The Capital Puzzle

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Abstract

Can a central bank tighten and real interest rates fall under monetary dominance? Rupert and Šustek (2019) showed that adding endogenous capital to the New Keynesian model allows real interest rates to move in any direction after a positive monetary shock. This would suggest that the real interest rate channel of monetary policy is observational rather than structural in these models. This paper demonstrates that the puzzle arises when the capital elasticity to the shock is so high that inflation expectations overreact, causing the endogenous, or systematic, component of the monetary policy rule to more than offset its exogenous one. The channel is indeed structural, but comparing actual rates to a counterfactual without nominal rigidities may be misleading, particularly after a shock that significantly impacts investment, such as a pandemic or a financial crisis. Alternatively, a sign-consistent measure of the monetary policy stance is the real interest rate gap that neutralizes the effect of shocks on endogenous state variables. Such an identification problem can also be mitigated by incorporating interest-rate smoothing into the Taylor rule.

Keywords: Monetary Policy, New Keynesian Model, Real Interest Rate Gap, Interest-Rate Smoothing

JEL Classification: E43; E52; E58.

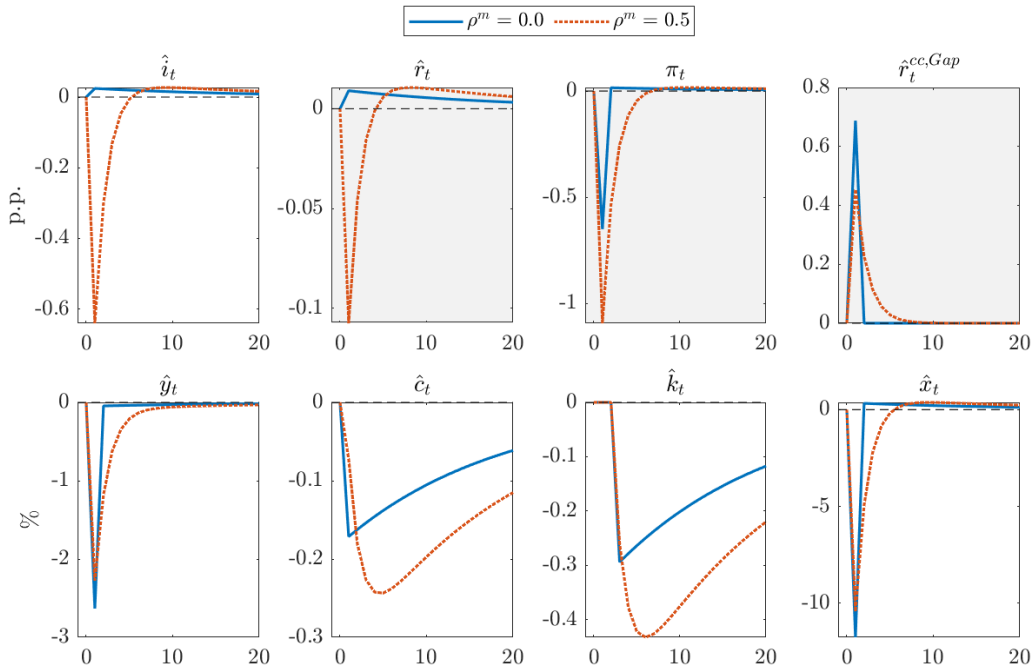
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1 Introduction

Can a central bank tighten and real interest rates fall under monetary dominance? In a recent paper, Rupert and Šustek (2019) challenged the existence of a real interest rate channel of monetary policy transmission in textbook New Keynesian models, e.g., Woodford (2003a) and Galí (2015). They showed that introducing endogenous capital into such models allows the real interest rate to move in any direction after a positive monetary shock, ε_t^m.[1] To illustrate this, Figure 1 displays the effect of a positive monetary shock under two different specifications for its persistence coefficient, ρ^m, a policy parameter. The real interest rate rises immediately after a transitory shock ($\rho^m = 0.0$) but falls when the latter is just mildly persistent ($\rho^m = 0.5$), while inflation falls and investment slumps in both cases.

[1] Woodford (2003a, sec. 5.3.3) calls the lack of any effect of variations in private spending on the economy's productive capacity one of the more obvious omissions in the baseline New Keynesian model. He observes that although there are calibrations for which introducing endogenous capital results in similar dynamics for output and inflation after a monetary shock, the mechanisms within each model that generate these results are not too closely parallel.



Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), real interest rate gap with constant capital ($\hat{r}_t^{cc,Gap}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

Figure 1: Impulse response function to a positive monetary shock in a canonical New Keynesian model augmented with endogenous capital

According to Rupert and Šustek (2019), this would prove that the real interest rate channel is only observational — not structural — in that class of models. A channel not robust to the Lucas (1976) critique raises serious concerns about the use of these models for policy recommendations. For example, the very interpretation of their impulse response functions becomes debatable, and it would be quite problematic to assume such a channel for the identification of vector autoregression (VAR) models, whether through sign restrictions, as in the method proposed by Uhlig (2005); through sequentially ordering nominal and real rates, as in a Cholesky decomposition (Benoit (1924)); or by selecting real rates instead of nominal ones as part of the model's endogenous variables.

The importance of Rupert and Šustek (2019)'s result is straightforward yet significant to the extent that it has recently been evoked by Holden (2024) as one of the reasons to motivate a radical shift in central banking from nominal to real interest rate rules. Although in general equilibrium all variables are determined simultaneously, every model needs a story to tell, and the common view on the transmission of monetary policy in textbook New Keynesian

models relies on the real interest rate channel, which Ireland (2010) describes as follows:

> "A monetary tightening in the form of a shock to the Taylor rule that increases the short-term nominal interest rate translates into an increase in the real interest rate as well when nominal prices move sluggishly due to costly or staggered price setting. This rise in the real interest rate then causes households to cut back on their spending, as summarized by the IS curve. Finally, through the Phillips curve, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock."

Galí (2015, p. 5) also emphasizes the real interest rate channel when describing the short-run non-neutrality of monetary policy in this class of models:

> "As a consequence of the presence of nominal rigidities, changes in short-term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter bring about changes in consumption and investment and, as a result, in output and employment, because firms find it optimal to adjust the quantity of goods supplied to the new level of demand. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium."

Schematically, a positive monetary shock (ε_t^m) should increase the nominal interest rate (i_t) and then the real interest rate (r_t), *because of sticky prices*, causing the reduction of consumption (c_t), output (y_t), and, finally, inflation (π_t).

$$\uparrow \varepsilon_t^m \quad \Rightarrow \quad \uparrow i_t \quad \Rightarrow \quad \underbrace{\uparrow r_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow c_t \quad \Rightarrow \quad \downarrow y_t \quad \Rightarrow \quad \downarrow \pi_t$$

However, Rupert and Šustek (2019) propose a different story, which they argue is more consistent with the actual mechanics of the model. The transmission does not operate through a real interest rate channel. First, equilibrium inflation is approximately determined as in a flexible-price model.[2] Second, output is pinned down by the New Keynesian Phillips curve, interpreted here to mean that, given the expected inflation trajectory, firms that cannot

[2] In Chapter 2 of Galí (2015), a canonical real business cycle (RBC) model is augmented with a fixed-intercept interest rate rule to pin down inflation and, thus, a trajectory for the price level. Current inflation, as a deviation from its steady-state value, is determined by the expected path of real interest rate deviations

adjust prices will change output. Finally, the real rate only reflects the feasibility of keeping consumption smooth when income changes, whereas consistency with the real interest rate channel depends on the persistence of the monetary shock. The canonical model, with fixed capital, is simply a limiting case where the latter adjustment costs are infinite. According to this view, monetary transmission should work as follows:

$$\uparrow \varepsilon_t^m \quad \Rightarrow \quad \uparrow i_t \quad \Rightarrow \quad \downarrow \pi_t \quad \Rightarrow \quad \underbrace{\downarrow y_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow c_t \quad \Rightarrow \quad \underbrace{?r_t}_{\substack{\text{depends on the presence} \\ \text{of capital and calibration}}}$$

In this paper, I show that this puzzle stems from a problem of definition. In the canonical New Keynesian model, monetary policy stance is measured by the real interest rate gap, $\hat{r}_t^{Gap}$, which compares the actual real interest rate of a sticky economy with the one of an identical counterpart except for the absence of nominal rigidities (1). This last rate is called the natural real interest rate, r_t^n. If there is only a history of monetary shocks in the economy, r_t^n should not move, implying that $\hat{r}^{Gap}$ is equal to r_t.

$$\hat{r}_t^{Gap} = \hat{r}_t - \hat{r}_t^n \tag{1}$$

Since the canonical model has no endogenous state variables, such as capital, this rate is strictly connected to the notion of equilibrium determination in that economy. In this case, a zero real interest rate gap implies that output is at its potential level and inflation is at the central bank's target. This is a short-run equilibrium, but it is also a long-run one in the absence of further exogenous shocks, where a single persistent exogenous shock is just a sequence of exogenous shocks. Woodford (2003a, Ch. 5 sec 3.4) points out that this definition of the real interest rate gap turns out to be odd once we introduce capital in the model. That is so because, out of the steady state, the stock of capital of the sticky economy will probably differ from the stock of the counterpart economy without nominal rigidities. He suggests defining the natural rate of this economy as the one that would result from the lack of nominal rigidities now and in the future, given all exogenous and predetermined state variables.[3]

from the steady state, as long as the Taylor principle is obeyed. It is important to note that the steady-state value of the real interest rate and the intercept of the monetary policy rule coincide, assuming a zero-inflation target. Chapter 1 of Woodford (2003a) shows the same idea in a partial-equilibrium monetary model where the sequence of real interest rates is exogenous.

[3]Woodford (2003a, Ch. 5 sec 3.4)'s refers only to price flexibility, probably, due to the context of the simple model he is analyzing.

Thus, for the real interest rate gap to represent the current monetary policy stance, it must control for the effect of shocks on state variables. The natural rate that matters for measuring the monetary policy stance is the one that assumes state variables are at the same level as the real rate, which I call *state-invariant definition*, and not the definition employed by, eg., Neiss and Nelson (2003), which assumes the non-existence of nominal rigidities neither in the past, present, or future, the *state-variant definition*. As a matter of fact, this last definition has been ubiquitously employed in dynamic stochastic general equilibrium (DSGE) models, such as Smets and Wouters (2003), whereas the state-invariant definition was to great extent overlooked in the New Keynesian literature. The finding that the state-variant definition is only observationally consistent with the real interest rate channel leads to a theoretical argument in favor of the state-invariant definition.

Here, I incorporate that important caveat in a different approach. I decompose the natural real interest into an exogenous component, $\hat{r}_t^{ncc}$, that corresponds to a counterfactual that keeps the capital stock constant, and a remaining component, $\eta_{nk}\hat{k}_t^n + \boldsymbol{\eta_{nkk}\varepsilon}_t$, that depends on the state variable and the shocks, where η_{nk} is a constant, $\boldsymbol{\eta_{nkk}}$ is a vector of constants, $\hat{k}_t^n$ is the log-deviation from the steady state of the capital stock in the economy without nominal rigidities, and $\boldsymbol{\varepsilon}_t$ is a vector of shocks.[4]

$$\hat{r}_t^n = \underbrace{\hat{r}_t^{ncc}}_{\text{= 0 if only shocks are monetary}} + \eta_{nk}\hat{k}_t^n + \underbrace{\boldsymbol{\eta_{nkk}\varepsilon}_t}_{\text{= 0 if only shocks are monetary}} \tag{2}$$

I proceed then by decomposing $\hat{r}_t$ as the sum of a counterfactual where capital is fixed, $\hat{r}_t^{cc}$, and a residual, $\left(\eta_k\hat{k}_t + \eta_{kk}\varepsilon_t^m\right)$, where η_k and η_{kk} are constants, and defining the state-invariant real interest rate gap, $\hat{r}_t^{cc,Gap}$, as the gap of real interest rates that controls for the endogenous state variables.

$$\hat{r}_t = \hat{r}_t^{cc} + \left(\eta_k\hat{k}_t + \eta_{kk}\varepsilon_t^m\right) \tag{3}$$

$$\hat{r}_t^{cc,Gap} = \hat{r}_t^{cc} - \hat{r}_t^{ncc} \tag{4}$$

As can be seen in Figure 1 by comparing $\hat{r}_t$ and $\hat{r}_t^{cc,Gap}$, the latter is the real interest rate gap measure whose sign is consistent with the New Keynesian theory at the shock period. Note that while Woodford (2003a)'s decomposition does not preclude that $\hat{r}_t$ reacts negatively to a positive monetary shock, the here proposed decomposition isolates in coefficients η_k

[4]Note that the remaining component does not depend on the monetary shock due to the lack of nominal rigidities in the counterfactual economy.

and η_{kk} any deep parameter sensitive to the speed of the adjustment of the state variable. I propose the following explanation for the New Keynesian capital puzzle: a monetary tightening in the form of a shock to the Taylor rule increases the short-term nominal interest rate (policy rule), causing an increase in the real interest rate when nominal prices move sluggishly (Fisher equation). This rise in the real interest rate causes households to cut back on their spending (IS). If investment sinks too much, households cut consumption even further (IS with capital). The large decline in output puts significant downward pressure on inflation (Phillips Curve), amplifying the endogenous negative response of the policy rule. This results in the nominal interest rate reversing its sign and dropping when its endogenous response is numerically larger that the original monetary shock, ultimately causing a drop in the real interest rate because prices are sticky. Then, onward, this flow continues with shrinking and oscillating amplitude until convergence to the in-period equilibrium, as induced by monetary dominance.

$$\uparrow \varepsilon_t^m \;\Rightarrow\; \uparrow i_t \;\Rightarrow\; \underbrace{\uparrow r_t}_{\text{if prices are sticky}} \;\Rightarrow\; \underbrace{\downarrow\downarrow c_t}_{\text{if capital sinks}} \;\Rightarrow\; \downarrow\downarrow y_t \;\Rightarrow\; \downarrow\downarrow \pi_t \;\Rightarrow\; \downarrow i_t \;\Rightarrow\; \downarrow r_t$$

From an empirical perspective, this paper also shows that the identification problem of the real interest rate channel of monetary policy can be largely mitigated in the relevant parameter range by adding interest-rate smoothing to the Taylor rule — a feature as prevalent as capital in medium-scale New Keynesian dynamic stochastic general equilibrium (DSGE) models, e.g., Smets and Wouters (2003, 2007). Smoothing interest rates narrows down the numerical difference between the real interest rate gaps measured under the state-invariant and state-variant definitions, while it also makes more likely that the state-variant real interest rate gap reflects the sign of the monetary shock.

I adopt the following modeling strategy to disentangle the observational consistency of the real interest rate channel from its structural validity. First, I solve the textbook New Keynesian model of Galí (2015) with capital and decompose the effect of shocks on the real interest rate gap into two components: one that only depends on the exogenous shocks and another one that only depends on the endogenous state variables of the model. This decomposition exposes that the first component retains the sign predicted by the New Keynesian theory. The sign of the endogenous component, by its turn, will depend on the interaction between the structure of the model and the monetary policy rule. If the endogenous component happens to have the opposite sign of the exogenous component and it is larger in magnitude

than the latter, a positive monetary shock can lead to an immediate decrease of the real interest rate. In this case, yes, a central bank can tighten and real interest rates fall, but that reflects the endogenous feedback of the structure of that economy (i.e., capital adjustment costs, consumption habits) and not the first-round effect of monetary policy. Second, I add smoothing in the Taylor rule to show that the latter can mitigate the identification problem revealed by Rupert and Šustek (2019). This finding that the real interest rate channel of monetary policy is reestablished with a common ingredient of medium-scale New Keynesian models weakens the concerns about its correct identification in policy-oriented DSGE or VAR models, where the latter are mostly immune to the problem because lagging terms are ubiquitous in their specification. Third, I better qualify my results by exploring different combinations of interest-rate smoothing and also capital adjustment costs, since making capital adjustment sluggish is warranted to prevent output from overreacting after a monetary shock. Finally, I show how smoothing solves the identification problem in a medium-scale DSGE model.

Either by focusing on the state-invariant real interest rate gap or accepting that at least some smoothing is the rule in central banking and that capital adjustment costs are never negligible in the real world leads to an even broader conclusion. Regarding sign identification, the relationship between inflation and real interest rates is actually more robust than that between the former and nominal interest rates. That is because, after a monetary shock, there is no univocal direction for the observed nominal interest rate response, whose rule is composed of both an endogenous (i.e., response to inflation) and an exogenous component (the shock). A sufficiently persistent positive shock can depress inflation expectations to such an extent that current inflation subsides to the point that current nominal interest rate must also decrease. This textbook result has been explored by both Woodford (2003a, sec. 4.2.4) and Galí (2015).

The next sections of this paper are structured as follows. Section 2 presents the related literature. Section 3 describes, solves, and analyzes the New Keynesian model before and after introducing endogenous capital. Section 4 add interest-rate smoothing into the model. Section 5 takes the proposed solutions to a medium-scale model. Finally, Section 6 concludes.

2 Related literature

This paper builds on Rupert and Šustek (2019), who scrutinize the mechanics of canonical New Keynesian models, i.e., those in Woodford (2003a) and Galí (2015). They argue that the monetary policy transmission mechanism in this class of models does not operate through the real interest rate channel, contrary to the conventional view. The observational similitude with the real interest rate channel would come from an implicit assumption of infinite capital adjustment costs.

Suspicion about the real interest rate channel of New Keynesian models is not new. The seminal work of Kimball (1995) on the derivation of a real business cycle model with sticky prices – he called it Neo-Monetarist – dedicates a whole section to discussing the unlikelihood of that channel. He concludes that, even when investment adjustment costs are introduced, parameter values perceived by him as "plausible" would imply that the real interest rate increases in response to a monetary expansion.[5] The "implausible" scenario would occur if either adjustment costs were "too high" or convergence back to the long-run equilibrium after a monetary shock was "too fast", not unlike what Rupert and Šustek (2019) find. Nonetheless, here lie two distinctions between the Neo-Monetarist model and most of the New Keynesian models that followed. First, while Kimball (1995) insisted on portraying monetary policy through a quantity equation with exogenous shocks to the money supply, the New Keynesian literature has followed the real-world trend of adopting nominal interest rate rules with an endogenous response to inflation. Especially when augmented with smoothing, as I propose in this paper, these last rules put in sharp relief the speed of the convergence back to a long-run equilibrium. Second, the parameterization he deems as "plausible" – an investment adjustment cost elasticity of 0.2 and a (labor-constant) elasticity of intertemporal substitution (EIS) for consumption of 0.2 – does not match modern estimations of these models, which find higher values for the EIS.[6]

Brault and Khan (2022) modify Rupert and Šustek (2019)'s work by including frictions on changes in the flow of investments, like Christiano, Eichenbaum and Evans (2005) do, rather than on capital adjustment. They find that the real interest rate moves in the same direction

[5]The model is linearized and, therefore, I assume a symmetrical response in the case of a monetary contraction.

[6]Using Bayesian methods, Smets and Wouters (2003) estimate the EIS to be 0.74 for the Euro Area, and Smets and Wouters (2007) estimate it to be 0.68-0.72 for the U.S. All values are posterior modes.

as the monetary policy shock, regardless of adjustment cost size or shock persistence. The authors argue that, at least in contemporary (medium-scale) New Keynesian models, the real interest rate channel is present.

On the literature about gauging monetary policy stance through interest rate gaps, Wicksell (1898, 1907) builds a whole monetary theory over the gap between the money rate and the natural interest rate in a mostly frictionless environment. More recently, despite relying on different definitions of the real interest rate gap when there is endogenous capital, Woodford (2003a) makes the theoretical case, while Neiss and Nelson (2003) present evidence in favor of its empirical relevance. Laubach and Williams (2003) estimate the gap in a semi-structural model, defining the natural rate as a trend that depends on output growth. Cour-Thimann, Pilegaard and Stracca (2006) specify the natural rate as the unobservable component of the real interest rate which has no contemporaneous and lagged correlation with the output gap. Lubik, Matthes et al. (2015) employ a time-varying parameter vector autoregressive model to calculate r-star without assuming economic relationships between key macroeconomic variables, a largely agnostic approach. In actual central banking practice, the use of the real interest rate gap has been pervasive (e.g., Waller (2024); Schnabel (2024)).

Concerning interest-rate smoothing, its addition to the canonical model is empirically motivated. The presence of a significant level of it in the response function of the Federal Reserve is found by Clarida, Galí and Gertler (1999), for both the pre-Volcker (1960:1-1979:2) and the Volcker-Greenspan (1979:3-1996:4) eras, as well as by Coibion and Gorodnichenko (2012), whose results employing both hard and narrative real-time data favor that source of purposeful policy inertia over serially correlated monetary shocks, either arbitrary or motivated by, perhaps, inherited persistence from the underlying data generating process of omitted variables to which the Federal Reserve also responds. For their part, Smets and Wouters (2007) estimate a medium-scale New Keynesian DSGE model using Bayesian methods for the United States and find considerable coefficients for interest-rate smoothing (above 0.7) as well as small coefficients for monetary shock persistence (below 0.3). These papers suggest the empirical presence of smoothing through the estimation of either single or multiple equation models, that is, by imposing only a little or a lot of informational restriction on the estimation.

Nonetheless, contrasting results can still be found depending on the estimation strategy as Rudebusch (2006) and Carrillo, Fève and Matheron (2018) demonstrate, favoring the

modeling of serially correlated monetary shocks employed by Rupert and Šustek (2019). Both approaches to monetary policy are not mutually exclusive, though, since data-driven monetary policy and contingency on new information do not preclude mild forms of forward guidance and policy inertia. This paper shows that different combinations of these two features are enough to restore the canonical identification of the real interest rate channel.

My modification is also theoretically justified. Sack and Wieland (2000) and Woodford (2003b) show that smoothing policy interest rates may be optimal from a welfare perspective, a concern already presented in Goodfriend (1987) in terms of a central bank's preference to maintain "orderly money markets" by minimizing unexpected asset price movements that otherwise could raise the risk of bankruptcies and banking crises. Thus, although smoothing is a policy choice, high levels of it are generally optimal, and low levels are empirically rare, which warrants the case of this paper.

3 New-Keynesian model before and after capital

In this section, I propose and solve a New Keynesian toy model: first the canonical version, then the model augmented with endogenous capital, and then I add interest-rate smoothing. This exposition strategy facilitates the comparison across the different versions of the model. The reader can find in Appendix A a glossary of all symbols used.

3.1 Canonical closed economy (with omitted or fixed capital)

Consider a closed economy without fiscal policy, where a one-period risk-free nominal bond is available in zero net supply and the central bank adopts a fixed-intercept Taylor rule. I expand here on the simplified presentation made by Rupert and Šustek (2019) of the canonical New Keynesian model of Galí (2015), with minor notational changes.

The simple model starts with seven variables: real consumption, c_t; labor l_t; real output, y_t; real wage, w_t; real marginal cost, χ_t; nominal interest rate, i_t; and inflation, π_t. Overlined variables represent their steady-state values. There are six parameters: the subjective discount factor, β; the inverse of the elasticity of labor supply, η; the elasticity of substitution between intermediate goods, ε; the fraction of producers not adjusting prices at any given period, θ; the intercept of the Taylor rule, i; and the Taylor-rule coefficient that gauges the central bank's reaction to current inflation, ν. There is also an exogenous monetary shock

variable, ξ_t^m.

Assuming a per-period utility function of the form

$$u_t = \log(c_t) - \frac{l_t^{1+\eta}}{1+\eta} \tag{5}$$

and an intermediate goods aggregator like

$$y_t = \left[\int y(j)^{\varepsilon}\,\mathrm{d}j\right]^{\frac{1}{\varepsilon}} \tag{6}$$

the equilibrium conditions of that economy are given by the Euler equation (7) in conjunction with equations (8) to (13), namely the first-order conditions (FOC) of labor, the production function, the real marginal cost, the New Keynesian Phillips curve under Calvo pricing already linearized around a zero steady-state inflation, a Taylor rule, and the market-clearing condition.

$$\left.\begin{aligned}
\frac{1}{c_t} &= \beta\,\mathbb{E}_t\left(\frac{1}{c_{t+1}}\frac{1+i_t}{1+\pi_{t+1}}\right) && (7)\\
\frac{w_t}{c_t} &= l_t^{\eta} && (8)\\
y_t &= l_t && (9)\\
\chi_t &= w_t && (10)\\
\pi_t &= \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\mathrm{mc}_t - \overline{\mathrm{mc}}\right) + \beta\,\mathbb{E}_t\,\pi_{t+1} && (11)\\
i_t &= i + \nu\pi_t + \xi_t^m && (12)\\
y_t &= c_t && (13)
\end{aligned}\right\}$$

As usual, the equilibrium conditions above can be simplified to a three-equation system linearized around a nonstochastic steady state ($\overline{\pi} = 0$, $\overline{y} = 1$). This is possible by first linearizing (7) and substituting the market-clearing condition (13) into it, then eliminating (13), (9), and (10) through the substitution of their respective expressions for c_t, l_t, and w_t into (8), which is later linearized such that $\hat{y}_t \equiv \frac{y_t - \overline{y}}{\overline{y}}$. Finally, the Taylor Rule (12) is rewritten as deviations of the interest rate from its steady-state value such that $\hat{i}_t = i_t - i$.

$$\left.\begin{aligned}
-\hat{y} &= -\mathbb{E}_t\,\hat{y}_{t+1} + \hat{i}_t - \mathbb{E}_t\,\pi_{t+1} && (14)\\
\pi_t &= \Omega\hat{y}_t + \beta\,\mathbb{E}_t\,\pi_{t+1} && (15)\\
\hat{i}_t &= \nu\pi_t + \xi_t^m && (16)
\end{aligned}\right\}$$

where

$$\Omega \equiv \frac{(1+\eta)(1-\theta)(1-\theta\beta)}{\theta} > 0 \tag{17}$$

Notice that when prices are fully flexible, $\theta \to 0$, then $\Omega \to \infty$, whereas when prices are fixed, $\theta \to 1$, then $\Omega \to 0$. As Rupert and Šustek (2019) comment, it is helpful to think of Ω as a weight that gauges the solution coefficients of the system between the fully flexible and the fixed price regime.

I can proceed further by substituting the policy rule (16) into (14) so that I reduce the system to only two equations:

$$\left.\begin{aligned} -\hat{y} = -\mathbb{E}_t\,\hat{y}_{t+1} + \nu\pi_t + \xi_t^m - \mathbb{E}_t\,\pi_{t+1} \quad (18)\\ \pi_t = \Omega\hat{y}_t + \beta\mathbb{E}_t\,\pi_{t+1} \quad (19)\end{aligned}\right\}$$

I assume the monetary shock follows an AR(1) process given by $\xi_t^m = \rho^m\xi_{t-1}^m + \epsilon_t^m$, where $\rho^m \in [0,1)$ and ϵ_t^m is i.i.d. N(0,1). Solving the model with the method of undetermined coefficients — also known as guess-and-verify — by conjecturing $\hat{y}_t = a\xi_t^m$ and $\pi_t = b\xi_t^m$, where a and b are the coefficients I want to obtain, and discarding explosive paths for output and inflation leads to

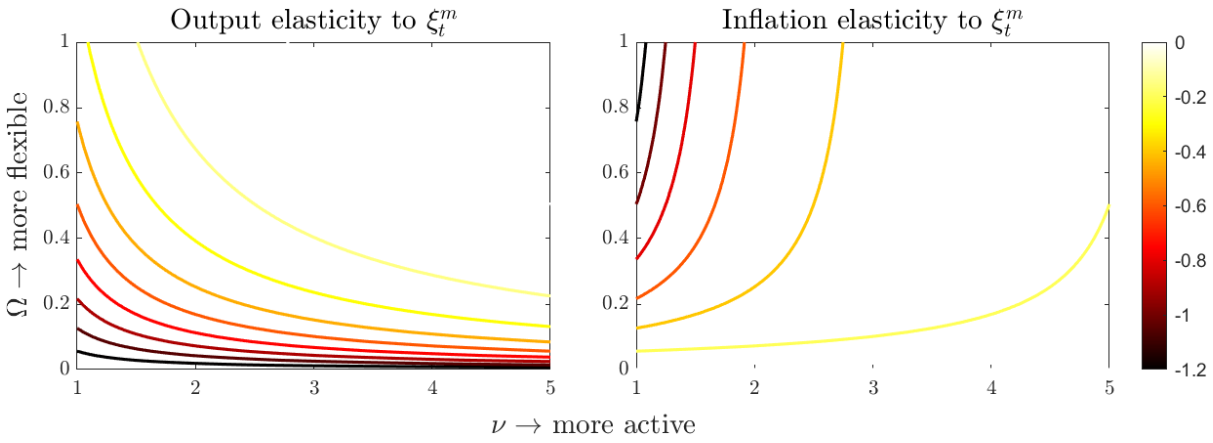
$$a = -\frac{1-\beta\rho^m}{(1-\rho^m)(1-\beta\rho^m)+\Omega(\nu-\rho^m)} < 0 \quad (20)$$

$$b = -\frac{1}{(1-\rho^m)(1-\beta\rho^m)\Omega^{-1}+(\nu-\rho^m)} < 0 \quad (21)$$

where both coefficients imply that a positive monetary shock always reduces inflation and output in the canonical New Keynesian model.

Figure 2 plots coefficients a and b for different values of ν and Ω, under the calibration of Rupert and Šustek (2019)[7]. As expected, flexible prices reduce output elasticity to zero at the same time that inflation elasticity is at its maximum. Moreover, a more active monetary policy reduces both elasticities.

[7]The following calibration includes parameters that will be incorporated into the model later in this paper: $\beta = 0.99$, $\eta = 1$, $\varepsilon = 0.83$, $\theta = 0.7$, $\nu = 1.5$, $\rho^m = 0.5$, $\alpha = 0.3$, $\delta = 0.025$.



Note: Darker colors imply higher (absolute) elasticity values.

Figure 2: Output and inflation elasticity to a monetary shock

The real interest rate as a deviation from its steady-state value can be obtained from the Fisher identity, $\hat{r}_t = \hat{i}_t - \mathbb{E}_t \pi_{t+1}$. Substituting my solution, I have

$$\hat{r}_t = \underbrace{\left(1 - \frac{1}{1 + \underbrace{\frac{1-\rho^m}{\nu-\rho^m}\frac{1-\beta\rho^m}{\Omega}}_{\in[0,1]}}\right)}_{\geq 0} \xi_t^m \tag{22}$$

whose coefficient on ξ_t^m, which is never negative, implies that the real interest rate always increases/decreases right after a positive/negative monetary shock. This is consistent with monetary policy being transmitted through a real interest rate channel.

3.2 Endogenous capital

I now incorporate endogenous capital and show that the real interest rate channel is reestablished once we use the state-invariant real interest rate gap definition.

I build on the model of Rupert and Šustek (2019), which assumes there is an economy-wide rental market of capital so that firms can rent capital in every period. In that sense, capital is not firm-specific.[8] Moreover, they assume that whenever households change their

[8] Altig et al. (2011) estimate a New Keynesian DSGE model for the U.S. and find that this modeling choice for introducing endogenous capital results in firms enduring long spells before readjusting prices, up to 9 quarters on average. They show that firm-specific capital can align that spell more with empirical evidence from micro data to, say, once a year.

stock of capital, there is a quadratic adjustment cost, $-\frac{\kappa}{2}\left(k_{t+1}-k_t\right)^2$, where k_t is the stock of capital inherited from the previous period and $\kappa \geq 0$ is a parameter that governs the size of the adjustment cost in terms of foregone real income.

Resuming from the canonical model of Section 3.1, there is a new Euler equation for the capital asset (23), where $\delta \in (0,1)$ is a depreciation rate, and q_t is the price of capital in terms of current consumption, Tobin's q, such that $q_t \equiv 1+\kappa\left(k_{t+1}-k_t\right)$. The production function (9) is replaced by (24), which incorporates capital and labor proportionate to constant returns to scale, where α is the Cobb-Douglas coefficient of capital. Equation (25) is the condition for the optimal mix of capital and labor in production, which comes from the FOC of the firm. The marginal cost (10) is updated to include the rent on capital, r_t^k, (26). The resources constraint (13) must now account for the investment flow so markets can clear (27).

$$\left.\begin{aligned}
\frac{1}{c_t} &= \beta \mathbb{E}_t\left(\frac{1}{c_{t+1}}\left(\frac{r_{t+1}^k-\delta}{q_t}+\frac{q_{t+1}}{q_t}\right)\right) && (23)\\
y_t &= k_t^{\alpha} l_t^{1-\alpha} && (24)\\
\frac{w_t}{r_t^k} &= \frac{1-\alpha}{\alpha}\left(\frac{k_t}{l_t}\right) && (25)\\
\chi_t &= \left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} && (26)\\
y_t &= c_t + k_{t+1} - (1-\delta)\,k_t + \frac{\kappa}{2}\left(k_{t+1}-k_t\right)^2 && (27)
\end{aligned}\right\}$$

After substituting equation (25) into (26) by eliminating r_t^k, and substituting equation (8) into (24) so as to eliminate l_t, the model is log-linearized around the zero-inflation non-stochastic steady state (Appendix B). For any variable X, $\hat{X} \equiv \frac{X_t-\overline{X}}{\overline{X}}$, with the exception of $\hat{i}_t \equiv i_t - \overline{i}$, $\hat{r}_t \equiv r_t - \overline{r}$, and $\hat{r}_t^k \equiv r_t^k - \overline{r}^k$. After that, it is possible to eliminate $\hat{r}_t^k$, $\hat{\chi}_t$, $\hat{w}_t$, and $\hat{l}_t$ to obtain the following reduced system with only four equations, which depends on the specification of a policy rule for $\hat{i}_t$.

$$\left.\begin{aligned}
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \hat{i}_t - \mathbb{E}_t\,\pi_{t+1} && (28)\\
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \mathbb{E}_t\,\hat{g}_{t+1} + \overline{r}^k\,\mathbb{E}_t\left(\hat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\hat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\hat{k}_{t+1}\right) && (29)\\
\pi_t &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t - \alpha\frac{1+\eta}{1-\alpha}\hat{k}_t + \hat{c}_t\right) + \beta\,\mathbb{E}_t\,\pi_{t+1} && (30)\\
\hat{y}_t &= \frac{\overline{c}}{\overline{y}}\hat{c}_t + \frac{\overline{k}}{\overline{y}}\hat{k}_{t+1} - (1-\delta)\frac{\overline{k}}{\overline{y}}\hat{k}_t && (31)
\end{aligned}\right\}$$

where $\Psi \equiv \overline{\chi}\frac{(1-\theta)(1-\theta\beta)}{\theta}$, such that when prices are flexible, $\Psi \to \infty$. Moreover, $G_{t+1} \equiv \frac{q_{t+1}}{q_t}$ is the capital gain, so $\hat{g}_t = \hat{q}_t - \hat{q}_{t-1} = \overline{\kappa}\left(\hat{k}_{t+1} - \hat{k}_t\right) - \overline{\kappa}\left(\hat{k}_t - \hat{k}_{t-1}\right)$, where $\overline{\kappa} = \kappa\overline{k}$.

3.3 The real interest rate gap

In the New Keynesian theory, the gauge of monetary policy stance is the real interest rate gap, $\hat{r}^{Gap}$. Neiss and Nelson (2003) calculate the gap by taking the difference between the actual real interest rate, $\hat{r}_t$, and the real interest rate of a counterfactual economy without nominal rigidities, $\hat{r}_t^n$, as in (1). This approach does not take into account the fluctuation of state variables, such as capital, which, I show next, is the reason why Rupert and Šustek (2019) find that the real interest rate channel of monetary policy transmission is only observational and not structural in New Keynesian models. Once we decompose the effect of the shock into a purely exogenous component that emerges from the counterfactual with fixed state variables, and one which depends on the existence of the state variable, (33), we can calculate a state-consistent real interest rate gap, like in Woodford (2003a), but whose sign reflects the actual monetary policy stance. This last measure is consistent with the New Keynesian theory and shows that the transmission of monetary policy is truly through a real interest rate channel, although deviations of state variables from their long-run values blur that same mechanism. A steady-state economy inflicted with a positive monetary shock will always exhibit a positive state-consistent real interest rate gap at the period of the shock.

We start by recalling some definitions introduced in this paper, (1) to (4). r_t^{cc} is the real interest rate of a counterfactual economy in which capital is kept fixed at its long-run (steady-state) value, r_t^{ncc} is the same counterfactual but with no nominal rigidities, $\hat{r}_t^{cc,Gap}$ is the gap between them, while η_{nk}, η_{nkk}, η_k, and η_{kk} are constants.

$$\hat{r}_t^{Gap} = \hat{r}_t - \hat{r}_t^n \tag{1}$$

$$\hat{r}_t^n = \hat{r}_t^{ncc} + \left(\eta_{nk}\hat{k}_t^n + \eta_{nkk}\varepsilon_t^m\right) \tag{2}$$

$$\hat{r}_t = \hat{r}_t^{cc} + \left(\eta_k\hat{k}_t + \eta_{kk}\varepsilon_t^m\right) \tag{3}$$

$$\hat{r}_t^{cc,Gap} = \hat{r}_t^{cc} - \hat{r}_t^{ncc} \tag{4}$$

We can now proceed with the decomposition. We substitute (3) and (2) into (4), and then (1) into its result to obtain (33).

$$\hat{r}_t^{cc,Gap} = \left(\hat{r}_t - \eta_k\hat{k}_t - \eta_{kk}\varepsilon_t^m\right) - \left(\hat{r}_t^n - \eta_{nk}\hat{k}_t^n - \eta_{nkk}\varepsilon_t^m\right)$$

$$\hat{r}_t^{cc,Gap} = \hat{r}_t^{Gap} - \eta_k \hat{k}_t + \eta_{nk} \hat{k}_t^n - \eta_{kk} \varepsilon_t^m + \eta_{nkk} \varepsilon_t^m$$

$$\hat{r}_t^{Gap} = \underbrace{\hat{r}_t^{cc,Gap}}_{\text{state-invariant effect}} + \underbrace{\eta_k \hat{k}_t - \eta_{nk} \hat{k}_t^n + \eta_{kk} \varepsilon_t^m - \eta_{nkk} \varepsilon_t^m}_{\text{state-variant effect}} \tag{32}$$

As the only shock of the model is monetary, we know that $\hat{r}_t^{cc,Gap} = \hat{r}_t^{cc}$, $\eta_{nkk} = 0$, and $\hat{k}_t^n = 0$.

$$\hat{r}_t^{Gap} = \underbrace{\hat{r}_t^{cc}}_{\text{state-invariant effect}} + \underbrace{\eta_k \hat{k}_t + \eta_{kk} \varepsilon_t^m}_{\text{state-variant effect}} \tag{33}$$

Returning to the model equations, after fixing capital, we can get rid of (29). Assuming the same policy rule as before, we have:

$$\left.\begin{aligned} -\hat{c}_t^{cc} &= -\mathbb{E}_t \hat{c}_{t+1}^{cc} + \nu \pi_t^{cc} + \xi_t^m - \mathbb{E}_t \pi_{t+1}^{cc} && (34)\\ \pi_t^{cc} &= \Psi \left(\frac{\eta + \alpha}{1 - \alpha} \hat{y}_t^{cc} + \hat{c}_t^{cc} \right) + \beta \mathbb{E}_t \pi_{t+1}^{cc} && (35)\\ \hat{y}_t^{cc} &= \frac{\overline{c}}{\overline{y}} \hat{c}_t^{cc} && (36) \end{aligned}\right\}$$

We substitute (36) in (35) to eliminate $\hat{y}_t$. Using the undetermined coefficients methods, we solve next.

$$\hat{c}_t^{cc} = a\xi_t^m \quad \Rightarrow \quad \mathbb{E}_t \hat{c}_{t+1}^{cc} = a\mathbb{E}_t \xi_{t+1}^m \quad \Rightarrow \quad \hat{c}_t = a\epsilon_t^m \quad \Rightarrow \quad \mathbb{E}_t \hat{c}_{t+1}^{cc} = a\rho^m \epsilon_t^m$$

$$\hat{\pi}_t^{cc} = b\xi_t^m \quad \Rightarrow \quad \mathbb{E}_t \hat{\pi}_{t+1}^{cc} = b\mathbb{E}_t \xi_{t+1}^m \quad \Rightarrow \quad \hat{\pi}_t^{cc} = b\epsilon_t^m \quad \Rightarrow \quad \mathbb{E}_t \hat{\pi}_{t+1}^{cc} = b\rho^m \epsilon_t^m$$

$$\left.\begin{aligned} -\hat{c}_t^{cc} &= -\mathbb{E}_t \hat{c}_{t+1}^{cc} + \nu \pi_t^{cc} + \xi_t^m - \mathbb{E}_t \pi_{t+1}^{cc}\\ \pi_t^{cc} &= \Psi \left(\frac{\eta + \alpha}{1 - \alpha} \frac{\overline{c}}{\overline{y}} + 1 \right) \hat{c}_t^{cc} + \beta \mathbb{E}_t \pi_{t+1}^{cc} \end{aligned}\right\}$$

$$\left.\begin{aligned} -a\epsilon_t^m &= -a\rho^m \epsilon_t^m + \nu b \epsilon_t^m + \epsilon_t^m - b\rho^m \epsilon_t^m\\ b\epsilon_t^m &= \Psi \left(\frac{\eta + \alpha}{1 - \alpha} \frac{\overline{c}}{\overline{y}} + 1 \right) a\epsilon_t^m + \beta b \rho^m \epsilon_t^m \end{aligned}\right\}$$

$$a = \frac{1}{\rho^m - 1} \left(\frac{\left(\nu - \rho^m\right)\Omega}{1 - \left(\nu - \rho^m\right)\Omega} + 1 \right) \tag{37}$$

$$b = \frac{\Omega}{1 - \left(\nu - \rho^m\right)\Omega} \tag{38}$$

where $\Omega = \frac{\Psi \left(\frac{\eta + \alpha}{1 - \alpha} \frac{\overline{C}}{\overline{Y}} + 1 \right)}{(1 - \beta\rho^m)(\rho^m - 1)} < 0$.

Since $\Omega < 0$ and $\left(\nu - \rho^m\right) > 0$, $-1 < \frac{(\nu - \rho^m)\Omega}{1 - (\nu - \rho^m)\Omega} < 0$, so that $a < 0$, which means that consumption always decreases with respect to the state-invariant effect of the shock. Moreover, the numerator of b is negative and the denominator is positive, so that $b < 0$, which means

that inflation also falls as a consequence of the state-invariant effect of the shock. $\hat{r}_t^{cc}$ and $\hat{r}_t^{ncc}$ can now be analytically solved, and therefore, also $\hat{r}_t^{cc,Gap}$.

$$\hat{r}_t^{cc} = i_t^{cc} - \mathbb{E}_t \pi_{t+1}^{cc} = \nu \pi_t^{cc} + \epsilon_t^m - \mathbb{E}_t \pi_{t+1}^{cc}$$

$$\hat{r}_t^{cc} = \nu b \epsilon_t^m + \epsilon_t^m - b\rho^m \epsilon_t^m = \left(\nu b + 1 - b\rho^m\right) \epsilon_t^m$$

$$\hat{r}_t^{cc} = \left(1 + (\nu - \rho^m) \frac{\Omega}{1 - \left(\nu - \rho^m\right)\Omega}\right) \epsilon_t^m$$

$$\hat{r}_t^{cc} = \underbrace{\left(1 + \frac{1}{\underbrace{\frac{1}{(\nu-\rho^m)\Omega} - 1}_{> -1}}\right)}_{> 0} \epsilon_t^m$$

A positive monetary shock will always result in a positive state-invariant effect, $\hat{r}_t^{cc} > 0$ and therefore $\hat{r}_t^{cc,Gap} > 0$, but the sign of the state-variant effect will depend on the interaction between the structure of the model economy and the policy rule. Therefore, $\hat{r}_t^{Gap}$ will be negative if the state-variant effect is negative and has a higher magnitude than the state-invariant effect. Tables (1) to (4) sweep the sign of the impulse response functions of r_t^{Gap} and $r_t^{cc,Gap}$ right after a positive monetary shock for different values of κ ranging from 0 to 0.5. As one can see, r_t^{Gap} turns negative the more persistent is the shock, but higher capital adjustment costs make more likely that the change in sign occurs at higher persistence levels. What capital adjustment costs do in the model is to introduce a friction on its state variable (i.e. capital), turning the latter less sensitive to exogenous shocks in general, and therefore attenuating the magnitude of the state-variant component of the real interest rate gap. Overall, the real interest rate gap consistent with the real interest rate channel of transmission of monetary policy is the state-invariant.

Table 1: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.0$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	$\rho^m = 0.95$	$\rho^m = 0.99$
r^{Gap}	+	-	-	-	-	-	-	-	-	-	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the specific real interest rate gap increases right after a positive monetary shock; - indicates that it decreases.

Table 2: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.1$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	ρ^m =0.95	ρ^m =0.99
r^{Gap}	+	+	+	+	+	+	+	+	-	-	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the specific real interest rate gap increases right after a positive monetary shock; - indicates that it decreases.

Table 3: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.2$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	ρ^m =0.95	ρ^m =0.99
r^{Gap}	+	+	+	+	+	+	+	+	+	-	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the specific real interest rate gap increases right after a positive monetary shock; - indicates that it decreases.

Table 4: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.5$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	ρ^m =0.95	ρ^m =0.99
r^{Gap}	+	+	+	+	+	+	+	+	+	+	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the specific real interest rate gap increases right after a positive monetary shock; - indicates that it decreases.

4 Interest-rate smoothing

I now show that adopting interest-rate smoothing in the Taylor Rule can deliver impulse-response functions with the sign consistent with the real interest rate channel — at least within the empirically relevant parameter range — even for the state-variant real interest rate gap, r_t^{Gap}. This finding largely minimizes the identification problem from an empirical perspective and provides new insight into its mechanics.

4.1 Smoothing the policy rule

I substitute the Taylor rule (12), adopted by Rupert and Šustek (2019), with one that includes interest-rate smoothing (39), whose persistence is governed by $\rho^i \in [0,1)$.

$$i_t = \rho^i i_{t-1} + \left(1-\rho^i\right)(i + \nu\pi_t) + \xi_t^m \Big\} \tag{39}$$

The reduced 4-equation system with the new policy rule becomes:

$$\left.\begin{aligned}
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \rho^i\hat{i}_{t-1} + \left(1-\rho^i\right)\nu\pi_t - \mathbb{E}_t\,\pi_{t+1} + \xi_t^m && (40)\\
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \mathbb{E}_t\,\hat{g}_{t+1} + \overline{r}^k\,\mathbb{E}_t\left(\hat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\hat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\hat{k}_{t+1}\right) && (41)\\
\pi_t &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t - \alpha\frac{1+\eta}{1-\alpha}\hat{k}_t + \hat{c}_t\right) + \beta\,\mathbb{E}_t\,\pi_{t+1} && (42)\\
\hat{y}_t &= \frac{\overline{c}}{\overline{y}}\hat{c}_t + \frac{\overline{k}}{\overline{y}}\hat{k}_{t+1} - (1-\delta)\frac{\overline{k}}{\overline{y}}\hat{k}_t && (43)
\end{aligned}\right\}$$

where $\Psi \equiv \frac{\overline{\chi}(1-\theta)(1-\theta\beta)}{\theta}$, such that when prices are flexible, $\Psi \to \infty$. Moreover, $G_{t+1} \equiv \frac{q_{t+1}}{q_t}$ is the capital gain, so $\hat{g}_t = \hat{q}_t - \hat{q}_{t-1} = \overline{\kappa}\left(\hat{k}_{t+1} - \hat{k}_t\right) - \overline{\kappa}\left(\hat{k}_t - \hat{k}_{t-1}\right)$, where $\overline{\kappa} = \kappa\overline{k}$.

To check whether the negative response of real interest rates to a positive monetary shock remains an identification problem, I sweep the combinations of parameter values for $\rho^m \in [0:0.1:0.9, 0.95, 0.99]$ and $\rho^i \in [0:0.1:0.9, 0.95, 0.99]$. Table 5 displays the sign of the reaction of the real interest rate right after the shock for all combinations under $\delta = 0.025$ and $\kappa = 0.0$. Tables 6 and 7 increase κ to 0.1 and 0.5, respectively. As one can see, under the hypothesis of no adjustment costs, ρ^i must be at least 0.95 to guarantee a positive response under all values of ρ^m. However, even a small adjustment cost, like $\kappa = 0.1$, is enough to largely increase the parameter range consistent with a real interest rate channel of monetary policy.

Table 5: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.0$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	ρ^i =0.95	ρ^i =0.99
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	-	-	-	-	-	-	-	+	+	+	+	+
$\rho^m = 0.2$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.3$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.4$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.5$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.6$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.7$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.8$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.9$	-	-	-	-	-	-	-	-	-	+	+	+
ρ^m =0.95	-	-	-	-	-	-	-	-	+	+	+	+
ρ^m =0.99	-	-	-	-	-	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Table 6: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.1$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	ρ^i =0.95	ρ^i =0.99
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.8$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.9$	-	+	+	+	+	+	+	+	+	+	+	+
ρ^m =0.95	-	+	+	+	+	+	+	+	+	+	+	+
ρ^m =0.99	-	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Table 7: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.5$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	ρ^i =0.95	ρ^i =0.99
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.8$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.9$	+	+	+	+	+	+	+	+	+	+	+	+
ρ^m =0.95	-	+	+	+	+	+	+	+	+	+	+	+
ρ^m =0.99	-	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

So, how restricting is the direction-switching behavior of the real interest rate in response to a monetary shock for the estimation of VARs and DSGEs? We have seen that, in the presence of interest-rate smoothing, an empirically validated (Coibion and Gorodnichenko, 2012), theoretically desirable (Woodford (2003b), Sack and Wieland (2000)), and prevalent feature of medium-scale DSGE models (Smets and Wouters (2003) estimates $\rho^i = 0.956$ for the Euro Area; Smets and Wouters (2007) estimates $\rho^i = 0.75 - 0.84$ for the United States), a plausibly small adjustment cost is enough to reestablish the sign consistency with the real interest rate channel. Thus, it is due to realistic combinations of endogenous state variable rigidities that observational consistency is obtained in New Keynesian models augmented with capital.

4.2 The mechanics of smoothing

Now, using the method of undetermined coefficients, I make explicit the solution for the real interest rate and compare it to the exposition of Rupert and Šustek (2019).

Originally, there are three state variables ($\hat{k}_t$, ξ^m_{t-1}, $\hat{i}_{t-1}$) and one shock (ϵ^m_t).[9] To reduce the number of coefficients I have to solve for, this representation can be simplified to just

[9] In Dynare code, there is an additional state variable, k_{t-1}, that is used just for plotting capital at the beginning of the period.

three state variables ($\hat{k}_t$, ξ_t^m, $\hat{i}_{t-1}$) using the monetary shock process equation. For the four jump variables, I assume $\hat{c}_t = a_0\hat{k}_t + a_1\xi_t^m + a_2\hat{i}_{t-1}$; $\pi_t = b_0\hat{k}_t + b_1\xi_t^m + b_2\hat{i}_{t-1}$; $\hat{y}_t = d_0\hat{k}_t + d_1\xi_t^m + d_2\hat{i}_{t-1}$; $\hat{k}_{t+1} = f_0\hat{k}_t + f_1\xi_t^m + f_2\hat{i}_{t-1}$. The set of coefficients to be determined for the solution of the full system is $\{a_0, a_1, a_2, b_0, b_1, b_2, d_0, d_1, d_2, f_0, f_1, f_2\}$.

With the log-linearized Fisher relation, $\hat{r}_t = \hat{i}_t - \mathbb{E}_t\pi_{t+1}$, and the Euler equation (40) I can write:

$$\hat{r}_t = \mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t$$

$$= \underbrace{\left(a_0 f_0 - a_0 + a_2\left(1-\rho^i\right)\nu b_0\right)\hat{k}_t + \left(a_0 f_2 - a_2 + a_2\rho^i + a_2\left(1-\rho^i\right)\nu b_2\right)\hat{i}_{t-1}}_{=0 \text{ at the shock}}$$

$$+ \left(\underbrace{\underbrace{\rho^m a_1 - a_1}_{\text{ex-smoothing}} + \underbrace{a_2\left(1-\rho^i\right)\nu b_1 + a_2}_{\text{smoothing}}}_{\text{indirect effect of capital}} \quad \underbrace{+a_0 f_1}_{\text{direct effect of capital}} \right)\xi_t^m$$

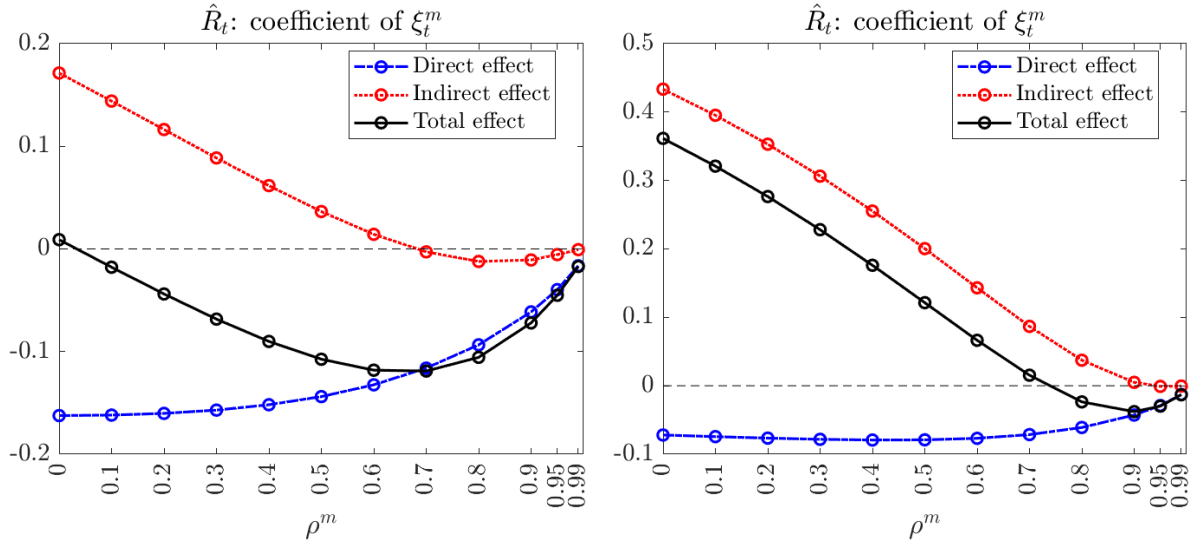
where $\hat{r}$ is the log deviation of the real interest rate. When I remove interest-rate smoothing, that is, when $\rho^i = 0$, $a_2 = 0$, $b_2 = 0$, $d_2 = 0$, and $f_2 = 0$, the model is the same as the one portrayed in Rupert and Šustek (2019).

The numerical coefficients can be extracted from the decision rules obtained for first-order solutions in Dynare (Adjemian et al. (2024)). Each coefficient is also a partial derivative with respect to a state variable or a shock (i.e., $a_0 \equiv \frac{\partial\hat{c}_t}{\partial\hat{k}_t}$). With that in mind, I can decompose the immediate effect of the shock on the real interest rate into a direct effect of capital and an indirect one. The direct effect is analytically the same as in Rupert and Šustek (2019) since it depends only on the existence of endogenous capital in the model. The indirect effect, on the other hand, can be decomposed into two components: ex-smoothing and smoothing. The ex-smoothing component is the full indirect effect in Rupert and Šustek (2019), whereas the smoothing component appears in my model whenever $\rho^i > 0$. Although the direct effect of capital is always negative, the indirect effect can switch signs depending on how much consumption smoothing is allowed. For that, the shock's persistence, the policy rate's smoothing, and capital adjustment costs are key.[10] I call the total effect the sum of the direct and indirect effects.

[10]The depreciation rate of capital, δ, is also important because it sets the $\overline{k} = \frac{\overline{K}}{\overline{Y}}$, but I prefer to keep it fixed to simplify the analysis.

$$\frac{\partial \hat{r}_t}{\partial \xi_t^m} = \underbrace{\underbrace{\left(\rho^m - 1\right)\frac{\partial \hat{c}_t}{\partial \xi_t^m}}_{\text{ex-smoothing}} + \underbrace{\left(1 + \left(1-\rho^i\right)\nu\frac{\partial \hat{\pi}_t}{\partial \xi_t^m}\right)\frac{\partial \hat{c}_t}{\partial i_{t-1}}}_{\text{smoothing}}}_{\text{indirect effect of capital}} \qquad \underbrace{+\frac{\partial \hat{c}_t}{\partial \hat{k}_t}\frac{\partial \hat{k}_{t+1}}{\partial \xi_t^m}}_{\text{direct effect of capital} < 0}$$

Under the benchmark calibration, with no adjustment costs and no interest-rate smoothing, the direct effect of capital on the real interest rate from a monetary shock is negative for all possible values of ρ^m, while the indirect effect is mostly positive. The absolute indirect effect is larger than the direct one only at the lowest range of ρ^m, as seen in Figure 3a. Note that for considerably persistent monetary shocks ($\rho^m > 0.7$), the indirect effect can be negative, which implies $\frac{\partial \hat{c}_t}{\partial \xi_t^m} > 0$, an atypical situation in which the prospect of a long spell of deflation motivates a consumption increase in the present due to income effect.[11] Meanwhile, in Figure 3b, I show that raising κ to 0.1 increases both components of the total effect, amplifying the range consistent with the real interest rate channel.



(a) $\kappa = 0$ (b) $\kappa = 0.1$

Figure 3: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary shock when $\rho^i = 0$

In Figure 4, I introduce interest-rate smoothing by setting $\rho^i = 0.5$, with no capital adjustment costs. In that case, the total effect curve becomes flatter near the zero axis. Raising the adjustment cost to $\kappa = 0.1$, as in Figure 5, is enough to turn the total effect curve positive for all possible values of ρ^m even with just a little interest-rate smoothing ($\rho^i = 0.1$).

[11] This case is explored in more detail in Section 4.3.3.

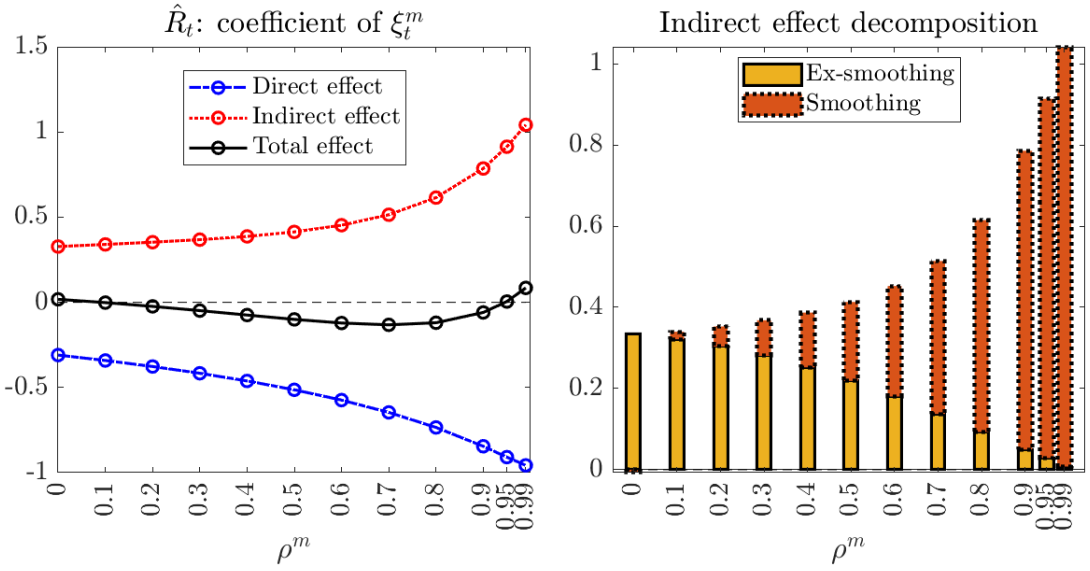


Figure 4: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary shock when $\rho^i = 0.5$ and $\kappa = 0$

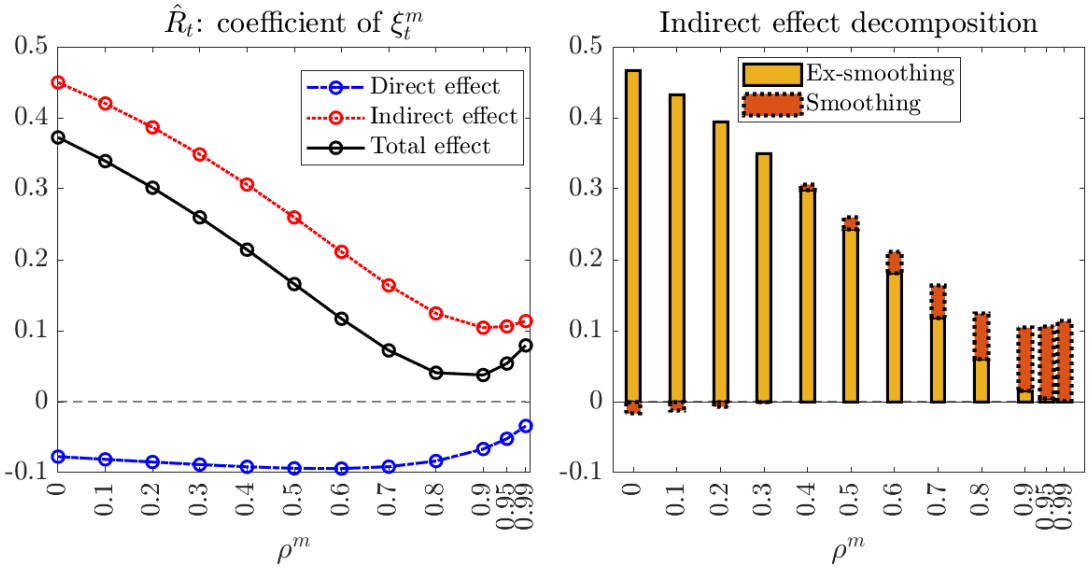


Figure 5: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary shock when $\rho^i = 0.1$ and $\kappa = 0.1$

4.3 Impulse response functions

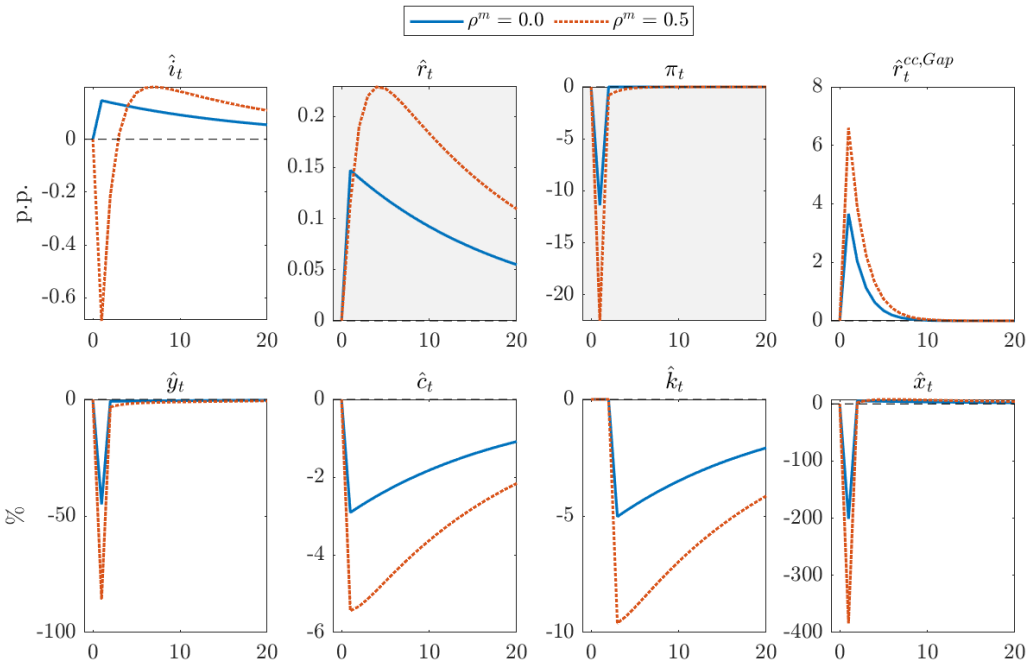
Next, I plot the impulse response functions of the New Keynesian model augmented with endogenous capital, adjustment costs, and interest-rate smoothing. I calibrate the standard deviation of the monetary shock to 1 p.p. The graphs display percentage deviations from steady-state values, except for interest rates, which are measured in p.p. deviations from

steady-state values. As expected, output, consumption, and inflation respond negatively in the event of a contractionary shock, except for the atypical case in which the income effect dominates the intertemporal substitution of consumption. In this last case, output and inflation fall, but consumption expands. The capital stock also decreases, but with a lag due to my timing convention. However, the nominal interest rate may react either positively or negatively, as the sign depends on inflation expectations and actual inflation, both of which may decrease significantly in the presence of persistence of the monetary shock, a well-documented pattern (Galí (2015) and Woodford (2003a, sec. 4.2.4)). Most importantly, the state-invariant real interest rate gap, $r_t^{cc,Gap}$, is always positive right after a positive monetary shock.[12]

4.3.1 Fixing with very high interest-rate smoothing

Figure 6 shows that a very high level of interest-rate smoothing ($\rho^i = 0.95$) reestablishes the observational consistency with the real interest rate channel of monetary policy transmission. However, without capital adjustment costs, output overreacts, becoming unrealistic.

[12] Interest-rate smoothing adds a new state variable to the model, which is lagged interest rate. Fixing that state would violate the determinacy condition of the model, therefore I refrain from fixing it in the calculation of $r_t^{cc,Gap}$ for the graphs of this section.

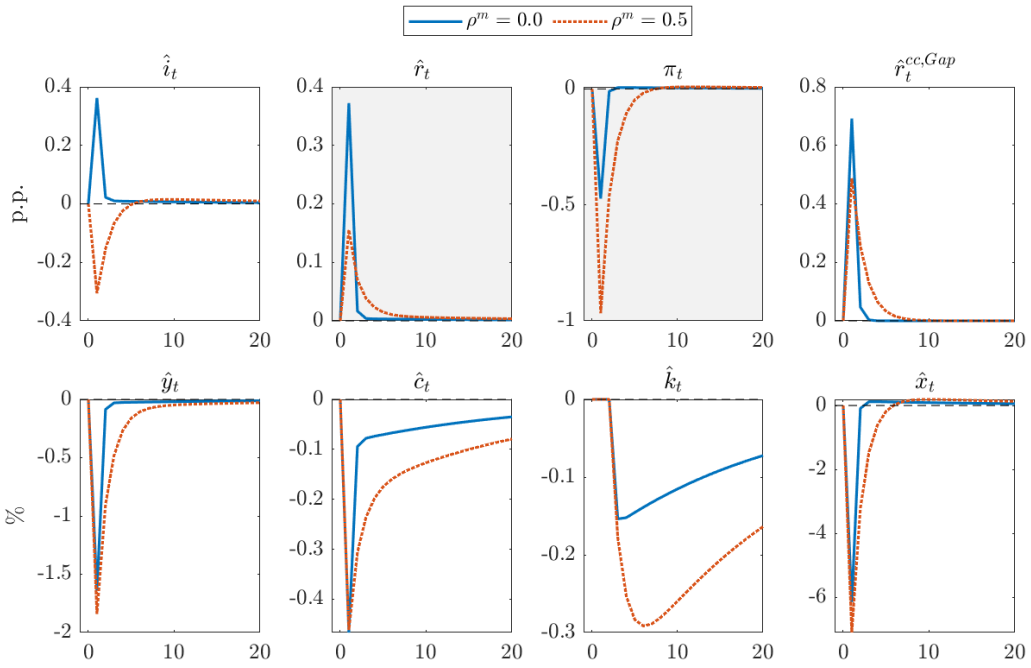


Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), real interest rate gap with constant capital ($\hat{r}_t^{cc,Gap}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

Figure 6: Impulse response function to a one-standard-deviation monetary shock under $\rho^i = 0.95$ and $\kappa = 0$

4.3.2 Fixing with very low interest-rate smoothing and small adjustment cost

Figure 7 shows that simply combining a very low level of smoothing ($\rho^i = 0.1$) with a small adjustment cost ($\kappa = 0.1$) resolves the identification problem. The adjustment cost still prevents output from overreacting right after the shock. Moreover, the negative association between changes in inflation and changes in the real interest rate does not depend on inflation expectations, differing from what is observed for changes in the nominal interest rate, whose sign depends on the persistence of the monetary shock.



Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), real interest rate gap with constant capital ($\hat{r}_t^{cc,Gap}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

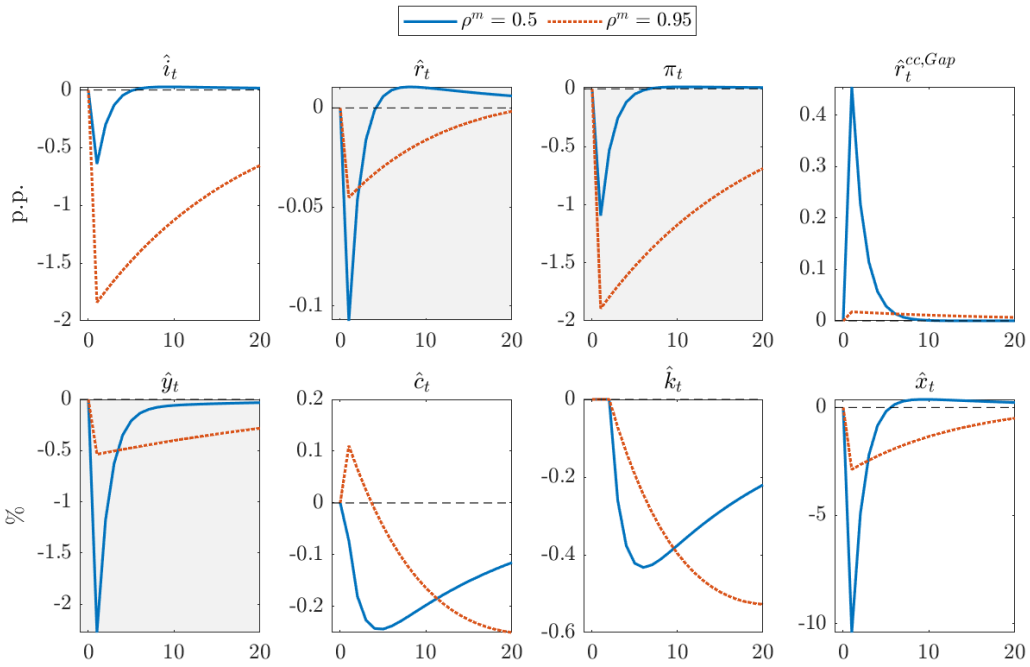
Figure 7: Impulse response function to a one-standard-deviation monetary shock under $\rho^i = 0.1$ and $\kappa = 0.1$

4.3.3 The atypical consumption situation

In Figure 3a, I showed that for considerably persistent monetary shocks, say $\rho^m > 0.7$, the indirect effect of capital can be negative, which implies $\frac{\partial \hat{c}_t}{\partial \xi_t^m} > 0$. This atypical situation in representative agent New Keynesian (RANK) models appears because the prospect of a long spell of deflation motivates a consumption increase in the present due to investment being much more elastic than output to a monetary shock in the absence of either capital or investment adjustment costs – an extreme case in which the income effect of the shock dominates its intertemporal substitution effect.[13] As can be seen in Figure 8, by comparing the impulse responses to a contractionary monetary shock for $\rho^m = 0.5$ and $\rho^m = 0.95$, the inconsistency with the real interest rate channel is present in this atypical case, but can also

[13]Kaplan, Moll and Violante (2018) point out that, for any reasonable parameterization, monetary policy in RANK models works almost exclusively through intertemporal substitution, in contrast with heterogeneous agent models, in which indirect effects, such as the ones that arise from changes in labor income, play a larger role.

be solved with interest-rate smoothing as shown in Figures 4 and 5. Here, the interpretation of $\hat{r}_t^{cc,Gap}$ is more challenging and raises a caveat. Although its sign at the impact of the monetary shock is consistent with theory, the total size of the tightening is smaller when ρ^m is higher and the income effect dominates.



Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), real interest rate gap with constant capital ($\hat{r}_t^{cc,Gap}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

Figure 8: Impulse response function to a one-standard-deviation monetary shock under $\rho^m = 0.5$ and $\rho^m = 0.95$

5 Can Smets and Wouters (2007) do it?

Moving from textbooks to real-world central banking practice, how likely is it that the real interest rate channel identification problem explored in this paper will appear? Quite unlikely. Medium-scale New Keynesian models often incorporate some additional ingredients that induce consumption and investment smoothing, complementing interest-rate smoothing in the Taylor rule. A more complete specification of the latter — responding to output change, output gap, and inflation expectation — suggests gradualism in monetary policy. Consumption habits, sticky wages, and investment adjustment costs all favor smooth consumption in general. In that sense, there are plenty of endogenous state variable frictions

which, at the end of the day, guarantee observational consistency with the real interest rate channel of monetary policy.

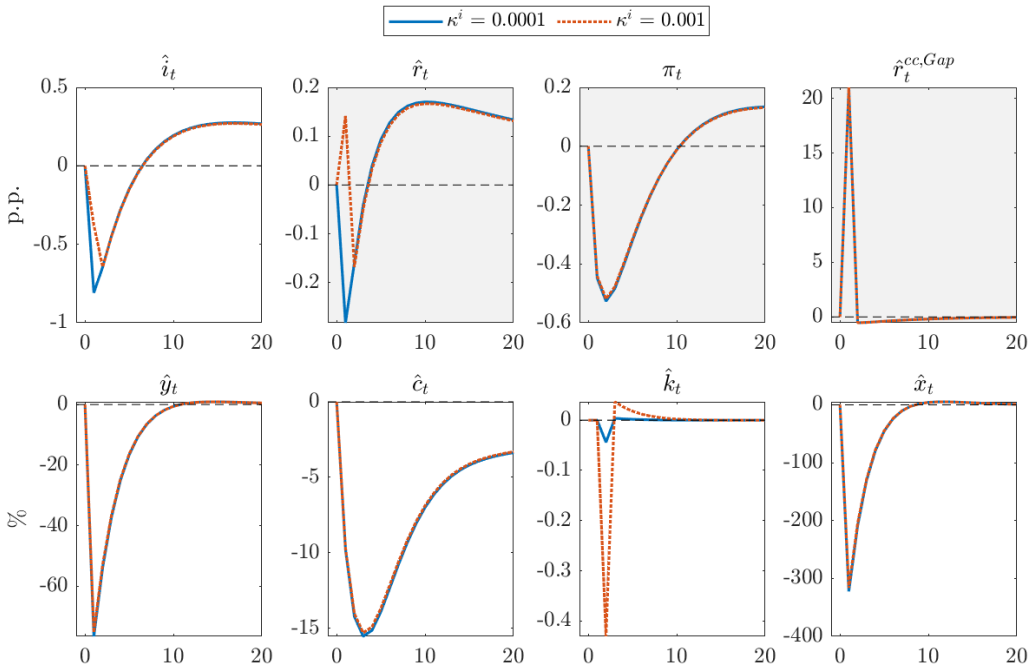
5.1 Impulse response functions

Taking Smets and Wouters (2007) as a reference and starting from calibration at the mode of the estimated parameters' posterior distribution, I can generate the real interest rate channel identification problem if I reduce the investment adjustment cost parameter (κ^i) from 5.4882 to as low as 0.0001 in addition to setting $\rho^m = 0$ and $\rho^i = 0$ (Figure 9).[14] Since κ^i is a structural parameter, not a policy choice like ρ^m and ρ^i, the negative association between changes in the real interest rate and changes in inflation is arguably much more robust than that between the latter and changes in the nominal interest rate, as it is immune to the Lucas (1976) critique. In this simulation, investment falls by more than 300%, exceeding the limits of the log-linear approximation of the model around a nonstochastic steady state. It is remarkable that the profound drops in output and consumption are not reflected in the real interest rate, but are clearly noticeable from the sign and the amplitude of $\hat{r}_t^{cc,Gap}$, which reaches more than 20 p.p.[15] This indicates that $\hat{r}_t^{cc,Gap}$ is not only sign-consistent with the stance of monetary policy, but it also may be a better gauge of its intensity.

In Figure 10, I change the policy parameter ρ^m, keeping $\rho^i = 0$, and apply the same monetary shock. Immediately after the shock, it is possible to make the nominal interest rate to move in either the same or the opposite direction as inflation. Nonetheless, $\hat{r}_t^{cc,Gap}$ is always positive on the impact. Note that both in Figure 9 and Figure 10, $\hat{r}_t^{cc,Gap}$ turns negative after some periods, which did not happen in the canonical model augmented with capital (Section 4.3). This happens in this medium-scale model due to the effect of the other tenths of state variables that exist in it and, for convenience, are still allowed to vary in our calculation of $\hat{r}_t^{cc,Gap}$.

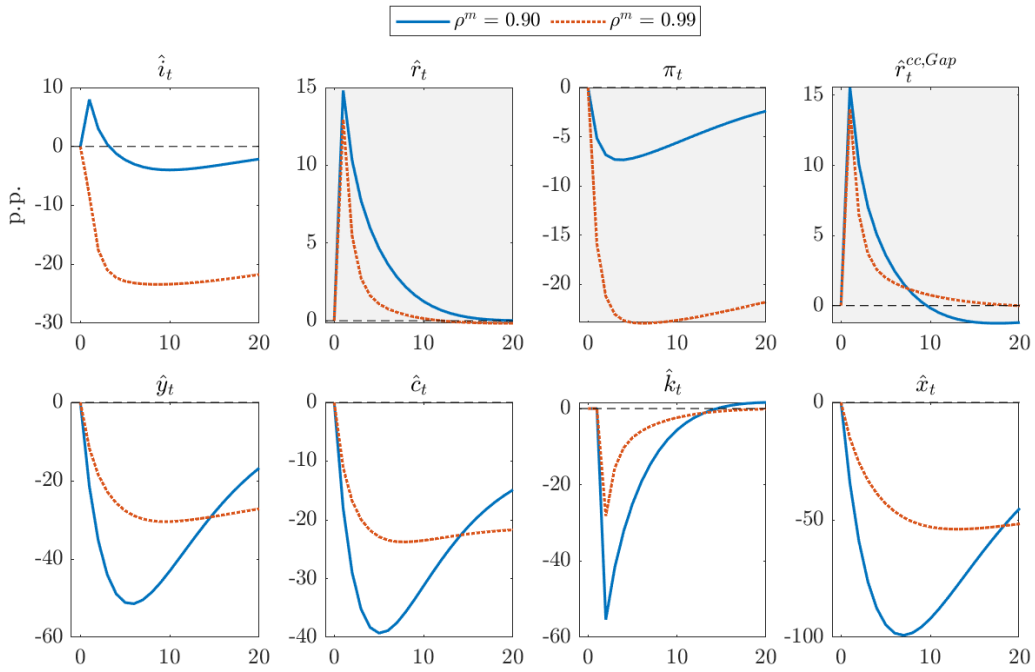
[14] At that calibration, the sign switching requires higher values of κ^i. I thank Johannes Pfeifer for providing Dynare codes for the replication of Smets and Wouters (2007).

[15] $\hat{r}_t^{cc,Gap}$ is calculated by fixing capital stock, capital services, and investment. The removed equations are the "Investment Euler Equation" (SW Equation 3), the "Definition of capital services" (SW Equation 6), and the "Law of motion for capital" (SW Equation 8). In the remaining equations, references to capital stock, capital services or investment are fixed at their steady-state levels.



Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), real interest rate gap with constant capital ($\hat{r}_t^{cc,Gap}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

Figure 9: Smets and Wouters (2007)'s impulse response function to a one-standard-deviation monetary shock under $\kappa^i = 0.0001$ and $\kappa^i = 0.001$. Both calibrations assume $\rho^m = 0$ and $\rho^i = 0$.



Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), real interest rate gap with constant capital ($\hat{r}_t^{cc,Gap}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

Figure 10: Smets and Wouters (2007)'s impulse response function to a one-standard-deviation monetary shock under $\rho^m = 0.90$ and $\rho^m = 0.99$. Both calibrations assume $\rho^i = 0$.

5.2 Interest-rate smoothing

To show how interest-rate smoothing also helps with the identification problem in the Smets and Wouters (2007) model, I raise the investment adjustment cost parameter from 0.001 to 0.005 in Table 8 and sweep for different values of ρ^m and ρ^i. Note that increasing interest-rate smoothing makes that channel more likely, just like in the textbook model. Finally, in Table 9, I double the investment adjustment costs parameter and reestablish the real interest rate channel.

Table 8: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.005$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	ρ^i =0.95	ρ^i =0.99
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.6$	+	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.7$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.8$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.9$	-	-	-	-	-	-	-	+	+	+	+	+
ρ^m =0.95	-	-	-	+	+	+	+	+	+	+	+	+
ρ^m =0.99	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Table 9: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.01$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	ρ^i =0.95	ρ^i =0.99
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.8$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.9$	+	+	+	+	+	+	+	+	+	+	+	+
ρ^m =0.95	+	+	+	+	+	+	+	+	+	+	+	+
ρ^m =0.99	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

5.3 Forecasting inflation with the real interest rate gap

Neiss and Nelson (2003) find that the state-variant real interest rate gap contains informational content that helps forecasting inflation, although the very own real interest rate is still a

better, and more readily available, indicator. I conduct a similar exercise by simulating the Smets and Wouters (2007)'s model with all its different types of shocks activated and find that the state-invariant gap has similar properties, also helping with forecasting inflation (Tables 10 to 17). Starting from the non-stochastic steady state, I simulate 5,200 periods and drop the first 200 periods.

When investment adjustment costs are meaningful, such as at the mode of the estimated parameters' posterior distribution, $\hat{r}_t^{Gap}$ is a slightly better predictor than $\hat{r}_t^{Gap,cc}$, as can be seen by the higher adjusted R-squared statistic achieved in Model 2 in comparison to Model 3. However, when investment adjustment costs are negligible, such as $\kappa^i = 0.0001$, leading to the appearance of the capital puzzle, the relation inverts and only $\hat{r}_t^{Gap,cc}$ is sign-consistent with actual monetary policy stance. Overall, $\hat{r}_t$ is still the variable which individually best improves forecasting, except for 4-period ahead annual inflation when investment adjustment costs are significant, in which case $\hat{r}_t^{Gap}$ is the variable that contains more information about the future. Combining all variables in the regression (Model 7) achieves the highest adjusted R-squared statistic across all forecasting set-ups.

Table 10: Forecasting π_{t+1} under realistic investment adjustment costs

Forecasting 1-period ahead inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\hat{\pi}_{t-1}$	0.833***	0.814***	0.820***	0.796***	0.798***	0.798***	0.730***
	(0.00784)	(0.00796)	(0.00790)	(0.00832)	(0.00833)	(0.00837)	(0.0103)
$\hat{r}_{t-1}^{Gap}$		-0.00344***			-0.00130***		-0.0833***
		(0.000332)			(0.000464)		(0.00743)
$\hat{r}_{t-1}^{Gap,cc}$			-0.00329***			-0.000982**	0.0816***
			(0.000344)			(0.000461)	(0.00738)
$\hat{r}_{t-1}$				0.0429***	0.0330***	0.0361***	-0.0195***
				(0.00358)	(0.00503)	(0.00482)	(0.00687)
Constant	0.00969**	0.0108**	0.0106**	0.0127***	0.0124***	0.0125***	0.0134***
	(0.00432)	(0.00427)	(0.00428)	(0.00427)	(0.00426)	(0.00426)	(0.00421)
Observations	4,999	4,999	4,999	4,999	4,999	4,999	4,999
Adjusted R-squared	0.693	0.699	0.698	0.701	0.702	0.702	0.709

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 11: Forecasting $\Delta_4\pi_{t+1}$ under realistic investment adjustment costs

Forecasting 1-period ahead annual inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\Delta_4\hat{\pi}_{t-1}$	0.967***	0.960***	0.964***	0.944***	0.950***	0.951***	0.909***
	(0.00367)	(0.00354)	(0.00354)	(0.00371)	(0.00375)	(0.00380)	(0.00453)
$\hat{r}_{t-1}^{Gap}$		-0.0109***			-0.00658***		-0.189***
		(0.000533)			(0.000773)		(0.0119)
$\hat{r}_{t-1}^{Gap,cc}$			-0.0108***			-0.00582***	0.182***
			(0.000559)			(0.000774)	(0.0119)
$\hat{r}_{t-1}$				0.117***	0.0647***	0.0744***	-0.0489***
				(0.00577)	(0.00836)	(0.00802)	(0.0110)
Constant	0.00840	0.0102	0.00960	0.0161**	0.0138**	0.0140**	0.0168**
	(0.00731)	(0.00703)	(0.00705)	(0.00704)	(0.00700)	(0.00701)	(0.00684)
Observations	4,996	4,996	4,996	4,996	4,996	4,996	4,996
Adjusted R-squared	0.933	0.938	0.938	0.938	0.939	0.939	0.942

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 12: Forecasting π_{t+4} under realistic investment adjustment costs

Forecasting 4-period ahead inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\hat{\pi}_{t-4}$	0.554***	0.531***	0.538***	0.498***	0.499***	0.498***	0.397***
	(0.0118)	(0.0121)	(0.0119)	(0.0125)	(0.0126)	(0.0126)	(0.0155)
$\hat{r}^{Gap}_{t-4}$		-0.00427***			-0.000130		-0.123***
		(0.000501)			(0.000698)		(0.0112)
$\hat{r}^{Gap,cc}_{t-4}$			-0.00397***			0.000335	0.122***
			(0.000520)			(0.000693)	(0.0111)
$\hat{r}_{t-4}$				0.0650***	0.0640***	0.0673***	-0.0146
				(0.00539)	(0.00757)	(0.00725)	(0.0104)
Constant	0.0260***	0.0274***	0.0271***	0.0305***	0.0305***	0.0306***	0.0320***
	(0.00650)	(0.00645)	(0.00646)	(0.00642)	(0.00642)	(0.00642)	(0.00635)
Observations	4,996	4,996	4,996	4,996	4,996	4,996	4,996
Adjusted R-squared	0.305	0.315	0.313	0.324	0.324	0.324	0.340

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 13: Forecasting $\Delta_4\pi_{t+4}$ under realistic investment adjustment costs

Forecasting 4-period ahead annual inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\Delta_4\hat{\pi}_{t-4}$	0.692***	0.673***	0.685***	0.628***	0.648***	0.650***	0.518***
	(0.0103)	(0.00986)	(0.00987)	(0.0104)	(0.0105)	(0.0106)	(0.0125)
$\hat{r}^{Gap}_{t-4}$		-0.0322***			-0.0213***		-0.600***
		(0.00148)			(0.00215)		(0.0328)
$\hat{r}^{Gap,cc}_{t-4}$			-0.0319***			-0.0189***	0.579***
			(0.00155)			(0.00215)	(0.0328)
$\hat{r}_{t-4}$				0.331***	0.163***	0.194***	-0.198***
				(0.0161)	(0.0233)	(0.0223)	(0.0304)
Constant	0.0713***	0.0769***	0.0749***	0.0933***	0.0858***	0.0863***	0.0952***
	(0.0204)	(0.0195)	(0.0196)	(0.0196)	(0.0195)	(0.0195)	(0.0189)
Observations	4,993	4,993	4,993	4,993	4,993	4,993	4,993
Adjusted R-squared	0.476	0.522	0.517	0.517	0.526	0.524	0.554

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 14: Forecasting π_{t+1} under negligible investment adjustment costs

Forecasting 1-period ahead inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\hat{\pi}_{t-1}$	0.926***	0.930***	0.895***	0.883***	0.889***	0.850***	0.858***
	(0.00536)	(0.00534)	(0.00548)	(0.00567)	(0.00572)	(0.00812)	(0.00826)
$\hat{r}_{t-1}^{Gap}$		0.125***			0.0820***		0.0732***
		(0.0140)			(0.0138)		(0.0139)
$\hat{r}_{t-1}^{Gap,cc}$			-0.0101***			0.0337***	0.0298***
			(0.000570)			(0.00586)	(0.00589)
$\hat{r}_{t-1}$				0.00970***	0.00912***	0.0407***	0.0366***
				(0.000525)	(0.000532)	(0.00541)	(0.00545)
Constant	0.00449	0.00225	0.00675	0.00722	0.00559	0.00846	0.00686
	(0.00701)	(0.00696)	(0.00680)	(0.00678)	(0.00676)	(0.00676)	(0.00675)
Observations	4,999	4,999	4,999	4,999	4,999	4,999	4,999
Adjusted R-squared	0.857	0.859	0.865	0.866	0.867	0.867	0.867

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 15: Forecasting $\Delta_4\pi_{t+1}$ under negligible investment adjustment costs

Forecasting 1-period ahead annual inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\Delta_4\hat{\pi}_{t-1}$	0.977***	0.990***	0.970***	0.958***	0.968***	0.884***	0.895***
	(0.00307)	(0.00301)	(0.00260)	(0.00254)	(0.00253)	(0.00293)	(0.00301)
$\hat{r}_{t-1}^{Gap}$		0.630***			0.431***		0.291***
		(0.0298)			(0.0251)		(0.0226)
$\hat{r}_{t-1}^{Gap,cc}$			-0.0457***			0.337***	0.318***
			(0.00102)			(0.00868)	(0.00867)
$\hat{r}_{t-1}$				0.0442***	0.0417***	0.345***	0.326***
				(0.000889)	(0.000876)	(0.00779)	(0.00780)
Constant	0.00673	-0.00647	0.0108	0.0126	0.00320	0.0221**	0.0153
	(0.0152)	(0.0146)	(0.0129)	(0.0124)	(0.0121)	(0.0109)	(0.0107)
Observations	4,996	4,996	4,996	4,996	4,996	4,996	4,996
Adjusted R-squared	0.953	0.957	0.966	0.969	0.970	0.976	0.977

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 16: Forecasting π_{t+4} under negligible investment adjustment costs

Forecasting 4-period ahead inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\hat{\pi}_{t-4}$	0.659***	0.670***	0.620***	0.603***	0.621***	0.524***	0.554***
	(0.0107)	(0.0106)	(0.0111)	(0.0115)	(0.0115)	(0.0164)	(0.0166)
$\hat{r}_{t-4}^{Gap}$		0.329***			0.278***		0.258***
		(0.0276)			(0.0278)		(0.0279)
$\hat{r}_{t-4}^{Gap,cc}$			-0.0131***			0.0796***	0.0659***
			(0.00115)			(0.0119)	(0.0119)
$\hat{r}_{t-4}$				0.0128***	0.0109***	0.0860***	0.0716***
				(0.00106)	(0.00107)	(0.0110)	(0.0110)
Constant	0.0203	0.0144	0.0232*	0.0239*	0.0184	0.0269**	0.0212
	(0.0139)	(0.0137)	(0.0138)	(0.0137)	(0.0136)	(0.0137)	(0.0136)
Observations	4,996	4,996	4,996	4,996	4,996	4,996	4,996
Adjusted R-squared	0.433	0.448	0.447	0.449	0.459	0.453	0.463

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 17: Forecasting $\Delta_4\pi_{t+4}$ under negligible investment adjustment costs

Forecasting 4-period ahead annual inflation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$\Delta_4\hat{\pi}_{t-4}$	0.739***	0.776***	0.719***	0.687***	0.716***	0.567***	0.606***
	(0.00958)	(0.00944)	(0.00834)	(0.00829)	(0.00832)	(0.0106)	(0.0109)
$\hat{r}^{Gap}_{t-4}$		1.804***			1.251***		1.038***
		(0.0935)			(0.0822)		(0.0815)
$\hat{r}^{Gap,cc}_{t-4}$			-0.132***			0.549***	0.482***
			(0.00328)			(0.0313)	(0.0313)
$\hat{r}_{t-4}$				0.124***	0.117***	0.614***	0.548***
				(0.00290)	(0.00287)	(0.0281)	(0.0282)
Constant	0.0608	0.0228	0.0732*	0.0778*	0.0505	0.0936**	0.0690*
	(0.0473)	(0.0457)	(0.0412)	(0.0405)	(0.0397)	(0.0393)	(0.0388)
Observations	4,993	4,993	4,993	4,993	4,993	4,993	4,993
Adjusted R-squared	0.544	0.575	0.655	0.666	0.681	0.685	0.695

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

6 Conclusion

Yes, a central bank can tighten and real interest rates fall under monetary dominance. The capital puzzle arises when the capital elasticity to the shock is so high that inflation expectations overreact, causing the endogenous, or systematic, component of the monetary policy rule to more than offset its exogenous one. This finding has important implications for the design and implementation of monetary policy, particularly in junctures when investment is significantly impacted, such as a financial crisis or a pandemic.

This paper demonstrates that the real interest rate channel is, indeed, structural in New Keynesian models and that the identification problem that emerges once one augments them with endogenous capital, as revealed by Rupert and Šustek (2019), stems from a problem of definition. The appropriate measure of the real interest rate gap, which accurately reflects the sign of the stance of monetary policy, compares the counterfactual ex-ante real interest rate as if state variables were invariant to its counterpart without nominal rigidities, called in this paper the state-invariant definition. Simply comparing the actual real interest rate to a counterfactual without nominal rigidities, the state-variant definition, as ubiquitously done in the literature (e.g., Neiss and Nelson (2003)), or isolating the exogenous from the endogenous component as in Woodford (2003a), is a sometimes misleading approximation, which can result in the wrong perception that monetary policy does not operate through the real interest rate channel.

Furthermore, the identification problem can be mitigated by including empirically validated interest-rate smoothing into the Taylor rule, a feature as prevalent in medium-scale New Keynesian models such as Smets and Wouters (2003, 2007) as capital itself. The sign of changes in the real interest rate right after a positive monetary shock is positive under realistic parameters, thereby reestablishing the observational consistency of the real interest rate channel of monetary policy transmission and weakening the empirical relevance of the New Keynesian capital puzzle. This finding suggests that it is acceptable to identify VAR models by imposing the same sign restriction on the real interest rate's response to a monetary shock. It is also acceptable to sequentially order nominal and real rates, as done in Cholesky decompositions. Moreover, using real rates instead of nominal ones still — at least partially — captures monetary policy shocks. Finally, interpretation of impulse responses from canonical DSGE models through the real interest rate channel remains consistent as

long as the monetary policy rule includes a sufficient degree of smoothing.

Ultimately, calculating the state-invariant real interest rate gap is enough for checking the sign of monetary policy stance. It may also be a better gauge of its intensity, in addition to improving the forecasting of future inflation. Alternatively, the state-variant real interest rate gap may be a good approximation if one incorporates a somewhat realistic combination of endogenous state variable rigidities, for example, acknowledging that at least some smoothing is the norm in central banking and that capital adjustment costs are never negligible in the real world. Overall, the negative association between changes in inflation and changes in the real interest rate in New Keynesian models is actually more robust than that between the former and changes in the nominal interest rate, which is more likely to be ambiguous.

References

Adjemian, Stéphane, Michel Juillard, Fréderic Karamé, Willi Mutschler, Johannes Pfeifer, Marco Ratto, Normann Rion, and Sébastien Villemot. 2024. "Dynare: Reference Manual, Version 6." CEPREMAP Dynare Working Papers 80.

Altig, David, Lawrence J Christiano, Martin Eichenbaum, and Jesper Lindé. 2011. "Firm-specific capital, nominal rigidities and the business cycle." Review of Economic Dynamics, 14(2): 225–247.

Amaral, Eduardo. 2024. "Smoothing the New-Keynesian Capital Puzzle." BCB Working Paper Series, 606.

Benoit. 1924. "Note Sur Une Méthode de Résolution des équations Normales Provenant de L'Application de la MéThode des Moindres Carrés a un Système D'équations Linéaires en Nombre Inférieur a Celui des Inconnues. — Application de la Méthode a la Résolution D'un Système Defini D'éQuations LinéAires." Bulletin géodésique, 2(1): 67–77.

Brault, Joshua, and Hashmat Khan. 2022. "The Real Interest Rate Channel Is Structural in Contemporary New-Keynesian Models: A Note." Journal of Money, Credit and Banking, 54(5): 1551–1563.

Carrillo, Julio, Patrick Fève, and Julien Matheron. 2018. "Monetary policy inertia or persistent shocks: a DSGE analysis." Eighth issue (March 2007) of the International Journal of Central Banking.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans. 2005. "Nominal rigidities and the dynamic effects of a shock to monetary policy." Journal of Political Economy, 113(1): 1–45.

Clarida, Richard, Jordi Galí, and Mark Gertler. 1999. "The science of monetary policy: a new Keynesian perspective." Journal of Economic Literature, 37(4): 1661–1707.

Coibion, Olivier, and Yuriy Gorodnichenko. 2012. "Why are target interest rate changes so persistent?" American Economic Journal: Macroeconomics, 4(4): 126–162.

Cour-Thimann, Philippine, Rasmus Pilegaard, and Livio Stracca. 2006. "The output gap and the real interest rate gap in the euro area, 1960–2003." Journal of Policy Modeling, 28(7): 775–790.

Galí, Jordi. 2015. Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. . 2nd ed., Princeton, USA:Princeton University Press.

Goodfriend, Marvin. 1987. "Interest rate smoothing and price level trend-stationarity." Journal of Monetary Economics, 19(3): 335–348.

Holden, Tom D. 2024. "Robust Real Rate Rules." Econometrica, 92(5): 1521–1551.

Ireland, Peter N. 2010. "Monetary transmission mechanism." In Monetary economics. 216–223. Springer.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante. 2018. "Monetary policy according to HANK." American Economic Review, 108(3): 697–743.

Kimball, Miles. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model." Journal of Money, Credit and Banking, 27(4): 1241–1277.

Laubach, Thomas, and John C Williams. 2003. "Measuring the natural rate of interest." Review of Economics and Statistics, 85(4): 1063–1070.

Lubik, Thomas A, Christian Matthes, et al. 2015. "Calculating the natural rate of interest: A comparison of two alternative approaches." Richmond Fed Economic Brief, , (Oct): 1–6.

Lucas, Robert. 1976. "Econometric policy evaluation: A critique." Vol. 1 of Carnegie-Rochester conference series on public policy, 19–46, North-Holland.

Neiss, Katharine S, and Edward Nelson. 2003. "The real-interest-rate gap as an inflation indicator." Macroeconomic dynamics, 7(2): 239–262.

Rudebusch, Glenn D. 2006. "Monetary Policy Inertia: Fact or Fiction?" International Journal of Central Banking.

Rupert, Peter, and Roman Šustek. 2019. "On the mechanics of New-Keynesian models." Journal of Monetary Economics, 102: 53–69.

Sack, Brian, and Volker Wieland. 2000. "Interest-rate smoothing and optimal monetary policy: a review of recent empirical evidence." Journal of Economics and Business, 52(1-2): 205–228.

Schnabel, Isabel. 2024. "R (ising) star?" SUERF Policy Note, 345(Apr).

Smets, Frank, and Rafael Wouters. 2003. "An estimated dynamic stochastic general equilibrium model of the Euro Area." Journal of the European Economic Association, 1(5): 1123–1175.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." American Economic Review, 97(3): 586–606.

Uhlig, Harald. 2005. "What are the effects of monetary policy on output? Results from an agnostic identification procedure." Journal of Monetary Economics, 52(2): 381–419.

Waller, Christopher J. 2024. "Some Thoughts on r*: Why Did It Fall and Will It Rise?" Speech at the Reykjavik Economic Conference, Reykjavik, Iceland.

Wicksell, Knut. 1898. Interest and Prices: A Study of the Causes Regulating the Value of Money. Originally published as Geldzins und Güterpreise in 1898. Translated to English in 1936 for the Royal Economic Society. Republished by Read Books Ltd. in 2013.

Wicksell, Knut. 1907. "The influence of the rate of interest on prices." The Economic Journal, 17(66): 213–220.

Woodford, Michael. 2003a. Interest and Prices: Foundations of a Theory of Monetary Policy. . 1st ed., Princeton, USA:Princeton University Press.

Woodford, Michael. 2003b. "Optimal interest-rate smoothing." The Review of Economic Studies, 70(4): 861–886.

A Glossary of symbols

Table 18: Model variables

Symbol	Definition/Description
Original Variables (Levels)	
c_t	Consumption
l_t	Labor
y_t	Output
w_t	Real wage
χ_t	Real marginal cost
i_t	Nominal interest rate
r_t	Real interest rate
π_t	Inflation
k_t	Capital stock (beginning of period)
r_t^k	Rental rate of capital
x_t	Investment
q_t	Tobin's q ($q_t \equiv 1 + \kappa(k_{t+1} - k_t)$)
G_{t+1}	Capital gain ($G_{t+1} \equiv q_{t+1}/q_t$)
Log-Deviations from Steady State (Hatted Variables)	
$\hat{c}_t$	$\log(c_t/\overline{c})$
$\hat{y}_t$	$\log(y_t/\overline{y})$
$\hat{k}_t$	$\log(k_t/\overline{k})$
$\hat{x}_t$	$\log(x_t/\overline{x})$
$\hat{r}_t$	$r_t - \overline{r}$ (real interest rate deviation)
$\hat{i}_t$	$i_t - \overline{i}$ (nominal interest rate deviation)
$\hat{r}_t^k$	$r_t^k - \overline{r}^k$ (rental rate deviation)
$\hat{\pi}_t$	$\pi_t - \overline{\pi}$ (inflation deviation, $\overline{\pi} = 0$)

Table 19: Model variables (part 2)

Symbol	Definition/Description
Shocks and Policy Variables	
ξ_t^m	Monetary policy shock (AR(1) process)
ε_t^m	Innovation to ξ_t^m (i.i.d. normal)
ρ^m	Persistence of ξ_t^m
Other Variables	
$\hat{r}_t^n$	Natural real interest rate (flex-price counterfactual)
$\hat{r}_t^{cc}$	Real interest rate with capital fixed
$\hat{r}_t^{ncc}$	Natural real interest rate with capital fixed (flex-price counterfactual)
$\hat{r}_t^{Gap}$	State-variant real interest rate gap: $\hat{r}_t - \hat{r}_t^n$
$\hat{r}_t^{cc,Gap}$	State-invariant real interest rate gap: $\hat{r}_t^{cc} - \hat{r}_t^{ncc}$
$\Delta_4 \hat{\pi}_t$	Annual inflation (4-period price level difference)

Table 20: Model deep parameters

Symbol	Name/Definition
β	Subjective discount factor
η	Inverse elasticity of labor supply
ε	Elasticity of substitution between intermediate goods
θ	Fraction of firms not adjusting prices (Calvo parameter)
ν	Taylor rule coefficient on inflation
ρ^m	Persistence of monetary policy shock
α	Capital share in production (Cobb-Douglas)
δ	Depreciation rate of capital
κ	Capital adjustment cost parameter
ρ^i	Interest-rate smoothing coefficient in Taylor rule
Smets-Wouters Model Additions	
κ^I	Investment adjustment cost parameter

Table 21: Model derived parameters

Symbol	Definition/Description
Ω	Slope of the Phillips curve in the canonical model: $\Omega \equiv \frac{(1+\eta)(1-\theta)(1-\theta\beta)}{\theta}$
Ψ	Slope of the Phillips curve with capital: $\Psi \equiv \overline{\chi}\frac{(1-\theta)(1-\theta\beta)}{\theta}$
$\overline{\kappa}$	Effective capital adjustment cost: $\overline{\kappa} \equiv \kappa\overline{K}$

Table 22: Additional coefficients

Symbol	Definition/Description
a, b	Canonical model decision rule coefficients for output and inflation
a_0, a_1, a_2	Consumption decision rule coefficients
b_0, b_1, b_2	Inflation decision rule coefficients
d_0, d_1, d_2	Output decision rule coefficients
f_0, f_1, f_2	Capital decision rule coefficients
$\eta_{nk}, \boldsymbol{\eta_{nkk}}$	Coefficients linking natural real rate to capital and shocks: $\hat{r}^n_t = \hat{r}^{ncc}_t + \eta_{nk}\hat{k}^n_t + \boldsymbol{\eta_{nkk}}\epsilon_t$
η_k, η_{kk}	Coefficients linking actual real rate to capital and shocks: $\hat{r}_t = \hat{r}^{cc}_t + \eta_k\hat{k}_t + \eta_{kk}\epsilon^m_t$

B Steady state

In this section, I derive the nonstochastic steady state of the canonical New Keynesian model augmented with endogenous capital.[16] From their definitions, capital gain and Tobin's q equal 1 at the steady state.

$$\overline{G} = 1 \tag{44}$$

[16]The steady state is the same with and without interest-rate smoothing.

$$\overline{Q} = 1 \tag{45}$$

I pick a zero-inflation steady state. The real rate of return on the implicit risk-free bond of the model is the one obtained from the Fisher equation. Through the no-arbitrage condition, I combine the two Euler equations of the model, (7) and (23), to get a relation between the two real rates.

$$\overline{\pi} = 0 \tag{46}$$

$$\overline{i} = \overline{r} + \overline{\pi} \quad \Rightarrow \quad \overline{i} = \overline{r} \tag{47}$$

$$\left(1 + \overline{i}\right) = \left(1 + \overline{\pi}\right)\left(1 + \frac{\overline{r}^k - \delta}{\overline{Q}}\right) \quad \Rightarrow \quad \overline{i} = \overline{r}^k - \delta \quad \Rightarrow \quad \overline{r} = \overline{r}^k - \delta \tag{48}$$

From the capital Euler equation (23) evaluated at the steady state I can isolate $\overline{r}^k$ as a function of the deep parameters:

$$\frac{1}{\overline{C}} = \beta\left(\frac{1}{\overline{C}}\left(\frac{\overline{r}^k - \delta}{\overline{Q}} + \frac{\overline{Q}}{\overline{Q}}\right)\right) \quad \Rightarrow \quad \overline{r}^k = \frac{1}{\beta} + \delta - 1 \tag{49}$$

Substituting $\overline{r}^k$ into the FOC of capital (25) at the steady state I get $\frac{\overline{K}}{\overline{L}}$:

$$\overline{r}^k = \alpha \overline{K}^{\alpha-1}\overline{L}^{1-\alpha} \quad \Rightarrow \quad \frac{\overline{K}}{\overline{L}} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right)^{\frac{1}{1-\alpha}} \tag{50}$$

The production function (24) at the steady state can be rewritten so as to put in evidence $\frac{\overline{K}}{\overline{L}}$ on the right side of the expression:

$$\overline{Y} = \left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha} \overline{L} \tag{51}$$

Investment at the steady state only compensates for depreciated capital.

$$\overline{I} = \delta \overline{K} \tag{52}$$

Now, the aggregate resource constraint (27) may be rewritten as auxiliary ratios by substituting the just derived expressions for $\overline{Y}$ and $\overline{I}$ and then dividing by $\overline{L}$.

$$\overline{Y} = \overline{C} + \overline{I} \quad \Rightarrow \quad \left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha} = \frac{\overline{C}}{\overline{L}} + \frac{\delta \overline{K}}{\overline{L}} \tag{53}$$

Combining the condition for the optimal mix of capital and labor in production (25) and the intratemporal condition at the steady state (8), I can write an expression for $\frac{\overline{C}}{\overline{L}}$:

$$\overline{W} = (1-\alpha)\overline{K}^{\alpha}\overline{L}^{-\alpha} \quad \text{and} \quad \frac{\overline{W}}{\overline{C}} = \overline{L}^{\eta} \quad \Rightarrow \quad \frac{\overline{C}}{\overline{L}} = (1-\alpha)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha} \overline{L}^{-(\eta+1)} \tag{54}$$

By substituting previously derived expressions for $\frac{\overline{K}}{\overline{L}}$ and $\frac{\overline{C}}{\overline{L}}$ into the aggregate resource constraint, I obtain $\overline{L}$ as a function of the deep parameters of the model.

$$\overline{L} = \left(\left(\frac{1}{1-\alpha}\right)\left(\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}\right)\right)^{\frac{-1}{\eta+1}} \tag{55}$$

I get the remaining steady-state variables as functions of the parameters by recursively substituting (55) into my previously derived expressions: (50) $\Rightarrow$ $\overline{K}$, (51) $\Rightarrow$ $\overline{Y}$, (52) $\Rightarrow$ $\overline{I}$, and (54) $\Rightarrow$ $\overline{C}$ and $\overline{W}$. Finally, I obtain $\overline{\chi}$ by substituting $\overline{r}^k$ and $\overline{W}$ into (26) evaluated at the steady state.