Institutional Trust and Law Abidingness*

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Abstract

This paper investigates the relationship between institutional trust, law enforcement, and citizen compliance through a formal model of legal rule-setting and enforcement. The model features a government that balances social welfare and revenue considerations when determining legal restrictions and enforcement efforts, alongside a heterogeneous population with varying motivations for compliance. While the government sets a *de jure* legal rule, it retains discretion in enforcement, establishing a *de facto* rule that may diverge from the written law. Citizens, categorized as rule-followers, opportunists, or crowd-followers, assess both the official rule and enforcement signals before deciding how to act. The analysis demonstrates how the government's rule-setting and enforcement strategies influence citizens' compliance, highlighting the critical role of institutional trust in maintaining legal legitimacy. It emphasizes that consistent and transparent enforcement is key to fostering compliance, while discrepancies between de jure and de facto law can erode trust and undermine the effectiveness of legal institutions.

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1 Introduction

Understanding why individuals obey or disobey the law is a fundamental question in the study of legal compliance. Traditional explanations often revolve around two primary motivations: instrumental reasons, where individuals comply to avoid punishment, and normative reasons, where individuals follow the law out of moral duty or respect for the legal system. The instrumental view is epitomized by Holmes's (1896) concept of the "badman," or called "opportunist," who follows the law solely to avoid consequences, while the normative view is embodied by the "rule-follower," who obeys the law because they believe in the inherent fairness of the law and feel guilt when they do not comply. As Tyler (2006) argues "lawmakers and law enforcers would do much better to make legal systems worthy of respect than to try to instill fear of punishment." The normative and instrumental perspectives overlook the role of social norms and perceived enforcement in shaping behavior. Many individuals are neither strict rule-followers nor pure opportunists but instead adjust their behavior based on how others act, forming what I call "crowd-followers." This paper develops a model to analyze how government enforcement strategies influence citizen compliance, especially when enforcement is imperfect due to cost, or selectively applied. It examines the government's trade-off between social welfare and fine revenue, showing how strategic enforcement shapes deterrence, social norms, and institutional trust.

Crowd-followers, a third type of citizen introduced in this paper, provide an additional layer to this understanding. Unlike rule-followers, who obey the law due to moral duty, or opportunists, who comply to avoid punishment, crowd-followers' behavior is shaped by social norms. These norms are influenced by the distribution of rule-followers and opportunists in the population. The government's decision on where to set the rule directly impacts rule-followers, while its enforcement strategy, both the *de facto* rule and the level of enforcement, affects opportunists. As rule-followers and opportunists adjust their behavior in response to these factors, crowd-followers, who tend to mimic the actions of others, also change their behavior. This cascading effect leads to the erosion of institutional trust, as the mismatch between the written law and its enforcement result undermines the perceived legitimacy of the legal system.

Institutional trust plays a pivotal role in shaping individuals' willingness to comply with the law. When citizens perceive their government as trustworthy, fair, and capable of effectively enforcing laws, compliance with legal rules increases. Trust in institutions enables governments to achieve higher levels of social welfare and stability, as citizens are more likely to accept governmental authority and follow its mandates. Conversely, gaps between *de jure* and *de facto* law can erode institutional trust, leading to diminished compliance and, in some cases, active opposition to the legal system. This paper explores the relationship between institutional trust and compliance, with a particular attention on how rule-setting and enforcement strategies shape citizens' willingness to obey the law.

A central theme of this analysis is the reciprocal nature of the relationship between governments and citizens. Legal systems are not static, top-down structures where governments impose rules on passive populations. Instead, they are dynamic and interactive, with government actions and citizen responses continuously shaping each other. Citizens' willingness to comply depends on their perception of whether the government, balancing social welfare and revenue considerations, enforces the law with sufficient effort and consistency. This perception forms the foundation of what we call *institutional trust* here. When enforcement is transparent and consistent, trust strengthens, reinforcing compliance and the legitimacy of legal institutions.

A second key theme is the distinction between *de jure* and *de facto* law. As Pound (1910) observed, "the law upon the statute books will be far from representing what takes place actually." This gap can arise from practical limitations, such as resource constraints, political pressures, or enforcement challenges. For instance, speed limits may go unenforced in remote areas due to high monitoring costs, or environmental regulations may be weakened by insufficient resources. While some discrepancies are unavoidable, they can also result from strategic manipulation. Governments may enact insincere rules to create the appearance of control while selectively enforcing laws to maximize compliance or revenue. A notable case occurred in 2013 when New Jersey legislator David Levinsky proposed doubling fines for speeding and lowering speed limits to improve pedestrian and bicyclist safety. While the committee rejected the increased fines, they approved the lower speed limits, illustrating

how legal adjustments can serve as a strategic instrument.[1]

The model features a government that sets a *de jure* legal rule, specifying permissible behavior, and an associated fine for violations. However, the government retains discretion in enforcement, determining a *de facto* rule that may differ from the written law. The government's objective is twofold: it seeks to align citizen behavior with its preferred level of restriction while also generating revenue from fines. Citizens, in turn, are heterogeneous in their motivations. Some follow the law out of intrinsic respect, others comply only when enforcement is strict, and a third group is influenced by prevailing social norms. These distinctions allow us to analyze how different enforcement strategies shape overall compliance. This framework provides insights into the conditions under which legal institutions maintain compliance and legitimacy.

This model shows that consistent enforcement builds institutional trust and encourages compliance. It also reveals a trade-off between generating revenue through fines and promoting social welfare through enforcement. These findings have broader implications for understanding the stability of legal systems and the role of institutional design in promoting law-abiding behavior. A reciprocal, trust-based relationship between governments and citizens is essential for sustaining legal legitimacy and social order.

The rest of the paper is organized as follows. The next section discusses some related literature. Section 3 introduces the baseline model, examining how enforcement behavior and citizen compliance interact, and how discrepancies between *de jure* and *de facto* law can destabilize this reciprocal relationship. Section 4 provides a preliminary analysis of the sincere rule in both extreme cases and general cases. Section 5 concludes with a discussion and possible extensions.

2 Related Literature

This paper relates to several strands of distinct literature. There is a large philosophy of law literature focusing on the morality of law (e.g. Hart & Green (2012); Dworkin (1986)).

[1]David Levinsky, Asking Motorists to Slow It Down, Burlington County Times, May 16, 2013.

As discussed in the introduction, this work is inspired by Fuller's *The Morality of Law.* Fuller defines law as "the purposive human enterprise of subjecting human conduct to the governance and control of general rules," emphasizing the communal and interactive nature of law. Fuller's principle that "there must be congruence between declared rule and official action" (Fuller, 1964, p 106.), which forms the core focus of our paper. Waldron (1994) considers law as a complex network of social institutions, and illustrates more on this principle, which "requires the government to lay down rules and to insist that its officials not depart from them, at least where they impact upon individual citizens, even if a departure from the rules would actually aid the pursuit of the goal" that the legal rule intends to achieve.

This paper also relates to several works in discussing legitimacy. One dimension of legitimacy is perceived through procedural justice. Tyler & Mitchell (1993) summarize a review indicating that the key factor affecting the perceived legitimacy of authorities is procedural fairness. Procedural judgments have been found to be more important than either outcome favorability - whether the person won or lost - or judgments about outcome fairness. Pettit (2023) proposes the "Tough-Luck Test," which argues that "for any law or measure that is initiated, those whose approval or interest it fails to match have good, if defeasible reason to think that, robustly over variations in the procedures employed, the choice was guided by the public good and it is just tough luck that it went against them."

The second layer of legitimacy comes from the citizens' everyday experience interacting with authorities, like police officer and judges. Tyler (2001) looks at how members of the public experience the social regulatory activities of the police and the courts. He examines the influence of personal experience on views about the legitimacy of law and legal authorities, and shows that personal experiences generalize to shape broad judgements about the legitimacy of legal authorities. He considers the behavior if the legal authorities during a personal encounter will influence the overall confidence in the legitimacy of legal authorities, which further becomes the basis on which people decide whether defer to particular authorities during a subsequent personal encounter. In his later work, based on a survey conducted in Chicago, Tyler (2006) challenges many assumptions about why people obey the law. He rejects the instrumental view that people obey the law because they fear the

potential punishment or anticipated payoffs. Instead, Tyler believes in a normative view that people are more concerned about if a legal system, including both the law itself and the authorities, is fair rather than they have won or lost. The normative elements, like fairness, respect, dignity, are more critical in obtaining legal compliance than often imagined, and this realization should force us to reconsider how the legal system is operated and the legal goods of society are delivered.

Following Tyler (2006)'s definition of legitimacy that "authorities, institutions, and social arrangements are appropriate, proper, and just," Patty & Penn (2011) develop a formal model of legitimate collective choice. They define a policy to be legitimate if the process through which the final choice determined is consistent with some principle that can be used to rank the potential policy choices. Set in complete information environments, instead of finding out which policy is optimal, their theory focuses on why the selected policy is legitimately claimed to be so. They characterize the class of procedures that are legitimate, and prove that legitimate policy decisions consistent with principles always exits. Penn & Patty (2023) continues the work and presents an optimal classification model comparing two cities: one issue speeding tickets to boost operating revenue by ticketing indiscriminately, while the other issues speeding tickets to improve public safety by only ticketing speeders. They show that people in the revenue-motivated city will have no incentive to slow down to avoid a ticket as they can expect to be ticketed regardless of their speed. This city will observe lower levels of public safety.

In addition to the algorithmic perspective offered by Penn and Patty, legitimacy can also be influenced by other factors perceived by citizens, such as the polarization of court, the demand of justice, or a sense of accountability. Gibson (2024) examines the impact of the U.S. Supreme Courts decision in *Dobbs v. Jackson Womens Health Organization* on institutional legitimacy. He argues that *Dobbs*, which overturned *Roe v. Wade*, represents a significant departure from prior Supreme Court rulings due to its unique erosion of diffuse support for the Court, driven by the moral polarization of the abortion issue. Besides, Benabou & Tirole (2006) proposes a theory of why people may feel such a need to believe in a just world, and of its implications for redistributive policies, which provides a good parallel for the

necessity of institutional trust in the legal system. In addition, the legitimacy of the legal authorities may relate to some literature on accountability, especially models considering the candidate's competence and effort. Canes-Wrone et al. (2001) develop a model under incomplete information that the voter has some probability of learning the incumbent and the challenger's executive competence, and analyze conditions under which a reelection-seeking executive will act in the public interest. Acharya et al. (2024) studies the interplay between the incumbent's ability and effort, and examines how political careers are shaped by the incentives that voters provide incumbents to work in the public interest.

There is some law and economics literature focusing on the design of rules, punishments, and monitoring structures in order to discourage certain types of behaviors. This literature, pioneered by Becker (1968) and Becker & Stigler (1974), and typically does not investigate on how private attitudes affect enforcement. Milgrom et al. (1990) argues that the emerging of institutions, such as a system of judges, complements the effectiveness of reputation mechanisms in large communities, emphasizing past interactions can be an effective bond for honest behavior. Also, Shavell (2004) provides an in-depth analysis and synthesis of the economic approach to the building blocks of our legal system.

This paper also engages with literature that examines the discrepancy between *de facto* and *de jure* rules. Huber & McCarty (2004) studies how bureaucracy's attempted and realized policies are shaped by capacity constraints and policy shocks. Gilbert (2015) explores how insincere rules can improve compliance through both punitive and deceptive mechanisms. Holland (2016) introduces the concept of forbearance, defined as the intentional and revocable nonenforcement of law. Holland argues that politicians often withhold sanctions to maximize both electoral votes and rents. Using multiple methods, she distinguishes between cases where politicians are unable to enforce the law and those where they are unwilling, showing how forbearance serves as a tool to mobilize voters and signal distributive commitments. Further, our paper is closely related to a small literature on law enforcement and law-abidingness. Acemoglu & Jackson (2017) consider the situation that agents have the option of whistle-blowing, and examines the interplay between social norms and the enforcement of laws. They show that laws that are in strong conflict with prevailing social

norms may backfire and lead to a significant decline in law-abiding behavior in society. In addition, Benabou & Tirole (2011) develop a model in which norms encourage certain types of behavior because of agents' desire to signal their intrinsic types to others or to themselves, and laws in that context both interact with this signaling role of norms and may themselves signal the society's attitude to individuals.

3 Baseline Model

This model is grounded on four key assumptions. First, we consider a public enforcement framework in which the government is responsible for both creating and enforcing the law. Second, for a more realistic perspective, we assume that rule-makers have "mixed" objectives: while they aim to promote social welfare, they are also motivated by practical considerations, particularly the need to offset enforcement costs and generate revenue from fines imposed on violators. Third, we use the terms "rule" and "law" interchangeably and do not delve into the specifies or ambiguities of their definitions for the purpose of this analysis. Fourth, the focus is limited to single-dimensional everyday behaviors, such as speeding, parking, or pollution emissions, that can be represented by a continuous range.

For example, a speed limit can be set anywhere between 0 and 85 mph[2], which we normalize to the interval $[0, 1]$. In this framework, the rule establishes an upper bound on permissible behavior: any action exceeding r is prohibited, where $r = 0$ represents a complete ban, and $r = 1$ signifies no restriction. In the baseline model, we assume that citizens do not have a strong intrinsic preference for a specific restriction level. Instead, their preference is shaped by the written r, indicating a desire for greater freedom within the range $[r, 1]$, where they prefer more freedom relative to the imposed limit. Simply put, citizens aim to maximize their freedom and efficiency, such as driving as fast as possible, without triggering accidents and enforcement.

[2]The highest posted speed limit in the country is 85 mph (137 km/h) and can be found only on Texas State Highway 130, a toll road that bypasses the Austin metropolitan area for long-distance traffic.

3.1 Setup

The model features two primary actors: the Government (G), and a representative citizen (C). For clarity, the female pronoun is used for the Citizen, while the neutral pronoun is used for the Government.

At $t = 0$, Nature determines the Government's type, reflecting its preferred level of restriction, $\theta_G \sim U[0, 1]$, as well as the distribution of citizen types. The population consists of three types of citizens:

A. A proportion α are **rule-followers**, who comply with the law out of duty or respect.

B. A proportion β are **opportunists**, who seek to maximize personal gain while minimizing penalties.

C. The remaining proportion $1 - \alpha - \beta$ are **crowd-followers**, whose preferences are influenced by social norms. Their type is given by $\theta_C \sim U[r, 1]$, reflecting a preference for more freedom relative to the imposed limit.

Given its type and the population distribution, the Government sets and publicly announces the written law $L(r, f)$, which consists of:

i. A legal rule $r \in [0, 1]$, specifying the maximum permissible behavior.

ii. A fine $f > 0$, representing the penalty for violations.

At $t = 1$, the Government exercises its discretion by setting the *de facto* legal rule $r' = r + \epsilon$, where $\epsilon \geq 0$. It then chooses an enforcement rate $e \in [0, 1]$, incurring an enforcement cost κe, where κ represents the marginal cost of enforcement. Simultaneously, the Citizen observes the *de jure* rule r and a signal of the *de facto* rule r' before choosing her behavior $b \in [r, 1]$. If her behavior exceeds the *de facto* rule ($b > r'$), she is in violation and faces detection with probability e, resulting in a fine.

3.2 Citizen's Payoffs

In the baseline model, the Citizen belongs to one of the three types:

A. **Rule-follower**: Obeys the law out of duty, complying strictly with the written rule, and her optimal behavior can be expressed as

$$b_A^* := \arg\max_{b_A} \quad -(b-r)^2 - \mathbb{1}_{b>r'} ef.$$

B. **Opportunist**: Violate the law as much as possible while minimizing penalties, and her optimal behavior can be expressed as

$$b_B^* := \arg\max_{b_B} \quad -(b-1)^2 - \mathbb{1}_{b>r'} ef.$$

These two types represent opposite extremes: rule-followers comply out reason of law, while opportunists obey only out of fear of punishment. However, most citizens are crowd-followers, whose behavior is shaped by social norms. Their optimal behavior depends on the population distribution of rule-followers and opportunists.

C. **Crowd-follower**: Given that the population consists of α rule-followers and β opportunists, her behavior is determined by

$$vb_C^* := \arg\max_{b_C} \quad -(b-\delta)^2 - \mathbb{1}_{b>r'} ef,$$

where $\delta := \alpha b_A^* + \beta b_B^* + \theta_C$, and θ_C is uniformly distributed in $[r, 1]$.

3.3 Government's Payoffs

The Government makes a choice of (e, r') to maximize its objectives, which balances social welfare and revenue. It payoffs is given by

$$\max_e \quad \underbrace{-\alpha(b_A^* - \theta_G)^2 - \beta(b_B^* - \theta_G)^2 - (1-\alpha-\beta)(b_C^* - \theta_G)^2}_{social welfare} \underbrace{-\kappa e}_{cost} \underbrace{+\lambda \mathbb{1}_{b>r'} ef}_{fine},$$

where the first three terms capture the social welfare loss from the gap between citizens' behavior and the Government's preferred restriction level θ_G, κe represents the enforcement cost, and $\lambda \mathbb{1}_{b>r'}$ accounts for the revenue generated from fines. The parameter $\lambda \geq 0$ reflects

the weight that the Government places on revenue relative to social welfare.

4 Preliminary Analysis

Both the extreme and general cases are analyzed to better understand the strategies the government can use to maximize its payoffs. The model aims to combine two key dimensions: whether the rule is sincere, and whether the rule is consistent. A *sincere* rule is defined as $r = \theta$, whereas an *insincere* rule is characterize by $r < \theta$, which can be written as $r + x = \theta$ with $x > 0$. As the *de facto* rule is defined as $r' := r + \epsilon$, where $\epsilon \geq 0$, a *congruent* means $r = r'$ and conversely, an *incongruent* rule arises when $r < r' = r + \epsilon$, with $\epsilon \geq 0$ chosen by the Government.

In the next three subsections, we consider the extreme cases ($\alpha = 0$, or $\beta = 0$, or both), the general cases with sincere and congruent rule ($r = \theta = r'$), and the general cases with sincere and incongruent rule ($r = \theta \leq r'$).

4.1 Extreme Cases

The Government's payoffs are

$$\max_{e} \quad -\alpha(b_A^* - r)^2 - \beta(b_B^* - r)^2 - (1 - \alpha - \beta)(b_C^* - r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef,$$

and the Citizen's payoffs can be written as

$$\max_{b} \quad -(b - \delta)^2 - \mathbb{1}_{b>r} ef,$$

with $\delta_A = r$ (rule-follower), $\delta_B = 1$ (opportunist), and $\delta_C = \theta_C$ (crowd-follower).

Proposition 1. *In a society where all citizens are rule-followers ($\alpha = 1$ and $\beta = 1$), the unique equilibrium is characterized by full compliance with the rule, $b^* = r$, and zero enforcement effort by the government, $e^* = 0$.*

This proposition captures the ideal scenario where all individuals comply with the law purely out of intrinsic respect. Since rule-followers align their behavior directly with the

written law, there is no need for enforcement mechanisms. The government, therefore, does not expend resources on enforcement because the law is naturally followed. The proof follows from the fact that rule-followers optimize their utility by choosing $b_A = r$, leading to $e^* = 0$ in equilibrium. This represents the scenario where legal norms are internalized, and compliance is costless to the state.

Proposition 2. *In a society where all citizens are opportunists ($\alpha = 0$ and $\beta = 1$), an equilibrium exists such that*

i. if $\kappa \geq \lambda f$, $e^* = 0$ and $b_B^* = 1$.

ii. if $\kappa < \lambda f$ and $\lambda \leq 1$, $e^* = \frac{(1-r)^2}{f}$ and $b_B^* = r$.

iii. if $\kappa < \lambda f$ and $\lambda > 1$, $e^* \nearrow \frac{(1-r)^2}{f}$ and $b_B^* = 1$.

This proposition highlights the enforcement challenge when all individuals are opportunists, obeying only when faced with significant penalties. The government must decide whether to invest in enforcement based on cost κ and revenue incentives λ. When enforcement costs exceed the potential fine revenue, $\kappa \geq \lambda f$, the government rationally chooses not to enforce the law, leading to full noncompliance. However, when $\kappa < \lambda f$, it becomes optimal to exert some enforcement effort, deterring violations partially or fully. The government's optimization problem, therefore, involves balancing enforcement costs against compliance incentives. In cases where the government prioritizes revenue over social welfare, the optimal strategy is to enforce the rule just enough to sustain violations, maximizing fine revenue rather than fully deterring opportunistic behavior.

Proposition 3. *In a society where all citizens are crowd-followers ($\alpha = 0$ and $\beta = 0$), an equilibrium exists such that*

i. if $\kappa \geq \lambda f$, $e^* = 0$ and $b_C^* = \theta_C$.

ii. if $\kappa < \lambda f$ and $\lambda \leq 1$, $e^* = \frac{(1-r)^2}{3f}$ and $b_C^* = r$.

iii. if $\kappa < \lambda f$ and $\lambda > 1$, $e^* \nearrow \frac{(1-r)^2}{3f}$ and $b_C^* = \theta_C$.

This proposition illustrates how compliance in a crowd-follower society is shaped more by social influence than by legal authority or punishment, playing a crucial role in the formation of institutional trust. When enforcement is costly or inconsistent, individuals tend to mirror the average behavior of others rather than strictly adhering to the law, resulting in a wide range of driving speeds rather than uniform compliance. If violations are widespread, speeding becomes the norm, weakening the laws deterrent effect and reinforcing noncompliance. Conversely, when the government enforces the law effectively and consistently, crowd-followers adjust their behavior to align more closely with legal rules, gradually strengthening institutional trust. This dynamic highlights the interaction between social norms and governance, where the effectiveness of laws depends not just on their existence but on the ability of institutions to shape and sustain compliance through credible enforcement.

4.2 Sincere and Congruent Rule

The Government's payoffs,

$$\max_{e} \quad -\alpha(b_A^* - r)^2 - \beta(b_B^* - r)^2 - (1-\alpha-\beta)(b_C^* - r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef,$$

and the Citizen's payoffs can be written as

$$\max_{b} \quad -(b-\delta)^2 - \mathbb{1}_{b>r} ef,$$

with $\delta_A = r$ (rule-follower), $\delta_B = 1$ (opportunist), and $\delta_C = \theta_C$ (crowd-follower).

Proposition 4. *Citizen's behavior depends on enforcement effort. If it exceeds the threshold* $\frac{(\delta-r)^2}{f}$*, their best response is* r*; otherwise, they follow their ideal point* δ*.*

Building on the previous results, this proposition identifies the tipping point at which enforcement becomes effective in driving compliance. When enforcement effort e is too low, individuals rely on personal preferences and social norms rather than legal rules. However, once e surpasses the necessary deterrence threshold, compliance rises sharply. The proof follows from solving the individual optimization problem for each citizen type, demonstrating

how even marginal increases in enforcement can lead to substantial behavioral shifts. When enforcement effort e falls into one of the following three categories, it leads to

I. no deterrence.

II. partial deterrence (affecting only crowd-followers).

III. full deterrence (affecting both crow-followers and opportunists).

The threshold can be summarized as the table below.

effort (e)	$I := \left[0, \frac{(r'-\delta)^2}{f}\right)$	$II := \left[\frac{(r'-\delta)^2}{f}, \frac{(r'-1)^2}{f}\right)$	$III := \left[\frac{(r'-1)^2}{f}, 1\right]$
A (rule-follower)	r	r	r
B (opportunists)	1	1	r'
C (crowd-follower)	δ	r'	r'

Proposition 5. *The government's enforcement decision depends on the enforcement cost, composition of the population, and the government's type.*

- *If $\kappa > (1-\alpha)\lambda f$ and $\lambda > 1$, then the government does not enforce the rule ($e^* \in I$), and all citizens act according to their ideal points ($b_A^* = r, b_B^* = 1, b_C^* = \delta$).*

- *If $\kappa < \lambda\beta f$ and $\lambda < 1$, then the government fully enforces the rule ($e^* \in III$), and all citizens comply in equilibrium $b_A^* = r, b_B^* = r, b_C^* = r$.*

This proposition outlines the conditions under which governments either enforce rules strictly or allow opportunists to prevail. The key takeaway is that when enforcement costs exceed the government's revenue incentives, the government opts to under-enforce the law, resulting in selective compliance. Conversely, when enforcement is cost-effective, full compliance can be achieved. The proof involves computing the equilibrium effort e^* by balancing enforcement costs and fine revenue. This reflects broader regulatory trade-offs: some jurisdictions enforce laws rigorously to maximize compliance, while others tolerate rule-breaking due to resource constraints.

4.3 Sincere and Incongruent Rule

The Government chooses (r, r'), which are both observable and $r' = r + \epsilon$ with $\epsilon \geq 0$, and then chooses effort e to maximize

$$\max_{e} \quad -\alpha(b_A^* - r)^2 - \beta(b_B^* - r)^2 - (1 - \alpha - \beta)(b_C^* - r)^2 - \kappa e + \lambda \mathbb{1}_{b>r'} ef,$$

and recall the Citizen's payoffs can be written as

$$\max_{b} \quad -(b - \delta)^2 - \mathbb{1}_{b>r'} ef,$$

with $\delta_A = r$ (rule-follower), $\delta_B = 1$ (opportunist), and $\delta_C = \theta_C$ (crowd-follower).

Proposition 6. *The government's enforcement decision depends on the enforcement cost, composition of the population, and the government's type.*

- *If $\kappa > (1 - \alpha)\lambda f$ and $\lambda > 1$, then the government does not enforce the rule ($e^* \in I$), and all citizens act according to their ideal points ($b_A^* = r, b_B^* = 1, b_C^* = \delta$).*
- *If $\kappa < \lambda\beta f$ and $\lambda < 1$, then the government does not find full deterrence optimal ($e^* \in II$), and only crowd-followers are deterred ($b_A^* = r, b_B^* = 1, b_C^* = r'$).*

This proposition demonstrates that even when governments prioritize social welfare over revenue, full deterrence may not be the optimal strategy. The proof is derived by analyzing the enforcement trade-offs between crowd-followers and opportunists. This result is aligned with theories of optimal law enforcement, where governments must strike a balance between deterrence and economic efficiency. In practice, this means that governments may implement stricter rules but enforce them selectively, deterring only those who respond to moderate enforcement, while opportunists continue to violate the law, ultimately undermining public safety.

Speeding on U.S. highways is a persistent issue driven by time-saving incentives, weak enforcement perceptions, and prevailing social norms. Route 130 in New Jersey exemplifies the challenge: despite posted speed limits of 45-55 MPH, drivers frequently exceed 80 MPH,

creating significant public safety risks. In response, policymakers enacted "Antwans Law," permanently lowering speed limits and tripling fines for violations. However, compliance remains limited because the effectiveness of these laws depends on more than just their existence but also actual enforcement. This aligns with the model's prediction that without sufficient deterrence, opportunistic drivers continue to speed, and social norms remain unchanged, which corresponds to this proposition.

Moreover, this model illustrates a broader challenge that extends to other regulatory domains, such as pollution emission or censorship issues in medicine and media. In these areas, laws alone do not guarantee compliance. Without consistent and sufficient enforcement, citizens may perceive the laws as either irrelevant or unenforceable, leading to continued violations despite the presence of written rules. This highlights the importance of building trust in legal institutions, which is central to fostering law-abiding behavior.

5 Concluding Remarks

The model examines how the government's rule-setting and enforcement strategies shape citizens' willingness to comply with the law. Beyond instrumental factors such as incentives and penalties, we incorporate normative considerations—specifically, citizens' perceptions of whether the government, given its dual focus on social welfare and fine revenue, exerts sufficient and consistent effort in law enforcement. This perception forms the basis of what we call *institutional trust*. Our concept of institutional trust is rooted in Fuller's distinction between internal and external morality. External morality refers to moral judgments about legal decisions, whether they are just, fair, or well-intentioned, based on ideological evaluation independent from personal payoffs. However, even well-intentioned policies can be poorly implemented, leading to unintended consequences. Internal morality, on the other hand, assesses the effectiveness of legal enforcement, focusing on factors such as the clarity, consistency, and timeliness of rules, often judged through direct interactions with legal authorities. This internal dimension is central to institutional trust, as it determines whether citizens perceive the law as legitimate and worth following.

5.1 Discussion

An insincere rule mandates behavior that differs from the rule-maker's true preferences. Instead of directly guiding citizens toward the ideal behavior, it strategically encourages a different response to improve compliance. Although this approach balances maximizing social welfare, by shaping citizen behavior, while minimizing enforcement costs and maximizing revenue. This strategy will be directly punished by rule-followers and further copied by the crowd-followers, making the citizen's behavior more deviated from the government's true preferences. Further, the success of an insincere rule also depends on whether citizens recognize the gap between the rule and the rule-maker's true intentions. If citizens detect the discrepancy, the rule can send misleading signals, eroding trust, reducing compliance, and weakening the law's legitimacy. In contrast, sincere rules clearly communicate the rule-maker's intentions and expectations. While insincere rules may be effective in the short term, repeated interactions shape citizens' expectations about enforcement, making this strategy harder to sustain over time.

Institutional trust is essential for government legitimacy. Trust strengthens when the gap between the written law and the enforced is minimized and when the government applies consistent and adequate enforcement efforts. Consistency between the written law and the executed rule requires the government to establish clear rules and ensure that officials not depart from them, at least where they impact upon individual citizens, even if a departure from the rules would actually aid the pursuit of the goal or intention of the rule. This approach ensures that each citizen knows that her situation is being taken into account in official decision-making to this extent: that officials are required not to surprise her with decisions not in accord with the rules (Waldron, 1994), thereby fostering stable expectations. With this in mind, citizens might feel more inclined to do what is required of them in the push towards the goal of the legal rule, even when that is costly or irksome, for they know that their own sacrifices in that regard are reciprocated by a certain forbearance on the part of the officials for their sake.

It is worth emphasizing how law on the books influences action, rather than solely focusing on why it diverges from law in action. Because penalties often depend on the extent

to which a citizen's behavior deviates from the written law, the law on the books not only defines illegality but also determines its severity. For example, whether driving 85 mph is classified as a serious or minor offense depends on the speed limit, and when punishment is tied to the severity of the violation, the law on the books plays a critical role in shaping the expected consequences for various behaviors. This dynamic explains the incentives for adopting insincere rules. Beyond shaping behavior through enforcement, written laws influence enforcement decisions, which then shape expectations and guide future actions. This reciprocal process highlights the evolving interaction between governments and citizens, showing how legal rules and their enforcement co-develop over time.

5.2 Possible Extensions

There are several possible ways of interpreting the difference between the written law (*de jure*) and actual law (*de facto*), including government's capability, leniency, and effort. Another factor contributing to the discrepancy can be the inconsistency between the government's intention and the citizens' interpretation. For example, the erosion of confidence in the legal system can result from polarization. The legal system becomes more politicized or even partisan, and is deciding important issues that should be left to the political branches. When people losing trust in the legal system, the legal decisions will not be interpreted as a product of impartial deliberation based on the facts and the law for each particular case, but as in favor of or against a particular party or position, and as predetermined by the composition of the decision-making panel.

In addition, we consider the law $L(r, f)$ as given in current setting, and one way to improve the model is to integrate the two-way interaction between government and citizens with collective decision-making in society. For example, add the voting or elected legislatures that reflect social preferences. Besides, we consider f to be fixed, but in reality, the fine usually depends on the exact level of disobedience. For instance, slight speeding will be fined differently comparing to serious speeding behavior. Furthermore, in our current model, the government determines the shock, or discretion, ϵ. It would be intriguing to explore scenarios where the shock arises endogenously, influenced by factors such as the

structural characteristics of the system. Another valuable direction is to explore the model within a multi-period context, which would provide deeper insights into long-term behavioral patterns. This perspective also aligns with the concept that current legal practices set precedents, effectively shaping the legal framework of the future.

Furthermore, the model can be incorporated with morality preferences. Unlike the public enforcement of everyday behaviors, where we assume citizens prefer more freedom, moral-related issues often involve citizens having their own ideal levels of restriction. In such cases, external morality comes into play as citizens evaluate whether a rule is inherently good and designed for the common good. The government, in turn, tends to favor the design of moral laws, as citizens morally assess their appropriateness. However, some laws, though codified, are rarely enforced due to the high cost of enforcement. For example, in developing countries, citizens may have differing preferences regarding restrictions on pirated products — balancing respect for intellectual property against viewing such resources as free tools for education. Despite strict regulations and significant fines for pirated resources like books and movies on the Internet, the government seldom enforces these laws effectively.

Some citizens naturally support laws that align with their ideal points, requiring no additional institutional incentives to support the endeavor. However, for those whose preferences diverge from the codified rule, the perception that the government respects them as individuals and maintains consistency between the written rule and the enforced becomes critical for enforcement. If the government abandons this institutional principle of consistency, it risks losing the only foundation on which some citizens are willing to cooperate. As Fuller emphasizes, such cooperation is essential to the legitimacy and efficacy of the legal enterprise.

References

Acemoglu, D. & Jackson, M. O. (2017). Social norms and the enforcement of laws. *Journal of the European Economic Association*, 15(2), 245–295.

Acharya, A., Lipnowski, E., & Ramos, J. (2024). Political accountability under moral hazard. *American Journal of Political Science*.

Becker, G. S. (1968). Crime and punishment: An economic approach. *Journal of political economy*, 76(2), 169–217.

Becker, G. S. & Stigler, G. J. (1974). Law enforcement, malfeasance, and compensation of enforcers. *The Journal of Legal Studies*, 3(1), 1–18.

Benabou, R. & Tirole, J. (2006). Belief in a just world and redistributive politics. *The Quarterly journal of economics*, 121(2), 699–746.

Benabou, R. & Tirole, J. (2011). *Laws and norms*. Technical report, National Bureau of Economic Research.

Canes-Wrone, B., Herron, M. C., & Shotts, K. W. (2001). Leadership and pandering: A theory of executive policymaking. *American Journal of Political Science*, (pp. 532–550).

Dworkin, R. (1986). *Laws empire*. Harvard University Press.

Fuller, L. L. (1964). The morality of law.

Gibson, J. L. (2024). Losing legitimacy: The challenges of the dobbs ruling to conventional legitimacy theory. *American Journal of Political Science*.

Gilbert, M. D. (2015). Insincere rules. *Va. L. Rev.*, 101, 2185.

Hart, H. L. A. & Green, L. (2012). *The concept of law*. oxford university press.

Holland, A. C. (2016). Forbearance. *American political science review*, 110(2), 232–246.

Holmes, O. W. (1896). The path of the law. *Boston L. School Mag.*, 1, 1.

Huber, J. D. & McCarty, N. (2004). Bureaucratic capacity, delegation, and political reform. *American Political Science Review*, 98(3), 481–494.

Milgrom, P. R., North, D. C., & Weingast*, B. R. (1990). The role of institutions in the revival of trade: The law merchant, private judges, and the champagne fairs. *Economics & Politics*, 2(1), 1–23.

Patty, J. W. & Penn, E. M. (2011). A social choice theory of legitimacy. *Social Choice and Welfare*, 36(3), 365–382.

Penn, E. M. & Patty, J. W. (2023). Algorithms, incentives, and democracy. *arXiv preprint arXiv:2307.02319*.

Pettit, P. (2023). *The State*. Princeton: Princeton University Press.

Pound, R. (1910). Law in books and law in action. *Am. L. Rev.*, 44, 12.

Shavell, S. (2004). *Foundations of economic analysis of law*. Harvard University Press.

Tyler, T. R. (2001). Trust and law abidingness: A proactive model of social regulation. *BUL Rev.*, 81, 361.

Tyler, T. R. (2006). *Why people obey the law*. Princeton university press.

Tyler, T. R. & Mitchell, G. (1993). Legitimacy and the empowerment of discretionary legal authority: The united states supreme court and abortion rights. *Duke LJ*, 43, 703.

Waldron, J. (1994). Why law-efficacy, freedom, or fidelity? *Law & Phil.*, 13, 259.

Appendix

A Preliminary

Since $\theta_C \sim U[r,1]$, we have

$$\mathbb{E}[\theta_C] = \frac{r+1}{2}$$
$$\mathbb{E}[\theta_C^2] = Var(\theta_C) + (\mathbb{E}[\theta_C])^2 = \frac{(1-r)^2}{12} + (\frac{r+1}{2})^2 = \frac{r^2+r+1}{3}$$

so that

$$\begin{aligned}\mathbb{E}[\delta] &= \mathbb{E}[\alpha r + \beta + (1-\alpha-\beta)\theta_C] \\ &= \alpha r + \beta + (1-\alpha-\beta)\mathbb{E}[\theta_C] \\ &= \alpha r + \beta + \frac{r+1}{2}(1-\alpha-\beta)\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}[\delta^2] &= \mathbb{E}[[\alpha r + \beta + (1-\alpha-\beta)\theta_C]^2] \\ &= (\alpha r+\beta)^2 + 2(\alpha r+\beta)(1-\alpha-\beta)\mathbb{E}[\theta_C] + (1-\alpha-\beta)^2\mathbb{E}[\theta_C^2] \\ &= (\alpha r+\beta)^2 + (\alpha r+\beta)(1-\alpha-\beta)(r+1) + \frac{r^2+r+1}{3}(1-\alpha-\beta)^2 \\ &= (\alpha r+\beta)(r-\beta r+1-\alpha) + \frac{r^2+r+1}{3}(1-\alpha-\beta)^2.\end{aligned}$$

Thus, we have

$$\begin{aligned}&\mathbb{E}[(\delta-r)^2] = \mathbb{E}[r^2 - 2\delta r + \delta^2] = r^2 - 2r\mathbb{E}[\delta] + \mathbb{E}[\delta^2] \\ &= r^2 - 2r[\alpha r + \frac{1}{2}(1-\alpha+\beta)] + (\alpha r+\beta)^2 + (\alpha r+\beta)(1-\alpha-\beta)(1-r) + \frac{r^2+r+1}{3}(1-\alpha-\beta)^2 \\ &= (1-\alpha)[\beta r^2 - (1-\alpha)r + \beta] + \frac{1}{3}(r^2+r+1)(1-\alpha-\beta)^2 := E.\end{aligned}$$

B Extreme Cases

B.1 Proof for Proposition 1.

Proof. Recall

$$\begin{aligned}A: \max_b \quad & -(b-r)^2 - \mathbb{1}_{b>r} ef, \\ G: \max_e \quad & -(b_A^* - r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef.\end{aligned}$$

For A,

$$b_A^* = r \text{ and } u_A^*(r) = 0.$$

For G,

$$e^* = 0 \text{ and } v^*(e) = 0.$$

Thus, an equilibrium exists with $b_A^* = r$ and $e^* = 0$. □

B.2 Proof for Proposition 2.

Proof. Recall

$$\begin{aligned} B: \max_b \quad & -(b-1)^2 - \mathbb{1}_{b>r} ef, \\ G: \max_e \quad & -(b_B^* - r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef. \end{aligned}$$

For B,

$$u_B^* = \begin{cases} u(r) = -(r-1)^2 & \text{if } e \geq \frac{(1-r)^2}{f}, \\ u(1) = -ef & \text{if } e < \frac{(1-r)^2}{f}. \end{cases}$$

For G,

$$v(e) = \begin{cases} -\kappa e & \text{if } e \geq \frac{(1-r)^2}{f}, \\ -(1-r)^2 - \kappa e + \lambda ef & \text{if } e < \frac{(1-r)^2}{f}. \end{cases}$$

It follows

i. if $e \geq \frac{(1-r)^2}{f}$, $v^*(e) = v(\frac{(1-r)^2}{f}) = -\frac{\kappa(1-r)^2}{f}$.

ii. if $e < \frac{(1-r)^2}{f}$, $v^*(e) = \begin{cases} v(0) = -(1-r)^2 & \text{if } \kappa \geq \lambda f, \\ \lim_{e \nearrow \frac{(1-r)^2}{f}} v(e) = (\lambda - 1 - \frac{\kappa}{f})(1-r)^2 & \text{if } \kappa < \lambda f. \end{cases}$

□

B.3 Proof for Proposition 3.

Proof. Recall

$$\begin{aligned} C: \max_b \quad & -(b-\theta_C)^2 - \mathbb{1}_{b>r} ef, \\ G: \max_e \quad & -(b_C^* - r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef. \end{aligned}$$

For C,

$$u_C^* = \begin{cases} u_C(r) = -(r-\theta_C)^2 & \text{if } e \geq \frac{(\theta_C - r)^2}{f}, \\ u_C(\theta) = -ef & \text{if } e < \frac{(\theta_C - r)^2}{f}. \end{cases}$$

For G,

$$v(e) = -(b_C^* - r)^2 - \kappa e + \mathbb{1}_{b>r} ef = \begin{cases} -\kappa e & \text{if } e \geq \frac{(\theta_C - r)^2}{f}, \\ -(\theta_C - r)^2 + (\lambda f - \kappa)e & \text{if } e < \frac{(\theta_C - r)^2}{f}. \end{cases}$$

Given $\theta_C \sim U[r, 1]$, we have

$$\mathbb{E}[(\theta_C - r)^2] = \mathbb{E}[\theta_C^2] - 2r\mathbb{E}[\theta_C] + r^2 = \frac{(1-r)^2}{3}.$$

It follows

i. if $e \geq \frac{(\theta_C - r)^2}{f}$, $v^*(e) = \mathbb{E}[v(\frac{(\theta_C - r)^2}{f})] = -\frac{\kappa(1-r)^2}{3f}$.

ii. if $e < \frac{(\theta_C - r)^2}{f}$, $v^*(e) = \begin{cases} v(0) = \mathbb{E}[-(\theta_C - r)^2] = -\frac{(1-r)^2}{3f} & \text{if } \kappa \geq \lambda f, \\ \lim_{v \nearrow \frac{(\theta_C - r)^2}{f}} v(e) = (-f + \lambda f - \kappa)\frac{(1-r)^2}{3f} & \text{if } \kappa < \lambda f. \end{cases}$

□

C General Cases

C.1 Proof for Proposition 5.

Proof. From proposition 4, the Government chooses its effort with the three cases below.

Choosing Effort/Strategy

I. $e \in \left[0, \frac{(r-\delta^2)}{f}\right)$

$$\begin{aligned} v_1(e) &= -\alpha(r-r)^2 - \beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef \\ &= -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 - \kappa e + \lambda(1-\alpha)ef \\ &= -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 + [\lambda(1-\alpha)f - \kappa]e. \end{aligned}$$

i. if $\lambda(1-\alpha)f - \kappa \geq 0$, i.e., $\kappa \leq \lambda(1-\alpha)f$,

$$\begin{aligned} \lim_e v_1^*(e) &= v\left(\frac{(\delta - r)^2}{f}\right) \\ &= -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 + [\lambda(1-\alpha)f - \kappa]\frac{(\delta - r)^2}{f} \\ &= -\beta(1-r)^2 + \frac{1}{f}[\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]\mathbb{E}[(\delta - r)^2] \end{aligned}$$

ii. if $\lambda(1-\alpha)f - \kappa < 0$, i.e., $\kappa > \lambda(1-\alpha)f$,

$$\begin{aligned}\mathbb{E}[v_1^*(e)] &= \mathbb{E}[v(0)] \\ &= \mathbb{E}[-\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2] \\ &= -\beta(1-r)^2 - (1-\alpha-\beta)\mathbb{E}[(\delta-r)^2]\end{aligned}$$

II. $e \in \left[\frac{(\delta-r)^2}{f}, \frac{(1-r)^2}{f}\right)$

$$\begin{aligned}v_2(e) &= -\alpha(r-r)^2 - \beta(1-r)^2 - (1-\alpha-\beta)(r-r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef \\ &= -\beta(1-r)^2 - \kappa e + \lambda\beta ef \\ &= -\beta(1-r)^2 + (\lambda\beta f - \kappa)e.\end{aligned}$$

i. if $\lambda\beta f - \kappa \geq 0$, i.e., $\kappa \leq \lambda\beta f$,

$$\begin{aligned}\lim_e v_2^*(e) &= v\left(\frac{(1-r)^2}{f}\right) \\ &= -\beta(1-r)^2 + (\lambda\beta f - \kappa)\frac{(1-r)^2}{f} \\ &= \left[\frac{\lambda\beta f - \kappa}{f} - \beta\right](1-r)^2\end{aligned}$$

ii. if $\lambda\beta f - \kappa < 0$, i.e., $\kappa > \lambda\beta f$,

$$\begin{aligned}\mathbb{E}[v_2^*(e)] &= \mathbb{E}[v\left(\frac{(\delta-r)^2}{f}\right)] \\ &= -\beta(1-r)^2 + (\lambda\beta f - \kappa)\frac{\mathbb{E}[(\delta-r)^2]}{f}.\end{aligned}$$

III. $e \in \left[\frac{(1-r)^2}{f}, 1\right]$

$$\begin{aligned}v_3(e) &= -\alpha(r-r)^2 - \beta(r-r)^2 - (1-\alpha-\beta)(r-r)^2 - \kappa e + \lambda \mathbb{1}_{b>r} ef \\ &= -\kappa e\end{aligned}$$

so that

$$v_3^*(e) = v\left(\frac{(1-r)^2}{f}\right) = -\frac{\kappa(1-r)^2}{f}.$$

Payoff Comparison

I. if $\kappa > \lambda(1-\alpha)f$, we compare

$$\begin{aligned}
v_1(e) &= v(0) = -\beta(1-r)^2 - (1-\alpha-\beta)\mathbb{E}[(\delta-r)^2] \\
v_2(e) &= v\left(\frac{(\delta-r)^2}{f}\right) = -\beta(1-r)^2 + (\lambda\beta f - \kappa)\frac{\mathbb{E}[(\delta-r)^2]}{f} \\
v_3(e) &= v\left(\frac{(1-r)^2}{f}\right) = \frac{-\kappa(1-r)^2}{f}
\end{aligned}$$

It follows

$$\begin{aligned}
v_2 - v_1 &= \frac{\lambda\beta f - \kappa}{f}E + (1-\alpha-\beta)E \\
&= \frac{E}{f}[\lambda\beta f - \kappa + (1-\alpha)f - \beta f] \\
&< \frac{E}{f}[\lambda\beta f - \lambda(1-\alpha)f + (1-\alpha)f - \beta f] \\
&= (1-\lambda)(1-\alpha-\beta)E
\end{aligned}$$

so that if $1 \leq \lambda$, we have $v_2 < v_1$.

$$\begin{aligned}
v_1 - v_3 &= -\beta(1-r)^2 - (1-\alpha-\beta)E + \frac{\kappa}{f}(1-r)^2 \\
&= (\frac{\kappa}{f} - \beta)(1-r)^2 - (1-\alpha-\beta)E \\
&> (\frac{\kappa}{f} - \beta - (1-\alpha-\beta))(1-r)^2 \\
&= \frac{1}{f}[\kappa - \beta f - (1-\alpha)f + \beta f](1-r)^2 \\
&= \frac{1}{f}[\kappa - (1-\alpha)f](1-r)^2 > 0
\end{aligned}$$

so that $v_1 > v_3$.
Thus, if $\kappa > \lambda(1-\alpha)f$ and $\lambda \geq 1$, we have $v^*(e) = v_1^*(0)$.

II. if $\lambda\beta f \leq \kappa \leq \lambda(1-\alpha)f$, we compare

$$\begin{aligned}
v_1(e) &= \lim_{e \to \frac{(\delta-r)^2}{f}} v(e) = -\beta(1-r)^2 + \frac{1}{f}[\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]\mathbb{E}[(\delta-r)^2] \\
v_2(e) &= v\left(\frac{(\delta-r)^2}{f}\right) = -\beta(1-r)^2 + (\lambda\beta f - \kappa)\frac{\mathbb{E}[(\delta-r)^2]}{f} \\
v_3(e) &= v\left(\frac{(1-r)^2}{f}\right) = \frac{-\kappa(1-r)^2}{f}
\end{aligned}$$

It follows

$$\begin{aligned} v_2 - v_1 &= \frac{\lambda\beta f - \kappa}{f}E - \frac{\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f}{f}E \\ &= \frac{E}{f}[\lambda\beta f - \kappa - \lambda(1-\alpha)f + \kappa + (1-\alpha-\beta)f] \\ &= (1-\lambda)(1-\alpha-\beta)E \end{aligned}$$

so that if $\lambda \geq 1$, we have $v_2 \leq v_1$.

$$\begin{aligned} v_1 - v_3 &= [\frac{\kappa}{f} - \beta](1-r)^2 + \frac{1}{f}[\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]E \\ &> [\frac{\kappa}{f} - \beta]E + \frac{1}{f}[\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]E \\ &= \frac{1}{f}[\kappa - \beta f + \lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]E \\ &= (\lambda-1)(1-\alpha)E \end{aligned}$$

so that if $\lambda \geq 1$, we have $v_1 > v_3$.
Thus, if $\lambda\beta f \leq \kappa \leq \lambda(1-\alpha)f$ and $\lambda \geq 1$, we have $v^*(e) = \lim_{e \to \frac{(\delta-r)^2}{f}} v_1(e)$.

III. if $\kappa < \lambda\beta f$, we compare

$$\begin{aligned} v_1(e) &= \lim_{e \to \frac{(\delta-r)^2}{f}} v(e) = -\beta(1-r)^2 + \frac{1}{f}[\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]\mathbb{E}[(\delta-r)^2] \\ v_2(e) &= \lim_{e \to \frac{(1-r)^2}{f}} v(e) = \left[\frac{\lambda\beta f - \kappa}{f} - \beta\right](1-r)^2 \\ v_3(e) &= v\left(\frac{(1-r)^2}{f}\right) = \frac{-\kappa(1-r)^2}{f} \end{aligned}$$

It follows

$$\begin{aligned} v_2 - v_1 &= \frac{\lambda\beta f - \kappa}{f}(1-r)^2 - \frac{1}{f}[\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]E \\ &> \frac{1}{f}(\lambda\beta f - \kappa)E - \frac{1}{f}[\lambda(1-\alpha)f - \kappa - (1-\alpha-\beta)f]E \\ &= \frac{E}{f}[\lambda\beta f - \kappa - \lambda(1-\alpha)f + \kappa + (1-\alpha-\beta)f] \\ &= (1-\lambda)(1-\alpha-\beta)E \end{aligned}$$

so that if $1 \geq \lambda$, we have $v_2 > v_1$.

$$\begin{aligned} v_2 - v_3 &= \left[\frac{\lambda\beta f - \kappa}{f} - \beta\right](1-r)^2 + \frac{\kappa}{f}(1-r)^2 \\ &= \left(\lambda\beta - \frac{\kappa}{f} - \beta + \frac{\kappa}{f}\right)(1-r)^2 \\ &= -(1-\lambda)\beta(1-r)^2 \end{aligned}$$

so that if $1 \geq \lambda$, we have $v_2 \leq v_3$.

Thus, if $\kappa < \lambda\beta f$ and $\lambda \leq 1$, we have $v^*(e) = v_3^*\left(\frac{(1-r)^2}{f}\right)$.

This completes the proof for proposition 5. □

C.2 Proof for Proposition 6.

Proof. From previous results in proposition 4, we have

I. if $\delta \leq r'$, then we have

e	$\left[0, \frac{(r'-\delta)^2}{f}\right)$	$\left[\frac{(r'-\delta)^2}{f}, \frac{(r'-1)^2}{f}\right)$	$\left[\frac{(r'-1)^2}{f}, 1\right]$
A	r	r	r
B	1	1	r'
C	δ	δ	δ

II. if $\delta > r'$

e	$\left[0, \frac{(r'-\delta)^2}{f}\right)$	$\left[\frac{(r'-\delta)^2}{f}, \frac{(r'-1)^2}{f}\right)$	$\left[\frac{(r'-1)^2}{f}, 1\right]$
A	r	r	r
B	1	1	r'
C	δ	r'	r'

The Government's payoffs

1. if $e \in [0, \frac{(r'-\delta)^2}{f})$, we have

$$\delta = \alpha r + \beta + (1-\alpha-\beta)\theta \leq r + \epsilon,$$

so that

$$\begin{aligned} (1-\alpha-\beta)\theta &\leq (1-\alpha)r - \beta + \epsilon \\ \theta &\leq \frac{(1-\alpha)r - \beta + \epsilon}{1-\alpha-\beta} := M \end{aligned}$$

i. if $r \leq M$, that is $r \geq 1 - \frac{\epsilon}{\beta}$,

$$Pr(\delta \leq r') = Pr(r \leq \theta \leq R_1) = \frac{R_1 - r}{1-r}.$$

ii. if $r > M$, that is $r < 1 - \frac{\epsilon}{\beta}$,

$$Pr(\delta \leq r') = Pr(r \leq \theta \leq R_1) = 0.$$

Then, the Government's payoffs can be written as

$$\begin{aligned} v_1(e) =& -\alpha(r-r)^2 - \beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 - \kappa e + \lambda \mathbb{1}_{b>r'} ef \\ =& \begin{cases} -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 + (\lambda\beta f - \kappa)e & \text{if } \delta \leq r' \\ -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 + [\lambda(1-\alpha)f - \kappa]e & \text{if } \delta > r' \end{cases} \\ =& \begin{cases} -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 + (\lambda\beta f - \kappa)e & \text{if } r \geq 1 - \frac{\epsilon}{\beta} \\ -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 + [\lambda(1-\alpha)f - \kappa]e & \text{if } r < 1 - \frac{\epsilon}{\beta} \end{cases} \end{aligned}$$

so that if $r \geq 1 - \frac{\epsilon}{\beta}$,

$$v_1^*(e) = \begin{cases} v(0) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 & \text{if } \kappa \geq \lambda\beta f, \\ \lim_{e \nearrow \frac{(r'-\delta)^2}{f}} v(e) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 + (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} & \text{if } \kappa < \lambda\beta f. \end{cases}$$

and if $r < 1 - \frac{\epsilon}{\beta}$,

$$v_1^*(e) = \begin{cases} v(0) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 & \text{if } \kappa \geq \lambda(1 - \\ \lim_{e \nearrow \frac{(r'-\delta)^2}{f}} v(e) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta-r)^2 + [\lambda(1-\alpha)f - \kappa]\frac{(r'-\delta)^2}{f} & \text{if } \kappa < \lambda(1 - \end{cases}$$

2. if $e \in [\frac{(r'-\delta)^2}{f}, \frac{(r'-1)^2}{f})$, we have

$$\begin{aligned} v_2(e) =& -\alpha(r-r)^2 - \beta(1-r)^2 - (1-\alpha-\beta)(r'-r)^2 - \kappa e + \lambda \mathbb{1}_{b>r'} ef \\ =& -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)e \end{aligned}$$

so that

$$v_2^*(e) = \begin{cases} v(\frac{(r'-\delta)^2}{f}) = -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} & \text{if } \kappa \geq \lambda\beta f, \\ \lim_{e \nearrow \frac{(r'-1)^2}{f}} v(e) = -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)\frac{(r'-1)^2}{f} & \text{if } \kappa < \lambda\beta f. \end{cases}$$

3. if $e \in [\frac{(r'-1)^2}{f}, 1]$, we have

$$\begin{aligned} v_3(e) =& -\alpha(r-r)^2 - \beta(r'-r)^2 - (1-\alpha-\beta)(r'-r)^2 - \kappa e + \lambda \mathbb{1}_{b>r'} ef \\ =& -(1-\alpha)\epsilon^2 - \kappa e \end{aligned}$$

so that

$$v_3^*(e) = v_3\left(\frac{(r'-1)^2}{f}\right) = -(1-\alpha)\epsilon^2 - \frac{\kappa(r'-1)^2}{f}.$$

Comparison Consider $r < 1 - \frac{\epsilon}{\beta}$,

1. if $\kappa > \lambda(1-\alpha)f$, we compare

$$
\begin{aligned}
v_1(e) &= v(0) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 \\
v_2(e) &= v(\frac{(\delta - r')^2}{f}) = -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} \\
v_3(e) &= v(\frac{(1-r')^2}{f}) = -(1-\alpha)\epsilon^2 - \kappa\frac{(1-r')^2}{f}
\end{aligned}
$$

if $\lambda < f$, then $v_2 - v_3 < 0$;
if $\lambda > 1$, then $v_3 - v_1 < 0$;
It follows

$$
\begin{aligned}
v_3 - v_2 &= -(1-\alpha)\epsilon^2 - \kappa\frac{(1-r')^2}{f} + \beta(1-r)^2 + (1-\alpha-\beta)\epsilon^2 - (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} \\
&< -\beta\epsilon^2 - \frac{\kappa}{f}(\delta - r')^2 + \beta(1-r)^2 - \lambda\beta(r'-\delta)^2 + \frac{\kappa}{f}(r'-\delta)^2 \\
&= -\beta\epsilon^2 + \beta(1-r)^2 - \lambda\beta(r'-\delta)^2 \\
&< -\beta\epsilon^2 + \beta(1-r')^2 - \lambda\beta(r'-\delta)^2 \\
&< -\beta\epsilon^2 + \beta(\delta - r')^2 - \lambda\beta(r'-\delta)^2 \\
&= -\beta\epsilon^2 + \beta(1-\lambda)(r'-\delta)^2
\end{aligned}
$$

so that $v_3 < v_2$ if $\lambda \geq 1$.

$$
\begin{aligned}
v_1 - v_2 &= -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 + \beta(1-r)^2 + (1-\alpha-\beta)\epsilon^2 - (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} \\
&= -(1-\alpha-\beta)(\delta - r)^2 + (1-\alpha-\beta)\epsilon^2 - \frac{\lambda\beta f - \kappa}{f}(r'-\delta)^2 \\
&= -(1-\alpha-\beta)(\delta - r)^2 + (1-\alpha-\beta)\epsilon^2 - \frac{\lambda\beta f - \kappa}{f}[(r-\delta)^2 + 2\epsilon(r-\delta) + \epsilon^2] \\
&= [-1 + \alpha + \beta - \lambda\beta + \frac{\kappa}{f}](\delta - r)^2 + [1 - \alpha - \beta - \frac{\lambda\beta f - \kappa}{f}]\epsilon^2 - \frac{\lambda\beta f - \kappa}{f}\epsilon^2 \\
&= [\frac{\kappa}{f} - (1-\alpha) + \beta(1-\lambda)](\delta - r)^2 + [1 - \alpha - \beta - \frac{\lambda\beta f - \kappa}{f}]\epsilon^2 - \frac{\lambda\beta f - \kappa}{f}\epsilon^2
\end{aligned}
$$

so that $v_1 \geq v_2$ if $\lambda \geq 1$.
Thus, $v_1 \geq v_2 > v_3$ and when $\kappa > \lambda(1-\alpha)f$,

$$
v^*(e) = v_1(e) = v_1(0) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2.
$$

2. if $\lambda\beta f \leq \kappa \leq \lambda(1-\alpha)f$, we compare

$$v_1(e) = \lim_{e \to \frac{(\delta - r')^2}{f}} v(e) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 + [\lambda(1-\alpha)f - \kappa]\frac{(r'-\delta)^2}{f}$$
$$v_2(e) = v\left(\frac{(\delta - r')^2}{f}\right) = -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f}$$
$$v_3(e) = v\left(\frac{(1 - r')^2}{f}\right) = -(1-\alpha)\epsilon^2 - \kappa\frac{(r'-1)^2}{f}$$

It follows

$$\begin{aligned}
v_1 - v_2 &= -(1-\alpha-\beta)(\delta - r)^2 + [\lambda(1-\alpha)f - \kappa]\frac{(r'-\delta)^2}{f} + (1-\alpha-\beta)\epsilon^2 - (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} \\
&= -(1-\alpha-\beta)(\delta - r)^2 + (1-\alpha-\beta)\epsilon^2 + [\lambda(1-\alpha)f - \kappa]\frac{(r'-\delta)^2}{f} - (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} \\
&= -(1-\alpha-\beta)(\delta - r)^2 + (1-\alpha-\beta)\epsilon^2 + (1-\alpha-\beta)\lambda(r'-\delta)^2 \\
&= (1-\alpha-\beta)[(r'-\delta)^2 - (\delta - r)^2 + \epsilon^2] \\
&= (1-\alpha-\beta)[((r-\delta)+\epsilon)^2 - (\delta - r)^2 + \epsilon^2] \\
&= (1-\alpha-\beta)[(r-\delta)^2 + 2\epsilon(r-\delta) + \epsilon^2 - (\delta - r)^2 + \epsilon^2] \\
&= (1-\alpha-\beta)[2\epsilon(r-\delta) + 2\epsilon^2] \\
&= (1-\alpha-\beta)2\epsilon(r - \delta + \epsilon) \\
&= (1-\alpha-\beta)2\epsilon(r' - \delta)
\end{aligned}$$

so that $v_1 \leq v_2$.

$$\begin{aligned}
v_2 - v_3 &= -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} + (1-\alpha)\epsilon^2 + \frac{\kappa}{f}(r'-1)^2 \\
&= \beta\epsilon^2 - \beta(1-r)^2 + (\lambda\beta f - \kappa)\frac{(r'-\delta)^2}{f} + \frac{\kappa}{f}(r'-1)^2 \\
&> \beta\epsilon^2 - \beta(1-r)^2 + (\lambda\beta f - \kappa)\frac{(r'-1)^2}{f} + \frac{\kappa}{f}(r'-1)^2 \\
&= \beta\epsilon^2 - \beta(1-r)^2 + \lambda\beta(r'-1)^2
\end{aligned}$$

3. if $\kappa < \lambda\beta f$, we compare

$$v_1(e) = \lim_{e \to \frac{(\delta - r')^2}{f}} v(e) = -\beta(1-r)^2 - (1-\alpha-\beta)(\delta - r)^2 + [\lambda(1-\alpha)f - \kappa]\frac{(r'-\delta)^2}{f}$$

$$v_2(e) = \lim_{e \to \frac{(1-r')^2}{f}} v(e) = -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)\frac{(r'-1)^2}{f}$$

$$v_3(e) = v\left(\frac{(1-r')^2}{f}\right) = -(1-\alpha)\epsilon^2 - \kappa\frac{(r'-1)^2}{f}$$

It follows

$$\begin{aligned} v_2 - v_3 &= -\beta(1-r)^2 - (1-\alpha-\beta)\epsilon^2 + (\lambda\beta f - \kappa)\frac{(r'-1)^2}{f} + (1-\alpha)\epsilon^2 + \kappa\frac{(r'-1)^2}{f} \\ &= -\beta(1-r)^2 + \beta\epsilon^2 + \lambda\beta(r'-1)^2 \\ &= \lambda\beta[(r-1+\epsilon)^2] - \beta(1-r)^2 + \beta\epsilon^2 \\ &= \lambda\beta[(r-1)^2 + 2\epsilon(r-1) + \epsilon^2] - \beta(1-r)^2 + \beta\epsilon^2 \\ &= \beta(\lambda+1)\epsilon^2 + -2\lambda\beta(1-r)\epsilon + (\lambda-1)\beta(1-r)^2 \end{aligned}$$

Since $\beta(\lambda+1) > 0$, and

$$\epsilon^* = \frac{2\lambda\beta(1-r)}{2\beta(\lambda+1)} = \frac{\lambda}{\lambda+1}(1-r) \text{ and } V(\epsilon^*) = \frac{-\beta}{1+\lambda}(1-r)^2 < 0.$$

When

$$\beta(\lambda+1)\epsilon^2 + -2\lambda\beta(1-r)\epsilon + (\lambda-1)\beta(1-r)^2 = 0,$$

we have

$$\epsilon = \frac{2\lambda\beta(1-r) \pm \sqrt{\Delta}}{2\beta(\lambda+1)},$$

where

$$\Delta := 4\lambda^2\beta^2(1-r)^2 - 4\beta(\lambda+1)(\lambda-1)\beta(1-r)^2 = 4\beta^2(1-r)^2[\lambda^2 - \lambda^2 + 1] = 4\beta^2(1-r)^2,$$

so that

$$\epsilon = \frac{2\lambda\beta(1-r) \pm 2\beta(1-r)}{2\beta(\lambda+1)} = \frac{(\lambda \pm 1)(1-r)}{\lambda+1}.$$

It follows

$$\epsilon_1 = \frac{\lambda-1}{\lambda+1}(1-r), \quad \epsilon_2 = 1-r.$$

When $\lambda \leq 1$, $\epsilon_1 \leq 0$, and since $\epsilon > 0$, so that $v_2 - v_3 < 0$ if $(0, 1-r)$.
Thus, when $\lambda \leq 1$, $\epsilon \in (0, 1-r)$, we have $v_2 - v_3$, reversing the result for $\epsilon = 0$ in the previous section.

This completes the proof for proposition 6. □