# Non-Parametric Markup Correction*

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March 2025

**Abstract**

Firm-level market power leads to substantial resource misallocation across firms and industries, providing a standard economic rationale for pro-competitive industrial policies. However, the effectiveness of such policies is often doubted, as welfare-improving interventions require detailed knowledge of underlying economic parameters. This paper introduces a non-parametric policy approach that relies solely on ex post public information about firm-level prices and sales. The proposed method leverages the insight that optimal firm-specific interventions can be closely approximated using sales data from similar competitor firms. Drawing on detailed firm-level and input-output data from the US, we estimate that this non-parametric approach achieves 80% of the welfare gains possible from fully eliminating the allocative inefficiency introduced by market power.

## 1 Introduction

Firm market power is a ubiquitous feature of modern economies, as evidenced by a vibrant empirical literature at the intersection of international trade, industrial organization, and macroeconomics. However, the design of industrial policies aimed at curbing

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the inefficiencies arising from such market power has been hindered by two key challenges. First, measuring the extent of market power in the economy is a daunting empirical task that requires accounting for complex input-output linkages within and across industries while ideally providing credible estimates of market power (e.g., price markups) without imposing overly restrictive parametric assumptions on production technologies and demand curves. Second, even if such estimates are available, designing and implementing policy could lead to a complex system of firm-specific subsidies that would be difficult to administer and impossible to enforce if firms possess private information (e.g., about their idiosyncratic productivities). In this paper, we advance the design and implementation of industrial policies that circumvent these challenges, enabling policymakers to more effectively mitigate the inefficiencies associated with market power.

We begin by using a general model of heterogeneous firms with product market power to derive a theoretical benchmark policy that restores marginal cost pricing and achieves an efficient allocation. This policy is a revenue-neutral transfer function that compensates firms with a transfer precisely offsetting the marginal loss of revenue on inframarginal units when a firm increases its sales by lowering its price. The negative effect on revenue from increasing supply is the source of markup distortions, leading firms to underproduce and charge a price above marginal cost. By fully offsetting this negative revenue effect, the transfer policy incentivizes firms to expand production until price equals marginal cost, thereby restoring efficiency. This benchmark transfer policy satisfies the following property: the derivative of the transfer function with respect to the firm's price is equal to the firm's production. Unlike existing approaches, this policy does not require knowledge of the distribution of markups within an industry and can be implemented using only information about the shape of the demand curve.

Requiring knowledge of the demand curve makes implementing the policy discussed above challenging, either due to insufficient information on demand schedules or because the optimal policy is sensitive to the exact details of the model specified by the policy maker. We therefore develop a transfer policy that depends only on data available to the policy maker. We call such policies *non-parametric*. In particular, we assume that the only data available to the policy maker are ex post prices and quantities set by each firm in the economy.

Inspired by our theoretical benchmark, we present a revenue-neutral transfer policy that sets the marginal transfer equal to the quantity produced by a "synthetic" closest neighbor. The quantity produced by this "synthetic" closest neighbor is defined as a weighted average of all other firms' quantities, with greater weights assigned to firms whose prices are closer to the firm's own price. This transfer policy implies that reducing

the firm's price by one unit increases the subsidy by an amount equal to the synthetic neighbor's quantity. This ensures that the marginal transfer closely follows the infra-marginal revenue loss and incentivizes firms to charge a price closer to their marginal cost. Our non-parametric policy bears some similarities to the "yardstick policy" discussed in Shleifer (1985) in the context of efficient cost setting by franchised monopolies.

The quantitative effectiveness of this non-parametric policy in achieving an efficient allocation is an empirical question. To address this, we employ a workhorse model commonly used in the international trade and macroeconomic literature. Specifically, we consider a model with a nested CES demand structured, input-output linkages, heterogeneous firms, and oligopolistic competition. We assume that firm-level markups are the only source of inefficiency in the economy. We calibrate the model using Compustat data for the U.S. To this end, we first estimate firm-level markups using the production-function approach as in De Loecker and Warzynski (2012) and then calibrate the demand parameters in the model to match estimated markups.

We use our calibrated model to compute the welfare gains from restoring marginal-cost pricing and compare them to the gains from implementing our non-parametric transfer policy. We show that our non-parametric policy increases welfare by almost 23% in 2015, which is about 80% of the welfare gains achieved under the first best (29%). Our proposed policy is therefore highly effective in reducing markup-induced misallocation and improving welfare. This is a remarkable result, because our simple policy rule can be implemented using public information on sales and prices and is agnostic to both the underlying demand parameters and the input-output structure.

Furthermore, we show that the presence of input-output linkages slightly reduces the relative efficacy of our transfer policy. Specifically, our non-parametric policy can achieve approximately 87% of the gains from implementing the first-best outcome in a model without input-output linkages. This is slightly higher than in a model with input-output linkages because distortions in one sector propagate to others when industries are interconnected through production networks, making it more difficult for our non-parametric policy to restore efficiency.

We also examine how the gains from the first best and the non-parametric transfer policy evolve over time from 1997 to 2015. Our results indicate a clear upward trend in both gains over time, implying that the extent of misallocation has increased, a finding consistent with the existing literature. The increasing misallocation makes efficiency-enhancing policies even more impactful in recent years. Interestingly, despite the general increase in misallocation, the *relative* efficacy of our transfer policy remains approximately unchanged over time at about 80%. This suggests that while the absolute gains from these

transfers increase as misallocation worsens, their effectiveness in replicating the first-best allocation does not change significantly.

Our model abstracts from the extensive margin by fixing the number of operating firms. One potential concern about the welfare implications of our transfer policy is that the policy may reduce firms' profits disproportionately in high-markup sectors, leading to less entry in these sectors and potentially welfare losses. We therefore evaluate changes in profits across sectors in response to our transfer policy and show that profits do not disproportionately fall and instead slightly rise in high-markup sectors. Hence, in a model featuring extensive margin, the transfer policy would likely encourage entry in high-markup sectors which might be potentially welfare improving.

# 2 Theoretical Model

The economy is populated by $L$ individuals each of whom supplies one unit of labor inelastically. There are multiple industries indexed by $s = 1, ..., S \in \mathbb{S}$. Each industry $s$ is populated by $i = 1, ..., n_s$ firms, with $\mathbb{I}_s$ denoting the set of all firms serving industry $s$. Correspondingly, $\mathbb{I} \equiv \cup_s \mathbb{I}_s$ denotes the set of all firms operating in the economy. Each firm produces a differentiated variety that can be used for either final consumption or as an intermediate input in production. The subscript "$i, s$" is used to denote variables associated with firm $i$ in industry $s$.

## 2.1 Preferences

The representative consumer's utility function combines firm-level goods from various industries. We assume that the utility aggregator is weakly separable across industries. Namely, $U = U(C_1, ..., C_S)$, where $C_s \equiv C_s(c_s)$ is an industry-level bundle that aggregates over all firm-level varieties in industry $s$, $c_s \equiv \{c_{1,s}, ..., c_{n_s,s}\}$. The consumer maximizes her utility by choosing quantities $c \equiv \{c_1, ...., c_S\}$, subject to the budget constraint. This choice yields a indirect utility function,

$$v(Y, \mathbf{p}) = \max_{c} \; U(C_1, ..., C_S) \quad s.t. \quad \sum_{s \in \mathbb{S}} \sum_{i \in \mathbb{I}_s} p_{i,s} c_{i,s} \leq Y$$

which summarizes social welfare as a function of aggregate income, $Y$, and the entire vector of prices, $\mathbf{p} \equiv \{p_1, ..., p_S\}$, where $p_s \equiv \{p_{1,s}, ..., p_{n_s,s}\}$ is the set of prices within in industry $s$.[1]

[1] Aggregate income consists of labor income and firm profits net of any lump sum tax payments.

The consumer's utility maximization problem yields a two-tier demand system. The upper tier describes how total demand is allocated across industries. Given our assumption on separability, the total demand for industry $s$ goods can be expressed as as function of income, $Y$, and industry-level price indexes. Formally, total demand for goods in industry $s$ satisfies

$$C_s = D_s(P_s, P_{-s}, Y) \tag{1}$$

where $P_s \equiv P_s(p_s)$ is an index that aggregates over the set of firm-level prices in industry $s$. Correspondingly, $P_{-s} = \{P_1, ..., P_{s-1}, P_{s+1}, ..., P_S\}$ denotes the vector of industry-level price indexes excluding $P_s$.

The lower-tier demand function describes demand for each firm's good in industry $s$, given total demand $C_s$. We assume that the within-industry utility aggregator is such that the Marshallian demand function for firm-level variety $i \in \mathbb{I}_s$ depends on own price, $p_{i,s}$, an idiosyncratic demand shifter $\omega_{i,s}$, total industry demand, $C_s$, and industry-wide indexes that aggregate information on firm-level prices within industry $s$ and income.

**Assumption 1.** *The final demand for good $i \in \mathbb{I}_s$ is fully determined by own price $p_{i,s}$, an idiosyncratic demand shifter $\omega_{i,s}$, total final demand for industry s goods, $C_s$, and industry-level index $\Lambda_s \equiv \Lambda_s(p_s, Y)$, that aggregates information on all prices in industry s, $p_s = \{p_{1,s}, ..., p_{n_s,s}\}$, and net income, $Y$:*

$$c_{i,s} = \mathcal{D}(p_{i,s}, \omega_{i,s}, \Lambda_s)\, C_s, \qquad i \in \mathbb{I}_s. \tag{2}$$

## 2.2 Production

Production in each industry combines labor with intermediate inputs sourced from multiple firms in the economy. Formally, firm-level variety $i \in \mathbb{I}_s$ is produced using a constant-returns technology summarized by the marginal cost

$$MC_{i,s} = \frac{1}{z_{i,s}} \mathcal{C}_s(w, \mathbf{p}), \tag{3}$$

where $z_{i,s}$ is a firm-level Hicks-neutral productivity shifter, $w$ denotes the economy-wide wage rate, and $\mathbf{p}$ denotes the entire vector of input prices. We assume that input demand has a similar structure to final good demand. Specifically, total input demand by industry $s'$ for firm-level variety $i \in \mathbb{I}_s$ is described by $\widetilde{\mathcal{D}}\left(p_{i,s}, \omega_{i,s}, \widetilde{\Lambda}_s\right) \widetilde{Q}_{s,s'}$, where $\widetilde{Q}_{s,s'}$ denotes the total input demand by industry $s'$ for industry $s$ inputs. For ease of notation, we use the function notation $Q_{i,s}(p)$ to specify the sum of final and input demand facing firm

$i \in \mathbb{I}_s$, as a function of price,

$$Q_{i,s}(p) \equiv \mathcal{D}(p, \omega_{i,s}, \Lambda_s) C_s + \sum_{s'=1}^{S} \left[ \widetilde{\mathcal{D}} \left( p, \omega_{i,s}, \widetilde{\Lambda}_s \right) \widetilde{Q}_{s,s'} \right], \tag{4}$$

where the last sum accounts for the input demand from other industries.

Our next key assumption concerns market structure.

**Assumption 2.** *Individual firms in industry k engage in (Bertrand) price competition. That is, firm $i \in \mathbb{I}_s$ choose its price, $p_{i,s}$, taking the prices chosen by other firms, $p_{-i,s}$, as given.*

## 2.3 Profit Maximization and Firm-Level Transfer Policies

We now describe firms' optimal pricing decisions in the presence of firm-level transfers. We let $T_{i,s}(p)$ denote the transfer offered to firm $i \in \mathbb{I}_s$ as a function of the price, $p$, charged by that firm. This dependence ensures incentive compatibility since price will be observable by the policy maker. After a transfer rule is announced, firm $i \in \mathbb{I}_s$ maximizes total profits by solving

$$\max_p [p - MC_{i,s}] Q_{i,s}(p) + T_{i,s}(p).$$

If the transfer function is continuously differentiable, the first-order condition yields the optimal pricing rule,

$$p_{i,s} = MC_{i,s} + \frac{Q_{i,s}(p_{i,s}) + T'_{i,s}(p_{i,s})}{-Q'_{i,s}(p_{i,s})}. \tag{5}$$

From this expression we can derive a simple formula for firm $i$'s optimal markup. To this end, define the Marshallian price elasticity of demand facing firm $i$ as $\varepsilon_{i,s} \equiv \left| \frac{\partial \ln Q_{i,s}(p)}{\partial \ln p} \right|_{p=p_{i,s}}$, and let $q_{i,s} \equiv Q_{i,s}(p_{i,s})$ denote firm $i$'s total output given its optimal price choice. Invoking this choice of notation and reorganizing Equation (5), delivers the following expression for firm $i$'s optimal markup

$$\mu_{i,s} \equiv \frac{p_{i,s}}{MC_{i,s}} = \left(1 - \frac{1 + \Delta_{i,s}}{\varepsilon_{i,s}}\right)^{-1}, \quad \text{where} \quad \Delta_{i,s} \equiv T'_{i,s}(p_{i,s}) / q_{i,s}.$$

Notice that without transfers, the optimal markup reduces to the standard monopoly markup. Namely,

$$T_{i,s}(.) = 0 \implies \mu_{i,s} = \left(1 - \frac{1}{\varepsilon_{i,s}}\right)^{-1}. \tag{6}$$

As we elaborate shortly, dispersion in the firm-level markups creates misallocation in our economy. The above equations highlight that contingent transfers can impact optimal

markup choices, reduce markup dispersion, and alleviate misallocation.

## 2.4 General Equilibrium

Given demand shifters, productivities, and transfers, $\{\omega_{i,s}, z_{i,s}, T_{i,s}(.)\}_{\mathbb{I}}$, a general equilibrium is a set of prices, quantities, and marginal costs, $\{p_{i,s}, q_{i,s}, MC_{i,s}\}_{\mathbb{I}}$, such that final and input demand choices are chosen optimally to satisfy Equations (1), (2), and (4), the marginal cost is given by Equation (3) to conforms with the firms's cost minimization problem, prices are set according to Equation (5) to maximize profits, and markets clear wherein total income equals the wage bill plus total profits (net of transfers):[2]

$$Y = wL + \sum_{s=1}^{S} \sum_{i=1}^{n_s} \left[ \left( p_{i,s} - MC_{i,s} \right) q_{i,s} \right].$$

Since the good market clear, the labor market clearing condition necessarily holds by Walras' Law. We, accordingly, normalize the wage rate to one ($w = 1$) by choice of numeraire hereafter.

## 2.5 Misallocation from Market Power

Without transfers, i.e., $T_{i,s}(.) = 0$, the market equilibrium is inefficient due to heterogeneity in firm-level markups. To elaborate, efficiency requires that the marginal rate of substitution between goods equal the marginal rate of transformation ($MRST = MRT$). For this condition to hold, the relative price of goods $\omega$ and $\omega' \in \Omega_k$ should equal their relative marginal cost. Namely,

$$\frac{p^{\star}_{i,s}}{p^{\star}_{i',s'}} = \frac{MC^{\star}_{i,s}}{MC^{\star}_{i',s'}},$$

where the superscript ($\star$) denotes variables corresponding to the efficient allocation. If production were competitive, this condition would arise naturally following the logic of the First Welfare Theorem. In the present framework, however, firms possess market power and hence charge a markup over marginal cost, causing in relative prices to diverge from their efficient levels. In particular, following Equation 6,

$$\frac{p_{i,s}}{p_{i,'s'}} = \frac{\mu_{i,s}}{\mu_{i',s'}} \frac{MC_{i,s}}{MC_{i',s'}} \neq \frac{MC_{i,s}}{MC_{i',s'}}.$$

[2]Transfers have a net-zero effect on the representative consumer's budget constraint because they are rebated back to shareholders as part of profits. To elucidate this point, consider the following fiscal scenario. The government finances $T = \sum_k \sum_\omega T_k(\omega)$ via a uniform tax on the wage bill. This tax deduction is offset when $T$ is rebated back to the consumers who are also the shareholders.

where recall that $\mu_{i,s} = \left(1 - \frac{1}{\varepsilon_{i,s}}\right)^{-1}$. We can dig deeper and derive a simple formula for the degree of misallocation net of input-output effects. The metric we use is ***distance from the Pareto-efficient frontier***, $\mathfrak{D}$, which is the welfare differential between the efficient and the market allocations. More formally,

$$\mathfrak{D} \equiv \log v\left(Y^{\star}, \mathbf{p}^{\star}\right) - \log v\left(Y, \mathbf{p}\right),$$

where $v(.)$ denotes the indirect utility function and the superscript, $\star$, denotes outcomes corresponding to the efficient (i.e., marginal-cost pricing) equilibrium. Using demand-side envelope conditions, Jensen's Inequality, and Hölder (1889)'s Hölder (1889) defect, we can derive a first-order approximation for distance from the efficient frontier for small markups. [define markup]

**Proposition 1.** *Distance from the Pareto-efficient frontier is approximated by the variance of firm-level markups. Namely,*

$$\mathfrak{D} \simeq \frac{\bar{\mu}}{2}\text{Var}\left(1/\mu_{i,s}\right),$$

*where* $\frac{1}{\bar{\mu}} \equiv \mathbb{E}\left(\frac{1}{\mu_{i,s}}\right)$ *denotes the average markup. The above approximation is exact when goods with dissimilar markups exhibit a unitary elasticity of substitution.*

To be clear, the above formula approximates the degree of misallocation for small markups net of input-output effects. It, nonetheless, provides a useful lens to think about the efficacy of policy at curbing misallocation. A basic implication of Proposition 1 is that the policy maker must eliminate markup heterogeneity to restore the first best outcome. To achieve this goal, she needs to implement a transfer policy that alters each firm's optimal pricing decision, prompting them to choose a price that equals their marginal cost. In the next section we present a set of policies that are designed to accomplish this exact goal.

# 3 First-Best Benchmark: Optimal Parametric Policy

To motivate our discussion of non-parametric policies, we first describe a family of parametric policies that completely corrects the distortions from market power and implements the first best allocation.

**Proposition 2.** *Any transfer function satisfying* $T'_{i,s}(p) = -Q_{i,s}(p)$ *restores allocative efficiency in the economy.*

The proof follows directly from the first and second order conditions of a firm's profit-maximization problem. To build intuition for why such a transfer function works, we consider the first-order condition of a firm $\omega$ and interpret it in terms of social and private marginal benefits and costs,

$$p_{i,s} - \frac{Q_{i,s}\left(p_{i,s}\right) + T'_{i,s}\left(p_{i,s}\right)}{-Q'_{i,s}\left(p_{i,s}\right)} = MC_{i,s}.$$

Selling one more unit of the good yields a private marginal benefit equal to the price at which the good is sold, $p_{i,s}$. Note that this benefit coincides with the social marginal benefit since consumers and other firms take this price as given when optimizing their utility and input costs. Producing the additional unit of the good also incurs a marginal cost. For a firm with market power, this marginal cost has two components. The first is the real cost of production $MC_{i,s}$, which coincides with the social marginal cost of producing the good. The second component captures the loss in revenue on inframarginal units caused by the price reduction necessary to sell the additional unit. In the absence of any transfer policies, this component is measured by $\frac{Q_{i,s}\left(p_{i,s}\right)}{-Q'_{i,s}\left(p_{i,s}\right)}$, and forces private marginal cost above the social marginal cost. The term $-\frac{1}{Q'_{i,s}\left(p_{i,s}\right)}$ measures the decline in price required to sell another unit as dictated by the demand curve, while $Q_{i,s}\left(p_{i,s}\right)$ measures the number of inframarginal units to which this price decline must be applied.

In the presence of the transfer function, however, the decline in inframarginal revenue is offset by an amount $\frac{T'_{i,s}\left(p_{i,s}\right)}{-Q'_{i,s}\left(p_{i,s}\right)}$, which measures the marginal transfer received by the firm when she lowers her price to sell one more unit. The transfer incentivizes the firm to sell more units at a lower price than she otherwise would. Under Proposition 2, $T'_{i,s}\left(p_{i,s}\right) = -Q_{i,s}\left(p_{i,s}\right)$ and the marginal transfer exactly offsets the loss in inframarginal revenue, resulting in marginal cost pricing and a first best allocation.

To the best of our knowledge, the family of policies described by Proposition 2 is new to the literature. Existing approaches to the regulation of firms with price-setting power typically take two forms. The first assumes complete information and the availability of unrestricted linear instruments, which the policy maker can then tailor to each firm, for example in the form of sales subsidies. While such policies can restore the first best, they are unrealistic from an informational perspective and require an infeasible amount of customization across firms. An alternative approach takes incomplete information seriously, and assumes that the policy maker does not observe the heterogeneity in firm productivities. Hence, any policy must respect incentive compatibility constraints. Depending on the social objective, these constraints can make the first best unattainable.

In contrast, the policies in Proposition 2 neither require customized linear instruments across firms nor are derived using incentive compatibility constraints. Instead, the policy achieves the first best purely via knowledge of the demand curve, which is a maintained assumption in both existing approaches. Of course, such knowledge still requires the specification of a parametric model of demand and so remains information-intensive.

### Special Case: *Surplus Subsidy Scheme*

We have specified the transfer function in terms of its first derivative. Solving the differential equation presented under Proposition 2 yields a family of efficient transfer functions.

**Corollary 1.** *Let $\bar{T}_{i,s}$ and $\bar{p}_{i,s}$ be constants. Then $T_{i,s}$ can take the form*

$$T_{i,s}(p) = \begin{cases} \bar{T}_{i,s} + \int_{p}^{\bar{p}_{i,s}} Q_{i,s}(x)\,dx & p \leq \bar{p}_{i,s} \\ \\ \bar{T}_{i,s} - \int_{\bar{p}_{i,s}}^{p} Q_{i,s}(x)\,dx & p \geq \bar{p}_{i,s} \end{cases} \tag{7}$$

The functional form of $T_{i,s}$ shows that the optimal transfer function is closely linked to the notion of consumer surplus, or the "area under the demand curve." Intuitively, by offering firms surplus in exchange for selling more units at a lower price, the policy compensates firms for the loss in profit. The presence of free constants $\bar{T}_i$ and $\bar{p}_{i,s}$ indicates that the size of these surplus transfers is, in principle, arbitrary and this policy is therefore revenue-neutral. What matters is that the marginal surplus transfer exactly offsets the marginal loss in revenue from charging a lower price on the inframarginal units, as summarized by Proposition 2.

## 4 Non-Parametric Industrial Policy

We are interested in policies that depend only on data available to the policy maker. We call such policies *"non-parametric"*. In this paper, we assume that the only data available to the policy maker are the (ex post) prices and quantities set by each firm in the economy. We let $\mathbb{D}_s = \{p_{i,s}, q_{i,s}\}_{i \in \mathbb{I}_s}$ denote the set of observed price and quantity data in industry $s$.

Our focus on non-parametric policies contrasts sharply with the usual approach to studying optimal policy problems that relies on the calibration or estimation of a parametric structural model. While this approach has useful theoretical advantages, it often

faces severe implementation frictions, either due to insufficient information or because the optimal policy is sensitive to the exact details of the model specified by the policy maker. Non-parametric policies are constructed to avoid these issues.

## 4.1 A Non-Parametric Policy

Recall that the key role of the optimal parametric transfer function is to compensate firms for the lost revenue on inframarginal units if they were to lower their prices toward marginal cost. Inspired by the first-order condition for firms' profit maximization problem, an important property for a non-parametric policy is to set $T'_{i,s}(p)$ to minimize the distance $\left|Q_{i,s}(p) - T'_{i,s}(p)\right|$ subject to using only the information in $\mathbb{D}_s$. Therefore, fix an industry $s$, a firm $i \in \mathbb{I}_s$, and a price $p$, and let $\mathbb{D}_{s,-i} = \left\{p_{j,s}, q_{j,s}\right\}_{j \in \mathbb{I}_s \setminus i}$ denote the set of price and quantity data excluding firm $i$. Define the quantity of firm $i$'s synthetic counterpart as

$$Q^T_{i,s}(p) = \begin{cases} \frac{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s}) q_{n,s}}{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s})} & \text{if } p \neq p_{n,s} \text{ for all } n \\ q_{n,s} & \text{if } p = p_{n,s} \text{ for some } n \end{cases}$$

Here, $Q^T_{i,s}(p)$ is interpolated from the revealed data $\mathbb{D}_{s,-i} = \left\{p_{j,s}, q_{j,s}\right\}_{j \in \mathbb{I}_s \setminus i}$, about other firms' posted prices and quantities. Similarly, define the price of firm's synthetic counterpart as:

$$P^T_{i,s}(p) = \begin{cases} \frac{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s}) p_{n,s}}{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s})} & \text{if } p \neq p_{n,s} \text{ for all } n \\ p_{n,s} & \text{if } p = p_{n,s} \text{ for some } n \end{cases}$$

The weighting function $\omega(p - p_{n,s}) = \| p - p_{n,s} \|^{-\theta}$, with parameter $\theta \gg 1$, assigns greater weight to firms whose prices are closer to $p$. Thus, $Q^T_{i,s}(.)$ and $P^T_{i,s}(.)$ provide the quantity and price of the firm's synthetic counterpart based on ex post observable data $\mathbb{D}_{s,-i}$ on the quantity and price of other firms in segment $s$. These functions are commonly known as Shephard's inverse-weighting interpolations and have well-studied properties. When $\theta \to \infty$, the functions $Q^T_{i,s}(p)$ and $P^T_{i,s}(p)$ provide the quantity and price of the nearest neighbor in terms of pricing.

Our proposed non-parametric policy provides each firm with a transfer based on its posted price $p$ and the corresponding price and quantity of its synthetic neighbor, as follows:

$$T_{i,s}(p) = \left(P^T_{i,s}(p) - p\right) Q^T_{i,s}(p). \tag{8}$$

Under this policy, each firm's subsidy consists of two parts. The first is a lump-sum trans-

fer equivalent to the revenue of the nearest synthetic competitor. The second indicates that raising the firm's price by one unit decreases the subsidy by an amount equal to the synthetic neighbor's quantity. This ensures that the marginal transfer closely matches the inframarginal revenue loss, using only information contained in $\mathbb{D}_s$.

### 4.1.1 How does the non-parametric policy affect firm markups?

To understand how this policy affects firm behavior, fix a firm $i$ in industry $s$. Under the non-parametric transfer scheme, firm $i$ maximizes the total profit net of transfers:

$$\pi_i(p) = (p - MC_{i,s}) Q_{i,s}(p) + \left(P^T_{i,s}(p) - p\right) Q^T_{i,s}(p)$$

Profit maximization given $\mathbb{D}_{s,-i} = \left\{p_{j,s}, q_{j,s}\right\}_{j \in \mathbb{I}_s \setminus i}$ implies

$$\mu_{i,s} = \frac{\varepsilon_{i,s}}{\varepsilon_{i,s} - 1 + \frac{q^T_{i,s}}{q_{i,s}} \left[ \frac{\partial \ln Q^T_{i,s}(p)}{\partial \ln p} \left(\frac{p^T_{i,s}}{p_{i,s}} - 1\right) - \left(\frac{\partial P^T_{i,s}(p)}{\partial p} - 1\right) \right]}$$

With granular firms and a sufficiently large $\theta$, $q^T_{i,s} \equiv Q^T_{i,s}(p_{i,s})$ corresponds to the quantity of closest price neighbor. Hence, $\frac{\partial Q^T_{i,s}(p)}{\partial p} \approx \frac{\partial P^T_{i,s}(p)}{\partial p} \approx 0$, and the optimal markup reduces to

$$\mu_{i,s} \approx \frac{\varepsilon_{i,s}}{\varepsilon_{i,s} - 1 + q^T_{i,s}/q_{i,s}} \approx \left(1 - \frac{1 - q^T_{i,s}/q_{i,s}}{\varepsilon_{i,s}}\right)^{-1}$$

where $q^T_{i,s} \equiv Q^T_{i,s}(p_{i,s})$, with $p_{i,s} = \mu_{i,s} \mathrm{MC}_{i,s}$. The more similar the synthetic neighbor to firm $i$, the lower the markup. In the limit where the nearest neighbor exactly mimics firm $i$'s pricing and sales, the markup will be eliminated altogether.

**Why does this policy work?**

The non-parametric policy has two properties that make it potentially effective. First, it mimics the optimal transfer, which following Proposition 2, has a derivative equal to the quantity supplied by the firm. Second, this policy uses a *synthetic* closest neighbor rather than a true closest neighbor, since using the closest neighbor might lead firms to engage in limit pricing, which in turn makes the profit functions non-differentiable. The next section employs a workhorse model commonly used in the literature to investigate the quantitative effectiveness of this non-parametric policy in achieving the efficient allocation.

**Link to Yardstick Competition (Shleifer, 1985).** The mechanics of our non-parametric policy bear some similarities to the workings of the "yardstick policy" discussed by Shleifer (1985) in the context of efficient cost setting by franchised monopolies. The key parallel is the idea that when a policy maker wants to regulate firms who have market power, she can use the existence of other similar firms operating in different markets to design a first best policy. While Shleifer (1985) designs a policy that mandates prices and lump sum transfers to each firm to incentivize efficient cost reduction when costs are observable, we show how to design a linear transfer scheme that induces efficient price setting when costs are unobservable. In addition, we now discuss how our policy extends to the case when firms have heterogeneous costs that are unobservable to the policy maker.

# 5 Ex-Ante Policy Evaluation

This section develops a simple methodology for *ex ante* evaluation of the policies identified in Section 3. Our conceptual framework, thus far, relied on little-to-no parametric assumptions. To conduct quantitative policy evaluation, however, we must rely on a parametric model—which we present below.

## 5.1 Parametric Model

On the demand side, we assume that the representative consumer's utility function has a nested CES parameterization:

$$U(C_1,..,C_S) = \left[\sum_{s=1}^{S} C_s^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \text{where} \quad C_s = \left[\sum_{i=1}^{n_s} c_{i,s}^{\frac{\gamma_s-1}{\gamma_s}}\right]^{\frac{\gamma_s}{\gamma_s-1}}. \tag{9}$$

The above structure implies a standard CES demand which is a special case of the general non-parametric demand system introduced in Section 2. In particular,

$$\mathcal{D}_i(p_{i,s}, p_{-i,s}) = (p_{i,s}/P_s)^{-\gamma_s} C_s, \quad \text{where} \quad C_s = P_s^{-\sigma} P^{\sigma-1} Y,$$

with $P_s$ and $P$ denoting the industry-level and economy-wide CES price indices:

$$P_s = \left[\sum_{i=1}^{n_s} p_{i,s}^{1-\gamma_s}\right]^{\frac{1}{1-\gamma_s}}, \qquad P = \left(\sum_{s=1}^{S} P_s^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{10}$$

The cost function facing firms in industry $s$ is specified as

$$\mathcal{C}_s(w, \mathbf{p}) = w^{\nu_s} \prod_{s'=1}^{S} P_{s'}^{(1-\nu_s)\alpha_{s's}}, \quad \text{where} \qquad P_s = \left[ \sum_{i=1}^{n_s} p_{i,s}^{1-\gamma_s} \right]^{\frac{1}{1-\gamma_s}}. \tag{11}$$

In this specification $\nu_s$ denotes the constant value-added share in industry $s$ and $\alpha_{s's}$ is the $(s', s)$ entry in the $S \times S$ input-output table. Notice that the CES input aggregator in each industry is identical to the CES utility aggregator for that industry. Hence, gross demand for variety $i \in \mathbb{I}_s$ is $q_{i,s} = \mathrm{q}_{i,s}(p_{i,s})$, where

$$\mathrm{q}_{i,s}(p_{i,s}) = \left( \frac{p_{i,s}^{-\gamma_s}}{P_s^{1-\gamma_s}} \right) E_s,$$

with $E_s = \sum_{i=1}^{n_s} [p_{i,s} q_{i,s}]$ denoting gross expenditure on industry $s$ goods. We use $\lambda_{i,s}$ and $e_{i,s}$ to denote the within-industry and overall gross expenditure shares:

$$\lambda_{i,s} \equiv \frac{p_{i,s} q_{i,s}}{E_s} \sim \text{within-industry exp. share}, \qquad e_{i,s} \equiv \lambda_{i,s} \times \frac{E_s}{E} \sim \text{overll exp. share}$$

Profit maximization implies the following optimal markup over marginal cost for each firm industry $k$:[3]

$$\mu_{i,s} = \frac{\varepsilon_{i,s}}{\varepsilon_{i,s} - 1}; \quad \text{where} \quad \varepsilon_{i,s} = \gamma_s [1 - \lambda_{i,s}] + \sigma \lambda_{i,s}. \tag{12}$$

Notice that the markup depends on the gross firm-level expenditure share, $\lambda_{i,s}$, but does not depend on what fraction of sales come from final versus intermediate goods. Next, we describe the general equilibrium in our parametric model.

**General Equilibrium.** For a vector of firm-level productivities, demand parameters, input-output shares, and population size, $\Theta = \{z_{i,s}, \gamma_s, \sigma, \nu_s, \alpha_{ss'}, L\}$, equilibrium is a vector of firm-level prices and expenditure shares, industry-level expenditure levels and price indexes, and net income, $\mathcal{E} = \{p_{i,s}, \lambda_{i,s}, E_s, P_s, Y\}$, such that gross expenditure on industry $s$ goods equals the sum of final consumption and intermediate input expenditure,

$$E_s = (P_s / P)^{1-\sigma} Y + \sum_{s'=1}^{S} \left( (1 - \nu_{s'}) \alpha_{ss'} \sum_{i=1}^{n_{s'}} \frac{q_{i,s'} p_{i,s'}}{\mu_{i,s'}} \right), \tag{13}$$

[3]The above formula is analogous to the optimal markup formula in Atkeson and Burstein (2008) in the case of price competition.

where $P_s$ and $P$ are industry-level and aggregate price indexes as specified by (10); the price and gross expenditure share associated with firm $\omega \in \Omega_k$ are given by

$$\lambda_{i,s} = \left(\frac{p_{i,s}}{P_s}\right)^{1-\gamma_s}, \qquad p_{i,s} = \frac{\mu_{i,s}}{z_{i,s}} w^{\nu_s} \prod_{s'=1}^{S} P_{s'}^{(1-\nu_s)\alpha_{s's}},$$

where the firm-level markup, $\mu_{i,s}$, is described by (12); and net income equals the wage bill, $wL$, plus total profits in the economy:

$$Y = wL + \sum_{s=1}^{S} \sum_{i=1}^{n_s} \left[\left(1 - \frac{1}{\mu_{i,s}}\right) \lambda_{i,s} E_s\right]. \tag{14}$$

The labor market clearing condition necessarily holds by Walras' Law and the wage rate can accordingly be normalized to one ($w = 1$).

## 5.2 Gains From Implementing the First Best

First, we characterize the change in welfare if marginal-cost pricing was restored by the first-best policy. We present our approach using the hat-algebra notation, which is summarized as follows for a generic variables $x$:

$$x \sim \text{factual values;} \qquad x' \sim \text{counterfactual value;} \qquad \hat{x} \equiv x'/x.$$

To be clear "counterfactual" in this subsection corresponds to first best equilibrium which can be implemented via the transfer scheme in Corollary 1. Given the CES demand system, the change in welfare after restoring efficiency can be expresses as follows

$$\hat{W} = \hat{Y}/\hat{P},$$

where $\hat{Y} = Y'/Y$ denotes the change in the representative consumer's net income (as specified by 14), while $\hat{P} = P'/P$ denotes the change in the consumer price index (as specified by 10). Since profits are drawn to zero in the efficient equilibrium, $Y' = w'L$. Using this observation and treating wage as the numeriare, $w' = w = 1$, we can calculate

change in net income as[4]

$$\hat{Y} = \frac{\sum_s \sum_{i \in \mathbb{I}_s} \left[ \frac{\nu_s}{\mu_{i,s}} e_{i,s} \right]}{1 - \sum_s \sum_{i \in \mathbb{I}_s} \left[ \frac{1-\nu_s}{\mu_{i,s}} e_{i,s} \right]},$$

where $e(\omega)$ denotes the overall gross expenditure share on firm $\omega \in \Omega_k$ under the status quo. Taking logs from the above equation we can express the per-cent change in income as

$$\Delta \log Y = -\log \left( 1 + \frac{\sum_s \sum_{i \in \mathbb{I}_s} \left[ \frac{\mu_{i,s}-1}{\mu_{i,s}} e_{i,s} \right]}{\sum_s \sum_{i \in \mathbb{I}_s} \left[ \frac{\nu_s}{\mu_{i,s}} e_{i,s} \right]} \right) < 0.$$

We can appeal to the CES demand structure to specify the change in the consumer price index. Specifically, denoting by $e_s \equiv E_s/E$ the gross expenditure share on industry $k$,

$$\hat{P} = \left[ \sum_{s=1}^{S} \beta_s \hat{P}_s^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad \text{where} \qquad \hat{P}_s = \frac{\mathcal{C}_s(w', \mathbf{p}')}{\overline{\mu}_s \mathcal{C}_s(w, \mathbf{p})} = \frac{1}{\overline{\mu}_s} \prod_{s'=1}^{S} \hat{P}_{s'}^{(1-\nu_s)\alpha_{s's}}. \tag{15}$$

The last line follows from the normalization, $w' = w = 1$, with $\overline{\mu}_s$ denoting the CES weighted average markup in industry $s$. Namely,

$$\overline{\mu}_s \equiv \left[ \sum_{i \in \mathbb{I}_s} \mu_{i,s}^{\gamma_s - 1} \lambda_{i,s} \right]^{\frac{1}{\gamma_s - 1}}. \tag{16}$$

The expression for $\hat{P}_s$ in Equation (15) follows from setting $\hat{w} = 1$ by choice of numeraire. It specifies a system of interdependent equations in terms of $\hat{P}_1, ..., \hat{P}_S$. Taking logs from this yields $\ln \hat{P}_s = -\ln \overline{\mu}_s + (1 - \nu_s) \sum_{s'} \left[ \alpha_{s's} \ln \hat{P}_{s'} \right]$. Inverting this system of equations delivers the following expression for $\hat{P}_s$

$$\hat{P}_s \equiv \prod_{s'=1}^{S} \overline{\mu}_{s'}^{-a_{ss'}}, \quad \text{where} \qquad [a_{ss'}]_{s,s'} = (\mathbf{I} - \mathbf{\Omega})^{-1}.$$

Stated verbally, exponent $a_{ss'}$is the $(s, s')$ entry of the inverse Leontief matrix, $(\mathbf{I} - \mathbf{\Omega})^{-1}$, where $\mathbf{\Omega} \equiv [(1 - \nu_s)\alpha_{s's}]_{s,s'}$ denotes the input-output matrix. Plugging the expression

[4] To elaborate on the derivation, we can appeal to Equation (14) and write change in net income as

$$\hat{Y} = \frac{Y'}{Y} = \frac{w'L}{wL + \sum_s \sum_{i \in \mathbb{I}_s} \left[ \frac{\mu_{i,s}-1}{\mu_{i,s}} p_{i,s} q_{i,s} \right]}.$$

Noting that $w' = w = 1$ by choice of numeraire and $e_{i,s} = p_{i,s} q_{i,s}/E$ by definition, yields the final expression for $\hat{Y}$.

back into Equation 15, implies

$$\Delta \log P = -\frac{1}{\sigma - 1} \log \left( \sum_{s=1}^{S} \beta_s \prod_{s'=1}^{S} \overline{\mu}_{s'}^{a_{ss'}(\sigma-1)} \right).$$

Combining the expressions for $\Delta \log P$ and $\Delta \log Y$ determines the change in welfare $\Delta \log W = \Delta \log Y - \Delta \log P$, as outlined by the following proposition.

**Proposition 3.** *Under functional forms 9 and 11, the welfare gains from implementing the first-best (i.e., marginal-cost pricing) allocation can be calculated as*

$$\Delta \log W \text{ (1st-best)} = \frac{1}{\sigma - 1} \log \left( \sum_{s} \beta_s \prod_{s'} \overline{\mu}_{s'}^{a_{ss'}(\sigma-1)} \right) - \log \left( 1 + \frac{\sum_s \sum_{i \in \mathbb{I}_s} \left[ \frac{\mu_{i,s}-1}{\mu_{i,s}} e_{i,s} \right]}{\sum_s \sum_{i \in \mathbb{I}_s} \left[ \frac{\nu_s}{\mu_{i,s}} e_{i,s} \right]} \right)$$

*with ex ante information on firm-level markups as well as information on observed expenditure, value added, and input-output shares. The weighted average markup, $\overline{\mu}_s$, is give by (16).*[5]

To give some perspective, Proposition 3 presents a simple formula for *ex ante* policy evaluation that depends on three sets of sufficient statistics: firm-level markups, the cross-industry substitution parameter $\sigma$, and observable shares ($e_{i,s}$, $\nu_s$, and $a_{s's}$). This formula will guide our quantitative policy evaluation in the subsequent section. Before moving on to quantification, we devise a simple approach to (*ex ante*) compute the gains from second best transfers.

## 5.3 Gains from Non-Parametric Policy

Second best transfers differ from the first best policy in that they (typically) do not eliminate the full markup on marginal cost. They instead induce a firm-specific reduction in markups that depends on firm- and industry-level characteristics. As such, we cannot derive a closed-from expression for the gains from second best transfer. We can, however, compute the second-best gains using a system of equations, the solution of which requires a minimal set of sufficient statistics. We present this approach below.

To present our approach, we reuse the hat-algebra notation, wherein $\hat{x} = x'/x$ denotes the change in a generic variable, $x$. Though, $x'$ now denotes counterfactual value

[5] Without input-output linkages, in which case $\nu_s = 1$, the welfare formula specified by Proposition 2 reduces to

$$\Delta \log W = \frac{1}{\sigma - 1} \log \left( \sum_{s} e_s \overline{\mu}_s^{(\sigma-1)} \right) + \log \left( \sum_{s} \sum_{i \in \mathbb{I}_s} \left[ \frac{1}{\mu_{i,s}} e_{i,s} \right] \right).$$

assumed by $x$ after the implementation of *second-best* transfers. Our goal is to determine the change in key equilibrium variables with information on markups and observable shares. To this end, we express all equilibrium conditions corresponding to optimal demand and production choices and market clearing conditions in changes. We then solve these equation under one system to uncover the effect of second-best transfers.

We start with gross firm-level expenditure shares, the change in which given by

$$\hat{\lambda}_{i,s} = \left(\hat{p}_{i,s}/\hat{P}_s\right)^{1-\gamma_s}, \tag{17}$$

where $\hat{p}_{i,s}$ denotes the change in firm $i'$ s price in response to second-best transfers and $\hat{P}_s$ denotes the corresponding change in industry $s$'s price index:

$$\hat{P}_s = \left[\sum_{i\in\mathbb{I}_s} \lambda_{i,s}\hat{p}_{i,s}^{1-\gamma_s}\right]^{\frac{1}{1-\gamma_s}}. \tag{18}$$

To determine the change in firm-level price, $\hat{p}_{i,s}$, note that $p_{i,s} = \mu_{i,s}\mathcal{C}_s(w,\mathbf{p})/z_{i,s}$. Since $z_{i,s}$ is invariant to policy and wage is normalize to one (i.e., $\hat{w} = 1$), we can appeal to Equation 11 to arrive at the following expression:

$$\hat{p}_{i,s} = \hat{\mu}_{i,s}\prod_{s'=1}^{S}\hat{P}_{s'}^{(1-\nu_s)\alpha_{s's}}. \tag{19}$$

The change in markup can be inferred from the optimal pricing rule under second best transfers. In the case of our parametric model this equation implies,

$$\mu'_{i,s} = \frac{p'_{i,s}}{\mathcal{C}_s(w',\mathbf{p}')/z_{i,s}} = \left(1 - \frac{1 - q'_{i^*,s}/q'_{i,s}}{\varepsilon'_{i,s}}\right)^{-1}, \quad \text{where} \qquad i^* = \arg\min_{i'\in\mathbb{I}_s\setminus i}\left\{\mid q'_{i',s} - q'_{i,s}\mid\right\}$$

for any firm $i$ in industry $s$. Noting that $\varepsilon_{i,s} = \gamma_s\left(1-\lambda_{i,s}\right) + \sigma\lambda_{i,s}$, we can write the above equation in changes to derive the following expression for the change in firm-level markups:

$$\hat{\mu}_{i,s} = \frac{1}{\mu_{i,s}}\left(1 - \frac{1 - \frac{q_{i^*,s}}{q_{i,s}}\left(\frac{\hat{p}_{i^*,s}}{\hat{p}_{i,s}}\right)^{-\gamma_s}}{\gamma_s\left[1-\hat{\lambda}_{i,s}\lambda_{i,s}\right] + \sigma\hat{\lambda}_{i,s}\lambda_{i,s}}\right)^{-1}. \tag{20}$$

The above equation invokes the fact that $\lambda'_{i,s} = \hat{\lambda}_{i,s}\lambda_{i,s}$ and $\hat{q}_{i^*,s}/\hat{q}_{i,s} = \left(\hat{p}_{i^*,s}/\hat{p}_{i,s}\right)^{-\gamma_s}$. Solving the system of Equations (17)-(20) determines $\mathbb{X}_1 = \left\{\hat{p}_{i,s}, \hat{\lambda}_{i,s}, \hat{\mu}_{i,s}, \hat{P}_s\right\}$ with information on $\mathbb{S} = \{q_{i,s}, \lambda_{i,s}, \mu_{i,s}, \alpha_{s's}, \nu_s, \gamma_s, \sigma\}$. After determining $\mathbb{X}_1$ , we can compute the change in

the aggregate consumer price index as

$$\hat{P} = \left[ \sum_{s=1}^{S} \beta_s \hat{P}_s^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

where $\beta_s$ denotes the share industry $s$ goods in total consumption spending under the status quo.

Next, we must compute the change in net income, $\hat{Y}$. This can be done by combining the already-derived values for $\mathbb{X} = \{\hat{p}_{i,s}, \hat{\lambda}_{i,s}, \hat{\mu}_{i,s}, \hat{P}_s\}$ with the good market clearing and balanced budget conditions. Using information on $\hat{P}_s$ and $\hat{P}$, we can write the good market clearing condition for industry $s$ (Equation 13) in changes as

$$E_s \hat{E}_s = \beta_s \left( \frac{\hat{P}_s}{\hat{P}} \right)^{1-\sigma} \hat{Y} Y + \sum_{s'} \left( (1-\nu_{s'}) \alpha_{ss'} \hat{E}_{s'} E_{s'} \sum_{i \in \mathbb{I}_{s'}} \left[ \frac{1}{\hat{\mu}_{i,s'} \mu_{i,s'}} \left( \frac{\hat{p}_{i,s'} p_{i,s'}}{\hat{P}_{s'} P_{s'}} \right)^{1-\gamma_{s'}} \right] \right). \quad (21)$$

The left-hand side in the above equation, $E_s \hat{E}_s = E_s'$, denotes to the gross industry-level expenditure in the counterfactual second-best equilibrium. The right-hand side denotes the sum of consumption and input expenditure, both in counterfactual equilibrium. Likewise, taking note that $w'L' = wL = L$, we can express the balanced budget condition in changes as

$$\hat{Y} Y = L + \sum_{s} \sum_{i \in \mathbb{I}_s} \left( \left[ 1 - \frac{1}{\hat{\mu}_{i,s} \mu_{i,s}} \right] \hat{\lambda}_{i,s} \lambda_{i,s} \hat{E}_s E_s \right). \quad (22)$$

Again, the left-hand side in the above equation denote net income, $\hat{Y} Y = Y'$, in the counterfactual equilibrium. The right-hand side denotes the wage bill plus total profits in counterfactual second best equilibrium. With information on $\mathbb{X}_1 = \{\hat{p}_{i,s}, \hat{\lambda}_{i,s}, \hat{\mu}_{i,s}, \hat{P}_s\}$ from the previous step, we can jointly solve the system of Equation (21)-22 to pin down $\mathbb{X}_2 = \{\hat{E}_s, \hat{Y}\}$. Then, we can use values for $\hat{Y}$ and $\hat{P}$ to determine the welfare gains from second best transfer.

**Proposition 4.** *The impact of second-best transfers, $\mathbb{X} = \{\hat{p}_{i,s}, \hat{\lambda}_{i,s}, \hat{\mu}_{i,s}, \hat{P}_s, \hat{E}_s, \hat{P}, \hat{Y}\}$, can be calculated by jointly solving Equations (17)-(22), with information on firm-level markups, demand parameters, and observable shares, $\mathbb{S} = \{q_{i,s}, \lambda_{i,s}, \mu_{i,s}, \alpha_{ss'}, \nu_s, \gamma_s, \sigma\}$. After determining $\mathbb{X}$, the welfare gains from second-best transfer can be calculated as*

$$\Delta \log W \text{ (non-parametric policy)} = \log \hat{Y} - \log \hat{P}.$$

While Proposition 3 does not provide an exact welfare formula, it outlines a tractable

approach for quantifying the gains from second best transfers. More importantly, the quantification requires information on a minimal set of sufficient statistics—namely, $\mathbb{S} = \{q_{i,s}, \lambda_{i,s}, \mu_{i,s}, \alpha_{ss'}, \nu_s, \gamma_s, \sigma\}$. In the next section, we infer $\mathbb{S}$ from firm-level data and use Propositions 3 and 4 to quantify the gains from implementing the first best as well as the second-best non-parametric transfer policy.

# 6 Quantitative Analysis

**Data Description.** We apply our parametric model to data from the U.S. Our application assumes that firm-level markups are the only source of inefficiency in the U.S. economy and we ask to what extent our non-parametric policy is effective to achieve the first-best allocation. We take data on firm-level variables from Compustat. For each NAICS industry $k$ in a given year, we observe data on gross firm-level sales, $p(\omega)q(\omega)$ plus a range of additional balance sheet information. From firm-level data on sales, we can infer gross firm-level expenditure shares as well as industry-level gross expenditures, namely, $\lambda(\omega)$, $e(\omega)$, and $E_k$. We construct these values for all years from 1997 to 2015. Herein, we use 2015 as our baseline year and present result for this year, unless otherwise noted.

Data on annual U.S. input-output tables are taken from the BEA. To merge the input-output data with Compustat, we crosswalk BEA industries to NAICS industries. This step compiles data on industry-level value-added and input-output shares, namely, $\nu_k$ and $\alpha_{kg}$. The input-output tables also identify net consumption spending per industry, which allows us to infer total net income and industry-level consumption share, namely, $Y$ and $\beta_k$. These data pin down all elements of $\mathbb{S}$ (the vector of sufficient statics for conducting welfare analysis) other than firm-level markups and demand parameters.

**Estimating Firm-Level Markups and Inferring $\sigma$ and $\{\gamma_k\}_k$.** We estimate firm-level markups using the production-function approach as in De Loecker and Warzynski (2012) (see the appendix for estimation details). Table 1 reports some basic summary statics for the estimated markups. The median firm-level markup, $\mu(\omega)$, is equal to 1.63 which is broadly consistent with the existing literature. More importantly, there is significant heterogeneity in markups, both within and across industries. The estimated markups exhibit a *within-industry* standard deviation of 0.05 and a *cross-industry* standard deviation of 0.70.

We can infer the demand parameters, $\sigma$ and $\{\gamma_k\}_k$, from the estimated markups. To this end, we can invoke Equation (12), which describes firm-level markups as a function of $\lambda(\omega)$, $\gamma_k$, and $\sigma$. We estimate $\sigma$ and $\{\gamma_k\}_k$ to minimize the squared distance between

Table 1: Estimated markups: summary statistics

| Statistic | Value |
|---|---|
| Median firm-level markup | 1.63 |
| Mean firm-level markup | 2.02 |
| Within-industry standard deviation | 0.05 |
| Cross-industry standard deviation | 0.70 |

Table 2: Estimated Elasticities: $\sigma$ and $\gamma_k$

| Parameter | Estimated Value |
|---|---|
| $\hat{\sigma}$ | 1.665 |
| Mean of $\gamma_k$ | 5.030 |
| Median of $\gamma_k$ | 3.129 |
| Standard deviation of $\gamma_k$ | 12.745 |
| Number of $\gamma_k$ | 58 |

estimated and model-implied markups given $\lambda(\omega)$:

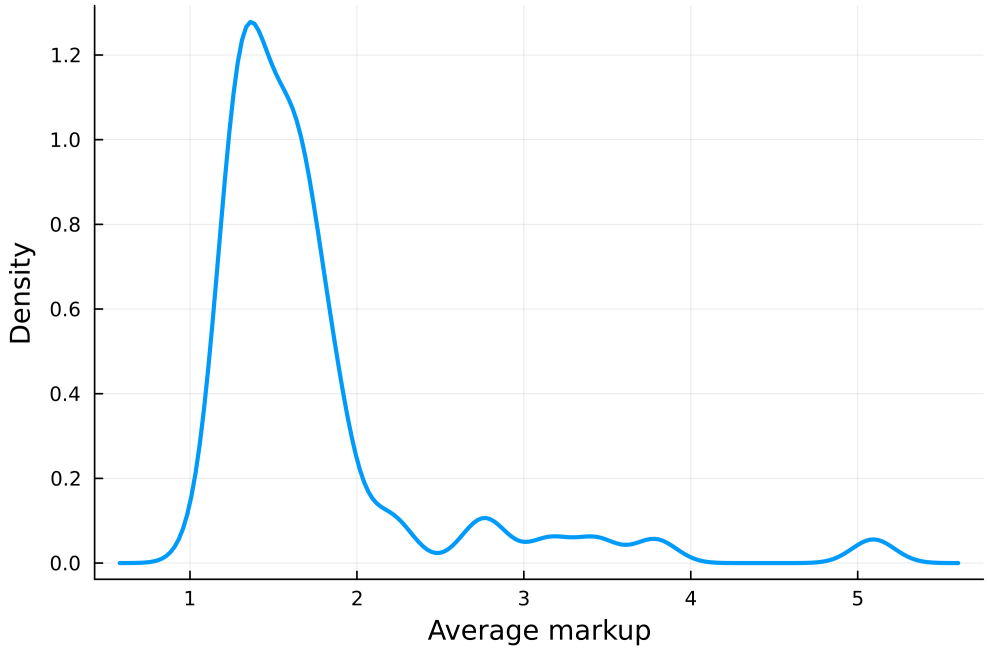
$$\min_{\sigma,\{\gamma_k\}_k} \sum_{\omega\in\Omega_k} \left[ \left( \mu(\omega) - \frac{\gamma_k \left[1-\lambda(\omega)\right] + \sigma\lambda(\omega)}{\gamma_k \left[1-\lambda(\omega)\right] + \sigma\lambda(\omega) - 1} \right)^2 \right].$$

Table 2 reports the estimated elasticity of substitution, $\hat{\sigma}$, along with the summary statistics for industry-level elasticities, $\gamma_k$. Specifically, we report the number of industries in the dataset, as well as the mean, median, and standard deviation of the estimated $\gamma_k$.

To conduct policy analysis using Propositions 2 and 3, firm-level markups (under the status quo) should be consistent with our parametric model. For many reasons (e.g., measurement error or model imperfections) our parametric model cannot *perfectly* replicate the estimated vector of firm-level markups given firm-level expenditure shares. So, we hereon use model-implied markups as true ones to conduct our analysis and compute welfare gains from different policies. Figure 1 depicts the cross-industry heterogeneity in predicted markups using the average of firm-level markups in each industry.

**Recovering Prices from Sales Data.** As noted earlier, our data reports firm-level sales, $p(\omega)q(\omega)$ (or expenditure share, $\lambda(\omega)$) but not the breakdown between price and quantity. Following Proposition 3, we need information on quantities to evaluate the second-best transfer policy. We therefore recover prices and quantities from sales by appealing to the structure of our model. We normalize the marginal cost of firm $\omega_0$ which records the lowest sales in industry $k$ to one—i.e., $\mathrm{C}_k(w,\mathbf{p})/z(\omega_0) = 1$. Using the nested-

Figure 1: Inter-industry distribution: Average markup




CES demand system, we can write $\lambda(\omega)/\lambda(\omega_0) = (p(\omega)/p(\omega_0))^{1-\sigma_k}$. Moreover, noting that $p(\omega_0) = \mu(\omega_0)\mathrm{C}_k(w, \mathbf{p})/z(\omega_0)$, we can recover the price of every firm-level variety $\omega \in \Omega_k$ as

$$p(\omega) = \mu(\omega_0)\left(\frac{\lambda(\omega)}{\lambda(\omega_0)}\right)^{\frac{1}{1-\gamma_k}}.$$

After we determine $p(\omega)$, we can immediately compute output quantities, $q(\omega)$, from sales data. With information on quantities, we now determined every element in $\mathrm{S} = \{q(\omega), \lambda(\omega), \mu(\omega), \alpha_{kg}, \nu_k, \gamma_k, \sigma\}$, allowing us to conduct an ex-ante policy evaluation based on Propositions 2 and 3.

## 6.1 The Efficacy of Non-Parametric Transfers

Table 3 reports the welfare gains from restoring marginal-cost pricing, $\%\Delta W$ (1st best), and compares them to the gains from implementing our non-parametric transfer policy, $\%\Delta W$ (non-parametric policy). Panel A presents the results using our baseline model with no input-output linkages, whereas Panel B employs the full model with IO linkages. A notable outcome is that the simple non-parametric transfer policy proposed in Section 3 can achieve about 87% of the gains from implementing the first best in our baseline model. Our proposed transfer scheme remains effective even after we account for input-

Table 3: The welfare gains from the first and second-best transfers

| **Panel A: Baseline Model** | |
|---|---|
| % $\Delta W$ (1st best) | 13.7% |
| % $\Delta W$ (non-parametric policy) | 11.9% |
| Relative efficacy of non-parametric policy | 0.87 |
| **Panel B: Model with Input-Output Linkages** | |
| % $\Delta W$ (1st best) | 29.1% |
| % $\Delta W$ (non-parametric policy) | 22.7% |
| Relative efficacy of non-parametric policy | 0.78 |

output linkages, achieving about 78% of the gains from implementing the first best. These results reveal that input-output linkages reduce the relative efficacy of parameter-blind transfers. To elaborate, when industries are interconnected through production networks, distortions in one sector propagate to others, making it harder for our non-parametric transfer policy to restore efficiency. As a result, although it remains substantial, the relative efficacy of these transfers declines when accounting for input-output linkages. Table 3 also corroborates with the conventional view that misallocation is magnified with input-output linkages, leading to larger welfare gains from either the first best or our non-parametric transfer policy.

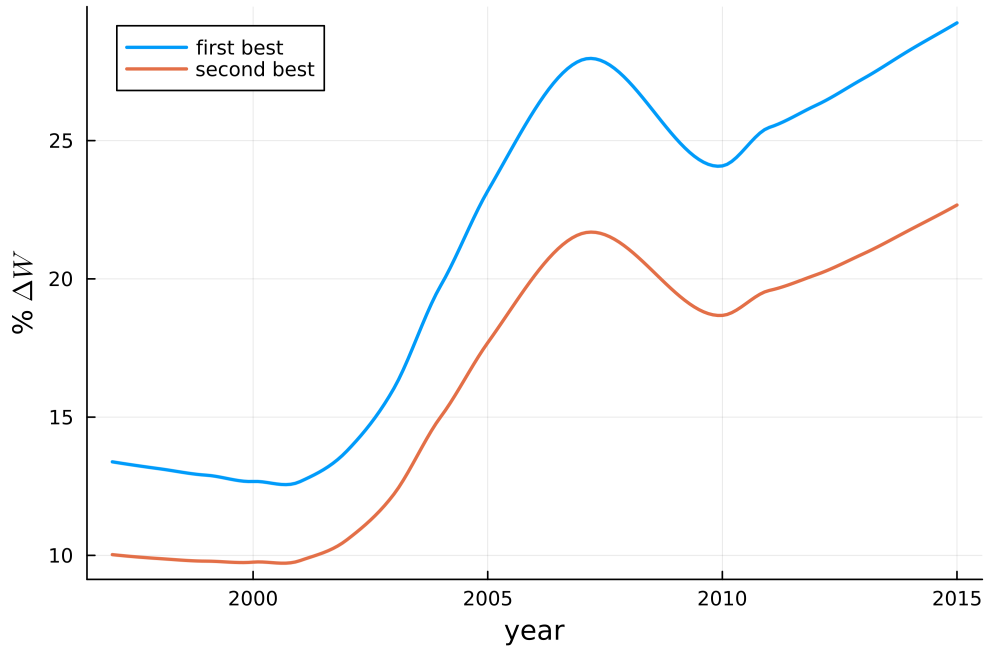
Taken together, our results indicate that our proposed transfer policy is highly effective in reducing markup-induced misallocation and improving welfare. This is a remarkable result, because our simple policy rule can be implemented using public information on sales and prices and is agnostic to both the underlying demand parameters and the input-output structure.

To ensure that the efficacy of our non-parametric transfer policy is not sensitive to the estimated demand parameters, Table C.1 performs a robustness check by employing various values for the parameter $\sigma$ and then re-estimating the parameters $\{\gamma_k\}_k$. As this table shows, the effectiveness of our non-parametric policy remains robust cross different parameters, although both the extent of misallocation and the efficacy of our policy rises with $\sigma$.

We now examine how the gains from the first best and the non-parametric transfer policy evolve over time in the model with input-output linkages.[6] Figure 2 presents the welfare gains from restoring marginal cost pricing (first best) and from implement-

[6] A similar pattern arises in the baseline model with non IO linkages.

Figure 2: The gains from the first- and second-best policies over time




ing non-parametric transfers (second best) for years 1997 to 2015. The results indicate a clear upward trend in both gains over time, implying that the extent of misallocation has increased over time, a finding consistent with the existing literature. The increasing misallocation makes efficiency-enhancing policies even more impactful in more recent years.

To analyze the effectiveness of our non-parametric policy, Figure 3 plots the percentage change in welfare under our proposed non-parametric policy *relative to* that under the first best in each year. Interestingly, despite the overall rise in misallocation discussed above, the relative efficacy of our transfer policy remains approximately constant over time at about 80%. This suggests that while the absolute gains from these transfers increase as misallocation worsens, their effectiveness in replicating the first-best allocation does not change significantly. This stability in relative efficacy implies that the mechanisms governing the policy's impact are not heavily influenced by time-varying economic factors, reinforcing the robustness of the proposed transfer policy.

Our model abstracts from the extensive margin by fixing the number of operating firms. One potential concern about the welfare implications of our transfer policy is that our policy may reduce firms' profits disproportionately in high-markup sectors, leading to less entry in these sectors and potentially welfare losses. We therefore evaluate changes in profits across sectors in response to our transfer policy. Figure 4 presents relative changes in profits after imposing the transfer policy (including the transfer it-

Figure 3: Relative efficacy of non-parametric transfer policy over time

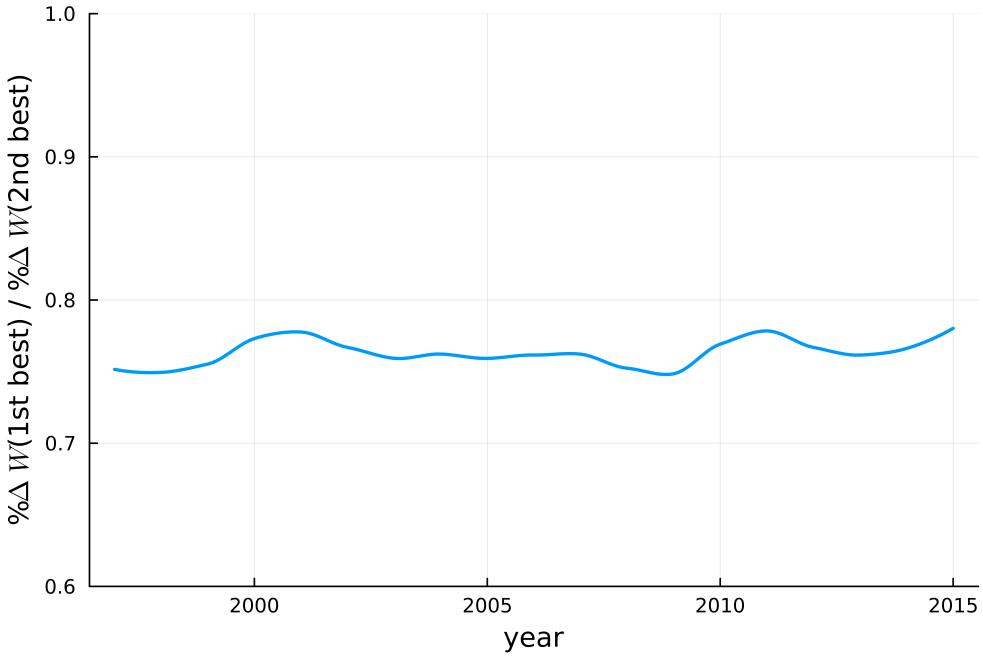
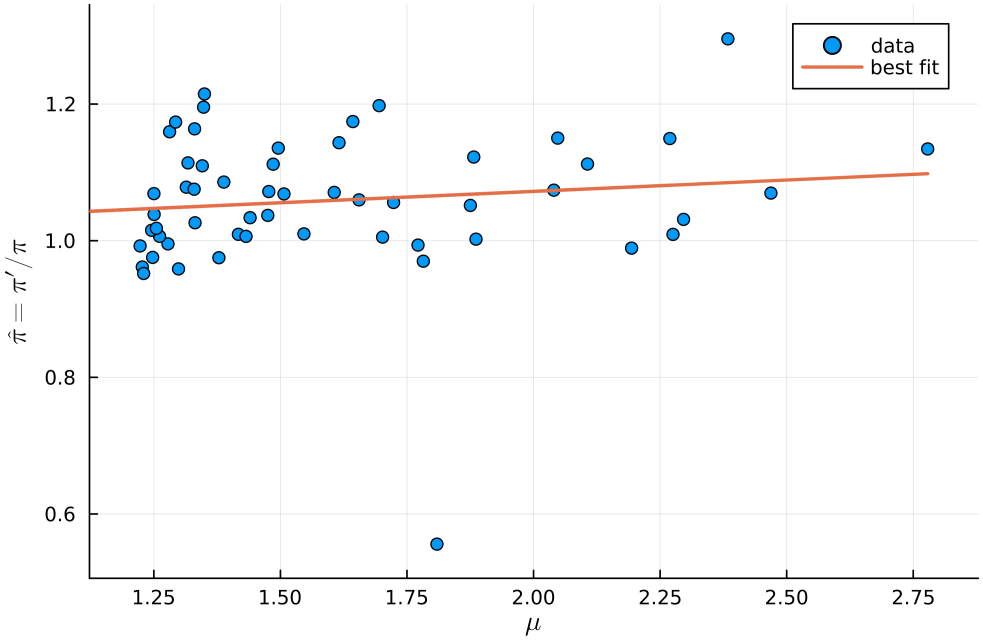



Figure 4: Relative change in profits versus markups




self) against markups. Each point in this figure represents a sector in the economy. The markup for each sector is a cost-weighted average of all firm-level markups within the sector. We sum over firm-level profits within each sector to compute sectoral profits.

As Figure 4 shows, profits do not disproportionately fall and instead slightly rise in

high-markup sectors. Hence, in a model featuring extensive margin, the transfer policy would likely encourage entry in high-markup sectors which might be potentially welfare improving.

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# Appendices

## A Proof of Proposition 1

We herein use $W \equiv v(Y, \mathbf{p})$ to denote welfare. Noting that $\mathfrak{D} \equiv \Delta \log W$, we can write the first-order Taylor approximation for $\mathfrak{D}$ as

$$\mathfrak{D} \equiv \Delta \log W \simeq \frac{\partial \log \nu(Y, \mathbf{p})}{\partial Y} \Delta Y + \frac{\partial \log \nu(Y, \mathbf{p})}{\partial \mathbf{p}} \cdot \Delta \mathbf{p},$$

where "$\cdot$" denotes the inner product operator. Rearranging and rewriting the above expression in summation notation yields

$$\begin{aligned} \mathfrak{D} &\simeq \frac{\partial \log \nu(.)}{\partial Y} \left[ \Delta Y + \sum_k \sum_{\omega \in \Omega_k} \left( \frac{\partial \log \nu(.)}{\partial p(\omega)} \left( \frac{\partial \log \nu(.)}{\partial Y} \right)^{-1} \Delta p(\omega) \right) \right] \\ &= \frac{\partial \log \nu(.)}{\partial Y} \left[ \Delta Y - \sum_k \sum_{\omega \in \Omega_k} \left( q(\omega) \Delta p(\omega) \right) \right], \end{aligned}$$

where the second line follows from Roy's identity, $\frac{\partial \log \nu(.)}{\partial p(\omega)} \left( \frac{\partial \log \nu(.)}{\partial Y} \right)^{-1} = -q(\omega)$. Rearranging the above equation and observing that $\frac{\Delta Y}{Y} \simeq \Delta \log Y$ and $\frac{\Delta p(\omega)}{p(\omega)} \simeq \Delta \log p(\omega)$, yields

$$\begin{aligned} \mathfrak{D} &\simeq \frac{\partial \log \nu(.)}{\partial \log Y} \left[ \frac{\Delta Y}{Y} - \sum_k \sum_{\omega \in \Omega_k} \left( \frac{p(\omega) q(\omega)}{Y} \frac{\Delta p(\omega)}{p(\omega)} \right) \right] \\ &\simeq \frac{\partial \log \nu(.)}{\partial \log Y} \left[ \Delta \log Y - \sum_k \sum_{\omega \in \Omega_k} \left( \frac{p(\omega) q(\omega)}{Y} \Delta \log p(\omega) \right) \right]. \end{aligned}$$

To be clear, $\Delta \log Y = \log Y^\star - \log Y$ and $\Delta \log p(\omega) = \log p^\star(\omega) - \log p(\omega)$, where $\star$ signifies that variables are evaluated under the efficient (i.e., marginal cost pricing) equilibrium. Focusing on the class of preferences that are semi-Gorman representable, we can set $\frac{\partial \log \nu(.)}{\partial \log Y} \simeq 1$ and arrive at the following simplified expression for $\mathfrak{D}$

$$\mathfrak{D} \simeq \Delta \log Y - \sum_k \sum_{\omega \in \Omega_k} \left( e(\omega) \Delta \log p(\omega) \right). \tag{23}$$

Since we are focusing on an economy without input-output linkages, $p(\omega) = \mu(\omega) w / z(\omega)$ and $p^\star(\omega) = w^\star / z(\omega)$. Hence, setting $w^\star = w$ by choice numeriare, the change in price

is $\Delta \log p(\omega) = \log\left(\frac{1}{\mu(\omega)}\right)$. The change in income can be specified as follows. Since there are zero profits under the optimal allocation (i.e., $\Pi^\star = 0$), then $Y^\star = w^\star L$. Noting our choice of numeraire wherein $w^\star = w$, we can express the change in income as

$$\Delta \log Y = \log\left(\frac{Y^\star}{Y}\right) = \log\left(\frac{w^\star L}{wL + \Pi}\right) = \log\left(\frac{wL}{wL + \Pi}\right) \sim \log\left(\frac{wL}{Y}\right).$$

Wage payments under the status quo are given by $wL = \sum_k \sum_{\omega \in \Omega_k} \left[\frac{1}{\mu(\omega)} p(\omega) q(\omega)\right]$, delivering the following expression for $\Delta \log Y$:

$$\Delta \log Y = \log\left(\frac{1}{Y} \sum_k \sum_{\omega \in \Omega_k} \left[\frac{1}{\mu(\omega)} p(\omega) q(\omega)\right]\right) = \log\left(\sum_k \sum_{\omega \in \Omega_k} \frac{1}{\mu(\omega)} e(\omega)\right).$$

Plugging the expressions for $\Delta \log p(\omega)$ and $\Delta \log Y$ back into Equation 23, updates the expression for $\mathfrak{D}$ as follows:

$$\mathfrak{D} \simeq \log\left(\sum_k \sum_{\omega \in \Omega_k} \frac{1}{\mu(\omega)} e(\omega)\right) - \sum_k \sum_{\omega \in \Omega_k} \left[e(\omega) \log\left(\frac{1}{\mu(\omega)}\right)\right]. \tag{24}$$

The above expression amounts to difference between the Arithmetic Mean (AM) and Geometric Mean (GM) of $\log\left(\frac{1}{\mu(\omega)}\right)$. Following Proposition 5 in Becker (2012), which derives from Hölder's (1889) Defect formula, there exists a constant $\kappa \in (\min_\omega \mu(\omega), \max_\omega \mu(\omega))$ such that

$$\log\left(\sum_k \sum_{\omega \in \Omega_k} \frac{1}{\mu(\omega)} e(\omega)\right) - \sum_k \sum_{\omega \in \Omega_k} \left[e(\omega) \log\left(\frac{1}{\mu(\omega)}\right)\right] = \frac{1}{2}\kappa \text{Var}(\mu(\omega)),$$

where $\text{Var}(\mu(\omega)) = \sum_k \sum_{\omega \in \Omega_k} \left[e(\omega) \mu(\omega)^2\right] - \left[\sum_k \sum_{\omega \in \Omega_k} e(\omega) \mu(\omega)\right]^2$ is the variance operator. Combining the above equation with Equation 24 completes the proof.

# B Proof of Proposition 3

To assess our non-parametric policy, it is useful to start with the case in which all firms are homogeneous and face the same marginal cost, $c(\omega) \equiv c$. In this setting, our non-parametric policy performs just as well as its parametric counterpart, and restores the first best.

**Proposition 5.** *When all firms have the same marginal cost, the non-parametric policy* $T_{\omega,k}(p) = p^*(p,\omega) q^*(p,\omega) - pq^*(p,\omega)$ *uniquely implements the efficient allocation.*

Intuitively, when all firms are homogeneous, the construction of $q^*(p, \omega)$ ensures that the marginal transfer exactly offsets the inframarginal revenue loss for each firm, resulting in marginal cost pricing and the first best allocation.

It is instructive to discuss the proof of Proposition 5. To show that the first best pricing rule $p(\omega) = c$ is the unique equilibrium, we rule out any other equilibrium with heterogeneous prices. First, suppose there is an equilibrium in which a firm $\omega_H$ charges the highest price $p_H$, and that this price is strictly greater than $c$. Then $\mathcal{Q}_k(p_H) \leq q^*(p_H, \omega_H)$ and profits are $\Pi(\omega_H) = (p_H - c)\mathcal{Q}_k(p_H) + T_{\omega_H,k}(p_H)$. Hence, a small change in price does not affect $q^*(p_H, \omega_H)$ and $p^*(p_H, \omega_H)$, but changes profits by

$$\frac{\partial \Pi(\omega_H)}{\partial p_H} = \mathcal{Q}_k(p_H) + (p_H - c)\mathcal{Q}_k'(p_H) - q^*(p_H, \omega) < 0$$

so that the firm is better off lowering her price, contradicting the existence of an equilibrium. Therefore, the highest price satisfies $p_H \leq c$ in any equilibrium. Second, suppose there is an equilibrium in which a firm $\omega_L$ charges the lowest price $p_L$, and that this price is strictly smaller than $c$. Then $\mathcal{Q}_k(p_L) \geq q^*(p_L, \omega_L)$ and profits are $\Pi(\omega_L) = (p_L - c)\mathcal{Q}_k(p_L) + T_{\omega_L,k}(p_L)$. Hence, a small change in price does not affect $q^*(p_L, \omega_L)$ and $p^*(p_L, \omega_L)$, but changes profits by

$$\frac{\partial \Pi(\omega_L)}{\partial p_L} = \mathcal{Q}_k(p_L) + (p_L - c)\mathcal{Q}_k'(p_L) - q^*(p_L, \omega_L) > 0$$

so that the firm is better off raising her price, contradicting the existence of an equilibrium. Therefore, the lowest price satisfies $p_L \geq c$ in any equilibrium. Finally, because $p_H \geq p_L$ in any equilibrium, we have $p_H \leq c \leq p_L \leq p_H$, which can only be satisified when $p_L = p_H = c$. Hence, the only equilibrium consists of all firms charging the same price $p(\omega) = c$.

Finally, it is useful to note that under this policy, $T_{\omega,k}(c) = 0$ for all firms at the first best. Hence, the policy maker can undo the distortions of monopoly pricing without having to redistribute income from wages to profits through the transfer policy. This contrasts with existing theoretical policies such as sales subsidies which both require far more information to implement, and redistribute income from workers to firms to encourage efficient pricing decisions.

We now turn to the empirically relevant case of heterogeneous productivities and marginal costs. Following the homogeneous firm case, we focus on symmetric equilibria in which firms with the same productivities choose the same price: $z(\omega') = z(\omega)$ implies $p(\omega') = p(\omega)$. This focus is also consistent with our empirical strategy that at-

tributes observed heterogeneity in prices and quantities to unobserved heterogeneity in productivities and hence in marginal costs.

Let $\mathbb{Z}_k$ denote the set of latent productivities underlying $\mathbb{D}_k$. If for each $z \in \mathbb{Z}_k$, there are at least two firms with productivity $z$ so that $|\{\omega \in \Omega_k : z(\omega) = z\}| \geq 2$ for all $z \in \mathbb{Z}_k$, then our policy implements the first best.

**Proposition 6.** *Suppose that $|\{\omega \in \Omega_k : z(\omega) = z\}| \geq 2$ for each $z \in \mathbb{Z}_k$ and suppose that in equilibrium, $z(\omega') = z(\omega)$ implies $p(\omega') = p(\omega)$ for all $\omega, \omega' \in \Omega_k$. Then, the non-parametric policy $T_{\omega,k}(p) = p^*(p,\omega) q^*(p,\omega) - pq^*(p,\omega)$ uniquely implements the efficient allocation.*

When equally productive firms set the same prices, we have $q^*(p(\omega), \omega) = \mathcal{Q}_k(p(\omega))$ so that the marginal transfer exactly offsets the inframarginal revenue loss on additional output, just as in the homogeneous firm case. Intuitively, the policy endogenously divides firms into groups based on productivity, and then treats each group as a set of homogeneous firms, and thus implements the first best allocation. From an empirical perspective, Proposition 6 suggests that the policy will perform well when the data indicates that firms can be split into homogeneous subgroups.

This leaves the case of how the policy performs when the data indicate that some firms do not belong to any subgroups, and instead have unique productivities. In this case, there will exist a firm for whom $q^*(p(\omega), \omega) \neq \mathcal{Q}_k(p(\omega))$, so that the policy will not implement the first best allocation in equilibrium.

**Uniqueness of $q^*(p,\omega)$** Consider the behavior of an individual firm $\omega$, holding the prices and quantities of other firms $\{p(\omega'), \mathcal{Q}_k(p(\omega')) : \omega' \neq \omega\}$ fixed. Let firm $\omega$ set a price $p$ with quantity $\mathcal{Q}_k(p)$. Suppose that there exist quantities $q_L$ and $q_H$ such that $q_L < \mathcal{Q}_k(p) < q_H$ and $|q_L - \mathcal{Q}_k(p)| = |q_H - \mathcal{Q}_k(p)|$. Under the policy, profits are

$$\Pi_L(\omega) = (p - c)\mathcal{Q}_k(p) + p_L q_L - pq_L$$

$$\Pi_H(\omega) = (p - c)\mathcal{Q}_k(p) + p_H q_H - pq_H$$

If $\Pi_L \neq \Pi_H$ then the firm is not indifferent between the closest quantities and so should adjust her price to make $q^*(p,\omega)$ unique. If $\Pi_L = \Pi_H$, then we can differentiate to obtain

$$\frac{\partial \Pi_L(\omega)}{\partial p} = \mathcal{Q}_k(p) + (p - c)\mathcal{Q}_k'(p) - q_L$$

$$\frac{\partial \Pi_H(\omega)}{\partial p} = \mathcal{Q}_k(p) + (p - c)\mathcal{Q}_k'(p) - q_H < \frac{\partial \Pi_L(\omega)}{\partial p}$$

Hence if the price maximizes $\Pi_L$ so that $\frac{\partial \Pi_L(\omega)}{\partial p} = 0$, then choosing $\Pi_H$ cannot also be optimal because $\frac{\partial \Pi_H(\omega)}{\partial p} < 0$ and the firm should lower her price. Similarly, if the price maximizes $\Pi_H$ so that $\frac{\partial \Pi_H(\omega)}{\partial p} = 0$, then choosing $\Pi_L$ cannot also be optimal because $\frac{\partial \Pi_L(\omega)}{\partial p} > 0$ and the firm should raise her price. In both cases, the firm is not indifferent between both closest quantities and so should adjust her price to make $q^*(p, \omega)$ unique.

# C Additional Tables and Figures

Table C.1: Robustness: The welfare gains from the first and second-best transfers

| | Cobb-Douglas | CES | | |
|---|---|---|---|---|
| | $\sigma = 1$ | $\sigma = 1.5$ | $\sigma = 2$ | $\sigma = 2.5$ |
| **Panel A: Baseline Model** | | | | |
| % $\Delta W$ (1st best) | 6.9% | 11.9% | 17.9% | 26.3% |
| % $\Delta W$ (non-parametric policy) | 5.5% | 10.1% | 16.1% | 23.9% |
| Relative efficacy of non-parametric policy | 0.80 | 0.85 | 0.90 | 0.91 |
| **Panel B: Model with Input-Output Linkages** | | | | |
| % $\Delta W$ (1st best) | 22.3% | 27.3% | 33.1% | 39.9% |
| % $\Delta W$ (non-parametric policy) | 16.3% | 21.0% | 26.1% | 32.3% |
| Relative efficacy of non-parametric policy | 0.73 | 0.77 | 0.79 | 0.81 |

# D Derivatives for the Non-Parametric Policy

Under our parametric model, we can calculate the exact derivatives of the second-best transfer, therefore calculating the new markups. Recall that the transfer for the firm $i$ in sector $s$ is

$$T_{i,s}(p) = \left(P_{i,s}^T(p) - p\right) Q_{i,s}^T(p). \tag{25}$$

where

$$Q_{i,s}^T(p) = \begin{cases} \frac{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s}) q_{n,s}}{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s})} & \text{if } p \neq p_{n,s} \text{ for all } n \in \mathbb{I}_s \\ q_{n,s} & \text{if } p = p_{n,s} \text{ for some } n \in \mathbb{I}_s \end{cases}$$

and

$$P_{i,s}^T(p) = \begin{cases} \frac{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s}) p_{n,s}}{\sum_{n \in \mathbb{I}_s \setminus i} \omega(p - p_{n,s})} & \text{if } p \neq p_{n,s} \text{ for all } n \in \mathbb{I}_s \\ p_{n,s} & \text{if } p = p_{n,s} \text{ for some } n \in \mathbb{I}_s \end{cases}$$

The weighting function $\omega(p - p_{n,s}) = \| p - p_{n,s} \|^{-\theta}$, with parameter $\theta \gg 1$, assigns greater weight to firms whose prices are closer to $p$. Therefore, the profit of the firm will be

$$\pi_{i,s}(p) = (p - MC_{i,s}) Q_{i,s}(p) + \left(P_{i,s}^T(p) - p\right) Q_{i,s}^T(p)$$

and the profit maximization implies that

$$\mu_{i,s} = \frac{\varepsilon_{i,s}}{\varepsilon_{i,s} - 1 + \frac{q_{i,s}^T}{q_{i,s}} \left[ \frac{\partial \ln Q_{i,s}^T(p)}{\partial \ln p} \left( \frac{p_{i,s}^T}{p_{i,s}} - 1 \right) - \left( \frac{\partial P_{i,s}^T(p)}{\partial p} - 1 \right) \right]}$$

Thus, in order to calculate the new markups, we need to find $\frac{\partial \ln Q_{i,s}^T(p)}{\partial \ln p} = \frac{\partial Q_{i,s}^T(p)}{\partial p}\frac{p}{Q_{i,s}^T(p)}$ and $\frac{\partial P_{i,s}^T(p)}{\partial p}$. If $p \neq p_{n,s}$ for all $n \in \mathbb{I}_s$, we have

$$\frac{\partial P_{i,s}^T(p)}{\partial p} = \frac{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega'(p-p_{n,s})\,p_{n,s}\right]\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\right]}{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\right]^2} - \frac{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega'(p-p_{n,s})\right]\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\,p_{n,s}\right]}{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\right]^2}, \tag{26}$$

$$\frac{\partial Q_{i,s}^T(p)}{\partial p} = \frac{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega'(p-p_{n,s})\,q_{n,s}\right]\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\right]}{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\right]^2} - \frac{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega'(p-p_{n,s})\right]\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\,q_{n,s}\right]}{\left[\sum_{n\in\mathbb{I}_s\setminus i}\omega(p-p_{n,s})\right]^2}. \tag{27}$$

Otherwise (i.e. $p = p_{n,s}$for some $n \in \mathbb{I}_s$), we have

$$\frac{\partial Q_{i,s}^T(p)}{\partial p} = \frac{\partial P_{i,s}^T(p)}{\partial p} = 0.$$

Now, we only need to find $\omega'(p-p_{n,s})$ which is

$$\omega'(p-p_{n,s}) = \theta \parallel p-p_{n,s} \parallel^{-\theta-1}\left(\frac{p-p_{n,s}}{\parallel p-p_{n,s} \parallel}\right)$$

Recal that with granular firms and a sufficiently large $\theta$, $q_{i,s}^T \equiv Q_{i,s}^T(p_{i,s})$ corresponds to the quantity of the closest price neighbor. Hence,

$$\frac{\partial Q_{i,s}^T(p)}{\partial p} \approx \frac{\partial P_{i,s}^T(p)}{\partial p} \approx 0,$$

and the optimal markup reduces to

$$\mu_{i,s} \approx \frac{\varepsilon_{i,s}}{\varepsilon_{i,s}-1+q_{i,s}^T/q_{i,s}} \approx \left(1-\frac{1-q_{i,s}^T/q_{i,s}}{\varepsilon_{i,s}}\right)^{-1}$$

where $q_{i,s}^T \equiv Q_{i,s}^T(p_{i,s})$, with $p_{i,s} = \mu_{i,s}\mathrm{MC}_{i,s}$.