When Does Idiosyncratic Risk Generate Asset Bubbles?

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Abstract

Heterogeneous-agent economies with idiosyncratic risk provide a realistic setting for asset valuation. This paper investigates whether and when idiosyncratic risk generates asset bubbles. We show that the key determinant is wealth inequality: asset bubbles emerge only if agents expect to become extremely poor in some states. When they evaluate an asset unaffected by their idiosyncratic risk, this asset appears infinitely larger than their size *ex-post*, and is therefore deemed very valuable *ex-ante*. This mechanism is fundamentally different from the classic rational bubble, and is potentially consistent with U.S. inequality data. Using this insight, we re-examine how long-lived assets like government debt are priced in workhorse heterogeneous-agent models, and show that some prior analysis may be incomplete due to overlooked possibility of transversality condition violation.

Keywords: Transversality Condition, Heterogeneous Agents, Wealth Distribution, Public Debt Valuation

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1 Introduction

Heterogeneous-agent economies with idiosyncratic risk constitute an important class of workhorse models in macroeconomics and finance. It is well understood that uninsurable idiosyncratic risk lowers the risk-free discount rate due to a precautionary saving motive. However, whether and when this mechanism lowers the discount rate enough to generate asset bubbles remains an open question.

In this paper, we characterize conditions under which idiosyncratic risk generates bubbles for long-lived assets. We focus on cases in which the present discounted value of asset price in distant future approaches infinity, resulting in a violation of the transversality condition (TVC). In such cases, the current asset price exceeds the present discounted value of future cash flows, i.e., $P > PV(CF)$, leading to a persistent deviation of asset price from its fundamentals.

We find that the TVC depends crucially on the degree of inequality in the wealth distribution, which is in turn shaped by properties of idiosyncratic risk. We uncover a new intuition for TVC violation in heterogeneous-agent economies, which is completely different from the classic rational bubble in overlapping-generation models: in our setting, TVC violation requires the wealth of a significant fraction of agents to become sufficiently small relative to the average wealth in the economy. When these agents evaluate an asset that is unaffected by their idiosyncratic risk, this asset appears infinitely large relative to their own wealth *ex-post*. As a result, they find this asset very valuable *ex-ante*, leading to TVC violation even in the absence of aggregate uncertainty and economic growth.

This result offers a new perspective for understanding how long-lived assets are priced in heterogeneous-agent economies. Through the lens of our characterization, we revisit asset pricing implications of models like Constantinides and Duffie (1996); Krueger and Lustig (2010), and show that some prior analyses may be incomplete due to the possibility of TVC violation.

We begin our analysis with the model of Brunnermeier, Merkel, and Sannikov (2022), which provides a tractable framework for modeling households facing uninsurable idiosyncratic risk. In this setting, the idiosyncratic risk is *permanent* and follows a random walk. As the wealth

distribution evolves over time, the vast majority of households become very poor, while a vanishing share of households accumulates nearly all the wealth. In other words, this economy exhibits an exploding degree of wealth inequality.

We consider how a household in this economy prices a long-lived aggregate asset such as government debt. The asset's valuation depends on its cash flow and discount rate, which is determined by the household's marginal utility growth and subjective discount rate. In standard settings including most representative-agent models, the asset's cash flow and the household's consumption level are cointegrated, making their covariance capturing risk premium the primary determinant of valuation. In our setting, as the household anticipates its wealth to decline far below the population average in almost all states of the world, the asset's cash flow appears very large relative to the household's size.

Due to this divergence in scale between the household and the asset, while the household's marginal utility growth correctly prices the asset's return period by period, the asset's long-run value discounted by the household's marginal utility growth tends to infinity, leading to TVC violation for aggregate assets. In contrast, each household's TVC for its own consumption stream still holds, as it is discounted at higher rates due to the idiosyncratic risk. As a result, the household's perception of its own wealth is finite, so that it will not demand more consumption goods than available in the economy.

That said, permanent idiosyncratic risk does not always result in TVC violation. When the risk is permanent but sufficiently small, while the household still expects its wealth to eventually vanish relative to the population average, this process occurs gradually and is outweighed by the subjective discount rate. As a result, the asset's terminal value remains finite and converges to zero in the limit.

We can also assess the condition for TVC violation by comparing the discount rate r with the growth rate g. Since households are risk-averse, the relevant discount rate r for any asset is not the risk-free rate but one adjusted for risk premium. We show that $r - g$ is decreasing in the magnitude of the idiosyncratic risk, as higher idiosyncratic risk leads to larger dispersion in marginal utility

growth and magnifies the precautionary saving motive. Moreover, TVC violation occurs precisely when idiosyncratic risk is large enough to push r below g. Therefore, $r > g$ remains sufficient and necessary for the TVC to hold in this setting.

The above analysis suggests that TVC violation occurs if there is sufficiently large permanent idiosyncratic risk, which leads to an exploding amount of wealth inequality. A natural question is whether TVC violation can still occur when households face transitory idiosyncratic risk and a stationary wealth distribution—a setting that may be more realistic for studying asset prices.

To address this question, we introduce two additional ingredients to the model: wealth reset and death. In the first model extension, a redistribution shock resets a fixed fraction of households' wealth to the average wealth level in each period, which preserves the total amount of resources in the economy. This setup generates a stationary wealth distribution while maintaining tractability. To our surprise, even when wealth inequality does not explode, TVC violation for aggregate assets can still occur. As before, the key lies in understanding the properties of the wealth distribution.

Specifically, when the intensity of the wealth reset shock is small, each household's wealth can deviate from the average level for an extended period before returning to the mean. Despite the wealth distribution remaining stationary, a household's wealth may be much lower than the population average in a significant fraction of states. When the left tail of the wealth distribution is sufficiently heavy, the household anticipates large dispersion in its marginal utility growth, which depresses the discount rate and leads to TVC violation much like the case with permanent idiosyncratic risk.

Conversely, when the intensity of the wealth reset shock is sufficiently large, the wealth distribution becomes more concentrated around its average. In this case, households expect their wealth levels to be comparable to the size of the economy in most states, and their marginal utility growth to be finite on average. As a result, the TVC for long-lived assets holds. This property of wealth distribution is again reflected in the $r - g$ condition. $r > g$ remains a necessary and sufficient condition for the TVC to hold.

These results show that TVC violation for aggregate assets occurs not only when wealth in-

equality explodes, but also when the wealth distribution is stationary yet sufficiently left-skewed. Quantitatively, when we compare the model-implied wealth distribution with U.S. post-war data, we find that the amount of wealth inequality in the U.S. data is potentially consistent with the case where TVC violation does arise, making asset bubbles an empirically relevant possibility.

In the second model extension, we explore how our conclusions change when we introduce death to our model. We assume that households die at a constant rate, and new households are born to maintain a constant population. This ingredient always makes households more impatient, leading them to consume a larger fraction of their wealth and discount future cash flows more heavily.

We consider two variants of the model with death. First, if each newborn household inherits the wealth of a specific dying household, the wealth distribution will diverge over time. We obtain results identical to our baseline case with permanent idiosyncratic risk: if the magnitude of idiosyncratic risk is large enough relative to the subjective discount rate, households are concerned about their vanishing wealth shares in scenarios where they do survive, leading to TVC violation. The only difference is that the subjective discount rate must be adjusted upwards to account for the death rate.

Second, if each newborn household starts with the average wealth of the dying households, the model behaves similarly to the case with wealth reset. In this case, the wealth distribution remains stationary, but whether the TVC holds depends on the degree of wealth inequality, with death playing a role similar to wealth reset. These results show that introducing death does not change our main conclusions.

In summary, our results show that the degree of wealth inequality has surprisingly important implications for asset valuation in heterogeneous-agent models. For example, through the lens of our characterization, the model of Constantinides and Duffie (1996) belongs to the case with permanent idiosyncratic risk, which implies a diverging wealth distribution. If the idiosyncratic risk is sufficiently large, the TVC for aggregate assets may fail even when the model has well-defined pricing relationships for one-period returns. Constantinides and Duffie (1996) also consider a

model variant resembling our case with wealth reset shocks. Even in this setting, TVC violations can arise if the wealth distribution is sufficiently left-skewed. These results underscore that the TVC and hence the commonly used present value relationship $P = PV(CF)$ cannot be taken for granted in heterogeneous-agent economies.

Our characterizations of the TVC also extend to Bewley models such as Krueger and Lustig (2010) and McKay, Nakamura, and Steinsson (2016); Kaplan, Moll, and Violante (2018), which have occasionally binding borrowing constraints. These borrowing constraints introduce an important caveat: our results characterizing the valuation equation $P = PV(CF) + TVC$ apply only to settings where the borrowing constraint never binds. In contrast, an occasionally binding borrowing constraint alters the standard valuation equation into an inequality: $P > PV(CF) + TVC$. Consequently, even when the TVC term is zero, asset prices can still exceed the present value of cash flows, i.e., $P > PV(CF)$.

We regard this inequality as a different mechanism for asset bubbles, one that arises even in models without truly idiosyncratic risk, such as Townsend (1980); Woodford (1990). Even in this case, our characterization of the TVC term still applies. TVC violation offers a sufficient, though not necessary condition for asset bubbles in the form of $P > PV(CF)$ to arise.

Our paper proceeds as follows. Section 2 studies the TVC in a baseline model with permanent idiosyncratic risk. Section 3 introduces wealth reset to model transitory idiosyncratic risk, and discusses this model's empirical reelvance. Section 4 studies models with death. Section 5 discusses the implications for workhorse heterogeneous-agent models. Section 6 concludes. All proofs are in the appendix.

Literature.

Our paper is related to Brunnermeier, Merkel, and Sannikov (2022), who study the pricing of government debt in a model with idiosyncratic risk. Their *dynamic trading perspective* prices the cash flows each individual receives from the government bond market, which, due to their trading and rebalancing, are not the same as the aggregate cash flows paid out by the government. They show that this perspective leads to a well-defined TVC *for individuals' wealth and asset*

holdings, which offers important insights about the valuation of government debt. Our results do not contradict with this perspective, as it also applies in all model variants we study. Instead, we focus on what Brunnermeier et al. (2022) calls the *buy and hold perspective*. This perspective, which prices the aggregate government cash flows discounted by households' marginal utilities, may or may not lead to well-defined TVC *for aggregate cash flows*. Our paper develops a precise characterization of this valuation question, which is central to standard asset pricing approaches.

In particular, we show that TVC violation can arise in heterogeneous-agent economies even when we take a reasonable calibration for investor preferences and wealth inequality. As a result, we need to be careful to apply the present value formula for aggregate cash flows. The possibility of asset bubbles provides a potential resolution for the public debt valuation puzzle in Jiang et al. (2024); Chen et al. (2022).

Our results parallel the literature on asset bubbles in overlapping-generation (OLG) models (Samuelson, 1958; Diamond, 1965; Blanchard and Watson, 1982). Just as Tirole (1985) characterizes conditions under which bubbly equilibrium exists in OLG models, we characterize conditions under which the TVC is violated in heterogeneous-agent models. We show that TVC violation depends on the degree of wealth inequality. To our surprise, bubbles can arise even in models with stationary wealth distribution, while the TVC can hold in models with exploding wealth inequality. These results can be applied to any aggregate assets including government debt, which have been studied by Blanchard (2019); Abel and Panageas (2022) in the OLG tradition and by Jiang et al. (2020); Reis (2021); Mian et al. (2022) in models with long-lived agents.

2 Transversality Condition with Permanent Idiosyncratic Risk

2.1 Model Set-Up

We begin with the set-up developed by Brunnermeier et al. (2022), which provides a tractable example of heterogeneous-agent economies with idiosyncratic risk. The details of the model are presented in Appendix A.1, and we will consider variations and generalizations in the next section.

In this set-up, there is a continuum of households indexed by $i \in [0,1]$. They have infinite lives and maximize their lifetime utility:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt\right],$$

with the risk aversion parameter $\gamma > 0$.

Each household manages its own capital, whose growth is driven by investment ι_t^i, depreciation δ, trading in capital market $d\Delta_t^{k,i}$, and an idiosyncratic risk $d\tilde{Z}_t^i$ with exogenously specified volatility $\tilde{\sigma}_t$:

$$\frac{dk^i}{k^i} = \left(\Phi(\iota_t^i) - \delta\right)dt + d\Delta_t^{k,i} + \tilde{\sigma}_t d\tilde{Z}_t^i. \tag{1}$$

To focus on the key intuition, we consider the case without aggregate shocks in the main text. In Appendix B, we additionally consider an aggregate shock to each household's capital and derive analytical solutions under log utility. We show that the magnitude of the aggregate shock does not affect our main results, and the TVC violation is only driven by the idiosyncratic risk.

Each household is subject to a skin-in-the-game constraint, which requires it to hold at least $\bar{\chi}$ of its own capital. The households can, and do, sell the remaining shares in exchange for aggregate bond and equity assets that are not exposed to their idiosyncratic risk. In equilibrium, the law of motion for individual household's wealth n_t^i is given by

$$\frac{dn_t^i}{n_t^i} = g_t dt + (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i, \tag{2}$$

which has a common drift term g_t, and is exposed to the idiosyncratic shock $d\tilde{Z}_t^i$ with loading $(1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t$. This loading captures the amount of idiosyncratic risk that the household has to bear in equilibrium, which is the product of its own capital's risk exposure $\bar{\chi}\tilde{\sigma}_t$ and the share $(1-\vartheta_t)$ of this capital in the household's total wealth.

In this setting, the household's consumption-wealth ratio is a constant, i.e., $c_t^i/n_t^i = \hat{c}$, which

implies the following marginal utility:

$$m_t^i = e^{-\rho t}(c_t^i)^{-\gamma} = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}. \tag{3}$$

Plugging in the law of motion for wealth into the marginal utility (3), we obtain the following dynamics

$$\frac{dm_t^i}{m_t^i} = -r_t^f dt - \gamma(1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i, \tag{4}$$

where the real risk-free rate is

$$r_t^f = \hat{c} + g - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2. \tag{5}$$

In this economy, the risk-free rate r_t^f is given by the growth rate g_t plus the subjective discount rate ρ minus the idiosyncratic risk's variance. A more volatile idiosyncratic risk lowers the interest rate due to the precautionary saving motive, which is standard in a large class of heterogeneous-agent models.

2.2 Transversality Condition for Individual Wealth

Iterating forward the households' budget constraint, we obtain that the household's wealth is equal to the present discounted value of its future consumption plus a transversality term:

$$n_t^i = \mathbb{E}_t\left[\int_0^\infty m_{t+j}^i c_{t+j}^i dj\right] + \lim_{T\to\infty}\mathbb{E}_t\left[m_T^i n_T^i\right]. \tag{6}$$

The transversality condition (TVC) for individual wealth is given by

$$\lim_{T\to\infty}\mathbb{E}_t\left[m_T^i n_T^i\right] = 0. \tag{7}$$

If this TVC holds, we can evaluate the household's wealth portfolio n_t^i based on the present dis-

counted value of consumption. If the TVC is violated, the household's net worth n_t^i can exceed the present value of consumption.

In this setting, the individual-level TVC always holds for all households. To see this, given the law of motion (2) and (4) for individual wealth and marginal utility, we obtain

$$\frac{d(m_t^i n_t^i)}{m_t^i n_t^i} = -[\underbrace{r_t^f}_{\text{risk-free rate}} + \underbrace{\gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2}_{\text{risk premium for idiosyncratic risk}} - \underbrace{g_t}_{\text{growth rate}}]dt, \tag{8}$$

which expresses the dynamics of the transversality term $m_t^i n_t^i$ in terms of the discount rate and the growth rate. Plugging in the expression (5) for the risk-free rate, we have

$$(r_t^f + \gamma(1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2) - g_t = \hat{c} > 0, \tag{9}$$

which implies that the appropriate discount rate for individual wealth, which accounts for the risk premium for idiosyncratic risk, is higher than the growth rate: r is always above g. This guarantees that the discounted wealth $m_t^i n_t^i$ tends to zero in the limit.

2.3 Transversality Condition for Aggregate Wealth

Next, we evaluate the TVC associated with the aggregate wealth $\bar{n}_t = \int_0^1 n_t^i di$. Its law of motion is given by

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt,$$

which, compared to the law of motion (2) for individual wealth, does not load on any idiosyncratic risk.

A claim to this aggregate wealth $\bar{n}_t$ pays it out at time t. We are interested in studying the value of such a claim in the distant future. If we evaluate this claim using an individual household's

marginal utility m_t^i, we obtain

$$\lim_{T\to\infty} \mathbb{E}_0 \left[m_T^i \bar{n}_T \right], \tag{10}$$

which answers how much the household would pay today in exchange for receiving the aggregate wealth in the distant future.

Why should the households care about this quantity? Consider, for example, their valuation of government risk-free debt. Just like the individual household's budget constraint (6), the government faces the following intertemporal budget condition:

$$b_t = \mathbb{E}_t \left[\int_0^\infty m_{t+j}^i s_{t+j} dj \right] + \lim_{T\to\infty} \mathbb{E}_t \left[m_T^i b_T \right], \tag{11}$$

where b_t denotes the market value of government debt and s_t denotes the primary surplus. In this set-up, each household allocates ϑ_t fraction of its wealth to government debt. So, in aggregate, $b_t = \vartheta_t \bar{n}_t$. The government debt's transversality condition is given by

$$\lim_{T\to\infty} \mathbb{E}_t \left[m_T^i \vartheta_T \bar{n}_T \right].$$

Since $\vartheta_t \in (0,1)$, $0 < \mathbb{E}_t \left[m_T^i \vartheta_T \bar{n}_T \right] < \mathbb{E}_t \left[m_T^i \bar{n}_T \right]$ for all time T. So, the TVC (10) for aggregate wealth is also a sufficient condition for the TVC for government debt to hold.[1] More broadly, for any asset whose cash flow is tied to aggregate quantities, the TVC (10) for aggregate wealth also sheds light on its TVC.

Equivalently, we can consider this TVC from the perspective of an average household. We use the equal-weighted average marginal utility growth $\bar{m}_t = \int_0^1 (m_t^i / m_0^i) di$, which has the following law of motion:

$$\frac{d\bar{m}_t}{\bar{m}_t} = -r_t^f dt.$$

[1] If ϑ_t is a constant, so that the market value of debt is cointegrated with the aggregate wealth, then, Eq. (10) is also a necessary condition.

Compared with the individual household's stochastic discount factor (SDF) m_t^i, the average SDF $\bar{m}_t$ has the same drift, but does not load on the idiosyncratic risk. Because the aggregate wealth $\bar{n}_T$ does not load on the idiosyncratic risk, either, the two TVCs are identical:

$$\lim_{T\to\infty} \mathbb{E}_0\left[m_T^i \bar{n}_T\right] = 0 \quad \Leftrightarrow \quad \lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right] = 0.$$

More precisely, if one is zero, the other is zero as well. If one is infinite, the other is infinite as well. These are the two possibilities in our set-up. Since it is more convenient to consider the average household's valuation, we will focus on the TVC discounted by the average SDF $\bar{m}_t$.

Unlike the TVC (7) for individual wealth n_t^i, this TVC for aggregate wealth $\bar{n}_t$ does not necessarily hold. Below, we provide two complementary perspectives to characterize this TVC, one focusing on the inequality in the wealth distribution and the other focusing on the comparison between the discount rate (r) and the growth rate (g).

The Wealth Distribution Perspective. Let $\eta_t^i = n_t^i/\bar{n}_t$ denote the wealth share of individual i. By definition, the wealth share has an average of 1: $\int_0^1 \eta_t^i di = 1$. We have the following result.

Proposition 1. *The transversality condition for aggregate wealth can be expressed as*

$$\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} (e^{-\rho T} \bar{n}_T^{1-\gamma}) \mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]. \tag{12}$$

The proof is presented in Appendix A.2. Eq. (12) is the central equation of our paper: whether the aggregate TVC holds or not depends on the distribution of the wealth share as summarized by $\int_0^1 (\eta_T^i)^{-\gamma} di$. This is an inequality measure, as a more unequal wealth distribution implies more households with low values of η_T^i and hence high values of $(\eta_T^i)^{-\gamma}$. If inequality is exploding over time, some households' wealth share η_T^i will become infinitely small and $(\eta_T^i)^{-\gamma}$ infinitely large in the limit. In this case, Eq. (12) may diverge to infinity. On the other hand, if the wealth share η_T^i is bounded from below, then, the TVC always holds.

As in Brunnermeier et al. (2022)'s set-up, Eq. (2) implies that idiosyncratic risk is permanent

in this model, with the wealth share following a random walk:

$$\frac{d\eta_t^i}{\eta_t^i} = (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i.$$

In this setting, as time passes, almost everyone becomes relatively poor, and wealth is concentrated in a vanishing measure of rich households. As a result, the share of households with above average wealth approaches zero: $\mathbb{P}(\eta_T^i \geq 1) \to 0$, which leads to an exploding amount of inequity and hence violation of the TVC for aggregate wealth.

To illustrate this point graphically, we simulate the model under a simple calibration. We report the parameter values in Appendix Table A.1. In particular, we assume that the risk aversion parameter is $\gamma = 2$, in line with a large body of asset pricing literature.

We also make the following simplifying assumptions:

Assumption 1. *(a) The idiosyncratic risk has a constant volatility: $\tilde{\sigma}_t = \tilde{\sigma}$.*

(b) The economy starts at the steady state, so that the portfolio share of equity $\vartheta_t = \vartheta$ remains constant.

(c) All households start with equal wealth shares: $\eta_0^i = 1$.

We illustrate the properties of the wealth distribution in this calibration in Figure 1. Panel (a) reports the distribution of the wealth share η_t^i for different time t. As time passes, while the average wealth share remains at 1, its distribution becomes increasingly dispersed. The figure shows that the median and the mode of the wealth distribution shift to the left, indicating that most households become poorer over time, while a vanishing share of households become richer. Panel (b) reports the cumulative distribution of the wealth share, which makes a similar point. As time passes, most households' wealth share η_t^i approaches zero.

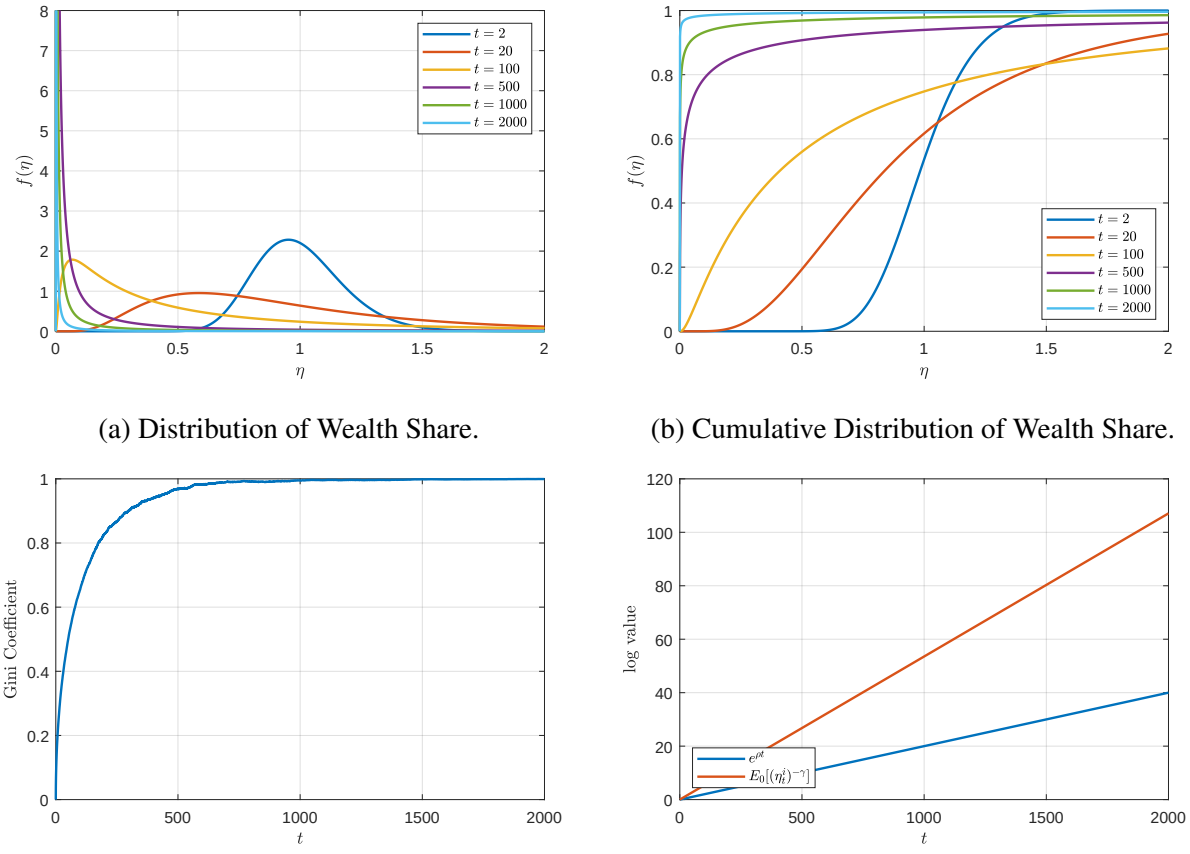
These results imply that, when we ask households to price the claim to aggregate wealth $\bar{n}_T$, all but a vanishing share of the households will become infinitesimally small relative to the size of

this asset, thereby imputing an infinite valuation for the aggregate wealth claim. Algebraically,

$$\lim_{T\to\infty} \mathbb{E}_0\left[m_T^i \bar{n}_T\right] = \lim_{T\to\infty} \hat{c}^{-\gamma}\mathbb{E}_0\left[\exp(-\rho T)(\bar{n}_T)^{1-\gamma}(\eta_T^i)^{-\gamma}\right].$$

On the right-hand side, $\exp(-\rho T)$ captures the subjective discount factor, whereas the expected inverse wealth share $(\eta_T^i)^{-\gamma}$ captures the growth of the marginal utility. If the marginal utility grows to infinity faster than the subjective discount factor approaching zero, this transversality term diverges to infinity.

Panel (c) of the figure reports the expected Gini coefficient of the wealth distribution over time, which measures the degree of inequality in the wealth distribution. As the wealth distribution diverges, the Gini coefficient also approaches one, indicating an exploding amount of inequality.



(a) Distribution of Wealth Share.

(b) Cumulative Distribution of Wealth Share.

(c) Gini Coefficient.

(d) TVC for Aggregate Wealth.

Figure 1: Dynamics with Permanent Idiosyncratic Risk.

Panel (d) reports the average path of the discounted marginal utility growth in Eq. (12) over time, which determines the TVC. The figure breaks down its two components: this variable is equal to the ratio between the integral of inverse wealth share $\int_0^1 (\eta_T^i)^{-\gamma} di$ and the subjective discount factor $\exp(\rho T)$. Both components approach infinity in the limit. In our calibration, the magnitude of the idiosyncratic risk $\tilde{\sigma}$ is sufficiently large, so that the term capturing wealth inequality increases faster than the time discount, leading to an infinite value of the discounted marginal utility growth in the limit.

It is worth noting that this argument is not driven by a force which, as a household becomes poorer, increases the magnitude of its idiosyncratic risk and thereby strengthens the precautionary saving motive. In our setting, poor and rich households hold the same portfolio, and their main difference is the size of their wealth, which determines how large any aggregate asset appears to them. The transversality condition is violated because all but a vanishing share of the households become infinitesimally small relative to the size of the aggregate asset, which is different from the mechanism due to risk premium.

The r Minus g Perspective. We can also understand our main result in light of the "r minus g" argument. Note that the discounted aggregate wealth has the following dynamics:

$$\frac{d(\bar{m}_t \bar{n}_t)}{\bar{m}_t \bar{n}_t} = [- \underbrace{r_t^f}_{\text{risk-free rate}} + \underbrace{g_t}_{\text{growth rate}}]dt.$$

Compared to the discounted individual wealth in Eq. (8), the discounted aggregate wealth does not contain the risk premium for idiosyncratic risk. The relevant $r - g$ condition becomes

$$r_t^f - g_t = \hat{c} - \gamma \bar{\chi}^2 (1 - \vartheta_t)^2 \tilde{\sigma}_t^2,$$

which is no longer guaranteed to be positive. This condition determines the aggregate TVC, which holds if and only if

$$\lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right] = \lim_{T\to\infty} \bar{n}_0 \mathbb{E}_0\left[\exp\left(-\int_0^T [\hat{c} - \gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2]\, dt\right)\right] = 0. \tag{13}$$

In other words, the aggregate TVC is violated when the magnitude of the idiosyncratic risk $\gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2$ is sufficiently large relative to the consumption wealth ratio $\hat{c}$, which is increasing in the subjective discount rate ρ. In this case, because inequality explodes fast enough, households lower their discount rate for the aggregate wealth, leading to $r < g$.

On the other hand, if the variance of the permanent idiosyncratic risk is small enough, i.e., $\gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 < \hat{c}$, the discount rate r can be above the growth rate g. In this case, the aggregate TVC holds: $\lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right] = 0$. This result is also consistent with the wealth distribution perspective. In this case, while the wealth distribution is still diverging, it is slow enough that this risk premium effect is dominated by the household's subjective discount rate.

This discussion shows that the TVC for aggregate wealth is violated only when wealth inequality explodes sufficiently quickly, which drives the (risk-adjusted) discount rate below the growth rate, i.e., $r < g$. Conversely, what r minus g captures in this setting is how fast permanent idiosyncratic risk drives the wealth distribution to explode.

What Does It Mean to Violate the TVC for Aggregate Wealth? When the TVC for aggregate wealth is violated, the government intertemporal budget condition, reproduced below,

$$b_t = \mathbb{E}_t\left[\int_0^\infty m_{t+j}^i s_{t+j} dj\right] + \lim_{T\to\infty} \mathbb{E}_t\left[m_T^i b_T\right],$$

has an infinite TVC term $\lim_{T\to\infty} \mathbb{E}_t\left[m_T^i b_T\right] = \infty$. Since b_t is well-defined and finite, it must be the case that the government runs deficits in enough states of the world to generate an infinitely negative present value of surpluses: $\mathbb{E}_t\left[\int_0^\infty m_{t+j}^i s_{t+j} dj\right] = -\infty$. While this is consistent with the discussion in Brunnermeier et al. (2022), which proposes a different dynamic trading perspective

to decompose government debt valuation b_t to inflows and outflows of individual households, our analysis offers a precise characterization of these aggregate terms on the right-hand side of the government budget condition.

3 Transversality Condition with Transitory Idiosyncratic Risk

So far, we have considered a case with permanent idiosyncratic risk, which gives rise to the possibility of TVC violation for aggregate assets. As we established in Proposition 1, the key to generate TVC violation is the behavior of the left tail in the wealth distribution. The case with permanent idiosyncratic risk implies an infinitely heavy left tail, which blows up the households' expected marginal utility growth in the limit.

In reality, it is more plausible to consider wealth distribution that does not diverge over time, which requires the idiosyncratic risk to be transitory. In this section, we explore the implications for the TVC under this condition. To our surprise, TVC violation can still occur. In fact, standard mechanisms that generate stationary wealth distribution lead to heavy left tails, which can likewise blow up the households' expected marginal utility growth.

3.1 Model Set-Up

We modify one assumption in the previous section to generate transitory idiosyncratic risk. For each household, their wealth n_t^i is reset with Poisson intensity ζ. We denote the jump process as $\tilde{J}_t^i$, which is i.i.d. across households.[2] For simplicity, we assume the wealth is always reset to the average wealth level $\bar{n}_t$, which preserves the total amount of wealth in this economy and can thus be understood as a form of wealth redistribution. Later, we will show that our result is general enough to accommodate a richer post-reset distribution of wealth.

We present detailed derivations of this model variant in Appendix A.1. In this set-up, individ-

[2]Modeling mean-reversion in this fashion allows us to generate a stationary wealth distribution in a tractable way. This is similar to the setting in Irie (2024), in which agents face a Poisson shock that switches their types. If, instead, we assume that the individual wealth faces a proportional tax that depends on the wealth level, the solution is much more complicated even with log utility.

ual wealth satisfies

$$\frac{dn_t^i}{n_t^i} = g_t dt + (1-\vartheta_t)\tilde{\sigma}_t \bar{\chi} d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i} d\tilde{J}_t^i,$$

which differs from Eq. (2) in the baseline model with no jumps in three ways. First, due to the household's exposure to the jump risk, the wealth n_t^i reverts to the mean level $\bar{n}_t$ when the jump $\tilde{J}_t^i$ occurs.

Second, the household internalizes the jump risk in its consumption decision. Because the household may lose its savings when the jump occurs, it becomes effectively more impatient and consumes a higher share of its wealth: the consumption-wealth ratio $\hat{c} = c_t^i/n_t^i$ is increasing in ζ. Since the household consumes more and saves less, the growth rate in this economy is also different from the baseline model.

Third, the risk-free rate is given by

$$r_t^f = \hat{c} + g - \gamma \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2.$$

While this equation is similar to Eq. (5) in the baseline case with permanent idiosyncratic risk, the $\hat{c}$ implicitly depends on the jump intensity ζ. A higher jump intensity implies a risk-free rate, again reflecting the household's higher impatience.

Throughout this section, we assume that the magnitude of the idiosyncratic risk is large enough, so that the TVC fails when the jump risk turns zero. Otherwise, the TVC holds even when there is no jump risk that shrinks the wealth distribution.

3.2 Transversality Condition for Aggregate Wealth

Next, we revisit the TVC for aggregate assets in the presence of wealth reset. Similar to our analysis of the baseline case in Section 2, we can use either an individual household's SDF or the average SDF to characterize the TVC for aggregate wealth. We find it more convenient to use the

average SDF $\bar{m}_t = \int_0^1 (m_t^i/m_0^i)di$. The dynamics of the average SDF is given by

$$d\bar{m}_t = -\left[\rho + \gamma g_t - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2\right]\bar{m}_t dt + \int_0^1 e^{-\rho t}\hat{c}\left(\bar{n}_t^{-\gamma} - (n_{t-}^i)^{-\gamma}\right)\frac{1}{m_0^i}d\tilde{J}_t^i di,$$

where the last term reflects the effect of the wealth reset shock. While this shock has no effect on the aggregate wealth $\bar{n}_t$, it affects the average marginal utility $\bar{m}_t$ since individual marginal utilities are non-linear in wealth.

Our analysis applies to any aggregate assets whose cash flows are unaffected by idiosyncratic risk. Here, we take the government debt as an example. Let $b_t = B_t/P_t$ denote the aggregate market value of government debt in real terms. We obtain the following result which generalizes Eq. (11):

Proposition 2. *In the setting with wealth reset, the intertemporal government budget condition is given by*

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right] - \zeta\mathbb{E}_t\left[\int_0^T e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt\right].$$

The proof is presented in Appendix A.5. This proposition characterizes the intertemporal government budget condition from the perspective of the average household, which has the following interpretation: the government debt is backed by future surpluses (the first term on the right-hand side), the standard TVC term (the second term), and an additional term capturing the effect of wealth reset.

From an individual household's perspective, when the jump happens, its own wealth is reset to the average level regardless of how much and what assets it holds. This means that the household effectively loses any assets it holds when the jump happens. The last term reflects this loss of value, which is equal to the jump intensity ζ times the present value of the asset holding b_t. This loss of value lowers the household's valuation b_0 of the government debt below the entire present discounted value of government surpluses s_t and bond terminal value b_T.

Since the market value of government debt is bounded by the aggregate wealth, i.e., $b_t < \bar{n}_t$,

the last term is bounded. Proposition 2 implies

$$b_0 \leq \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right],$$

which shows that the discounted aggregate wealth $\bar{m}_t \bar{n}_t$ again arises as the key object determining the TVC for aggregate assets. Below, we offer two perspectives to understand its property.

The Wealth Distribution Perspective. For simplicity of exposition, we maintain the same assumption that the households begin with identical wealth: $\eta_0^i = 1$. Then, the TVC term (12), reproduced below, also arises in this setting:

$$\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} \mathbb{E}_0\left[(\hat{c}^{-\gamma}\bar{n}_T^{1-\gamma})e^{-\rho T}\int_0^1 (\eta_T^i)^{-\gamma} di\right]. \tag{14}$$

Like the baseline case with permanent idiosyncratic risk, whether the TVC holds for the aggregate wealth depends on how the integral of inverse wealth share $\int_0^1 (\eta_T^i)^{-\gamma} di$ evolves over time and compares to the subjective discount rate $e^{-\rho T}$. The difference is that transitory idiosyncratic risk makes wealth dynamics mean-reverting, whereas permanent idiosyncratic risk leads to a diverging wealth distribution.

For a concrete example, we again consider a simple calibration under Assumption 1. In this case, the government commits to a constant market value share of debt: $\vartheta_t = \vartheta$. Starting from any initial distribution, the wealth distribution converges to the distribution with density function f that satisfies the following Kolmogorov forward equation:

$$0 = \frac{1}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(\eta^2 f''(\eta) + 4\eta f'(\eta) + 2f(\eta)) - \zeta f(\eta). \tag{15}$$

This equation has the following solution (see Appendix A.6):

$$g(\eta) = \begin{cases} C\eta^{-\frac{3}{2}+\frac{\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}, & 0 < \eta < 1, \\ C\eta^{-\frac{3}{2}-\frac{\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}, & \eta > 1, \end{cases} \tag{16}$$

where C is a constant that ensures that the distribution integrates to one. Even though this distribution is obtained by introducing wealth reset to an expanding log-normal wealth distribution, the stationary distribution is not log-normal—a important feature that we will revisit shortly.

We obtain two distinct cases. First, if

$$\zeta > \frac{\gamma(1+\gamma)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{17}$$

i.e., the jump intensity is sufficiently high relative to the magnitude of the idiosyncratic risk, then, the left tail is light enough to ensure that the integral of inverse wealth share is well-defined:

$$\lim_{T\to\infty} \mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right] < \infty.$$

This condition is also equivalent to $\hat{c} > \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \rho + (\gamma-1)g$, which we use for the discussion of Table 1.

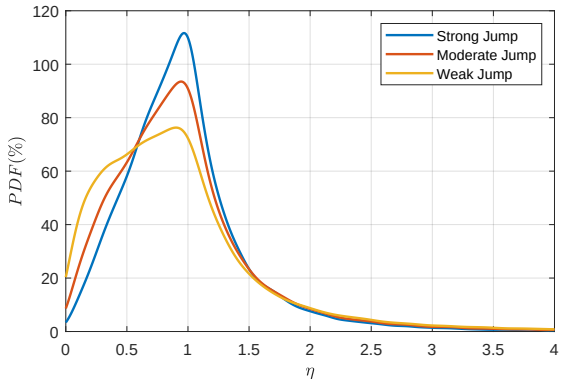
Conversely, if $\zeta \leq \frac{\gamma(1+\gamma)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$, i.e., the jump intensity is lower than the magnitude of the idiosyncratic risk, while we still obtain a stationary wealth distribution in the limit, its left tail is too heavy so that $\lim_{T\to\infty} \mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right] = \infty$.

We consider three different calibrations to illustrate these cases: a case with a high jump intensity $\zeta = 5.4\%$, a case with an intermediate jump intensity $\zeta = 3.8\%$, and a case with a low jump intensity $\zeta = 2.5\%$. $\lim_{T\to\infty} \mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ is finite in the first case, and infinite in the second and third cases. All other variables are identical. In all three cases, we start with identical wealth shares for all households. The wealth distribution quickly converges to a well-defined wealth distribution after a few periods. We illustrate the properties of these three cases in Figure 2.

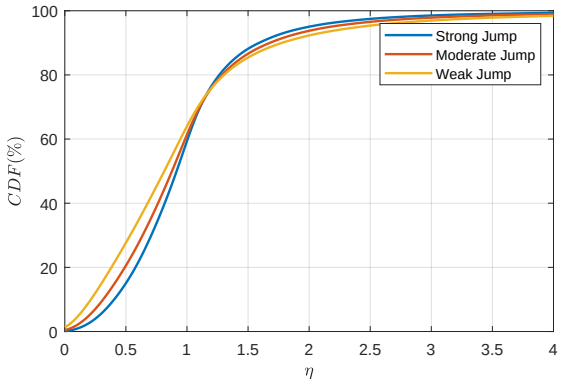
Panel (a) plots the stationary distribution of wealth share η_t^i described by Eq. (16). The case with a higher jump intensity has a greater mass around the mean and a thinner left tail, while the case with a lower jump intensity has a heavier left tail. In all three cases, the left tail of the wealth distribution declines with a power function. While it is not visually obvious, this tail is heavier than the one implied by log-normal distribution which guarantees a finite $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$.

Panel (b) plots the cumulative distribution of the wealth share η_t^i. Both panels show that there is no qualitative difference between the distributions obtained in these three cases, as they come from the same family of distributions with slightly different jump intensity ζ.

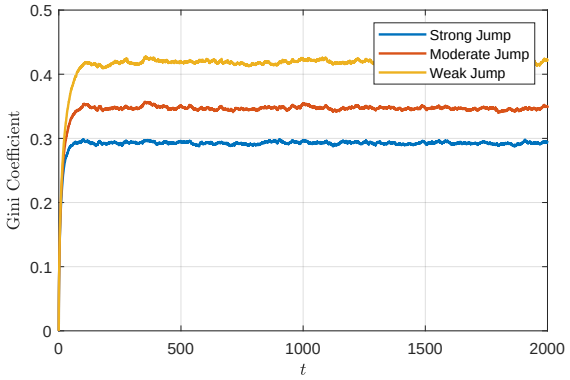
Panel (c) plots how the Gini coefficient evolves over time. Consistent with the convergent behavior of the wealth distribution, the Gini coefficient also stabilizes quickly. The Gini coefficients



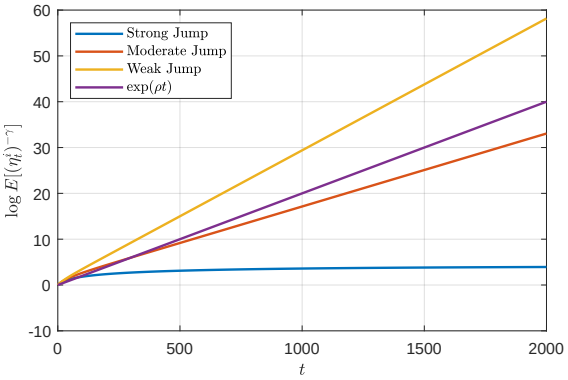
(a) Distribution of Wealth Share.



(b) Cumulative Distribution of Wealth Share.



(c) Gini Coefficient.



(d) TVC for Aggregate Wealth.

Figure 2: Dynamics with Transitory Idiosyncratic Risk

are also similar across the three cases, which range from 0.3 to 0.4.

Finally, panel (d) reports the average path of marginal utility growth in Eq. (14) over time, which determines the TVC for aggregate wealth. We break down this path into two components: the integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ and the subjective discount factor $\exp(\rho T)$. The expected marginal utility growth is the ratio between the two components. In this panel, we see meaningful differences between the three cases.

In the case with sufficiently high jump intensity, the integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ converges to a finite value. In comparison, in the cases with lower or intermediate jump intensity, this integral diverges to infinity, as there is enough downside in each household's expected consumption path, leading to an infinite expected value of the marginal utility growth in the limit.

Comparing these values of expected marginal utility growth to the subjective discount factor $\exp(\rho T)$ leads to further differentiation. In the case with a high jump intensity, the finite expected value of marginal utility growth is dominated by the exponentially growing subjective discount factor, leading to a finite TVC term. In the case with intermediate jump intensity, while the expected marginal utility growth diverges to infinity, it does so at a slower rate than the subjective discount factor. As a result, the TVC still holds. In contrast, in the case with a low jump intensity, the expected marginal utility growth diverges to infinity at a faster rate than the subjective discount factor, leading to a violation of the TVC for aggregate wealth.

Table 1(a) provides a summary of these cases. In the case with a high jump intensity, the TVC holds and $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ converges to a finite value. In the case with a low jump intensity, the

Table 1: Summary of the TVC for Aggregate Wealth.

(a) With Wealth Reset, Transitory Idiosyncratic Risk				
Jump Intensity	Parameter Region	TVC holds?	$\mathbb{E}[\int(\eta_\infty^i)^{-\gamma}di]=\infty$?	$r>g$?
High	$\hat{c}>\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2+\rho+(\gamma-1)g$	Yes	No	Yes
Moderate	$\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2<\hat{c}\leq\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2+\rho+(\gamma-1)g$	Yes	Yes	Yes
Low	$0<\hat{c}\leq\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	No	Yes	No
(b) Without Wealth Reset, Permanent Idiosyncratic Risk				
Idiosyncratic Volatility	Parameter Region	TVC holds?	$\mathbb{E}[\int(\eta_\infty^i)^{-\gamma}di]=\infty$?	$r>g$?
Low	$\hat{c}>\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	Yes	Yes	Yes
High	$\hat{c}\leq\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	No	Yes	No

TVC is violated and $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ diverges to infinity. In the case with an intermediate jump intensity, while $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ still diverges to infinity, the TVC holds. Despite their different implications for TVC, these cases have similar wealth distribution. The key is how the expected marginal utility growth compares to the subjective discount factor, as described by Eq. (14).

Table 1(b) also summarizes the TVC in cases without wealth reset, where the idiosyncratic risk is permanent. In this case, the wealth distribution is diverging and $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ always diverges to infinity. However, the TVC can still hold if the idiosyncratic risk is small enough, in which case the subjective discount factor dominates the expected marginal utility growth.

Comparison to Data. These results show that a stationary wealth distribution does not guarantee the TVC for aggregate assets. TVC violation can still occur when the left tail of the wealth distribution is heavy enough, so that the households expect to experience a very high marginal utility upon negative realizations of the idiosyncratic shock. In this case, the integral of inverse wealth share, which measures the degree of inequality in the wealth distribution, still diverges to infinity just like the case with permanent idiosyncratic risk.

A natural question is whether the wealth distribution generated by models with TVC violation is consistent with the data. If the wealth distribution has to be incredibly skewed in order to generate TVC violation, then TVC violation is nothing but a theoretical curiosity.

Figure 3 compares these model variants with the data. We use the Gini coefficient based on income data to measure the degree of inequality in the U.S.[3] From 1963 to 2022, the Gini coefficient has remained stable, fluctuating around 0.4. In the baseline model with permanent idiosyncratic risk, if we start with the same degree of inequality, the wealth distribution will diverge over time. Under our calibration, motivated by the empirical moments considered in Brunnermeier et al. (2022), we expect the Gini coefficient to rise to 0.6 around 2020 and approach 0.7 after 2050.

In contrast, models with transitory idiosyncratic risk imply a stable Gini coefficient. We again

[3]The wealth in our model encompasses not only financial wealth but also human capital. Greenwald et al. (2021)'s estimates show that the Gini for financial wealth is approximately 0.74 in the U.S., whereas the adjusted total wealth Gini is 0.41, which aligns closely with the figures used here. 0.4 is also close to the Gini index for income and consumption, which is relevant in marginal utility dispersion.

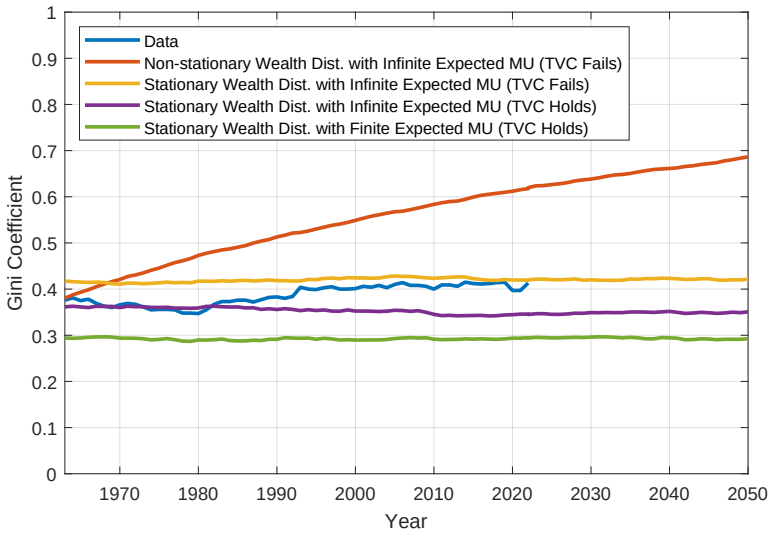


Figure 3: Gini Coefficient of Consumption Inequality.

consider three cases with different values of jump intensity ζ. Both the case with moderate jump intensity and the case with low jump intensity yields a Gini coefficient around 0.4, even though the TVC holds in the former case and is violated in the latter.

This result suggests that we do not need an extremely skewed wealth distribution to generate a case with TVC violation, as long as households have reasonably high risk aversion (we set $\gamma = 2$ in our calibration). In this case, the market value of long-lived assets may be higher than the discounted present value of its cash flows.

The r Minus g Perspective. Next, we turn to the $r - g$ perspective, which allows us to obtain a more general result characterizing the dynamics of $\bar{m}_t \bar{n}_t$ without relying on Assumption 1 and the specific wealth distribution.

Proposition 3. *The TVC term for aggregate wealth has the following dynamics:*

$$d[\bar{m}_t \bar{n}_t] = -\left[\hat{c} - \gamma \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2\right] \bar{m}_t \bar{n}_t dt + \zeta e^{-\rho t} \bar{n}_t^{1-\gamma} \bar{n}_0^{\gamma} dt.$$

The aggregate TVC holds if and only if

$$\hat{c} > \gamma \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2. \tag{18}$$

The proof is presented in Appendix A.7. This proposition shows that the sign of $\hat{c} - \gamma \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2$ plays a crucial role in determining the TVC for aggregate wealth. This term corresponds to the relevant r minus g condition, which is

$$r_t^f - g_t = \hat{c} - \gamma \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2.$$

Thus, this proposition shows that the relevant condition for the TVC is again "$r-g>0$." To satisfy the TVC for aggregate wealth, we need either a sufficiently high discount rate, which requires a small enough volatility $\tilde{\sigma}_t$ of the idiosyncratic risk, or a sufficiently high jump risk ζ so that the households' wealth tends to revert to the mean.

Table 1(a) also reports the $r-g$ condition in the last column. Eq. (18) is satisfied when the jump intensity is in the high or moderate range, which ensures that $r > g$ and the TVC holds. In contrast, this condition rules out the case with a low jump intensity, in which the left-tail of wealth distribution explodes fast enough to dominate the subjective discount rate in the expression (14) from the wealth distribution perspective.

As shown in Appendix A.8, this result is more general than the wealth distribution perspective in our previous discussion, which assumes a specific wealth distribution. In fact, we can impose an arbitrary wealth distribution for households after experiencing the wealth reset, including a log-normal one. In this case, while the stationary wealth distribution may no longer take the form of Eq. (16), Proposition 3 still holds. In particular, with a low enough jump intensity ζ, the TVC for aggregate wealth is still violated, which implies that the left tail of the wealth distribution has to be heavier than the log-normal distribution in the limit.

Summary. Summarizing our discussions of the cases with permanent and transitory idiosyncratic risk, we arrive at the following conclusions from our theoretical characterizations:

First, non-stationary wealth distribution does not necessarily imply that aggregate TVC fails. When the wealth distribution explodes slower than the subjective discount rate, the TVC still holds. When the wealth distribution diverges fast enough, the TVC is violated.

Second, stationary wealth distribution does not guarantee that aggregate TVC holds. The left tail of the wealth distribution matters. When the left tail is heavy enough, the TVC is violated. In this case, enough households expect to become much smaller than the size of the economy, and aggregate assets will appear infinitely large to them, which is the source of TVC violation.

Third, similar to the overlapping generation literature, $r - g > 0$ remains a sufficient and necessary condition for the TVC to hold in our setting. Our analysis shows that $r - g$ is driven by the subjective discount rate and the magnitude of the idiosyncratic risk.

Fourth, the degree of wealth inequality as we observe in the U.S. post-war data do not rule out cases with TVC violation. When households have a reasonably high risk aversion, the amount of idiosyncratic risk in the left tail required to generate TVC violation is potentially consistent with the Gini coefficient observed in the data. In this case, idiosyncratic risk gives rise to asset bubbles, and the long-lived assets may have market value above present value of their future cash flows.

4 Transversality Condition with Stochastic Death

Our characterizations of the TVC remain valid in a model where the households die with a constant probability ζ. For concreteness, let us first consider the following argument in discrete time. Let c_t^i denote the household's consumption at time t if it survives till this time, and suppose the household's utility is 0 in periods after death. Consider a long-lived asset with price p_t and dividend d_t at time t. The household alive at time t has the following first-order condition

$$u'(c_t^i)p_t = (1-\zeta)\mathbb{E}_t\left[\delta u'(c_{t+1}^i)(p_{t+1}+d_{t+1})\right] + \zeta \times 0,$$

which means that the asset is valuable to the household in period t only because there is a chance that the household survives in period $t+1$ and receives the payoff.

Iterating this equation forward, we obtain

$$u'(c_t^i)p_t = \mathbb{E}_t\left[\sum_{k=1}^{\infty}((1-\zeta)\delta)^k u'(c_{t+k}^i)d_{t+k}\right] + \lim_{k\to\infty}\mathbb{E}_t\left[((1-\zeta)\delta)^k u'(c_{t+k}^i)p_{t+k}\right],$$

which is very similar to the case without death, except that the subjective discount rate δ is augmented with the survival probability $(1-\zeta)$. This is very intuitive: death increases the household's effective discount rate, making them act more impatiently.

Other than this difference, the valuation of this long-lived asset still depends on the distribution of the household's marginal utility growth condition on survival. If the wealth and consumption distributions are sufficiently left-skewed, then the TVC can still fail. In this setting, we recover our key result that the TVC for aggregate assets depends on the thickness of the left tail of the wealth distribution.

In Appendix A.9, we make this point more rigorously in a continuous-time model that is similar to our previous setting. Households die at a rate of ζ, while a fraction ζ of new households are born and receive the wealth of the deceased households, so that the total amounts of wealth and financial assets are preserved. We consider two possibilities. First, if each new household simply inherits the wealth from a deceased household, then, this setting is identical to the baseline case without the jump risk as presented in Section 2. There is an exploding degree of inequality, and the TVC for aggregate assets can be violated if the magnitude of the idiosyncratic risk is large enough. The only difference is that the subjective discount rate ρ is augmented by the death rate ζ just like the discrete-time example above. Specifically, the sufficient and necessary condition for the TVC to hold is

$$\hat{c} - \gamma\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 > 0,$$

which is identical to the baseline case with permanent idiosyncratic risk except the subject discount rate ρ is augmented by the death rate ζ.

Second, if the dying households' wealth is pooled together and distributed equally to newborn households, which also preserves the total amount of wealth, then, this setting is identical to our case with the wealth reset shock in Section 3. The wealth distribution is stationary, but the TVC for aggregate assets still depends on the thickness of its left tail. Similarly, the only difference is that the subjective discount rate ρ is augmented by the death rate ζ. The sufficient and necessary condition for the TVC to hold is again

$$\hat{c} - \gamma \bar{\chi}^2 (1 - \vartheta_t)^2 \tilde{\sigma}_t^2 > 0,$$

which guarantees that the left tail of the stationary wealth distribution is thin enough so that the households do not expect to become infinitely smaller than the aggregate asset.

This discussion shows that introducing stochastic death is not a quick fix to the issue of TVC for long-lived assets. Whether the TVC holds or not still depends on the degree of inequality in the wealth distribution, or, equivalently, the relative magnitude of the subjective discount rate and the idiosyncratic risk. Introducing stochastic death only increases the effective subjective discount rate, which makes the TVC more likely to hold if the model has the same values for other parameters.

5 Implications for Heterogeneous-Agent Models

Our results show that the inequality in the wealth distribution is the key to understand TVCs in heterogeneous-agent models. In this section, we apply this insight to several workhorse models in macroeconomic and finance literature, and evaluate its implications for asset valuation.

5.1 Constantinides-Duffie

Constantinides and Duffie (1996) study an economy with heterogeneous agents and idiosyncratic risk, which features a no-trade equilibrium. In their baseline model, each agent's labor income is subject to permanent idiosyncratic risk. This risk can generate negative labor income for some households, and reduce their total wealth and consumption to zero in the limit. Specifically, household i's one-period pricing kernel is

$$m_{t+1}^i = \exp(-\rho - \alpha \Delta c_{t+1}^i),$$

where the consumption growth is given by

$$\Delta c_{t+1}^i = \Delta c_{t+1} + y_{t+1}\eta_{t+1}^i - \frac{y_{t+1}^2}{2}.$$

As the idiosyncratic shock η_{t+1}^i is i.i.d., the household's consumption share follows a random walk. As a result, the distribution of consumption and wealth features an exploding degree of inequality.

This model is very similar to our baseline model without jump risk. If the permanent idiosyncratic risk is large enough, then the TVC is violated for the long-lived asset such as the equity market.[4] In this case, while the households' SDF correctly prices the one-period return of a long-lived asset like equity with price p_t and dividend d_t, i.e.,

$$\mathbb{E}_t[m_{t+1}^i(p_{t+1} + d_{t+1})] = p_t,$$

[4]One detail is that Constantinides and Duffie (1996) impose an assumption that the cumulative SDF M_t satisfies $\lim_{t\to\infty} \mathbb{E}[M_t] = 0$ (see Eq. (5) in the paper). However, this condition is not enough to rule out TVC violation. If the aggregate wealth grows at a constant rate $g > 0$, which is sufficiently large relative to the magnitude of the idiosyncratic risk, then, the cumulative SDF $\bar{m}_t$ in our baseline model will converge to zero. However, the rate at which the SDF converges to zero can be dominated by the rate at which the cash flow grows, so that the discounted present value of the asset is infinite, which leads to TVC violation.

the equity market will be infinitely valuable as the transversality condition is violated, i.e.,

$$\lim_{T\to\infty} \mathbb{E}_0\left[m_T^i p_T\right] = \infty.$$

In this sense, the characterization of the one-period return provides an incomplete characterization of the asset's value.

Constantinides and Duffie (1996) also consider a model variant in their appendix which features wealth reset. This model can be replicated by our model with wealth reset. As we showed in Section 3, even though the wealth distribution is stationary, the TVC for aggregate wealth still depends on the thickness of the left tail of the wealth distribution. As a result, we cannot take the TVC for granted in this model.

5.2 Brunnermeier-Sannikov-Merkel

Our results also clarify the characterizations of the intertemporal government budget condition in Brunnermeier et al. (2022). As we discussed in the literature review, Brunnermeier et al. (2022) propose a *dynamic trading perspective* which focuses on a household's individual wealth process and its time-varying holding of the Treasury portfolio. As the TVC for individual wealth always holds, this perspective is always valid, and offers a novel way of understanding how individual households value government debt. Our analysis does not contradict with this perspective. In fact, since the TVC for individual wealth always holds in our settings, this perspective is always applicable.

In comparison, the *buy and hold perspective* evaluates the aggregate government budget. As Brunnermeier et al. (2022) point out, the TVC for individual wealth process does not necessarily imply the TVC for aggregate government debt portfolio. The latter is not part of households' optimality condition because they do not buy and hold a fixed fraction of outstanding government debt. However, since standard asset pricing approaches rely on the TVC for aggregate assets to evaluate government debt, the wealth portfolio, and other long-lived assets, it is still important to

understand the conditions under which the TVC for aggregate assets holds.

Our results provide a general framework to consider this issue, as well as a concrete example with TVC violation. Specifically, we find that the key is to understand the properties of the wealth distribution, and its left tail in particular. When the left tail is sufficiently heavy, the TVC for aggregate assets may be violated. In this setting, we may not be able to evaluate these assets based on their aggregate cash flows using the standard discounted present value formula.

5.3 Aiyagari-Bewley-Huggett

Compared to Brunnermeier et al. (2022); Constantinides and Duffie (1996) and the settings considered in this paper, classic Aiyagari-Bewley-Huggett models additionally feature a borrowing constraint which prevents each household's asset holding from going below a certain level. This constraint is fairly general: if the minimal level is positive, households are required to hold certain assets in a way similar to financial repression.

When the borrowing constraint is binding, the household would like to shift consumption from the future to the present, but they cannot do so by borrowing. If we use the discrete-time notation in this literature, the Euler equation becomes an inequality:

$$m_t^i \geq \mathbb{E}_t \left[m_{t+1}^i (1+r) \right],$$

where the marginal utility $m_t^i = \rho^t / c_t^i$ if households have log utility.

If this borrowing constraint binds with non-zero probability, the government's intertemporal budget condition (11) also becomes an inequality. In discrete-time notation,

$$b_t \geq \lim_{T\to\infty} \mathbb{E}_t \left[\sum_{k=1}^{T} \frac{m_{t+k}^i}{m_t^i} s_{t+k} \right] + \lim_{T\to\infty} \mathbb{E}_t \left[\frac{m_T^i}{m_t^i} b_T \right]. \tag{19}$$

The proof is presented in Appendix C.1. In this setting, the asset price b_t can exceed the present value of surpluses for two distinct reasons. First, TVC violation may still arise and lead to an

infinite $\lim_{T\to\infty} \mathbb{E}_t \left[\frac{m_T^i}{m_t^i} b_T\right]$ term. Second, even when the TVC holds, the present value of surpluses only provides a lower bound for the valuation of government debt. In this sense, TVC violation for long-lived assets provide a sufficient, though not necessary, condition for asset bubbles.

Our characterizations of the TVC still apply to these settings. Consider Krueger and Lustig (2010) for a concrete example. This paper proposes a heterogeneous-agent model in which a fraction of the total endowment is distributed as equity dividend while the rest is labor income in each period. On the equity side, every household holds the market index, although they differ in their positions. On the labor income side, each household faces an idiosyncratic risk driven by a Markov process.

In this environment, the distribution of the individual household's marginal utility growth, i.e., m_T^i/m_t^i, is still the key to understand the TVC for aggregate assets. If the labor income is bounded from below by a positive number, then, the households' consumption shares are also bounded from below, which provides a finite upper bound on the households' marginal utility. Appendix C.2 provides the details. In this case, the households do not expect to become infinitely poorer than the population average, and the TVC for aggregate assets holds.

5.4 Townsend and Woodford

Townsend (1980) offers another example to illustrate the effects of borrowing constraints on asset valuation. Here, we consider a simplified version of the model in Woodford (1990). We consider an economy with two types of infinitely-lived households. Type A households have endowment e_1 in even periods and e_2 in odd periods, while type B households have endowment e_2 in even periods and e_1 in odd periods. We assume $e_1 > e_2 > 0$, which creates incentives to trade between the two types of households.

The households can buy government debt with supply d, which is financed by lump-sum taxes τ equally split between the two types of households. The interest rate is r. Households face a

borrowing constraint: $b_{i,t} \geq 0$, for $i \in \{A, B\}$. The households' budget constraints are

$$c_{i,t} + \tau/2 + b_{i,t} = e_{i,t} + (1+r)b_{i,t-1},$$

and the tax is equal to the interest on the government debt, i.e., $\tau = rd$.

The households maximize lifetime utility $\sum_{t=0}^{\infty} \rho^t u(c_{i,t})$. We assume that the endowment level e_2 is sufficiently lower than e_1 so that the borrowing constraint is binding for the household receiving endowment e_2. By market clearing, the other household must hold all the government debt. By symmetry, the household receiving e_2 must hold all the government debt from the previous period. Thus, in period 1, household A will choose to hold no government and consume all available resources:

$$c_{A,1} = e_2 + (1+r)d - \tau/2 =: \underline{c},$$
$$c_{B,1} = e_1 - d - \tau/2 =: \bar{c}.$$

Now, we are in a position to evaluate the government debt. Since the borrowing constraint always binds for the household receiving the lower endowment, Eq. (19) applies. Consider this equation for household A:

$$d > \lim_{T\to\infty} \mathbb{E}_0 \left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right] + \lim_{T\to\infty} \mathbb{E}_0 \left[\rho^T \frac{u'(c_{A,T})}{u'(c_{A,0})} d \right] = \lim_{T\to\infty} \mathbb{E}_0 \left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right].$$

The proof is presented in Appendix C.3. In this example, as the household's marginal utility is bounded, the TVC term converges to 0. However, the borrowing constraint implies that the government debt value still exceeds the present value of the future tax revenue.

5.5 HANK Models

HANK models also feature heterogeneous agents facing idiosyncratic risk and borrowing constraints. As a result, the TVC for long-lived assets also depends on the wealth distribution. For

example, in McKay, Nakamura, and Steinsson (2016), each household receives an equal share of the dividend from monopolistically competitive firms, and faces a non-negative borrowing constraint. In this case, the present value relationship again becomes an inequality as in Eq. (19). Moreover, as household wealth is bounded from below by the dividend-output ratio, which limits the degree of wealth inequality and hence the expected growth rate of household marginal utility.

Similarly, in Kaplan, Moll, and Violante (2018), households receive lump-sum government transfer of constant proportion to the output, and also face a borrowing constraint. In the steady state, household income is bounded from below by the government transfer minus borrowing limit times the exogenous borrowing rate, which is positive under the paper's calibration.[5] As a result, household wealth share is bounded from below despite an extremely tailed financial asset distribution. Through the lens of our characterization, we can see that bounded wealth distribution implies that the TVC for aggregate assets holds in these models.

6 Conclusion

This paper characterizes conditions under which the bubbles arise and TVC fails for long-lived assets in heterogeneous-agent economies. We find that the degree of inequality in the wealth distribution is the key to understanding this question. We show examples in which an exploding amount of inequality, or a heavy left tail in a stationary wealth distribution leads to TVC violation. In comparison, a stationary and more balanced wealth distribution satisfies the TVC. In heterogeneous-agent models, the r minus g condition remains a useful guide to understand the TVC.

These results have important implications for asset valuation in heterogeneous-agent models. For example, the TVC may fail in settings considered by Constantinides and Duffie (1996) and Brunnermeier et al. (2022), where the wealth distribution explodes over time. In contrast, the

[5] Under the steady state in Kaplan et al. (2018), the borrowing limit is one times quarterly labor income. The labor share of the economy is 60% and the unsecured borrowing rate is 8% annually, which caps households' quarterly interest payment at 1.2% of quarterly output. The government transfer is calibrated to 6% of output. Overall, individual's income is bounded by 4.8% of output.

TVC holds in settings considered by Krueger and Lustig (2010) and McKay et al. (2016); Kaplan et al. (2018), when the wealth distribution is bounded from below. Our results provide a general framework to understand the TVC in these models, and offer a new perspective on how to evaluate long-lived assets in heterogeneous-agent models.

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A Model Appendix

In this Appendix, we provide derivation details and additional explanations for models in Section 3 and 4. Note that the permanent risk model in Section 2 can be viewed as a special case of the transitory risk model in Section 3 with wealth reset intensity $\zeta = 0$.

A.1 Model Derivation

A.1.1 Set-up

Households Household's lifetime utility is given by

$$V_0^i = \mathbb{E}\left[\int_0^\infty e^{-\rho t}(c_t^i)^{-\gamma} dt\right].$$

The capital evolution follows

$$\frac{d\tilde{k}^i}{\tilde{k}^i} = \left(\Phi(\iota_t^i) - \delta\right) dt + \tilde{\sigma}_t d\tilde{Z}_t^i + d\Delta_t^{k,i},$$

where $d\Delta_t^{k,i}$ is the physical capital transactions between households, and

$$\Phi(\iota) = \frac{1}{\phi}\log(1 + \phi\iota).$$

The idiosyncratic risk processes $\tilde{Z}_t^i$ are independent across households. Besides, households are subject to wealth reset shock with intensity ζ. In the permanent shock case, $\zeta = 0$. Formally, let $\tilde{J}_t^i$ be a Poisson process with intensity ζ, the wealth dynamic for household i is given by

$$\frac{dn_t^i}{n_{t-}^i} = -\frac{c_t^i}{n_{t-}^i}dt + dr_t^B + \theta_t^{K,i}\left(dr_t^{K,i}(\iota_t^i) - dr_t^B\right) + \theta^{E,i}\left(dr_t^{E,i} - dr_t^B\right) + \theta_t^{\bar{E},i}\left(dr_t^{\bar{E},i} - dr_t^B\right) + \frac{\bar{n}_t - n_{t-}^i}{n_{t-}^i}d\tilde{J}_t^i,$$

where $\tilde{J}_t^i$ and $\tilde{J}_t^j$ are stochastically independent if $i \neq j$. Households face a skin-in-the-game constraint when issuing equity, i.e.,

$$-\theta_t^{E,i} \leq (1-\bar{\chi})\theta_t^{K,i}.$$

Government The nominal growth of government debt follows

$$dB_t = \mu_t^B B_t dt.$$

Government budget constraint is given by

$$i_t B_t + P_t \mathfrak{g} K_t = \mu_t^B B_t + P_t \tau_t a_t K_t,$$

where nominal rate i_t, spending and tax rate $\mathfrak{g}$, productivity a_t are exogenous processes adapted to the Brownian filtration Z_t. Tax rate τ_t is endogenous. With q_t^B defined below, the budget constraint can be restated as

$$\check{\mu}_t^B q_t^B + s_t = 0, \tag{A.1}$$

where $\check{\mu}_t^B := \mu_t^B - i_t$ is the bond issuance (net of interest payment) rate. $s_t := \tau_t a_t - \mathfrak{g}$ is the real surplus-capital ratio.

A.1.2 Symmetric Equilibrium

Aggregation Under symmetric equilibrium,

$$\forall i \in [0,1], \quad (\theta_t^{K,i}, \theta_t^{E,i}, \theta_t^{\bar{E},i}, \iota_t^i, \hat{c}_t^i) = (\theta_t^K, \theta_t^E, \theta_t^{\bar{E}}, \iota_t, \hat{c}_t),$$

where $\hat{c}_t^i = c_t^i / n_t^i$ is the consumption-wealth ratio.

Under symmetric equilibrium, the returns adjust such that no one trades physical capital with

others, i.e., $\Delta_t^{k,i} = 0$. Aggregate capital evolves according to

$$dK_t = (\Phi(\iota_t) - \delta) K_t dt.$$

Define the real price of physical capital as q_t^K and the real bond-capital ratio as $q_t^B = \frac{B_t/P_t}{K_t}$. The state variable ϑ_t is defined as the share of wealth in government bond, i.e.,

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K}. \tag{A.2}$$

The difference of drift $\mu_t^{q,B} - \mu_t^{q,K}$ can be derived as follows,

$$\begin{aligned}\mu_t^{\vartheta}\vartheta_t &= \frac{q_t^B}{q_t^B + q_t^K}\left(\mu_t^{q,B} - \frac{q_t^B\mu_t^{q,B} + q_t^K\mu_t^{q,K}}{q_t^B + q_t^K}\right) \\ &= \frac{q_t^B}{q_t^B + q_t^K}(1-\vartheta_t)\left(\mu_t^{q,B} - \mu_t^{q,K}\right),\end{aligned}$$

i.e.,

$$\frac{\mu_t^{\vartheta}}{1-\vartheta_t} = \mu_t^{q,B} - \mu_t^{q,K}. \tag{A.3}$$

Return Processes Here, we express the return process with the drift terms of q_t^B, q_t^K and ϑ_t.

Inflation:

$$\frac{dP_t}{P_t} = \frac{d(B_t/(q_t^B K_t))}{B_t/(q_t^B K_t)} = \left(\mu_t^B - \mu_t^{q,B} - \Phi(\iota_t) + \delta\right) dt.$$

Bond return:

$$dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^B K_t/B_t)}{q_t^B K_t/B_t} = \left(\Phi(\iota_t) - \delta - \breve{\mu}_t^B + \mu_t^{q,B}\right) dt. \tag{A.4}$$

Capital return:

$$\begin{aligned} dr_t^K &= \frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K}dt + \frac{d(q_t^K \tilde{k}_t^i)}{q_t^K \tilde{k}_t^i} \\ &= \left(\frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i) - \delta)\right)dt + \tilde{\sigma}_t d\tilde{Z}_t^i. \end{aligned} \tag{A.5}$$

Equity has the same risk loading as capital, but the expected return may be different when skin-in-the-game constraint is binding. Outside equity has the same aggregate risk loading but idiosyncratic loadings average out.

Market Clearing Goods market clearing:

$$\hat{c}_t(q_t^B + q_t^K) + \mathfrak{g} + \iota_t = a_t. \tag{A.6}$$

Asset market clearing:

$$\theta_t^K = \frac{q_t^K K_t}{(q_t^B + q_t^K)K_t} = 1 - \vartheta_t, \tag{A.7}$$

$$\theta^E = \theta^{\bar{E}}. \tag{A.8}$$

A.1.3 Solution

Households Problem We use dynamic programming to solve the households' portfolio choice problem. Guess that the value function takes the form $V(n_t^i, \omega_t) = \frac{\omega_t}{\rho+\zeta}\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + v_t$, where ω_t and v_t depends on investment opportunity in aggregate level. Heuristically,

$$\begin{aligned} V(n_t^i, \omega_t) &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + e^{-\rho dt}(1_{\text{jump}}V(n_{t+dt}^i, v_{t+dt}) + 1_{\text{nojump}}V(n_{t+dt}^i, v_{t+dt}))\right] \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + (1-\rho dt)(\zeta dt V(\bar{n}_t, v_{t+dt}) + (1-\zeta dt)V(n_{t+dt}^i, v_{t+dt}))\right] \end{aligned}$$

The Hamilton-Jacobi-Bellman equation is

$$(\rho+\zeta)V(n_t^i,\omega_t) = \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \zeta V(\bar{n}_t,\omega_t) + V_t + V_n(n_t^i,\omega_t)dn_t^i + \frac{1}{2}V_{nn}(n_t^i,\omega_t)\langle dn_t^i, dn_t^i\rangle\right]$$

$$\rho v_t + \omega_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} - \frac{\zeta}{\rho+\zeta}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma}$$
$$= \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}\left\{\frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \frac{\dot{w}_t}{\rho+\zeta}\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \dot{v}_t + \omega_t\frac{(n_t^i)^{1-\gamma}}{\rho+\zeta}\frac{dn_t^i}{n_t^i}\frac{1}{dt} - \omega_t\frac{\gamma}{2}\frac{(n_t^i)^{1-\gamma}}{\rho+\zeta}\left\langle\frac{dn_t^i}{n_t^i},\frac{dn_t^i}{n_t^i}\right\rangle\frac{1}{dt}\right\}.$$

For simplicity, we denote $\hat{\rho} := \rho+\zeta$. The HJB equation can be restated as

$$\begin{aligned}0 = \max_{c_t^i,\iota_t^i,\theta_t^i}\Bigg\{&-\rho v_t + \dot{v}_t + \frac{\zeta}{\hat{\rho}}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma} - \omega_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{1}{\hat{\rho}}\dot{\omega}_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{(c_t^i)^{1-\gamma}}{1-\gamma}\\
&-\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\frac{c_t^i}{n_t^i} + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\left(\Phi(\iota_t)-\delta-\check{\mu}_t^B+\mu_t^{q,B}\right)\\
&+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{K,i}\left(\frac{(1-\tau_t)a_t-\iota_t^i}{q_t^K}+\mu_t^{q,K}+(\Phi(\iota_t^i)-\delta)\right)+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt}+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt}\\
&-\frac{\gamma}{2}\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(\theta_t^{K,i}+\theta_t^{E,i})^2\tilde{\sigma}_t^2\Bigg\}. \end{aligned} \tag{A.9}$$

s.t.

$$-\theta_t^{E,i} \le (1-\bar{\chi})\theta_t^{K,i}$$

The lagrangian is hence given by

$$\begin{aligned}\mathcal{L}_t^i = &-\rho v_t + \dot{v}_t + \frac{\zeta}{\hat{\rho}}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma} - \omega_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{1}{\hat{\rho}}\dot{\omega}_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \frac{(c_t^i)^{1-\gamma}}{1-\gamma}\\
&-\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\frac{c_t^i}{n_t^i} + \omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\left(\Phi(\iota_t)-\delta-\check{\mu}_t^B+\mu_t^{q,B}\right)\\
&+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{K,i}\left(\frac{(1-\tau_t)a_t-\iota_t^i}{q_t^K}+\mu_t^{q,K}+(\Phi(\iota_t^i)-\delta)\right)+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt}+\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt}\\
&-\omega_t\frac{\gamma}{2}\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}}(\theta_t^{K,i}+\theta_t^{E,i})^2\tilde{\sigma}_t^2\\
&+\lambda_t^i(\theta_t^{E,i}+(1-\bar{\chi})\theta_t^{K,i}).\end{aligned}$$

The first order conditions w.r.t. c_t^i, ι_t^i, $\theta_t^{K,i}$, $\theta_t^{E,i}$ and $\theta_t^{\bar{E},i}$ are

$$-\omega_t \frac{(n_t^i)^{1-\gamma}}{\hat{\rho} n_t^i} + (c_t^i)^{-\gamma} = 0, \tag{A.10}$$

$$-\frac{1}{q_t^K} + \frac{1}{1+\phi\iota_t^i} = 0, \tag{A.11}$$

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \gamma(\theta_t^{K,i} + \theta_t^{E,i})\tilde{\sigma}_t^2 - (1-\bar{\chi})\lambda_t^i, \tag{A.12}$$

$$\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \gamma(\theta_t^{K,i} + \theta_t^{E,i})\tilde{\sigma}_t^2 - \lambda_t^i, \tag{A.13}$$

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = 0. \tag{A.14}$$

Characterizing Consumption, Investment q_t^B and q_t^K The first order condition Eq. (A.10) implies the consumption-wealth ratio

$$c_t^i = \left(\frac{\hat{\rho}}{\omega_t}\right)^{1/\gamma} n_t^i =: \hat{c}_t n_t^i. \tag{A.15}$$

Following the same steps in B, we obtain

$$\iota_t = \frac{(a_t - \mathfrak{g})(1-\vartheta_t) - \hat{c}_t}{1 + \hat{c}_t\phi - \vartheta_t}. \tag{A.16}$$

$$q_t^K = \frac{1 - \vartheta_t + \phi(a_t - \mathfrak{g})(1-\vartheta_t)}{1 + \hat{c}_t\phi - \vartheta_t} = (1-\vartheta_t)\frac{1 + \phi(a_t - \mathfrak{g})}{1 + \hat{c}_t\phi - \vartheta_t}. \tag{A.17}$$

$$q_t^B = \vartheta_t \frac{1 + \phi(a_t - \mathfrak{g})}{1 + \hat{c}_t\phi - \vartheta_t}. \tag{A.18}$$

Portfolio Weights Again, under symmetric equilibrium, market clearing implies

$$\theta_t^K = 1 - \vartheta_t,$$

$$\theta_t^E = -(1-\bar{\chi})(1-\vartheta_t),$$

$$\theta_t^{\bar{E}} = (1-\bar{\chi})(1-\vartheta_t).$$

Return Processes Under symmetric equilibrium, the outside equity has the same expected return as individual equity, i.e.,

$$\mathbb{E}_t[dr_t^{E,i}] = \mathbb{E}_t[dr_t^{\bar{E},i}], \tag{A.19}$$

with which we can pin down the lagrangian multiplier from Eq. (A.13) and (A.14)

$$\lambda_t^i = \gamma(\theta_t^{K,i} + \theta_t^{E,i})\tilde{\sigma}_t^2 = \gamma\bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t^2.$$

Note that $\theta_t^{E,i} + \theta_t^{K,i} \geq \chi\theta_t^{K,i}$, hence λ_t^i is always positive and the skin-in-the-game constraint is always binding. In the absence of aggregate risks, equity and real bond have the same expected return, which equals the risk-free rate r^f, i.e.,

$$\mathbb{E}_t[dr_t^{E,i}] = \mathbb{E}_t[dr_t^{\bar{E},i}] = \mathbb{E}_t[dr_t^B] = r_t^f dt. \tag{A.20}$$

The expected return of capital in excess of equity is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} = \chi\lambda_t^i = \gamma(1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2.$$

Equilibrium: Discipline Drift of the State Variable We discipline μ_t^ϑ as Proposition 1. By Eq. (**??**) and (**??**), the expected return of capital in excess of real bond return is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \frac{(1-\tau_t)a_t - \iota_t}{q_t^K} + \mu_t^{q,K} + \check{\mu}_t^B - \mu_t^{q,B}.$$

Plugging Eq. (**??**) and (**??**) yields

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \frac{\check{\mu}_t^B - \mu_t^\vartheta}{1-\vartheta_t} + \frac{a_t - \mathfrak{g} - \iota_t}{q_t^K}.$$

By Eq. (A.12) and (A.16),

$$\gamma(1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2 = \frac{\check{\mu}_t^B - \mu_t^\vartheta}{1-\vartheta_t} + \frac{a_t - \mathfrak{g} - \iota_t}{q_t^K}$$
$$\Rightarrow \mu_t^\vartheta = \check{\mu}_t^B + \hat{c}_t - \gamma(1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2. \tag{A.21}$$

Verifying Value Function I verify the value function by plugging endogenous variables into the HJB Equation (A.9). The coefficient of $(n_t^i)^{1-\gamma}$ should be zero, i.e.,

$$\begin{aligned}
0 =& -\omega_t\frac{1}{1-\gamma} + \frac{1}{\hat{\rho}}\dot{\omega}_t\frac{1}{1-\gamma} + \left(\frac{\omega_t}{\hat{\rho}}\right)^{-\frac{1-\gamma}{\gamma}}\frac{1}{1-\gamma} - \omega_t\frac{1}{\hat{\rho}}\left(\frac{\omega_t}{\hat{\rho}}\right)^{-\frac{1}{\gamma}} \\
&+ \omega_t\frac{1}{\hat{\rho}}\vartheta_t\mathbb{E}[r_t^B]/dt + \omega_t\frac{1}{\hat{\rho}}(1-\vartheta_t)\mathbb{E}[r_t^{K,i}]/dt - \frac{\gamma}{2}\omega_t\frac{1}{\hat{\rho}}\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2, \\
0 =& -\rho v_t + \dot{v}_t + \frac{\zeta}{\hat{\rho}}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma}.
\end{aligned}$$

which are ODEs for ω_t and v_t. Note that $\vartheta_t, \mu_t^\vartheta$ and $\bar{n}_t$ depend on aggregate level variables only, so do ω_t and v_t. The value function is verified. Besides, in the absence of aggregate shocks, $\omega_t = \omega$ is constant. Plug in $\mathbb{E}[r_t^B]/dt = r^f$, $\mathbb{E}[r_t^{K,i}]/dt = r^f + \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$ and $\omega_t = \hat{\rho}\hat{c}^{-\gamma}$ to get

$$0 = -\hat{\rho}\hat{c}^{-\gamma}\frac{1}{1-\gamma} + \hat{c}^{1-\gamma}\frac{1}{1-\gamma} - \hat{c}^{1-\gamma} + \hat{c}^{-\gamma}r^f + \hat{c}^{-\gamma}\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \hat{c}^{-\gamma}\frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2,$$

i.e.,

$$r^f = \frac{-\gamma\hat{c} + \hat{\rho}}{1-\gamma} - \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{A.22}$$

In the mean time, according to Eq. (A.4), (A.20) and (A.21), r^f is given by

$$r^f = \Phi(\iota) - \delta + \hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

$\hat{c}$ can be solved numerically by

$$\hat{c} - (\gamma - 1)\Phi(\iota) = \hat{\rho} - (\gamma - 1)\delta - \frac{\gamma(\gamma - 1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 \tag{A.23}$$

where

$$\iota = \frac{(a - \mathfrak{g})(1 - \vartheta) - \hat{c}}{1 + \hat{c}\phi - \vartheta}$$
$$\Phi(\iota) = \frac{1}{\phi}\log(1 + \phi\iota).$$

Since

$$\frac{d\iota}{d\hat{c}} = \frac{-(1 + \hat{c}\phi - \vartheta) - ((a - \mathfrak{g})(1 - \vartheta) - \hat{c})\phi}{(1 + \hat{c}\phi - \vartheta)^2} < 0,$$

ι and $\Phi(\iota)$ are decreasing in $\hat{c}$. The LHS is monotonically increasing in $\hat{c}$. There is a unique positive solution of $\hat{c}$ if

$$\frac{\gamma}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 - \frac{\hat{\rho}}{1 - \gamma} + \delta < \Phi(a - \mathfrak{g}).$$

How does $\hat{c}$ change when we marginally increase γ? Their relationship is not monotone. In Eq. (A.23), a higher γ decreases the right-hand side, but also decreases the $-(\gamma - 1)\Phi(\iota)$ term on the left-hand side. Either term could dominate, leading to an ambiguous effect of γ on $\hat{c}$.

Having said that, if γ is sufficiently high or $\tilde{\sigma}^2$ is sufficiently high, the effect on the right-hand side dominates, leading to a negative relationship between γ and the consumption response $\hat{c}$. Specifically, the derivative $dc/d\gamma$ is given by

$$d\hat{c}\left(1 - (\gamma - 1)\Phi'(\iota)\frac{d\iota}{dc}\right) = \left(\frac{\hat{c} - \hat{\rho}}{\gamma - 1} - \frac{\gamma - 1}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2\right)d\gamma,$$

where the LHS is always positive, while RHS tends to be negative with higher γ or $\tilde{\sigma}^2$. Moreover,

note that

$$\frac{d\hat{c}}{d\hat{\rho}} = \left(1 - (\gamma - 1)\Phi'(\iota)\frac{d\iota}{dc}\right)^{-1} > 0.$$

Consumption rate increases when subjective discount rate ρ or wealth reset shock intensity ζ goes up.

A.2 Proof for Proposition 1

Recall that

$$m_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}.$$

We obtain

$$\begin{aligned}\lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] &= \lim_{T\to\infty} \mathbb{E}_0\left[\left(\int_0^1 e^{-\rho T}\left(\frac{n_T^i}{n_0^i}\right)^{-\gamma} di\right)\bar{n}_T\right] \\ &= \lim_{T\to\infty} \mathbb{E}_0\left[e^{-\rho T}\bar{n}_0\left(\frac{\bar{n}_T}{\bar{n}_0}\right)^{1-\gamma}\left(\int_0^1\left(\frac{\eta_T^i}{\eta_0^i}\right)^{-\gamma} di\right)\right].\end{aligned}$$

Normalizing $\eta_0^i = 1$ and $\bar{n}_0 = 1$ yields

$$\lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] = \lim_{T\to\infty} \mathbb{E}_0\left[e^{-\rho T}\bar{n}_T^{1-\gamma}\left(\int_0^1\left(\eta_T^i\right)^{-\gamma} di\right)\right].$$

A.3 Transversality Conditions with Permanent Idiosyncratic Risk

A.3.1 Individual Level TVC

The r Minus g Perspective Individual TVC is satisfied iff $\mathbb{E}_t[d(m_t^i n_t^i)] < 0$, i.e.,

$$-\rho - \gamma g + \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + g - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 < 0,$$

i.e.,

$$\rho > (1-\gamma)g + \frac{\gamma}{2}(\gamma-1)\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2. \tag{A.24}$$

Note that

$$r^f = \rho + \gamma g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{A.25}$$

$$g = -\hat{c} + r^f + \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 \tag{A.26}$$

The first equation is from the drift term of the SDF. The second equation is obtained from the wealth dynamics. The equations jointly solve

$$g = \frac{-\hat{c}+\rho}{1-\gamma} + \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{A.27}$$

$$r^f = \frac{-\gamma\hat{c}+\rho}{1-\gamma} - \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2. \tag{A.28}$$

Plug Eq. (A.25) into (A.24), individual TVC is satisfied iff

$$r^f + \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 > g,$$

which together with Eq. (A.26) implies individual TVC holds if and only if $\hat{c} > 0$. As a result, individual TVC always hold. The intuition here is that the growth rate of individual wealth is the return of, aka the discount rate for, individual wealth portfolio, minus consumption rate. As long as consumption rate is positive, the growth rate of individual wealth is always smaller than the discount rate.

The Wealth Distribution Perspective The individual level TVC can be stated as Wealth inequality view:

$$\lim_{T\to\infty}\mathbb{E}[m_T^i n_T^i] = \lim_{T\to\infty}\hat{c}^{-\gamma}\mathbb{E}\left[e^{-\rho T}\bar{n}_T^{1-\gamma}(\eta_T^i)^{1-\gamma}\right] = \lim_{T\to\infty}(\hat{c}^{-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma})\mathbb{E}\left[(\eta_T^i)^{1-\gamma}\right].$$

Recall that

$$\frac{d\eta_t^i}{\eta_t^i} = \bar{\chi}(1-\vartheta)\tilde{\sigma}d\tilde{Z}_t^i,$$

by Itô's lemma,

$$d(\eta_t^i)^{1-\gamma} = \frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(\eta_t^i)^{1-\gamma}dt + (1-\gamma)(\eta_t^i)^{1-\gamma}d\tilde{Z}_t^i,$$

hence

$$d\mathbb{E}[(\eta_t^i)^{1-\gamma}] = \frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\mathbb{E}[(\eta_t^i)^{1-\gamma}]dt,$$

which implies

$$\mathbb{E}[(\eta_T^i)^{1-\gamma}] = (\eta_0^i)^{1-\gamma}\exp\left(\frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2T\right).$$

The term can be further decomposed to

$$\begin{aligned}\mathbb{E}[(\eta_T^i)^{1-\gamma}] &= \mathbb{E}[(\eta_T^i)^{-\gamma}]\mathbb{E}[\eta_T^i] + Cov[(\eta_T^i)^{-\gamma},\eta_T^i]\\ &= (\eta_0^i)^{1-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2T\right)\\ &+ (\eta_0^i)^{1-\gamma}\left[\exp\left(\frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2T\right) - \exp\left(\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2T\right)\right].\end{aligned}$$

The first term reflects how long-run idiosyncratic risk (the marginal utility inequality) increase the valuation of future wealth portfolio by shrinking the long-term risk-free rate through the precau-

tionary saving channel[6], while the second term reflects how this force is largely offseted by the risk premium of the individual's own wealth portfolio in the long run. As risk-aversion γ goes up, while the preceived long-run idiosyncratic risk depresses the long-run risk-free rate by magnitude of $\exp((\gamma(\gamma+1)\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2/2)T)$, the associated long-term risk premium also explodes at the same speed. In the mean time, marginal utility decreases faster on average due to idiosyncratic risk premia. In equilibrium, the sum of the three forces ensures that the valuation of individual wealth portfolio in the long-run does not explode and individual TVC holds. Finally, recall that $\bar{n}_T$ grows at rate g, which is given in Eq. (A.27), i.e.,

$$\begin{aligned}(\bar{n}_T)^{1-\gamma} &= (\bar{n}_0)^{1-\gamma}\exp\left((1-\gamma)gT\right)\\ &= (\bar{n}_0)^{1-\gamma}\exp\left(\left(-\hat{c}+\rho-\frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right)T\right)\end{aligned}$$

The three terms $e^{-\rho T}$, $(\bar{n}_T)^{1-\gamma}$ and $\mathbb{E}[(\eta_T^i)^{1-\gamma}]$ echo the three terms in (A.24). Together, we obtain

$$\lim_{T\to\infty}\mathbb{E}[m_T^i n_T^i] = \lim_{T\to\infty}(\hat{c}^{-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma})\mathbb{E}\left[(\eta_T^i)^{1-\gamma}\right] = \lim_{T\to\infty}\hat{c}^{-\gamma}(\bar{n}_0)^{1-\gamma}\exp(-\hat{c}T).$$

A.3.2 Aggregate Level TVC

The r Minus g Perspective Similarly, individual TVC is satisfied iff $\mathbb{E}_t[d(m_t^i\bar{n}_t)] < 0$, i.e.,

$$-\rho-\gamma g+\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2+g<0,$$

i.e.,

$$\rho > (1-\gamma)g+\frac{\gamma}{2}(\gamma+1)\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

[6]In the absence of idiosyncratic risks, the long-run risk-free rate is given by $\exp(-r^f T) = e^{-\rho T}\bar{n}_T^{-\gamma}$. With idiosyncratic risk, $\exp(-r^f T) = e^{-\rho T}\bar{n}_T^{-\gamma}\mathbb{E}[(\eta_T^i)^{-\gamma}]$. r^f shrinks by $\frac{1}{T}\log\mathbb{E}[(\eta_T^i)^{-\gamma}]$.

Plug in Eq. (A.25) and (A.26), we obtain that aggregate level TVC is satisfied if and only if

$$r^f > g.$$

Together with Eq. (A.28) and (A.27), the aggregate TVC holds if and only if

$$\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 < \hat{c}.$$

The Wealth Distribution Perspective The transversality condition is given by

$$\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] = \lim_{T\to\infty}\hat{c}^{-\gamma}\mathbb{E}\left[e^{-\rho T}\bar{n}_T^{1-\gamma}(\eta_T^i)^{-\gamma}\right] = \lim_{T\to\infty}(\hat{c}^{-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma})\mathbb{E}\left[(\eta_T^i)^{-\gamma}\right].$$

Following the similar steps when we discussed the individual TVC, the wealth dynamics imply

$$\mathbb{E}[(\eta_T^i)^{-\gamma}] = (\eta_0^i)^{-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2T\right).$$

Hence,

$$\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] = \lim_{T\to\infty}\hat{c}^{-\gamma}(\eta_0^i)^{-\gamma}\bar{n}_0^{1-\gamma}\exp\left(\left(-\rho+(1-\gamma)g+\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right)T\right).$$

Plug in g from Eq. (A.27) to obtain

$$\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] = \lim_{T\to\infty}\hat{c}^{-\gamma}(\eta_0^i)^{-\gamma}\bar{n}_0^{1-\gamma}\exp\left(\left(\hat{c}-\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right)T\right).$$

The aggregate TVC holds if $\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 < \hat{c}$.

A.4 Transversality Conditions with Transitory Idiosyncratic Risk

A.4.1 Aggregate Level TVC

We show the wealth distribution perspective here only, and leave the r minus g perspective in Appendix A.7 which also serves as the proof for Proposition 3.

The Wealth Distribution Perspective Consider the wealth reset shock with intensity ζ. The wealth dynamics become

$$d\eta_t^i = \bar{\chi}(1-\vartheta)\tilde{\sigma}\eta_{t-}^i d\tilde{Z}_t^i + (1-\eta_{t-}^i)d\tilde{J}_t^i.$$

By Itô's lemma,

$$\begin{aligned} d(\eta_t^i)^{-\gamma} = &\left[\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(\eta_{t-}^i)^{-\gamma} + \zeta(1-(\eta_{t-}^i)^{-\gamma})\right]dt \\ &- \gamma\bar{\chi}(1-\vartheta)\tilde{\sigma}(\eta_{t-}^i)^{-\gamma}d\tilde{Z}_t^i + (1-(\eta_{t-}^i)^{-\gamma})(d\tilde{J}_t^i - \zeta dt). \end{aligned}$$

Take expectation on both sides yields an ODE w.r.t. $\mathbb{E}[(\eta_t^i)^{-\gamma}]$[7], i.e.,

$$d\mathbb{E}[(\eta_t^i)^{-\gamma}] = \left[\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\mathbb{E}[(\eta_t^i)^{-\gamma}] + \zeta(1-\mathbb{E}[(\eta_t^i)^{-\gamma}])\right]dt.$$

The solution to the ODE is

$$\mathbb{E}_0\left[(\eta_t^i)^{-\gamma}\right] = \left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta} + (\eta_0^i)^{-\gamma}\right)e^{[\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta]t} - \frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2-\zeta}.$$

Recall that growth rate of aggregate wealth is

$$g = \frac{-\hat{c}+\rho+\zeta}{1-\gamma} + \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2,$$

[7]Note that the difference between $\mathbb{E}[(\eta_t^i)^{-\gamma}]$ and $\mathbb{E}[(\eta_{t-}^i)^{-\gamma}]$ is of magnitude dt, and vanishes after multiplying dt outside the bracket.

Denote $\hat{c} = \hat{\mathfrak{c}}(\rho)$, which is given implicitly by

$$\hat{c} - (\gamma - 1)\Phi(\iota) = \hat{\rho} - (\gamma - 1)\delta - \frac{\gamma(\gamma - 1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2,$$

where

$$\iota = \frac{(a - \mathfrak{g})(1 - \vartheta) - \hat{c}}{1 + \hat{c}\phi - \vartheta}, \quad \Phi(\iota) = \frac{1}{\phi}\log(1 + \phi\iota).$$

The aggregate TVC condition can be stated as

$$\begin{aligned}
\lim_{T\to\infty} \mathbb{E}[m_T^i \bar{n}_T] &= \lim_{T\to\infty} (\hat{c}^{-\gamma} e^{-\rho T} \bar{n}_T^{1-\gamma}) \mathbb{E}\left[(\eta_T^i)^{-\gamma}\right] \\
&= \lim_{T\to\infty} \hat{c}^{-\gamma} \bar{n}_0^{1-\gamma} e^{-\rho T} \exp\left(\left(-\hat{c} + \rho + \zeta - \frac{\gamma(\gamma - 1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2\right) T\right) \\
&\times \exp\left(\left(\frac{\gamma(\gamma + 1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 - \zeta\right) T\right) \left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 - \zeta} + (\eta_0^i)^{-\gamma}\right) \\
&- \lim_{T\to\infty} \hat{c}^{-\gamma} \bar{n}_0^{1-\gamma} e^{-\rho T} \bar{n}_T^{1-\gamma} \frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 - \zeta} \\
&= \lim_{T\to\infty} \hat{c}^{-\gamma} \bar{n}_0^{1-\gamma} \exp(-(\hat{c} - \gamma\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2)T) \left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 - \zeta} + (\eta_0^i)^{-\gamma}\right) \\
&- \lim_{T\to\infty} \hat{c}^{-\gamma} \bar{n}_0^{1-\gamma} e^{-\rho T} \bar{n}_T^{1-\gamma} \frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2 - \zeta}.
\end{aligned}$$

Hence, the aggregate TVC holds if and only if

$$\hat{c} > \gamma\bar{\chi}^2(1 - \vartheta)^2\tilde{\sigma}^2$$

and

$$-\rho + (1 - \gamma)g < 0.$$

The first condition states that the wealth reset intensity should be big enough such that the marginal utility dispersion does not explode too fast, hence risk free rate will not be lower than the growth rate. The second condition is imposed as we are evaluating long-lived assets with the realized marginal utility process, we want to rule out the case where the whole economy shrinks and marginal utilities explode on average. In other word, we assume transversality condition hold had the economy been populated by one representative agent instead of heterogeneous agents. In this way, we shed light on the effect of heterogeneity and idiosyncratic risk.

A.5 Proof for Proposition 2

Denote the nominal government bond outstanding as B_t and real government bond as b_t. We obtain

$$dB_t = i_t B_t dt - P_t s_t dt.$$

Hence,

$$\begin{aligned}
d(m_t^i b_t) &= b_t dm_t^i + m_{t-}^i / P_t dB_t + m_{t-}^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\
&= b_t \left(-(r_t^f - \zeta) m_{t-}^i dt - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_{t-}^i d\tilde{Z}_t^i + \left(e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma} - m_{t-}^i \right) d\tilde{J}_t^i \right) \\
&+ i_t b_t m_{t-}^i dt - s_t m_t^i dt + m_{t-}^i b_t (dr_t^B - i_t dt) \\
&= -m_{t-}^i s_t dt + m_{t-}^i \zeta b_t dt - m_{t-}^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i + b_t \left(e^{-\rho t} \frac{1}{(\rho+\zeta)\bar{n}_t} - m_{t-}^i \right) d\tilde{J}_t^i \\
&= -m_{t-}^i s_t dt + \zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma} b_t dt - m_{t-}^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i \\
&+ b_t \left(e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma} - m_{t-}^i \right) d(\tilde{J}_t^i - \zeta t),
\end{aligned}$$

where we plugged in the definition of bond return, i.e.,

$$dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^B K_t / B_t)}{q_t^B K_t / B_t} = r_t^f dt$$

Integrate $d(m_t^i b_t)$ from $t=0$ to T and take expectation to obtain[8]

$$\mathbb{E}_0[m_T^i b_t] - m_0^i b_0 = -\mathbb{E}_0\left[\int_0^T m_t^i s_t\, dt\right] + \zeta \mathbb{E}_0\left[\int_0^T e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma}\, dt\right].$$

Divide both sides by m_0^i to obtain

$$b_0 = \mathbb{E}_0\left[\int_0^T \frac{m_t^i}{m_0^i} s_t\, dt\right] - \zeta \mathbb{E}_0\left[\int_0^T e^{-\rho t}\left(\frac{\bar{n}_T}{\bar{n}_0}\right)^{-\gamma} b_t\, dt\right] + \mathbb{E}_0\left[\frac{m_T^i}{m_0^i} b_T\right].$$

To derive the bond valuation equation from the aggregate SDF, we start by dividing $d(m_t^i B_t/P_t)$ by m_0^i, i.e.,

$$\begin{aligned} d[(m_t^i/m_0^i)b_t] = &-\frac{m_{t-}^i}{m_0^i} s_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt - \frac{m_{t-}^i}{m_0^i} b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i \\ &+ b_t\left(e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} - \frac{m_{t-}^i}{m_0^i}\right) d(\tilde{J}_t^i - \zeta t). \end{aligned}$$

Intergrate by i to obtain

$$d[\bar{m}_t b_t] = -\bar{m}_t s_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt.$$

Integrate by t and take expectation to obtain

$$\mathbb{E}_0[\bar{m}_T b_T] - b_0 = -\mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_t\left[\int_0^T \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt\right],$$

i.e.,

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] - \mathbb{E}_t\left[\int_0^T \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-\gamma} b_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right].$$

[8]Note that the set of jump time $\mathcal{T} = \{t | m_t^i \neq m_{t-}^i\}$ has zero measure, and we can replace $t-$ by t in the intergrals. Heuristically, $\int_0^T m_t^i s_t\, dt = \int_0^T m_{t-}^i s_t\, dt + \int_0^T (m_t^i - m_{t-}^i) s_t\, dt = \int_0^T m_{t-}^i s_t\, dt$.

A.6 Wealth Distribution Dynamics

Recall that the individual wealth satisfies

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t^m dZ_t + (1-\vartheta_t)\tilde{\sigma}_t \bar{\chi} d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i} d\tilde{J}_t^i.$$

For aggregate wealth,

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt + \sigma_t^m dZ_t.$$

Hence,

$$\frac{d\eta_t^i}{\eta_t^i} = \frac{d(n_t^i/\bar{n}_t)}{n_t^i/\bar{n}_t} = \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i + \frac{1-\eta_t^i}{\eta_t^i} d\tilde{J}_t^i. \tag{A.29}$$

We obtain the probability density function for η_T^i from the Kolmogorov Forward Equation, i.e.,

$$\frac{\partial g(\eta,t)}{\partial t} = -\frac{\partial}{\partial \eta}(\beta(\eta,t)g(\eta,t)) + \frac{1}{2}\frac{\partial^2}{\partial \eta^2}(\gamma(\eta,t)^2 g(\eta,t)) - \zeta g(\eta,t),$$

where $\beta(\eta,t) = 0$, $\gamma(\eta,t) := \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t\eta$. [9] Under Assumption 1, let $\vartheta_t = \vartheta, \tilde{\sigma}_t = \tilde{\sigma}$, we obtain

$$\frac{\partial g(\eta,t)}{\partial t} = \frac{(\bar{\chi}(1-\vartheta)\tilde{\sigma})^2}{2}\frac{\partial^2}{\partial \eta^2}(\eta^2 g(\eta,t)) - \zeta g(\eta,t).$$

The boundary condition is that $g(\eta,t)$ is always a valid pdf at any time t, i.e.,

$$\int_0^\infty g(\eta,t)\, d\eta = 1, \quad \forall t > 0.$$

For initial condition, we require

$$\lim_{t\to 0} g(\eta,t) = \delta(\eta - 1),$$

[9] Note that there is a kink at $\eta = 1$, and the KFE holds on $(0,1)$ and $(1,+\infty)$.

where $\delta(.)$ denotes the Dirac function. Generally, the Kolmogorov Forward Equation PDE has no analytical solution. However, we can obtain the explicit expression of wealth distribution at the steady state. Let $\partial g(\eta, t)/\partial t = 0$, we obtain

$$0 = \frac{1}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(\eta^2 g''(\eta) + 4\eta g'(\eta) + 2g(\eta)) - \zeta g(\eta),$$

i.e.,

$$0 = \eta^2 g''(\eta) + 4\eta g'(\eta) + 2 - \frac{2\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2} g(\eta).$$

The ODE is solved separately on $(0, 1)$ and $(1, +\infty)$. The general solution is

$$g(\eta) = C_1\eta^{\alpha_1} + C_2\eta^{\alpha_2}, \qquad \eta < 1,$$
$$g(\eta) = C_3\eta^{\alpha_1} + C_4\eta^{\alpha_2}, \qquad \eta > 1,$$

where

$$\alpha_1 = -\frac{3}{2} + \frac{\sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}, \qquad \alpha_2 = -\frac{3}{2} - \frac{\sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}.$$

The steady state probability density function $g(\eta)$ must satisfy

$$\int_0^\infty g(\eta)\, d\eta = 1.$$

Note that $\alpha_1 > -1$ and $\alpha_2 < -1$. Hence, we require $C_2 = 0$ and $C_3 = 0$ to ensure $|\int_0^\infty g(\eta)\, d\eta| < \infty$. Moreover, we require $C_1 = C_4 = C$ such that $g(\eta)$ is continuous at $\eta = 1$. The integral is

computed as

$$\begin{aligned}\int_0^\infty g(\eta)\,d\eta &= \frac{2C}{-1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}\,\eta^{\frac{-1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_0^1 + \frac{2C}{-1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}\,\eta^{\frac{-1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_1^\infty \\ &= \frac{2C}{-1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} - \frac{2C}{-1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} \\ &= \frac{4\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}C,\end{aligned}$$

which implies $C_1 = C_4 = C = \frac{2\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}$. Note that

$$\begin{aligned}\int_0^\infty \eta g(\eta)\,d\eta &= \frac{2C}{1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}\,\eta^{\frac{1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_0^1 + \frac{2C}{1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}\,\eta^{\frac{1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_1^\infty \\ &= \frac{2C}{1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} + \frac{2C}{-1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} \\ &= \frac{4\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}C = 1.\end{aligned}$$

Hence we verify that $g(\eta)$ is a valid wealth distribution function, i.e., $\int_0^1 \eta^i\,di = \int_0^\infty \eta g(\eta)\,d\eta = 1$.

A.7 Proof for Proposition 3

Note that

$$\begin{aligned}d(m_t^i\bar{n}_t) = &-\left[\rho-\gamma g+\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2+g\right]m_{t-}^i\bar{n}_t dt \\ &+\zeta[e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}-m_{t-}^i\bar{n}_t]dt \\ &-\gamma\bar{\chi}(1-\vartheta)\tilde{\sigma}m_{t-}^i\bar{n}_t d\tilde{Z}_t^i+[e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}-m_{t-}^i\bar{n}_t]d(\tilde{J}_t^i-\zeta t).\end{aligned}$$

By Eq. (A.22) and (A.26)

$$g = \frac{-\hat{c} + \rho + \zeta}{1-\gamma} + \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

Plug in to obtain

$$\begin{aligned} d(m_t^i \bar{n}_t) = &- \left[\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\right] m_{t-}^i \bar{n}_t dt \\ &+ \zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma} dt \\ &- \gamma\bar{\chi}(1-\vartheta)\tilde{\sigma} m_{t-}^i \bar{n}_t d\tilde{Z}_t^i + [e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma} - m_{t-}^i \bar{n}_t] d(\tilde{J}_t^i - \zeta t). \end{aligned}$$

Divide by m_0^i and integrate across i to obtain

$$d(\bar{m}_t \bar{n}_t) = -(\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2)\bar{m}_t\bar{n}_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{1-\gamma}\bar{n}_0 dt.$$

The ODE admits the following solution

$$\begin{aligned} \bar{m}_t\bar{n}_t = &\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}\exp(-(\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2)t)\left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \zeta} + 1\right) \\ &- \hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}e^{-\rho t}\bar{n}_t^{1-\gamma}\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \zeta}. \end{aligned}$$

Given $-\rho - (\gamma-1)g < 0$, the aggregate TVC holds if and only if

$$\hat{c} > \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2.$$

A.8 Arbitrary Wealth Distribution After Wealth Reset

We show that the main results are preserved even if we impose arbitrary wealth distribution after wealth resets. Denote the pdf for wealth resets as $h(\tilde{\eta})$. We require that $\int_0^\infty \tilde{\eta}^{-\gamma}h(\tilde{\eta})\, d\tilde{\eta} < \infty$, i.e., the expected marginal utility immediately after the wealth reset is well-defined. The previous specification can be viewed as a special case where $h(\tilde{\eta}) = \delta(\tilde{\eta} - 1)$, where $\delta(.)$ is the dirac

function. First, we show that this general case admits the same solution of aggregate variables in the reset-to-average case. Heuristically, household's HJB equation is given by

$$\begin{aligned} V(n_t^i, \omega_t) &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + e^{-\rho dt} (1_{\text{jump}} V(n_{t+dt}^i, v_{t+dt}) + 1_{\text{nojump}} V(n_{t+dt}^i, v_{t+dt})) \right] \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + (1-\rho dt)(\zeta dt \mathbb{E}^\eta [V(h(\tilde{\eta})\bar{n}_t, v_{t+dt})] + (1-\zeta dt) V(n_{t+dt}^i, v_{t+dt})) \right]. \end{aligned}$$

$\mathbb{E}^\eta[.]$ denotes taking expectation w.r.t. $\tilde{\eta}$. The only difference is that the after-jump value function changed from $V(\bar{n}_t, v_{t+dt})$ to $V(h(\tilde{\eta})\bar{n}_t, v_{t+dt})$. Since the assignment of after-jump wealth is independent with agent's pre-jump wealth, the expectation depends on aggregate variables only. As a result, $h(\tilde{\eta})$ only affects v_t in the value function, which does not matter household's consumption or investment choices.

Next, we show that $h(\tilde{\eta})$ does not matter for the transversality condition either. Note that the dynamics of SDF is

$$\begin{aligned} dm_t^i = &- \left[\rho + \gamma g - \frac{\gamma(\gamma+1)}{2} \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2 + \zeta \right] m_{t-}^i dt \\ &+ \zeta e^{-\rho t} \hat{c}^{-\gamma} (\tilde{\eta})^{-\gamma} \bar{n}_t^{-\gamma} dt \\ &- \gamma \bar{\chi} (1-\vartheta) \tilde{\sigma} m_{t-}^i d\tilde{Z}_t^i \\ &+ \left(e^{-\rho t} \hat{c}^{-\gamma} (\tilde{\eta})^{-\gamma} \bar{n}_t^{-\gamma} - m_{t-}^i \right) d(\tilde{J}_t^i - \zeta t). \end{aligned}$$

Note that $\tilde{\eta}$ denotes the wealth share after jump, which is a random variable with pdf $h(\tilde{\eta})$. Hence,

$$\begin{aligned} dm_t^i \bar{n}_t = &- \left[\rho + (\gamma-1) g - \frac{\gamma(\gamma+1)}{2} \bar{\chi}^2 (1-\vartheta)^2 \tilde{\sigma}^2 + \zeta \right] m_{t-}^i \bar{n}_t dt \\ &+ \zeta e^{-\rho t} \hat{c}^{-\gamma} (\tilde{\eta})^{-\gamma} \bar{n}_t^{1-\gamma} dt \\ &- \gamma \bar{\chi} (1-\vartheta) \tilde{\sigma} m_{t-}^i \bar{n}_t d\tilde{Z}_t^i \\ &+ \left(e^{-\rho t} \hat{c}^{-\gamma} (\tilde{\eta})^{-\gamma} \bar{n}_t^{1-\gamma} - m_{t-}^i \bar{n}_t \right) d(\tilde{J}_t^i - \zeta t). \end{aligned}$$

Take expectation to obtain

$$d\mathbb{E}\left[m_t^i n_t^i\right] = -\left[\rho + (\gamma-1)g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta\right]\mathbb{E}[m_{t-}^i \bar{n}_t]dt + \zeta e^{-\rho t}\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_t^{1-\gamma}dt.$$

Note that $[\rho + (\gamma-1)g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta = r^f - g = \hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$. The solution is

$$e^{(r^f-g)t}\mathbb{E}[m_t^i\bar{n}_t] - m_0^i\bar{n}_0 = \frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}(e^{[\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2]t} - 1),$$

i.e.,

$$\mathbb{E}[m_t^i\bar{n}_t] = \left(m_0^i\bar{n}_0 - \frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}\right)\exp(-(\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2)t) + \frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}\exp(-(\rho + (\gamma-1)g)t).$$

For all $h(\tilde{\eta})$ such that $\mathbb{E}[(\tilde{\eta})^{-\gamma}]$ exists, aggregate TVC depends on $\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$, i.e., $r^f - g$. In the first part we show that $h(\tilde{\eta})$ does not matter for any aggregate variables. Hence, $h(\tilde{\eta})$ does not matter for aggregate TVC.

A.9 Model with Stochastic Death

We model the stochastic death as follows. Define $\{d_t^i\}$ as a Poisson process with intensity ζ. Death events are independent across agents, i.e., $\{d_t^i\}$ and $\{d_t^j\}$ are stochastically independent for $i \neq j$. Let $d_0^i = 0$. The stochastic death time τ^i is defined as

$$\tau^i := \inf\{t > 0 \mid d_t^i \geq 1\}$$

The death process is hence defined by

$$death_t^i = d^i_{\{t\wedge\tau^i\}},$$

i.e., when agent i is alive, $death_t^i = 0$. Otherwise, $death_t^i = 1$.

Consider household's problem. Heuristically,

$$\begin{aligned} V(n_t^i, v_t) &= \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + e^{-\rho dt}(1_{\text{death}} V(n^i_{t+dt}, v_{t+dt}) + 1_{\text{nodeath}} V(n^i_{t+dt}, v_{t+dt})) \right] \\ &= \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + (1-\rho dt)(\zeta dt \times 0 + (1-\zeta dt) V(n^i_{t+dt}, v_{t+dt})) \right] \end{aligned}$$

The Hamilton-Jacobi-Bellman equation is

$$\begin{aligned} &(\rho+\zeta)V(n_t^i,\omega_t) = \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t \left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \zeta \times 0 + V_t + V_n(n_t^i,\omega_t) dn_t^i + \frac{1}{2} V_{nn}(n_t^i,\omega_t) \langle dn_t^i, dn_t^i \rangle \right] \\ &\rho v_t + \omega_t \frac{(n_t^i)^{1-\gamma}}{1-\gamma} - 0 \\ &= \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E} \left\{ \frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \frac{\dot{w}_t}{\rho+\zeta} \frac{(n_t^i)^{1-\gamma}}{1-\gamma} + \dot{v}_t + \omega_t \frac{(n_t^i)^{1-\gamma}}{\rho+\zeta} \frac{dn_t^i}{n_t^i} \frac{1}{dt} - \omega_t \frac{\gamma}{2} \frac{(n_t^i)^{1-\gamma}}{\rho+\zeta} \left\langle \frac{dn_t^i}{n_t^i}, \frac{dn_t^i}{n_t^i} \right\rangle \frac{1}{dt} \right\}. \end{aligned}$$

Note that the HJB is essetianlly the same as the wealth reset model, except that the utility after death is zero while the utility after wealth reset is $V(\bar{n}_t, v_t)$. The first order conditions are the same, though the solution for v_t is different. Hence, we obtain the same equilibrium in the wealth reset model.

Below, we discuss household's decision and the relevant transversality conditions in a different perspective. Let n_t^i denote the wealth of agent i at time t. For $t > \tau^i$, i.e., after the agent's death, we assume that n_t^i denotes the wealth following the agent's consumption and investment plan if it were still alive. We denote $\check{m}_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}$ as the hypothetical marginal utility if the agent were still alive.

We denote m_t^i as the agent's actual marginal utility, which is zero after death:

$$m_t^i := \begin{cases} \check{m}_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}, & death_t^i = 0, \\ 0, & death_t^i = 1. \end{cases}$$

We obtain

$$dm_t^i = \begin{cases} -\left[\rho + \gamma g_t - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2\right] m_t^i dt - \gamma\bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i, & t < \tau^i, \\ -m_{\tau^i-}^i, & t = \tau, \\ 0 & t > \tau^i. \end{cases}$$

Aggregate TVC For long-lived asset, e.g. government bond at $t < \tau^i$,

$$\begin{aligned} d(m_t^i b_t) &= b_t dm_t^i + m_t^i/P_t dB_t + m_t^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\ &= b_t\left(-(r_t^f - \zeta)m_t^i dt - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i\right) \\ &+ i_t b_t m_t^i dt - s_t m_t^i dt + m_t^i b_t(dr_t^B - i_t dt) \\ &= -m_t^i s_t dt + m_t^i \zeta b_t dt - m_t^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i. \end{aligned}$$

At $t = \tau^i$, $d(m_t^i b_t) = -m_{\tau^i-}^i b_{\tau^i}$. At $t \geq \tau^i$, $d(m_t^i b_t) = 0$. Integrate from $t = 0$ to $t = T$ and take expectation. Recall that τ^i is independent of n_t^i and b_t, and

$$\mathbb{E}_0\left[1_{\{\tau^i \leq u\}}\right] = 1 - e^{-\zeta u}.$$

Hence,

$$\begin{aligned} m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right] \\ &= (1 - e^{-\zeta T})\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \leq T\right] + e^{-\zeta T}\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right]. \end{aligned}$$

The first term is computed as

$$\begin{aligned}
&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \le T\right] \\
=&\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] - \mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] + \mathbb{E}_0\left[\check{m}_{\tau^i-}^i b_{\tau i} \middle| \tau^i \le T\right],
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] &= \mathbb{E}_0\left[\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t \, dt \middle| \tau^i\right] \middle| \tau^i \le T\right] \\
&= \frac{1}{1-e^{-\zeta T}} \int_0^T \zeta e^{-\zeta u} \mathbb{E}_0\left[\int_0^u \check{m}_t^i s_t \, dt\right] du \\
&= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \left(\int_0^u \zeta e^{-\zeta u} \check{m}_t^i s_t \, dt\right) du\right] \\
&= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \left(\int_t^T \zeta e^{-\zeta u} \check{m}_t^i s_t \, du\right) dt\right] \\
&= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T}) \check{m}_t^i s_t dt\right],
\end{aligned}$$

and similarly

$$\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \zeta (e^{-\zeta t} - e^{-\zeta T}) \check{m}_t^i b_t dt\right], \\
\mathbb{E}_0\left[\check{m}_{\tau^i-}^i b_{\tau i} | \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta u} \check{m}_u^i b_u du\right].
\end{aligned}$$

The second term is computed as

$$\begin{aligned}
&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right] \\
=&\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - \mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + \mathbb{E}_0\left[\check{m}_T^i b_T\right].
\end{aligned}$$

Finally,

$$\begin{aligned}
m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right] \\
&= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i s_t dt\right] + \mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta T}\check{m}_t^i b_t dt\right] \\
&+ e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + e^{-\zeta T}\mathbb{E}_0\left[\check{m}_T^i b_T\right] \\
&= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t}\check{m}_t^i) s_t dt\right] + \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i) b_T\right].
\end{aligned}$$

Recall that $\check{m}_t^i$ is the marginal utility if the agent were alive at time t. The TVC term for this aggregate asset is the discounted asset value $m_T^i b_T$ times the probability of surviving until T, which is $e^{-\zeta T}$. The aggregate TVC holds if and only if $\hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 > 0$.

Individual TVC Similarly, for individual wealth, we obtain

$$\begin{aligned}
d(m_t^i n_t^i) &= n_t^i dm_t^i + m_t^i dn_t^i + dm_t^i dn_t^i \\
&= n_t^i(-(r_t^f - \zeta)m_t^i dt - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i) \\
&+ m_t^i(-c_t^i dt + (r_t^f + \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2)n_t^i dt + \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t n_t^i d\tilde{Z}_t^i) - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 m_t^i n_t^i dt \\
&= -c_t^i m_t^i dt + \zeta m_t^i n_t^i dt
\end{aligned}$$

for $t < \tau^i$, i.e., if the agent were still alive. Following the same steps above, we obtain

$$m_0^i n_0^i = \mathbb{E}_0\left[\int_0^T (e^{-\zeta t}\check{m}_t^i) c_t^i dt\right] + \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i)\check{n}_T^i\right].$$

The TVC for individual household's wealth is

$$\lim_{T\to\infty} \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i\check{n}_T^i)\right] = 0.$$

Again, the analysis is the same as that in A.1 without wealth reset shock, but the subjective discount rate is augmented by ζ.

A.10 Calibration

Table A.1: Parameter Values.

Variable	Notation	Value	Notes
Baseline Model with Permanent Idiosyncratic Risk			
risk aversion	γ	2	
subjective discount rate	ρ	0.02	
capital adjustment cost	ϕ	8.1	
Depreciation rate	δ	0.055	
Government spending	$\mathfrak{g}$	0.15	
real growth rate	g	0%	Targeted by choosing a
idiosyncratic risk variance	$\tilde{\sigma}$	0.54	From Brunnermeier et al. (2022)
share of undiversifiable risk	$\bar{\chi}$	0.3	From Brunnermeier et al. (2022)
share of equity	ϑ	0.18	From Brunnermeier et al. (2022)
Model Variant with Transitory Idiosyncratic Risk			
weak wealth reset intensity	ζ	2.5%	
moderate wealth reset intensity	ζ	3.8%	
strong wealth reset intensity	ζ	5.4%	

B Model with Aggregate Risk

In this section, we consider a model with aggregate risk. We make two modifications to the model in the main text. First, following Brunnermeier et al. (2022), we assume that the production process is subject to an additional aggregate risk dZ_t that is common to all households. The law of motion for the wealth of household i is given by

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t^m dZ_t + (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i$$

which additionally loads on the aggregate risk dZ_t.

Second, for tractability, households have log preference, i.e., $\gamma = 1$, which allows us to solve the model in closed form.

Other assumptions are identical to those in the main text. We solve the model using the same steps. In this appendix, we report the key results in this setting, and confirm our claim that introducing aggregate risk does not change the main results of the paper.

Permanent Idiosyncratic Risk. First, we consider the case with permanent idiosyncratic risk. Under the wealth distribution perspective, the transversality condition for aggregate wealth can be expressed as

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T\right] = \bar{n}_0 \lim_{T\to\infty} \mathbb{E}_0 \left[e^{-\rho T} \int_0^1 \frac{\eta_0^i}{\eta_T^i} di\right].$$

Under the r minus g perspective, we can further write the discounted aggregate wealth as

$$\frac{d(\bar{m}_t \bar{n}_t)}{\bar{m}_t \bar{n}_t} = [- \underbrace{r_t^f}_{\text{risk-free rate}} - \underbrace{(\sigma_t^m)^2}_{\text{risk premium for aggregate risk}} + \underbrace{g_t}_{\text{growth rate}}]dt,$$

with the following $r - g$ condition

$$(r_t^f + (\sigma_t^m)^2) - g_t = \rho - \bar{\chi}^2 (1 - \vartheta_t)^2 \tilde{\sigma}_t^2.$$

Transitory Idiosyncratic Risk. Similarly, in the case with wealth reset shock, the TVC for aggregate wealth can be expressed as

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T\right] = \lim_{T\to\infty} e^{-\rho T} \bar{n}_0 \mathbb{E}_0 \left[\int_0^1 \frac{1}{\eta_T^i} di\right].$$

The TVC term for aggregate wealth has the following dynamics:

$$d[\bar{m}_t \bar{n}_t] = -[\rho - \bar{\chi}^2 (1 - \vartheta_t)^2 \tilde{\sigma}_t^2 + \zeta] \bar{m}_t \bar{n}_t dt + \zeta e^{-\rho t} \bar{n}_0 dt$$

which implies the following r minus g condition for the aggregate wealth:

$$(r_t^f + (\sigma_t^m)^2) - g_t = \rho - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 + \zeta.$$

All these results are special cases of our main results in the paper, which are obtained when we set $\gamma = 1$. In this case with log utility, the magnitude of the aggregate risk does not enter the transversality condition, because it does not affect the relative distribution of household wealth.

C Model Extension with Borrowing Constraints

In this appendix, we provide more details for the heterogeneous-agent models with borrowing constraints, which we discussed in Section 5.3.

C.1 Intertemporal Government Budget Condition

First, we show that with borrowing constraint, the Euler equation becomes an inequality:

$$\mathbb{E}_t\left[\frac{m_{t+1}^i}{m_t^i}\exp(r_t)\right] \leq 1.$$

Suppose the household maximizes lifetime utility:

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0\left[\exp(-\rho t)u(c_t)\right],$$

such that

$$y_t + b_{t-1}\exp(r_{t-1}) \geq c_t + b_t,$$
$$b_t \geq 0.$$

The Lagrangian is

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\exp(-\rho t) u(c_t) + \zeta_t (y_t + b_{t-1} \exp(r_{t-1}) - c_t - b_t) + \mu_t b_t \right].$$

The first-order conditions are

$$\exp(-\rho t) u'(c_t) = \zeta_t,$$
$$\zeta_t - \mu_t = \mathbb{E}_t[\zeta_{t+1} \exp(r_t)],$$

which implies

$$\mathbb{E}_t \left[\frac{\exp(-\rho) u'(c_{t+1})}{u'(c_t)} \exp(r_t) \right] = 1 - \frac{\mu_t}{u'(c_t)} \leq 1.$$

with $\mu_t > 0$ if and only if $b_t = 0$, i.e., the borrowing constraint is binding.

Next, We evaluate government bond in the presence of borrowing constraint. Recall that

$$b_t \exp(r_t) = s_{t+1} + b_{t+1}.$$

Multiplying both sides by m^i_{t+1}/m^i_t and take expectation to obtain

$$b_t \geq b_t \mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} \exp(r_t) \right] = \mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} s_{t+1} \right] + \mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} b_{t+1} \right].$$

The inequality follows $\mathbb{E}_t \left[\frac{m^i_{t+1}}{m^i_t} \exp(r_t) \right] \leq 1$, which we proved above. Similarly,

$$b_{t+1} \geq \mathbb{E}_{t+1} \left[\frac{m^i_{t+2}}{m^i_{t+1}} s_{t+2} \right] + \mathbb{E}_{t+1} \left[\frac{m^i_{t+2}}{m^i_{t+1}} b_{t+2} \right],$$

which implies

$$b_t \geq \sum_{k=1}^{2} \mathbb{E}_t \left[\frac{m_{t+k}^i}{m_t^i} s_{t+k} \right] + \mathbb{E}_t \left[\frac{m_{t+2}^i}{m_t^i} b_{t+2} \right].$$

In this way, we can iterate forward to obtain

$$b_t \geq \sum_{k=1}^{\infty} \mathbb{E}_t \left[\frac{m_{t+k}^i}{m_t^i} s_{t+k} \right] + \lim_{k \to \infty} \mathbb{E}_t \left[\frac{m_{t+k}^i}{m_t^i} b_{t+k} \right].$$

C.2 Consumption Lower Bound in Bewley Model

Here we feature a simple model without aggregate growth or aggregate uncertainty but with uninsured idiosyncratic risk. There is a continuum of agents on $[0, 1]$. Agent i get endowment y_t^i each period, with $y_t^i \geq \underline{y} > 0$. Aggregate (after-tax) endowment is $y = \int_0^1 y_t^i \, di$. Agents can invest into risk-free bonds subject to borrowing constraints, i.e., $b_t^i \geq 0$. The supply of bond is fixed, $b = \int_0^1 b_t^i \, di$. Under steady state, interest rate is constant r. Agents maximize lifetime utility

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\exp(-\rho t) u(c_t^i) \right].$$

We assume that the utility function satisfies the Inada condition, i.e., $\lim_{c \to 0} u'(c) = \infty$ and $u'(c)$ decreases in c.

In this setting, we show that consumption has a lower bound $\underline{c} > 0$. Denote $z_t^i = b_{t-1}^i \exp(r) + y_t^i$ as the resource available at time t. $z_t^i \geq \underline{y}$. The value function is

$$V(z_t^i, y_t^i) = \max_{c_t^i} \{ u(c_t^i) + \exp(-\rho) \mathbb{E}_t [V(z_{t+1}^i, y_{t+1}^i)] \}.$$

If y_t^i is i.i.d., the expectation term on the right-hand side does not depend on y_t^i, and we can rewrite value function as $V(z_t^i)$, i.e., a function of available resources. In the case, Aiyagari (1994) proves that the consumption has a positive lower bound, at which household consumes all available

resources. The Euler equation is

$$u'(c_t^i) \geq \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)], \tag{C.1}$$

with equality obtained when $b_t^i > 0$.

Next, we show that the consumption has a positive lower bound. Assume that the marginal value of wealth $V_z(z_t^i, y_t^i)$ is decreasing in wealth z_t^i for any given income y_t^i. Note that

$$V_z(z_{t+1}^i, y_{t+1}^i) \leq V_z(\underline{y}, y_{t+1}^i) \leq \overline{V}_z := \sup_{y_t^i} V_z(\underline{y}, y_{t+1}^i).$$

The first inequality follows that $z_{t+1}^i \geq \underline{y}$ no matter how much agent saves or consumes at time t. The second inequality assumes that given a positive level of resource available ($\underline{y}$), the marginal gain of extra resource $V_z(\underline{y}, y_{t+1}^i)$ is always finite across every possible y_{t+1}^i. Two special cases are (1) i.i.d. idiosyncratic shock case in Aiyagari (1994), where y_{t+1}^i is i.i.d., and (2) finite space Markov chain in Krueger and Lustig (2010), where y_{t+1}^i has finite numbers of value, and the upper bound of $V_z(\underline{y}, y_{t+1}^i)$ exists.

Now suppose to the contrary that c_t^i does not have a positive lower bound. On the one hand, there exists $\underline{c} > 0$, such that

$$u'(c_t^i) > \exp(r-\rho)\overline{V}_z \geq \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)], \quad \forall c_t^i < \underline{c}.$$

On the other hand, since c_t^i does not have a positive lower bound. There must be some states where $c_t^i < \underline{y}$. Note that since $c_t^i + b_t^i \geq \underline{y}$, $c_t^i < \underline{y}$ must imply $b_t^i > 0$ and equality in the Euler equation (C.1), i.e.,

$$u'(c_t^i) = \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)].$$

By contradiction, c_t^i must have a positive lower bound.

Intuitively, $c_t^i < \underline{y}$ implies that the agent must be saving and hence the borrowing constraint is not binding. In this case, the marginal utility from consumption at time t must equate the marginal utility from saving to the next period. Since the resource available at $t+1$ is bounded from below by $\underline{y}$, the marginal utility of saving extra resource from t to $t+1$ is bounded from above. The marginal utility of consumption cannot be too high at time t, otherwise the agent could obtain a higher lifetime utility by saving less and consume more at time t. This provides a lower bound for consumption.

C.3 Townsend and Woodford Model

The government budget constraint is

$$d(1+r) = \tau + d,$$

which implies

$$d = \frac{\tau}{r(1+r)}$$

for $r > 0$ cases. Since household purchase government bonds only at good times, we obtain the following Euler (in)equalities:

$$\rho \frac{u'(\underline{c})}{u'(\bar{c})} = (1+r)^{-1},$$
$$\rho \frac{u'(\bar{c})}{u'(\underline{c})} \leq (1+r)^{-1},$$

which imply

$$\mathbb{E}_0 \left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right] \leq \mathbb{E}_0 \left[\sum_{t=1}^{T} (1+r)^t \tau \right].$$

In the meantime,

$$0 \leq \lim_{T\to\infty} \mathbb{E}_0 \left[\rho^T \frac{u'(c_{A,T})}{u'(c_{A,0})} d \right] \leq \lim_{T\to\infty} \mathbb{E}_0 \left[\rho^T \frac{u'(\underline{c})}{u'(\bar{c})} d \right] = \lim_{T\to\infty} \rho^{T-1}(1+r)^{-1} d = 0$$