Monetary Policy in Financial Oligopolies *

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Abstract

In light of rising bank concentration, we examine how intermediary market power affects the transmission of monetary policy. To this end, we incorporate a finite number of strategically interacting banks into an otherwise standard New Keynesian framework. Banks operate as dynamic platforms, exerting both oligopoly power in the loan market and oligopsony power in the deposit market. We characterize the strategic behavior of the banks using coupled generalized Euler equations (GEE), which generate endogenous, state-dependent markups and markdowns. Using this framework, we show that increasing bank concentration dampens and shortens the persistence of monetary policy effects on inflation, output, and consumption by restricting credit supply and dampening investment in physical capital.

JEL Codes: D43, E44, G12, G21, L11.

Keywords: Dynamic Financial Oligopoly and Oligopsony, Dynamic Platforms, Endogenous Financial Markups, Financial Intermediaries, Market Power, NK models.

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1 Introduction

The transmission of monetary policy (MP) shocks is central to macroeconomic analysis, yet the role of financial market structure in shaping this transmission and its impact on macroeconomic outcomes remains underexplored. In view of the rising concentration in the banking sector, in this paper, we examine how bank market power alters the general equilibrium effects of MP shocks by embedding a finite number of banks (i.e., banks are non-atomistic) into a standard New Keynesian (NK) framework with nominal rigidities. Unlike models with monopolistic competition in the banking sector, where banks are atomistic price setters, our approach explicitly accounts for strategic interactions among a finite number of banks. In this setting, lending markups and borrowing markdowns emerge endogenously as a function of both market structure and macroeconomic state variables. We formalize these dynamics through a Markov-Perfect Equilibrium (MPE) framework (e.g., Maskin and Tirole, 2001), similar in spirit to that proposed by Villa (2025), which allows banks to dynamically adjust their pricing behavior in response to idiosyncratic and aggregate conditions. We build on Villa (2025) and characterize banks' strategic behavior using generalized Euler equations that incorporate both oligopsony (deposit market) and oligopoly (loan market) power, thereby modeling banks as dynamic platforms. The structure of these generalized Euler equations parallels those found in the optimal fiscal policy literature (see, for instance, Klein and Ríos-Rull, 2003; Krusell, Martin, and Ríos-Rull, 2004; Klein, Krusell, and Ríos-Rull, 2008; and Clymo and Lanteri, 2020).

We view the general equilibrium nature of our MPE as essential for understanding the macroeconomic implications of bank market power. By explicitly modeling the interaction between the banking sector and the real economy, we analyze how the dynamics of output, inflation, consumption, and capital accumulation respond to monetary policy shocks when the financial sector consists of a few strategic banks. Crucially, the dampening effect of bank market power on MP transmission arises endogenously from equilibrium interactions. In a standard NK setting, a monetary policy rate cut lowers household savings incentives, raising consumption and increasing firms' demand for capital. However, in a market where banks strategically internalize changes in credit demand, a higher demand for loans leads to persistently higher lending markups, raising the cost of investment relative to a perfect competition benchmark. As a result, firms substitute away from capital toward labor, altering the factor composition of production and significantly muting the expansionary effects of monetary policy.

This shift in equilibrium responses has important macroeconomic consequences. Unlike standard NK models, where capital accumulation amplifies the effects of monetary policy

over time, the presence of bank market power significantly weakens the persistence of MP shocks. Since investment is constrained, the long-run propagation of demand-side stimulus is reduced, leading to a structurally dampened response to monetary interventions. In a fully general equilibrium setting, this mechanism feeds back into inflation, consumption, and labor markets, further weakening the impact of MP shocks. Moreover, banks reduce deposit rates more aggressively than under perfect competition, further dampening household saving behavior and limiting the overall supply of credit. As a consequence, much of the stimulus-driven increase in demand is absorbed by higher labor input rather than capital deepening, shifting the composition of economic adjustment.

By integrating an oligopolistic banking sector with a finite number of strategic banks into a general equilibrium framework, this paper provides a novel perspective on how financial market structure conditions the effectiveness of monetary policy. Our results underscore that banking concentration is not merely a microeconomic phenomenon but has first-order macroeconomic implications, significantly altering both the magnitude and persistence of policy shocks.

Related Literature A growing body of research highlights the crucial role of financial intermediaries' market power in shaping the transmission of monetary policy. Traditional models often assume a frictionless financial sector, yet recent studies underscore that banks' strategic interactions in both loan and deposit markets significantly influence macroeconomic dynamics.

Our framework builds on Villa (2025) who studies how few strategic banks affects the transmission of macroeconomic shocks in a real economy with heterogenous firms. Villa (2025) develops a model where a finite number of strategic banks interact with heterogeneous firms, showing that oligopolistic banks extract higher markups from smaller firms with fewer outside financing options. This distortion dampens firms growth and becomes particularly severe during recessions or in the event of a major bank failure. Our study builds on this framework by embedding strategic banks into a New Keynesian economy and explicitly modeling their dual market power on both the deposit and lending sides. A related methodological approach is found in Bertolotti, Lanteri, and Villa (2025), who examines strategic suppliers of durable goods in a macroeconomic setting. The parallels between banking and durable goods markets source from the intrinsic dynamic nature of these problems and suggest that insights from industrial organization models and recursive contracts (e.g., optimal fiscal policy) can be fruitfully applied to monetary policy analysis. One of the first attempts to integrate a small number of strategic banks into a macroeconomic model is Corbae and D'Erasmo (2021), who develop a rich quantitative model of banking industry

dynamics to study the effects of financial regulations.

From a structural and monetary policy perspective, Wang, Whited, Wu, and Xiao (2022) quantitatively assesses the role of bank market power relative to other frictions in monetary transmission. The study finds that banking sector oligopoly effects are as influential as regulatory constraints and financial frictions in shaping policy outcomes. This result suggests that central banks must account for banks' strategic responses when designing policy interventions. Empirical studies also support the notion that bank concentration weakens monetary policy transmission. Drechsler, Savov, and Schnabl (2017); Li, Loutskina, and Strahan (2019) focus on deposit market power and provide evidence that larger, more concentrated banking systems exhibit lower interest rate pass-through, dampening the real effects of monetary policy. Wang, Whited, Wu, and Xiao (2022) further highlight that bank market power influences the cyclicality of credit supply, with stronger dampening effects during downturns. These findings align with our theoretical prediction that increased banking sector concentration in the EU leads to a muted and less persistent monetary policy response in inflation, output, and investment.

A number of other recent papers emphasize that an imperfectly competitive banking sector alters the effectiveness of monetary policy. Drechsler, Savov, and Schnabl (2017) show that banks with market power can amplify financial market instabilities in response to monetary policy shocks, potentially leading to abrupt increases in equity premium volatility—so-called "market tantrums" (Feroli, Kashyap, Kashyap, and Shin, 2014; Sahay, Arora, Arvanitis, Faruqee, N'Diaye, and Griffoli, 2014). The presence of bank market power can thus create nonlinearities in policy transmission, where financial conditions deteriorate sharply under contractionary shocks.

Our paper contributes to the literature by integrating a finite number of non-atomistic financial intermediaries into an otherwise standard NK framework with nominal rigidities à la Rotemberg (1982). In this environment, banks act as strategic intermediaries, exerting simultaneous oligopoly power in the loan market and oligopsony power in the deposit market, where markups and markdowns arise endogenously as functions of economic fundamentals. Consequently, these markups and markdowns depend on both the banking sector's market structure and the broader macroeconomic state variables. This framework enables us to analyze how endogenous pricing distortions shape the transmission of monetary policy shocks in a general equilibrium setting with nominal rigidities.

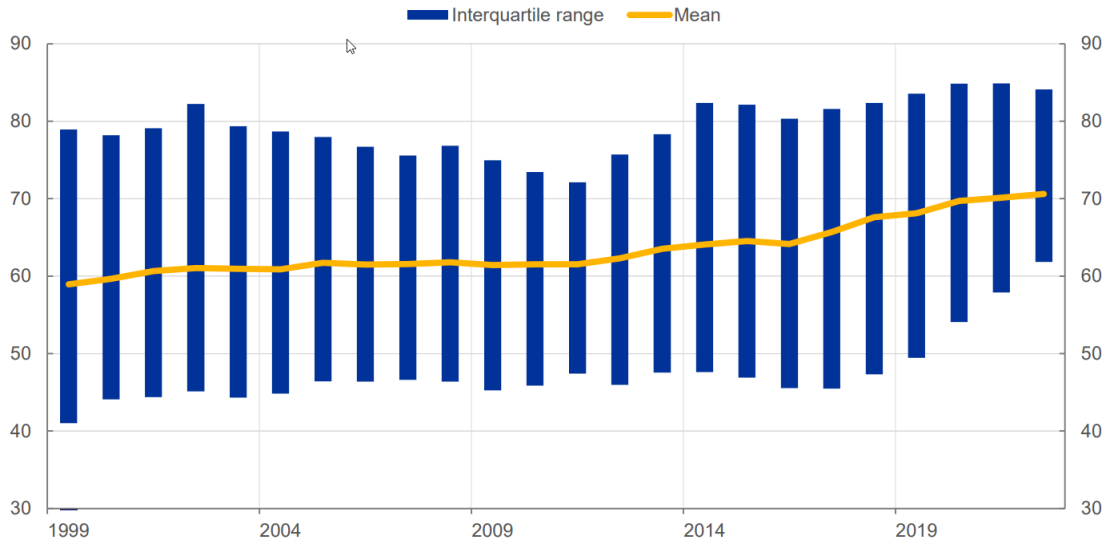
Outline. The rest of the paper is organized as follows. Section 2 presents stylized facts that motivate the analysis. Section 3 describes the model and discusses the main mechanism and its effects in detail. Section 4 explains the calibration and illustrates the results. Section

6 concludes.

2 Stylized Facts

The paper is motivated by two stylized observations. First, as shown in figure 1, the EU banking sector concentration has been growing steadily since 1999. While the average market share of the top five banks in a given EU country was around 60% in 1999, it has increased to 70% despite the EU's efforts to promote a banking union. At the same time, there is a considerable degree of variation across countries, with the current interquartile range between 60 and 85%.

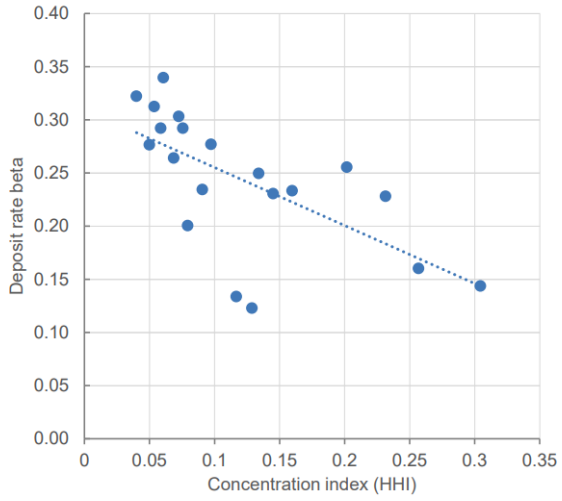
Figure 1. EU Bank Concentration



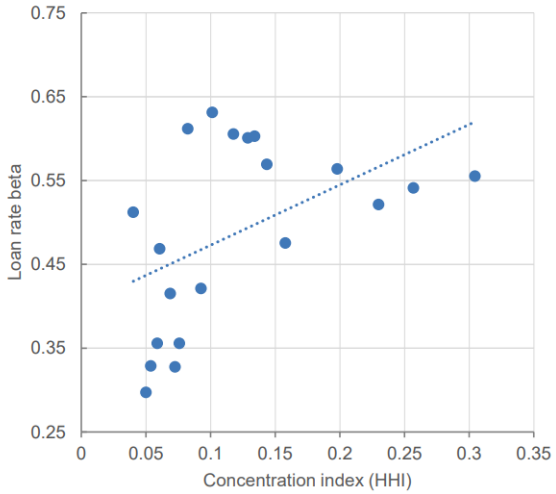
Notes: Share of the top five credit institutions in total bank assets for individual euro area countries. The yellow line is a simple average of individual country values. The blue bars show the interquartile range of individual country values. Sources: Banking structural statistical indicators (SDW) and ECB staff calculations https://www.ecb.europa.eu/press/key/date/2024/html/ecb.sp240425 e762e980c3.en.pdf.

Figure 2 shows that heterogeneity in banking sector concentration has real economic effects. It shows that countries with a more concentrated banking sector, measured with the Herfindahl–Hirschman Index (HH) index, have smaller deposit rate beta and larger loan rate beta. In other words, the elasticity of deposit and loan rates with respect to changes in the interbank rate depends on the banking sector concentration. Consequently, the banking sector market power is relevant in understanding the transmission of monetary policy, especially, its differential effects across the EU.

Figure 2. Deposit and Loan Markets Pass-Through



(a) Stronger pass-through in more competitive deposit marketssub@(a) Stronger pass-through in more competitive deposit markets



(b) Stronger pass-through in less competitive loan marketssub@(b) Stronger pass-through in less competitive loan markets

Notes: This binned scatter plot shows the relation between bank-level deposit/loan betas (obtained from a regression of the monthly, year-over-year change in average deposit/loan rates on the year-over-year change in Deposit facility rate (DFR) between Jan 2022 and December 2023) and a country-level HHI index for the deposit market. The HHI is grouped into 20 equally sized bins, and then the average HHI and Deposit beta is plotted for each bin.Sources: Banking structural statistical indicators (SDW) and ECB staff calculations https://www.ecb.europa.eu/press/key/date/2024/html/ecb.sp240425 e762e980c3.en.pdf.

3 Model

In this section we present the model and the definition of equilibrium. The equilibrium concept used in this section is a Markov perfect equilibrium (e.g., Maskin and Tirole, 2001). Specifically, we characterize the equilibrium using coupled generalized Euler equations, thereby characterizing banks as dynamic platforms. The generalized Euler equation appears in a similar fashion in the optimal fiscal policy literature (see, for instance, Klein and Ríos-Rull, 2003; Krusell, Martin, and Ríos-Rull, 2004; Klein, Krusell, and Ríos-Rull, 2008; and Clymo and Lanteri, 2020). Succinctly, in the infinite-horizon model, each bank faces a dynamic problem that: (i) depends on the same bank's future strategies and other banks' current and future strategies, and is subject (ii) to all firms' dynamic demand for loans and (iii) to all households' dynamic supply of deposits.

3.1 Environment

Time is discrete $t = 0, 1, \ldots$ and the horizon is infinite. The economy features a continuum of households, equivalent to one representative household called *depositor*, B identical non-atomistic banks with mass $\mathtt{w}_b$, a continuum of banks called *fringe* with mass $\mathtt{w}_f$, and a continuum of identical firms $j \in [0, 1]$. The total mass of the banking sector is normalized to one, i.e. $B\mathtt{w}_b + \mathtt{w}_f = 1$. Both non-atomistic and atomistic banks are owned by a separate continuum of identical and infinitely lived households with mass one, equivalent to one representative household called *banker*. Firms are owned by a separate continuum of identical and infinitely lived households with mass one, equivalent to one representative household called *entrepreneur*.

Preferences. The depositor, the banker, and the entrepreneur rank streams of consumption C_t^χ and labor consumption N_t^χ, with $\chi = \{D, B, E\}$, according to the following lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t \cdot \left[u(C_t^\chi) - v(N_t^\chi) \right], \tag{1}$$

where $\beta \in (0, 1)$ is the household's discount factor, $u_c > 0$, $u_{cc} < 0$, $v_n > 0$, and $v_{nn} > 0$.

Markets. There are seven markets in the economy: banks' deposit, banks' equity, firms' loans, firms' equity, interbank market, the labor market, and the market for the representative good.

Banks' equity and deposit markets. The banker owns the banks and consumes their dividends. This is reflected in the banker's budget constraint (in real terms):

$$\begin{aligned} C_t^B + Q_{B,t} B_{t+1}^G + p_{B,t} \sum_{b=1}^{B_t} \mathtt{w}_b S_{b,t+1} + \int_0^{\mathtt{w}_f} p_{f,t} S_{f,t+1} df = \\ W_t N_t^B + \frac{B_t^G}{\Pi_t} + \sum_{b=1}^{B_t} \mathtt{w}_b \left(p_{b,t} + \mathrm{Div}_{b,t} \right) S_{b,t} + \int_0^{\mathtt{w}_f} \left(p_{f,t} + \mathrm{Div}_{f,t} \right) S_{f,t} df, \end{aligned} \tag{2}$$

where $\mathrm{Div}_{b,t}$ is the bank's b dividend (with corresponding share price $p_{B,t}$), $\mathrm{Div}_{f,t}$ is the fringe's f dividend (with corresponding share price $p_{f,t}$), B_{t+1}^G denotes government's debt holdings, $Q_{B,t}$ is the corresponding government debt price, W_t denotes real wage, and N_t^B indicates hour worked of the banker. Each firm can set prices $P_{j,t}$, hence inflation is defined as $\Pi_t \equiv P_{j,t}/P_{j,t-1}$. Note that all debts instruments are not inflation-protected, hence their returns are divided by Π_t since the budget constraint is expressed in real terms. The depositor

can invest in the production sector through the banks, by supplying deposits. Hence, the depositor's budget constraint (in real terms) reads:

$$C_t^D + Q_{D,t}\left(\sum_{b=1}^{B_{t+1}} \mathtt{w}_b D_{b,t+1} + \int_0^{\mathtt{w}_f} D_{f,t+1} df\right) = W_t N_t^D + \sum_{b=1}^{B_t} \mathtt{w}_b \frac{D_{b,t}}{\Pi_t} + \int_0^{\mathtt{w}_f} \frac{D_{f,t}}{\Pi_t} df, \tag{3}$$

where $D_{b,t+1}$ denotes deposits in big banks, $D_{f,t+1}$ denotes deposits in the fringe banks, $Q_{D,t}$ is the associated deposits price, and N_t^D denotes hour worked of the depositors.

Firms' equity market. The entrepreneur owns the firms and consumes their dividends. This is reflected in the entrepreneur's budget constraint (in real terms):

$$C_t^E + \int_0^1 p_{j,t} S_{j,t+1} dj = W_t N_t^E + \int_0^1 \left(p_{j,t} + \mathrm{Div}_{j,t}\right) S_{j,t} dj, \tag{4}$$

where $\mathrm{Div}_{j,t}$ is the firm's real dividend. Since all firms are identical in equilibrium, and the entrepreneur's utility is concave, $\mathrm{Div}_{j,t} > 0$ in equilibrium.

Firms' loan market. The banks supply loans to the continuum of identical firms. Each bank $b = 1 \ldots B_t$ can issue non-state-contingent loans $L_{b,j,t+1}$ to each firm. Similarly the fring can issue non-state-contingent loans $L_{f,j,t+1}$. Banks take each firm's demand and the depositor's supply schedules as given and compete dynamically à la Cournot, i.e, simultaneously and independently choose their loan portfolios. As discussed previously, the Cournot model has the handy property that the number of firms (banks in my case) is a sufficient statistic to determine market power, rendering it a parsimonious and tractable modeling choice. Alternatively, one could consider a Bertrand model where banks face capacity constraints, in order to rule out perfectly competitive outcomes. As shown by Kreps and Scheinkman (1983), quantity precommitment followed by Bertrand competition would induce Cournot outcomes. The process determines the total amount of loans banks supply to each firm which, together with the firm's demand schedule, pins down the loan interest rate $R_{L,t+1} \equiv 1/Q_{L,t}$. At time t, each firm commits to repay such an interest rate at $t+1$. We assume each dollar of loan must be covered with one unit of deposit. Hence, simultaneously with the loan supply, banks determine deposit demand pinning down the deposit interest rate $R_{D,t+1} \equiv 1/Q_{D,t}$

Interbank market. A big bank b can lend $M_{b,t}$ to other big banks that will be repaid in the following period at rate $r_{M,t+1}$. Since, in equilibrium, all big banks are identical, $\forall b : M_{b,t} = 0$.

Labor market. Hours worked supplied by the three households need to equate the labor demanded by the firm, i.e. $N_t^D + N_t^B + N_t^E = N_t$.

Goods market. All households demand goods supplied by all firms. The resource constraint

of the economy is given by

$$C_t^D + C_t^B + C_t^E + X_t + \mathcal{R}(\Pi_t) = Y_t, \tag{5}$$

where $X_t = K_{t+1} - (1-\delta)K_t$ is investment in physical capital, which depreciates at rate δ, and Y_t denotes GDP. Furthermore, we follow Rotemberg (1982) and assume each firm can set prices $P_{j,t}$ incurring the following convex quadratic reduced-form adjustment cost

$$\mathcal{R}(\Pi_t) = \frac{\varphi}{2}\left(\Pi_t - \Pi\right)^2 Y_t.$$

Technology. In each period t, each firm utilize capital K_t, and hires labor to produce output with a constant-returns to scale production function $Y_{j,t} = f(Z_t, K_{j,t}, N_{j,t})$, where Z_t is a stochastic TFP process specified below.

Shocks. There are two sources of aggregate uncertainty in the model. First, we assume aggregate productivity Z_t follows an AR(1) process in logs: $\log(Z_t) = \rho_Z \log(Z_{t-1}) + \eta_{Z,t}$, where $\eta_{Z,t} \sim N(0, \sigma^2_{\eta_Z})$. Second, we assume that monetary policy shocks ζ_t follows an AR(1) process in logs: $\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + \eta_{\zeta,t}$, where $\eta_{\zeta,t} \sim N(0, \sigma^2_{\eta_\zeta})$.

Government. The government balance its budget which, in real terms, reads:

$$B_{t+1}^G = R_{B,t}\frac{B_t^G}{\Pi_t} + G_t.$$

For simplicity, we assume government bonds are in zero net supply, i.e. $B_t^G = 0$, hence also $G_t = 0$.

Central Bank We assume the central bank seeks to achieve an inflation target Π by setting the nominal rate according to the following Taylor Rule:

$$\frac{1}{Q_{B,t}} = \frac{1}{\beta}\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} e^{\zeta_t}. \tag{6}$$

3.2 Households

Depositor We now describe the depositor's problem in recursive form. Let $V_D(S_t^D)$ be the value function of the depositor with holdings $S_t^D \equiv \{\{D_{b,t}\}_{b=1}^B, D_{f,t}\}$. This function satisfies

the following functional equation:

$$V_D(S_t^D, Z_t, \zeta_t) = \max_{C_t^D, S_{t+1}^D, N_t^D} \quad u(C_t^D) - v(N_t^D) + \beta \cdot \mathbb{E}_t[V_D(S_{t+1}^D, Z_{t+1}, \zeta_{t+1})], \tag{7}$$

subject to the budget constraint (3). The depositor takes all interest rates as given, and chooses assets such that:

$$Q_{D,t} = \mathbb{E}\left[\mathcal{M}_{t,t+1}^D \cdot \Pi_{t+1}^{-1}\right], \tag{8}$$

where $\mathcal{M}_{t,t+1}^D \equiv \beta \frac{u_c(C_{t+1}^H)}{u_c(C_t^H)}$. The intratemporal labor-consumption margin reads:

$$u_c(C_t^D) W_t = v_n(N_t^D).$$

Banker We now describe the banker's problem in recursive form. Let $V_B(S_t^B)$ be the value function of the banker with holdings $S_t^B \equiv \{\{S_{b,t}\}_{b=1}^B, B_t^G\}$. This function satisfies the following functional equation:

$$V_B(S_t^B, Z_t, \zeta_t) = \max_{C_t^B, S_{t+1}^B, N_t^B} \quad u(C_t^B) - v(N_t^B) + \beta \cdot \mathbb{E}_t[V_B(S_{t+1}^B, Z_{t+1}, \zeta_{t+1})], \tag{9}$$

subject to the budget constraint (2). The banker takes all interest rates as given, and chooses government bonds such that:

$$Q_{B,t} = \mathbb{E}\left[\mathcal{M}_{t,t+1}^B \cdot \Pi_{t+1}^{-1}\right], \tag{10}$$

where $\mathcal{M}_{t,t+1}^B \equiv \beta \frac{u_c(C_{t+1}^B)}{u_c(C_t^B)}$. Moreover, banks' equities are priced according to:

$$\forall b : 1 = \mathbb{E}\left[\mathcal{M}_{t,t+1}^B \cdot \frac{p_{b,t+1} + \text{Div}_{b,t+1}}{p_{b,t}}\right]. \tag{11}$$

The intratemporal labor-consumption margin reads:

$$u_c(C_t^B) W_t = v_n(N_t^B).$$

Entrepreneur We now describe the entrepreneur's problem in recursive form. Let $V_E(S_t^E)$ be the value function of the entrepreneur with holdings $S_t^E \equiv \{S_{j,t}\}$. This function satisfies

the following functional equation:

$$V_E(S_t^E, Z_t, \zeta_t) = \max_{C_t^E, S_{t+1}^E, N_t^E} \quad u(C_t^E) - v(N_t^E) + \beta \cdot \mathbb{E}_t[V_E(S_{t+1}^E, Z_{t+1}, \zeta_{t+1})], \tag{12}$$

subject to the budget constraint (4). Firms' equities are priced according to:

$$\forall j : 1 = \mathbb{E}\left[\mathcal{M}_{t,t+1}^E \cdot \frac{p_{j,t+1} + \text{Div}_{j,t+1}}{p_{j,t}}\right], \tag{13}$$

where $\mathcal{M}_{t,t+1}^E \equiv \beta \frac{u_c(C_{t+1}^E)}{u_c(C_t^E)}$. The intratemporal labor-consumption margin reads:

$$u_c(C_t^E) W_t = v_n(N_t^E).$$

3.3 Final and Intermediate Firms (Loan Demand)

The demand for intermediate goods is given by a standard static profit maximization of the final good producer

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t. \tag{14}$$

We now characterize the intermediate firm j's problem in recursive form. Let $V_F(X_{j,t})$ be the value function of the intermediate firm j with holdings $S_{j,t}^F \equiv \{\{L_{b,j,t}\}_{b=1}^B, \{L_{b,j,t}^s\}_{b=1}^{\mathcal{B}}, K_{j,t}\}$. This function satisfies the following functional equation:

$$V_F(S_{j,t}^F, Z_t, \zeta_t) \;= \max_{S_{j,t+1}^F, P_{j,t}, N_{j,t}} \text{Div}_{j,t} + \mathbb{E}_t\left[\mathcal{M}_{t,t+1}^E \cdot V_F(S_{j,t+1}^F, Z_{t+1}, \zeta_{t+1})\right],$$

subject to

$$\begin{aligned}
K_{j,t+1} &= K_{j,t}(1-\delta) + X_t \\
\text{Div}_{j,t} &= \frac{P_{j,t}}{P_t} Y_{j,t} - W_t N_{j,t} - X_t - \mathcal{R}\left(\frac{P_{j,t}}{P_{j,t-1}}\right) + \\
&\quad + Q_{L,t}\left(\sum_{b=1}^B \mathtt{w}_b L_{b,j,t+1} + \int_0^{\mathtt{w}_f} L_{f,j,t+1} df\right) - \sum_{b=1}^B \mathtt{w}_b \frac{L_{b,j,t}}{\Pi_t} - \int_0^{\mathtt{w}_f} \frac{L_{f,j,t}}{\Pi_t} df,
\end{aligned}$$

and subject to the demand for intermediate goods (14). Each firm takes the loans' market price $Q_{L,t}$ as given and finances itself through internal financing (production) and external financing (loans from banks). The first-order condition with respect to $N_{j,t}$ (after imposing

symmetry among firms $P_{j,t} = P_t$) is

$$W_t = \Gamma_t f_N(Z_t, K_t, N_t), \tag{15}$$

where Γ_t is the Lagrange multiplier on the demand constraint (14). The first-order condition with respect to $K_{j,t+1}$ (after imposing symmetry among firms) is

$$1 = \mathbb{E}_t \left[\mathcal{M}^E_{t,t+1} \cdot (1 + \Gamma_{t+1} f_K(Z_{t+1}, K_{t+1}, N_{t+1}) - \delta) \right]. \tag{16}$$

The first-order condition with respect to $L_{b,j,t+1}$ (or $L_{f,j,t+1}$) (after imposing symmetry among firms), is

$$Q_{L,t} = \mathbb{E}_t \left[\mathcal{M}^E_{t,t+1} \cdot \Pi^{-1}_{t+1} \right]. \tag{17}$$

The first-order condition with respect to $P_{j,t}$ (after imposing symmetry) yields the following expression for Γ_t

$$\left[1 - \theta + \theta \Gamma_t\right] Y_t - \mathcal{R}_\pi(\Pi_t)\Pi_t + \mathbb{E}_t \left[\mathcal{M}^E_{t,t+1} \mathcal{R}_\pi(\Pi_{t+1})\Pi_{t+1} \right] = 0, \tag{18}$$

which is a standard New Keynesian Phillips curve (NKPC).

3.4 Banks (Deposit Demand and Loans Supply)

The economy is populated by a fringe and B non-zero mass banks. The fringe takes interest rates $Q_{D,t}$ and $Q_{L,t}$ as given and allocate deposits $D_{f,t}$ to firms in the form of loans $L_{f,t}$ up to a capacity constraint $L_{f,t} \leq \bar{L}_f$ where, in equilibrium, this constraint always binds. We view the big banks as a succession of decision makers—one bank b at each time t—with *No Commitment* to future realized policies. Each bank at time t determines the current level of loans and deposits issued – by best responding to the other banks – that will be inherited by the future banks as state variables. In this setting, the banks thus faces two sources of strategic interactions, namely they play a game against their future selves and against the other banks. In this context, each bank b's real dividend is given by:

$$\text{Div}_{b,t} = \mathtt{w}_b \left(\frac{L_{b,t}}{\Pi_t} - Q_{L,t} L_{b,t+1} + \frac{M_{b,t}}{\Pi_t} - Q_{M,t} M_{b,t+1} - \frac{D_{b,t}}{\Pi_t} + Q_{D,t} D_{b,t+1} \right). \tag{19}$$

Hence, for each bank b, the net present value of real dividend net of quadratic equity issuance cost $\Lambda^B(\text{Div}_{b,t}) = \mathbb{I}[\text{Div}_{b,t} < 0]\lambda_B \text{Div}_{b,t}^2$ reads:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \mathcal{M}_{t,t+1}^B \cdot \left(\text{Div}_{b,t} - \Lambda^B(\text{Div}_{b,t})\right). \tag{20}$$

We focus on a symmetric Markov-perfect equilibrium and denote the state of the economy at time t by $\mathcal{X}_t \equiv (\{D_{b,t}\}_{b=1}^B, \{L_{b,t}\}_{b=1}^B, K_t, Z_t, \zeta_t)$. In this environment, let all future banks set their policy according to functions $\tilde{C}^H(\mathcal{X}_t)$, $\tilde{C}^B(\mathcal{X}_t)$, $\tilde{C}^E(\mathcal{X}_t)$, $\tilde{N}(\mathcal{X}_t)$, $\tilde{\Gamma}(\mathcal{X}_t)$, $\tilde{D}_b(\mathcal{X}_t)$, $\tilde{L}_b(\mathcal{X}_t)$, and $\tilde{K}(\mathcal{X}_t)$. Each current bank b takes these policies of future players as given, and understands that it can only influence future players' actions through its current policies (which will be state variables for the future players). As these functions are unknown, we are looking for the solutions of a fixed-point problem. By construction then, as the policies are time-invariant functions of the current state $\mathcal{X}_t$, these policies are time-consistent.

In this context, let $\tilde{W}_b(\mathcal{X}_t)$ be the present discounted value of each bank real dividend (20) as associated with the policy functions introduced above, given the state of the economy $\mathcal{X}_t$. With this notation at hand, each bank b at time t "directly" chooses deposits demand and loan supply $(D_{b,t+1}, L_{b,t+1})$ – which "indirectly" pin down $(Q_{D,t}, Q_{L,t}, K_{t+1}, \Gamma_t)$ through all households and firms constraints – to maximize

$$\text{Div}_{b,t} - \Lambda^B(\text{Div}_{b,t}) + \mathbb{E}\left[\mathcal{M}_{t,t+1}^B \tilde{W}_b(\mathcal{X}_{t+1})\right], \tag{21}$$

subject to the dynamic deposit supply and the dynamic loan demand systems specified below and given that inflation is pinned down by the Taylor rule (6), where we assume that banks take as given their shareholders current and future stochastic discount factors $\{\mathcal{M}_{t+k,t+k+1}^B\}_{k=0}^{\infty}$.

Deposit Demand System The dynamic deposit demand system is given by the deposit supply Eulers:

$$u_c(C_t^D)Q_{D,t} - \beta \mathbb{E}_t\left[\mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})\right] = 0 \tag{22}$$

$$u_c(C_t^D)W_t - v_n(N_t^D) = 0, \tag{23}$$

with attached Lagrange multipliers $(\mathcal{D}_t, \mathcal{D}_t^n)$ and where we define $\mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1}) \equiv u_c(\tilde{C}^E(\mathcal{X}_{t+1})) \cdot \Pi_{t+1}^{-1}$. Furthermore, C_t^D and W_t can be substituted away using the entrepreneur resource

constraint and the firm labor demand:

$$C_t^D = W_t N_t^D + \sum_{b=1}^{B_t} \mathtt{w}_b \frac{D_{b,t}}{\Pi_t} - Q_{D,t} \sum_{b=1}^{B_{t+1}} \mathtt{w}_b D_{b,t+1} + \mathtt{w}_f \frac{D_{f,t}}{\Pi_t} - Q_{D,t} \mathtt{w}_f D_{f,t+1}, \tag{24}$$

$$W_t = \Gamma_t f_N(Z_t, K_t, N_t). \tag{25}$$

Loan Demand System The dynamic loan demand system is given by the loan demand Eulers, the investment Euler, and the NKPC:

$$u_c(C_t^E) Q_{L,t} - \beta \mathbb{E}_t \left[\mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1}) \right] = 0 \tag{26}$$

$$u_c(C_t^E) W_t - v_n(N_t^E) = 0 \tag{27}$$

$$u_c(C_t^E) - \beta \mathbb{E}_t \left[\mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1}) \right] = 0 \tag{28}$$

$$u_c(C_t^E) \left[(1 - \theta + \theta \Gamma_t) Y_t - \mathcal{R}_\pi(\Pi_t) \Pi_t \right] + \beta \mathbb{E}_t \left[\mathcal{S}^{\mathcal{L}^\Pi}(\mathcal{X}_{t+1}) \right] = 0, \tag{29}$$

with attached Lagrange multipliers ($\mathcal{L}_t$,$\mathcal{L}_t^n$, $\mathcal{L}_t^K$, $\mathcal{L}_t^\Pi$) and where we define

$$\mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1}) \equiv u_c(\tilde{C}^E(\mathcal{X}_{t+1})) \cdot \Pi_{t+1}^{-1}$$

$$\mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1}) \equiv u_c(\tilde{C}^E(\mathcal{X}_{t+1})) \cdot \left(1 + \tilde{\Gamma}(\mathcal{X}_{t+1}) f_K(Z_{t+1}, K_{t+1}, \tilde{N}(\mathcal{X}_{t+1})) - \delta \right)$$

$$\mathcal{S}^{\mathcal{L}^\Pi}(\mathcal{X}_{t+1}) \equiv u_c(\tilde{C}^E(\mathcal{X}_{t+1})) \mathcal{R}_\pi(\Pi_{t+1}) \Pi_{t+1}.$$

Furthermore, C_t^E and W_t can be substituted away using the entrepreneur resource constraint

$$C_t^E = W_t N_t^E + Y_t + Q_{L,t} \sum_{b=1}^{B} \mathtt{w}_b L_{b,t+1} - \sum_{b=1}^{B} \mathtt{w}_b \frac{L_{b,t}}{\Pi_t} + Q_{L,t} \mathtt{w}_f L_{f,t+1} - \mathtt{w}_f \frac{L_{f,t}}{\Pi_t} - W_t N_t - X_t - \Phi(\Pi_t) \tag{30}$$

and the firm labor demand. Given these substitutions, there are nine first-order conditions with respect to $M_{b_{t+1}}$ and

$$(D_{b,t+1}, L_{b,t+1}, Q_{D,t}, Q_{L,t}, K_{t+1}, \Gamma_t, N_t^D, N_t^E).$$

The first-order condition with respect to $M_{b,t+1}$ reads

$$Q_{M,t}(1 - \mathcal{E}_t) = \mathbb{E} \left[\mathcal{M}_{t,t+1}^B \cdot \Pi_{t+1}^{-1} (1 - \mathcal{E}_{t+1}) \right], \tag{31}$$

where $\mathcal{E}_t \equiv \Lambda_{\text{Div}}^B(\text{Div}_{b,t})$. The remaining eigth optimality conditions are Generalized Euler Equations as they feature derivatives of future policies ($\tilde{C}^H(\mathcal{X}_t)$, $\tilde{C}^B(\mathcal{X}_t)$, $\tilde{C}^E(\mathcal{X}_t)$, $\tilde{N}(\mathcal{X}_t)$,

$\tilde{\Gamma}(\mathcal{X}_t)$, $\tilde{D}_b(\mathcal{X}_t)$, $\tilde{D}_b(\mathcal{X}_t)$, $\tilde{L}_b(\mathcal{X}_t)$) that the current banks take as given, with respect to the current strategies. All eigth first-order conditions are reported in Appendix A.

The six first order conditions with respect to $(Q_{D,t}, Q_{L,t}, K_{t+1}, \Gamma_t, N_t^D, N_t^E)$ yield analytical expressions for the six Lagrange multipliers $(\mathcal{D}_t, \mathcal{L}_t, \mathcal{L}_t^K, \mathcal{L}_t^\Pi, \mathcal{D}_t^n, \mathcal{L}_t^n)$. Substituting these six Lagrange multipliers in the first-order condition with respect to $L_{b,t+1}$ yields an implied best response function for bank b, which we denote as $\tilde{L}_b^I(\mathcal{X}_t)$, given all other bank current strategies $\tilde{L}_{-b}^I(\mathcal{X}_t)$ and given all banks future strategies $\tilde{L}_b(\mathcal{X}_{t+1})$. A similar logic can be applied for deposits. Given that all banks are symmetric in equilibrium, we impose symmetry ex-post to find the fixed point such that

$$\forall b: \ \tilde{D}_b^I(\mathcal{X}_t) = \tilde{D}_{-b}^I(\mathcal{X}_t) = \tilde{D}_b(\mathcal{X}_{t+1}) \quad \text{and} \quad \tilde{L}_b^I(\mathcal{X}_t) = \tilde{L}_{-b}^I(\mathcal{X}_t) = \tilde{L}_b(\mathcal{X}_{t+1}). \tag{32}$$

Markov-Perfect Equilibrium In this context we focus on a Symmetric Markov Perfect Equilibrium in the sense that we look for symmetric strategies not only among banks within the same time period but also throughout time, i.e. equation (32) holds. More formally, a symmetric MPE consists of a set of six policy functions

$$(\tilde{D}_b(\mathcal{X}_t), \tilde{L}_b(\mathcal{X}_t), \tilde{Q}_{D,t}(\mathcal{X}_t), \tilde{Q}_{L,t}(\mathcal{X}_t), \tilde{K}(\mathcal{X}_t), \tilde{\Gamma}(\mathcal{X}_t), \tilde{N}(\mathcal{X}_t)) \tag{33}$$

that satisfies the GEEs (34)-(36) and, simultaneously, all constraints of problem (40).

Proposition 3.1. *In a Symmetric Markov Perfect Equilibrium, the price of deposits satisfies the following Generalized Euler equation:*

$$Q_{D,t}\left(1 - \mathcal{E}_t + \eta_t^D\right) = \underbrace{\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^B \Pi_t^{-1}(1 - \mathcal{E}_{t+1})\right]}_{Q_{M,t}(1-\mathcal{E}_t)}, \tag{34}$$

which can be interpreted as a dynamic markdown rule where η^D is the inverse price elasticity

of deposit supply, defined as

$$\begin{aligned}\eta_t^D \equiv \frac{\partial Q_{D,t}}{\partial D_{b,t+1}}\frac{D_{b,t+1}}{Q_{D,t}} =& -\mathcal{D}_t\left(u_{cc}(C_t^D)Q_{D,t} + Q_{D,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right]\right)\\ &- \mathcal{L}_t Q_{D,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right]\\ &- \mathcal{L}_t^K Q_{D,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right]\\ &+ \mathcal{L}_t^\Pi Q_{D,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^\Pi}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right]\\ &- \mathcal{D}_t^n W_t u_{cc}(C_t^D),\end{aligned} \tag{35}$$

where $\bar{D}_{b,t+1} \equiv \mathtt{w}_b D_{b,t+1}$. *Similarly, the price of loans satisfies the following Generalized Euler equation:*

$$Q_{L,t}\left(1-\mathcal{E}_t+\eta_t^L\right) = \underbrace{\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^B \Pi_t^{-1}(1-\mathcal{E}_{t+1})\right]}_{Q_{M,t}(1-\mathcal{E}_t)}, \tag{36}$$

and where η^L *is the inverse price elasticity of loan demand, defined as*

$$\begin{aligned}\eta_t^L \equiv \frac{\partial Q_{L,t}}{\partial L_{b,t+1}}\frac{L_{b,t+1}}{Q_{L,t}} =& -\mathcal{L}_t\left(u_{cc}(C_t^E)Q_{L,t} - Q_{L,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right]\right)\\ &- \mathcal{L}_t^K\left(u_{cc}(C_t^E) - Q_{L,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right]\right)\\ &- \mathcal{L}_t^\Pi\left(u_{cc}(C_t^E)\kappa_t + Q_{L,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^\Pi}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right]\right)\\ &+ \mathcal{D}_t Q_{L,t}^{-1}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right]\\ &- \mathcal{L}_t^n W_t u_{cc}(C_t^E),\end{aligned} \tag{37}$$

where $\bar{L}_{b,t+1} \equiv \mathtt{w}_b L_{b,t+1}$, $\kappa_t \equiv (\theta - 1 + \theta\Gamma_t)Y_t$, *and all Lagrange multipliers* $(\mathcal{D}_t, \mathcal{L}_t, \mathcal{L}_t^K, \mathcal{L}_t^\Pi, \mathcal{D}_t^n, \mathcal{L}_t^n)$ *are given by equations* (49)-(54) *reported in Appendix* A.

Risk Sharing and Perfect Competition In the perfect competition case, we introduce an infinite number of banks, $B \to \infty$, each with mass $\lim_{B\to\infty} \mathtt{w}_b = B^{-1} = 0$, hence we implicitly normalize the total mass of banks to 1. In this limiting setting, banks do not internalize the effects of their actions on other agents but instead take them as given. As a result, we obtain $\lim_{B\to\infty} \eta_t^D = \lim_{B\to\infty} \eta_t^L = 0$. The GEEs (34)-(36) then simplifies to

$Q_{D,t} = Q_{M,t} = Q_{L,t}$, which—through the Euler equations (8) and (17)—yields a standard risk-sharing condition:

$$\mathbb{E}_t \left[\mathcal{M}^D_{t,t+1}\Pi^{-1}_{t+1}\right] = \mathbb{E}_t \left[\mathcal{M}^B_{t,t+1}\Pi^{-1}_{t+1}\right] = \mathbb{E}_t \left[\mathcal{M}^E_{t,t+1}\Pi^{-1}_{t+1}\right] \tag{38}$$

between depositors, bankers, and entrepreneurs. We use this perfect competition case as a benchmark in the quantitative experiments presented in Section 4.

In light of this consideration, a natural question arises: How is the risk-sharing mechanism distorted in the case of imperfect competition in the financial intermediation sector? We can use the GEEs (34)-(36), combined with the Euler equations (8) and (17), to derive the following general expression:

$$\mathbb{E}_t \left[\mathcal{M}^D_{t,t+1}\Pi^{-1}_{t+1}\right] \cdot (1 + \eta^D_t) = \mathbb{E}_t \left[\mathcal{M}^D_{t,t+1}\Pi^{-1}_{t+1}\right] = \mathbb{E}_t \left[\mathcal{M}^E_{t,t+1}\Pi^{-1}_{t+1}\right] (1 + \eta^L_t),$$

which reveals that the risk-sharing mechanism is distorted by the elasticities (35) and (37).

In our model, banks act as intermediaries, exercising market power on both sides. On the deposit side, they hold oligopsony power ($\eta^D_t \leq 0$)—that is, because there are few banks competing for deposits, they can offer lower deposit rates. On the lending side, they exert oligopoly power ($\eta^L_t \geq 0$), meaning that the few banks that lend can charge higher rates to entrepreneurs compared to the perfect competition benchmark. Although these elasticities are difficult to sign analytically, it is always the case in our simulations that $\eta^D_t \leq 0$ and $\eta^L_t \geq 0$. Assuming these signs hold, in a setting with few strategic banks, the risk-sharing condition becomes:

$$\mathbb{E}_t \left[\mathcal{M}^D_{t,t+1}\Pi^{-1}_{t+1}\right] \geq \mathbb{E}_t \left[\mathcal{M}^B_{t,t+1}\Pi^{-1}_{t+1}\right] \geq \mathbb{E}_t \left[\mathcal{M}^E_{t,t+1}\Pi^{-1}_{t+1}\right]. \tag{39}$$

The practical implication is that depositors are effectively under-compensated for the liquidity they provide, while entrepreneurs are over-charged for the liquidity they are provided.

4 Quantitative Analysis

In this section, we leverage our equilibrium definitions and characterization to solve the model numerically. We calibrate the model and explore the implications of double-sided market power for the transmission of monetary policy shocks.

4.1 Solution Method

We begin this section by briefly discussing our global solution method.

Markov Perfect Equilibrium. First we solve for the deterministic steady state without market power. In the deterministic steady-state the level of loans $\bar{L}$ is undetermined so we just calibrate the value as explained below. We build grids for $\mathcal{X}_t$ around the steady state and approximate the shocks processes with Markov chains using the Rouwenhorst method. Hence, we solve for the Markov Perfect Equilibrium using projections methods – a grid-based approximation for each policy function combined with piece-wise trilinear interpolation– and the fixed-point iteration algorithm. In particular, we start with a guess for the policy functions $(\tilde{D}_b(\mathcal{X}_t), \tilde{L}_b(\mathcal{X}_t), \tilde{Q}_{D,t}(\mathcal{X}_t), \tilde{Q}_{L,t}(\mathcal{X}_t), \tilde{K}(\mathcal{X}_t), \tilde{\Gamma}(\mathcal{X}_t), \tilde{N}(\mathcal{X}_t))$ that coincides with the solution of the policy functions that solve the perfect competition case. Given this candidate policy functions, we obtain the implied policy functions by solving at each point in the grid all constraints of problem (40) and the GEEs (34)-(36). We approximate the derivatives of the future policy functions contained in the GEEs (34)-(36) using finite-difference methods. We repeat these steps until all policy functions converge.

4.2 Calibration

We proceed to describe our choices of functional forms and parameter values, which we report in Table I. The length of a period is one year with $\beta = 0.96$ to target an annual real inflation rate of 2%. We parameterize the utility function as follows: $u(C) \equiv \frac{C^{1-\sigma}}{1-\sigma}$ and $v(N) \equiv \chi \frac{N^{1-\eta}}{1-\eta}$, with $\sigma = 1$ to have log-utility of consumption and $\eta = 1$ to match a unitary Frisch elasticity of labor supply. The production function is Cobb-Douglas $f(Z_t, K_t, N_t) \equiv Z_t K_t^{\alpha} N_t^{1-\alpha}$, where the capital share $\alpha = .4$ is set to match a labor share of 60%.

We set the price elasticity of demand θ to 7, which is a standard value used in the literature. The Taylor rule responds only to deviations from the steady state inflation rate. We set the steady state inflation rate to 2 percent, which is the a standard annual inflation target. The parameter φ, which governs the strength of the nominal rigidity, is set to 50 to render the slope of the NKPC 0.0413 in line with the range of values in Clarida, Gali, and Gertler (1999).

We calibrate the number of banks B to 5 to reflect the elevated concentration of the banking sector in Europe and the fringe capacity constraint parameters $\bar{L}_f$ to .2 to match a steady-state market share of the fringe of 20%. Given that the total value of loans $B\bar{L} + \bar{L}_f$ is set to 1.2 – to match a 30% loan-to-GDP ratio – this yields $\bar{L} = 0.2$. Hence, the big 5 banks have roughly 80% of the total market share in the steady state.

Table I. Parameter Values

		Parameter	Value	Target/Source
HHs	Discount factor	β	0.96	2% annual real interest rate
	Risk aversion	γ	1	Standard value
	Frisch elasticity	η	1	Standard value
Banks	Number of banks	B	5	High concentration in Europe
	Fringe capacity constraint	$\bar{L}_f$	.2	20% steady-state market share
	Steady state loans	$B\bar{L} + \bar{L}_f$	1.2	30% loan-to-GDP ratio
Firms	Capital depreciation	δ	.1	Standard value
	Capital share	α	.4	60% labor share of income
	Nominal rigidity	φ	50	Slope of the NKPC
	Price elasticity of demand	θ	7	Standard value
	Persistence of TFP shocks	ρ_Z	.909	Khan and Thomas (2013)
	Std. dev. of TFP shocks	σ_{η_Z}	.014	Khan and Thomas (2013)
Central bank	Annual Inflation Target	$\bar{\Pi}$	1.02	2% annual inflation
	Monetary policy response to inflation	ϕ_π	1.5	Taylor (1993)
	Persistence of monetary policy shocks	ρ_ζ	.1	Smets and Wouters (2007)
	Std. dev. of monetary policy shocks	σ_{η_ζ}	.003	Smets and Wouters (2007)

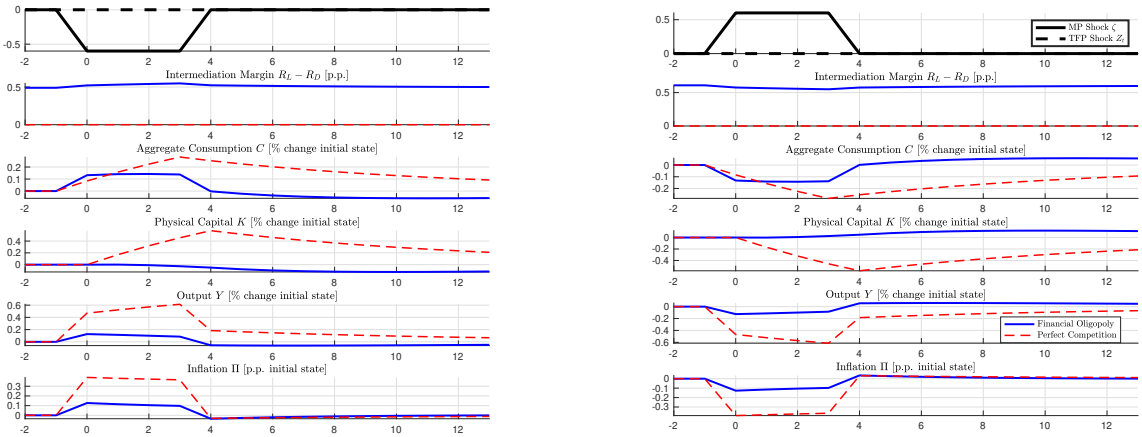
Notes: The table reports the parameter values.

4.3 Transmission of Monetary Policy Shocks

We illustrate the model workings through conditional dynamics following negative and positive monetary policy shocks. The left panel of figure 3 shows the effects of monetary expansion followed by a reversion back to the original level and the right panel shows the analogous monetary contraction dynamics. We contrast this to the perfect-competition benchmark, which is nested in the baseline model as the number of banks (B) goes to infinity. In both cases, a dynamic oligopoly structure dampens the effects of monetary policy, both on inflation and on output. As the monetary policy mainly acts through the intertemporal substitution channel in the benchmark model, the bank's market power, as well, affects the monetary policy transmission to the extent that it shapes household consumption-saving choices. In the perfect competition benchmark, output increases immediately following the monetary expansion (left panel) and there is a gradual capital buildup during the low interest rate period, which leads to the propagation of the shock over time. In the benchmark model, the cut in the interbank rate is completely passed through to the deposit and loan rates. A fall in the deposit rate boosts aggregate demand to which firms respond by initially increasing employment and by gradually building up capital. In the baseline model, such a cut in the interbank rate is not passed through one-for-one. Instead, banks cut the deposit rates by more and the loan rates by less (figure 4), leading to an increase in the intermediation margin. The relative drop in the deposit and the loan rate depends on the respective demand elasticities, which is an endogenous object in this model. Following the rate cut both output

and inflation increase by less than in the perfect bank competition case. Yet, aggregate consumption has a large initial boost, which is mainly due to large banker's consumption enjoying higher profit margins. Over time, the bank's market power in the loan market results in higher loan rates, which depresses firm investment and capital accumulation.

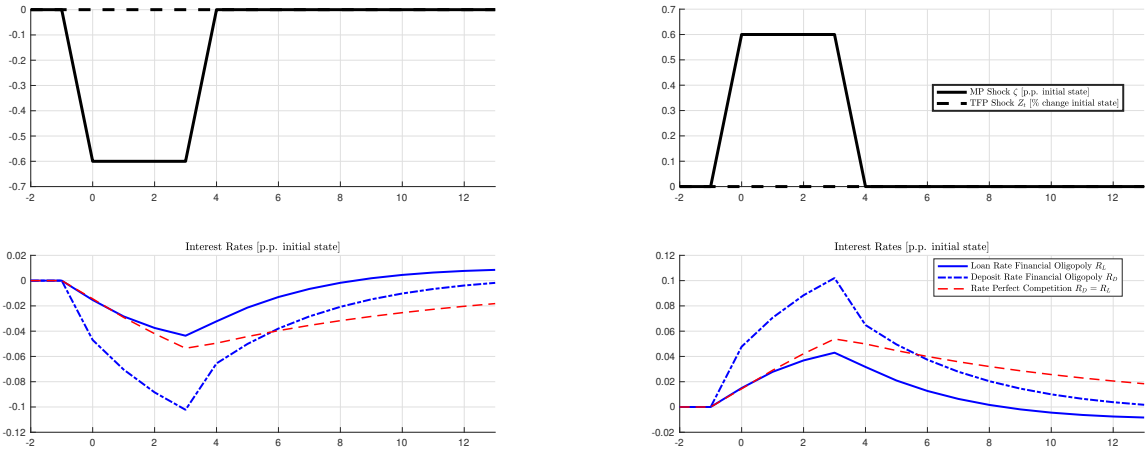
Figure 3. Conditional Dynamics Responses to MP Shocks



(a) Period of low TFP with rate cut (b) Period of high TFP with rate hike

Notes: The figure displays the dynamics of relevant variables around positive and negative monetary shocks that increase/decrease interest rates, at $t = 0$ starting from different levels of TFP.

Figure 4. Intermediation Margins



(a) Period of low TFP with rate cut (b) Period of high TFP with rate hike

Notes: The figure displays the interest rates under the calibrated financial oligopoly around positive and negative monetary shocks that increase/decrease interest rates, at $t = 0$ starting from different levels of TFP.

5 Extension: Market Power with Regulatory Constraints

We use the same notation as section where each bank b at time t "directly" chooses deposits demand and loan supply $(D_{b,t+1}, L_{b,t+1})$ – which "indirectly" pin down $(Q_{D,t}, Q_{L,t}, K_{t+1}, \Gamma_t)$ through all households and firms constraints – to maximize

$$\text{Div}_{b,t} + \mathbb{E}\left[\mathcal{M}^B_{t,t+1}\tilde{W}_b(\mathcal{X}_{t+1})\right], \tag{40}$$

subject to the Taylor rule (6), the dynamic deposit supply, and the dynamic loan demand systems exactly as specified in section 3.4.

In this section, we extend the model to study the interaction between bank market power and regulatory constraints. Specifically, we assume that banks are subject to a Loan-to-Deposit Ratio (LDR):

$$\forall b : L_{b,t+1} \leq \psi \cdot D_{b,t+1}, \tag{41}$$

with attach Lagrange multiplier $\mathcal{L}^\psi$ and where $0 < \psi < 1$. Implicitly we assume that banks cannot lend out all their deposits, ensuring they maintain some liquidity. A strict LDR cap (e.g., $\psi = 0.8$) limits credit expansion but makes banks more resilient to liquidity shocks. This regulatory constraint is effectively an occasionally binding constraint depending on the state of the economy. In our context, this constraint limits the bank's ability to create loans endogenously based on deposit inflows thereby dampening banks market power. Under these circumstnces, the GEEs (34)-(36) assume the following generalized form.

Proposition 5.1. *In a Symmetric Markov Perfect Equilibrium, the price of deposits satisfies the following Generalized Euler equation:*

$$Q_{D,t}\left(1 - \mathcal{E}_t + \eta^D_t\right) = Q_{M,t}(1 - \mathcal{E}_t) + \psi\mathcal{L}^\psi_t, \tag{42}$$

where the inverse price elasticity of deposit supply η^D_t is given by equation (35) and the price of loans satisfies the following Generalized Euler equation:

$$Q_{L,t}\left(1 - \mathcal{E}_t + \eta^L_t\right) = Q_{M,t}(1 - \mathcal{E}_t) + \mathcal{L}^\psi_t, \tag{43}$$

and where η^L_t is given by equation (37) and, as in the baseline, all Lagrange multipliers $(\mathcal{D}_t, \mathcal{L}_t, \mathcal{L}^K_t, \mathcal{L}^\Pi_t, \mathcal{D}^n_t, \mathcal{L}^n_t)$ are given by equations (49)-(54) reported in Appendix A.

We now analyze the case of perfect competition and risk-sharing mechanism as we did for the baseline.

Risk Sharing and Perfect Competition In the perfect competition case, we introduce an infinite number of banks, $B \to \infty$, each with mass $\lim_{B\to\infty} \mathtt{w}_b = B^{-1} = 0$, hence we implicitly normalize the total mass of banks to 1. In this limiting setting, banks do not internalize the effects of their actions on other agents but instead take them as given. As a result, we obtain $\lim_{B\to\infty} \eta_t^D = \lim_{B\to\infty} \eta_t^L = 0$. The GEEs (42)-(43) then simplifies to

$$(1-\mathcal{E}_t)Q_{D,t} - \psi\mathcal{L}_t^\psi = (1-\mathcal{E}_t)Q_{M,t} = (1-\mathcal{E}_t)Q_{L,t} - \mathcal{L}_t^\psi,$$

which—through the Euler equations (8) and (17), the fact that the Karush–Kuhn–Tucker conditions require $\mathcal{L}_t^\psi \geq 0$, and that we assume ψ to be strictly smaller than 1 and that $(1-\mathcal{E}_t) > 0$—yields the risk-sharing condition:

$$\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^E \Pi_{t+1}^{-1}\right] \geq \mathbb{E}_t\left[\mathcal{M}_{t,t+1}^D \Pi_{t+1}^{-1}\right] \geq \mathbb{E}_t\left[\mathcal{M}_{t,t+1}^B \Pi_{t+1}^{-1}\right] \tag{44}$$

We use this perfect competition case as a benchmark in the quantitative experiments presented in Section 4.

How is the risk-sharing mechanism distorted in the case of imperfect competition that interacts with regulatory constraints? In general, the GEEs (42)-(43) imply that

$$Q_{D,t}\cdot(1-\mathcal{E}_t+\eta_t^D) - \psi\mathcal{L}_t^\psi = Q_{M,t} = Q_{L,t}(1-\mathcal{E}_t+\eta_t^L) - \mathcal{L}_t^\psi,$$

which encapsulates how the risk-sharing mechanism is distorted by both market power – through the elasticities (35) and (37) – and through the regulatory constraint captured by the term Γ_t. In our simulation, it remains true that $\eta_t^D \leq 0$ (oligopsony power) and $\eta_t^L \geq 0$ (oligopoly power). Assuming these signs hold, in a setting with few strategic banks, the risk-sharing condition is:

$$\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^D \Pi_{t+1}^{-1}\right] \geq \mathbb{E}_t\left[\mathcal{M}_{t,t+1}^B \Pi_{t+1}^{-1}\right], \tag{45}$$

The practical implication is that depositors reamain effectively under-compensated for the liquidity they provide, while entrepreneurs are not necessarily over-charged for the liquidity they are provided. If the regulatory constraint term $\mathcal{L}_t^\psi$ does not exceed the market power term $Q_{L,t}\eta_t^L$ –i.e. $\mathcal{L}_t^\psi \leq Q_{L,t}\eta_t^L$ – then

$$\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^B \Pi_{t+1}^{-1}\right] \geq \mathbb{E}_t\left[\mathcal{M}_{t,t+1}^E \Pi_{t+1}^{-1}\right],$$

which yields a risk-sharing condition similar to that of the baseline in ordinal terms. Vicev-

ersa happens when $\mathcal{L}_t^{\psi} \leq Q_{L,t}\eta_t^L$.

6 Conclusion

In this paper, we examine how intermediary market power influences the transmission of monetary policy within a New-Keynesian framework featuring a finite number of banks with two-sided market power. By modeling banks as dynamic platforms with both oligopoly power in the loan market and oligopsony power in the deposit market, we characterize equilibrium through coupled generalized Euler equations, leading to endogenous mark-ups and mark-downs. Our findings highlight that increased banking sector concentration in the EU dampens and shortens the effects of monetary policy shocks on inflation and output. These results underscore the importance of market structure in shaping policy effectiveness, with implications for financial regulation and competition policy.

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APPENDIX

A Mathematical Appendix

We derive the first-order conditions in generic form for both the baseline and the extension. In the baseline, $\mathcal{L}_t^{\psi} = 0$ since there is no regulatory constraint. In the extension, $\mathcal{L}_t^{\psi}$ needs to be determined endogenously state-by-state using the Karush–Kuhn–Tucker conditions. Note that the envelope conditions read $\tilde{W}_{b,D_b}(\mathcal{X}_t) = -\Pi_t^{-1}(1-\mathcal{E}_t)$, $\tilde{W}_{b,L_b}(\mathcal{X}_t) = \Pi_t^{-1}(1-\mathcal{E}_t)$, and $\tilde{W}_{b,K}(\mathcal{X}_t) = 0$.

The first-order condition with respect to $\bar{D}_{b,t+1} \equiv \mathtt{w}_b D_{b,t+1}$ is:

$$Q_{D,t}(1-\mathcal{E}_t) + \mathbb{E}_t\left[\mathcal{M}_{t,t+1}^{B}\tilde{W}_{b,D_b}(\mathcal{X}_{t+1})\right] - \mathcal{D}_t u_{cc}(C_t^D)Q_{D,t}^2 - \mathcal{D}_t^n u_{cc}(C_t^D)W_t Q_{D,t} - \mathcal{L}_t^{\psi}\psi \quad (46)$$
$$- \mathcal{D}_t\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right] - \mathcal{L}_t\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right] - \mathcal{L}_t^K\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right] + \mathcal{L}_t^{\Pi}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^{\Pi}}(\mathcal{X}_{t+1})}{\partial \bar{D}_{b,t+1}}\right] = 0.$$

The first-order condition with respect to $\bar{L}_{b,t+1} \equiv \mathtt{w}_b L_{b,t+1}$ is:

$$-Q_{L,t}(1-\mathcal{E}_t) + \mathbb{E}_t\left[\mathcal{M}_{t,t+1}^{B}\tilde{W}_{b,L_b}(\mathcal{X}_{t+1})\right] + \left(\mathcal{L}_t Q_{L,t} + \mathcal{L}_t^K + \mathcal{L}_t^{\Pi}\kappa_t + \mathcal{L}_t^n W_t\right)u_{cc}(C_t^E)Q_{L,t} + \mathcal{L}_t^{\psi} \quad (47)$$
$$- \mathcal{D}_t\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right] - \mathcal{L}_t\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right] - \mathcal{L}_t^K\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right] + \mathcal{L}_t^{\Pi}\beta\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^{\Pi}}(\mathcal{X}_{t+1})}{\partial \bar{L}_{b,t+1}}\right] = 0, \quad (48)$$

with $\kappa_t \equiv (1-\theta+\theta\Gamma_t)Y_t$.

The first-order condition with respect to $Q_{D,t}$ is:

$$\bar{D}_{b,t+1}(1-\mathcal{E}_t) + \mathcal{D}_t\left(u_c(C_t^D) - u_{cc}(C_t^D)\sum_{b=1}^{B}\bar{D}_{b,t+1}\right) - \mathcal{D}_t^n u_{cc}(C_t^D)W_t\sum_{b=1}^{B}\bar{D}_{b,t+1} = 0. \quad (49)$$

The first-order condition with respect to $Q_{L,t}$ is:

$$-\bar{L}_{b,t+1}(1-\mathcal{E}_t) + \mathcal{L}_t\left(u_c(C_t^E) + u_{cc}(C_t^E)\sum_{b=1}^{B}\bar{L}_{b,t+1}\right) + \mathcal{L}_t^n u_{cc}(C_t^E)W_t\sum_{b=1}^{B}\bar{L}_{b,t+1} = 0. \quad (50)$$

The first-order condition with respect to K_{t+1} is:

$$
\begin{aligned}
&-\left(\mathcal{L}_t Q_{L,t}+\mathcal{L}_t^K+\mathcal{L}_t^\Pi\kappa_t+\mathcal{L}_t^n W_t\right)u_{cc}(C_t^E)+\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^E\tilde{W}_{b,K}(\mathcal{X}_{t+1})\right] \qquad (51)\\
&-\mathcal{D}_t\beta\mathbb{E}_t\left[\frac{\partial\mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]-\mathcal{L}_t\beta\mathbb{E}_t\left[\frac{\partial\mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]-\mathcal{L}_t^K\beta\mathbb{E}_t\left[\frac{\partial\mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]+\mathcal{L}_t^\Pi\beta\mathbb{E}_t\left[\frac{\partial\mathcal{S}^{\mathcal{L}^\Pi}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]=0.
\end{aligned}
$$

The first-order condition with respect to Γ_t is:

$$
\begin{aligned}
&\mathcal{D}_t u_{cc}(C_t^D)Q_{D,t}f_N(Z_t,K_t,N_t)N_t^D+\mathcal{D}_t^n u_{cc}(C_t^D)W_t f_N(Z_t,K_t,N_t)N_t^D+\mathcal{D}_t^n u_c(C_t^D)f_N(Z_t,K_t,N_t) \qquad (52)\\
&+\left(\mathcal{L}_t Q_{L,t}+\mathcal{L}_t^K+\mathcal{L}_t^\Pi\kappa_t+\mathcal{L}_t^n W_t\right)u_{cc}(C_t^E)f_N(Z_t,K_t,N_t)N_t^E\\
&+\mathcal{L}_t^n u_c(C_t^E)f_N(Z_t,K_t,N_t)+\mathcal{L}_t^\Pi\theta Y_t u_c(C_t^E)=0.
\end{aligned}
$$

The first-order condition with respect to N_t^D is:

$$
\begin{aligned}
&\mathcal{D}_t u_{cc}(C_t^D)Q_{D,t}\Gamma_t f_N(Z_t,K_t,N_t)+\mathcal{D}_t u_{cc}(C_t^D)Q_{D,t}\Gamma_t f_{NN}(Z_t,K_t,N_t)N_t^D \qquad (53)\\
&+\mathcal{D}_t^n u_{cc}(C_t^D)W_t\Gamma_t f_N(Z_t,K_t,N_t)+\mathcal{D}_t^n u_{cc}(C_t^D)W_t\Gamma_t f_{NN}(Z_t,K_t,N_t)N_t^D\\
&+\mathcal{D}_t^n u_c(C_t^D)\Gamma_t f_{NN}(Z_t,K_t,N_t)-\mathcal{D}_t^n v_{nn}(N_t^D)+\frac{\partial C_t^E}{\partial N_t^D}\left(\mathcal{L}_t Q_{L,t}+\mathcal{L}_t^K+\mathcal{L}_t^\Pi\kappa_t+\mathcal{L}_t^n W_t\right)u_{cc}(C_t^E)\\
&+\mathcal{L}_t^\Pi\kappa_t Y_t^{-1}u_c(C_t^E)\frac{\partial Y_t}{\partial N_t^D}+\mathcal{L}_t^n u_c(C_t^E)\frac{\partial W_t}{\partial N_t^D}=0.
\end{aligned}
$$

The first-order condition with respect to N_t^E is:

$$
\begin{aligned}
&\mathcal{D}_t u_{cc}(C_t^D)Q_{D,t}\Gamma_t f_{NN}(Z_t,K_t,N_t)N_t^D \qquad (54)\\
&+\mathcal{D}_t^n u_{cc}(C_t^D)W_t\Gamma_t f_{NN}(Z_t,K_t,N_t)N_t^D\\
&+\mathcal{D}_t^n u_c(C_t^D)\Gamma_t f_{NN}(Z_t,K_t,N_t)-\mathcal{L}_t^n v_{nn}(N_t^E)+\frac{\partial C_t^E}{\partial N_t^E}\left(\mathcal{L}_t Q_{L,t}+\mathcal{L}_t^K+\mathcal{L}_t^\Pi\kappa_t+\mathcal{L}_t^n W_t\right)u_{cc}(C_t^E)\\
&+\mathcal{L}_t^\Pi\kappa_t Y_t^{-1}u_c(C_t^E)\frac{\partial Y_t}{\partial N_t^E}=0.
\end{aligned}
$$

Derivation of the Multipliers

We rearrange the last six first-order conditions to get closed form expression for the multipliers $(\mathcal{D}_t,\mathcal{L}_t,\mathcal{L}_t^K,\mathcal{L}_t^\Pi,\mathcal{D}_t^n,\mathcal{L}_t^n)$.

(D) FOC with respect to $Q_{D,t}$:

$$\bar{D}_{b,t+1}(1-\mathcal{E}_t) + \mathcal{D}_t\Big(u_c(C_t^D) - u_{cc}(C_t^D)\sum_{b=1}^{B}\bar{D}_{b,t+1}\Big) - \mathcal{D}_t^n\, u_{cc}(C_t^D)W_t\sum_{b=1}^{B}\bar{D}_{b,t+1} = 0. \tag{D}$$

Rearranging (D), we have

$$\Big(u_c(C_t^D) - u_{cc}(C_t^D)\sum_{b=1}^{B}\bar{D}_{b,t+1}\Big)\,\mathcal{D}_t - u_{cc}(C_t^D)W_t\Big(\sum_{b=1}^{B}\bar{D}_{b,t+1}\Big)\,\mathcal{D}_t^n = -\bar{D}_{b,t+1}\,.$$

Thus, if we define

$$c_{11} = u_c(C_t^D) - u_{cc}(C_t^D)\sum_{b=1}^{B}\bar{D}_{b,t+1}, \quad c_{15} = -u_{cc}(C_t^D)W_t\sum_{b=1}^{B}\bar{D}_{b,t+1}, \quad b_1 = -\bar{D}_{b,t+1}(1-\mathcal{E}_t),$$

the first row of the system is

$$c_{11}\,\mathcal{D}_t + c_{15}\,\mathcal{D}_t^n = b_1\,.$$

(L) FOC with respect to $Q_{L,t}$:

$$-\bar{L}_{b,t+1}(1-\mathcal{E}_t) + \mathcal{L}_t\Big(u_c(C_t^E) + u_{cc}(C_t^E)\sum_{b=1}^{B}\bar{L}_{b,t+1}\Big) + \mathcal{L}_t^n\, u_{cc}(C_t^E)W_t\sum_{b=1}^{B}\bar{L}_{b,t+1} = 0. \tag{L}$$

Rearranging (L), we obtain

$$\Big(u_c(C_t^E) + u_{cc}(C_t^E)\sum_{b=1}^{B}\bar{L}_{b,t+1}\Big)\,\mathcal{L}_t + u_{cc}(C_t^E)W_t\Big(\sum_{b=1}^{B}\bar{L}_{b,t+1}\Big)\,\mathcal{L}_t^n = \bar{L}_{b,t+1}\,.$$

Thus, with

$$c_{22} = u_c(C_t^E) + u_{cc}(C_t^E)\sum_{b=1}^{B}\bar{L}_{b,t+1}, \quad c_{26} = u_{cc}(C_t^E)W_t\sum_{b=1}^{B}\bar{L}_{b,t+1}, \quad b_2 = \bar{L}_{b,t+1}(1-\mathcal{E}_t),$$

the second row of the system is

$$c_{22}\,\mathcal{L}_t + c_{26}\,\mathcal{L}_t^n = b_2\,.$$

(K) FOC with respect to K_{t+1}:

$$
\begin{aligned}
&-\Big(\mathcal{L}_t Q_{L,t} + \mathcal{L}_t^K + \mathcal{L}_t^{\Pi}\,\kappa_t + \mathcal{L}_t^n W_t\Big) u_{cc}(C_t^E) + \mathbb{E}_t\Big[\mathcal{M}_{t,t+1}^E\,\tilde{W}_{b,K}(\mathcal{X}_{t+1})\Big] \qquad (55)\\
&-\mathcal{D}_t\,\beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right] - \mathcal{L}_t\,\beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right] - \mathcal{L}_t^K\,\beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]\\
&\quad + \mathcal{L}_t^{\Pi}\,\beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^{\Pi}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right] = 0.
\end{aligned}
$$

Rearranging (55), we write

$$
\begin{aligned}
&-\beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]\mathcal{D}_t\\
&-\Big(Q_{L,t}\,u_{cc}(C_t^E) + \beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]\Big)\mathcal{L}_t\\
&-\Big(u_{cc}(C_t^E) + \beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]\Big)\mathcal{L}_t^K\\
&+\Big(\beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^{\Pi}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right] - \kappa_t\,u_{cc}(C_t^E)\Big)\mathcal{L}_t^{\Pi}\\
&-W_t\,u_{cc}(C_t^E)\,\mathcal{L}_t^n + \mathbb{E}_t\Big[\mathcal{M}_{t,t+1}^E\,\tilde{W}_{b,K}(\mathcal{X}_{t+1})\Big] = 0.
\end{aligned}
$$

Thus, the nonzero coefficients here are

$$
\begin{aligned}
c_{31} &= -\beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{D}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right],\\
c_{32} &= -\Big(Q_{L,t}\,u_{cc}(C_t^E) + \beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]\Big),\\
c_{33} &= -\Big(u_{cc}(C_t^E) + \beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^K}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right]\Big),\\
c_{34} &= \beta\,\mathbb{E}_t\left[\frac{\partial \mathcal{S}^{\mathcal{L}^{\Pi}}(\mathcal{X}_{t+1})}{\partial K_{t+1}}\right] - \kappa_t\,u_{cc}(C_t^E),\\
c_{36} &= -W_t\,u_{cc}(C_t^E),
\end{aligned}
$$

and we set

$$
b_3 = -\mathbb{E}_t\Big[\mathcal{M}_{t,t+1}^E\,\tilde{W}_{b,K}(\mathcal{X}_{t+1})\Big].
$$

(LK) FOC with respect to Γ_t:

$$\begin{aligned}
&\mathcal{D}_t\, u_{cc}(C_t^D) Q_{D,t}\, f_N(Z_t, K_t, N_t)\, N_t^D + \mathcal{D}_t^n\, u_{cc}(C_t^D) W_t\, f_N(Z_t, K_t, N_t)\, N_t^D \\
&+ \mathcal{D}_t^n\, u_c(C_t^D)\, f_N(Z_t, K_t, N_t) \\
&+ \Big(\mathcal{L}_t Q_{L,t} + \mathcal{L}_t^K + \mathcal{L}_t^\Pi\, \kappa_t + \mathcal{L}_t^n W_t\Big) u_{cc}(C_t^E)\, f_N(Z_t, K_t, N_t)\, N_t^E \\
&+ \mathcal{L}_t^n\, u_c(C_t^E)\, f_N(Z_t, K_t, N_t) + \mathcal{L}_t^\Pi\, \theta\, Y_t\, u_c(C_t^E) = 0. \qquad \text{(LK)}
\end{aligned}$$

Dividing (LK) by $f_N(Z_t, K_t, N_t)$ (which we assume is nonzero) we write

$$\begin{aligned}
u_{cc}(C_t^D) Q_{D,t} N_t^D\, \mathcal{D}_t + \Big(u_{cc}(C_t^D) W_t N_t^D + u_c(C_t^D)\Big)\mathcal{D}_t^n \\
+ u_{cc}(C_t^E) Q_{L,t} N_t^E\, \mathcal{L}_t + u_{cc}(C_t^E) N_t^E\, \mathcal{L}_t^K \\
+ \big(\kappa_t u_{cc}(C_t^E) N_t^E + \theta\, Y_t\, u_c(C_t^E)/f_N\big)\, \mathcal{L}_t^\Pi + \big(u_{cc}(C_t^E) W_t N_t^E + u_c(C_t^E)\big)\, \mathcal{L}_t^n = 0.
\end{aligned}$$

Thus, the nonzero coefficients are

$$\begin{aligned}
c_{41} &= u_{cc}(C_t^D) Q_{D,t} N_t^D, \\
c_{42} &= u_{cc}(C_t^E) Q_{L,t} N_t^E, \\
c_{43} &= u_{cc}(C_t^E) N_t^E, \\
c_{44} &= \kappa_t u_{cc}(C_t^E) N_t^E + \theta\, Y_t\, u_c(C_t^E)/f_N, \\
c_{45} &= u_{cc}(C_t^D) W_t N_t^D + u_c(C_t^D), \\
c_{46} &= u_{cc}(C_t^E) W_t N_t^E + u_c(C_t^E),
\end{aligned}$$

with $b_4 = 0$.

(ND) FOC with respect to N_t^D:

$$\begin{aligned}
&\mathcal{D}_t\, u_{cc}(C_t^D) Q_{D,t} \Gamma_t\, f_N(Z_t, K_t, N_t) \\
+\;& \mathcal{D}_t\, u_{cc}(C_t^D) Q_{D,t} \Gamma_t\, f_{NN}(Z_t, K_t, N_t)\, N_t^D \\
+\;& \mathcal{D}_t^n\, u_{cc}(C_t^D) W_t \Gamma_t\, f_N(Z_t, K_t, N_t) \\
+\;& \mathcal{D}_t^n\, u_{cc}(C_t^D) W_t \Gamma_t\, f_{NN}(Z_t, K_t, N_t)\, N_t^D \\
+\;& \mathcal{D}_t^n\, u_c(C_t^D) \Gamma_t\, f_{NN}(Z_t, K_t, N_t) - \mathcal{D}_t^n\, v_{nn}(N_t^D) \\
+\;& \tfrac{\partial C_t^E}{\partial N_t^D} \Big(\mathcal{L}_t Q_{L,t} + \mathcal{L}_t^K + \mathcal{L}_t^\Pi\, \kappa_t + \mathcal{L}_t^n W_t\Big) u_{cc}(C_t^E) \\
+\;& \mathcal{L}_t^\Pi\, \kappa_t\, Y_t^{-1}\, u_c(C_t^E) \tfrac{\partial Y_t}{\partial N_t^D} + \mathcal{L}_t^n\, u_c(C_t^E) \tfrac{\partial W_t}{\partial N_t^D} = 0.
\end{aligned}$$

Thus, we identify

$$\begin{aligned}
c_{51} &= u_{cc}(C_t^D)Q_{D,t}\Gamma_t\Big(f_N + f_{NN}N_t^D\Big),\\
c_{55} &= u_{cc}(C_t^D)W_t\Gamma_t\Big(f_N + f_{NN}N_t^D\Big) + u_c(C_t^D)\Gamma_t f_{NN} - v_{nn}(N_t^D),\\
c_{52} &= \tfrac{\partial C_t^E}{\partial N_t^D}\, Q_{L,t}\, u_{cc}(C_t^E),\\
c_{53} &= \tfrac{\partial C_t^E}{\partial N_t^D}\, u_{cc}(C_t^E),\\
c_{54} &= \tfrac{\partial C_t^E}{\partial N_t^D}\, \kappa_t\, u_{cc}(C_t^E) + \kappa_t\, Y_t^{-1}\, u_c(C_t^E)\tfrac{\partial Y_t}{\partial N_t^D},\\
c_{56} &= \tfrac{\partial C_t^E}{\partial N_t^D}\, W_t\, u_{cc}(C_t^E) + u_c(C_t^E)\tfrac{\partial W_t}{\partial N_t^D},
\end{aligned}$$

with $b_5 = 0$.

(NE) FOC with respect to N_t^E:

$$\begin{aligned}
&\mathcal{D}_t\, u_{cc}(C_t^D)Q_{D,t}\Gamma_t\, f_{NN}(Z_t,K_t,N_t)\, N_t^D\\
+\;&\mathcal{D}_t^n\, u_{cc}(C_t^D)W_t\Gamma_t\, f_{NN}(Z_t,K_t,N_t)\, N_t^D\\
+\;&\mathcal{D}_t^n\, u_c(C_t^D)\Gamma_t\, f_{NN}(Z_t,K_t,N_t)\\
+\;&\mathcal{L}_t^n\, \Big(\tfrac{\partial C_t^E}{\partial N_t^E}W_t u_{cc}(C_t^E) - v_{nn}(N_t^E)\Big)\\
+\;&\tfrac{\partial C_t^E}{\partial N_t^E}\Big(\mathcal{L}_t Q_{L,t} + \mathcal{L}_t^K + \mathcal{L}_t^\Pi\,\kappa_t + \mathcal{L}_t^n W_t\Big)u_{cc}(C_t^E)\\
+\;&\mathcal{L}_t^\Pi\,\kappa_t\, Y_t^{-1}\, u_c(C_t^E)\tfrac{\partial Y_t}{\partial N_t^E} = 0.
\end{aligned}$$

Hence, we define

$$\begin{aligned}
c_{61} &= u_{cc}(C_t^D)Q_{D,t}\Gamma_t\, f_{NN}(Z_t,K_t,N_t)\, N_t^D,\\
c_{65} &= u_{cc}(C_t^D)W_t\Gamma_t\, f_{NN}(Z_t,K_t,N_t)\, N_t^D + u_c(C_t^D)\Gamma_t\, f_{NN}(Z_t,K_t,N_t),\\
c_{62} &= \tfrac{\partial C_t^E}{\partial N_t^E}\, Q_{L,t}\, u_{cc}(C_t^E),\\
c_{63} &= \tfrac{\partial C_t^E}{\partial N_t^E}\, u_{cc}(C_t^E),\\
c_{64} &= \tfrac{\partial C_t^E}{\partial N_t^E}\, \kappa_t\, u_{cc}(C_t^E) + \tfrac{\kappa_t}{Y_t}\, u_c(C_t^E)\tfrac{\partial Y_t}{\partial N_t^E},\\
c_{66} &= \tfrac{\partial C_t^E}{\partial N_t^E}W_t u_{cc}(C_t^E) - \, v_{nn}(N_t^E),
\end{aligned}$$

with $b_6 = 0$.

Collecting all six rows we obtain

$$\begin{pmatrix} c_{11} & 0 & 0 & 0 & c_{15} & 0 \\ 0 & c_{22} & 0 & 0 & 0 & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & 0 & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix} \begin{pmatrix} \mathcal{D}_t \\ \mathcal{L}_t \\ \mathcal{L}_t^K \\ \mathcal{L}_t^\Pi \\ \mathcal{D}_t^n \\ \mathcal{L}_t^n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} .$$

That is, if we define

$$\mathbf{x} = \begin{pmatrix} \mathcal{D}_t \\ \mathcal{L}_t \\ \mathcal{L}_t^K \\ \mathcal{L}_t^\Pi \\ \mathcal{D}_t^n \\ \mathcal{L}_t^n \end{pmatrix} ,$$

then we have

$$A\,\mathbf{x} = \mathbf{b}\,,$$

with

$$\mathbf{b} = \begin{pmatrix} -\bar{D}_{b,t+1} \\ \bar{L}_{b,t+1} \\ -\mathbb{E}_t\Big[\mathcal{M}_{t,t+1}^E\, \tilde{W}_{b,K}(\mathcal{X}_{t+1})\Big] \\ 0 \\ 0 \\ 0 \end{pmatrix} .$$