# Dynamic Models, New Gains from Trade?*

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June 2025

**Abstract**

Yes. We state closed-form expressions for steady state gains from trade that apply in a class of dynamic trade models that includes dynamic versions of the Krugman (1980), Melitz (2003), and customer capital (e.g., Arkolakis, 2010) models. The gains are a function of the domestic trade share and the long-run elasticity of trade with respect to iceberg trade costs, similar to Arkolakis, Costinot, and Rodríguez-Clare (2012). In a dynamic setting this long-run elasticity cannot be estimated in one step by relying on tariff variation as shifters of trade costs. Instead it can be recovered by combining two tariff elasticity estimates: the long- and the short-run. Thus, the short-run tariff elasticity indirectly enters the formula for the steady state gains from trade. Our main substantive finding is that the gains from trade are large. They depend crucially on the short-run tariff elasticity, and can be arbitrarily large even if the long-run tariff elasticity is high. Accounting for the transition path has a modest impact on the magnitude of the gains from trade, relative to simply comparing steady states.

*Keywords:* Dynamic Gains from Trade, Trade Elasticities, Sufficient Statistics
*JEL Codes:* F12, F15, F62

*We are grateful to Costas Arkolakis, Yan Bai, Kei-Mu Yi, Jing Zhang, as well as seminar and conference participants at Bank of Canada, CEPR ERWIT, Chicago, Columbia-GCAP, CEMFI, Dallas Fed, NYU, Penn State, UC-Boulder, UCSD, and Wisconsin for helpful comments. Email: cboehm@utexas.edu, alev@umich.edu, npnayar@utexas.edu and htoma@umich.edu.

# 1. Introduction

The last decade has seen a veritable explosion of work employing dynamic quantitative trade models. These models are useful for studying a number of salient mechanisms and phenomena, including the gradual adjustment of trade flows to trade cost shocks, the interaction between factor accumulation and trade, and the role of firms' forward-looking export entry decisions.[1] However, studying the determinants of the gains from trade in dynamic environments is often challenging. There are currently few analytical characterizations of the dynamic gains from trade, as these models are typically solved numerically and are often computationally intensive. In particular, we currently lack compact and intuitive gains from trade formulas in the spirit of Arkolakis, Costinot, and Rodríguez-Clare (2012, henceforth ACR) for dynamic economies.

This paper makes three contributions. Our theoretical contribution is to state closed-form expressions for the gains from trade (GFT) that apply in a class of dynamic models in steady state. The formula circumvents the need to solve computationally intensive dynamic trade models, and thus provides a useful benchmark for assessing the dynamic gains from trade. Similar to ACR, welfare gains from trade are a function of the domestic absorption share and the long-run iceberg trade elasticity. Different from ACR, the long-run iceberg elasticity is a function of multiple structural parameters. One of these parameters is most naturally associated with the short-run trade elasticity. The second captures the difference between the long-run and short-run adjustment to trade cost shocks. Our measurement contribution is to show how empirical elasticity estimates at different time horizons can be used to recover the structural parameters required to calculate the GFT in a dynamic setting. Lastly, we quantify the GFT. Under our preferred tariff elasticity estimates, the gains from trade in this class of models are large. We highlight the importance of the short-run tariff elasticity, and show that gains from trade can be arbitrarily large under a low short-run elasticity, regardless of the value of the long-run tariff elasticity. We compute a multi-country general equilibrium dynamic trade model, and compare our formula-implied steady state gains to those that explicitly account for the transition path. Accounting for the transition has a modest impact on the GFT, regardless of whether countries have access to international bond markets.

To illustrate the model features important for our main theoretical result, we start with a simple dynamic Krugman (1980) model. There are multiple countries and firms. Firms face downward-sloping demand in destination markets and earn positive flow profits each period. In order to enter a destination market, a firm has to pay a stochastic sunk cost. Firms enter a market if the net present value of their expected profits from selling there cover the sunk costs of entry. These features introduce forward-looking behavior and gradual adjustment to shocks. Following a reduction in trade costs, two forces will act on the welfare of the domestic agents: the gain from imported varieties, captured by the domestic trade share as in ACR; and the loss in domestic varieties. It turns out that under the

[1] Dynamic trade models have a tradition going back to at least the 1960s (e.g., Bardhan, 1965, 1966; Oniki and Uzawa, 1965; Inada, 1968; Stiglitz, 1970).

assumption that the sunk costs of entry follow an inverse Pareto distribution, the loss of domestic varieties is a power function of the domestic trade share. Thus, the domestic trade share is a sufficient statistic for the welfare change, modulo the relevant elasticity. This elasticity is a function of the Dixit-Stiglitz substitution elasticity between firms' goods $\sigma$, and the dispersion parameter of the sunk cost distribution. Intuitively, this parameter controls how strongly domestic variety responds to foreign competition.

We then state a general set of conditions under which the closed-form expression for the gains from trade applies. The first two conditions coincide with ACR: trade is balanced *in steady state*; and the ratio of aggregate profits to aggregate sales is constant. The third condition puts restrictions on supply and demand. Total bilateral exports can without loss of generality be written as a product of the mass of firms (or customers) and sales per unit mass. The result requires that (i) domestic demand per unit mass of firms has a constant elasticity of substitution functional form (closely related to ACR's third assumption), and (ii) the mass of firms is a power function of sales per firm normalized by the source country wage. The latter is an additional restriction required in our dynamic environment. Qualitatively, it is intuitive that the mass of firms increases in per-firm sales relative to factor cost, as it follows from the logic of common entry problems. Sales relative to factor cost is a reflection of profits; when profits are higher, more firms enter. However, the power functional form of this relationship is a non-trivial restriction. In the Krugman model example above it is micro-founded with a distributional assumption on sunk cost draws, but that is not the only possible microfoundation as we show below.

Our main theoretical contribution is to derive a closed-form GFT formula in a general dynamic environment satisfying these assumptions.[2] The steady state real consumption level under trade relative to autarky is given by

$$\lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0(1+\chi)}}, \tag{1.1}$$

where $\lambda_{jj}$ is the share of domestically-produced goods in total spending, $\varepsilon_\kappa^0$ is the elasticity of the CES demand per unit mass to unit costs, and $\chi$ is the exponent governing the relationship between the mass of firms and per-firm sales. To fix ideas, in the dynamic Krugman model, $\varepsilon_\kappa^0$ is simply $1-\sigma$. Importantly, $\varepsilon_\kappa^0(1+\chi)$ is also the *long-run* elasticity of trade with respect to the iceberg trade costs.

A critical difference with respect to ACR is that the long-run trade elasticity is governed by multiple structural parameters. The reason is that in our setting, both the long-run trade elasticity and the GFT reflect an additional margin of adjustment. In the dynamic Krugman model, this margin captures changes in the mass of domestically-produced varieties.[3] This margin is switched off in ACR, and thus both the trade elasticity and the formula's exponent reflect only the consumption gain from foreign varieties ($1/(1-\sigma)$ in the Krugman case). Indeed we show that if the mass of domestic varieties were constant, our formula coincides with ACR. The welfare impact of the loss of domestic varieties scales

[2]The formula compares steady state consumption levels, and requires balanced trade only in steady state. Countries opening up to trade can borrow and lend internationally along the transition path.

[3]In other dynamic models that satisfy our assumptions, the additional margin of adjustment captures changes in the mass of *potential* exporters or changes in the mass of customers.

with the domestic trade share with elasticity $-\frac{1}{1-\sigma}\frac{\chi}{1+\chi}$. The first term in this composite fraction reflects the welfare losses from lower domestic variety. The second captures the responsiveness of domestic variety to the changes in profit opportunities brought about by trade cost changes, and is governed by the parameter $\chi$. The gains from foreign variety and losses from domestic variety add up to the overall elasticity of welfare with respect to the domestic trade share: $\frac{1}{1-\sigma} - \frac{1}{1-\sigma}\frac{\chi}{1+\chi} = \frac{1}{(1-\sigma)(1+\chi)}$ as in (1.1).

We show that the conditions for our theoretical result are satisfied by two additional dynamic models: (i) the customer base model à la Arkolakis (2010) with the cost of acquiring customers taking a power form and gradual customer base adjustment to trade cost shocks; and (ii) a dynamic version of the Melitz (2003) model with Pareto productivity and inverse Pareto sunk cost distributions, in which the set of firms selling to each destination adjusts gradually in response to shocks. We also develop an extension to include physical capital accumulation following, e.g., Alvarez (2017) or Ravikumar, Santacreu, and Sposi (2019), and discuss how firm entry interacts with capital accumulation.

We then turn to measurement. While domestic trade shares are easy to obtain, the long-run elasticity of trade with respect to iceberg costs is not. The main reason is that we typically do not observe iceberg trade costs. Instead, the predominant approach in the literature is to use tariff variation, as tariffs are often the only directly observed component of *ad valorem* trade costs.[4]

The distinction between the iceberg and the tariff elasticity is innocuous in conventional static settings, as one can be easily recovered from the other by adding or subtracting 1. This is no longer the case in dynamic environments, which feature the additional adjustment margin discussed above. To make this point explicit, we state a generalization of the main proposition to an environment with both iceberg costs and tariffs. The GFT formula still requires the long-run elasticity of trade with respect to iceberg trade costs. However, in a dynamic world the long-run elasticity of trade with respect to iceberg trade costs cannot be recovered from the long-run elasticity with respect to tariffs alone. In addition, the formula now features an adjustment for tariff revenue. This type of adjustment was derived in a static setting by Felbermayr, Jung, and Larch (2015). In the class of models we consider, computing this adjustment requires not only the long-run trade elasticity, but also knowledge of $\varepsilon_\kappa^0$ and $\chi$ separately.

To summarize, implementing the dynamic GFT formula faces two hurdles: (i) the long-run iceberg trade cost elasticity cannot be directly computed from the estimated long-run tariff elasticity; and (ii) implementing the tariff adjustment requires knowledge of not only the long-run iceberg elasticity, but $\varepsilon_\kappa^0$ and $\chi$ separately. We propose a solution: these two parameters can be inferred from estimates of two tariff elasticities at different time horizons: the short- and the long-run. Intuitively, the short-run tariff elasticity is a function of only $\varepsilon_\kappa^0$, while the long-run tariff elasticity is a function of both $\varepsilon_\kappa^0$ and $\chi$. Thus, with two empirical estimates – the short- and the long-run – one can recover both parameters.

Equipped with estimates for these parameters, we turn to quantification. First, we report the

[4] At least 20 papers have used tariff variation to estimate the trade elasticity in the past 25 years. See Head and Mayer (2014) and Boehm, Levchenko, and Pandalai-Nayar (2023) for references.

dynamic gains from trade in steady state for a large set of countries according to our formula and accounting for tariffs. Our preferred short– and long-run tariff elasticity estimates are taken from the dynamic gravity estimates at different horizons in Boehm, Levchenko, and Pandalai-Nayar (2023).[5] The GFT are substantial, with gains of 25-30% for even the largest and most closed countries such as the US, Brazil, and China, and gains of over 100% for several countries. At the same time, the tariff revenue adjustment plays a small role in all but a handful of economies.

Second, we highlight the role of the short-run trade elasticity by setting the long-run tariff elasticity at a high (in absolute terms) value while varying the short-run elasticity. It turns out that conditional on a fixed long-run elasticity, the short-run elasticity is decisive for the magnitude of the GFT. In fact, as the short-run elasticity approaches $-1$ from below, the gains from trade become infinite even with a high long-run tariff elasticity. The intuition is that the short-run tariff elasticity is closely related to the elasticity of substitution between domestic and foreign varieties, which is critical for the GFT. Most available estimates of the short-run elasticity are low (Fitzgerald and Haller, 2018; Boehm, Levchenko, and Pandalai-Nayar, 2023; Auer, Burstein, and Lein, 2021), suggesting that gains from trade are likely quite large, regardless of the long-run elasticity.

Third, we compute the GFT in a 30-country general equilibrium dynamic Krugman model, accounting for the transition path from one trade regime to another. In our baseline scenario a single country transitions from autarky to trade or from trade to autarky. All countries have access to international bond markets. The length of the transition is disciplined in part by the time it takes for the trade elasticity to converge to its long-run value, which Boehm, Levchenko, and Pandalai-Nayar (2023) estimate to be around 7-10 years. We also consider a variety of additional scenarios, including cases in which all countries transition simultaneously and cases in which there is no trade in international bond markets. While interesting in their own right, these exercises also help evaluate the usefulness of the formula as an approximation for the GFT in computationally challenging dynamic models under a range of assumptions.

There are three main findings. First, the disparity between the formula-implied steady state welfare gains and the gains taking into account the transition path is modest, with the difference of about $7-12\%$. Second, the steady state formula overstates the dynamic gains of the transition from autarky to trade, but understates the gains from staying open to trade compared to the transition from trade to autarky. The reason is as follows. In the opening scenario, the country begins in the autarky steady state and transitions to the trade steady state slowly. Over this transition, consumption is lower than the eventual steady state consumption, because firms need to invest in setting up exporting operations, and doing so requires forgoing consumption over the transition path. As a result, the dynamic gains of going from autarky to trade are below the steady state comparison. When moving from trade to autarky, countries' accumulated exporting capital has become useless. At the same time the mass of domestic varieties is lower than in the autarky steady state and the country must

[5]The class of models covered by our analytical GFT formula is consistent with the dynamic gravity estimation of short- and long-run elasticities implemented in that paper, without taking a stance on the specific microfoundation.

accumulate domestic firms to replace imports. Thus, when shocked with an unanticipated increase in trade costs, countries also temporarily decrease consumption below the level obtained in the eventual autarky steady state. This reduces the value of the consumption path towards autarky – effectively the denominator of the GFT – relative to steady state, and thus raises the implied GFT. Finally, access to international bond markets during the transition and whether other countries are already open or everyone opens simultaneously has essentially no effect on the GFT. This suggests that the analytical formula has potentially wide applicability across different asset market structures and trade cost disturbances.

**Literature.** While the field of international trade has always been interested in the gains from trade, the literature on the quantification of GFT was given fresh impetus by the landmark contribution of Arkolakis, Costinot, and Rodríguez-Clare (2012), who stated closed-form expressions for the GFT in a wide class of static trade models.[6] This led to an active literature exploring various analytical and quantitative properties of the sufficient statistics formulas, such as sectoral comparative advantage (Costinot and Rodríguez-Clare, 2014; Levchenko and Zhang, 2014) or trade elasticities (Ossa, 2015; Imbs and Mejean, 2017). The formulas have also been extended in a variety of directions, such as variable markups (Arkolakis et al., 2019; Heid and Stähler, 2024), non-constant trade elasticities (Melitz and Redding, 2015; Feenstra, 2018; Adão, Arkolakis, and Ganapati, 2020), gains from multinational production (Ramondo and Rodríguez-Clare, 2013), non-representative agent settings (Galle, Rodríguez-Clare, and Yi, 2023; Waugh, 2023), and accounting for tariff revenue (Felbermayr, Jung, and Larch, 2015; Lashkaripour, 2021) and distortions (Bai, Jin, and Lu, 2024), to name a few. In static settings, Melitz and Redding (2015) and Feenstra and Weinstein (2017) highlight that allowing for changes in the mass of (potential) firms leads to welfare gains that differ from the ACR formula, implying that the GFT can then be sensitive to microfoundations. In our dynamic trade setting the mass of firms also changes, contributing to the gains from trade. Aside from the fact that ours is a dynamic setting, our theoretical contributions relative to these papers are to (i) analytically characterize the mapping between the mass of firms and the domestic trade share, yielding ACR-like GFT welfare formulas that account for endogenous mass adjustment; and (ii) establish this mapping in a class of models that covers multiple microfoundations.

The literature on analytical GFT characterizations in dynamic environments is more limited. Arkolakis, Eaton, and Kortum (2011) and Chen et al. (2024) develop results for a dynamic version of the Eaton-Kortum model, Atkeson and Burstein (2010) and Alessandria, Choi, and Ruhl (2021a) for a dynamic heterogeneous firm model, and Fitzgerald (2024) for the Armington model. On the quantitative side, a number of papers numerically compute the GFT in dynamic models, including accounting for the transition path (see, among others, Alvarez, 2017; Brooks and Pujolas, 2018; Mutreja, Ravikumar, and Sposi, 2018; Ravikumar, Santacreu, and Sposi, 2019, 2024; Anderson, Larch, and Yotov,

[6] Antecedents that stated similar formulas in specific settings include Eaton and Kortum (2002) for the Ricardian model, Eaton and Kortum (2005) for the Armington model, and Arkolakis et al. (2008) for the Melitz model.

2020).[7] We provide a relatively general characterization that applies to steady state comparisons in a broad class of dynamic trade models. Unlike the Eaton-Kortum setting, our analytical results cover cases in which there is net firm entry and profits. We also emphasize the importance of measurement, in particular the information contained in trade elasticities at multiple horizons in conditioning the steady state gains from trade.

The remainder of the paper is organized as follows. Section 2 fully lays out the simplest dynamic model to illustrate the mechanics behind the result. Section 3 states several general results and establishes the mappings to other dynamic models. Section 4 quantifies the gains from trade. Section 5 concludes.

## 2. Warmup: Welfare Gains in a Dynamic Krugman Model

This section derives the gains from trade formula in the simplest possible setup: a dynamic version of the Krugman (1980) model. It serves to introduce the notation maintained throughout the paper, and to demonstrate what features are essential for our main result.

### 2.1 Model Setup

Consider a dynamic economy with $J$ countries indexed by $i$ and $j$, and discrete time indexed by $t$. Each country is populated by a representative consumer who consumes $C_{jt}$ and inelastically supplies labor $L_j$.

**Households.** Consumers in country $j$ maximize

$$\max_{\{C_{jt}\}} \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\gamma}}{1-\gamma}$$

subject to the budget constraint

$$P_{jt}C_{jt} + B_{jt} + B_{jt}^* + w_{jt}\psi\left(\frac{B_{jt}^*}{P_{jt}}\right) = w_{jt}L_j + \Pi_{jt} + R_{jt}^g + B_{jt-1}\left(1 + r_{jt-1}^n\right) + B_{jt-1}^*\left(1 + r_{t-1}^{n*}\right), \tag{2.1}$$

and a no-Ponzi game condition. Here, $P_{jt}$ is the consumption price index in country $j$, $w_{jt}$ the nominal wage, $\Pi_{jt}$ aggregate profits, and $R_{jt}^g$ are government tariff revenues rebated to the household. Domestic bond holdings $B_{jt}$, satisfying $B_{jt} = 0$ in equilibrium, are included in the budget constraint only to price country $j$'s nominal interest rate $r_{jt}^n$. Only the international bond $B_{jt}^*$ is traded in

[7] Beyond the focus on the GFT, the vast and constantly growing literature on dynamic trade cannot be comprehensively summarized here (see, among many others, Costantini and Melitz, 2007; Ruhl, 2008; Drozd and Nosal, 2012; Burstein and Melitz, 2013; Alessandria and Choi, 2014; Ruhl and Willis, 2017; Leibovici and Waugh, 2019; Alessandria, Arkolakis, and Ruhl, 2021; Alessandria, Choi, and Ruhl, 2021b; Kleinman, Liu, and Redding, 2023; Steinberg, 2022; Blaum, 2024; Fitzgerald, Haller, and Yedid-Levi, 2024).

equilibrium. It yields nominal interest rate $r_t^{n*}$ and is subject to holding costs $\psi\left(B_{jt}^*/P_{jt}\right)$ (Schmitt-Grohé and Uribe, 2003), denominated in units of domestic labor. The bond holding cost function $\psi$ satisfies $\psi(0) = 0$, $\psi'(0) = 0$, and $\psi''(0) = \psi$ in the steady state. We assume that firms producing in country $j$ are exclusively owned by the consumer in $j$, and hence the consumer receives all profits as income. The parameters $\beta$ and $\gamma$ denote the household's discount factor and the coefficient of relative risk aversion, respectively.

Optimal behavior implies that consumption follows the Euler equations

$$\begin{aligned} C_{jt}^{-\gamma} &= \left(1 + r_{jt}\right)\beta C_{jt+1}^{-\gamma} \\ C_{jt}^{-\gamma} &= \frac{1 + r_{jt}^*}{1 + \frac{w_{jt}}{P_{jt}}\psi'\left(\frac{B_{jt}^*}{P_{jt}}\right)}\beta C_{jt+1}^{-\gamma}, \end{aligned}$$

where $1 + r_{jt} = \left(1 + r_{jt}^n\right)\frac{P_{jt}}{P_{jt+1}}$ and $\left(1 + r_{jt}^*\right) = \left(1 + r_t^{n*}\right)\frac{P_{jt}}{P_{jt+1}}$ are the real interest rates earned on holding the domestic and international bond.

The consumption bundle $C_{jt}$ is a CES aggregate of quantities $q_{ijt}(\omega)$ supplied by firms indexed by $\omega$, from all countries $i$ serving market $j$:

$$C_{jt} = \left(\sum_i \int_{\Omega_{ijt-1}} q_{ijt}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the demand elasticity and $\Omega_{ijt-1}$ denotes the endogenous set of varieties produced in country $i$ and available for purchase in country $j$ at time $t$. Since this set is determined by firms' exporting decisions in the previous period, it is indexed with subscript $t-1$. Demand for each variety $\omega$ and the ideal price index satisfy:

$$q_{ijt}(\omega) = C_{jt}\left(\frac{p_{ijt}^c(\omega)}{P_{jt}}\right)^{-\sigma}, \tag{2.2}$$

$$P_{jt} = \left(\sum_i \int_{\Omega_{ijt-1}} \left(p_{ijt}^c(\omega)\right)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}},$$

where $p_{ijt}^c(\omega)$ is the price faced by the consumer in country $j$.

**Firms.** Firms are monopolistically competitive, face the downward-sloping demand curve given by (2.3), and take the ideal price index as given. The production function is linear in labor. Shipments from country $i$ to $j$ are subject to iceberg transport costs $\kappa_{ijt}$, so that

$$q_{ijt}(\omega) = \frac{1}{\kappa_{ijt}} l_{ijt}(\omega), \tag{2.3}$$

where $l_{ijt}(\omega)$ is the firm's labor input for producing for market $j$. The marginal cost of serving market $j$ is therefore $\kappa_{ijt} w_{it}$. Profit-maximizing firms charge a constant markup over marginal cost:

$$p^x_{ijt}(\omega) = \frac{\sigma}{\sigma - 1} \kappa_{ijt} w_{it}, \tag{2.4}$$

where $p^x_{ijt}(\omega)$ is the price received by the exporter. As a result, per-period profits are a constant fraction of firm revenue:

$$\pi_{ijt}(\omega) = \frac{1}{\sigma} p^x_{ijt}(\omega) q_{ijt}(\omega) = \frac{1}{\sigma} x_{ijt}(\omega). \tag{2.5}$$

**Entry.** Every period there is a unit mass of potential firms that can enter market $j$ from $i$. Entry is subject to a stochastic sunk cost of $\xi^s_{ijt}(\omega)$ units of country $i$'s labor. A firm $\omega$ from $i$ that pays the sunk costs in period $t$ sells to $j$ from $t+1$ onwards until it exits. Exit is random and occurs with probability $\delta$. The value of exporting is therefore

$$v_{ijt}(\omega) = \frac{1}{1 + r^n_{it}} \left( \pi_{ijt+1}(\omega) + (1 - \delta) v_{ijt+1}(\omega) \right). \tag{2.6}$$

A potential entrant enters if the value of exporting exceeds the sunk cost of entry. The marginal firm's sunk costs $\bar{\xi}^s_{ijt}$ satisfy

$$v_{ijt}(\omega) = w_{it} \bar{\xi}^s_{ijt}(\omega). \tag{2.7}$$

Denote by $n_{ijt}$ the mass of exporters from $i$ to $j$. Its law of motion is

$$n_{ijt} = (1 - \delta) n_{ijt-1} + G\left( \bar{\xi}^s_{ijt} \right),$$

where $G$ denotes the cumulative distribution function of $\xi^s_{ijt}$.

**Tariffs, aggregation, and market clearing.** Let $\tau_{ijt}$ denote gross *ad valorem* tariffs.[8] Then the prices paid by the consumers and prices received by the exporters satisfy

$$p^c_{ijt}(\omega) = \tau_{ijt} p^x_{ijt}(\omega), \tag{2.8}$$

and the government collects $(\tau_{ijt} - 1) p^x_{ijt}(\omega)$ in revenue per unit sold.

Total exports from $i$ to $j$ non-inclusive of tariff payments are:

$$X_{ijt} = \int_{\Omega_{ijt-1}} x_{ijt}(\omega) \, d\omega = n_{ijt-1} x_{ijt}. \tag{2.9}$$

[8] In this notation, a 5% *ad valorem* tariff implies $\tau_{ijt} = 1.05$.

The tariff revenue of government $j$ is $R_{jt}^g = \sum_i \left(\tau_{ijt} - 1\right) X_{ijt}$, and profits in country $j$ are

$$\Pi_{jt} = \sum_i \int_{\Omega_{jit-1}} \pi_{jit}(\omega)\, d\omega - \sum_i \int_{\Omega_{jit}^e} w_{jt} \xi_{jit}^s(\omega)\, d\omega, \tag{2.10}$$

where $\Omega_{jit}^e = \left\{\omega \in [0,1] : \bar{\xi}_{jit}^s \geq \xi_{jit}^s(\omega)\right\}$ is the set of entrants.

Lastly, all countries' labor markets clear:

$$L_i = \sum_j \int_{\Omega_{ijt-1}} l_{ijt}(\omega)\, d\omega + \sum_j \int_{\Omega_{ijt}^e} \xi_{ijt}^s(\omega)\, d\omega + \psi\left(\frac{B_{jt}^*}{P_{jt}}\right), \tag{2.11}$$

and the market for international bonds clears, $\sum_j B_{jt}^* = 0$.

## 2.2 Steady State Welfare Gains from Trade

In this subsection, we abstract from tariff revenues: $\tau_{ijt} = 1$ for all $i$ and $j$, implying that $R_{jt}^g = 0$. Since all operating firms in the model have identical quantities and prices, we will drop the firm index $\omega$ going forward. Steady state objects are identified by dropping the time subscripts. Bond holdings are zero in steady state and hence the budget constraint (2.1) implies that real consumption is:

$$C_j = \frac{w_j L_j + \Pi_j}{P_j}. \tag{2.12}$$

We will denote the gross proportional gains from trade as the ratio of real consumption under the current trade regime relative to autarky:

$$GFT = \frac{C_j}{C_j^{AUT}}.$$

In the tradition following Eaton and Kortum (2002) and Arkolakis, Costinot, and Rodríguez-Clare (2012), we seek to express (2.12) as a function of the domestic trade share and exogenous parameters. We start with the standard step that the domestic trade share is:

$$\lambda_{jj} \equiv \frac{n_{jj} x_{jj}}{Y_j} = \frac{n_{jj}\left(\frac{\sigma}{\sigma-1} w_j\right)^{1-\sigma}}{P_j^{1-\sigma}}, \tag{2.13}$$

where $Y_j \equiv P_j C_j$ is total expenditure. Solving this expression for the price index and combining the result with equation (2.12) implies that real consumption satisfies:

$$C_j \propto \frac{w_j L_j + \Pi_j}{w_j \lambda_{jj}^{-\frac{1}{1-\sigma}} n_{jj}^{\frac{1}{1-\sigma}}}. \tag{2.14}$$

From here, we proceed to show that (i) aggregate profits are a constant fraction of labor income; and that (ii) the mass of domestic firms $n_{jj}$ is a power function of $\lambda_{jj}$. To compute profits and the mass of entrants, we must make a distributional assumption on the sunk costs of entry. We assume that the sunk costs are drawn from an inverse Pareto distribution:

$$G\left(\xi^s\right) = \left(b\xi^s\right)^{\chi}, \tag{2.15}$$

where $\chi > 0$ is the Pareto dispersion parameter and $b > 0$ is the location parameter, that defines the domain of this distribution: $0 < \xi^s \leq \frac{1}{b}$. We assume throughout that $b$ is sufficiently small to ensure that not all potential entrants find it worthwhile to enter in any given period ($\bar{\xi}^s_{ijt} < \frac{1}{b}$ for all $t$). Under this assumption the steady state mass of firms becomes

$$n_{ji} = \frac{1}{\delta}\left(b\bar{\xi}^s_{ji}\right)^{\chi}. \tag{2.16}$$

Since $1 + r_j = 1/\beta$ in the steady state, the value of selling to $i$ is:

$$v_{ji} = \frac{\beta}{1-\beta\left(1-\delta\right)}\pi_{ji} = \frac{\beta}{1-\beta\left(1-\delta\right)}\frac{1}{\sigma}x_{ji},$$

and the threshold sunk cost of entry is:

$$\bar{\xi}^s_{ji} = \frac{\beta}{1-\beta\left(1-\delta\right)}\frac{1}{\sigma}\frac{x_{ji}}{w_j}. \tag{2.17}$$

Equations (2.16) and (2.17) imply that the mass of firms is a power function of per-unit sales normalized by the cost of production:

$$n_{ji} \propto \left(\frac{x_{ji}}{w_j}\right)^{\chi}. \tag{2.18}$$

Combining (2.10), (2.11), (2.16), and (2.17), while noting that both bond holdings and bond holding costs are zero in steady state, leads to the desired result that total profits are a constant multiple of labor income:

$$\Pi_j = \frac{\frac{1}{\sigma}\left(1 - \frac{\chi}{\chi+1}\frac{\beta}{1-\beta(1-\delta)}\delta\right)}{1 - \frac{1}{\sigma}\left(1 - \frac{\chi}{\chi+1}\frac{\beta}{1-\beta(1-\delta)}\delta\right)}w_j L_j. \tag{2.19}$$

Finally, starting with the expression for steady state $n_{jj}$ in (2.16), and combining it with (2.17), (2.19), and the expression for domestic sales $x_{jj}$ leads to:

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}. \tag{2.20}$$

Combining (2.14) with (2.19) and (2.20) then yields the result that real consumption is proportional

to the domestic trade share:

$$C_j \propto \lambda_{jj}^{\frac{1}{1-\sigma}} n_{jj}^{-\frac{1}{1-\sigma}} = \lambda_{jj}^{\frac{1}{(1-\sigma)(1+\chi)}}. \tag{2.21}$$

Since in autarky $\lambda_{jj} = 1$, (2.21) is also the gains from trade.

Note the difference with the ACR formula for the static Krugman model, $\lambda_{jj}^{\frac{1}{1-\sigma}}$, which would obtain in a setting in which $n_{jj}$ is either exogenously fixed or constant across equilibria. Compared to the classic case and holding $\sigma$ fixed across models, the gains from trade are moderated because international trade leads to the reduction in domestic varieties. The log change in real consumption following a change in the domestic trade share can be written as:

$$\begin{aligned} d\ln C_j &= \frac{1}{1-\sigma} d\ln \lambda_{jj} - \frac{1}{1-\sigma}\frac{d\ln n_{jj}}{d\ln \lambda_{jj}} d\ln \lambda_{jj} \\ &= \underbrace{\frac{1}{1-\sigma} d\ln \lambda_{jj}}_{\text{Gain from foreign varieties}} \underbrace{-\frac{1}{1-\sigma}\frac{\chi}{1+\chi} d\ln \lambda_{jj}}_{\text{Loss of domestic varieties}}. \end{aligned} \tag{2.22}$$

The first term is the usual direct effect of the change in the interior trade share, interpreted as the consumption gains from the availability of foreign goods. It increases with trade openness (recall that an increase in trade openness is a fall in $\lambda_{jj}$). The second term is the welfare reduction from the loss of domestic varieties, as an increase in trade openness unambiguously lowers $n_{jj}$. It contributes negatively to the gains from trade. In this model, however, the net gain from openness is positive.

Two further points are worth noting. First, the loss of domestic varieties was modeled and quantified by Melitz and Redding (2015) and Feenstra and Weinstein (2017) in specific static models. We build on these contributions by deriving a parsimonious functional form (2.20) that relates domestic variety to the domestic trade share, which in turn leads to the closed-form GFT expression (2.21) requiring only data on $\lambda_{jj}$. As we show below, this property extends to several alternative micro-foundations, implying these models admit the same analytical GFT formula. Second, (2.22) together with (2.18) highlight the role of the Pareto dispersion parameter $\chi$. As evident from (2.18), $\chi$ is the elasticity of the mass of domestic varieties to the domestic profit opportunities in steady state. When $\chi$ is high, domestic variety is very sensitive to the profit opportunities, and so the fall in profits due to import competition leads to a large decline in domestic variety, and hence a large second term in (2.22). When $\chi$ is low, the opposite is true: import competition does not move domestic variety much, and thus the second term in (2.22) is smaller in magnitude.

**The long-run trade elasticity.** A key reason for the appeal of ACR's result is that the exponent on the domestic trade share is the inverse of the trade elasticity. We now show that the dynamic GFT formula shares this feature. Recall that bilateral trade flows are given by (2.9). The long-run trade

elasticity with respect to iceberg trade costs therefore has the following components:

$$\frac{d \ln X_{ij}}{d \ln \kappa_{ij}} = \frac{d \ln x_{ij}}{d \ln \kappa_{ij}} + \frac{d \ln n_{ij}}{d \ln \kappa_{ij}}.$$

It is immediate that the $\frac{d \ln x_{ij}}{d \ln \kappa_{ij}} = 1 - \sigma$, as usual. From (2.16) and (2.17), $\frac{d \ln n_{ij}}{d \ln \kappa_{ij}} = \chi(1 - \sigma)$. Together, the long-run trade elasticity is

$$\frac{d \ln X_{ij}}{d \ln \kappa_{ij}} = (1 - \sigma)(1 + \chi),$$

and thus the gains from trade formula (2.21) features the inverse of the trade elasticity. Note that as in ACR and everywhere else in the literature, this is a partial elasticity, that ignores the general-equilibrium changes in expenditures, wages, and prices.

## 3. General Result

We now state the set of conditions under which the dynamic GFT formula applies.

**Proposition 3.1.** *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

*A.1 For all countries $j$, trade is balanced (expenditure = revenue):*

$$Y_j = w_j L_j + \Pi_j,$$

*where $Y_j = C_j P_j$.*

*A.2 For all countries $j$, profits are a constant share of GDP:*

$$\frac{\Pi_j}{Y_j} = const$$

*A.3 For all country pairs $(i, j)$ trade flows satisfy*

$$X_{ij} = n_{ij} x_{ij} \tag{3.1}$$

*where*

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi} \tag{3.2}$$

*and domestic per-unit-mass sales satisfy*

$$x_{jj} \propto Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{3.3}$$

*for some constant $\chi > 0$ and where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.*

*Then*

$$C_j \propto \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0(1+\chi)}} \tag{3.4}$$

*where $\lambda_{jj} = \frac{X_{jj}}{Y_j}$, and $\varepsilon_\kappa^0(1+\chi)$ is the long-run elasticity of trade flows with respect to iceberg trade costs.*

*Proof.* See Appendix A. □

Note that since $\lambda_{jj} = 1$ in autarky, (3.4) is also the gross proportional gains from trade in steady state. Assumptions A.1 and A.2 are identical to R1 and R2 in ACR. Assumption A.3 puts structure on supply and demand. Condition (3.1) stipulates that total exports from $i$ to $j$ can be written as a product of some generic mass $n_{ij}$ and sales per unit mass $x_{ij}$. This is without loss of generality, as we can always express total exports as some (average) sales per firm/variety/HS code/etc. times the total number/mass of those units. Condition (3.3) states that domestic demand per unit mass is CES. It essentially corresponds to ACR's R3. Note that the proposition is stated in terms of the functional form for domestic sales. By doing so the proposition covers a greater range of models. In some models, such as Krugman and customer capital, international trade flows $x_{ij}$ take the same form, modulo iceberg costs $\kappa_{ij}$. In the Melitz model, domestic sales per unit mass satisfy (3.3), while export sales contain additional terms, as will become clear below.

Finally, condition (3.2) is an additional restriction required in our dynamic environment. Qualitatively, it is intuitive: entry increases in the ratio of sales to unit costs. Section 2, in particular equations (2.5) and (2.7), illustrate how this can arise: the value of exporting scales with per period profits, which are in turn proportional to sales (numerator). Sunk costs are paid in terms of domestic labor (denominator). However, the proposition requires more than an increasing relationship: it requires that entry is a power function of this ratio. Section 2 shows how the inverse Pareto distribution of sunk costs leads to (3.2) in the Krugman case.

### 3.1 Mapping from specific models

Section 2 shows that the dynamic Krugman model satisfies the conditions of Proposition 3.1, with $\varepsilon_\kappa^0 = 1 - \sigma$. We now discuss two additional commonly used dynamic models: the customer base model and the Melitz-Pareto model.

**Customer base model.** In the customer base model (e.g. Arkolakis, 2010; Drozd and Nosal, 2012; Gourio and Rudanko, 2014; Fitzgerald, Haller, and Yedid-Levi, 2024), firms gradually build up the mass of customers they serve. Let there be a country $i$ representative firm that faces downward-sloping demand (2.3) per unit mass of customers in country $j$. As above, its profits per unit mass of customers are given by (2.5). Let $n_{ijt}$ be the mass of customers that the firm serves. This mass

depreciates at rate $\delta$ and can be built up by customer acquisition (or investment) $a_{ijt}$. Thus, the customer mass evolves according to:

$$n_{ijt} = (1-\delta)\, n_{ijt-1} + a_{ijt}. \tag{3.5}$$

Investment of $a_{ijt}$ requires $f(a_{ijt})$ units of labor. The firm chooses the path of customer base investment to maximize the present value of profits:

$$\max_{\{a_{ijt+s}\}} \sum_{s=0}^{\infty} m^n_{it,t+s} \left[ n_{ijt+s-1} \pi_{ijt+s} - w_{it} f\left(a_{ijt+s}\right) \right] \tag{3.6}$$

subject to (3.5), where $m^n_{it,t+s}$ is the firm's discount factor. The first-order conditions of this problem can be manipulated to yield:

$$w_{it} f'\left(a_{ijt}\right) = v_{ijt} \tag{3.7}$$

$$v_{ijt} = \frac{1}{1+r^n_{it}} \left( \pi_{ijt+1} + (1-\delta)\, v_{ijt+1} \right), \tag{3.8}$$

where we assumed that the discount factor of the firm coincides with that of the representative consumer. Let the cost of accessing customers be given by:

$$f\left(a_{ijt}\right) = \frac{\chi}{(1+\chi)\,\zeta} \left(a_{ijt}\right)^{\frac{1}{\chi}+1}. \tag{3.9}$$

Then, in steady state:

$$n_{ij} = \frac{1}{\delta} a_{ij} = \frac{1}{\delta} \left( \zeta \frac{v_{ij}}{w_i} \right)^{\chi}. \tag{3.10}$$

In turn, combining (2.5) and (3.8) yields the proportionality of $v_{ij}$ to $x_{ij}$, verifying assumption A.3 in Proposition 3.1.

To see that Assumption A.2 is satisfied, note that aggregate profits can be written as:

$$\Pi_i = \sum_j \left( \frac{1}{\sigma} n_{ij} x_{ij} - w_i \frac{\chi}{(1+\chi)\,\zeta} \left(a_{ij}\right)^{\frac{1}{\chi}+1} \right). \tag{3.11}$$

Since $a_{ij}$ is proportional to $n_{ij}$, and $a_{ij}^{\frac{1}{\chi}}$ is proportional to $x_{ij}/w_i$ (see 3.10 for both), $\left(a_{ijt}\right)^{\frac{1}{\chi}+1}$ is proportional to $n_{ij}x_{ij}$, and $w_i$ cancels out in the consumer base cost term.

The deeper microfoundation, and thus the interpretation of some equilibrium quantities (e.g., $n_{ij}$) or parameters (e.g., $\chi$) are different from the Krugman model. However, this model is isomorphic to the dynamic Krugman model in its predictions for aggregate trade flows and the functional forms of the trade elasticities.

**Melitz-Pareto.** The dynamic Melitz (2003) model differs from the Krugman model in Section 2 in two ways. First, firms are heterogeneous in productivity, denoted $\varphi(\omega)$. Continuing to assume

constant Dixit-Stiglitz markups, a firm $\omega$'s price becomes:

$$p_{ijt}^{x}(\omega) = \frac{\sigma}{\sigma-1}\frac{\kappa_{ijt}w_{it}}{\varphi(\omega)}. \tag{3.12}$$

We assume that $\varphi(\omega)$ is distributed Pareto:

$$F(\varphi) = 1 - \left(\frac{\varphi_L}{\varphi}\right)^{\theta}. \tag{3.13}$$

Second, the firm in $i$ needs to pay a per-period fixed cost $\xi_{ij}$ denominated in units of $i$'s labor in order to serve market $j$.

As in Section 2, each firm must pay a stochastic sunk cost $\xi_{ijt}^{s}(\omega)$ to enter market $j$, drawn from an inverse Pareto distribution (2.15). Paying this sunk cost also reveals to the firm its productivity for serving market $j$. Thus, the entry decision is made based on expected profits. Further, due to the per-period fixed cost not all firms $n_{ijt-1}$ that have paid the sunk cost will end up exporting. The marginal firm earns variable profits that just cover the per-period fixed costs: $\frac{1}{\sigma}x_{ijt}(\omega) = w_{it}\xi_{ij}$. Combining (2.3) and (3.12) (and noting that without tariffs $p_{ijt}^{x}(\omega) = p_{ijt}^{c}(\omega)$) leads to the productivity cutoff for selling from $i$ to $j$:

$$\varphi_{ijt}^{m} = \frac{\sigma}{\sigma-1}\kappa_{ijt}w_{it}\left(\frac{\sigma w_{it}\xi_{ij}}{C_{jt}\left(P_{jt}\right)^{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{3.14}$$

Total sales from $i$ to $j$ are:

$$\begin{aligned} X_{ijt} &= \int x_{ijt}(\omega)\, d\omega \\ &= n_{ijt-1}\int_{\varphi_{ijt}^{m}}^{\infty} x_{ijt}(\varphi)\, dF(\varphi) \\ &= n_{ijt-1}\, C_{jt}\left(P_{jt}\right)^{\sigma} \underbrace{\left(\left(\frac{\theta\varphi_L^{\theta}}{\theta-(\sigma-1)}\right)^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\frac{\kappa_{ijt}w_{it}}{\left(\varphi_{ijt}^{m}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}}\right)^{1-\sigma}}_{x_{ijt}}, \end{aligned} \tag{3.15}$$

where the last line comes from applying the Pareto distribution. Relative to the Krugman model, average sales per firm, $x_{ijt}$, are affected by entry/exit of the marginal firms – movements in $\varphi_{ijt}^{m}$. Combining (3.14) and (3.15) leads to the following expression for $\varphi_{ijt}^{m}$:

$$\varphi_{ijt}^{m} = \left(\frac{\theta\varphi_L^{\theta}\xi_{ij}\sigma}{\theta-(\sigma-1)}\frac{w_{it}}{x_{ijt}}\right)^{\frac{1}{\theta}}. \tag{3.16}$$

In turn, combining (3.15) and (3.16) produces the following expression for $x_{ijt}$:

$$x_{ijt} \propto \left(\frac{Y_{jt}}{w_{it}}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}} Y_{jt} \left(\frac{\kappa_{ijt} w_{it}}{P_{jt}}\right)^{-\theta}. \tag{3.17}$$

Equation (3.17) clarifies that in the Melitz model, sales involve an additional term $\left(Y_{jt}/w_{it}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}}$ that is absent from Krugman and customer capital models. This term arises due to the extensive margin, whereby the cutoff for serving a market is a function of market size $Y_{jt}$, scaled by the domestic unit costs: if market size increases, more and more firms will enter, increasing sales per unit mass.[9] Even though foreign sales do not follow a simple CES demand functional form, domestic sales do. If A.2 holds, then the ratio $Y_{jt}/w_{jt}$ is constant and $x_{jj}$ conforms to (3.3) in Proposition 3.1. We show below that A.2 holds.

In steady state, at the time sunk costs are paid, expected profits are:

$$E\left[\pi_{ij}(\omega)\right] = \frac{1}{\sigma} x_{ij} - w_i \xi_{ij} \left(\frac{\varphi_L}{\varphi_{ij}^m}\right)^{\theta}. \tag{3.18}$$

Combining with (3.16) leads to the familiar result that expected profits are a constant fraction of expected sales: $E\left[\pi_{ij}(\omega)\right] = \frac{\sigma-1}{\theta}\frac{x_{ij}}{\sigma}$. Since (2.16) and (2.17) hold unchanged in the Melitz model (with the qualification that here, $x_{ij}$ is sales per unit mass of firms rather than representative firm sales), they lead to (3.2), and Assumption A.3 is satisfied.

To see that A.2 is satisfied, note that the steady state profits to country $i$ firms from selling to $j$ are:

$$\begin{aligned}
\Pi_{ij} =& \frac{\sigma-1}{\theta}\frac{1}{\sigma} n_{ij} x_{ij} - w_i \int_0^{\bar{\xi^s}_{ij}} \xi^s g\left(\xi^s\right) d\xi^s \\
=& \frac{\sigma-1}{\theta}\frac{1}{\sigma} n_{ij} x_{ij} - \frac{\chi}{\chi+1}\beta\frac{\sigma-1}{\theta\sigma} x_{ij} n_{ij} \\
=& \left(1 - \frac{\chi}{\chi+1}\frac{\beta\delta}{1-\beta(1-\delta)}\right)\frac{\sigma-1}{\theta\sigma} X_{ij},
\end{aligned}$$

where the second line uses the distributional assumption on the sunk costs $\xi^s$. Summing across destinations and imposing trade balance delivers Assumption A.2.

We obtain the familiar result that the elasticity of $x_{ij}$ with respect to trade costs $\varepsilon_\kappa^0$ is no longer a function of the elasticity of substitution, but of the dispersion parameter in the Pareto productivity distribution. Relative to the Krugman model, following a change in trade costs, average sales per unit mass $x_{ijt}$ will change both because of the intensive margin (all firms' sales change) and the extensive

[9] Note that sales per unit mass of potential sellers, $x_{ij} = \int_{\varphi_{ijt}^m}^{\infty} x_{ijt}(\varphi)\, dF(\varphi)$, is not the same as the average sales of firms serving a market, which is $x_{ij}/(1 - F(\varphi_{ijt}^m))$. When market size increases, $\varphi_{ijt}^m$ falls – less productive firms enter. This increases $x_{ij}$ since a higher fraction of firms per unit mass sell to the market. At the same time, the average sales fall, as less productive firms can serve larger markets.

margin (marginal firms entering/exiting). As in Arkolakis et al. (2008) and ACR, when it comes to $x_{ij}$, those two margins' net effect is captured by $-\theta$.

Differently from those static models, and as in the Krugman model in Section 2, the gains from trade are conditioned not just by $\theta$, but also by the parameter of the sunk cost distribution $\chi$, due to the adjustment of the mass of firms $n_{jj}$ that pay the sunk costs to obtain productivity draws. Thus, the Melitz extension retains the intuitions laid out in Section 2.

### 3.2 Generalization to Tariffs

Often, trade elasticities are estimated using variation in tariffs. To build up towards measurement and quantification, we state a generalization of Proposition 3.1 to a case with tariffs.

**Proposition 3.2.** *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

*A.1′ For all countries $j$, trade is balanced (expenditure = revenue):*

$$Y_j = w_j L_j + \Pi_j + R_j^g,$$

*where $Y_j = C_j P_j$ and $R_j^g = \sum_i \left(\tau_{ij} - 1\right) X_{ij}$.*

*A.2′ For all countries $j$, profits are a constant share of labor income:*

$$\frac{\Pi_j}{w_j L_j} = const$$

*A.3′ For all country pairs $(i, j)$ trade flows satisfy*

$$X_{ij} = n_{ij} x_{ij} \tag{3.19}$$

*where*

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi} \tag{3.20}$$

*and domestic per-unit-mass sales satisfy*

$$x_{jj} \propto Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{3.21}$$

*for some constant $\chi > 0$ and where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.*

*Then*

$$C_j \propto \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0(1+\chi)}} \left(1 - \frac{R_j^g}{Y_j}\right)^{-\left(1-\frac{\chi}{1+\chi}\frac{1}{\varepsilon_\kappa^0}\right)}, \tag{3.22}$$

*and* $\varepsilon_\kappa^0(1+\chi)$ *is the long-run elasticity of trade flows with respect to iceberg trade costs.*

*Proof.* See Appendix A. □

Note that $\lambda_{ij} \equiv \frac{\tau_{ij} X_{ij}}{Y_j}$ is now the tariff-inclusive expenditure shares on goods from $i$. Since $\lambda_{jj} = 1$ and $R_j^g = 0$ in autarky, (3.22) is also the gross proportional gains from trade in steady state.

Because tariffs generate revenue, (3.22) differs from (3.4) by the multiplicative factor that is a function of one minus the tariff revenue share in final expenditure. This multiplicative factor is greater than 1 as long as tariff revenue is positive. Thus, it amplifies the gains from trade relative to the no-tariff formula, conditional on the same $\lambda_{jj}$. In static models, this tariff adjustment to the ACR formula was to our knowledge first stated by Felbermayr, Jung, and Larch (2015). We show that it operates in a similar way in a dynamic setting. As in Felbermayr, Jung, and Larch (2015), the exponent on the tariff adjustment term cannot be recovered from the long-run trade elasticity alone. We show below how to recover this exponent from estimates of short- and long-run trade elasticities.

The data requirements for computing (3.22) are low. In addition to the domestic trade share, all it additionally requires is the total tariff revenue as share of GDP. This information is often available from statistical authorities. For the quantification below, we will require bilateral *ad valorem* tariff rates. Thus, it will be convenient to state the following alternative functional form for this adjustment factor:

$$1 - \frac{R_j^g}{Y_j} = \sum_i \frac{1}{\tau_{ij}} \lambda_{ij}.$$

We note that the Melitz-Pareto model with tariffs is not covered by Proposition 3.2, because tariffs also affect the extensive margin conditional on drawing the sunk cost, in a way that is not captured by (3.21). Proposition A.1 in Appendix A is an extension of Proposition 3.2 that also covers the Melitz-Pareto model with tariffs. The extended proposition is identical to Proposition 3.2 except for a strictly more general functional form for sales per unit mass $x_{jj}$. This generalization only affects the exponent on the tariff adjustment term $\left(1 - \frac{R_j^g}{Y_j}\right)$, and leaves the component of the GFT related to $\lambda_{jj}$ unaffected. As we show in the quantification below, the tariff adjustment term is not quantitatively important. In addition, the change to the exponent introduced by the extensive margin in the Melitz-Pareto model vanishes as the firm size distribution approaches a power law with exponent close to $-1$, the empirically relevant case (Axtell, 2001; Di Giovanni, Levchenko, and Rancière, 2011; Di Giovanni and Levchenko, 2013). Appendix A contains the detailed discussion.

### 3.3 Ex-ante analysis

The analysis thus far has considered how much welfare changes for a given change in openness as measured by $\lambda_{jj}$. We next consider an ex-ante analysis, asking how expenditure shares change for a given change in trade costs. For any variable $x$, denote by $\widehat{x} \equiv x'/x$ the gross proportional change between the pre-shock steady state value $x$ and the post-shock steady state value $x'$.

**Proposition 3.3.** *Consider a class of dynamic models that satisfy the Assumptions A.1'-A.3' in Proposition 3.2, and in addition the trade shares satisfy:*

$$\lambda_{ij} = \frac{n_{ij} \left(\tau_{ij}\kappa_{ij}w_i\right)^{\varepsilon_\kappa^0}}{\sum_k n_{kj} \left(\tau_{kj}\kappa_{kj}w_k\right)^{\varepsilon_\kappa^0}} \tag{3.23}$$

*for all $i, j$.*

*Then, following changes in iceberg trade costs $\hat{\kappa}_{ij}$ and tariffs $\hat{\tau}_{ij}$ for all $i \neq j$, the steady state change in expenditure shares is given by*

$$\hat{\lambda}_{ij} = \frac{\left(\hat{\kappa}_{ij}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{ij}\hat{w}_i\right)^{\chi\left(\varepsilon_\kappa^0-1\right)+\varepsilon_\kappa^0}}{\sum_k \lambda_{kj} \left(\hat{\kappa}_{kj}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{kj}\hat{w}_k\right)^{\chi\left(\varepsilon_\kappa^0-1\right)+\varepsilon_\kappa^0}} \tag{3.24}$$

*for all $i$ and $j$, where the wage changes are determined by the system*

$$\hat{w}_i = \sum_j \frac{\frac{1}{\tau_{ij}}\lambda_{ij} \left(\hat{\kappa}_{ij}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{ij}\right)^{(\chi+1)\left(\varepsilon_\kappa^0-1\right)} \left(\hat{w}_i\right)^{\chi\left(\varepsilon_\kappa^0-1\right)+\varepsilon_\kappa^0} \hat{w}_j \frac{w_j L_j}{w_i L_i}}{\sum_k \frac{1}{\tau_{kj}}\lambda_{kj} \left(\hat{\kappa}_{kj}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{kj}\right)^{(\chi+1)\left(\varepsilon_\kappa^0-1\right)} \left(\hat{w}_k\right)^{\chi\left(\varepsilon_\kappa^0-1\right)+\varepsilon_\kappa^0}} \tag{3.25}$$

*for all $i$.*

*Proof.* See Appendix A. □

Proposition 3.3 is analogous to the ex ante analysis in ACR, but once again with the important difference that in our framework, the masses of firms $n_{ji}$ will adjust following the trade cost shock. Because of the adjustment in $n_{ji}$, and in contrast to Proposition 3.1, ex ante analysis requires knowledge of $\varepsilon_\kappa^0$ and $\chi$ separately, even if the long-run iceberg trade cost elasticity $\varepsilon_\kappa^0(1+\chi)$ is known.

Note also that while the Krugman and the customer capital model laid out in Section 3.1 satisfy the restriction (3.23) needed for the proposition, for the Melitz model it depends on whether the per-period fixed costs are paid in the source or destination country's labor. When those costs are paid in the destination country's labor, the trade shares comply with (3.23) up to a vector of multiplicative constants irrelevant for the proof, and thus Proposition 3.3 applies. When the fixed costs are paid in

the source country labor, as is more commonly assumed, the trade shares are instead:

$$\lambda_{ij} = \frac{n_{ij}\left(\tau_{ij}\kappa_{ij}w_i\right)^{-\theta}\left(w_i\xi_{ij}\right)^{-\frac{\theta-(\sigma-1)}{\sigma-1}}}{\sum_k n_{kj}\left(\tau_{kj}\kappa_{kj}w_k\right)^{-\theta}\left(w_k\xi_{kj}\right)^{-\frac{\theta-(\sigma-1)}{\sigma-1}}}.$$

While these shares do not comply with (3.23), they can still be used to perform an ex-ante analysis in a way similar to Proposition 3.3. This analysis requires knowledge of one more parameter, $\sigma$, which cannot be inferred from trade elasticity estimation alone, for any horizon. In addition, these Melitz trade shares converge to the trade shares (3.23) as $\frac{\theta}{\sigma-1} \to 1$. As noted above and elaborated in the Appendix, in the Melitz-Pareto model $-\frac{\theta}{\sigma-1}$ is the slope of the power law in firm size. In the data, firm size follows a power law with an exponent close to $-1$, making $\frac{\theta}{\sigma-1} \approx 1$ an appropriate calibration, and thus (3.23) a good approximation of the trade shares in the Melitz model.

### 3.4 Endogenous Capital Accumulation

Adding physical capital accumulation to the class of models considered above is not common, as the masses of firms $n_{ijt}$ are themselves a type of capital stock, subject to forward-looking accumulation decisions and depreciation. Nonetheless, to bridge our framework with a wider range of dynamic trade models, we now extend the model to endogenous capital $K_{jt}$.

Households own capital and invest to accumulate it. The budget constraint and the capital accumulation equation are:

$$P_{jt}\left(C_{jt}+I_{jt}\right)+B_{jt}+B^*_{jt}+w_{jt}\psi\left(\frac{B^*_{jt}}{P_{jt}}\right)=w_{jt}L_j+r^k_{jt}K_{jt}+\Pi_{jt}+R^g_{jt}+B_{jt-1}\left(1+r^n_{jt-1}\right)+B^*_{jt-1}\left(1+r^{n*}_{t-1}\right)$$

and

$$K_{jt+1}=(1-\delta)K_{jt}+I_{jt},$$

where $I_{jt}$ is investment and the rental price of capital is $r^k_{jt}$. The Euler equation for capital is:

$$\left(\frac{C_{jt+1}}{C_{jt}}\right)^{\gamma}=\beta\left(\frac{r^k_{jt+1}}{P_{jt+1}}+1-\delta\right).$$

Production is Cobb-Douglas in capital and labor. Competitive producers assemble capital-labor bundles according to the following production function:

$$b_{it}=\alpha^{\alpha}(1-\alpha)^{1-\alpha}l^{\alpha}_{it}k^{1-\alpha}_{it},$$

where $l_{it}$ and $k_{it}$ are the labor and capital employed by the bundle producers. As a result, the cost of

the input bundle is

$$c_{it} = w_{it}^{\alpha}(r_{it}^{k})^{1-\alpha}.$$

For expositional clarity we specialize the production side to the Krugman model. In that case, production quantities, prices, and per-period profits are given by (2.3)-(2.5), replacing $w_{it}$ by $c_{it}$ and $l_{it}$ by $b_{it}$. The value of exporting is still given by (2.6), and the marginal firm's sunk costs $\bar{\xi}_{ijt}^{s}$ satisfy (2.7), once again with $w_{it}$ replaced by $c_{it}$.

Under these conditions and in steady state total profits are proportional to total factor income: $\Pi_j \propto w_j L_j + r_j^k K_j$. In addition, cost minimization and factor market clearing imply that

$$\frac{w_{jt} L_j}{r_{jt}^{r} K_{jt}} = \frac{\alpha}{1-\alpha}.$$

Combining these two properties yields the result that in steady state, capital and investment are proportional to consumption:

$$I_j = \delta K_j \propto C_j.$$

This means that real consumption is proportional to the total real income, which is in turn proportional to the real wage income:

$$\begin{aligned} C_j \propto & \frac{w_j L_j + r_j^k K_j + \Pi_j}{P_j} \\ \propto & \frac{w_j L_j}{P_j}. \end{aligned}$$

In turn, the price index is (see 2.13):

$$P_j = \frac{\sigma}{\sigma - 1} c_j \left( \frac{\lambda_{jj}}{n_{jj}} \right)^{\frac{1}{\sigma - 1}}.$$

Putting these together, real consumption is proportional to:

$$C_j \propto \left( \frac{\lambda_{jj}}{n_{jj}} \right)^{\frac{1}{1-\sigma}} K_j^{1-\alpha}.$$

Note that when $K_j$ is fixed, the same steps with respect to $n_{jj}$ yield the GFT formula (2.21)/(3.4) as in Sections 2 and 3. When $n_{jj}$ is fixed, we are in a model with constant variety but endogenous capital accumulation, reminiscent of e.g., Alvarez (2017) and Ravikumar, Santacreu, and Sposi (2019). In that case, we can use the proportionality of steady state capital to consumption to arrive at the following GFT formula:

$$C_j \propto \lambda_{jj}^{\frac{1}{1-\sigma}\frac{1}{\alpha}}.$$

Trade leads to capital accumulation, raising the GFT above the "static" level (as $\alpha < 1$), a channel emphasized by Ravikumar, Santacreu, and Sposi (2019) among others.

Finally, when both $n_{jj}$ and $K_j$ adjust following trade opening, as would be the case in the complete model, the GFT formula is

$$GFT = \lambda_{jj}^{\frac{\frac{1}{1+\chi}\frac{1}{1-\sigma}}{1-(1-\alpha)\left(1-\frac{\chi}{1+\chi}\frac{1}{1-\sigma}\right)}}.$$

Relative to the formula in the labor-only model (2.21)/(3.4), the GFT are amplified by $1-(1-\alpha)\left(1-\frac{\chi}{1+\chi}\frac{1}{1-\sigma}\right)$, which comes from the fact that the physical capital stock $K_j$ reacts to trade opening. Note that it is no longer guaranteed that the GFT are positive. The gains are positive when $\sigma$ is large enough:

$$\sigma > 1 + \frac{\chi}{1+\chi}\frac{1-\alpha}{\alpha}.$$

Under this parameter restriction, the GFT are strictly larger than in the labor-only model (2.21)/(3.4), as the country accumulates $K_j$ following trade opening.

On the other hand, the combination of accumulable $K_j$ and accumulable $n_{jj}$ leads to negative GFT under some parameter values. This is because the complementarity between $K_j$ and $n_{jj}$ leads to an agglomeration force: a larger capital stock lowers the cost of accumulating $n_{jj}$, and vice versa. Combining expressions for $n_{jj}$, $x_{jj}$, and $c_j$ leads to the following steady state relationship:

$$K_j \propto \left(\frac{n_{jj}^{\frac{1+\chi}{\chi}}}{\lambda_{jj}}\right)^{\frac{1}{1-\alpha}}.$$

Domestic capital accumulation has a positive relationship with $n_{jj}$, and a negative relationship with $\lambda_{jj}$, both of which are intuitive. When the loss of domestic variety following trade opening is significant enough relative to the export opportunities encapsulated by $\lambda_{jj}$, trade opening reduces the steady state level of capital, and thereby welfare.

## 4. Measurement and Quantification

In this section we turn to measurement and quantify the welfare gains from trade. We make four main points. The first is that in a dynamic world the long-run trade elasticity with respect to iceberg costs required by the formula cannot be recovered from a single empirical estimate of the elasticity of trade with respect to tariffs. Second, we compute the gains from trade under our preferred estimates of the trade elasticities, taken from Boehm, Levchenko, and Pandalai-Nayar (2023). This exercise shows that the gains from trade are large, and that the quantitative impact of the tariff adjustment to the GFT formula in (3.22) is generally minor. Third, we highlight that in the dynamic world, the long-run tariff elasticity is not sufficient for computing the gains from trade, and that GFT can vary widely

even conditional on the same long-run tariff elasticity. Along the way, we also compare the dynamic gains from trade to those obtained from the static ACR models. Finally, the fourth part of the section compares the GFT implied by the formula to those computed numerically in a multi-country general equilibrium dynamic setting taking into account the transition path.

### 4.1 Data

The quantification relies on several sources of data. First, computing the gains from trade using (3.4) requires the domestic absorption share $\lambda_{jj}$. Typically, domestic absorption is measured from standard datasets such as the OECD Inter-Country Input Output tables (ICIO). The ICIO contains information on all bilateral sectoral expenditures, covering manufacturing and services, and intermediate and final goods. Importantly, it also contains information on expenditure on domestic sectors. However, the ICIO does not contain information on bilateral tariff revenues, so aggregate expenditure and expenditure shares constructed from this source are not tariff-inclusive.

Computing the gains from trade when *ad valorem* tariffs are non-zero (3.22) requires the total tariff revenue as a share of total (tariff-inclusive) spending. Aggregate tariff revenues are available from the World Bank. However, the full quantitative implementation of the dynamic model additionally requires all tariff-inclusive bilateral expenditure shares $\lambda_{ij}$. Therefore, aggregate tariff revenues are not sufficient for our purposes.

To construct bilateral tariff revenue, we obtain tariff data from the TRAINS dataset. This database reports the applied tariff by country pair at the Harmonized System (HS) 6-digit level. We link these data to trade flows at the HS-6 level from the BACI version of UN-COMTRADE. To compute tariff revenue, we multiply the bilateral, product-level applied tariffs obtained from TRAINS with bilateral trade flows from BACI:

$$R_{ij}^g = \sum_p X_{ijp}^{\text{BACI}} \left( \tau_{ijp}^{\text{TRAINS}} - 1 \right),$$

where $R_{ij}^g$ is bilateral tariff revenue from goods trade and $p$ is an HS-6 product. BACI does not contain information on services trade flows. Services trade flows are subject to no tariff, so the aggregate bilateral tariff rate imposed by $j$ on $i$ consistent with goods tariff revenues $R_{ij}^g$ is:

$$\tau_{ij} - 1 = \frac{R_{ij}^g}{X_{ij}^{\text{ICIO}}},$$

where $X_{ij}^{\text{ICIO}}$ is total expenditure of $j$ on goods and services from $i$, sourced from the OECD ICIO database. We can then calculate all tariff-adjusted trade shares $\lambda_{ij}$:

$$\lambda_{ij} = \frac{\tau_{ij} X_{ij}^{\text{ICIO}}}{\sum_k \tau_{kj} X_{kj}^{\text{ICIO}}}.$$

Our baseline sample includes 67 countries and a rest-of-the-world aggregate in 2006.[10] We validate our tariff revenue measures by comparing $R_j^g = \sum_i R_{ij}^g$ with national tariff revenue obtained from the World Bank. As the World Bank tariff revenue data are provided in local currency, we convert them to US dollars using an annual exchange rate obtained from the same source. Appendix Figure C1 illustrates that our baseline tariff revenue measures are similar to those obtained from the World Bank. The calibration of the full quantitative model in Section 4.4 additionally requires data on real GDP. We obtain these from the Penn World Tables.

### 4.2 Measurement: Trade Elasticities

As in ACR, the gains from trade in this class of dynamic models is a function of the domestic absorption share and exogenous parameters. Propositions 3.1-3.2 state that the domestic share is raised to the power of the inverse of the long-run iceberg trade elasticity. In dynamic models, this long-run trade elasticity is a function of different structural parameters than the "trade elasticity" in static models. We now show that this has implications for how this long-run elasticity can be recovered from econometric estimates.

The exponent in the gains from trade formula is the inverse of the long-run elasticity of trade with respect to iceberg trade costs $\kappa_{ij}$:

$$\varepsilon_\kappa \equiv \frac{d \ln X_{ij}}{d \ln \kappa_{ij}} = \frac{d \ln n_{ij}}{d \ln x_{ij}} \frac{d \ln x_{ij}}{d \ln \kappa_{ij}} + \frac{d \ln x_{ij}}{d \ln \kappa_{ij}} = \frac{d \ln x_{ij}}{d \ln \kappa_{ij}} (1 + \chi) = \varepsilon_\kappa^0 (1 + \chi). \tag{4.1}$$

Though a few papers have used shipping cost data to compute the trade elasticity (e.g., Hummels, 2001; Shapiro, 2016; Adão, Costinot, and Donaldson, 2017), the large majority of existing trade elasticity estimates use tariffs. However, the trade elasticity with respect to tariffs differs from that with respect to iceberg costs. The tariff elasticity is:

$$\varepsilon_\tau \equiv \frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \frac{d \ln n_{ij}}{d \ln x_{ij}} \frac{d \ln x_{ij}}{d \ln \tau_{ij}} + \frac{d \ln x_{ij}}{d \ln \tau_{ij}} = \frac{d \ln x_{ij}}{d \ln \tau_{ij}} (1 + \chi) = (\varepsilon_\kappa^0 - 1)(1 + \chi). \tag{4.2}$$

The reason for the difference is that iceberg trade costs and tariffs have distinct effects on sales net of tariff payments $x_{ij}$ (see e.g. 2.5), and therefore on flow profits and entry $n_{ij}$. As highlighted in (4.1)-(4.2), the elasticities of $n_{ij}$ to $\tau_{ij}$ and $\kappa_{ij}$ are composites of two elasticities: $\frac{d \ln n_{ij}}{d \ln \kappa_{ij}} = \frac{d \ln n_{ij}}{d \ln x_{ij}} \frac{d \ln x_{ij}}{d \ln \kappa_{ij}}$, and analogously for $\tau_{ij}$. The first term is always simply $\chi$: $\frac{d \ln n_{ij}}{d \ln x_{ij}} = \chi$. But the elasticity of sales per unit mass with respect to $\tau_{ij}$ is $\varepsilon_\kappa^0 - 1$ (e.g. $-\sigma$ in the Krugman model), whereas the elasticity of sales to $\kappa_{ij}$ is $\varepsilon_\kappa^0$ (e.g. $1 - \sigma$ in the Krugman model). This is immediate from (2.3), (2.4), and (2.8).

The intuition can be built up as follows. When iceberg trade costs are modeled as part of firms' technologies for serving foreign markets – by far the most common approach in the trade literature,

[10] Three percent of the observations show positive bilateral goods trade flows in the ICIO but have no tariffs declared in TRAINS. In these cases, we assume there is 0 tariff revenue associated with these pairs.

also adopted throughout this paper – iceberg trade costs affect the price received by the exporter $p_{ij}^x$ while tariffs do not (see equation 2.4). Specifically, a given increase in the iceberg trade cost translates into a higher price received by the exporter. By contrast, a higher tariff does not raise the price received by the exporter.

At the same time, the prices faced by consumers include both tariffs and iceberg trade costs (to see this, combine equations 2.4 and 2.8). Changes in tariffs and changes iceberg trade costs therefore affect demand identically, but have different effects on producer prices and thus sales and flow profits. A given increase in the iceberg trade cost is not as damaging to the exporter's profits as an identical increase in the tariff: $\varepsilon_\kappa^0$ is smaller in absolute value than $\varepsilon_\kappa^0 - 1$ when comparing (4.1) and (4.2). Since changes in flow profits drive the firms' entry decisions and ultimately the mass of firms in steady state, $n_{ij}$, an increase in iceberg costs reduces $n_{ij}$ by less than an equal increase in tariffs.[11]

Most (though not all) of the literature that estimates trade elasticities in the context of static models recognizes the distinction between tariff and iceberg elasticities. When $n_{ij}$ does not respond to trade costs, as is the case in practice in most static models such as Armington or Krugman, this distinction is fairly innocuous. To account for it, one could either add 1 to the tariff elasticity to recover the iceberg cost elasticity, or use trade flows inclusive of tariff payments in estimation. In our dynamic setting, or more generally if the mass of exporters responds to trade costs in the long run, neither of these simple adjustments work, requiring another strategy to recover the iceberg elasticity.[12]

Fortunately, it is possible to use tariff elasticity estimates at different horizons to construct the long-run elasticity that enters the gains from trade formula. The key is to use estimates of both short- and long-run tariff elasticities to separately pin down $\varepsilon_\kappa^0$ and $1 + \chi$. Knowledge of $\varepsilon_\kappa^0$ and $1 + \chi$ separately is also required to compute the tariff revenue adjustment to the GFT formula as in (3.22).

We will call the "short run" a time period over which $x_{ij}$ can adjust but $n_{ij}$ cannot. This is consistent with the models laid out above, in which $n_{ij}$ only starts adjusting with a one-period lag. The short-run tariff elasticity is then:

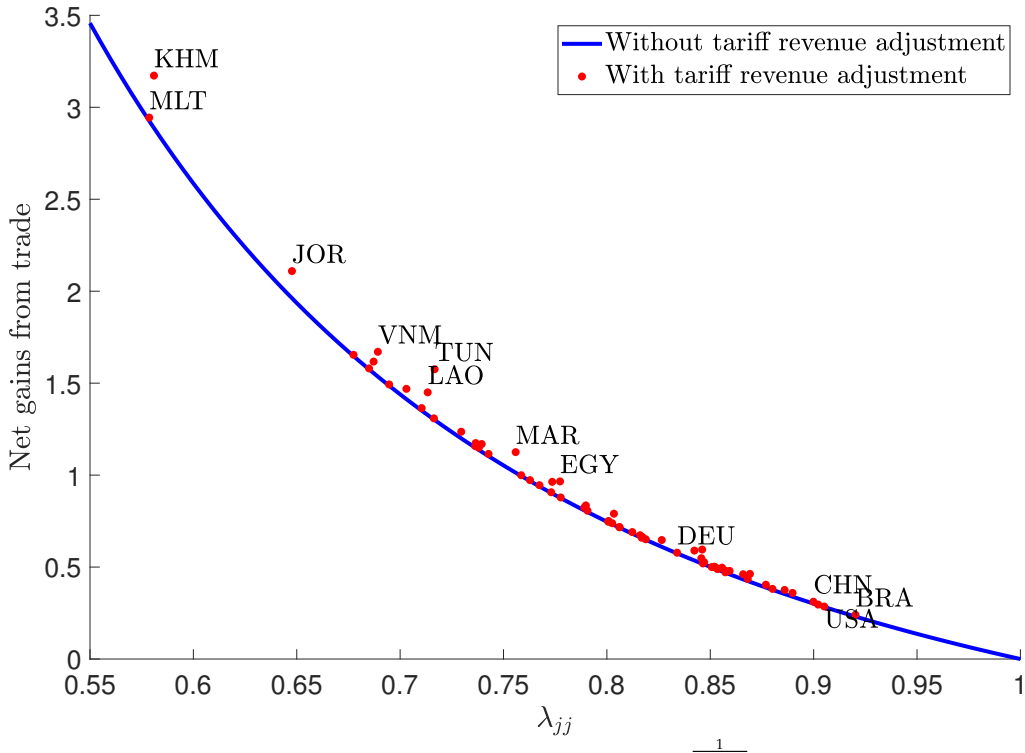
$$\varepsilon_\tau^0 \equiv \left.\frac{d \ln X_{ijt}}{d \ln \tau_{ijt}}\right|_{n_{ijt-1} \text{ fixed}} = \frac{d \ln x_{ijt}}{d \ln \tau_{ijt}} = \varepsilon_\kappa^0 - 1. \tag{4.3}$$

It is immediate that with both the short- and long-run tariff elasticities (4.2)-(4.3) in hand, one can

[11] An alternative but for our purposes equivalent way of modeling iceberg trade costs is to assume that the difference between consumer and producer prices includes both tariffs and trade costs, because some products "melt" in transit. In this case, neither tariffs nor iceberg trade costs are part of firms' marginal costs and hence neither affects producer prices. However, changes in iceberg trade costs alter the amount that needs to be produced and shipped. Consider again the two thought experiments of increasing tariffs or increasing iceberg trade costs by an equal amount. Producer prices are not affected in this case while consumer prices rise with equal elasticities. As a result, demand falls by an equal amount in both thought experiments. Since higher iceberg trade costs imply that firms must now produce more per unit that arrives in the destination, the drop in the quantity demanded is partially offset by an increase in the amount that ultimately melts. Production therefore falls by less after a rise in iceberg trade costs. Firm sales, which equal the producer price times the quantity produced, and profits therefore also fall by less in the case of an increase in iceberg trade costs compared to an equal increase in tariffs.

[12] It is immediate from (4.1)-(4.2) that adding 1 to $\varepsilon_\tau$ does not recover $\varepsilon_\kappa$. It is also easy to verify that the long-run tariff elasticity of tariff-inclusive trade flows $d \ln\left(\tau_{ij} n_{ij} x_{ij}\right) / d \ln \tau_{ij}$ also does not equal $\varepsilon_\kappa$, once again because of changes in $n_{ij}$.

Figure 1: Steady State Gains from Trade




**Notes:** The figure depicts the net proportional welfare gains from trade, $\lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0(1+\chi)}} - 1$ as a function of the domestic absorption share $\lambda_{jj}$. The blue line implements the formula (3.4). The red dots implement the formula that adjusts for tariff revenue, (3.22).

recover both $\varepsilon_\kappa^0$ and $1 + \chi$, and construct the object needed for the welfare gains formula, (4.1). We pursue this strategy in the quantification that follows.

### 4.3 Steady State Gains from Trade

Figure 1 plots the gains from trade for a sample of countries based on (3.22). It uses the long-run tariff elasticity estimate $\varepsilon_\tau = -2$, and a short-run tariff elasticity estimate $\varepsilon_\tau^0 = -1.25$ (Boehm, Levchenko, and Pandalai-Nayar, 2023). The blue line is simply the formula (3.4) that ignores tariffs and only uses information on the domestic trade share. The red dots are (3.22), and thus make the tariff adjustment using country-specific tariff revenue data. Conditional on a fixed $\lambda_{jj}$, the tariff adjustment unambiguously increases the gains from trade. However, the tariff adjustment is small quantitatively for all but a few countries.

The gains from trade are large. Even the most closed countries – the US, Brazil, China – gain on the order of 25-30% from trade. Jordan's welfare triples, and Malta's quadruples, when they go from autarky to trade.

To highlight the role of dynamics and the short-run elasticity in conditioning the gains from trade, we now perform the following thought experiment. Suppose the value of the long-run tariff elasticity $\varepsilon_\tau$ is known. This is the object most commonly estimated (or, at least, intended to be estimated) in

the literature. To make the results stark, suppose that the value of this long-run elasticity is high, for example $-5$ (Costinot and Rodríguez-Clare, 2014).

A researcher working on static models covered by ACR would simply add 1 to yield a long-run elasticity with respect to iceberg costs of $-4$, and compute the gains from trade based on this. As ACR and Costinot and Rodríguez-Clare (2014) highlight, the basic single-sector ACR formula under this level of trade elasticity yields small gains from trade. These are depicted by the red line in Figure 2.

However, even holding $\varepsilon_\tau$ fixed at $-5$, in a dynamic world an additional piece of information is required, that we obtain from the short-run tariff elasticity. Specifically, the iceberg elasticity required in Propositions 3.1-3.2 can be expressed in terms of the estimable tariff elasticities as:

$$\begin{aligned}\varepsilon_\kappa^0(1+\chi) &= \varepsilon_\kappa^0(1+\chi)\frac{\varepsilon_\kappa^0-1}{\varepsilon_\kappa^0-1}\\ &= \underbrace{(\varepsilon_\kappa^0-1)(1+\chi)}_{\varepsilon_\tau}\frac{\varepsilon_\kappa^0}{\varepsilon_\kappa^0-1}\\ &= \varepsilon_\tau\frac{\varepsilon_\tau^0+1}{\varepsilon_\tau^0}.\end{aligned}$$

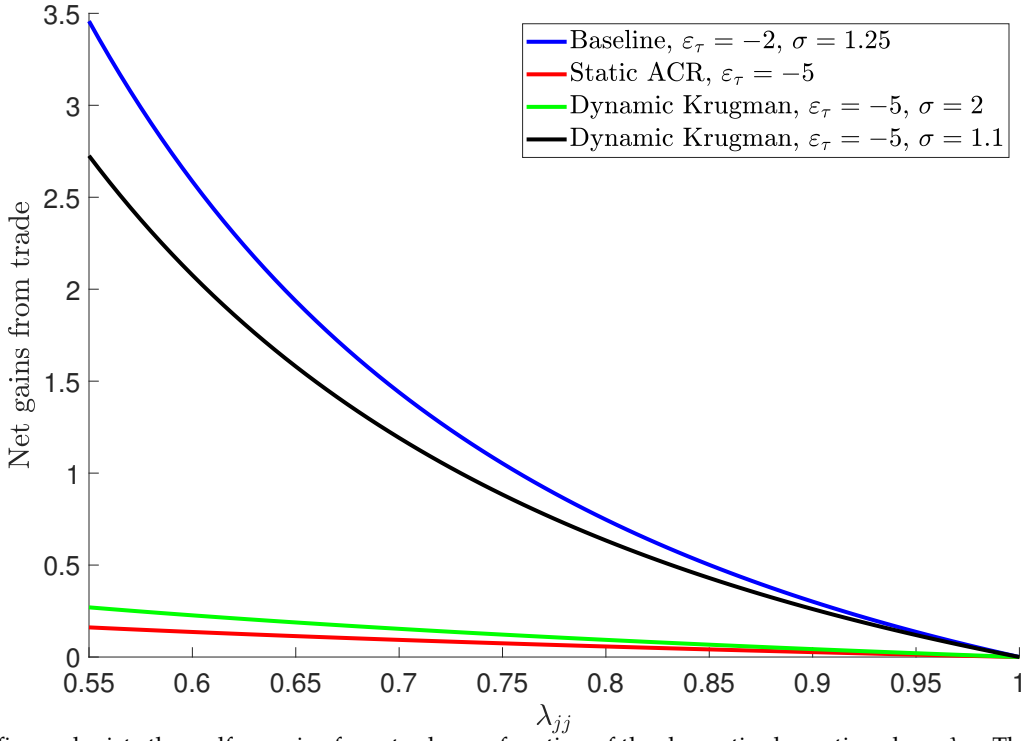
This expression makes it clear that a high long-run tariff elasticity $\varepsilon_\tau$ is consistent with arbitrarily large gains from trade if the short-run tariff elasticity is sufficiently low. Indeed, as $\varepsilon_\tau^0 \uparrow -1$, the gains from trade become infinite. The green and black lines in Figure 2 plot the gains from trade according to (3.4) under an identical long-run elasticity $\varepsilon_\tau = -5$, but for two values of $\varepsilon_\tau^0$, $-1.1$ and $-2$. The difference in the gains from trade is drastic. Indeed, the black line is not too far from our baseline gains from trade plotted under the long-run tariff elasticity of $-2$, despite a much higher (in absolute value) long-run tariff elasticity that it uses, $-5$. However, even with an unreasonably high short-run elasticity of $-2$, the dynamic gains from trade are higher than in the static ACR implementation.

### 4.4 The Transition Path and Welfare Gains

In the final exercise, we answer the question of how costly it is that the formula compares steady states, and thus ignores the transition path. To do this, we compute welfare taking into account the transition between trade regimes. This requires calibrating the full model, and thus taking a stand on all the parameters.

We employ the Krugman model from Section 2. The equilibrium definition, all model equations and the solution approach are provided in Appendix B.1. In addition to $\sigma$ and $\chi$, calibrated as above using short- and long-run tariff elasticity estimates, we require the depreciation rate $\delta$, risk aversion $\gamma$, the discount factor $\beta$, the Inverse Pareto scale parameter $b$, and the adjustment cost function $\psi$. Of these, the most important one is $\delta$, as it controls the speed of transition. The lower is $\delta$, the slower the transition, and the greater the discrepancy between steady state and fully dynamic gains. When depreciation is full ($\delta = 1$), transition occurs in 1 period. This parameter is disciplined

Figure 2: Steady State Gains from Trade: the Role of the Short-Run Elasticity




**Notes:** The figure depicts the welfare gains from trade as a function of the domestic absorption share $\lambda_{jj}$. The red line depicts the ACR formula with elasticity $-4$, $\lambda_{jj}^{-1/4}$. The other lines implement the formula (3.4) for different values of $\varepsilon_\tau$ and $\varepsilon_\tau^0$.

by the speed of convergence of the trade elasticity to the long-run. We set $\delta = 0.25$ to match the convergence speed estimated in Boehm, Levchenko, and Pandalai-Nayar (2023). As robustness we also consider higher and lower values of $\delta$ below. Further, we choose the adjustment cost function $\psi\left(B_{jt}^*/P_{jt}\right) = \psi/2\left(B_{jt}^*/P_{jt}\right)^2$ and calibrate the parameter $\psi = 0.05$ to ensure that the bond holding costs are small. The remaining parameter choices are standard. Table 1 summarizes the baseline calibration. We consider alternative parameter choices for robustness below. We solve the model for the 29 largest countries in the world by total GDP and a rest-of-the-world aggregate.

In addition to the observed trade equilibrium our exercises require computation of new steady states that occur if, say, country $i$ moves to autarky. When doing so, we set all pairs of bilateral trade costs $\kappa_{ij}$ and $\kappa_{ji}$ for country $i$ to infinity for all $j \neq i$.[13] All other parameters remain unchanged, including the calibrated tariff rates and countries' labor endowments. We then apply Proposition 3.3 to obtain the trade shares $\lambda_{ij}$ for all country pairs in the new steady state.

While steady state comparisons are unambiguous, in a dynamic setting there are multiple ways in which a country can transition between autarky and trade. First, the direction matters. The consumption stream of transitioning from autarky to trade will differ from the consumption stream associated with transitioning from trade to autarky. Second, assumptions on a country's access to

[13] In practice, we choose a very large number.

Table 1: Baseline Calibration

| Parameters | Value / Target / Source | Notes |
|---|---|---|
| $\sigma$ | 1.25 | Short-run tariff elasticity |
| $\chi$ | 0.6 | Inverse Pareto shape parameter |
| $\beta$ | 0.97 | Discount factor |
| $\gamma$ | 2 | Relative risk aversion |
| $\delta$ | 0.25 | Exit rate |
| $b$ | 1 | Inverse Pareto scale parameter |
| $\psi$ | 0.05 | Bond holding cost parameter |
| $\tau_{ij}$ | BACI, TRAINS | Average bilateral tariff |
| $\kappa_{ij}$ | $\lambda_{ij}$ from BACI, ICIO, TRAINS | Non-tariff trade costs |
| $L_i$ | Relative real GDP from PWT | Labor endowment |

**Notes:** The table shows the baseline calibration.

Table 2: Quantitative Exercises

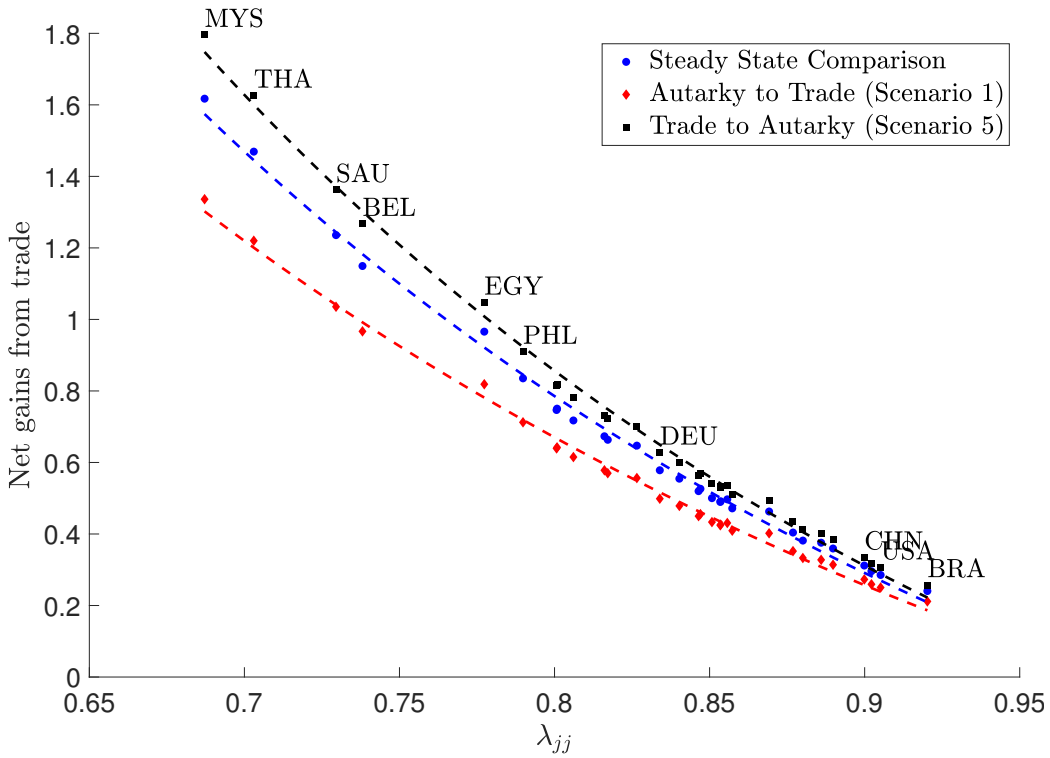
| (1) Scenario | (2) Transition | (3) Access to bond markets | (4) Other countries | (5) Net gains from trade (country average) | (6) Cross-sectional correlation with Scenario 1 |
|---|---|---|---|---|---|
| 1 (baseline) | Autarky to Trade | Yes | Open throughout | 0.5473 | 1 |
| 2 | Autarky to Trade | Yes | Also transitioning | 0.5456 | 1.0000 |
| 3 | Autarky to Trade | No | Open throughout | 0.5451 | 1.0000 |
| 4 | Autarky to Trade | No | Also transitioning | 0.5451 | 1.0000 |
| 5 (baseline) | Trade to Autarky | *Irrelevant* | *Irrelevant* | 0.6979 | 0.9997 |

**Notes:** The table shows the various scenarios considered in the quantitative exercises, the average gains from trade in each scenario, and the cross-sectional correlation of the gains from trade with the baseline scenario 1.

international bond markets will affect its consumption trajectory when opening up to trade. Finally, the trade openness of other countries in the world during the transition could impact the consumption stream of an individual country opening to trade. Therefore, we will simulate both the transition from autarky to trade of a single country alone (while all other countries are open throughout) and the transition of all countries from autarky to trade simultaneously. For the transition from trade to autarky, as trade costs are infinite from the first period onwards, both the assumptions on international asset markets and on openness trajectories of other countries are irrelevant. Table 2 provides an overview of these scenarios. In our baseline scenarios we consider (i) the transition of a single country from autarky to trade, assuming that all countries have access to international bond markets and (ii) the transition from trade to autarky.

Figure 3 reports three sets of gains from trade: (i) comparing autarky and trade steady states according to the formula (3.22) (blue); (ii) transitioning from autarky to the current levels of trade openness (scenario 1 from Table 2, in red); and (iii) transitioning from the current levels of openness

Figure 3: Steady State Gains vs. Gains over the Transition Path




**Notes:** The blue dots depict the GFT computed using formula (3.22) and hence represent steady state comparisons. The red dots depict the GFT for a single country starting in autarky and moving to the observed levels of trade, relative to remaining in autarky forever. The country has access to bond markets and all other countries' trade costs remain unchanged in this exercise (Scenario 1). The black dots depict the GFT for a single country trading forever relative to transitioning to autarky (Scenario 5). Dashed lines represent an exponential fit between the gains from trade and the domestic absorption shares.

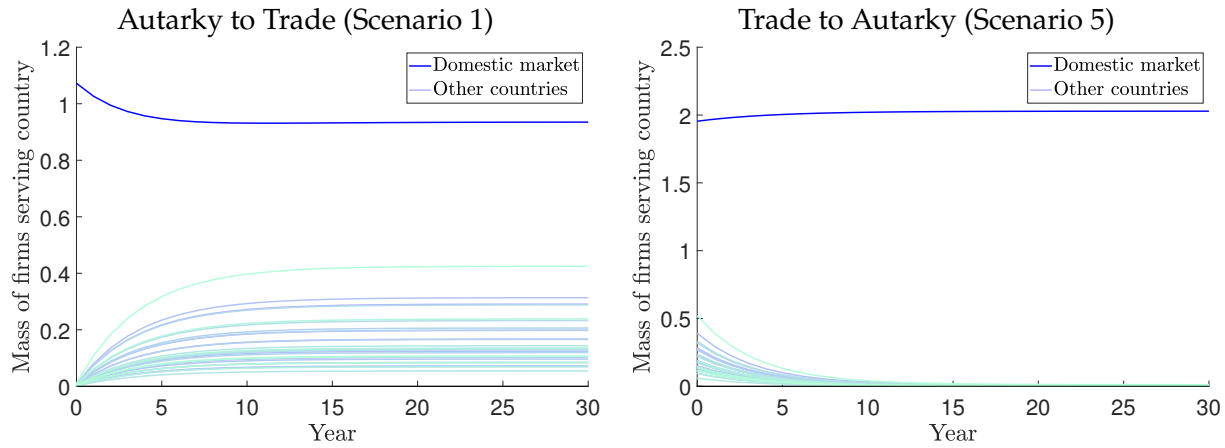
to autarky (scenario 5 from Table 2, in black). For scenarios (ii) and (iii) we begin in the initial steady state and then unexpectedly and permanently change trade costs $\kappa_{ij}$ at time 1 to the value in the terminal steady state. When computing the welfare gains for country $i$, we make this adjustment to trade costs for all $j \neq i$ and leave trade costs for country pairs that do not involve $i$ unchanged. To take changes in consumption over the transition path into account when computing the welfare gains, we first compute steady state consumption equivalents (see Appendix B.2 for details) and then take the ratio of these consumption equivalents.

Two conclusions stand out from the figure. First, the disparity between steady state gains and the full dynamic gains is relatively modest. On average in this sample, the autarky-to-trade gains are 14 percent smaller, and trade-to-autarky gains are 8 percent larger. Second, the steady state gains are always in between those two.

To illustrate the intuition for this ranking of gains, Appendix Figure C2 plots the dynamic paths of consumption, and Figure 4 plots the evolution of the masses of firms for one country, Malaysia.[14] When moving from autarky to trade, the country starts in the autarky steady state, and transitions to

[14] Appendix Figure C3 plots the consumption paths for Scenarios 1 and 5 for all countries.

Figure 4: Mass of Firms in the Transition Path, Malaysia




**Notes:** This figure shows the transition paths of the masses of Malaysian firms after a surprise permanent change in the trade regime. The left figure depicts the transition in Scenario 1 and the right figure the transition in Scenario 5. The dark line denotes the mass of Malaysian firms serving the domestic market $n_{jjt}$ and the light lines denote the masses of Malaysian firms serving the other countries $n_{jit}$, $j \neq i$.

the trade steady state slowly. Over this transition, consumption is lower than in the terminal steady state. This is because agents need to invest in "exporting capital" $n_{ijt}$, $j \neq i$ starting from a level of 0, as illustrated in the left panel of Figure 4. Consumption only reaches the new steady state level once these masses of firms have converged to the higher level of the new steady state. As a result, the dynamic gains of going from autarky to trade are below the steady state comparison.

Moving from trade to autarky, countries' accumulated exporting capital $n_{ijt}$ has become useless, because flow exports along the intensive margin $x_{ijt}$ is zero under infinite trade costs. At the same time, firms invest in their domestic operations, increasing the mass of domestic firms $n_{iit}$ as shown in the right panel of Figure 4. The result is an immediate drop in consumption *below* the level of the autarky steady state, and a gradual convergence of consumption to the autarky steady state level from below (Appendix Figure C2). This reduces the consumption equivalent relative to the steady state – effectively the denominator of the GFT formula – and thus raises the implied GFT relative to the steady state comparison. The GFT numbers for each country and each scenario are listed in Appendix Table C1.

**International asset markets and solo vs. simultaneous transitions.** Table 2 investigates the importance of the international asset markets, and whether the country opens to trade alone or simultaneously with all other countries. It presents results for every combination of assumptions, listed in columns 3-4. Column 5 reports the mean GFT, and column 6 the correlation in GFT across countries between that row and the baseline (scenario 1).

Both assumptions make very little difference, delivering virtually identical GFT. The GFT are a tiny bit larger when other countries are also transitioning compared to a solo transition, but the disparities are minuscule. Evidently, quantitatively there is limited impact of changes in firm creation

in other countries on the NPV of consumption in the opening country. Similarly, whether during the transition the country has access to an international bond or is in financial autarky makes virtually no difference for the resulting GFT. Partial equilibrium reasoning might suggest that the introduction of the international bond expands the set of options available to agents, and thus should lead to higher welfare (since they can always choose not to trade the bond). However, this reasoning breaks down in general equilibrium environments. In general equilibrium, the introduction of an international bond changes all the prices in the economy, most notably the path of domestic interest rates. As a result, it is not clear that adding financial instruments will unambiguously increase all participants' welfare.

This is not to say that these assumptions are completely immaterial. Appendix Figure C2 plots the path of Malaysia's consumption following trade opening under the 5 scenarios. The blue and orange lines depict the consumption paths with international bonds, and the purple and yellow lines under financial autarky. Appendix Figure C4 shows the debt-to-GDP ratio of each opening country along its transition when all other countries are already open. The amount of borrowing is substantial, with debt-to-GDP ratio peaking at around 40% in the case of Malaysia. While there is no clear benchmark of how much a country transitioning out of autarky would be expected to borrow (as we never observe such a controlled scenario in practice), these amounts of debt are substantial relative to the total debt-to-GDP ratios over the period 2000-present of about 100% in advanced economies and 40% in emerging markets (International Monetary Fund, 2025). Thus, international borrowing enables the country to move consumption forward in a meaningful way. However, when a country borrows, it must repay, and its consumption stream is below the steady state value for much longer compared to financial autarky. The net effect of the ability to borrow on life-time utility and hence steady state consumption equivalents following trade opening turns out to be negligible quantitatively. The lack of impact of financial integration echoes the finding of Gourinchas and Jeanne (2006) that the gains from financial integration are small when they come exclusively from speeding up the transition to the same steady state, as is the case in our exercise.

Relatedly, when international borrowing is allowed, the time profile of consumption depends meaningfully on whether other countries are opening at the same time (blue vs. orange lines). When other countries are already open, the opening country can move consumption forward to a greater extent. This is sensible: when other countries are already in steady state, they do not need to undertake large investments in exporting capital required by trade opening. Thus, the solo opening country can borrow more and consume more earlier. By contrast, in financial autarky it makes no difference for the profile of consumption whether other countries are already open or not (the purple and yellow lines are virtually indistinguishable from each other).

**Robustness.** We recompute the quantitative model with alternative parameter values to evaluate the robustness of our conclusions. As discussed above, the most important parameter is $\delta$, governing the speed of the transition. We consider alternative values of $\delta = 0.35$ and $\delta = 0.15$. Additionally, we consider a higher demand elasticity of $\sigma = 1.5$. We further vary the curvature of adjustment costs

Table 3: Robustness: Alternative Parameter Values

| Calibration | Average Steady State Gains | Average Dynamic Gains | | | |
|---|---|---|---|---|---|
| | | Autarky to Trade | | Trade to Autarky | |
| | | | difference | | difference |
| | (1) | (2) | (3) | (4) | (5) |
| Baseline | 0.641 | 0.547 | -14.6% | 0.698 | 9.0% |
| $\delta = 0.35$ | 0.641 | 0.570 | -11.0% | 0.683 | 6.6% |
| $\delta = 0.15$ | 0.641 | 0.500 | -21.9% | 0.730 | 13.9% |
| $\sigma = 1.5$ | 0.278 | 0.242 | -12.8% | 0.298 | 7.4% |
| $\chi = 1$ | 0.488 | 0.420 | -14.0% | 0.558 | 14.4% |
| $\chi = 0.3$ | 0.840 | 0.729 | -13.2% | 0.878 | 4.6% |
| $\beta = 0.9$ | 0.641 | 0.401 | -37.4% | 0.803 | 25.4% |
| $\gamma = 3$ | 0.641 | 0.542 | -15.4% | 0.702 | 9.5% |

**Notes:** The table presents results from robustness checks that explore the implications of alternative calibrations. Column (1) shows the average gains from trade relative to autarky in steady state, using the formula in (3.4). Columns (2) and (3) show the absolute and relative gains in the quantitative dynamic Krugman model with transition when moving from autarky to trade (Scenario 1). Columns (4) and (5) show the absolute and relative gains in the dynamic Krugman model with transition when moving from trade to autarky (Scenario 5). The rows show alternative parameter choices. The first row shows the baseline calibration in Table 1. The next two rows change the speed of transition to a fast transition ($\delta = 0.35$) and a slow transition ($\delta = 0.15$). The next row changes the short-run iceberg elasticity $\sigma$. The subsequent two rows alter the convexity of adjustment $\chi$. The second to last row explores the role of a lower discount factor $\beta$. The last row changes the relative risk aversion $\gamma$.

$\chi$, allowing for high curvature $\chi = 1$ and low curvature $\chi = 0.3$. Lastly, we show results for a lower discount factor $\beta = 0.9$ and a higher relative risk aversion $\gamma = 3$. The results are shown in Table 3. Across all calibrations, the steady state gains from trade implied by the formula in (3.4) remains a good approximation of quantitative gains from trade including transition dynamics, with average differences ranging from 4.6% to −37..4%. As expected, the largest average difference arises when households discount the future more ($\beta = 0.9$) and with a substantially slower transition ($\delta = 0.15$), and when moving from autarky to trade. Even here, the steady state gains from trade are a reasonable approximation. In all cases, the steady state gains from trade remain in between those computed in the full model going from autarky to trade and trade to autarky. We show the GFT numbers for each country and alternative parameter values in Appendix Tables C2 and C3.

## 5. Conclusion

Research employing dynamic trade and spatial models has exploded in recent years. We provide closed-form gains from trade formulas that apply in a wide class of such models with different microfoundations. Our results emphasize the important role of measurement. We show that the short-run tariff elasticity is a crucial object even in evaluating the long-run steady state gains. In our

quantification, the gains from trade are large, because the short-run elasticity is typically estimated to be small. Finally, we show that accounting for the transition path has a modest effect on the magnitude of the gains, even if countries can borrow and lend. Whether the steady-state formula over- or under-states the transition path gains depends on whether the transition is from autarky to trade or in the opposite direction.

Our main quantitative result, that the gains from trade in dynamic trade models are large, is derived in models without extensions such as intermediate inputs, or multiple sectors. Costinot and Rodríguez-Clare (2014) quantify several of these extensions to the static ACR framework, and show they increase static gains from trade by as much as an order of magnitude. Incorporating such extensions would further amplify the already large GFT in dynamic settings and thus highlight the dramatic welfare impacts of changes in trade barriers.

# References


Adão, Rodrigo, Costas Arkolakis, and Sharat Ganapati. 2020. "Aggregate Implications of Firm Heterogeneity: A Nonparametric Analysis of Monopolistic Competition Trade Models." Mimeo, Chicago Booth, Yale, and Georgetown.

Adão, Rodrigo, Arnaud Costinot, and Dave Donaldson. 2017. "Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade." *American Economic Review* 107 (3):633–89.

Alessandria, George, Costas Arkolakis, and Kim J. Ruhl. 2021. "Firm Dynamics and Trade." *Annual Review of Economics* 13 (1):253–280.

Alessandria, George and Horag Choi. 2014. "Establishment heterogeneity, exporter dynamics, and the effects of trade liberalization." *Journal of International Economics* 94 (2):207 – 223.

Alessandria, George, Horag Choi, and Kim Ruhl. 2021a. "Trade adjustment dynamics and the welfare gains from trade." *Journal of International Economics* 131:103458.

———. 2021b. "Trade adjustment dynamics and the welfare gains from trade." *Journal of International Economics* 131:103458.

Alvarez, Fernando. 2017. "Capital accumulation and international trade." *Journal of Monetary Economics* 91:1 – 18.

Anderson, James E, Mario Larch, and Yoto V Yotov. 2020. "Transitional Growth and Trade with Frictions: A Structural Estimation Framework." *The Economic Journal* 130 (630):1583–1607.

Arkolakis, Costas. 2010. "Market Penetration Costs and the New Consumers Margin in International Trade." *Journal of Political Economy* 118 (6):1151–1199.

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson, and Andrés Rodríguez-Clare. 2019. "The Elusive Pro-Competitive Effects of Trade." *Review of Economic Studies* 86 (1):46–80.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. "New Trade Models, Same Old Gains?" *American Economic Review* 102 (1):94–130.

Arkolakis, Costas, Svetlana Demidova, Peter J. Klenow, and Andres Rodriguez-Clare. 2008. "Endogenous Variety and the Gains from Trade." *American Economic Review* 98 (2):444–50.

Arkolakis, Costas, Jonathan Eaton, and Samuel S. Kortum. 2011. "Staggered Adjustment and Trade Dynamics." Mimeo, Yale and Penn State.

Atkeson, Andrew and Ariel Burstein. 2010. "Innovation, Firm Dynamics, and International Trade." *Journal of Political Economy* 118 (3):433–484.

Auer, Raphael, Ariel Burstein, and Sarah M. Lein. 2021. "Exchange Rates and Prices: Evidence from the 2015 Swiss Franc Appreciation." *American Economic Review* 111 (2):652–86.

Axtell, Robert L. 2001. "Zipf Distribution of U.S. Firm Sizes." *Science* 293 (5536):1818–1820.

Bai, Yan, Keyu Jin, and Dan Lu. 2024. "Misallocation under Trade Liberalization." *American Economic Review* 114 (7):1949–85.

Bardhan, P. K. 1965. "Equilibrium Growth in the International Economy." *Quarterly Journal of Economics* 79 (3):455–464.

———. 1966. "On Factor Accumulation and the Pattern of International Specialisation." *Review of Economic Studies* 33 (1):39–44.

Blaum, Joaquin. 2024. "Global Firms in Large Devaluations." *The Quarterly Journal of Economics* 139 (4):2427–2474.

Boehm, Christoph, Andrei A. Levchenko, and Nitya Pandalai-Nayar. 2023. "The Long and Short (Run) of Trade Elasticities." *American Economic Review* 113 (4):861–905.

Brooks, Wyatt J. and Pau S. Pujolas. 2018. "Capital accumulation and the welfare gains from trade." *Economic Theory* 66 (2):491–523.

Burstein, Ariel and Marc Melitz. 2013. "Trade Liberalization and Firm Dynamics." In *Advances in Economics and Econometrics Tenth World Congress. Applied Economics*, vol. 2. Cambridge, UK: Cambridge University Press.

Chen, Junyuan, Carlos Góes, Marc-Andreas Muendler, and Fabian Trottner. 2024. "Dynamic Adjustment to Trade Shocks." Mimeo, UC San Diego.

Costantini, James and Marc Melitz. 2007. "The Dynamics of Firm-Level Adjustment to Trade Liberalization." In *The Organization of Firms in a Global Economy*, edited by E. Helpman, D. Marin, and T. Verdier. Cambridge, MA: Harvard University Press.

Costinot, Arnaud and Andrés Rodríguez-Clare. 2014. "Chapter 4 - Trade Theory with Numbers: Quantifying the Consequences of Globalization." In *Handbook of International Economics*, *Handbook of International Economics*, vol. 4, edited by Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff. Elsevier, 197–261.

Costinot, Arnaud and Andrés Rodríguez-Clare. 2014. "Trade Theory with Numbers: Quantifying the Consequences of Globalization." *Handbook of International Economics* 4:197–261.

di Giovanni, Julian and Andrei A. Levchenko. 2013. "Firm Entry, Trade, and Welfare in Zipf's World." *Journal of International Economics* 89 (2):283–296.

di Giovanni, Julian, Andrei A. Levchenko, and Romain Rancière. 2011. "Power Laws in Firm Size and Openness to Trade: Measurement and Implications." *Journal of International Economics* 85 (1):42–52.

Drozd, Lukasz A. and Jaromir B. Nosal. 2012. "Understanding International Prices: Customers as Capital." *American Economic Review* 102 (1):364–95.

Eaton, Jonathan and Samuel S. Kortum. 2002. "Technology, Geography, and Trade." *Econometrica* 70 (5):1741–1779.

———. 2005. "Technology in the Global Economy: A Framework for Quantitative Analysis." Book manuscript.

Feenstra, Robert C. 2018. "Restoring the product variety and pro-competitive gains from trade with heterogeneous firms and bounded productivity." *Journal of International Economics* 110:16–27.

Feenstra, Robert C. and David E. Weinstein. 2017. "Globalization, Markups, and US Welfare." *Journal of Political Economy* 125 (4):1040–1074.

Felbermayr, Gabriel, Benjamin Jung, and Mario Larch. 2015. "The welfare consequences of import tariffs: A quantitative perspective." *Journal of International Economics* 97 (2):295–309.

Fitzgerald, Doireann. 2024. "3-D Gains from Trade." Mimeo, Minneapolis Fed.

Fitzgerald, Doireann and Stefanie Haller. 2018. "Exporters and shocks." *Journal of International Economics* 113:154–171.

Fitzgerald, Doireann, Stefanie Haller, and Yaniv Yedid-Levi. 2024. "How Exporters Grow." *Review of Economic Studies* 91 (4):2276–2306.

Galle, Simon, Andrés Rodríguez-Clare, and Moises Yi. 2023. "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade." *Review of Economic Studies* 90 (1):331–375.

Gourinchas, Pierre-Olivier and Olivier Jeanne. 2006. "The Elusive Gains from International Financial Integration." *The Review of Economic Studies* 73 (3):715–741.

Gourio, François and Leena Rudanko. 2014. "Customer Capital." *Review of Economic Studies* 81 (2):1102–1136.

Head, Keith and Thierry Mayer. 2014. "Gravity Equations: Workhorse, Toolkit, and Cookbook." In *Handbook of International Economics*, vol. 4, edited by Elhanan Helpman, Kenneth Rogoff, and Gita Gopinath. Elsevier B.V., 131–195.

Heid, Benedikt and Frank Stähler. 2024. "Structural gravity and the gains from trade under imperfect competition: Quantifying the effects of the European Single Market." *Economic Modelling* 131:106604.

Hummels, David. 2001. "Toward a Geography of Trade Costs." Gtap working papers, Center for Global Trade Analysis, Department of Agricultural Economics, Purdue University.

Imbs, Jean and Isabelle Mejean. 2017. "Trade Elasticities." *Review of International Economics* 25 (2):383–402.

Inada, Ken-ichi. 1968. "Free Trade, Capital Accumulation and Factor Price Equalization." *Economic Record* 44 (3):322–341.

International Monetary Fund. 2025. "General government gross debt database." https://www.imf.org/external/datamapper/GGXWDG_NGDP@WEO/OEMDC/ADVEC/WEOWORLD, accessed 19 May 2025.

Kleinman, Benny, Ernest Liu, and Stephen J. Redding. 2023. "Dynamic Spatial General Equilibrium." *Econometrica* 91 (2):385–424.

Krugman, Paul. 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade." *American Economic Review* 70 (5):950–59.

Lashkaripour, Ahmad. 2021. "The cost of a global tariff war: A sufficient statistics approach." *Journal of International Economics* 131:103419.

Leibovici, Fernando and Michael E. Waugh. 2019. "International trade and intertemporal substitution." *Journal of International Economics* 117:158–174.

Levchenko, Andrei A. and Jing Zhang. 2014. "Ricardian productivity differences and the gains from trade." *European Economic Review* 65:45–65.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71 (6):1695–1725.

Melitz, Marc J. and Stephen J. Redding. 2015. "New Trade Models, New Welfare Implications." *American Economic Review* 105 (3):1105–46.

Mutreja, Piyusha, B. Ravikumar, and Michael Sposi. 2018. "Capital goods trade, relative prices, and economic development." *Review of Economic Dynamics* 27:101–122.

Oniki, H. and H. Uzawa. 1965. "Patterns of Trade and Investment in a Dynamic Model of International Trade." *Review of Economic Studies* 32 (1):15–38.

Ossa, Ralph. 2015. "Why trade matters after all." *Journal of International Economics* 97 (2):266–277.

Ramondo, Natalia and Andrés Rodríguez-Clare. 2013. "Trade, Multinational Production, and the Gains from Openness." *Journal of Political Economy* 121 (2):273–322.

Ravikumar, B., Ana Maria Santacreu, and Michael Sposi. 2019. "Capital accumulation and dynamic gains from trade." *Journal of International Economics* 119:93–110.

———. 2024. "Trade liberalization versus protectionism: Dynamic welfare asymmetries." *European Economic Review* 163:104692.

Ruhl, Kim J. 2008. "The International Elasticity Puzzle." Working Paper.

Ruhl, Kim J. and Jonathan L. Willis. 2017. "New Exporter Dynamics." *International Economic Review* 58 (3):703–726.

Schmitt-Grohé, Stephanie and Martín Uribe. 2003. "Closing small open economy models." *Journal of International Economics* 61 (1):163–185.

Shapiro, Joseph S. 2016. "Trade Costs, $CO^2$, and the Environment." *American Economic Journal: Economic Policy* 8 (4):220–54.

Steinberg, Joseph B. 2022. "Export Market Penetration Dynamics." *Journal of International Economics* 145:103807.

Stiglitz, Joseph E. 1970. "Factor Price Equalization in a Dynamic Economy." *Journal of Political Economy* 78 (3):456–488.

Waugh, Michael E. 2023. "Heterogeneous Agent Trade." NBER Working Papers 31810, National Bureau of Economic Research, Inc.

# A. Theory Appendix

## A.1 Proofs

**Proof of Proposition 3.1.** From A.1 and A.2, real consumption is proportional to the real wage:

$$C_j \propto \frac{w_j}{P_j}. \tag{A.1}$$

From A.3, the price index

$$P_j \propto w_j \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{-\frac{1}{\varepsilon_\kappa^0}}. \tag{A.2}$$

From A.3, the mass of firms

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}, \tag{A.3}$$

where we also used A.1. Putting (A.1)-(A.3) together yields the first result.

To derive the last claim, note that:

$$\frac{d \ln X_{ij}}{d \ln \kappa_{ij}} = \frac{d \ln n_{ij}}{d \ln x_{ij}} \frac{d \ln x_{ij}}{d \ln \kappa_{ij}} + \frac{d \ln x_{ij}}{d \ln \kappa_{ij}}.$$

It is immediate from A.3 that $\frac{d \ln x_{ij}}{d \ln \kappa_{ij}} = -\varepsilon_\kappa^0$, and $\frac{d \ln n_{ij}}{d \ln x_{ij}} = \chi$, which gives the result.

**Proof of Proposition 3.2.** From A.1′,

$$Y_j = \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \left(w_j L_j + \Pi_j\right) \tag{A.4}$$

From A.2′,

$$C_j \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \frac{w_j}{P_j} \tag{A.5}$$

From A.3′,

$$\frac{w_j}{P_j} = \lambda_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{\frac{1}{\varepsilon_\kappa^0}} \tag{A.6}$$

Also from A.3′,

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}} \left(1 - \frac{R_j^g}{Y_j}\right)^{-\frac{\chi}{1+\chi}}, \tag{A.7}$$

Putting (A.5)-(A.7) together yields the first result. This last step also uses the fact that (A.4) and A.2′ imply that $\frac{Y_j}{w_j} \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1}$. The proof of the claim about the trade elasticity is identical to Proposition 3.1.

**Proof of Proposition 3.3.** Begin with combining $n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^\chi$ with $X_{ij} = n_{ij} x_{ij}$ both from A.3′ of Proposition 3.2 and the definition of expenditure shares $\lambda_{ij} = \frac{\tau_{ij} X_{ij}}{Y_j}$ to obtain

$$n_{ij} \propto \left(\frac{\frac{1}{\tau_{ij}} \lambda_{ij} Y_j}{w_i}\right)^{\frac{\chi}{1+\chi}}$$

for all $i$ and $j$. Plugging this expression into the share (3.23) gives

$$\lambda_{ij} = \frac{\left(\lambda_{ij}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{ij}\right)^{\varepsilon_\kappa^0} \left(\tau_{ij} w_i\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}}{\sum_k \left(\lambda_{kj}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{kj}\right)^{\varepsilon_\kappa^0} \left(\tau_{kj} w_k\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}},$$

and solving for $\lambda_{ij}$ yields

$$\lambda_{ij} = \frac{\left(\kappa_{ij}\right)^{\varepsilon_\kappa^0(1+\chi)} \left(\tau_{ij} w_i\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}}{\left(\sum_k \left(\lambda_{kj}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{kj}\right)^{\varepsilon_\kappa^0} \left(\tau_{kj} w_k\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}\right)^{(1+\chi)}}. \tag{A.8}$$

Now consider the expenditure share of $j$ from $\ell$, $\lambda_{\ell j}$, and sum over $\ell$ to get

$$\frac{\sum_\ell \left(\kappa_{\ell j}\right)^{\varepsilon_\kappa^0(1+\chi)} \left(\tau_{\ell j} w_\ell\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}}{\left(\sum_k \left(\lambda_{kj}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{kj}\right)^{\varepsilon_\kappa^0} \left(\tau_{kj} w_k\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}\right)^{(1+\chi)}} = \sum_\ell \lambda_{\ell j} = 1.$$

Using this relationship to substitute out the denominator in equation (A.8) gives

$$\lambda_{ij} = \frac{\left(\kappa_{ij}\right)^{\varepsilon_\kappa^0(1+\chi)} \left(\tau_{ij} w_i\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}}{\sum_\ell \left(\kappa_{\ell j}\right)^{\varepsilon_\kappa^0(1+\chi)} \left(\tau_{\ell j} w_\ell\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}}.$$

Since this relationship holds before and after the change in trade costs, we have, for $\hat{\lambda}_{ij} = \lambda'_{ij}/\lambda_{ij}$, that

$$\hat{\lambda}_{ij} = \frac{\left(\hat{\kappa}_{ij}\right)^{\varepsilon_\kappa^0(1+\chi)} \left(\hat{\tau}_{ij} \hat{w}_i\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}}{\sum_\ell \lambda_{\ell j} \left(\hat{\kappa}_{\ell j}\right)^{\varepsilon_\kappa^0(1+\chi)} \left(\hat{\tau}_{\ell j} \hat{w}_\ell\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}},$$

which is equation (3.24) in the proposition.

Next, note that tariff revenues can be written as

$$R_j^g = Y_j \left(1 - \sum_i \frac{1}{\tau_{ij}} \lambda_{ij}\right).$$

Plugging this into the budget constraint in A.1′ of Proposition 3.2 gives

$$Y_j = \frac{1}{\sum_i \frac{1}{\tau_{ij}} \lambda_{ij}} \left(w_j L_j + \Pi_j\right).$$

Now using A.2′ of Proposition 3.2 gives

$$Y_j \propto \frac{1}{\sum_k \frac{1}{\tau_{kj}} \lambda_{kj}} w_j L_j. \tag{A.9}$$

Next, combining trade balance from A.1′ of Proposition 3.2, $\sum_i X_{ij} = \sum_i X_{ji}$, with the definition of expenditure shares $\lambda_{ij} = \frac{\tau_{ij} X_{ij}}{Y_j}$, gives

$$Y_j \left(\sum_i \frac{1}{\tau_{ij}} \lambda_{ij}\right) = \sum_i \frac{1}{\tau_{ji}} \lambda_{ji} Y_i.$$

Combining this expression with (A.9) yields

$$w_j L_j = \sum_i \frac{\frac{1}{\tau_{ji}}\lambda_{ji}}{\sum_k \frac{1}{\tau_{ki}}\lambda_{ki}} w_i L_i.$$

Again, this relationship holds before and after the change in trade costs, so we have, for $\hat{w}_j = w_j'/w_j$, that

$$\hat{w}_j w_j L_j = \sum_i \frac{\frac{1}{\hat{\tau}_{ji}}\hat{\lambda}_{ji}\frac{1}{\tau_{ji}}\lambda_{ji}}{\sum_k \frac{1}{\hat{\tau}_{ki}}\hat{\lambda}_{ki}\frac{1}{\tau_{ki}}\lambda_{ki}} \hat{w}_i w_i L_i.$$

Now substituting for $\hat{\lambda}_{ji}$ and $\hat{\lambda}_{ki}$ using relationship (3.24) gives

$$\hat{w}_j = \sum_i \frac{\frac{1}{\tau_{ji}}\lambda_{ji}\left(\hat{\kappa}_{ji}\right)^{\varepsilon_\kappa^0(1+\chi)}\left(\hat{\tau}_{ji}\right)^{\left(\varepsilon_\kappa^0-1\right)(1+\chi)}\left(\hat{w}_j\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}\hat{w}_i\frac{w_i L_i}{w_j L_j}}{\sum_k \frac{1}{\tau_{ki}}\lambda_{ki}\left(\hat{\kappa}_{ki}\right)^{\varepsilon_\kappa^0(1+\chi)}\left(\hat{\tau}_{ki}\right)^{\left(\varepsilon_\kappa^0-\right)1(1+\chi)}\left(\hat{w}_k\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}}.$$

This is equation (3.25) in the proposition.

## A.2 Appendix Propositions

**Proposition A.1.** *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

*A.1′ For all countries $j$, trade is balanced (expenditure = revenue):*

$$P_j C_j = w_j L_j + \Pi_j + R_j^g$$

*where*

$$R_j^g = \sum_i \left(\tau_{ij} - 1\right) X_{ij}$$

*and trade balance holds $\sum_i X_{ij} = \sum_i X_{ji}$.*

*A.2′ For all countries $j$, profits are a constant share of labor income:*

$$\frac{\Pi_j}{w_j L_j} = const$$

*A.3″ For all country pairs $(i, j)$ trade flows satisfy*

$$X_{ij} = n_{ij} x_{ij}$$

*where*

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi}$$

*and domestic per-unit-mass sales satisfy*

$$x_{jj} \propto \left(\frac{Y_j}{w_j}\right)^{\varepsilon^1} Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{A.10}$$

*for some constants $\varepsilon^1 > 0$ and $\chi > 0$, and where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij}/\partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.*

*Then*

$$C_j \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-\left(1-\frac{1}{\varepsilon_\kappa^0}\frac{\chi}{1+\chi}-\frac{\varepsilon^1}{\varepsilon_\kappa^0}\right)} \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}\frac{1}{1+\chi}} \tag{A.11}$$

*where* $\lambda_{jj} = \frac{X_{jj}}{Y_j}$*, and* $\varepsilon_\kappa^0(1+\chi)$ *is the long-run elasticity of trade flows with respect to iceberg trade costs.*

*Proof.* Derivations of (A.4) and (A.5) are identical to the steps in the proof of Proposition 3.2. From A.3″,

$$\frac{w_j}{P_j} = \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} \left(\frac{Y_j}{w_j}\right)^{-\frac{\varepsilon^1}{\varepsilon_\kappa^0}}.$$

Also from A.3″,

$$n_{jj} \propto \left(\lambda_{jj}\frac{Y_j}{w_j}\right)^{\frac{\chi}{1+\chi}}.$$

Thus,

$$\frac{w_j}{P_j} \propto \left(\lambda_{jj}^{\frac{1}{1+\chi}} \left(\frac{Y_j}{w_j}\right)^{-\varepsilon^1-\frac{\chi}{1+\chi}}\right)^{\frac{1}{\varepsilon_\kappa^0}}. \tag{A.12}$$

Putting (A.5) and (A.12) together yields the first result. This last step also uses the fact that (A.4) and A.2′ imply that $\frac{Y_j}{w_j} \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1}$. The proof of the claim about the trade elasticity is identical to Proposition 3.1.

□

**Discussion.** The conditions required for Proposition A.1 are identical to the conditions in Proposition 3.2 in every way except the per-firm sales (A.10). This functional form for sales is a strict generalization of (3.21), that allows per-firm sales to depend non-linearly on home market size and bilateral tariffs (recall from (A.4), $Y_j$ is a function of total tariff revenue). The resulting gains from trade formula (A.11) differs from (3.22) by $\left(1 - \frac{R_j^g}{Y_j}\right)^{\frac{\varepsilon^1}{\varepsilon_\kappa^0}}$. Note that the alternative formulation for per-firm sales only affects the tariff adjustment component of the GFT formula. The non-tariff component is unchanged, and $\lambda_{jj}$ is still raised to the power of the trade elasticity.

Proposition A.1 covers the Melitz (2003) model with tariffs. In that case, firm $\omega$'s sales are given by

$$x_{ijt}(\omega) = \frac{1}{\tau_{ijt}} C_{jt} \left(P_{jt}\right)^{\sigma} \left(\frac{\sigma}{\sigma-1}\frac{\tau_{ijt}\kappa_{ijt}}{\varphi(\omega)} w_{it}\right)^{1-\sigma}, \tag{A.13}$$

and the cutoff firm has productivity

$$\varphi_{ijt}^m = \frac{\sigma}{\sigma-1}\tau_{ijt}\kappa_{ijt}w_{it}\left(\frac{\sigma\tau_{ijt}w_{it}\xi_{ij}}{C_{jt}\left(P_{jt}\right)^{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{A.14}$$

Combining these, the average firm sales are:

$$x_{ijt} \propto \left(\frac{1}{\tau_{ijt}}\frac{Y_{jt}}{w_{it}}\right)^{\frac{\theta}{\sigma-1}-1} \frac{1}{\tau_{ijt}} Y_{jt} \left(\frac{\tau_{ijt}\kappa_{ijt}w_{it}}{P_{jt}}\right)^{-\theta}. \tag{A.15}$$

Intuitively, tariffs and market size in the Melitz model affect the extensive margin, and thus appear non-linearly in the average firm sales. This property of the Melitz model with tariffs was pointed out by Felbermayr, Jung, and Larch (2015). It is easy to verify that the Melitz model with tariffs satisfies all the conditions for Proposition

A.1 to hold. As equation (A.15) makes clear, the Melitz model satisfies A.3″ for $\varepsilon^1 = \frac{\theta}{\sigma-1} - 1$.

What is notable about this functional form for $\varepsilon^1$ is that it goes to zero as $\frac{\theta}{\sigma-1} \to 1$. Di Giovanni, Levchenko, and Rancière (2011) and di Giovanni and Levchenko (2013) show that the distribution of sales to any destination in the Melitz-Pareto model follows a power law with exponent $-\frac{\theta}{\sigma-1}$. Further, these papers document that in the data, firm sales follow a power law with exponent close to $-1$, known as Zipf's Law (see also Axtell, 2001). This implies that when calibrated to the observed firm size distribution, $\frac{\theta}{\sigma-1} \approx 1$ and therefore $\varepsilon^1 \approx 0$. Intuitively, $\varepsilon^1$ appears because tariffs affect the extensive margin of exports conditional on drawing the sunk cost. As the firm size distribution approaches Zipf's Law, the extensive margin plays no role in the aggregate outcomes (see di Giovanni and Levchenko, 2013, for a detailed treatment of this result).

# B. Quantitative Appendix

## B.1 Equilibrium and Model Equations

For given sequences of trade costs $\{\kappa_{ijt}\}_{t=1}^{\infty}$ and tariffs $\{\tau_{ijt}\}_{t=1}^{\infty}$, as well as initial conditions $n_{ij0}$ and $\frac{B^*_{j0}}{P_{US0}}$, an equilibrium of this economy are sequences of individual trade flows $\left\{\frac{x_{ijt}}{P_{it}}\right\}_{t=1}^{\infty}$ ($J^2$ variables), aggregate trade flows $\left\{\frac{X_{ijt}}{P_{it}}\right\}_{t=1}^{\infty}$ ($J^2$ variables), masses of firms $\{n_{ijt}\}_{t=1}^{\infty}$ ($J^2$ variables), firm values $\left\{\frac{v_{ijt}}{P_{it}}\right\}_{t=1}^{\infty}$ ($J^2$ variables), real wages $\left\{\frac{w_{it}}{P_{it}}\right\}_{t=1}^{\infty}$ ($J$ variables), consumption $\{C_{it}\}_{t=1}^{\infty}$ ($J$ variables), real exchange rates $\left\{\frac{P_{it}}{P_{USt}}\right\}_{t=1}^{\infty}$ ($J-1$ variables), bond holdings $\left\{\frac{B^*_{jt}}{P_{USt}}\right\}_{t=1}^{\infty}$ ($J$ variables), and the interest rate $\left\{1+r^*_t\right\}_{t=1}^{\infty}$ (1 variable) such that the following equations hold:

- Current account ($J-1$ equations, one is redundant by Walras' law):

$$\frac{B^*_{jt}}{P_{USt}} - \left(1+r^*_{t-1}\right)\frac{B^*_{jt-1}}{P_{USt-1}} = \frac{P_{jt}}{P_{USt}}\sum_{i=1}^{J}\frac{X_{jit}}{P_{jt}} - \sum_{i=1}^{J}\frac{P_{it}}{P_{USt}}\frac{X_{ijt}}{P_{it}}$$

- Euler equation of internationally-traded bond ($J$ equations):

$$C_{jt}^{-\gamma} = \frac{1+r^*_t}{1+\frac{w_{jt}}{P_{jt}}\psi'\left(\left(\frac{P_{jt}}{P_{USt}}\right)^{-1}\frac{B^*_{jt}}{P_{USt}}\right)}\frac{\frac{P_{jt}}{P_{USt}}}{\frac{P_{jt+1}}{P_{USt+1}}}\beta C_{jt+1}^{-\gamma}$$

- Price index ($J$ equations):

$$1 = \left(\sum_i n_{ijt-1}\left(\frac{\sigma}{\sigma-1}\tau_{ijt}\kappa_{ijt}\frac{\frac{P_{it}}{P_{USt}}}{\frac{P_{jt}}{P_{USt}}}\frac{w_{it}}{P_{it}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

- Mass of traded goods ($J^2$ equations):

$$n_{ijt} = (1-\delta)\,n_{ijt-1} + \left(b\frac{\frac{v_{ijt}}{P_{it}}}{\frac{w_{it}}{P_{it}}}\right)^{\chi}$$

- Value of firms ($J^2$ equations):

$$\frac{v_{ijt}}{P_{it}} = \beta\frac{C_{it+1}^{-\gamma}}{C_{it}^{-\gamma}}\left(\frac{1}{\sigma}\frac{x_{ijt+1}}{P_{it+1}} + (1-\delta)\frac{v_{ijt+1}}{P_{it+1}}\right)$$

- Labor market clearing ($J$ equations):

$$L_i = \frac{\sigma-1}{\sigma}\frac{1}{\frac{w_{it}}{P_{it}}}\sum_{j=1}^{J}n_{ijt-1}\frac{x_{ijt}}{P_{it}} + b^{\chi}\frac{\chi}{\chi+1}\sum_{j=1}^{J}\left(\frac{\frac{v_{ijt}}{P_{it}}}{\frac{w_{it}}{P_{it}}}\right)^{\chi+1} + \psi\left(\frac{1}{\frac{P_{it}}{P_{USt}}}\frac{B^*_{it}}{P_{USt}}\right)$$

- Aggregate trade flows ($J^2$ equations):

$$\frac{X_{ijt}}{P_{it}} = n_{ijt-1}\frac{x_{ijt}}{P_{it}}$$

- Individual trade flows ($J^2$ equations):

$$\frac{x_{ijt}}{P_{it}} = \frac{1}{\tau_{ijt}} C_{jt} \left( \frac{\frac{P_{jt}}{P_{USt}}}{\frac{P_{it}}{P_{USt}}} \right)^{\sigma} \left( \frac{\sigma}{\sigma - 1} \tau_{ijt} \kappa_{ijt} \frac{w_{it}}{P_{it}} \right)^{1-\sigma}$$

- Market clearing for international bond (1 equation):

$$\sum_{j=1}^{J} \frac{B_{jt}^{*}}{P_{US,t}} = 0$$

and that all households' transversality conditions are satisfied.

## B.2 Dynamic Transitions

This section details the procedure to compute the dynamic welfare gains presented in Figure 3 and Table C1, among others. We use the 30-country calibration listed in Table C1 and compute the five different scenarios listed in Table 2.

We first compute the steady states of the model under trade and under autarky. The steady state under trade matches the observed expenditure shares and tariffs for 2006. The steady state under autarky depends on the scenario and is computed as discussed in Section 4.4. In all scenarios, we consider an unexpected permanent shock to the non-tariff trade costs in period 1.

We use the Newton algorithm in order to compute the transition path of the model variables, where period 0 represents the initial steady state. Depending on the scenario, convergence to the terminal steady state requires computing the transition path for up to 4000 periods. All parameters other than a particular set of non-tariff iceberg trade costs $\kappa_{ij}$ (which depend on the scenario) remain constant throughout the simulations.

We base the gains from trade calculations over the transition path on consumption equivalent variation. Define the present value of consumption in period 1 $V_{j1}$ as

$$V_{j1} = \sum_{t=1}^{\infty} \beta^t \frac{\left(C_{jt}\right)^{1-\gamma}}{1-\gamma}.$$

**Autarky to trade.** Consider the transition path from autarky to trade. Let the superscript $T$ denote the transition path under trade and superscript $A$ denote the initial steady state under autarky. We then compute the present value of consumption under the transition path to trade as

$$V_{j1}^{T} = \sum_{t=1}^{\infty} \beta^t \frac{\left(C_{jt}^{T}\right)^{1-\gamma}}{1-\gamma}.$$

Now, assume a case where the household receives a constant consumption equivalent $C_j^{T,e}$ in every period, such that

$$V_{j1}^{T,e} = \sum_{t=1}^{\infty} \beta^t \frac{\left(C_{j}^{T,e}\right)^{1-\gamma}}{1-\gamma},$$

where the superscript $e$ denotes the consumption equivalent.

Setting $V_{j1}^{T} = V_{j1}^{T,e}$ gives

$$C_j^{T,e} = \left( (1-\beta) \sum_{t=1}^{\infty} \beta^t \left( C_{jt}^{T} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}},$$

which is our measure of welfare associated with the transition path to trade. The dynamic gains from trade for a scenario involving the transition from autarky to trade are defined as

$$DGFT_j^{A \to T} = \frac{C_j^{T,e}}{C_j^{A}}.$$

**Trade to autarky.** We also analyze the transition path from trade to autarky. In this case, the superscript $A$ denotes the transition path under autarky and superscript $T$ denotes the initial steady state under trade. We compute the present value of consumption under the autarky transition path as

$$V_{j1}^{A} = \sum_{t=1}^{\infty} \beta^t \frac{\left( C_{jt}^{A} \right)^{1-\gamma}}{1-\gamma}.$$

Following similar steps as above, the welfare measure associated with the transition to autarky is

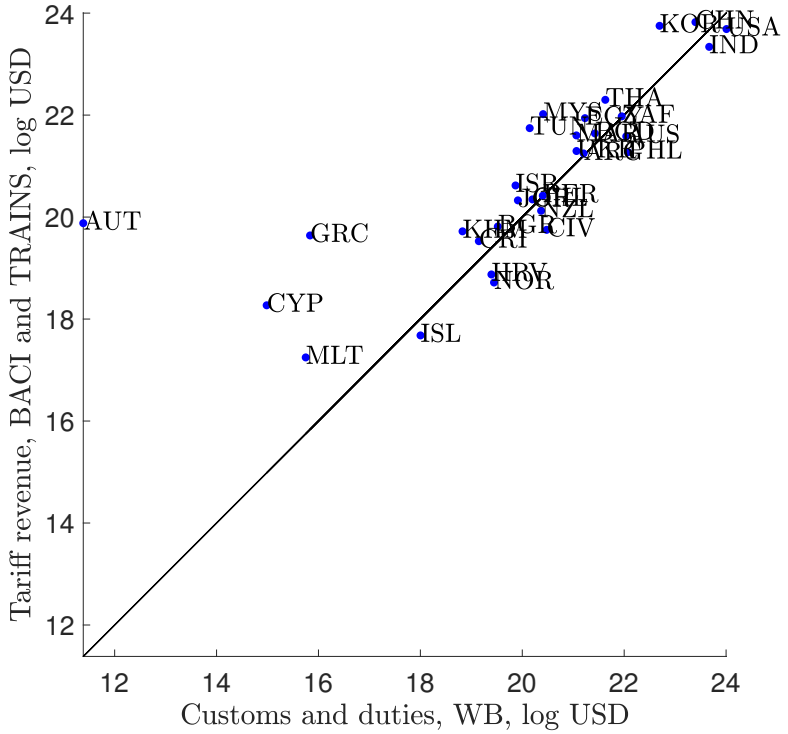
$$C_j^{A,e} = \left( (1-\beta) \sum_{t=1}^{\infty} \beta^t \left( C_{jt}^{A} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

The dynamic gains from trade are then

$$DGFT_j^{T \to A} = \frac{C_j^{T}}{C_j^{A,e}}.$$

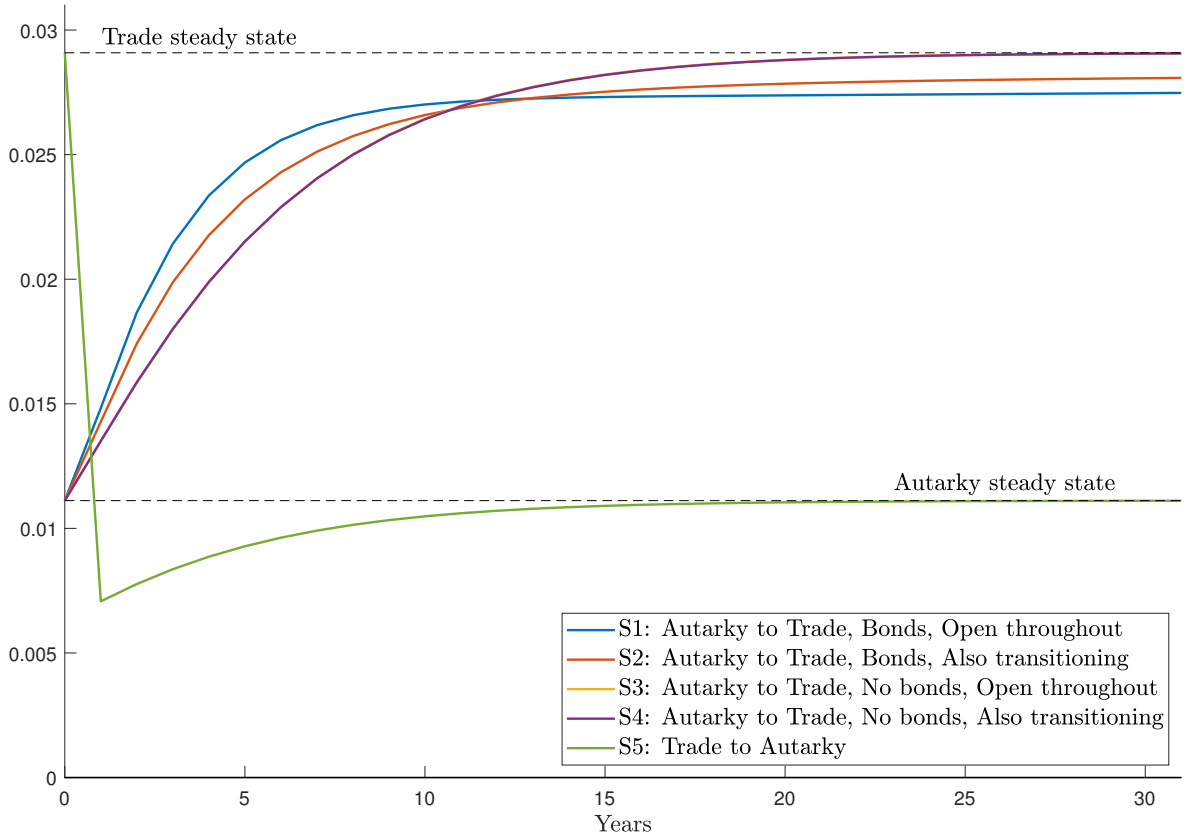
# C. Additional Results

Figure C1: Tariff Revenue Comparison



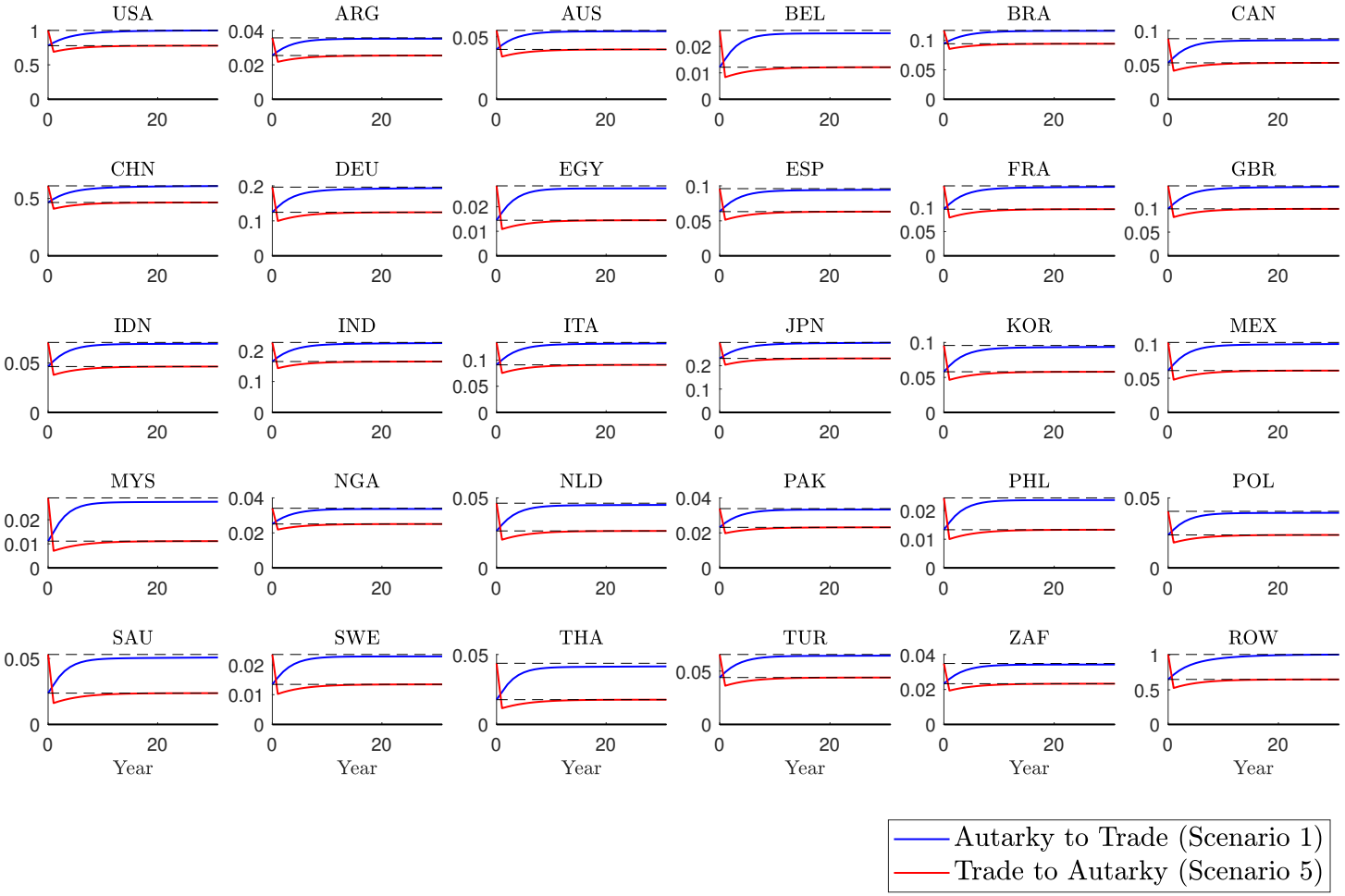

**Notes:** The figure compares the log of tariff revenues calculated from BACI-TRAINS with the log of customs and duties from the World Bank (WB) for the year 2006.

Figure C2: Consumption Paths in Different Scenarios: Malaysia



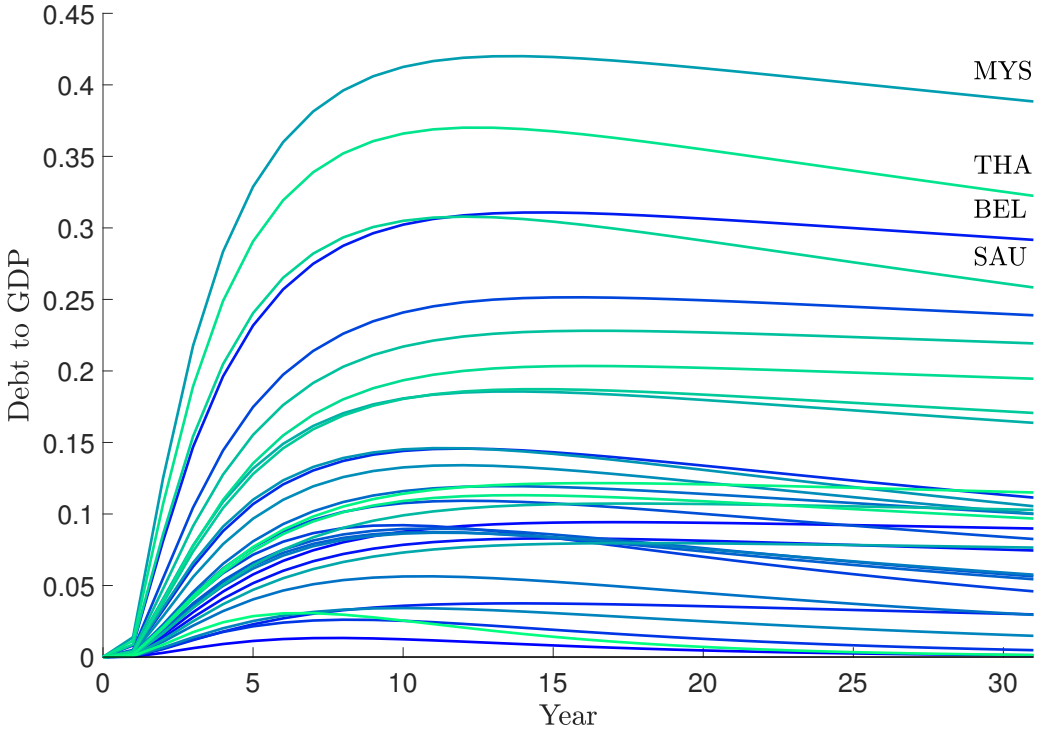

**Notes:** This figure shows the consumption paths following trade opening and closing under the different scenarios summarized in Table 2.

Figure C3: Consumption along the Transition Paths




**Notes:** The figure shows the transition paths of consumption for all countries included in our baseline calibration. Each plot shows two different scenarios, Scenario 1 and 5. In Scenario 1 only the country in the title of the subplot transitions. The shock is unanticipated and occurs in period 1. Dashed lines mark the steady state values under trade and autarky. Consumption is normalized to one for the USA in the trade steady state.

Figure C4: Transition Paths of Debt-to-GDP Ratios




**Notes:** The figure shows the debt-to-GDP ratios for all countries included in our baseline calibration along the transition from autarky to trade (Scenario 1). The shock is unanticipated and occurs in period 1.

Table C1: Net Dynamic Gains from Trade by Country

| Country | Steady state comp. | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
|---|---|---|---|---|---|---|
| MYS | 1.617 | 1.336 | 1.326 | 1.321 | 1.322 | 1.796 |
| THA | 1.469 | 1.220 | 1.212 | 1.209 | 1.209 | 1.626 |
| SAU | 1.236 | 1.035 | 1.030 | 1.028 | 1.028 | 1.362 |
| BEL | 1.149 | 0.967 | 0.961 | 0.960 | 0.960 | 1.268 |
| EGY | 0.966 | 0.819 | 0.815 | 0.814 | 0.814 | 1.047 |
| PHL | 0.835 | 0.712 | 0.709 | 0.709 | 0.709 | 0.910 |
| NLD | 0.750 | 0.642 | 0.640 | 0.640 | 0.640 | 0.819 |
| SWE | 0.747 | 0.639 | 0.637 | 0.637 | 0.637 | 0.815 |
| POL | 0.717 | 0.615 | 0.613 | 0.613 | 0.613 | 0.783 |
| MEX | 0.673 | 0.578 | 0.577 | 0.577 | 0.577 | 0.732 |
| CAN | 0.663 | 0.570 | 0.569 | 0.569 | 0.569 | 0.722 |
| KOR | 0.647 | 0.556 | 0.555 | 0.555 | 0.555 | 0.699 |
| DEU | 0.578 | 0.499 | 0.498 | 0.498 | 0.498 | 0.628 |
| ROW | 0.555 | 0.479 | 0.478 | 0.478 | 0.478 | 0.601 |
| IDN | 0.526 | 0.456 | 0.455 | 0.455 | 0.455 | 0.570 |
| ESP | 0.520 | 0.450 | 0.449 | 0.449 | 0.449 | 0.564 |
| FRA | 0.500 | 0.433 | 0.433 | 0.433 | 0.433 | 0.542 |
| ZAF | 0.497 | 0.431 | 0.430 | 0.430 | 0.430 | 0.536 |
| TUR | 0.491 | 0.426 | 0.425 | 0.425 | 0.425 | 0.532 |
| GBR | 0.490 | 0.424 | 0.424 | 0.424 | 0.424 | 0.530 |
| ITA | 0.472 | 0.409 | 0.409 | 0.409 | 0.409 | 0.511 |
| PAK | 0.463 | 0.402 | 0.401 | 0.401 | 0.401 | 0.495 |
| ARG | 0.404 | 0.352 | 0.352 | 0.352 | 0.352 | 0.435 |
| AUS | 0.382 | 0.333 | 0.333 | 0.333 | 0.332 | 0.411 |
| IND | 0.376 | 0.328 | 0.328 | 0.327 | 0.327 | 0.403 |
| NGA | 0.359 | 0.314 | 0.314 | 0.314 | 0.314 | 0.385 |
| CHN | 0.312 | 0.273 | 0.273 | 0.273 | 0.273 | 0.335 |
| JPN | 0.296 | 0.259 | 0.259 | 0.259 | 0.259 | 0.318 |
| USA | 0.285 | 0.250 | 0.250 | 0.250 | 0.250 | 0.307 |
| BRA | 0.240 | 0.212 | 0.212 | 0.211 | 0.211 | 0.257 |

**Notes:** The table presents the net dynamic GFT underlying Table 2. Steady state gains are based on equation (3.22). For details in the scenarios, see Section 4.4 and Appendix B.2. Countries are ordered in descending order of their GFT.

Table C2: Net Dynamic Gains from Trade by Country, Autarky to Trade, Robustness

| Country | Steady state comp. | $\delta = 0.35$ | $\delta = 0.15$ | $\sigma = 1.5$ | $\chi = 1$ | $\chi = 0.3$ | $\beta = 0.9$ | $\gamma = 3$ |
|---|---|---|---|---|---|---|---|---|
| MYS | 1.617 | 1.403 | 1.202 | 0.537 | 0.989 | 1.869 | 0.928 | 1.314 |
| THA | 1.469 | 1.280 | 1.100 | 0.498 | 0.908 | 1.692 | 0.855 | 1.201 |
| SAU | 1.236 | 1.084 | 0.937 | 0.432 | 0.776 | 1.422 | 0.735 | 1.021 |
| BEL | 1.149 | 1.011 | 0.877 | 0.405 | 0.721 | 1.330 | 0.690 | 0.954 |
| EGY | 0.966 | 0.855 | 0.746 | 0.360 | 0.640 | 1.077 | 0.591 | 0.810 |
| PHL | 0.835 | 0.743 | 0.651 | 0.313 | 0.546 | 0.949 | 0.519 | 0.706 |
| NLD | 0.750 | 0.669 | 0.587 | 0.283 | 0.489 | 0.859 | 0.471 | 0.636 |
| SWE | 0.747 | 0.666 | 0.585 | 0.281 | 0.485 | 0.857 | 0.469 | 0.634 |
| POL | 0.717 | 0.641 | 0.563 | 0.271 | 0.468 | 0.823 | 0.452 | 0.610 |
| MEX | 0.673 | 0.602 | 0.529 | 0.258 | 0.443 | 0.767 | 0.427 | 0.573 |
| CAN | 0.663 | 0.593 | 0.522 | 0.254 | 0.436 | 0.758 | 0.421 | 0.565 |
| KOR | 0.647 | 0.579 | 0.510 | 0.253 | 0.435 | 0.725 | 0.411 | 0.552 |
| DEU | 0.578 | 0.519 | 0.458 | 0.225 | 0.382 | 0.660 | 0.370 | 0.494 |
| ROW | 0.555 | 0.498 | 0.439 | 0.217 | 0.369 | 0.630 | 0.356 | 0.473 |
| IDN | 0.526 | 0.473 | 0.419 | 0.208 | 0.353 | 0.596 | 0.340 | 0.453 |
| ESP | 0.520 | 0.468 | 0.414 | 0.204 | 0.346 | 0.593 | 0.336 | 0.447 |
| FRA | 0.500 | 0.450 | 0.398 | 0.197 | 0.334 | 0.570 | 0.324 | 0.430 |
| ZAF | 0.497 | 0.447 | 0.396 | 0.199 | 0.338 | 0.558 | 0.322 | 0.428 |
| TUR | 0.491 | 0.443 | 0.392 | 0.195 | 0.329 | 0.558 | 0.319 | 0.423 |
| GBR | 0.490 | 0.441 | 0.390 | 0.194 | 0.327 | 0.557 | 0.318 | 0.421 |
| ITA | 0.472 | 0.425 | 0.377 | 0.187 | 0.316 | 0.537 | 0.307 | 0.406 |
| PAK | 0.463 | 0.417 | 0.370 | 0.190 | 0.324 | 0.509 | 0.302 | 0.400 |
| ARG | 0.404 | 0.365 | 0.325 | 0.165 | 0.277 | 0.453 | 0.266 | 0.350 |
| AUS | 0.382 | 0.345 | 0.307 | 0.155 | 0.259 | 0.432 | 0.252 | 0.331 |
| IND | 0.376 | 0.340 | 0.302 | 0.155 | 0.261 | 0.417 | 0.248 | 0.326 |
| NGA | 0.359 | 0.326 | 0.290 | 0.149 | 0.250 | 0.399 | 0.238 | 0.312 |
| CHN | 0.312 | 0.283 | 0.252 | 0.130 | 0.216 | 0.349 | 0.208 | 0.272 |
| JPN | 0.296 | 0.269 | 0.240 | 0.122 | 0.203 | 0.335 | 0.198 | 0.258 |
| USA | 0.285 | 0.259 | 0.232 | 0.118 | 0.196 | 0.323 | 0.191 | 0.249 |
| BRA | 0.240 | 0.219 | 0.196 | 0.102 | 0.169 | 0.268 | 0.162 | 0.211 |

**Notes:** The table presents the net dynamic GFT underlying Table 3. Steady state gains are based on equation (3.22). For details in the scenarios, see Section 4.4 and Appendix B.2. Countries are ordered in descending order of their GFT.

Table C3: Net Dynamic Gains from Trade, Trade to Autarky, Robustness

| Country | Steady state comp. | $\delta = 0.35$ | $\delta = 0.15$ | $\sigma = 1.5$ | $\chi = 1$ | $\chi = 0.3$ | $\beta = 0.9$ | $\gamma = 3$ |
|---|---|---|---|---|---|---|---|---|
| MYS | 1.617 | 1.750 | 1.894 | 0.677 | 1.388 | 2.359 | 2.129 | 1.815 |
| THA | 1.469 | 1.586 | 1.713 | 0.626 | 1.264 | 2.121 | 1.918 | 1.642 |
| SAU | 1.236 | 1.329 | 1.432 | 0.540 | 1.066 | 1.760 | 1.596 | 1.373 |
| BEL | 1.149 | 1.237 | 1.333 | 0.506 | 0.989 | 1.640 | 1.486 | 1.278 |
| EGY | 0.966 | 1.026 | 1.092 | 0.443 | 0.849 | 1.304 | 1.198 | 1.053 |
| PHL | 0.835 | 0.890 | 0.951 | 0.386 | 0.728 | 1.145 | 1.047 | 0.915 |
| NLD | 0.750 | 0.801 | 0.857 | 0.349 | 0.652 | 1.034 | 0.944 | 0.823 |
| SWE | 0.747 | 0.797 | 0.853 | 0.347 | 0.648 | 1.031 | 0.941 | 0.820 |
| POL | 0.717 | 0.766 | 0.819 | 0.335 | 0.623 | 0.988 | 0.902 | 0.787 |
| MEX | 0.673 | 0.717 | 0.765 | 0.317 | 0.587 | 0.918 | 0.840 | 0.736 |
| CAN | 0.663 | 0.707 | 0.754 | 0.313 | 0.577 | 0.906 | 0.829 | 0.725 |
| KOR | 0.647 | 0.686 | 0.728 | 0.309 | 0.569 | 0.863 | 0.795 | 0.702 |
| DEU | 0.578 | 0.615 | 0.656 | 0.276 | 0.504 | 0.784 | 0.719 | 0.630 |
| ROW | 0.555 | 0.589 | 0.627 | 0.267 | 0.485 | 0.747 | 0.686 | 0.603 |
| IDN | 0.526 | 0.558 | 0.593 | 0.254 | 0.461 | 0.705 | 0.648 | 0.571 |
| ESP | 0.520 | 0.552 | 0.588 | 0.251 | 0.454 | 0.701 | 0.644 | 0.566 |
| FRA | 0.500 | 0.531 | 0.565 | 0.242 | 0.437 | 0.674 | 0.619 | 0.544 |
| ZAF | 0.497 | 0.526 | 0.557 | 0.242 | 0.438 | 0.658 | 0.607 | 0.537 |
| TUR | 0.491 | 0.521 | 0.554 | 0.238 | 0.430 | 0.659 | 0.606 | 0.534 |
| GBR | 0.490 | 0.520 | 0.553 | 0.237 | 0.428 | 0.658 | 0.604 | 0.532 |
| ITA | 0.472 | 0.501 | 0.532 | 0.229 | 0.412 | 0.633 | 0.582 | 0.512 |
| PAK | 0.463 | 0.487 | 0.513 | 0.230 | 0.413 | 0.596 | 0.554 | 0.496 |
| ARG | 0.404 | 0.427 | 0.451 | 0.200 | 0.357 | 0.530 | 0.490 | 0.436 |
| AUS | 0.382 | 0.404 | 0.428 | 0.189 | 0.335 | 0.505 | 0.466 | 0.412 |
| IND | 0.376 | 0.395 | 0.417 | 0.188 | 0.333 | 0.487 | 0.451 | 0.403 |
| NGA | 0.359 | 0.378 | 0.399 | 0.180 | 0.319 | 0.465 | 0.432 | 0.386 |
| CHN | 0.312 | 0.329 | 0.347 | 0.157 | 0.276 | 0.406 | 0.376 | 0.335 |
| JPN | 0.296 | 0.313 | 0.331 | 0.149 | 0.260 | 0.389 | 0.359 | 0.319 |
| USA | 0.285 | 0.301 | 0.319 | 0.144 | 0.251 | 0.375 | 0.346 | 0.308 |
| BRA | 0.240 | 0.253 | 0.267 | 0.123 | 0.213 | 0.310 | 0.288 | 0.258 |

**Notes:** The table presents the net dynamic GFT underlying Table 3. Steady state gains are based on equation (3.22). For details in the scenarios, see Section 4.4 and Appendix B.2. Countries are ordered in descending order of their GFT.