Hotelling Meets Keynes: Aggregate Adjustment with Spatial Competition and Nominal Rigidity*

Stephanie Schmitt-Grohé† Martín Uribe‡

March 20, 2025
Preliminary and Incomplete

Abstract

This paper embeds a circular Hotelling model of spatial competition into a new-Keynesian model with staggered price setting. The resulting framework provides micro-foundations for a cost-push shock, taking the form of random variations in transportation costs. An increase in transportation costs raises price markups and is contractionary and inflationary. Spatial frictions also have consequences for the propagation of aggregate demand disturbances (including monetary shocks), as they dampen their output effects and amplify their inflationary effects.

JEL Classification: E31, E32, L13.
Keywords: Hotelling model, new-Keynesian model, spatial frictions, market power, cost-push shocks.

*We thank for comments seminar participants at the University of Wisconsin.
†Columbia University, CEPR, and NBER. E-mail: stephanie.schmittgrohe@columbia.edu.
‡Columbia University and NBER. E-mail: martin.uribe@columbia.edu.

1 Introduction

Markup shocks are elusive objects. Estimated new-Keynesian models attribute the vast majority of the variance of inflation to exogenous variations in the intratemporal elasticity of substitution (the reciprocal of the markup) that are unrelated to other standard sources of aggregate fluctuations, such as monetary, preference, or technology shocks (Smets and Wouters, 2007). A number of authors have noted that this is an unappealing property of the mainstream framework for macroeconomic policy evaluation (King and Watson, 2012; Del Negro, Giannoni, and Schorfheide, 2015). A natural question, then, is what are the microfoundations of these random movements in the elasticity of substitution. This paper contributes to addressing this question by adding a spatial dimension to the new-Keynesian framework. Specifically, it embeds a circular Hotelling model of spatial competition into a new-Keynesian model with staggered price setting. The resulting framework gives rise to a cost-push shock, taking the form of random variations in transportation costs.

In the proposed model, firms compete with other firms along two dimensions. As in the new-Keynesian model, they compete in a monopolistic fashion with other firms producing imperfectly substitutable goods and located in the same marketplace. But they also compete with firms producing the identical good that are located in geographically distant markets. Along this dimension, product differentiation is determined by distance. The price elasticity of the demand function faced by an individual firm is then influenced by two features of the economic environment: First, transportation costs introduce a wedge between the price that is relevant for the firm and the price that is relevant for the consumer. And second, when the firm increases the price of the good it produces, it loses sales not only to firms located in the same market (as in the new-Keynesian framework), but also to firms located elsewhere. Thus, the intratemporal elasticity of substitution ceases to be the single determinant of the price elasticity of demand. Other factors, such as transportation costs and the form of competition with firms in other markets become relevant determinants of market power.

The model economy features a continuum of households evenly distributed around a

circle. Households belong to an extended family that centrally assigns the same level of consumption of a composite good to every member. The composite good is made of a continuum of varieties. Households face transportation costs to reach product markets that are proportional to the distance separating them from the market. Each household chooses how much to consume of each good variety. On the supply side, there is a fixed finite number of shopping centers located evenly over the circle. In each shopping center there is a continuum of firms producing and selling the different varieties of goods. Nominal prices are sticky à la Calvo.

To understand how variations in transportation costs can induce endogenous changes in markups, imagine a firm whose market extends for some distance to the left and to the right of its location. Consider two types of households. One living in the interior of the firm's market and the other at the border of the firm's market with that of a spatial competitor. The firm producing variety i in period t sells it at price P_{it}. However, for a household living in the interior but at a certain distance from the firm, the relevant price is $P_{it} + TC_t$, where TC_t denotes the cost of transporting the good. Thus, transportation costs represent a wedge between the price that is relevant for the firm and the price that is relevant for the household. Suppose the demand function is isoelastic in the price relevant for the household. Specifically, suppose it is of the form $A(P_{it}+TC_t)^{-\eta}$. From the perspective of the firm, the price elasticity of demand is not η, but $\eta \times \frac{P_{it}}{P_{it}+TC_t}$. It follows that an increase in transportation costs makes the demand function less elastic. In turn, faced with a less elastic demand function, the firm raises its markup of price over marginal cost and reduces the supply of goods. The intuition is that when transportation costs increase, customers living at a non-zero distance from the firm become less sensitive to changes in the price since the overall cost they face becomes more dominated by transportaton costs. The firm recognizes this and understands that it can increase its price without losing as many customers as it would if transportation costs had not gone up.

Consider now the effect of a change in the price on consumers living at themarket's

border. The size of the firm's market, that is, the distance between the location of the firm and the market's border in eigher direction, naturaly depends on the difference between the firm's price and the price of its spatial competititor . The higher the the firm's price relative to that of its spatial competitor, the smaller the size of its market will be. However the price sensitivity of the firm's market size falls with transportation costs. The reason is that for customers living near the border, it takes a larger change in price to switch seller the more costly it is to commute. It follows that when transportation costs increase, the extensive margine of demand also becomes less elastic.

Thus, when transportation costs increase, the firm gains market power both because the demand of customers in the interior of the market becomes less elastic and because the size of the market becomes less sensitive to price changes.

In general equilibrium, the mechanism just described gives rise to contractions in output in response to increases in transportation costs that are mediated by increases in markups. With sticky prices, transportation costs are passed on to consumer prices, causing inflation. Thus, transportation costs generate stagflation. It is in sense that spatial frictions offers microfoundations to cost-push shocks. We show that if the central bank follows a Taylor type monetary policy rule, faced with a recession and inflation cause by an increase in transportation costs, leans on the side of price stability an tightens money market conditions. As a result, monetary policy exacerbates the contractionary effects of the cost-push shock.

The spatial dimension of the model also has consequences for the propagation of aggregate demand shocks, such as monetary policy shocks and preference shocks. We find that spatial frictions dampen the output effects of these shocks and amplify their inflationary effects. The intuition behind this result is that because spatial frictions reduces the price elasticity of demand firms have more leeway to adjust prices in response to shocks without significantly altering demand.

The paper is related to two strands of literature. There is a large number of papers that incorporate cost-push shocks into the new-Keynesian model. Pioneer works in this area

are Clarida, Galí, and Gertler (1999) and Woodford (2003). Smets and Wouters (2007) and Del Negro, Gianonni, and Schorfheide (2015) estimate that cost-push shocks taking the form of exogenous variations in the elasticity of substitution explain more than two thirds of observed movements in inflation. King and Watson (2012) argue that this finding represents a weakness of the new-Keynesian framework. Christiano, Eichenbaum, and Evans (2005) and Ravenna and Walsh (2006) introduce a cost channel into a new-Keynesian model by assuming that marginal costs are increasing in the nominal interest rate. We contribute to this literature by microfounding cost-push shocks that arise from the realistic assumption that consumers and markets are separated by distance broadly interpreted.

The second branch of literature to which the paper is related originates in the seminal work of Hotelling (1929) and is concerned with spatial frictions as a source of market power. Hotelling's model assumes that firms are distributed along a line instead of a circle and that individual demands for goods are perfectly inelastic. A circular version of Hotelling's model was introduced by Salop (1979). Barro (2024) introduces a price elastic demand within the Salop framework. He finds that if transportation costs are small relative to the price of the good, then the markup is unrelated to the price elasticity of demand. A specific contribution of the present paper to this line of literature is to show that shopping costs represent a non-negligible fraction of the price. Specifically, using data from the American Time Use Survey and model restrictions, we find that shopping costs represent 13 percent of the product price. As a result, for plausible calibrations of the elasticity of substitution, the equilibrium markup is not delinked from the price elasticity. More importantly, the present paper departs from this literature in focus and scope. In particular, the models cited above are static and in partial equilibrium, and their focus is to understand the level of the equilibrium markup. By contrast, the present model embeds the Hotelling model in a dynamic, stochastic, general equilibrium framework with nominal rigidity to show that random variations in transportation costs provide microfoundations to cost-push shocks. In this regard, the key element of the Hotelling model is not so much the level of the markup,

but its elasticity with respect to transportation costs.

The remainder of the paper is organized as follows. Section 2 embeds a circular model of spatial competition in a multi-product general equilibrium setting with nominal rigidity. Section 3 shows how to aggregate the model and presents a recursive representation of its equilibrium conditions. Section 4 characterizes the equilibrium markup and its elasticity with respect to transportation costs. Section 5 characterizes the equilibrium dynamics implied by transportation cost shocks, and shows that they are akin to those implied by cost-push shocks. Section 6 shows that spatial competition dampens the output effect of demand shocks and amplifies their inflationary effects. Finally, section 7 concludes.

2 The Model

The key ingredients of the model are a spatial friction in the form of a circular space that is costly to traverse and nominal price rigidity.

2.1 Households

The economy is populated by a continuum of households of measure 1 evenly distributed over the circumference of a circle with perimeter equal to 1. To facilitate aggregation, we assume that these households belong to a large family, which pools income and assigns the same level of consumption and leisure to all its members. Preferences of the family are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \chi \frac{h_t^{1+\gamma}}{1+\gamma} \right], \tag{1}$$

where c_t denotes consumption of each household, h_t denotes hours worked by each household, ξ_t is an exogenous and stochastic preference shock, and $\beta \in (0,1)$ and $\sigma, \chi, \gamma > 0$ are parameters.

The family is subject to the budget constraint

$$P_t^c c_t + \frac{B_{t+1}}{1+i_t} + T_t = W_t h_t + \Phi_t + B_t, \tag{2}$$

where P_t^c denotes the unit cost of the consumption good to the household, B_{t+1} denotes units of a nominal bond purchased in period t and paying the interest rate i_t in period $t+1$, T_t denotes nominal lump-sum taxes, W_t denotes the nominal wage rate, and Φ_t denotes nominal profits received from firms.

The family chooses processes $\{c_t, h_t, B_{t+1}\}_{t=0}^{\infty}$ to maximize the utility function (1) subject to the budget constraint (2) and to some borrowing limit that prevents it from engaging in Ponzi schemes, taking as given profits Φ_t, lump-sum taxes T_t, prices P_t^c, interest rates i_t, wages W_t, the stochastic process followed by the preference shock ξ_t, and the initial condition B_0. Letting $\beta^t \lambda_t / P_t^c$ denote the Lagrange multiplier associated with the budget constraint (2), the first-order conditions of the family's optimization problem are

$$e^{\xi_t} c_t^{-\sigma} = \lambda_t, \tag{3}$$

$$\chi c_t^{\sigma} h_t^{\gamma} = \frac{W_t}{P_t^c}, \tag{4}$$

and

$$\lambda_t = \beta(1+i_t) E_t \frac{\lambda_{t+1}}{1+\pi_{t+1}^c}, \tag{5}$$

where $\pi_t^c \equiv P_t^c / P_{t-1}^c - 1$ denotes the growth rate of P_t^c in period t.

2.2 The Demand for Variety i in Location δ

There are N shopping centers in the economy, each selling a continuum of goods indexed by $i \in [0,1]$. Consumers are price takers. As will become clear later, in equilibrium, households make all of their purchases at the closest shopping center. Given the level of consumption, c_t, assigned by its family, a household living at a distance δ of the shopping center it patronizes

chooses the consumption of varieties $i \in [0,1]$, denoted $c_{\delta it}$, to minimize total expenditure,

$$\int_0^1 (P_{it} + \tau_t \delta W_t) c_{\delta it} di,$$

subject to the aggregation technology

$$c_t = \left[\int_0^1 c_{\delta it}^{1-1/\eta} di \right]^{\frac{1}{1-1/\eta}}, \tag{6}$$

where P_{it} denotes the nominal price of variety i, τ_t represents the transportation time per unit of good for each unit of distance traveled, and the parameter $\eta > 1$ denotes the elasticity of substitution across varieties.

We assume that transportation time can be buffeted by random shocks. Thus, τ_t is exogenous and stochastic. This shock is a new type of disturbance in the new-Keynesian framework and embodies random fluctuations in spatial frictions. This kind of frictions can have a variety of natures. For example, they could encompass factors that affect the time required for consumers to access markets, such as disruptions or technological changes in transportation or variations in the quality and cost of access to e-commerce platforms. As we will see shortly, transportation cost shocks give rise to endigenous variations in markups and trigger stagflationary dynamics akin to cost-push shocks.

The household's cost minimization problem delivers the following demand for individual varieties:

$$c_{\delta it} = c_t \left(\frac{P_{it} + \tau_t \delta W_t}{P_{\delta t}} \right)^{-\eta}, \tag{7}$$

where $P_{\delta t}$ is given by

$$P_{\delta t} \equiv \left[\int_0^1 (P_{it} + \tau_t \delta W_t)^{1-\eta} di \right]^{\frac{1}{1-\eta}} \tag{8}$$

and represents the minimum cost of one unit of the composite consumption good for a household located at a distance δ of the shops it patronizes. Thus, $P_{\delta t}$ is the cost of consumption relevant for a household living at a distance δ of the location where it shops for goods.

Equation (7) suggests that the elasticity of substitution across varieties falls with distance. To see this, consider any pair of varieties, i and j. The relevant relative price for a consumer located at a distance δ of the nearest shopping center is $(P_{it} + \tau_t \delta W_t)/(P_{jt} + \tau_t \delta W_t)$. As δ increases, the transportation component in both prices gains relative importance, and, as a result, a given increase in the relative price P_{it}/P_{jt} has a smaller effect on the relative demand $c_{\delta it}/c_{\delta jt}$. This means that the spatial dimension of competition dampens the importance of competition across the variety dimension. This insight builds on traditional treatments of the Hotelling model that assume a single type of good and is important because it shows that spatial competition mitigates the misallocation due to price dispersion originating from price stickiness.

2.3 Firms

The N shopping centers are evenly distributed on the circle. In each commercial center there is a continuum of firms indexed by $i \in [0, 1]$, each producing a different variety of good. Firms compete monopolistically with firms located in the same shopping center producing other varieties. In addition, firms face Bertrand competition with firms located in bordering markets that produce the same variety of good. Let b_{it} be the distance from the location of firm i to the border of its market. The distance b_{it} can change endogenously with the price set by the firm. Specifically, let P_{it}^{sc} denote the price posted by a spatial competitor of the firm producing variety i. Then, at the border, P_{it} must satisfy the no-arbitrage condition

$$P_{it} + \tau_t b_{it} W_t = P_{it}^{sc} + \tau_t (1/N - b_{it}) W_t.$$

This condition says that at the border between two markets the price of a given variety of good including transportation costs must be the same in both markets. Thus, the customer living at the border is indifferent between shopping in either market. If the firm increases its price above this level, the customer chooses to buy the good in the bordering market. The

firm takes this restriction into account in deciding what price to charge.

Firm i produces variety i with the production technology $e^{z_t}h_{it}$, where h_{it} is employment at firm i and z_t is an aggregate productivity shock.

Prices are sticky à la Calvo. Each period, with probability $1-\theta$, firms producing variety i in all locations get to set their price optimally. At the posted prices, firms must satisfy demand at all times.

The demand faced by a firm producing variety i is the integral of the demands from households located at a distance less than or equal to b_{it}, that is, $2\int_0^{b_{it}} c_{\delta it}d\delta$. If the last time firm i could reoptimize prices was in period t, then in period $t+s$, for $s \geq 0$, its price is still P_{it}. Using the demand for good i by a household located at a distance δ from the shopping center given in equation (7), the total demand firm i faces in period $t+s$ is equal to

$$\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s}) \equiv 2c_{t+s}\int_0^{b_{it+s}} \left(\frac{P_{it}+\tau_{t+s}\delta W_{t+s}}{P_{\delta t+s}}\right)^{-\eta} d\delta. \tag{9}$$

To alleviate the notation, we omit in the arguments of the function $\mathcal{D}(\cdot,\cdot;\cdot)$ the variables W_{t+s} and $P_{\delta t+s}$. The profit maximization problem of a firm i that gets to change its price in period t is then given by

$$\max_{\{P_{it},\{b_{it+s},h_{it+s}\}_{s\geq 0}\}} E_t \sum_{s=0}^{\infty}(\beta\theta)^s \frac{\lambda_{t+s}}{P^c_{t+s}}\left[P_{it}\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s}) - W_{t+s}h_{it+s}\right]$$

subject to

$$e^{z_{t+s}}h_{it+s} \geq \mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s}) \tag{10}$$

and

$$P_{it} + \tau_{t+s}b_{it+s}W_{t+s} = P^{sc}_{it} + \tau_{t+s}\left(\frac{1}{N} - b_{it+s}\right)W_{t+s}. \tag{11}$$

The choice of the stochastic discount factor, $\beta^s\lambda_{t+s}/P^c_{t+s}$, reflects the fact that profits belong to the central family.

Letting $(\beta\theta)^s\lambda_{t+s}mc_{it+s}$ denote the Lagrange multiplier associated with constraint (10),

the first-order conditions associated with this problem are (10) holding with equality, (11),

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left[\frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{P^c_{t+s}} + \left(\frac{P_{it}}{P^c_{t+s}} - mc_{it+s} \right) \left(\mathcal{D}_1(P_{it}, b_{it+s}; \tau_{t+s}) + \mathcal{D}_2(P_{it}, b_{it+s}; \tau_{t+s}) \frac{\partial b_{it+s}}{\partial P_{it}} \right) \right] = \tag{12}$$

and

$$mc_{it} e^{z_t} = \frac{W_t}{P^c_t}, \tag{13}$$

where $\mathcal{D}_j(\cdot,\cdot;\cdot)$ for $j = 1, 2$ denotes the partial derivative of $\mathcal{D}(\cdot,\cdot;\cdot)$ with respect to its jth argument and

$$\frac{\partial b_{it+s}}{\partial P_{it}} = \frac{-1}{2\tau_{t+s} W_{t+s}}. \tag{14}$$

This last expression reflects the fact that the firm internalizes that when it changes the price, it affects the extensive margin of its market. First-order condition (13) implies that mc_{it} is independent of i, so we drop the subscript i from this variable.

Equation (12) is the core pricing condition for a firm that gets to reset prices in period t. It is of interest to compare it with its counterpart in the standard new-Keynesian model without spatial frictions. It can be shown that equation (12) can be written in the same form as in a standard new-Keynesian model, that is, as a present discounted value of differences between the firm's marginal revenue and its marginal cost, weighted by the size of its demand. The key difference between the two formulations is that while in the standard new-Keynesian model the elasticity of demand is constant (and equal to η), in the model with spatial frictions it is time varying. Formally, Appendix A shows that equation (12) can be rewritten as

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \tilde{\eta}_{it+s} \mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s}) \left[\frac{P_{it}}{P^c_{t+s}} \left(1 - \frac{1}{\tilde{\eta}_{it+s}} \right) - mc_{t+s} \right] = 0. \tag{15}$$

The expression in square brackets is the difference between marginal revenue and marginal cost and $\tilde{\eta}_{it+s}$ is the price elasticity of demand faced by a firm i in period $t+s$ that last

reoptimized its price in period t and is given by

$$\tilde{\eta}_{it+s} \equiv -\frac{\mathcal{D}_1(P_{it}, b_{it+s}; \tau_{t+s})P_{it}}{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})} - \frac{\mathcal{D}_2(P_{it}, b_{it+s}; \tau_{t+s})\frac{\partial b_{it+s}}{\partial P_{it}}P_{it}}{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}. \tag{16}$$

The difference between first-order condition (15) and the corresponding first-order condition in a new-Keynesian model without spatial competition is that in the latter the elasticity of demand is constant (NK models without markup shocks) or exogenous (NK models with markup shocks), whereas in the present formulation it is an endogenously determined variable.

The elasticity of demand $\tilde{\eta}_{it+s}$ has an intensive margin component, given by $\frac{\mathcal{D}_1(P_{it}, b_{it+s}; \tau_{t+s})P_{it}}{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}$, and an extensive margin component, given by $\mathcal{D}_2(P_{it}, b_{it+s}; \tau_{t+s})\frac{\partial b_{it+s}}{\partial P_{it}}\frac{P_{it}}{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}$. The intensive margin component is the standard notion of a demand elasticity. The difference is that in the present formulation, this component varies with market conditions. For example, an increase in wages or in the time it takes to travel a unit of distance, τ_t, raises the cost of shopping and increases the market power of the firm over customers that are relatively close to its location relative to its nearest spatial competitors. The extensive margin component, which also varies with market conditions, increases the overall price elasticity. This term captures the fact that when the firm increases its price it loses customers to spatial competitors located in bordering markets. As we will see, in general, the elasticity $\tilde{\eta}_{it+s}$ can be larger or smaller than η, depending on the sizes of the intensive- and extensive-margin components.

2.4 The Government

We assume that the government follows a Taylor-type interest rate feedback rule with smoothing of the form

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_t}{1+\pi}\right)^{\alpha_\pi} \left(\frac{y_t}{y}\right)^{\alpha_y} e^{\nu_t}, \tag{17}$$

where π_t denotes inflation and y_t denotes the measure of aggregate activity to which the central bank responds, both to be defined shortly, ν_t is a monetary shock, α_π and α_y are parameters, and variables without a time subscript refer to steady-state values.

The government's budget constraint is of the form

$$T_t + \frac{B_{t+1}}{1+i_t} = B_t.$$

We assume that fiscal policy is Ricardian in the sense that lump-sum taxes, T_t, adjust endogenously to ensure government solvency independently of the path of the price level.

3 Aggregation and Equilibrium

To facilitate aggregation, we linearize the object $\left(\frac{P_{it}+\tau_t\delta W_t}{P_{\delta t}}\right)^{-\eta}$ around $\tau_t = 0$. We show in Appendix B that this linearization yields a demand function for variety i at a location situated at a distance δ of a nearest shopping center of the form

$$c_{\delta it} \approx c_t \left(\frac{P_{it}}{P_t}\right)^{-\eta} \left[1 - \tau_t \delta \eta w_t \left(\frac{P_t}{P_{it}} - s_t\right)\right], \tag{18}$$

where

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}, \tag{19}$$

is the consumer price level,

$$s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di, \tag{20}$$

is a measure of price dispersion across varieties, and $w_t \equiv W_t/P_t$ is the real wage. The first factor on the right-hand side of (18), $c_t \left(\frac{P_{it}}{P_t}\right)^{-\eta}$, is the demand faced by the firm producing variety i in period t from a household located at distance δ if transportation costs were nil. The expression within square brackets is the correction to this demand due to transportation costs. This correction reflects the fact that for the consumer living at a distance $\delta \in [0, b_{it}]$

from the shopping center, transportation costs, τ_t, affect both the cost of good i, $P_{it}+\tau_t\delta W_t$, and the cost of living, $P_{\delta t}$. The variable s_t is a measure of the misallocation caused by price dispersion due to nominal price rigidity. In the new-Keynesian model without spatial competition this object appears only in the resource constraint of the economy. In the present model price dispersion due to price rigidity varies with distance δ. That is, two different locations will in general display different dispersions of prices across varieties. For this reason, in the present model s_t appears not only in the resource constraint of the economy but also in instances involving aggregation of prices or quantities across varieties or space.

With the approximation given in (18), the total demand for good i at a given shopping center, $\mathcal{D}(P_{it}, b_{it}; \tau_t) \equiv \int_{-b_{it}}^{b_{it}} c_{\delta it} d\delta$ (equation (9)), is given by

$$\mathcal{D}(P_{it}, b_{it}; \tau_t) \approx 2c_t \left(\frac{P_{it}}{P_t}\right)^{-\eta} b_{it} \left[1 - \tau_t \eta w_t \frac{b_{it}}{2} \left(\frac{P_t}{P_{it}} - s_t\right)\right]. \tag{21}$$

In what follows, $\mathcal{D}(P_{it}, b_{it}; \tau_t)$ refers to the firm's demand function linearized around $\tau_t = 0$ as given in equation (21).

We consider a symmetric equilibrium in which all firms producing variety i, regardless of the location of the retail store, charge the same price. That is, in equilibrium $P_{it} = P_{it}^{sc}$ for all i, sc. From the no-arbitrage condition (11), we then have that

$$b_{it} = \frac{1}{2N} \tag{22}$$

for all $i \in [0, 1]$ and all $t \geq 0$.

To obtain the equilibrium value of the unit cost of consumption at the household level, P_t^c, note that the nominal amount of resources the family must distribute to its members to ensure that all consume the same amount of the composite good c_t is $c_t 2N \int_0^{\frac{1}{2N}} P_{\delta t} d\delta$. By definition, this nominal quantity must equal $P_t^c c_t$. Thus, we have that

$$P_t^c = 2N \int_0^{\frac{1}{2N}} P_{\delta t} d\delta. \tag{23}$$

In Appendix C, we show that linearizing P_t^c around $\tau_t = 0$ one can write

$$P_t^c = P_t + \frac{\tau_t}{4N} s_t W_t. \tag{24}$$

This expression says that the unit cost of consumption, P_t^c, equals the price level, P_t, plus total distribution costs per unit of aggregate consumption, $\frac{\tau_t}{4N} s_t W_t$.

Market clearing in the labor market requires that the supply of labor, h_t, equal the sum of the demand for labor for the production of goods, $N \int_0^1 h_{it} di$, and the demand for labor for transportation services, $2N\tau_t \int_0^1 \int_0^{1/(2N)} c_{\delta it} \delta d\delta di$. So we have

$$h_t = N \int_0^1 h_{it} di + 2N\tau_t \int_0^1 \int_0^{1/(2N)} c_{\delta it} \delta \, d\delta di.$$

Integrating the left- and right-hand sides of (10) holding with equality and using (21), we have that the demand for labor for the production of goods is

$$N \int_0^1 h_{it} di = e^{-z_t} c_t \left[s_t + \frac{\eta \tau_t w_t}{4N} (s_t^2 - v_t) \right], \tag{25}$$

where

$$v_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\eta - 1} di. \tag{26}$$

As mentioned earlier, s_t measures misallocation due to price rigidity, which is a standard feature of the new-Keynesian model. The present model features an adjustment to this misallocation proportional to $s_t^2 - v_t$. Appendix D shows that $s_t^2 - v_t$ is negative, which means that the amount of misallocation due to price rigidity is smaller in the model with spatial frictions. The misallocation is smaller because as shown in section 2.2 distance reduces the dispersion of relative prices inclusive of transportation costs across varieties.

To obtain the demand for labor for transportation purposes, $2N\tau_t \int_0^1 \int_0^{1/(2N)} c_{\delta it} \delta \, d\delta di$,

use equation (18) to eliminate $c_{\delta it}$ and then perform the double integral. This yields

$$2N\tau_t \int_0^1 \int_0^{1/(2N)} c_{\delta it}\delta \, d\delta di = c_t \left[\frac{\tau_t}{4N} s_t + \frac{1}{12} \left(\frac{\tau_t}{N} \right)^2 \eta w_t \left(s_t^2 - v_t \right) \right]. \tag{27}$$

Thus the market clearing condition in the labor market is

$$h_t = e^{-z_t} c_t \left[s_t + \frac{\eta \tau_t w_t}{4N} (s_t^2 - v_t) \right] + c_t \left[\frac{\tau_t}{4N} s_t + \frac{1}{12} \left(\frac{\tau_t}{N} \right)^2 \eta w_t \left(s_t^2 - v_t \right) \right]. \tag{28}$$

Note that when τ_t/N goes to zero, the labor market clearing condition (28) reduces to $e^{z_t} h_t = s_t c_t$, which is the market clearing condition in the new-Keynesian model. This is the case when transportation costs are zero either because there are no transportation costs ($\tau_t = 0$) or because the number of shopping centers is arbitrarily large ($N \to \infty$), so that shoppers do not need to travel to reach a seller.

Letting $p_t^c \equiv P_t^c / P_t$, equations (4), (13), and (24) can be written as

$$\chi c_t^\sigma h_t^\gamma = \frac{w_t}{p_t^c}, \tag{29}$$

$$m c_t e^{z_t} = \frac{w_t}{p_t^c}, \tag{30}$$

and

$$p_t^c = 1 + \frac{\tau_t}{4N} s_t w_t. \tag{31}$$

Let $\pi_t \equiv P_t / P_{t-1} - 1$ denote the inflation rate and $\tilde{p}_t \equiv P_{it}/P_t$ denote the relative price of variety i for firms that get to reoptimize their price in period t. Then equation (19) implies that

$$1 = \theta (1 + \pi_t)^{\eta - 1} + (1 - \theta) \tilde{p}_t^{1-\eta}. \tag{32}$$

And from equations (20) and (26), the production inefficiency factors s_t and v_t can be written recursively as

$$s_t = \theta (1 + \pi_t)^\eta s_{t-1} + (1 - \theta) \tilde{p}_t^{-\eta} \tag{33}$$

and

$$v_t = \theta(1+\pi_t)^{1+\eta} v_{t-1} + (1-\theta)\tilde{p}_t^{-(1+\eta)}. \tag{34}$$

The growth rates of P_t^c and P_t are linked by the relationship

$$1 + \pi_t^c = (1+\pi_t)\frac{p_t^c}{p_{t-1}^c}. \tag{35}$$

We define the measure of aggregate activity to which the central bank responds, y_t, as the market value of all final goods produced in the economy deflated using the price index, P_t. That is, $y_t \equiv N\int_0^1 (P_{it}/P_t)\mathcal{D}(P_{it},(2N)^{-1};\tau_t)di$.Using equation (21) to eliminate $\mathcal{D}(P_{it},(2N)^{-1};\tau_t)$ and taking into account the definition of s_t given in equation (20), we obtain

$$y_t = c_t. \tag{36}$$

We show in Appendix E that in equilibrium equation (12) can be written recursively as

$$x_{1,t} + x_{2,t} + x_{3,t} + x_{4,t} = 0, \tag{37}$$

where

$$x_{1,t} = \lambda_t \tilde{p}_t^{1-\eta} X_{1,t} + \beta\theta E_t \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}(1+\pi_{t+1})}\right)^{1-\eta} x_{1,t+1}, \tag{38}$$

$$x_{2,t} = \lambda_t \tilde{p}_t^{-\eta} X_{2,t} + \beta\theta E_t \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}(1+\pi_{t+1})}\right)^{-\eta} x_{2,t+1}, \tag{39}$$

$$x_{3,t} = \lambda_t \tilde{p}_t^{-\eta-1} X_{3,t} + \beta\theta E_t \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}(1+\pi_{t+1})}\right)^{-\eta-1} x_{3,t+1}, \tag{40}$$

$$x_{4,t} = \lambda_t \tilde{p}_t^{2-\eta} X_{4,t} + \beta\theta E_t \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}(1+\pi_{t+1})}\right)^{2-\eta} x_{4,t+1}, \tag{41}$$

$$X_{1,t} = \left[\left((1-\eta)\left(2+\frac{\tau_t}{2N}\eta w_t s_t\right)+\eta\right)\left(\frac{P_t}{P_t^c}\right) - \left(\frac{2+\frac{\tau_t}{N}\eta w_t s_t}{-\frac{\tau_t}{N}w_t}\right) mc_t\right] c_t \tag{42}$$

$$X_{2,t} = \left[\left(\eta + \eta\left(\frac{\tau_t}{2N}\eta w_t s_t\right)\right) mc_t + \eta\left(\eta\frac{\tau_t}{2N}w_t\right)\left(\frac{P_t}{P_t^c}\right)\right] c_t \tag{43}$$

$$X_{3,t} = -(1+\eta)\left(\eta\frac{\tau_t}{2N}w_t\right)mc_t c_t, \tag{44}$$

$$X_{4,t} = \left[\frac{\left(2+\frac{\tau_t}{N}\eta w_t s_t\right)}{-\frac{\tau_t}{N}w_t}\right]\left(\frac{P_t}{P_t^c}\right)c_t. \tag{45}$$

We are ready to define the competitive equilibrium.

Definition 1 (Competitive Equilibrium)**.** *A competitive equilibrium is a set of processes* $\{c_t,\ h_t,\ \lambda_t,\ w_t,\ p_t^c,\ i_t,\ \pi_t^c,\ mc_t,\ y_t,\ \pi_t,\ \tilde{p}_t,\ s_t,\ v_t,\ x_{1t},\ x_{2t},\ x_{3t},\ x_{4t},\ X_{1t},\ X_{2t},\ X_{3t},\ X_{4t}\}$ *satisfying* (3), (5), (17), *and* (28)–(45)*, given exogenous processes for the transportation cost shock* τ_t*, the preference shock* ξ_t*, the productivity shock* z_t*, and the monetary policy shock* ν_t*, and the initial conditions* i_{-1}, p_{-1}^c, s_{-1} *and* v_{-1}.

An inspection of the complete set of equilibrium conditions given in Definition 1 reveals that the equilibrium depends on the object $\tau_t/(4N)$, which is the average time devoted to purchasing goods per unit of consumption. In particular, the equilibrium does not depend separately on the transportation cost τ_t or the number of shopping centers, N. We summarize this result in the following proposition:

Proposition 1. *Let* $\tau_t > 0$ *and* $N > 1$*. Then, the equilibrium values of all endogenous variables of the model depend on the average time spent shopping,* $\tau_t/(4N)$*, but not on* τ_t *or* N *separately.*

This result is of use for the calibration of the model, as it implies that one need not calibrate τ and N separately. Furthermore, one can show that when the production function is linear in labor, as maintained here (see constraint (10)), in a zero-inflation steady state the ratio $\tau/(4N)$ equals the ratio of hours allocated to shopping to hours allocated to working, for which data is available, in the case of the United States, from the American Time Use Survey (ATUS). We summarize this result in the following proposition:

Proposition 2. *In a zero-inflation steady state, the ratio of hours allocated to shopping to hours allocated to working is given by* $\tau/(4N)$.

Proof. See Appendix F. □

4 The Price Markup

Both spatial competition and nominal rigidity affect the equilibrium markup of prices over marginal costs. Because the production technology is assumed to be linear in labor, the nominal marginal cost of production for any firm $i \in [0,1]$ is given by W_t/e^{z_t}. Here, e^{z_t} represents the marginal product of labor. The gross markup of price over marginal cost charged by firm i is therefore given by $(P_{it}/W_t)e^{z_t}$. Using equation (19) to aggregate prices across firms, the average markup is given by $(P_t/W_t)e^{z_t}$. Letting μ_t denote the gross markup, we can then write

$$\mu_t = \frac{e^{z_t}}{w_t}. \tag{46}$$

In the absence of both nominal rigidity ($\theta = 0$) and the spatial dimension ($\tau_t = 0$ and $N = 1$), the gross markup becomes constant and equal to $\eta/(\eta - 1)$. In the absence of nominal rigidity ($\theta = 0$) but with a spatial dimension ($\tau_t > 0$ and $N > 1$), all firms $i \in [0,1]$ charge the same price and therefore the same markup. In such environment, first-order condition (15) implies that the gross markup is endogenously determined and given by $\tilde{\eta}_t/(\tilde{\eta}_t - 1)$, where $\tilde{\eta}_t$ is the elasticity of demand $\tilde{\eta}_{it+s}$ (see equation (16)) evaluated at $P_{it} = P_t$, $s = 0$, and $b_t = 1/(2N)$. In the presence of both nominal rigidity ($\theta > 0$) and spatial frictions ($\tau_t > 0$ and $N > 1$), every firm $i \in [0,1]$ that gets to reoptimize its price in period t, sets its price such that a weighted present value of the difference between its markup, μ_{it+s}, and $\tilde{\eta}_{it+s}/(\tilde{\eta}_{it+s} - 1)$ is zero (see first-order condition (15)).

In this section, we focus on the steady-state markup and in the next section we study dynamics. As in much of the related literature, we consider a zero-inflation steady state. Since in the zero-inflation steady state nominal rigidity plays no role in determining relative prices or quantities, the analysis in this section applies to economies with spatial frictions and with or without sticky prices.

Appendix G shows that the steady-state markup, μ, and the steady-state elasticity of

demand, $\tilde{\eta}$, are given by the economically relevant solution to the following two equations

$$\tilde{\eta} = \frac{\mu}{\mu - 1} \tag{47}$$

and

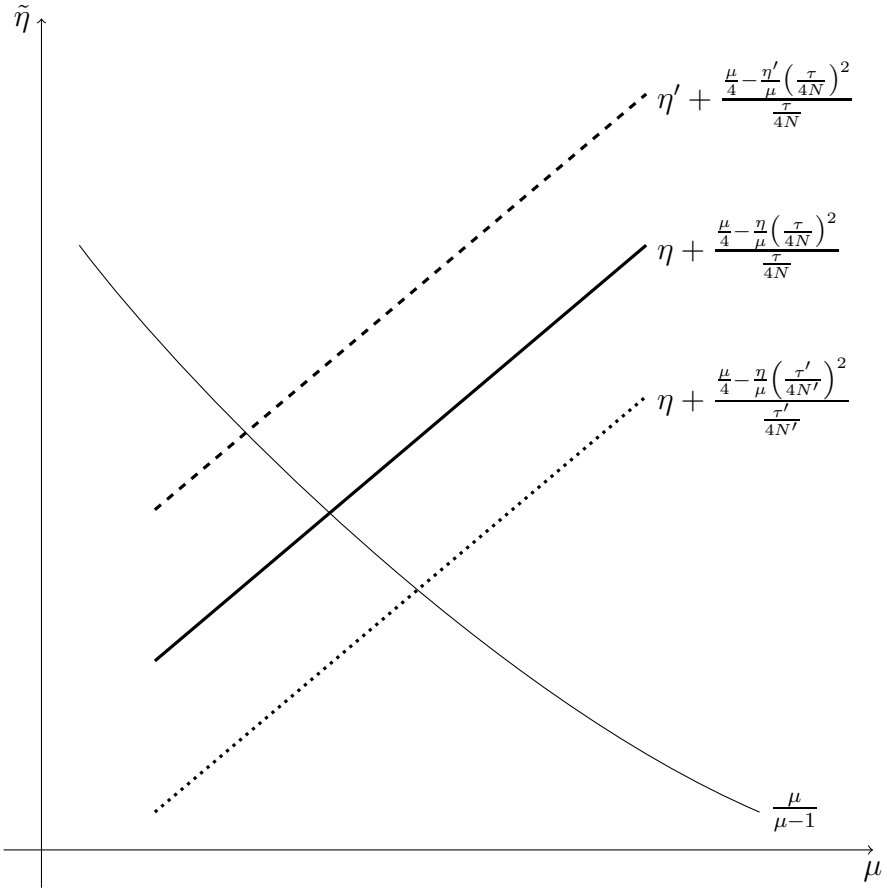
$$\tilde{\eta} = \eta + \frac{\frac{\mu}{4} - \frac{\eta}{\mu}\left(\frac{\tau}{4N}\right)^2}{\frac{\tau}{4N}}. \tag{48}$$

Figure 1 depicts the two equations. In the space $(\mu, \tilde{\eta})$ equation (47) is downward sloping and equation (48) is upward sloping. The steady-state markup and the steady-state price elasticity depend only on the average transportation cost, $\tau/(4N)$, and on the elasticity of substitution across varieties, η.

Given μ, an increase in the average transportation cost, $\tau/(4N)$, reduces the right-hand side of (48). In Figure 1, the increase in transportation cost from $\tau/(4N)$ to $\tau'/(4N')$ shifts the upward sloping curve down and to the right, as shown by the dotted line. In the new steady state, the price elasticity falls and the markup increases. The intuition for why the steady-state markup is increasing in $\tau/(4N)$ is as follows. In the present model, firms compete not only across varieties, but also across space. The spatial dimension of competition affects market power through two channels. One operates at the border of the market and the other operates in the interior of the market. Consider first how the border affects market power. The arbitrage condition (11), which states that at the margin changes in the price charged by the firm can affect the size of its market, limits the market power of the firm. But this limit to market power weakens with the average transportation cost, $\tau/(4N)$, because this cost governs the cost of switching stores.

Consider now the intra-border channel through which spatial competition affects market power. As $\tau/(4N)$ increases, the firm gains market power over all customers living between its store location and the market's border, as it becomes more expensive for customers to travel to the adjacent market. Thus, both the border and the intra-border channels imply that market power is increasing in transportation costs, $\tau/(4N)$.

Figure 1: Characterization of the Steady-State Markup



Notes. The solid downward sloping line represents equation (47) and the solid upward sloping line represents equation (48). The intersection of the two lines determines the steady state markup, μ, and the steady-state price elasticity $\tilde{\eta}$. An increase in the average transportation cost from $\tau/(4N)$ to $\tau'/(4N') > \tau/(4N)$ shifts the upward sloping line down and to the right (dotted line), causing a fall in the price elasticity $\tilde{\eta}$ and an increase in the markup μ. An increase in the elasticity of substitution from η to $\eta' > \eta$ shifts the upward sloping line up and to the left (dashed line), provided $\tau/(4N) < 1$, causing an increase in the price elasticity $\tilde{\eta}$ and a fall in the markup μ.

Let's now analyze the relationship between the steady-state markup and the elasticity of substitution, η. Given μ, an increase in η reduces the right-hand side of (48), provided $\tau/(4N) < 1$. As we will show shortly, $\tau/(4N) < 1$ is the case of greatest empirical interest. Thus an increase in η shifts the upward sloping line in Figure 1 up and to the left as shown by the dashed line. As a result the steady-state markup, μ, falls and the steady-state price elasticity, $\tilde{\eta}$, increases. The intuition behind this result is that an increase in η raises the elasticity of substitution across varieties, which passes through to the overall price elasticity of demand. Facing a more price elastic demand, firms find it optimal to charge lower markups.

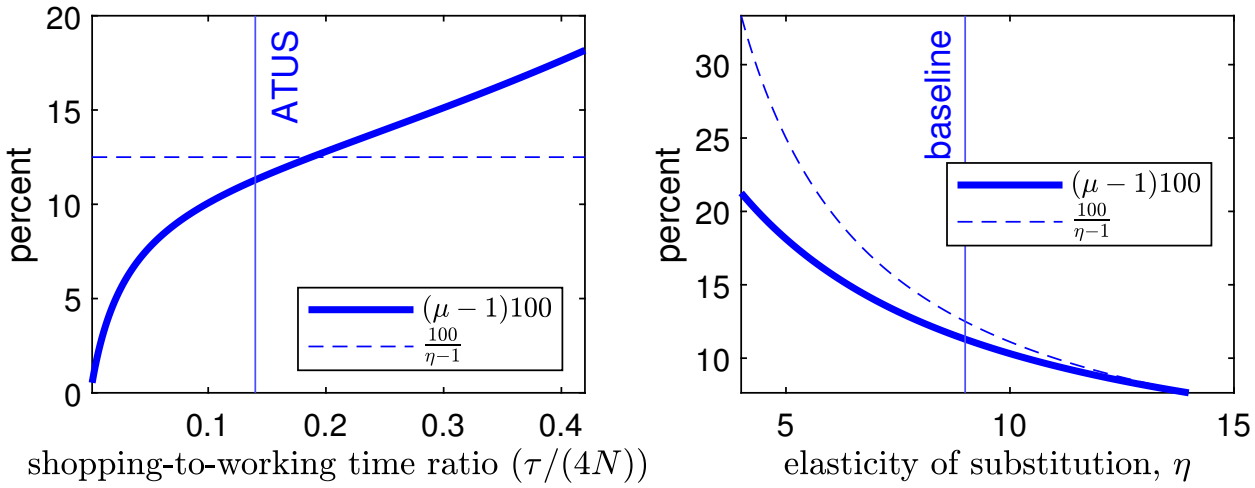
We now turn to a quantitative characterization of the steady-state markup. To this end we calibrate the parameters η and $\tau/(4N)$. Proposition 2 shows that the average transportation cost, $\tau/(4N)$, equals the ratio of time spent purchasing goods and services to the time spent working. Using data from the American Time Use Survey for the year 2023, we find that this ratio equals 0.14 (Bureau of Labor Statistics, 2024).[1] Following Galí (2015), we set $\eta = 9$. The resulting steady-state markup is 11.3 percent. This is slightly smaller than the markup implied by the Lerner formula $\eta/(\eta - 1)$, which for the assumed value of η equals 12.5 percent. Thus, for the calibrated value of $\tau/(4N)$, spatial frictions dampen market power but not significantly.

The left panel of Figure 2 plots the net steady-state markup in percent, $(\mu - 1) \times 100$, as a function of the average transportation cost, $\tau/(4N)$. For comparison, it also displays the net markup implied by the Lerner condition in percent, which is constant and equal to $100/(\eta - 1)$. The steady-state markup in the model with spatial competition, shown with a solid line, is monotonically increasing in $\tau/(4N)$ and, in particular, can be smaller or larger than then markup implied by the Lerner condition.

The left panel of Figure 2 also suggests that as the average transportation cost, $\tau/(4N)$, approaches zero, the markup converges to zero. The reason is that in the absence of trans-

[1]The ratio of 0.14 is the average time spent purchasing goods and services relative to the average time spent working for the civilian-population age group 25 to 54 years. The 2023 observation is conservative, as the series displays a slight negative time trend. Using a broader age bracket increases the ratio; for example, for the civilian population aged 18 and over, the ratio was 0.18 in 2023.

Figure 2: The Steady-State Price Markup



Notes. The figure displays with a solid line the steady-state net markup expressed in percent, $(\mu - 1) \times 100$, and with a broken line the net markup in percent implied by the Lerner condition, $100/(\eta - 1)$. In the left panel, the vertical line marks the observed shopping-to-working time ratio reported by the American Time Use Survey (ATUS) in 2023, $\tau/(4N) = 0.14$. In the right panel, the vertical line marks the calibrated value of the elasticity of substitution across varieties, $\eta = 9$. In the left panel, η takes its calibrated value of 9 and in the right panel, $\tau/(4N)$ takes its calibrated value of 0.14.

portation costs product differentiation due to distance disappears. And, since firms compete with neighboring firms in a Bertrand fashion, the equilibrium is marginal cost pricing, or markups equal to zero. It follows that as transportation costs converge to zero, the markup does not converge to the one implied by the Lerner formula. At the other extreme, as transportation costs become arbitrarily large, so does the markup. The intuition behind this result is that when transportation costs become arbitrarily large, they completely dominate the price of the good inclusive of transportation costs, which is the one relevant for consumers. As a result, demand becomes insensitive to changes in the product price. Faced with a price inelastic demand, firms charge an arbitrarily high markup.

The right panel of Figure 2 displays the markup implied by the model with a solid line and the markup implied by the Lerner condition with a broken line as a function of the elasticity of substitution η. In both cases, the markup is decreasing in η. For relatively low values of η, the markup is noticeably lower than the one implied by the Lerner condition. For example, when η equals 4, the markup is 21 percent and the one implied by the Lerner condition is 33 percent. For values of η greater than or equal to the calibrated value of 9, there is little difference in the two markups.

The finding that under a plausible calibration of the transportation cost, $\tau/(4N)$, the net markup is close to $100/(\eta - 1)$ and decreasing in η in an economically meaningful way contrasts with the findings of Barro (2024). In the context of a static partial equilibrium Hotelling model he argues that with spatial frictions, as an approximation, the markup is largely independent of η. The discrepancy arises from his hypothesis that transportation costs constitute a negligible fraction of the product price, P. The left panel of Figure 2 confirms that when transportation costs are small, the markup can be significantly different from $100/(\eta - 1)$. However, for empirically plausible values of transportation costs, this is not the case. The fact that, as mentioned above, the ratio of time spent shopping to time spent working, $\tau/(4N)$, is 0.14 is an indication that transportation costs are not small. More precisely, in the present model, the ratio of transportation costs to price is equal to the ratio

of time spent shopping to time spent working multiplied by the real wage, $\tau/(4N) \times w$, which under the maintained calibration is 0.13, which is not a negligible number.

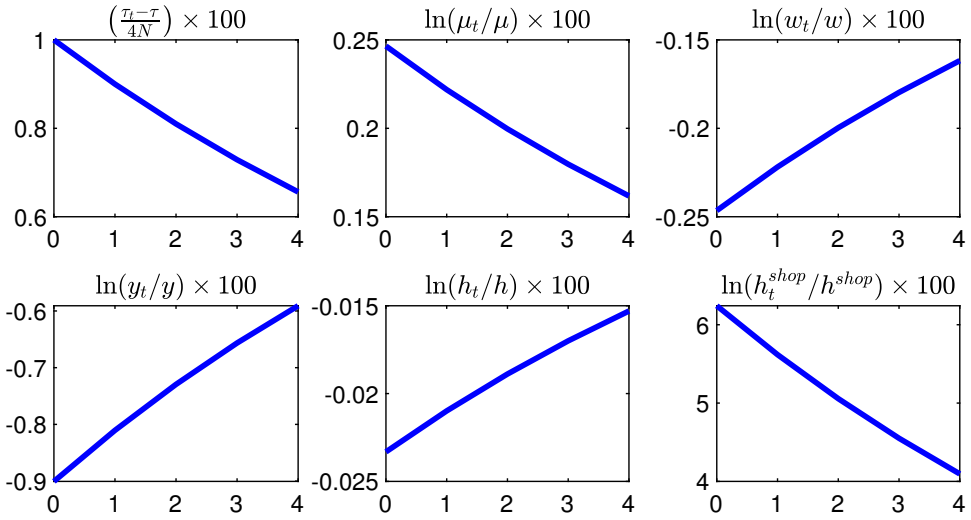
5 Transportation Cost Shocks As Cost-Push Shocks

In this section, we study the equilibrium dynamics implied by the model. To this end we start by calibrating its structural parameters. As in section 4, we set $\tau/(4N)$ to 0.14 to match the observed ratio of time spent shopping to time spent working, and the elasticity of substitution across product varieties η to 9. The time unit is meant to be one quarter. Following Galí (2015), we set the subjective discount factor β to 1 percent per quarter ($\beta = 0.99$), the intertemporal elasticity of substitution to 1 ($\sigma = 1$), the Frisch elasticity of labor supply to 0.2 ($\gamma = 5$), the scaler χ to 1, the probability of not being able to reoptimize prices to 0.75 ($\theta = 0.75$), the steady-state rate of inflation to 0 ($\pi = 0$), the inflation coefficient of the Taylor rule to 1.5 ($\alpha_\pi = 1.5$), and the output coefficient of the Taylor rule to 0.125 ($\alpha_y = 0.125$). We assume that all shocks follow autoregressive processes of order 1. We approximate the model's dynamics by log-linearizing its equilibrium conditions around the nonstochastic steady state.

To gather intuition about the effects induced by spatial frictions in general equilibrium, consider first a version of the model with Hotelling-type frictions but with flexible prices ($\theta = 0$). For notational convenience we refer to the baseline model, which includes Hotellian and new-Keynesian features, as the HOTNK model, and to the version of the HOTNK model with flexible prices as the HOT model. So we begin with a characterization of the equilibrium dynamics induced by the HOT model.

Figure 3 displays impulse responses to an innovation in τ_t that increases the shopping-to-working time ratio, $\tau_t/(4N)$, by one percentage point on impact (from its steady-state value of 0.14 to 0.15). The transportation cost shock, τ_t, is assumed to have a serial correlation of 0.9. The increase in transportation cost produces an increase in the markup of prices

Figure 3: Impulse Response to a Transportation Cost Shock: HOT Model



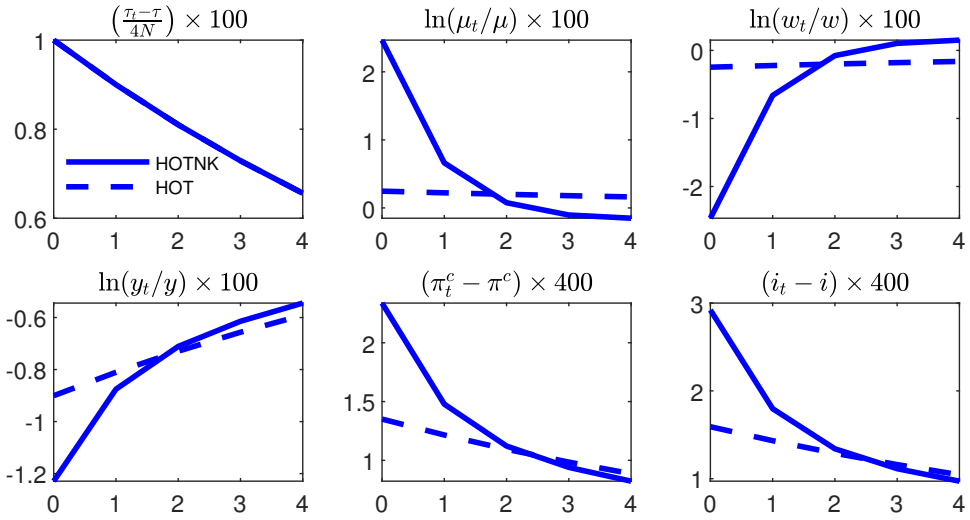
Notes. The HOT model features spatial frictions and flexible prices. The shock is a one percentage point increase in the shopping-to-working time ratio, $\tau_t/(4N)$, from 0.14 to 0.15. The logarithm of τ_t is assumed to follow an AR(1) process with persistence 0.9. The horizontal axis measures quarters after the shock. Variables without a time subscript represent steady-state values. The variable h_t^{shop} denotes time allocated to shopping.

over marginal cost, μ_t. In turn, the markup increases because of a fall in the price elasticity of demand, $\tilde{\eta}_t$. To understand why the price elasticity of demand falls, one must take into account the intra-border and border effects of an increase in transportation costs.

Consider first the intra-border effect. The price faced by the consumer living at the average distance $1/(4N)$ from the closest shopping center inclusive of transportation costs is given by $P_{it} + \tau_t W_t/(4N)$. All else equal, an increase in τ_t reduces the share of the product price P_{it} in the total price relevant for the consumer. As a result, an increase in P_{it} causes a smaller percentage increase in $P_{it} + \tau_t W_t/(4N)$ the higher τ_t is. This makes the demand for good i less sensitive to changes in the product price, P_{it}. Facing a less elastic demand, it is optimal for the firm to increase the product markup.

The border effect of an increase in τ_t goes in the same direction, that is, it makes the demand less price elastic. This is so because an increase in transportation cost makes the border of the firm's market less sensitive to changes in the product price, P_{it}, as consumers

Figure 4: Impulse Response to a Transportation Cost Shock in the HOTNK and HOT Models



Notes. The HOTNK model features spatial frictions and sticky prices. The shock is a one percentage point increase in the shopping-to-working time ratio, $\tau_t/(4N)$, from 0.14 to 0.15. The logarithm of τ_t is assumed to follow an AR(1) process with persistence 0.9. The horizontal axis measures quarters after the shock. Variables without a time subscript represent steady-state values. The variable h_t^{shop} denotes time allocated to shopping.

are willing to tolerate a larger increase in P_{it} before switching shopping location.

The increase in the markup reduces the real wage that firms are willing to pay at any level of employment. Consequently, holding constant the labor supply schedule to the production sector, the increase in the markup causes a contraction in employment. But the supply of labor to the goods-producing sector also contracts because the transportation cost raises the demand for labor for transportation services. This shift in the labor supply schedule in the production sector further reduces employment and the production of final goods.

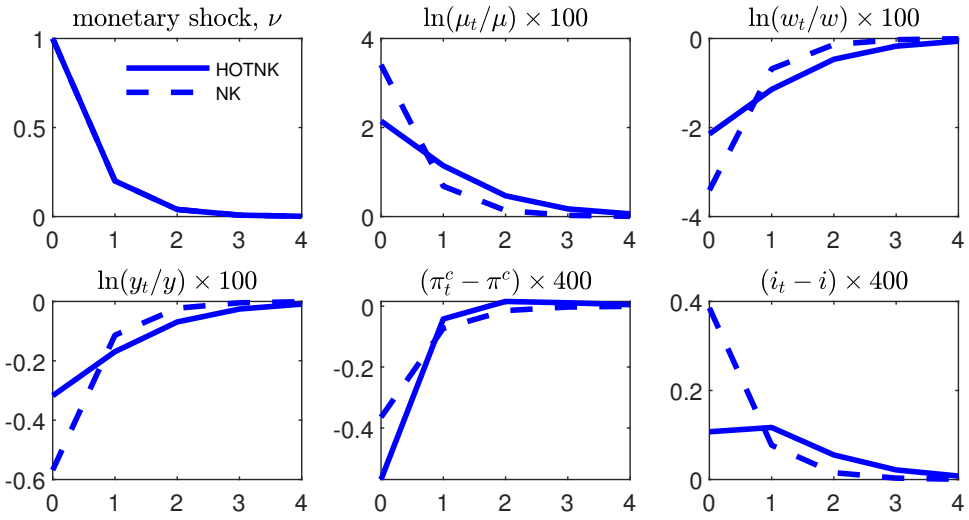
Nominal rigidity can amplify the macroeconomic effects of transportation cost shocks. Figure 4 displays with solid lines impulse responses to an innovation in τ_t that increases the shopping-to-working time ratio, $\tau_t/(4N)$, by one percentage point from its steady state value of 0.14 to 0.15 in the HOTNK model. For comparison the figure also displays with dashed lines the corresponding impulse responses in the HOT model. In the HOTNK model, the

transportation cost shock produces an increase in the markup that is nearly ten times as large as the one predicted by the HOT model. The contraction in output and employment in the production sector is 35 percent larger in the HOTNK model than in the HOT model. The shock is stagflationary as the contraction is accompanied by a significant increase in inflation of about 2.5 percentage points. The fact that the transportation shock lowers output while driving up prices suggests that it behaves like a cost-push shock. Faced with inflation and recession, the Taylor rule leans on the side of price stability. The central bank tightens nominal rates by 3 percentage points. This increase in interest rates is responsible for the amplification effect. In other words, amplification is a consequence of the central bank following a Taylor-type interest rate feedback. To see this, note that, —as is well known,— in the presence of a single nominal rigidity, which here takes the form of sticky product prices, the central bank can achieve the flexible price allocation by stabilizing the price that suffers from nominal rigidity. Specifically, in the HOTNK model the monetary policy that achieves the flexible price allocation is $\pi_t = 0$ for all t. It can be shown, however, that even in this case the transportation cost shock is stagflationary, as the stabilization of π_t cannot prevent an increase in consumer-price inflation, π_t^c. In this sense, the transportation shock τ_t can robustly be interpreted as a cost-push shock.

6 Spatial Frictions as Dampeners of Demand Shocks

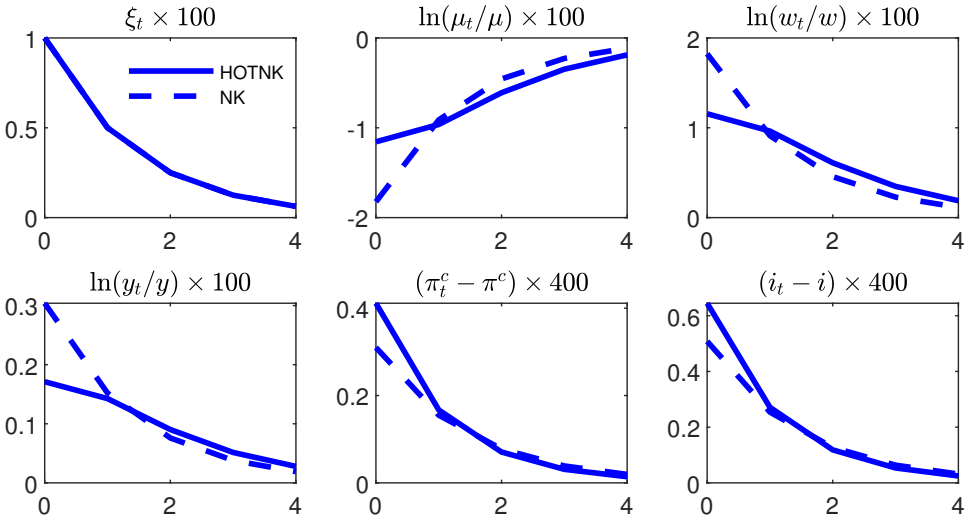
The presence of spatial frictions dampens the output effects of demand shocks and amplifies their inflationary effects. Figures 5 and 6 display impulse responses to monetary and preference shocks in the HOTNK model. For comparison, the figures also include the corresponding impulse responses in the new-Keynesian model, denoted NK, which features nominal rigidity, but no spatial frictions. In response to both shocks, the response of output is more muted in the HOTNK model than in the NK model, but the response of inflation is more pronounced.

Figure 5: Impulse Response to a Monetary Shoc in the HOTNK and NK Modelsk



Notes. The shock is a 1 percentage point increase in the exogenous component of the Taylor rule, e^{ν_t}. The monetary shock, v_t, is assumed to follow an AR(1) process with persistence 0.2.

Figure 6: Impulse Response to a Preference Shock



Notes. Solid lines correspond to the HOTNK model and dashed lines to the NK model. The shock is a 1 percentage point increase in e^{ξ_t}.

The intuition behind this effect has to do with the role that the spatial friction has on the price elasticity of demand. Because spatial frictions make the demand function less elastic, firms have more space to change prices without significantly affecting quantities. To see this, recall that the price that a consumer living at a distance δ from the nearest market pays for any variety $i \in [0, 1]$ is $P_{it} + \tau_t \delta W_t$. From the perspective of the firm, the transportation component $\tau_t \delta W_t$ is perfectly inelastic. Thus, a given change in P_{it} has a smaller effect on demand than in the absence of transportation frictions. Specifically, a one-percent increase in P_{it} reduces demand by η percent in the absence of transportation costs but only by $\eta \frac{P_{it}}{P_{it}+\tau_t \delta W_t}$ with transportation costs. So with transportation costs, firms that get to reoptimize prices have more leeway to set prices without affecting demand.

7 Conclusion

TBA.

Appendix

A Derivation of Equation (15)

In this appendix, we show how to derive equation (15) from equation (12). We start by reproducing equation (12) for convenience:

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left[\frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{P^c_{t+s}} + \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left(\mathcal{D}_1(P_{it}, b_{it+s}; \tau_{t+s}) + \mathcal{D}_2(P_{it}, b_{it+s}; \tau_{t+s}) \frac{\partial b_{it+s}}{\partial P_{it}} \right) \right] = 0 \tag{12}$$

Letting

$$\mathcal{D}' \equiv \left(\mathcal{D}_1(P_{it}, b_{it+s}; \tau_{t+s}) + \mathcal{D}_2(P_{it}, b_{it+s}; \tau_{t+s}) \frac{\partial b_{it+s}}{\partial P_{it}} \right)$$

denote the total derivative of $\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})$ with respect to P_{it}, we can rewrite (12) as

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left[\frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{P^c_{t+s}} + \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \mathcal{D}' \right] = 0.$$

Pull $\mathcal{D}'$ outside of the squared brackets to get

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \mathcal{D}' \left[\frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{\mathcal{D}' P^c_{t+s}} + \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \right] = 0.$$

Rearrange by collecting terms in $\frac{P_{it}}{P^c_{t+s}}$ to write

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \mathcal{D}' \left[\frac{P_{it}}{P^c_{t+s}} \left(1 + \frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{\mathcal{D}' P_{it}} \right) - mc_{t+s} \right] = 0.$$

Use the definition of $\tilde{\eta}_{it+s}$ given in equation (16) to replace $\frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{\mathcal{D}' P_{it}}$ to obtain

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \mathcal{D}' \left[\frac{P_{it}}{P^c_{t+s}} \left(1 - \frac{1}{\tilde{\eta}_{it+s}} \right) - mc_{t+s} \right] = 0.$$

Finally, use the defintion of $\tilde{\eta}_{it+s}$ again, but now to replace $\mathcal{D}'$, and multiply by $-P_{it}$. This yields

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \tilde{\eta}_{it+s} \mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s}) \left[\frac{P_{it}}{P^c_{t+s}} \left(1 - \frac{1}{\tilde{\eta}_{it+s}} \right) - mc_{t+s} \right] = 0,$$

which is equation (15).

B First-Order Approximation of the Demand for Good i Around $\tau_t = 0$

This appendix derives equations (18) and (21).

Let

$$f(\tau_t, \delta) \equiv \left(\frac{P_{it} + \tau_t \delta W_t}{P_{\delta t}} \right)^{-\eta}.$$

Let's write $f(\tau_t, \delta)$ as

$$f(\tau_t, \delta) \equiv \left[\frac{N(\tau_t, \delta)}{D(\tau_t, \delta)} \right]^{\frac{-\eta}{1-\eta}},$$

where

$$N(\tau_t, \delta) = (P_{it} + \tau_t \delta W_t)^{1-\eta}$$

and

$$D(\tau_t, \delta) = P_{\delta t}^{1-\eta} = \int_0^1 (P_{it} + \tau_t \delta W_t)^{1-\eta} di.$$

Then, we have that

$$N(0, \delta) = P_{it}^{1-\eta}$$

$$N_1(0, \delta) = (1-\eta) W_t P_{it}^{-\eta} \delta$$

$$D(0, \delta) = P_t^{1-\eta}$$

$$D_1(0, \delta) = (1-\eta) W_t \delta \int_0^1 P_{it}^{-\eta} di,$$

where

$$P_t \equiv \left[\int_0^1 P_{jt}^{1-\eta} dj \right]^{\frac{1}{1-\eta}}.$$

The first-order Taylor expansion of $f(\tau_t, \delta)$ around $\tau_t = 0$ is

$$f(\tau_t, \delta) \approx \left(\frac{N(0, \delta)}{D(0, \delta)} \right)^{\frac{-\eta}{1-\eta}} - \tau_t \frac{\eta}{1-\eta} \left(\frac{N(0, \delta)}{D(0, \delta)} \right)^{\frac{-\eta}{1-\eta} - 1} \left[\frac{N_1(0, \delta) D(0, \delta) - N(0, \delta) D_1(0, \delta)}{D(0, \delta)^2} \right].$$

Substitute $N(0,\delta)$, $D(0,\delta)$, $N_1(0,\delta)$, and $D_1(0,\delta)$ in this expression to get

$$f(\tau_t,\delta) \approx \left(\frac{P_{it}}{P_t}\right)^{-\eta} - \tau_t \delta \eta W_t \left(\frac{P_{it}}{P_t}\right)^{-1} \left[\frac{P_{it}^{-\eta} P_t^{1-\eta} - P_{it}^{1-\eta} \int_0^1 P_{jt}^{-\eta} dj}{P_t^{2(1-\eta)}}\right].$$

Letting $w_t \equiv W_t/P_t$, $s_t \equiv \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\eta} dj$, and rearranging, we have

$$f(\tau_t,\delta) \approx \left(\frac{P_{it}}{P_t}\right)^{-\eta} - \tau_t \delta \eta w_t \left(\frac{P_{it}}{P_t}\right)^{-1} \left[\left(\frac{P_{it}}{P_t}\right)^{-\eta} - \left(\frac{P_{it}}{P_t}\right)^{1-\eta} s_t\right].$$

The demand for good i from a location situated at a distance δ of a shipping center is given, is $c_{\delta it} = c_t f(\tau_t,\delta)$. Thus, we have that

$$c_{\delta it} \approx c_t \left(\frac{P_{it}}{P_t}\right)^{-\eta} \left[1 - \tau_t \delta \eta w_t \left(\frac{P_t}{P_{it}} - s_t\right)\right],$$

which is equation (18). And the demand for good i for a firm with a market of size b_{it} is given by $\mathcal{D}(P_{it}, b_{it}; \tau_t) = 2\int_0^{b_{it}} c_{\delta it} d\delta$. Thus, we have that

$$\mathcal{D}(P_{it}, b_{it}; \tau_t) \approx 2c_t \left(\frac{P_{it}}{P_t}\right)^{-\eta} b_{it} \left[1 - \tau_t \eta w_t \frac{b_{it}}{2} \left(\frac{P_t}{P_{it}} - s_t\right)\right],$$

which is equation (21). For the remainder of the appendix, $\mathcal{D}(P_{it}, b_{it}; \tau_t)$ refers to this linearization of the firm's demand function around $\tau_t = 0$.

C First-Order Approximation of P_t^c Around $\tau_t = 0$

This appendix derives equation (24).

For convenience, we reproduce here equation (23):

$$P_t^c = 2N \int_0^{\frac{1}{2N}} P_{\delta t} d\delta. \tag{23}$$

Write P_t^c as

$$P_t^c = 2N \int_0^{\frac{1}{2N}} D(\tau_t, \delta)^{\frac{1}{1-\eta}} d\delta,$$

where $D(\tau_t, \delta) \equiv P_{\delta t}^{1-\eta}$. Linearizing around $\tau_t = 0$, we have

$$P_t^c \approx D(0,\delta)^{\frac{1}{1-\eta}} + \left[\frac{2N}{1-\eta} \int_0^{\frac{1}{2N}} D(0,\delta)^{\frac{\eta}{1-\eta}} D_1(0,\delta) d\delta \right] \tau_t.$$

Recalling from Appendix B that $D(0,\delta) = P_t^{1-\eta}$, $D_1(0,\delta) = (1-\eta)\delta W_t \int_0^1 P_{it}^{-\eta} di$, and that $s_t = \int_0^1 (P_{it}/P_t)^{-\eta} di$, we obtain

$$P_t^c \approx P_t + \frac{\tau_t}{4N} s_t W_t,$$

which is equation (24).

D Proof that $s_t^2 - v_t \leq 0$

Let $p_{it} \equiv P_{it}/P_t$ and $\omega_{it} \equiv p_{it}^{1-\eta} > 0$. Then, from (19) we have that $\int_0^1 \omega_{it} di = 1$. From (20) and (26), we have that $s_t = \int_0^1 (1/p_{it})\omega_{it} di$ and $v_t = \int_0^1 (1/p_{it})^2 \omega_{it} di$. Thus,

$$s_t^2 - v_t = \left[\int_0^1 \left(\frac{1}{p_{it}}\right) \omega_{it} di\right]^2 - \int_0^1 \left(\frac{1}{p_{it}}\right)^2 \omega_{it} di,$$

which, by Jensen's inequality, is in general negative and zero in the absence of price dispersion.

E Recursive Representation of Equation (12) in Equilibrium

In this appendix, we show that in equilibrium optimality condition (12) can be expressed recursively.

From equation (21), we have that the partial derivatives of $\mathcal{D}$ are

$$\mathcal{D}_1(P_{it}, b_{it+s}; \tau_{t+s}) = -\eta \frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{P_{it}} + 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} b_{it+s} \tau_{t+s} \eta w_{t+s} \frac{b_{it+s}}{2} \frac{P_{t+s}}{P_{it}^2}$$

and

$$\mathcal{D}_2(P_{it}, b_{it+s}; \tau_{t+s}) = \frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{b_{it+s}} - 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} \tau_{t+s} \eta w_{t+s} \frac{b_{it+s}}{2} \left(\frac{P_{t+s}}{P_{it}} - s_{t+s} \right).$$

In equilibrium, $b_{it} = 1/(2N)$ (equation (22)). To alleviate notation, we define $b = 1/(2N)$. Then we can express $\mathcal{D}$, $\mathcal{D}_1$, and $\mathcal{D}_2$ as:

$$\mathcal{D}(P_{it}, b; \tau_{t+s}) = 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} b \left[1 - \tau_{t+s} \eta w_{t+s} \frac{b}{2} \left(\frac{P_{t+s}}{P_{it}} - s_{t+s} \right) \right], \tag{E1}$$

$$\mathcal{D}_1(P_{it}, b; \tau_t) = -\eta \frac{\mathcal{D}(P_{it}, b; \tau_{t+s})}{P_{it}} + 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} b \tau_{t+s} \eta w_{t+s} \frac{b}{2} \frac{P_{t+s}}{P_{it}^2}, \tag{E2}$$

and

$$\mathcal{D}_2(P_{it}, b; \tau_t) = \frac{\mathcal{D}(P_{it}, b; \tau_{t+s})}{b} - 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} \tau_{t+s} \eta w_{t+s} \frac{b}{2} \left(\frac{P_{t+s}}{P_{it}} - s_{t+s} \right). \tag{E3}$$

Using these expressions to replace $\mathcal{D}_1$, and $\mathcal{D}_2$ and (14) to replace $\partial b_{it+s}/\partial P_{it}$ in equation (12)

yields

$$
\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left[\frac{\mathcal{D}(P_{it}, b_{it+s}; \tau_{t+s})}{P^c_{t+s}} + \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left(\mathcal{D}_1(P_{it}, b_{it+s}; \tau_{t+s}) + \mathcal{D}_2(P_{it}, b_{it+s}; \tau_{t+s}) \frac{\partial b_{it+s}}{\partial P_{it}} \right. \right. \\
&= E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \frac{\mathcal{D}(P_{it}, b; \tau_{t+s})}{P^c_{t+s}} \\
&\quad + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left(-\eta \frac{\mathcal{D}(P_{it}, b; \tau_{t+s})}{P_{it}} + 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} b \tau_{t+s} \eta w_{t+s} \frac{b}{2} \frac{P_{t+s}}{P^2_{it}} \right) \\
&\quad + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \frac{\left(\frac{\mathcal{D}(P_{it}, b; \tau_{t+s})}{b} - 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} \tau_{t+s} \eta w_{t+s} \frac{b}{2} \left(\frac{P_{t+s}}{P_{it}} - s_{t+s} \right) \right)}{(-2\tau_{t+s} W_{t+s})}.
\end{aligned}
$$

Multiply this expression by P_{it} and recall that $w_{t+s} = W_{t+s}/P_{t+s}$ to obtain

$$
\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \frac{P_{it}}{P^c_{t+s}} \mathcal{D}(P_{it}, b; \tau_{t+s}) \qquad \text{(E4} \\
&\quad + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left(-\eta \mathcal{D}(P_{it}, b; \tau_{t+s}) + 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta-1} b \tau_{t+s} \eta w_{t+s} \frac{b}{2} \right) \\
&\quad + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left(\frac{P_{it}}{P_{t+s}} \right) \left[\frac{\frac{\mathcal{D}(P_{it}, b; \tau_{t+s})}{b} - 2c_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} \tau_{t+s} \eta w_{t+s} \frac{b}{2} \left(\frac{P_{t+s}}{P_{it}} - s_{t+s} \right)}{(-2\tau_{t+s} w_{t+s})} \right]
\end{aligned}
$$

To facilitate the collection of terms involving P_{it}, let

$$
d_{1,t+s} \equiv 2c_{t+s} \left[b + \tau_{t+s} \eta w_{t+s} \frac{b^2}{2} s_{t+s} \right] \qquad \text{(E5)}
$$

$$
d_{2,t+s} \equiv 2c_{t+s} \frac{b^2}{2} \tau_{t+s} \eta w_{t+s}. \qquad \text{(E6)}
$$

So that we can write:

$$
\mathcal{D}(P_{it}, b; \tau_{t+s}) = d_{1,t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} - d_{2,t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta-1}.
$$

With these definitions in hand we can write (E4) as:

$$
\begin{aligned}
0 \;=\; & E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \frac{P_{it}}{P^c_{t+s}} \left(d_{1,t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} - d_{2,t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta-1} \right) \\
& + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left(-\eta d_{1,t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} + (1+\eta) d_{2,t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta-1} \right) \\
& + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left[\frac{\frac{d_{1,t+s}}{b} \left(\frac{P_{it}}{P_{t+s}} \right)^{1-\eta} - \frac{d_{2,t+s}}{b} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta}}{(-2\tau_{t+s} w_{t+s})} \right] \\
& - E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left[\frac{\left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} \frac{d_{2,t+s}}{b}}{(-2\tau_{t+s} w_{t+s})} \right] \\
& + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P^c_{t+s}} - mc_{t+s} \right) \left[\frac{\left(\frac{P_{it}}{P_{t+s}} \right)^{1-\eta} \frac{d_{2,t+s}}{b} s_{t+s}}{(-2\tau_{t+s} w_{t+s})} \right].
\end{aligned} \tag{E7}
$$

Notice that P_{it} appears either as $\frac{P_{it}}{P^c_{t+s}}$, $\left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta}$, $\left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta-1}$, or $\left(\frac{P_{it}}{P_{t+s}}\right)^{1-\eta}$. Next rearrange to collect terms in these expressions and use $\frac{P_{it}}{P^c_{t+s}} = \frac{P_{it}}{P_{t+s}} \frac{P_{t+s}}{P^c_{t+s}}$ to have a system in which P_{it} appears always as P_{it}/P_{t+s}.

$$
\begin{aligned}
0 \;=\; & E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{1-\eta} \left[(1-\eta) d_{1,t+s} + \frac{\frac{d_{2,t+s}}{b}}{\tau_{t+s} w_{t+s}} \right] \left(\frac{P_{t+s}}{P^c_{t+s}} \right) \\
& + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} \left[\eta d_{1,t+s} - \frac{\frac{d_{2,t+s}}{b}}{\tau_{t+s} w_{t+s}} \right] mc_{t+s} \\
& + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta} \left[\eta d_{2,t+s} \right] \left(\frac{P_{t+s}}{P^c_{t+s}} \right) \\
& - E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{-\eta-1} \left[(1+\eta) d_{2,t+s} \right] mc_{t+s} \\
& + E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{2-\eta} \left[\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b} s_{t+s}}{-2\tau_{t+s} w_{t+s}} \right] \left(\frac{P_{t+s}}{P^c_{t+s}} \right) \\
& - E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{t+s}} \right)^{1-\eta} \left[\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b} s_{t+s}}{-2\tau_{t+s} w_{t+s}} \right] mc_{t+s}.
\end{aligned} \tag{E8}
$$

Finally, we can express this first-order condition as the sum of terms in $\left(\frac{P_{it}}{P_{t+s}}\right)^{k-\eta}$ for $k = -1, 0, 1, 2$:

$$
\begin{aligned}
0 \;=\; & E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \{ \\
& \left(\frac{P_{it}}{P_{t+s}}\right)^{1-\eta} \left[\left((1-\eta) d_{1,t+s} + \frac{\frac{d_{2,t+s}}{b}}{\tau_{t+s} w_{t+s}} \right) \left(\frac{P_{t+s}}{P^c_{t+s}}\right) - \left(\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b} s_{t+s}}{-2\tau_{t+s} w_{t+s}} \right) mc_{t+s} \right] \\
& + \left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta} \left[\left(\eta d_{1,t+s} - \frac{\frac{d_{2,t+s}}{b}}{\tau_{t+s} w_{t+s}} \right) mc_{t+s} + (\eta d_{2,t+s}) \left(\frac{P_{t+s}}{P^c_{t+s}}\right) \right] \\
& - \left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta-1} \left[(1+\eta) d_{2,t+s} \right] mc_{t+s} \\
& + \left(\frac{P_{it}}{P_{t+s}}\right)^{2-\eta} \left[\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b} s_{t+s}}{-2\tau_{t+s} w_{t+s}} \right] \left(\frac{P_{t+s}}{P^c_{t+s}}\right) \Bigg\}.
\end{aligned}
\tag{E9}
$$

Divide equation (E9) by b. The resulting expression can be written so that $d_{i,t+s}$ for $i = 1, 2$ appears always as $d_{i,t+s}/b$ and τ_t always appears as $b\tau_t$

$$
\begin{aligned}
0 \;=\; & E_t \sum_{s=0}^{\infty} (\beta\theta)^s \lambda_{t+s} \{ \\
& \left(\frac{P_{it}}{P_{t+s}}\right)^{1-\eta} \left[\left((1-\eta) \frac{d_{1,t+s}}{b} + \frac{\frac{d_{2,t+s}}{b}}{b\tau_{t+s} w_{t+s}} \right) \left(\frac{P_{t+s}}{P^c_{t+s}}\right) - \left(\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b} s_{t+s}}{-2b\tau_{t+s} w_{t+s}} \right) mc_{t+s} \right] \\
& + \left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta} \left[\left(\eta \frac{d_{1,t+s}}{b} - \frac{\frac{d_{2,t+s}}{b}}{b\tau_{t+s} w_{t+s}} \right) mc_{t+s} + \eta \left(\frac{d_{2,t+s}}{b}\right) \left(\frac{P_{t+s}}{P^c_{t+s}}\right) \right] \\
& - \left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta-1} \left[(1+\eta) \frac{d_{2,t+s}}{b} \right] mc_{t+s} \\
& + \left(\frac{P_{it}}{P_{t+s}}\right)^{2-\eta} \left[\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b} s_{t+s}}{-2b\tau_{t+s} w_{t+s}} \right] \left(\frac{P_{t+s}}{P^c_{t+s}}\right) \Bigg\}.
\end{aligned}
\tag{E10}
$$

Let

$$X_{1,t+s} = \left[\left((1-\eta)\frac{d_{1,t+s}}{b} + \frac{\frac{d_{2,t+s}}{b}}{b\tau_{t+s}w_{t+s}}\right)\left(\frac{P_{t+s}}{P^c_{t+s}}\right) - \left(\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b}s_{t+s}}{-2b\tau_{t+s}w_{t+s}}\right)mc_{t+s}\right] \quad \text{(E11)}$$

$$X_{2,t+s} = \left[\left(\eta\frac{d_{1,t+s}}{b} - \frac{\frac{d_{2,t+s}}{b}}{b\tau_{t+s}w_{t+s}}\right)mc_{t+s} + \eta\left(\frac{d_{2,t+s}}{b}\right)\left(\frac{P_{t+s}}{P^c_{t+s}}\right)\right] \quad \text{(E12)}$$

$$X_{3,t+s} = -(1+\eta)\left(\frac{d_{2,t+s}}{b}\right)mc_{t+s} \quad \text{(E13)}$$

$$X_{4,t+s} = \left[\frac{\frac{d_{1,t+s}}{b} + \frac{d_{2,t+s}}{b}s_{t+s}}{-2b\tau_{t+s}w_{t+s}}\right]\left(\frac{P_{t+s}}{P^c_{t+s}}\right). \quad \text{(E14)}$$

Then (E10) becomes

$$0 = E_t\sum_{s=0}^{\infty}(\beta\theta)^s\lambda_{t+s}\left\{\left(\frac{P_{it}}{P_{t+s}}\right)^{1-\eta}X_{1,t+s} + \left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta}X_{2,t+s} + \left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta-1}X_{3,t+s} + \left(\frac{P_{it}}{P_{t+s}}\right)^{2-\eta}X_{4,t+s}\right\}.$$

Let

$$x_{1t} = E_t\sum_{s=0}^{\infty}(\beta\theta)^s\lambda_{t+s}\left(\frac{P_{it}}{P_{t+s}}\right)^{1-\eta}X_{1,t+s}$$

$$x_{2t} = E_t\sum_{s=0}^{\infty}(\beta\theta)^s\lambda_{t+s}\left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta}X_{2,t+s}$$

$$x_{3t} = E_t\sum_{s=0}^{\infty}(\beta\theta)^s\lambda_{t+s}\left(\frac{P_{it}}{P_{t+s}}\right)^{-\eta-1}X_{3,t+s}$$

$$x_{4t} = E_t\sum_{s=0}^{\infty}(\beta\theta)^s\lambda_{t+s}\left(\frac{P_{it}}{P_{t+s}}\right)^{2-\eta}X_{4,t+s}.$$

Express x_{it} for $i = 1, 2, 3, 4$ recursively.

$$\begin{aligned} x_{1t} &= \lambda_t \left(\frac{P_{it}}{P_t}\right)^{1-\eta} X_{1,t} + E_t \sum_{s=1}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{t+s}}\right)^{1-\eta} X_{1,t+s} \\ &= \lambda_t \left(\frac{P_{it}}{P_t}\right)^{1-\eta} X_{1,t} + E_t \sum_{s=1}^{\infty} (\beta\theta)^s \lambda_{t+s} \left(\frac{P_{it}}{P_{it+1}}\right)^{1-\eta} \left(\frac{P_{it+1}}{P_{t+s}}\right)^{1-\eta} X_{1,t+s} \\ &= \lambda_t \left(\frac{P_{it}}{P_t}\right)^{1-\eta} X_{1,t} + (\beta\theta) E_t \left(\frac{P_{it}}{P_{it+1}}\right)^{1-\eta} E_{t+1} \sum_{h=0}^{\infty} (\beta\theta)^c \lambda_{t+1+h} \left(\frac{P_{it+1}}{P_{t+1+h}}\right)^{1-\eta} X_{1,t+1+h} \\ &= \lambda_t \left(\frac{P_{it}}{P_t}\right)^{1-\eta} X_{1,t} + \beta\theta E_t \left(\frac{P_{it}}{P_{it+1}}\right)^{1-\eta} x_{1,t+1} \end{aligned} \tag{E15}$$

Following similar steps we obtain

$$x_{2,t} = \lambda_t \left(\frac{P_{it}}{P_t}\right)^{-\eta} X_{2,t} + \beta\theta E_t \left(\frac{P_{it}}{P_{it+1}}\right)^{-\eta} x_{2,t+1} \tag{E16}$$

$$x_{3,t} = \lambda_t \left(\frac{P_{it}}{P_t}\right)^{-\eta-1} X_{3,t} + \beta\theta E_t \left(\frac{P_{it}}{P_{it+1}}\right)^{-\eta-1} x_{3,t+1} \tag{E17}$$

$$x_{4,t} = \lambda_t \left(\frac{P_{it}}{P_t}\right)^{2-\eta} X_{4,t} + \beta\theta E_t \left(\frac{P_{it}}{P_{it+1}}\right)^{2-\eta} x_{4,t+1} \tag{E18}$$

To simplify the notation, divide (E5) and (E6) by b and then replace b by $1/(2N)$. This yields

$$\frac{d_{1,t}}{b} = 2c_t \left[1 + \frac{\tau_t}{4N} \eta w_t s_t\right] \tag{E19}$$

$$\frac{d_{2,t}}{b} = c_t \frac{\tau_t}{2N} \eta w_t. \tag{E20}$$

Now use (E19), (E20), and $b\tau = \frac{\tau}{2N}$ to eliminate $\frac{d_{1,t}}{b}$, $\frac{d_{2,t}}{b}$, and $b\tau$ from (E11), (E12), (E13),

(E14) to obtain

$$X_{1,t} = \left[\left((1-\eta)\left(2+\frac{\tau_t}{2N}\eta w_t s_t\right)+\eta\right)\left(\frac{P_t}{P_t^c}\right)-\left(\frac{2+\frac{\tau_t}{N}\eta w_t s_t}{-\frac{\tau_t}{N}w_t}\right)mc_t\right]c_t \quad \text{(E21)}$$

$$X_{2,t} = \left[\left(\eta+\eta\left(\frac{\tau_t}{2N}\eta w_t s_t\right)\right)mc_t+\eta\left(\eta\frac{\tau_t}{2N}w_t\right)\left(\frac{P_t}{P_t^c}\right)\right]c_t \quad \text{(E22)}$$

$$X_{3,t} = -(1+\eta)\left(\eta\frac{\tau_t}{2N}w_t\right)mc_t c_t \quad \text{(E23)}$$

$$X_{4,t} = \left[\frac{\left(2+\frac{\tau_t}{N}\eta w_t s_t\right)}{-\frac{\tau_t}{N}w_t}\right]\left(\frac{P_t}{P_t^c}\right)c_t. \quad \text{(E24)}$$

In sum, equilibrium condition (12) can be written recursively as

$$\begin{aligned}
0 &= x_{1,t}+x_{2,t}+x_{3,t}+x_{4,t}\\
x_{1,t} &= \lambda_t\left(\frac{P_{it}}{P_t}\right)^{1-\eta}X_{1,t}+\beta\theta E_t\left(\frac{P_{it}}{P_{it+1}}\right)^{1-\eta}x_{1,t+1}\\
x_{2,t} &= \lambda_t\left(\frac{P_{it}}{P_t}\right)^{-\eta}X_{2,t}+\beta\theta E_t\left(\frac{P_{it}}{P_{it+1}}\right)^{-\eta}x_{2,t+1}\\
x_{3,t} &= \lambda_t\left(\frac{P_{it}}{P_t}\right)^{-\eta-1}X_{3,t}+\beta\theta E_t\left(\frac{P_{it}}{P_{it+1}}\right)^{-\eta-1}x_{3,t+1}\\
x_{4,t} &= \lambda_t\left(\frac{P_{it}}{P_t}\right)^{2-\eta}X_{4,t}+\beta\theta E_t\left(\frac{P_{it}}{P_{it+1}}\right)^{2-\eta}x_{4,t+1}\\
X_{1,t} &= \left[\left((1-\eta)\left(2+\frac{\tau_t}{2N}\eta w_t s_t\right)+\eta\right)\left(\frac{P_t}{P_t^c}\right)-\left(\frac{2+\frac{\tau_t}{N}\eta w_t s_t}{-\frac{\tau_t}{N}w_t}\right)mc_t\right]c_t\\
X_{2,t} &= \left[\left(\eta+\eta\left(\frac{\tau_t}{2N}\eta w_t s_t\right)\right)mc_t+\eta\left(\eta\frac{\tau_t}{2N}w_t\right)\left(\frac{P_t}{P_t^c}\right)\right]c_t\\
X_{3,t} &= -(1+\eta)\left(\eta\frac{\tau_t}{2N}w_t\right)mc_t c_t\\
X_{4,t} &= \left[\frac{\left(2+\frac{\tau_t}{N}\eta w_t s_t\right)}{-\frac{\tau_t}{N}w_t}\right]\left(\frac{P_t}{P_t^c}\right)c_t,
\end{aligned}$$

which are equilibrium conditions (37)–(45).

F Proof of Proposition 2

We drop the time subscript to indicate steady-state values. By equation (32), in a zero-inflation steady state, $\tilde{p} = 1$. It follows from (33) and (34) that $s = v = 1$. Then, setting $z = 0$, equation (25) implies that hours working is equal to c, and equation (27) implies that hours shopping is equal to $c\tau/(4N)$. The ratio of hours spent shopping to hours spent working is then $\tau/(4N)$, which is what we set out to prove.

G Derivation of the Steady-State Markup

This appendix derives equations (47) and (48), which we repeat here for convenience:

$$\tilde{\eta} = \frac{\mu}{\mu - 1} \tag{47}$$

and

$$\tilde{\eta} = \eta + \frac{\frac{\mu}{4} - \frac{\eta}{\mu}\left(\frac{\tau}{4N}\right)^2}{\frac{\tau}{4N}} \tag{48}$$

In the steady state inflation is zero and thus $P_{it} = P_t = P$ for all i. Evaluating equation (13) at the steady state yields

$$mc = \frac{w}{p^c}. \tag{G1}$$

In the steady state $\tilde{\eta}_{it} = \tilde{\eta}$ for all i, where $\tilde{\eta}$is given by the right-hand side of equation (16) evaluated at $P_{it} = P$, $b_{it} = b$ and $s = 0$. Evaluate (46) at the steady state to obtain

$$\mu = \frac{1}{w}. \tag{G2}$$

Evaluating (15) at the steady state yields

$$\left(1 - \frac{1}{\tilde{\eta}}\right) = mcp^c = w$$

where the last equality follows from (G1). Combining this expression with (G2) gives (47).

Evaluate equation (16) at the steady state. This gives

$$\tilde{\eta} = -\left[\mathcal{D}_1(P, b; \tau) + \mathcal{D}_2(P, b; \tau)\frac{-1}{2\tau W}\right]\frac{P}{\mathcal{D}(P, b; \tau)}. \tag{G3}$$

In the steady state, by (32) $\tilde{p} = 1$, by (33), $s = 1$, and by (34), $v = 1$. Evaluate (E1), (E2)

and (E3) at the steady state. This yields

$$\mathcal{D}(P,b;\tau) = 2cb,$$

$$\mathcal{D}_1(P,b;\tau) = -\eta\frac{\mathcal{D}(P,b;\tau)}{P} + 2cb\tau\eta w\frac{b}{2}\frac{1}{P},$$

and

$$\mathcal{D}_2(P,b;\tau) = \frac{\mathcal{D}(P,b;\tau)}{b}.$$

Then we have that

$$\frac{\mathcal{D}_1(P,b;\tau)P}{\mathcal{D}(P,b;\tau)} = -\eta + \tau\eta w\frac{b}{2}$$

and

$$\frac{\mathcal{D}_2(P,b;\tau)P}{\mathcal{D}(P,b;\tau)}\frac{-1}{2\tau W} = \frac{-1}{2\tau wb}$$

Use these expressions in (G3):

$$\begin{aligned}\tilde{\eta} &= -\left[\frac{\mathcal{D}_1(P,b;\tau)P}{\mathcal{D}(P,b;\tau)} + \frac{\mathcal{D}_2(P,b;\tau)P}{\mathcal{D}(P,b;\tau)}\frac{-1}{2\tau W}\right] \\ &= -\left[-\eta + \tau\eta w\frac{b}{2} - \frac{1}{2\tau wb}\right] \\ &= \eta - \tau\eta w\frac{b}{2} + \frac{1}{2\tau wb}\end{aligned}$$

Use (G2) to replace w and $b = 1/(2N)$ to replace b. This yields

$$\tilde{\eta} = \eta - \eta\frac{\tau}{\mu 4N} + \frac{N\mu}{\tau} = \eta + \frac{\frac{\mu}{4} - \frac{\eta}{\mu}\left(\frac{\tau}{4N}\right)^2}{\frac{\tau}{4N}},$$

which is equation (48).

References

Barro, Robert J., "Markups and Entry in a Circular Hotelling Model," NBER working paper 32660, December 2024.

Bureau of Labor Statistics, "American Time Use Survey," Washington, DC, United States Department of Labor, 2024. https://www.bls.gov/tus/tables.htm (accessed March 1, 2025)

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy 113,* February 2005, 1-45.

Clarida, Richard, Jordí Gali, and Mark Gertler, "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature 37,* December 1999, 1661-1707.

Del Negro, Marco, Marc P. Giannoni, and Frank Schorfheide, "Inflation in the Great Recession and New Keynesian Models," *American Economic Journal: Macroeconomics 7,* 2015, 168-196.

Hotelling, Harold , "Stability in Competition," *Economic Journal 39,* March 1929, 41-57.

King, Robert G., and Mark W. Watson, "Inflation and Unit Labor Cost," *Journal of Money, Credit and Banking 44,* December 2012, 111-149.

Smets, Frank, and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review 97,* June 2007, 586-606.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy,* Princeton, NJ: Princeton University Press, 2003.