Collateralized Loan Obligations as Fire-Sale Insulation*

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Abstract

We develop a model where CLOs are the optimal financial structure for securitizing assets that trade in illiquid markets. As is true empirically, CLOs hold portfolios of risky loans that they actively rebalance and finance themselves with safe, long-maturity debt. CLOs face restrictions on their equity payouts, but these restrictions ignore the market-to-market value of CLO assets. Banks that hold CLOs' safe debt are insulated from fire-sales, and a financial system where CLOs take risk instead of banks is more efficient than traditional banking. CLOs are most useful in moderately illiquid markets with significant gains from trade and fire-sale costs.

Keywords: Collateralized Loan Obligations, Financial Intermediation, Safe Assets

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1 Introduction

Collateralized Loan Obligations (CLOs) are one of the most novel and most rapidly growing segments of the US securitization market. CLOs have grown from roughly $200 billion in U.S. assets in 2006 to $1 trillion in 2023 and own 60-70 percent[1] of all U.S. leveraged loans: loans to firms that are too risky to quality for traditional bank loans (Walsh et al., 2023). CLOs hold portfolios (or pools) of loans against which they issue "tranches:" securities of varying degrees of risk that they can sell to more or less risk-tolerant investors. While this "pooling and tranching" structure is the standard approach to securitization (DeMarzo, 2005), CLOs are unique in that they are allowed to buy and sell the loans that they hold over time, subject to a detailed set of rules. The markets where CLOs buy and sell loans are considerably less liquid than equity markets, and sales of loans in this market often result in "fire-sale discounts" that push loan prices below their fundamental value (Elkamhi and Nozawa, 2022; Kundu, 2023, 2024). This paper presents a model in which CLOs are the optimal financial structure for securitizing assets which can be traded in illiquid markets.

In our model, financial intermediaries face a classic problem of transforming risky, long-maturity, illiquid assets into safe, liquid, short-maturity liabilities. Intermediaries can issue short-maturity safe debt (e.g. bank deposits) at a low interest rate R_d and raise any additional from investors at a higher rate of return R_e in equity markets. This spread is due to investors' demand for holding safe, "money-like" assets (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016; Van Binsbergen et al., 2022). Financial intermediaries that can perform "safety transformation," financing the purchase of risky assets in part by issuing safe debt, profit from borrowing at the low rate R_d (Diamond, 2020). However, such intermediaries may have to sell the risky assets they hold during a crisis in order not to default (Stein, 2012). Such "fire-sales" can depress asset prices, exacerbating the crisis (Shleifer and Vishny, 2011; Dávila and Korinek, 2018). To avoid bank runs, intermediaries that are funded

[1] Guggenheim Investments, S&P LCD, Bank of America, page 8, under figure titled "CLOs Are the Largest Leveraged Loan Investor."

with short-term debt must hold assets whose market value is higher than their outstanding stock of debt. Such intermediaries are therefore vulnerable to the impact of fire sales.

CLOs are an ideal solution for insulating financial intermediaries funded with short-maturity debt from the impact of fire sales. As we document empirically, CLOs hold portfolios of risky loans funded by issuing long-maturity safe debt. A second intermediary, which we call a bank, then holds the long-maturity safe debt of CLOs in order to issue the short-maturity safe debt demanded by depositors. During a financial crisis, CLOs sell their worst-performing loans and use the proceeds to pay back part of their debt, as documented empirically by Elkamhi and Nozawa (2022). However, the long-term funding of CLOs protects them from the impact of any fire-sale discounts cause by this selling. CLOs still earn high returns on investment (Cordell et al., 2021) by waiting for asset prices to recover after a crisis . Banks, in contrast, can simply hold the long-term debt of CLOs to maturity. Because the assets banks hold are never sold off during a crisis, the banks are insulated from the impact of the fire-sales performed by CLOs.

Our model is consistent with a range of stylized facts about the structure of CLO balance sheets. As mentioned above, CLOs are funded by long-term debt and hold a portfolio of loans that they rebalance over time. Another key feature of real-world CLOs is that their payouts to equity investors are restricted by financial covenants. The so-called "overcollateralization test" and "interest coverage test" require CLOs to hold a portfolio of loans with a sufficiently high face value and that generates a sufficiently high amount of interest income in order to make payments to equity investors. However, these restrictions make no reference to the market price of loans that CLOs hold. Because CLOs invest in an asset class that tends to strongly recover after financial crises, this lack of reference to market prices allows CLOs to profit from holding under-priced loans at the trough of a crisis. In our model, the efficiency of CLOs as investors comes precisely from the fact that they are able to hold loans to maturity without worrying about temporary fluctuations in loan prices.

We compare the equilibrium of a financial system with CLOs to a traditional system

where deposit-issuing banks lend directly to firms. Compare to a traditional banking system, loans sell for strictly higher prices in a system with CLOs. This is because the same loan can back a greater quantity of risk-free debt when CLOs can insulate banks from the negative impact of fire sales. The size of this benefit is pinned down by the extent to which CLOs can sustain market-to-market insolvency during the most severe financial crisis while holding a loan to maturity. A bank which could not become temporarily insolvent would have to reduce its debt issuance to avoid insolvency. The additional present value CLOs get from being able to issue more debt at the rate R_d instead of paying dividends discounted at the rate R_e determines how much more valuable loans are to the CLO than to a bank.

In addition, because a CLO finds it more attractive to hold an illiquid loan to maturity than a bank does, the financial system with CLOs must also experience smaller fire-sale discounts than occur in a traditional banking system. This is because in each economy, the bank or CLO must be indifferent between selling a bad loan or holding it to maturity at an interior solution. If the bank finds it less attractive to hold a loan to maturity, it must also in equilibrium get a worse price from selling the loan to an outside investor. CLOs therefore improve financial stability by reducing the equilibrium quantity of fire sales that occur during a financial crisis.

Finally, we perform a simple calibration to quantify the benefits of CLOs. We match CLO's return on equity and safe debt to estimates from Cordell et al. (2021), CLO leverage from a dataset of CLO contracts, estimates on the impact of CLO fire sales from Elkamhi and Nozawa (2022), and the returns on CLO assets to returns on LSTA index for the leveraged loan market. We find that CLOs can pay a 74 basis point higher rate of return than banks to buy the same loans. We also find that CLOs strictly dominate an economy with securitization vehicles that only hold assets to maturity. Next, we perform a sensitivity analysis that varies the liquidity of the assets CLOs hold. When assets are perfectly liquid, a traditional banking system is equally efficient as an economy with CLOs, since no fire-sale discounts occur. When assets are perfectly illiquid, CLOs create the same amount of value as buy-and-

hold securitization. Assets with an intermediate degree of illiquidity, like leveraged loans, are precisely the assets which are best matched to the CLO's financial structure.

Our model provides a fundamental explanation for the benefits of the CLO's financial structure. There is a large empirical literature on CLOs examining topics ranging from credit ratings (Griffin and Nickerson, 2023), relationships with banks (Peristiani and Santos, 2019), the possibility of adverse selection(Benmelech et al., 2012), asset prices (Cordell et al., 2021; Elkamhi et al., 2024), and particularly the impact of CLO's firesales, (Elkamhi and Nozawa, 2022; Kundu, 2023, 2024). The closest paper to ours is Xu (2023), which theoretically demonstrates the advantage of portfolio rebalancing in securitization as performed by CLOs. Our paper is the first, however, to jointly explain why many of the distinct features of the CLO financial structure are jointly optimized for securitizing assets that trade in an illiquid market. We are also the first to theoretically compare CLOs to more traditional financial structures and document the relative advantage CLOs provide.

Our model also illustrates how to understand the modern financial system in particular as organized around the creation of safe assets. Theories of financial intermediation that emphasize persistent relationships between banks and firms do not explain the role of intermediaries for assets that are traded anonymously in markets. Recent work has shown how financial institutions can be insulated against price fluctuations in illiquid markets (Chodorow-Reich et al., 2021; Hanson et al., 2015) and that intermediaries beyond just banks can help with the creation of safe assets (Sunderam, 2015). Our paper shows that these features of modern markets naturally arise when the financial sector is designed to create safe assets backed by risky loans that trade in an illiquid market. In our model, CLOs and banks each play distinct important roles in financial intermediation. Their separation is not merely an *ad hoc* response to arbitrary regulation, even if their growth may be a response to policies reducing banks' lending capacity (Buchak et al., 2018; Diamond et al., 2024). As financial development makes more assets tradable in securities markets, the current design of CLO balance sheets may therefore play a greater and greater role in the future of financial intermediation.

2 Motivating Facts

This section presents stylized empirical facts about CLOs. We use a database from Semeris Documents that provides detailed information about the contractual structure ("indentures") of CLOs. We also reference existing facts in the literature.

Fact 1: CLO's raise the majority of their funding in the form of long-maturity AAA debt, and banks are the largest investor in AAA CLO debt. In our database of CLO indentures, the average CLO raises 63.1% of their initial funding by issuing debt with a AAA credit rating. The remaining funding comes from a mix of equity financing and more junior debt securities, which have lower credit ratings and pay higher interest rates than the AAA tranche. The average maturity of the AAA debt issues by CLOs in our dataset is 12.36 years. CLO indentures often have a "weighted average life" restriction that sets a maximum on the maturity of the loans they hold, ensuring that there is only modest maturity mismatch between the assets and liabilities of CLOs.

DeMarco et al. (2020) provides summary statistics on the investors who fund CLOs in 2020. The largest investor in the AAA debt of CLOs are banks, who own 28 % of al AAA CLO debt observed in their sample. Other large investors include pensions, insurance companies, "other financial institutions" (including bank holding companies), and mutual funds. 97% of banks' investments in CLOs are in the safest AAA tranches.

Fact 2: CLOs are the largest investors in the leveraged loan market and actively re-balance their loan holdings over times Leveraged loans refer to the riskier segment of the market for corporate loans, akin to junk bonds. The LSTA leveraged loan index which tracks this market had $1.4 trillion of assets in 2024. CLOs in 2024 owned $ 1.3 trillion ($1 trillion in the U.S.) in leveraged loans and tend to own 65-70 % of all outstanding leveraged loans (Walsh et al., 2023).

Unlike traditional securitization vehicles, CLOs trade their loan holdings over time. In our database, the median CLO is given 5 years when they can actively re-balance their loan holdings before they must hold their portfolio to maturity. In a traditional securitization,

a fixed pool of assets produces cash flows which are then divided among investors. CLOs, in contrast trade their leveraged loans in a somewhat liquid secondary market. CLOs tend to sell loans after severe drops in loan performance. Several empirical papers have shown that these sales by CLOS result in "fire-sale discounts" that push the prices of sold loans temporarily below their fundamental value (Elkamhi and Nozawa, 2022; Kundu, 2023, 2024). In addition, figure 1 shows the performance of the overall leveraged loan market since 2000. Both in the 2008 and 2020 crises, crashes in loan prices were followed by rapid recoveries, so investors who stayed invested during crises were able to profit *ex post*.

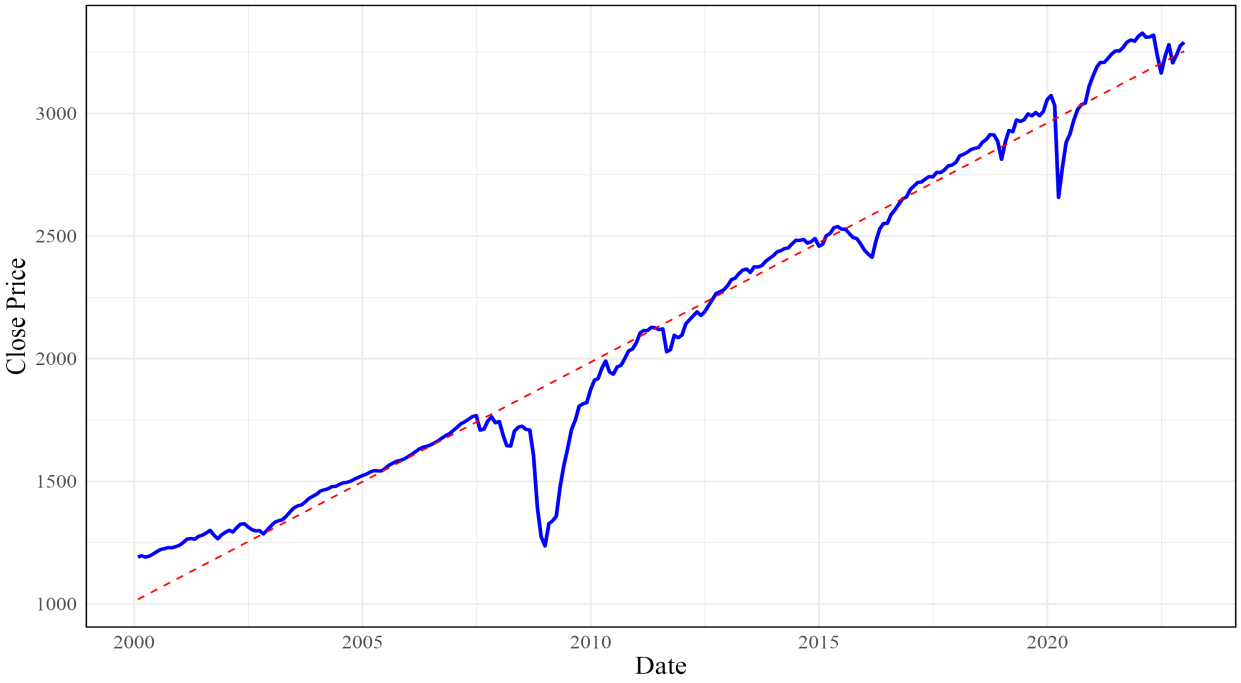


Figure 1: Performance of LSTA Leveraged Loan Index, 2000-2024.
The dotted line shows the best linear fit to the index's performance over this period.

Fact 3: CLO payouts to equity investors are restricted by financial covenants, but these covenants do not reference the market price of CLO loan portfolios. The payouts CLOs make to their equity investors follow strict contractual rules. The interest collected from the loans held by a CLO is placed in a "interest collection account," and all interest payments the CLO owes to its debt investors are then paid out of this account. If the

CLO's balance sheet then satisfies a list of financial covenants, the remainder of this account can then be paid to the equity investors. If there is a covenant violation, the remaining interest in this account is instead used to pay down the CLO's outstanding debt.

CLO's have two main financial covenants: the "overcollateralization test" and the "interest coverage test," which appear in all CLO indentures in our database. The overcollateralization test examines the ratio between the face value of loans held by a CLO to the face value of outstanding debt it owes to its debt investors. The test is only satisfied if this ratio is sufficiently high, with an average cutoff of 124% percent in our data on CLO indentures. The interest coverage test examines the ratio between the interest payments the CLO receives on loans it holds to the amount of interest it owes to its debt holders. This covenant is only satisfied when the ratio is sufficiently high, with an average cutoff of 120% percent in our data on CLO indentures. Neither covenant references the market price of loans the CLO holds in even a single deal in our database. In addition, while a violation of either covenant stops the CLO from paying dividends to equity investors, neither forces the CLO to liquidate its assets. Instead, CLOs are forced to use payouts that would have gone to equity to pay down the principal of outstanding debt. As a result, CLOs are not forced to sell their assets in a severe loan market downturn as in 2008 or 2020. This allows the CLO to stay invested and profit from a possible recovery in loan values, as happened after both crises. As documented by Cordell et al. (2021), CLOs created in 2007 had the highest average return on equity, showing that CLOs can even benefit from crises.

3 Model

This section presents our model of the role of CLOs in financial intermediation. We describe the environment in section 3.1. We consider the optimal decisions of a bank who raises funds directly from depositors and equity investors in section 3.3. With then consider the investment decisions of a CLO whose long-maturity debt is held by banks instead of by

depositors in section 3.4. Finally, we demonstrate the benefits a financial system with CLOs to one only with banks in section 3.7.

3.1 Setup

The model has four periods $t = 0, 1, 2, 3$ and three types of investors: depositors, equity investors, and outside financiers. Deposit holders are active in all periods, will only hold short-term risk-free debt, and have a required return R_d. Equity investors are only active at time 0, are risk neutral, and have a required return $R_e > R_d$. Any risky asset sold after time 0 must be bought by an outside financier, who is risk neutral with required return $R_a > R_d$. Assets sold to the outside financier also face an illiquidity cost that depresses its price.

There is one type of asset, loans, and all loans are ex ante identical at t=0. Loan i pays a dividend $\delta_{i,3}$ at time 3, and shocks that occur at times 1 and 2 make this payoff risky. There is a binary "good" or "bad" shock at time 1, followed by a second good or bad shock at time 2, with a probability $Pr(B_t)$ of a bad shock at time t. Each loan i pays $\delta_{i,3} = 1$ at time 3 except when both bad shocks are realized. Loan i's payoff at time 3 in the bad/bad is state is $f_h = 1$ if the loan is "high quality" and $f_l < 1$ if the loan is "low quality." Loan quality is drawn i.i.d. across loans and is realized at time 1, with each loan having a quality f with probability π_f. Loans are also illiquid, so trading them after time 0 depresses their price. Let $p_{f,t}$ be the price at time t of a loan with quality f at which a loan can be purchased. Because of illiquidity, there may be also be a strictly lower "fire-sale" price $p_{f,t}^{FS}$ at which a loan can be sold to outside financiers.

Fire-Sale Discounts. Assets which are purchased by the outside financiers after time 0 suffer a fire-sale discount. These fire-sales can only occur at time 1, since all assets become risk free at time 2 and therefore can be sold to the depositor whose required return R_d is below the outside financier's required return R_a.

Assets sold to outside financier u at time 1 can be re-sold by the outside financiers at

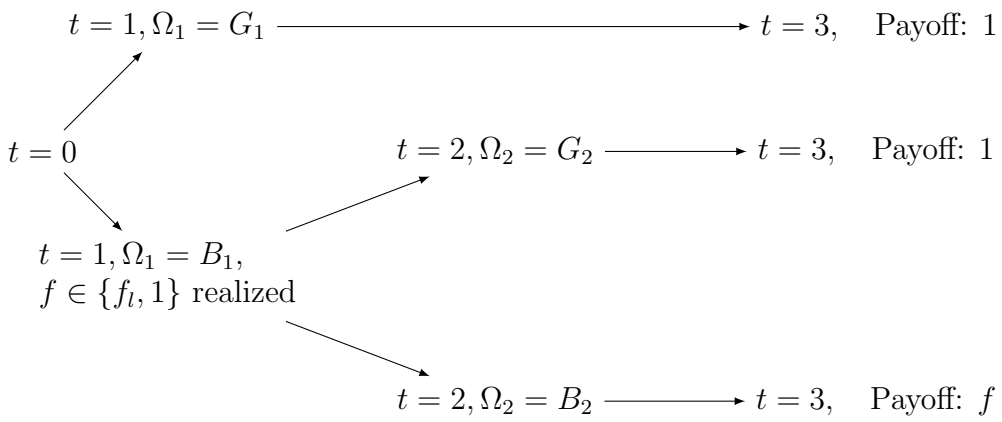


Figure 2: **Loan payoffs and timing.** Capital letters G_t, B_t, for $t = 1, 2$ denote aggregate good or bad states Ω_t in period 1 and 2. The quality of an individual loan $f \in \{f_l, 1\}$ is realized at time 1 and determines its payoff in state B_2. All payoffs occur at time 3.

time 2 with a transaction cost $D_1(u)$, where u is uniformly distributed on $[0, 1]$. To sell $s_{f,1}$ units of the asset, the marginal outside financier with $u = s_{f,1}$ must be indifferent to trading. If loan trades at price $p_{f,2}$ at time 2, the marginal outside financier only gets $(1 - D_1(s_{f,1})p_{f,2}$ when re-selling. Because outside financiers have a discount rate R_a, asset prices must satisfy

$$p_{f,1} = \frac{(1 - D(s_{f,1}))\mathrm{E}\left[p_{f,2}\right]}{(1 + R_a)}. \tag{1}$$

Let

$$p_{f,2}^{FS} = (1 - D(s_{f,1}))p_{f,2} \tag{2}$$

be the price of asset f at time 2 inclusive of the fire-sale discount the outside financier faces, which is determined by the quantity $s_{f,1}$ of this asset sold at time 1.

3.2 Financial Intermediaries

Financial intermediaries are profit-maximizing firms that raise funding from debt and equity investors, invest in loans, and can trade with outside financiers. Because depositors are infinitely risk-averse, we require that intermediaries have enough assets in all states of the world to repay existing deposits, even if they would have to sell assets at the fire-sale price in

equation 2. In addition, securities issued by intermediaries face the same illiquidity frictions as loans if they are sold to outside financiers.

If an intermediary raises d_0 of debt and e_0 of equity at time 0 and invests I_0, its objective function is to maximize its time 0 profit

$$d_0 + e_0 - I_0, \tag{3}$$

subject to making investments with enough future payoffs to compensate investors for the funding d_0, e_0.

The purpose of financial intermediation in this economy is to transform risky, long-maturity, illiquid loans into safe, short-term, liquid debt that can be held by depositors. Because depositors' required return R_d is lower than R_e, an intermediary's cost of funding is lowest when it is able to issue as many deposits as possible and relying as little as possible on equity.

We allow intermediaries to invest in securities issued by other intermediaries. This allows for "credit chains" where intermediaries borrow from investors and then invest in securities issued by other intermediaries that hold loans. A financial system with both banks and CLOs is precisely such a credit chain.

We first analyze the decisions of an intermediary which raises all of its funding from the outside debt and equity investors, which we refer to as a bank. Then, we examine the problem of a second intermediary who may also raise funding from the bank, which will endogenously have a structure that matches our stylized facts about CLOs.

3.3 Bank's Problem

Banks raise all of their funding from outside debt and equity investors and maximize profits as given by equation 3. Let Q_0^B be the quantity of loans the bank buys at time 0, d_0^B be the quantity of deposits it issues, and δ_t^B be the dividend it pays to equity at time $t > 0$.

Because the equity investor is risk neutral with required return R_e, the equity e_0^B it is able to raise at time 0 is given by $e_0^B = \mathrm{E}\left[\sum_{t=1}^{3} \frac{\delta_t^B}{(1+R_e)^t}\right]$. The bank's objective function is therefore

$$-p_0 Q_0^B + d_0^B + \mathrm{E}\left[\sum_{t=1}^{3} \frac{\delta_t^B}{(1+R_e)^t}\right]. \tag{4}$$

The bank chooses to hold a quantity $q_{f,t}^B$ of loans of quality f at time. The bank is only able to issue equity at time 0 but can continue to raise additional deposit funding d_t^B at times $t > 0$, subject to a constraint. Because depositors are infinitely risk-averse, they will only lend to a bank that has enough assets to pay them all off in a potential run. If the bank sold off their assets at the fire-sale price, they would need to be able to repay depositors, resulting in the solvency constraint

$$\overbrace{(1+R_d)d_t^B}^{\text{Payment due to depositors at t+1}} \quad \leq \quad \overbrace{\sum_f p_{f,t+1}^{FS} q_{f,t}^B}^{\text{Market value of bank's assets}} \quad , \quad \forall t = 0,1,2,3\ . \tag{5}$$

In addition, the bank has budget constraints

$$\delta_t^B = d_t^B + \sum_f p_{f,t}(q_{f,t-1}^B - q_{f,t}^B) - (1+R_d)d_{t-1}^B \geq 0 \tag{6}$$

at times $t = 1, 2$ and at time $t = 3$

$$\delta_3^B = \sum_f \delta_f^B q_{f,2}^B - (1+R_d)d_2^B \geq 0. \tag{7}$$

The payoff of a loan of quality f is $\delta_f = f$ if both aggregate bad shocks are realized, and $\delta_f = 1$ otherwise. In addition, $q_{f,0}^B = Q_0^B \pi(f)$ since the quality of loans is not known at time 0 and a fraction $\pi(f)$ of the initial loans Q_0^B gets a quality f at time 1. The bank's problem is to maximize its profits in equation 4 subject to its deposit issuance constraints (equation 5) and budget constraints (equations 6-7). Its choice variables are loan quantities $Q_0^B, q_{f,1}^B, q_{f,2}^B$, deposits d_t^B, and dividends δ_t^B.

Decisions after uncertainty is realized. In period 2, all uncertainty is realized about the payoffs δ_f of loans. At this point, these loans are short-term risk-free assets which can be held by the depositor. The prices of such loans are therefore

$$p^B_{f,2} = \frac{\delta_f}{1 + R_d}. \tag{8}$$

Because $R_e > R_d$, the bank now optimally replaces all of its equity funding with debt finance and pays the largest possible dividend to equity.

Similarly, if the good state occurs at time 1, all uncertainty is again realized and $\delta_f = 1$ for all loans. Again, all loans can be financed entirely by risk-free deposits and have a price

$$p^B_{f,1}(G_1) = \frac{1}{(1 + R_d)^2}, \tag{9}$$

and the bank optimally pays the largest possible dividend. This implies that $\delta^B_3 = 0$, $\delta^B_2 = \sum_f p^B_{f,2} q_{f,1} - (1 + R_d) d^B_1$, and $\delta^B_1(G_1) = Q_0 p^B_{1,1}(G_1) - (1 + R_d) d_0$.

Funding a portfolio most efficiently. Next, we consider how a bank chooses its mix of debt and equity financing, holding fixed its investments $(Q_0, q^B_{f,t})$. Because the cost of equity R_e is greater than the cost of deposits R_d, the bank increases its profits by relying more on deposit financing whenever possible. The following lemma follows straightforwardly from this preference for deposit financing.

Lemma 1. *1. Suppose the bank reaches the bad state B_1 at time 1. Given its choice of assets $q_{f,1}$, it issues the largest possible quantity of deposits $d^B_1(B_1) that it is able to repay.*

2. The bank chooses to pay no dividends in the bad state at time 1, since this payoff could have been used to issue more deposits at time 0 instead.

Proof. Suppose the bank is able to reduce its dividend payments in all states of the world at time t+1, taking as given the assets that it holds at time t. The bank therefore can subtract a risk-free payoff ϵ from its time t+1 dividends in order to issue $\frac{\epsilon}{1+R_d}$ more deposits at time

t that it uses to fund a dividend to equity investors. The derivative of the bank's objective function with respect to such a decrease ϵ in dividends is

$$\frac{1}{(1+R_d)(1+R_e)^t} - \frac{1}{(1+R_e)^{t+1}} > 0 \tag{10}$$

As a result, both at time t=1 and at time t=0, the bank will choose to increase its deposit issuance until its non-negative dividend constraint at time t+1 begins to bind. □

Using this lemma, the bank's deposit issuance $d_1^B(B_1)$ in the bad state is given by

$$d_1^B(B_1) = \frac{1}{1+R_d}\sum_f p_{f,2}^{FS}(B_2) q_{f,1}^B = \frac{1}{(1+R_d)^2}\sum_f (1 - D(s_{f,1})) f q_{f,1}^B. \tag{11}$$

The bank's problem therefore reduces to

$$\begin{aligned}
\max_{Q_0^B, d_0^B, q_1^B(h), q_1^B(l)} & -p_0^B Q_0^B + d_0^B + \Pr(G_1)\frac{1}{1+R_e}\left[\frac{Q_0^B}{(1+R_d)^2} - d_0^B(1+R_d)\right] \\
& + \Pr(B_1)\Pr(G_2)\frac{1}{(1+R_e)^2}\left[\frac{\sum_f q_{f,1}^B}{1+R_d} - \frac{\sum_f (1-D(s_{f,1}))f q_{f,1}^B}{1+R_d}\right] \\
& + \Pr(B_1)\Pr(B_2)\frac{1}{(1+R_e)^2}\left[\frac{\sum_f f q_{f,1}^B}{1+R_d} - \frac{\sum_f (1-D(s_{f,1}))f q_{f,1}^B}{1+R_d}\right],
\end{aligned} \tag{12}$$

subject to the non-negativity of all $Q_0, q_{f,1}^B$ and the budget constraint

$$\sum_f p_{f,1}^B q_{f,1}^B - \frac{\sum_f (1-D(s_{f,1}))f q_{f,1}}{(1+R_d)^2} = Q_0^B \sum_f p_{f,1}^B(B_1)\pi(f) - (1+R_d)d_0^B. \tag{13}$$

This reduction of the bank's problem illustrates the basic economics of financial intermediation in our model. First, the goal of the bank is to finance its purchases of loans Q_0^B with "cheap" deposit financing with a required return R_d instead of "expensive" equity financing with a required return $R_e > R_d$. Second, the bank's financial constraints in the bad states of the world are what limit this deposit issuance. After uncertainty is realized (either at time 2 or if a good shock occurs at time 1), the bank immediately pays the largest possible

dividend. Only in the bad state in time 1, when loan payoffs are still risky, does the bank continue keep a buffer of equity. This is because deposits must be risk-free, so some equity is required to finance the bank's holding of risky assets.

The bank's ability to adjust its loan holdings $q^B_{f,1}(B_1)$ gives it the choice to reduce the risk of its portfolio, potentially allowing it to rely more on risk-free deposit financing. However, potential fire-sale discounts caused by trading restrict the bank's issuance of deposits. For an asset that pays off f in the worst state of the world B_2, the bank is only able to pledge a fraction $(1 - D(s_{f,1}))\, f$ of this payoff to back deposits. This is because the bank needs to keep the market value of its assets above the amount of deposits it must repay, even when fire sales depress the prices of its assets. The bank's willingness to pay for an asset at time 0 is impacted not only by the asset's cashflow but also by the fire sale discounts the asset may face later on.

The bank's willingness to pay for assets. First, equation 11 shows that if the bank buys a risk-free asset with $f = 1$ and no fire-sale discount ($D(s_{f,1}) = 0$) at time 1, the bank is able to issue deposits that are worth $\frac{1}{(1+R_d)^2}$ at time 1 to finance this purchase. The bank is therefore willing to buy such an asset at a price of $\frac{1}{(1+R_d)^2}$. If hypothetically some other agent sold such a risk-free asset at time 1, the bank's ability to issue deposits to fund the purchase of this asset ensures that the asset is never fire-sold to outside financiers. Since no investors' cost of capital is below R_d, the asset's equilibrium price must be $\frac{1}{(1+R_d)^2}$, which we formalize in the following lemma.

Lemma 2. *At time 1, a risk-free asset that pays off* 1 *at time 3 trades for a price of* $\frac{1}{(1+R_d)^2}$*, and such an asset will never be fire-sold to outside financiers.*

Next, we consider the bank's first-order condition of buying a loan, potentially selling it at time 1, and using the proceeds to back deposits. Because we know that the bank will not fire sale a loan with high quality $f = 1$, we consider only two possibilities. First, the bank holds the loan to maturity. Second, if a bank buys a loan at time 0, it chooses to sell it at time 1 if a low quality $f < 1$ is realized.

If the bank buys loans Q_0 at time 0 and sells them at time 1 if they turn out to be low quality in order to buy more risk-free assets, the bank's portfolio at time B_1 gets an additional risk-free payoff of $\delta_d^{B,FS} = \pi(f = 1) + (1 - \pi(f = 1))p_{fl,t}(1 + R_d)^2$ at time 3. Because these risk-free assets do not suffer from a fire-sale discount, it can use all of these cash flows to back deposits $d_0^B = \frac{\delta_d^{B,FS}}{(1+R_d)^3}$. The bank's first-order condition is therefore

$$p_0^{B,FS} = \frac{\delta_d^{B,FS}}{(1 + R_d)^3} + \frac{Pr(G_1)}{1 + R_e}[\frac{1 - \delta_d^{B,FS}}{(1 + R_d)^2}]. \tag{14}$$

If instead the bank buys loans Q_0 at time 0 and holds them to maturity, we have that the bank's time 1 loan holdings satisfy $\frac{\partial q_{f,1}^B}{\partial Q_0} = \pi(f)$. This provides the bank a payoff of $\sum_f f\pi(f)$ at time 3, of which only $\delta_d^{B,HTM} = \sum_f (1 - D(s_{f,1}))f\pi(f)$ can be used to back deposits due to the bank's market-to-market constraint. The bank's first order condition for an asset it is willing to hold to maturity is

$$\begin{aligned} p_0^{B,HTM} =& \frac{\delta_d^{B,HTM}}{(1 + R_d)^3} + \frac{\Pr(G_1)}{1 + R_e}\left[\frac{1 - \delta_d^{B,HTM}}{(1 + R_d)^2}\right] + \frac{\Pr(G_2)\Pr(B_1)}{(1 + R_e)^2}\left[\frac{1 - \delta_d^{B,HTM}}{1 + R_d}\right] \\ &+ \frac{\Pr(B_2)\Pr(B_1)}{(1 + R_e)^2}\left[\frac{\sum_f f\pi(f) - \delta_d^{B,HTM}}{1 + R_d}\right]. \end{aligned} \tag{15}$$

Here, $\delta_d^{B,HTM}$ is the largest possible risk-free payoff that the bank can promise to depositors at time 1 backed by a portfolio of loans held to maturity. This payment depends both on the payoff $\sum_f f\pi(f)$ these loans pay in the worst state of the world and on potential fire sale discounts $D(s_{f,1})$ that may impact the loan's resale prices. As a result, even in the worst state of the world, the bank must promise $\sum_f D(s_{f,1})f\pi(f)$

Equations 14-15 illustrate the costs and benefits of a bank choosing to hold a loan bought at time 0 on its balance sheet instead of selling it at time 1. First, for a loan held to maturity, only its worst realized cashflow f can be used to back deposits, while potentially it could have the choice to sell the asset at time 1 for a greater resale value. Second, the bank's solvency constraint implies that only $(1 - D(s_{f,1}))f$ of this lowest cashflow can be pledged

to depositors, since otherwise the bank would be vulnerable to a potential run. The bank's ability to back deposits then depends not only on the fundamental risk of the assets it holds but also on the possibility of firesales which depress the assets' prices. We summarize our results in the following proposition.

Proposition 1. *1. If a bank buys an asset at time 0 and sells it at time 1, the bank's willingness to pay for this asset at time 0 is given by equation 14. The bank can issue deposits up to the lowest possible market value of the asset at time 1 and must fund the remainder of the asset's purchase with equity.*

2. If a bank buys an asset at time 0 and holds it to maturity, the bank's willingness to pay for this asset at time 0 is given by equation 15. The bank's ability to fund this investment with deposits is strictly less than the amount of risk-free debt the loan's cashflows are able to back. Potential fire sales that depress the loan's price at time 2 reduce the bank's ability to borrow against the asset.

Because we know that risk-free assets would never experience a fire sale at time 1, equation 15 in the special case of a risk-free asset with $\pi(f = 1) = 1$ becomes

$$p_0^{B,HTM,riskfree} = \frac{1}{(1 + R_d)^3} \tag{16}$$

since here $\delta_d^{B,HTM} = \sum_f (1 - D(s_{f,1}))f\pi(f) = 1$. As a result, if another financial institution other than the bank were willing to issue risk-free long term debt, its cost of capital would be the same as the rate R_d at which the bank borrows from depositors. Because this long-term debt would not then be exposed to runs, this raises the possibility of the bank investing in another financial intermediary's long-term debt, which could increase the overal stability of the financial system. We summarize our result in the following lemma. This lemma implies that if a financial intermediary other than the bank were able to issue long-maturity risk-free debt, the intermediary would be able to borrow at the same rate R_d as the bank.

Lemma 3. *The price at time 0 of a risk-free asset that pays 1 at time 3 is* $\frac{1}{(1+R_d)^3}$. *As a result, if a financial institution were able to issue long-maturity risk-free debt, it would borrow at the same rate* R_d *as the bank borrows from depositors.*

3.4 Collateralized Loan Obligations

This section considers the optimal decisions made by a financial intermediary funded with long-maturity risk-free debt that we call a collateralized loan obligation (CLO). It chooses a quantity of risk-free debt d_0^C to issue at time 0, which it sells to the bank at interest rate R_d. The CLO otherwise has the same funding and investment opportunities as the bank. Like the bank, the CLO maximizes profit as given in equation 3. Let Q_0^C be the quantity of loans the CLO buys at time 0, d_0^C be the quantity of long-term debt it issues, and δ_t^C be the dividend it pays to equity at time $t > 0$. Because the equity investor is risk neutral with required return R_e, the equity e_0^C it is able to raise at time 0 is given by $e_0^C = \mathrm{E}\left[\sum_{t=1}^{3} \frac{\delta_t^C}{(1+R_e)^t}\right]$. The CLO's objective function is therefore

$$-p_0^C Q_0^C + d_0^C + \mathrm{E}\left[\sum_{t=1}^{3} \frac{\delta_t^C}{(1+R_e)^t}\right]. \tag{17}$$

The CLO chooses to hold a quantity $q_{f,t}^C$ of loans of quality f at time $t > 0$. The CLO is also only able to issue equity at time 0. Different from the bank, however, CLO debt is long-term. The CLO has budget constraints

$$\delta_t^C = \sum_f p_{f,t}^C \left(q_{f,t-1}^C - q_{f,t}^C\right) \geq 0 \tag{18}$$

at times $t = 1, 2$, and at time $t = 3$

$$\delta_3^C = \sum_f \delta_{f,3} q_{f,2}^C - (1+R_d)^3 d_0^C \geq 0\,. \tag{19}$$

The CLO's problem is to maximize its profits in equation 17 by choosing $d_0^C, Q_0^C, q_{f,t}^C$ subject to its budget constraints (equations 18 - 19).

Given its debt issuance d_0^C, the CLO optimally chooses to pay the largest possible dividend to equityholders after all uncertainty has been realized either at time 2 or at time 1 if state G_1 is realized, subject to replaying its debt. Since all assets compound at the deposit rate R_d once all uncertainty is realized, and the CLO's equityholders have a required return $R_e > R_d$, equity financing is unprofitable once there is no remaining risk. Just like the bank, the CLO wants to remove its reliance on expensive equity financing once uncertainty is resolved.

From this, we first know that $\delta_3^C = 0$, since any dividend paid at time 3 could instead have financed a dividend paid at time 2. The CLO needs a minimum portfolio that pays $(1+R_d)^3 d_0^C$ to repay its debt, so it follows that the CLO's largest possible dividend at time 2 is $\delta_2^C = \sum_f \frac{\delta_f}{1+R_d} q_{f,1}^C(B_1) - (1+R_d)^2 d_0^C$. Finally, to ensure in the time 1 good state that the CLO's assets can pay at least $(1+R_d)^3 d_0^C$, it needs assets at time 1 worth $(1+R_d) d_0^C$. It follows that the CLO's time 1 dividend in the good state is $\delta_1^C(G_1) = \frac{Q_0^C}{(1+R_d)^2} - (1+R_d) d_0^C$.

The CLO's problem therefore reduces to

$$\max_{Q_0^C, q_1^C(h), q_1^C(l), d_0^C} d_0^C - p_0^C Q_0^C + \frac{\Pr(G_1)}{1+R_e}\left[\frac{Q_0^C}{(1+R_d)^2} - (1+R_d) d_0^C\right]$$

$$+\frac{Pr(B_1)\delta_1^C(B_1)}{1+R_e} + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{1}{1+R_d}\sum_f q_{f,1}^C - (1+R_d)^2 d_0^C\right] \tag{20}$$

$$\frac{\Pr(B_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{1}{1+R_d}\sum_f f q_{f,1}^C - (1+R_d)^2 d_0^C\right] \tag{21}$$

subject non-negativity of its portfolio holdings and its dividends and to the budget constraint

$$\sum_f p_{f,1}^C q_{f,1}^C + \delta_1^C(B_1) = Q_0^C \sum_f p_{f,1}^C(B_1)\pi(f).$$

Holding fixed the CLO's asset portfolio and $\delta(B_1)$, the derivative of its objective function

with respect to d_0^C is

$$1 - Pr(G_1)\frac{1+R_d}{1+R_e} - [Pr(B_1)Pr(G_2) + Pr(B_1)Pr(B_2)]\frac{(1+R_d)^2}{(1+R_e)^2} > 0, \tag{22}$$

so the CLO chooses to borrow the maximum quantity of risk-free debt $d_0^C = \frac{\sum_f f q_{f,1}^C}{(1+R_d)^3}$ that its portfolio can back. As with the bank, the CLO prefers to finance itself with "cheap" debt finance and minimizes its realiance on "expensive" equity finance. In addition, if the CLO takes a time 1 dividend $\delta^C(B_1)$ and invests it in a risk-free asset at the rate R_d while issuing this maximum quantity of debt, the derivative of its objective function is

$$\frac{1}{1+R_d} - [Pr(G_1) + Pr(B_1)]\frac{1}{1+R_e} > 0. \tag{23}$$

The CLO therefore chooses not to pay a dividend in the bad state at time 1, so $\delta_1^C(B_1) = 0$. The CLO's problem reduces even further then to

$$\begin{aligned}\max_{Q_0^C, q_{f,1}^C} \frac{1}{(1+R_d)^3}\sum_f q_{f,1}^C f - p_0^C Q_0^C + \frac{\Pr(G_1)}{1+R_e}\left[\frac{Q_0^C - \sum_f q_{f,1}^C f}{(1+R_d)^2}\right] \\ + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{\sum_f (1-f) q_{f,1}^C}{1+R_d}\right].\end{aligned} \tag{24}$$

subject to the non-negativity of all Q_0^C, $q_{f,1}^B$ and the budget constraint

$$\sum_f p_{f,1}^C q_{f,1}^C = Q_0^C \sum_f p_{f,1}^C(B_1)\pi(f).$$

The CLO's objective function in equation 24 illustrates its advantage over the bank as a holder of loans. The key fact is that the CLO is able to issue a risk-free debt claim whose value depends on the present value $\frac{\sum_f q_{f,1}^C f}{(1+R_d)^3}$ of the largest possible risk-free cashflow the CLO can promise to ints investors. Because the CLO's debt is long term and is not vulnerable to runs, fluctuations in the price of the assets it holds do not prevent is from issuing risk-free

debt. Other than this difference, the CLO has the same cost of debt and equity financing as the bank. As we show below, this makes a CLO where the CLO holds risky loans on its balance sheet superior to one where the loans are held directly by banks.

The CLO's willingness to pay for assets. We now examine the CLO's willingness to pay for loans that it either holds to maturity or sells at time 1. If the CLO holds a portfolio of loans to maturity, each unit of loans has a cash flow of 1 in any state except the time 2 bad state and $\sum_f f\pi(f)$ in this bad state. The CLO can therefore issue risk-free debt paying $\sum_f f\pi(f)$ backed by this asset. The remaining loan payoffs must be funded by equity. The CLO's first-order condition for buying loans that it holds to maturity is therefore

$$p_0^{C,HTM} = \frac{\sum_f f\pi(f)}{(1+R_d)^3} + \frac{\Pr(G_1)}{1+R_e}\left[\frac{1-\sum_f f\pi(f)}{(1+R_d)^2}\right] + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{1-\sum_f f\pi(f)}{1+R_d}\right]. \quad (25)$$

Suppose instead that the CLO buys loans at time 0 and then sells its low quality loans (quality $f = f_l < 1$) at time 1 in the bad state. If the CLO re-invests the proceeds into risk-free assets, this would allow it to hold risk-free assets paying $\delta_d^{C,FS} = \pi(f=1) + (1-\pi(f=1))p_{f_l,1}(1+R_d)^2$. The CLO can therefore use this investment to back $\frac{\delta_d^{C,FS}}{(1+R_d)^3}$ at time 0 and finance the remainder of this investment with equity. The price $p_0^{C,FS}$ it would be willing to pay for a loan at time 0 is then

$$p_0^{C,FS} = \frac{\delta_d^{C,FS}}{(1+R_d)^3} + \frac{\Pr(G_1)}{1+R_e}\left[\frac{1-\delta_d^{C,FS}}{(1+R_d)^2}\right]. \quad (26)$$

The CLO's wllingness to pay for a loan at time 0 is similar to that of a bank. Indeed, for loans which are sold at time 1, equations 14 and 26 are identical. In addition, for a loan which is held to maturity, the CLO's willingness to pay is equal to what a bank's would be (in equation 15) if the fire-sale discount $D(s_{f,1})$ were equal to 0. As a result, the CLO is willing to pay either weakly or strictly more than the bank to buy loans at time 0. The model therefore has an equilibrium where the CLO holds all risky assets, and the bank only holds the CLO's risk-free debt. We summarize these results in the proposition below.

Proposition 2. *1. For a loan held to maturity, the CLO's willingness to pay for the loan at time 0 is strictly greater than the bank's willingness to pay if the loan experiences a fire-sale discount* $D(s_{f,1}) > 0$.

2. For loans which are either sold at time 1 or experience no fire-sale discount, the bank and CLO have the same willingness to pay at time 0.

3. As a result, one equilibrium of the model has the CLO holding all risky assets, while the bank only holds the CLO's risk-free debt. Any other equilibrium has the same asset prices and payoffs to debt and equity investors as this equilibrium.

We illustrate the organization of the financial system in figure 3. Without CLOs, banks would hold risky, long-maturity loans financed with both debt and equity. In a financial system where CLOs grow to their steady state size, they entirely crowd out the need for the bank to take risk. CLO equity, instead of bank equity, bears all financial risk. Because CLO debt is long-term, the CLO cannot experience a run. The bank, in contrast, is entirely involved in maturity transformation without holding any risky assets. It holds long-maturity risk-free assets financed entirely by short-maturity deposits. Because the bank never has to sell any of the debt it holds, its asset portfolio never experiences any fire-sale discounts. The CLO performs safety transformation betweek risky assets and safe debt, while the bank performs maturity transformation between long-term assets and short-term liabilities. This division of roles insulates the bank from the negative impact of loan fire sales.

3.5 Equilibrium prices and CLO leverage

We complete our characterization of the model's equilibrium by solving for asset prices. If the CLO holds some low quality loans to maturity and sells others, then both equation 25 and 26 must describe the equilibrium price p_0 of loans at time 0. Loans held to maturity can only back $\sum_f \pi(f)f$ units of deposits, while loans that are sold at time one back $\delta_d^{C,FS} = \pi(f_l)(f_l - p_{f_l,1}(1+R_d)^2)$. If we define the difference between these quantities as $\Delta_d^{CLO} = \sum_f \pi(f)f - \delta_d^{C,FS} = \pi(f_l)(f_l - p_{f_l,1}(1+R_d)^2)$, then we must have by setting 25 and 26 equal

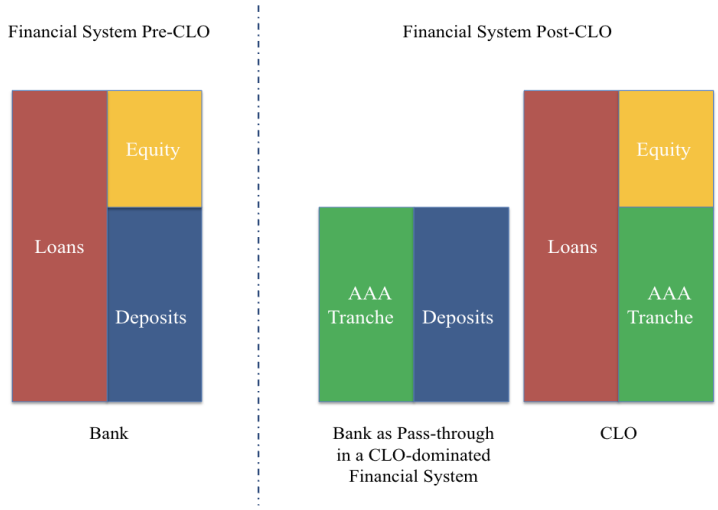


Figure 3: Organization of financial system with and without CLOs.
For each financial institution, we present its assets on the left-hand side of its balance sheet and liabilites on the right.

that

$$0 = (f_l - p_{f_l,1}(1+R_d)^2)[\frac{1}{(1+R_d)^3} - \frac{\Pr(G_1)}{(1+R_e)(1+R_d)^2}] + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{(1-f_l)}{1+R_d}\right]. \quad (27)$$

Equation 27 pins down the equilibrium price of a low-quality loan $p_{f_l,1}$ sold in the bad state at time 1. These sold loans are purchased by the outside financier, so this loan price must also satisfy by equation 1

$$p_{f_l,1} = \frac{(1-D(s_{f_l,1}))E(p_{f_l,2})}{1+R_a} = \frac{(1-D(s_{f_l,1}))(Pr(G_2)+f_l Pr(B_2))}{(1+R_a)(1+R_d)}. \quad (28)$$

Given a price $p_{f_l,1}$, equation 28 determines the quantity $s_{f_l,1}$ of loans the CLO sells at time 1.

Finally, we can use these prices and sale quantities $s_{f,1}$ to determin the total supply of risk-free debt the CLO can issue. In the bad state at time 1, the CLO holds $Q_0\pi(f_h)$ units of high-quality loans that are risk-free. In addition, it sells $s_{f_l,1}$ of its low quality loans at a

price $p_{f_l,1}$, which it invests in additional risk-free assets at the rate R_d for a risk-free payoff of $s_{f_l,1}p_{f_l,1}(1+R_d)^2$ at time 3. Finally, it continues to hold $Q_0\pi(f_l) - s_{f_l,1}$ of low-quality loans to maturty, which each have a minimum payoff of f_l. Summing across all of these loans gives the CLO the ability to back debt of face value d_0^C given by

$$d_0^C = \frac{1}{(1+R_d)^3}\left[s_{f_l,1}p_{f_l,1}(1+R_d)^2 + \left(Q_0\pi(f_l) - s_{f_l,1}\right)f_l + Q_0\pi(f_h)\right] . \tag{29}$$

We provide explicit expressions for $p_{f_l,1}$ and $s_{f_l,1}$ in appendix A.1.

One feature of this equilibrium is that the CLO holds all of the high-quality loans $Q_0\pi(f_h)$ plus an additional investment at the risk-free rate in the bad state at time 1. Because the only other risk-free asset than high-quality loans is the CLO's own debt, this can be interpreted as the CLO buying back (or repaying) its debt in the bad state of the world at time 1.

3.6 Empirical realism of the model's CLOs

This section explains how the CLOs that emerge in our model's equilibrium are consistent with our motivating facts in section 2. Our model is consistent with the first fact, that CLO's raise the majority of their funding in the form of long-maturity AAA debt that is held by banks. Our CLO does in fact issue long-maturity safe debt, and all of it is held by banks. While our model so far has no parameters, so we cannot pin down a specific amount of leverage, our quantitative analysis below is able to match the fact that CLOs are funded with (65 percent) AAA rated debt (Cordell et al., 2021).

Our second fact, that CLOs actively rebalance their portfolios of risky loans, is also consistent with the model. CLO's are able to issue more safe debt at time 0 by committing themselves to selling low-quality loans at time 1. In addition, these sales cause fire-sale discounts that temporarily depress loan prices, consistent with empirical work on CLOs. As empirically documented by Elkamhi and Nozawa (2022), CLOs voluntarily pay down their outstanding debt in states of the world where they face financial distress. This "callability"

of CLO debt, giving the CLO the option to voluntarily prepay, is a common feature of CLO indentures.

The model also is consistent with fact 3, that the contractual restrictions on CLO dividend payments do not reference the market prices of CLO loan portfolios. The CLO in our model is restricted from paying a dividend in the bad state at time 1, while it can pay a dividend after good news is realized at times 1 or 2. The fact that CLOs issue long-term rather than short-term debt means that they are able to violate the market-to-market solvency constraint that a bank must satisfy at time 1. It is precisely the CLO's ability to allow the market value of its asset holdings in a crisis to fall below the value of its outstanding debt that makes it more efficient than a bank. Like the financial covenants we observe in real-world CLO contracts, our model features CLOs whose dividend payments are restricted in bad states of the world without any reference to the secondary market prices of the assets the CLO holds.

The model illustrates how the many unique features of CLOs are jointly designed to improve the securitization process. CLOs long-term debt financing allows them to survive when the market value of their assets temporarily crashes. As a result, CLOs are ideally designed to be able rebalance a portfolio of illiquid assets over time, since the CLOs asset sales can depress the market-to-market value of the assets it continues to hold.

3.7 Quantifying the Benefit of CLOs

We now use our model to quantify the benefits of a CLO-based financial over a traditional financial system where banks lend directly to firms. The advantage of CLOs over a traditional banking system is due to the fact that CLOs only have to worry about the cashflows paid by their investment portfolio. This is because CLO's long-term debt financing insulates them from possible bank runs. The CLO therefore can survive when the market value of the assets it holds during a fire-sale is less than the value of its debt liabilities. As the fire-sale discount caused by selling loans to outside financiers converges to 0, so does the advantage of CLOs over traditional banking.

To compare CLOs to a traditional banking system, we first compute the amount of leverage the CLO is able to take on by avoiding a bank's market-to market solvency constraint. If the CLO buys loan and holds them to maturity, the CLO is able to have outstanding risk-free debt that pays

$$\sum_f f\pi(f). \tag{30}$$

Instead, if the bank held this same loan portfolio, because of the bank's mark-to-market solvency constraint, it would only be able to promise a risk-free payoff to its depositors of

$$\sum_f (1 - D(s_{f,1}))f\pi(f). \tag{31}$$

The CLO is therefore able to promise additional risk-free cash flows of

$$\Delta_d = \sum_f D(s_{f,1})f\pi(f). \tag{32}$$

Δ_d is the additional the CLO can pay to its risk-free debtholders above what the bank is able to pay. $\frac{\Delta_d}{1+R_d}$ is also the difference between the fire-sale price of the CLO's assets in the time 2 bad state and the value of its outstanding liabilities. It is precisely because the CLO is able to be market-to-market insolvent during a financial crisis that it is a more cost-effective investor than a bank. This shows why it is crucial for a CLO's financial covenants that restrict its dividends to equity holders not to depend on the market value of its asset holdings.

Based on this greater ability to borrow against loans it holds to maturity, the CLO is also willing to pay more for such loans than a bank. This extra willingness to pay is equal to the value of using Δ_d to fund debt issuance instead of dividends to equity, resulting in the following proposition.

Proposition 3. *Compared to a traditional banking system where banks lend directly to firms, the price p_0 of loans at time 1 in a system with CLOs is strictly greater. The difference in*

prices is given by

$$\Delta^{RR} \sum_{f} D(s^{B}_{f,1}) f \pi(f)\,, \tag{33}$$

where

$$\Delta^{RR} = \frac{1}{(1+R_d)^3} - \frac{\Pr(G_1)}{(1+R_e)(1+R_d)^2} - \frac{\Pr(B_1)\Pr(G_2)}{(1+R_e)^2(1+R_d)} - \frac{\Pr(B_2)\Pr(B_1)}{(1+R_d)(1+R_e)^2}\,. \tag{34}$$

and $s^{B}_{f,1}$ is the quantity of loan f that is sold to outside financiers in the equilibrium of the traditional banking economy.

Financial stability implications. Next, we examine the impact of adding CLOs to a traditional banking system on financial stability. To do so, we consider two parallel economies: one with a traditional banking system and another where CLOs are active. In either economy, some quantity of lows of low quality f_l is liquidated in a fire sale in the time 1 bad state. If a bank or CLO buys loans at time 0 and sells the fraction $\pi(f_l)$ of these loans that become low quality at time 1 in exchange for risk-free assets, they are able to buy a risk-free payoff of $\pi(f_l)p_{f_l,1}(B_1)(1+R_d)^2$ at time 3. In addition, the remaining fraction $\pi(f_h)$ of loans that end up to be high quality each make a payoff of 1 at time 3. As a result, in either economy, equations 14 and 26 imply that loan prices at time 0 must satisfy

$$p_0 = \frac{\pi(f_h) + \pi(f_l)p_{f_l,1}(B_1)(1+R_d)^2}{(1+R_d)^3} + \frac{\Pr(G_1)}{1+R_e}\left[\frac{1-[\pi(f_h)+\pi(f_l)p_{f_l,1}(B_1)(1+R_d)^2]}{(1+R_d)^2}\right]. \tag{35}$$

If we denote $p_0^{bank}, p_{f,1}^{bank}$ the prices of loans at time 0 and 1 in the traditional banking economy and $p_0^{CLO}, p_{f,1}^{CLO}$, then equation 35 implies

$$p_0^{CLO} - p_0^{bank} = \pi(f_l) \left[\frac{1}{1+R_d} - \frac{Pr(G_1)}{1+R_e} \right] (p_{f_l,1}^{CLO} - p_{f_l,1}^{bank}). \tag{36}$$

Based on the CLO and bank's willingness to hold loans to maturity, we already have an expression for this difference in prices based on proposition 3, so we have that

$$\frac{\Delta^{RR} \sum_f D(s_{f,1}^B) f \pi(f)}{\pi(f_l)[\frac{1}{1+R_d} - \frac{Pr(G_1)}{1+R_e}]} = p_{f_l,1}^{CLO} - p_{f_l,1}^{bank} > 0, \tag{37}$$

where Δ^{RR} is given in equation 34.

As a result, the price of a low-quality loan in the time 1 bad state is strictly higher in the financial system with CLOs than in a traditional banking system. This implies that fewer loans $s_{f,1}$ are sold to the outside financier, resulting in a smaller fire-sale discount. We summarize our results in the following proposition, which implies that CLOs make the financial system more stable.

Theorem 1. *The price at time 1 of a loan of low quality f_l is strictly higher in a fianancial system with CLOs than a traditional banking system where banks hold loans directly. The difference in prices of such a loan across these two financial systems is given in equation 37. This implies that in a traditional banking system, a larger quantity of these low-quality loans are sold to outside financiers, resulting in a larger fire-sale discount.*

4 Numerical Example

In this section, we present a simple calibration of our model to quantify the impact of CLOs on the financial system. As we describe in the appendix, we extend the model to incorporate a continuous probability distribution for idiosyncratic payoffs f, improving the quantitative fit of the model. The complete optimization problems faced by both the CLO and the bank,

are provided in Appendix B, with full characterizations in Sections **??** and B.1.2.

In Section 4.1, we calibrate the model to key metrics, including CLO debt and equity returns, the probability of a bad state (which we see as a financial crisis), empirical measures of fire-sale discounts, CLO capital structure, and the returns on a leveraged loan portfolio. Section 4.2 then presents a counterfactual analysis to quantify the efficiency of a CLO-based financial structure in supporting safe debt levels and generating shareholder value.

4.1 Calibration

The model parameters and the model fit to the data targets are presented in Tables 1 and 2. Further details on the calculation of model moments are provided in Appendix B.

We set the return on the CLO equity tranche, R_e, and the AAA debt tranche, R_d, to 9.88% and 2.33%, respectively, following Cordell et al. (2021). We target the average AAA-debt CLO leverage to 63% as we calculate in Semeris.

The fire sales discount follows the functional form

$$D(s_{f,1}) = 1 - \exp(-\kappa s_{f,1}).$$

This exponential functional form implies that $\frac{1}{s_{f,1}} \frac{d(1-D(s_{f,1}))}{ds_{f,1}}$ is constant, so that selling a given fraction of the outstanding stock of an asset decreases its price by a fixed fraction of its current value. We use this functional form to match the estimates of Elkamhi and Nozawa (2022). They find that selling 1% of the total outstanding stock of CCC-rated loans by CLOS decreases their price by 2.34%, and 40% of such loans are owned by CLOs. As a result, a sale of 1% of CLO holdings of a loan should depress its price by $.4 * 2.34\% = .936\%$. We therefore set $\kappa = .936$.

Despite its simplicity, the model aligns well with the data, particularly in achieving a CLO leverage level of 60%, even with a significant fire-sale discount that limits the CLO's ability to maintain risk-free debt. The model nearly replicates the price change of the LSTA

Leveraged Loans Index in and out of crises, achieving values of 0.80 and 1.31 respectively, and captures the unconditional average return on the loan index at 3.48% is only somewhat below the observed 5.06%.

Table 1: Model Parameters

Targets	**Definition**	**Value**	**Source**
Return on CLO equity tranche	R_e	0.0988	Cordell et al. (2021)
Return on CLO AAA debt tranche	R_d	0.0233	Cordell et al. (2021)
Probability of crisis	$\Pr(B)$	0.083	LSTA US Leveraged Loan (2000-2024)
CLO fire sale discount	κ	0.936	Elkamhi and Nozawa (2022)
Outside Discount rate	R_a	0.0435	Calibrated to Fit Table 2
Mean idiosyncratic shocks	μ	0.54726	Calibrated to Fit Table 2
St.dev. idiosyncratic shocks	σ	0.12938	Calibrated to Fit Table 2

Baseline parameterization of the model.

Table 2: Model Fit

Targets	**Data**	**Model**	**Source**
CLO Leverage	0.63	0.60	Semeris
Annual Expected Return on Leveraged Loan	1.051	1.035	LSTA US Leveraged Loan (2000-2024)
Price Change on Leveraged Loan into crisis	0.79	0.80	LSTA US Leveraged Loan (2000-2024)
Price Change on Leveraged Loan out of crisis	1.36	1.31	LSTA US Leveraged Loan (2000-2024)

Data targets used to calibrate the model as well as the corresponding model values. Returns are reported in annualized levels.

4.2 Results

Counterfactual analysis. Table 3 compares leverage, and asset values, and social welfare across across three distinct financial systems. First, we examine a financial system with CLOs as described in our theoretical model. Next, we consider a financial system where all loans

are held by securitization vehiles that cannot rebalance their portfolios. Finally, we consider a traditional financial system, where banks that issue deposits and equity directly hold risky loans.

Our social welfare metric is

$$\mathcal{W} = p_0 Q_0 + \frac{Pr(B_1)}{1+R_a} \int \int_0^{s_{f,1}} [D(s_{f,1}) - D(u)] \frac{f + Pr(G_2)(1-f)}{1+R_a} du d\mu(f). \tag{38}$$

To justify this metric, note that both equity and debt investors have linear utility and hence obtain zero welfare whenever their first-order conditions are satisfied. The first term $p_0 Q_0$ equals the funding these agents are willing to pay to the providers of loans held within the financial system. The second term is the present value at time 0 of future gains earned by the outside financiers who buy assets. Because these outside financiers have heterogeneous costs $D(u)$ of trading assets, agents with a cost below the market-clearing value $D(s_{f,1})$ obtain consumer surplus. The difference $[D(u) - D(s_{f,1})] \frac{f+Pr(G_2)(1-f)}{1+R_a}$ is the difference between their valuation of asset f and the market-clearing price. Integrating this profit across all outside financiers with $u < s_{f,1}$ who buy asset f and then across all assets f yields their total welfare gain when state B_1 occurs. This quantity times $\frac{Pr(B_1)}{1+R_a}$ gives the present value at time 0 of their future gains from trading at time 1, since they only trade when state B_1 occurs and their discount rate is R_a.

We compare these financial systems at our calibrated parameter values in table 3. Leverage d_0 represents the quatity of risk-free debt issued at time 0 in each financial system. The baseline CLO shows the highest leverage ratio at 61%, benefiting from its ability to rebalance its asset holdings. The CLO without rebalancing and traditional bank both have lower leverage ratios of 51% and 56%, respectively. We then consider the price p_0 at which loans trade in each financial system. The CLO achieves the highest asset value at 0.8994 is 74 basis points higher than the price of a loan when only banks exist. As a quick bank-of-the-envelope calculation, the 74 basis point higher return CLOs can offer for loans above banks

times their $1.3 trillion total portfolio is $9.6 billion of total additional profit.

Buy-and-hold securitization yields the same price as a CLO because the CLO holds a positive quantity of loans of any quality f. As a result, the buy-and-hold securitzation vehicle's first-order conditions hold in the CLO equilibrium. Because debt and equity investors have knife-edge linear objective functions in our model, this implies that the buy-and-hold equilibrium must have the same loan price. Once we include the welfare gains of outside financiers, whose preferences have curvative, we see that in terms of welfare, CLOs strictly dominate both buy-and-hold securitization and a traditional banking system.

Table 3: Loan Prices and Intermediary Leverage Across Financial Systems

Variables	**CLOs**	**Buy-and-Hold Securitization**	**Traditional Banks**
Safe Debt Quantity d_0	0.605	0.510	0.564
Loan Price p_0	0.899	0.899	0.892
Welfare $\mathcal{W}$	0.9039	0.899	0.896

We perform a sensitivity analysis of these safe debt quantities and welfare in figures 4 and 5. First, we vary the parameter κ that determines the cost associated with fire sales in figure 4. As κ increases, safe debt issuance d_0 decreases for all models. The baseline CLO, represented by the blue line, initially holds higher leverage when fire sale costs are low, reflecting its ability to rebalance its portfolio. As we showed theoretically, CLOs only have an advantage over banks when holding assets that experience fire-sale dsicounts. As a result, banks and CLOs converge to the same equilibrium allocation as κ approaches 0. However, as κ rises and fire sales become more costly, leverage gradually declines. As κ becomes sufficiently large, the leverage of the CLO approaches that of a buy-and-hold securitization vehicle. This illustrates why CLOs are active in the moderately liquid market of leveraged loans to firms. Treasury bonds have no need to be securitized, and mortgages which are too small to trade in a securities market are securitized by vehicles that hold assets to maturity.

The second figure explores the impact of changes in the mean of idiosyncratic shocks,

μ. Here, as μ increases, leverage d_0 rises across all models, since the amount of risk in the economy is decreasing. We consider only the range of μ from .2 to .8 since our standard deviation of .36 places bounds on the possible mean of a random variable contained within the unit interval.[2] When μ is low, loans cannot back a large quantity of risk-free assets when held to maturity. As a result, bank and CLO leverage is far above that of buy-and-hold securitization. As μ increases and the economy becomes low risk, there are fewer low-quality assets which a bank or CLO would benefit from selling.

Welfare also increases for all three financial systems as the mean of bad-state asset payoffs increase. Interestingly, the buy-and-hold securitization vehicle eventually provides welfare that surpasses the welfare of the bank, even though banks have the option to hold their assets to maturity. This is because banks face a pecuniary externality, where the fire sales of one bank tighten the solvency constraints of other banks. This illustrates why even in markets where banks could hold loans to maturity, there is even a benefit of creating securitization vehicles that cannot trade assets over time.

In both comparative static exercises, we uniformly see that CLOs can maintain higher leverage and provide greater overall welfare than either a traditional banking system or a system with buy-and-hold securitization. The benefits of a CLO are greatest when the illiquidity of their assets is moderate, and the calibrated parameter $\kappa = .936$ we inferred from Elkamhi and Nozawa (2022) that measures the illiquidity of loans is in range where CLOs add the most value. In this range, CLOs can both benefit from the ability to rebalance their loan portfolios and also benefit from their long-term financial that insulates them against the impact of fire-sales.

[2]These actual bounds are .153 to .847, but the distribution becomes too singular for our method to accurately integrate near these cutoffs.

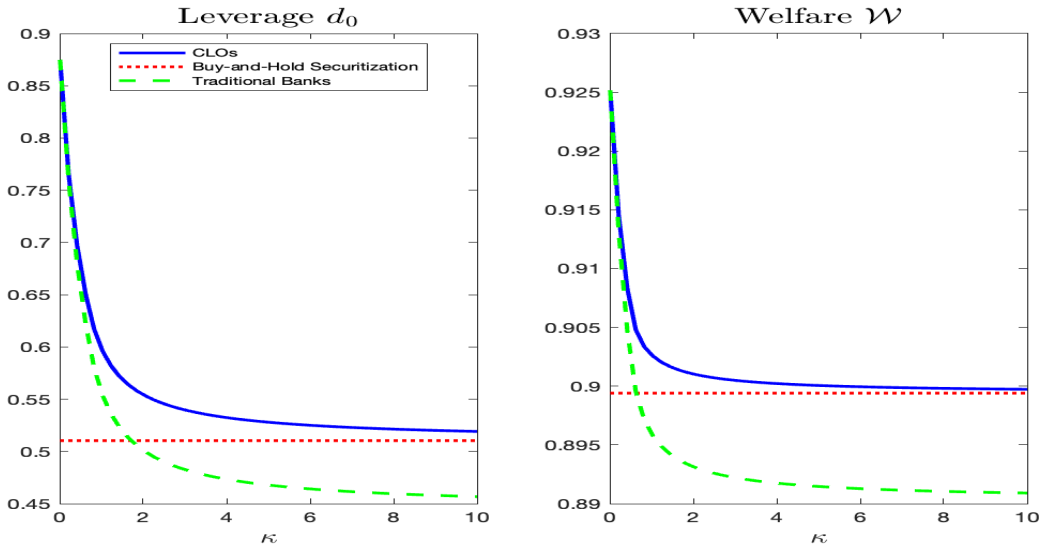


Figure 4: Comparative statics with respect to fire sale cost κ in baseline CLO model, model without rebalancing, and traditional banking model. The blue line represents the equilibrium choices of the CLO, the red line represents the equilibrium choices of the CLO without rebalancing and the green line represents the choices of the bank.

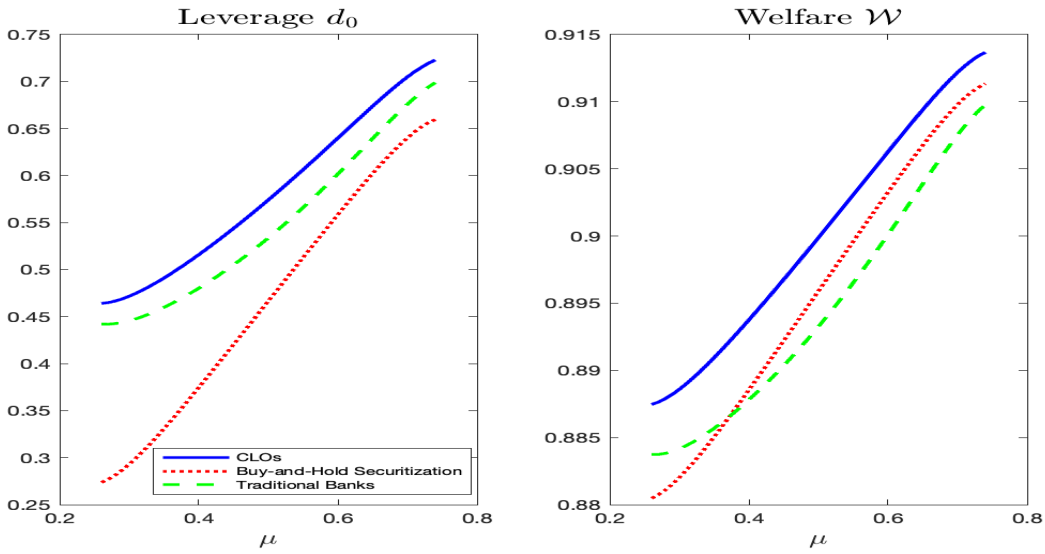


Figure 5: Comparative statics with respect to the mean of idiosyncratic shocks μ in baseline model, model without rebalancing and bank model. The blue line represents the equilibrium choices of the CLO, the red line represents the equilibrium choices of the CLO without rebalancing and the green line represents the choices of the bank.

5 Conclusion

This paper shows how the contractual structure of CLOs is ideally designed for securitizing assets that trade in illiquid markets. CLOs are able to sell poorly-performing assets during crises, and this increases the amount of safe debt CLOs are able to issue before crises. Because CLO debt is long term and CLO covenants allow CLOs to become insolvent on a mark-to-market basis during crises, CLOs are shielded from the negative effects of fire sales. Banks which are funded with short-maturity deposits that hold the safe long-term debt of CLOs are therefore insulated from the effects of fire sales in risky asset markets. This financial structure's advantage over traditional banking is greatest when assets are the least liquid, since fire-sales would then harm banks the most. Similarly, this financial structure's advantage over buy-and-hold securitization are the greatest when assets are the most liquid, since illiquidity reduces the gains from trade. Compared to traditional banking and buy-and-hold securitization, CLOs are therefore the most valuable in markets that are moderately illiquid. This explains why leveraged loans are an ideal asset class for CLOs. As financial innovation expands the list of assets which can be traded, one might expect CLOs to be a useful financial structure for a growing share of the overall financial system.

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A Appendix

A.1 Explicit expressions for loan prices and quantities

From equation 27, we know that

$$0 = \pi(f_l)(f_l - p_{f_l,1}(1+R_d)^2)[\frac{1}{(1+R_d)^3} - \frac{\Pr(G_1)}{(1+R_e)(1+R_d)^2}] + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{(1-f_l)\pi(f_l)}{1+R_d}\right]. \tag{A.1}$$

The implied loan sale price is then

$$p_{f_l,1} = \frac{f_l[\frac{1}{(1+R_d)^2} - \frac{\Pr(G_1)}{(1+R_e)(1+R_d)}] + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}[(1-f_l)]}{(1+R_d)^2[\frac{1}{(1+R_d)^3} - \frac{\Pr(G_1)}{(1+R_e)(1+R_d)^2}]} \tag{A.2}$$

Given this loan price, we also have from the outside financier's first-order condition (equation 1)

$$p_{f_l,1} = \frac{(1-D(s_{f,1}))\mathrm{E}\left[p_{f_l,2}\right]}{(1+R_a)} = (1-D(s_{f,1}))\frac{f_l + Pr(G_2)(1-f_l)}{1+R_a} \tag{A.3}$$

$$s_{f_l,1} = D^{-1}(1 - \frac{(1+R_a)p_{f_l,1}}{f_l + Pr(G_2)(1-f_l)}). \tag{A.4}$$

The economy has a unique equilibrium with an interior solution for the quantity $s_{f_l,1}$ of loans sold if equation A.4 has a solution with $0 < s_{f_l,1} < 1$ under the assumption that D is a strictly increasing function.

B Numerical Exercise Appendix

B.1 Model Appendix

B.1.1 CLO

The model we use in our numerical exercise is identical to the one in the main text, except that the quality of loans f that can be drawn at time 1 is continuously distributed according to a measure μ. As we derived in the main text in equation 24, the CLO's objective function

can be written as

$$\max_{Q_0^C, q_{f,1}^C(B_1)} - p_0^C Q_0^C + \frac{1}{(1+R_d)^3}\int q_{f,1}^C(B_1) f df + \frac{\Pr(G_1)}{1+R_e}\left[\frac{Q_0^C}{(1+R_d)^2} - \frac{1}{(1+R_d)^2}\int q_{f,1}^C(B_1) f df\right] + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{1}{1+R_d}\int q_{f,1}^C(B_1) df - \frac{1}{1+R_d}\int q_{f,1}^C(B_1) f df\right], \tag{B.5}$$

subject the nonnegativity of dividends paid to equity at time $t > 0$ and

$$Q_0^C \int p_{f,1}^C(B_1) d\mu(f) = \int p_{f,1}^C(B_1) q_{f,1}^C(B_1) df\,. \tag{B.6}$$

Given the CLO's quantity Q_0^C of loans at time 0, it will choose some qualities f where it sells all loans at time 1, others where it sells a fraction of loans, and some where all loans of quality f are held to maturity.

We consider the CLO's first-order condition of buying some loans at time 0 and adjusting its loan holdings at time 1 in the direction $\frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C}$ which must satisfy

$$\int p_{f,1}^C(B_1) d\mu(f) = \int p_{f,1}^C(B_1) \frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C} df\,.$$

The CLO is indifferent on the margin to buying or selling loans of any quality f where it transacts at the market-clearing price, so any such $\frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C}$ is admissible so long as $\frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C} = 0$ for the qualities f that the CLO entirely sells.

The CLO's first-order condition is

$$p_0^C = \frac{1}{(1+R_d)^3}\left[\int \frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C} f df\right] + \frac{\Pr(G_1)}{(1+R_e)(1+R_d)^2}\left[1 - \left(\int \frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C} f df\right)\right] + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2(1+R_d)}\left[\int \frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C}(1-f) df\right].$$

This can be rewritten as

$$\int \frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C}[f[\frac{1}{(1+R_d)^3} - \frac{Pr(G_1)}{(1+R_e)(1+R_d)^2}] + \frac{Pr(B_1)Pr(G_2)(1-f)}{(1+R_e)^2(1+R_d)}]] df = p_0^C - \frac{Pr(G_1)}{(1+R_e)(1+R_d)^2} \tag{B.7}$$

.

The left hand side of equation B.7 must be equalized across all admissible perturbations $\frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C}$. If we take a sequence of $\frac{\partial q_{f,1}^C(B_1)}{\partial Q_0^C}$ that converge to a point mass of spending all of $\int p_{f,1}^C(B_1) d\mu(f)$ only on loans of quality f, we get

$$\frac{\int p^C_{f',1}(B_1)d\mu(f')}{p^C_{f,1}}[f[\frac{1}{(1+R_d)^3}-\frac{Pr(G_1)}{(1+R_e)(1+R_d)^2}]+\frac{Pr(B_1)Pr(G_2)(1-f)}{(1+R_e)^2(1+R_d)}]=$$
$$p^C_0-\frac{Pr(G_1)}{(1+R_e)(1+R_d)^2} \tag{B.8}$$

. The effective discount rate for a cashflow f on the right hand side is strictly lower than the discount rate for the remaining risky cashflow $1-f$. It follows that the CLO requires a higher expected return for loans with a lower value of f. Because the outside financier is risk neutral, this implies that: 1. the CLO will pay strictly more than the outside financier for $f > f^*$ for some f^*. Then the CLO and outside financier are both indifferent to holding loans at a quality f^* with no firesale discount. After this, as the expected return for the CLO decreases with f, the outside investor needs a growing firesale discount to not strictly prefer buying the asset. This implies that $s^C_{f,1}$ is greater for lower levels of f. Finally, there is some value f^- at which the entire stock of a loan is sold to the outside financier at a discount given by $D(1)$. We also know that $f^* < 1$ since risk-free assets are discounted at the rate of return $R_d > R_a$. The value of f^- may or may not be greater than 0.

For any $f^C \in [f^{-1}, f^*]$, we can therefore consider an admissable perturbation $\frac{\partial q^C_{f,1}(B_1)}{\partial Q^C_0}$ where all assets up to a quality f are sold to buy risk-free assets (or equivalently pay down the CLO's debt) and all assets above quality f^C are held to maturity. This yields the first-order condition

$$\begin{aligned}
&-p^C_0+\frac{1}{(1+R_d)^3}\left[\int_{f^C}^1 f d\mu(f)+\int_0^{f^C} p^C_{f,1}(B_1)(1+R_d)^2 d\mu(f)\right]\\
&\quad+\frac{\Pr(G_1)}{1+R_e}\left[\frac{1}{(1+R_d)^2}-\frac{1}{(1+R_d)^2}\left(\int_{f^C}^1 f d\mu(f)+\int_0^{f^C} p^C_{f,1}(B_1)(1+R_d)^2 d\mu(f)\right)\right]\\
&\quad+\frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{\int_{f^C}^1(1-f)d\mu(f)}{1+R_d}\right]=0\,.
\end{aligned}$$

Differentiating this expression with respect to f^C yields for any $f \in [f^-, f^*]$

$$\underbrace{\frac{(p^C_{f,1}(B_1)(1+R_d)^2-f)}{(1+R_d)^3}}_{\text{benefit: add'l. deposits gained}}=\underbrace{\frac{\Pr(G_1)}{1+R_e}\frac{(p^C_{f,1}(B_1)(1+R_d)^2-f)}{(1+R_d)^2}+\frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\frac{(1-f)}{1+R_d}}_{\text{cost: dividends lost}}\,.$$

This expression represents the additional deposits the CLO can back by re-balancing portfolio. The left hand side (LHS) captures the amount of deposits gained, while the two terms on the right hand side (RHS) reflect the present value of dividends forfeited when

transitioning from holding assets to maturity to fire selling them. Solving for $p^C_{f,1}(B_1)$ yields

$$p^C_{f,1}(B_1) = \frac{\frac{f}{(1+R_d)^2}\left[\frac{1}{1+R_d} - \frac{\Pr(G_1)}{1+R_e}\right] + \frac{1-f}{1+R_d}\frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}}{\left[\frac{1}{1+R_d} - \frac{\Pr(G_1)}{1+R_e}\right]},$$

which gives

$$p^C_{f,1}(B_1) = \frac{f}{(1+R_d)^2} + \frac{\frac{1-f}{1+R_d}\frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}}{\left[\frac{1}{1+R_d} - \frac{\Pr(G_1)}{1+R_e}\right]}. \tag{B.9}$$

The quantity $s_{f,1}$ sold of loans of quality f for $f \in [f^{-1}, f^*]$ is given by

$$p^C_{f,1}(B_1) = (1 - D(s_{f,1}))\frac{f + \Pr(G)(1-f)}{(1+R_a)(1+R_d)}. \tag{B.10}$$

$$s_{f,1} = D^{-1}\left[1 - \left(\frac{f}{(1+R_d)^2} + \frac{\frac{1-f}{1+R_d}\frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}}{\left[\frac{1}{1+R_d} - \frac{\Pr(G_1)}{1+R_e}\right]}\right)\frac{(1+R_a)(1+R_d)}{f + \Pr(G)(1-f)}\right] \tag{B.11}$$

The cutoff f^* is given by the value of f which solves equation B.11 with $s_{f^*,1} = 0$. The cutoff f^- is given by the value of f which has a solution with $s_{f^*,1} = 1$.

Finally, we can rewrite the total amount of debt the CLO is able to back as

$$\begin{aligned} d^C(f^*, f^-) =& \frac{\int \left((1-s_{f,1})f + s_{f,1}p_{f,1}(B_1)(1+R_d)^2\right) d\mu(f)}{(1+R_d)^3} \\ =& \frac{\int_{f^*}^1 f d\mu(f)}{(1+R_d)^3} + \frac{\int_0^{f^-} p^{FS}_{f,1}(B_1)(1+R_d)^2 d\mu(f)}{(1+R_d)^3} \\ &+ \frac{\int_{f^-}^{f^*}\left((1-s_{f,1})f + s_{f,1}p^C_{f,1}(B_1)(1+R_d)^2\right) d\mu(f)}{(1+R_d)^3}, \end{aligned} \tag{B.12}$$

B.1.2 Traditional Banking Economy

Here, we charecterize the equilibrium of the economy where only banks exist without CLOs.

The bank's problem reduces to (as we showed when deriving equation 12)

$$\begin{aligned} \max_{Q^B_0, d^B_0, q^B_{f,1}(B_1)} & -p^B_0 Q^B_0 + d^B_0 + \frac{\Pr(G_1)}{1+R_e}\left[\frac{Q^B_0}{(1+R^B_d)^2} - d^B_0(1+R^B_d)\right] \\ &+ \frac{\Pr(B_1)\Pr(G_2)}{(1+R_e)^2(1+R^B_d)}\left[\int q^B_{f,1}\, d\mu(f) - \int q^B_{f,1}\left(1 - D(s_{f,1})\right) f\, d\mu(f)\right] \\ &+ \frac{\Pr(B_1)\Pr(B_2)}{(1+R_e)^2(1+R^B_d)}\left[\int q^B_{f,1}\left(1 - (1 - D(s_{f,1}))\, f\, d\mu(f)\right]\right., \end{aligned}$$

subject to

$$
\begin{aligned}
&Q_0^B \int p_{f,1}^B(B_1)\, d\mu(f) - d_0^B(1+R_d) \\
=&\int p_{f,1}^B(B_1) q_{f,1}^B(B_1)\, d\mu(f) - \frac{1}{(1+R_d)^2} \int q_{f,1}^B(B_1)(1-D(s_{f,1}))f\, d\mu(f)\,.
\end{aligned}
$$

Following the same approach as for the CLO, one can show that there is a cutoff f^- below which the bank sells all loans of quality f, a cutoff f^* above which they hold all loans to maturity, and an interval in between with an interior solution. For any $f^B \in [f^-, f^*]$ we have the first-order condition

$$
\begin{aligned}
p_0^B =& \frac{1}{(1+R_d)^3}\left[\int_{f^B}^1 f\left[1-D(s_{f,1})\right] d\mu(f) + \int_0^{f^B} p_{f,1}^B(B_1)(1+R_d^B)^2 d\mu(f)\right] \\
&+ \frac{\Pr(G_1)}{1+R_e}\left[\frac{1}{(1+R_d)^2} - \frac{1}{(1+R_d)^2}\left(\int_{f^B}^1 f\left(1-D(s_{f,1})\right) d\mu(f) + \int_0^{f^B} p_{f,1}^B(B_1)(1+R_d^B)^2 d\mu(f)\right)\right] \\
&+ \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{\int_{f^B}^1 (1-f\left[1-D(s_{f,1})\right]) d\mu(f)}{1+R_d}\right] \\
&+ \frac{\Pr(B_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{\int_{f^B}^1 f(1-\left[1-D(s_{f,1})\right]) d\mu(f)}{1+R_d}\right].
\end{aligned}
$$

Notice the first bracketed term of the expression for p_0^B is the total amount of deposits the bank can issue, we denote that by $d(Q_0^B, f^B)$

$$
d(Q_0^B, f^B) \coloneqq \underbrace{\frac{\int_{f^B}^1 f\left(1-D(s_{f,1})\right) d\mu(f)}{(1+R_d)^3}}_{\text{worst payoff of assets HTM}} + \underbrace{\frac{\int_0^{f^B} p_{f,1}^B(B_1)(1+R_d^B)^2 d\mu(f)}{(1+R_d^B)^3}}_{\text{payoff of asset fire sold}},
$$

we can write the FOC with respect to Q_0^B more succinctly as,

$$
\begin{aligned}
p_0^B(f^B) = d(Q_0^B, f^B) &+ \frac{\Pr(G_1)}{1+R_e}\left[\frac{1}{(1+R_d)^2} - (1+R_d) d(Q_0^B, f^*)\right] \\
&+ \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{\int_{f^B}^1 \left(1-f\left(1-D(s_{f,1})\right)\right) d\mu(f)}{1+R_d}\right] \\
&+ \frac{\Pr(B_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{\int_{f^B}^1 f\left(1-\left(1-D(s_{f,1})\right)\right) d\mu(f)}{1+R_d}\right]. \qquad \text{(B.13)}
\end{aligned}
$$

Differentiating this expression with respect to f^B yields for all $f \in [f^-, f^*]$

$$\begin{aligned}
&\frac{\left(p^B_{f,1}(B_1)(1+R_d)^2 - f\left(1-D(s_{f,1})\right)\right)}{(1+R_d)^3} \\
=&\frac{\Pr(G_1)}{1+R_e}\frac{\left(p^B_{f,1}(B_1)(1+R_d)^2 - f\left(1-D(s_{f,1})\right)\right)}{(1+R_d)^2} + \frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2}\frac{\left(1-f\left(1-D(s_{f,1})\right)\right)}{1+R_d} \\
&+ \frac{\Pr(B_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{f\left(1-(1-D(s_{f,1}))\right)}{1+R_d}\right].
\end{aligned}$$

Solving for $p^B_{f,1}(B_1)$ yields

$$p^B_{f,1}(B_1) = \frac{f\left(1-D(s_{f,1})\right)}{(1+R_d)^2} + \frac{\frac{1-f\left(1-D(s_{f,1})\right)}{1+R_d}\frac{\Pr(G_2)\Pr(B_1)}{(1+R_e)^2} + \frac{\Pr(B_2)\Pr(B_1)}{(1+R_e)^2}\left[\frac{f\left(1-\left(1-D(s_{f,1})\right)\right)}{1+R_d}\right]}{\left[\frac{1}{1+R_d} - \frac{\Pr(G_1)}{1+R_e}\right]}. \tag{B.14}$$

The outside financier's first-order condition implies that prices must also satisfy

$$p^B_{f,1}(B_1) = \left[1-D(s_{f,1})\right]\frac{f+\Pr(G)(1-f)}{(1+R_a)(1+R_d)}. \tag{B.15}$$

The cutoff f^* is pinned down by solving equations B.14 and B.15 with $s_{f^*,1}=0$. Similarly, the cutoff f^- solves the same equations with $s_{f^-,1}=1$. The interior value of $s_{f,1}$ which solves these two equations for f between these two cutoffs pins down the bank's portfolio rebalancing choices. We verify numerically that these soltutions are unique in our numerical example. Finally, we can rewrite the total amount of debt the bank is able to back as

$$\begin{aligned}
d^B(f^*, f^-) =&\frac{\int\left((1-s_{f,1})f\left(1-D(s_{f,1})\right) + s_{f,1}p^B_{f,1}(B_1)(1+R^B_d)^2\right)d\mu(f)}{(1+R^B_d)^3} \\
=&\frac{\int_{f^*}^1 f\left(1-D(s_{f,1})\right)d\mu(f)}{(1+R_d)^3} + \frac{\int_0^{f^-} p^{FS}_{f,1}(B_1)(1+R^B_d)^2 d\mu(f)}{(1+R^B_d)^3} \\
&+ \frac{\int_{f^-}^{f^*}\left((1-s_{f,1})f\left[1-D(s_{f,1})\right] + s_{f,1}p^B_{f,1}(B_1)(1+R^B_d)^2\right)d\mu(f)}{(1+R^B_d)^3}.
\end{aligned} \tag{B.16}$$

B.2 Calibration Appendix

Our calibration targets for loan returns come from the time series of the Morningstar LSTA Leveraged Loans Index spanning 2000 to 2024. We calculate the unconditional probability of a crisis as $\frac{1}{12}$, considering the Global Financial Crisis (GFC) and the Covid crisis as the only two crises over the 24-year sample. The model matches the annualized average return

on the index, 5.06%, with

$$\frac{1-\Pr(B)\Pr(B)+\Pr(B)\Pr(B)\int f\,d\mu(f)}{p_0}=(1+0.0506)^3.$$

We compute the price change of the leveraged loan index into (top tick to bottom tick) and out (bottom tick to top tick) of the two crises, averaging them to obtain 0.79 and 1.40, respectively. The model counterparts are:

$$\frac{\int p_{f,1}(B)\,d\mu(f)}{p_0}=0.79,$$
$$\frac{\frac{1}{1+R_d}\left(\Pr(G)+\int f\,d\mu(f)\Pr(B)\right)}{\int p_{f,1}(B)\,d\mu(f)}=1.4.$$

The idiosyncratic shocks follow a generalized Beta distribution with support $[0,1]$ and are parameterized by mean μ and standard deviation σ. These parameters, together with the outside financier rate of return R_a, allow us to match the average return, as well as the returns entering and exiting crises, alongside the CLO leverage ratio.

Fire Sale Cost. We start from the expression for the fire sale price

$$p_{f,1}^{FS}(s_{f,1})=(1-D(s_{f,1}))\,p_{f,1}^{FS}(0),$$

where $p_{f,1}^{FS}(0)=\frac{f+\Pr(G)(1-f)}{(1+R_a)(1+R_d)}$. Taking the derivative of the price (change) with respect to $s_{f,1}$ we get

$$p_{f,1}^{'FS}(s_{f,1})=-D'(s_{f,1})p_{f,1}^{FS}(0).$$

Dividing both sides by $p_{f,1}^{FS}$ we get the percent change in price (given that $s_{f,1}\in[0,1]$

$$\frac{p_{f,1}^{'FS}(s_{f,1})}{p_{f,1}^{FS}(s_{f,1})}=-D'(s_{f,1})\frac{p_{f,1}^{FS}(0)}{p_{f,1}^{FS}(s_{f,1})}=\frac{-D'(s_{f,1})}{1-D(s_{f,1})}.$$

Notice that the left-hand side represents exactly $d\log p_{f,1}^{FS}(s_{f,1})/ds_{f,1}$ and so maps into the definition of the (negative) price impact per unit of quantity sold (percentage change) as provided in Elkamhi and Nozawa (2022) and equal to 2.34%. We time this number by the share of loans held by CLO which is roughly 40%. Hence a function for $(1-D(s_{f,1}))$ that satisfies this equation is $\exp(-2.34*0.4s_{f,1})$. Hence we can impute $\kappa=2.34*0.4$.

Leveraged Loans Index Returns. Our main source for calibration targets is the time series of the Morningstar LSTA Leveraged Loans Index from 2000 to 2024. We compute returns as

$$Ret_t=\frac{\text{Close Price}_t}{\text{Close Price}_{t-1}}-1$$

- We calculate the average monthly return and annualized it. The resulting target is

0.0506.

- We approximate the unconditional probability of a crisis as 1/12, which means once every 12 years.
- We compute the change in price between $2008-06-30$ (GFC peak) and $2008-12-31$ (GFC through) and between $2020-01-31$ (Covid crisis peak) and $2020-03-31$ (Covid through) and average the two. The resulting target is 0.79.
- We compute the change in price $2008-12-31$ (GFC through) and $2009-12-31$ (GFC recovery) and between $2020-03-31$ (Covid through) and $2021-02-28$ (Covid recovery) and average the two. The resulting target is 1.36.