Optimal Dynamic Spatial Policy*

Eric Donald† *University of Pittsburgh*

Masao Fukui‡ *Boston University*

Yuhei Miyauchi§ *Boston University*

March 21, 2025

Preliminary and Incomplete

Abstract

We study optimal transfer policy in dynamic spatial equilibrium models with frictional migration and incomplete financial markets. A key policy trade-off is to provide consumption insurance while minimizing the distortion of migration flows. We derive a recursive formula for optimal spatial transfers that strikes this balance. We calibrate our model to U.S. states and find that the U.S. economy would benefit from increased transfers to low-income-growth states. Welfare gains from optimal transfers are substantial but smaller than in a framework abstracting from slow migration adjustment.

*We thank Costas Arkolakis, Lorenzo Caliendo, Jonathan Dingel, David Lagakos, Michael Peters, and Pascual Restrepo as well as seminar participants at Barcelona Summer Forum, Green Line Macro Meeting, University of Pittsburgh, and Yale for their helpful comments.

†University of Pittsburgh (e-mail: eric.donald@pitt.edu).

‡Boston University (e-mail: mfukui@bu.edu).

§Boston University (e-mail: miyauchi@bu.edu).

1 Introduction

In recent decades, there has been a notable divergence in the trajectories of economic activity among different regions in the United States (Moretti 2012). Regions specialized in the manufacturing sector have been particularly affected by the increasing import competition (Autor, Dorn, and Hanson 2013, Caliendo, Dvorkin, and Parro 2019) and the rise of automation (Acemoglu and Restrepo 2020). Looking ahead, the evolving natural resource landscape, influenced by anticipated climate change, is expected to further reshape regional economic activity (Cruz and Rossi-Hansberg 2023, Hanson 2023, Arkolakis and Walsh 2023).

The divergent trajectories of regional economic activity have sparked discussions among policymakers and researchers regarding place-based policies. These discussions revolve around the need to support declining regions or those facing adverse conditions. Critics argue that such policies inadvertently incentivize individuals to stay in, or relocate to, unproductive regions (Glaeser and Gottlieb 2008, Kline and Moretti 2014b, Fajgelbaum and Gaubert 2020). According to this perspective, place-based policies are only justified if they address market failures by internalizing productivity spillovers or other local externalities. The disincentive effect is potentially more problematic if agents make forward-looking migration decisions anticipating future policies. On the other hand, proponents of place-based policies highlight the slow and frictional nature of people's migration responses. They argue that these policies can effectively provide insurance against adverse regional shocks without distorting migration decisions (Bartik 2020, Gaubert, Kline, and Yagan 2021). Despite the contentious debate, there is currently limited theoretical and empirical understanding of how place-based policies should strike a balance between these two perspectives, particularly in the context of dynamically evolving regional economic activity.

This paper addresses this ongoing debate by examining optimal place-based transfer policy using dynamic quantitative spatial equilibrium models that account for frictional migration and incomplete financial markets. Our main theoretical result is a recursive formula for the optimal spatial transfers that balances the benefit of providing consumption insurance while minimizing distortions from migration responses. We use this theory to assess the optimality of the observed patterns of spatial transfers across U.S. states since 1980.

We begin by establishing a model environment that captures the dynamic evolution of spatial economic activity under forward-looking migration decisions. Locations may differ in their fundamental components, including productivity and amenities, as well as bilateral trade and migration frictions. These fundamental components can arbitrarily evolve over time, capturing the heterogeneous trajectories of regional economic activity. These fundamental factors, along with localized production and amenity externalities, shape the dynamics of wages and amenities in equilibrium. We model forward-looking migration decisions as a dynamic discrete choice

problem following the approach of Artuç, Chaudhuri, and McLaren (2010), Kennan and Walker (2011), and Caliendo et al. (2019). However, unlike these previous papers, which assume a specific functional form of idiosyncratic preference shocks, our framework allows for flexible functional forms and arbitrary correlation across potential destinations.

Using this framework, we study the problem of optimal spatial transfers across locations and over time. The key restriction of the policy is that the Planner cannot directly regulate migration flows (i.e., directly specify the location decisions based on the individual idiosyncratic preference shocks). Therefore, the optimal transfer must overcome the dynamic moral hazard problem associated with agents' migration decision. Given this restriction, the optimal transfers balance two forces. On the one hand, due to incomplete financial markets, the Planner aims to equalize the dispersion in marginal utility through transfers. On the other hand, the Planner seeks to use transfers to incentivize migration towards areas that improve aggregate efficiency.

Our main theoretical result is a recursive expression of the optimal spatial transfers that strikes this balance. Our formula equates the marginal benefit of providing consumption insurance and smoothing, which depends on the marginal utility from a transfer in each location and time, with the marginal efficiency loss, which depends on a fiscal externality arising from distorting migration decisions. Importantly, the fiscal externality depends not only on contemporaneous economic conditions and externalities, but also on future fiscal externalities in a recursive manner. This implies that transfer policies are influenced not only by current economic activity but also by its dynamic evolution. For example, in response to persistent negative shocks in a region, the Planner gradually decreases transfers to provide dynamic incentives for relocation.

To provide further intuition, we consider a special case with a complete financial market. In this case, marginal utility is equalized across agents in different locations at every period. Therefore, the only market failures the Planner aims to address are production and amenity externalities. In this scenario, a simple period-by-period Pigouvian externality correction is sufficient to achieve efficiency. Notably, since future externalities are fully addressed by future transfer policies, contemporaneous policies do not need to directly respond to the future evolution of economic activity and externalities. Therefore, despite considering a dynamic environment, our optimal spatial transfer formula aligns with those of the static spatial equilibrium environment (Fajgelbaum and Gaubert 2020).

We show that our results can be generalized in many different directions. For one thing, we consider the case with a dynamic form of agglomeration externality, i.e., local population affects not only contemporaneous but also future productivity, as well as cross-region agglomeration spillovers (e.g., Desmet, Nagy, and Rossi-Hansberg 2018, Allen and Donaldson 2020, Peters 2022, Cai, Caliendo, Parro, and Xiang 2022). Our expression remains to hold by simply incorporating these forces into the recursive expression for the fiscal externality. For another, we also show

that our framework can be extended to the case with heterogeneous types of households with respect to preference, productivity, and migration cost (e.g., Fajgelbaum and Gaubert 2020, Rossi-Hansberg, Sarte, and Schwartzman 2019).

Equip with this theoretical framework, we calibrate our model to the evolution of economic activity across U.S. states to assess the optimality of the observed patterns of spatial transfers. In particular, we calibrate our model to exactly replicate the evolution of pre- and post-tax-and-transfer income (hereafter post-tax income in short), consumption expenditure, and population movement across the U.S. states since 1980. Using the calibrated model, we resolve the model by replacing these spatial transfers with the optimal ones prescribed by our framework.

In the data, we observe that the net public transfer, defined as the difference between post- and pre-tax income, is positive for low-income states and negative for high-income states, consistent with the previous research emphasizing the role of spatial transfers for redistribution (e.g., Gaubert, Kline, Vergara, and Yagan 2021). We also find that these public transfers flow from high-income-*growth* states to low-income-*growth* states. These results are consistent with the observation that public transfers play an important role in mitigating regional income shocks as documented by Asdrubali, Sørensen, and Yosha (1996). Interestingly, we also find that private transfers, defined by consumption expenditure minus post-tax income, tend to flow from high-income states to low-income states, and particularly from high-income-*growth* states to low-income-*growth* states. These results are consistent with the observation that there are pre-existing mechanisms aside from public transfers that mitigate regional income shocks as documented by Asdrubali et al. (1996). Our calibration takes into account the presence of these non-public regional risk-sharing mechanisms.

Our calibrated model reveals substantial and systematic differences between the optimal spatial transfers prescribed by our theoretical framework and the observed ones. In particular, in our baseline specification, the slope between the log consumption expenditure and log pre-tax income should be 0.52 under optimal spatial transfers in 2000, instead of 0.63 as observed in the data. These results indicate that optimal transfers entail more redistribution toward low-income states. Moreover, the slope between the growth rate of consumption expenditure and the growth rate of pre-tax income should be 0.07 from 1980 to 2000 under optimal spatial transfers, instead of 0.41 as observed in the data. Therefore, optimal transfers entail more redistribution toward low-income-growth states. Such large redistribution is desirable in our context because of the large and persistent location decisions across U.S. states as observed in data. Therefore, the benefits of providing additional consumption insurance outweigh the costs of distorting migration decisions.

We find that implementing the optimal transfer policy from 1980 onwards leads to a 0.05 percent increase in consumption equivalent welfare. While this is a significant gain considering

the size of the U.S. economy and population, the welfare gains are smaller compared to the range of estimates provided using a static spatial framework (Fajgelbaum and Gaubert 2020, Rossi-Hansberg et al. 2019). This difference arises because migration is frictional and it takes time for the effects and gains of the policy to materialize. Therefore, using a dynamic framework is important not only to understand the structure of the optimal policy but also to understand the welfare gains resulting from the policy.

Our paper is related to several strands of literature. First, our paper contributes to the literature on place-based policy. A majority of the academic research on place-based policies has focused on its role in ensuring aggregate efficiency by correcting localized externalities (e.g., Glaeser and Gottlieb 2008, Kline and Moretti 2014b, Fajgelbaum and Gaubert 2020, Rossi-Hansberg et al. 2019). More recent work highlights the redistributive motive of place-based policy. Gaubert et al. (2021) advocate the role of place-based policy as a tool for redistribution across different types of households. Davis and Gregory (2021) focus on its role in equalizing the dispersion of marginal utility within types and discuss a similar policy trade-off as in our paper in a static partial equilibrium environment.[1] To date, little effort has been devoted to analyzing place-based policy in a dynamic environment with heterogeneous and divergent regional economic activity with slow and frictional migration responses (Bartik 2020). We contribute to this literature by characterizing optimal policies that strike the trade-off between consumption insurance and efficiency in a dynamic general equilibrium environment.

Second, our paper contributes to the literature on the dynamic evolution of spatial economic activity with population mobility. In particular, our framework builds on the approach of modeling forward-looking migration decisions as a dynamic discrete choice problem (Artuç et al. 2010, Kennan and Walker 2011, Caliendo et al. 2019). Such a framework is used and extended to study various aspects of the dynamics of regional economic activities, such as the regional incidence of trade shocks (Caliendo et al. 2019), spatial development and growth (Desmet et al. 2018, Allen and Donaldson 2020, Kleinman, Liu, and Redding 2023, Cai et al. 2022), regional depopulation and aging (Giannone, Miyauchi, Paixão, Pang, and Suzuki 2023), climate change and natural disasters (Balboni 2019, Cruz 2021, Pang and Sun 2023, Bilal and Rossi-Hansberg 2023), and the role of financial frictions (Giannone, Li, Paixão, and Pang 2023, Greaney 2023, Dvorkin 2023). Despite the popularity of the framework and the breadth of the applications, there have only been limited attempts to characterize optimal policy in this framework. The closest contribution is by Cameron, Chaudhuri, and McLaren (2007), who show that market equilibria are efficient in a setting where there are no trade frictions, households are risk neutral, and there are no externalities. We show that there is room for policy intervention in a more general setting and explicitly characterize the optimal spatial transfers.

[1] The insight that marginal utility is not equalized in spatial equilibrium model dates back to Mirrlees (1972).

2 Model

We consider a world consisting of J locations. Time is discrete and indexed by t. There is a unit measure of households in the economy. Each location features a competitive labor market. In each location, competitive firms produce differentiated goods that can be traded across locations subject to iceberg trade costs. The productivity and amenity levels of each location are influenced by localized agglomeration externalities.

Households are forward-looking, have perfect foresight, and optimally decide where to move given some initial distribution of labor across locations. Households face costs to move across locations and experience an idiosyncratic shock that affects their moving decision. This formulation of forward-looking migration decisions closely follows the approach of Artuç et al. (2010) and Kennan and Walker (2011) in partial equilibrium and Caliendo et al. (2019) in general equilibrium. However, unlike these previous studies, which assume specific functional forms for idiosyncratic preference shocks, our framework allows for flexible functional forms and arbitrary correlation across potential destinations.

In the following sections, we first outline how households make decisions based on equilibrium prices, wages, and local amenities. We then describe how these prices, wages, and amenities are determined in competitive equilibrium.

2.1 Household Decisions

In each period t, households consume a basket of goods produced in various locations. We denote the consumption of goods produced in location i and consumed by households in location j as q_{ijt}.[2] The bundle of products consumed by households is denoted as $c_{jt} = c_{jt}\left(\{q_{ijt}\}\right)$, where $c_{jt}\left(\cdot\right)$ represents a function with constant returns to scale.[3] Additionally, households derive utility from location-specific amenities, denoted as a_{jt}. Amenity a_{jt} is influenced by both exogenous factors (e.g., rivers, beaches) and agglomeration externalities, as elaborated in the next section. We denote the flow utility of households in period t in location j by $u\left(c_{jt}, a_{jt}\right)$.

In each period t, households in location j provide one unit of labor at a wage rate of w_{jt}. In addition to labor income, households receive a location-specific public transfer denoted as τ_{jt}. This transfer aims to capture various forms of subsidies and taxes that are specific to different locations and times. The goal of Section 3 is to characterize the optimal choice of $\{\tau_{jt}\}$ by the

[2] In Appendix B.2, we provide an extension that incorporates heterogeneous household types with varying income, consumption, and migration decisions even conditional on location and time.

[3] Although we do not explicitly model nontradables (e.g., services, housing) or intermediate inputs for production, these factors can be captured by reinterpreting $c_{jt}(\cdot)$ as "reduced factor demand" in Adao, Costinot, and Donaldson (2017), who show that any trade models under perfect competition can be represented as a factor exchange economy.

Planner. Furthermore, households also receive private transfers s_{jt}. We include s_{jt} to account for transfers occurring across regions, encompassing elements such as factor income owned in other locations, pure altruistic transfers (e.g., charities), or payments from private insurance associated with regional income shocks. Together, the household consumption is given by

$$c_{jt} = \frac{w_{jt} + \tau_{jt} + s_{jt}}{P_{jt}}, \tag{1}$$

where P_{jt} is the ideal price index of the consumption bundle given by

$$P_{jt} = \min_{\{q_{ijt}\}} \Big[\sum_i p_{ijt} q_{ijt} \Big] \;\; s.t. \;\; c_{jt}\big(\{q_{ijt}\}\big) \geq 1, \tag{2}$$

where p_{ijt} is the price of good produced in i sold in location j at period t. The consumption of each variety is also given by

$$\{q_{ijt}\} = \arg\min_{\{q_{ijt}\}} \Big[\sum_i p_{ijt} q_{ijt} \Big] \;\; s.t. \;\; c_{jt}\big(\{q_{ijt}\}\big) \geq c_{jt}. \tag{3}$$

A key feature of our environment is the presence of dispersion in the marginal utility of consumption, $\partial u_{jt}/\partial c_{jt}$, across locations and over time. In particular, if agents are risk averse, i.e., the utility function is concave, the Planner can enhance households' welfare by offering consumption smoothing through public transfers $\{\tau_{jt}\}$. In other words, a key rationale for the policy is to address the incompleteness of the financial market. We further explore this aspect when analyzing the optimal transfer policy in Section 3.

At the end of each period t, households make decisions regarding their next place of residence in the following period. Relocating from location j to location k incurs migration costs $\tilde{\chi}_{jkt}$ in utility. Additionally, each household experiences an idiosyncratic preference shock ε_{jkt} when considering a move from location j to location k in period t. We assume that the vector of idiosyncratic shocks $\varepsilon_{jt} = [\varepsilon_{j1t}, \ldots, \varepsilon_{jJt}]$ follows the cumulative distribution function $G_{jt}(\cdot)$. Our only requirements for $G_{jt}(\cdot)$ are that it is continuous and differentiable. It is worth noting that this specification encompasses the common parametric assumption where ε_{jkt} is assumed to follow an i.i.d. extreme-value distribution (Artuç et al. 2010, Kennan and Walker 2011, and Caliendo et al. 2019). By allowing for flexible functional forms and arbitrary correlation of shocks across destinations, our model captures flexible substitution patterns of migration choices.[4] Given

[4]Our formulation does not account for the case where individual preference shocks are serially correlated, as explored by Howard and Shao (2022). In such cases, the Planner must infer the previous idiosyncratic shocks conditional on location and time, adding additional complication to the optimal transfer policy problem. In an extreme scenario where preference shocks are perfectly correlated over time and observable, the model is isomorphic to our extension with multiple types of households as described in Appendix B.2.

these assumptions, households' idiosyncratic lifetime utility from residing in location j at time t is given by

$$v_{jt} = u\left(c_{jt}, a_{jt}\right) + \max_{k}\Big[\beta V_{kt+1} - \tilde{\chi}_{jkt} + \varepsilon_{jkt}\Big], \tag{4}$$

where β represents the discount factor, and $V_{jt} \equiv \mathbb{E}(v_{jt})$ denotes the expected lifetime utility from residing in location j at time t. Note that the uncertainty of v_{jt} arises because the sequence of future preference shocks has not yet been realized at the time of making movement choices in period t. Note also that households anticipate future public transfers and equilibrium wages when making location decisions, as captured by V_{jt}. The fraction of households that move from location j to location k in period t is given by

$$\mu_{jkt} = Pr\Bigg(k = \arg\max_{k}\Big[\beta V_{kt+1} - \tilde{\chi}_{jkt} + \varepsilon_{jkt}\Big]\Bigg). \tag{5}$$

With these movement flows, the population in each location, denoted as L_{jt}, follows the law of motion

$$L_{jt} = \sum_{i} L_{it-1}\mu_{ijt-1}. \tag{6}$$

Before proceeding to the description of the equilibrium, we provide a useful reformulation of the aforementioned dynamic migration decision:

Lemma 1. *Given $\{V_{kt+1}\}$, the expected lifetime utility and aggregate behavior of this economy's heterogeneous households at time t coincide with the solution to the following representative agent recursive formulation:*

$$V_{jt} = u(c_{jt}, a_{jt}) + \max_{\{\mu_{jkt}\}_k}\Big[-\psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta\sum_{k}\mu_{jkt}V_{kt+1}\Big] \tag{7}$$
$$\text{s.t.} \quad \sum_{k}\mu_{jkt} = 1,$$

where $\psi_{jt}(\cdot)$ is an exogenous function that depends on the migration costs $\{\tilde{\chi}_{jkt}\}$ and the distribution of idiosyncratic preference shocks $G_{jt}(\cdot)$.

The proof and the exact expression for $\psi_{jt}(\cdot)$ is in Appendix A.1. This lemma demonstrates that the dynamic discrete choice problem faced by agents with heterogeneous idiosyncratic shocks can be effectively represented as a representative agent's problem, wherein the migration flows μ_{jkt} are directly chosen. This formulation builds upon prior research that formulated static discrete choice problems as representative agent problems (e.g., Anderson, De Palma, and Thisse

1992, Dubé, Joo, and Kim 2022), but we extend it to a dynamic context and explicitly derive the corresponding direct (as opposed to indirect) utility function under arbitrary preference shocks. While this lemma simply provides an alternative expression for the dynamic migration decision, it significantly aids in formulating the optimal transfer problem. It is worth noting that in the special case where $G_{jkt}(\cdot)$ follows an i.i.d. Type-I Extreme Value distribution with dispersion parameter θ, we have $\psi_{jt}\big(\{\mu_{jkt}\}\big) = \frac{1}{\theta}\sum_k \mu_{jkt} \ln\big(\mu_{jkt}/\chi_{jkt}\big)$, and migration flows follow the familiar expression that $\mu_{ijt} = \frac{\chi_{ijt}\exp(\beta V_{jt+1})^\theta}{\sum_k \chi_{ikt}\exp(\beta V_{kt+1})^\theta}$, where we define $\chi_{ijt} \equiv -\exp(\tilde{\chi}_{ijt})$. We refer to the functions $\psi_{ijt}(\cdot)$, which capture both the idiosyncratic shocks and migration costs, as "migration frictions".

2.2 Competitive Equilibrium

We now discuss how wages, amenities, price indices, and production are determined in competitive equilibrium.

In each location, there exists a representative and competitive producer that utilizes labor as its sole factor of production. Firms operate under a linear production technology with labor productivity denoted as z_{jt}. As we specify below, we allow this productivity to be influenced by both exogenous location-specific factors and agglomeration externalities. When goods are shipped from location i to location j during period t, they incur iceberg trade costs represented by $\kappa_{ijt} \geq 1$. Specifically, to enable residents of location j to consume q_{ijt} units of goods, $\kappa_{ijt}q_{ijt}$ units of goods must be dispatched from location i. We also normalize within-region trade cost such that $\kappa_{iit} = 1$. Given perfect competition in the labor and product markets, the price of good produced in i and sold in j at period t is given by

$$p_{ijt} = \kappa_{ijt} w_{it}/z_{it}. \tag{8}$$

The goods market clearing condition at each location and time is given by

$$\sum_j \kappa_{ijt} q_{ijt} L_{jt} = z_{it} L_{it}. \tag{9}$$

The productivity and amenities of locations are influenced by both location-specific factors and agglomeration externalities driven by the local population size:

$$z_{jt} = Z_{jt}(L_{jt}), \qquad a_{jt} = A_{jt}\left(L_{jt}\right). \tag{10}$$

Here, $Z_{jt}(\cdot)$ and $A_{jt}(\cdot)$ represent functions that are specific to the location and time and depend on the population size. It is worth noting that households do not internalize the impact of their

migration decisions on the productivity and amenities of their region. Previous studies have highlighted that, in a static context, addressing these externalities is one important motivation for implementing place-based policies (Glaeser and Gottlieb 2008, Kline and Moretti 2014a, Fajgelbaum and Gaubert 2020). In Section 3, we show how these agglomeration externalities influence the optimal spatial transfers.[5]

We assume that public transfers $\{\tau_{jt}\}$ are set to satisfy the government budget constraint:

$$\sum_j \tau_{jt} L_{jt} = 0. \tag{11}$$

We also assume that private transfers $\{s_{jt}\}$ have zero net supply:

$$\sum_j s_{jt} L_{jt} = 0. \tag{12}$$

Given these conditions, the competitive equilibrium is defined as follows.

Definition 1 (Competitive Equilibrium)**.** Given fundamentals for labor productivity and amenities $\{Z_{jt}(\cdot), A_{jt}(\cdot)\}$, migration frictions $\{\psi_{ijt}(\cdot)\}$, iceberg trade costs $\{\kappa_{ijt}\}$, and the initial population distribution $\{L_{j0}\}$, the competitive equilibrium of this economy with transfers consists of movement flows $\{\mu_{ijt}\}$, continuation values $\{V_{jt}\}$, population distributions $\{L_{jt}\}$, consumption decisions $\{c_{jt}, \{q_{ijt}\}\}$, prices $\{P_{jt}, \{p_{ijt}\}\}$, amenities $\{a_{jt}\}$, public transfers $\{\tau_{jt}\}$, private transfers $\{s_{jt}\}$, interest rates $\{R_t\}$, government bonds $\{B_t\}$, and wages $\{w_{jt}\}$ such that (i) Households' consumption decisions $\{c_{jt}, \{q_{ijt}\}\}$ are given by Equations (1) and (3), (ii) Movement flows $\{\mu_{ijt}\}$ and continuation values $\{V_{jt}\}$ solve the representative household problem (7), (iii) Population distributions $\{L_{jt}\}$ follow the law of motion (6), (iv) Goods prices $\{P_{jt}, \{p_{ijt}\}\}$ satisfy Equations (2) and (8), (v) Goods markets clear by Equation (9), (vi) Amenities $\{a_{jt}\}$ and productivity $\{z_{jt}\}$ satisfy Equation (10), (vii) Government budget constraint holds (11), and (viii) Private transfers $\{s_{jt}\}$ are in zero net supply (12).

3 Optimal Dynamic Spatial Transfers

In this section, we define and characterize the solution to the optimal spatial transfer problem.

[5] Some studies have examined more general forms of externalities where past population size, as well as population sizes in other locations, generate externalities on contemporaneous productivities and amenities (e.g., Desmet et al. 2018, Allen and Donaldson 2020, Peters 2022, Cai et al. 2022). In Appendix B.1, we demonstrate how our expression for optimal spatial transfers can be extended to such environments.

3.1 Planning Problem

This section sets up a planning problem for the optimal transfer policy. At $t = 0$, households are distributed across locations with measure $\{L_{j0}\}$. We consider a planning problem that sketches the Pareto frontier across households in different initial locations.[6]

Definition 2 (Planning Problem)**.** Given Pareto weights $\{\varkappa_i\}$ attached to individuals in location i in period 0, the Planner solves

$$\max_{\{\{q_{kjt},\mu_{jkt},p_{kjt}\},c_{jt},L_{jt+1},V_{jt},P_{jt},a_{jt},B_{t+1},R_{t+1},w_{jt},\tau_{jt},s_{jt}\}_{t\geq 0}} \sum_i \varkappa_i L_{i0} V_{i0}$$

$$\text{s.t.} \quad V_{jt} = u(c_{jt}, a_{jt}) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1} \tag{PK}$$

$$\{\mu_{jkt}\} \in \arg \max_{\{\mu_{jkt}\}:\sum_k \mu_{jkt}=1} u(c_{jt}, a_{jt}) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1} \tag{IC}$$

(1)-(3), (6)-(12).

The set of solutions to this problem given an arbitrary Pareto weights $\{\varkappa_i\}$ defines the Pareto frontier.

A crucial constraint for the Planner is that transfers $\{\tau_{jt}\}$ cannot be contingent on the idiosyncratic preference shocks $\{\varepsilon_{ijt}\}$. This restriction is evident in the incentive compatibility constraints for migration decisions (IC). If transfers were allowed to depend on idiosyncratic shocks ε_{ijt}, the Planner could directly manipulate migration flows by providing negative consumption unless individuals choose the desired location. This restriction gives rise to a dynamic moral hazard problem, where the Planner must utilize spatial transfers $\{\tau_{jt}\}$ to influence migration flows while satisfying (IC) and the promise-keeping constraint (PK) that specifies the evolution of households' continuation values.

We view this restriction as capturing a realistic constraint for place-based policies. Firstly, in most democratic countries, regulating mobility would violate constitutional rights. Secondly, even in contexts where there are quotas for migration flows, such as the Fukou policies in China, it is unrealistic to assume that the government can observe an individual's idiosyncratic preferences for location decisions. If the government cannot do so, the policy may lead to misallocation of people with respect to these idiosyncratic shocks.[7,8]

[6] Note that our solution encompasses any solution to a concave and differentiable social welfare function by appropriately setting the Pareto weights.

[7] See Glaeser and Luttmer (2003) for evidence that rent control policy functioned as an effective quota for the residential population in New York City, and it may have led to misallocation of residents in terms of willingness to pay to reside in the city.

[8] In Appendix A.3, we characterize the solution to the first-best problem and demonstrate that the transfer policies cannot achieve the first-best allocation.

We also assume that the Planner can perfectly commit to future policies. Commitment is relevant in our setting because once a location choice has been made, it becomes a sunk decision for households. Therefore, without commitment, the Planner would expropriate all income and equalize marginal utility in each period.[9]

3.2 Characterizing Optimal Spatial Transfers

This section focuses on solving for the optimal sequence of spatial transfers. To accomplish this, we follow the primal approach in the public finance literature to focus on a relaxed planning problem where the Planner picks an incentive-compatible sequence of consumption and migration flows, and show that the solution to the original problem coincides with the original one. The relaxed planning problem is defined as follows.

Definition 3 (Relaxed Planning Problem)**.** Given Pareto weights $\{\varkappa_i\}$ attached to individuals in location i in period 0, the Planner solves

$$\max_{\{\{q_{kjt},\mu_{jkt}\},L_{jt+1},V_{jt}\}_{t\geq 0}} \sum_i \varkappa_i L_{i0} V_{i0}$$

$$\text{s.t.} \quad Z_{it}(L_{it})L_{it} = \sum_j \kappa_{ijt} q_{ijt} L_{jt}, \qquad L_{jt+1} = \sum_i L_{it}\mu_{ijt} \tag{RC}$$

$$V_{jt} = u\big(c_{jt}(\{q_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1} \tag{PK}$$

$$\{\mu_{jkt}\} \in \arg \max_{\{\mu_{jkt}\}:\sum_k \mu_{jkt}=1} u\big(c_{jt}(\{q_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1}. \tag{IC}$$

The relaxed planning problem differs from the original in that the Planner directly specifies consumption and migration flows without considering the equilibrium determination of prices. Therefore, the Planner's choice set is larger in the relaxed problem than in the original. However, we show in Appendix A.2 that there exists supporting prices and transfers which implement the solution to the relaxed problem as a competitive equilibrium. Thus, the solution to the relaxed planning problem is also a solution to the original planning problem.

This relaxed planning problem also highlights the fundamental trade-off faced by the Planner. In the presence of a location experiencing a negative productivity shock $Z_{jt}(\cdot)$, the concave utility function implies higher marginal utility, creating an incentive for the Planner to increase

[9] This mirrors the commitment problem in the capital taxation literature (Kydland and Prescott 1977, Chari and Kehoe 1990, Klein, Krusell, and Rios-Rull 2008, Farhi, Sleet, Werning, and Yeltekin 2012), where investment decisions are sunk at the beginning of each period and the Planner has an incentive to expropriate the entire capital stock to facilitate redistribution.

consumption c_{jt}. However, this action increases migration flows μ_{ijt} into the location experiencing negative productivity shock through the constraint (IC). The optimal transfer policy must strike a balance between these two forces.

To characterize the solution to the planning problem, it is useful to introduce the concept of the fiscal externality in location j in period t: Δ_{jt}. Formally, we define Δ_{jt} as the Lagrange multiplier associated with the population law of motion in (RC) in the dual of the Relaxed Planning Problem 3. In simple terms, Δ_{jt} represents the marginal benefit to the government budget of relocating one unit of population to location j in period t. In Appendix A.2, we demonstrate that Δ_{jt} can be recursively expressed as follows:

$$\Delta_{jt} = w_{jt} - P_{jt}c_{jt} + w_{jt}\gamma^z_{jt} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}} P_{jt} A_{jt} \gamma^a_{jt} + \frac{1}{R_{t+1}} \sum_k \mu_{jkt} \Delta_{kt+1}, \tag{13}$$

where $\gamma^Z_{jt} \equiv \frac{\partial \log z_{jt}}{\partial \log L_{jt}}$ and $\gamma^A_{jt} \equiv \frac{\partial \log a_{jt}}{\partial \log L_{jt}}$ represent the elasticities of the agglomeration externalities for productivity and amenity, respectively; and $\partial_c u_{jt} \equiv \frac{\partial u_{jt}}{\partial c_{jt}}$ and $\partial_a u_{jt} \equiv \frac{\partial u_{jt}}{\partial a_{jt}}$ represent the marginal utility of consumption and amenity, respectively; and R_t is the marginal rate of substitution for the Planner between period t and $t-1$. In this expression, the first two terms, $w_{jt} - P_{jt}c_{jt}$, capture the net resources utilized by households in location j and time t, accounting for labor productivity minus consumption expenditure. The next two terms correspond to the agglomeration externalities for productivity and amenity. Finally, the last term captures future fiscal externalities. Specifically, locating an individual in location j generates fiscal externalities in the future, depending on the probability of migration from j to other locations.

It is worth noting that, if $\Delta_{jt} > \Delta_{kt}$, the Planner can enhance the total budget by relocating individuals from location k to location j. However, the Planner may choose to maintain this gap if the marginal utility from consumption is lower in location j. Our main proposition characterizes the consumption levels $\{c_{jt}\}$ that characterize this trade-off.

Proposition 1. *If a competitive equilibrium with transfers solves the Planner's problem, then the following condition is satisfied for all locations j and time $t \geq 0$:*

$$-\sum_i L_{it} \sum_k \Delta_{kt+1} \frac{1}{P_{jt+1}} \frac{\partial \mu_{ikt}}{\partial c_{jt+1}} = \sum_i L_{it} \mu_{ijt} \left(\frac{\beta R_{t+1} \partial_c u_{jt+1} / P_{jt+1}}{\partial_c u_{it} / P_{it}} - 1 \right), \qquad \forall j, t \tag{14}$$

where Δ_{jt} is defined recursively by Equation (13), and R_{t+1} is the marginal rate of substitution for the Planner between period $t+1$ and t, implicitly determined to satisfy the government budget constraint (11).

The proof of Proposition 1 can be found in Appendix A.2. This proposition implicitly specifies the consumption levels $\{c_{jt+1}\}$ that equalize the marginal cost and benefit of increasing trans-

fers τ_{jt+1}. On the left-hand side of Equation (14), we summarize the marginal cost of increasing τ_{jt+1} by distorting migration decisions. An incremental increase in τ_{jt+1} leads to a consumption increase of $1/P_{jt+1}$. This, in turn, affects the migration shares to destinations k by $\frac{1}{P_{jt+1}}\frac{\partial \mu_{ikt}}{\partial c_{jt+1}}$. It is important to note that the migration patterns change not only for destination j but also for all other destinations $k \neq j$. For each destination k, a unit change in migration share is associated with a fiscal externality Δ_{kt+1}, which encompasses the net resource cost and agglomeration externality in location k at period $t+1$ (Equation (13)). Summing up these effects across all origin locations i, multiplied by L_{it}, yields the total efficiency loss resulting from the marginal increase in transfers τ_{jt+1}. Note that the efficiency loss would be zero if fiscal externalities Δ_{jt} were equalized across locations since a marginal reallocation of the population across locations would have no effect on the government's budget.

On the right-hand side of Equation (14), we summarize the marginal benefit of increasing τ_{jt+1} from consumption insurance. Marginal transfers in τ_{jt+1} provide consumption smoothing benefits if the marginal utility of income in location j at $t+1$, $\partial_c u_{jt+1}/P_{jt+1}$, is relatively high compared to that of the origin location i at t, $\partial_c u_{it}/P_{it}$, adjusted by the marginal rate of intertemporal substitution: βR_{t+1}. It is worth noting that if perfect consumption smoothing is achieved, i.e., the usual Euler equation is satisfied so that $\beta R_{t+1}\partial_c u_{jt+1}/P_{jt+1} = \partial_c u_{it}/P_{it}$, the right-hand side of Equation (14) becomes zero, implying no benefit from public transfers for consumption insurance and smoothing. We will revisit this scenario in the next section when discussing a special case of a complete financial markets.

The trade-off between providing insurance and incentivizing efficient migration, as presented in Proposition 1, resonates with the incentive-insurance trade-off emphasized in the unemployment insurance literature (Baily 1978 and Chetty 2006) and the dynamic labor contract literature (Burdett and Coles 2003 and Balke and Lamadon 2022). In the unemployment insurance literature, the trade-off arises between the benefits of unemployment insurance and the fiscal externality resulting from changes in the probability of transitioning out of unemployment. In the dynamic labor contract literature, firms face a trade-off in tenure contracts between providing consumption smoothing and altering workers' effort and churning probability.

Proposition 1 further highlights the forward-looking nature of optimal spatial transfers. The recursive nature of fiscal externality, as presented in Equation (13), demonstrates that altering migration patterns not only generates fiscal externalities in the present but also has implications for the future. In Appendix C, we show through a numerical simulation that these dynamic considerations lead to frontloading transfers to locations with negative permanent shocks, i.e., provide a large transfer and gradually decrease transfers to provide additional incentive for outmigration.

It is worth noting that the expression in Proposition 1 does not depend on the Pareto weights

$\{\varkappa_{jt}\}$. This is because the Pareto weights are encapsulated in the consumption levels in period t. Conditional on those values, Proposition 1 specifies the growth rates of the marginal utility of consumption at every period after $t = 0$ irrespective of the Pareto weights. The initial period consumption level at $t = 0$, in turn, is determined to simply equalize the marginal utility of consumption up to Pareto weights, i.e., $\varkappa_i \partial_c u_{i0}/P_{io} = \varkappa_j \partial_c u_{j0}/P_{j0}$ for all i, j, since the migration decision occurs after the consumption decision within a period.

Finally, we turn our attention to the construction of the public transfers τ_{jt} necessary to achieve the optimal consumption levels identified in Proposition 1. Once we have determined the optimal consumption values, we can establish the corresponding optimal public transfers by utilizing the households' budget constraint (1). Specifically, the optimal transfers are given by the expression:

$$\tau_{jt} = P_{jt} c_{jt} - w_{jt} - s_{jt}. \tag{15}$$

If private transfers $\{s_{jt}\}$ are exogenous, this equation provides a direct characterization of the optimal public transfers. However, if private transfers s_{jt} respond to equilibrium conditions (including $\{\tau_{jt}\}$), the determination of $\{\tau_{jt}\}$ becomes a fixed-point problem that needs to be solved.

3.3 Special Case: Complete Financial Markets

In this subsection, we explore a special case of our model that assumes a complete financial market, providing further insight into Proposition 1.

There are two settings that attain a complete financial market in our setting. The first case involves the introduction of Arrow-Debreu securities that pay out for each location, time, *and idiosyncratic preference shocks of each individual.* Note that, without the availability of securities contingent on idiosyncratic shocks, the financial market is still incomplete. We formally present this model extension in Appendix A.4. The second case assumes risk neutrality (constant marginal utility $\partial_c u_{jt}$ across locations and time) and equalized consumption prices across locations ($P_{it} = P_{jt}$ for all $i, j \in J$). It is important to note that risk neutrality alone does not guarantee the completeness of financial market due to differential growth rates of prices faced by individuals moving across locations. In both scenarios, we assume that private transfers s_{jt} are set to zero for simplicity, as they would introduce additional distortions in migration flows under the complete financial market.[10]

We acknowledge that both of these settings rely on unrealistic and highly specific assumptions. Therefore, the purpose of this discussion is not to present empirically relevant cases, but rather to elucidate the intuition behind our main result in Proposition 1.

[10] If s_{jt} is not equal to zero, the expression for τ_{jt} in Corollary 1 is simply replaced by $\tau_{jt} + s_{jt}$, meaning that the net amount of public and private transfers serve as a Pigouvian correction.

In the scenario of a complete financial market, there are no benefits from providing additional consumption smoothing. Consequently, the sole objective of policy becomes correcting production and amenity agglomeration externalities. More formally, in the case of complete financial markets, the right-hand side of Proposition 1 becomes zero. To equate the left-hand side with zero, it is sufficient to equalize fiscal externalities Δ_{jt} across locations for each period t. The Planner can achieve this via a Pigouvian correction, i.e., setting transfers to offset the agglomeration externalities that enter as a part of fiscal externality (13), as demonstrated in the following corollary.

Corollary 1. *Under complete financial markets, the optimal set of public transfer* $\{\tau_{jt}\}$ *is given by*

$$\tau_{jt} = w_{jt}\gamma^z_{jt} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}} P_{jt} A_{jt} \gamma^a_{jt} - K_t, \tag{16}$$

where K_t *is a lump-sum set to satisfy the government budget constraint (11).*

The proof of Corollary 1 is in Appendix A.4. As mentioned earlier, it is intuitive that the optimal transfer only depends on agglomeration externalities due to the absence of objectives related to providing consumption insurance. An interesting result is that future fundamentals and externalities do not directly influence the optimal transfers. This is because future transfer policies fully address future externalities. Therefore, despite the consideration of a dynamic environment, our optimal spatial transfer formula recovers that of a static environment (Fajgelbaum and Gaubert 2020).[11,12]

3.4 Extensions

In this section, we lay out several extensions to our optimal policy framework.

Dynamic and Cross-Regional Externalities First, we consider an extension with a dynamic form of agglomeration externality, i.e., local population affects not only contemporaneous but also future productivity, as well as cross-region agglomeration spillovers, as pursued by Desmet et al. (2018), Allen and Donaldson (2020), Peters (2022), Cai et al. (2022). In particular, we assume that the productivity in location j in time t takes the form: $Z_{jt} = g_{jt}\big(\bar{Z}_{jt}, \{L_{kt}\}\big)$, where

[11] See Golosov, Hassler, Krusell, and Tsyvinski (2014) for a related result in the context of a pollution externality in a dynamic general equilibrium setting. When dynamic externalities arise, such that the current population distribution affects future externalities, Corollary 1 can be adjusted to incorporate these dynamic externalities, as further discussed in Section 3.4.

[12] It is also worth noting that, in the special case of complete financial markets, the transfers of Corollary 1 implement the utilitarian first-best allocation as a competitive equilibrium.

$g_{jt}(\cdot)$ is an arbitrary function, $\{L_{kt}\}$ is the profile of population size across locations (including those outside location j), and $\bar{Z}_{jt}$ is a fundamental component of the productivity. $\bar{Z}_{jt}$, in turn, evolves according to $\bar{Z}_{jt} = h_{jt}(\{\bar{Z}_{kt-1}\}, \{L_{kt-1}\})$, where $h_{jt}(\cdot)$ is an arbitrary function, which can depend on the profiles of fundamental productivity in the previous period and the profiles of population sizes in the previous period. This specification allows changes in fundamental productivity to follow a dynamic, endogenous process that is itself dependent on the population distribution. For instance, one could allow the production of knowledge to have a spatial component. Furthermore, we can allow for the possibility of a lagged agglomeration effect.

We show in Appendix B.1 that the basic features of Proposition 1 carry over to this case. The optimization condition of Equation (14) continues to hold with straightforward adjustments accounting for the additional externalities. The only substantive change is that the fiscal effects of movement choice, previously summarized by Δ_{jt} in Equation (13), must now account for a recursive term that describes the dynamic effect of altering fundamental productivity. Nonetheless, the basic logic of the formula, where the Planner equalizes the distortion cost with the consumption smoothing benefit of a marginal transfer, is unchanged with the inclusion of a general, dynamic relationship between labor productivity and the population distribution across space.

Heterogeneous Household Types In Appendix B.2, we provide an extension that incorporates heterogeneous household types with varying income, consumption, and migration decisions even conditional on location and time, as pursued by Fajgelbaum and Gaubert (2020) and Rossi-Hansberg et al. (2019) in a static environment. The basic features of Proposition 1 carry over to this case, except that the fiscal externality Δ_{jt} is now specific to each household type. In particular, the type-specific Δ_{jt} depends on the cross-type agglomeration spillovers in this extension.

Origin-Destination Specific Transfers In the baseline model, we have restricted ourselves to location-specific transfers. In some countries or contexts, one may argue for a richer implementation of transfers that depends on households' past location as well. In Appendix B.3, we extend our model to an environment where transfers can depend on households' origin and destination location. We show that the basics of our formula remain unchanged in such cases except that now the formula holds for each origin-destination pair.

4 Data and Calibration

To take the model to the data, we use as an empirical setting the distribution of economic activity across U.S. states since 1980. This section discusses our main data sources and how we calibrate

our model to data. We also document salient patterns of spatial transfer across U.S. states over time.

4.1 Data

Our quantitative analysis focuses on the 48 contiguous U.S. states plus the District of Columbia, excluding Alaska and Hawaii.

Pre-Tax Income, Taxes, and Public Transfers We follow Gaubert et al. (2021) to obtain the measure of average pre-tax income, taxes and transfer in each state from the Bureau of Economic Analysis (BEA)'s Regional Account. Pre-tax income equals wages, employer-provided benefits, proprietors' income, dividends, interest, and rent and excludes capital gains and thereby corporate retained earnings. Taxes include all major federal, state, and local taxes except sales taxes. Transfer includes social security payment including Social Security Disability Insurance, as well as all major government transfers including Social Security and Medicaid. For brevity, in what follows, we refer to the pre-tax income plus the net tax and transfers as "post-tax income".

Consumption Expenditure We obtain personal consumption expenditure from Bureau of Economic Analysis (BEA)'s Regional Account. Unlike the aforementioned data on income, taxes, and public transfers, this data only start from 1997. In the subsequent section, we explain how we impute these values before 1997.

Consumer Price Index We first obtain Regional price parities (RPPs) from BEA to measure the differences in price levels across states in 2008. We then use the state-level inflation rates from Hazell, Herreño, Nakamura, and Steinsson (2022) to impute the price indices since 1980.

Migration Flows We use data on bilateral five-year migration flows between U.S. states from the U.S. population census from 1980-2000 and from the American Community Survey (ACS) after 2000. We define a period in the model as equal to five years to match these observed data. We interpolate between census decades to obtain five-year migration flows for each year of our sample period. Following Kleinman et al. (2023), to take account of international migration to each state and fertility/mortality differences across states, we adjust these migration flows by a scalar for each origin and destination state, such that origin population in year t pre-multiplied by the migration matrix equals destination population in year $t+1$, as required for internal consistency.

Trade Flows In the U.S., the Commodity Flow Survey (CFS) reports the flow of manufacturing goods shipped between CFS zones in the US every five years. There are two key limitations in this data set. First, the CFS only started in 1997. Second, the CFS does not report the service trade flows. To overcome this data limitation, we follow the approach of Fajgelbaum and Gaubert (2020) to use the gravity equation of trade flows to find the unique estimates of trade flows between states that are consistent with the actual distance between states, observed pre- and post-tax income, under the assumption that trade frictions are isoelastic with respect to distance.[13] In our baseline analysis, we set the elasticity of distance to trade flow as $\psi = -1$. In Section 5, we show that our results are robust to alternate assumptions of this elasticity.

4.2 Observed Patterns of Spatial Transfers over Time

Before proceeding with the analysis for the optimal transfers, we present salient patterns of spatial transfer across U.S. states over time. In particular, we demonstrate that both public and private transfers involve positive net transfers from high-income states to low-income states, as well as from high-income-growth states to low-income-growth states.

To investigate this question, we run the following regression:

$$\log(w_{jt} + \tau_{jt}) = \beta^{\tau} \log(w_{jt}) + \xi_t^{\tau} + \eta_j^{\tau} + \epsilon_{jt}^{\tau}. \tag{17}$$

Here, j is the state, t is year, w_{jt} is pre-tax income, τ_{jt} is the net transfer (so that $w_{jt} + \tau_{jt}$ is the post-tax income), ξ_t^{τ} is the year fixed effect, η_j^{τ} is the state fixed effect, and ϵ_{jt}^{τ} is the residual. In our analysis below, we investigate the results both without and with state fixed effects; the former specification informs the patterns of tax-and-transfer between low- and high-income states, and the latter specification informs those between slower- and faster-income growth states. We run this regression from 1997 to 2019 at an annual frequency.

Columns (1)-(4) of Table 1 present the results. Column (1) starts with the specification without state fixed effects. The result shows that a one percent increase in the pre-tax income is associated with a 0.74 percent increase in the post-tax income. These results are consistent with Gaubert et al. (2021), who document that there is a net transfer from high-income states to low-income states. In Column (2), we present the results with the specification with state fixed effects. The result indicates that a state experiencing a one percent increase in the growth rate of pre-tax income is associated with a 0.75 percent increase in the growth rate of post-tax income. These results are consistent with the observation that public transfer plays a role in mitigating regional income shocks as documented by Asdrubali et al. (1996). These results are robust to solely focusing on before 2005 (Column 3) and by focusing on low-income states (Column 4).

[13] See Appendix E.1 for further details on this calibration procedure.

Table 1: Observed Transfers vs Income

	log (Post-Tax-and-Transfer Income)				log (Consumption Expenditure)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log (Pre-Tax-and-Transfer Income)	0.74***	0.75***	0.81***	0.77***				
	(0.02)	(0.01)	(0.04)	(0.02)				
log (Post-Tax-and-Transfer Income)					0.89***	0.52***	0.41***	0.45***
					(0.08)	(0.06)	(0.07)	(0.08)
Observations	1173	1173	459	575	1173	1173	459	575
Adjusted R^2	0.992	0.999	0.999	0.999	0.951	0.995	0.996	0.995
Year FE	X	X	X	X	X	X	X	X
State FE		X	X	X		X	X	X
Sample			Before 2005	Low Income			Before 2005	Low Income

Notes: This table reports the results of the regression specification (17) for Columns (1)-(4) and (18) for Columns (5)-(8). The sample of this regression is the U.S. states from 1997 to 2019 at an annual frequency.

Columns (5)-(8), in turn, focus on the patterns of private transfers using the following regression specification:

$$\log(w_{jt} + \tau_{jt} + s_{jt}) = \beta^s \log(w_{jt} + \tau_{jt}) + \xi_t^s + \eta_j^s + \epsilon_{jt}^s, \tag{18}$$

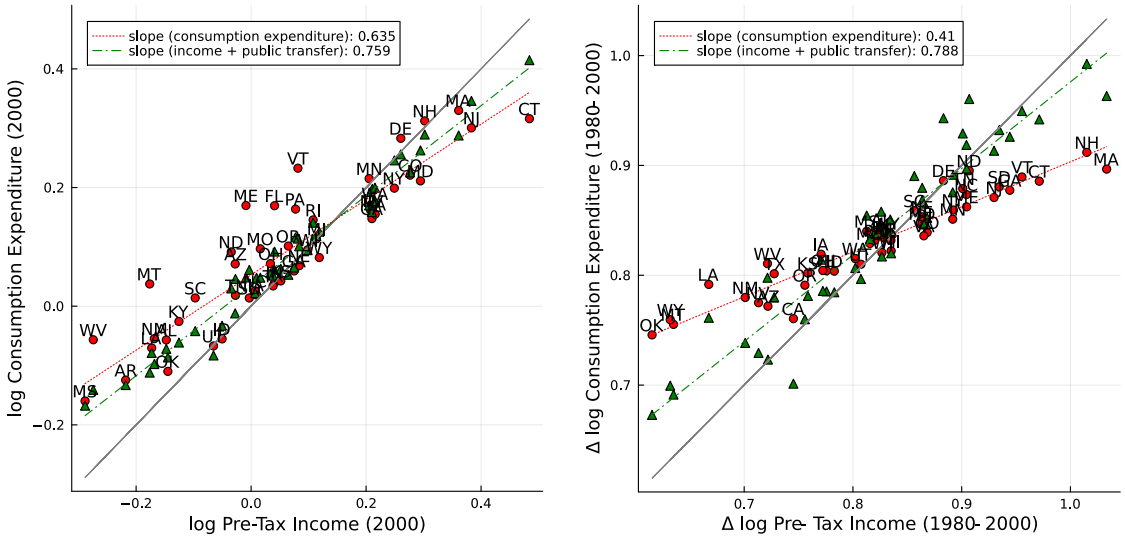
where the dependent variable $(w_{jt}+\tau_{jt}+s_{jt})$ is the consumption expenditure (so that s_{jt} is defined as consumption expenditure minus post-tax income). Column (5) starts with the specification without state fixed effects. The result shows that a one percent increase in the post-tax income is associated with a 0.89 percent increase in the post-tax income. Therefore, while the magnitude is smaller compared to the public transfer, private transfer flows from high-income to low-income states. In Column (6), we present the results with the specification with state fixed effects. The result indicates that a state experiencing a one percent increase in the growth rate of post-tax income is associated with a 0.52 percent increase in the growth rate of consumption expenditure. These results are consistent with the observation that there are pre-existing mechanisms aside from public transfers that mitigate regional income shocks as documented by Asdrubali et al. (1996). These results are robust to solely focusing before 2005 (Column 7) and by focusing on low-income states (Column 8).

To further illustrate the patterns of public and private transfers, in Figure 1, we present the patterns of pre-tax income, post-tax income, and consumption expenditure, for 2000 (Panel a) and the growth from 1980-2000 (Panel b). Since we only observe the consumption expenditure after 1997, we use regression (18) to predict the patterns of private consumption (and private transfer as its difference from the observed post-tax income). More specifically, we use the estimated regression coefficients β^s and state fixed effects η_j^s to predict the value of consumption expenditure, where we adjust ξ_t so that the net private transfer is zero, i.e., $\sum_j s_{jt} L_{jt} = 0$. Panel (a) shows that,

in 2000, states with one percent higher pre-tax income are associated with 0.76 percent higher post-tax income and 0.63 percent higher consumption expenditure. Panel (b) shows that states with one percent higher pre-tax income growth are associated with 0.78 percent higher post-tax income growth and 0.41 percent higher consumption expenditure.

Figure 1: Observed Transfers in the U.S. Over Time

(a) 2000 (b) Change from 1980-2000



Notes: These figures plot the patterns of pre-tax income, post-tax income, and consumption expenditure, for 2000 (Panel a) and the growth from 1980-2000 (Panel b). To construct the consumption expenditure in 1980, we use the estimated regression coefficients β^s and state fixed effects η_j^s to predict the value of consumption expenditure, where we adjust ξ_t so that the net private transfer is zero, i.e., $\sum_j s_{jt} L_{jt} = 0$.

Overall, the evidence suggests that both public and private transfers play a role in mitigating the differences in nominal income levels across states, as well as the differences in the growth rates of nominal income across years. In what follows, we assess to what extent these patterns align with the optimal transfers through the lens of our framework.

4.3 Parametric Model

To facilitate our quantitative analysis, we parameterize our model using common specifications found in the existing literature. We specify a utility function that is additively separable between consumption and amenity, where the consumption component follows a constant relative risk aversion (CRRA) form:

$$u(c_{jt}, a_{jt}) = \begin{cases} \frac{c_{jt}^{1-\rho}}{1-\rho} + \log a_{jt} & \rho \neq 1 \\ \log c_{jt} + \log a_{jt} & \rho = 1. \end{cases} \tag{19}$$

We further assume that the consumption bundle takes the form of a constant elasticity of substitution aggregator across varieties produced in different locations:

$$c_{jt}\left(\{q_{ijt}\}\right) = \left(\sum_i q_{ijt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{20}$$

We assume that idiosyncratic preference shocks for migration decisions follow the i.i.d. type-I extreme value distribution following Artuç et al. (2010) and Caliendo et al. (2019), so the migration friction function $\psi_{it}(\cdot)$ is given by

$$\psi_{it}\left(\{\mu_{ijt}\}\right) = \frac{1}{\theta}\sum_j \mu_{ijt}\log\left(\mu_{ijt}/\chi_{ijt}\right). \tag{21}$$

We assume that productivity and amenity spillovers take an iso-elastic form such that

$$Z_{jt}\left(L_{jt}\right) = \bar{Z}_{jt}L_{jt}^{\gamma^Z}, \quad A_{jt}\left(L_{jt}\right) = \bar{A}_{jt}L_{jt}^{\gamma^A}, \tag{22}$$

where γ^Z and γ^A are parameters that govern the spillover elasticities, and $\bar{Z}_{jt}$ and $\bar{A}_{jt}$ are productivity and amenity fundamentals.[14]

4.4 Calibration

To take our model to the data, we use the parametric model discussed in the previous section. Doing so requires the values of structural parameters $\{\beta, \rho, \theta, \sigma, \gamma^A, \gamma^Z\}$ and location fundamentals, including productivities $\{\overline{Z}_{jt}\}$, amenities $\{\overline{A}_{jt}\}$, bilateral migration costs $\{\chi_{ijt}\}$, and bilateral trade costs $\{\kappa_{ijt}\}$. We discuss the calibration of these parameters in turn.

4.4.1 Structural Parameters $\{\beta, \rho, \theta, \sigma, \gamma^A, \gamma^Z\}$

Table 2 summarizes our baseline calibration of the structural parameters. We set the 5-year discount factor equal to the conventional value of $\beta = (0.95)^5$. We set the relative risk aversion to $\rho = 1$, which corresponds to logarithmic utility. We assume a value for the migration elasticity of $\theta = 0.5$, which is within the range of parameters estimated and calibrated in the literature (Kleinman et al. 2023, Caliendo et al. 2019, Cruz 2021). We set the agglomeration spillovers for amenity and productivity to $\gamma^Z = \gamma^A = 0.05$, which is within the range of parameters found in the literature (e.g., Melo, Graham, and Noland 2009). For all parameters, we provide sensitivity

[14] In Appendix D, we list up the system of equations that determine the equilibrium and optimal allocation in this parametric setting.

analysis and discuss how our quantitative results are affected by the choices of these parameter values.

Table 2: Baseline Parameter Calibration

Parameter	Description	Value
β	Discount Factor	0.95^5
ρ	Relative Risk Aversion	1
θ	Migration Elasticity	0.5
σ	Trade Elasticity	5
γ^A	Amenity Spillover Elasticity	0.05
γ^P	Productivity Spillover Elasticity	0.05

4.4.2 Fundamentals and Transfers

We now discuss how we calibrate productivities $\{\overline{Z}_{jt}\}$, amenities $\{\overline{A}_{jt}\}$, bilateral migration costs $\{\chi_{ijt}\}$, bilateral trade costs $\{\kappa_{ijt}\}$, public transfers $\{\tau_{jt}\}$, and private transfers $\{s_{jt}\}$ from the data on trade flows, pre- and post-tax income, prices, and migration flows. Since we only observe the data until 2015, we need to make an assumption regarding the evolution of fundamentals and transfers after 2015. Below, we show how to calibrate fundamentals to exactly replicate the observed data between 1980 to 2015.[15]

Productivity $\{\overline{Z}_{jt}\}$ To recover productivity, we first back out the variety-specific prices sold within the region using the CES demand in equation (20), $p_{jjt}^{1-\sigma} = P_{jt}^{1-\sigma} x_{jjt} / \sum_{\ell} x_{\ell jt}$, where P_{jt} is the observed consumer price index and x_{ijt} is the nominal trade flow from state i to j in period t. Furthermore, using equation (8) and the normalization of within-region trade cost at one ($\kappa_{iit} = 1$), we can back out $z_{jt} = w_{jt}/p_{jjt}$. Together with the parametric assumption on the productivity spillover (22), we can back out $\overline{Z}_{jt} = z_{jt} L_{jt}^{-\gamma^Z}$. We also assume that the exogenous component of productivity remains constant after 2015, i.e., $\overline{Z}_{jt} = \overline{Z}_{jT}$ for all $t > T$.

Bilateral Trade Costs $\{\kappa_{ijt}\}$ Using the CES demand in equation (20), the ratio of nominal trade flows satisfies $x_{ijt}/x_{jjt} = (\kappa_{ijt} z_{it} w_{it})^{1-\sigma} / (z_{jt} w_{jt})^{1-\sigma}$. Using this equation, together with the value of $\{z_{jt}\}$ obtained in the previous procedure and the observed pre-tax income $\{w_{jt}\}$,

[15] Our approach of recovering the fundamentals to exactly fit the observed data patterns is isomorphic to the "dynamic-hat algebra" approach in Caliendo et al. (2019). Explicitly recovering fundamentals has the additional benefit of clarifying identifiable objects from the data, as well as clarifying the possibility of relaxing parametric assumptions, such as the non-log utility in consumption.

we recover $\{\kappa_{ijt}\}$. We also assume that bilateral trade costs remain constant after 2015, i.e., $\kappa_{ijt} = \kappa_{ijT}$ for all $t > T$.

Amenities $\{\overline{A}_{jt}\}$ and Bilateral Migration Frictions $\{\chi_{ijt}\}$ We start by introducing the normalization of these variables such that $\overline{A}_{jt} = 1$ and $\chi_{jjt} = 1$. Note that these normalizations do not affect the subsequent estimation and simulation of the optimal spatial transfers.[16] We back out off-diagonal elements of migration frictions, $\{\chi_{ijt}\}$ in two steps. We first back out those in the last data period, $t = T$, using the migration flow in period T, $\{\mu_{ijT}\}$, and the future evolution of fundamentals and transfers. Here, we also assume that bilateral migration frictions do not change after 2015, such that $\chi_{ijt} = \chi_{ijT}$ for all $t > T$. Once we obtain $\{\mu_{ijT}\}$, we sequentially back out $\{\chi_{ijt}\}$ for $t < T$ backward starting from $t = T - 1$ using migration flows $\{\mu_{ijt}\}$ in each period. Appendix E.2 describes the details of this procedure.

Public Transfers $\{\tau_{jt}\}$ For the time period when we observe the data, we calibrate them as the difference between pre-tax and post-tax income. To calibrate anticipated public transfers for future periods ($t > T$), we assume that the marginal rate of public transfer with respect to income is unchanged from T. In particular, we assume that $\tau_{it} = \varphi_{it} w_{it} L_{it} + \varrho_t$, where φ_{it} is the marginal rate of public transfer and ϱ_t is a lump-sum component to satisfy the government budget constraint ($\sum_i \tau_{it} L_{it} = 0$). We assume that marginal rate of public transfer is unchanged from the last observed period, such that $\varphi_{it} = \varphi_{iT}$ for all $t > T$.

Private Transfers $\{s_{jt}\}$ For the time period that we observe the data, we calibrate them as the difference between post-tax income and consumption expenditure. Outside these periods, we assume that private transfers are determined based on time-invariant state-specific components and the fluctuation of post-tax income. More specifically, we use the estimated regression coefficients β^s and state fixed effects η^s_j from regression (18) to predict the value of consumption expenditure, where we adjust ξ_t so that the net private transfer is zero, i.e., $\sum_j s_{jt} L_{jt} = 0$. We also use the same relationship between private transfer and the post-tax income when we solve for optimal tranfers.

Pareto Weights $\{\varkappa_j\}$ Our framework presented in Section 3 allows us to trace the Pareto frontier using an arbitrary set of Pareto weights attached for individuals in each location in the

[16] More specifically, under the assumption of additively separable utility (19), we cannot separately identify amenity and migration frictions. Similarly, we cannot separately identify the level of migration frictions *from* location j at period t and the migration frictions *to* location j in $t - 1$. Note that, although these variables cannot be separately identified, the normalization does not affect any equilibrium predictions.

initial period. For quantitative illustration, we use a particular value for $\{\varkappa_j\}$ given its high dimensionality. In our baseline specification, we calibrate the Pareto weights such that the optimal consumption patterns in the initial data period perfectly coincide with those observed in the data. Namely, by normalizing the Pareto weights in location 1 by $\varkappa_1 = 1$, we set the Pareto weights for j, $\varkappa_j = \frac{\partial_c u_{10}/P_{10}}{\partial_c u_{j0}/P_{j0}}$, so that the marginal utility from transfers is equalized across locations in the initial period up to Pareto weights. We also present how our results are affected by alternatively assuming a utilitarian Planner, i.e., $\varkappa_j = 1$ for all location j.

5 Optimal Dynamic Spatial Transfers in the U.S.

In this section, we use our calibrated model to compute the optimal spatial transfers in the U.S. since 1980. In particular, using the calibrated model in Section 4.4, which exactly matches the observed spatial transfers in the data between 1980-2015, we solve the model by replacing these observed public transfers with the optimal ones prescribed under Proposition 1 (see Appendix D for the full system of equations). In this section, we assess the differences between optimal and observed transfers, population distribution, and welfare gains from implementing the optimal transfer.

5.1 Optimal vs. Observed Transfers

Figure 2 presents the patterns of consumption expenditure and pre-tax income across locations and over time, thereby highlighting the patterns of spatial transfer. In Figure 2a, we plot the log of observed consumption expenditure against the log of observed pre-tax income for each state in 1980 (labeled "observed"). As discussed in Section 4.2, the regression slope is 0.62 and flatter than one, reflecting the fact that both public and private transfers contribute to the reallocation of income from high- to low-income states. The figure overlays the same relationship under our optimal allocation (labeled "optimal"). The observed and optimal consumption expenditures perfectly coincide in 1980, which is true by construction from our calibration of Pareto weights (Section 4.4).

In Figure 2b, we present the same relationships in 2000. We find that the slope of the log of consumption expenditure to the log of pre-tax income under the optimal allocation (0.525) is flatter than the same slope under the observed allocation (0.635). These results indicate that optimal transfer entails more redistribution from high- to low-income states in 2000.

In Figure 2c, we focus on the changes in consumption expenditure and pre-tax income over time. In the data, a one percent increase in the growth rates of the pre-tax income between 1980 to 2000 is associated with a 0.41 percent increase in the growth rates of consumption expenditure.

As documented in Figure 1b of Section 4.2, these patterns are driven by the fact that both public and private transfers play a role in regional risk sharing. In contrast, under optimal transfers, we find a slope of 0.07. In other words, under our parametrization, more than 90 percent of the differences in the state-level income growth should be redistributed from high-income-growth to low-income-growth states. These patterns indicate that, in our context, the benefits of providing additional consumption insurance outweigh the costs of distorting migration decisions.

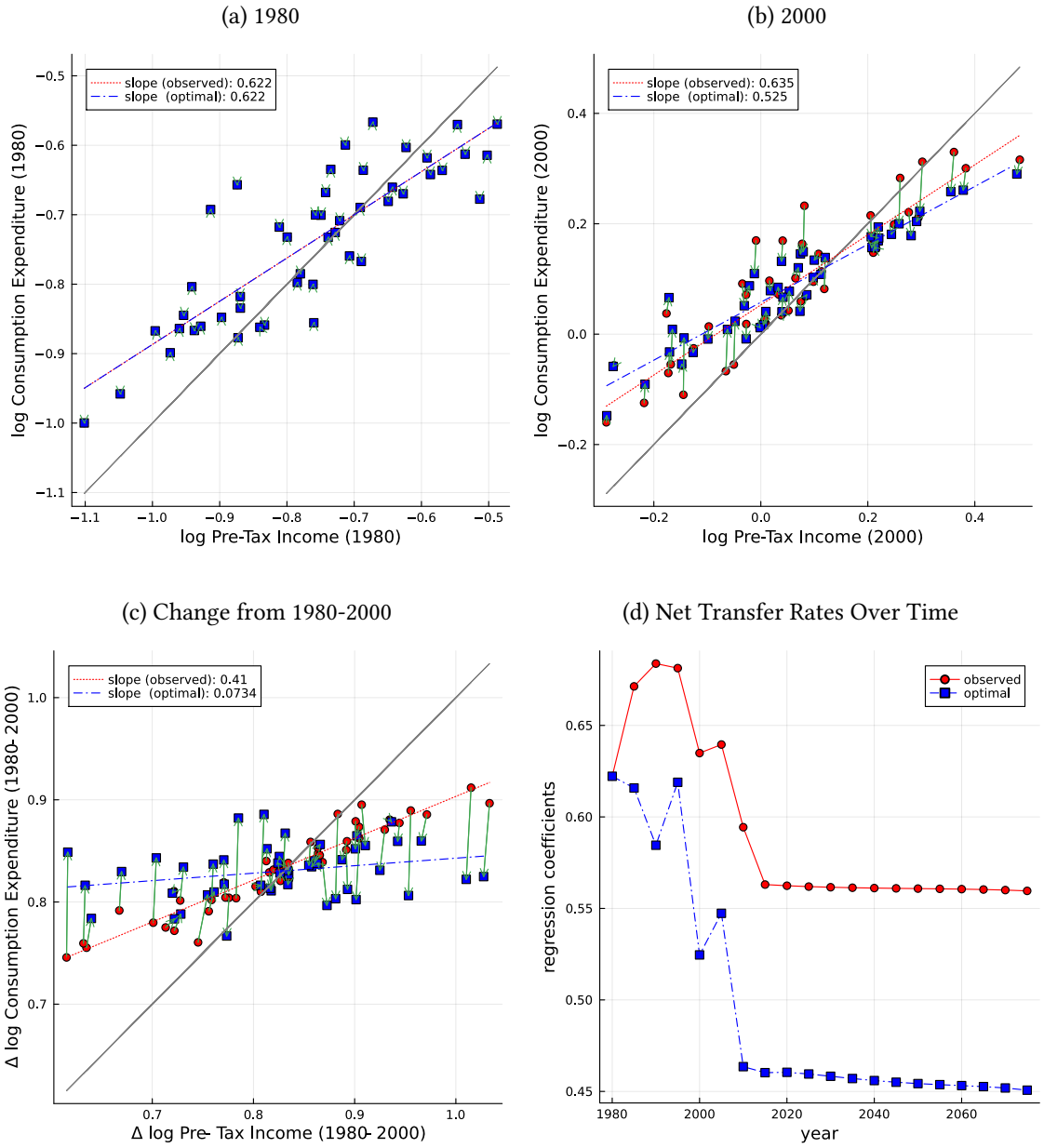
Finally, in Figure 2d, we present the regression coefficients of log of consumption expenditure on log of pre-tax income period-by-period under the observed and optimal allocations. As already mentioned, these regression coefficients are identical between the observed and optimal transfers in 1980. However, from 1980 onward, the regression coefficients under the observed allocation diverge and stay higher than those under the optimal allocation. In the long run, the coefficient in the observed equilibrium is around 0.55, while the coefficient under the optimal transfer is around 0.45. Therefore, optimal transfer entails more redistribution from high- to low-income states in the long run.

In Table 3, we provide a sensitivity analysis of these results under alternative calibration. In Row (1), we report the regression coefficients of the log of consumption expenditure on the log of observed pre-tax income in 2000 and 2075. For 2075, we use our assumption about the evolution of fundamentals and transfers as we described in Section 4.4. We also report the regression coefficients of the log difference of consumption expenditure on the log difference of observed pre-tax income from 1980 to 2000.

In Row (2), we report these regression coefficients under optimal transfers prescribed from our baseline specification. Consistent with Figure 2, we find that the optimal transfer entails more redistribution from high-income states to low-income states, and particularly more so from high-income-growth states to low-income-growth states.

In Rows (3) and (4), we report our results under alternative values for the relative risk aversion parameter ρ instead of the log utility ($\rho = 1$). When we assume risk neutrality ($\rho = 0$; Row 3), we find that the regression coefficients are significantly larger than the observed ones, consistent with the interpretation that the insurance benefit arises because of the concavity of the utility function. It is worth noting that the regression coefficients are less than one even under risk neutrality. This pattern arises because of the spatial differences in the consumer price index. That is, in a location where the price index is low, the marginal value of transfer is higher. In our context, poorer states tend to have lower price indexes (Diamond and Moretti 2021), which supports positive net transfers toward these areas even under risk neutrality. When we assume the relative risk aversion of $\rho = 3$ (Row 4), we find that the regression coefficients are smaller than our baseline specification. This finding is consistent with the interpretation that the insurance benefit is higher if agents are more risk-averse.

Figure 2: Observed vs Optimal Transfer in the U.S. Over Time



Notes: These figures compare the patterns of the log of consumption expenditure against the log of pre-tax income for each state, under observed allocation (labeled "observed") and under optimal allocation (labeled "optimal"). Figure 2d presents the regression coefficients of log of consumption expenditure on log of pre-tax income period-by-period under observed and optimal allocation.

In Rows (5) and (6), we report our results under alternative values for the migration elasticity θ instead of our baseline calibration ($\theta = 0.5$). As we increase the migration elasticity to $\theta = 1$ (Row 5), $\theta = 2$ (Row 6), and $\theta = 5$ (Row 7), the regression coefficients of log of consumption expenditure on log of pre-tax income in 2000 and 2075 increase and exceed the coefficients under

observed allocation. This finding is consistent with the interpretation that a higher migration elasticity implies a larger distortion in migration decisions for the same amount of transfer. At the same time, even if we set the migration elasticity to $\theta = 5$ (Row 7), the regression coefficient of log consumption expenditure growth on log pre-tax income growth from 1980 to 2000 is 0.31, which is below the coefficient under the observed allocation (0.41). Therefore, even if we set the migration elasticity to a much higher value than the typical estimates in the existing literature, optimal transfers still entail more redistribution from high-income-growth states to low-income-growth states.

In Rows (8) and (9), we shut down the productivity and amenity spillover, respectively. In both scenarios, we find that the regression coefficients are slightly smaller than the ones in our specification, indicating that the presence of agglomeration externalities favors less redistribution toward low-income states. At the same time, the differences in regression coefficients from our baseline specification are relatively modest, highlighting a larger role of consumption insurance and migration responses in shaping the optimal transfers in our dynamic context.

In Rows (10) and (11), we find that alternative values for the trade elasticities σ have minimal effects on our results. In Rows (12) and (13), we find that the alternative values of the distance elasticity on trade cost used for calibrating trade flows (ϕ) play a minimal role in the regression coefficients. Therefore, conditional on the observed differences of price indices (which are always calibrated at the same value across all specifications), trade frictions across states play a minimal role in governing the shape of optimal transfers.

In Row (13), we find that a higher value of the discount factor ($\beta = 0.98^5$) leads to lower regression coefficients than our baseline specification, especially in the long run (2075). This pattern is consistent with the interpretation that a larger discount factor implies a greater benefit from consumption smoothing.

Finally, in Row (14), we consider a utilitarian Planner (i.e., equal Pareto weights $\varkappa_i = 1$ for all i) instead of imposing the implied Pareto weights to rationalize the observed patterns of transfer in 1980. We find a substantially smaller regression coefficient in 2000 at 0.18, which increases to 0.41 in 2075. These patterns are consistent with the interpretation that a utilitarian Planner will redistribute more toward low-income states starting from the initial period than our baseline specification. However, the gap decreases over time, because the initial location in 1980 does not affect each worker's location in the long run.

5.2 Optimal vs. Observed Population Distribution

Figure 3 shows how the optimal spatial transfer changes the spatial distribution of population across the U.S. In Figure 3a, we show the log change in population size between 1980 and 2000

Table 3: Optimal Net Transfer Rates under Alternative Specifications

		Slope: log Cons. Exp. on log Pre-Tax-and-Transfer Income		
	Specification	2000	2075	Growth 1980-2000
(1)	Observed	0.63	0.56	0.41
(2)	Baseline	0.52	0.45	0.07
(3)	Lower Relative Risk Aversion $\rho = 0$	0.98	0.77	0.82
(4)	Higher Relative Risk Aversion $\rho = 3$	0.48	0.30	-0.02
(5)	Higher Migration Elasticity $\theta = 1$	0.62	0.66	0.15
(6)	Higher Migration Elasticity $\theta = 2$	0.75	0.90	0.25
(7)	Higher Migration Elasticity $\theta = 5$	0.96	1.2	0.31
(8)	No Productivity Spillover $\gamma^Z = 0$	0.52	0.43	0.06
(9)	No Amenity Spillover $\gamma^A = 0$	0.49	0.42	0.05
(10)	Lower Trade Elasticity $\sigma = 3$	0.53	0.45	0.07
(11)	Higher Trade Elasticity $\sigma = 8$	0.52	0.45	0.07
(12)	Lower Dist. Elas. on Trade Cost $\phi = -0.5$	0.52	0.45	0.07
(13)	Higher Dist. Elas. on Trade Cost $\phi = -2$	0.53	0.45	0.07
(14)	Higher Discount Factor $\beta = 0.98^5$	0.52	0.39	0.06
(15)	Utilitarian Planner	0.18	0.41	0.14

Notes: This table reports the regression coefficients of the log of consumption expenditure on the log pre-tax income, in 2000 and 2075, as well as those of the growth rates from 1980 to 2000, under observed allocation (Row 1) and under optimal allocation (Row 2 onwards).

against the log change in pre-tax income during the same period, as directly observed in data ("observed") and under optimal spatial transfers ("optimal"). Figure 3b presents the same patterns for log changes from 1980 to 2075.

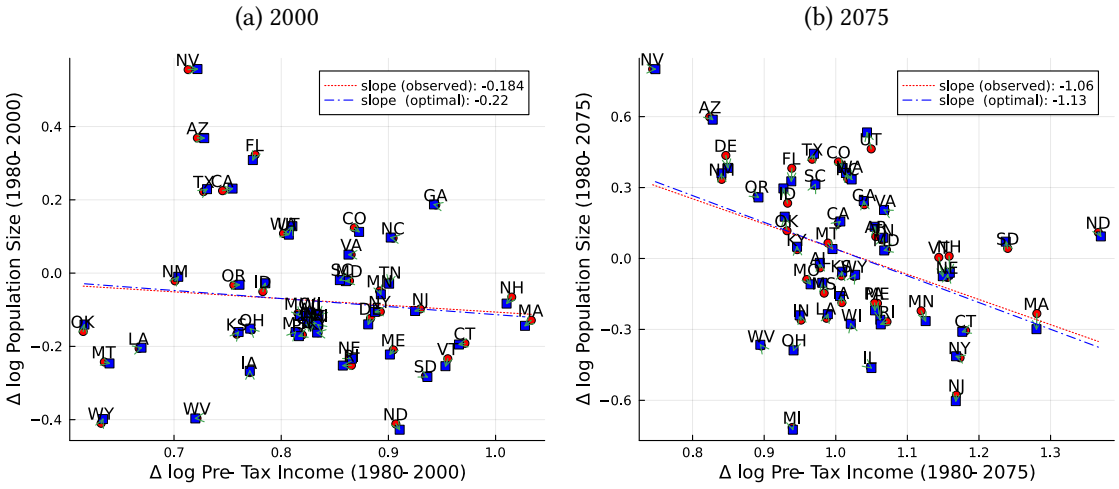
In the data, we find that there is a weak negative correlation between these two variables. In particular, sun-belt states (e.g., Nevada, Arizona, Florida, Texas) have experienced more modest growth in pre-tax income than the average between 1980 and 2000, while increasing their population size at the fastest rates. These patterns are consistent with an interpretation that these sun-belt states experienced a relative decline in housing costs and attracted population, as documented by Glaeser and Gottlieb (2009).

We now compare the observed growth in population size and pre-tax income against those under the optimal policy. In both 2000 and 2075, we find that states with faster growth in nominal income tend to show slower population growth relative to those under the observed spatial transfers. These patterns are consistent with the interpretation that optimal consumption insurance toward regions of slower income growth indeed reallocates the population toward these states.

5.3 Welfare Gains from Optimal Transfers

Table 4 presents the consumption-equivalent welfare gains by implementing optimal spatial transfer relative to those under the observed allocation. In our baseline calibration (Row 1), we find

Figure 3: Observed vs Optimal Population Size in the U.S. Over Time



Notes: These figures compare the patterns of the population size changes as observed in the data (labeled "observed") and under the optimal transfer (labeled "optimal").

welfare gains of 0.05 percent by implementing the optimal spatial transfer. While this is a significant gain considering the size of the U.S. economy and population, the welfare gains are smaller by an order of magnitude compared to the welfare gains from optimal policy in a static framework (Fajgelbaum and Gaubert 2020, Rossi-Hansberg et al. 2019). As we further argue below, this difference arises because migration is frictional and it takes time for the effects and the gain of the policy to materialize.

In Rows (2)-(4), we report the welfare gains under higher values for the migration elasticity θ. We find that the welfare gains increase to 0.076 percent with $\theta = 1$ (Row 2), 0.16 percent with $\theta = 2$ (Row 3), and 0.42 percent with $\theta = 5$ (Row 4). These results are consistent with the interpretation that a larger migration elasticity implies that the population reallocates more quickly as a response to spatial transfers and hence there is significantly less discounting of the welfare gains.

In Rows (5) and (6), we report how our results change by shutting down productivity and amenity spillovers. We find that the welfare gains are 0.05 percent without productivity spillovers and 0.058 percent without amenity spillovers, which are similar to 0.05 percent in our baseline scenario. Therefore, in our context, we find that the welfare gains arising from the agglomeration externality are substantially smaller than those resulting from the benefit of consumption insurance.

In Rows (7) and (8), we find that alternative values for the trade elasticities σ have minimal effects on our results. In Rows (9) and (10), we also find that alternative values for ϕ, the elasticity of trade flows on geographic distance, have modest effects on our results.

In Row (11), we find that a larger discount factor ($\beta = 0.98^5$) leads to 0.055 percent welfare gains, which is slightly higher than our baseline specification. This result is consistent with the interpretation that a higher discount factor leads to more weight in the future after the population responds enough to the policy change.

Table 4: Optimal Net Transfer Rates under Alternative Specifications

	Specification	Welfare Gains (%)
(1)	Baseline	0.050
(2)	Higher Migration Elasticity $\theta = 1$	0.076
(3)	Higher Migration Elasticity $\theta = 2$	0.16
(4)	Higher Migration Elasticity $\theta = 5$	0.42
(5)	No Productivity Spillover $\gamma^Z = 0$	0.050
(6)	No Amenity Spillover $\gamma^A = 0$	0.058
(7)	Lower Trade Elasticity $\sigma = 3$	0.049
(8)	Higher Trade Elasticity $\sigma = 8$	0.050
(9)	Lower Dist. Elas. on Trade Cost $\phi = -0.5$	0.051
(10)	Higher Dist. Elas. on Trade Cost $\phi = -2$	0.039
(11)	Higher Discount Factor $\beta = 0.98^5$	0.055

Notes: This table reports the consumption-equivalent welfare gains in 1980 under optimal spatial transfer after 1980 relative to those under the observed spatial transfer. We assume Pareto weights calibrated in 1980 to rationalize the observed transfers in 1980.

6 Conclusion

We study optimal transfer policy in dynamic spatial equilibrium models with frictional migration and incomplete financial markets. A key policy trade-off is to provide consumption insurance while minimizing the distortion of migration flows. We derive a recursive formula for optimal spatial transfers that strikes this balance. We calibrate our model to U.S. states and find that the U.S. economy would benefit from increased transfers to low-income-growth states. Welfare gains from optimal transfers are substantial but smaller than in a framework abstracting from slow migration adjustment.

Our framework can be applied to many different applications. In ongoing work, we plan to apply our framework to particular sources of regional shocks, such as import competition shocks from China, and examine optimal policy in the presence and absence of those shocks. Such an exercise will clarify how the government should manage regional shocks dynamically, such as gradually decreasing transfers as a response to negative persistent shocks to provide dynamic incentives for out-migration.

References

Acemoglu, D. and P. Restrepo (2020). Robots and Jobs: Evidence from US Labor Markets. *Journal of Political Economy 128*(6), 2188–2244.

Adao, R., A. Costinot, and D. Donaldson (2017). Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade. *American Economic Review 107*(3), 633–689.

Allen, T. and D. Donaldson (2020). Persistence and Path Dependence in the Spatial Economy. Working Paper 28059, National Bureau of Economic Research.

Anderson, S., A. De Palma, and J.-F. Thisse (1992). *Discrete Choice Theory of Product Differentiation*. MIT Press.

Arkolakis, C. and C. Walsh (2023). Clean Growth. *Working Paper*.

Artuç, E., S. Chaudhuri, and J. McLaren (2010). Trade Shocks and Labor Adjustment: A Structural Empirical Approach. *American Economic Review 100*(3), 1008–1045.

Asdrubali, P., B. Sørensen, and O. Yosha (1996). Channels of Interstate Risk Sharing: United States 1963–1990. *The Quarterly Journal of Economics 111*(4), 1081–1110.

Autor, D., D. Dorn, and G. Hanson (2013). The China Syndrome: Local Labor Market Effects of Import Competition in the United States. *American Economic Review 103*(6), 2121–2168.

Baily, M. (1978). Some Aspects of Optimal Unemployment Insurance. *Journal of Public Economics 10*(3), 379–402.

Balboni, C. (2019). In Harm's Way? Infrastructure Investments and the Persistence of Coastal Cities. *Working Paper*.

Balke, N. and T. Lamadon (2022). Productivity Shocks, Long-Term Contracts, and Earnings Dynamics. *American Economic Review 112*(7), 2139–77.

Bartik, T. (2020). Using Place-Based Jobs Policies to Help Distressed Communities. *Journal of Economic Perspectives 34*(3), 99–127.

Bilal, A. and E. Rossi-Hansberg (2023). Anticipating Climate Change Across the United States. Working Paper 31323, National Bureau of Economic Research.

Burdett, K. and M. Coles (2003). Equilibrium Wage-Tenure Contracts. *Econometrica 71*(5), 1377–1404.

Cai, S., L. Caliendo, F. Parro, and W. Xiang (2022). Mechanics of Spatial Growth. Working Paper 30579, National Bureau of Economic Research.

Caliendo, L., M. Dvorkin, and F. Parro (2019). Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock. *Econometrica 87*(3), 741–835.

Cameron, S., S. Chaudhuri, and J. McLaren (2007). Trade Shocks and Labor Adjustment: Theory. Working Paper 13463, National Bureau of Economic Research.

Chari, V. and P. Kehoe (1990). Sustainable Plans. *Journal of Political Economy 98*(4), 783–802.

Chetty, R. (2006). A General Formula for the Optimal Level of Social Insurance. *Journal of Public Economics 90*(10-11), 1879–1901.

Cruz, J.-L. (2021). Global Warming and Labor Market Reallocation. *Working Paper*.

Cruz, J.-L. and E. Rossi-Hansberg (2023). The Economic Geography of Global Warming. *The Review of Economic Studies 91*(2), 899–939.

Davis, M. and J. Gregory (2021). Place-Based Redistribution in Location Choice Models. Working Paper 29045, National Bureau of Economic Research.

Desmet, K., D. K. Nagy, and E. Rossi-Hansberg (2018). The Geography of Development. *Journal of Political Economy 126*(3), 903–983.

Diamond, R. and E. Moretti (2021). Where is Standard of Living the Highest? Local Prices and the Geography of Consumption. Working Paper 29533, National Bureau of Economic Research.

Dubé, J.-P., J. Joo, and K. Kim (2022). Discrete-Choice Models and Representative Consumer Theory. Working Paper 30130, National Bureau of Economic Research.

Dvorkin, M. (2023). Heterogeneous Agents Dynamic Spatial General Equilibrium. *Working Paper*.

Fajgelbaum, P. and C. Gaubert (2020). Optimal Spatial Policies, Geography, and Sorting. *The Quarterly Journal of Economics 135*(2), 959–1036.

Farhi, E., C. Sleet, I. Werning, and S. Yeltekin (2012). Non-Linear Capital Taxation Without Commitment. *The Review of Economic Studies 79*(4), 1469–1493.

Gaubert, C., P. Kline, D. Vergara, and D. Yagan (2021). Trends in US Spatial Inequality: Concentrating Affluence and a Democratization of Poverty. *AEA Papers and Proceedings 111*, 520–525.

Gaubert, C., P. Kline, and D. Yagan (2021). Place-Based Redistribution. Working Paper 28337, National Bureau of Economic Research.

Giannone, E., Q. Li, N. Paixão, and X. Pang (2023). Unpacking Moving. *Working Paper*.

Giannone, E., Y. Miyauchi, N. Paixão, X. Pang, and Y. Suzuki (2023). Living in a Ghost Town: The Geography of Depopulation and Aging. *Working Paper*.

Glaeser, E. and J. Gottlieb (2008). The Economics of Place-Making Policies. Working Paper 14373, National Bureau of Economic Research.

Glaeser, E. and J. Gottlieb (2009). The Wealth of Cities: Agglomeration Economies and Spatial Equilibrium in the United States. *Journal of Economic Literature 47*(4), 983–1028.

Glaeser, E. and E. Luttmer (2003). The Misallocation of Housing Under Rent Control. *American Economic Review 93*(4), 1027–1046.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014). Optimal Taxes on Fossil Fuel in General Equilibrium. *Econometrica 82*(1), 41–88.

Golosov, M., A. Tsyvinski, and N. Werquin (2016). Recursive Contracts and Endogenously Incomplete Markets. *Handbook of Macroeconomics 2*, 725–841.

Greaney, B. (2023). Homeownership and the Distributional Effects of Uneven Regional Growth. *Working Paper*.

Hanson, G. (2023). Local Labor Market Impacts of the Energy Transition: Prospects and Policies. Working Paper 30871, National Bureau of Economic Research.

Hazell, J., J. Herreño, E. Nakamura, and J. Steinsson (2022). The Slope of the Phillips Curve: Evidence from U.S. States. *The Quarterly Journal of Economics 137*(3), 1299–1344.

Howard, G. and H. Shao (2022). Internal Migration and the Microfoundations of Gravity. *Working Paper*.

Kennan, J. and J. Walker (2011). The Effect of Expected Income on Individual Migration Decisions. *Econometrica 79*(1), 211–251.

Klein, P., P. Krusell, and J.-V. Rios-Rull (2008). Time-Consistent Public Policy. *The Review of Economic Studies 75*(3), 789–808.

Kleinman, B., E. Liu, and S. Redding (2023). Dynamic Spatial General Equilibrium. *Econometrica 91*(2), 385–424.

Kline, P. and E. Moretti (2014a). Local Economic Development, Agglomeration Economies, and the Big Push: 100 Years of Evidence from the Tennessee Valley Authority. *The Quarterly Journal of Economics 129*(1), 275–331.

Kline, P. and E. Moretti (2014b). People, Places, and Public Policy: Some Simple Welfare Economics of Local Economic Development Programs. *Annual Review of Economics 6*(1), 629–662.

Kydland, F. and E. Prescott (1977). Rules Rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy 85*(3), 473–491.

Melo, P., D. Graham, and R. Noland (2009). A Meta-Analysis of Estimates of Urban Agglomeration Economies. *Regional Science and Urban Economics 39*(3), 332–342.

Mirrlees, J. (1972). The Optimum Town. *The Swedish Journal of Economics*, 114–135.

Moretti, E. (2012). *The New Geography of Jobs*. Houghton Mifflin Harcourt.

Pang, X. and P. Sun (2023). Moving into Risky Floodplains: The Spatial Implications of Flood Relief Policies. *Working Paper*.

Peters, M. (2022). Market Size and Spatial Growth—Evidence From Germany's Post-War Population Expulsions. *Econometrica 90*(5), 2357–2396.

Rossi-Hansberg, E., P.-D. Sarte, and F. Schwartzman (2019). Cognitive Hubs and Spatial Redistribution. Working Paper 26267, National Bureau of Economic Research.

Online Appendix for "Optimal Dynamic Spatial Policy" (Not for Publication)

Eric Donald, Masao Fukui, Yuhei Miyauchi

June 2023

A Theory Derivations and Proofs **1**
A.1 Proof of Lemma 1 . . . 1
A.2 Proof of Proposition 1 . . . 3
A.3 First-Best Allocation . . . 6
A.4 Complete Financial Markets and Proof of Corollary 1 . . . 9

B Theory Extensions **13**
B.1 Dynamic and Cross-Regional Agglomeration Externalities . . . 13
B.2 Heterogeneous Household Types . . . 14
B.3 Origin-Destination Specific Transfer . . . 20

C Numerical Illustrations **24**

D Appendix for Parametric Model **27**

E Data and Calibration Appendix **28**
E.1 Calibration of Trade Flows . . . 28
E.2 Calibration of Bilateral Migration Frictions . . . 29

A Theory Derivations and Proofs

A.1 Proof of Lemma 1

Economy with Heterogeneous Preferences. Consider a problem of households living in location j at time t, deciding where to migrate. We index each individual by $\omega \in [0,1]$, and $\{\epsilon_{jk}(\omega)\}_k$ denote the preference draw of individual ω. Each individual solves the following problem:

$$\begin{aligned} v_{jt}(\omega) = u(C_{jt}, a_{jt}) + \max_{\{\mathbb{I}_{jkt}(\omega)\}_k} \sum_k \mathbb{I}_{jkt}(\omega) \left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] \\ \text{s.t.} \quad \sum_k \mathbb{I}_{jkt}(\omega) = 1, \end{aligned} \tag{A.1}$$

where $\mathbb{I}_{jkt}(\omega) \in \{0,1\}$ is an indicator function for location choice of individual ω. The fraction of individuals migrating from j to k is given by

$$\mu_{jkt} = \int_0^1 \mathbb{I}_{jkt}(\omega) d\omega. \tag{A.2}$$

Economy with Representative Agent. Define the following function:

$$\begin{aligned} \psi_{jkt}(\{\mu_{jkt}\}_k) = - \max_{\{\mathbb{I}_{jkt}(\omega)\}_{\omega,k}} \int_0^1 \sum_k (\chi_{jkt+1} + \epsilon_{jk}(\omega)) \mathbb{I}_{jkt}(\omega) d\omega \\ \text{s.t.} \quad \int_0^1 \mathbb{I}_{jkt}(\omega) d\omega = \mu_{jkt} \\ \sum_k \mathbb{I}_{jkt}(\omega) = 1. \end{aligned} \tag{A.3}$$

The representative agent solves

$$V_{jt} = u(C_{jt}, a_{jt}) + \max_{\{\mu_{jkt}\}_k} \sum_k \mu_{jkt} \beta V_{kt+1} - \psi_{jkt}(\{\mu_{jkt}\}_k) \tag{A.4}$$

Equivalence Result. We formally restate the equivalence result of Lemma 1 as follows.

Lemma. *Suppose $\{\mathbb{I}_{jkt}(\omega)\}_k$ solves (A.1) for all ω. Then, $\{\mu_{jkt}\}_k$, given by (A.2), solves (A.4). Conversely, suppose $\{\mu_{jkt}\}_k$ solves (A.4). Then $\{\mathbb{I}_{jkt}(\omega)\}_{\omega,k}$, given by the solution to (A.3) associated with $\{\mu_{jkt}\}_k$, solves (A.1) for almost all ω. Moreover, the expected utility in the economy with*

heterogeneous preferences equals the utility of the representative agent:

$$\int_0^1 v_{jt}(\omega)d\omega = V_{jt}$$

Proof. We prove the first part. Suppose to the contrary, there exists $\{\tilde{\mu}_{jkt}\}_k$ such that

$$\sum_k \tilde{\mu}_{jkt}\beta V_{kt+1} - \psi_{jkt}(\{\tilde{\mu}_{jkt}\}_k) > \sum_k \mu_{jkt}V_{kt+1} - \psi_{jk}(\{\mu_{jkt}\}_k). \tag{A.5}$$

Let $\{\tilde{\mathbb{I}}_{jkt}(\omega)\}_{\omega,k}$ denote the solution to (A.3) associated with $\{\tilde{\mu}_{jkt}\}_k$. Plugging into (A.5),

$$\int_0^1 \sum_k \tilde{\mathbb{I}}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] d\omega > \int_0^1 \sum_k \mathbb{I}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] d\omega, \tag{A.6}$$

where $\sum_k \tilde{\mathbb{I}}_{jkt}(\omega) = 1$ and $\sum_k \mathbb{I}_{jkt}(\omega) = 1$ for all ω. However, this is a contradiction because by our presumption, for any ω,

$$\sum_k \mathbb{I}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] \geq \sum_k \tilde{\mathbb{I}}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right]$$

for all $\tilde{\mathbb{I}}_{jkt}(\omega)$, which would imply

$$\int_0^1 \sum_k \tilde{\mathbb{I}}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] d\omega \leq \int_0^1 \sum_k \mathbb{I}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] d\omega. \tag{A.7}$$

Now we prove the converse. Suppose to the contrary, there exists $\{\tilde{\mathbb{I}}_{jkt}(\omega)\}_k$ such that

$$\sum_k \tilde{\mathbb{I}}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] > \sum_k \mathbb{I}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] \tag{A.8}$$

and $\sum_k \tilde{\mathbb{I}}_{jkt}(\omega) = 1$ hold for all $\omega \in \Omega$, where $\Omega \subset [0,1]$ and $|\Omega| > 0$. Define

$$\tilde{\mu}_{jkt} = \int_0^1 \tilde{\mathbb{I}}_{jkt}(\omega)d\omega. \tag{A.9}$$

Then

$$\begin{aligned}\sum_k \mu_{jkt}\beta V_{kt+1} - \psi_{jkt}(\{\mu_{jkt}\}_k) &= \int_0^1 \sum_k \mathbb{I}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] d\omega \\ &< \int_0^1 \sum_k \tilde{\mathbb{I}}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] d\omega \\ &\leq \sum_k \tilde{\mu}_{jkt}\beta V_{kt+1} - \psi_{jkt}(\{\tilde{\mu}_{jkt}\}_k).\end{aligned}$$

This is a contradiction that $\{\mu_{jkt}\}_k$ is a solution to (A.4).

We need to show that the expected utility coincides with each other in the two economies. This immediately follows given the above result. Let $\{\mathbb{I}_{jkt}(\omega)\}_{\omega,k}$ be the solution to (A.1) for all ω, and let $\{\mu_{jkt}\}_k$ denote the solution to (A.4). Then

$$\int_0^1 \sum_k \mathbb{I}_{jkt}(\omega)\left[\beta V_{kt+1} + \chi_{jkt+1} + \epsilon_{jk}(\omega)\right] d\omega = \sum_k \mu_{jkt}\beta V_{kt+1} - \psi_{jkt}(\{\mu_{jkt}\}_k). \tag{A.10}$$

□

A.2 Proof of Proposition 1

Instead of solving the original planning problem, we consider the following relaxed version of the problem:

$$\max_{\{\{q_{kjt},\mu_{jkt}\},L_{jt+1},V_{jt}\}_{t\geq 0}} \sum_i \varkappa_i L_{i0} V_{i0} \tag{A.11}$$

$$\text{s.t.} \quad Z_{it}(L_{it})L_{it} = \sum_j \kappa_{ijt} q_{ijt} L_{jt}$$

$$L_{jt+1} = \sum_i L_{it}\mu_{ijt}$$

$$V_{jt} = u\big(c_{jt}(\{q_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1}$$

$$\{\mu_{jkt}\} \in \arg \max_{\{\mu_{jkt}\}:\sum_k \mu_{jkt}=1} u\big(c_{jt}(\{q_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1}.$$

Here, we dropped constraints (1), (3), (2), (8), (11), and (12). We later show that the solution to the above problem can be implemented as an equilibrium that satisfies all these dropped constraints. Let $\tilde{\Lambda}_{it}$ denote the Lagrange multiplier on the resource constraint. Let $\hat{\mu}_{jkt}(\{V_{it+1}\})$ denote the solution to the household's migration problem. Then the dual of the above problem can be written

as

$$\max_{\{\{q_{kjt},\mu_{jkt}\},L_{jt+1},V_{jt}\}_{t\geq 0}} \sum_{t=0}^{\infty}\prod_{s\leq t}\frac{1}{R_s}\sum_i \Lambda_{it}\Big(Z_{it}(L_{it})L_{it}-\sum_j \kappa_{ijt}q_{ijt}L_{jt}\Big) \tag{A.12}$$

$$\begin{aligned}
\text{s.t.}\quad & V_{j0}\geq \underline{V}_{j0}\\
& L_{jt+1}=\sum_i L_{it}\mu_{ijt}\\
& V_{jt}=u\big(c_{jt}(\{q_{ijt}\}),A_{jt}(L_{jt})\big)-\psi_{jt}\big(\{\mu_{jkt}\}\big)+\beta\sum_k \mu_{jkt}V_{kt+1}\\
& \mu_{jkt}=\hat{\mu}_{jkt}(\{V_{it+1}\}),
\end{aligned}$$

where we consider the normalized multiplier $\Lambda_{it}\equiv\tilde{\Lambda}_{it}/\tilde{\Lambda}_{1t}$ for all t and denote $R_t\equiv\tilde{\Lambda}_{1t-1}/\tilde{\Lambda}_{1t}$.[1] Note that this problem has been normalized by $1/\tilde{\Lambda}_{1,0}$.

We solve the recursive form of the above problem as in Golosov, Tsyvinski, and Werquin (2016). The dual of the optimal transfer problem can be recursively rewritten as

$$\mathcal{S}_t\big(\{L_{jt},V_{jt}\}\big)=\max_{\{\{q_{ijt},\mu_{ijt}\},L_{jt+1},V_{jt+1}\}}\sum_i \Lambda_{it}\Big(Z_{it}(L_{it})L_{it}-\sum_j \kappa_{ijt}q_{ijt}L_{jt}\Big)+\frac{1}{R_{t+1}}\mathcal{S}_{t+1}\big(\{L_{jt+1},V_{jt+1}\}\big) \tag{A.13}$$

$$\begin{aligned}
\text{s.t.}\quad & L_{jt+1}=\sum_i L_{it}\mu_{ijt}: \quad \tilde{\Delta}_{jt+1}\\
& V_{jt}\leq u\big(c_{jt}(\{q_{ijt}\}),A_{jt}(L_{jt})\big)-\psi_{jt}\big(\{\mu_{jkt}\}\big)+\beta\sum_k \mu_{jkt}V_{kt+1}: \quad \Xi_{jt}\\
& \mu_{ijt}=\hat{\mu}_{ijt}\big(\{V_{kt+1}\}\big): \quad \Omega_{ijt},
\end{aligned}$$

where next to each constraint is the corresponding multiplier.

First, taking the FOC with respect to q_{ijt}, we have

$$\Xi_{jt}\partial_c u_{jt}\frac{\partial c_{jt}}{\partial q_{ijt}}=\Lambda_{it}\kappa_{ijt}L_{jt}. \tag{A.14}$$

Denote by $\tilde{P}_{jt}$ the Planner's ideal price index, defined with respect to shadow prices, for consumption in location j, [2] then we have

$$\Xi_{jt}=\frac{\tilde{P}_{jt}L_{jt}}{\partial_c u_{jt}}. \tag{A.15}$$

[1] We also take the normalization that $R_0=1$.

[2] That is, $\tilde{P}_{jt}=\min_{\{q_{ijt}\}}\big[\sum_i \Lambda_{it}\kappa_{ijt}q_{ijt}\big]$ $s.t.$ $c_{jt}\big(\{q_{ijt}\}\big)\geq 1$.

Next, taking the FOC with respect to L_{jt+1}, we have

$$\frac{1}{R_{t+1}}\frac{\partial S_{t+1}}{\partial L_{jt+1}} = \tilde{\Delta}_{jt+1} \equiv \frac{1}{R_{t+1}}\Delta_{jt+1}. \tag{A.16}$$

Taking the FOC with respect to V_{jt+1}, we have

$$\sum_i \sum_k \Omega_{ikt}\frac{\partial \hat{\mu}_{ikt}}{\partial V_{jt+1}} + \beta \sum_i \Xi_{it}\mu_{ijt} = -\frac{1}{R_{t+1}}\frac{\partial S_{t+1}}{\partial V_{jt+1}}. \tag{A.17}$$

Taking the FOC with respect to μ_{ijt}, we have

$$\begin{aligned}\Omega_{ijt} &= \frac{1}{R_{t+1}}\Delta_{jt+1}L_{it} + \Xi_{it}\Big(\beta V_{jt+1} - \frac{\partial \psi_{it}}{\partial \mu_{ijt}}\Big) \\ &= \frac{1}{R_{t+1}}\Delta_{jt+1}L_{it} + \Xi_{it}\zeta_{it},\end{aligned} \tag{A.18}$$

where the second line comes from household optimization Equation (A.26).

The envelope conditions are

$$\frac{\partial \mathcal{S}_t}{\partial L_{jt}} = \Lambda_{jt}\big(Z_{jt} + \frac{dZ_{jt}}{dL_{jt}}L_{jt}\big) - \sum_i \Lambda_{it}\kappa_{ijt}q_{ijt} + \Xi_{jt}\partial_a u_{jt}\frac{dA_{jt}}{dL_{jt}} + \frac{1}{R_{t+1}}\sum_k \mu_{jkt}\Delta_{kt+1} \tag{A.19}$$

$$\frac{\partial \mathcal{S}_t}{\partial V_{jt}} = -\Xi_{jt}. \tag{A.20}$$

Combining Equations (A.15), (A.17), (A.18), and (A.20), as well as noting that $\sum_k \frac{\partial \hat{\mu}_{ikt}}{\partial V_{jt}} = 0$, we have

$$\begin{aligned}&\frac{1}{R_{t+1}}\sum_i\sum_k \Delta_{kt+1}L_{it}\frac{\partial \hat{\mu}_{ikt}}{\partial V_{jt+1}} + \beta\sum_i \frac{P_{it}L_{it}}{\partial_c u_{it}}\mu_{ijt} = \frac{1}{R_{t+1}}\frac{P_{jt+1}L_{jt+1}}{\partial_c u_{jt+1}} \\ &\qquad\Leftrightarrow \quad \sum_i\sum_k \Delta_{kt+1}L_{it}\frac{\partial \hat{\mu}_{ikt}}{\partial V_{jt+1}} = \sum_i L_{it}\mu_{ijt}\Big(\frac{P_{jt+1}}{\partial_c u_{jt+1}} - \frac{\beta R_{t+1}P_{it}}{\partial_c u_{it}}\Big) \\ &\qquad\Leftrightarrow \quad -\sum_i L_{it}\sum_k \Delta_{kt+1}\frac{1}{P_{jt+1}}\frac{\partial \hat{\mu}_{ikt}}{\partial c_{jt+1}} = \sum_i L_{it}\mu_{ijt}\Big(\frac{\beta R_{t+1}\partial_c u_{jt+1}/P_{jt+1}}{\partial_c u_{it}/P_{it}} - 1\Big),\end{aligned} \tag{A.21}$$

where $\frac{\partial \hat{\mu}_{ikt}}{\partial c_{jt+1}} \equiv \frac{\partial \hat{\mu}_{ikt}}{\partial V_{jt+1}}\partial_c u_{jt}$. This gives us Equation (14) of Proposition 1. We remove the hat notation for movement flows in Proposition 1 to simplify notation.

Now we show that the above solution can be supported as an equilibrium with transfers $\tau_{jt} = P_{jt}c_{jt} - w_{jt} - s_{jt}$. Supporting wages and prices are set according to $w_{it} = \Lambda_{it}Z_{it}$ and $p_{ijt} = \kappa_{ijt}w_{it}/Z_{it}$, so the producer price of location 1, w_{1t}/Z_{1t}, is set as the numeraire in each period. Thus, the Planner and household ideal price indices coincide, and since (A.14) is equiv-

alent to the household's consumption FOC, the household's equilibrium consumption allocation coincides with that of the Planner. Therefore, (1), (2), and (8) are satisfied. Since all of the resource constraints and household budget constraints are satisfied by construction, the government budget balances by Walras' law, implying (11) and (12) are satisfied. We can then interpret R_t as the Planner's marginal rate of substitution with respect to the numeraire good across time.

To derive the formula for Δ_{jt}, simply combine Equations (A.15), (A.16), and (A.19) and evaluate at equilibrium prices to achieve

$$\Delta_{jt} = w_{jt} - P_{jt}c_{jt} + w_{jt}\gamma^z_{jt} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}} P_{jt} A_{jt} \gamma^a_{jt} + \frac{1}{R_{t+1}} \sum_k \mu_{jkt} \Delta_{kt+1}. \tag{A.22}$$

□

A.3 First-Best Allocation

In this section, we characterize the first-best planning problem, in which the Planner can directly control consumption and migration flows. For the sake of exposition, we will consider the case of a utilitarian Planner. Utilitarianism allows us to economize on notation while still illustrating the basic features of the first-best allocation.[3] Unlike in the planning problem of Section 3, direct control over the allocation allows the Planner to separate migration flows from the consumption offered in each location. While the information requirements of the first-best are not realistic, it provides us a useful benchmark with an important lesson: when the Planner can choose consumption separately from migration flows, they equalize the marginal resource cost of consumption across locations. When this separation is impossible, as in Section 3, this introduces a trade-off between equalizing marginal utility and efficient migration.

Definition A.1 (Utilitarian First-Best Planning Problem)**.** Starting with initial state variables

[3] In the more general case, where we sketch the entire initial Pareto frontier, the Planner may allocate different consumption and migration choices on the basis of initial location to otherwise identical members of the population. Thus, this requires the Planner to keep track of everyone's initial location, which substantially complicates the notation of the planning problem.

$\{L_{j0}\}$, the Planner solves

$$\begin{aligned}
\max_{\{\{q_{kjt},\mu_{jkt}\},L_{jt+1},V_{jt}\}_{t\geq 0}} \quad & \sum_j L_{j0}V_{j0} \\
\text{s.t.} \quad & \sum_j \kappa_{ijt}q_{ijt}L_{jt} = Z_{it}(L_{it})L_{it} \\
& L_{jt} = \sum_i L_{it-1}\mu_{ijt-1} \\
& V_{jt} = u\big(c_{jt}(\{q_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta\sum_k \mu_{jkt}V_{kt+1} \\
& \sum_j \mu_{ijt} = 1.
\end{aligned} \tag{A.23}$$

The solution to this problem has the following characteristics.

Proposition A.1. *The utilitarian first-best allocation satisfies:*

(i) Perfect insurance. That is, the marginal utility of consumption in time t is set according to

$$\frac{\partial_c u_{jt}}{\tilde{P}_{jt}} = \frac{\partial_c u_{kt}}{\tilde{P}_{kt}} \quad \forall j,k, \tag{A.24}$$

where $\tilde{P}_{jt}$ is the Planner's ideal shadow price index for consumption in location j.

(ii) The movement flow condition for sending location i is set according to

$$\tilde{\beta}_{t+1}\Big(V_{jt+1} - \frac{1}{\beta}\frac{\partial\psi_{it}}{\partial\mu_{ijt}}\Big) + \Delta_{jt+1} = \tilde{\beta}_{t+1}\Big(V_{kt+1} - \frac{1}{\beta}\frac{\partial\psi_{it}}{\partial\mu_{ikt}}\Big) + \Delta_{kt+1} \quad \forall j,k, \tag{A.25}$$

where $\tilde{\beta}_{t+1}$ is a discount term defined below, and Δ_{jt} is given recursively by Equation (13), defined with respect to the Planner's shadow prices.

The proof of Proposition A.1 is below, but we have established the most important feature of the first-best allocation: perfect insurance. The Planner's ability to smooth consumption comes from the ability to control migration flows separately from consumption. Once migration flows have taken place, the distribution of consumption across locations is simply a static problem to maximize a concave objective. In that setting, the objective's margins will be equalized up to real differences in marginal cost, as a failure to do so is akin to misallocation in the social objective.

For migration flows, the Planner equalizes, for a given sending location, the marginal benefit of additional population across prospective locations. This is a similar condition to that of the household (A.26) where the marginal mover is indifferent between prospective locations. However, the Planner's condition includes the marginal fiscal externality of additional population Δ_{jt}.

Thus, the household decision has a wedge in that they do not internalize the fiscal externality of their location choices.

It is also clear that this first-best allocation cannot be generally achieved under the spatial transfer policy. To see this, note that the first-order condition of the migration decision (7), which the Planning problem in Definition 2 takes as an incentive compatibility constraint, is given by

$$\beta V_{kt+1} - \frac{\partial \psi_{it}}{\partial \mu_{ikt}} = \beta V_{jt+1} - \frac{\partial \psi_{it}}{\partial \mu_{ijt}}. \tag{A.26}$$

This equation is not compatible with Equation (A.25) unless $\Delta_{jt} = \Delta_{kt}$ for all j, k.

Finally, a couple of comments are in order. In principle, the first-best choice set includes the possibility of consumption and movement flows that depend on the entire history of idiosyncratic shocks for each member of the household. However, given our assumption that preference shocks are separable from consumption and iid, consumption only needs to condition on contemporaneous location, and migration decisions only need to condition on contemporaneous preference shocks. Next, because the first-best describes an allocation, not an equilibrium, the conditions of Proposition A.1 are written in terms of the Planner's shadow prices. However, if the first-best allocation were to be implemented as an equilibrium, these shadow prices would suffice as supporting prices, just as in Proposition 1.

Proof. We follow the same strategy as in Appendix A.2 and solve the Planner's recursive dual problem, except now we can drop the incentive compatibility constraint and replace it with an accounting constraint. The Planner solves

$$\mathcal{S}_t\big(\{L_{jt}, V_{jt}\}\big) = \max_{\{\{q_{ijt}, \mu_{ijt}\}, L_{jt+1}, V_{jt+1}\}} \sum_i \Lambda_{it}\Big(Z_{it}(L_{it})L_{it} - \sum_j \kappa_{ijt} q_{ijt} L_{jt}\Big) + \frac{1}{R_{t+1}} \mathcal{S}_{t+1}\big(\{L_{jt+1}, V_{jt+1}\}\big) \tag{A.27}$$

$$\text{s.t.} \quad L_{jt+1} = \sum_i L_{it}\mu_{ijt} : \quad \tilde{\Delta}_{jt+1}$$

$$V_{jt} \leq u\big(c_{jt}(\{q_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1} : \quad \Xi_{jt}$$

$$\sum_j \mu_{ijt} = 1 : \quad \Omega_{it}$$

where next to each constraint is the corresponding multiplier.

As before, taking the FOCs with respect to q_{ijt}, L_{jt+1}, V_{jt+1}, and μ_{ijt}, we have

$$\begin{aligned}
\Xi_{jt} &= \frac{\tilde{P}_{jt} L_{jt}}{\partial_c u_{jt}} \\
\frac{1}{R_{t+1}} \frac{\partial S_{t+1}}{\partial L_{jt+1}} &= \tilde{\Delta}_{jt+1} \equiv \frac{1}{R_{t+1}} \Delta_{jt+1} \\
\beta \sum_i \Xi_{it} \mu_{ijt} &= -\frac{1}{R_{t+1}} \frac{\partial S_{t+1}}{\partial V_{jt+1}} \\
\Omega_{it} &= \frac{1}{R_{t+1}} \Delta_{jt+1} L_{it} + \Xi_{it} \big(\beta V_{jt+1} - \frac{\partial \psi_{it}}{\partial \mu_{ijt}} \big),
\end{aligned}$$

where $\tilde{P}_{jt}$ is the Planner's ideal shadow price index for consumption of location j. The envelope conditions again follow Equations (A.19) and (A.20).

Combining the FOC and envelope condition for V_{jt+1} with the law of motion for population, as well as iterating backward, gives us

$$\Xi_{jt} = \Big[\prod_{s \leq t} R_s \Big] \beta^t \frac{L_{jt}}{\tilde{\Lambda}_{10}}. \tag{A.28}$$

Here we have used the result from the primal problem that $\Xi_{j0} = L_{j0}/\tilde{\Lambda}_{10}$, where the factor of proportionality $\tilde{\Lambda}_{10}$ comes from the normalization applied to the dual problem (A.12). Plugging this into the FOC for consumption gives us Equation (A.24) of Proposition A.1.

If we define $\tilde{\beta}_{t+1} \equiv \big[\prod_{s \leq t+1} R_s \big] \beta^{t+1} / \tilde{\Lambda}_{10}$, then plugging equation (A.28) into the FOC for migration flows gives us Equation (A.25) of Proposition A.1.

□

A.4 Complete Financial Markets and Proof of Corollary 1

In this section, we study the case where the regional risk-sharing mechanism is complete. There exists a representative, competitive financial intermediary that offers state-contingent contracts to households. Crucially, the financial intermediary has a perfect enforcement technology and can observe the entire history of households' idiosyncratic preference shocks, so contracts are complete. These contracts specify a sequence of consumption and migration decisions which conditions on households' history of preference shocks. In exchange, households are guaranteed an expected utility level of $\underline{V}$.

We assume that contracting takes place before the realization of any preference shocks and before households learn their initial location. There is a known distribution of initial locations, but households do not know their own initial location. This guarantees our assumption that pay-

ments $\{s_{jt}\}$ do not vary within location as all households undertake contracts from the same position. Finally, given our assumption that preference shocks are separable from consumption and iid, consumption only needs to condition on contemporaneous location, and migration decisions only need to condition on contemporaneous preference shocks. The financial intermediary then faces the same movement cost function as the representative household from Lemma 1 as they select movers from the population so as to minimize the overall opportunity cost of moving for a given array of movement flows.

The optimal contract solves

$$\max_{\{\{\mu_{ijt}\},c_{jt},L_{jt+1},V_{jt}\}_{t\geq 0}} \sum_{t=0}^{\infty} \prod_{s\leq t} \frac{1}{R_s} \sum_j \big(w_{jt} + \tau_{jt} - P_{jt}c_{jt}\big) L_{jt} \tag{A.29}$$

$$\text{s.t.} \quad \sum_j L_{j0}V_{j0} \geq \underline{V} : \quad \hat{\Psi}$$

$$L_{jt+1} = \sum_i L_{it}\mu_{ijt} : \quad \Big[\prod_{s\leq t} \frac{1}{R_s}\Big]\hat{\Delta}_{jt+1}$$

$$V_{jt} = u(c_{jt}, a_{jt}) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1} : \quad \Big[\prod_{s\leq t} \frac{1}{R_s}\Big]\hat{\Xi}_{jt}$$

$$\sum_j \mu_{ijt} = 1 : \quad \Big[\prod_{s\leq t} \frac{1}{R_s}\Big]\hat{\Omega}_{it},$$

where next to each constraint is the corresponding multiplier with any normalization factor in square brackets. Hat notation signifies that the multiplier corresponds to the financial intermediary's problem, as opposed to the Planner's problem.

First, taking the FOC with respect to c_{jt}, we have

$$P_{jt}L_{jt} = \hat{\Xi}_{jt}\partial_c u_{jt}. \tag{A.30}$$

Next, taking the FOC with respect to L_{jt}, we have

$$\hat{\Delta}_{jt} = w_{jt} + \tau_{jt} - P_{jt}c_{jt} + \frac{1}{R_{t+1}} \sum_k \mu_{jkt}\hat{\Delta}_{kt+1}. \tag{A.31}$$

Taking the FOC with respect to V_{jt}, we have

$$\frac{1}{R_t}\hat{\Xi}_{jt} = \beta \sum_i \hat{\Xi}_{it-1}\mu_{ijt-1} \tag{A.32}$$

$$\hat{\Xi}_{j0} = L_{j0}\hat{\Psi}. \tag{A.33}$$

Taking the FOC with respect to μ_{ijt}, we have

$$\hat{\Omega}_{it} = \frac{1}{R_{t+1}} \hat{\Delta}_{jt+1} L_{it} + \hat{\Xi}_{it} \Big(\beta V_{jt+1} - \frac{\partial \psi_{it}}{\partial \mu_{ijt}} \Big). \tag{A.34}$$

Combining Equations (A.32) and (A.33) with the law of motion for population gives us

$$\hat{\Xi}_{jt} = \Big[\prod_{s \leq t} R_s \Big] \beta^t \hat{\Psi} L_{jt}. \tag{A.35}$$

Plugging Equation (A.35) into Equation (A.30) gives us complete risk sharing:

$$\frac{\partial_c u_{jt}}{P_{jt}} = \frac{1}{\big[\prod_{s \leq t} R_s \big] \beta^t} \frac{1}{\hat{\Psi}}. \tag{A.36}$$

To determine optimal transfers in the context of complete markets, the Planner must satisfy an incentive compatibility constraint that reflects complete risk sharing. However, we will solve a relaxed problem that drops this IC constraint and show that the Planner's solution still satisfies Equation (A.36).

As in Appendix A.2, the Planner solves the problem (B.40). The FOCs for q_{ijt}, L_{jt+1}, and V_{jt+1} again follow Equations (A.15), (A.14), (A.16), and (A.17). The envelope conditions again follow Equations (A.19) and (A.20). Comparing the Planner's consumption FOC (A.14) and the financial intermediary's consumption FOC (A.30), we have that $\Xi_{jt} = \hat{\Xi}_{jt}$ if supporting prices are set such that private price indices equal the Planner's shadow prices.

We will guess that $\tau_{jt} = w_{jt} \gamma^z_{jt} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}} P_{jt} A_{jt} \gamma^a_{jt} - K_t$, as in Corollary 1, and verify that this provides a solution to the Planner's problem.[4] Comparing the Planner's population FOC (A.16) and the financial intermediary's population FOC (A.31), we have that $\hat{\Delta}_{jt} = \Delta_{jt} - \mathcal{K}_t$, where $\mathcal{K}_t \equiv K_t + \sum_{\hat{t} \geq 1} \prod_{s=1}^{\hat{t}} \frac{1}{R_{t+s}} K_{t+\hat{t}}$ is the present value of lump-sum taxes. Taking the FOC with respect to μ_{ijt}, we have

$$\begin{aligned} \Omega_{ijt} &= \frac{1}{R_{t+1}} \Delta_{jt+1} L_{it} + \Xi_{it} \Big(\beta V_{jt+1} - \frac{\partial \psi_{it}}{\partial \mu_{ijt}} \Big) && \text{(A.37)} \\ &= \frac{1}{R_{t+1}} \Delta_{jt+1} L_{it} + \hat{\Xi}_{it} \Big(\beta V_{jt+1} - \frac{\partial \psi_{it}}{\partial \mu_{ijt}} \Big) \\ &= \hat{\Omega}_{it} + \frac{1}{R_{t+1}} \mathcal{K}_{t+1} L_{it}, && \text{(A.38)} \end{aligned}$$

where the last line comes from the financial intermediary's optimization Equation (A.34).

[4] By government budget balance (11), the lump-sum tax is equal to the average externality: $K_t = \sum_k L_{kt} \big(w_{kt} \gamma^z_{kt} + \frac{\partial_a u_{kt}}{\partial_c u_{kt}} P_{kt} A_{kt} \gamma^a_{kt} \big)$.

Combining Equations (A.15), (A.17), (A.37), and (A.20), as well as noting that $\sum_k \frac{\partial \hat{\mu}_{ikt}}{\partial V_{jt}} = 0$, we have

$$\sum_i L_{it}\mu_{ijt}\left(\frac{\beta R_{t+1}\partial_c u_{jt+1}/P_{jt+1}}{\partial_c u_{it}/P_{it}} - 1\right) = 0, \tag{A.39}$$

which will be satisfied if the Planner selects consumption according to complete risk sharing (A.36). Hence, the requirement that consumption provides complete risk sharing is slack, so our relaxation was valid.

Therefore, the transfers described in Corollary 1 solve the Planner's problem with supporting wages and prices set according to $w_{it} = \Lambda_{it}Z_{it}$ and $p_{ijt} = \kappa_{ijt}w_{it}/Z_{it}$. Furthermore, this transfer policy implements the utilitarian first-best allocation as a competitive equilibrium. That is, the utilitarian Planner's first-best optimality conditions (A.24) and (A.25) coincide with those of the financial intermediary when transfers correct externalities.

Risk Neutrality and Equalization of Consumption Prices An alternative setup for the complete market is to assume risk neutrality (constant marginal utility $\partial_c u_{jt}$ across locations and time) and equalized consumption prices across locations ($P_{it} = P_{jt}$ for all $i, j \in J$). As noted in the main text, linear utility functions and equalized price indices across locations imply that the RHS of Equation (14) is equal to zero. Policy should then equalize the fiscal externality across locations as this guarantees that the LHS of Equation (14) is equal to zero.

Equalization of the fiscal externality at all points in time implies that

$$\tau_{jt} = \tau_{1t} + w_{jt}\gamma^z_{jt} - w_{1t}\gamma^z_{1t} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}}P_{jt}A_{jt}\gamma^a_{jt} - \frac{\partial_a u_{1t}}{\partial_c u_{1t}}P_{1t}A_{1t}\gamma^a_{1t}. \tag{A.40}$$

Plugging this into the government budget constraint (11), we have

$$\tau_{jt} = w_{jt}\gamma^z_{jt} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}}P_{jt}A_{jt}\gamma^a_{jt} - \sum_k L_{kt}\left(w_{kt}\gamma^z_{kt} + \frac{\partial_a u_{kt}}{\partial_c u_{kt}}P_{kt}A_{kt}\gamma^a_{kt}\right), \tag{A.41}$$

which gives us Equation (16) of Corollary 1. One can see that the budget-balancing transfer K_t is equal to the average externality in each period.

To see that this transfer policy implements the utilitarian first-best allocation as a competitive equilibrium, consider the utilitarian Planner's first-best optimality conditions (A.24) and (A.25). Equation (A.24) is satisfied by assumption. Plugging the condition for household movement flows (A.26) into (A.25), we have equalization of the fiscal externality $\Delta_{jt} = \Delta_t \ \forall t$. This is the exact condition the transfers of Equation (A.41) are designed to achieve.

□

B Theory Extensions

B.1 Dynamic and Cross-Regional Agglomeration Externalities

We follow the same strategy as in Appendix A.2 and solve a relaxation of the Planner's recursive dual problem. Again, one can back out supporting prices that implement the solution as an equilibrium. The Planner solves

$$\mathcal{S}_t\big(\{L_{jt}, V_{jt}, \bar{Z}_{jt}\}\big) = \max_{\{\{q_{ijt},\mu_{ijt}\},L_{jt+1},V_{jt+1}\}} \sum_i \Lambda_{it}\Big(Z_{it}L_{it} - \sum_j \kappa_{ijt} q_{ijt} L_{jt}\Big) + \frac{1}{R_{t+1}}\mathcal{S}_{t+1}\big(\{L_{jt+1}, V_{jt+1}, \bar{Z}_{jt+1}\}\big) \tag{B.1}$$

$$\begin{aligned}
\text{s.t.} \quad L_{jt+1} &= \sum_i L_{it}\mu_{ijt} : \quad \tilde{\Delta}_{jt+1} \\
V_{jt} &\leq u\big(c_{jt}(\{q_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu_{jkt}\}\big) + \beta \sum_k \mu_{jkt} V_{kt+1} : \quad \Xi_{jt} \\
\mu_{ijt} &= \hat{\mu}_{ijt}\big(\{V_{kt+1}\}\big) : \quad \Omega_{ijt},
\end{aligned}$$

where labor productivity $Z_{jt} = g_{jt}\big(\bar{Z}_{jt}, \{L_{kt}\}\big)$ and fundamental productivity $\bar{Z}_{jt} = h_{jt}\big(\{\bar{Z}_{kt-1}, L_{kt-1}\}\big)$ allow for general externalities. Again, next to each constraint is the corresponding multiplier.

As before, taking the FOCs with respect to q_{ijt}, L_{jt+1}, V_{jt+1}, and μ_{ijt}, we have

$$\begin{aligned}
\Xi_{jt} &= \frac{\tilde{P}_{jt} L_{jt}}{\partial_c u_{jt}} \\
\frac{1}{R_{t+1}}\frac{\partial S_{t+1}}{\partial L_{jt+1}} &= \tilde{\Delta}_{jt+1} \equiv \frac{1}{R_{t+1}}\Delta_{jt+1} \\
\sum_i \sum_k \Omega_{ikt}\frac{\partial \hat{\mu}_{ikt}}{\partial V_{jt+1}} + \beta \sum_i \Xi_{it}\mu_{ijt} &= -\frac{1}{R_{t+1}}\frac{\partial S_{t+1}}{\partial V_{jt+1}} \\
\Omega_{ijt} &= \frac{1}{R_{t+1}}\Delta_{jt+1} L_{it} + \Xi_{it}\zeta_{it},
\end{aligned}$$

where $\tilde{P}_{jt}$ is the Planner's ideal shadow price index for consumption of location j and ζ_{it} is the household's multiplier on their accounting constraint for movement flows.

For the envelope conditions, we have

$$\frac{\partial \mathcal{S}_t}{\partial L_{jt}} = \Lambda_{jt} Z_{jt} - \sum_i \Lambda_{it} \kappa_{ijt} q_{ijt} + \sum_k \Lambda_{kt} \frac{\partial Z_{kt}}{\partial L_{jt}} L_{kt} + \Xi_{jt} \partial_a u_{jt} \frac{\partial A_{jt}}{\partial L_{jt}} + \frac{1}{R_{t+1}} \sum_k \left(\frac{\partial \mathcal{S}_{t+1}}{\partial \bar{Z}_{kt+1}} \frac{\partial \bar{Z}_{kt+1}}{\partial L_{jt}} + \mu_{jkt} \Delta_{kt+1} \right)$$

$$\frac{\partial \mathcal{S}_t}{\partial V_{jt}} = -\Xi_{jt}$$

$$\frac{\partial \mathcal{S}_t}{\partial \bar{Z}_{jt}} = \Lambda_{jt} \frac{\partial Z_{jt}}{\partial \bar{Z}_{jt}} L_{jt} + \frac{1}{R_{t+1}} \sum_k \frac{\partial \mathcal{S}_{t+1}}{\partial \bar{Z}_{kt+1}} \frac{\partial \bar{Z}_{kt+1}}{\partial \bar{Z}_{jt}}.$$

Denote by $\varsigma_{jt} \equiv \frac{\partial \mathcal{S}_t}{\partial \ln(\bar{Z}_{jt})}$, $\gamma_{jt}^{Z,k} \equiv \frac{\partial \ln(z_{jt})}{\partial \ln(L_{kt})}$, $\gamma_{jt}^{Z\bar{Z}} \equiv \frac{\partial \ln(z_{jt})}{\partial \ln(\bar{Z}_{jt})}$, $\gamma_{jt}^{\bar{Z},k} \equiv \frac{\partial \ln(\bar{Z}_{jt})}{\partial \ln(L_{kt-1})}$, and $\gamma_{jt}^{\bar{Z}\bar{Z},k} \equiv \frac{\partial \ln(\bar{Z}_{jt})}{\partial \ln(\bar{Z}_{kt-1})}$. Combining equations in a similar manner as that used to derive Equation (A.21), we have

$$-\sum_i L_{it} \sum_k \Delta_{kt+1} \frac{1}{P_{jt+1}} \frac{\partial \mu_{ikt}}{\partial c_{jt+1}} = \sum_i L_{it} \mu_{ijt} \left(\frac{\beta R_{t+1} \partial_c u_{jt+1} / P_{jt+1}}{\partial_c u_{it} / P_{it}} - 1 \right) \tag{B.2}$$

$$\Delta_{jt} = w_{jt} - P_{jt} c_{jt} + \sum_k \frac{L_{kt}}{L_{jt}} w_{kt} \gamma_{kt}^{Z,j} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}} P_{jt} a_{jt} \gamma_{jt}^{A} + \frac{1}{R_{t+1}} \sum_k \left(\varsigma_{kt+1} \frac{\gamma_{kt+1}^{\bar{Z},j}}{L_{jt}} + \mu_{jkt} \Delta_{kt+1} \right)$$

$$\varsigma_{jt} = w_{jt} L_{jt} \gamma_{jt}^{Z\bar{Z}} + \frac{1}{R_{t+1}} \sum_k \varsigma_{kt+1} \gamma_{kt+1}^{\bar{Z}\bar{Z},j}.$$

Just as in Appendix A.2, setting supporting transfers, wages, and prices according to $\tau_{jt} = P_{jt} c_{jt} - w_{jt} - s_{jt}$, $w_{it} = \Lambda_{it} Z_{it}$, and $p_{ijt} = \kappa_{ijt} w_{it} / Z_{it}$ implements the Planner's allocation as an equilibrium with the producer price of location 1 set as the numeraire.

As argued in the main text, one can see that the logic of Proposition 1 extends to a setting with a more general structure of externalities. The optimality condition for this more general setting, Equation (B.2), has the same interpretation as before; the only substantive difference comes from the additional recursive terms in the fiscal externality which reflect the dynamic nature of the fundamental productivity process.

B.2 Heterogeneous Household Types

We consider an extension to the baseline model by allowing for the presence of ex-ante heterogeneous household types. We assume each household is indexed by type $\theta \in \Theta$.

B.2.1 Competitive Equilibrium

The value function of each household, which is analogous to (1), is now given by

$$V_{jt}^{\theta} = \max_{\{\mu_{jkt}^{\theta}\}} u^{\theta}(c_{jt}^{\theta}, a_{jt}^{\theta}) - \psi_{jt}^{\theta}\big(\{\mu_{jkt}^{\theta}\}\big) + \beta^{\theta} \sum_{k} \mu_{jkt}^{\theta} V_{kt+1}^{\theta}$$
$$\text{s.t.} \quad \sum_{k} \mu_{jkt}^{\theta} = 1.$$

Therefore, the optimal migration choices solve

$$\{\mu_{jkt}^{\theta}\} \in \arg\max_{\{\mu_{jkt}^{\theta}\}} u^{\theta}(c_{jt}^{\theta}, a_{jt}^{\theta}) - \psi_{jt}^{\theta}\big(\{\mu_{jkt}^{\theta}\}\big) + \beta^{\theta} \sum_{k} \mu_{jkt}^{\theta} V_{kt+1}^{\theta}. \tag{B.3}$$
$$\text{s.t.} \quad \sum_{k} \mu_{jkt}^{\theta} = 1.$$

The household's consumption basket is $c_{jt}^{\theta} = c_{jt}^{\theta}\big(\{q_{ijt}^{\theta}\}\big)$, and we assume $c_{jt}^{\theta}\big(\{q_{ijt}^{\theta}\}\big)$ is homogeneous of degree one. The households solve

$$\begin{aligned} \{q_{ijt}^{\theta}\}_i &\in \arg\max_{\{q_{ijt}^{\theta}\}} c_{jt}^{\theta}\big(\{q_{ijt}^{\theta}\}\big) \\ \text{s.t.} \quad \sum_{i} p_{ijt} q_{ijt}^{\theta} &\leq w_{jt}^{\theta} + \tau_{jt}^{\theta} + s_{jt}^{\theta}. \end{aligned} \tag{B.4}$$

The associated price index is

$$P_{jt}^{\theta} = \min_{\{q_{ijt}^{\theta}\}} \big[\sum_{i} p_{ijt} q_{ijt}^{\theta} \big] \quad \text{s.t.} \quad c_{jt}^{\theta}\big(\{q_{ijt}^{\theta}\}\big) \geq 1. \tag{B.5}$$

We allow the spatial transfer of the government to depend on the household's type, τ_{jt}^{θ}. The budget constraint of each household is

$$c_{jt}^{\theta} = \frac{w_{jt}^{\theta} + \tau_{jt}^{\theta} + s_{jt}^{\theta}}{P_{jt}^{\theta}}. \tag{B.6}$$

On the production side, we postulate the production function of each location i as $F_{it}\big(\{Z_{it}^{\theta} L_{it}^{\theta}\}_{\theta}\big)$, where F_{it} is constant returns to scale. The firms choose the labor inputs to maximize profits:

$$\max_{L_{it}^{\theta}} p_{it} F_{it}(\{Z_{it}^{\theta} L_{it}^{\theta}\}_{\theta}) - \sum_{\theta} w_{it}^{\theta} L_{it}^{\theta},$$

where p_{it} denotes the price of locally produced goods. Under perfect competition and constant

returns to scale, the price is equal to the unit cost of production:

$$p_{it} = \min_{\{L_{it}^{\theta}\}_{\theta}} \sum_{\theta} w_{it}^{\theta} L_{it}^{\theta} \quad \text{s.t.} \quad F_{it}\big(\{Z_{it}^{\theta} L_{it}^{\theta}\}_{\theta}\big) \geq 1, \tag{B.7}$$

and the optimality conditions are

$$p_{it} \frac{\partial F_{it}\big(\{Z_{it}^{\theta} L_{it}^{\theta}\}_{\theta}\big)}{\partial (Z_{it}^{\theta} L_{it}^{\theta})} Z_{it}^{\theta} = w_{it}^{\theta}. \tag{B.8}$$

As before, we assume that the presence of iceberg trade costs, κ_{ijt}, when shipped from location i to j at time t. The price of the goods produced in location i faced by the consumers in location j is, therefore,

$$p_{ijt} = p_{it} \kappa_{ijt}. \tag{B.9}$$

Following Fajgelbaum and Gaubert (2020), we allow a general form of within-location spillovers both for amenity and productivity:

$$a_{it}^{\theta} = A_{it}^{\theta}\big(\{L_{it}^{\theta'}\}_{\theta'}\big), \tag{B.10}$$

$$Z_{it}^{\theta} = Z_{it}^{\theta}\big(\{L_{it}^{\theta'}\}_{\theta'}\big). \tag{B.11}$$

The market clearing conditions are

$$\sum_{j} \sum_{\theta} \kappa_{ijt} q_{ijt}^{\theta} L_{jt}^{\theta} = F_{it}\big(\{Z_{it}^{\theta} L_{it}^{\theta}\}_{\theta}\big). \tag{B.12}$$

The population distribution evolves according to

$$L_{jt+1}^{\theta} = \sum_{i} \mu_{ijt}^{\theta} L_{it}^{\theta}. \tag{B.13}$$

The government budget constraint is

$$\sum_{j} \sum_{\theta} \tau_{jt}^{\theta} L_{jt}^{\theta} = 0. \tag{B.14}$$

Likewise, the private transfer satisfies

$$\sum_{j} \sum_{\theta} s_{jt}^{\theta} L_{jt}^{\theta} = 0. \tag{B.15}$$

Given public and private transfers, $\{\tau_{jt}^{\theta}, s_{jt}^{\theta}\}$ that satisfy (B.14) and (B.15) Competitive equilibrium consists of

- prices: $\{\{w_{jt}^{\theta}, P_{jt}^{\theta}\}_{\theta}, \{p_{ijt}\}_i, p_{jt},\},$
- quantities: $\{\{\mu_{ijt}^{\theta}, q_{ijt}^{\theta}\}_i, L_{jt}^{\theta}, a_{jt}^{\theta}, Z_{jt}^{\theta}, c_{jt}^{\theta}\},$

such that equations (B.3)-(B.15) hold.

B.2.2 Planning Problem

The planner's problem in this economy is analogous to Definition 2:

Definition B.1 (Planning Problem with Heterogeneous Households)**.** Given Pareto weights $\{\varkappa_i^{\theta}\}$ attached to individuals of type θ in location i in period 0, the Planner solves

$$\max_{\left\{\{q_{kjt}^{\theta}, \mu_{jkt}^{\theta}, p_{kjt}p_{kt}\}, c_{jt}^{\theta}, L_{jt+1}^{\theta}, V_{jt}^{\theta}, P_{jt}, a_{jt}^{\theta}, w_{jt}^{\theta}, \tau_{jt}^{\theta}, s_{jt}^{\theta}\right\}_{t\geq 0}} \quad \sum_i \sum_{\theta} \varkappa_i^{\theta} L_{i0}^{\theta} V_{i0}^{\theta}$$

$$\text{s.t.} \quad V_{jt}^{\theta} = u^{\theta}(c_{jt}^{\theta}, a_{jt}^{\theta}) - \psi_{jt}^{\theta}\left(\{\mu_{jkt}^{\theta}\}\right) + \beta^{\theta} \sum_k \mu_{jkt}^{\theta} V_{kt+1}^{\theta} \tag{PK}$$

$$\{\mu_{jkt}^{\theta}\} \in \arg \max_{\{\mu_{jkt}^{\theta}\}: \sum_k \mu_{jkt}^{\theta} = 1} u^{\theta}(c_{jt}^{\theta}, a_{jt}^{\theta}) - \psi_{jt}^{\theta}\left(\{\mu_{jkt}^{\theta}\}\right) + \beta^{\theta} \sum_k \mu_{jkt}^{\theta} V_{kt+1}^{\theta} \tag{IC}$$

(B.4) - (B.15).

The set of solutions to this problem given arbitrary Pareto weights $\{\varkappa_i^{\theta}\}$ defines the Pareto frontier.

As in the baseline, we consider the following relaxed dual problem in tackling the above problem:

$$\max_{\left\{\{q_{kjt}^{\theta}, \mu_{jkt}^{\theta}\}, L_{jt+1}^{\theta}, V_{jt}^{\theta}\right\}_{t\geq 0}} \sum_{t=0}^{\infty} \prod_{s\leq t} \frac{1}{R_s} \sum_i \Lambda_{it} \left(F_{it}(\{Z_{it}^{\theta}(\{L_{it}^{\theta'}\}_{\theta'}) L_{it}^{\theta}\}_{\theta}) - \sum_j \sum_{\theta} \kappa_{ijt} q_{ijt}^{\theta} L_{jt}^{\theta} \right) \tag{B.16}$$

$$\text{s.t.} \quad V_{j0} \geq \underline{V}_{j0}$$

$$L_{jt+1}^{\theta} = \sum_{i,\theta} L_{it}^{\theta} \mu_{ijt}^{\theta}$$

$$V_{jt}^{\theta} = u\big(c_{jt}^{\theta}(\{q_{ijt}^{\theta}\}), A_{jt}^{\theta}(L_{jt}^{\theta})\big) - \psi_{jt}\left(\{\mu_{jkt}^{\theta}\}\right) + \beta^{\theta} \sum_k \mu_{jkt}^{\theta} V_{kt+1}^{\theta}$$

$$\mu_{jkt}^{\theta} = \hat{\mu}_{jkt}^{\theta}(\{V_{it+1}^{\theta}\}),$$

Recursive formulation is

$$\begin{aligned}
\mathcal{S}_t\big(\{L^\theta_{jt}, V^\theta_{jt}\}\big) = &\max_{\{\{q^\theta_{ijt},\mu^\theta_{ijt}\},L^\theta_{jt+1},V^\theta_{jt+1}\}} \sum_i \Lambda_{it}\Big(F_{it}(\{Z^\theta_{it}(\{L^{\theta'}_{it}\}_{\theta'})L^\theta_{it}\}_\theta) - \sum_{j,\theta}\kappa_{ijt}q^\theta_{ijt}L^\theta_{jt}\Big) \\
&+ \frac{1}{R_{t+1}}\mathcal{S}_{t+1}\big(\{L^\theta_{jt+1}, V^\theta_{jt+1}\}\big) \qquad \text{(B.17)} \\
\text{s.t.} \quad L^\theta_{jt+1} &= \sum_i L^\theta_{it}\mu^\theta_{ijt} : \quad \tilde{\Delta}^\theta_{jt+1} \\
V^\theta_{jt} &\le u^\theta\big(c^\theta_{jt}(\{q^\theta_{ijt}\}), A^\theta_{jt}(\{L^{\theta'}_{jt}\}_{\theta'})\big) - \psi^\theta_{jt}\big(\{\mu^\theta_{jkt}\}\big) + \beta^\theta \sum_k \mu^\theta_{jkt}V^\theta_{kt+1} : \quad \Xi^\theta_{jt} \\
\mu^\theta_{ijt} &= \hat{\mu}^\theta_{ijt}\big(\{V^\theta_{kt+1}\}\big) : \quad \Omega^\theta_{ijt},
\end{aligned}$$

The first-order conditions are

$$\Lambda_{it}\kappa_{ijt} = \frac{\partial u^\theta(c^\theta_{jt}, a^\theta_{jt})}{\partial c^\theta_{jt}}\frac{\partial c^\theta_{jt}(\{q^\theta_{ijt}\})}{\partial q^\theta_{ijt}}\Xi^\theta_{jt}/L^\theta_{jt} \qquad \text{(B.18)}$$

$$\tilde{\Delta}^\theta_{jt+1}L^\theta_{it} = \Omega^\theta_{ijt} \qquad \text{(B.19)}$$

$$\tilde{\Delta}^\theta_{jt+1} = \frac{1}{R_t}\frac{\partial \mathcal{S}_{t+1}}{\partial L^\theta_{jt+1}} \qquad \text{(B.20)}$$

$$\frac{1}{R_t}\frac{\partial \mathcal{S}_{t+1}}{\partial V^\theta_{jt+1}} + \sum_k\sum_l \Omega^\theta_{klt+1}\frac{\partial \hat{\mu}^\theta_{klt}}{\partial V^\theta_{jt+1}} + \beta^\theta\sum_k \Xi^\theta_{kt}\mu^\theta_{kjt} = 0 \qquad \text{(B.21)}$$

Define the planner's price index for location j for type θ at time t as

$$P^\theta_{jt}c^\theta_{jt} = \min_{\{q^\theta_{ijt}\}} \sum_i \Lambda_{it}\kappa_{ijt}q^\theta_{ijt} \quad \text{s.t.} \quad c^\theta_{jt}(\{q^\theta_{ijt}\}) \ge c^\theta_{jt} \qquad \text{(B.22)}$$

Given this definition, we can multiply both sides by q^θ_{ijt} and sum over i to rewrite equation (B.18) as

$$1 = \frac{\partial u^\theta(c^\theta_{jt}, a^\theta_{jt})}{\partial c^\theta_{jt}}\frac{1}{P^\theta_{jt}}\Xi^\theta_{jt}/L^\theta_{jt}, \qquad \text{(B.23)}$$

where we used the fact that c^θ_{jt} is homogenous of degree one.

The envelope conditions are

$$\frac{\partial \mathcal{S}_t}{\partial L_{jt}^{\theta}} = \Lambda_{jt}\left[\frac{\partial F_{jt}(\{Z_{jt}^{\theta}(\{L_{jt}^{\theta'}\}_{\theta'})L_{jt}^{\theta}\}_{\theta})}{\partial(Z_{jt}^{\theta}L_{jt}^{\theta})}Z_{jt}^{\theta} + \sum_{\tilde{\theta}}\frac{\partial F_{jt}(\{Z_{jt}^{\theta}(\{L_{jt}^{\theta'}\}_{\theta'})L_{jt}^{\theta}\}_{\theta})}{\partial(Z_{jt}^{\tilde{\theta}}L_{jt}^{\tilde{\theta}})}L_{jt}^{\tilde{\theta}}\frac{\partial Z_{jt}^{\theta}(\{L_{jt}^{\theta'}\}_{\theta'})}{\partial L_{jt}^{\tilde{\theta}}}\right]$$
$$-P_{jt}^{\theta}c_{jt}^{\theta} + \sum_{\tilde{\theta}}\Xi_{jt}^{\tilde{\theta}}\frac{\partial u^{\tilde{\theta}}(c_{jt}^{\tilde{\theta}}, a_t^{\tilde{\theta}})}{\partial a^{\tilde{\theta}}}\frac{\partial a^{\tilde{\theta}}(\{L_{jt}^{\theta'}\}_{\theta'})}{\partial L_{jt}^{\theta}} + \sum_{k}\mu_{jkt}^{\theta}\tilde{\Delta}_{jt+1}^{\theta} = 0 \tag{B.24}$$

$$\frac{\partial \mathcal{S}_t}{\partial V_{jt}^{\theta}} = -\Xi_{jt}^{\theta} \tag{B.25}$$

Substituting (B.19), (B.23), and (B.25) into (B.21), and defining $\Delta_{jt} \equiv \frac{\partial \mathcal{S}_t}{\partial L_{jt}^{\theta}}$, we obtain

$$-\sum_{k}\sum_{l}\Delta_{lt+1}^{\theta}L_{kt}^{\theta}\frac{\partial \hat{\mu}_{klt}^{\theta}}{\partial c_{jt+1}^{\theta}}\frac{1}{P_{jt+1}^{\theta}} = \sum_{k}L_{kt}^{\theta}\mu_{kjt}^{\theta}\left(\beta^{\theta}R_t\frac{\frac{\partial u^{\theta}(c_{jt+1}^{\theta}, a_{jt+1}^{\theta})}{\partial c_{jt+1}^{\theta}}\frac{1}{P_{jt+1}^{\theta}}}{\frac{\partial u^{\theta}(c_{kt}^{\theta}, a_{kt}^{\theta})}{\partial c_{kt}^{\theta}}\frac{1}{P_{kt}^{\theta}}} - 1\right) \tag{B.26}$$

Plugging (B.18) and (B.20) into (B.24) and rearranging,

$$\Delta_{jt}^{\theta} = \Lambda_{jt}\left[\frac{\partial F_{jt}(\{Z_{jt}^{\theta}(\{L_{jt}^{\theta'}\}_{\theta'})L_{jt}^{\theta}\}_{\theta})}{\partial(Z_{jt}^{\theta}L_{jt}^{\theta})}Z_{jt}^{\theta} + \sum_{\tilde{\theta}}\frac{\partial F_{jt}(\{Z_{jt}^{\theta}(\{L_{jt}^{\theta'}\}_{\theta'})L_{jt}^{\theta}\}_{\theta})}{\partial(Z_{jt}^{\tilde{\theta}}L_{jt}^{\tilde{\theta}})}Z_{jt}^{\tilde{\theta}}\gamma_{jt}^{z,\tilde{\theta},\theta}\right]$$
$$-P_{jt}^{\theta}c_{jt}^{\theta} + \sum_{\tilde{\theta}}\frac{\frac{\partial u^{\tilde{\theta}}(c_{jt}^{\tilde{\theta}}, a_t^{\tilde{\theta}})}{\partial a^{\tilde{\theta}}}}{\frac{\partial u^{\tilde{\theta}}(c_{jt}^{\tilde{\theta}}, a_{jt}^{\tilde{\theta}})}{\partial c_{jt}^{\tilde{\theta}}}}P_{jt}^{\tilde{\theta}}A_{jt}^{\tilde{\theta}}\gamma_{jt}^{a,\tilde{\theta},\theta} + \frac{1}{R_t}\sum_{k}\mu_{jkt}^{\theta}\Delta_{jt+1}^{\theta}, \tag{B.27}$$

where

$$\gamma_{jt}^{z,\tilde{\theta},\theta} \equiv \frac{\partial \ln Z_{jt}^{\tilde{\theta}}(\{L_{jt}^{\theta'}\}_{\theta'})}{\partial \ln L_{jt}^{\theta}} \tag{B.28}$$

$$\gamma_{jt}^{a,\tilde{\theta},\theta} \equiv \frac{\partial \ln A_{jt}^{\tilde{\theta}}(\{L_{jt}^{\theta'}\}_{\theta'})}{\partial \ln L_{jt}^{\theta}}. \tag{B.29}$$

Define the wage of type θ in location j at time t in the planner's solution to be

$$w_{jt}^{\theta} = \Lambda_{jt}\frac{\partial F_{jt}(\{Z_{jt}^{\theta}(\{L_{jt}^{\theta'}\}_{\theta'})L_{jt}^{\theta}\}_{\theta})}{\partial(Z_{jt}^{\theta}L_{jt}^{\theta})}Z_{jt}^{\theta}. \tag{B.30}$$

Then (B.27) can be rewritten as

$$\Delta_{jt}^{\theta} = w_{jt}^{\theta} + \sum_{\tilde{\theta}} w_{jt}^{\tilde{\theta}} \gamma_{jt}^{z,\tilde{\theta},\theta} - P_{jt}^{\theta} c_{jt}^{\theta} + \sum_{\tilde{\theta}} \frac{\frac{\partial u^{\tilde{\theta}}(c_{jt}^{\tilde{\theta}}, a_t^{\tilde{\theta}})}{\partial a^{\tilde{\theta}}}}{\frac{\partial u^{\tilde{\theta}}(c_{jt}^{\tilde{\theta}}, a_{jt}^{\tilde{\theta}})}{\partial c_{jt}^{\tilde{\theta}}}} P_{jt}^{\tilde{\theta}} A_{jt}^{\tilde{\theta}} \gamma_{jt}^{a,\tilde{\theta},\theta} + \frac{1}{R_t} \sum_k \mu_{jkt}^{\theta} \Delta_{jt+1}^{\theta}. \tag{B.31}$$

One can easily verify, as in the baseline model, that the above allocation can be supported as a competitive equilibrium with the following transfer policies:

$$\tau_{jt}^{\theta} = P_{jt}^{\theta} c_{jt}^{\theta} - w_{jt}^{\theta} - s_{jt}^{\theta}. \tag{B.32}$$

B.3 Origin-Destination Specific Transfer

In the main text, we restricted ourselves to location-specific transfers. Such policy is arguably realistic, it is natural to consider richer policy tools. Here, we consider a transfer policy that can depend on the households' origin and destination location. We denote τ_{jt}^{o} as the transfer to households living in location j at time t and in location o at time $t-1$. With such a policy, the consumption of households with past location o and current location j at time t is

$$c_{jt}^{o} = \frac{w_{jt} + \tau_{jt}^{o} + s_{jt}^{o}}{P_{jt}}, \tag{B.33}$$

which replaces equation (1). Here, we allow the private transfer to also depend on origin and destination.

B.3.1 Competitive Equilibrium

The value function of a household currently living in location j with past location o is

$$V_{jt}^{o} = \max_{\{\mu_{jkt}^{o}\}} u(c_{jt}^{o}, a_{jt}) - \psi_{jt}\big(\{\mu_{jkt}^{o}\}\big) + \beta \sum_k \mu_{jkt}^{o} V_{kt+1}^{j}$$
$$\text{s.t.} \quad \sum_k \mu_{jkt}^{o} = 1.$$

We denote the solution to the above problem as

$$\hat{\mu}_{ijt}^{o}\big(\{V_{kt+1}^{\theta}\}\big) \equiv \arg\max_{\{\mu_{jkt}^{o}\}} u(c_{jt}^{o}, a_{jt}) - \psi_{jt}\big(\{\mu_{jkt}^{o}\}\big) + \beta \sum_k \mu_{jkt}^{o} V_{kt+1}^{j} \tag{B.34}$$
$$\text{s.t.} \quad \sum_k \mu_{jkt}^{o} = 1. \tag{B.35}$$

The consumption choices for each location's goods and the associated price indices remain

the same as in the baseline model for each o:

$$\begin{aligned} \{q_{ijt}^o\}_i &\in \arg\max_{\{q_{ijt}^o\}} c_{jt}\big(\{q_{ijt}^o\}\big) \\ \text{s.t.} \quad \sum_i p_{ijt} q_{ijt}^o &\leq w_{jt} + \tau_{jt}^o + s_{jt}^o. \end{aligned} \tag{B.36}$$

The associated price index remains the same as in (2).

The production side of the economy remains identical to the main text with the population in each location given by

$$L_{jt} = \sum_o L_{jt}^o,$$

and L_{jt}^o evolves according to

$$L_{jt+1}^o = \sum_i \mu_{ijt}^o L_{it}^o. \tag{B.37}$$

The government budget constraint is

$$\sum_j \sum_o \tau_{jt}^o L_{jt}^o = 0. \tag{B.38}$$

Likewise, the private transfer satisfies

$$\sum_j \sum_o s_{jt}^\theta L_{jt}^o = 0. \tag{B.39}$$

B.3.2 Planning Problem

The planning problem can be recursively written as

$$\begin{aligned}
\mathcal{S}_t\big(\{L^o_{jt}, V^o_{jt}\}\big) = &\max_{\{\{q^o_{ijt},\mu^o_{ijt}\}\}_{i,j,o},\{L^i_{jt+1},V^i_{jt+1}\}_{i,j}} \sum_i \Lambda_{it}\Big(Z_{it}(L_{it})L_{it} - \sum_{j,o}\kappa_{ijt}q^o_{ijt}L^o_{jt}\Big) \\
&+ \frac{1}{R_{t+1}}\mathcal{S}_{t+1}\big(\{L^i_{jt+1}, V^i_{jt+1}\}\big) \qquad \text{(B.40)} \\
\text{s.t.} \quad L_{it} &= \sum_o L^o_{it} \\
L^i_{jt+1} &= \sum_o \mu^o_{ijt}L^o_{it} \cdot : \quad \tilde{\Delta}^i_{jt+1} \\
V^o_{jt} &\le u\big(c_{jt}(\{q^o_{ijt}\}), A_{jt}(L_{jt})\big) - \psi_{jt}\big(\{\mu^o_{jkt}\}\big) + \beta\sum_k \mu^o_{jkt}V^j_{kt+1} : \quad \Xi^o_{jt} \\
\mu^o_{ijt} &= \hat{\mu}^o_{ijt}\big(\{V^i_{kt+1}\}\big) : \quad \Omega^o_{ijt},
\end{aligned}$$

The first-order conditions are

$$1 = \frac{\partial u(c^o_{jt}, A_{jt})}{\partial c^o_{jt}}\frac{1}{P_{jt}}\Xi^o_{jt}/L^o_{jt} \qquad \text{(B.41)}$$

$$\tilde{\Delta}^i_{jt+1}L^o_{it} = \Omega^o_{ijt} \qquad \text{(B.42)}$$

$$\tilde{\Delta}^i_{jt+1} = \frac{1}{R_t}\frac{\partial \mathcal{S}_{t+1}}{\partial L^i_{jt+1}} \qquad \text{(B.43)}$$

$$\frac{1}{R_t}\frac{\partial \mathcal{S}_{t+1}}{\partial V^i_{jt+1}} + \sum_o\sum_l \Omega^o_{ilt}\frac{\partial \hat{\mu}^o_{ilt}}{\partial V^i_{jt+1}} + \beta\sum_o \Xi^o_{it}\mu^o_{ijt} = 0 \qquad \text{(B.44)}$$

where

$$P_{jt} = \min_{\{q^o_{ijt}\}}[\sum_i \Lambda_{it}\kappa_{ijt}q^o_{ijt}] \quad \text{s.t.} \quad c_{jt}(\{q^o_{ijt}\}) \ge 1. \qquad \text{(B.45)}$$

The envelope conditions are

$$\frac{\partial \mathcal{S}_t\big(\{L^o_{jt}, V^o_{jt}\}\big)}{\partial L^o_{jt}} = \Lambda_{jt}Z_{jt}(1+\gamma^z_{jt}) - P_{jt}c^o_{jt} + \Xi^o_{jt}\frac{\partial u(c^o_{jt}, A_{jt})}{\partial A_{jt}}A'_{jt}(L_{jt}) + \sum_k \mu^o_{jkt}\tilde{\Delta}^j_{kt+1} \qquad \text{(B.46)}$$

$$\frac{\partial \mathcal{S}_t\big(\{L^o_{jt}, V^o_{jt}\}\big)}{\partial V^o_{jt}} = -\Xi^o_{jt} \qquad \text{(B.47)}$$

Defining $\Delta^o_{jt} \equiv \frac{\partial S_t}{\partial L^o_{jt}}$, we can combine above equations to obtain

$$-L_{it}\sum_l \Delta^i_{lt+1}\frac{\partial \hat{\mu}_{ilt}}{\partial c^i_{jt+1}}\frac{1}{P_{jt+1}} = \beta R_t \sum_o L^o_{it}\mu^o_{ijt}\left(\frac{\frac{\partial u(c^i_{jt+1},A_{jt+1})}{\partial c^i_{jt+1}}\frac{1}{P_{jt+1}}}{\frac{\partial u(c^o_{it},A_{it})}{\partial c^o_{it}}\frac{1}{P_{it}}}-1\right) \tag{B.48}$$

$$\Delta^o_{jt} = w_{jt}(1+\gamma^z_{jt}\frac{L^o_{jt}}{L_{jt}}) - P_{jt}c^o_{jt} + \frac{\frac{\partial u(c^o_{jt},A_{jt})}{\partial A_{jt}}}{\frac{\partial u(c^o_{jt},A_{jt})}{\partial c_{jt}}}P_{jt}A_{jt}\gamma^A_{jt}\frac{L^o_{jt}}{L_{jt}} + \frac{1}{R_t}\sum_k \mu^o_{jkt}\Delta^j_{kt+1} \tag{B.49}$$

where $w_{jt} = \Lambda_{jt}Z_{jt}$. One can verify that the above allocation can be supported as a competitive equilibrium with the following transfer policies:

$$\tau^o_{jt} = P_{jt}c^o_{jt} - w_{jt} - s^o_{jt}. \tag{B.50}$$

C Numerical Illustrations

In this section, we consider the setting with only two locations, $\{1, 2\}$. We set the structural parameters $\{\beta, \rho, \theta, \sigma, \gamma^A, \gamma^Z\}$ using our calibration for the U.S. economy in Section 4, as summarized by Table 2. For location fundamentals, we set the following values: $\bar{Z}_{jt} = \bar{A}_{jt} = 1$ for all j, t; $\kappa_{ijt} = 1.2$ for $i \neq j$ and $\kappa_{ijt} = 1$ for $i = j$; $\chi_{ijt} = 0.1$ for $i \neq j$ and $\chi_{ij} = 1$ for $i = j$. We always choose the consumption basket in location 1 in each period as numeraire.

Permanent and Transient Shocks In Figures C.1 and C.2, we show the impulse responses to the permanent and transitory negative shock of 30% to location 1, respectively. To simulate these impulse responses, we assume that the economy is in a steady state up to $t = 0$. At $t = 0$, an unanticipated permanent and temporary change of productivity in location 1 materializes, $\overline{Z}_{1t}$, whose process is given by the top left panel ("Productivity") of Figures C.1 and C.2, respectively. The solid lines in each panel show the equilibrium responses of the employment, consumption, real wages, optimal transfers, and consumer prices for locations 1 and 2, under the optimal transfer characterized by Proposition 1. To benchmark our results, we also simulate our model under an alternative assumption that transfer is set to zero in the dashed lines.

Right after the negative productivity shocks to location 1, net transfer sharply increases in location 1 to mitigate the differences in consumption between the two locations. Interestingly, even if the shock is permanent, net transfer decreases over time until it reaches the steady-state level. This back-loading of incentives is a natural consequence of the insurance-incentive trade-off highlighted in Proposition 1. On the one hand, the Planner wants to provide consumption insurance to location 1. On the other hand, the Planner wants to incentivize out-migration from location 1. The most effective way of ensuring incentives for the Planner is to commit to cut transfer in the future. Reflecting this back-loaded pattern of transfer, the consumption in location 1 is substantially higher under optimal transfer than the no-transfer benchmark, while the gap decreases over time. Lastly, given the transfer, there is less migration than no transfer benchmark, and a larger size of the population stays in location 1 under the optimal transfer.

Responses to Anticipated Shocks In Figure C.3, we alternatively consider the impulse responses to anticipated permanent and transitory shocks in the future, respectively. To simulate these impulse responses, we assume that the economy is in a steady state up to $t = 0$. At $t = 0$, agents learn that the productivity in location 1, $\overline{Z}_{1t}$, will drop by 30 percent at $t = 10$ and for all subsequent period, as shown in the top left panel ("Productivity") of Figures C.1.

In both cases, net transfer in location 1 is positive and increasing up to $t = 10$, and turns negative after $t = 10$. Before the negative shock materializes, the Planner wants to encourage outmigration from location 1. However, after the shock hits, the Planner has the incentive to

Figure C.1: Permanent Shock

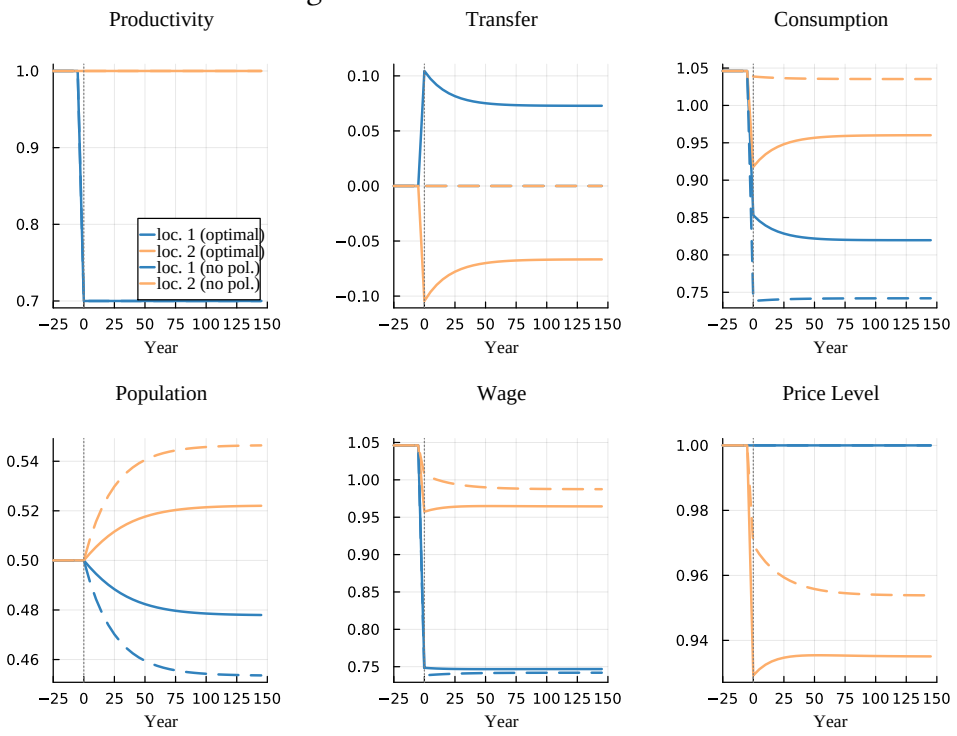
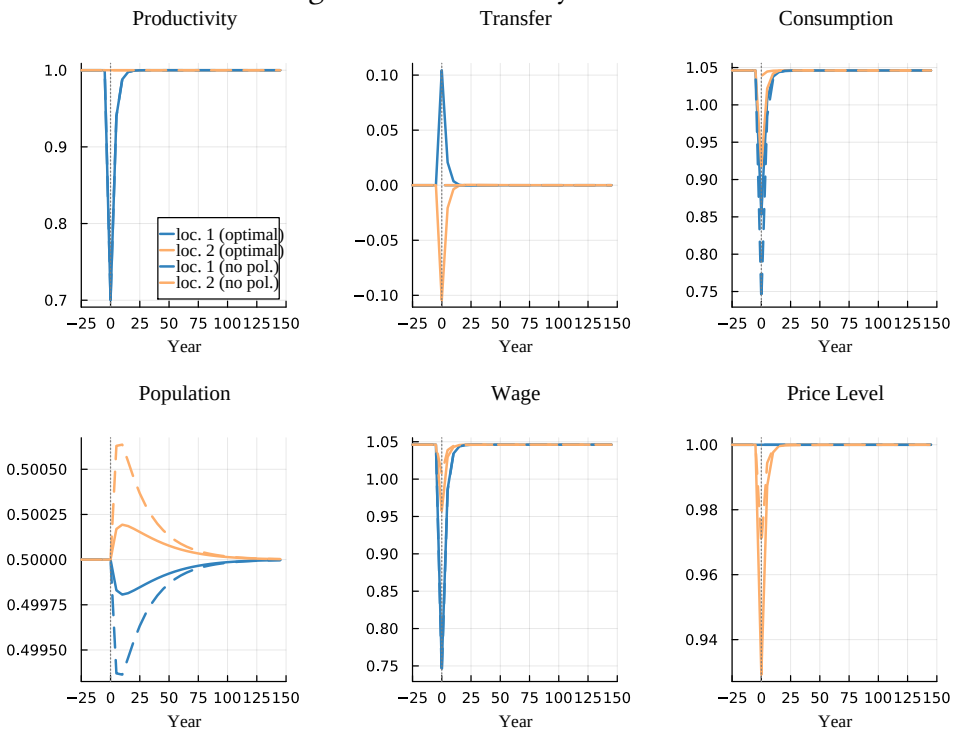
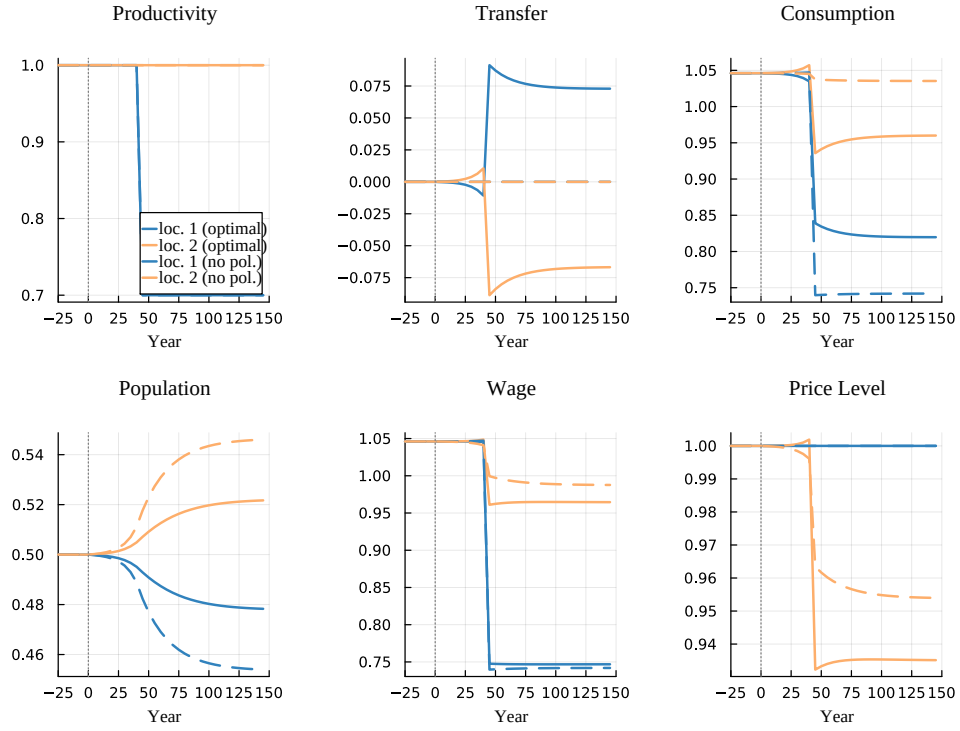


Figure C.2: Transitory Shock



provide consumption insurance to mitigate the negative shock. At the same time, for the same reasons as the unanticipated permanent shock in Figure C.1, the transfer gradually decreases over time to mitigate the moral-hazard problem of migration decisions.

Figure C.3: Permanent Anticipated Shock



D Appendix for Parametric Model

Given productivities $\{Z_{jt}(\cdot)\}$, amenities $\{A_{jt}(\cdot)\}$, migration frictions $\{\psi_{ijt}(\cdot)\}$, trade frictions $\{\kappa_{ijt}\}$, private and public transfers $\{s_{jt}, \tau_{jt}\}$, and initial population distribution $\{l_{j0}\}$,

- Consumption $\{c_{jt}, q_{jt}\}$:

$$c_{jt} = \frac{w_{jt} + \tau_{jt} + s_{jt}}{P_{jt}} \tag{D.1}$$

  and

$$q_{ijt} = \frac{P_{jt} c_{jt} \Psi_{ijt}}{p_{it}}, \qquad \Psi_{ijt} = \frac{p_{ijt}^{1-\sigma}}{\sum p_{ijt}^{1-\sigma}} \tag{D.2}$$

- Migration decisions $\{\mu_{ijt}, V_{jt}\}$:

$$\mu_{jk} = \frac{\chi_{jk} \exp(\theta V_k)}{\sum_l \chi_{jl} \exp(\theta V_l)} \tag{D.3}$$

$$V_{jt} = u\big(c_{jt}, a_{jt}\big) + \beta \frac{1}{\theta} \log \sum_k \chi_{jkt} \exp(\theta V_{kt+1}) \tag{D.4}$$

- Population evolution $\{l_{jt}\}$:

$$L_{jt} = \sum_i \mu_{ijt} L_{it-1} \tag{D.5}$$

- Prices $\{p_{jt}, P_{jt}\}$:

$$P_{jt} = \left(\sum_l (\kappa_{ljt} w_{lt} / Z_{lt})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{D.6}$$

$$p_{ijt} = \kappa_{ijt} w_{it} / z_{it} \tag{D.7}$$

  where we take location 1's price index as numeraire such that $P_{1t} = 1$ for all t.

- Market clearing $\{w_{jt}\}$:[5]

$$w_i L_i = \sum_j \Psi_{ijt} P_{jt} c_{jt} L_{jt} \tag{D.8}$$

- Amenity and productivity $\{a_{jt}, z_{jt}\}$:

$$z_{jt} = \bar{Z}_{jt} L_{jt}^{\gamma^Z}, \quad a_{jt} = \bar{A}_{jt} L_{jt}^{\gamma^A} \tag{D.9}$$

Optimal Allocation. Under optimal allocation, we replace the consumption equation (D.1) with the one in Proposition 1 for $t > 0$:

[5] We combine equation (D.7) and goods market clearing ($\sum_j \kappa_{ijt} q_{ijt} L_{jt} = z_{it} L_{it}$) yields the above expression.

$$-\sum_i L_{it} \sum_k \Delta_{kt+1} \frac{1}{P_{jt+1}} \frac{\partial \mu_{ikt}}{\partial c_{jt+1}} = \sum_i L_{it} \mu_{ijt} \left(\frac{\beta R_{t+1} \partial_c u_{jt+1} / P_{jt+1}}{\partial_c u_{it} / P_{it}} - 1 \right) \tag{D.10}$$

where for the i.i.d. logit case,

$$\frac{\partial \mu_{ikt}}{\partial c_{jt+1}} = \partial_c u_{jt+1} \times \begin{cases} -\theta \mu_{jit} & \text{if } k \neq i \\ \theta \left(1 - \mu_{jit}\right) & \text{if } k = i \end{cases}$$

and $\{\Delta_{jt}\}$ is determined by

$$\Delta_{jt} = w_{jt} - P_{jt} c_{jt} + w_{jt} \gamma^z_{jt} + \frac{\partial_a u_{jt}}{\partial_c u_{jt}} P_{jt} A_{jt} \gamma^a_{jt} + \frac{1}{R_{t+1}} \sum_k \mu_{jkt} \Delta_{kt+1} \tag{D.11}$$

and R_t is determined so that the government budget constraint is satisfied, i.e.,

$$\sum_i P_{it} c_{it} L_{it} = \sum_i w_{it} L_{it} \tag{D.12}$$

For $t = 0$, the level of consumption is $\{c_{j0}\}$ is determined by

$$\frac{\varkappa_j \partial_c u_{j0}}{P_{j0}} = \frac{\varkappa_i \partial_c u_{i0}}{P_{i0}} \tag{D.13}$$

where $\{\varkappa_j\}$ are the Pareto weights.

E Data and Calibration Appendix

E.1 Calibration of Trade Flows

This appendix provides a detail of the calibration of trade flows. We we follow the approach of Fajgelbaum and Gaubert (2020) to use the gravity equation of trade flows to find the unique estimates of trade flows between states that are consistent with the actual distance between states, observed pre-tax income, post-tax-and-transfer income, under the assumption of trade frictions with respect to distance.

Under the parametric model (CES goods demand), trade flows from i to j in period t follow the gravity structure:

$$\Psi_{ijt} = \frac{\delta^O_{it} \kappa_{ijt}}{\sum \delta^O_{jt} \kappa_{ijt}},$$

where δ_{jt}^O and δ_{it}^D are endogenous variables that depend on equilibrium variables such as wages and prices. Furthermore, from market clearing condition, we have

$$w_i L_i = \sum_j \Psi_{ijt} P_{jt} c_{jt} L_{jt}.$$

Finally, we impose a parametric restriction of the trade cost so that it is an iso-elastic function of geodesic distance between i and j such that

$$\kappa_{ijt} = \text{dist}_{ij}^{\psi},$$

where ψ is a parameter. Given the assumption of ψ and the data $\{\text{dist}_{ij}, w_{it}, P_{it}, c_{it}, L_{it}\}$, we can use the above equations to identify $\left\{\delta_{it}^O\right\}$ (up to scale), and hence trade flows $\{\Psi_{ijt}\}$.

E.2 Calibration of Bilateral Migration Frictions

We first back out the migration frictions, $\{\chi_{ijT}\}$, for the terminal period in the data $t = T$. To do so, we follow the following steps:

1. We guess $\{V_{jT+1}\}_j$. Using the migration flow data at T, we recover

$$\chi_{ijT} = \frac{\mu_{ijT}}{\mu_{iiT}} \exp(\theta(V_{iT+1} - V_{jT+1})),$$

   where recall our normalize $\chi_{iiT} = 1$.

2. Using the future fundamentals and assumptions of public and transfers for $t > T$, including the assumption that $\chi_{ijt} = \chi_{ijT}$ for $t > T$, we simulate the model forward starting at $t = T$ all the way toward $t = \infty$. This simulation yields the equilibrium paths of continuation value, including those at $t = T + 1$, $\{V_{jT+1}^*\}_j$.

3. We repeat the step 1 and 2 by iteratively updating $\{V_{jT+1}\}_j$ with $\{V_{jT+1}^*\}_j$.

4. Stop if this procedure converges.

Once we obtain $\{\chi_{ijT}\}$, we sequentially back out $\{\chi_{ijt}\}$ for $t < T$ backward starting from $t = T - 1$ using the following procedure:

1. Using $\{V_{jt+1}\}_j$, we obtain $\{V_{jt}\}_j$ using the Bellman equation:

$$V_{jt} = u\big(c_{jt}, a_{jt}\big) + \beta \frac{1}{\theta} \log \sum_k \chi_{jkt} \exp(\theta V_{kt+1})$$

2. Using $\{V_{jt}\}_j$, we back out $\{\chi_{ijt-1}\}$

$$\chi_{ijt-1} = \frac{\mu_{ijt-1}}{\mu_{iit-1}} \exp(\theta(V_{it} - V_{jt}))$$