Expectations and the Term Structure of Interest Rates*

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Abstract

This paper examines the relationship between investors' subjective expectations and the term structure of interest rates. We develop regression-based tests of the hypothesis that bond risk premia are constant, without imposing assumptions on investors' expectation, such as rational expectations or alternative models of expectations formation. We then introduce structured restrictions on investors' expectations by assuming that they understand the relationships between various interest rates and that this understanding is common knowledge. We derive a second set of regressions that leverage survey data to test whether investors' expectations of different interest rates are consistent with common knowledge of rationality. Empirically, we find that (i) short-term bonds do not exhibit time-varying risk premia, yet (ii) investors' expectations and the implied bond yields remain inconsistent with the predictions of expectations hypothesis.

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1 Introduction

According to the expectations hypothesis of the term structure of interest rates, the yield on a long-term bond equals the average of expected short-term interest rates over the bond's maturity, plus a risk premium. This formulation, however, is only one of several algebraically equivalent expressions of the expectations hypothesis. For instance, under expectations hypothesis, forward rates should equal expected future spot rates (plus a risk premium), and expected holding-period returns should be constant across maturities, (again, plus a term representing risk premium).

A large body of empirical research has documented strong evidence against various formulations of the expectations hypothesis, highlighting systematic deviations from its predictions. For example, Fama (1984) shows that forward rates have strong predictive power for future spot rates, while Campbell and Shiller (1991) provide evidence against the implication that short- and long-term bond returns should be equalized. The standard interpretation of these empirical findings is to attribute the failure of the expectations hypothesis to the presence of time-varying risk premia.

However, as far back as Friedman (1979), it has been well understood that standard regression tests of the expectations hypothesis—such as those mentioned above—are inherently *joint* tests of the expectations hypothesis and the rational expectations hypothesis. This is because these regressions proxy expected interest rates with their realized values—an exercise that is valid only if investors' expectations are consistent with (i) the process that drives the short-term interest rate and (ii) the equilibrium structural relationships between different interest rates. Consequently, it remains unclear whether a rejection based on these regressions reflects a failure of the expectations hypothesis, the assumption of rational expectations, or both.

In fact, even the "algebraic equivalence" among different formulations of the expectations hypothesis hinges on the rational expectations assumption. When investors' subjective expectations deviate from the rational expectations benchmark, forward rates may no longer coincide with expected future spot rates, even if expected one-period holding returns remain uniform across maturities. For instance, if investors misunderstand the relationship between short- and long-term interest rates—or believe that other market participants do—then prevailing market expectations of future spot rates need not coincide with the current forward rates.

In this paper, we examine how investors' subjective expectations shape the term structure of interest rates. Rather than imposing rational expectations, we begin by allowing investors to hold arbitrary subjective beliefs and derive a relationship between realized and expected interest rates while assuming risk premia are constant. We then show how survey data can be used to construct regression-based tests of the hypothesis that risk premia are constant, without imposing any restrictions on how investors form their expectations. We then introduce restrictions on investors' expectation formation by assuming that they understand the relationship between various interest rates and that this understanding is common knowledge. Leveraging this assumption, we derive a second set of regressions that enables us to test, using survey data, whether investors' expectations of different interest rates are consistent with common knowledge of rationality. Empirically, we find that (i) short-term bonds do not exhibit time-varying risk premia, and yet (ii) investors' expectations remain inconsistent with the expectations hypothesis, even when we do not require their subjective expectations to coincide with those implied by the true data-

generating process.

We derive our theoretical results in the context of a simple discrete-time economy in which investors can hold positions in bonds of different maturities. Importantly, we do not restrict these investors' expectations; instead, we allow investors' expectations to be arbitrary and potentially heterogenous. This heterogeneity can reflect the fact that investors may be subject to different behavioral biases, may have different models for how various (endogenous or exogenous) variables evolve, different understandings of the economy's structural equations, or a mix of the above.

Naturally, when investors' expectations are arbitrary, the set of relationships that various interest rates must satisfy is relatively limited. Nonetheless, we show that, when risk premia are constant—and irrespective of investors' expectation formation process—bond yields are determined by the short-term interest rate and the average (consensus) of investors' one-step-ahead forecasts of bond yields. We then develop two families of regressions that allow us to test whether bond risk premia are constant using survey data. The first family of regressions is the survey-based counterpart to the textbook regression of Campbell and Shiller (1991), which tests the relationship between the yield spread of a bond that matures in n-periods and the one-period-ahead expected change in the bond's yield. The second family of regressions is based on the observation that, even with arbitrary expectations, one-step-ahead forecasts of forward rates are equal to current forward rates, plus a constant risk premium term.

In our second set of results, we explore how various assumptions on investors' expectations shape the term structure of interest rates. We focus on two assumptions. First, we assume that all investors understand that bond returns have to be equalized, and that this understanding is common knowledge among all investors. Second, we assume that the consensus expectations of the short-term interest rate satisfy the law of iterated expectations. This latter assumption is a restriction on investors' higher-order expectations about the short-term interest rate, i.e., what each investor believes about other investors' beliefs. It is satisfied, for example, when investors believe (correctly or not) that other investors share their own beliefs about the short-term interest rate.

We show that under these assumptions, realized and expected short-term interest rates satisfy a generalization of the expectations hypothesis, according to which the yield of a long-term bond is equal to the consensus subjective expectation of the short-term bond over the bond's lifetime. More importantly for our purposes however, we use this characterization to obtain two new families of regressions to test the two aforementioned assumptions using survey data.

We then use survey data from Blue Chip Financial Forecasts (BCFF) to empirically test for the various relationships between realized and expected bond we derived in prior sections. Using a panel of forecasts for yields of different maturities and horizons, we document the following results. As our first empirical finding, we do not find evidence for time-varying risk premia at the short end of the yield curve. In contrast, we document that longer maturity bond exhibit significant risk premia. Specifically, we find that when the yield spread rises, expected longer maturity yields tend to fall rather than rise, an outcome that is indicative of time-varying risk premia. While this echoes findings of earlier literature, recall that our results hold in the absence of assumptions about the underlying expectation formation process such as rational expectations.

We then turn to testing whether investors understand the relationship between short and long-maturity yields, and whether this understanding is common knowledge among them. We find that,

unlike what is predicated by the expectations hypothesis, we strongly reject the notion that forecasters understand the relationship between short- and long-term interest rates, this understanding may not be common among them, or the consensus estimate may not satisfy the law of iterated expectations. In summary, our findings document strong evidence for behavioral biases in agents' expectations in the bond market.

Related Literature. Our paper belongs to the literature that uses survey data to understand the relationship between investors' subjective expectations and the term structure of interest rates. Early works in this literature include Friedman (1979) and Froot (1987), who document that the rejection of the expectations hypothesis documented in the empirical literature may stem from the failure of rational expectations rather than the expectations hypothesis itself. More recently, Piazzesi, Salomao, and Schneider (2015) study subjective bond risk premia implied by surveys and compare them with bond risk premia implied by statistical models. Relatedly, Nagel and Xu (2025) uncover acyclical subjective bond risk premiums, a pattern consistent with real-time learning of professional forecasters. We contribute to this literature by developing regression-based tests that allow us to test for (i) whether expected and realized interest rate are consistent with constant risk premia and (ii) whether investors' expectations of yields of various maturities are consistent with one another.

A more recent literature explores whether the failure of the expectations hypothesis can be attributed to misspecification in investors' beliefs about the process driving the short-term interest rate (Crump, Eusepi, and Moench, 2024), or to investors' inability to learn the parameters governing that process (Farmer, Nakamura, and Steinsson, 2024). A key maintained assumption in these studies is that, even when investors hold misspecified or uncertain models of the dynamics of the short-term interest rate, long- and short-term rate are still linked through a generalized form of the expectations hypothesis—specifically, that long-term rates reflect the average of investors' *subjective* expectations of future short rates. We contribute to this literature by developing regression-based tests of whether realized and expected interest rates satisfy the above mentioned relationship.

Our paper also belongs to the literature that explores over and underreaction of expectations in macroeconomics and finance (Coibion and Gorodnichenko, 2015; Bordalo, Gennaioli, Ma, and Shleifer, 2020). Within the context of the bond market—and hence more closely related to our work—d'Arienzo (2020) and Wang (2021) document that whereas expectations of short-term interest rates underreact to news, expectations of longer-term interest rates overreact. In contrast to these papers, which study over- and underreaction of expectations with respect to news in the true data-generating process, we test for whether investors' expectations of the short- and long-term interest rates are consistent with one another.

Finally, our paper is also related to recent work of Shue, Townsend, and Wang (2025), who document a form of categorical thinking about interest rates, in that investors lump short- and long- term interest rates into the coarse category of interest rates. They document this behavioral bias by studying the contemporaneous comovement between expected changes in short and long rates. Our work differs from this paper along two dimensions. First, whereas Shue, Townsend, and Wang (2025) interpret their results under the assumption that risk premia are constant, we develop a family of regressions to directly test for time variation in risk premia using survey data. Second, using our theoretical framework, we

clarify that unless one takes a position on investors' higher-order expectations, it is not possible to separately test whether investors have mistaken beliefs about the relationship between short and long-term interest rates, or whether the consensus expectations operator does not satisfy the law of iterated expectations.

Outline. The rest of the paper is organized as follows. We present our conceptual framework in Section 2, and develop our theoretical results in Section 3. Section 4 contains our main empirical findings. The proofs and some additional results are provided in the Appendix.

Notation. Throughout, we denote the price of a bond that matures in n period and its yield by $P_t^{(n)}$ and $y_t^{(n)} = -(1/n)\log P_t^{(n)}$, respectively. The short-term interest rate is thus represented by $y_t^{(1)}$. We denote the (log) excess return of the n-period bond between periods t and $t+1$ by $\text{rx}_{t+1}^{(n)} = \log P_{t+1}^{(n-1)} - \log P_t^{(n)} + \log P_t^{(1)}$. Finally, we use $f_t^{(n,m)} = (1+m/n)y_t^{(n+m)} - (m/n)y_t^{(m)}$ to express the current yield of an n-period bond maturing in $n+m$ periods (the forward rate).

2 Framework

In this section, we present a conceptual framework that forms the basis of our subsequent analysis. For concreteness, we focus on a simple economic environment, but as it will become clear, our results continue to hold more generally.

Consider a discrete-time, infinite-horizon economy, consisting of a unit mass of investors, indexed $i \in [0,1]$, with potentially different expectations. The difference in investors' expectations may be due to different information sets, different models for how various (endogenous or exogenous) variables evolve, different understandings of the economy's structural equations, or a mix of the above. At any given time t, investor i has access to zero-coupon, default-free bonds with face value one that mature in $n \geq 1$ periods. All bonds are in zero net supply, and investors can rebalance their bond portfolios in all periods. Investor i thus faces the following optimization problem:

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{it}[u_i(c_{it})] \\ \text{s.t.} \quad & c_{it} + \sum_{n=1}^{\infty} X_{it}^{(n)} P_t^{(n)} + \frac{1}{2}\sum_{n=1}^{\infty}(1/\delta_i^{(n)})|X_{it}^{(n)}|^2 + Q_{it} = X_{it-1}^{(1)} + \sum_{n=2}^{\infty} X_{it-1}^{(n)} P_t^{(n)} + I_{it}, \end{aligned} \tag{1}$$

where $P_t^{(n)}$ is the time-t price of the bond that matures in n periods, $X_{it}^{(n)}$ denotes investor i's position in the bond, and $\delta_i^{(n)} \geq 0$ parameterizes a maturity-specific transaction cost. In the above expression, Q_{it} denotes the investor's positions in any other potential asset he may have access to, while I_{it} represents i's sources of income other than the returns on his bond positions (such as endowments and returns on other asset holdings).

Operator $\mathbb{E}_{it}[\cdot]$ in (1) represents investor i's subjective expectations, which we allow to be arbitrary: $\mathbb{E}_{it}[\cdot]$ may be inconsistent with the the economy's structural equations (such as market clearing) as well

as with the processes that drive the fundamental shocks to the economy (such as shocks to the short-term interest rate). It may also be distinct from other investors' expectations. In what follows, we use

$$\overline{\mathbb{E}}_t[\cdot] = \int_0^1 \hat{\delta}_i^{(n)} \mathbb{E}_{it}[\cdot] \mathrm{d}i$$

to denote the cross-sectional (weighted) average of investors' expectations, where $\hat{\delta}_i^{(n)} = \delta_i^{(n)} / \int_0^1 \delta_j^{(n)} \mathrm{d}j$. With some abuse of terminology, we refer to $\overline{\mathbb{E}}_t[\cdot]$ as the consensus expectations operator.[1] We have the following result.

Lemma 1. *The log price of the n-period bond is given by*

$$\log P_t^{(n)} = \log P_t^{(1)} + \overline{\mathbb{E}}_t[\log P_{t+1}^{(n-1)}] + \underbrace{\frac{1}{2}\overline{\mathbb{V}\mathrm{ar}}_t(\log R_{t+1}^{(n)})}_{\text{convexity adjustment}} + \underbrace{\overline{\mathbb{C}\mathrm{ov}}_t(\log M_{it+1}, \log R_{t+1}^{(n)})}_{\text{risk premium}} \tag{2}$$

to a second-order approximation, where

$$\overline{\mathbb{V}\mathrm{ar}}_t(\log R_{t+1}^{(n)}) = \int_0^1 \hat{\delta}_i^{(n)} \mathbb{E}_{it}[\log^2 R_{t+1}^{(n)}] \mathrm{d}i - \left(\int_0^1 \hat{\delta}_i^{(n)} \mathbb{E}_{it}[\log R_{t+1}^{(n)}] \mathrm{d}i \right)^2$$

$$\overline{\mathbb{C}\mathrm{ov}}_t(\log M_{it+1}, \log R_{t+1}^{(n)}) = \int_0^1 \hat{\delta}_i^{(n)} \mathbb{E}_{it}[\log M_{it+1} \log R_{t+1}^{(n)}] \mathrm{d}i - \int_0^1 \hat{\delta}_i^{(n)} \mathbb{E}_{it}[\log R_{t+1}^{(n)}] \mathrm{d}i \int_0^1 \hat{\delta}_i^{(n)} \mathbb{E}_{it}[\log M_{it+1}] \mathrm{d}i,$$

and $M_{it+1} = \beta u_i'(c_{it+1}) / u_i'(c_{it})$ is investor i's stochastic discount factor.

This is a straightforward result that expresses the price of the n-period bond in terms of the price of the short-term bond, the bond's risk premium, and the weighted average of investors' expectations of bond prices next period, with higher weights assigned to expectations of investors with lower transaction costs. More importantly for our purposes, however, Lemma 1 holds irrespective of how investors form their expectations, as it is simply a consequence of investors' individual portfolio optimization and market clearing. In other words, equation (2) remains valid whether investors share identical or heterogeneous beliefs, regardless of how their expectations relate (if at all) to the process driving the short-term interest rate, and irrespective of their understanding of the relationship between different bond prices—including equation (2) itself.

3 Theoretical Results

In this section, we take equation (2) as our starting point and examine how different assumptions about risk premia and investors' expectations affect the term structure of interest rates. Throughout, we also maintain two additional assumptions. First, we assume the interest rate on short-term bonds follows an exogenous process, though as already mentioned, investors may have misspecified and heterogenous beliefs about this process. Second, we assume that the convexity adjustment term in equation (2) is either small or constant over time. This ensures that fluctuations in bond prices arise solely from changes in risk premia, investors' expectations of future prices, or both.

[1] In the special case that all investors have identical trading costs, this operator reduces to the unweighted cross-sectional average of investors' expectations. This is the assumption we maintain in our empirical analysis.

3.1 Constant Risk Premia

As a first step, we derive the relationship between bond yields and investors' expectations under the assumption that risk premia are constant.

Assumption 1. Bond risk premia are constant over time.

Lemma 2. *If Assumption 1 is satisfied, then*

$$y_t^{(n)} = \frac{1}{n} y_t^{(1)} + \frac{n-1}{n} \overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] + \textit{constant}, \tag{3}$$

for all maturities $n \geq 1$*, where the constant may depend on the bond's maturity,* n*, but not on time,* t*.*

This result, which is a simple restatement of equation (2) under Assumption 1, relates the yield of the n-period bond to the short-term interest rate, the average of investors' one-period-ahead expectations of the yield of a bond with maturity $n-1$, and a constant risk premium term. As in Lemma 1, equation (3) holds regardless of how investors form their expectations, as it follows solely from individual portfolio optimization and market clearing. This observation also implies that any other relationship between various interest rates and/or their expectations—such as the the textbook statement of expectations hypothesis—necessarily relies on additional assumptions on investors' expectation formation process. In particular, any statement that involves $\overline{\mathbb{E}}_t[y_{t+h}^{(n)}]$ for $h > 1$ implicitly or explicitly relies on assumptions on how investors' long-horizon forecasts relate to their one-step-ahead forecasts.

Before proceeding further, it is worth pointing out that equation (3) has equivalent statements in terms of excess returns and forward rates. First, equation (3) implies that subjective excess returns must be unpredictable, i.e., $\overline{\mathbb{E}}_t[\mathrm{rx}_{t+1}^{(n)}]$ is a constant for all n. In other words, apart from a constant risk premium, the consensus expected one-period holding returns should be identical across all maturities. Second, it is straightforward to verify that forward rates satisfy

$$\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] = f_t^{(n,m+1)} + \text{constant}. \tag{4}$$

This relationship is akin to the martingale property of forward rates (plus a constant drift) dating back to Samuelson (1965) and Sargent (1972). However, it differs from these classical results along two dimensions. First, it generalizes those results by showing that away from rational expectations, forward rates are martingales with respect to the investors' consensus subjective expectations, as opposed to the objective expectations corresponding to the true data-generating process. Second, the "martingale property" only holds for one-step-ahead forecasts, in the sense that, in general, the difference between $\overline{\mathbb{E}}_t[f_{t+h}^{(n,m)}]$ and $f_t^{(n,m+h)}$ may not be a constant when $h \geq 2$. This is a consequence of the fact that, unless one imposes other restrictions on how investors' form their expectations, the mere fact that risk premia are constant does not induce a relationship between realized bond yields and investors' $h \geq 2$ step-ahead forecasts.

We now use Lemma 2 and the various equivalent statements of equation (3) to obtain regression-based tests for whether risk premia are constant.

Proposition 1. *Suppose Assumption 1 is satisfied. Then,*

(a) the slope coefficient of the following regression is equal to 1 for all $n \geq 2$*:*

$$\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] - y_t^{(n)} = \alpha + \frac{1}{n-1}\beta(y_t^{(n)} - y_t^{(1)}) + \epsilon_t \tag{5}$$

(b) the slope coefficient of the following regression is equal to 1 for all $n \geq 1$ *and* $m \geq 0$*:*

$$\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] - y_t^{(n)} = \alpha + \beta(f_t^{(n,m+1)} - y_t^{(n)}) + \epsilon_t. \tag{6}$$

Therefore, to the extent that one-step-ahead forecasts of yields $\overline{\mathbb{E}}_t[y_{t+1}^{(n)}]$ are observable (say, in survey data), one can use the two families of regressions in (5) and (6) to directly test for the hypothesis that bond risk premia are constant without imposing any assumptions on how investors form their expectations. Notably, these regressions are similar to some of the standard tests of the expectations hypothesis, but with the important difference that they rely on expected yields and forward rates as opposed to their realizations.

For example, note that (5) is similar to the well-known Campbell and Shiller (1991) regression, $y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha + \frac{1}{n-1}\beta(y_t^{(n)} - y_t^{(1)}) + \epsilon_t$, but with the realization $y_{t+1}^{(n-1)}$ on the left-hand side replaced by its time-t consensus forecast, $\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}]$. Naturally, it also has a similar interpretation: under the null hypothesis that risk premia are constant (i.e., $\beta = 1$ in (5)), an increase in the yield spread $y_t^{(n)} - y_t^{(1)}$ has to be associated with a higher *expected* yield of the long-term bond to ensure that expected returns of long- and short-term bonds are equalized. However, whereas the standard Campbell-Shiller regression is a valid test of Assumption 1 only under rational expectations, regression (5) provides a valid test of this assumption irrespective of investors' expectation formation process (including how they relate to the true data-generating process).

Similarly, regression (6) tests the Samuelson-Sargent martingale property of forward rates (equation (4)) by testing the relationship between forward premia and their subjective one-step-ahead forecasts. It is also straightforward to verify that in the special case of $m = 0$, regression (6) can be equivalently expressed in terms of excess returns as $\overline{\mathbb{E}}_t[\text{rx}_{t+1}^{(n+1)}] = \alpha + \beta(f_t^{(1,n)} - y_t^{(1)}) + \epsilon_t$, with the slope coefficient $\beta = 0$ when risk premia are constant. This is the survey-based counterpart to the Fama and Bliss (1987) regression, but with one-step-ahead consensus forecasts of excess returns replacing the realized excess returns.

We conclude this discussion with two remarks. First, while families of regressions (5) and (6) are equivalent under the null hypothesis that Assumption 1 is satisfied, they represent distinct tests of this assumption. Specifically, neither regression can be derived from the others without explicitly imposing Assumption 1. Second, although prior work has relied on survey data to test Assumption 1, to the best of our knowledge, these tests inherently rely on additional implicit assumptions about how investors form their expectations. For example, Friedman (1979) tests for Assumption 1 by examining whether $\overline{\mathbb{E}}_t[y_{t+h}^{(1)}] = f_t^{(1,h)}$. However, except for the special case that $h = 1$, such a test uses forecasts for horizons beyond the immediate next period. Therefore, as discussed above—and as we will revisit later—such a test is a valid test of Assumption 1 only under additional assumptions about investors' expectation formation processes.

3.2 Expectations and Interest Rates

As discussed above, equation (3) (in its various forms) represents the sole relationship between bond yields and investors' expectations when risk premia are constant. We now explore how various assumptions on investors' expectations shape the term structure of interest rates.

Assumption 2. All investors understand that bond prices satisfy equation (2). Furthermore, this understanding is common knowledge among all investors.

This assumption consists of two parts. The first part asserts that all investors understand the key structural equation governing the relationship between bond prices—specifically, that bond prices must be linked to the price of the short-term bond, investors' consensus expectations of future prices, and a term representing risk premia. This condition holds when investors recognize that all other investors optimize their portfolios and that bond markets must clear. The second part of the assumption imposes an additional restriction: not only does each investor understand that prices must satisfy equation (2), but each also understands that all other investors share this understanding, that they know others share it, that they know others know they share it, and so on and so forth. However, note that while the common knowledge assumption imposed by Assumption 2 constrains investors' beliefs about others' understanding of the structural equation governing bond prices, it does not impose any restrictions on their beliefs about the fundamental shocks to the economy—namely, the process driving the short-term interest rate. Our next assumption imposes such a restriction.

Assumption 3. The consensus expectations of the short-term interest rate satisfy the law of iterated expectations, i.e., $\overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+h}[y^{(1)}_{t+h+s}] = \overline{\mathbb{E}}_t[y^{(1)}_{t+h+s}]$ for all $h, s \geq 0$.

This assumption is a restriction on investors' higher-order expectations about the short-term interest rate, i.e., what each investor believes about other investors' beliefs. It is satisfied, for example, when investors believe (correctly or not) that other investors share their own beliefs about the short-term interest rate. Crucially, Assumption 3 does not impose any restrictions on investors' first-order beliefs: different investors may agree or disagree about the short-term interest rate, and their forecasts may or may not coincide with objective forecasts obtained by an econometrician who knows the process that drives the short-term interest rate. We have the following result.

Lemma 3. *If Assumptions 1 and 2 are satisfied, then bond yields are given by*

$$y^{(n)}_t = \frac{1}{n}\sum_{s=0}^{n-1} \overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+1}\ldots\overline{\mathbb{E}}_{t+s-1}[y^{(1)}_{t+s}] + \textit{constant}. \tag{7}$$

Furthermore, if Assumption 3 is also satisfied, then

$$y^{(n)}_t = \frac{1}{n}\sum_{s=0}^{n-1} \overline{\mathbb{E}}_t[y^{(1)}_{t+s}] + \textit{constant}. \tag{8}$$

Statement (a) is in line with earlier results by Allen, Morris, and Shin (2006) and Barillas and Nimark (2017), who show that under heterogenous expectations, asset prices depend on iterated consensus expectations of fundamental economic shocks—in this case, the short-term interest rate. The result

above makes it explicit that equation (7) follows from the assumption that the relationship between bond prices and investors' one-step-ahead forecasts of future prices is common knowledge.

Statement (b) of Lemma 3 then follows directly from imposing Assumption 3 on equation (7). The resulting equation, (8), takes the familiar form of the expectations hypothesis of interest rates, linking the yield of long-maturity bonds to the average expected short-term interest rate over the bond's lifetime—though in this case, what matters are investors' consensus subjective expectations. An immediate implication of equation (8) is a generalization of the martingale property of forward rates in (4) to forecast horizons beyond the next period, namely,

$$\overline{\mathbb{E}}_t[f_{t+h}^{(n,m)}] = f_t^{(n,m+h)} + \text{constant} \qquad \text{for all } h \geq 1. \tag{9}$$

Lemma 3 serves a dual purpose in our analysis. First, it clarifies the explicit assumptions necessary to obtain the version of the expectations hypothesis presented in equation (8), which establishes a tight relationship between various interest rates. More specifically, while Assumption 1 links the realized yield of an n-period bond to investors' one-step-ahead forecasts of the yield on a bond maturing in $n-1$ periods (as captured by equations (3) and (4)), it is the restrictions imposed by Assumptions 2 and 3 that link expected and realized yields of bonds across different maturities and forecast horizons (as captured by equations (8) and (9)). Second, and more importantly, Lemma 3 serves as the basis for obtaining regressions to test Assumptions 2 and 3 using data on realized bond yields and survey data, as formalized in the next result.

Proposition 2. *Suppose Assumptions 1–3 are satisfied. Then,*

(a) the coefficient of the following regression has to be equal to 1 for all $n \geq 1$, $m \geq 0$, and $h \geq 2$:

$$\overline{\mathbb{E}}_t[f_{t+h}^{(n,m)}] - y_t^{(n)} = \alpha + \beta(f_t^{(n,m+h)} - y_t^{(n)}) + \epsilon_t. \tag{10}$$

(b) the coefficient of the following regression has to be equal to 1 for all n:

$$\overline{\mathbb{E}}_t[y_{t+1}^{(n)}] - y_t^{(n)} = \alpha + \beta\frac{1}{n}(\overline{\mathbb{E}}_t[y_{t+n}^{(1)}] - y_t^{(1)}) + \epsilon_t. \tag{11}$$

The two families of regressions in (10) and (11) test the relationship between short- and long-term interest rates as expressed in equation (8). Specifically, under the assumption of constant bond risk premia, a slope coefficient in (10) and (11) that deviates from 1 signals a joint rejection of Assumptions 2 and 3. This rejection could be due to one or more of the following three reasons: investors may not fully understand the relationship between different interest rates (violation of Assumption 2(a)), this understanding may not be common knowledge (violation of Assumption 2(b)), the consensus expectations of the short-term interest rate may violate the law of iterated expectations (violation of Assumption 3).

The specification in (10) is a direct test of the martingale property of forward rates in (9) for all horizons $h \geq 2$. Note that we do not include $h = 1$ corresponding to one-step-ahead forecasts of forward rates, as such a regression does not provide a test of Assumptions 2 and 3. Rather, it reduces to equation (6), which can only be used as a test of Assumption 1.

Turning next to regression (11), note that this specification follows from the observation that if interest rates satisfy equation (8), then an expected increase in the short-term interest rate should translate into an expected increase in long-term interest rates.[2]

We conclude this discussion by pointing out that while Proposition 2 provides a joint test of Assumptions 2 and 3 using survey data, testing for the two assumptions separately is not possible by relying on data on first-order expectations. Specifically, recall from Lemma 3(a) that if one only imposes Assumption 2, bond yields satisfy expression (7). But testing this relationship requires either (i) data on investors' beliefs about other investors' beliefs (i.e., higher-order beliefs), or (ii) a model for how investors' higher-order expectations depend on their first-order expectations.

4 Empirical Analysis

In this section, we use survey and interest rate data to empirically test for the various relationships between realized and expected bond derived in the previous section. We start by describing our datasets and then turn to presenting our empirical findings.

4.1 Data

We use data from various sources.

Survey data. We use survey data from Blue Chip Financial Forecasts (BCFF). Towards the end of each month, around 40 financial professionals provide forecasts about interest rates (as well as other macroeconomic variables). These forecasts are made available on the first day of the following month. For our analysis, we use forecasts on 1, 2, 4, 8, 20, and 40 quarter yields. Forecasters are asked to provide their estimate for the current quarter as well as up to five quarters ahead. From these, we calculate a consensus forecast as the (unweighted) average of all forecasts. We label these forecasts $\overline{\mathbb{E}}_t[y_{t+h}^{(n)}]$ for $h = 1, \ldots, 6$. We sample our data at the quarterly frequency, spanning 1988:Q1 to 2024:Q4.

Interest rates. We use zero-coupon nominal Treasury yields with maturities ranging from one to 41 quarters from Liu and Wu (2021) available from Cynthia Wu's website. We rely on this dataset because it provides a daily zero coupon yield curve with maturities ranging between 1 and 360 months. We convert the daily data to quarterly frequency by taking quarterly averages. To align the yield data with the survey data, we start our sample in 1988:Q1 and end it in 2024:Q4.

4.2 Are Risk Premia Constant?

As our first empirical exercise, we use regression families (5) and (6) to test Assumption 1, i.e., whether risk premia are constant or time varying.

We start with regression (5), which recall is the survey-based counterpart to the textbook Campbell-Shiller regression, with future realized yields replaced by their one-step-ahead consensus forecasts from

[2] This specification is similar to the specification in Shue, Townsend, and Wang (2025). However, rather than considering the expected change in short-term interest rate in the long run, $\overline{\mathbb{E}}_t[y_{t+n}^{(1)}] - y_t^{(1)}$, they use the short-run change expected change of the short rate, $\overline{\mathbb{E}}_t[y_{t+1}^{(1)}] - y_t^{(1)}$, as their main independent variable.

Table 1. Testing for Constant Risk Premia using Regression (5)

	$\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] - y_t^{(n)}$					
	$n=2$	$n=3$	$n=5$	$n=9$	$n=21$	$n=41$
$\frac{1}{n-1}(y_t^{(n)} - y_t^{(1)})$	0.83	0.97	0.72	0.84	-0.22^{**}	-2.97^{***}
	(0.31)	(0.29)	(0.31)	(0.30)	(0.47)	(0.95)

Notes: This table presents OLS regression coefficients from equation (5) regressing (consensus) expected change in yields, $\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] - y_t^{(n)}$, on yield spreads, $\frac{1}{n-1}(y_t^{(n)} - y_t^{(1)})$. The consensus forecast $\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}]$ is constructed as the average of the forecasters in the dataset. Bond maturities are $n = 2, 3, 6, 9, 21$, and 41 quarters. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis that $\beta = 1$, with *, **, and *** indicating statistical significance at 10%, 5%, and 1% levels, respectively.

survey data. This regression tests the relationship between the yield spread of a bond that matures in n-periods and the one-period-ahead expected change in the bond's yield. When risk premia are constant, a higher yield spread should lead to higher expected yields on long-term bonds. Since the BCFF survey only contains forecasts for bond yields of maturities 1, 2, 4, 8, 20, and 40 quarter, we can run regression (5) for $n = 2, 3, 5, 9, 21$, and 41 quarters.

Table 1 presents the results. For shorter maturities, we do not find evidence for presence of time-varying risk premia. Specifically, for $n = 2, 3, 5$ and 9 quarters, we cannot reject the null hypothesis that $\beta = 1$. The estimated coefficients are around one. In contrast, for longer maturities—specifically, $n = 21$ and 41—we find evidence for significant deviations from the null hypothesis that $\beta = 1$. The estimated coefficients are negative at the long end, indicating that when the yield spread rises, expected longer maturity yields tend to fall rather than rise. This result on the behavior of expected yields aligns with earlier papers that test this regression in the data for realized yields, such as Shiller (1979), Shiller et al. (1983), and Campbell and Shiller (1991).

Next, we turn to regression family (6) which tests the Samuelson-Sargent martingale property of forward rates. Recall that under the null hypothesis that risk premia are constant, the estimated slope coefficients of these regressions have to be equal to one for all $n \geq 1$ and all $m \geq 0$. Table 2 reports the results. Estimated coefficients echo the results from Table 1. For shorter maturities (i.e., when both n and $n + m$ are small) the estimated coefficients are near one, and we cannot reject the null hypothesis that $\beta = 1$, indicating that when forward premia rise, the expected one-step-ahead forecasts $\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] - y_t^{(n)}$ rise one-for-one. In contrast, for longer maturities (that is, when either m or $n + m$ are large), we resoundingly reject $\beta = 1$.

4.3 Do Investors Understand the Relationship Between Interest Rates?

Our results in Tables 1 and 2 indicate that the absence of risk premia at the short end of the yield curve, while at the same time pointing towards time-varying risk premia for longer maturity bonds. While these results align with earlier literature, recall that our regressions did not rely on of rational expectations nor any assumptions about the expectation formation process.

Table 2. Testing for Constant Risk Premia using Regression (6)

$n+m\backslash m$	0	1	2	4	8	20
1	0.92					
	(0.15)					
2	0.94	1.03				
	(0.14)	(0.07)				
4	0.70*	0.83*	0.86			
	(0.17)	(0.09)	(0.09)			
8	0.65**		0.87**	0.95		
	(0.14)		(0.06)	(0.04)		
20	0.01***	0.46***	0.62***	0.76***	0.86***	
	(0.25)	(0.12)	(0.08)	(0.05)	(0.03)	
40	−2.11***	−0.60***	−0.09***	0.31***	0.58***	0.76***
	(0.66)	(0.32)	(0.21)	(0.13)	(0.09)	(0.06)

Notes: This table presents OLS regression estimates from equation (6) regressing the difference between the (consensus) forecasts of the forward rate and the realized yield, $\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] - y_t^{(n)}$, on the realized forward spread, $(f_t^{(n,m+1)} - y_t^{(n)})$.The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis that $\beta = 1$. *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.

Table 3. Testing for Constant Risk Premia using Regression (10)

$n+m\backslash m$	0	1	2	4	8	20
1	0.83**					
	(0.07)					
2	0.83**	0.87**				
	(0.07)	(0.05)				
4	0.82**	0.84***	0.86***			
	(0.07)	(0.05)	(0.04)			
8	0.89		0.91**	0.95		
	(0.07)		(0.04)	(0.03)		
20	0.90	0.88**	0.88**	0.90***	0.92***	
	(0.06)	(0.05)	(0.04)	(0.03)	(0.02)	
40	0.71**	0.68***	0.68***	0.71***	0.76***	0.83***
	(0.11)	(0.09)	(0.07)	(0.06)	(0.04)	(0.02)

Notes: This table presents OLS regression coefficients from equation (10) regressing the difference between the (consensus) expected forward rate and the realized yield, $\overline{\mathbb{E}}_t[f_{t+h}^{(n,m)}] - y_t^{(n)}$, on the forward spread, $f_t^{(n,m+h)} - y_t^{(n)}$, pooled across different forecast horizons $h = 2, 3, 4, 5$, and 6 quarters. Consensus forecasts are constructed as the average of the forecasters in the dataset. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis that $\beta = 1$. *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.

As our next exercise, we use regression families (10) and (11) to test Assumptions 2 and 3: that is, whether investors understand the relationship between short- and long-term interest rates and whether

Table 4. Testing for Relationship Between Interest Rates using Regression (11)

	$\overline{\mathbb{E}}_t[y_{t+1}^{(n)}] - y_t^{(n)}$	
	$n = 2$	$n = 4$
$\frac{1}{n}(\overline{\mathbb{E}}_t[y_{t+n}^{(1)}] - y_t^{(1)})$	1.25^{***}	1.21
	(0.05)	(0.13)

Notes: This figure reports OLS regression coefficients from equation (11) regressing (consensus) expected change in yields, $\overline{\mathbb{E}}_t[y_{t+1}^{(n)}] - y_t^{(n)}$, on (consensus) expected change in short-term yields, $\frac{1}{n}(\overline{\mathbb{E}}_t[y_{t+n}^{(1)}] - y_t^{(1)})$. Consensus forecasts are constructed as the average of the forecasters in the dataset. Bond maturities are $n = 2, 4$, quarters. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis that $\beta = 1$. *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.

the consensus expectations operator satisfies the law of iterated expectations.

Table 3 presents OLS coefficients for regression (10) together with their respective standard errors, pooled across forecast horizons $h = 2, 3, 4, 5$, and 6 quarters available in the survey data. We exclude the $h = 1$ quarter forecast horizon, as recall that the relationship between realized forward rates and their one-period-ahead forecasts is only a test of Assumption 1, as opposed to a joint test of Assumptions 1–3. As the table indicates, the estimate slope coefficient is consistently below 1, and statistically significant for the vast majority of (m, n) pairs available in the survey data. This indicates a joint rejection of Assumptions 1–3 in our dataset. Furthermore, in view of our findings in Tables 1 and 2, which did not indicate time-varying risk premia for shorter-term interest rates, we interpret the rejection of $\beta = 1$ for short maturity bonds in Table 3 as a rejection of Assumptions 1 and 3. Thus, either investors do not understand the relationship between different interest rates (violation of Assumption 2(a)), this understanding may not be common knowledge among them (violation of Assumption 2(b)), or the consensus expectations of the short-term interest rate may violate the law of iterated expectations (violation of Assumption 3).

Finally, we turn to regression (11) as another test of whether investors understand the relationship between short- and long-term interest rates. This family of regressions tests whether expected changes in the short-rate translate to higher expected changes in long-term interest rates. The results are presented in Table 4. Notice that since we only have forecasts up to five quarters ahead, we can only run this regression for bonds of maturity $n = 2$ and 4 quarters. In both cases, we find a coefficient that is greater than 1, though only the estimate for $n = 2$ quarters is statistically different from 1.

5 Conclusion

A large body of empirical research has documented strong evidence against the expectations hypothesis of interest rate, with the standard interpretation that the failure of the expectations hypothesis to the presence of time-varying risk premia. it has been well understood that standard tests of the expectations hypothesis are inherently joint tests of the expectations hypothesis and the rational expectations hypothesis, which imposes tight restrictions on investors' expectations.

In this paper, we examine how investors' subjective expectations shape the term structure of interest rates. We begin by allowing investors to hold arbitrary subjective beliefs and derive the relationship between realized and expected interest rates must satisfy when risk premia are constant. We then show how survey data can be used to construct regression-based tests of the hypothesis that risk premia are constant, without imposing any restrictions on how investors form their expectations. We then introduce restrictions on investors' expectation formation by assuming that they understand the relationship between various interest rates and that this understanding is common knowledge. Leveraging this assumption, we derive a second set of regressions that enables us to test, using survey data, whether investors' expectations of different interest rates are consistent with common knowledge of rationality. Empirically, we find that (i) short-term bonds do not exhibit time-varying risk premia, and yet (ii) investors' expectations remain inconsistent with the expectations hypothesis, even when we do not require their subjective expectations to coincide with those implied by the true data-generating process.

A Appendix

Proof of Lemma 1

The first-order optimality conditions corresponding to investor i's optimization problem (1) imply that

$$P_t^{(n)} + X_{it}^{(n)}/\delta_i^{(n)} = \mathbb{E}_{it}[M_{it+1}P_{t+1}^{(n-1)}]$$
$$P_t^{(1)} + X_{it}^{(1)}/\delta_i^{(1)} = \mathbb{E}_{it}[M_{it+1}],$$

where $M_{it+1} = \beta u_i'(c_{it+1})/u'(c_{it})$. Multiplying both sides of the equations by $\delta_i^{(n)}$ and integrating over all i, we obtain

$$P_t^{(n)} = \int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[M_{it+1}P_{t+1}^{(n-1)}]\mathrm{d}i$$
$$P_t^{(1)} = \int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[M_{it+1}]\mathrm{d}i,$$

where $\hat{\delta}_i^{(n)} = \delta_i^{(n)}/\int_0^1 \delta_j^{(n)}\mathrm{d}j$ and we are using the fact that the bonds are in zero net supply. Consequently,

$$\log P_t^{(n)} - \log P_t^{(1)} = \log\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[M_{it+1}P_{t+1}^{(n-1)}]\mathrm{d}i - \log\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[M_{it+1}]\mathrm{d}i,$$

which in turn implies that

$$\begin{aligned}\log P_t^{(n)} - \log P_t^{(1)} \approx{}& \int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1} + \log P_{t+1}^{(n-1)}]\mathrm{d}i + \frac{1}{2}\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[(\log M_{it+1} + \log P_{t+1}^{(n-1)})^2]\mathrm{d}i\\ &- \frac{1}{2}\left(\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1} + \log P_{t+1}^{(n-1)}]\mathrm{d}i\right)^2 - \int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1}]\mathrm{d}i\\ &- \frac{1}{2}\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log^2 M_{it+1}]\mathrm{d}i + \frac{1}{2}\left(\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1}]\mathrm{d}i\right)^2\end{aligned}$$

to a second-order approximation. As a result,

$$\begin{aligned}\log P_t^{(n)} - \log P_t^{(1)} \approx{}& \overline{\mathbb{E}}_t[\log P_{t+1}^{(n-1)}] + \frac{1}{2}\overline{\mathbb{E}}_t[\log^2 P_{t+1}^{(n-1)}] + \int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1}\log P_{t+1}^{(n-1)}]\mathrm{d}i\\ &- \frac{1}{2}\left(\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1} + \log P_{t+1}^{(n-1)}]\mathrm{d}i\right)^2 + \frac{1}{2}\left(\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1}]\mathrm{d}i\right)^2,\end{aligned}$$

which can be rewritten as

$$\begin{aligned}\log P_t^{(n)} - \log P_t^{(1)} \approx{}& \overline{\mathbb{E}}_t[\log P_{t+1}^{(n-1)}] + \frac{1}{2}\overline{\mathbb{E}}_t[\log^2 P_{t+1}^{(n-1)}] + \int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log M_{it+1}\log P_{t+1}^{(n-1)}]\mathrm{d}i\\ &- \frac{1}{2}\left(\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[\log P_{t+1}^{(n-1)}]\mathrm{d}i\right)\left(\int_0^1 \hat{\delta}_i^{(n)}\mathbb{E}_{it}[2\log M_{it+1} + \log P_{t+1}^{(n-1)}]\mathrm{d}i\right).\end{aligned}$$

Rearranging and simplifying the above expression establishes equation (2). □

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