Aggregate and Cross-Sectional Spending Multipliers

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Abstract

This paper relates the industry composition of government spending with aggregate multipliers and income redistribution across occupations and income quintiles. In an economy with multiple segmented factor markets, aggregate multipliers are larger whenever spending targets industries which employ elastically supplied factors or face more nominal rigidities. Quantitatively, multipliers can be up to 4 times larger than in textbook aggregated models. Spending has an asymmetric incidence on factor demand – and hence on income across households – whenever the government bundle uses primary factors in different proportions compared to the private bundle. The direct incidence of spending is countervailed by a cross-sectional crowding out of private consumption. Given its current composition, spending increases labor income the most for households in lower quintiles, while households in the top quintile benefit from higher returns to non-labor factors. Employment responses exhibit substantial heterogeneity across occupations within income quintiles.

1 Introduction

The allocation of government spending across industries has important aggregate and redistributive implications, which remain largely unexplored so far.

Empirical estimates of output, consumption, and employment multipliers vary widely across studies, both in absolute magnitude and relative to each other (Ramey (2019)). Besides methodological differences, estimates may vary across sample periods because government spending has a stronger effect on aggregate demand in times of recession, or when monetary policy is constrained by the zero lower bound on interest rates (Auerbach and Gorodnichenko (2011, 2012)). This paper formalizes and quantifies a complementary explanation, namely that government spending has a time-varying effect on aggregate supply depending on its industry composition.[1] Specifically, the output multiplier is larger when government spending is targeted toward industries with flatter supply curves. Moreover,

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[1] Cox et al. (2024) make a similar argument. I contribute a fully analytical characterization of spending multipliers with an arbitrary number of industries and primary factors, and input-output linkages.

changes in the industry composition of government spending can rationalize variation in the relative size of output, consumption, and employment multipliers across sample periods.

Besides its role in shaping aggregate multipliers, the industry composition of government spending also has redistributive implications. This paper is the first to provide a comprehensive analysis of the incidence of spending across households through the production side of the economy. Earlier studies highlighted a correlation between the incidence of spending on labor demand and marginal propensities to consume, which is important for aggregate multipliers (Bilbiie and Straub (2004); Dupor et al. (2018); Flynn et al. (2021); Beraldi and Malgieri (2024)). I provide a full characterization of income incidences across households, depending on their differential exposure through labor demand, consumption prices, and asset income. I go beyond measuring direct incidences, and emphasize how the response of prices and employment is shaped by a cross-sectional crowding-out of private demand, which happens at every node of the production network. Consistent with my focus on the production side of the economy, I abstract from heterogeneity in marginal propensities to consume.

I begin by deriving a set of analytical results in a quantitatively realistic, yet analytically tractable model economy, introduced in section 2. The economy features multiple industries, which use different combinations of intermediate inputs and primary factors (consisting in differentiated labor occupations and semi-fixed assets), and sell to multiple final users (differentiated households, asset utilization producers, and the government). Final users own shares in the industries and primary factors, earning income rebates accordingly, and households are taxed to finance government spending. The baseline model assumes complete asset markets, to focus on the supply-side implications of government spending. I later present extensions where households cannot insure against spending shocks, and face borrowing constraints.

Section 3 considers a simplified setting, to illustrate which model elements determine aggregate and redistributive effects of government spending. Two fundamental features of the framework are segmented factor markets, paired with heterogeneous factor-level supply elasticities and/or network-adjusted industry-level nominal rigidities. Together, these two elements are necessary and sufficient to generate a connection between the industry composition of government spending and aggregate multipliers. A third important feature are unrestricted substitution elasticities, which govern expenditure switching at each node of the production and consumption network. These shape the incidence of government spending on relative factor employment and income.

Section 4 begins by analyzing how the industry composition of government spending shapes aggregate multipliers. In conventional textbook economies – with labor as the only primary factor of production – an increase in government spending raises aggregate supply through a wealth effect,[2] and raises aggregate demand by increasing output given private consumption. In a realistic economy with multiple segmented factor markets, government spending further changes aggregate supply through a reallocation effect. The reallocation effect is positive whenever spending increases the relative demand for industries with flatter supply curves. Notably, factor market segmentation is

[2] As households are taxed to finance government spending, they feel poorer and supply more labor at a given real wage.

necessary for relative demand changes to affect relative prices.[3] Moreover, increasing demand for industries with flatter supply curves has a smaller incidence on aggregate prices, resulting in a larger spending multiplier. In turn, the model allows us to trace differences across industry-level supply slopes back to heterogenous factor supply elasticities and network-adjusted nominal rigidities.

The theory also provides a natural framework to compare output, employment, and consumption multipliers. As expected, reallocation effects imply larger employment and consumption multipliers relative to output whenver the government bundle is more labor intensive than the private bundle. Consequently, there is no tradeoff between maximizing output and maximizing employment or consumption whenever labor intensive industries also face more nominal rigidities, or when labor is more elastically supplied than other primary factors.

Section 4 also analyzes the redistributive implications of government spending. Income responses across households depend on the incidence of spending on labor income, asset income, and consumption prices. In studying the incidence on the demand for labor and other productive assets, I isolate a direct effect and a countervailing cross-sectional crowding out effect. The direct effect is proportional to the factor content[4] of the spending bundle relative to the aggregate private bundle. This direct effect is partially offset by a crowding out effect, due to a reduction of private demand, as the relative price of goods purchased by the government increases. The crowding out effect depends on substitution elasticities – which govern expenditure switching at each node of the production and consumption networks – in a way that the model makes precise.

Finally, section 5 presents quantitative and empirical results. It first documents variation in the composition of government spending between 1963 and 2023, showing a shift away from construction and durable manufacturing, toward professional services and information technology. These changes mirror a similar shift in private consumption, so that there is no clear time trend in the relative price stickiness and labor share of spending relative to private consumption, although there is time variation. In the calibrated model, variation in the relative price stickiness of spending versus private consumption explains most of the variation in calibrated output multipliers. Likewise, variation in relative labor intensity explains most of the time-varying difference between calibrated output and employment multipliers.

Time variation in the nominal rigidity and labor share of spending relative to private consumption also allows me to empirically validate the theory predictions. I find that – consistent with the theory – both are positively and significantly related with estimated output multipliers.

I conclude by quantifying the redistributive implications of government spending. Given the current spending composition (as measured in 2018), the overall incidence is progressive across income quintiles. The lowest quintiles benefit the most in terms of labor income, whereas the top quintile benefits the most from higher returns to semi-fixed assets. Nonetheless, the incidence of spending on labor income varies greatly across occupations within

[3]Relative demand changes can affect relative and aggregate prices also when there are decreasing returns to scale at the industry level. Conceptually, decreasing returns to scale production functions are isomorphic to constant-returns production functions, which use unobserved industry-specific fixed assets.

[4]The factor content is computed by adding up the direct content, and the indirect content through intermediate inputs.

quintiles. Spending has the highest incidence on Protective Service occupations (with a cumulative multiplier on labor income of about 7), and the lowest incidence on Food Service occupations (with a multiplier of -0.1).

1.1 Related Literature

Estimates of spending multipliers are notoriously inconsistent across studies (Ramey (2019)). This can be due to a time-varying effect of spending on aggregate demand, which is more powerful under the zero lower bound on interest rates (Christiano et al. (2011); Farhi and Werning (2016); Auerbach and Gorodnichenko (2011, 2012); Ramey and Zubairy (2018)), or if marginal propensities to consume are high (Bilbiie and Straub (2004); Auclert et al. (2018); Hagedorn et al. (2019)). I argue that spending also has a time-varying effect on aggregate supply, which is more powerful when it reallocates demand toward industries with flatter supply curves.

Other papers pointed out that government spending is concentrated on a small set of industries, and explored the implications for aggregate multipliers. Closest to my work, Bouakez et al. (2018, 2023); Barattieri et al. (2023); Cox et al. (2024) set up quantitative input-output models to relate the industry composition of spending with aggregate output multipliers.[5] A recurring insight in this literature is that targeting sticky-priced industries yields a larger multiplier. I contribute by providing an analytical characterization of spending multipliers in a large scale, quantitatively realistic input-output model. This allows me to connect aggregate multipliers with factor-level supply elasticities, industry-level nomianal rigidities and subsitution elasticities, and the industry composition of government spending. I also establish that factor market segmentation, paired with heterogeneity in either nominal rigidities and/or factor supply elasticities, is necessary and sufficient for the industry composition of government spending to affect aggregate multipliers. As a further contribution, my model also provides a natural framework to study the relationship between output, employment and consumption multipliers.

A growing literature highlights that the incidence of government spending on labor demand is correlated with marginal propensities to consume across households, implying a larger aggregate multiplier (Dupor et al. (2018), Flynn et al. (2021), Beraldi and Malgieri (2024)). I contribute a more comprehensive analysis of the redistributive incidence of spending through the production side of the economy. To focus on the production side, I abstract from heterogeneous marginal propensities to consume in the baseline framework. My analysis goes beyond previous studies along three dimensions. First, I not only account for the incidence of spending on labor demand, but also for its incidence on consumption prices and income rebates from profits and other productive assets. Second, I allow for good prices and factor employment to respond to demand changes with heterogeneous elasticities. Third, I place no restriction on substitution elasticities in production and consumption. This allows me to account for a

[5] Ramey and Shapiro (1998) and Proebsting (2022) instead consider environements where capital cannot be freely reallocated, and spending is concentrated in a subset of industries. The capital reallocation frictions in Ramey and Shapiro (1998) generate a non-linear effect, whereby the goods toward which government spending is directed endogenously become more inelastically supplied. They discuss implications for the aggregate spending multiplier, as well as for the dynamics of consumption, investment, and real wages.

Other related papers are Monacelli et al. (2010), who study output and unemployment multipliers with search and matching frictions, Monacelli and Perotti (2010), who study the spending multiplier in open economies, Slavtchev and Wiederhold (2016), who study the effect of spending on R&D activity.

cross-sectional crowding out of private expenditure, which partially offsets the direct effect of spending on relative factor demand.

My framework also speaks to a growing empirical literature, which exploits variation in the spending composition across industries (Nekarda and Ramey (2011)) and across locations (Nakamura and Steinsson (2014); Chodorow-Reich (2019); Coviello et al. (2022); Auerbach et al. (2019, 2024)) to learn about aggregate multipliers. This empirical strategy is motivated based on model economies where regions face identical supply, demand, and pricing elasticities, and a symmetric trade network. As I discuss in a companion paper (Rubbo (2025)), accounting for asymmetric trade linkages and heterogeneous elasticities is fundamental to avoid mis-specification of cross-sectional moments, and to correctly map cross-sectional estimates into aggregate multipliers.

2 Environment

This section sets up the model economy, consisting of multiple heterogeneous industries (indexed by $i, j \in \mathcal{N}$), primary factors (indexed by $f \in \mathcal{F}$), and final users (indexed by $z \in \mathcal{Z}$). Primary factors consist in differentiated labor occupations and semi-fixed assets, while final users consist in households, asset utilization, and the government. Households own shares in firms and primary factors, and optimally allocate the returns from semi-fixed assets between consumption and utilization. The government chooses the amount of spending and its allocation across industries. Spending is financed by taxing households lump-sum, proportional to their income shares. Sections 2.1 and 2.2 outline the assumptions about preferences, production, and policy instruments, deriving optimality conditions for consumers and producers. Section 2.3 defines the general equilibrium, and section 2.4 introduces notation for the log-linearized model, later analyzed in section **??**.

2.1 Production

There is a set $\mathcal{N}$ of good-producing industries in the economy, indexed by i, j. Within each industry there is a continuum of firms, producing differentiated varieties.

All firms x in industry i have the same constant returns to scale production function, taking as inputs primary factors (differentiated labor and semi-fixed assets) and intermediate goods:

$$Y_{ixt} = G_i(\{L_{ifxt}\}_{f\in\mathcal{F}}, \{X_{ijxt}\}_{j=1}^{N}),$$

where L_{ifxt} is the quantity of primary factor f hired by firm x in industry i at time t, and X_{ijxt} is the quantity of intermediate input j used by the firm. The elasticity of substitution between goods j and k in the production of good i is denoted by

$$\theta^i_{jk} \equiv -\frac{d\log\frac{X_{ij}}{X_{ik}}}{d\log\frac{P_j}{P_k}}$$

Customers (final users and other producers) buy a CES bundle of sectoral varieties, with elasticity of substitution ϵ_i. The industry output is given by

$$Y_i = \left(\int Y_{if}^{\frac{\epsilon_i - 1}{\epsilon_i}} df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}$$

and the implied sectoral price index is

$$P_i = \left(\int P_{if}^{1-\epsilon_i} df \right)^{\frac{1}{1-\epsilon_i}}$$

The government provides proportional input subsidies, fully financed through lump-sum taxes on profits. I follow a standard practice in the literature, and assume that steady-state subsidies are chosen to eliminate the markup distortions arising from monopolistic competition.

All producers minimize costs given input prices. Due to constant returns to scale all firms within an industry i use inputs in the same proportions, so that heir marginal costs are equalized. The marginal cost of industry i, denoted by MC_i, is the solution of the cost minimization problem (omitting time subscripts for legibility)

$$MC_i = min_{\{X_{ij}\},\{L_{if}\}} \sum_{f \in \mathcal{F}} W_f L_{if} + \sum_{j \in \mathcal{N}} P_j X_{ij} \quad s.t.\ G_i \left(\{L_{if}\}_{f \in \mathcal{F}}, \{X_{ij}\}_{j \in \mathcal{N}} \right) = 1 \tag{1}$$

Producers are subject to nominal rigidities, modeled à la Calvo. In every sector i and at each period t, only a randomly-selected fraction δ_i of the firms can update their price. This setting nests a model with flexible prices, when $\delta_i = 1$ for all industries i. The firms x who can adjust their price set it to maximize the present discounted value of future profits, conditional on being unable to further update their price:

$$P_{ixt}^* = \frac{\epsilon_{it}}{\epsilon_{it} - 1} \frac{\mathbb{E} \sum_s \left[SDF_{t+s} \left(1 - \delta_i\right)^s Y_{ixt+s}(P_{ixt}^*) \left(1 - \tau_{it+s}\right) MC_{it+s} \right]}{\mathbb{E} \sum_s \left[SDF_{t+s} \left(1 - \delta_i\right)^s Y_{ixt+s}(P_{ixt}^*) \right]} \tag{2}$$

where SDF_{t+s} is the economy-wide stochastic discount factor between t and $t+s$, and demand functions are given by $Y_{ixt+s}(P_{xt}) = Y_{it+s} \left(\frac{P_{ixt}^*}{P_{it+s}} \right)^{-\epsilon_{it}}$. The firms x who cannot adjust their price instead keep it unchanged ($P_{ixt} = P_{ixt-1}$), and must absorb any cost changes in their markup $\mathcal{M}_{ixt}$.

Below I approximate the model around an efficient initial equilibrium, which satisfies Assumption 1.

Assumption 1. *All producers charge unit (post-subsidy) markups in the initial equilibrium.*

2.2 Primary factors and final users

The economy has a set $\mathcal{F} = \mathcal{L} \cup \mathcal{K}$ of primary factors, indexed by f, consisting in differentiated labor occupations ($f \in \mathcal{L}$) and semi-fixed assets ($f \in \mathcal{K}$, such as land, equipment, structures....). Factor markets are fully segmented. Every industry can use a combination of all factors, although potentially in different proportions. There is also a set $\mathcal{Z} = \mathcal{H} \cup \mathcal{U} \cup \mathcal{G}$ of final users, indexed by z, consisting in households ($z \in \mathcal{H}$), asset utilization producers ($z \in \mathcal{U}$), and the government (G). Households can represent different income or demographic groups, or geografic

locations. Household types own different shares in industries and semi-fixed assets. Household types also supply labor to multiple occupations, and occupations have different prevalence across household types.[6]

2.2.1 Households

Preferences Households $z \in \mathcal{H}$ maximize the present discounted value of utility flows

$$\sum_{t=0}^{\infty} e^{-\rho t} \left[\frac{C_{zt}^{1-\gamma}}{1-\gamma} - \sum_{f \in \mathcal{L}} \frac{L_{zft}^{1+\varphi_f}}{1+\varphi_f} + G_{zt} \right] \tag{3}$$

where $G_{zt} \equiv \mathcal{G}_z \left(g_{z1t}, ..., g_{zNt} \right)$ is the utility that household z derives from government spending.[7] Households enjoy consumption (C) and dislike labor (L), with separable disutility of labor supply across occupations $f \in \mathcal{L}$. The parameter γ governs both the intertemporal elasticity of substitution and the wealth effects in labor supply. The parameter φ_f is the inverse Frisch elasticity, which determines the slope of occupation-specific labor supply curves. The consumption aggregator $C_{zt} \equiv \mathcal{C}_{zt} \left(c_{z1t}, ..., c_{zNt} \right)$ is homothetic over the N goods produced in the economy. The elasticity of substitution between goods i and j in consumption is denoted by

$$\theta_{ij}^{C} \equiv -\frac{d \log \frac{c_i}{c_j}}{d \log \frac{P_i}{P_j}}.$$

Households are subject to the budget constraint

$$P_{zt}^{C} C_{zt} \leq \sum_{f \in \mathcal{F}} \hat{\Xi}_{zf} W_f L_f + \sum_{i \in \mathcal{N}} \hat{\Xi}_{zi} \Pi_{it} + T_{zt} \tag{4}$$

where P_{zt}^{C} is the price index implied by the consumption aggregator $\mathcal{C}_z$, W_{ft} is the nominal wage earned by factor f, $\hat{\Xi}_{zf}$ and $\hat{\Xi}_{zi}$ are the ownership shares of household z in primary factor f and industry i, Π_{it} are profits in industry i, and T_{zt} includes the household's net borrowing and a lump-sum transfer to the government. I use the same notation W_f and L_f for the price and employment of labor and semi-fixed assets, to emphasize that both are primary factors of production. For labor occupations $f \in \mathcal{L}$, $\hat{\Xi}_{zf}$ simply represent the share of household z in the headcount of occupation f. Section 2.2.2 below derives the income rebates $\hat{\Xi}_{zf}$ of semi-fixed assets $f \in \mathcal{K}$.

Euler equations. Households trade a risk-free asset, in zero net supply, which pays a nominal interest rate i_{t+1}. The optimal consumption-saving choice determines the net borrowing component of T_{zt}, and follows a standard

[6] In the model, there isn't necessarily a one-to-one mapping between household types and occupations. For example, in the calibration household types correspond to income quintiles. Households in the first quintile may be predominantly employed as service workers, but they can also work in manufacturing. Households in the second quintile may be primarily employed in manufacturing, but they could also be employed as service workers, and in other occupations as well.

[7] Given my focus on the positive properties of spending multipliers, I simplify the analysis by assuming that spending enters utility separably.

Euler equation

$$\frac{C_{z,t+1}}{C_{z,t}} = \left[\frac{1+i_{t+1}}{e^{-\rho}}\frac{P^C_{z,t}}{P^C_{z,t+1}}\right]^{\frac{1}{\gamma}}, \ \forall z \in \mathcal{H}. \tag{5}$$

Households can also trade shares in firms and semi-fixed assets, whose rates of return follow analogous Euler equations.In the baseline version of the framework, households can also insure against changes in the amount and industry composition of government spending. This is formally stated in assumption 2, which I relax in section 4.4.

Assumption 2. *Households can fully insure against changes in the amount and industry composition of government spending. Therefore at all times t is holds that*

$$\frac{C_{z,t}}{C_{z',t}} = \left(\frac{P^C_{z',t}}{P^C_{z,t}}\right)^{\frac{1}{\gamma}}, \ \forall z, z' \in \mathcal{H}.$$

Consumption-leisure tradeoff. Workers are hired by occupation-specific labor unions, which pay an occupation-specific, flexible wage W_{ft}. They optimally supply labor according to the consumption-leisure tradeoff

$$C^{\gamma}_{zt} L^{\varphi_f}_{zft} = \frac{W_{ft}}{P^C_{zt}}, \ \forall z \in \mathcal{H}, f \in \mathcal{L}. \tag{6}$$

Labor markets are fully segmented, hence wages differ across occupations. Labor unions sell labor services to producers in all industries, and are treated like any other industry. In particular there is a continuum of unions for each occupation, with fixed unit mass, facing Calvo-style price rigidities. Due to the Calvo friction, different unions hiring the same occupation $f \in \mathcal{L}$ might charge different prices. For simplicity I assume that ownership of each f-specific union is shared among households proportionately to their headcount in occupation f. This guarantees that all units of f-type labor are paid the same price, equal to the average price charged by f-specific unions. Notably, the flexible wage W_{ft} has no direct data counterpart. The model counterpart of wages in the data is the price paid by firms to the labor unions.

2.2.2 Semi-fixed assets

I model the supply of semi-fixed assets in a stylized way, with the objective to obtain tractable supply curves with constant elasticity φ_f. In particular, I abstract from the intertemporal dimension of investment and rely instead on a model of asset utilization. Each asset $f \in \mathcal{K}$ is produced by combining a fixed endowment ($\bar{K}_f$) with utilization U_f. At each period, the supply of asset f is given by

$$L_{ft} = U_{ft}\bar{K}_f.$$

For simplicity I assume that the utilization component U_{ft} fully depreciates from one period to the next, while the endowment component $\bar{K}_f$ never depreciates.

The production function for utilization is given by

$$U_{ft} \equiv \left[\left(1+\varphi_f\right) C_{ft}\right]^{\frac{1}{1+\varphi_f}},$$

where the utilization bundle C_f is produced with constant returns to scale, using as inputs a combination of goods $i \in \mathcal{N}$:

$$C_{ft} = \mathcal{C}_f(c_{f1,t}, ..., c_{fN,t}).$$

The elasticity of substitution between goods i and j in the utilization bundle of factor f is

$$\theta_{ij}^f \equiv -\frac{d\log\frac{c_{fi}}{c_{fj}}}{d\log\frac{P_i}{P_J}}$$

Utilization is produced by asset retailers, with marginal cost P_f^C. Retailers purchase asset endowments from the households, combine them with utilization, and sell their services to firms at a rental rate W_f in a perfectly competitive market. Asset retailers are owned by the households in proportion to their ownership shares in the asset endowments, and rebate their profits accordingly.

Profit maximization by utilization producers yields supply curves with constant elasticity $\frac{1}{\varphi_f}$:

$$U_f^{\varphi_f} = \frac{W_f}{P_f^C}\bar{K}_f, \ \forall f \in \mathcal{K}. \tag{7}$$

Inverse supply elasticities φ_f can vary across assets f, and also between labor and semi-fixed assets. Profit maximization by utilization producers implies that the payments $W_f L_f$ to factor f are divided between utilization expenditures, with share $\frac{1}{1+\varphi_f}$, and rebates to the households, with share $\frac{\varphi_f}{1+\varphi_f}$. Denoting by ζ_{hf} the ownership share of household h in asset f, this yields $\hat{\Xi}_{hf} = \frac{\varphi_f}{1+\varphi_f}\zeta_{hf}$ in equation (4). Likewise we have

$$P_{ft}^C C_{ft} = \hat{\Xi}_{ff} W_{ft} L_{ft},$$

where $\hat{\Xi}_{ff} = \frac{1}{1+\varphi_f}$ and $\hat{\Xi}_{ff'} = 0$ for $f \in \mathcal{U}, f' \in \mathcal{F}, f' \neq f$.

Like for labor, utilization producers also sell the assets to factor marketplaces, who then resell them to the various industries. Marketplaces are owned by households and utilization producers according to the shares $\hat{\Xi}$, and rebate their profits accordingly. Similar to the labor unions, factor marketplaces can have sticky prices, and effective asset returns are given by the price charged by marketplaces.

2.2.3 Aggregation and income shares

Denote steady-state variables with a star. Definition 1 introduces aggregate nominal and real GDP, the GDP deflator, and the income shares of final users and primary factors.

Definition 1. Aggregate nominal GDP is given by total final expenditures:

$$GDP = \sum_{z \in \mathcal{Z}} P_z^C C_z.$$

The steady-state income shares of final users in total GDP are denoted by

$$s_z \equiv \frac{P_z^{*C} C_z^*}{GDP^*},\ z \in \mathcal{Z},$$

while the steady-state income shares of primary factors are denoted by

$$\bar{\Psi}_f \equiv \frac{W_f^* L_f^*}{GDP^*},\ f \in \mathcal{F}.$$

Infinitesimal changes in real GDP around the initial steady-state are denoted by $d \log Y_t$, and are equal to the share-weighted sum of changes in aggregate consumption and utilization:

$$d \log Y_t = \sum_{z \in \mathcal{Z}} s_z d \log C_{zt}.$$

Infinitesimal changes in the GDP deflator around the initial steady-state are denoted by $d \log P_t^Y$, and are equal to the share-weighted change in the price indices of final users:

$$d \log P_t^Y = \sum_{z \in \mathcal{Z}} s_z d \log P_{zt}^C.$$

Remark 1. Around an efficient equilibrium, changes in real GDP equal the income-weighted sum of changes in the quantities of primary factors:

$$d \log Y_t = \sum_{f \in \mathcal{F}} \bar{\Psi}_f d \log L_{ft}.$$

2.2.4 Policy

The central bank sets the nominal interest rate i_t, according to a conventional Taylor rule:

$$i_t - \log \rho = \boldsymbol{\phi}_\pi \boldsymbol{\pi}_t + \phi_y y_t, \tag{8}$$

where $\boldsymbol{\pi}_t$ is a vector encoding industry-level inflation rates, y_t is the log-deviation of real GDP from its steady-state value, and $\boldsymbol{\phi}_\pi$, ϕ_y are reaction coefficients.

The government chooses the share of nominal spending in total nominal GDP, denoted by $s_G = \frac{P^G C_G}{GDP}$, and the allocation of spending across industries. The expenditure share of the government on industry i is denoted by Ω_{Gi}.

In order to finance spending the government taxes households proportionally to their income shares, setting

$$\frac{\tilde{T}_{ht}}{GDP} = -\frac{s_h}{\sum_{z\in\mathcal{H}} s_z} s_G, \ \forall h \in \mathcal{H}.$$

The government also chooses industry-level cost subsidies, and fully finances them with lump-sum taxes on profits.

2.3 Equilibrium

The equilibrium concept adapts the definition in Baqaee and Farhi (2020), to account for the endogenous determination of markups given pricing frictions and shocks. Given sectoral markups, all markets must clear; and the evolution of markups must be consistent with Calvo pricing and the realization of shocks and policies.

Definition 2. At each period t, for given sectoral probabilities of price adjustment δ_i, and given monetary and fiscal policies, the general equilibrium is given by a vector of firm-level markups $\mathcal{M}_{ixt}$, a vector of sectoral prices P_{it}, a vector of factor-specific nominal wages W_{ft}, a vector of factor supplies L_{ft}, a vector of sectoral outputs Y_{it}, a matrix of intermediate input quantities X_{ijt}, and a matrix of final demands C_{izt}, such that: (i) a fraction δ_i of firms in each sector i charges the profit-maximizing price given by (2); (ii) the markup charged by adjusting firms is given by the ratio of the profit-maximizing price and marginal costs, while the markups of non-adjusting firms are such that their price remains constant; (iii) agents maximize utility subject to their budget constraint; (iv) producers in each sector i minimize costs and charge the relevant markup; and (v) markets for all goods and factors clear.

This equilibrium concept nests the standard one with flexible prices, which is obtained as a special case when $\delta_i = 1$ for every sector i.

2.4 Notation

This Section introduces notation for the variables and parameters that characterize the log-linearized model. Steady-state variables are denoted with a star. I use lower case letters to denote log-deviations from the initial steady state. Vectors are boldfaced.

Model variables. Table 1 summarizes notation for the endogenous variables (aggregate output, factor-level employment, the final users' consumption, industry-level prices and inflation, the nominal interest rate) and for the shocks (spending share and composition).

Aggregate output	$\bar{y}_t \equiv \log \frac{Y_t}{Y^*}$
Factor employment	$\boldsymbol{\ell}_t = \begin{pmatrix} \ell_{1t} & ... & \ell_{Ft} \end{pmatrix}^T$, $\ell_{ft} \equiv \log \frac{L_{f,t}}{L_f^*}$
Final users consumption	$\boldsymbol{c}_t = \begin{pmatrix} c_{1t} & ... & c_{Zt} \end{pmatrix}^T$, $c_{zt} \equiv \log \frac{C_{zt}}{C_z^*}$
Industry-level prices	$\boldsymbol{p}_t = \begin{pmatrix} p_{1t} & ... & p_{Nt} \end{pmatrix}^T$, $p_{it} \equiv \log \frac{P_{i,t}}{P_i^*}$
Industry-level inflation	$\boldsymbol{\pi}_t = \begin{pmatrix} \pi_{1t} & ... & \pi_{Nt} \end{pmatrix}^T$, $\pi_{it} \equiv p_{it} - p_{i,t-1}$
Nominal interest rate	i_{t+1}
Spending share in GDP	$s_G \equiv \frac{G}{GDP}$
Spending composition	$\boldsymbol{\Omega}_{G,1} = \begin{pmatrix} \Omega_{G1,t} & ... & \Omega_{GN,t} \end{pmatrix}$

Table 1: Model variables

Income flows and income shares. The income flows in the economy consist of the industries' expenditures on primary factors and intermediate inputs, the final users' expenditures, the income rebates from primary factors and firms to final users, and lump-sum transfers.

I encode direct expenditure shares in the matrix Ω, whose ij-the element corresponds to the expenditure share of agent i on j. I define Ω to include factor marketplaces and final users as well, and extend the set $\mathcal{N}$ accordingly. The total expenditure of i on j (directly and through intermediate inputs) is denoted by $\Psi_{ij} \equiv (I - \Omega)^{-1}_{ij}$. As a convention, I denote by $\Omega_{\mathcal{Z},:}$ and $\Psi_{\mathcal{Z},:}$ the rows corresponding to final users, and by $\Omega_{:,\mathcal{F}}$ and $\Psi_{:,\mathcal{F}}$ the columns corresponding to labor unions and factor marketplaces. In particular, the block $\Psi_{\mathcal{Z}\mathcal{F}}$ represents the total content of primary factors in final uses. I adopt the same notation for the rows and columns corresponding to final users and primary factors in all matrices defined in Tables 2 and 3 .

Table 2 summarizes the notation for income flows and income shares.

Expenditure shares	$\Omega \in \mathbb{R}^{\lvert\mathcal{N}\rvert \times \lvert\mathcal{N}\rvert}$	$\Omega_{ij} = \frac{P_j^* X_{ij}^*}{P_i^* Y_i^*}$
Leontief inverse	$\Psi \equiv (I - \Omega)^{-1}$	
Income shares of final users	$\mathbf{s} \in \mathbb{R}^{\lvert\mathcal{Z}\rvert}$, $\mathcal{S} \equiv diag(\boldsymbol{s})$	$s_h \equiv \frac{P_h^{C*} C_h^*}{GDP^*}$
Income shares of primary factors	$\bar{\boldsymbol{\Psi}}_{\mathcal{F}} \in \mathbb{R}^{\lvert\mathcal{F}\rvert}$	$\bar{\Psi}_f \equiv \frac{W_f^* L_f^*}{GDP^*}$
Domar weights	$\bar{\boldsymbol{\Psi}} \in \mathbb{R}^{\lvert\mathcal{N}\rvert}$, $\mathcal{T} \equiv diag\left(\bar{\boldsymbol{\Psi}}\right)$	$\bar{\Psi}_i \equiv \frac{P_i^* Y_i^*}{GDP^*}$
Ownership shares	$\Xi \in \mathbb{R}^{\lvert\mathcal{Z}\rvert \times \lvert\mathcal{N}\rvert}$	$\Xi \equiv \mathcal{S}^{-1}\left(\hat{\Xi} + \frac{\boldsymbol{T^*}}{GDP^*}\mathbf{1}^T\right)\mathcal{T}$
Industry shares in the GDP deflator	$\bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \in \mathbb{R}^{\lvert\mathcal{N}\rvert}$	$\bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \equiv \boldsymbol{s}^T \Omega_{\mathcal{Z},:}$

Table 2: Input-output definitions and income shares

Elasticities. Table 3 introduces the notation used below to represent demand, supply, and pricing elasticities in a compact way.

Wealth effects in factor supply	$\Gamma \in \mathbb{R}^{\lvert\mathcal{F}\rvert\times\lvert\mathcal{Z}\rvert}$	$\Gamma = \hat{\Xi}^T_{:\mathcal{F}} diag(\boldsymbol{\gamma}),\ \boldsymbol{\gamma} \equiv \begin{pmatrix} \gamma \mathbf{1}_{\mathcal{H}} & \mathbf{0}_{\mathcal{U}} & 0 \end{pmatrix}$
Factor supply elasticities	$\Phi \in \mathbb{R}^{\lvert\mathcal{F}\rvert\times\lvert\mathcal{F}\rvert}$	$\Phi \equiv diag(\boldsymbol{\varphi}) + \Gamma \begin{pmatrix} \mathbf{1}_{\mathcal{H}}\bar{\boldsymbol{\Xi}}_{\mathcal{HF}} & \Xi_{\mathcal{UF}} & \mathbf{0} \end{pmatrix}^T$
Industry shares in factor price deflators	$\beta \in \mathbb{R}^{\lvert\mathcal{N}\rvert\times\lvert\mathcal{F}\rvert}$	$\beta^T \equiv (\mathcal{I} - \Gamma)\Omega_{C:} + \Gamma \begin{pmatrix} \mathbf{1}_{\mathcal{H}}\bar{\boldsymbol{\Xi}}_{\mathcal{HF}} & \Xi_{\mathcal{UF}} & \mathbf{0} \end{pmatrix}^T \Psi^{-1}$, $\mathcal{I}_{hf} \equiv \hat{\Xi}^T_{hf}\mathbb{I}_{f\in\mathcal{L}} + \mathbb{I}_{h=f, f\in\mathcal{K}}$
Industry shares in nominal rate deflator	$\pi^R \equiv \hat{\boldsymbol{\beta}}^T \boldsymbol{\pi}$	$\hat{\boldsymbol{\beta}}^T \equiv (1-\gamma)\bar{\boldsymbol{\Omega}}_{\mathcal{H}:} + \gamma\left[\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\Psi^{-1} - \bar{\boldsymbol{\Xi}}_{\mathcal{HF}}\mathcal{D}_p\right]$
Substitution elasticities	θ^i_{jk}, for $i, j, k \in \mathcal{N}$	$\theta^i_{jk} \equiv -\frac{d\log\frac{X_{ij}}{X_{ik}}}{d\log\frac{P_j}{P_k}}$
Slope of real marginal cost Phillips curves	$\Delta \equiv diag\left(\hat{\delta}_1, ..., \hat{\delta}_{\bar{N}}\right)$	$\hat{\delta}_i \equiv \frac{\delta_i(1-\rho(1-\delta_i))}{1-\rho\delta_i(1-\delta_i)}$
Price pass-through	$\mathcal{P} \equiv \Delta(I - \Omega\Delta)^{-1}$	
Demand elasticity	$\mathcal{D}_p \equiv \frac{\partial \ell_f}{\partial p_i} \in \mathbb{R}^{\lvert\mathcal{F}\rvert\times\lvert\mathcal{N}\rvert}$	Defined in Appendix A

Table 3: Elasticities

3 A Simple Model of Demand Reallocation

This section familiarizes readers with notation and fundamental economic concepts. It starts by recalling the textbook analysis of government spending, in an aggregated economy featuring only one industry, one factor of production, and a representative household. It then extends the analysis to a two-sector economy, illustrating how the slopes of industry-level supply curves interact with the industry composition of government spending to determine the response of aggregate supply.

Benchmark aggregated model. Consider a special case of the framework introduced in section 2, featuring one household and one industry, which turns a single primary factor (labor) into the consumption good with constant returns to scale.

Using the notation introduced in section 2, the household's optimal consumption-leisure tradeoff is

$$w_t = (\gamma + \varphi)\,\ell_t - \gamma\frac{ds_{Gt}}{1 - s^*_G} + p_t. \tag{9}$$

where φ is the inverse Frish elasticity and γ governs wealth effects in labor supply. In equation (9), increasing governemnt spending raises labor supply through a wealth effect: as households are taxed to finance spending, they feel poorer and hence are willing to work more at any given real wage.

Log-linearizing the optimal price setting equation (2) yields

$$p_t = \hat{\delta} w_t + \left(1 - \hat{\delta}\right)\left(p_{t-1} + \rho\mathbb{E}\pi_{t+1}\right), \tag{10}$$

Equations (9) and (10) then yield an aggregate supply curve

$$\left(1 - \hat{\delta}\right)\pi_t = \hat{\delta}\left[(\gamma + \varphi)\,y_t - \gamma\frac{ds_{Gt}}{1 - s^*_G}\right] + \left(1 - \hat{\delta}\right)\rho\mathbb{E}\pi_{t+1}. \tag{11}$$

Government spending shifts the supply curve, meaning that it changes output for constant price dynamics. The supply shift $\varsigma_S \equiv \frac{\partial y_t}{\partial s_G}$ satisfies

$$(1 - s_G^*)\,\varsigma_S = \frac{\gamma}{\gamma + \varphi}. \tag{12}$$

When prices are flexible it holds $\hat{\delta} = 1$, implying a vertical aggregate supply curve. Therefore the equilibrium output response $\frac{dy_t}{ds_{Gt}}$ coincides with the supply shift. In particular, equilibrium output does not depend on aggregate demand. Moreover, output at each time t only depends on spending at time t, regardless of the persistence of the spending shock.

With sticky prices, instead, the supply curve is no longer vertical ($\frac{1}{1-\kappa^{p,sticky}} < \infty$), and thus the equilibrium response of output and prices is jointly determined by the aggregate supply curve (**??**) and the aggregate demand curve.

The aggregate demand curve in turn is obtained by combining the household's Euler equation with market clearing for the final good, and solving for the nominal interest rate using the Taylor rule (8):

$$\pi_t = -\frac{\gamma + \phi_y}{\phi_\pi} y_t + \frac{\gamma}{\phi_\pi} \frac{ds_{Gt} - ds_{Gt+1}}{1 - s_G^*} + \frac{\gamma}{\phi_\pi} y_{t+1} + \frac{1}{\phi_\pi} \pi_{t+1}, \tag{13}$$

I define the demand shift $\varsigma_D \equiv \frac{\partial y_t}{\partial s_{Gt}}$ as the change in current output for constant prices and constant future variables:

$$(1 - s_G^*)\,\varsigma_D = \frac{\gamma}{\gamma + \phi_y}.$$

For a spending shock with zero persistence, intersecting the demand and supply curves implies a contemporaneous multiplier of

$$\frac{dy_t}{ds_{Gt}} = \frac{\varsigma_D + \frac{\gamma+\varphi}{1-\hat{\delta}}\varsigma_S}{1 + \frac{\gamma+\varphi}{1-\hat{\delta}}}, \tag{14}$$

and a multiplier of $\frac{dy_{t+s}}{ds_{Gt}} \equiv 0$ thereafter. Comparing equations (12) and (14) we see that the sticky-price multiplier is larger than the flex-price one whenever the demand shift is larger than the supply shift ($\varsigma_D > \varsigma_S$), which is the case as long as the policy rule does not react too strongly to current output ($\phi_y < \varphi$).

Industry-specific supply slopes and the aggregate multiplier. Consider now an economy with two industries, each using labor to produce a final good with constant returns to scale. Labor markets are fully segmented across the two indsutries. For this example only, assume for simplicity that all households and the government have equal expenditure shares on the two goods in steady-state.

Under assumption 2, and given that households consume identical baskets of goods, this economy has the same aggregate demand equation (13) as the aggregated model. What changes instead are the supply relations. Under

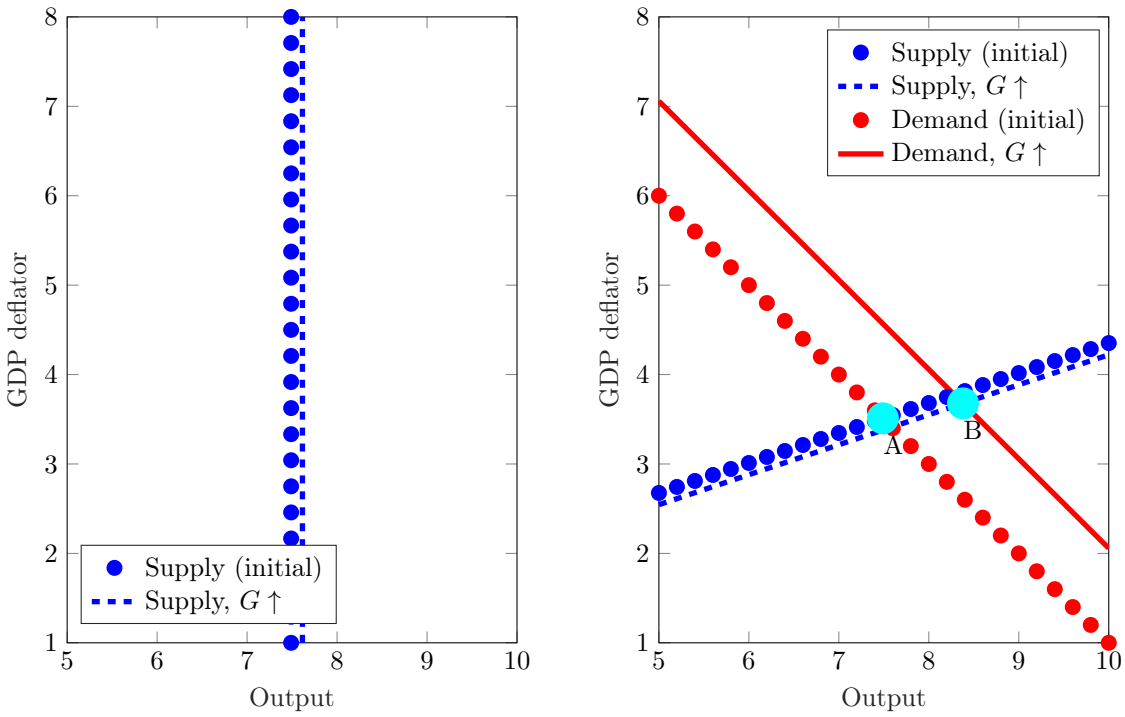


Figure 1: Demand and supply shifts in the flex-price model (left) and sticky-price model (right).

our assumptions, the price of industry i is given by

$$p_{it} = \hat{\delta}_i \left[\varphi_i \ell_{it} + \gamma \left(y_t - \frac{ds_{Gt}}{1 - s_G^*} \right) + \bar{p}_t \right] + \Sigma_{it},$$

where Σ_i is a function of lagged prices and expected inflation, and I denote by $\bar{x} = \frac{1}{2}(x_i + x_j)$ the average between the values of a variable x pertaining to the two industries i and j. With some abuse of notation, I index by i and j the employment of workers in each industry.[8] Note that industries now have potentially different supply slopes, $\hat{\delta}_i \varphi_i$. The industry with more flexible prices (higher $\hat{\delta}_i$) or more inelastic labor supply (higher φ_i) faces a steeper supply curve.

Denoting by θ the common substitution elasticity in private and government consumption, the relative demand for labor types is given by

$$\ell_{it} - \ell_{jt} = -\theta (p_{it} - p_{jt}) + s_G d\bar{\Omega}_{Git}. \tag{15}$$

For any variable x, further denote by

$$\begin{aligned} \underline{x} &\equiv \bar{x} \left(1 - \frac{\theta \bar{\delta\varphi}}{1 + \theta \bar{\delta\varphi}} \frac{\hat{\delta}_i \varphi_i - \bar{\delta\varphi}}{\bar{\delta\varphi}} \frac{x_i - \bar{x}}{\bar{x}} \right) \\ \tilde{x} &\equiv \bar{x} \left(1 + \theta \bar{\delta\varphi} \left(1 - \frac{\hat{\delta}_i \varphi_i - \bar{\delta\varphi}}{\bar{\delta\varphi}} \frac{x_i - \bar{x}}{\bar{x}} \right) \right). \end{aligned} \tag{16}$$

Combining the supply and relative demand equations allows us to derive an aggregate supply curve, which relates aggregate prices with aggregate output:

$$\left(1 - \underline{\hat{\delta}}\right) \bar{p}_t = \left(\underline{\hat{\delta}\gamma} + \underline{\hat{\delta}\varphi} \right) y_t - \underline{\hat{\delta}\gamma} ds_{Gt} + s_G \frac{Cov\left(\hat{\delta}\varphi, d\bar{\Omega}_{Gt}\right)}{1 + \theta \bar{\delta\varphi}} + \Sigma_t.$$

The aggregate supply shift is then given by $\frac{\varsigma_S^{s_G}}{1 - s_G^*} ds_G + \varsigma_S^{\bar{\Omega}_{Gi}}$, where

$$\varsigma_S^{\bar{\Omega}_{Gi}} \varsigma_S^{s_G} \equiv \frac{\underline{\hat{\delta}\gamma}}{\underline{\hat{\delta}\gamma} + \underline{\hat{\delta}\varphi}}, \equiv -s_G \frac{Cov\left(\hat{\delta}\varphi, d\bar{\Omega}_{Gt}\right)}{\tilde{\delta\gamma} + \tilde{\delta\varphi}}. \tag{17}$$

The term $\varsigma_S^{s_G}$ is the counterpart of ς_S in the aggregated model. The term $\varsigma_S^{\bar{\Omega}_{Gi}}$ captures a reallocation effect, indicating that the supply shift is larger when spending increases by more on the industry with a flatter supply slope ($Cov\left(\hat{\delta}\varphi, d\bar{\Omega}_{Gt}\right) < 0$). This is because, when the government reallocates resources toward this industry, its price increases by less than the other industry's price declines. Hence the average final price declines for given output. An analogous reallocation effect would be present in response to an increase in the spending share s_G, if the steady-state spending bundle was biased towards the industry with a flatter supply slope, compared to the

[8] In the general case – discussed in section **??** – there will no longer be a one-to-one correspondence between industries and primary factors, hence the slopes κ will need to be defined for each industry-factor pair.

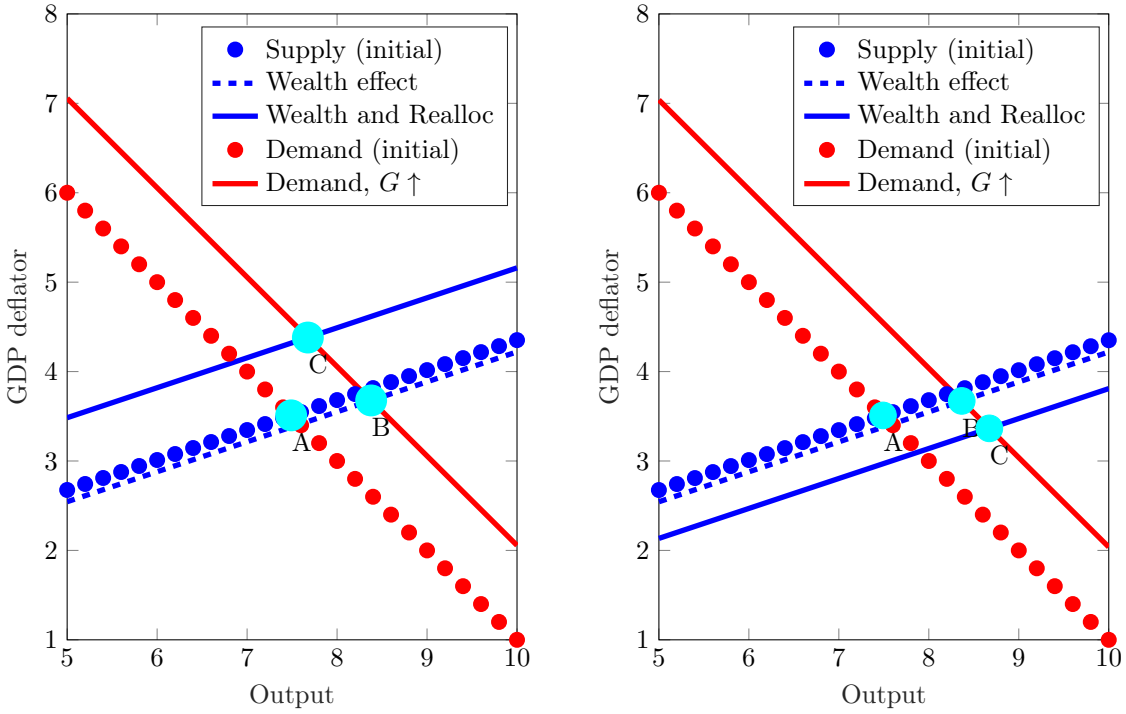


Figure 2: Demand and Supply Shifts, Utilities (left) and Prof services (right)

private bundle.

The reallocation effect $\varsigma_S^{\bar{\Omega}_{Gi}}$ in equation (17) is increasing in the dispersion between supply slopes across industries, as it is intuitive. Moreover, for given dispersion, the reallocation effect also gets amplified when the average slope $\delta\bar{\varphi}$ is flatter. In fact, when prices are less responsive to changes in demand, the expenditure switching channel – mediated by the subsitution elasticity θ in the expression for $\delta\tilde{\gamma}$ and $\delta\tilde{\varphi}$ – is muted, and fewer resources are crowded out from the industry where spending is targeted.

Figure 2 compares demand and supply shifts in the calibrated model, when spending is targeted on the Utilities sector – an industry with flexible pries (high $\hat{\delta}_i$) and inelastic factor supply (high φ_i) – versus the Professional Services sector – an industry with sticky pries and elastic labor supply.

4 Aggregate and Cross-Sectional Spending Multipliers

This section generalizes the methods and insigths introduced in section **??**. Section 4.1 first characterizes the log-linearized equilibrium of the economy, highlighting how the relative demand for primary factors is determined by government spending, and how the presence of multiple heterogeneous primary factors shapes industry-level supply curves. Section 4.2 then moves on to relate the industry composition of government spending with the response of

aggregate output, employment, and consumption. Finally, section 4.3 relates the spending composition with the redistributive incidence of spending on income across households.

4.1 Log-Linearized Equilibrium

Lemma 1 characterizes the equilibrium of the log-linearized economy.

Lemma 1. *Define all variables as in table 1, and Φ, β, Γ, $\mathcal{P}$, $\mathcal{D}_p$ as in table 3. The log-linearized equilibrium is pinned down by the Taylor rule (8) and the following four equations:*

$$\begin{cases} \boldsymbol{w}_t = & \Phi \boldsymbol{\ell}_t + \beta^T \boldsymbol{p}_t - \Gamma \mathbf{1} \frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}} \\ \boldsymbol{p}_t = & \mathcal{P}_{:\mathcal{F}} \boldsymbol{w}_t + \left(I - \mathcal{P}\Psi^{-1}\right)\left(\boldsymbol{p}_{t-1} + \rho \mathbb{E}\boldsymbol{\pi}_{t+1}\right) \\ \boldsymbol{\ell}_t = & \mathbf{1} y_t - \mathcal{D}_p \boldsymbol{p}_t + \boldsymbol{\mathcal{D}}_{Gt} \\ \boldsymbol{c}_{\mathcal{H},t+1} = & \boldsymbol{c}_{\mathcal{H},t} + \frac{1}{\gamma}\left(\mathbf{1} i_t - \Omega_{\mathcal{H}:} \boldsymbol{\pi}_{t+1}\right). \end{cases} \tag{18}$$

Proof. The system (**??**) is obtained by log-linearizing equations (1), (2), (4), (5), (6) and (7), and by solving out for consumption using the final users' budget constraints. The full derivation is in Appendix *A*. The vector $\boldsymbol{\mathcal{D}}_G$ is defined below. □

The first equation in (18) describes factor supply, the second describes pricing, while the fourth is the households' Euler equation. These equations are similar to aggregated models, just replicated across multiple primary factors, industries, and households – all stacked in vector notation. The third equation instead is present only in models with segmented factor markets, and it generalizes the relative demand equation (15) in the simplified environment of section 3.

The factor supply equation is akin to the familiar consumption-leisure tradeoff. The matrix Φ is a generalized inverse supply elasticity, which combines Frish elasticities, wealth effects in labor supply, and asset supply elasticities. The factor price deflators $\beta^T \boldsymbol{p}$ are combinations of consumption prices and wealth effects from factor income and profit rebates. Like in aggregated models, the spending share in aggregate GDP ds_{Gt} affects labor supply through a wealth effect (governed by the elasticity Γ): as households are taxed to finance spending, they feel poorer and are willing to supply more labor at any given real wage.[9]

In the pricing equation, the elasticity $\mathcal{P} = (\mathcal{P}_{ij})$ describes the pass-through of a desired price change in industry j into the price of industry i. In turn, current price changes relative to steady-state may be due to factor price changes ($\boldsymbol{w}$), price changes inherited from previous periods ($\boldsymbol{p}_{t-1}$), and expected future inflation ($\mathbb{E}\boldsymbol{\pi}_{t+1}$).

The relative demand equation expresses factor demand as a function of relative prices and government demand. The matrix $\mathcal{D}_p$ (derived in Appendix *A*) is a generalized elasticity of substitution – which maps changes in relative

[9] The spending share does not affect the supply of non-labor factors, as captured by $\Gamma_{:z} = \mathbf{0}$ for $z \notin \mathcal{H}$.

prices into changes in relative factor demand – while accounting for expenditure switching at each node of the production network and for income reallocation across final users with heterogeneous consumption baskets. The next paragraph illustrates the incidence of spending on relative factor demand ($\boldsymbol{\mathcal{D}}_G$).

Incidence of spending on relative factor demand Define $\bar{\boldsymbol{\Psi}}_{\mathcal{HF}} \equiv \frac{\boldsymbol{s}_{\mathcal{H}}^T}{\boldsymbol{s}_{\mathcal{H}}^T \mathbf{1}} \Psi_{\mathcal{HF}}$, and denote exogenous changes in government spending shares by $d\bar{\boldsymbol{\Omega}}_G$.[10] The incidence of spending on the relative demand for primary factors ($\boldsymbol{\mathcal{D}}_G$ in equation (18)) is given by

$$\boldsymbol{\mathcal{D}}_G \equiv \mathcal{D}_\ell \left[\left(\boldsymbol{\Psi}_{G\mathcal{F}}^T - \bar{\boldsymbol{\Psi}}_{\mathcal{HF}}^T \right) ds_G + s_G Cov_{\boldsymbol{\Omega}_{G:}} \left(\Psi_{:\mathcal{F}}, d \log \bar{\boldsymbol{\Omega}}_{G:t}^T \right) \right].$$

In the expression for $\boldsymbol{\mathcal{D}}_G$, government spending affects relative factor demand through two channels. First, whenever $\boldsymbol{\Psi}_{G\mathcal{F}} \neq \bar{\boldsymbol{\Psi}}_{\mathcal{HF}}$, the government bundle has a different factor content from the average consumption bundle of the households who are taxed to finance the government's purchases. Therefore increasing the spending share s_G reallocates demand toward the factors used more intensively in the government bundle. Second, changes in the industry composition of government spending ($d\bar{\boldsymbol{\Omega}}_G$) reallocate demand toward the factors used more intensively by industries where spending is redirected, as captured by the covariance between factor contents $\Psi_{:\mathcal{F}}$ and the change in government expenditure shares.

The income reallocation multiplier $\mathcal{D}_\ell$ (derived in Appendix A) captures a feedback effect, whereby changes in relative factor demand affect the relative income of final users who consume different baskets of goods, which in turn feeds back into relative factor demand.

Industry-level supply curves As a preliminary step toward the analysis of spending multipliers, proposition 2 derives the incidence of aggregate output and government spending on industry-level inflation rates $\boldsymbol{\pi}$.

Lemma 2. *Industry-level inflation rates are given by*

$$\left(I - \kappa \Phi^{-1} \mathbf{1} \bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right) \boldsymbol{\pi}_t = \kappa \left[\mathbf{1} y_t - \Phi^{-1} \Gamma \mathbf{1} \frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}} + \boldsymbol{\mathcal{D}}_{Gt} \right] + \boldsymbol{\mathcal{X}}_t, \tag{19}$$

where $\boldsymbol{\mathcal{X}}_t$ is a function of lagged prices and expected inflation (derived in Appendix A), and

$$\kappa \equiv \mathcal{W}^{-1} \mathcal{P}_{:\mathcal{F}} \Phi, \;\; \mathcal{W} \equiv I + \mathcal{P}_{:\mathcal{F}} \left(\Phi \mathcal{D}_p - \left(\beta^T - \mathbf{1} \bar{\boldsymbol{\Omega}}_{\mathcal{Z}:} \right) \right) \tag{20}$$

All prices are flexible if and only if $\kappa \Phi^{-1} \mathbf{1} = \mathbf{1}$ and $\boldsymbol{\mathcal{X}}_t \equiv \mathbf{0}$ $\forall t$,[11] otherwise $\kappa \Phi^{-1} \mathbf{1} \leq \mathbf{1}$ (with at least one strict inequality).

[10] Expenditure shares can also change endogenously, in response to price changes.

[11] When prices are flexible the coefficient on inflation in (19) is not invertible, meaning that the price level is indeterminate.

Proof. Equation (19) is obtained by combining the factor supply, relative factor demand, and pricing equations in (18). The full derivation is reported in Appendix A. □

Equation (19) is the multi-industry counterpart of the supply equation (**??**) in the aggregated model of section 3. The matrix $\kappa \in \mathbb{R}^{N \times F}$ is a supply slope, relating industry-level prices to changes in the employment of primary factors. Inspecting the slope κ reveals that, unlike in section 3, prices in each industry potentially depend on the full distribution of employment across all primary factors – either directly, through input-output linkages, or through expenditure switching and/or a price-wage multiplier.

Industry-by-factor slopes are increasing in factor supply elasticities Φ – which govern by how much factor prices increase as factor employment increases – and in the pass-through of factor prices into good prices $\mathcal{P}_{:\mathcal{F}}$. In turn, $\mathcal{P}_{if}$ is high for industry-factor pairs such that industry i has flexible prices, is upstream, has upstream suppliers with flexible prices, and has a high total content of factor f.

The multiplier $\mathcal{W}$ accounts for equilibrium adjustments in relative factor demand and supply, due to expenditure switching and a price-wage feedback. Expenditure switching happens whenever there is a change in relative demand across goods. As the goods in higher demand become more expensive, customers reallocate expenditure away from these goods, partially offsetting the initial change in demand. This is described by the term $\mathcal{P}_{:\mathcal{F}}\Phi\mathcal{D}_p$. The price-wage feedback instead is described by the term $\mathcal{P}_{:\mathcal{F}}\left(\beta^T - \mathbf{1}\bar{\mathbf{\Omega}}_{C:}\right)$, which tells us that primary factors whose wage deflator increases by more also become more expensive, thereby raising the price of industries which use these factors.

The right hand side of equation (19) describes the supply shift caused by government spending, discussed in detail in proposition 1. The left hand side is a price-wage multiplier, capturing a feedback loop whereby higher prices imply higher required nominal wages, which in turn feeds back into higher prices, etc. Aggregating both sides of equation (19) according to the industries' shares in the GDP deflator, we obtain the aggregate supply curve

$$\left(1 - \bar{\boldsymbol{\kappa}}\Phi^{-1}\mathbf{1}\right)\bar{\pi}_t = \bar{\boldsymbol{\kappa}}\left[\mathbf{1}y_t - \Phi^{-1}\Gamma\mathbf{1}\frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}} + \boldsymbol{\mathcal{D}}_{Gt}\right] + \bar{\mathcal{X}}_t, \tag{21}$$

where $\bar{\boldsymbol{\kappa}} \equiv \bar{\mathbf{\Omega}}_{\mathcal{Z}:}\kappa$. Following lemma 2, when prices are flexible we have $\bar{\boldsymbol{\kappa}}\Phi^{-1}\mathbf{1} = 1$, meaning that the price level is indeterminate and the aggregate supply curve (21) is vertical.

The incidence of each factor f on the aggregate GDP deflator is proportional to $\bar{\kappa}_f$, which averages the incidence of f across all industries, weighted by final use shares. Corollary (1) discusses which primitive elasticities determine a heterogeneous incidence of primary factors on the aggregate GDP deflator.

Corollary 1. *Denote by* $\bar{\Phi} \equiv \bar{\mathbf{\Psi}}_{\mathcal{F}}^T\Phi\mathbf{1}$, $\bar{\boldsymbol{\beta}} \equiv \beta\bar{\mathbf{\Psi}}_L$, $\bar{\boldsymbol{\mathcal{P}}} \equiv \boldsymbol{s}^T\mathcal{P}_{\mathcal{Z}:}$, $\bar{\mathcal{P}} \equiv \boldsymbol{s}^T\mathcal{P}_{\mathcal{Z}\mathcal{F}}\mathbf{1}$, *and define* $\hat{\mathcal{D}}_p \equiv \mathcal{D}_p\left(I - \frac{\kappa\mathbf{1}\bar{\mathbf{\Omega}}_C}{\boldsymbol{\kappa}^Y\mathbf{1}}\right)$. *The incidence of primary factors on the GDP deflator is proportional to their income shares* ($\boldsymbol{\kappa}^Y \propto \bar{\mathbf{\Psi}}_{\mathcal{F}}^T$) *whenever (i) the factor price pass-through into the GDP deflator is proportional to factor income shares* ($\bar{\boldsymbol{\mathcal{P}}}_{\mathcal{F}} = \bar{\mathcal{P}}\bar{\mathbf{\Psi}}_{\mathcal{F}}^T$); *(ii) factor supply elasticities are such that* $\bar{\mathbf{\Psi}}_{\mathcal{F}}^T\Phi = \bar{\Phi}\bar{\mathbf{\Psi}}_{\mathcal{F}}^T$; *and (iii) the aggregate factor price deflator is equal to the GDP deflator* ($\bar{\boldsymbol{\beta}}^T = \bar{\mathbf{\Omega}}_{\mathcal{Z}:}$). *Otherwise, primary factors* f *that are inelastically supplied* ($\bar{\mathbf{\Psi}}_{\mathcal{F}}^T\Phi_{:f} > \bar{\Phi}\bar{\Psi}_f$),

have high price pass-through ($\bar{\mathcal{P}}_{\mathcal{Z}f} > \bar{\mathcal{P}}\bar{\Psi}_f$), or have a high incidence on factor price deflators ($\left(\bar{\boldsymbol{\beta}}^T\kappa\right)_f > \left(\bar{\boldsymbol{\beta}}^T\kappa\mathbf{1}\right)\bar{\Psi}_f$), affect the aggregate GDP deflator more than proportionally to their income share. In addition, factors f which are substituted for by inelastically supplied or flex-priced factors f', as captured by the difference $\left[\bar{\mathcal{P}}\left(\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\Phi - \bar{\Phi}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right) + \bar{\Phi}\left(\bar{\boldsymbol{\mathcal{P}}}_{\mathcal{F}} - \bar{\mathcal{P}}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\right]\hat{\mathcal{D}}_p\kappa_{:f} > 0$, also affect the aggregate GDP deflator more than proportionally to their income share.

Proof. Using the definition of the slope κ, we can compute the incidence of each primary factor on the GDP deflator, relative to the incidence of a proportional increase in the employment of all primary factors:

$$\begin{aligned}\bar{\boldsymbol{\kappa}} - (\bar{\boldsymbol{\kappa}}\mathbf{1})\,\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T \propto &\left[\bar{\mathcal{P}}\left(\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\Phi - \bar{\Phi}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right) + \left(\bar{\boldsymbol{\mathcal{P}}}_{\mathcal{F}} - \bar{\mathcal{P}}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\bar{\Phi}\right]\left(I - \hat{\mathcal{D}}_p\kappa\right) + \bar{\mathcal{P}}\left(\bar{\boldsymbol{\beta}}^T\kappa - \left(\bar{\boldsymbol{\beta}}^T\kappa\mathbf{1}\right)\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\\ &+ \left(\bar{\boldsymbol{\mathcal{P}}}_{\mathcal{F}} - \bar{\mathcal{P}}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\left[\left(\Phi - \bar{\Phi}I\right)\left(I - \mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T - \hat{\mathcal{D}}_p\kappa\right) + \beta^T\left(\kappa - (\kappa\mathbf{1})\,\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\right]\end{aligned}$$

The terms in the first line are explained in the statement of the proposition. The second line is a covariance term, capturing the notion that factors which are both inelastically supplied and have a high pass-through $\bar{\mathcal{P}}_f$ have an even larger incidence on the GDP deflator. Likewise, factors f' which have a higher incidence on the price deflators of factors f, which in turn have a high passthrough $\bar{\mathcal{P}}_f$, have an even larger incidence on the GDP deflator. □

4.2 Aggregate Multipliers

In order to connect the industry composition of government spending with aggregate multipliers, I start by discussing how it affects aggregate supply and demand shifts. The spending composition has important implications for aggregate supply, while the effect of spending on aggregate demand is similar to aggregated models.

Aggregate supply. Proposition 1 moves from equation (**??**) and the definition of $\boldsymbol{\mathcal{D}}_G$ to derive the aggregate supply shift induced by government spending. Prior to stating the proposition, definition 3 introduces notation for cross-sectional mean and covariance operators.

Definition 3. Given a unit-sum vector $\boldsymbol{w} \in \mathbb{R}^N$ and a vector $\boldsymbol{x} \in \mathbb{R}^N$, the average $\mathbb{E}_{\boldsymbol{w}}(\boldsymbol{x})$ is given by

$$\mathbb{E}_{\boldsymbol{w}}(\boldsymbol{x}) \equiv \sum_i w_i x_i.$$

The covariance operator $Cov_{\boldsymbol{w}}(X,Y) \in \mathbb{R}^{M\times P}$ maps matrices $X \in \mathbb{R}^{M\times N}$ and $Y \in \mathbb{R}^{N\times P}$ into

$$Cov_{\boldsymbol{w}}(X,Y) \equiv \sum_i w_i X_{:i}Y_{i:} - \left[\sum_i w_i X_{:i}\right]\left[\sum_i w_i Y_{i:}\right].$$

Proposition 1. *The partial elasticity of aggregate output to government spending, for constant GDP deflator, is*

given by

$$\bar{s}_{\mathcal{H}} \frac{\partial y_t}{\partial s_{Gt}} = \mathbb{E}_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left[\Phi^{-1}\Gamma\mathbf{1}\right] + \frac{Cov_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left[\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}_{\mathcal{F}}}, \Phi^{-1}\Gamma\mathbf{1} - \mathcal{D}_\ell\left(\mathbf{\Psi}_{GL}^T - \mathbf{\Psi}_{HHL}^T\right)\right]}{\mathbb{E}_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left[\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\right]} \tag{22}$$

$$\frac{\partial y_t}{\partial \mathbf{\Omega}_{Gt}} = -s_G \frac{Cov_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left[\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}_{\mathcal{F}}}, \mathcal{D}_\ell \Psi_{:\mathcal{F}}^T\right]}{\mathbb{E}_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left[\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\right]}.$$

Proof. The result follows immediately from equation (21), definition 3, and the definition of $\boldsymbol{\mathcal{D}}_G$. □

The first term in equation (1) is similar to aggregated models, replacing the aggregate labor supply elasticity with a geometric average across labor factors. The additional terms are new, and capture reallocation effects. Reallocation effects depend on the covariance between the changes in factor demand induced by government spending, and the factors' incidences on the GDP deflator (discussed in Lemma 1). The second term in $\frac{\partial y_t}{\partial s_{Gt}}$ tells us that increasing the spending share in GDP increaes aggregate supply by more when the spending bundle uses primary factors that have a smaller incidence on the GDP deflator compared to the private consumption bundle. Likewise, $\frac{\partial y_t}{\partial \Omega_{iGt}}$ shows that reallocating demand towards an industry i increases aggregate supply whenever good i has a high content Ψ_{fi} of primary factors f which have a smal incidence $\bar{\kappa}_f$ on the GDP deflator.

Furthermore, changing the spending share has a larger effect on aggregate supply when labor-intensive industries also have a larger cost pass-through into the GDP deflator. This is captured by the term $Cov_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left[\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}_{\mathcal{F}}}, \Phi^{-1}\Gamma\mathbf{1}\right]$. In fact, the wealth effects generated by higher taxes only reduce the price of labor, not of semi-fixed assets ($\Gamma_{f,:} = 0$ for $f \in \mathcal{K}$).

Notably, the first term in equation (1) does not depend on the degree of nominal rigidity. The demand reallocation terms instead depend on the degree of nominal rigidity through the ratio $\frac{\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}_{\mathcal{F}}}}{\mathbb{E}_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left[\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\right]}$. In general, reallocation effects get amplified when prices are stickier. Intuitively, nominal rigidities dampen relative price adjustment, thereby preventing a crowding out of private expenditure from the industries where spending is targeted, which would otherwise offset the direct reallocation effects. More formally, the multiplier $\mathcal{W}^{-1}$ in the expression for $\bar{\boldsymbol{\kappa}}$ is decreasing in the elements of the cost pass-through matrix $\mathcal{P}_{:\mathcal{F}}$. While this property cannot be proved in general, the section 3 illustrated the logic why this is true in most cases.

Directly following the discussion above, Corollary 2 summarizes the conditions under which government spending has no reallocation effect on the aggregate supply curve.

Corollary 2. *In an economy with multiple segmented factor markets ($\Psi_{:\mathcal{F}} \neq \mathbf{1}\bar{\mathbf{\Psi}}_{\mathcal{F}}^T$), changing the spending share s_G has the same effect on aggregate supply as in an economy with only one unsegmented factor market if and only if either (a) the spending bundle is identical to the private consumption bundle, and all primary factors face the same supply elasticities; or (b) primary factors have identical incidences on the aggregate GDP deflator ($\boldsymbol{\kappa}^Y \propto \bar{\mathbf{\Psi}}_{\mathcal{F}}^T$). Changing the composition of spending has no effect on the aggregate supply curve if and only if primary factors have*

identical incidences on the aggregate GDP deflator ($\boldsymbol{\kappa}^Y \propto \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T$), or factor markets are not segmented ($\Psi_{:\mathcal{F}} = \mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T$)

.

Remark 2. With multiple industries, there are many possible ways to define aggregate inflation and hence many possible aggregate supply curves. Lemma 1 focuses on the GDP deflator, to build intuition. However one could derive analogous expressions for any other aggregate inflation index $\boldsymbol{\phi}^T\boldsymbol{\pi}$, by replacing the slope $\bar{\boldsymbol{\kappa}}$ with $\bar{\boldsymbol{\kappa}}^{\phi} \equiv \boldsymbol{\phi}^T\kappa$. If prices are flexible the choice of a price index is simply a normalization, and it is easy to verify that all price indices imply the same elasticity of aggregate output to govenrment spending in equation (1). If instead prices are sticky, the partial elasticity in (1) depends on which aggregate price index is assumed to be constant. In general no price index is a sufficient statistic for the aggregate spending multiplier, which depends on the dynamic evolution of the full price distribution (as discussed below).

Aggregate demand Proposition 3 derives the aggregate demand equation.

Lemma 3. *The dynamics of aggregate real GDP are described by the aggregate demand equation*

$$\begin{aligned}\mathbb{E}y_{t+1} - y_t = \frac{1}{\gamma}\left(i_t - \mathbb{E}\bar{\pi}^R_{t+1}\right) + \frac{\mathbb{E}ds_{G,t+1} - ds_{Gt}}{\bar{s}_{\mathcal{H}}} \\ - \bar{\boldsymbol{\Xi}}_{\mathcal{H},\mathcal{F}}\left(\mathbb{E}\boldsymbol{\mathcal{D}}_{G,t+1} - \boldsymbol{\mathcal{D}}_{G,t}\right),\end{aligned} \tag{23}$$

where $\bar{\pi}^R$ is a nominal rate deflator defined in Table 3.

The first two terms in (23) are similar to the textbook aggregated model. The third term tells us that, for given consumption, spending increases aggregate output by more when it reallocates income towards asset utilization – resulting in a larger demand shift.

Price and output dynamics If all prices are flexible ($\kappa\Phi^{-1}\mathbf{1} = \mathbf{1}$), equation (**??**) pins down aggregate output given government spending. In other words, the partial derivatives in proposition (1) are also total derivatives. If instead some industries face nominal rigidities ($\kappa\Phi^{-1}\mathbf{1} \neq \mathbf{1}$), the joint dynamics of industry-level prices and aggregate output – given government spending – are described by equations (19), (23) and (8).

4.2.1 Employment and Consumption Multipliers

So far, we defined supply and demand curves in terms of aggregate real GDP (y). The equilibrium equations (18) can be combined to derive supply and demand curves in terms of different aggregates as well, such as total labor employment and total household consumption. The same method as above can then be used to compute employment and consumption multipliers.

Employment multiplier Proposition **??** derives the counterparts of equations (19) and (23) in terms of aggregate labor employment, denoted $\bar{\ell} \equiv \frac{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T \boldsymbol{\ell}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T \mathbf{1}}$.

Proposition 2. *The aggregate GDP deflator, expressed as a function of aggregate employment, is given by*

$$\left(1 - \bar{\boldsymbol{\kappa}}^{\mathcal{L}} \Phi^{-1} \mathbf{1}\right) \bar{\pi}_t = \bar{\boldsymbol{\kappa}}^{\mathcal{L}} \left[\mathbf{1} \bar{\ell}_t - \Phi^{-1} \Gamma \mathbf{1} \frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}} + \boldsymbol{\mathcal{D}}_{Gt}^{\mathcal{L}}\right] + \bar{\mathcal{X}}_t^{\mathcal{L}}, \tag{24}$$

where $\mathcal{X}^{\mathcal{H}}$ is a function of lagged prices and expected inflation, and $\kappa^{\mathcal{L}}$ is defined as in (20), but rescaling all demand elasticities relative to aggregate employment:

$$\mathcal{D}_x^{\mathcal{L}} \equiv \left(I - \frac{\mathbf{1} \bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T \mathbf{1}}\right) \mathcal{D}_x, \ x = p, G. \tag{25}$$

The dynamics of aggregate employment follow

$$\bar{\ell}_{t+1} - \bar{\ell}_t = \frac{1}{\gamma} \left(i_t - \bar{\pi}_{t+1}^{R,\mathcal{L}}\right) + \frac{ds_{Gt+1} - ds_{Gt}}{s_{\mathcal{H}}} - \bar{\boldsymbol{\Xi}}_{\mathcal{H},\mathcal{F}} \left(\boldsymbol{\mathcal{D}}_{Gt+1}^{\mathcal{L}} - \boldsymbol{\mathcal{D}}_{Gt}^{\mathcal{L}}\right),$$

where the nominal rate deflator $\bar{\pi}_{t+1}^{R,\mathcal{L}}$ is computed in the same way as $\bar{\pi}_{t+1}^{R}$ in (23), but using the re-scaled demand elasticity $\mathcal{D}_p^{\mathcal{L}}$. The partial elasticities of aggregate employment through the supply and demand relations (24) and (25) are larger than the partial output whenever spending is targeted toward labor-intensive goods. Specifically, the employment based demand curve is shifted by more whenever $\left(\frac{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T}{\bar{\Psi}_{\mathcal{L}}} - \frac{\bar{\boldsymbol{\Psi}}_{\mathcal{K}}^T}{\bar{\Psi}_{\mathcal{K}}}\right) \boldsymbol{\mathcal{D}}_G > 0$, while the reallocation effect in the employment-based supply curve is larger than the output-based one whenever $\left(\frac{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T}{\bar{\Psi}_{\mathcal{L}}} - \frac{\bar{\boldsymbol{\Psi}}_{\mathcal{K}}^T}{\bar{\Psi}_{\mathcal{K}}}\right) (I - \mathcal{D}_p \kappa) \boldsymbol{\mathcal{D}}_G > 0$.[12]

Proof. The full derivation is in Appendix **??**. The first step consists in re-normalizing the relative demand equation in (18) to express factor employment relative to aggregate labor employment $\bar{\ell}$ (instead of aggregate real GDP y) and hence modifying the demand elasticities as in (25). The supply elasticities and the aggregate demand curve are then computed in the same way as in propositions 2, 1 and 3. □

Consumption multiplier Proposition 3 derives the counterparts of equations (19) and (23) in terms of aggregate household consumption, denoted $\bar{c} \equiv \frac{\boldsymbol{s}_{\mathcal{H}}^T \boldsymbol{c}}{\boldsymbol{s}_{\mathcal{H}}^T \mathbf{1}}$.

Proposition 3. *The aggregate GDP deflator, expressed as a function of aggregate consumption, is given by*

$$\left(1 - \bar{\boldsymbol{\kappa}}^{\mathcal{H}} \Phi^{-1} \mathbf{1}\right) \bar{\pi}_t = \bar{\boldsymbol{\kappa}}^{\mathcal{H}} \left[\mathbf{1} \bar{c}_t + \left(I - \Phi^{-1} \Gamma\right) \mathbf{1} \frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}} + \boldsymbol{\mathcal{D}}_{Gt}^{\mathcal{H}}\right] + \mathcal{X}^{\mathcal{H}}, \tag{26}$$

[12] The term $(I - \mathcal{D}_p \kappa) \boldsymbol{\mathcal{D}}_G$ captures the total incidence of government spending on factor demand, including both direct and substitution effects, as explained in detail in proposition 4 below.

where

$$\begin{aligned}
\mathcal{D}_x^{\mathcal{H}} &\equiv \left(I - \mathbf{1}\bar{\Xi}_{\mathcal{H}\mathcal{F}}\right)\mathcal{D}_x,\ x = p, G \\
\beta_{\mathcal{H}} &\equiv \beta + \Phi\mathbf{1}\left(\bar{\Omega}_{\mathcal{H}} - \bar{\Xi}_{\mathcal{H}:}\Psi^{-1}\right) \\
\kappa^{\mathcal{H}} &\equiv \left(I + \mathcal{P}_{:L}\left(\Phi\mathcal{D}_p^{\mathcal{H}} - \left(\beta_{\mathcal{H}}^T - \mathbf{1}\bar{\Omega}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:\mathcal{F}}\Phi
\end{aligned} \tag{27}$$

and $\mathcal{X}^{\mathcal{H}}$ is a function of lagged prices and expected inflation. The reallocation effect in the consumption-based supply curve (26) is larger than in the output-based one whenever $\left[\left(\bar{\Xi}_{\mathcal{H}:}\Psi^{-1} - \bar{\Omega}_{\mathcal{H}}\right)\kappa - \frac{\bar{\Psi}_{\mathcal{K}}^T}{1+\varphi_{\mathcal{K}}}\left(I - \mathcal{D}_p\kappa\right)\right]\boldsymbol{\mathcal{D}}_G > 0$. The aggregate demand curve expressed in terms of aggregate consumption is simply the aggregate Euler equation

$$\bar{c}_{t+1} = \bar{c}_t + \frac{1}{\gamma}\left(i_t - \pi_{t+1}^{CPI}\right), \tag{28}$$

where we denoted $\pi_t^{CPI} \equiv \bar{\Omega}_{\mathcal{H}}\boldsymbol{\pi}_t$.

Proof. The full derivation is in Appendix **??**. Like in proposition 2, the first step is re-normalize the relative demand equation relative to aggregate consumption, and hence modify the demand elasticities as in (27). The supply slope $\kappa^{\mathcal{H}}$ is then computed following the same steps as in proposition 2, noting that the price deflators in the factor supply equation also need to be adjusted. □

The comparison between output and consumption responses is similar to aggregated models. Unlike for output and employment, government spending does not shift the consumption-based aggregate demand curve (28). Moreover, the consumption-based supply curve is shifted in the opposite direction ($\left(\left(I - \Phi^{-1}\Gamma\right)\mathbf{1} > \mathbf{0}\right)$, because households are taxed to finance spending. Like for aggregate labor employment, the consumption-based supply curve has a larger reallocation effect than the output-based one when spending is directed towards labor intensive industries ($\frac{\bar{\Psi}_{\mathcal{K}}^T}{1+\varphi_{\mathcal{K}}}\left(I - \mathcal{D}_p\kappa\right)\boldsymbol{\mathcal{D}}_G < 0$), because the income of semi-fixed assets is partly spent on utilization rather than fully rebated to households. Moreover, the reallocation effect is also larger when spending is targeted toward goods used more intensively in utilization than in consumption ($\left(\bar{\Xi}_{\mathcal{H}:}\Psi^{-1} - \bar{\Omega}_{\mathcal{H}}\right)\kappa\boldsymbol{\mathcal{D}}_G > 0$), so that profit income increases by more than consumption prices.

4.3 Cross-Sectional Employment

Proposition 4 derives the effect of government spending on occupation employment and asset utilization, relative to aggregate output.

Proposition 4. *Denote cross-sectional factor employment by $\boldsymbol{\ell}^{XS} \equiv \boldsymbol{\ell} - \mathbf{1}y$, and define $\hat{\mathcal{D}}_p$ as in proposition 2.*[13]

[13]The matrix $\hat{\mathcal{D}}_p$ describes the response of relative employment to changes in industry-level prices, relative to the aggregate price change implied by a proportional employment change across all factors.

It holds that

$$\boldsymbol{\ell}_t^{XS} = \left(I - \hat{\mathcal{D}}_p\kappa\right)\boldsymbol{\mathcal{D}}_{Gt} + \hat{\mathcal{D}}_p\kappa\Phi^{-1}\Gamma\mathbf{1}\frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}} + \hat{\mathcal{D}}_p\left(\kappa\Phi^{-1}\mathbf{1} - \mathbf{1}\right)\bar{\pi}_t + \boldsymbol{\mathcal{G}}_t^{\mathcal{F}} \tag{29}$$

where $\boldsymbol{\mathcal{G}}^{\mathcal{F}}$ is a function of lagged prices and expected inflation. When prices are flexible we have $\kappa\Phi^{-1}\mathbf{1} = \mathbf{1}$, so that cross-sectional employment does not depend on the price level $\bar{\pi}$.

Proof. Equation (29) is obtained by combining the factor supply, pricing, and relative factor demand equations in (18). The full derivation is reported in Appendix **??**. □

The industry composition of government spending has a direct incidence on factor demand ($\boldsymbol{\mathcal{D}}_{Gt}$), dampened by a cross-sectional crowding out of private expenditure ($-\hat{\mathcal{D}}_p\kappa\boldsymbol{\mathcal{D}}_{Gt}$). As the relative demand for primary factors changes, relative prices also adjust (as described by the supply slopes κ), triggering expenditure switching (governed by $\hat{\mathcal{D}}_p$) and hence counterbalancing the initial change in demand.

In addition, increasing s_G also raises the relative supply of labor compared to non-labor factors, due to wealth effects in labor supply (governed by the matrix Γ). Labor intensive goods thereore becomes cheaper (as described by the supply slope $\kappa\Phi^{-1}$), and thus producers and final users substitute toward theses goods (as governed by $\hat{\mathcal{D}}_p$).

When prices are sticky and factor price pass-through is heterogeneous ($\kappa\Phi^{-1}\mathbf{1} \not\propto \mathbf{1}$), cross-sectional employment also depends on aggregate demand (here parameterized by the GDP deflator). As aggregate demand increases, industries i that are more downstream or have stickier prices have a smaller price pass-through $\kappa_{i,:}\Phi^{-1}\mathbf{1}$. Therefore producers and final users substitute toward factors used more intensively by these industries, as governed by the demand elasticity $\hat{\mathcal{D}}_p$.

4.4 Uninsurable Spending Shocks

The equilibrium system (18) is derived under the assumption that households can fully insure against changes in the amount and industry composition of government spending (assumption 2). Proposition **??** relaxes this assumption, and derives the equilibrium of an economy where households can trade a risk-free bond, but cannot insure against spending shocks.

Lemma 4. *Assume that households cannot insure against changes in the amount and composition of government spending, while they can trade a risk-free bond and shares in industries and semi-fixed assets. Then the equilibrium system becomes*

$$\begin{cases} \boldsymbol{w}_t = & \Phi\boldsymbol{\ell}_t + \beta^T\boldsymbol{p}_t + \Gamma\left(\boldsymbol{\mathcal{C}} - \mathbf{1}\frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}}\right) \\ \boldsymbol{p}_t = & \mathcal{P}_{:L}\boldsymbol{w}_t + \left(I - \mathcal{P}\Psi^{-1}\right)\left(\boldsymbol{p}_{t-1} + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) \\ \boldsymbol{\ell}_t = & \mathbf{1}y_t - \mathcal{D}_p\boldsymbol{p}_t + \boldsymbol{\mathcal{D}}_{Gt} + \bar{s}_{\mathcal{H}}\mathcal{D}_\ell Cov_{\boldsymbol{s}_{\mathcal{H}}}\left(\Psi_{:\mathcal{F}}, \boldsymbol{\mathcal{C}}\right) \\ \boldsymbol{c}_{\mathcal{H},t+1} = & \boldsymbol{c}_{\mathcal{H},t} + \frac{1}{\gamma}\left(\mathbf{1}i_t - \Omega_{\mathcal{H}:}\boldsymbol{\pi}_{t+1}\right), \end{cases} \tag{30}$$

where $\boldsymbol{\mathcal{C}} \equiv \begin{pmatrix} \boldsymbol{\mathcal{C}}_{\mathcal{H}} & \mathbf{0}_{\mathcal{U}\cup G} \end{pmatrix}^T$ *is equal to the present discounted value of changes in each households' income relative to the average household:*

$$\boldsymbol{\mathcal{C}} = (1-\rho)\sum_{t=0}^{\infty}\rho^t\left[\left(\Xi_{\mathcal{HF}} - \mathbf{1}\bar{\boldsymbol{\Xi}}_{\mathcal{HF}}\right)\boldsymbol{\ell}_t + \frac{1}{\gamma}\left[\gamma\left(\Xi_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\right)\Psi^{-1} + (1-\gamma)\left(\Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H},:}\right)\right]\boldsymbol{p}_t\right].$$

Proof. See Appendix *A*. □

The system (30) features the same growth rates of consumption as the full-insurance equilibrium (18), but with a zero-sum level shift $\boldsymbol{\mathcal{C}}$ across households. This level shift reflects changes in the present discounted value of income across households, depending on the incidence of spending on the demand for production factors that they own. This adjustment in relative consumption levels causes additional wealth effects in the factor supply equation, and a reallocation of demand toward factors uses more intensively by the households whose income increases relative to the average, as described by the relative demand equation.

5 Quantitative and Empirical Results

This section illustrates the quantitative and empirical predictions of the model. Section 5.1 presents the data used to calibrate the model and test the theory predictions. Section 5.2 describes how the composition of government spending has changed over time. Section 5.3 then estimates the empirical relationship between aggregate output multipliers and the industry composition of government spending. It finds that the multiplier is increasing in the labor share and degree of nominal rigidity of the spending bundle relative to private consumption, as predicted by the model. Finally, sections 5.4 and 5.5 calibrate the model to the US economy, showing that the industry composition of government spending has a quantitatively large effect on aggregate multipliers, and government spending has significantly heterogeneous incidences on labor demand and total income across households.

5.1 Data

Table 4 displays the data sources used for the empirical analysis.

Government expenditure shares of intermediate inputs	BEA input-output tables
Government expenditure shares on labor occupations	BEA IO tables, American Community Survey
Government expenditure shares on semi-fixed assets[14]	NIPA tables

Table 4: Government expenditure data over time.

[14] I follow the methodology in Vom Lehn and Winberry (2021) to construct expenditure shares on semi-fixed assets and expenditure shares in asset utilization.

Tables 5 and 6 display the data sources used to measure income flows in the calibrated model, and the calibrated values of elasticities.[15]

Expenditure shares on intermediate inputs	BEA input-output tables
Expenditure shares on labor occupations	BEA IO tables, American Community Survey
Expenditure shares on semi-fixed assets[16]	NIPA tables
Expenditure shares in asset utilization bundles	Capital Flow Tables
Expenditure shares in consumption	BEA IO tables, Consumer Expenditure Survey
Ownership shares in industries and semi-fixed assets[17]	American Community Survey

Table 5: Calibration of income flows.

Labor supply elasticity	$\varphi_o = 2 \ \forall o \in \mathcal{L}$
Supply elasticity of semi-fixed assets[18]	$\varphi_K = XXX$
Wealth effects in labor supply, intertermporal elasticity of substitution	$\gamma = 1$
Substitution elasticities[19]	Consumption: 0.9 Intermediates vs labor: 0.5 Intermediates vs semi-fixed assets: 0.5 Labor vs semi-fixed assets: 0.5 Across intermediate inputs: 0.1 Across labor types: 0.1 Across asset types: 0.1
Price adjustment frequencies	Pasten et al. (2019)

Table 6: Calibration of model elasticities.

5.2 Industry Composition of Government Spending over Time

Total government spending consists in good purchases and expenditure on labor and semi-fixed assets. Figure 3 plots the industry composition of the government's good purchases over time. Figures 4 and 5 instead plot the government's expenditure share on labor occupations and semi-fixed assets, as a fraction of total government spending. Given changes in the composition of government and private expenditures over time, figures 6 and 7 plot the average price adjustment frequency and the average labor share of government spending relative to private consumption.

[15] The calibration is described in detail in Appendix XXX.

[16] I follow the methodology in Vom Lehn and Winberry (2021) to construct expenditure shares on semi-fixed assets and expenditure shares in asset utilization.

[17] In the calibrated model I assume that households have the same ownership share in all industries and semi-fixed assets. Formally, $\hat{\Xi}_{hi} = \hat{\Xi}_{jh} \ \forall i, j \in \mathcal{K} \cup \mathcal{N}$.

[18] I calibrate the supply elasticity of fixed assets relative to labor in order to replicate the relative impulse response of high- and low-labor share industries to identified monetary shocks. Additional detail is reported in Appendix XXX.

[19] The calibration of substitution elasticities follows Atalay (2017)

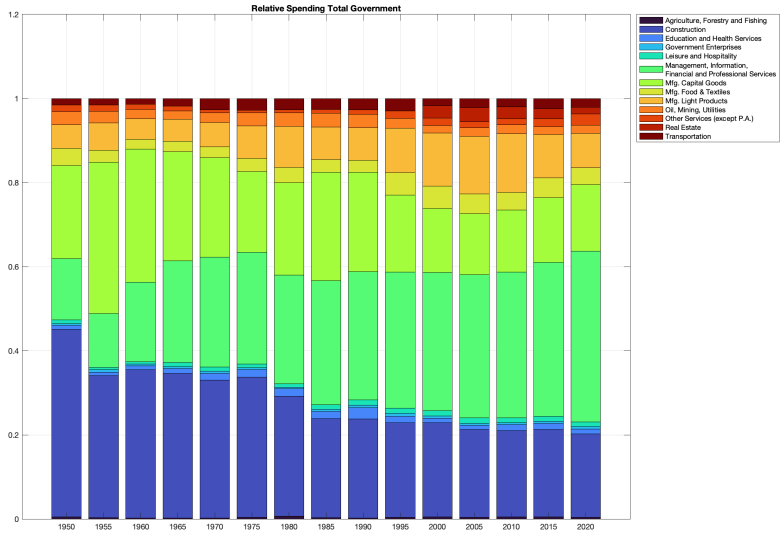


Figure 3: Spending composition over time

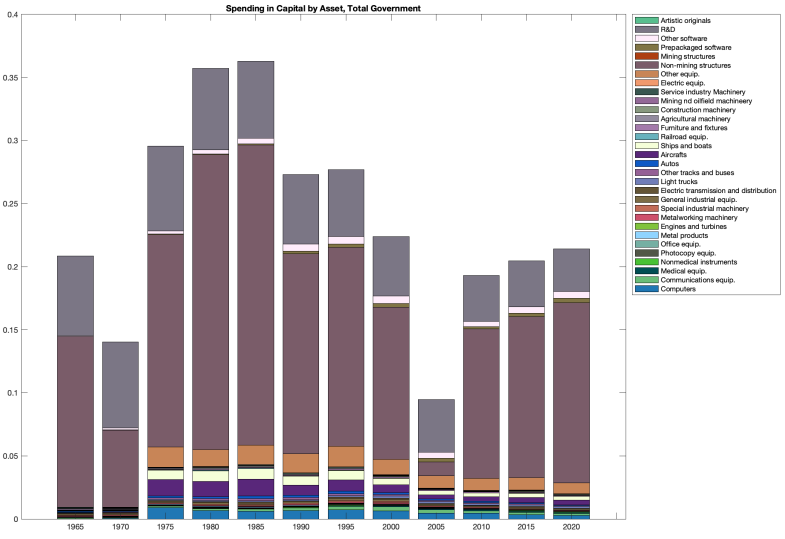


Figure 4: Spending composition over time (use of semi-fixed assets)

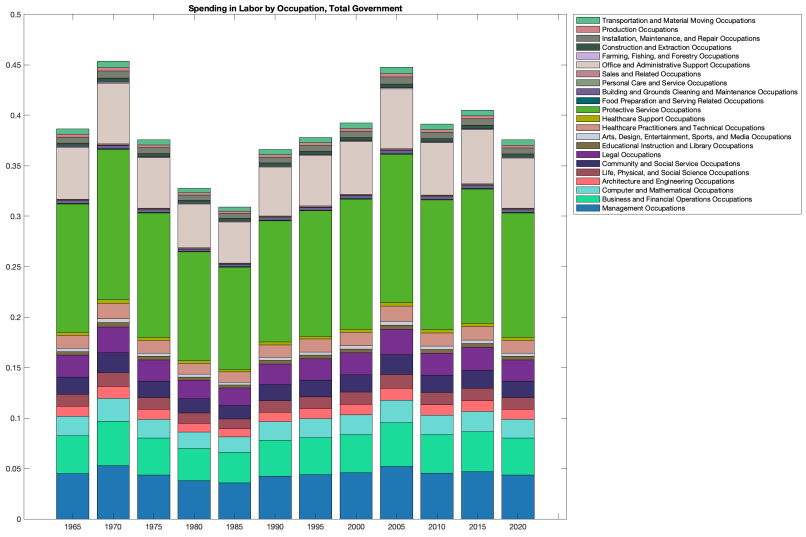


Figure 5: Spending composition over time (labor employment)

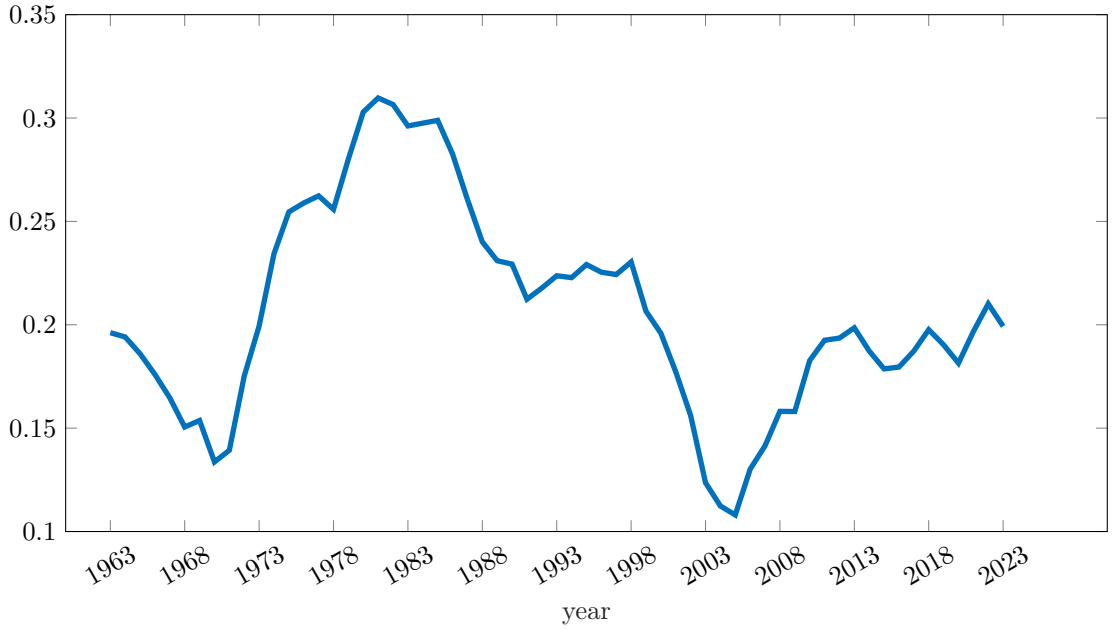


Figure 6: Average price adjustment frequency of government bundle relative to private consumption.

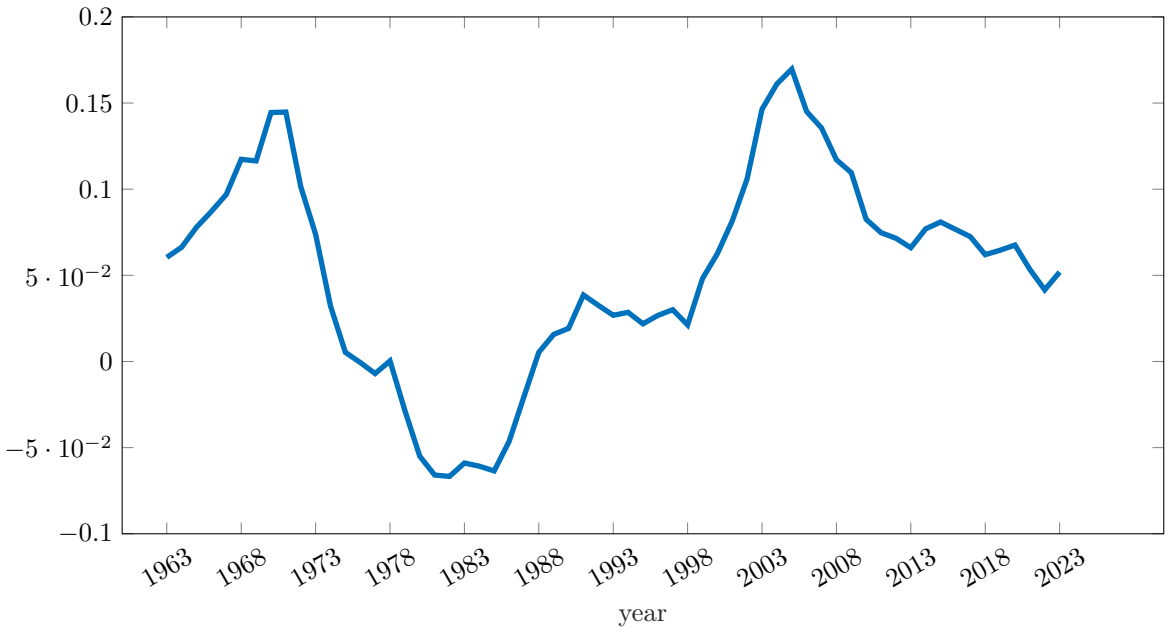


Figure 7: Labor share of government spending relative to private consumption.

5.3 Estimation: Local Projections

As figures 6 and 7 illustrate, there is variation over time in the degree of nominal rigidity and labor intensity of government spending relative to private consumption, even though these two variables do not exhibit a time trend. Figures 8 and 9 estimate the relation between these variables and aggregate spending multipliers.

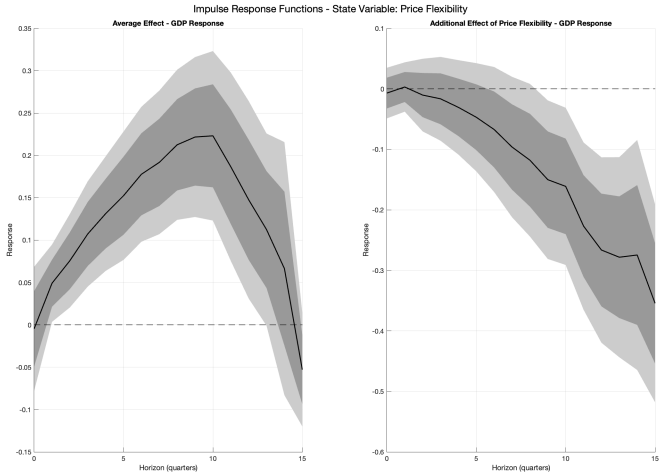


Figure 8: Impulse responses of output to governement spending

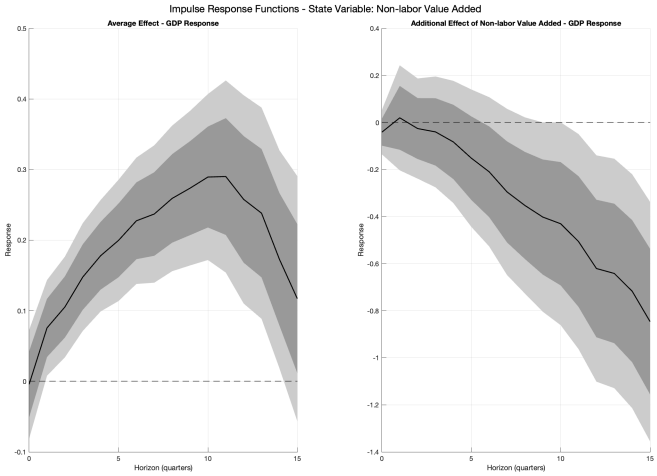


Figure 9: Impulse responses of output to governement spending

As the figures illustrate, there is a significant negative relationship between the aggregate output multiplier and both price flexibility and non-labor share in value added of the government bundle, relative to private consumption. Hence the empirical results are consistent with model predictions.

5.4 Calibration: Aggregate Multipliers

Figures 10 through 14 illustrate aggregate output, employment and consumption multipliers in the calibrated model.

Following our discussion in sections 3 and 4, each bar in figure 10 plots the aggregate supply shifter which results from concentrating government spending entirely on the corresponding industry. Aggregate supply shifts are correlated with output multipliers, plotted in the blue bars of figure 11. The red and yellow bars in figure 11 plot employment and consumption multipliers.

Figures 12, 13 and 14 instead study how spending multipliers changed over time, depending on the industry composition of government spending relative to private consumption. Figure 12 compares output and employment multipliers over time. Figure 13 then shows that model-implied variation in the output multiplier is almost entirely explained by changes in the price stickiness of government spending relative to private consumption. Figure 14 illustrates that the difference between model-implied output and employment multipliers is almost entirely explained by variation in the labor intensity of government spending relative to private consumption.

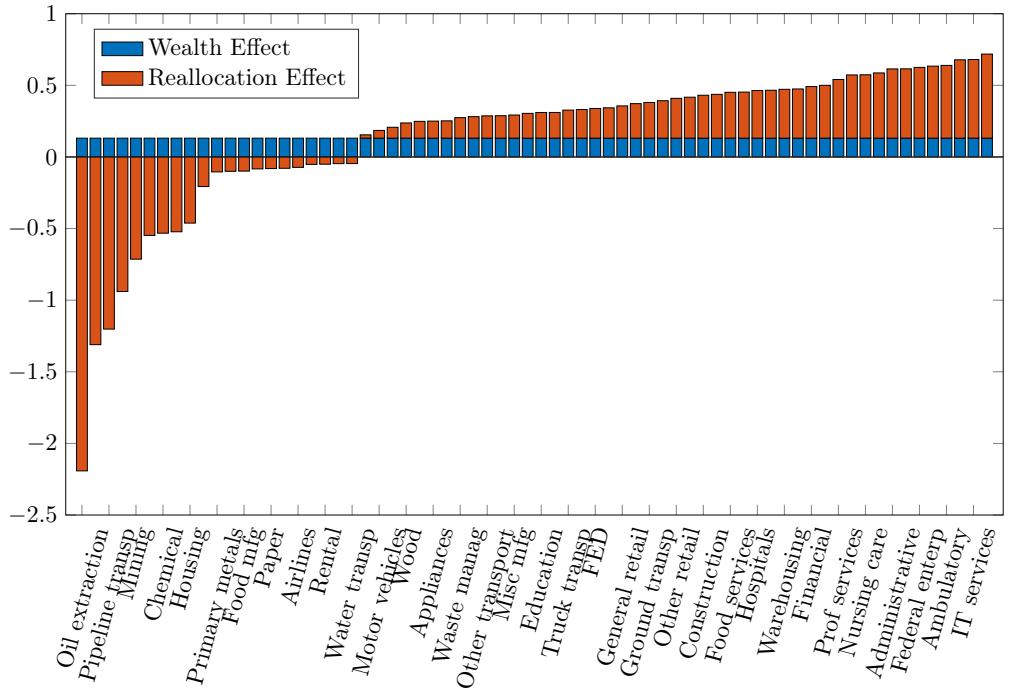


Figure 10: Aggregate supply shifters according to the industry where spending is targeted.

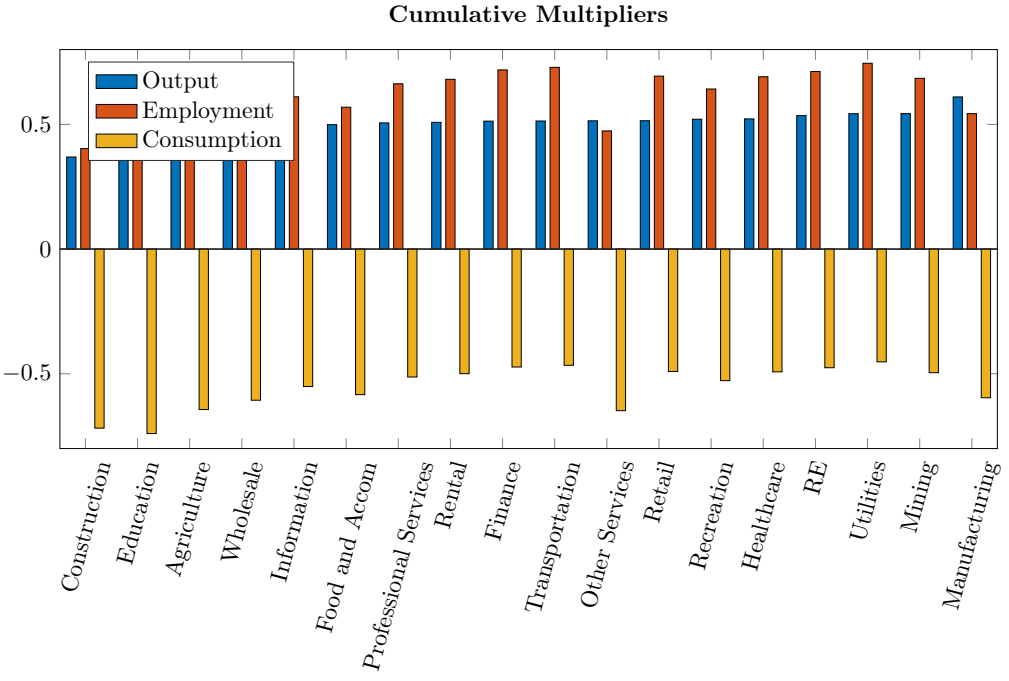


Figure 11: Aggregate output, employment, and consumption multiplier, by industry where spending is targeted.

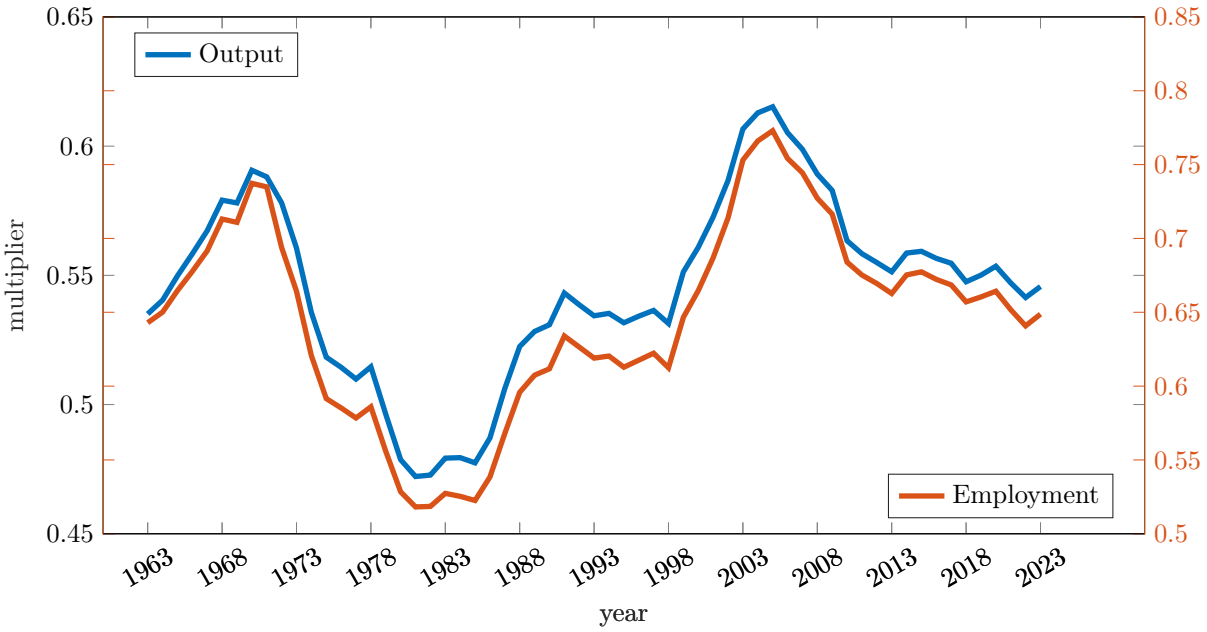


Figure 12: Output and Employment Multipliers Over Time.

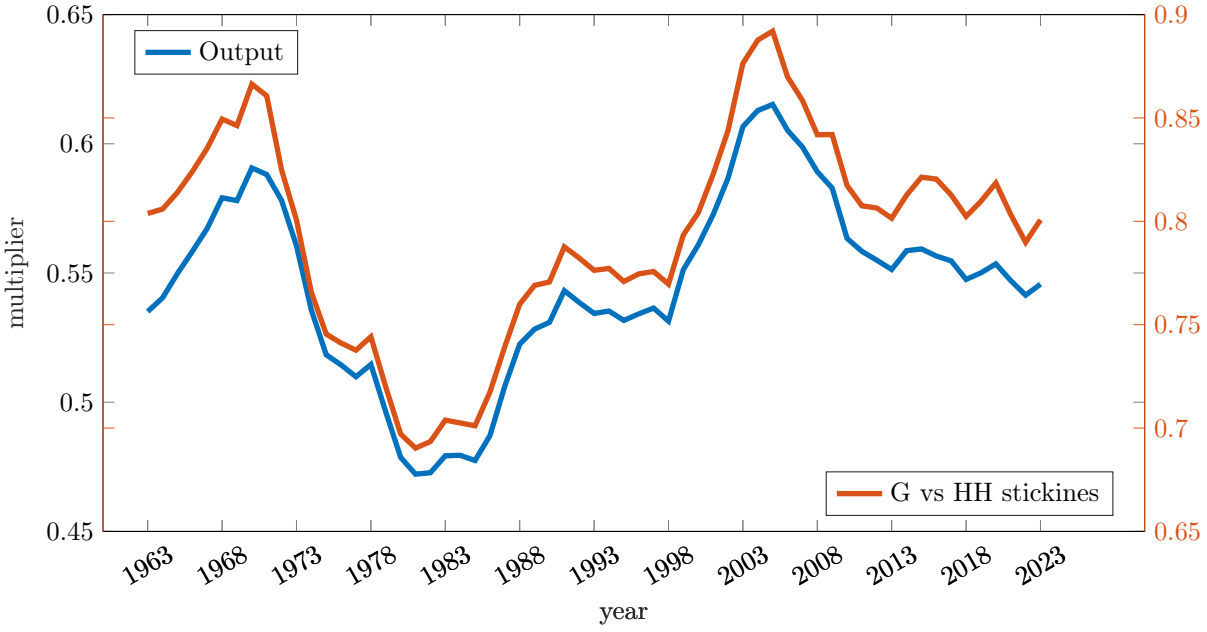


Figure 13: Aggregate output multiplier and average price stickiness of government spending relative to private consumption

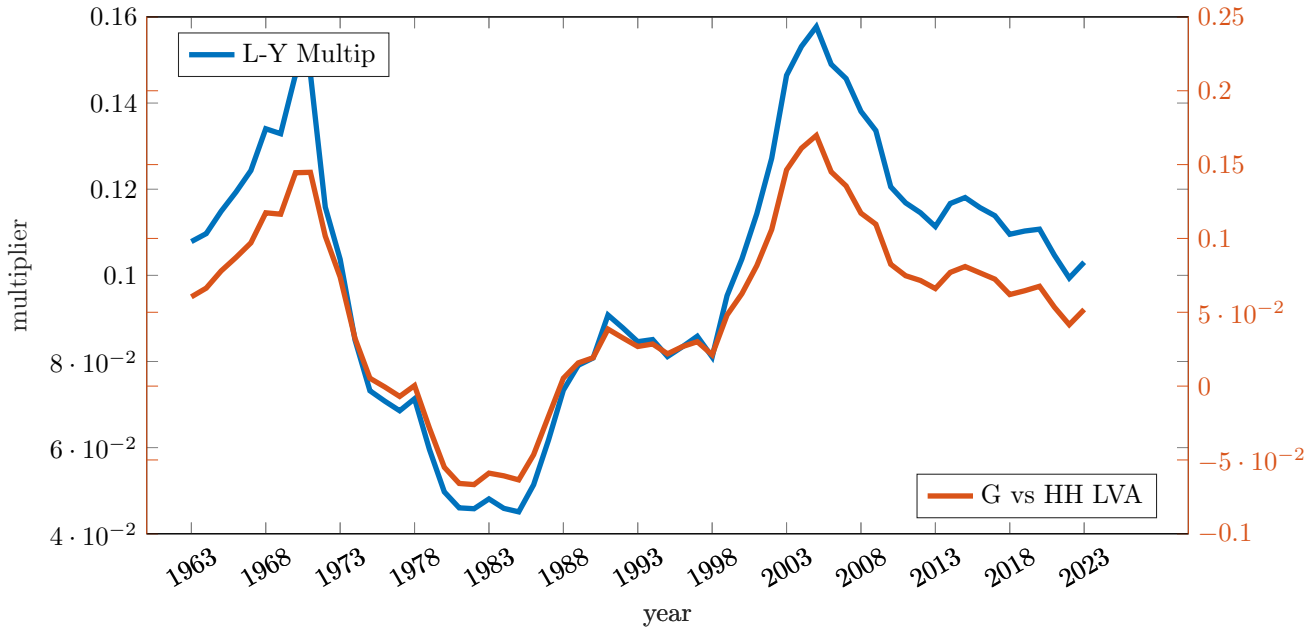


Figure 14: Difference between employment and output multiplier, and average labor share of government spending relative to private consumption

5.5 Calibration: Redistributive Effects

Figures 15 through 17 illustrate the redistributive implications of government spending, through the lens of the calibrated model.

Figure 15 plots the incidence of government spending across income quintiles, given its industry composition in 2018 and assuming that households cannot insure against spending shocks (as in the model of section 4.4). As the figure shows, government spending benefits households in the first four quintiles the most, by increasing their labor income. Households in the top quintile benefit from higher returns to semi-fixed assets, however this does not fully compensate for a smaller increase in labor income. Next, figure 16 plots the effect of spending on labor income across occupations. Darker bars correspond to lower-income occupations. Comparing figures 15 and 16 reveals that focusing on income quintiles masks significant underlying heterogeneity in labor income responses across occupations.

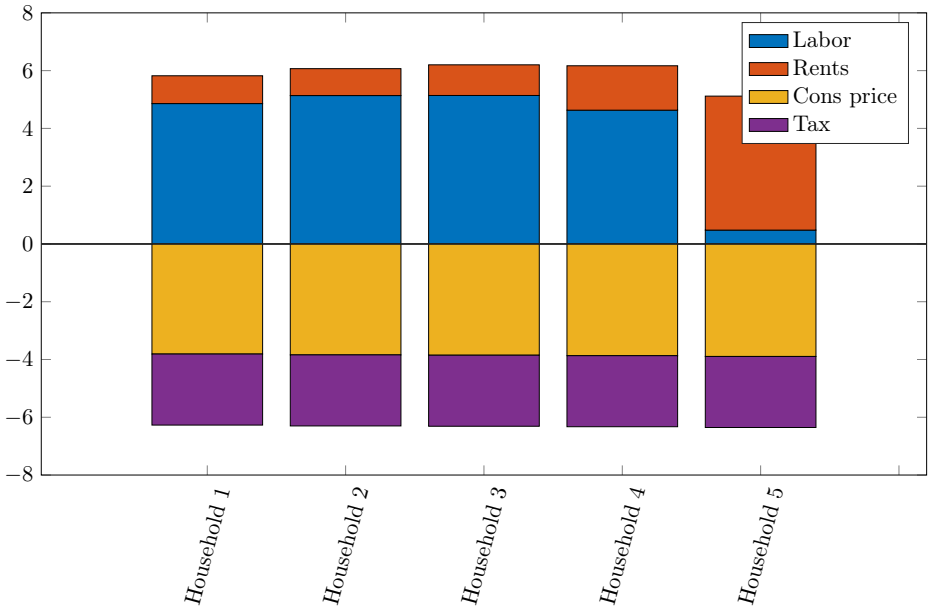


Figure 15:

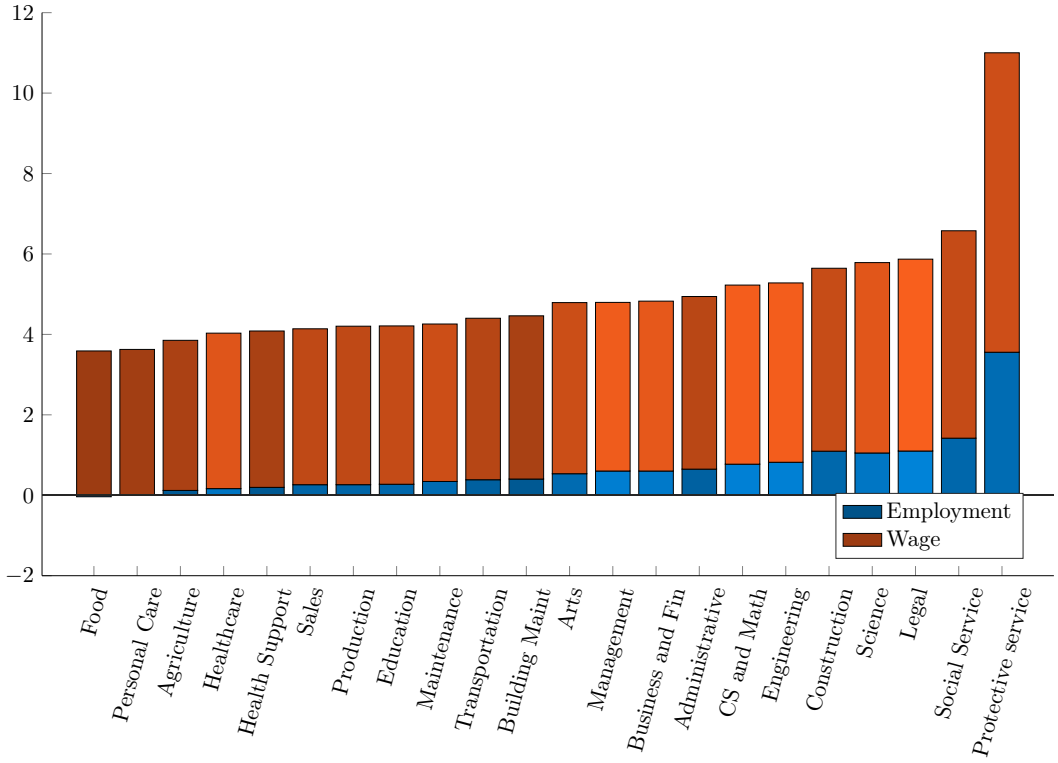


Figure 16:

Finally, figure 17 plots industry-by-occupation employment multipliers, corresponding to the response of em-

ployment across occupations (on the columns) to a 1% increase in spending, entirely concentrated in one given industry (on the rows).

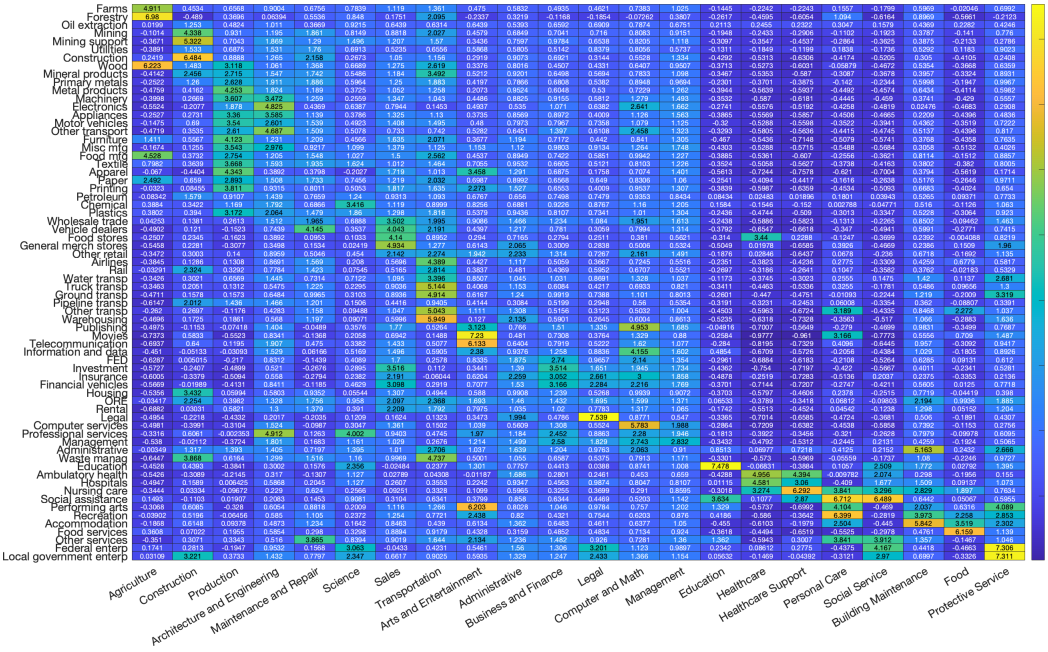


Figure 17: Industry-by-occupation incidence of spending

6 Conclusion

This paper studies how the industry composition of government spending shapes its effect on aggregate variables (output, employment, and consumption), and its redistributive incidence across households.

Building on a model economy with multiple industries and primary factors, I establish that spending increases aggregate output by more when it is targeted toward industries with flatter supply curves, meaning that they face more elastic factor supply, or lower network-adjusted price reset probabilities. Moreover, spending raises aggregate employment relative to aggregate output whenever it is targeted toward labor intensive industries. Hence there is no tradeoff between maximizing output and maximizing employment whenever labor is more elastically supplied than other production factors, and labor intensive industries adjust prices less frequently.

I then turn to the redistributive implications of government spending across households, and decompose them into a consumption price incidence and a factor income incidence. Spending direclty affects relative factor income, whenever the spending bundle has a different factor content than the private bundle. The direct effect is counterbalanced by a cross-sectional crowding out effect, whereby the relative price of goods consumed more intensively by the government increases, and hence private demand declines.

Quantitatively, the calibrated model implies a wide range of output multipliers depending on the industry

composition of the spending bundle. There is variation over time in the average price adjustment frequency and labor share of the spending bundle relative to private consumption, although there is no clear time trend. Exploiting this time variation, I estimate a positive and significant relationship between the aggregate output multiplier and both price stickiness and labor share of the spending bundle relative to private consumption.

Given its industry composition in 2018, government spending is mildly progressive according to the calibrated model, because it increases labor income by more for households in lower quintiles. Households in the top quintile also gain from higher asset returns, but this does not fully offset the smaller labor income gains.

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Appendix

A: Derivations

Proof of Lemma 1

Under full insurance, the households' consumption expenditure is given by

$$\boldsymbol{p}_{\mathcal{H}} + \boldsymbol{c}_{\mathcal{H}} = \mathbf{1}\left(\bar{c}_{\mathcal{H}} + \bar{p}_{\mathcal{H}}\right) - \left(\Gamma^{-1} - I\right)\left(\Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}}\right)\boldsymbol{p}.$$

Therefore we have

$$\Gamma \boldsymbol{c}_{\mathcal{H}} = \Gamma \mathbf{1}\bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\boldsymbol{\ell} - \left[\Omega_{\mathcal{H}:} - (I - \Gamma)\,\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}} - \Gamma \mathbf{1}\bar{\Xi}_{\mathcal{H}:}\right]\boldsymbol{p} - \Gamma \frac{ds_G}{\bar{s}_{\mathcal{H}}}. \tag{31}$$

Plugging into the log-linearized consumption-leisure tradeoff,

$$\Gamma \boldsymbol{c} + diag\left(\boldsymbol{\varphi}\right)\boldsymbol{\ell} = \boldsymbol{w} - \Omega_{C:}\boldsymbol{p},$$

yields the factor supply equation.

To derive the relative demand equation, start by approximating the accounting relation

$$\bar{\mathbf{\Psi}}_{\mathcal{F}} = \Psi_{\mathcal{Z}\mathcal{F}}^T \mathbf{s}$$

to obtain

$$d\bar{\mathbf{\Psi}}_{\mathcal{F}} = d\left(\Psi_{\mathcal{Z}\mathcal{F}}^T\right)\mathbf{s} + \Psi_{\mathcal{Z}\mathcal{F}}^T d\boldsymbol{s}.$$

Next denote by $\boldsymbol{\mu}$ the vector of log-changes in industry-level markups, and use the definition of $\Psi \equiv (I-\Omega)^{-1}$ to solve

$$d\left(\Psi_{\mathcal{Z}\mathcal{F}}^T\right)\mathbf{s} = \Psi_{:\mathcal{F}}^T d\Omega^T|_{Sub}\ \bar{\mathbf{\Psi}} - \Psi_{:\mathcal{F}}^T \mathcal{T}\boldsymbol{\mu} + s_G Cov_{\mathbf{\Omega}_G}\left(\Psi_{:\mathcal{F}}^T, d\log\mathbf{\Omega}_G\right),$$

where

$$\Psi_{:\mathcal{F}}^T d\Omega^T|_{Sub}\ \bar{\mathbf{\Psi}} \equiv \sum_{i,j,k} \bar{\Psi}_i \omega_{ij}\omega_{ik}\left(1-\theta_{jk}^i\right)\left(\Psi_{:\mathcal{F}}\right)_{j,:}(p_j - p_k) =$$

$$= \sum_{i,j,k} \bar{\Psi}_i \omega_{ij}\omega_{ik}\left(\Psi_{:\mathcal{F}}\right)_{j,:}(p_j - p_k) - \Theta\left(\Psi_{:\mathcal{F}}, \mathbf{p}\right) =$$

$$= \left[\Psi_{:\mathcal{F}}^T\left(I - \bar{\mathbf{\Omega}}_{\mathcal{Z}}\mathbf{1}^T\right)\mathcal{T}\Psi^{-1} - Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}, \Omega_{\mathcal{Z}:}\right) - \Theta\left(\Psi_{:\mathcal{F}}, I\right)\right]\mathbf{p}$$

and we defined

$$\left[\Theta\left(X,Y\right)\right]_{fi} \equiv \sum_j \bar{\mathbf{\Psi}}_j \sum_{nq} \Omega_{jn}\Omega_{jq}\theta_{nq}^j\left(X_{fn} - X_{fq}\right)\left(Y_{nj} - Y_{qj}\right).$$

Solving for markups as a function of prices and wages,

$$\boldsymbol{\mu} = \Psi^{-1}\boldsymbol{p} - \Omega_{:\mathcal{F}}\boldsymbol{w},$$

yields

$$d\left(\Psi_{\mathcal{Z}\mathcal{F}}^T\right)\mathbf{s} = -\left[\bar{\mathbf{\Psi}}_{\mathcal{F}}\bar{\mathbf{\Omega}}_{\mathcal{Z}} + Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}, \Omega_{\mathcal{Z}:}\right) + \Theta\left(\Psi_{:\mathcal{F}}, I\right)\right]\mathbf{p} - \mathcal{T}_{\mathcal{F}}\boldsymbol{w} + s_G Cov_{\mathbf{\Omega}_G}\left(\Psi_{:\mathcal{F}}^T, d\log\mathbf{\Omega}_G\right)$$

and hence

$$d\bar{\mathbf{\Psi}}_{\mathcal{F}} = \mathcal{T}_{\mathcal{F}}\left(\boldsymbol{w} + \boldsymbol{\ell} - \mathbf{1}y\right) - \bar{\mathbf{\Psi}}_{\mathcal{F}}\bar{\mathbf{\Omega}}_{\mathcal{Z}}\boldsymbol{p} = -\left[\bar{\mathbf{\Psi}}_{\mathcal{F}}\bar{\mathbf{\Omega}}_{\mathcal{Z}} + Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}, \Omega_{\mathcal{Z}:}\right) + \Theta\left(\Psi_{:\mathcal{F}}, I\right)\right]\mathbf{p} - \mathcal{T}_{\mathcal{F}}\boldsymbol{w} + \Psi_{\mathcal{Z}\mathcal{F}}^T d\boldsymbol{s} + s_G Cov_{\mathbf{\Omega}_G}\left(\Psi_{:\mathcal{F}}^T, d\log\mathbf{\Omega}_G\right)$$

so that

$$\mathcal{T}_{\mathcal{F}}\left(\boldsymbol{\ell} - \mathbf{1}y\right) = -\left[Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}, \Omega_{\mathcal{Z}:}\right) + \Theta\left(\Psi_{:\mathcal{F}}, I\right)\right]\mathbf{p} + \Psi_{\mathcal{Z}\mathcal{F}}^T d\boldsymbol{s} + s_G Cov_{\mathbf{\Omega}_G}\left(\Psi_{:\mathcal{F}}^T, d\log\mathbf{\Omega}_G\right)$$

Moreover we have

$$\Psi_{\mathcal{Z}\mathcal{F}}^T d\boldsymbol{s} = Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}^T, d\log \boldsymbol{s}\right)$$

and

$$d\log \boldsymbol{s} = \Xi\left(\boldsymbol{\ell} - \mathbf{1}y\right) + \left[\Xi\Psi^{-1} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}} - \left(\Gamma^{-1} - I\right)\begin{pmatrix} \Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:} \\ \mathbb{O} \end{pmatrix}\right]\boldsymbol{p} - \begin{pmatrix} \mathbf{1}_{\mathcal{H}} \\ \mathbf{0}_{\mathcal{U}} \end{pmatrix}\frac{ds_G}{\bar{s}_{\mathcal{H}}}$$

which yields

$$\boldsymbol{\ell} - \mathbf{1}y = -\mathcal{D}_p\boldsymbol{p} + \boldsymbol{\mathcal{D}}_G,$$

where

$$\begin{aligned}
\mathcal{D}_p &\equiv \mathcal{D}_\ell\left[\Theta(\Psi_{:L}^T, I) + Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}^T, \begin{pmatrix} \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:} \\ \Omega_{\mathcal{U},:} \end{pmatrix} - \Xi\Psi^{-1} + \Gamma^{-1}\begin{pmatrix} \Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:} \\ \mathbb{O} \end{pmatrix}\right)\right] \\
\mathcal{D}_\ell &\equiv \mathcal{T}_{\mathcal{F}}^{-1}\left(I - Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}^T, \Xi_{:\mathcal{F}}\mathcal{T}_{\mathcal{F}}^{-1}\right)\right).
\end{aligned}$$

Proof of Lemma 2 Combining the factor supply and relative demand equations in (18) yields

$$\boldsymbol{w}_t = \Phi\left[\mathbf{1}y_t + \boldsymbol{\mathcal{D}}_{Gt} - \Phi^{-1}\Gamma\mathbf{1}\frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}}\right] + \left(\beta^T - \Phi\mathcal{D}_p\right)\boldsymbol{p}_t.$$

Plugging into the pricing equation then gives

$$\left(I - \kappa\Phi^{-1}\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}\right)\boldsymbol{p} = \kappa\left[\mathbf{1}y + \boldsymbol{\mathcal{D}}_G - \Phi^{-1}\Gamma\mathbf{1}\frac{ds_G}{\bar{s}_{\mathcal{H}}}\right] + (I - \mathcal{V})\left(\boldsymbol{p}_{-1} - \rho\mathbb{E}\boldsymbol{\pi}_{+1}\right) \tag{32}$$

where

$$\begin{aligned}
\kappa &\equiv \left(I + \mathcal{P}_{:L}\left(\Phi\mathcal{D}_p - \left(\beta^T - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi \\
\mathcal{V} &\equiv \left(I + \mathcal{P}_{:L}\left(\Phi\mathcal{D}_p - \left(\beta^T - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}\right)\right)\right)^{-1}\mathcal{P}\Psi^{-1}\left[I - \Psi_{:L}\left(\left(\beta^T - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}\right) - \Phi\mathcal{D}_p\right)\right].
\end{aligned}$$

Equation (19) is obtained from (32) noting that $\boldsymbol{p} = \boldsymbol{\pi} + \boldsymbol{p}_{-1}$.

Proof of Proposition 1 Equation (21) implies

$$\begin{aligned}
\frac{\partial y}{\partial s_G} &= \frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}\mathbf{1}}\left[\frac{\partial \boldsymbol{\mathcal{D}}_G}{\partial s_G} - \frac{\Phi^{-1}\Gamma\mathbf{1}}{\bar{\boldsymbol{s}}_{\mathcal{H}}}\right] \\
\frac{\partial y}{\partial \bar{\boldsymbol{\Omega}}_G} &= \frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}\mathbf{1}}\frac{\partial \boldsymbol{\mathcal{D}}_G}{\partial \bar{\boldsymbol{\Omega}}_G}.
\end{aligned}$$

Equation (22) follows from the equalities $\bar{\boldsymbol{\kappa}}\mathbf{1} = \mathbb{E}_{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}}\left[\frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}}\right]$, $\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\boldsymbol{\mathcal{D}}_G = \mathbf{0}$, and $\bar{\boldsymbol{\kappa}}\frac{\partial \boldsymbol{\mathcal{D}}_G}{\partial X} = Cov_{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}}\left(\frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}}, \frac{\partial \boldsymbol{\mathcal{D}}_G}{\partial X}\right)$ for $X = s_G, \bar{\boldsymbol{\Omega}}_G$.

Proof of Lemma 3 The households' Euler equations in (18), together with equation (31), imply

$$\bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\boldsymbol{\ell}_{t+1} = \bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\boldsymbol{\ell}_t + \frac{1}{\gamma}\left[i_t - \left[(1-\gamma)\,\bar{\boldsymbol{\Omega}}_{\mathcal{H}} + \gamma\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\right]\mathbb{E}\boldsymbol{\pi}_{t+1}\right] + \frac{ds_{Gt+1} - ds_{Gt}}{\bar{s}_{\mathcal{H}}}.$$

Combining with the relative demand equation then yields

$$\begin{aligned} y_{t+1} &= y_t + \frac{1}{\gamma}\left[i_t - \hat{\boldsymbol{\beta}}^T\mathbb{E}\boldsymbol{\pi}_{t+1}\right] + \frac{ds_{Gt+1} - ds_{Gt}}{\bar{s}_{\mathcal{H}}} - \bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\left(\boldsymbol{\mathcal{D}}_{Gt+1} - \boldsymbol{\mathcal{D}}_{Gt}\right) \\ \hat{\boldsymbol{\beta}}^T &\equiv (1-\gamma)\,\bar{\boldsymbol{\Omega}}_{\mathcal{H}} + \gamma\left(\bar{\boldsymbol{\Xi}}_{\mathcal{H}:} - \bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\mathcal{D}_p\right). \end{aligned}$$

Proof of Proposition 2 The relative demand equation in (18) implies

$$\boldsymbol{\ell} - \mathbf{1}\bar{\ell} = \left(I - \frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^T\mathbf{1}}\right)(\boldsymbol{\ell} - \mathbf{1}y) = -\mathcal{D}_p^{\mathcal{L}}\boldsymbol{p} + \boldsymbol{\mathcal{D}}_G^{\mathcal{L}}. \tag{33}$$

Combining equation (33) with the consumption-leisure tradeoff and the pricing equation, as in the proof of Lemma **??**, yields

$$\begin{aligned} \left(1 - \kappa^{\mathcal{L}}\Phi^{-1}\mathbf{1}\right)\boldsymbol{p}_t &= \kappa^{\mathcal{L}}\left[\mathbf{1}\bar{\ell}_t - \Phi^{-1}\Gamma\mathbf{1}\frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}} + \boldsymbol{\mathcal{D}}_{Gt}^{\mathcal{L}}\right] + \left(I - \mathcal{V}^{\mathcal{L}}\right)\left(\boldsymbol{p}_{t-1} + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) \\ \mathcal{V}^{\mathcal{L}} &\equiv \left(I + \mathcal{P}_{:L}\left(\Phi\mathcal{D}_p^{\mathcal{L}} - \left(\beta^T - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}\right)\right)\right)^{-1}\mathcal{P}\Psi^{-1}\left[I - \Psi_{:L}\left(\left(\beta^T - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}\right) - \Phi\mathcal{D}_p^{\mathcal{L}}\right)\right]. \end{aligned}$$

Let's now solve for aggregate demand. Aggregating the households' Euler equations yields

$$\bar{c}_{\mathcal{H},t+1} = \bar{c}_{\mathcal{H},t} + \frac{1}{\gamma}\left(i_t - \bar{\boldsymbol{\Omega}}_{\mathcal{H}:}\boldsymbol{\pi}_{t+1}\right),$$

while the households' budget constraints imply

$$\bar{c}_{\mathcal{H}t} = \bar{\ell} + \bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\boldsymbol{\mathcal{D}}_{Gt}^{\mathcal{L}} + \left(\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\Psi^{-1} - \bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\mathcal{D}_p^{\mathcal{L}} - \bar{\boldsymbol{\Omega}}_{\mathcal{H}:}\right)\boldsymbol{p} - \frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}}.$$

Therefore we have

$$\begin{aligned} \bar{\ell}_{t+1} &= \bar{\ell}_t + \frac{1}{\gamma}\left(\mathbf{1}i_t - \bar{\boldsymbol{\beta}}_{\mathcal{L}}^T\boldsymbol{\pi}_{t+1}\right) + \frac{ds_{Gt+1} - ds_{Gt}}{\bar{s}_{\mathcal{H}}} - \bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\left(\boldsymbol{\mathcal{D}}_{Gt+1}^{\mathcal{L}} - \boldsymbol{\mathcal{D}}_{Gt}^{\mathcal{L}}\right) \\ \beta_{\mathcal{L}}^T &\equiv (1-\gamma)\,\Omega_{\mathcal{H}:} + \gamma\left(\Xi_{\mathcal{H}:}\Psi^{-1} - \Xi_{\mathcal{H}\mathcal{F}}\mathcal{D}_p^{\mathcal{L}}\right). \end{aligned}$$

The reallocation effect on aggregate employment is larger than the reallocation effect on aggregate output whenever

$$\begin{aligned}
&\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)\boldsymbol{\mathcal{D}}_{Gt}\\
&<\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\boldsymbol{\mathcal{D}}_{Gt}\iff
\end{aligned}$$

$$\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\left[\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)-\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\right]$$

$$\begin{aligned}
&\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\left[\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)-\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\right.\\
&\left.+\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)-\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\mathcal{D}_{p}-\left(\beta^{T}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:L}\Phi\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\right]\boldsymbol{\mathcal{D}}
\end{aligned}$$

$$\bar{\boldsymbol{\kappa}}^{\mathcal{L}}\mathbf{1}\left(\frac{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}-\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\left(I-\mathcal{D}_{p}\kappa\right)\boldsymbol{\mathcal{D}}_{Gt}>0\iff\begin{pmatrix}\frac{1-\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T} & -\bar{\boldsymbol{\Psi}}_{\mathcal{K}}^{T}\end{pmatrix}\left(I-\mathcal{D}_{p}\kappa\right)\boldsymbol{\mathcal{D}}_{Gt}>0$$

Similarly, the aggregate demand shift in terms of employment is larger than in terms of output whenever

$$\begin{aligned}
&\bar{\boldsymbol{\Xi}}_{\mathcal{HF}}\left(I-\frac{\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}\right)\left(\boldsymbol{\mathcal{D}}_{Gt+1}-\boldsymbol{\mathcal{D}}_{Gt}\right)<\bar{\boldsymbol{\Xi}}_{\mathcal{HF}}\left(I-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\left(\boldsymbol{\mathcal{D}}_{Gt+1}-\boldsymbol{\mathcal{D}}_{Gt}\right)\iff\\
&\left(\frac{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}}{\bar{\boldsymbol{\Psi}}_{\mathcal{L}}^{T}\mathbf{1}}-\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\right)\left(\boldsymbol{\mathcal{D}}_{Gt+1}-\boldsymbol{\mathcal{D}}_{Gt}\right)>0.
\end{aligned}$$

Proof of Proposition 3 The relative demand equation in (18) implies

$$\begin{aligned}
\boldsymbol{\ell}-\mathbf{1}\bar{c}&=\left(I-\mathbf{1}\bar{\boldsymbol{\Xi}}_{\mathcal{HF}}\right)\left(\boldsymbol{\ell}-\mathbf{1}y\right)+\mathbf{1}\left[\bar{\boldsymbol{\Omega}}_{\mathcal{H}}-\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\Psi^{-1}\right]\boldsymbol{p}+\mathbf{1}\frac{ds_{G}}{\bar{s}_{\mathcal{H}}}\\
&=-\mathcal{D}_{p}^{\mathcal{H}}\boldsymbol{p}+\boldsymbol{\mathcal{D}}_{G}^{\mathcal{H}}+\mathbf{1}\left[\bar{\boldsymbol{\Omega}}_{\mathcal{H}}-\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\Psi^{-1}\right]\boldsymbol{p}+\mathbf{1}\frac{ds_{G}}{\bar{s}_{\mathcal{H}}},
\end{aligned}$$

where we used the aggregated household budget constraint

$$\bar{c}=\bar{\boldsymbol{\Xi}}_{\mathcal{HF}}\boldsymbol{\ell}+\left[\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\Psi^{-1}-\bar{\boldsymbol{\Omega}}_{\mathcal{H}}\right]\boldsymbol{p}-\frac{ds_{G}}{\bar{s}_{\mathcal{H}}}.$$

Plugging into the factor supply equation yields

$$\begin{aligned}
w&=\Phi\mathbf{1}\bar{c}+\Phi\left[\boldsymbol{\mathcal{D}}_{G}^{\mathcal{H}}+\left(I-\Phi^{-1}\Gamma\right)\mathbf{1}\frac{ds_{G}}{\bar{s}_{\mathcal{H}}}\right]+\left[\beta_{\mathcal{H}}^{T}-\Phi\mathcal{D}_{p}^{\mathcal{H}}\right]\boldsymbol{p}\\
\beta_{\mathcal{H}}^{T}&\equiv\beta^{T}+\Phi\mathbf{1}\left[\bar{\boldsymbol{\Omega}}_{\mathcal{H}}-\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\Psi^{-1}\right].
\end{aligned}$$

Further combining with the pricing equation yields the expression for the GDP deflator in proposition 3. Following a similar argument as for proposition 2, the reallocation effect on aggregate consumption is larger than on aggregate output whenever

$$\bar{\boldsymbol{\kappa}}^{\mathcal{H}}\boldsymbol{\mathcal{D}}_{G}^{\mathcal{H}}-\bar{\boldsymbol{\kappa}}\boldsymbol{\mathcal{D}}_{G}>0\iff$$

$$\bar{\mathbf{\Omega}}_{\mathcal{Z}:}\left[\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\mathbf{1}\bar{\mathbf{\Xi}}_{\mathcal{H}\mathcal{F}}\right)\mathcal{D}_p^{\mathcal{H}}-\left(\beta_{\mathcal{H}}^T-\mathbf{1}\bar{\mathbf{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:\mathcal{F}}\Phi\left(I-\mathbf{1}\bar{\mathbf{\Xi}}_{\mathcal{H}\mathcal{F}}\right)-\left(I+\mathcal{P}_{:L}\left(\Phi\left(I-\mathbf{1}\bar{\mathbf{\Psi}}_{\mathcal{F}}^T\right)\mathcal{D}_p-\left(\beta^T-\mathbf{1}\bar{\mathbf{\Omega}}_{\mathcal{Z}:}\right)\right)\right)^{-1}\mathcal{P}_{:\mathcal{F}}$$

$$\bar{\boldsymbol{\kappa}}^{\mathcal{H}}\mathbf{1}\left[\left(\bar{\mathbf{\Xi}}_{\mathcal{H}\mathcal{F}}-\bar{\mathbf{\Psi}}_{\mathcal{F}}^T\right)\left(I-\mathcal{D}_p\kappa\right)+\left(\bar{\mathbf{\Xi}}_{\mathcal{H}:}\Psi^{-1}-\bar{\mathbf{\Omega}}_{\mathcal{H}}\right)\kappa\right]\boldsymbol{\mathcal{D}}_{Gt}>0,$$

and profit maximization by utilization producers implies $\bar{\mathbf{\Xi}}_{\mathcal{H}\mathcal{F}}-\bar{\mathbf{\Psi}}_{\mathcal{F}}^T=-\frac{\bar{\mathbf{\Psi}}_{\mathcal{K}}^T}{1+\boldsymbol{\varphi}_K}$.

Proof of Proposition 4 Aggregating both sides of equation (32) according to final use shares $\bar{\mathbf{\Omega}}_{\mathcal{Z}}$ and solving for aggregate output yields

$$y=\frac{1-\boldsymbol{\kappa}^Y\Phi^{-1}\mathbf{1}}{\boldsymbol{\kappa}^Y\mathbf{1}}\bar{p}+\frac{\boldsymbol{\kappa}^Y}{\boldsymbol{\kappa}^Y\mathbf{1}}\left[\Phi^{-1}\boldsymbol{\mathcal{S}}_G ds_G-\boldsymbol{\mathcal{D}}_G\right]-\frac{\bar{\mathbf{\Omega}}_{\mathcal{Z}}}{\boldsymbol{\kappa}^Y\mathbf{1}}\left(I-\mathcal{V}\right)\left(p_{-1}-\rho\mathbb{E}\pi_{+1}\right).$$

Plugging the expression for y back into (32) then gives

$$\left(I-\kappa\Phi^{-1}\mathbf{1}\bar{\mathbf{\Omega}}_{\mathcal{Z}}\right)\boldsymbol{p}=\left(1-\boldsymbol{\kappa}^Y\Phi^{-1}\mathbf{1}\right)\frac{\kappa\mathbf{1}}{\boldsymbol{\kappa}^Y\mathbf{1}}\bar{p}+\left(I-\frac{\kappa\mathbf{1}\bar{\mathbf{\Omega}}_{\mathcal{Z}}}{\boldsymbol{\kappa}^Y\mathbf{1}}\right)\kappa\left[\mathcal{D}_G-\Phi^{-1}\mathcal{S}_G ds_G\right]+\left(I-\frac{\kappa\mathbf{1}\bar{\mathbf{\Omega}}_{\mathcal{Z}}}{\boldsymbol{\kappa}^Y\mathbf{1}}\right)\left(I-\mathcal{V}\right)\left(p_{-1}-\rho\mathbb{E}\pi_{+1}\right).$$

Noting that $\mathcal{D}_p\mathbf{1}=\mathbf{0}$ then allows us to write

$$\mathcal{D}_p\boldsymbol{p}=\mathcal{D}_p\left(I-\frac{\kappa\mathbf{1}\bar{\mathbf{\Omega}}_{\mathcal{Z}}}{\boldsymbol{\kappa}^Y\mathbf{1}}\right)\left[\kappa\Phi^{-1}\mathbf{1}-\mathbf{1}\right]\bar{p}+\mathcal{D}_p\left(I-\frac{\kappa\mathbf{1}\bar{\mathbf{\Omega}}_{\mathcal{Z}}}{\boldsymbol{\kappa}^Y\mathbf{1}}\right)\left[\kappa\left[\mathcal{D}_G-\Phi^{-1}\mathcal{S}_G ds_G\right]+\left(I-\mathcal{V}\right)\left(p_{-1}-\rho\mathbb{E}\pi_{+1}\right)\right],$$

which, combined with the relative demand equation, yields

$$\boldsymbol{\ell}-\mathbf{1}y=\hat{\mathcal{D}}_p\left(\mathbf{1}-\kappa\Phi^{-1}\mathbf{1}\right)p_t^Y+\left(I-\hat{\mathcal{D}}_p\kappa\right)\boldsymbol{\mathcal{D}}_{Gt}+\hat{\mathcal{D}}_p\kappa\Phi^{-1}\Gamma\mathbf{1}\frac{ds_{Gt}}{\bar{s}_{\mathcal{H}}}-\hat{\mathcal{D}}_p\left(I-\mathcal{V}\right)\left(p_{-1}-\rho\mathbb{E}\pi_{+1}\right).$$

Proof of Lemma 4 The households' Euler equations imply

$$\Gamma\boldsymbol{c}_{\mathcal{H}t}=\Gamma\boldsymbol{\mathcal{C}}+\Gamma\mathbf{1}\bar{c}_{\mathcal{H}t}-\left(\boldsymbol{p}_{\mathcal{H}t}-\bar{p}_{\mathcal{H}t}\right)\ \forall t>0,$$

where $\boldsymbol{\mathcal{C}}=\left(\mathcal{C}_1,...,\mathcal{C}_H\right)^T$ is a vector of level shifts in the households' consumption, that are constant over time and satisfy $\sum_{h\in\mathcal{H}}s_h\mathcal{C}_h=0$. Combining this relationship with the households' budget constraints then yields

$$\Gamma\boldsymbol{c}_{\mathcal{H}t}=\Gamma\boldsymbol{\mathcal{C}}+\Gamma\mathbf{1}\bar{\mathbf{\Xi}}_{\mathcal{H}:}\boldsymbol{\ell}+\Gamma\mathbf{1}\bar{\mathbf{\Xi}}\Psi^{-1}\boldsymbol{p}-\Gamma\mathbf{1}\bar{p}_{\mathcal{H}}-\left(\boldsymbol{p}_{\mathcal{H}t}-\bar{p}_{\mathcal{H}t}\right)\ \forall t>0.$$

Further combining with the consmption-leisure tradeoff gives

$$\boldsymbol{w}=\Phi\boldsymbol{\ell}+\Gamma\left(\boldsymbol{\mathcal{C}}-\mathbf{1}\frac{ds_G}{\bar{s}_{\mathcal{H}}}\right)+\beta^T\boldsymbol{p}$$

where all elasticities are defined as in the ex-ante complete market case. The relative demand equation can be derived as in the proof of lemma 1, just noting that

$$d\log \boldsymbol{s} = \Xi\left(\boldsymbol{\ell} - \mathbf{1}y\right) + \left[\Xi\Psi^{-1} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}} - \begin{pmatrix} \left(\Gamma^{-1} - I\right)\left(\Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:}\right) \\ \mathbb{O} \end{pmatrix}\right]\boldsymbol{p} + \boldsymbol{\mathcal{C}} - \begin{pmatrix} \mathbf{1}_{\mathcal{H}} \\ \mathbf{0}_{\mathcal{U}} \end{pmatrix}\frac{ds_G}{\bar{s}_{\mathcal{H}}}$$

so that

$$\boldsymbol{\ell} - \mathbf{1}y = -\mathcal{D}_p\boldsymbol{p} + \boldsymbol{\mathcal{D}}_G + \bar{s}_{\mathcal{H}}Cov_{\boldsymbol{s}_{\mathcal{H}}}\left(\Psi^T_{\mathcal{H}\mathcal{F}}, \boldsymbol{\mathcal{C}}\right).$$

Finally, the aggregate Euler equation is the same as in the complete market case, because the growth rates of consumption are unaffected by the level shifts $\boldsymbol{\mathcal{C}}$ for $t > 0$.

The households' intertemporal budget constraints then pin down the level shifts $\boldsymbol{\mathcal{C}}$, by imposing that the present value of consumption equals the present value of factor and profit income, minus taxes:

$$\sum_{t=0}^{\infty}\rho^t\left(\boldsymbol{c}_{\mathcal{H}t} + \boldsymbol{p}_{\mathcal{H}t} - \Xi_{\mathcal{H}\mathcal{F}}\boldsymbol{\ell} - \Xi_{\mathcal{H}:}\Psi^{-1}\boldsymbol{p}\right) = \sum_{t=0}^{\infty}\rho^t\left(\boldsymbol{\mathcal{C}} + \mathbf{1}\left(\bar{c}_{\mathcal{H}t} + \bar{p}_{\mathcal{H}t}\right) + \left(I - \Gamma^{-1}\right)\left(\boldsymbol{p}_{\mathcal{H}t} - \bar{p}_{\mathcal{H}t}\right) - \Xi_{\mathcal{H}\mathcal{F}}\boldsymbol{\ell} - \Xi_{\mathcal{H}:}\Psi^{-1}\boldsymbol{p}\right) = -\mathbf{1}\sum_{t=0}^{\infty}\rho^t\frac{ds}{\bar{s}}$$

Therefore

$$\boldsymbol{\mathcal{C}} = (1-\rho)\left(I - \frac{\mathbf{1}\boldsymbol{s}^T_{\mathcal{H}}}{\boldsymbol{s}^T_{\mathcal{H}}\mathbf{1}}\right)\sum_{t=0}^{\infty}\rho^t\left[\Xi_{\mathcal{H}\mathcal{F}}\boldsymbol{\ell}_t + \frac{1}{\gamma}\left[\gamma\Xi_{\mathcal{H}:}\Psi^{-1} + (1-\gamma)\,\Omega_{\mathcal{H}:}\right]\boldsymbol{p}_t\right].$$