Strategic Money and Credit Ledgers

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Abstract

A sufficiently dominant BigTech trading platform can set up a ledger that allows firms to purchase inputs by issuing unsecured IOUs (possibly in the form of digital tokens), which then serve as a savings vehicle and medium of exchange. The platform, unlike a stand-alone ledger, can incentivize the use of the ledger and enforce repayment by compromising the universality of the liquidity of public funds. A universal CBDC, which ensures universality of payment, undermines this credit enforcement. In GE, IOU ledger lending lowers the equilibrium interest rate, but also increases the rent extraction of the platform. A higher rate of inflation undermines the public money alternative, enabling platform-operated ledger credit enforcement, but also leading to higher trading mark-ups and rent extraction.

Keywords: Ledgers, currency competition, private currencies, "Industrial Organization of Money", platforms, Bigtech.

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1 Introduction

Historically, payment and credit services have been provided differently from the current bank-centric arrangement. For example, in early modern London a credit system emerged that involved tradable, uncollateralized IOUs. Merchants bought grain from farmers in exchange of IOUs, sold the grain in London and subsequently repaid the IOU holders. This meant that the merchants became active participants in both the grain and secondary IOU markets and so a "bills-of-exchange" system emerged. However, scaling this system proved challenging because of limitations on record keeping and IOU enforcement. Many proposals were put forward to address these challenges (e.g. Smith (1776)) but what emerged was a collateral-dependent bank lending system that suffers from financial exclusion. Recent advancements in ledger and trading platform technologies have led to renewed interest outside of the banking sector in scaling up an uncollateralized credit system. BigTech platforms have taken steps to overcome the record keeping and enforcement challenges. In China, Alibaba's MyBank offers members of their ecosystem uncollateralized loans and supply chains platforms organize tradable account receivables (e.g. Liu et al. (2022)). In India, FinTech startups have attempted to offer similar services without the "backing" of large trading platforms and with less success, Rishabh and Schäublin (2021).

This paper investigates why a large private BigTech trading platform can emerge as the provider of the settlement ledger that records IOUs (possibly represented by a digital token). Although the underlying IOUs contracts are uncollateralized, issuers have no incentive to default. To do this, we build a general equilibrium production model in which settlement assets and ledgers are privately provided. In contrast to Kocherlakota (1998) in which a benevolent planner organizes the monopolistic ledger, we consider a private profit-motivated intermediary which offers a centralized ledger that competes with standard cash payments. Consequently, the intermediary has to incentivize to use the ledger instead of the cash payment instrument. More specifically, we start with a two-period real model and then extend it to an infinite-horizon

monetary macro-model. In our two-period real model, agents issue IOUs to purchase inputs in the initial period and then trade their output in the second period.

First, consider a standalone ledger operator. It adds a second payment option through a centralized ledger to the standard "spot exchanges" (which requires "cash input goods" to be held in advance). These ledger payments are directly recorded and IOU contracts are executed without requiring a resource in advance constraint. In our model, contracts are automatically enforced when payments are received through the ledger because the ledger automatically redirects payments to settle IOUs. Conceptually, this is similar to a worker taking a loan from their employer where a portion of his wages is diverted to repay the loan. By contrast, when agents receive a spot payment, then the seller may default and therefore he offers a better price. As a result, in equilibrium all agents prefer to store the cash endowment good and plan to default on IOUs. In anticipation of this, no IOUs are accepted and funding dries up.

Next, consider a BigTech platform that operates both a trading platform and a centralized ledger. The trading platform insists that on-platform trades must be settled on the ledger. In doing so, it destroys the universality of cash endowment goods. Now, agents who stored the endowment good, instead of lending it out in exchange for IOUs, may be stuck with an unusable payment instrument if they need to trade through the platform. If a sufficiently large fraction of trades occur on the platform and the platform's markup charges are not too high, agents accept IOUs and use them (or the digital token representing them) as payment instrument on the ledgers. Consequently, IOUs are perfectly enforced and defaults do not occur. Note that the platform only finds it attractive to set up a ledger that incentivizes contract repayment if it controls a large fraction of the goods trading, That is, we have a "natural" monopoly on contract enforcement.

Our finding that the platform's ability to destroy the universality of cash liquidity is essential for the emergence of an unsecured credit market has important implications for the introduction of a central bank digital currency

(CBDC). CBDC is a public government ledger. If CBDC is universal, so that the platform is forced to accept CBDC payments in addition to platform ledger payments, then our unsecured private credit market is not viable. However, the public ledger may be able to enforce repayment. In the alternative case where the government does not force the adoption of the public CBDC ledger, then it also potentially incentivizes the platform to create a "black market" with its own private tokens. This is because previously the government provided the unmonitored external cash option and the platform provided the contract enforcement, whereas now the platform is the one that can provide the hidden side trading option.

In Section 3, we extend our analysis to an infinite-horizon monetary model to study feedback in the payment and trading systems. We introduce settlement through two possible financial assets: government money and IOUs on the ledger. Again, this could be interpreted as a "bills-of-exchange" model where tradable IOUs (possibly represented in form of digital tokens) are used as the medium of exchange on the platform. We show that, in our monetary model, the ledger-operating platform can and will ensure IOU repayment under certain conditions, generalizing our results from Section 2.

Our dynamic, monetary model derives two main additional insights. First, we show that the platform markup affects the equilibrium interest rate. When markups are low, then more trades occur on the platform and more agents save in the form of ledger IOUs, which increases the loan supply. By contrast, when markups are high, then agents tend to hold government money until they want to consume, which limits loan supply and translates into a higher equilibrium interest rate. The higher earned interest rate partially offsets the markup disincentive to trade through the platform, and so allows the platform to charge high markups without significantly losing customers. Second, whether the platform ledger credit and payment system is viable depends on the inflation rate on public money. Public money has to be sufficiently unattractive outside option. Also, the higher the inflation, the higher the markups the platform charges its clients for trading on it.

We conclude our analysis by studying competition between two *private*

platforms, each providing its own, possibly interoperable, trading and settlement technologies. We show that competing private platforms that bundle ledger and trading technologies will cooperate on contract enforcement as long as the gap between their respective trading technologies is not too large and financial frictions do not prevent the less efficient platform from committing to pay the more efficient platform. Otherwise, a dominant platform will emerge that will attract more trades and extract higher rents.

Literature Review. Our paper is related to several branches of research. First, it connects to the literature concerning the role of ledgers and settlement assets in organizing trading systems. Aiyagari and Wallace (1991) and Kocherlakota (1998) study how a planner can increase the contracting space by updating a common ledger with trading histories. Freeman (1996b,a) studies how the choice of settlement asset creates or mitigates trading frictions in the currency market. Our model shares many features with these papers. However, we consider an environment where a private, profit-maximizing agent controls the ledger. This brings an industrial organization perspective to the literature on ledgers and settlement assets. In Brunnermeier and Payne (2023), we extend our model to study strategic information portability decisions in a contested market setting.

Second, we relate to the literature studying how FinTech can expand uncollateralized debt limits. In our model, the BigTech ledger operator can incentivize the repayment of debt contracts by threatening to seize assets and exclude agents if they don't use platform tokens (representing IOUs recorded on the ledger). In this sense, forcing the use of the ledger creates a form of "digital collateral" that can be used in automated contracting (e.g. Garber et al. (2021), Kahn and van Oordt (2022)). This would resolve the supply chain contracting issues presented in Bigio (2023). Empirically, Liu et al. (2022) documents BigTech uncollateralized lending successes in China. Rishabh and Schäublin (2021) documents for India that, without a coordinating BigTech platform, borrowers' non-cash revenue drops after fintech companies disbursed "digitally collateralized" loans.

Third, we relate to the growing field of digital currencies. We are most closely related to the papers that study private Tech platforms providing centralized currencies, (e.g. Chiu and Wong (2020), Chiu and Koeppel (2025), Cong et al. (2020), Ahnert et al. (2022)). More generally, we touch upon the growing literature on central bank digital currency (e.g. Fernández-Villaverde et al. (2020), Keister and Sanches (2019), Kahn et al. (2019)) and decentralized, programmable cryptocurrencies (e.g., Fernández-Villaverde and Sanches (2018), Benigno et al. (2019), Abadi and Brunnermeier (2024), Schilling and Uhlig (2019), Cong et al. (2021)).

Forth, we relate to the literature on currency competition (e.g., Hayek (1976), Kareken and Wallace (1981), Brunnermeier and Sannikov (2019)). Formally, our dynamic model in section 3 expands on the two currency cash-in-advance model from Svensson (1985) and endogenizes currency demand using search and trading frictions in the tradition of the new monetarist literature (e.g. Lagos and Wright (2005), Lagos et al. (2017)).

We structure the paper in the following way. Section 2 solves the two-period version of the model with a monopoly ledger controller and exogenous platform demand. Section 3 extends the analysis to an infinite-horizon macroeconomic model in which agents endogenously choose whether to trade on or off the platform. Section 4 concludes.

2 An Illustrative Model of Ledgers, Platforms, and Contract Enforcement

In this section, we outline a two-period version of our model. We use this model to highlight how a large trading platform can expand uncollateralized lending in the economy, but other market arrangements cannot. To keep the focus on enforcement issues, we start with a real model where the ledger manages physical trades as well as record keeping. In the next section, we consider a monetary economy where the ledger manages the trade of tokenized claims to production output and financial assets in the economy.

2.1 Environment and First Best Allocations

Setting, production, and preferences: Time lasts for two periods: $t \in \{0, 1\}$. The economy contains a collection of storable "endowment goods" and a collection of perishable "output goods". The economy is populated by a unit continuum of savers and a unit continuum of producers. Each saver is born with one unit of an endowment good at $t = 0$. Each producer can transform 1 endowment good at $t = 0$ into $z \in (1, 2)$ output goods at $t = 1$. Each saver can store 1 endowment good at $t = 0$ to get 1 endowment good at $t = 1$. Savers get linear utility from consuming goods at the start of $t = 1$ while producers get linear utility from consuming goods at the end of $t = 1$. No agents get utility from consuming the goods that they are endowed with or have produced. Instead, they only derive utility from goods endowed to or produced by other agents. Ultimately, this means that agents need to trade endowment goods in order to produce at $t = 0$ and output or endowment goods in order to consume at $t = 1$.

Information, commitment, and matching frictions: Agents have publicly verifiable identities and can therefore be identified in $t = 1$. However, agents' actions are not publicly observable, agents cannot commit, and there is no public legal system for contract enforcement.

A benevolent planner who cares equally about all agents, reallocates 1 endowment good to each producer at $t = 0$ to maximize production and reallocates output goods across agents. We refer to this as the **first-best allocation**. The goal of this section is to explore which private payment, record-keeping, and trading technologies are able to implement the best allocation despite information and commitment frictions in the economy. We first show that introducing an independent stand-alone common record keeping ledger into the economy is not sufficient to achieve the first best. We then show that a large trading platform may choose to provide the ledger technology that leads to contract enforcement and the first best allocation.

2.2 Payment Technologies and a Common Ledger

In order to consider whether the private sector can implement the first-best allocation, we need to specify the trading and payment arrangements in the economy. In this section, we introduce a ledger technology that (i) collects and records trades and contracts and (ii) settles all trades and/or contracts at the end of each period. Next section, we introduce a platform that controls the trading system.

Trading arrangements and payment technologies: At $t = 0$, a competitive market opens for IOU contracts between savers and producers that promise R goods in period $t = 1$ for each endowment good given in period $t = 0$.

At $t = 1$, producers enter with output goods and savers enter with either stored endowment goods or IOUs. At the beginning of the period, a competitive market opens for trading goods and IOUs but is subject to payment frictions. There are two payment technologies in the economy for settling goods trades at $t = 1$: spot payments and ledger payments. **Spot payments** (indexed by s) are not recorded and are settled immediately. Im impose that spots trades can only be undertaken in trades which involve endowment goods. We interpret this as saying that endowment goods are universally liquid whereas output goods are not. **Ledger payments** (indexed by l) are recorded and can be made with any goods. At the time of the transaction, agents give their output goods to the ledger. At the end of the period, agents can return to the ledger and the ledger settles all transactions and IOUs with seniority given to earlier claims. This means the ledger automatically uses revenue from ledger trades to settle contracts but cannot use revenue from spot trades. Since all trades occurs before IOU settlement, this means that contracts are only enforced when the output producer trades through the ledger. We let ϕ denote the endogenous probability that an IOU is repaid. We use output goods as the numeraire and let (p, q) denote the number of endowment goods and IOUs required to purchase one output good.

Producer's problem: At $t = 1$, a producer chooses whether to trade their

output good for other output goods, IOUs, or endowment goods to solve:

$$V_1^p = \max\{z - R,\ (1-\phi)Rqz - R,\ pz\} \tag{2.1}$$

where the first, second, and third terms are the payoff when the producer trades for output goods, IOUs, and endowment goods respectively. In the first two cases, the producer must pay through the ledger and so cannot default while in the third case they can use spot trade and default. At $t = 0$, producers issue IOUs if

$$V_1^p \geq 0, \tag{2.2}$$

Saver's problem: At $t = 1$, a saver trades for either output goods or other endowment goods. If they hold an endowment good, then they solve:

$$V_1^{s,e} = \max\{1,\ (1-\phi)Rq/p,\ 1/p\} \tag{2.3}$$

where the first, second, and third terms are the payoff when they trade for other endowment goods, IOUs, or output goods. If they hold an IOU, then they solve:

$$V_1^{s,i} = \max\{p/q,\ (1-\phi)R,\ 1/q\} \tag{2.4}$$

where first, second, and third terms are the payoffs when they trade their IOU for endowment goods, other IOUs, or output goods. At $t = 0$, the saver's problem is to decide whether to store endowment goods or exchange endowment goods for IOUs. They make this choice by solving:

$$\max\{V_1^{s,e}, V_1^{s,i}\} \tag{2.5}$$

where the first term is the expected payoff from storing endowment goods and the second term is the payoff for an IOU.

Equilibrium: An equilibrium is a collection of prices (R, p, q) such that savers

and borrower make optimal choices satisfying (2.1), (2.2), (2.3), (2.4), and (2.5) and markets clear at $t = 0$ and $t = 1$.

Proposition 1. *The only equilibrium is that savers store endowment goods, no IOUs are issued, and no production takes place.*

Proof. See Appendix 1. □

Proposition 1 highlights that introducing a stand-alone record keeping ledger does not resolve the contracting problems in the economy. This is because agents have insufficient disincentive from taking the outside option and trading using the spot market. Relative to Kocherlakota (1998), the ledger in our environment has competition from another payment technology and so the economy needs an institution to incentivize the use of the ledger.

2.3 Trading Platform

We now introduce a large trading platform into the economy. This means the economy now has a large institution that can force trade onto the ledger.

Trading technologies: Suppose there are now two trading technologies for connecting agents in the economy, indexed by $n \in \{o, p\}$. Trading technology $n = o$ is not controlled by anyone and is referred to as the **"open" public marketplace**. Trading technology $n = p$ is controlled by a profit maximizing organization, which we refer to as the **private platform**. Agents are randomly allocated to the technologies each period. With probability $1 - \eta$ producers are allocated to posting on the public marketplace and with probability η they post on the the private platform. We endogenize η in Section 3. The platform charges a markup μ on the profit received by savers using their trading technology. We impose that all profits from markups collected by the platform at t are redistributed lump sum to the agents as dividends.

We assume that the platform provides both the trading technology and the common settlement ledger for the economy. We assume the platform can control which payment technology is used for its trades, forces agents using

the platform to make payments using their ledger, and blocks agents from using endowment goods for trades on the ledger. We also allow the platform to guarantee IOUs (act as a discounter for the financial system).

Producer's problem: At $t = 1$, if the producer goes to the public market place, then their decision is the same as in Subsection (2.2) while if they go to the platform, then they are only allowed to post trades using the ledger so they can only trade for output goods and IOUs. This means their value at time $t = 1$ becomes:

$$\begin{aligned} V_1^p = \eta \max\{z - R,\ (1-\phi)Rqz - R\} \\ + (1-\eta)\max\{z - R,\ (1-\phi)Rqz - R,\ pz\} \end{aligned}$$

At $t = 0$, producers issue again issue IOUs if $V_1^p \geq 0$.

Saver's problem: At $t = 1$, if the saver is allocated to the public marketplace, then their problem is the same as in (2.2) while, if they go to the platform, then they can only trade if they accepted IOUs. Thus,

$$\begin{aligned} V_1^{s,e} &= (1-\eta)\max\{1,\ (1-\phi)Rq/p,\ 1/p\} \\ V_1^{s,i} &= \eta(1-\mu)\max\{(1-\phi)R,\ 1/q\} \\ &\quad + (1-\eta)\max\{p/q,\ (1-\phi)R,\ 1/q\} \end{aligned}$$

At $t = 0$, their optimization problem is once again $\max\{V_1^{s,e}, V_1^{s,i}\}$.

Proposition 2. *For sufficiently large η, the platform operates the ledger, offers to discount IOUs, and sets the maximum markups that are incentive compatible with contract enforcement:*

$$\mu \leq 1 - \left(\frac{4}{z} - 1\right)\frac{1-\eta}{\eta} =: \bar{\mu} \tag{2.6}$$

In this case, the first best allocation is implemented. For sufficiently low η, the platform does not set up a ledger and there is no production in the economy.

Proof. See Appendix A.2. □

If $\eta = 1$, then it is clear that the platform can ensure contract enforcement because it can force all trades in the economy through their ledger. The reason why they can potentially ensure contract enforcement when $\eta < 1$ is more subtle. If the platform penalizes bringing endowment goods to the platform ledger by setting high $\mu(i)$, then it discourages agents from storing endowment goods and instead encourages them to produce. If fewer agents are storing endowment goods, then it is hard for agents to find spot trades and default at $t = 1$. That is, making endowment goods unattractive on the platform dries up the market for side-spot trades at $t = 1$. Ultimately, this decreases the interest rate, and so makes agents more willing to produce and work through the ledger. In this sense there is feedback: if the platform forces agents to use the ledger for settlement, then it becomes more attractive to produce and use the ledger for settlement.

Theorem 2 also shows that the dominance of the trading platform, as measured by η, characterizes whether the economy has a problem with monopoly "rent" extraction or a problem with credit "fragility". The intuition for this is the following. The platform derives profit from charging markups on trade. However, it also incentivizes late consumer repayment by threatening to exclude them from trade. This means that increasing the markup to increase profits will also decrease the severity of the exclusion punishment. Together, these forces mean that the maximum markup the platform can charge and maintain contract enforcement is given by (2.6). When η is high, the platform can maintain a positive markup while still incentivizing no-default, and so is willing to set up the ledger. However, when η is low, the platform would have to offer a negative markup (i.e. a subsidy) to make exclusion from trade sufficiently costly to incentivize no-default. In this case, it prefers to not set up the common ledger.

Corollary 1. *If the common settlement ledger and trading technology are operated independently by separate institutions and one of the institutions does*

not cooperate on exclusion, then all output good producers default and there is no production in the economy.

Why is cooperation between the ledger operator and the platform required to ensure contract enforcement? If the ledger automatically enforces contracts but the platform allows agents to undertake spot payments regardless of whether they have defaulted, then all agents use spot trades and default. If the platform attempts to ensure no-default, then it needs cooperation from the ledger. Agents trade before contract settlement, and so the platform needs to force agents to use the ledger so that revenue can be seized to fulfill the contract. But, if the ledger operator does not allow the ledger to be used for payment on the platform, then the platform cannot do this.

Taking stock, we have the following key lessons from our stylized two-period model that we explore further in subsequent sections.

(i) **Ledgers are only useful if they are "backed":** The ledger record keeping technology is potentially very powerful in the economy but only if agents use it. This means that platforms controlling the trading technologies in the economy need to "back" the ledger by forcing agents to use it. Otherwise, introducing the ledger technology will not change the equilibrium. In this sense, "BigTech" platforms are natural providers of currency ledgers. For example, the dominance of Alibaba and WeChat in the Chinese payment system might reflect their underlying advantage in providing payments.

(ii) **The payment technology can collateralize sales revenue:** In this economy, the type of payment technology matters for the collateralizability of future sales revenue. Trades settled using the common ledger can always be used for the repayment of contracts and so essentially act as digital "collateral" for borrowing. Trades not settled using the ledger are not automatically used for the repayment of contracts and so can only be used as collateral if the agents coordinate on reporting and excluding defaulting agents. In this sense, the model is set up so the platform can choose to what extent future sales revenue can be collateralized across

the economy. We extend this in the monetary model in Section 3, where we allow collateralized sales revenue to be traded as "bills-of-exchange" that relax the cash-in-advance constraint.

(iii) **The economy can have monopoly rents or credit fragility:** When the platform is very dominant in the economy we see a key trade-off with having the platform provide a common ledger. On the one hand, the platform takes actions to get agents to cooperate on enforcing default by threatening to seize their resources and exclude them from the ledger. In this sense, we get a "fortuitous" alignment of incentives where the platform wants to force the agent to behave in the way that they would choose if they could agree to coordinate. On the other hand, the ledger controller has a monopoly over the provision of ledger services and so is able to extract rents by charging a high transaction fee.

When the platform is not very dominant in the economy, the alignment of incentives breaks down and the platform is unwilling to provide the common ledger for the economy. This is because the cost of making exclusion from the platform a sufficiently harsh punishment becomes too much for the platform to pay. In this sense, the uncollateralized credit equilibrium is "fragile"; a weak platform finds it too costly for the platform to set up a no-default ledger. We explore these tradeoffs in more detail in the dynamic general equilibrium model in Section 3.

(iv) **A natural monopoly dilemma:** We can also see that there is a type of natural monopoly force in this economy. The more trade that uses the ledger (the higher is η), the more easily the ledger controller can enforce contracts. For example, suppose that the minimum η for which the no-default incentive compatibility constraint holds and $\bar{\mu} > 0$ is greater than $1/2$. In this case, there is no way for multiple ledgers to operate in the economy and enforce contracts unless they cooperate on enforcement. In other words, one large ledger provider can better enforce contracts than a collection of non-cooperative smaller ledger providers. So, a regulator in this environment needs to find a way to get a monopoly ledger provider

to behave more competitively or have multiple large ledgers compete on markups while coordinating on contract enforcement. We take up these questions in section 3.7.

2.4 Remark: Other Potential Ledger Providers

So far, we have shown that if it has sufficient market power, it will implement the first-best allocation. We close this section by discussing why other institutional arrangements would be less successful in implementing the first best.

Banks as ledger and uncollateralized credit providers: There is a large literature showing that the threat of exclusion from future access to the loan market can incentivize the repayment of uncollateralized loans (e.g. Kehoe and Levine (1993)). This might suggest that traditional banks should be able to provide the ledger for uncollateralized loans instead of platforms. The reason this is not possible in our model is because agents only need to access the loan market once, and so exclusion from future credit is not costly. Instead, the agents need to interact repeatedly with the goods market, and so exclusion from the goods market is the costly exclusion. This makes the platform the natural institution for threatening punishment for default. If we expanded the model to have additional periods and repeated borrowing, then there could be equilibria with uncollateralized credit supported by exclusion from future lending, as in many finance models. We do not consider such equilibria for two reasons. First, this credit could be provided by *either* the bank or platform (or any other large agent in the economy). In this sense, the bank is not better at excluding agents from future credit than the platform. Second, exclusion from future credit has already been successfully studied in the literature and we want to focus on new mechanisms related to the emergence of platforms.

Industry supply chain as ledger and uncollateralized credit providers: We could also ask whether a platform on which all goods are traded is a superior ledger provider compared to a supply chain platform that is restricted to a subset

of goods, e.g., everything related to cars for an automobile supply chain platform. Answering this question goes beyond our analysis of our formal model, as the model does not distinguish between utility from different types of goods. Our insight that the power of a platform to exclude agents from trading after defaulting on debt is essential to make ledger participation attractive. If the platform covers all goods, the IOUs are denominated in the overall consumption basket, and hence neither the platform nor the agents face any "exchange rate risk". In contrast, in a setting with a platform that covers only a subset of goods, agents face an exchange rate risk, which makes the threat of exclusion less powerful.

3 Dynamic Model of Platform & Ledger Provision

The model in Section 2 illustrated the value of having a large trading platform that provides a settlement ledger. However, the model has some important limitations. First, the platform can extract all producer profits (subject to incentivizing no-default) without distorting production. Second, agent decisions about where to trade are exogenous even though platform trading is what generates ledger demand. Third, all payment is made in terms of goods, despite contemporary ledgers using digital tokens.

In this section, we address these limitations by integrating the model from Section 2 into an infinite-horizon macroeconomic model where projects have flexible size, agents choose where to trade goods, payments are settled using financial assets (money or IOUs) and agents need to save resources through financial intermediaries. This means that there are now two key endogenous prices: the real exchange rate between marketplaces and the spread between the return on IOUs and money. We use this dynamic model to understand feedback in the payment and trading systems. We show that agents can be locked into repayment when they can only receive revenue in financial assets recorded through the ledger system. More generally, we show that the will-

ingness of the platform to enforce contracts depends upon agents having a low elasticity of substitution between trading platforms. Finally, we show that platform markup choices impact both the real exchange rate between marketplaces and the interest rate in the economy. High markups discourage trade on the platform, which increases money demand and decreases IOU demand, leading to high interest rates in the economy.

3.1 Environment Changes

Time is discrete with an infinite horizon. There is a collection of goods that can be used for production and consumption. The economy contains a continuum of agents, mutual funds, and a private platform that operates a ledger for the economy. There is a money asset provided by the government, an interperiod IOUs created through the ledger, and equity in platform. Each period is divided into a morning subperiod when a goods market opens and an evening subperiod when the asset market opens. Agents need money or tokens to trade in the morning subperiod, as in Svensson (1985) and Lagos and Wright (2005).

Production, preferences, and life-cycle: Each agent follows a "life-cycle" where they start as producers and then become consumers. An agent born in the morning of time t (at age 0) arrives without resources but with a technology to convert x_t goods at time t into $y_{t+1} = zx_t^{\alpha}$ goods at time $t+1$, where $z > 0$ is productivity and $\alpha \in (0,1)$. At $t+1$ (at age 1), they sell their goods, consume, and save. At time $t+2$ (at age 2), they consume again and exit. Agents of generation t rank consumption allocations according to $(1-\beta)u(c_{1,t+1}) + \beta u(c_{2,t+2})$, where $u(c) = \log(c)$, $c_{\tau,t}$ is consumption of goods produced by other agents at time t when the agent's age is τ, and $\beta \in [0,1]$.

Goods trading frictions: As before, there is both a public marketplace and a private platform to connect buyers and sellers, indexed by $n \in \{o,p\}$ respectively. However, now agents must choose where to trade. In addition to the pecuniary benefits from trading, each time period t, for each technology

$n \in \{o, p\}$, each agent i, gets an idiosyncratic, independent amenity draw for trading on that platform.[1] The draw for agents of age τ is distributed according to $\tilde{\zeta}^{ni}_{\tau,t} \sim \log(\zeta^n_\tau) + \text{Gu}(1/\gamma_\tau, -(1/\gamma_\tau)\mathcal{E})$, where Gu denotes the Gumbel distribution, $\mathcal{E}$ is the Euler–Mascheroni constant, ζ^n_τ is a technology specific component that characterizes the average service quality provided by the platform to sellers, and γ_τ is the elasticity of substitution. For convenience, we normalize $\zeta^o_\tau = 1$ and denote $\zeta^p_\tau = \zeta_\tau$. We do not impose a physical interpretation on the amenity values but they could be modeled as idiosyncratic search costs or good quality.[2] At age 0, in the morning subperiod, the agents observe their amenity shock for trading at age 0 and 1.[3] At age 2, in the morning subperiod, the agents observe their amenity shock for trading at age 2. As before, the platform charges a markup μ on buyers.

Assets and funds: At each time t, newly born agents issue IOUs that promise to pay 1 good in the morning market at $t + 1$. At time 0, the platform issues equity shares that pay firm profits π_t each period t. The government issues public money ("dollars"), in supply $\overline{M}_t$ each period t, and rebates seigniorage to the mutual funds proportional to their wealth.

There is a continuum of competitive mutual funds that pool resources across agents to purchase assets.[4] On the liability side, the fund issues one-period deposits that allow agents to withdraw money in the morning subperiod. We assume the fund faces a "money-in-advance" constraint that it must purchase money in the afternoon market to fullfil withdrawals in the following

[1] We introduce idiosyncratic risk in order to avoid "bang-bang" solutions to the platform choice problem. Our model uses tools from the discrete choice literature. This is analogous to assuming a CES preference function across the trading technologies.

[2] For the cost interpretation, note that the Gumbel distribution takes values across the real line and so ζ^{ni}_τ would represent a normalized cost. For the good quality interpretation, observe that we can write the total utility of a buyer receives as: $\log(e^{\zeta^{ni}_\tau}\zeta^n_\tau c)$ and so $e^{\zeta^{ni}_\tau}\zeta^n_\tau$ is essentially scaling the utility that the buyer gets from the good they consume.

[3] We make the assumption that agents observe their amenity shock for age 1 at age 0 for mathematical convenience so that we can get a closed form solution to the agent problem.

[4] Having the mutual funds simplify asset pricing. An equivalent structure would be to have a "Lucas family" that pools resources together but penalizes agents based on where they choose to trade or what type of currency they bring back to family.

morning market. There are two types of funds: those that accept defaulting agents and those that do not.

Morning markets and payment technologies: Each morning subperiod, a competitive goods market opens on each trading technology among the sellers and buyers who chose to go to that trading technology. Once again, there are two payment technologies in the economy: spot trade and ledger trade. However, now currencies and financial assets are used for payment. Spot transactions are not recorded and are subject to a resource-in-advance constraint: the payment must be made using goods and/or public money ("dollars") issued by the government. Let P_t^m denote the units of money required to purchase a good through the public marketplace in the morning market in period t.

The other payment technology is the digital ledger provided by the platform, which is not subject to a resource-in-advance constraint. During the morning market, the digital ledger allows agents to make payments on the platform up to the value of the assets they hold on the ledger. This means that agents can use certain future ledger income to purchase goods so the ledger is essentially creating "bills of exchange" for revenue inside the ledger ecosystem. Let P_t^b and P_t^s respectively denote the number of $(t+1)$-maturity IOUs and shares required to purchase a good on the private platform in the morning market in period t. Since t-maturity IOUs are settled in goods at the end the period t morning market, they can be traded for goods at rate 1-1 so long as there is no default. The fund discounts IOUs issued by newly born agents wanting to trade on the public marketplace at rate $\breve{E}_t$ IOUs per unit of money.

As in Section 2, agents will choose spot trade and default, even on the platform, if they are not restricted to do otherwise. So, without loss of generality, we impose that all trade on the public marketplace is spot-trade, while the platform mandates that sellers can only accept tokens when trading on the private platform. In this sense, money is the currency for spot trade, while IOUs (and other financial assets on the ledger) are the currency for ledger trade.

Afternoon market: In the afternoon subperiod, a competitive asset market opens for trading all currencies and financial assets. Let E_t denote the number of $(t+1)$-maturity IOUs required to purchase 1 dollar in the afternoon market in period t. i.e. the "nominal exchange rate" between IOUs and money. Let Q_t^s denote the number of $(t+1)$-maturity IOUs required to purchase 1 share. Following the monetary literature, it will be helpful to distinguish between "money-goods" traded in dollar transactions and "token-goods" traded in ledger transactions. We typically use platform-goods as the numeraire and use "real prices" to refer to prices in terms of platform-goods. We define the real prices of money, $(t+1)$-maturity IOUs, and platform shares as $q_t^m := E_t/P_t^b$, $q_t^b := 1/P_t^b$ and $q_t^s := Q_t^s/P_t^b$. We define the real exchange rate between marketplace-goods and platform-goods as $\epsilon_t := E_t P_t^m/P_t^b$ and the real price of shares as q_t^s. Where appropriate, we use $R_{t,t+1}^m$, $R_{t,t+1}^b$, and $R_{t,t+1}^s$ to denote the real return on the holdings of money, bonds, and equity shares between t and $t+1$. We let $r_{t,t+1}^{bn}$ denote the effective real borrowing rate when the agent issues an IOU to trade on market n (after any fund discounting to convert it the required currency) and let $r_{t,t+1}^{dn}$ denote the effective real deposit return set by the fund when agents withdraw deposits in the medium of exchange on market n.

Information frictions: The environment has analogous information frictions to Section 2.1 except for two differences. First, if an agent defaults on an IOU, then the holder can recover a fraction $\chi \in [0,1)$ of the input good and the producer keeps the rest of their production. Second, the platform now excludes the funds accepting defaulting agents from the ledger technology rather than the individual agents.

Timing: We show the timeline visually in Figure 1 and describe it in detail below. In the morning new IOUs are issued and goods are traded. In the evening, there is a secondary market for money and financial securities. The timing in the morning market is the following:

(i) Age-0 agents are born and begin the period without any assets or inventory. They issue bonds that are discounted by the funds into the currency they need to use to buy input goods. Age-1 agents begin with inventory and a payment type that they have chosen to accept. Age-2 agents begin with holdings of deposits in the funds.

(ii) The goods markets open on each trading technology. Agents withdraw their wealth from the fund in the required currency. Agents trading in a particular currency participate in a competitive goods market.[5]

(iii) At the close of the goods market, producers repay loans (or default and face potential punishment). IOUs due are settled by the ledger. Depositors who purchase goods consume and exit.

The timing in the afternoon market is the following:

(i) Age-1 agents deposit revenue with the fund.

(ii) The currency and asset markets open. Funds choose their asset portfolio for the next period, including their currency reserves.

3.2 Comparison to Other Models

Our environment attempts to nest or reflect canonical models with currency and settlement frictions. We do not offer a novel theory of money or ledger demand. Instead, we study how large players in the economy strategically supply ledger technologies. To illustrate this, we discuss how our model relates to models in the literature and why we have made deviations.

(i) *Cash-in-advance models:* The timing of trade and construction of the "synthetic" real exchange rate between the two segmented markets is

[5]The search literature often studies models where pricing is determined through one-to-one matching and bargaining over prices. Throughout this paper, we instead consider segmented competitive markets. We believe this is a closer approximation to the markets we are studying, especially in later sections when we model trade taking place on platforms such as Amazon or Alibaba.

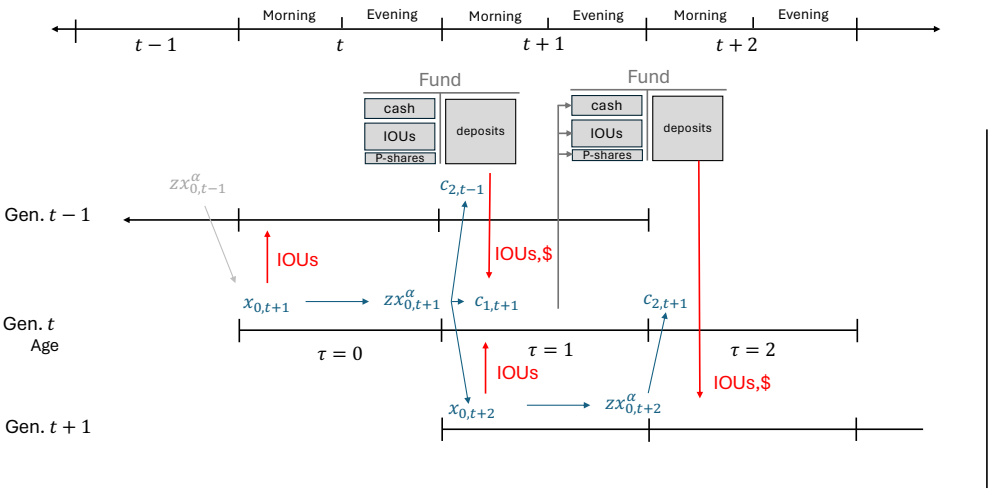


Figure 1: Timeline For The Infinite Horizon Model.

taken from the two-currency cash-in-advance model proposed by Svensson (1985). The difference in our setup is that the cash-in-advance constraint is only relevant on the public marketplace because the platform allows trade using claims to future revenue on the ledger. In this sense, trade on the platform is settled using digital bills of exchange that are automatically enforced at the end of the morning market. This is similar to the existence of both cash-good trades and credit-good trades in Lucas Jr and Stokey (1985). However, in our model, the creation of bills of exchange is not exogenous. Instead, the platform endogenously sets up trading rules to facilitate the creation of bills-of-exchange or credit-good trades in order to maximize their markup profit.

(ii) *Money search models:* Like in Lagos and Wright (2005), we adopt a morning-evening subperiod structure where search frictions in the morning market that require agents to hold money or have access to a currency ledger. Unlike many papers in this literature, for simplicity, we abstract from bargaining between agents and instead consider segmented com-

petitive markets. We also take the view that digital money trades occur with access to a currency ledger and so are necessarily monitored trades rather than anonymous trades in a decentralized markets. For this reason, we focus on how large institutions monitoring trades might supply digital currency ledgers rather than on how demand for digital money is different from demand for other monies.

(iii) *Social planner ledger provision:* Like in Kocherlakota (1998), we focus on how the introduction of a common record keeping technology can change equilibrium allocations. We have two main points of departure from Kocherlakota (1998): (a) we consider a ledger provided by a profit-maximizing platform and (b) agents choose whether to use the ledger or an outside payment technology. If we eliminated spot trade and treated the platform as a benevolent agent, we would get the results in Kocherlakota (1998).

(iv) *OLG models with financial assets:* Our model nests a classic OLG environment in which agents need an asset that they can buy when they are young and sell when they are old (e.g. Samuelson (1958), Diamond (1965)). In our environment, there are two assets available for storage that are differentiated by their usefulness in trade. In this sense, the financial assets in our model have both a role of storage and of a medium of exchange. Like in these models, the risk-free rate in our model will end up being distorted. However, unlike in these models, it is the strategic behavior of the platform controlling trade that leads to the interest rate distortion.

(v) *Tokenization:*

3.3 Market Equilibrium Without Default

In this subsection, we characterize equilibrium with agent choice about where to trade. We first characterize the buyer and seller problems under no-default. We then characterize market prices and show how platform markups affect

general equilibrium. We use our characterization to study how platform decisions interact with general equilibrium. In the next section, we return to the difficulties of loan enforcement and characterize the incentive compatibility constraint for no-default.

3.3.1 Agent Problem

We consider the problem of an agent in the generation born at time $t \geq 2$ and use the following terminology. Let n_τ denote the agent's choice of goods trading technology at age τ and let $\underline{\mathbf{n}}_\tau = (n_s : 0 \leq s \leq \tau)$ denote the history of technology choices up to age τ.

We now set up the budget constraints. Consider the agent at age 0. The agent chooses where to purchase input goods, n_0, and where to sell output goods, n_1. If the agent purchases $x_{0,t}$ input goods, then they issue IOUs discounted through the fund at real borrowing cost $r^{bn}_{t,t+1}(1+\mu^{n_0}_t)x_{0,t}$.

At age 1, the agent then produces $y_{1,t+1} = z(x_{0,t})^\alpha$ goods, consumes $c_{1,t+1}$ goods, repays the IOU, and deposits $d_{1,t+1}$ in a fund. If the agent sells on the private platform, i.e. $n_1 = p$, then their revenue in platform goods is $y_{1,t+1}$. If the agent sells on the public marketplace, i.e. $n_0 = o$, then their revenue in units of platform goods is $\epsilon_t y_{t+1}$. Thus, their budget constraint at $t = 1$ in real terms when they buy inputs on n_0 and sell on n_1 is:

$$d_{1,t+1} \leq \epsilon^{n_1}_{t+1}\left(z(x_{0,t})^\alpha - (1+\mu^{n_1}_{t+1})c_{1,t+1}\right) - r^{bn_0}_{t,t+1}(1+\mu^{n_0}_{t+1})x_{0,t} \tag{3.1}$$

where ϵ^n_t denotes the real exchange rate between goods on the trading technology $n \in \{o,p\}$ and goods on the private platform, i.e. is ϵ_t if $n = o$ and 1 if $n = p$.

At age 2, the agent chooses a marketplace on which to consume, n_2. If they choose n_2, then the real value of their withdrawals is $r^{dn_2}_{t+1,t+2}d_{1,t+1}$ and so the agent faces the budget constraint:

$$(1+\mu^{n_2}_{t+1})\epsilon^{n_2}_{t+2}c_{2,t+2} \leq r^{dn_2}_{t+1,t+2}d_{1,t+1} \tag{3.2}$$

Finally, taking price and return processes as given, at age 0, an agent in generation t solves problem (3.3) below:

$$\mathbb{E}_t\Big[\max_{x_0,c_1,c_2,d_1,\underline{\mathbf{n}}} \left\{\tilde{\zeta}_{0,t}^{n_0} + \tilde{\zeta}_{1,t+1}^{n_1} + (1-\beta)u(c_{1,t+1}) + \beta(\tilde{\zeta}_{2,t+2}^{n_2} + u(c_{2,t+2}))\right\}\Big] \tag{3.3}$$
$$s.t. \quad (3.1), (3.2).$$

Theorem 1. *An agent choosing trading technologies $\underline{\mathbf{n}}_2 = (n_0, n_1, n_2)$, undertakes production:*

$$x_{0,t} = \left(\frac{\epsilon_{t+1}^{n_1}\alpha z(1-\mu_{t+1}^{n_1})}{(1+\mu_t^{n_0})r_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}}, \qquad y_{1,t+1} = z\left(\frac{\epsilon_{t+1}^{n_1}\alpha z}{(1+\mu_t^{n_0})r_{t,t+1}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}}, \tag{3.4}$$

$$\pi_{1,t+1} = \left(\frac{\epsilon_{t+1}^{n_1}\alpha z(1-\mu_{t+1}^{n_1})}{((1+\mu_t^{n_0})r_{t,t+1}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

and chooses consumption and saving:

$$c_{1,t+1} = \frac{(1-\beta)\pi_{1,t+1}}{\epsilon_{t+1}^{n_1}(1+\mu_{t+1}^{n_1})}, \qquad d_{1,t+1} = \beta\pi_{1,t+1}, \qquad c_{2,t+2} = \frac{r_{t+1,t+2}^{dn_2}\beta\pi_{1,t+1}}{\epsilon_{t+2}^{n_2}(1+\mu_{t+2}^{n_2})} \tag{3.5}$$

The fraction of producers and consumers respectively choosing trading technologies n_0, n_1, and n_2 are:

$$\eta_{0,t}^{n_0} = \frac{\left(\zeta_0^{n_0}((1+\mu_t^{n_0})r_{t,t+1}^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta_0^{n_0'}((1+\mu_t^{n_0'})r_{t,t+1}^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \tag{3.6}$$

$$\eta_{1,t+1}^{n_1} = \frac{\left(\zeta_1^{n_1}\left(\epsilon_{t+1}^{n_1}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu_{t+1}^{n_1})^{\beta-1}\right)^{\gamma_1}}{\sum_{n_0'n_1'}\left(\zeta_1^{n_1'}\left(\epsilon_{t+1}^{n_1'}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu_{t+1}^{n_1'})^{\beta-1}\right)^{\gamma_1}} \tag{3.7}$$

$$\eta_{2,t+2}^{n_2} = \frac{\left(\zeta_2^{n_2}r_{t+1,t+2}^{dn_2}/((1+\mu_{t+2}^{n_2})\epsilon_{t+2}^{n_2})\right)^{\gamma_2}}{\sum_{n_2'}\left(\zeta_2^{n_2'}r_{t+1,t+2}^{dn_2'}/((1+\mu_{t+2}^{n_2'})\epsilon_{t+2}^{n_2'})\right)^{\gamma_2}} \tag{3.8}$$

Proof. See Appendix A. □

From Theorem 1, we can see the expected relationships between the vari-

ables. Holding everything else constant, an increase in the private platform's trading advantage, $\zeta_\tau^p/\zeta_\tau^o$, leads to more agents using the private platform at each age. An increase in the effective cost of borrowing to purchase on a trading technology, $r_{t,t+1}^{bn_0}$, leads to fewer agents purchasing input goods there. Likewise, an increase in the effective price of goods on a trading technology, $\epsilon_{t+2}^{n_2}$, leads to fewer agents purchasing consumption goods there. Finally, an increase in the markup, $(1+\mu_t)$, leads to fewer agents buying on the platform at all ages.

3.3.2 Fund Problem

In the evening of each period t, the funds take real deposit wealth A_t from agents. They then purchase money M_t to back deposit withdrawals and discount the issuance of IOUs in dollars, purchase bonds B_t, and purchase shares in the platform, S_t. So, their budget constraint at t is:

$$q_t^m M_t + q_t^b B_t + q_t^s S_t \leq A_t.$$

In the morning of $t+1$, depositors who trade on the public marketplace ($n = o$) withdraw money, and depositors who trade on the private platform ($n = p$) exchange the market value of the remaining assets in the fund. The fund offers depositors withdrawing in money the return on money and the other depositors the return on assets that can be used in exchange on the ledger. So the "money-in-advance" constraint on the fund is:

$$R_{t,t+1}^m \eta_{2,t+1}^m A_t \leq q_{t+1}^m M_t \quad \Leftrightarrow \quad \eta_{2,t+1}^m A_t \leq q_t^m M_t$$

Let $\breve{M}_t := M_t - \eta_{2,t+1}^m A_t/q_t^m$ denote the excess money holdings that can be used to discount new IOUs in the morning of $t+1$. Then, the real wealth available to depositors trading on the platform is:

$$\breve{A}_t = q_{t+1}^b \breve{E}_t \breve{M}_t + B_t + q_{t+1}^s S_t$$

Lemma 1 shows that in equilibrium the fund returns offset the opportunity

cost of holding money or bonds. So, for agents borrowing and withdrawing in IOUs, the effective interest rate is the return on IOUs $r^{bp}_{t,t+1} = r^{dp}_{t,t+1} R^b_{t,t+1}$. By contrast, for agents borrowing and withdrawing in money, the borrowing rate is $r^{bo}_{t,t+1} = \epsilon_t (R^b_{t-1,t} / R^m_{t-1,t}) R^b_{t,t+1}$ and the deposit rate is $r^{do}_{t,t+1} = R^m_{t,t+1}$.

Lemma 1. *If the money-in-advance constraint binds and the fund holds excess returns, then the borrowing rate and deposit rate faced by agents using trading technology n are given by:*

$$r^{bn}_{t,t+1} = \epsilon^n_t \frac{R^b_{t-1,t}}{R^n_{t-1,t}} R^b_{t,t+1}, \qquad r^{dn}_{t,t+1} = R^n_{t,t+1} \tag{3.9}$$

Proof. See Appendix A. □

3.3.3 Market Equilibrium

We now consider the market equilibrium. We define equilibrium for an arbitrary sequence of markup policies although, in numerical examples, we will ultimately focus on the steady-state limit with a fixed markup.

Definition 1. *Given a sequence of ledger policies, $\underline{\boldsymbol{\mu}}$, a competitive equilibrium is a collection of price and return sequences, $(\underline{\mathbf{R}}^b, \underline{\mathbf{R}}^m, \underline{\mathbf{r}}^d, \underline{\mathbf{r}}^b, \underline{\mathbf{q}}^s)$, and agent choice sequences, $(\{\underline{\boldsymbol{\eta}}_\tau\}_{\tau \leq 2}$, $\underline{\boldsymbol{x}}$, $\underline{\boldsymbol{y}}$, $\underline{\boldsymbol{c}}_1$, $\underline{\boldsymbol{c}}_2)$, such that: (i) given prices, the agent choices solve optimization problem* (3.3)*, (ii) given prices, the fund choices solve equation* (3.9)*, and (iii) market clearing is satisfied for the goods market on each trading technology, the IOU market, the money market, and the equity market:*

$$\begin{aligned} Y^n_t &= X^n_{0,t} + C^n_{1,t} + C^n_{2,t}, \quad \forall n \in \{o, p\} \\ B_t &= I_{0,t} \\ M_t &= \bar{M}_t \\ S_t &= 1 \end{aligned}$$

where $Y^n_t := \eta^n_{1,t} \sum_{n_0} \eta^{n_0}_{0,t-1} y^{(n_0,n)}_{1,t}$ is the aggregate output on trading technology n at t, $X^n_{0,t} := \eta^n_{0,t} \sum_{n_1} \eta^{n_1}_{1,t+1} x^{(n,n_1)}_{0,t}$ is aggregate input good purchases on trad-

ing technology n at t, $I_{0,t} := \sum_{n_0,n_1} \eta_{0,t}^{n_0}\eta_{1,t+1}^{n_1} r_{t,t+1}^{bm}(1+\mu_t^{n_0}) x_{0,t}^{(n_0,n_1)}$ is aggregate IOU issuance at t, $C_{1,t}^n := \eta_{1,t}^n \sum_{n_0} \eta_{0,t-1}^{n_0} c_{1,t}^{(n_0,n)}$ is aggregate consumption on n by agents aged 1 at t, and $C_{2,t}^n := \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0}\eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)}$ is aggregate consumption on n by agents aged 2 at t,

Theorem 2 characterizes the equilibrium under some parametric restrictions on the elasticities of substitution for buyers at ages 0 and 2. Our environment is sufficiently tractable that we get a closed-form expression for the real exchange rate ϵ_t. However, the expressions for $R_{t,t+1}^b$, $R_{t,t+1}^m$, and q_t^s are necessarily difference equations because previous agent choices discipline their current behavior.

Theorem 2. *Suppose that $\frac{1+\alpha\gamma_0}{1-\alpha} = 1+\gamma_2$ and $(\zeta_0^n)^{\gamma_0} = (\zeta_2^n)^{\gamma_2}$. Let the excess return on bonds over money be denoted $\mathcal{S}_{t,t+1} := R_{t,t+1}^b / R_{t,t+1}^m$. Then the real exchange rate, bond return, money return, and equity price satisfy:*

$$\epsilon_t = \left[\frac{\zeta_1^{\gamma_1}}{\zeta_2^{\gamma_2}} \frac{(1+\mu_t)^{\gamma_1(1-\beta)}}{\mathcal{S}_{t-1,t}^{1+\gamma_2}} \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t}\right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{1}{\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)}} \tag{3.10}$$

$$\begin{aligned} \eta_{2,t+1}^p \beta \Pi_{1,t} = \sum_{n_1} \Bigg(\sum_{n_0} \eta_{0,t}^{n_0}\eta_{1,t+1}^{n_1} \epsilon_t^{n_0} \mathcal{S}_{t-1,t}^{n_0} (1+\mu_{t+1}^{n_0}) x_{0,t}^{(n_0,n_1)} \\ + \eta_{0,t+1}^m \frac{\mathcal{S}_{t,t+1}\epsilon_{t+1}}{R_{t,t+1}^b} x_{0,t+1}^{(m,n_1)} \Bigg) + q_t^s \end{aligned} \tag{3.11}$$

$$R_{t,t+1}^m = \left(\frac{\bar{M}_t}{\bar{M}_{t+1}}\right) \left(\frac{\eta_{2,t+1}^o \beta \Pi_{1,t} + \frac{1}{R_{t,t+1}^m} X_{0,t+1}^o}{\eta_{2,t+2}^o \beta \Pi_{1,t+1} + \frac{1}{R_{t,t+2}^m} X_{0,t+2}^o} \right) \tag{3.12}$$

$$q_t^s = \frac{1}{R_{t,t+1}^b} \left(\pi_{t+1}^s + q_{t+1}^s \right) \tag{3.13}$$

where $\Pi_{1,t} = \sum_{n_0,n_1} \eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1} \pi_{1,t}^{(n_0,n_1)}$ is aggregate agent profit at time t and $X_{0,t}^o := \sum_{n_1} \eta_{0,t}^o \eta_{1,t+1}^{n_1} \epsilon_t x_{0,t}^{(o,n_1)}$ is aggregate input purchases on marketplace o at time t.

Proof. See Appendix A. □

Theorem 2 illuminates the how variables affect the equilibrium prices.

First, observe that the real exchange rate is decreasing in the excess return on bonds relative money, $\mathcal{S}_{t-1,t}$ and, holding asset returns constant, increasing in μ_t. This is because a decrease in $\mathcal{S}_{t-1,t}$ and an increase in μ both encourages buyers to choose the public public marketplace instead of the private platform, which increases demand on the public marketplace and so the relative price on the public marketplace.

The return on money is the ratio of money demand growth to money supply growth. This implies that, in the steady state, we have the familiar relationship that $R^m_{t,t+1} = 1/g_M$, where g^M is the growth rate of money. As is standard in "currency-in-advance" models, the environment has money neutrality in the sense that the level of money supply does not affect real variables. However, it does not have super-neutrality in the sense that the growth rate of money affects real variables by impacting agent borrowing decisions. The price of equity is given by future dividends discounted by the bond rate.

Finally, the return on IOUs satisfies (3.11). The left-hand-side of this equation denotes the supply of deposits than can be used to purchase ledger assets. The right-hand-side denotes the demand for ledger assets. Evidently, a decrease in the fraction of agents wanting to trade on the private platform, $\eta^p_{2,t+1}$, leads to decrease in the supply of deposits available to purchase ledger assets decreases (a decrease in supply in the loan market). Likewise, a decrease in input good purchases, $x^{n_0,n_1}_{0,t}$ leads to a decrease in the issuance of IOUs (a decrease in demand in the loan market).

To help illustrate these forces, we plot the steady state equilibrium in Figure 2 as a function of the platform markup. The blue dashed line shows the partial equilibrium allocations as μ varies when the interest rate is fixed at 3%. The black solid line shows general equilibrium when the IOU interest rate is allowed to vary. From the blue dashed lines we can see that a higher markup leads to fewer agents going to the platform and more agents going to the public marketplace, which ultimately increases the real exchange rate (i.e., makes the public marketplace relatively cheaper). From the black solid lines, we can see that in general equilibrium, an increase in μ also increases the loan rate R^b because the decrease in activity on the platform restricts the resources

the fund can use to purchase IOUs. This encourages agents to hold ledger assets and come to the platform, which partially offsets the impact of the the higher markups. Ultimately, this ends up causing the platform markups to have a much greater impact on output. In this sense, the platform trying to extract rents in the product market restricts the supply of credit in the loan market and so restricts production in the economy.

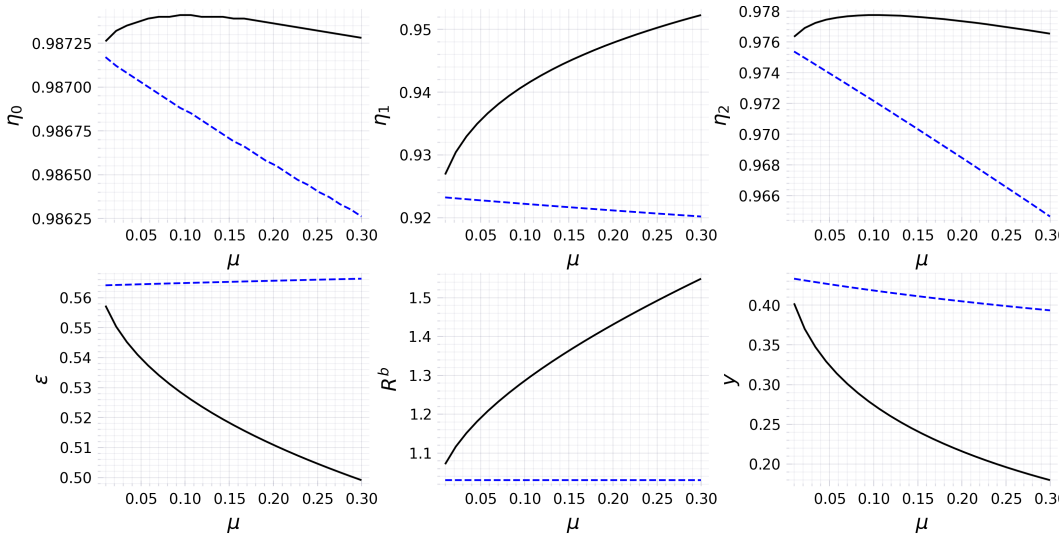


Figure 2: Equilibrium for $\mu \in [0, 0.3]$.

Other variables are $z = 1$, $\alpha = 0.45$, $\beta = 0.9$, $\gamma_1 = 1.9$, $\gamma_2 = 1.5$, $\zeta = 1.0$, and $\kappa = 0.1$. Black line denotes general equilibrium. The blue dashed line denotes partial equilibrium with fixed interest rate R^b.

3.4 Default and Incentive Compatibility

We now return to the question of incentive compatibility in the dynamic model. Funds that accept defaulting agents cannot use the payment ledger because as soon as they store wealth in the fund, their resources will be taken to fulfill contracts. So, if agents default and deposit into a fund, then they are restricted to the monetary system. The agent problems are very similar, so we defer the details to Appendix A. Theorem 3 states the main result: the incentive compatibility constraint on deterring agents from defaulting.

Theorem 3. *No agents default if the following incentive compatibility constraint is satisfied* $\forall t \geq 0$*:*

$$\frac{\check{\pi}_{1,t}}{\pi_{1,t}} \leq \frac{\bar{\nu}_{2,t+1}}{\check{\nu}_{2,t+1}}, \tag{3.14}$$

where $\pi_{1,t}$ *is agent profit at age* 1 *if they repay,* $\check{\pi}_{1,t}$ *is agent profit at age* 1 *if they default,* $\bar{\nu}_{2,t+1} := \left(\sum_{n_2} \left(\frac{\zeta_2^{n_2} r_{t,t+1}^{dn_2}}{(1+\mu_{t+1}^{n_2})\epsilon_{t+1}^{n_2}} \right)^{\gamma^b} \right)^{1/\gamma^b}$ *is the average marginal utility of wealth at age* 2 *if the agent repays at age* 1*, and* $\check{\nu}_{2,t+1} := \frac{\zeta_2^o r_{t,t+1}^{do}}{\epsilon_{t+1}}$ *is marginal utility of wealth at age* 2 *if the agent defaults at age* 1 *and is excluded from the platform. Imposing equilibrium, the incentive compatibility constraint becomes* $\forall t \geq 0$*:*

$$1 + \mu_{t+1} \leq \frac{\zeta_2 \epsilon_{t+1} \mathcal{S}_{t,t+1}}{\left(\left(\chi^{-1} \min\{\epsilon_{t-1} \mathcal{S}_{t-2,t-1}, 1\} R_{t-1,t}^b \right)^{\frac{\alpha\gamma^b}{1-\alpha}} - 1 \right)^{1/\gamma^b}}, \tag{3.15}$$

For $\chi > 0$*, the incentive compatibility constraint will be satisfied for sufficiently large* ζ_2*.*

Proof. See Appendix A. □

Contract enforcement is similar to that of the two-period model. If agents trade using the platform, they have to repay because the platform forces them to use the ledger. If they trade using the public marketplace, they can trade using money, default, and deposit into the shadow financial system. IC constraint (3.14) says that agents repay if the benefit of having access to the platform at age 2 is greater than the additional profit gained by defaulting. This ultimately places an upper bound on how large the markup can be, as described by equation (3.15). This is the dynamic analogue of equation (2.6) in Section (2).

To help illuminate the enforcement in the dynamic model, we consider a collection of special cases.

Case: $\beta = 1$ **and no money.** If there is no money in the economy and agents only value consumption at age 2, then the presence of a ledger is sufficient to incentivise repayment, regardless of what the platform does. Why? When $\beta = 1$, agents do not value consumption at $t = 1$ and do not engage in barter trade with other agents of their generation for perishable goods. Instead, they only trade goods to agents of other generations in exchange for financial assets. If there is no money, then the age-1 agents selling goods receive newly issued IOUs from the age-0 agents and old IOUs from the age-2 agents purchasing goods. Since all IOUs are on the ledger, there is no way for age-1 agents to default, regardless of whether the platform excludes defaulting agents or not. In this sense, the agents are locked into the ledger payment system because they need a way to store wealth.

Case: $\beta = 1$ **with money and ledgers.** Suppose that we introduce money into the model with $\beta = 1$. Agents now have an outside payment option and the platform is necessary for ensuring contract enforcement in the economy. The agents have no need to consume at $t = 1$ and so all trade occurs after contract settlement. In this case, platform exclusion from purchasing consumption goods at $t = 2$ can discipline no-default.

Case: $\beta = 0$ **with money and ledgers.** In this case, exclusion from trade at $t = 2$ is no longer relevant. In order to get no-default, the platform has to ensure that the platform price is sufficiently high that the opportunity cost of not being able to sell on the platform is a sufficiently high punishment for no-default. If ζ_1 is low, then this requires a negative markup μ, running the risk of credit fragility as in Section 2.

3.5 Platform Problem

We can now write down the problem of a private platform. Suppose the economy starts with an initial collection of age-1 agents with loans and goods

inventory $(b_{0,0}, y_{1,0})$ and a collection of age 0 agents with wealth $(a_{2,0})$ in a collection of funds. Given a belief about the interest rate processes, $\{\hat{R}^b, \hat{R}^m\}$, the platform chooses a sequence $\boldsymbol{\mu}$ to maximise their equity price by solving problem:

$$
\begin{aligned}
q_0^s = \max_{\boldsymbol{\mu}} \left\{ \sum_{t=0}^{\infty} \hat{\xi}_{0,t} \pi_t^s \right\} \quad s.t. \\
& \pi_t^s = \mu_t \eta_{1,t}^p \sum_{n_0} \eta_{0,t-1} y_{1,t}^{(n_0,p)}, \quad t \geq 1 \\
& \pi_0^s = \mu_0 \eta_{1,0} y_{1,0} \\
& \text{Agent choices: } (3.4), (3.5), (3.6), (3.7), (3.8), \\
& \text{Equilibrium prices: } (3.10), (3.12), (3.13), (3.11)
\end{aligned}
$$

where $\hat{\xi}_{0,t} = \prod_{j=0}^{t} (\hat{R}_{j,j+1})^{-1}$ is the household SDF.

We defer the first order conditions for this problem to Appendix A.5. Figure 3 plots the numerical solution to the equilibrium for different values of buyer elasticity. Evidently, for low γ^b, the constraint on incentivizing default is non-binding because it is easy for the platform to attract buyers. However, as γ^b increases, it becomes more difficult for the platform to attract buyers and so the constraint eventually binds. Once this occurs, the platform must significantly decrease μ in order to attract enough buyers to achieve the no-default equilibrium. In this sense, buyer elasticity is the key variable for understanding to what extent platform rent extraction is disciplined by the platform needing to attract traders.

Credit Fragility and Monetary Policy: Figure 3 depicts the platform's optimal markup charge for different supply growth of public money. The blue dashed line, μ^*, indicates the markup if the platform need not to care about the incentive constraint. The incentive constraint requires that the markup is below the red dashed line, $\bar{\mu}^{IC}$. If the incentive constraint require the markup to be negative, the platform derives negative value from running a non-default ledger and so would not set up the payment ledger. In this sense, the uncollateralized credit equilibrium is "fragile"; strong competition from

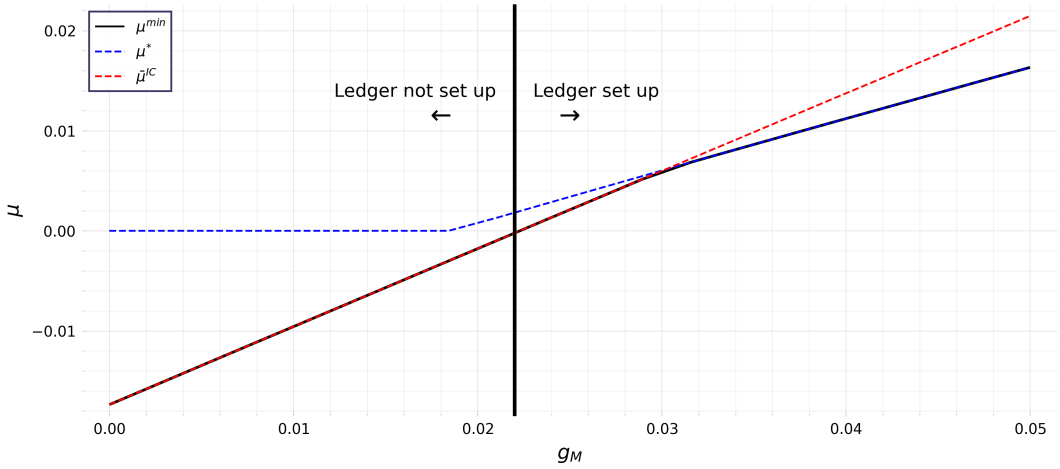


Figure 3: Markup as a function of public money supply growth rate

Panel 1: The blue dashed line depicts the platform's markup choice, μ^* if they are not constrained by having to ensure no-default. The red dashed line depicts the minimum value $\bar{\mu}$ required to deter funds from defaulting. The black line depicts the markup for the equilibrium chosen by the platform, μ.

the dollar – due to low dollar-inflation – makes it too costly for the platform to set up a no-default ledger.

3.6 Crowding Out Dollars

So far, we have assumed that only dollars are used on the public marketplace. We have also assumed that non-defaulting agents can bring dollars back to the platform. We now consider alternative arrangements. We suppose the platform can allow IOUs to be used on the public marketplace and prevents any agents from bringing dollars back to the platform. In this case, there is no reason to use cash. Accepting the tokens is always strictly better. So, we get tokenization.

3.7 Discussion of Regulatory Options

We have shown that a tech platform will provide and "back" a common settlement ledger in an unregulated economy if they have a sufficiently dominant

trading technology. This incentivizes the financial sector to coordinate on enforcement, but also gives the platform market power to extract rents. In this sense, regulators have a "natural monopoly" dilemma. We close the paper by discussing how some classic policy responses would play out in our model: regulated competition between platforms and a public ledger option. We do this by varying the outside option with which the platform competes.

3.7.1 Regulated Platform Competition

Environment changes: The environment is the same as in subsection 3.1 but with the following changes. There are now two private platforms, labeled $n \in \{1, 2\}$. There is no public marketplace or public currency. Both platforms manage their own ledger, charge a markup μ^n, and have average trading advantage ζ^n. For simplicity, we assume that the platforms choose a fixed μ at time $t = 0$ for all periods. We let $\eta_{\tau,t}$ denote the fraction of agents at age τ choosing platform 1.

Since there is no public dollar, agents cannot undertake side payments; all transactions are observed by one of the two platforms. In other words, in this new environment the only way producers can default is by writing a contract on the ledger provided by platform n, then defaulting and trading on the other platform n'. We also now assume that $1-\chi$ captures the dead-weight loss producers incur when they default and lenders get nothing when default occurs. We use the currency provided by ledger 1 as the numeraire for asset pricing. So, ϵ_t now refers to the real exchange rate from tokens provided by platform 1 to tokens provided by platform 2.

Regulation: The regulator allows the platforms to bargain at time $t = 0$ over committing to exclude funds who allow their depositors to default on contracts on the other ledger. We assume that funds face no borrowing constraints or commitment problems during this bargaining and the Nash bargaining protocol is followed. The regulator does not allow the platforms to collude on setting markups at times $t \geq 0$.

Platform competition at $t = 0$: For $t > 0$, the equilibrium is the same as in the subsection 3.3 but with $(1 - \mu_t)$ replaced by $(1 - \mu_t^1)/(1 - \mu_t^2)$. Let q_0^{En} denote the price of equity in platform n at $t = 0$ under cooperation on enforcement for $t \geq 0$ and let $\tilde{q}_0^{En}$ denote price of equity in platform n at $t = 0$ if there is no cooperation on enforcement for $t \geq 0$. The surplus from cooperation is $S = q_0^{E2} - \tilde{q}_0^{E2} + q_0^{E1} - \tilde{q}_0^{E1}$. If the surplus is positive, then the platforms bargain over coordination on contract enforcement. We assume that platform 1 makes a (positive or negative) transfer T to platform 2 at time 0 and the payment is determined by a Nash Bargaining protocol. In particular, we have:

$$\begin{aligned} T = & \arg\max_T \left\{ \left(q_0^{E2} - T - \tilde{q}_0^{E2} \right) \left(q_0^{E1} + T - \tilde{q}_0^{E1} \right) \right\} \\ & = q_0^{E2} - \tilde{q}_0^{E2} - (q_0^{E1} - \tilde{q}_0^{E1}) \end{aligned}$$

If the surplus is negative, then the platforms do not coordinate. Proposition 3 shows that, when platforms are symmetric, the outcome of the bargaining is contract enforcement on both ledgers whereas when platforms are asymmetric the dominant platform provides the ledger.

Proposition 3. *We have the following:*

(i) If the platforms are symmetric, then the outcome of the bargaining at time 0 is that contracts are enforced on both ledgers and no transfer is made.

(ii) If $\zeta := \zeta_\tau^1 / \zeta_\tau^2 > 1$, then for χ and ζ sufficiently high, the outcome of the bargaining at $t = 0$ is that platform 1 provides the monopoly ledger for the economy.

Proof. See Appendix A. □

The first part of the proposition says that contract enforcement coordination is straightforward when the platforms are similar. The second part of theorem reinforces the market structure result in Section 2. Ultimately, it shows that the only ledger operators that are viable are those that also possess a platform trading technology. In other words, there is a natural bundling

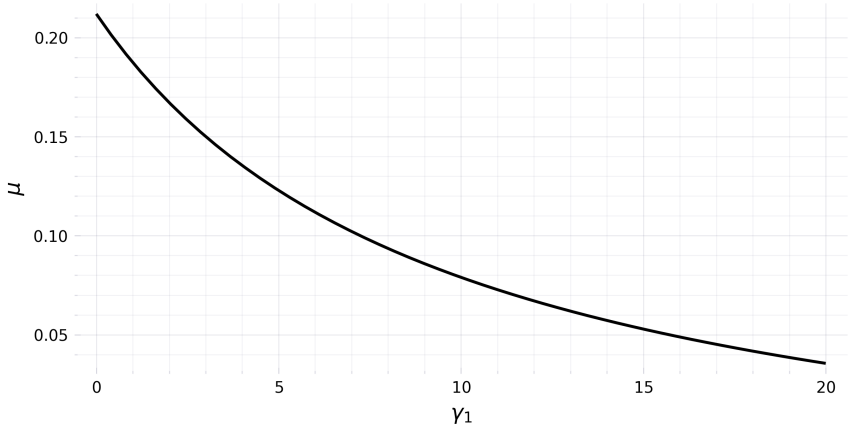


Figure 4: Ledger choice for $\gamma_1 \in [0, 20]$.

Other variables are $\gamma_2 = 0.5$, $z = 1.0$, $\alpha = 0.75$, $\rho = 1.0$, $\lambda = 1.0$, $\zeta = 1$, $\kappa = 0.1$, and $\chi = 0.5$.

between offering ledger and trading services. This implies that a financial intermediary with no trading technology (which would be modeled as $\zeta_2 = 0$ in our environment) would never provide the ledger in equilibrium.

Given the outcome of the bargaining over contract enforcement, we show the numerical solution to symmetric platform competition for $t > 0$ in Figure 4. Evidently, we recover the perfectly competitive market outcome as $\gamma_1 \to \infty$ and the agents become highly elastic. In this sense, seller elasticity governs the extent to which we recover Bertrand style competition. This suggests two major lessons for financial competition regulators:

(i) They should focus on understanding the elasticity with which buyers and sellers are able substitute between trading technologies.

(ii) Small platforms need to be able to commit to making payments to large platforms in order to get cooperation on an enforcement equilibrium.

3.7.2 Public Ledger Option

Environment changes: We now consider the environment from subsection 3.1 but with public money replaced by a digital ledger provided by the government

that can be used by all intermediaries and agents in the economy. This can be interpreted as giving common access to central bank reserves, introducing a public CBDC, or allowing tech platforms access the FedNow payment system. The government commits to using payment revenue to enforce contracts written on the ledger. This means that there is no longer a cash-in-advance constraint on the public marketplace and contracts are enforced on the public marketplace if agents use the public ledger.

The platform now has additional relevant strategic options. As before, it could help ensure contract enforcement by forcing traders to pay using the public settlement ledger or their own ledger. However, it could also choose to provide a private token that replaces public money for hidden trades.

Corollary 2. *Let (R, Y) denote the steady state equilibrium interest rate and aggregate output from 3.5 when the government provides money and the platform provides the settlement ledger. (R^*, Y^*) denote the steady state equilibrium interest rate and aggregate output when the government provides a common digital ledger and the platform optimizes markups.*

(i) If the government creates a forced tender ledger that must be used for all payment, then all contracts are enforced, the cash-in-advance constraint is eliminated, $R^* < R$, and $Y^* > Y$.

(ii) If the government allows the platform to choose any payment technology, then the platform response depends upon their trading advantage. For sufficiently large ζ, the platform forces agents to use public ledger. For sufficiently small ζ, the platform prefers to offer its own private token that is only used for trade on the ledger and allow agents to default.

If it feasible for the government to create a forced tender ledger, then the economy ends up back in the world of Kocherlakota (1998). All agents are using a ledger run by a benevolent central planner ensuring that all contracts are enforced.

However, if the platform provides public ledger as a voluntary payment technology, then the response of the platform is more complicated. In our

original model, the platform is offering the only payment technology where contracts can be enforced whereas the government is offering the payment technology that can be used for side-trading to avoid contract enforcement. We showed that the platform is strongly incentivized to ensure contract enforcement because otherwise there is no lending and production in the economy. When the government introduces a public ledger, this reverses. The government is now offering a ledger where contracts are always enforced and the platform is deciding whether to offer the side-trading option where contracts are not enforced. That is, the platform is now deciding whether or not set up a "black-market" with a private token. The platform faces the trade-off that allowing default increase the interest rate in the economy thereby decreasing output but allowing default also attracts more producers to trade on the platform. What the platform decides depends upon their market power.

In our environment, introducing a public ledger option is potentially powerful but also offers a few points of caution for regulators:

(i) A forced tender ledger is very powerful but raises privacy concerns.

(ii) Introducing a non-compulsory public ledger improves contract enforcement in the public marketplace but incentivizes the private platform to set up a "black-market" with its own private tokens where agents can default.

4 Conclusion

In this paper, we model the strategic decision making of a private controller of the currency ledger used for settling transactions and writing contracts. We find that in an unregulated economy "BigTech" platforms are likely to provide "FinTech" services. This brings both benefits and costs. Tech platforms can expand uncollateralized credit across a supply chain by exploiting their control of the payment system to better coordinate the financial system to enforce contracts. However, Tech platforms will also use their control of the ledger to increase their market power and charge high markups. We see these issues

playing out in China where tech platforms Alibaba and WeChat have created a well-functioning payment system with very limited competition.

Ultimately, our model suggests that currency ledgers may need to be regulated like other natural monopolies. This could include restrictions on when ledgers can cooperate on contract enforcement and compete on markups. It could also include a competing public option in the form a programmable Central Bank Digital Currency (CBDC) ledger. We consider further modeling of the government's regulatory options as important future work.

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A Supplementary Proofs (Online Appendix)

A.1 Proof of Proposition 1

Proof. We proceed by characterizing the $t = 1$ markets for different values of φ, the fraction of savers who store endowment goods. We then characterize the possible equilibria at $t = 0$.

The $t = 1$ market. Let φ denote the fraction of savers who store endowment goods. If $\varphi = 1$, then only endowment goods are stored and savers trade endowment goods with each other at 1:1. If $\varphi = 0$, then no endowment goods are stored so producers enter with output goods and lenders enter with IOUs. In this case, agents trade output goods or IOUs with each other for the price $q = 1/((1-\phi)R)$ at which producers are indifferent. Finally, if $\varphi \in (0, 1)$, then both endowment goods and output goods are potentially traded in equilibrium. Since the market is competitive, the price is determined by relative scarcity. If the quantity of production goods is greater than the quantity of endowment goods, $(1 - \varphi)z > \varphi$, then endowment goods are relatively scarce. In this case, the prices are $p = (z - R)/z < 1$ and $q = 1/((1 - \phi)R)$ so all savers trade endowment goods for output goods and producers are indifferent between trading output goods with each other, for IOUs, and for endowment goods. If the quantity of endowment goods is greater than the quantity of production goods, then $\varphi > (1 - \varphi)z$, then output goods are relatively scarce. In this case, the price is $p = 1$ and $q = 1/((1 - \phi)R)$ so all output producers trade output goods with savers and all savers are indifferent between trading with producers and with each other.

The $t = 0$ equilibrium. $\varphi = 1$ is an equilibrium since the saver optimization problem (2.5) is satisfied and producers no longer participate so their constraints are trivially satisfied. Now consider $\varphi \in [0, 1)$. If $(1 - \varphi)z > \varphi$, then, at $t = 1$, endowment goods are scarce and producer indifference prices the assets. Thus, $p = (z - R)/z < 1$, $q = 1/((1 - \phi)R)$ and all endowment savers trade for output goods, and at $t = 0$ the savers are indifferent in the IOU market. Producers default if they trade with endowment good savers and so the probability of default is $\phi = \varphi/((1 - \varphi)z)$. Equilibrium in the IOU

market implies that the following equation must be satisfied:

$$\begin{aligned} 1/q &= 1/p \\ (1-\phi)R &= z/(z-R) \\ \Rightarrow (1-\phi)R(z-R) &= z \\ \Rightarrow -R^2 + zR &= z/(1-\phi) \\ \Rightarrow R^2 - zR + z/(1-\phi) &= 0 \\ \Rightarrow R &= \frac{z + \sqrt{z^2 - 4z/(1-\phi)}}{2} \end{aligned}$$

This cannot have a solution if:

$$\begin{aligned} z^2 &< 4z/(1-\phi) \\ \Leftrightarrow z &< 4/(1-\phi) \\ \Leftrightarrow 1-\phi &< 4 \end{aligned}$$

which is necessarily true. Alternatively, if $(1-\varphi)z \leq \varphi$, then, at $t=1$, $p=1$, $q = 1/((1-\phi)R)$ and all producers trade for endowment goods and so $\phi = 1$. In this case, indifference in the $t=0$ IOU market requires:

$$(1-\phi)R = 1$$

but this cannot be satisfied because $\phi = 1$. □

A.2 Proof of Proposition 2

Proof. We need to find conditions on μ such that there is an equilibrium with $\phi = 0$ and no equilibrium with $\phi > 0$.

The $t=1$ market. The equilibrium on the public marketplace is the same as in Subsection 2.2. The equilibrium on the platform is that agents trade output goods and IOUs with price $q = 1/((1-\phi)R)$.

The $t=0$ market. Again, let φ denote the fraction of savers who store IOUs. We start by finding the condition of μ under which $\varphi = 0$ is and equilibrium. In this case, $\phi = 0$, endowment goods are scare so the price on the platform is $q = 1/R$ and the prices on the public marketplace are

$p = (z - R)/z$ and $q = 1/R$. So, the saver at time 0 chooses IOUs if:

$$\begin{aligned}
\eta(1-\mu)R + (1-\eta)R &\geq (1-\eta)z/(z-R) \\
\Rightarrow (\eta(1-\mu) + 1 - \eta)R(z-R) &\geq (1-\eta)z \\
\Rightarrow -R^2 + zR &\geq \frac{(1-\eta)z}{\eta(1-\mu)+1-\eta} \\
\Rightarrow R^2 - zR + \frac{(1-\eta)z}{\eta(1-\mu)+1-\eta} &\geq 0
\end{aligned}$$

which has a solution:

$$\frac{z \pm \sqrt{z^2 - \frac{4(1-\eta)z}{\eta(1-\mu)+1-\eta}}}{2}$$

which has a solution if:

$$\begin{aligned}
z &\geq \frac{4(1-\eta)}{\eta(1-\mu)+1-\eta} \\
\Rightarrow \eta(1-\mu) + 1 - \eta &\geq \frac{4}{z}(1-\eta) \\
\Rightarrow 1 - \mu &\geq \frac{4(1-\eta)}{z\eta} - \frac{1-\eta}{\eta} \\
\Rightarrow \mu &\leq 1 - \left(\frac{4}{z} - 1\right)\frac{1-\eta}{\eta}
\end{aligned}$$

Finally, we now show that there is no equilibrium with $\varphi > 0$. This follows for the platform's promise to discount IOUs.

□

A.3 Discrete Choice Problems

This section of the appendix contains working for the discrete choice problems. Since these are standard results, we provided limited detail.

Lemma 2. *Let $\{\zeta^n\}_{n \leq N}$ be a collection of independent draws from $Gu(\mu, \gamma)$, where $\gamma = -\mu\mathcal{E}$ and $\mathcal{E}$ represents the Euler–Mascheroni constant. Let $u(c) = \log(c)$. Then:*

$$\max_{n \leq N} \{\zeta^n + \varphi^n u(\pi^n)\} \sim Gu\left(\mu + \gamma \log\left(\sum_n (\pi^n)^{\varphi^n/\gamma}\right), \gamma\right) \tag{A.1}$$

and so we have:

$$\mathbb{E}[\max_n\{\zeta^n + \varphi^n \log(\pi^n)\}] = \gamma \log\left(\sum_n (\pi^n)^{\varphi^n/\gamma}\right),$$
$$\mathbb{P}\left(n = argmax_{n'}\left\{\zeta^{n'} + \varphi^{n'}\log(\pi^{n'})\right\}\right) = \frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'}(\pi^{n'})^{\varphi^{n'}/\gamma}}$$

Proof. Using the definition of the Gumbel distribution and the independence of the N draws, we have that:

$$\begin{aligned}\mathbb{P}(\max_n\{\zeta^n + \varphi^n u(\pi^n)\} \le k) &= \prod_n \mathbb{P}(\zeta^n + \varphi^n u(\pi^n) \le k)\\ &= \exp\left(\sum_n -e^{-(k-\mu)/\gamma} e^{\varphi^n u(\pi^n)/\gamma}\right)\\ &= \exp\left(-e^{-\left(k-\mu-\gamma\log\left(\sum_n e^{\varphi^n u(\pi^n)/\gamma}\right)\right)/\gamma}\right)\end{aligned}$$

which implies result (A.1). From the properties of the Gumbel distribution, the expectation is:

$$\begin{aligned}\mathbb{E}[\max_n\{\zeta^n + \varphi^n \log(\pi^n)\}] &= \left[\mu + \gamma\log\left(\sum_n (\pi^n)^{\varphi^n/\gamma}\right)\right] + \gamma\mathcal{E}\\ &= \gamma\log\left(\sum_n (\pi^n)^{\varphi^n/\gamma}\right)\end{aligned}$$

and the probability of choosing n is:

$$\begin{aligned}\mathbb{P}\left(n = \text{argmax}_{n'}\left\{\zeta^{n'} + \varphi^{n'}\log(\pi^{n'})\right\}\right) &= \frac{e^{\varphi^n u(\pi^n)/\gamma}}{\sum_{n'} e^{\varphi^{n'} u(\pi^{n'})/\gamma}}\\ &= \frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'}(\pi^{n'})^{\varphi^{n'}/\gamma}}\end{aligned}$$

□

A.4 Other Proofs

Proof of Theorem 1. We solve the problem recursively.

At age 2:, taking price processes as given, an agent with deposits d chooses on which platform to search to solve problem (A.6) below (dropping the explicit

i superscript and the time subscript on the choice n_2):

$$\begin{aligned} V_{2,t+2}(d) = \mathbb{E}\left[\max_{c,n}\left\{\tilde{\zeta}^n_{2,t+2} + u(c)\right\}\right] \\ s.t. \quad (1+\mu^n_{t+2})c \leq r^{dn}_{t+1,t+2}d/\epsilon^n_{t+2}, \quad \forall n \in \{0,1\}, \end{aligned} \tag{A.2}$$

where $V_{2,t+2}$ is the value function at the start of the agent's final period. Using standard discrete choice results (summarized in Lemma 2 in the Appendix), the value function satisfies:

$$V_{2,t+2}(d) = \log\left(\bar{\nu}_{2,t+2}d\right) \tag{A.3}$$

where the average purchasing power at time τ is:

$$\bar{\nu}_{2,t+2} := \left(\sum_{n_2}\left(\frac{\zeta_2^{n_2} r^{dn_2}_{t+1,t+2}}{(1+\mu^{n_2}_{t+2})\epsilon^{n_2}_{t+2}}\right)^{\gamma^b}\right)^{1/\gamma^b} \tag{A.4}$$

and the fraction of buyers who choose n at time $t+2$ is given by:

$$\eta^n_{2,t+2} = \frac{\left(\zeta^n_2 r^{dn}_{t+1,t+2}/((1+\mu^n_{t+2})\epsilon^n_{t+2})\right)^{\gamma_2}}{\sum_{n'}\left(\zeta^{n'}_2 r^{dn'}_{t+1,t+2}/((1+\mu^{n'}_{t+2})\epsilon^{n'}_{t+2}\right)^{\gamma_2}}$$

At age 1:, after selling production goods, taking price processes as given, an agent who has made profit π in token goods selling on platform n solves the problem:

$$\begin{aligned} V^n_{1,t+1}(\pi) = \max_{c,d}\left\{(1-\beta)u(c) + \beta V_{2,t+2}\left(d\right)\right\} \\ s.t. \quad d \leq \pi - \epsilon^n_{t+1}(1+\mu^n_{t+1})c, \end{aligned}$$

Substituting in the constraint gives the standard consumption saving decision:

$$\max_d\left\{(1-\beta)u\left(\frac{\pi-d}{\epsilon^n_{t+1}(1+\mu^n_{t+1})}\right) + \beta V_{2,t+2}(d)\right\}$$

The first order condition gives:

$$0 = -\frac{(1-\beta)u'(c_{1,t+1})}{\epsilon^n_{t+1}(1+\mu^n_{t+1})} + \beta V'_{2,t+2}(d)$$

Imposing the functional forms and rearranging gives:

$$\frac{1-\beta}{\epsilon^n_{t+1}(1+\mu^n_{t+1})c} = \frac{\beta}{d} = \frac{\beta}{\pi - \epsilon^n_{t+1}(1+\mu^n_{t+1})c}$$

and so:

$$c_{1,t+1} = \frac{(1-\beta)\pi}{\epsilon^n_{t+1}(1+\mu^n_{t+1})}, \qquad\qquad d_{1,t+1} = \beta\pi$$

And so we have:

$$\begin{aligned} V^n_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon^n_{t+1}(1+\mu^n_{t+1})}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^n_{t+1}(1+\mu^n_{t+1})\right) + \log(\pi) \end{aligned}$$

At age 0: Now consider the problem of an agent at age 0 during the morning market. They choose where to purchase input goods, where to sell output goods, and the quantity of input goods to solve (A.7) below:

$$\begin{aligned} V_{0,t} &= \mathbb{E}_t\left[\max_{n_0,n_1,x,\pi}\left\{\tilde{\zeta}^{n_0}_{0,t} + \tilde{\zeta}^{n_1}_{1,t+1} + V^{n_1}_{1,t+1}(\pi)\right\}\right] \quad s.t. \\ \pi &= \epsilon^{n_1}_{t+1} z x^{\alpha} - (1+\mu^{n_0}_t) r^{bn_0}_{t,t+1} x \end{aligned} \tag{A.5}$$

For a given choice of $\underline{\mathbf{n}}_1$, the agent choose x to maximize:

$$\max_x \left\{\epsilon^{n_1}_{t+1} z x^{\alpha} - (1+\mu^{n_0}_t) r^{bn_0}_{t,t+1} x\right\}$$

Taking the FOC gives that producer labor demand, output, and profit are given by:

$$x_{0,t} = \left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{(1+\mu^{n_0}_t) r^{bn_0}_{t,t+1}}\right)^{\frac{1}{1-\alpha}}, \qquad\qquad y^{n_0}_{1,t+1} = z\left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{(1+\mu^{n_0}_t) r^{bn_0}_{t,t+1}}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\pi_{1,t+1} = \left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{((1+\mu^{n_0}_t) r^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

And so we have:

$$
\begin{aligned}
V_{0,t} &= \mathbb{E}_t \left[\max_{n_0,n_1,x,\pi} \left\{ \tilde{\zeta}_{0,t}^{n_0} + \tilde{\zeta}_{1,t+1}^{n_1} + V_{1,t+1}^{n_1}(\pi_{1,t+1}^{(n_0,n_1)}) \right\} \right] \\
&= \mathbb{E}_t \Bigg[\max_{n_0,n_1} \Bigg\{ \tilde{\zeta}_{0,t}^{n_0} + \tilde{\zeta}_{1,t+1}^{n_1} - \log\left(\left(\epsilon_{t+1}^{n_1}(1+\mu_{t+1}^{n_1}) \right)^{1-\beta} \right) \\
&\quad + \log\left(\left(\frac{\epsilon_{t+1}^{n_1}\alpha z}{((1+\mu_t^{n_0})r_{t,t+1}^{bn_0})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \right) \Bigg\} \Bigg] \\
&\quad + \log\left((1-\beta)^{1-\beta}\beta^{\beta} \right) + \beta \log\left(\bar{\nu}_{2,t+2} \right) \\
&= \mathbb{E}_t \left[\max_{n_0,n_1} \left\{ \tilde{\zeta}_{0,t}^{n_0} + \tilde{\zeta}_{1,t+1}^{n_1} + \log\left(\frac{(\epsilon_{t+1}^{n_1})^{\frac{1}{1-\alpha}-(1-\beta)} (1+\mu_{t+1}^{n_1})^{-(1-\beta)}}{\left((1+\mu_t^{n_0})r_{t,t+1}^{bn_0}\right)^{\frac{\alpha}{1-\alpha}}} \right) \right\} \right] \\
&\quad + \log\left((\alpha z)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \right) \\
&\quad + \log\left((1-\beta)^{1-\beta}\beta^{\beta} \right) + \beta \log\left(\bar{\nu}_{2,t+2} \right)
\end{aligned}
$$

and Lemma 2 implies that the fraction of agents at age 0 choosing to purchase inputs on n_0 and at age 1 choosing to purchase on n_1 satisfies:

$$
\begin{aligned}
\eta_{0,t}^{n_0} &= \frac{\left(\zeta_0^{n_0} ((1+\mu_t^{n_0}) r_{t,t+1}^{bn_0})^{-\frac{\alpha}{1-\alpha}} \right)^{\gamma_0}}{\sum_{n_0'} \left(\zeta_0^{n_0'} ((1+\mu_t^{n_0'}) r_{t,t+1}^{bn_0'})^{-\frac{\alpha}{1-\alpha}} \right)^{\gamma_0}} \\
\eta_{1,t+1}^{n_1} &= \frac{\left(\zeta_1^{n_1} \left(\epsilon_{t+1}^{n_1} \right)^{\frac{1}{1-\alpha}+\beta-1} (1+\mu_{t+1}^{n_1})^{\beta-1} \right)^{\gamma_1}}{\sum_{n_0' n_1'} \left(\zeta_1^{n_1'} \left(\epsilon_{t+1}^{n_1'} \right)^{\frac{1}{1-\alpha}+\beta-1} (1+\mu_{t+1}^{n_1'})^{\beta-1} \right)^{\gamma_1}}
\end{aligned}
$$

and:

$$
\begin{aligned}
V_{0,t} = \; & \log(\bar{\nu}_{0,t}) + \log(\bar{\nu}_{1,t+1}) + \beta \log\left(\bar{\nu}_{2,t+2} \right) \\
& + \log\left((\alpha z)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \right) + \log\left((1-\beta)^{1-\beta}\beta^{\beta} \right)
\end{aligned}
$$

where:

$$\bar{\nu}_{0,t} := \left(\sum_{n_0} \left(\zeta_0^{n_0} \left((1+\mu_t^{n_0}) r_{t,t+1}^{bn_0} \right)^{-\frac{\alpha}{1-\alpha}} \right)^{\gamma_0} \right)^{1/\gamma_0}$$

$$\bar{\nu}_{1,t+1} := \left(\sum_{n_1} \left(\zeta_1^{n_1} \left(\epsilon_{t+1}^{n_1} \right)^{\frac{1}{1-\alpha}+\beta-1} (1+\mu_{t+1}^{n_1})^{\beta-1} \right)^{\gamma_1} \right)^{1/\gamma_1}$$

and $\bar{\nu}_{2,t+2}$ is given by (A.4). □

Proof of Lemma 1. Afternoon at time t: Let A_t denote the real value of deposits in the afternoon sub-period at time t. The fund invests the portfolio by purchasing B_t $(t+1)$-IOUs, M_t money, and S_t shares subject to the budget constraint:

$$q_t^b B_t + q_t^m M_t + q_t^s S_t \leq A_t$$

The fund offers the return on money for money withdrawals so it must hold:

$$\begin{aligned} & q_{t+1}^m M_t \geq R_{t,t+1}^m \eta_{2,t+1}^o A_t \\ \Rightarrow\ & q_t^m M_t \geq \eta_{2,t+1}^o A_t \end{aligned}$$

Morning at time t: Let $\breve{M}_t := M_t - \eta_{2,t+1}^o A_t / q_t^m$ denote excess holdings of money beyond the money-in-advance constraint. The fund uses its excess money to discount new IOUs. This leaves it with $\breve{E}_t \breve{M}_t$ IOUs. Incorporating discounting, the wealth available to depositors trading on the platform is $\breve{M}_t \breve{E}_t$ $(t+2)$-IOUs, B_t $(t+1)$-IOUs, and S_t shares. In real terms, this gives wealth:

$$\begin{aligned} \breve{a}_{t+1} &= \frac{1}{P_{t+1}^b} \breve{M}_t \breve{E}_t + B_t + \frac{1}{P_{t+1}^s} S_t \\ &= q_{t+1}^b \breve{M}_t \breve{E}_t + B_t + q_{t+1}^s S_t \\ &= \left(\frac{q_{t+1}^b \breve{E}_{t+1}}{q_t^m} \breve{\theta}_t^m + \frac{1}{q_t^b} \theta_t^b + \frac{q_{t+1}^s + \pi_{t+1}^s}{q_t^s} \theta_t^s \right) A_t \end{aligned}$$

where $\breve{\theta}_t^m := q_t^m \breve{\theta}_t^m / A_t$, $\theta_t^b := q_t^b B_t / A_t$, and $\theta_t^s := q_t^s S_t / A_t$.

Optimization: In an equilibrium with discounting of IOUs, the fund is indifferent across asset classes. That is:

$$\frac{q_{t+1}^b \breve{E}_{t+1}}{q_t^m} = \frac{1}{q_t^b} = \frac{q_{t+1}^s + \pi_{t+1}^s}{q_t^s}$$

so we have that:

$$\breve{E}_{t+1} = \frac{1}{q_t^b}\frac{q_t^m}{q_{t+1}^b} = \frac{1}{q_t^b}\frac{q_t^m}{q_{t+1}^m/E_t} = E_{t+1}\frac{R_{t,t+1}^b}{R_{t,t+1}^m}$$

and the depositor returns are:

$$r_{t,t+1}^{dp} = R_{t,t+1}^b$$
$$r_{t,t+1}^{do} = R_{t,t+1}^m$$

Entrepreneur borrowing costs: Now, we can return to the problem of the entrepreneur. The entrepreneur needs to purchase $x_{0,t}$ goods. If they trade on the platform, then they issue $P_t^b(1+\mu_t^{n_0})x_{0,t}$ IOUs. If they trade on the public marketplace, then they issue $\breve{E}_t P_t^m(1+\mu_t^{n_0})x_{0,t}$ IOUs. Each IOU promises 1 good so their total borrowing costs are:

$$(1+\mu_t^{n_0})\breve{E}_t^{n_0}P_t^{n_0}x.$$

That is, $r_{t,t+1}^{bn} := \breve{E}_t^{n_0}P_t^{n_0}$ is given by (imposing the indifference condition):

$$r_{t,t+1}^{bp} := P_t^b = \frac{1}{q_t^b} = R_{t,t+1}^b$$
$$r_{t,t+1}^{bo} := \breve{E}_t P_t^m = E_t\frac{R_{t-1,t}^b}{R_{t-1,t}^m}P_t^m = \frac{R_{t-1,t}^b}{R_{t-1,t}^m}\epsilon_t P_t^b = \frac{R_{t-1,t}^b}{R_{t-1,t}^m}\epsilon_t R_{t,t+1}$$

Equilibrium balance sheet: In equilibrium, the fund must hold enough money at t to discount all IOUs and fulfill withdrawal requests at $t+1$:

$$\begin{aligned}
M_t &= \frac{R_{t,t+1}^m}{q_{t+1}^m}\eta_{2,t+1}^o A_t + \sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}P_{t+1}^m(1+\mu_{t+1}^o)x_{0,t+1} \\
&= \frac{\eta_{2,t+1}^o}{q_t^m}A_t + \frac{1}{q_{t+1}^m}\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\epsilon_{t+1}x_{0,t+1} \\
&= \frac{\eta_{2,t+1}^o}{q_t^m}A_t + \frac{1}{q_t^m R_{t,t+1}^m}\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\epsilon_{t+1}x_{0,t+1}
\end{aligned}$$

So, in real terms the portfolio is:

$$q_t^m M_t = \eta_{2,t+1}^o A_t + \frac{1}{R_{t,t+1}^m}\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\epsilon_{t+1}x_{0,t+1}$$

□

Proof of Theorem 2. Summarising the equilibrium and optimization equations gives the following characterization of equilibrium. Given states $\{M_t, \mu_{t-2}, \mu_{t-1}, R_{t-1}\}$ and current policy μ_t, we can solve for the equilibrium variables at time t:

$$\left(\underline{\mathbf{c}}_{1,t}^{\mathbf{n}_t}, \underline{\mathbf{c}}_{2,t}^{\mathbf{n}_t}, \underline{\mathbf{x}}_t^{\mathbf{n}_{t+1}}, \underline{\mathbf{y}}_t^{\mathbf{n}_t}, \underline{\boldsymbol{\pi}}_t^{\mathbf{n}_t}, \underline{\mathbf{d}}_t^{\mathbf{n}_t}, \epsilon_t, R_t^{bn}, R_t^{dn}, \underline{\boldsymbol{\eta}}_t^{\mathbf{n}_t}\right)$$

using the equations for agent choices:

$$x_{0,t}^{\mathbf{n}_1} = \left(\frac{\epsilon_{t+1}^{n_1}\alpha z}{(1+\mu_t^{n_0})r_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}} \qquad y_{1,t}^{\mathbf{n}_1} = z\left(\frac{\epsilon_t^{n_1}\alpha z}{(1+\mu_{t-1}^{n_0})r_{t-1,t}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\pi_{1,t}^{\mathbf{n}_1} = \left(\frac{\epsilon_t^{n_1}\alpha z}{((1+\mu_{t-1}^{n_0})r_{t-1,t}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) \qquad c_{1,t}^{\mathbf{n}_1} = \frac{(1-\beta)\pi_{1,t}^{\mathbf{n}_1}}{(1+\mu_t^{n_1})\epsilon_t^{n_1}}$$

$$d_{1,t}^{\mathbf{n}_1} = \beta\pi_{1,t}^{\mathbf{n}_1} \qquad c_{2,t}^{\mathbf{n}_2} = \frac{r_{t-1,t}^{dn_2}\beta\pi_{1,t-1}^{\mathbf{n}_1}}{(1+\mu_t^{n_2})\epsilon_t^{n_2}}$$

$$\eta_{0,t}^{n_0} = \frac{\left(\zeta_0^{n_0}((1+\mu_t^{n_0})r_{t,t+1}^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta_0^{n_0'}((1+\mu_t^{n_0'})r_{t,t+1}^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \qquad \eta_{2,t}^{n_2} = \frac{\left(\zeta_2^{n}r_{t-1,t}^{dn}/((1+\mu_t^{n})\epsilon_t^{n})\right)^{\gamma_2}}{\sum_{n_2'}\left(\zeta_2^{n'}r_{t-1,t}^{dn'}/((1+\mu_t^{n'})\epsilon_t^{n'}\right)^{\gamma_2}}$$

$$\eta_{1,t}^{n_1} = \frac{\left(\zeta_1^{n_1}\left(\epsilon_{t+1}^{n_1}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu_{t+1}^{n_1})^{\beta-1}\right)^{\gamma_1}}{\sum_{n_0'n_1'}\left(\zeta_1^{n_1'}\left(\epsilon_{t+1}^{n_1'}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu_{t+1}^{n_1'})^{\beta-1}\right)^{\gamma_1}}$$

fund equations:

$$r_{t,t+1}^{bn} = \epsilon_t^n\frac{R_{t-1,t}^b}{R_{t-1,t}^n}R_{t,t+1}^b \qquad r_{t,t+1}^{dn} = R_{t,t+1}^n$$

$$A_t = q_t^m M_t + q_t^b B_t + q_t^s S_t \qquad \eta_{2,t+1}^m A_t + X_{0,t+1}^o \leq q_t^m M_t$$

$$A_t = \sum_{n_0,n_1}\eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}d_{1,t}^{(n_0,n_1)}$$

and the market clearing conditions:

$$\begin{aligned}
\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}y^{(n_0,n)}_{1,t} &= \eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t} + \eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}c^{(n_0,n)}_{1,t} \\
&\quad + \eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t} \\
q^b_t\breve{B}_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}r^{bn_0}_{t,t+1}(1+\mu^{n_0}_{t+1})x^{(n_0,n_1)}_{0,t} \\
\breve{B}_t &= B_t \\
q^m_t\bar{M}_t &= \eta^o_{2,t+1}A_t + \frac{1}{R^m_{t,t+1}}\sum_{n_1}\eta^o_{0,t+1}\eta^{n_1}_{1,t+2}\epsilon_{t+1}x_{0,t+1} \\
S_t &= 1
\end{aligned}$$

(i) Solve for ϵ_t: We start by solving for ϵ_t. Substituting the fund returns into the agent choices gives:

$$\eta^{n_0}_{0,t} = \frac{\left(\zeta^{n_0}_0(\epsilon^{n_0}_t(1+\mu^{n_0}_t)/R^{n_0}_{t-1,t})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n'_0}\left(\zeta^{n'_0}_0(\epsilon^{n'_0}_t(1+\mu^{n'_0}_t)/R^{n'_0}_{t-1,t})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \quad \eta^{n_2}_{2,t} = \frac{\left(\zeta^n_2 R^n_{t-1,t}/((1+\mu^n_t)\epsilon^n_t)\right)^{\gamma_2}}{\sum_{n'}\left(\zeta^{n'}_2 R^{n'}_{t-1,t}/((1+\mu^{n'}_t)\epsilon^{n'}_t\right)^{\gamma_2}}$$

Now, return to the goods market clearing condition. After rearranging, we have:

$$\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}\left(y^{(n_0,n)}_{1,t} - c^{(n_0,n)}_{1,t}\right) = \eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t} + \eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t}$$

where the LHS can be computed using:

$$\begin{aligned}
&y^{(n_0,n)}_{1,t} - c^{(n_0,n)}_{1,t} \\
&\quad = z\left(\frac{\alpha z\epsilon^n_t}{(1+\mu^{n_0}_{t-1})r^{bn_0}_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} - \frac{(1-\beta)}{(1+\mu^n_t)\epsilon^n_t}\left(\frac{\alpha z\epsilon^n_t}{((1+\mu^{n_0}_{t-1})r^{bn_0}_{t-1,t})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) \\
&\quad = \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu^n_t}\right)z\left(\frac{\alpha z\epsilon^n_t}{(1+\mu^{n_0}_{t-1})r^{bn_0}_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \\
&\quad = \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu^n_t}\right)y^{(n_0,n)}_{1,t}
\end{aligned}$$

and so:

$$\eta_{1,t}^n \sum_{n_0} \eta_{0,t-1}^{n_0} \left(y_{1,t}^{(n_0,n)} - c_{1,t}^{(n_0,n)} \right)$$

$$= \eta_{1,t}^n \sum_{n_0} \eta_{0,t-1}^{n_0} \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^n} \right) z \left(\frac{\alpha z \epsilon_t^n}{(1+\mu_{t-1}^{n_0}) r_{t-1,t}^{bn_0}} \right)^{\frac{\alpha}{1-\alpha}}$$

$$= \eta_{1,t}^n \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^n} \right) z (\alpha z \epsilon_t^n)^{\frac{\alpha}{1-\alpha}} \sum_{n_0} \frac{\eta_{0,t-1}^{n_0}}{((1+\mu_{t-1}^{n_0}) r_{t-1,t}^{bn_0})^{\frac{\alpha}{1-\alpha}}}$$

and where the RHS can be computed using:

$$\eta_{0,t}^n \sum_{n_1} \eta_{1,t+1}^{n_1} x_{0,t}^{(n,n_1)} + \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)}$$

$$= \eta_{0,t}^n \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\frac{\epsilon_{t+1}^{n_1} \alpha z}{(1+\mu_t^n) r_{t,t+1}^{bn}} \right)^{\frac{1}{1-\alpha}}$$

$$+ \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \frac{r_{t-1,t}^{dn} \beta}{(1+\mu_t^n)\epsilon_t^n} \left(\frac{\epsilon_t^{n_1} \alpha z}{((1+\mu_{t-1}^{n_0}) r_{t-1,t}^{bn_0})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right)$$

$$= \frac{\left(\zeta_0^n ((1+\mu_t^n) r_{t,t+1}^{bn})^{-\frac{\alpha}{1-\alpha}} \right)^{\gamma_0}}{\sum_{n'} \left(\zeta_0^{n'} ((1+\mu_t^{n'}) r_{t,t+1}^{bn'})^{-\frac{\alpha}{1-\alpha}} \right)^{\gamma_0}} \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\frac{\epsilon_{t+1}^{n_1} \alpha z}{(1+\mu_t^n) r_{t,t+1}^{bn}} \right)^{\frac{1}{1-\alpha}}$$

$$+ \frac{\left(\zeta_2^n r_{t-1,t}^{dn} / ((1+\mu_t^n)\epsilon_t^n) \right)^{\gamma_2}}{\sum_{n'} \left(\zeta_2^{n'} r_{t-1,t}^{dn'} / ((1+\mu_t^{n'})\epsilon_t^{n'} \right)^{\gamma_2}}$$

$$\sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \frac{r_{t-1,t}^{dn} \beta}{(1+\mu_t^n)\epsilon_t^n} \left(\frac{\epsilon_t^{n_1} \alpha z}{((1+\mu_{t-1}^{n_0}) r_{t-1,t}^{bn_0})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right)$$

$$= (\zeta_0^n)^{\gamma_0} \left((1+\mu_t^n) r_{t,t+1}^{bn} \right)^{-\frac{1+\alpha\gamma_0}{1-\alpha}} \frac{1}{\bar{\nu}_{0,t}^{\gamma_0}} \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\epsilon_{t+1}^{n_1} \alpha z \right)^{\frac{1}{1-\alpha}}$$

$$+ (\zeta_2^n)^{\gamma_2} \left(\frac{r_{t-1,t}^{dn}}{(1+\mu_t^n)\epsilon_t^n} \right)^{1+\gamma_2} \frac{1}{\bar{\nu}_{2,t}^{\gamma_2}} \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \beta \left(\frac{\epsilon_t^{n_1} \alpha z}{((1+\mu_{t-1}^{n_0}) r_{t-1,t}^{bn_0})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right)$$

$$= (\zeta_0^n)^{\gamma_0} \left((1+\mu_t^n)\epsilon_t^n \frac{R_{t-1,t}^b R_{t,t+1}^b}{R_{t-1,t}^n} \right)^{-\frac{1+\alpha\gamma_0}{1-\alpha}} \frac{1}{\bar{\nu}_{0,t}^{\gamma_0}} \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\epsilon_{t+1}^{n_1} \alpha z \right)^{\frac{1}{1-\alpha}}$$

$$+ (\zeta_2^n)^{\gamma_2} \left(\frac{R_{t-1,t}^n}{(1+\mu_t^n)\epsilon_t^n} \right)^{1+\gamma_2} \frac{1}{\bar{\nu}_{2,t}^{\gamma_2}} \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \beta \left(\frac{\epsilon_t^{n_1} \alpha z}{((1+\mu_{t-1}^{n_0}) r_{t-1,t}^{bn_0})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right)$$

Under the assumption that $\frac{1+\alpha\gamma_0}{1-\alpha} = 1+\gamma_2$ and $(\zeta_0^n)^{\gamma_0} = (\zeta_2^n)^{\gamma_2}$, we can take out the n specific component to get:

$$\begin{aligned}
&\eta_{0,t}^n \sum_{n_1} \eta_{1,t+1}^{n_1} x_{0,t}^{(n,n_1)} + \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)} \\
&\quad = (\zeta_2^n)^{\gamma_2} \left(\frac{R_{t-1,t}^n}{(1+\mu_t^n)\epsilon_t^n} \right)^{1+\gamma_2} \left[\frac{1}{\bar{\nu}_{0,t}^{\gamma_0}} \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\frac{\alpha z \epsilon_{t+1}^{n_1}}{(R_{t-1,t}^b R_{t,t+1}^b)^{1+\alpha\gamma_0}} \right)^{\frac{1}{1-\alpha}} \right. \\
&\qquad \left. + \frac{1}{\bar{\nu}_{2,t}^{\gamma_2}} \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \left(\frac{\alpha z \epsilon_{t-1}^{n_1}}{((1+\mu_{t-1}^{n_0}) r_{t-2,t-1}^{bn_0})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \right]
\end{aligned}$$

So, the market clearing condition in goods market n becomes:

$$\begin{aligned}
&\eta_{1,t}^n \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^n} \right) z (\alpha z \epsilon_t^n)^{\frac{\alpha}{1-\alpha}} \sum_{n_0} \frac{\eta_{0,t-1}^{n_0}}{((1+\mu_{t-1}^{n_0}) r_{t-1,t}^{bn_0})^{\frac{\alpha}{1-\alpha}}} \\
&\quad = (\zeta_2^n)^{\gamma_2} \left(\frac{R_{t-1,t}^n}{(1+\mu_t^n)\epsilon_t^n} \right)^{1+\gamma_2} \left[\frac{1}{\bar{\nu}_{0,t}^{\gamma_0}} \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\frac{\alpha z \epsilon_{t+1}^{n_1}}{(R_{t-1,t}^b R_{t,t+1}^b)^{1+\alpha\gamma_0}} \right)^{\frac{1}{1-\alpha}} \right. \\
&\quad \left. + \frac{1}{\bar{\nu}_{2,t}^{\gamma_2}} \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \left(\frac{\alpha z \epsilon_{t-1}^{n_1}}{((1+\mu_{t-1}^{n_0}) r_{t-2,t-1}^{bn_0})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \right]
\end{aligned}$$

Dividing the market clearing condition in market n by the market clearing condition in market n' gives:

$$\left(\frac{\zeta_1^n \left(\epsilon_t^n\right)^{\frac{1}{1-\alpha}+\beta-1} (1+\mu_t^n)^{\beta-1}}{\zeta_1^{n'} \left(\epsilon_t^{n'}\right)^{\frac{1}{1-\alpha}+\beta-1} (1+\mu_t^{n'})^{\beta-1}} \right)^{\gamma_1} \left(\frac{\left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^n}\right)\epsilon_t^n}{\left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^{n'}}\right)\epsilon_t^{n'}} \right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\zeta_2^n}{\zeta_2^{n'}} \right)^{\gamma_2} \left(\frac{\frac{R_{t-1,t}^n}{(1+\mu_t^n)\epsilon_t^n}}{\frac{R_{t-1,t}^{n'}}{(1+\mu_t^{n'})\epsilon_t^{n'}}} \right)^{1+\gamma_2}$$

which implies that:

$$\left(\frac{\epsilon_t^n}{\epsilon_t^{n'}} \right)^{\frac{\gamma_1+\alpha}{1-\alpha} - \gamma_1(1-\beta)+1+\gamma_2} \left(\frac{1+\mu_t^n}{1+\mu_t^{n'}} \right)^{-\gamma_1(1-\beta)} \left(\frac{1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^n}}{1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^{n'}}} \right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\zeta_2^n}{\zeta_2^{n'}} \right)^{\gamma_2} \left(\frac{\zeta_1^n}{\zeta_1^{n'}} \right)^{-\gamma_1} \left(\frac{R_{t-1,t}^n}{R_{t-1,t}^{n'}} \right)^{1+\gamma_2}$$

which gives:

$$\frac{\epsilon_t^n}{\epsilon_t^{n'}} = \left[\left(\frac{\zeta_2^n}{\zeta_2^{n'}} \right)^{\gamma_2} \left(\frac{\zeta_1^n}{\zeta_1^{n'}} \right)^{-\gamma_1} \left(\frac{R_{t-1,t}^n}{R_{t-1,t}^{n'}} \right)^{1+\gamma_2} \left(\frac{1+\mu_t^n}{1+\mu_t^{n'}} \right)^{-\gamma_1(1-\beta)} \left(\frac{1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^n}}{1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t^{n'}}} \right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{1}{\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)}}$$

and so for $n = o$ and $n' = p$ we have:

$$\epsilon_t = \left[\zeta_1^{\gamma_1} \zeta_2^{-\gamma_2} \left(\frac{R_{t-1,t}^m}{R_{t-1,t}^b} \right)^{1+\gamma_2} (1+\mu_t)^{\gamma_1(1-\beta)} \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t} \right)^{\frac{\alpha}{1-\alpha}} \right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$

(ii) Solve for $R_{t,t+1}^b$: We have the following equations for the morning "primary" IOU market, the morning deposit market, the afternoon fund budget constraint, the afternoon bond market, the afternoon money market, and the afternoon equity market.

$$\begin{aligned}
q_t^b \breve{B}_t &= \sum_{n_0,n_1} \eta_{0,t}^{n_0} \eta_{1,t+1}^{n_1} r_{t,t+1}^{bn_0} (1+\mu_{t+1}^{n_0}) x_{0,t}^{(n_0,n_1)} \\
A_t &= \sum_{n_0,n_1} \eta_{0,t-1}^{n_0} \eta_{1,t}^{n_1} d_{1,t}^{(n_0,n_1)} \\
A_t &= q_t^b B_t + q_t^m M_t + q_t^s S_t \\
B_t &= \breve{B}_t \\
q_t^m M_t &= \eta_{2,t+1}^o A_t + \frac{1}{R_{t,t+1}^m} \sum_{n_1} \eta_{0,t+1}^o \eta_{1,t+2}^{n_1} \epsilon_{t+1} x_{0,t+1} \\
S_t &= 1
\end{aligned}$$

Substituting the other equations into the fund budget constraint:

$$\begin{aligned}
\sum_{n_0,n_1} \eta_{0,t-1}^{n_0} \eta_{1,t}^{n_1} d_{1,t}^{(n_0,n_1)} = & \sum_{n_0,n_1} \eta_{0,t}^{n_0} \eta_{1,t+1}^{n_1} \epsilon_t^{n_0} \frac{R_{t-1,t}^b}{R_{t-1,t}^{n_0}} R_{t,t+1}^b (1+\mu_{t+1}^{n_0}) x_{0,t}^{(n_0,n_1)} \\
& + \eta_{2,t+1}^m \sum_{n_0,n_1} \eta_{0,t-1}^{n_0} \eta_{1,t}^{n_1} d_{1,t}^{(n_0,n_1)} \\
& + \sum_{n_1} \eta_{0,t+1}^m \eta_{1,t+2}^{n_1} \frac{\epsilon_{t+1}}{R_{t,t+1}^m} (1+\mu_{t+1}^{n_0}) x_{0,t+1}^{(m,n_1)} + q_t^s
\end{aligned}$$

and so after rearranging and imposed $d_{t,1} = \beta\pi_{1,t}$ we have:

$$\begin{aligned}
&(1-\eta^m_{2,t+1}) \sum_{n_0,n_1} \eta^{n_0}_{0,t-1}\eta^{n_1}_{1,t}\beta\pi^{(n_0,n_1)}_{1,t} \\
&= \sum_{n_1}\Bigg(\sum_{n_0}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\epsilon^{n_0}_t\frac{R^b_{t-1,t}}{R^{n_0}_{t-1,t}}R^b_{t,t+1}(1+\mu^{n_0}_{t+1})x^{(n_0,n_1)}_{0,t} \\
&\quad + \eta^m_{0,t+1}\frac{\epsilon_{t+1}}{R^m_{t,t+1}}x^{(m,n_1)}_{0,t+1}\Bigg) + q^s_t \\
&= \sum_{n_1}\Bigg(\sum_{n_0}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\epsilon^{n_0}_t\mathcal{S}^{n_0}_{t-1,t}R^b_{t,t+1}(1+\mu^{n_0}_{t+1})x^{(n_0,n_1)}_{0,t} \\
&\quad + \eta^m_{0,t+1}\frac{\mathcal{S}_{t,t+1}\epsilon_{t+1}}{R^b_{t,t+1}}x^{(m,n_1)}_{0,t+1}\Bigg) + q^s_t
\end{aligned}$$

which gives the expression in the main text.

Return on Money: Rearranging the money market clearing condition:

$$\begin{aligned}
q^m_t\bar{M}_t &= \eta^o_{2,t+1}A_t + \frac{1}{R^m_{t,t+1}}\sum_{n_1}\eta^o_{0,t+1}\eta^{n_1}_{1,t+2}\epsilon_{t+1}x_{0,t+1} \\
&= \eta^o_{2,t+1}\beta\Pi_{1,t} + \frac{1}{R^m_{t,t+1}}X^o_{0,t+1}
\end{aligned}$$

gives that:

$$R^m_{t,t+1} = \frac{q^m_{t+1}}{q^m_t} = \left(\frac{\bar{M}_t}{\bar{M}_{t+1}}\right)\left(\frac{\eta^o_{2,t+1}\beta\Pi_{1,t} + \frac{1}{R^m_{t,t+1}}X^o_{0,t+1}}{\eta^o_{2,t+2}\beta\Pi_{1,t+1} + \frac{1}{R^m_{t,t+2}}X^o_{0,t+2}}\right)$$

so in the steady state, this is:

$$\overline{R}^m = 1/\mu^M$$

Equity Price: The equity price satisfies:

$$q^s_t = \frac{1}{R_{t,t+1}}\left(\pi^s_{t+1} + q^s_{t+1}\right)$$

In the steady state, this implies that:

$$\bar{q}^s = \frac{\bar{\pi}^s}{\bar{R}^b - 1}$$

□

Proof of Theorem 3. We consider the value for an agent who chooses to default when other agents are choosing to not default. We solve the problem recursively.

At age 2: an agent who has defaulted and joined a fund taking defaulting agents can only hold cash and so only trade on the public marketplace. Taking price processes as given, an agent with deposits d chooses on which platform to search to solve problem (A.6) below (dropping the explicit i superscript and the time subscript on the choice n_2):

$$\begin{aligned} \check{V}_{2,t+2}(d) = \mathbb{E}\left[\max_{c}\left\{\tilde{\zeta}^{o}_{2,t+2} + u(c)\right\}\right] \\ s.t. \quad c \leq r^{o}_{t+1,t+2} d/\epsilon^{o}_{t+2}, \end{aligned} \tag{A.6}$$

where $\check{V}_{2,t+2}$ is the value function at the start of the agent's final period. Evaluating this expression gives:

$$\check{V}_{2,t+2}(d) = \log\left(\check{\nu}_{2,t+2} d\right)$$

where $\check{\nu}_{2,t+2} := r^{o}_{t+1,t+2}/\epsilon^{o}_{t+2}$.

At age 1: the agent cannot default if they end up trading on the private platform. So, their value is:

$$\begin{aligned} V^{p}_{1,t+1}(\pi) = \max_{c,d}\left\{(1-\beta)u(c) + \beta V_{2,t+2}\left(d\right)\right\} \\ s.t. \quad d \leq \pi - \epsilon^{p}_{t+1}(1+\mu^{p}_{t+1})c, \end{aligned}$$

and from the proof of Theorem 1 we have:

$$\begin{aligned} V^{p}_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon^{p}_{t+1}(1+\mu^{p}_{t+1})}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^{p}_{t+2}(1+\mu^{p}_{t+1})\right) + \log(\pi) \end{aligned}$$

However, if they trade on the public marketplace, then they can trade with cash, default, and go to a bank accepting cash trades without reporting them to the ledger. In this case, their value at $t = 1$ is given by:

$$\begin{aligned} \check{V}^{o}_{1,t+1}(\pi) = \max_{c,d}\left\{(1-\beta)u(c) + \beta\check{V}_{2,t+2}\left(d\right)\right\} \\ s.t. \quad d \leq \check{\pi} - \epsilon_{t+1}c, \end{aligned}$$

where $\check{\pi}$ is the profit if the agent defaults. As before, we have that:

$$c_{1,t+2} = \frac{(1-\beta)\check{\pi}}{\epsilon_{t+1}}, \qquad\qquad d_{1,t+1} = \beta\check{\pi}$$

And so we have:

$$\begin{aligned}\check{V}^{n}_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\check{\pi}}{\epsilon^{n}_{t+1}}\right) + \beta\log\left(\check{\nu}_{2,t+2}\beta\check{\pi}\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+2}\right) + \log(\check{\pi})\end{aligned}$$

At Age 0: Now consider the problem of an agent at age 0 conditional on choosing to sell on the public marketplace $(n_0, n_1) = (n_0, o)$. If they intend to repay at age 1, then they solve the same problem as before:

$$\begin{aligned}V_{0,t|n_1=o} = \max_{x,\pi}\left\{V^{o}_{1,t+1}(\pi)\right\} \quad s.t. \\ \pi = \epsilon^{o}_{t+1} z x^{\alpha} - (1+\mu^{n_0}_{t}) r^{bn_0}_{t,t+1} x\end{aligned} \tag{A.7}$$

Taking the FOCs gives profit:

$$\pi^{(n_0,o)}_{1,t+1} = \left(\frac{\epsilon^{o}_{t+1}\alpha z}{((1+\mu^{n_0}_{t})r^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

From the proof of Theorem 1, we have that:

$$\begin{aligned}V_{0,t|n_1=o} &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+1}\right) \\ &\quad + \log(\pi^{(n_0,o)}_{1,t+1}) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+1}\right) \\ &\quad + \log\left(\left(\frac{\epsilon_{t+1}\alpha z}{((1+\mu^{n_0}_{t})r^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right)\end{aligned}$$

If they intend to default at age 1, then they solve (A.8) below:

$$\begin{aligned}\check{V}_{0,t|n_1=o} = \mathbb{E}_t\left[\max_{x,\pi}\left\{\check{V}^{o}_{1,t+1}(\check{\pi})\right\}\right] \quad s.t. \\ \check{\pi} = \epsilon^{o}_{t+1} z x^{\alpha} - \chi(1+\mu^{n_0}_{t})x\end{aligned} \tag{A.8}$$

For a given choice of n_0, they choose x to maximize:

$$\max_x \left\{ \epsilon^o_{t+1} z x^\alpha - \chi(1+\mu^o_t)x \right\}$$

Taking the FOC gives profit:

$$\check{\pi}^{(n_0,o)}_{1,t+1} = \left(\frac{\epsilon^o_{t+1}\alpha z}{(\chi(1+\mu^o_t))^\alpha} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right).$$

So, their value is:

$$\begin{aligned}
\check{V}_{0,t|n_1=o} &= \log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+1}\right) \\
&\quad + \log(\check{\pi}^{(n_0,o)}_{1,t+1}) \\
&= \log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+1}\right) \\
&\quad + \log\left(\left(\frac{\epsilon_{t+1}\alpha z}{(\chi(1+\mu^{n_0}_t))^\alpha} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right)\right)
\end{aligned}$$

Incentive compatibility: An agent choosing (n_0, o) chooses to repay if:

$$\begin{aligned}
V_{0,t|n_1=o} &\geq \check{V}_{0,t|n_1=o} \\
\Rightarrow \beta\log\left(\bar{\nu}_{2,t+2}\right) + \log(\pi^{(n_0,o)}_{1,t+1}) &\geq \beta\log\left(\check{\nu}_{2,t+2}\right) + \log(\check{\pi}^{(n_0,o)}_{1,t+1}) \\
\Rightarrow \beta\log\left(\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}}\right) &\geq \log\left(\frac{\check{\pi}^{(n_0,o)}_{1,t+1}}{\pi^{(n_0,o)}_{1,t+1}}\right) \\
\Rightarrow \beta\log() &= \left(\frac{\alpha}{1-\alpha}\right)\log\left(\frac{r^{bn_0}_{t,t+1}}{\chi}\right)
\end{aligned}$$

where:

$$\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}} = \frac{\left(\sum_{n_2}\left(\frac{\zeta_2^{n_2} r^{dn_2}_{t+1,t+2}}{(1+\mu^{n_2}_{t+2})\epsilon^{n_2}_{t+2}}\right)^{\gamma^b}\right)^{1/\gamma^b}}{\frac{\zeta^o_2 r^{do}_{t+1,t+2}}{\epsilon_{t+2}}} = \left(1 + \left(\left(\frac{\zeta^p_2}{\zeta^o_2}\right)\left(\frac{r^p_{t+1,t+2}}{r^o_{t+1,t+2}}\right)\left(\frac{\epsilon_{t+2}}{1+\mu_{t+2}}\right)\right)^{\gamma^b}\right)^{1/\gamma^b}$$

and

$$\frac{\breve{\pi}_{1,t+1}^{(n_0,o)}}{\pi_{1,t+1}^{(n_0,o)}} = \frac{\left(\frac{\epsilon_{t+1}^{o}\alpha z}{(\chi(1+\mu_t^o))^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)}{\left(\frac{\epsilon_{t+1}^{o}\alpha z}{((1+\mu_t^{n0})r_{t,t+1}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)} = \left(\frac{r_{t,t+1}^{bn_0}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}$$

So, the incentive compatibility constraint becomes:

$$\left(1+\left(\left(\frac{r_{t+1,t+2}^{p}}{r_{t+1,t+2}^{o}}\right)\left(\frac{\zeta_2\epsilon_{t+2}}{1+\mu_{t+2}}\right)\right)^{\gamma^b}\right)^{1/\gamma^b} \geq \left(\frac{r_{t,t+1}^{bn_0}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}$$

Rearranging, this becomes:

$$\frac{\zeta_2\epsilon_{t+2}}{1+\mu_{t+2}} \geq \left(\frac{r_{t+1,t+2}^{o}}{r_{t+1,t+2}^{p}}\right)\left(\left(\frac{r_{t,t+1}^{bn_0}}{\chi}\right)^{\frac{\alpha\gamma^b}{1-\alpha}}-1\right)^{1/\gamma^b}$$

which implies:

$$\begin{aligned}
1+\mu_{t+2} &\leq \zeta_2\epsilon_{t+2}\left(\frac{r_{t+1,t+2}^{dp}}{r_{t+1,t+2}^{do}}\right)\left(\left(\frac{r_{t,t+1}^{bn_0}}{\chi}\right)^{\frac{\alpha\gamma^b}{1-\alpha}}-1\right)^{-1/\gamma^b} \\
&= \zeta_2\epsilon_{t+2}\left(\frac{R_{t+1,t+2}^{b}}{R_{t+1,t+2}^{m}}\right)\left(\left(\frac{\epsilon_t^{n_0}\frac{R_{t-1,t}^{b}}{R_{t-1,t}^{n0}}R_{t,t+1}^{b}}{\chi}\right)^{\frac{\alpha\gamma^b}{1-\alpha}}-1\right)^{-1/\gamma^b} \\
&= \zeta_2\epsilon_{t+2}\mathcal{S}_{t+1,t+2}\left(\left(\frac{\epsilon_t^{n_0}\mathcal{S}_{t-1,t}^{n_0}R_{t,t+1}^{b}}{\chi}\right)^{\frac{\alpha\gamma^b}{1-\alpha}}-1\right)^{-1/\gamma^b}
\end{aligned}$$

So, the condition is:

$$1+\mu_{t+2} \leq \zeta_2\epsilon_{t+2}\mathcal{S}_{t+1,t+2}\left(\left(\frac{\min\{\epsilon_t\mathcal{S}_{t-1,t},1\}R_{t,t+1}^{b}}{\chi}\right)^{\frac{\alpha\gamma^b}{1-\alpha}}-1\right)^{-1/\gamma^b}$$

□

A.5 Platform Problem

The platform earns profit:

$$\pi_t^s = \mu_t \Bigg(\eta_{0,t}^n \sum_{n_1} \eta_{1,t+1}^{n_1} x_{0,t}^{(n,n_1)} + \eta_{1,t}^n \sum_{n_0} \eta_{0,t-1}^{n_0} c_{1,t}^{(n_0,n)} + \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)} \Bigg).$$

Imposing the equilibrium objects that the platform takes as given (the goods market and the agent decisions about where to trade), the platform profit can be re-expressed as:

$$\begin{aligned}
\pi_t^s &= \mu_t \eta_{1,t}^p \sum_{n_0} \eta_{0,t-1}^{n_0} y_{1,t}^{(n_0,p)} \\
&= \mu_t \left(\frac{1}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}} \right) \\
&\quad \times \sum_{n_0} \left(\frac{1}{1 + \left(\frac{\zeta^{n_0'}}{\zeta^{n_0}}\right)^{\gamma_0} \left(\frac{(1+\mu_{t-1}^{n_0'}) r_{t-1,t}^{b n_0'}}{(1+\mu_{t-1}^{n_0}) r_{t-1,t}^{b n_0}} \right)^{-\frac{\alpha\gamma_0}{1-\alpha}}} \right) z \left(\frac{\alpha z}{(1+\mu_{t-1}^{n_0}) r_{t-1,t}^{b n_0}} \right)^{\frac{\alpha}{1-\alpha}} \\
&= \mu_t \left(\frac{1}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}} \right) z \left(\frac{\alpha z}{R_{t-1,t}^b} \right)^{\frac{\alpha}{1-\alpha}} \\
&\quad \times \left(\frac{\left(\frac{1}{\epsilon_{t-1} S_{t-2,t-1}}\right)^{\frac{\alpha}{1-\alpha}}}{1 + (\zeta)^{\gamma_0} \left(\frac{1+\mu_{t-1}}{\epsilon_{t-1} S_{t-2,t-1}} \right)^{-\frac{\alpha\gamma_0}{1-\alpha}}} + \frac{\left(\frac{1}{1+\mu_{t-1}}\right)^{\frac{\alpha}{1-\alpha}}}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_0} \left(\frac{\epsilon_{t-1} S_{t-2,t-1}}{1+\mu_{t-1}} \right)^{-\frac{\alpha\gamma_0}{1-\alpha}}} \right) \\
&=: \Pi(\mu_t, \epsilon_t, \mathbf{r}_t) \Gamma(\mu_{t-1}, \epsilon_{t-1}, \mathbf{r}_t) \\
&=: \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathbf{r}_t)
\end{aligned}$$

and ϵ_t is given by:

$$\epsilon_t = \left[\zeta_1^{\gamma_1}\zeta_2^{-\gamma_2}\left(\frac{R^m_{t-1,t}}{R^b_{t-1,t}}\right)^{1+\gamma_2}(1+\mu_t)^{\gamma_1(1-\beta)}\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t}\right)^{\frac{\alpha}{1-\alpha}}\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$

$$=: \epsilon(\mu_t, \mathbf{r}_t)$$

where $\mathbf{r}_t = (R^b_{t-1,t}, R^b_{t-2,t-1}, R^m_{t-1,t}, R^m_{t-2,t-1})$ is the set of returns that the platform does not internalize.

The Lagrangian for the platform is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \xi_{0,t}\pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathbf{r}_t)$$

So, for all $t \geq 0$, the FOC for μ_t is given by:

$$\begin{aligned}0 = {} & \xi_{0,t}\frac{\partial\pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathbf{r}_t)}{\partial\mu_t} + \xi_{0,t+1}\frac{\partial\pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathbf{r}_{t+1})}{\partial\mu_t} \\ & + \xi_{0,t}\frac{\partial\pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathbf{r}_t)}{\partial\epsilon_t}\frac{\partial\epsilon(\mu_t, R^b_{t-1,t}, R^{\$}_{t-1,t})}{\partial\mu_t} \\ & + \xi_{0,t+1}\frac{\partial\pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathbf{r}_{t+1})}{\partial\epsilon_t}\frac{\partial\epsilon(\mu_t, R^b_{t-1,t}, R^{\$}_{t-1,t})}{\partial\mu_t}\end{aligned}$$

where

$$\begin{aligned}&\frac{\partial\pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathbf{r}_t)}{\partial\mu_t} \\ &= \Gamma(\mu_{t-1}, \epsilon_{t-1}, \mathbf{r}_t)z\left(\frac{\alpha z}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}}\right. \\ &\quad \left. - \frac{\mu_t\left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(\beta-1)\gamma_1(1+\mu_t)^{(\beta-1)\gamma_1-1}}{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}\right)^2}\right) \\ &= \Gamma(\mu_{t-1}, \epsilon_{t-1}, \mathbf{r}_t)z\left(\frac{\alpha z}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}} \\ &\quad \times\left(1-\frac{\mu_t\left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(\beta-1)\gamma_1(1+\mu_t)^{(\beta-1)\gamma_1-1}}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}}\right)\end{aligned}$$

and

$$\frac{\partial \pi^s(\mu_{t+1},\mu_t,\epsilon_{t+1},\epsilon_t,\mathbf{r}_{t+1})}{\partial \mu_t}$$
$$= \Pi(\mu_{t+1},\epsilon_{t+1},\mathbf{r}_{t+1})$$
$$\times \left(\frac{\left(\frac{1}{\epsilon_t S_{t-1,t}}\right)^{\frac{\alpha(1-\gamma_0)}{1-\alpha}} (\zeta)^{\gamma_0} \left(\frac{\alpha\gamma_0}{1-\alpha}\right)(1+\mu_t)^{-\frac{\alpha\gamma_0}{1-\alpha}-1}}{\left(1+(\zeta)^{\gamma_0}\left(\frac{1+\mu_t}{\epsilon_t S_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} - \right.$$
$$\left. \frac{\left(\frac{\alpha}{1-\alpha}\right)(1+\mu_t)^{-\frac{\alpha}{1-\alpha}-1}\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\epsilon_t S_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}} + (1+\mu_t)^{\frac{\alpha\gamma_0}{1-\alpha}} \frac{\gamma_0(\epsilon_t S_{t-1,t})^{-\frac{\alpha\gamma_0}{1-\alpha}}}{\zeta^{\gamma_0}}\right)}{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\epsilon_t S_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} \right)$$

and

$$\frac{\partial \pi^s(\mu_t,\mu_{t-1},\epsilon_t,\epsilon_{t-1},\mathbf{r}_t)}{\partial \epsilon_t}$$
$$= -\frac{\Gamma(\mu_{t-1},\epsilon_{t-1},\mathbf{r}_t) z \mu_t \left(\frac{\alpha z}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \epsilon_t^{\frac{\alpha}{1-\alpha}-1}}{\left(1+\zeta^{-\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}\right)}$$
$$\times \left(\frac{\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1 \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}} \right)$$

and

$$\frac{\partial \pi^s(\mu_{t+1},\mu_t,\epsilon_{t+1},\epsilon_t,\mathbf{r}_{t+1})}{\partial \epsilon_t}$$
$$= \Pi(\mu_t,\epsilon_t,\mathbf{r}_t)\times$$
$$\left(\frac{-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\epsilon_t S_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{\epsilon_t}\left(1+(1+\gamma_0)\zeta^{\gamma_0}\left(\frac{1+\mu_t}{\epsilon_t S_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)}{\left(1+(\zeta)^{\gamma_0}\left(\frac{1+\mu_t}{\epsilon_t S_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} \right.$$
$$\left. + \frac{\left(\frac{1}{1+\mu_t}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\alpha\gamma_0}{1-\alpha}\right)\left(\frac{S_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}(\epsilon_t)^{-\frac{\alpha\gamma_0}{1-\alpha}-1}}{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\epsilon_t S_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} \right)$$

and

$$\frac{\partial \epsilon(\mu_t, R^b_{t-1,t}, R^{\$}_{t-1,t})}{\partial \mu_t}$$

$$= \left[\zeta_1^{\gamma_1}\zeta_2^{-\gamma_2}\left(\frac{R^m_{t-1,t}}{R^b_{t-1,t}}\right)^{1+\gamma_2}\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}} \times \left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}$$

$$\left[(1+\mu_t)^{\gamma_1(1-\beta)-1}\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t}\right)^{\frac{\alpha}{1-\alpha}}\left(\gamma_1(1-\beta)\right.\right.$$

$$\left.\left.+\left(\frac{\alpha(1-\beta)}{1+\mu_t}\right)\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t}\right)^{-1}\right)\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}-1}$$

We can now characterize steady-state equilibrium with an optimizing platform:

A.6 Proofs for Platform Competition (Preliminary)

Proof of Proposition 3. (i) The market equilibrium is the same as in subsection 3.3 except that now $(1-\mu_t)$ is replaced by $(1-\mu_t^1)/(1-\mu_t^2)$. If χ is sufficiently large that the threat of exclusion from either platform is sufficient to incentive funds to repay loans on that ledger, then $q^{E\mathcal{L}} = \tilde{q}^{E\mathcal{L}}$ and there is no need to bargain over enforcement because it doesn't require cooperation. If χ is sufficiently low that only exclusion from both platforms is sufficient to incentivize repayment, then for both $\mathcal{L}$, we have $q^{E\mathcal{L}} = q^E$ and $\tilde{q}^{E\mathcal{L}} = 0$ so outcome of the Nash Bargaining is cooperation on enforcement without a transfer $T = 0$.
(ii) For ζ close to 1, when the trading advantage of platform 1 is not too large, the possible outcomes look like those in subsection 3.3. That is, if χ is large, then both platforms are able to enforce contract without cooperation and if χ is small, then cooperation is required for any contract enforcement. However, when χ and ζ are large, it is possible that, under non-cooperation, platform 1 can enforce contracts while platform 2 cannot. In this case, ledger 1 becomes the dominant ledger and so the currency market frictions become relevant.

Platform bargaining at $t = 0$ is now more complicated because the outside option for platform 1 is more complicated. If χ and ζ are sufficiently large that ledger 1 can incentivize contract enforcement on their ledger without cooperation and $\varsigma\epsilon < 1$, then $\tilde{q}^{E1} > q^{E1}$ and so ledger 1 prefers the non-

cooperative outcome. This means that the transfer platform 2 would have to pay to get enforcement leads to negatives surplus:

$$q^{E0} - \tilde{q}^{E2} - T = q^{E1} - \tilde{q}^{E1} < 0$$

and so the bargaining breaks down.

□