i} \right) \sim \chi^2_{2\sum_{i=1}^N n_i - N} \tag{5.1}$$

This problem gets more complicated in our true model with on-the-job search, in which the separation rate $s(w)$ from a firm of wage w is

$$\begin{aligned} s(w) &= \delta(w) + (1 - F(w))\lambda_e + \lambda_r \\ &= e^{-\epsilon w} \left[\mu(w)\delta_1 + (1 - \mu(w))\delta_2 \right] + (1 - F(w))\lambda_e + \lambda_r \\ &= \mu(w)\delta_1 e^{-\epsilon w} + (1 - \mu(w))\delta_2 e^{-\epsilon w} + (1 - F(w))\lambda_e + \lambda_r \ . \end{aligned}$$

This is a competing risks problem such that the time before separation is distributed as an exponential with rate $\delta(w) + (1 - F(w))\lambda_e + \lambda_r$.

However, the endogenous function, $F(w)$, moving freely, presents two problems. First, the null hypothesis for each w_i becomes difficult to characterize, as a function of w_i itself. More difficult, however, is that each w_i is no longer independent. But, we can simplify this by invoking our empirical result that separations to non-employment are a constant fraction of all separations, which we will call $1 - \ell$. Under the null hypothesis that firm heterogeneity is the data generating process, $s(w) = \frac{\delta(w)}{1-\ell}$ and thus our null for the mean of tenure is

proportional to our earlier H_0.

Second-moment test: Similarly, we can devise a statistical test using the variance of tenure and not just the mean. This has a nice intuition that worker heterogeneity in separation rates tends to be largest at the bottom of the wage distribution, pushing up dispersion in tenure. However, these also have the highest separation rates, pushing down the dispersion of tenure. Hence, if there is no heterogeneity, only the latter force operates.

To formalize this logic, again suppose that separations to non-employment are the only determinant of tenure, i.e. no on-the-job search. Then, the variance of tenure as a function of wage is

$$\begin{aligned}\text{Var}(\tau(w)) &= \mu(w)^2 \frac{1}{(\delta_1 e^{-\epsilon w})^2} + (1-\mu(w))^2 \frac{1}{(\delta_2 e^{-\epsilon w})^2} \\ &= \mu(w)^2 (\delta_1 e^{-\epsilon w})^{-2} + (1-\mu(w))^2 (\delta_2 e^{-\epsilon w})^{-2}\end{aligned}$$

If $\mu(w) = 1$ so that there is no heterogeneity, then the variance of tenure follows as a simple exponential distribution at each w. Hence we can test against that null hypothesis that $\text{Var}(\tau(w)) = \frac{1}{(\delta_1 e^{-\epsilon w})^2}\ \forall w$. Of course, we can actually observe $\delta(w)$ in the data, so our null hypothesis is

$$H_0 : \text{Var}(\tau | w_i) = \frac{1}{\hat{\delta}_i^2}\ \forall w_i \tag{5.2}$$

We can again test on a finite set of $w_i \in \{w_1, \ldots, w_N\}$ and the empirical $\hat{\delta}_i$ estimated at each w_i. This yields a test statistic at each w_i that is the estimated variance at each i and the one constructed from separation rates at i using the estimated exponential distribution: $\widehat{var(\tau|w_i)} - \frac{1}{\hat{\delta}_i^2}$.

Incorporating on-the-job search, the variance of tenures is again somewhat more complicated:

$$\text{var}(\tau|w) = \frac{1}{(\delta(w) + (1-F(w))\lambda_e + \lambda_r)^2} = \frac{1}{(s(w))^2}$$

Incorporating for our empirical finding that $s(w) = \frac{\delta(w)}{1-\ell}$, our null hypothesis is now that

$\text{var}(\tau|w) = \frac{(1-\ell)^2}{\delta(w_i)^2}$ and the rest of the analysis is the same. The test statistic,

$$\widehat{\text{var}(\tau|w)} - \frac{(1-\ell)^2}{\hat{\delta}_i^2}$$

The distribution of this test statistic are difficult to characterize because it is the product of two $\chi^2_{2n_1}$ distributed variables. The joint hypothesis, is the distributed as the sum of such products. Hence, we bootstrap the distribution and use this to generate p-values.

In Figure 11, we present the test statistics for both second and first moment tests across the wage domain. The joint test statistic of each has a p-value indistinguishable from 0 both because our sample size is so large but also because the deviation at each point is so large. In the first moment, the actual tenure is considerably longer than that implied by a constant separation rate within firms. For the second moment, there is considerably more dispersion in tenure than would be predicted. Both are suggestive of workers with different separation rates.

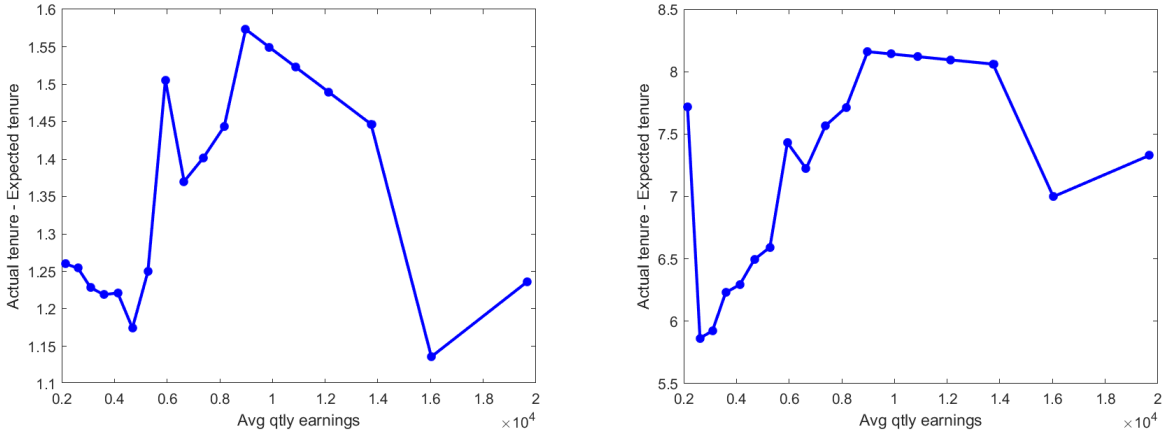


Figure 11: First moment (LEFT) and second moment (RIGHT) tests for the presence of worker heterogeneity in separation rates. By both tests we reject uniformity.

5.4 Discussion: different dimensions for measuring the job ladder

Theory suggests that on-the-job search creates a job ladder and shapes the relationship between firm size, wages, and churn. Here, we validate this theory, measuring how wages and churn vary at the worker and firm level in a manner consistent with the canonical job ladder model. However, the magnitudes are off. The theoretical relationship is driven by high-wage firms poaching from low-wage firms, increasing their churn. But this poaching mechanism yields a familiar finding in the search literature: the dispersion we observe in data is far greater than that predicted by the simple frictional labor market models. But still, in this case, frictions are key to generating the dispersion we observe, because they also facilitate sorting between firms by wage and workers by their separation rates. Together, these mechanisms drive the empirical patterns we observe.

The importance of nonemployment separations—and the unexpected finding that they comprise a near-constant fraction of separations across the job ladder—informs how job ladder models rank firms. On the one hand, job ladders create a natural ranking of firms and we suggest another dimension by which they can be measured. On the other hand, our results suggest caution in selecting the appropriate dimension to measure.

In foundational work like Bagger and Lentz (2018) or Sorkin (2018), firm rankings can be taken from either worker transition patterns or firm hiring patterns. Our proposed ordering of firms' separation patterns is also consistent in rank, but not magnitude with these approaches. Firms at the bottom of the ladder are both hiring less from other firms and also losing more workers, but the variation in these losses over the wage distribution is much larger than would be implied by poaching alone. That is, while a firm's poaching probability aligns with its churn rank, the overall churn ladder is cardinally much steeper than what poaching flows alone imply.

This extra steepness is important for papers that try to match the differences in the pattern of firm-level flows in the cross-section. Job-to-job transitions alone cannot create the heterogeneity in firm-level employee flows nor the stark contrast in churn rates between low- and high-wage firms. The literature working on the foundations of Burdett and Mortensen (1998) or Postel-Vinay and Robin (2002) implies cross-firm differences in churn that are too

low and insufficiently correlated with wages unless explicitly incorporating nonemployment separation heterogeneity as we did with our two mechanisms.

This exercise also has lessons for other methods of measuring rank, looking at firm characteristics like wage, productivity or size (see, e.g. Moscarini and Postel-Vinay (2012) or Haltiwanger et al. (2018)). Again, the position on the churn ladder does not change the rank order, but it provides a more direct measure than size. In a model, higher rank firms poach from lower rank firms and thereby become larger. However, size can also be affected by, e.g. production functions and the Coaseian contracting environment. So rather than looking at this second order implicate, we suggest looking at churn itself.

Our model included both firm and worker heterogeneity in unemployment separations and distinguished the contribution of each, which sheds light on several important lines of research that feature firm or worker separation heterogeneity. Of course, ours was a fairly parsimonious model so the micro foundations of firm-level differences are not settled in this paper.

Firm-level differences in separation rates have been crucial to the "slippery" job ladder mechanisms that try to explain persistent earnings losses after job loss. This literature uses a steep $e^{-\epsilon_\delta w}$ to explain the experience of displaced workers, watching them experience repeated unemployment spells (see Pinheiro and Visschers (2015) or Jarosch (2023)). The churn ladder offers a complementary perspective on this phenomenon: high separation rate firms at the bottom rungs of the job ladder, those that make earnings losses more persistent for separated workers, are one ingredient to the churn ladder. But the degree to which a steep gradient of $\delta(w)$ is because of firms themselves, $e^{-\epsilon_\delta w}$, or worker sorting, $\mu(w)$, informs the literature's notions of job ladder slipperiness. Our estimates suggest a sizeable portion of the high separation rates at these firms is because of the concentration of high separation workers they have. Thus slow earnings recovery is not only because of the firms displaced workers match with, but also because these workers were unlikely to climb the ladder in the first place.

Unobservable worker-level heterogeneity is a key driver for our churn ladder and shapes the cross-sectional distribution of firms. Similar worker-side differences in job separation

rates have also been central in the recent discussion of heterogeneous unemployment rates. This work has, with worker-side data, shown large differences in the separation and unemployment rates across workers (see Hall and Kudlyak (2019), Ahn et al. (2023), or Gregory et al. (2021)) to better understand business cycle fluctuations in unemployment.Again, the churn ladder provides another perspective on the role of worker heterogeneity. Not only does the high separation worker drive the unemployment rate, it also elevates worker churn rates at the bottom of the job ladder. In this context, because of endogenous sorting our model would predict that high separation firms play an analogously crucial role to recessionary increases in unemployment because of their concentration of these cyclically sensitive workers. Leaving this postulate for future work, our churn ladder also introduces evidence that corroborates Borovičková and Macaluso (2023), in which worker-heterogeneity is crucial to individuals' earnings dynamics.

6 Conclusions

In this paper, we combine new evidence on the empirical shape of the churn ladder from both worker- and firm-side observations. This evidence is qualitatively consistent with a broad class of models, that the churn ladder exists in tandem with the wage ladder. We estimate the firm-side negative relationship between earnings and churn and document their significant dispersion in the cross-section. Estimating the slope of both wage and churn over the job ladder gives a quantitative relationship that can be directly compared to calibrated workhorse models of on-the-job search.

The failure of the job ladder model to generate the dispersion in turnover that we see in the data is informative of an important missing mechanism. To match the turnover rates at the top of the firm ranking, firms must experience far fewer separations to both poaching and nonemployment. We demonstrate that there is substantial heterogeneity in separation rates across employers within our data, and more importantly, separations to nonemployment are highly correlated with the firm's wage rank. Heterogeneity in worker separations and their implied sorting is consistent with a growing literature identifying latent heterogeneity

in workers' attachment to employment and the reconciliation of worker flows. Our finding elaborates on this literature: heterogeneity in workers' separation rates seems to play a key role in the distribution of gross flows across the firm distribution as well.

Our paper confirms the job ladder model's relationship between turnover and earnings while also highlighting the need for more heterogeneity. What we observe empirically is excessive firm-level heterogeneity in the amount of turnover, which, while it qualitatively goes in the same direction as the workhorse job ladder model would predict, is much more dispersed. Our theory, however, shows that this firm-level heterogeneity can be created by worker-side heterogeneity. We think this is an important insight: that these firm-level patterns are equally shaped by worker-level differences and sorting.

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A Appendix: additional descriptive statistics on the churn ladder

A.1 Summary statistics by size and industry

In this section we present a few more facts about the distribution of these joint churn and wage ladders. There is a great amount of heterogeneity across firms in both churn and average wages, and most of it exists within observable characteristics.

First, we break down the wage and churn distributions across firm size and firm size changes. We show in Table 2 that annual churn is decreasing in size and lowest among firms that do not change their size.

Table 2: Mean and variance of churn by employment size and growth

	Mean Churn	Var. of churn within group
Small (< 50)	0.821	2.465
Med (50-1000)	0.794	1.358
Large (1000+)	0.616	0.706
Shrinking ($< -2\%$)	0.814	1.975
Stable $[-2\%, 2\%]$	0.567	0.508
Growing ($> 2\%$)	0.875	2.78

We also note that there is considerable variation in churn across industries. Table 3 lists the mean and variance of the annual churn rate for firms within each sector. Note that there is considerable variation across industries in both the mean and variance of churn rates. However, this variation in churn across industries appears to be largely driven by differences in wages. In Figure 12, we plot sector-level means of churn rates against sector-level means of firms' average quarterly earnings. We get a familiar negative relationship between wages and churn and similar dispersion in churn rates across the wage distribution, just as we see in the unconditional relationship in Figure 1.

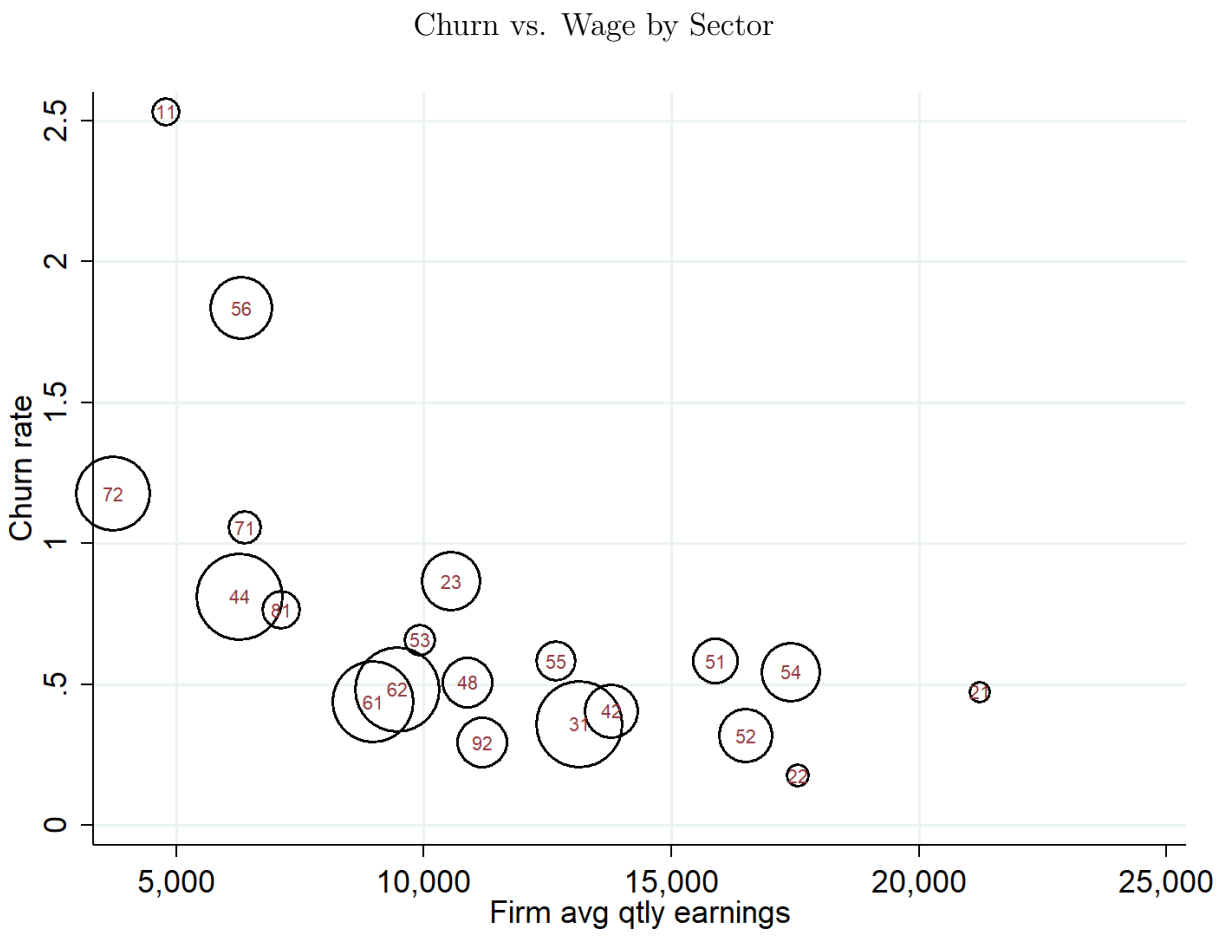


Figure 12: Employment-weighted mean of annual churn rate of firms within each sector, plotted against the employment-weighted mean of firms' average quarterly earnings. Bubble size represents the employment weight of the sector. Sector labeled 31 represents Manufacturing (31-33), Sector 44 represents Retail Trade (44-45), and Sector 48 represents Transportation and Warehousing (48-49).

Table 3: Mean and variance of churn by industry sector.

	Mean Churn	Var. of churn within group
NAICS 11: Ag., Forestry, Fishing, Hunting	2.443	21.76
NAICS 21: Mining, Quarrying, Oil & Gas Extr.	0.685	0.716
NAICS 22: Utilities	0.268	0.350
NAICS 23: Construction	0.980	1.239
NAICS 31-33: Manufacturing	0.515	0.647
NAICS 42: Wholesale Trade	0.458	0.576
NAICS 44-45: Retail Trade	0.812	1.202
NAICS 48-49: Transport. & Warehousing	0.728	1.099
NAICS 51: Information	0.597	8.649
NAICS 52: Finance & Insurance	0.400	1.148
NAICS 53: Real Estate & Rental & Leasing	0.662	1.063
NAICS 54: Prof., Sci., & Tech. Services	0.520	1.310
NAICS 55: Mgmt. of Companies & Enterprises	0.581	5.612
NAICS 56: Admin. & Supp. etc. Services	1.347	5.986
NAICS 61: Education	0.663	2.515
NAICS 62: Health Care & Social Assistance	0.620	1.218
NAICS 71: Arts, Entertainment, & Recreation	1.311	5.383
NAICS 72: Accommodation & Food Services	1.266	1.250
NAICS 81: Other Services	0.765	1.192
NAICS 99: Public Administration	0.484	0.653

B Appendix: Robustness of the worker churn ladder

In this section, we plot the worker-level churn ladder across different groups. The churn ladder is remarkably robust across observable characteristics of the worker. The decline in churn across job indexes within workers' continuous employment spells is consistent in slope across sex, race, education level, and age. There are significant variations in the levels of churn over these employment cycles, but the decline across job transitions is remarkably consistent.

Of note as well is that the slope of the worker churn ladder is very similar across workers' lifetime income profiles. In Figure 17, we plot the churn ladder for each tercile of lifetime earnings, conditioning on birth cohort. High and low lifetime earnings workers experience similar churn ladders.

We also note that the worker-level churn ladder holds across firm characteristics such as size and growth, despite the variation in churn levels in these firms that we found in Tables 2 and 3. While we see significant differences in the initial levels of churn across sectors, the worker-level coefficients at each job index within sectors displays very similar negative slopes in terms of percentage changes.

Churn Ladder: Sex

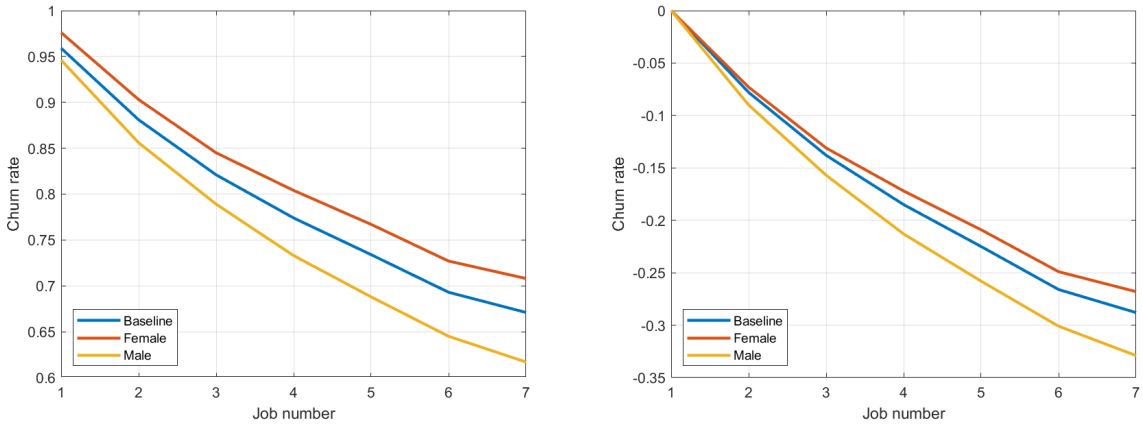


Figure 13: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

Churn Ladder: Race

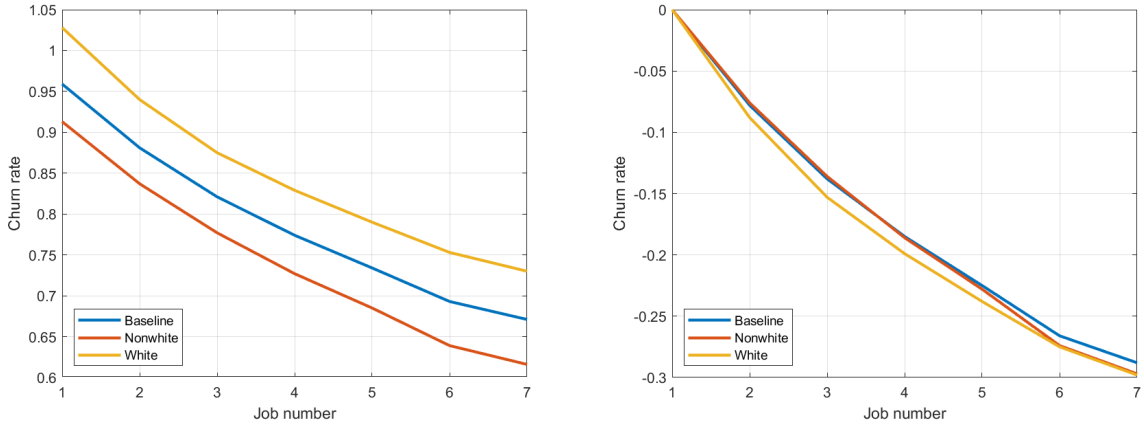


Figure 14: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

Churn Ladder: Age

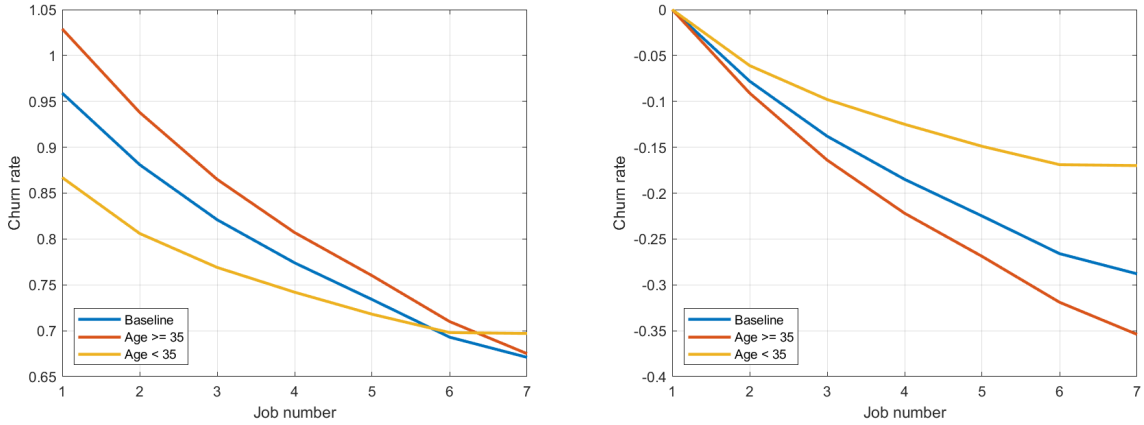


Figure 15: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

Churn Ladder: Education

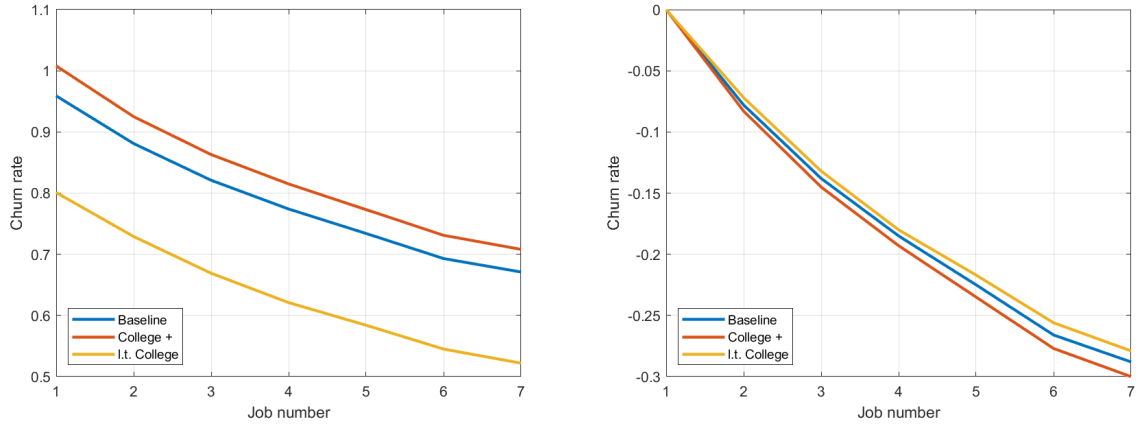


Figure 16: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

Churn Ladder: Lifetime Income Terciles (cond. on cohort)

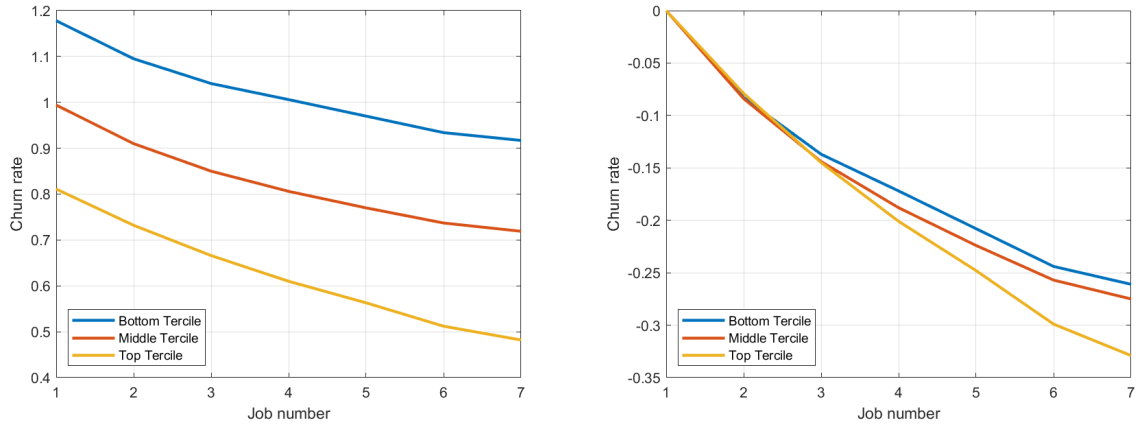


Figure 17: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

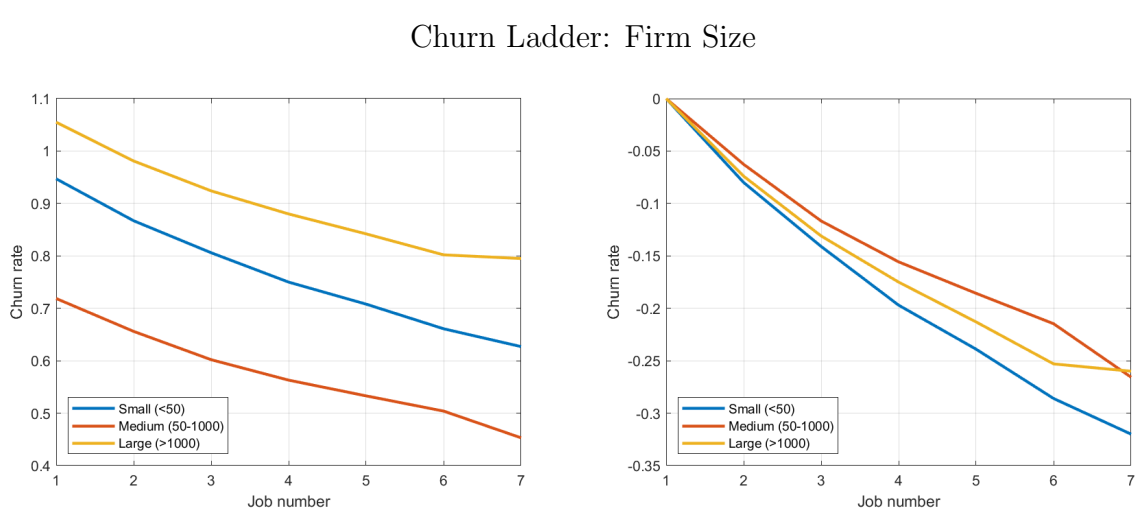


Figure 18: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

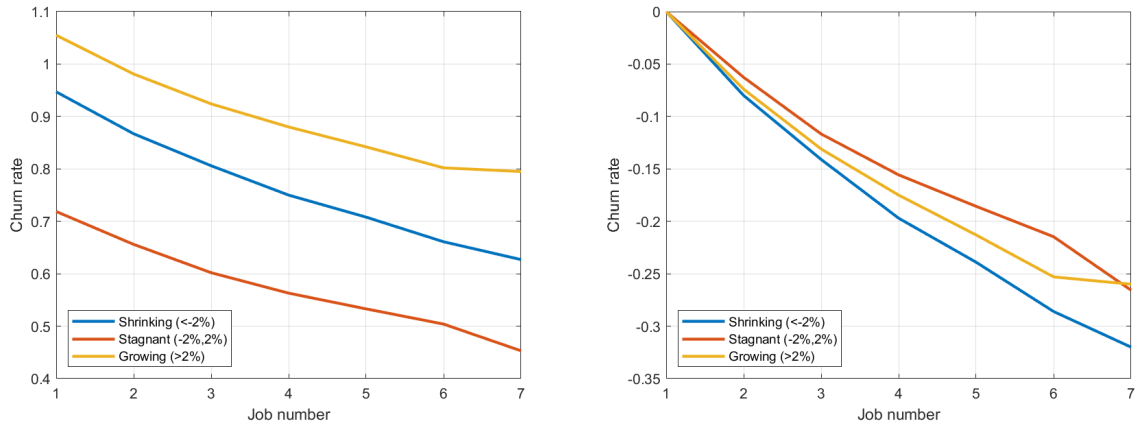


Figure 19: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

Churn Ladder: By Sectors

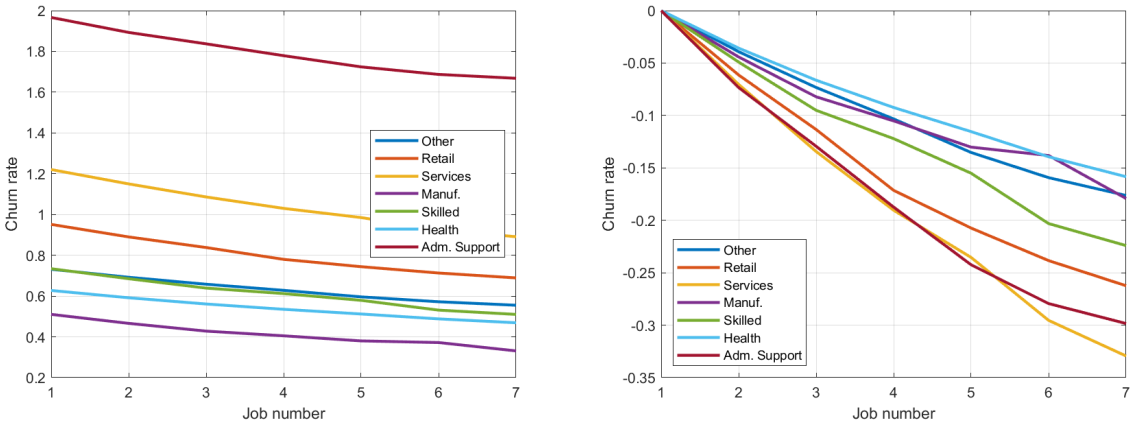


Figure 20: Worker-level churn ladder estimates in levels (LEFT) and relative to initial job (RIGHT).

C Appendix: deriving the identification with discrete types

C.1 Mapping model to data

Suppose wages are defined on a discrete interval, now with discrete densities $\tilde{h}$. Then for discrete wage value w_k from range $w \in \{w_1, \ldots, w_W\}$ and $\delta_j \in \{\delta_1, \ldots, \delta_\Delta\}$ we have the following equation (4.6):

$$\sum_{i=1}^{k}\sum_{j=1}^{\Delta}(\delta_j e^{\varepsilon_\delta w_i} + (\lambda_r + \lambda_e)(1 - F(w_k)))(1 - u_j)h(w_i, \delta_j) =$$
$$\lambda_u u F(w_k) + F(w_k)\sum_{i=k+1}^{W}\sum_{j=1}^{\Delta}\lambda_r(1 - u_j)h(w_i, \delta_j)$$

Where $\delta(w_k) = \sum_{j=1}^{\Delta}\delta_j\tilde{h}(w_k, \delta_j)$, that is the effective separation rate for a firm at wage W_k is the worker-weighted average of separation rates δ_j for each j type of worker at wage w_k. Suppose there are two types, δ_1, δ_2 with underlying probability $p_1, 1 - p_1$. we can write (4.6) as:

With two types, δ_1, δ_2 with underlying probability $p_1, 1-p_1$ our identification comes from solving a fairly straightforward system.

We can write (4.6) as:

$$\sum_{i=1}^{k}(\delta_1 e^{\epsilon_\delta w_k} + \lambda_r(1 - F(w_k)) + \lambda_e(1 - F(w_k)))(1 - u_1)h(w_i, \delta_1) +$$
$$(\delta_2 e^{\epsilon_\delta w_k} + \lambda_r(1 - F(w_k)) + \lambda_e(1 - F(w_k)))(1 - u_2)h(w_i, \delta_2) =$$
$$\lambda_u u F(w_k) + F(w_k)\sum_{i=k+1}^{W}\lambda_r(1 - u_1)h(w_i, \delta_1) + F(w_k)\sum_{i=k+1}^{W}\lambda_r(1 - u_2)h(w_i, \delta_2)\ .$$

Given a $G(w)$ and $\delta(w)$, our joint densities $h(w_i, \delta_j)$ are defined by the $2 \times N$ conditions:

$$\delta(w_i) = \left(\delta_1\frac{h(w_i, \delta_1)}{g(w_i)} + \delta_2\frac{h(w_i, \delta_2)}{g(w_i)}\right)e^{\varepsilon_\delta w_i}$$
$$g(w_i) = h(w_i, \delta_1) + h(w_i, \delta_2)$$
$$\text{for } i \in \{1, ..., N\}$$

If we fixed the types δ_j, then to solve for the masses p_1 and $(1 - p_1)$ of each worker type $j \in \{1, 2\}$, we can use the relationship between the aggregate unemployment rate u and unemployment rate for each type, u_j and their steady-state relationship to inflows and out-

flows.

Now to get the masses p_1 and $(1 - p_1)$ of each worker type $j \in \{1, 2\}$, we can use the relationship between the aggregate unemployment rate u and unemployment rate for each type, u_j and their steady-state relationship to inflows and outflows.

$$u = \sum_j u_j p_j = u_1 p_1 + u_2(1 - p_1) = \frac{\delta_1}{\delta_1 + \lambda_u} p_1 + \frac{\delta_2}{\delta_2 + \lambda_u}(1 - p_1)$$

$$u = \left(\frac{\delta_1}{\delta_1 + \lambda_u} - \frac{\delta_2}{\delta_2 + \lambda_u} \right) p_1 + \frac{\delta_2}{\delta_2 + \lambda_u}$$

And it should be that for the employed, the following holds:

$$\frac{p_j(1 - u_j)}{1 - u} = \sum_{i=1}^{N} h(w_i, \delta_j) \text{ for } j \in \{1, 2\}$$

D Monte Carlo evidence on empirically decomposing separation rate heterogeneity

Endogenous sorting places workers with high separation rates (δ) into low-wage firms. To analyze this, we can estimate empirical separation rates $d_{i,t}$ using a log-linear model that includes worker and firm fixed effects.

$$\log \hat{d}_{i,t} = \gamma_i + \phi_{j(i,t)} + \upsilon_{i,t} \ .$$

This model is correctly specified since the true separation rate is log-additively separable. The error term ($\upsilon_{i,t}$) represents randomness from a homogenous Poisson process.

Monte Carlo simulations show that sorting inflates the estimated variance of firm effects on separations $\phi_{j(i,t)}$ by 1.644 times, with the estimated variance at 4.49% compared to the true variance of 2.73%. This highlights how sorting biases our understanding of the role firms play in worker separations.