Certain vs. Uncertain Timing

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Abstract

Risk aversion is a foundational cornerstone of modern finance, and implies a preference for certainty to uncertainty. However, I show that standard financial pricing frameworks make strong predictions on the pricing of certain vs. uncertain timing, which do not always accord with empirical pricing. First, I show that the SDF framework - regardless of the choice of SDF - prices an asset with uncertain timing according to its marginals. I further show that there is no extension of the SDF that can accommodate a preference for certain timing without giving up theoretical properties that financial economists have historically found appealing. Second, comparing dividend-free American and European call options on the same underlying, offered by the same exchange, with identical contract specifications, I show that the short side requires higher compensation for the American than the European, over and above pricing effects of the early exercise premium. These results cannot be explained by hedging considerations, re-investment risk, liquidity effects, moneyness, informed trading, or microstructure differences, but are consistent with markets pricing an aversion to uncertain timing. Third, a supplemental set of tests elicits willingness-to-pay and pricing narratives in surveys and shows that these are consistent with the market results.

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1 Introduction

Which do you prefer: \$2 for sure, or the gamble {\$1 with 50% probability and \$3 with 50% probability}? Modern finance, from portfolio choice theory to the CAPM to methods of calculating discounted present value, builds on the idea of risk aversion. Risk aversion implies that individuals prefer certainty to uncertainty, when both the certain and uncertain options have the same expected value. According to standard theory, your preference should therefore at least weakly favor the certain \$2.

Now consider a different setting: suppose that the amount of money is fixed, but there is uncertainty is over the timing of payment. In particular, you know you will receive \$10, but you don't know when. Would you prefer that \$10 in two months for sure, or the gamble {50%, 1 month, 50% 3 months}?

Reasonable people may have different opinions - but I prove in this paper (Theorem 1) that the standard SDF framework is not able to price these lotteries. In addition, no extension of the SDF framework, even moving beyond standard models, can capture a preference for certain timing (Theorem 2) without giving up properties that financial economists have historically found appealing. These theoretical findings show that the standard way of teaching and understanding asset pricing - through the stochastic discount factor framework (Duffie, 2001; Cochrane, 2011; Campbell, 2017) - is either unable to price, or makes strong predictions for, certain vs. uncertain timing.

I next turn to data. I show that a preference for certain timing appears in real-world options markets. In particular, I compare American to European options on the same underlying – the S&P 100 – offered by the same exchange with identical contract specifications. The reason this is a test of certain vs. uncertain timing is that American options can be exercised at any time, while European options can be exercised only at expiry. When the probability of early exercise is nonzero, as shown in past work for both retail and institutional investors (Diz and Finucane, 1993; Jensen and Pedersen, 2016; McDonald, 2013), the short side of an American option is exposed to timing risk that the short side of an otherwise identical European option is not.

Comparing otherwise identical dividend-free American to European index call options, I find a pricing premium for American options, and further show in a difference-in-difference specification that this premium derives more from the short (ask) side than the long (bid) side. In particular, the specification I consider to separate out an

aversion to timing risk on the short side, from a willingness to pay for flexibility on the long side, is: $(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}) - (\text{bid}_{\text{American}} - \text{bid}_{\text{European}})$. I find that this difference is positive and statistically significant, and remains positive, throughout nearly the entire twelve-year minute-by-minute sample. Because all options in the main specification are dividend-free, these results constitute a violation of Merton (1973)'s theorem. The difference-in-difference specification also allows me to go further, removing any asset-specific fixed effects, and differencing out pricing effects from demand for early exercise, so that only excess pricing differences on the ask side remains. I then explicitly consider a number of alternative explanations and show that none of them can remove the effect of an additional demand for compensation on the short side.

For example, one alternative explanation is that the difference in pricing represents an adversarial response to informed trading on the long side. But I show that empirically, most trades in both these option classes are not informed (as we expect for index options). A second alternative explanation is that the short side might be demanding compensation for the complexity or inconvenience of dynamic hedging. But I show that the results hold in short-dated zero gamma and zero delta options, where hedging differences do not change over the option's lifetime. A third alterative explanation is that the options have liquidity differences. But I explicitly control for liquidity by restricting attention only to minutes and options which have similar bid-ask spreads (measured as %) and show that the results continue to hold. In addition, any symmetric liquidity premium gets differenced out in the specification. A fourth possible explanation is that the results are driven disproportionately by options with particular moneyness; but I show that the difference shows up for both in-the-money and out-of-the-money options. A fifth possible explanation is that the difference reflects re-investment risk. But I show that the pricing difference appears even in options with only a few days to expiry, in whose lifetime there are no FOMC meetings. A sixth possible explanation is that option quotes may be less informative than realized prices; but I show that the results continue to hold for options with strictly positive open interest. A seventh possible explanation is that microstructure differences between the two options may result in differential willingness to pay. But, by construction, the index options I consider have identical settlement time, contract size, strike prices, strike price intervals, position limits, margin rules, expiration date, and expiration months, and are offered by the same exchange – that is, they have

identical contract specifications, except that one is American and the other European. An eighth possible explanation is that inventory pressures can cause bids to adjust different from asks - but a more flexible specification that allows for this, continues to show a pricing difference in asks over and above what can be explained by bids. Cumulatively, these robustness checks, and the main specification, suggest an unwillingness in to be short the American over and above pricing of an early exercise premium on the long side, consistent with an aversion to holding uncertain timing.

Finally, through a survey, I directly elicit willingness to pay to further test the predictions of standard pricing theories. All are falsified: the empirical pricing of uncertain timing differs from the pricing of its marginals, and the willingness to pay for certain timing holds amongst those with the desirable theoretical properties we would have to give up to model a preference for certain timing. In addition, I elicit incentivized pricing narratives, which demonstrate that the majority of respondents price uncertainty in the time and money domains in a conceptually similar way.

The paper primarily contributes to a literature on theoretical properties of the stochastic discount factor, and to a debate over whether there is a right SDF to use (Hansen and Jagannathan (1991), Duffie (2001), Lettau and Ludvigson (2001), Daniel et al. (2002), Binsbergen et al. (2012), Campbell (2017), Vayanos and Vila (2021), among many others). It also contributes to a literature on empirical option pricing, particularly as deviations from theory (Gromb and Vayanos, 2010; McDonald, 2013; Bryzgalova et al., 2024). There are a number of other strands of literature the paper also speaks to, which are comprehensively reviewed in Section 6.

The paper proceeds as follows. Section 2 derives the theoretical results on the SDF. Section 3 shows that options markets have prices that are consistent with a preference for certain to uncertain timing, and tests for robustness to various alternative explanations. Section 4 elicits willingness-to-pay and pricing narratives in surveys and shows that the results are consistent with the market findings. Section 5 discussions potential implications across finance. Section 7 concludes.

2 Theoretical Results: Pricing Frameworks and Uncertain Timing

I start by showing in Theorem 1 that the standard stochastic discount factor framework in asset pricing is defined on a space that does not allow perfect negative correlation over time to be priced separately from the marginal distributions. In particular, while the SDF thinks of there being two kinds of substitution: substitution across states and over time - the empirical sections of the paper show that it is also worth pricing temporal risk, which the standard SDF framework is not able to do.

A natural response may be to extend the SDF to price temporal risk. Even such an extension, however, would not be able to capture a consistent preference for certain timing without giving up at least one property that financial economists have historically found appealing (Theorem 2).

2.1 The Stochastic Discount Factor Framework and Uncertain Timing

The stochastic discount factor framework in complete markets arises from Arrow-Debreu securities, e.g. state-contingent securities. I show here that the standard formalization of assets, as allowing for substitution either over (observable) states or over time, does not allow timing uncertainty to be uniquely priced.

To illustrate, let the state space Ω be divided into observable state components Ω_O (such as macroeconomic indicators) and unobservable state components Ω_U (such as each person's internal state, their likelihood of employment termination, their personal obligations, so on). Asset pricing studies often apply a stochastic discount factor framework to observable states (Campbell and Cochrane, 1999; Pastor and Veronesi, 2003; Lettau and Wachter, 2007; Verdelhan, 2018) with some per-period error term. The issue with this SDF framework as regards timing uncertainty is that unobservable state components need not collapse to observable state components in a way that respects the standard separation between substitution over states and over time.

As a practical example, empirically, exercise time for American options is given not only by macroeconomic variables but also by possibly idiosyncratic considerations such as institution-specific short-sale constraints (Jensen and Pedersen, 2016) or retail

investor proclivities (Diz and Finucane, 1993). As Section 3 discusses in its empirical exercise, this makes the time of exercise for the short side a probability measure which is negatively correlated over time (i.e., there can be only one time of exercise).

To gain an intuition for why standard asset pricing frameworks have difficulty with timing uncertainty, consider a simple multiperiod setup; following Campbell (2017), let $\Omega_O = 1, 2, .., |S|$ be a finite set of observable states, $T = 1, 2, ...,$ a set of time periods, $\Omega_O \times T$ with generic element (s, t) the set of state-time pairs, and $a_{s,t}$ an Arrow-Debreu security that pays \$1 in state s at time t and \$0 in all other states and times. Let $\pi_{s,t}$ be the physical probability that state s occurs at time t. An asset is a random variable with respect to Ω_O. The law of one price says that two assets with the same observable payoffs in every state must have the same price. In particular, the price of asset X is $\sum_{s,t} \pi(s,t) M_t(s) X(s,t)$, where M_t, $t \in T$ is a special random variable termed the stochastic discount factor.

Consider, however, two assets with respect to the entire state space Ω, one of which, after integrating out unobservable variables Ω_U (i.e., projecting on to Ω_O), has a 70% chance of paying \$1 at time 2 and a 30% chance of paying \$1 at time 10; that is, the asset pays out \$1 for sure across all observable states, but the timing of its payment is uncertain. The second asset, conditional on observable variables, similarly pays out \$1 for sure, but its time of payment is period 2 with 50% probability and period 3 with 50% probability. If these two assets both had to be represented by the SDF framework in the prior paragraph, it would not be possible to distinguish between their pricing. This is because when we project the assets onto Ω_O, both assets have identical payoffs in each observable state-time pair (s, t), and therefore by the law of one price must have identical pricing.

In the more general multi-period setup of Duffie (2001), it is possible to get around this difficulty by conditioning on a filtration to obtain an expected payoff, but another issue arises: it is not possible to separately price timing uncertainty from independent coin flips. Theorem 1 proves this formally.

Theorem 1. *Consider the standard multiperiod SDF framework (Duffie, 2001): there exists a probability space $(\Omega_O, \{\mathcal{F}_t\}_{t \in T}, P_O)$, where Ω_O is a finite set of states, $\{\mathcal{F}_t\}$ a filtration, T is a set of times (typically the natural numbers), and P_O a probability measure. In this framework, assets are adapted processes: $X = (X_1, X_2, ...)$ where each X_t is measurable with respect to $(\Omega_O, \mathcal{F}_t)$. Assume at time zero the existence of a pricing kernel $M = \{M_t\}_{t \in T}$: the price of asset X is given by $Price(X) =$*

$E_{P_O}[\sum_t M_t X_t]$.

Consider now an asset A defined as a random variable with respect to the richer state space $\Omega = \Omega_O \times \Omega_U$ and which, after integrating out Ω_U, pays out \$1 almost surely with respect to P_O, but the timing of payment is uncertain: it pays out \$1 at period t_1 with probability p_A and at period $t_2 > t_1$ with probability $1 - p_A$, and these probabilities are perfectly negatively correlated.

Consider also asset A' defined as a random variable with respect to Ω_O, which pays out \$1 at period t_1 with probability p_A, and independently pays out \$1 at period t_2 with probability $1 - p_A$.

Under the multi-period SDF framework for observable states $(\Omega_O, \{\mathcal{F}_t\}, P_O)$, it is not possible to separately price assets A and A'.

This result - that assets with timing uncertainty are difficult to fit into an observable SDF framework - may help rationalize why some assets, despite decades of research in asset pricing, lack standard pricing models. These include American options, mortgages with prepayment risk, and callable or convertible bonds - all assets in which the time of payment can be uncertain.

2.2 What if we expand the SDF framework?

A natural question that arises, is why not expand the SDF framework to use a novel utility function that can accommodate a preference for certain timing.

This section shows that it is not possible to accommodate a preference for certain timing without giving up at least one property that financial economists have historically found appealing. Intuitively, a preference for uncertain timing comes from the convexity of the time discounting function. Theorem 2 shows that there is no way to get around this tension while preserving properties historically considered desirable for financial models.

2.2.1 Space and Definitions

Per-period consumption lies in $C \subseteq \mathbb{R}^+ \cup \{0\}$ and time takes on values $T = 1, 2, ...$ The infinite dimensional space C^∞ represents deterministic consumption streams. Vector $\vec{c} \in C^\infty$ refers to receiving consumption $\bar{c}$ at each period t. Pair $(c, t) \in C \times T$ denotes receiving consumption c in period t, and zero consumption in all other periods. Let Δ be the set of simple lotteries over $C \times T$ endowed with the topology of weak

convergence. A typical element of Δ is denoted m and a generic realization of m denoted $c(\vec{m})$. We study a complete and transitive ordering $\succeq$ over Δ.

Money lotteries, defined as lotteries that pay out uncertain consumption at a specified date $\bar{t}$ are denoted $q^{(\bar{t})} \in \Delta$, and time lotteries, defined as lotteries that pay out certain consumption $x \in C$ at an uncertain date, are denoted $p^{(x)} \in \Delta$. Notation $\vec{\bar{c}} + (x, t)$ denotes the deterministic consumption stream of $\bar{c}$ in each period, except in period t where consumption is $\bar{c} + x$. Notation $\vec{\bar{c}} + p^{(x)}$ denotes the lottery which, with probability $p^{(x)}(t)$, gives consumption vector $\vec{\bar{c}} + (x, t)$. Unless otherwise specified, for both time and money lotteries, all uncertainty is resolved immediately. This assumption is made to focus on the risk aspects of the models, rather than on resolution period effects.

Definition 1 (Prefer Certain to Uncertain Timing)**.**
Say that the preference $\succeq$ ***prefers certain to uncertain timing*** *if, for any $\bar{c} \in C$, $x \in \mathbb{R}$, and time lottery $p^{(x)} \in \Delta$ with $\mathbb{E}\left[p^{(x)}\right] \in C \times T$, consumption stream $\vec{\bar{c}} + (x, \mathbb{E}\left[p^{(x)}\right]) \succ \vec{\bar{c}} + p^{(x)}$. Analogously, the preference prefers uncertain to certain timing or is indifferent over timing risk if the above holds with $\prec$ or $\sim$, respectively.*

In words, given any constant consumption stream, the agent prefers certain to uncertain timing if, for time lottery $p^{(x)}$, they prefer to receive consumption at its deterministic expected time rather than play the time lottery.

Definition 1 imposes restrictions on behavior only at certainty; it says nothing about mean-preserving spreads.

The restriction to constant consumption is to focus on the pure timing uncertainty aspect of the preference, rather than on deterministic consumption variability. This definition and framework generalizes ones used in the decision theory literature (Ebert, 2020) to account for shocks of any sign and non-zero baseline consumption, as is typically the case in finance.

Definition 2 (Convex Time Discounting)**.**
Say that a representation U has ***convex discounting in the absence of risk*** *if, for any deterministic $t, t', t'' \in T$ with $t'' = \frac{t+t'}{2}$, $\frac{1}{2}U((x,t)) + \frac{1}{2}U((x,t')) > U((x,t''))$.*

The next set of definitions, regarding recursive preferences, are standard (Chew and Epstein (1990)).

Definition 3 (Utility Certainty Equivalent)**.**
Say that function $I : M([0,1]) \to [0,1]$ is a utility certainty equivalent operator if it is continuous, increasing with respect to first-order stochastic dominance, and has $I(x) = x$ for every $x \in \mathbb{R}_+$.

Definition 4 (Image Measure)**.**
Given a function $U : \Delta \to [0,1]$ and a probability measure $m \in \Delta$, define the image measure $m \cdot U^{-1} \in M([0,1])$ by letting $[m \cdot U^{-1}](B) \coloneqq m\left(\left\{c(\vec{m}) : U(c(\vec{m})) \in B\right\}\right)$ for all $B \in \mathcal{B}([0,1])$, where $\mathcal{B}$ is the Borel measure.

Definition 5 (Recursive Representation)**.**
Say that representation $U : \Delta \to [0,1]$ is recursive if it takes the form:

$$U(c, m) = W(c, I(m \cdot U^{-1})),$$

where $I : M([0,1]) \to [0,1]$ is a utility certainty equivalent operator, and $W : C \times [0,1] \to [0,1]$ is a continuous function, strictly increasing in its second argument.

Definition 6 (Kreps-Porteus Recursive Representation)**.**
Say that representation $U : \Delta \to [0,1]$ is a Kreps-Porteus recursive representation if it is a recursive representation and $I(m \cdot U^{-1}) = \phi^{-1}\mathbb{E}[\phi(m \cdot U^{-1})]$ for some increasing and continuous $\phi : \mathbb{R} \to \mathbb{R}$.

Assumption (Differentiability)**.** *Assume that the Kreps-Porteus function U is twice differentiable and that U' and U'' are well-defined at all but at most a finite number of consumption paths in C^∞.*

2.2.2 Theoretical Tradeoffs

The Kreps-Porteus class is characterized by dynamic consistency, history independence, and no probability distortion (Kreps and Porteus, 1978; Strzalecki, 2023). Dynamic consistency helps ensure that the willingness to pay for an asset remains consistent if nothing else changes. History independence is useful for tractability and to avoid having to impose a starting value for initial consumption. No probability distortion allows for standard expectation taking. As a result, all three properties are arguably desirable.

Theorem 2 shows that it is not possible to have these properties alongside convex discounting and a preference for certain timing. As a result, even moving beyond

standard and exotic models used in the literature, it is not possible to easily accommodate a preference for certain timing.

Theorem 2. *Suppose U is a differentiable Kreps-Porteus representation with convex time discounting. Then U does not satisfy a preference for certain to uncertain timing.*

2.2.3 Habit Formation

Theorem 3 shows that giving up recursivity in the most common way, e.g. via habit formation, also does not help accommodate a preference for certain timing.

Theorem 3. *The Campbell and Cochrane (1999) habit formation model does not satisfy a preference for certain timing.*

2.2.4 Positive vs. Negative Shocks

Corollaries 1 and 2 show that additive utility models (e.g., expected utility and hyperbolic discounting) predict a preference for uncertain timing for positive shocks and a preference for certain timing for negative shocks. While the former is known, these results are included here for completeness because decision theoretic derivations tend to use only positive shocks and zero baseline consumption. Corollaries 1 and 2, in contrast, work with the full generality of Definition 1. The dichotomy between positive and negative shocks is one we will want to be aware of when testing for preferences in real-world markets, where shocks can take any sign.

Corollary 1. *Any expected utility model, $U(p) = \int_{c(\vec{p})\in p} \sum \delta^t u(c(p)_t) dp(c(\vec{p}))$ prefers uncertain timing for all time lotteries $p^{(x)}$ when $x > 0$, and certain timing when $x < 0$.*

Corollary 2. *Any hyperbolic discounting model $U(p) = \int_{c(\vec{p})\in p} \sum \beta\delta^t u(c(p)_t) dp(c(\vec{p}))$ prefers uncertain timing all time lotteries $p^{(x)}$ when $x > 0$ and $p^{(x)}(t = 0) = 0$, and certain timing when $x < 0$ and $p^{(x)}(t = 0) = 0$.*

3 Option Pricing and Timing Uncertainty

3.1 Overview

Options constitute one of the world's largest and most actively traded asset classes. In the first quarter of 2024 alone, total notional value in options traded in the US

was 40 trillion dollars (Office of the Comptroller of the Currency, 2024), larger than the US national debt (Department of the Treasury, 2024), and over 10 times larger than annual social security benefits (Social Security Administration, 2024).

There are two types of commonly traded options: American and European. European options may be exercised only at the designated expiration date and time. American options can be exercised any time up to and including expiry. One of the classical results in derivatives is Merton's theorem (Merton, 1973): absent dividend considerations, the value of an American call option is equal to the value of a European call option with the same strike price and expiration date.

Model-free tests of Merton's theorem have been limited because, in the real world, assets typically have only one option type associated with them, American or European. To the extent that the theorem has been challenged, it is in the idea that the long side of the option may exercise early due to irrationality, transaction costs, or short sale constraints (Finucane, 1997; Poteshman and Serbin, 2003; Jensen and Pedersen, 2016). The idea that early exercise is valuable for the long side of the option is called the early exercise premium (McDonald, 2013).

I introduce in this paper the idea of a further channel that challenges Merton's theorem: that the short side of the American option requires compensation to hold timing risk. This unwillingness to hold timing risk is conceptually similar to standard risk aversion. Standard theory says the short side holds no compensatable risk because if an option is idiosyncratically exercised early, unlinked to market variables, there is no re-investment risk. However, the idea that timing uncertainty is inherently undesirable, whether for practical or psychological reasons, would predict that such risk is compensated.

I test for an aversion to timing risk over and above an early exercise premium by finding a setting with otherwise identical American and European options, and adopting a difference-in-difference approach.

Such a setting is hard to find in general, because most underlyings have only one option type (American or European) on them. However, there is (at least) one exception. CBOE offers both American and European options on the S&P 100, which have the same contract size, trading fees, settlement times, and other contract-specific details.[1] These otherwise identical American and European options will be the focus

[1]There also exist both American and European options on the S&P 500; in particular, American-style contracts on SPDR and European-style contracts on the S&P 500 index. However, these differ

of analysis.

To isolate timing risk over and above an early exercise premium, I look at the following object:

$$(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}) - (\text{bid}_{\text{American}} - \text{bid}_{\text{European}}) \tag{1}$$

Using this differences in differences approach accomplishes several goals. First, it captures the timing risk premium over and above the early exercise premium. This is because the early exercise premium is thought to arise on the long side as a valuation of flexibility (whether rational (Jensen and Pedersen, 2016) or irrational (Poteshman and Serbin, 2003)). Therefore, the second difference (i.e., the bid difference) captures the value of flexibility to the buy side. And this second difference is differenced out.

The overall term thus finds the premium in the ask of the American not attributable to higher pricing from the early exercise premium. This approach allows me to remain agnostic as to the source of the early exercise premium (an interesting but separate question), and focus only on the goal of this paper, which is an aversion to timing risk on the short side of an American, over and above the pricing in of desires for flexibility on the long side.

The differences-in-differences approach also allows me to control for unobservable differences between the American and European options. While all contract specifications are the same (by construction) and controls for liquidity are included (Section 3.3.1, below), it is possible that the option classes differ in unobservable ways. To the extent that these unobservable differences result in structurally higher prices in one options class or the other, those differences should show up in both the bid and the ask. Subtracting out the bid difference therefore also allows me to control for any unobservable factors that would symmetrically affect the spread.

This exercise contributes to a small literature on the pricing of American options

in at least three ways that OEX and XEO do not. First, the most liquid of the S&P 500 options, SPY and SPX, have different contract sizes. In contrast, OEX and XEO have the same contract size. Second, all S&P 500 index options have different CBOE trading fees than SPY, meaning that even after matching on contract size, one would not be able to differentiate fees from pricing premia (CBOE, 2024*a*). This is because it is not possible to see participant identity in the data, and the fees differ by participant type. In contrast, OEX and XEO have the same CBOE trading fees for all participants. Third, SPY versus SPX (and other S&P 500 index options) settle 15 minutes apart; that difference in settlement time can contribute to what is known as a wildcard premium (Fleming and Whaley, 1994), further affecting pricing. In contrast, OEX and XEO have the same settlement time.

relative to European ones. The canonical paper in this literature is Merton (1973), which argues that a dividend-free American call option should be priced equivalently to a dividend-free European call, a theoretical prediction that I show my findings violate. Prior literature challenging Merton's theorem focuses on the early exercise premium (Sung, 1995; Zivney, 1991; Harvey and Whaley, 1991; Li et al., 2024), typically using midpoints; I focus here on timing risk and separate out bid and ask quotes to identify an aversion to timing risk.[2]

3.2 Linkage to Theory

The motivating example in the introduction considered with certainty {\$10 in 2 months for sure} against receiving receiving that \$10 in a timing gamble, which was {50%, 1 month, 50% 3 months}.

The lottery corresponding to the options analysis is even more egregious. In particular, from the perspective of the short side of an option, money can arrive or depart only at expiry for a European option but at any time up to and including expiry for an American option. As a result, an example that would conceptually correspond to the options comparison is \$x in, say, 7 days for sure ('Sure Later Timing'), against \$x in 1 day with some probability; 2 days with some probability; and so on, up to 7 days ('Uncertain Sooner Timing'). The second lottery time-dominates the first one, in that all payments are received or made weakly sooner.

Is \$x positive or negative? When the short side owns the underlying (i.e., a covered call), \$x is a positive cash flow akin to selling one's asset. When the short side does not own the underlying (i.e., a naked short call), \$x will generally be negative. We are not able to observe either identity or asset ownership in the data, so we will not be able to say with certainty whether \$x should be treated as positive or negative in this dataset.

The exercise is nevertheless informative because standard pricing models (e.g., Merton (1973)), do not account for a preference for certain timing. However, what

[2]To my knowledge, there is one prior paper with a reduced-form comparison of American to European options that considers bids versus asks; it finds that most of the premium shows up on the ask side (Dueker and Miller, 2002). The paper hypothesizes that it may be because the options in question were about to be discontinued leading to wider bid-ask spreads. Timing uncertainty provides a more general way to rationalize why differences in American versus European options may show up in ask quotes - the short side requires compensation to bear timing risk. Further, the options that I consider here are continuously traded and not discontinued at any point in the sample.

I will show is that the market appears to prefer 'Sure Later Timing' to 'Uncertain Sooner Timing.'

As a follow-up question, one could ask which representative preference models the findings theoretically map to. When $x is positive, finding a preference for 'Sure Later Timing' relative to 'Uncertain Sooner Timing' goes against standard theories (see Theorem 2 and Corollary 1), which would predict a preference for uncertainty. In fact, it strongly goes against standard theory, because 'Uncertain Sooner Timing' strictly dominates 'Sure Later Timing.' When $x is negative, finding a preference for 'Sure Later Timing' would be in line with standard preference theories, but not standard option pricing models which do not having a timing preference component.

3.3 Data

For the cleanest possible results, I use minute-by-minute quote data from CBOE, which offers both OEX and XEO.[3] I compare quotes within the same minute. CBOE treats both OEX and XEO identically for the purpose of fees, settlement dates and times, contract size, expiration dates, tick size, and other institutional features (CBOE, 2024*c*,*b*). The minute-by-minute quote data is available starting in 2012; the sample I use is therefore Jan 2012 - Nov 2024.

Data on dividends paid by companies in the S&P 100 comes from Compustat, whose series starts in 2002; data on end-of-day levels of the S&P 100 also comes from Compustat. Data on Federal Open Market Committee meeting dates comes from the Federal Reserve Board. For robustness, Appendix B.4 also collects dividend data by hand and shows that it maps well to Compustat.

3.3.1 Cleaning

Following standard data cleaning procedure for options data (Andersen et al., 2017), I drop any quotes with a bid of zero.

Both OEX and XEO options settle to the closing price on the day of exercise, regardless of the time in the day when the option is exercised. Timing uncertainty is therefore not present within a given day, but rather over different days. To study timing uncertainty, I therefore drop quotes where the option expires on the same day

[3]Past research has shown that minute-by-minute data is superior to end-of-day quotes (such as those offered by OptionMetrics), which may be subject to end-of-day volatility (Andersen et al., 2017).

as the quote, because such options are not subject to timing uncertainty - all parties know that they will be exercised that day and that the settlement value will be the day's closing value regardless of the time of exercise.

While most contract specifications are held fixed by CBOE, there are two features that differentiate the American and European options that we must control for. The first and most important is dividends. When a stock is known to start paying a dividend on a particular date, the stock price adjusts downwards by the dividend amount on that date (McDonald, 2013). The S&P 100 is a weighted average of stock prices. Consequently, if for example an American and European option expire on a date with a dividend payment, the holder of the American option may be willing to pay a premium - say, up to the anticipated dividend amount - to exercise their option before the stock price update for the dividend occurs. We should therefore restrict our attention to options in whose lifetime, including at initiation and expiry, there are no dividend considerations for any of the stocks in the S&P 100. This restriction substantially reduces our sample, but is also an important step towards identification, by removing the standard explanation for American option premia.[4]

I therefore focus attention on dates and options where, were a market participant to enter the option on that date, they would not face any dividend considerations for the lifetime of the option. To find such dates, I first find the set of dates on which S&P 100 dividends paid are zero.[5] I then fill in holidays and weekends (for which dividends paid are also zero, because these are not trading days).

The second feature we must rule out as a source of the premium is liquidity: in this case, the American option is the more traded product, perhaps because it was introduced before the European version (Harvey and Whaley, 1991). Following Amihud and Mendelson (1986) and Bryzgalova et al. (2022), I use the bid-ask spread, that is, (ask - bid)/midpoint, as the illiquidity measure. Restricting this is a little tricky because if, for example, the price premium for the American option arises from sellers demanding compensation for timing uncertainty, imposing a tight bid-ask spread means that it will look as though American bids are higher than European bids in the sample - although actually what is happening is that American asks are higher than European asks and the data restriction imposes tight bid-ask spreads.

[4]Li et al. (2024) also consider the options I study here, but (1) do not correct for dividend concerns and (2) do not adjust for liquidity differences.

[5]Compustat calculates dividends based on the ex-dividend date, which is also the relevant date for dividend-based American option strategies.

Thus, we want some liquidity threshold to compensate for liquidity premia, but not such a tight liquidity threshold that we are no longer able to differentiate between movement on the bid and on the ask side. I use a 5% threshold in the main text, e.g. restrict attention to minutes where the difference in bid-ask spreads between the OEX and XEO quotes is less than 5%. For robustness, Table 3 re-performs the analysis for other thresholds and also without liquidity corrections. As the liquidity correction becomes looser, the difference in asks further dominates the difference in bids, consistent with pricing premia for American options arising at least partially from compensation for timing uncertainty for the short side (rather than entirely from a willingness to pay for flexibility on the long side).

The notional value of all options in the final cleaned dataset is approximately $3.9 billion.

3.4 Summary Statistics

Table 1 provides summary statistics for moneyness, days to expiry, and bid-ask spreads for all options in the analysis dataset post cleaning (Section 3.3.1). The bid-ask spreads are similar because of the imposed liquidity controls. Because companies within the S&P 100 pay out dividends at different times (Harvey and Whaley, 1991), in this data, the longest lived contracts with strictly zero dividend considerations have a lifetime from quote date to expiry of seven days. The median quote is in-the-money, but the moneyness distribution skews left; as a robustness check (Section 3.6), I restrict consideration to options that are in-the-money and show that the results continue to hold. Appendix B.1 provides further descriptive statistics for these options.

3.5 Baseline Results

Table 2, Column 1 shows the baseline results. The mean ask difference (OEX - XEO) is $0.50; midpoint $0.32; and bid $0.15; all are statistically significant at the 1% level. The difference-in-difference, e.g. the ask difference between the two products less the bid difference between the two products, is a statistically significant $0.36, with standard error $0.01. Since the average bid-ask spread of OEX options in the sample is $1.96, this average difference-in-difference corresponds to 18% of the average bid-ask spread. These results show that (1) the market prices American call options

TABLE 1: Summary Statistics for OEX and XEO Options

	Both	**OEX**	**XEO**
Bid-Ask Spread (%):			
Mean		4.95%	4.81%
Median		1.99%	1.79%
Standard Deviation		10.91%	10.96%
Moneyness:			
Mean	-0.09		
Median	0.09		
Standard Deviation	0.59		
N. Contracts by Days to Expiry:			
1	1,389,723		
2	386,010		
3	186,894		
4	109,849		
5	26,822		
7	42,773		

Note: This table lists summary statistics for the main sample of OEX and XEO options, cleaned according to the procedure in Section 3.3. The bid-ask spread is calculated as $100 * (ask - bid)/midpoint$. Days to expiry is the number of days between the quote date and the expiration date of the option. Moneyness is calculated as $(underlying - strike)/strike$. The top line follows Equation 1, e.g. the ask difference between the American and European less the bid difference between the American and European.

without dividend considerations higher than otherwise identical European options, and (2) that this higher pricing comes both from the long and the short side, but more so from the short side.

Figure 1 plots the average ask, bid, and midpoint difference, aggregated by week, over time. This figure shows that the ask difference is usually higher than the bid difference across all time periods considered. This indicates that the results of Table 2 are largely consistent across time and not the result of a few outliers.

Timing uncertainty is the primary feature that distinguishes OEX from XEO in this dataset because, by construction, we have controlled for other contract characteristics: quote time, tick size, liquidity, type of settlement, tick size, expiration date, etc. One supply side story consistent with this evidence is that sellers require compensation for bearing the timing risk inherent in American options. Timing risk may interact with various considerations at different horizons and so the pattern of price premia for American options need not increase in the number of days to expiry

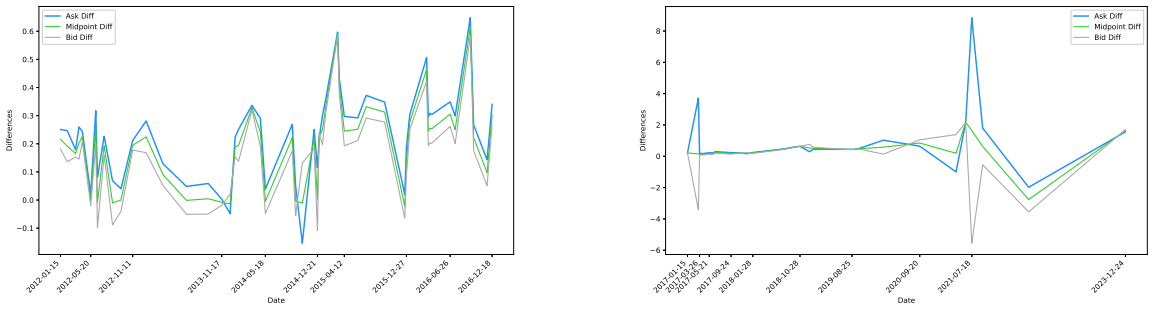
if other considerations dominate; nevertheless, Figure 2 runs the regression:

$$(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}) = \sum_i \mathbb{1}\{\text{Number of days to expiry} = i\} + \beta_1 (\text{bid}_{\text{American}} - \text{bid}_{\text{European}}) + \beta_2 C + \epsilon,$$

where C are various controls (for example, moneyness and bid-ask spread percentage differences) and shows that the difference in asks is monotonic in the time to expiry, even after controlling for the difference in bids, across a number of specifications.[6]

The robustness section, next, shows that these baseline results continue to hold in various subsamples of the data.

FIGURE 1: Ask Difference - Bid Difference is Positive Across Sample



(A) 2012-2016 (B) 2017-2024

Note: This figure plots average price differences for asks, bids, and midpoints, aggregated weekly. Weeks with data are shown as points on each line. Because some dates have large magnitude differences, for ease of visual processing, Figure 1a focuses on the period 2012-2016 and Figure 1b 2017-2024. The figure shows that ask differences (in blue) are generally positive and higher than bid differences (in gray) over time.

3.6 Assessing Possible Explanations

3.6.1 Out-of-the-Money Anomalies.

Prior studies show that pricing anomalies can occur for deeply out of the money options (Hull, 2018). The second column of Table 2 restricts analysis to the subsample of quotes that are in-the-money. The results continue to hold: on average, ask quotes

[6] One may ask why not regress Equation 1 on time to expiry instead. The reason is that the long side may react differently to information than the short side (Section 3.6.8), for example for institutional or microeconomic reasons. The specification used for monotonicity allows for asymmetric reactions.

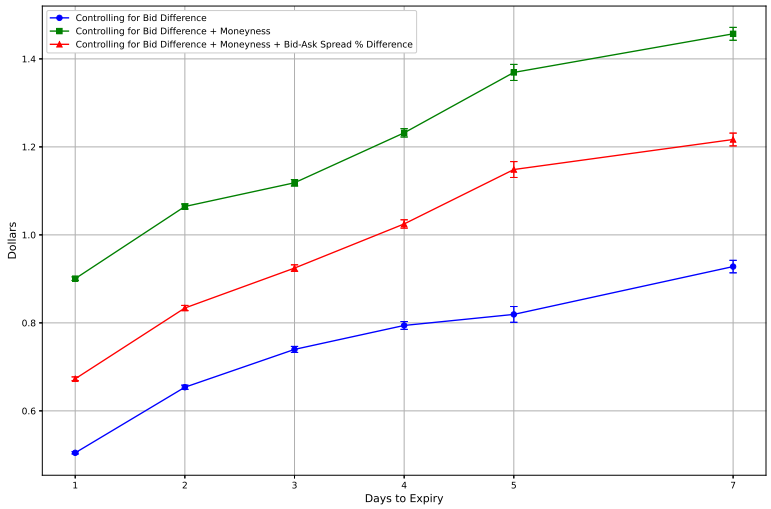
TABLE 2: Pricing Differences between Dividend-Free American (OEX) and European (XEO) Options

	Baseline	In the Money	No Fed Meetings	Positive Open Interest
Ask Difference - Bid Difference	\$0.36***	\$0.33***	\$0.36***	\$0.05***
	(0.00)	(0.00)	(0.00)	(0.00)
Ask Difference	\$0.50***	\$0.40***	\$0.50***	\$0.23***
	(0.00)	(0.00)	(0.00)	(0.00)
Midpoint Difference	\$0.32***	\$0.23***	\$0.32***	\$0.20***
	(0.00)	(0.00)	(0.00)	(0.00)
Bid Difference	\$0.15***	\$0.07***	\$0.15***	\$0.18***
	(0.00)	(0.00)	(0.00)	(0.00)
Ask Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.10	\$0.00
50%	\$0.30	\$0.30	\$0.30	\$0.20
75%	\$0.50	\$0.40	\$0.50	\$0.40
Midpoint Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.10	\$0.00
50%	\$0.25	\$0.25	\$0.25	\$0.15
75%	\$0.45	\$0.35	\$0.45	\$0.35
Bid Difference Quartiles:				
25%	\$0.10	\$0.00	\$0.10	\$0.00
50%	\$0.20	\$0.20	\$0.20	\$0.10
75%	\$0.40	\$0.30	\$0.40	\$0.30
Number of Observations	2,142,071	1,562,191	2,142,071	226,954
Number of Unique Days	109	86	109	104

*Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.*

Note: This table shows the difference in quoted prices (OEX (American) - XEO (European)) between otherwise identical OEX and XEO options in dollar terms. The baseline column performs this analysis on the data cleaned as described in Section 3.3. The in-the-money column restricts the cleaned data to only in-the-money options. The no-fed-in-lifetime column restricts the cleaned data to only options in whose lifetime there is no Fed meeting. The positive open interest column restricts the cleaned data to only option quotes and minutes for which both OEX and XEO have strictly positive open interest.

FIGURE 2: Ask Difference is Monotonic in Time to Expiry, Even After Controlling for Bid Difference



Note: This figure plots the regression coefficients when regressing ask difference on an indicator for number of days to expiry, controlling for bid difference and other variables. The blue line shows these coefficients on indicators for number of days to expiry, when bid difference is controlled for. The red line shows these coefficients when controlling for bid difference and in addition controlling for an indicator on whether or not the option is in-the-money. The green line shows these coefficients when controlling for bid difference, an indicator for whether the option is in-the-money, and the difference in bid-ask spread of OEX (expressed as a percentage) and bid-ask spread of XEO (expressed as a percentage) for each minute and option. The error bars represent the 95% confidence interval on each coefficient in each regression.

for the American option are \$0.40 higher than for the European, midpoints \$0.23 higher, and bid quotes \$0.07 higher.

3.6.2 Re-Investment Risk.

In addition to an aversion to uncertain timing, another reason that the short side for the American option may require additional compensation is if there is a risk of interest rates changing during the lifetime of the option. To check for this, I use Federal Reserve meeting dates to determine whether there is a Fed meeting during the lifetime of any of the options in the sample. Column 3 of Table 2 shows that restricting to no-dividend days of the S&P 100 is a stronger restriction than restricting to options in whose lifetime there are no Fed meetings, so that the baseline sample already includes only options with no re-investment risk.

TABLE 3: OEX - XEO Pricing Differences Under Different Liquidity Thresholds

	Baseline (Liquidity Difference < 5%)	Liquidity Difference < 1%	Liquidity Difference < 10%	No Liquidity Controls
Ask Difference - Bid Difference	\$0.36***	\$0.01***	\$0.58***	\$0.69***
	(0.00)	(0.00)	(0.00)	(0.00)
Ask Difference	\$0.50***	\$0.33***	\$0.64***	\$0.71***
	(0.00)	(0.00)	(0.00)	(0.00)
Midpoint Difference	\$0.32***	\$0.33***	\$0.35***	\$0.36***
	(0.00)	(0.00)	(0.00)	(0.00)
Bid Difference	\$0.15***	\$0.32***	\$0.06***	\$0.02***
	(0.00)	(0.00)	(0.00)	(0.00)
Ask Difference Quartiles:				
25%	\$0.10	\$0.20	\$0.10	\$0.10
50%	\$0.30	\$0.30	\$0.30	\$0.30
75%	\$0.50	\$0.50	\$0.50	\$0.50
Midpoint Difference Quartiles:				
25%	\$0.10	\$0.15	\$0.10	\$0.10
50%	\$0.25	\$0.25	\$0.25	\$0.25
75%	\$0.45	\$0.50	\$0.50	\$0.50
Bid Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.05	\$0.00
50%	\$0.20	\$0.20	\$0.20	\$0.20
75%	\$0.40	\$0.40	\$0.40	\$0.50
Number of Observations	2,142,071	1,562,191	2,142,071	226,954
Number of Unique Days	109	109	109	109

3.6.3 Strictly Positive Open Interest.

One concern may be that option quotes are not meaningful for options which do not have positive open interest. This is unlikely to explain the results because CBOE has designated market makers for option classes, who are required to uphold certain requirements, including minimum quote contract size requirements (Cboe, 2019). As a further explicit check, Column 4 of Table 2 restricts attention to options and minutes for which both OEX and XEO have strictly positive open interest. The result of timing uncertainty being priced continues to hold.

3.6.4 Wildcard Premium Differences.

Fleming and Whaley (1994) document that some options have wildcard premiums, meaning that the settlement price of the contract is established before the last time at which the option holder can decide to exercise. This would affect a comparison of options on the S&P 500, because PM-settled SPX (European) options are exercised at the close of the trading day, but holders of SPY (American) have the option to exercise after the closing price of the underlying - so one is subject to a wildcard premium

while the other is not. This is one of the reasons I chose not to use the S&P 500 for the reduced form comparison. A benefit of the options on the S&P 100 considered here is that both stop trading at the same time and have identical settlement specifications, and therefore there is no wildcard premium difference between them.

3.6.5 Informed Trading

The next set of explanations we will consider explores the idea that the short side of the market may be strategically reacting to actions from the long side of the market. We will consider these explanations one by one, but they can also be answered by showing that the options we consider are traded largely by uninformed investors. If the trading is uninformed, that alleviates strategic concerns that might affect pricing.

There are several reasons to believe that the trades corresponding to our main sample are uninformed. First, past research shows that trading in index options is generally uninformed, with informed investors typically locating in single-name stock options (Pan and Poteshman, 2006). But second, we can perform an explicit check for uninformed trading by looking at trade size, because Muravyev (2016) shows that small option trades tend to be uninformed in that they do not predict market movements. In addition, small option trades have been shown to be a valid (if noisy) proxy for retail trading in options, which past work shows is uninformed (Poteshman and Serbin, 2003; Barber and Odean, 2013; Bauer et al., 2009; Bryzgalova et al., 2022). Appendix B.6 checks the proportion of small trades (few contracts traded, small monetary amount transacted, or both) and shows that these small trades constitute over over 50% of trades corresponding to the main sample. Within buys - which is what the short side of the market would be responding to - small trades account for approximately 57-83% of trades, depending on the measure used. From past research, we conclude that a large portion of trades corresponding to the main sample are uninformed, which means strategic responses to them are less likely to be driving the results.

3.6.6 Adversarial Thinking.

Because of the constraint that these options not have dividend considerations in their lifetime, these are generally short-dated options (Table 1). One might ask why the long side of an option would choose to enter an American rather than European; in

particular, rationally a market participant should be willing to pay more for a short-dated American than European if either (1) the long side values the timing flexibility, or (2) the long side intends to exercise right away. If (2) holds, then the short side, anticipating this, may adjust their prices. However, standard theory predicts they would adjust prices in the opposite direction from what I find. This is because an American where the long side exercises immediately is equivalent to a European expiring today; but we know from both standard models and a large empirical literature that European options with more days to expiry are more expensive, because they have a higher implied volatility (Bakshi and Kapadia, 2003; McDonald, 2013). This kind of adversarial thinking would therefore predict that American options trade at a discount relative to European options, the opposite of what I find here.

Another type of adversarial thinking might posit that options are exercised only on days disadvantageous to the short side of the option. Robustness check 'Re-Investment Risk,' above, shows that the result holds even when the most common market variables related to this, interest rate considerations, are controlled for. In addition, such considerations should be minimal one day to expiry, but the effect shows up for such options too (Appendix B.2). Finally, arguments of this form should be minimal if trading on the long side is uninformed, as discussed in Section 3.6.5, above.

3.6.7 Hedging Considerations.

One reason American options may be harder to hold for the short side than European ones is that, when there is uncertainty over the day of exercise, hedging becomes more complicated, and the short side of the American may have a practical or psychological aversion to dealing with this complexity (Puri, 2024), or may have budgeting needs that make changes in hedging difficult.

To show that there is a pricing of timing uncertainty over and above differential hedging considerations, Appendix B.5 re-performs the analysis for only options whose hedging requirements are not expected to change over their lifetime. First, it reconducts the test for deep out-of-the money options (that is, those whose delta is equal to zero). This means that there is no practical need to hedge either the American or European option, particularly for these short-dated options close to expiry. Though the sample becomes an order of magnitude smaller, the qualitative result: a pricing premium on the short side, over and above that on the long side (Equation

1) continues to hold. Second, it reconducts the test for zero gamma options, that is, those which, under certain model-based assumptions, have a stable delta over their remaining lifetime. The results of a a pricing premium on the short side, over and above that on the long side, hold in this subsample as well.

3.6.8 Asymmetric Changes in Bids and Asks - For Example Due to Inventory

Specification 1 treats asks and bids symmetrically. But, it is possible that the long side moves differently than the short side. For example, under some models (Ho and Stoll, 1981), if a market maker has accumulated excessive long positions due to rising bids, they might lower ask prices to encourage sales and rebalance inventory. The main specification does not allow for that flexible difference in changes in bids relative to asks. To allow for this flexibility, I run the regression:

$$\left(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}\right)_{m,o} = \beta_0 + \beta_1 \left(\text{bid}_{\text{American}} - \text{bid}_{\text{European}}\right)_{m,o} + \epsilon,$$

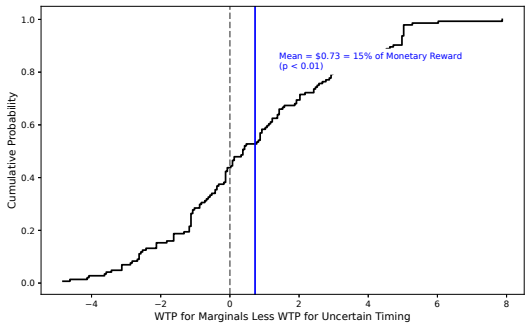
where m denotes the minute and o the option. We are interested in whether $\beta_0 > 0$ even after allowing β_1 to be flexible (e.g., not imposing that it is one). The regression yields a statistically significant $\beta_0 = 0.58$ ($p < 0.01$), showing that there is an unwillingness to be short the American over and above flexible pricing incorporation of the long side.

3.6.9 Further Liquidity Considerations

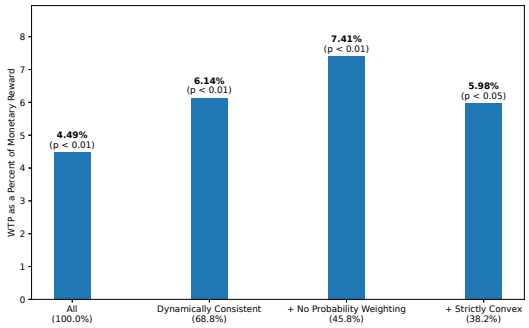
Figlewski (2022) shows theoretically that when the liquidity of the options market is highly variable, an American option can be more valuable than a European one because the American allows the long side to exercise at times when the market is relatively illiquid. However, the baseline analysis controls for liquidity, by construction (Section 3.3.1). In addition, this is a reason why the long side may value the American, but not a reason for the short side to price the American at a premium; per the 'Adversarial Thinking' section above, standard option value arguments imply that the short side should be willing to offer options at a discount if they believe early exercise is likely. In addition, the 'Hedging' section above shows that the results hold even for options where the short side has no need to change their hedging requirements, e.g. their own budgeting considerations are not affected.

4 Willingness-to-Pay and Pricing Narratives

FIGURE 3: Willingness-to-Pay Measures



(A) WTP for Marginals − WTP for Uncertain Timing (Theorem 1)



(B) WTP for Certain − Uncertain Timing and Theoretical Properties (Theorem 2)

Note: Panel (A) shows the cumulative density function for difference in willingness to pay for the marginals of the uncertain timing lottery less the willingness to pay for the uncertain timing lottery itself. Green color-codes responses where the difference was above zero, and coral responses where the difference was below zero. Panel (A) also provides a label for the mean difference and the p-value of that difference. Panel (B) shows, as a percentage of the money at stake ($5), the difference in willingness to pay for certain and uncertain timing. The first bar shows this overall; the second within the subset of respondents who are dynamically consistent; the third bar within the subset of respondents who are dynamically consistent and do not distort probabilities; and the fourth bar within the subset of respondents who are dynamically consistent, do not distort probabilities, and display locally convex deterministic discounting. The percentages on the bottom of the x-axis show the fraction of respondents within each group. For example, 66.7% of respondents are dynamically consistent, while 34.1% satisfy all three properties. The p value is shown underneath the mean value for each bar.

As a supplement to the market results, in this section I use a survey to elicit willingness to pay for lotteries with uncertain timing and measure that willingness to pay against the willingness to pay for the lottery's marginals, to directly test the prediction of Theorem 1. Consistent with the market-based results, Theorem 1 is falsified: the pricing of the marginal distributions differs from that of the lottery with uncertain timing. In addition, the positive and statistically significant difference in willingness to pay for certain vs. uncertain timing holds even amongst those who do not distort probabilities; who are dynamically consistent; and who discount deterministic time in a locally convex way, providing further evidence that the theoretical tradeoffs in Theorem 2 have empirical bite. Finally, an incentivized elicitation of internal algorithms used to make choices show that many respondents self-describe as approaching pricing of certainty in the time and money domains similarly.[7]

[7]Self-descriptions are often used in surveys (ex. Schotter (2003), Bailey et al. (2018)) – here I use them in the context of studying attitudes towards timing risk.

TABLE 4: Incentivized Narratives

Panel A: Sample Narratives	**Panel B: Classification**	
(1) I simply just went for less risks.	GENERAL PRICING RULE	50%
(2) I tried to play it relatively safe.	MONEY CERTAINTY PRICING, TIME UNCERTAINTY TOLERANT	23%
(3) I chose the safe option with no probability and was a guarantee up until \$4 when I changed to the probability option.	MONEY CERTAINTY PRICING, TIME CERTAINTY PRICING	15%
(4) For lotteries that include a 50% chance of receiving nothing, I elected to receive everything over a minimal amount. For lotteries that paid out and varied only in time, I elected to receive the deferred payment for anything less than the immediate award.	MONEY UNCERTAINTY TOLERANT, TIME UNCERTAINTY TOLERANT	10%
	MONEY UNCERTAINTY TOLERANT, TIME CERTAINTY PRICING	2%
(5) I chose to not look at the timeframe of when I would get the bonus because I knew that I would get it eventually. Instead, I only focused on the dollar amount that would be given to me. In the case of a lottery, I calculate the dollar amount by taking the expected payoff from the lottery.		

4.1 Data and Methodology

The survey was run on Prolific in March 2025; IRB approval was obtained from the New York University Institutional Review Board, under ID IRB-FY2024-8127. Respondents received an average payment of \$4.23, and the median time taken was 21 minutes. Willingness to pay was measured via standard incentivized multiple price lists (Holt and Laury, 2002), that is, a question and row were selected at random for 10% of respondents, who then received a bonus payment according to their response in that row. A series of eleven comprehension questions ensured that participants understood the difference between sure money and sure timing; sure money and uncertain timing; and uncertain money and sure timing (Online Appendix C). There were 145 respondents total.

The main three questions to test for attitudes towards uncertain timing were elicitations of willingness to pay (WTP) today for the lotteries: (1) {50%, \$5 in 2 days; 50%, \$5 in 8 days}; (2) {50%, \$0 in 2 days; 50%, \$5 in 2 days}; (3) {50%, \$0 in 8 days; 50%, \$5 in 14 days}; and (4) \$5 in 5 days. The prediction of the standard SDF framework is that, for any SDF, holding all else equal, WTP for (2) + WTP for

(3) = WTP for (1). The prediction of Definition 1 is that the WTP for (1) should be weakly lower than the WTP for (4).

In addition, to measure correlations of the answers to these questions with (a) dynamic consistency, (b) no probability distortion, and (c) convex discounting, that is, some of the properties at odds with a preference for certain timing in Theorem 2, the survey included additional standard questions for deterministic time discounting (Sprenger, 2015) and probability distortion vs. complexity aversion (Puri, 2024); these questions are described in detail in Online Appendix C.

The order of the questions was randomized within the survey for each participant. Directly after the uncertain timing question, the participant was surprised with a question which asks them to describe how they made their prior choices. The question was made incentive compatible by having, for 5% of respondents, a new participant read the original participant's description, and having the new participant attempt to replicate the original participant's choices on the timing risk questions. If the new participant correctly replicated the original participant's choice on a randomly selected question, both earn a bonus.

4.2 Results

The spread between the willingness to pay for the marginals vs. the willingness to pay for uncertain timing with the same marginals, is depicted in Figure 3a. In contrast to the marginalization approach of the standard SDF framework, these direct elicitations reveal an average difference of \$0.73 in willingness-to-pay, equivalent to 15% of the \$5 monetary award in the uncertain timing lottery.

Figure 3b shows that the statistically significant willingness to pay for certain timing remains even after applying properties that financial economists have historically found appealing; namely, dynamic consistency, no probability distortion, and locally convex discounting.[8]

Table 4, Panel A provides examples of incentivized narratives about how respondents approached uncertain timing and substitution across time and money. Table 4, Panel B classifies these into general choice rules (e.g., no differentiation between time and money in approach to choice); stated certainty preference in both domains;

[8]Online Appendix C performs a robustness check in which the sample is restricted only to high financial literacy respondents who satisfy monotonicity conditions, and shows that the results continue to hold.

certainty preference in money but uncertainty tolerant over time; etc. The classification is done using o3 with the prompt given in Online Appendix C. Overall, 65% of respondents report pricing narratives of either a general pricing rule across domains or a higher WTP for certainty in both domains.

5 Potential Applications Across Finance

Future work can build on the idea that (1) markets appear to price a preference certain timing but (2) finance models do not always capture that, in a variety of ways. Below, I sketch potential applications across various subfields of finance.

Bond Markets. Callable and convertible bonds are subject to timing uncertainty. Pricing models for these bonds typically conceptualize them as a standard bond plus an option (Brealey et al., 2020). To the extent that options are mispriced by standard models because they do not account for an aversion to timing uncertainty (Section 3), corporate bonds may be as well.

Climate Change. The time of a disaster is uncertain as well as the magnitude. Existing approaches to disaster that use Poisson processes to model climate risk may mis-estimate disutility arising from uncertain timing, if not accounting for the fact that the discount rate determines not only attitudes regarding the future but also attitudes regarding timing risk.

Corporate Decisions. Companies often have uncertainty not only over the magnitude of cash flows on an uncertain project, but also when they will occur. One puzzle in the corporate literature is that companies appear to use hurdle rates that are higher than their cost of capital as estimated by the CAPM (Graham, 2022). Since the intertemporal CAPM uses discounted expected utility, which Corollary 1 shows is temporally risk-seeking, it may be the case that one reason for these 'high' hurdle rates is that the cost of capital estimate does not account for the timing uncertainty inherent in some projects.

6 Connections to Literature

The contribution of this paper spans financial subfields. In showing that the standard stochastic discount factor framework is unable to capture timing uncertainty, and there is no debate-free extension that can do so even once one moves beyond existing models, the paper contributes to a literature on the stochastic discount factor and its properties (Ross (1978), Harrison and Kreps (1979), Hansen and Singleton (1982), Hansen and Jagannathan (1991), Duffie (2001), Hansen and Scheinkman (2017)).

By showing that pricing models do not accommodate timing uncertainty, but a preference for certain timing shows up empirically in real-world markets, the paper also contributes to literatures across financial subfields on pricing and decision making. In particular, the findings have implications for thinking about how to model the representative investor; a topic extensively thought about and discussed by, for example, Campbell and Cochrane (1999); Lettau and Ludvigson (2001); Daniel et al. (2002); Lewellen and Nagel (2006); Verdelhan (2010), and Barberis (2013).

In showing that standard frameworks, and their extensions, do not appear to be able to capture empirical pricing, the paper complements work on the term structure of the equity premium (Binsbergen et al., 2012; Binsbergen and Koijen, 2017; Lazarus, 2022), which shows that standard theory does not match models on dividend strips. In this case, the empirical pricing I document, and which does not map well to any SDF or its extensions, is certain vs. uncertain timing.

In studying empirical pricing of options, the paper contributes to a literature on actual prices of options and how these either adhere to, or deviate from, standard theory. Past work has studied a willingness to pay for flexibility on the long side (McDonald, 2013), which my specification explicitly distinguishes from; as well as frictions across markets (Bryzgalova et al., 2024); and the identification and behavior of retail trades (Bryzgalova et al., 2022). Here, I study the short side of an index option market, to show that, in a clean test, the equilibrium pricing is consistent with an aversion to bearing timing risk.

Because pricing appears throughout finance, the paper also has implications for how to think about discounting risky cash flows (Graham, 2022), when the time of project completion or return may be uncertain; welfare implications associated with climate finance, where the timing of subsidy or disaster may be uncertain (Bolton and Kacperczyk, 2020; Stroebel and Wurgler, 2021; Pastor et al., 2024); valuation

in venture capital, where the timing of return is uncertain (Da Rin et al., 2013); the pricing of corporate bonds, where the timing of conversation or call is uncertain (Brealey et al., 2020); and more generally, has implications for behavior in any setting with uncertain timing.

The paper also complements a large literature on the elasticity of intertemporal substitution, a component of preferences whose calibration is thought to be first order for financial decisions (Hansen and Singleton, 1982; Vissing-Jorgensen, 2002; Woodford, 2003). It also complements a smaller literature on the timing of uncertainty resolution (Epstein et al., 2014). Here, I show that no matter what the calibration of the EIS, and even when all uncertainty is resolved immediately, the SDF and its extensions struggle to capture uncertain timing.

The finance literature has not historically focused on attitudes towards timing risk in pricing models or empirical markets. This paper shows that such attitudes are relevant empirically and theoretically. In particular, the standard SDF framework does not allow for pricing of timing risk, but real-world markets with sophisticated participants feature pricing consistent with compensation for bearing uncertain timing. This is not, however, the first paper to think about these topics in the world. There is a small but rich interdisciplinary literature on this topic. In biology, Kacelnik and Bateson (1996) show that foraging animals like pigeons and rats are temporally risk seeking. In decision theory, Chesson and Viscusi (2003) show that expected utility is temporally risk-seeking over gains, but unincentivized experiments seem to show the opposite. Onay and Onculer (2007) generalize this result to time-separable preferences, and show that temporal risk aversion may be partially accommodated by a model that includes probability weighting. DeJarnette et al. (2020) leverage this past work to show that temporal risk aversion is axiomatically incompatible with a property they introduce called stochastic impatience, across a wide range of utility functions, and perform further experiments supporting certain timing; they and Dillenberger et al. (2024) also derive results for both properties within Epstein-Zin. Ebert (2020) introduces terminology and discusses possible ways to formalize temporal attitudes.

In addition to contributing to the finance literature, the paper contributes to this interdisciplinary literature by bringing the topic of timing risk to applications and theoretical frameworks relevant for finance. First, on the empirical side, I provide some of the first real-world evidence of the pricing of certain timing in large financial

markets with sophisticated participants. Second, on the theory side, this is a first paper across literatures to study temporal risk within the stochastic discount factor framework central to asset pricing. In addition, I expand the theory to show that even moving beyond commonly-used models, it is not possible to accommodate empirical pricing without giving up at least one property financial economists have historically found appealing. Third, I elicit willingness-to-pay and pricing narratives in surveys to supplement the market results.

Finally, the paper also has policy implications for the recent discussion around discount rates, especially as regards pricing of longer-term government policies (Joskow et al., 2023). Whereas prior work has calibrated a long-run discount rate parameter (Lucas, 2023), the present work shows that for any standard pricing framework, such calibration, and its application, simultaneously pins down not only deterministic discounting but also (perhaps inadvertently) prices uncertain timing through the marginals.

7 Conclusion

I show in this paper that attitudes towards certain vs. uncertain timing are of interest for financial economics, both theoretically and empirically. Standard pricing frameworks are not able to capture timing risk; and there is no debate-free extension, even moving beyond standard models, that can do so. Empirically, however, in a reduced-form test, holding all else equal, options markets' prices are consistent with compensation for bearing timing risk. The results are robust to tests for informed pricing, inventory pressure, hedging considerations, liquidity concerns, re-investment risk, microstructure differences, and other alternative explanations. A supplemental survey elicits willingness to pay and further confirms that the theoretical predictions of standard pricing models do not hold in the data; in addition pricing narratives treat uncertainty over time and over money similarly.

The paper has implications for both academia and policy. It implies that standard pricing models do not correctly price assets or financial decisions subject to uncertain timing. This finding has immediate relevance for a policy debate over the correct discount rate to use, by showing that any estimate of the discount rate in the face of timing uncertainty also simultaneously pins down risk attitudes towards timing risk. The paper further indicates that both the pricing and decision-making surrounding

any number of financial products - options, employee compensation, project valuation, callable bonds, climate policy, etc. - should be re-examined.

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