How Would a Rapid (Climate) Transition Affect Workers?*

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Abstract

We develop a life-cycle model with directed search and worker skill heterogeneity in which labor demand expands in a new sector and declines in an old sector that uses an aging technology. We show that worker welfare is strongly *non-linear* and even *non-monotonic* in the speed at which the new sector takes over. That is, workers' marginal welfare change can be positive or negative as a function of this speed and very sensitive to it. Two features are central to the shape of this relation: the joint distribution of worker skill transferability and age, and the responsiveness of layoffs and hires to firms' productivity which determines the response of structural unemployment. We estimate the model and apply it to study the transition from polluting to clean technologies.

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1 Introduction

As the world embarks on a climate transition, economic activity will shift from polluting to clean technologies. This transition will entail a substantial and potentially costly reallocation of labor. Indeed, high-emissions sectors employ millions of workers, job losses are expected to accelerate among these workers, and earnings losses after displacement are high for them (Walker, 2013; Curtis et al., 2023; Bassanini et al., 2024). The prospect of a painful transition has fueled a raging debate about how rapidly the climate transition should occur. While the benefits of curbing emissions are well understood and quantified, we know much less about the welfare cost imposed on workers in this context — or indeed for any technological transition where one set of jobs is replaced by another. In this paper, we study the labor market costs of technological transitions and how they vary as the transition *accelerates*. We quantify these costs in the context of the climate transition which is a necessary first step to decide how ambitious climate goals should be, e.g., whether to achieve zero-net emissions by 2050 or 2040 for instance.

We study this question using a life-cycle model with worker skill heterogeneity and a realistic labor market. The model features a transition between two sectors: the revenue product of labor grows exogenously in a new sector while an aging sector shrinks relative to trend. A parameter η governs the speed of this exogenous transition. Finite-lived risk-averse workers direct their search across sectors as in Menzio and Shi (2011) and differ in the units of human capital they can supply to the new sector. Unemployed workers face a risk of entering a long-term unemployment state, thereby losing access to generous unemployment benefits until they find a new job.

Before conducting a quantitative analysis, we first seek to understand qualitatively how workers' welfare depends on the speed of transition and which structural objects govern the shape of this relationship. To do so, we first consider a baseline model that features the minimal ingredients necessary without additional microfoundations that obscure the broader conceptual points. In particular, we initially assume a reduced-form markdown on wages and a reduced-form but endogenous layoff probability. Using this baseline model, we study how changes in the transition speed η affects worker welfare.

We show that the effect of technological transitions on worker welfare can be

highly *non-linear* or even *non-monotonic* in the speed of transition. In particular, workers' marginal welfare change can be positive or negative, extremely steep or flat as the transition speed changes. Depending on the features of the economy — most critically the joint distribution of skill specificity and age, and the sensitivity of layoffs and hires to firms' productivity — aggregate social welfare may feature an interior maximum or minimum as a function of speed of transition. This point holds for general technological transitions in which a new sector takes over an old one using an aging technology, of which the climate transition is just one example. Our insights apply more broadly, including to Schumpeterian industry dynamics where a new technology emerges over time (Aghion and Howitt, 1992), or when a burgeoning private sector takes over public activities in a formerly planned economy (Atkeson and Kehoe, 1996).

We find that non-linearities arise for two reasons: *heterogeneity* in worker skill specificity and age, and the emergence of *structural unemployment* along the transition. The importance of heterogeneity is most transparently illustrated in a Walrasian limit of our model in which search frictions disappear and workers are paid their marginal product. Here, a worker switches sectors only when the productivity in the new sector is sufficiently high relative to the old sector to compensate for any skill loss incurred upon switching. A worker's consumption falls relative to trend while they stay in the shrinking old sector, and rises after they move to the growing new sector. Workers have finite lives and do not benefit from further increases in productivity in the new sector after they have retired.

A faster transition has two opposing effects on a workers' welfare. On the one hand, it accentuates the decline in consumption while the worker remains in the old sector. On the other hand, it increases consumption after the worker switches to the new sector and ensures that a larger share of the transition is completed before retirement. A worker suffers the most for a *medium-speed* transition when the transition is fast enough that the worker experiences all of the earnings decline in the old sector, but slow enough that they retire before sufficiently benefiting from the subsequent recovery. The joint distribution of workers' age, which governs their horizon, and skill specificity, which determines their switching time and costs, is thus key for the aggregate welfare effects of faster transitions. In particular, we show analytically that aggregate welfare changes can be increasing, decreasing, U-shaped or even have inverse U-shaped regions depending on that distribution

of age and skill.

The crucial role of structural unemployment is most clearly demonstrated in a limiting economy at the opposite extreme: a model with homogeneous skills and search frictions. Here, the emergence of structural unemployment creates non-linearities of its own. To understand why, consider the effect of a faster transition. The value of matches in the old sector declines more quickly, which generates a spike in involuntary separations. At the same time, the job-finding rate rises in the new sector. These two opposing forces mirror the ones at play in the Walrasian benchmark. The presence of structural unemployment changes the picture, however. Workers who would have switched to the new sector in the Walrasian benchmark now face low job-finding rates in that sector. These workers can become "trapped" and transition very slowly across sectors when their skill transferability is small relative to the cost of hiring a new worker. In this case, they experience structural unemployment and large welfare losses along the way. This effect is particularly strong for medium-speed transitions that do not sufficiently incentivize firms to post new vacancies. In fact, this force is sufficiently potent that it can introduce its own form of non-monotonicity: welfare can be U-shaped as a function of the transition speed even in regions of the parameter space where welfare would be monotonic in a Walrasian economy.[1]

These limiting economies elucidate that two sets of objects are central to the shape and level of worker welfare changes as transitions get faster. First, the joint distribution of age and skill determine workers' horizon and cost of switching sector. Second, firms' propensity to fire and hire workers as the transition accelerates is governed by the average wage markdown – which affects the level of firm values – and the elasticity of layoffs with respect to firm values. The focus of our quantitative model is to microfound and quantify these objects.

Our quantitative model extends the baseline setup in five key directions. First, we microfound the distribution of markdowns by allowing for a job ladder as in Menzio and Shi (2011). Second, we microfound the probability of involuntary layoffs with random operating costs in the spirit of Clementi and Palazzo (2016). Third, we recognize that job loss carries scarring costs for workers. Specifically, we

[1] Our model with directed search does not feature the type of congestion of externalities present in random search models. Such externalities could, in principle, introduce an additional source of non-linearity.

follow Jarosch (2023) and suppose that unemployment results in a period of prolonged exogenous separations. Fourth, we introduce idiosyncratic amenity values in the search decision to rationalize the fact that workers often experience wage cuts when switching jobs (Sorkin, 2018). Finally, we introduce a third sector that neither grows nor declines relative to trend. This outside option is always available to workers but comes with a lower wage.

The quantitative model is calibrated to match various features of developed labor markets. Crucially, we match the quantities we showed were crucial in our benchmark model: the average markdown and the separation elasticity to productivity shocks. We also match the period of elevated separation rates following a job displacement which is important quantitatively. The joint distribution of skill specificity and age is inferred from job-switching behavior over the early stages of the climate transition using a novel estimation strategy leveraging the block recursivity of competitive search equilibria. Given a path for sectoral labor productivities, age-specific job flows across markets depend on two sets of independent objects: workers' policy functions and the distribution of skill-specificity. We are therefore able to invert the model and back out the weights on each skill type that match the realized job flows in the data. Having recovered the distribution of skill specificity by age, we can simulate our economy forward, update the paths of productivities to account for selection into reallocation, and repeat this procedure until convergence.

We use our quantitative model to study the transition towards clean technologies. We group sectors into two groups labeled "green" or "polluting" largely following the approach of Bontadini and Vona (2023) and Bassanini et al. (2024). The path of exogenous labor productivities is chosen to match the observed path of wages in these two groups of sectors.

<*...Results from the quantitative application will be added soon...*>

A large literature documents persistent and large earnings losses from job displacement and studies how human capital specificity affects the speed of labor reallocation following an aggregate shock. Building on the seminal work of Jacobson et al. (1993), Braxton and Taska (2023) show that workers displaced by technological change incur substantial losses when their skills make it difficult to adopt these new technologies. Dix-Carneiro (2014) estimates a dynamic equilibrium trade model with sector-specific human capital to study the speed of transi-

tion following a trade liberalization in Brazil. Traiberman (2020) argues that trade shocks destroy occupation-specific human capital, which slowed labor reallocation and generated substantial welfare costs for workers in Denmark. Adão et al. (2024) show that the adoption of new technologies takes more time if existing workers do not possess the right skills to operate these technologies, with younger generations playing an important role in this adoption. Human capital specificity is central to our analysis too. In contrast to this literature, however, we study a counterfactual question: how large would welfare costs be if the transition is accelerated or slowed? We show that the marginal cost to workers of speeding up transitions is far from constant and we identify the structural determinants of this non-linearity. We investigate this question in the context of the climate transition.

We develop a rich model that accounts for labor market frictions and involuntary employment. As such, our paper relates to a literature that studies how technological transitions affect unemployment and wages. The key takeaways in this literature are that faster technological change increases wage dispersion due to vintage human capital effects (Violante, 2002), tends to reduce long-run employment due to job obsolescence (Postel-Vinay, 2002) or higher costs of technology updating (Mortensen and Pissarides, 1998). Our analysis also considers transitions which make an old vintage of technology obsolete. Our focus is normative, however, and we highlight stark non-linearities in welfare as function of the speed of transition.

Our work is closely connected to a recent strand of the literature which asks whether policy should intervene to slow down the adoption of new technologies or assist workers during these transitions. Guerreiro et al. (2021), Costinot and Werning (2023), Thuemmel (2023) and Lehr and Restrepo (2023) argue that taxing new technologies can be optimal to mitigate the distributional consequences of automation. Beraja and Zorzi (2024b) show that technological transitions are inefficiently fast when they displace workers who are unable to perfectly smooth consumption when they are displaced. Braxton and Taska (2024) argue that governments should insure workers against technological change by expanding retraining subsidies. Relative to this literature, we emphasize strong non-linearities and non-monotonicities in welfare as transitions get faster, clarify the structural objects that underlie these effects, and quantify them in the context of the climate transition. In particular, we show that jointly modeling involuntary unemploy-

ment, on-the-job search, skill heterogeneity, and life cycles is key to capture these non-linearities.

Our quantitative exercise contributes to a growing body of work that studies the costs and benefits of a transition towards clean technologies (Bilal and Stock, 2025 for a review). A large literature estimates sizeable economic costs of climate change (Barrage and Nordhaus, 2023; Bilal and Rossi-Hansberg, 2023; Bilal and Känzig, 2024; Nath et al., 2024), suggesting that a fast transition away from polluting technologies could generate substantial societal gains. However, there has been comparatively little attention on the labor market costs of speeding up the transition to green technology. In other words, there are reliable estimates of the marginal benefit of speeding up the climate transition to curb emissions, but we know much less about the net marginal cost to workers. Curtis and Marinescu (2023) show that solar and wind job postings have more than tripled in the last decade, while Curtis et al. (2023) argue that the rate of transition from dirty to green jobs is rising rapidly and comes with potentially large costs to incumbent workers employed in polluting sectors. Vona et al. (2018) find that environmental regulation affects the demand for "green" skills while Hanson (2023) discusses various policies to ease the labor market transition away from fossil-fuel production. Känzig (2023) finds that the introduction of the European carbon market had sizable effects on unemployment.[2] Arkolakis and Walsh (2023, 2024) study the effect that increased availability of clean energy is likely to have on aggregate growth. While this work is positive in nature, our focus is on quantifying the welfare costs to workers of a faster transition.[3]

2 Baseline Model

We present our baseline framework here, and microfound all of its components in Section 4. We consider a technological transition consisting of an expansion of a new occupation at the expense of an old one, which leads to a reallocation of

[2] Similarly, Känzig (2021) finds that shocks to energy prices can have sizeable effect on unemployment.

[3] A complementary set of papers studies how policy can influence the speed of this transition (Acemoglu et al. (2012), Acemoglu et al. (2016), Acemoglu et al. (2023)).

labor between the two.[4] Workers are heterogeneous in their ability to transfer skills across occupations. Labor markets are imperfect which can generate structural unemployment.

2.1 Environment

Time is continuous and indexed by t. There are two jobs $k =$ "old","new" which produce the same final good – their output is perfectly substitutable with price normalized to 1. We refer to these jobs interchangeably as occupations or sectors. We assume that up to some initial period $t = 0$, the economy was in steady state with only the old sector active. The existence and growth of the new technology becomes common knowledge as of period 0. To maximize expositional clarity, we consider an economy with zero trend growth in the main text. This is without loss of generality: we show in Appendix A.1 that our insights apply exactly in an appropriately-calibrated economy with exogenous trend growth at constant rate g in which the transition governs deviations from this trend.[5] Therefore, our expressions should be interpreted as being relative to a constant trend growth rate.

2.1.1 Workers

Demographics and preferences. We assume an overlapping generation structure in which workers live for J periods and are replaced by new workers upon retirement. Workers supply labor and consume at all ages $j \in [0, J]$. Their preferences are represented by

$$\mathbb{E}_0 \left[\int_0^J \exp\left(-\rho t\right) u\left(c_t\right) dt \right] \tag{2.1}$$

where $u(c) \equiv c^{1-\sigma} / \left(1-\sigma\right)$ is the flow utility function, c_t is consumption in period t of life, and $0 < \rho < \infty$ is the discount rate. Let τ denote a worker's year of birth, so that incumbents' age as of period 0 is $-\tau$.

Skills. Workers' productivity $\psi\left(j, k, \omega\right)$ depends on their age j, their occupation of employment k as well as their skill type $\omega \in \Omega \subset \mathbb{R}_+$. Specifically, a worker's

[4] In the quantitative model of section 4, we introduce a third sector which does not undergo a transition and serves as an outside option.

[5] The isomorphism holds so long as workers have constant relative risk aversion preferences with coefficient σ, and firm owners die and are replaced at exogenous rate σg.

effective labor supply in occupation k is

$$\psi\left(j,\omega,k\right)\equiv\psi^{\text{age}}\left(j\right)\times\psi^{\text{comp}}\left(\omega,k\right), \tag{2.2}$$

where $\psi^{\text{age}}\left(j\right)$ captures the evolution of workers' earnings over the life cycle while $\psi^{\text{comp}}\left(\omega,k\right)$ captures workers' comparative advantage across sectors.[6] We suppose in the following that

$$\frac{\psi^{\text{comp}}\left(\omega_1,\text{new}\right)}{\psi^{\text{comp}}\left(\omega_1,\text{old}\right)}\geq\frac{\psi^{\text{comp}}\left(\omega_0,\text{new}\right)}{\psi^{\text{comp}}\left(\omega_0,\text{old}\right)}\quad\forall\omega_1\geq\omega_0$$

so high-ω workers have a stronger comparative advantage in the new sector. In this benchmark model, we set $\psi^{\text{age}}(j)=1$ for all ages j to illustrate the mechanisms as clearly as possible — we bring back productivity life cycles in the quantitative model. Therefore, we define $\psi(\omega,k)\equiv\psi^{\text{comp}}\left(\omega,k\right)$ for now. Let $\hat{\mu}\left(\tau,\omega\right)$ denote the distribution of skill ω and each cohort τ of workers. We normalize this distribution so $\hat{\mu}\left(\tau,d\omega\right)=1/J$ for every cohort.

2.1.2 Technology

There is a continuum of firms in each occupation. Each of these firms employs one worker and operates a linear technology

$$y_t\left(j,\omega,k\right)=A_t(k;\eta)\times\psi\left(j,\omega,k\right) \tag{2.3}$$

where ψ is the skill of the worker they employ, t is calendar time, η is a parameter governing the speed of transition between the sectors and $A_t(k;\eta)$ is the detrended productivity of sector k in period t. We assume that $A_t\left(\text{old};\eta\right)$ decreases over time while $A_t\left(\text{new};\eta\right)$ increases over time and faster so when the speed of transition η is large.[7] For illustrative purposes, we assume in our numerical examples in

[6] We abstract from skill accumulation so skill types correspond to present value production differences between sectors.

[7] Formally, our analytical results hold if, for all t and η, $\partial_t A_t(old;\eta)\leq 0\leq\partial_t A_t(new;\eta)$ (old sector is declining while the new sector is growing), $\partial_\eta\partial_t A_{new}(t;\eta)>0$ and $\partial_\eta\partial_t A_{old}(t;\eta)<0$ (η increases the speed of transition), and $\partial_t A_t(old;0)=\partial_t A_t(new;0)=0$ (if the transition speed is 0, there is no movement in productivity).

Section 3 that the transition is linear so

$$A_t(k;\eta) = A_{k0} + \min\{\eta t, 1\}\,\Delta_k \quad \text{where} \quad \Delta_k \equiv A_{k\infty} - A_{k0} \tag{2.4}$$

with $\Delta_{\text{new}} \geq 0$ and $\Delta_{\text{old}} \leq 0$ denoting the long-run changes in sectoral productivities.

We broadly interpret sectoral technology $A_t(k;\eta)$ as reflecting shifts in workers' marginal revenue product of labor. Note that the fact that the old sector is declining relative to trend does not imply the old sector is declining in absolute terms, simply that it begins growing at a slower rate than previously. The parameter η governing the speed of technology can be thought of as a stand-in for the scale of taxes and subsidies levied on clean versus polluting technologies in a climate application, or the rate at which a new technology overtakes an aging technology in a Schumpeterian model.

Firms created in period t are owned by infinitely-lived entrepreneurs with discount rate ρ and linear flow utility.[8]

2.1.3 Labor Markets and Incomes

Labor Markets. Workers and firms search for each other in a frictional labor market. Unemployed and employed workers receive the chance to search for a job at Poisson flow rate λ^U and λ^E respectively. When a search opportunity arrives, they may direct their search to either the new or old sector. We assume the existence of a constant returns to scale matching function which yields a job-finding rate $f(\theta)$ for θ market tightness defined as the ratio of vacancies to job-seekers, where f is increasing and concave and satisfies usual Inada conditions. In turn, firms fill the vacancy at rate $q(\theta) \equiv f(\theta)/\theta$ which decreases with θ.

To attract workers, firms may freely post vacancies at a constant cost ν. Letting $V_t(j,\omega,k)$ denote the value to a firm in sector k in period t of a job filled by a worker of age j and skill type ω, the free entry condition states that

$$q\left(\theta_t(j,\omega,k)\right) V_t(j,\omega,k) \leq \nu \tag{2.5}$$

[8] While profits are zero in steady state, *ex post* profits might not be zero during the transition to the new steady state. Assuming that profits are claimed by a separate group of investors is standard in search models. It also required to obtain block-recursivity in directed search models (Menzio and Shi, 2011) which we leverage as part of our structural estimation.

in every period t and market k, with equality if a positive measure of vacancies are posted. This condition implies that the probability that a worker searching in sector k is able to find a job is given by a function $\tilde{f}(V) \equiv f(q^{-1}(\nu/V))$. New cohorts ($\tau > 0$) are born unemployed.

Separations. Existing employment relationships are dissolved for three reasons. First, workers retire at age J. Second, workers may search on-the-job and seek employment in another sector. Finally, firms face operating costs with Poisson arrival rate χ_0. When incurred, the firm must pay the lesser of the cost ϕ and their full firm value. These costs lead workers to be fired with probability $\mathcal{F}(V;\phi)$ where V is the (continuation) value to the firm.[9] We assume $\mathcal{F}(V;\phi) = 1$ if $V \leq 0$: firms can freely layoff negative value workers. This reduced form specification for separations nests both the exogenous separations case, i.e., $\mathcal{F}(V;\phi)$ is constant, and the case in which firms terminate low-value matches.[10] The level and elasticity of this function will be micro-founded by the mean and variance of the random operating costs in the quantitative model.

Incomes. Firms pay workers a wage which is a reduced form markdown π on their marginal product. Therefore, worker wages when employed are given by $(1-\pi)y_t(j,\omega,k;\eta)$ and firms' flow profits are $\pi y_t(j,\omega,k;\eta)$. The distribution of markdowns will be microfounded in the quantitative model through directed search with a job-ladder.

Unemployed workers receive a flow unemployment benefit that scales with workers' life cycle productivity. Unemployed workers can be either short-term or long-term unemployed. Short-term unemployed workers (U) transition stochastically to a long-term unemployment state (D) at rate κ_d. A workers earns unemployment or disability flow benefits $b_t^d(\omega)\,\psi^{\text{age}}(j) > 0$ depending on their unemployment status $d \in \{U, D\}$, with $b_t^U(\omega) \geq b_t^D(\omega)$. We assume that these benefits

[9] The firing probability $\mathcal{F}(V)$ of course depends on the operating cost ϕ. We leave out this dependence to save on notation.

[10] These layoffs may be inefficient in that the sense that they depend only on firm value, not total match surplus. Jäger et al. (2023) and Davis and Krolikowski (2025) provide evidence for inefficient separations. In practice, firm's reluctance to decrease wages (Grigsby et al. (2021)) is an important obstacle to renegotiation. Accordingly, our quantitative model features fixed wage contracts. Blanco et al. (2024) present a model of inefficient separations arising from wage rigidity.

take the form of home production to abstract from taxation. Workers consume their income every period.

2.2 Equilibrium

Fixing a speed of transition η, the value of an employed worker in sector k is

$$\begin{aligned}\rho W_t^E(j,\omega,k) =& u\left(\left(1-\pi\right) y_t\left(j,\omega,k;\eta\right)\right) \\ &+ \lambda^E \max_{k'}\left\{\tilde{f}\left(V_t\left(j,\omega,k'\right)\right)\left[W_t^E\left(j,\omega,k'\right) - W_t^E\left(j,\omega,k\right)\right]\right\} \\ &+ \chi_0 \mathcal{F}\left(V_t\left(j,\omega,k\right);\phi\right)\left[\left(1-\xi\right)^{1-\sigma} W_t^U\left(j,\omega,U\right) - W_t^E\left(j,\omega,k\right)\right] \\ &+ \left.\frac{d}{d\epsilon} W_{t+\epsilon}^E\left(j+\epsilon,\omega,k\right)\right|_{\epsilon=0}, \end{aligned} \tag{2.6}$$

where $W_t^U(j,\omega,U)$ is the value of being (short-term) unemployed, which we define below. Workers discount at rate ρ. When employed, they receive a flow payoff equal to their utility from consuming their wage, which is the markdown $1-\pi$ times their marginal product $y_t(\cdot)$. They have the opportunity to search for another job at rate λ^E. At rate χ_0, the worker is involuntarily fired with probability $\mathcal{F}\left(V_t(j,\omega,k);\phi\right)$, in which case they exchange the value of employment for the value of being short-term unemployed. When they lose their job, they lose a fraction ξ of the present value of income, which scales down their value of unemployment by a factor $(1-\xi)^{1-\sigma}$. Finally, there is a drift in values both over time and over the life cycle. Since age advances one-for-one with time, the drift in both dimensions is characterized by the single index ϵ. In the following, we let

$$k_t^{E,\star}\left(j,\omega,k\right) \equiv \arg\max_{k'} \tilde{f}\left(V_t\left(j,\omega,k'\right)\right)\left[W_t^E\left(j,\omega,k'\right) - W_t^E\left(j,\omega,k\right)\right] \tag{2.7}$$

denote the optimally searched market of a worker (j,ω) employed in sector k in period t.

In turn, the value of an unemployed worker of duration type d, age j and skill type ω may be expressed as

$$\rho W_t^U(j,\omega,d) = u\left(b_t^d(\omega)\right) + \lambda^U \max_k \left\{\tilde{f}\left(V_t(j,\omega,k)\right)\left[W_t^E(j,\omega,k) - W_t^U(j,\omega,d)\right]\right\} + \kappa_d\left[W_t^U(j,\omega,D) - W_t^U(j,\omega,d)\right] + \frac{d}{d\epsilon}W_{t+\epsilon}^U(j+\epsilon,\omega)\Big|_{\epsilon=0} \tag{2.8}$$

where $W_t^E(j,\omega,k;\eta)$ is the value of a worker of age j and skill type ω employed in sector k in time t. Their flow payoff is the utility value of unemployment benefits. At rate λ^U, they have a chance to search at which point they choose which sector to search in and find a job with probability $\tilde{f}(V_t(j,\omega,k))$.[11] At rate κ_d, they become long-term unemployed if they were not already. Finally, they have drift in their values both over time and over the life cycle. We define $k_t^{U,\star}(j,\omega,d)$ for unemployed workers similarly to (2.7).

The value of a filled job to a firm in sector k is given by

$$\begin{aligned}\rho V_t(j,\omega,k) =& \pi y_t(j,\omega,k;\eta) - \chi_0 \min\{\phi, V_t(j,\omega,k)\} \\ &- \left[\chi_0 \mathcal{F}\left(V_t(j,\omega,k);\phi\right) + \lambda^E \tilde{f}\left(V_t\left(j,\omega,k_t^\star(j,\omega,k)\right)\right)\right] V_t(j,\omega,k) \\ &+ \frac{d}{d\epsilon}V_{t+\epsilon}(j+\epsilon,\omega,k)\Big|_{\epsilon=0}\end{aligned} \tag{2.9}$$

Firms' flow profits are $\pi y_t(\cdot)$ since wages are a constant markdown on production. Firms also discount at rate ρ and receive this flow payoff so long as the match continues. At rate χ_0, the firm must pay the lesser of the operating cost ϕ or their firm value. The match may dissolve before workers hit retirement age either because the firm lays off the worker, which occurs at rate $\chi_0\mathcal{F}(V)$, or because workers voluntarily quit due to on-the-job search. The opportunity for on-the-job search arrives at rate λ^E and the probability of separating conditional on searching is given by $\tilde{f}\left(V_t(\cdot)\right)$ in the sector in which the employed worker optimally searches $k_t^\star(\cdot)$. The firm's value also experiences drift both over time and over workers' life cycle. At retirement ($j = J$), all values are equal to zero.

An equilibrium consists of the value functions defined above and distributions

[11] We assume that $(1-\pi)A_t(old;\eta) \geq b_1$ for all t such that workers always prefer to search for employment. In the quantitative model, workers will always weakly prefer to search for employment without this assumption.

$\boldsymbol{\mu}_t(j,\omega) \equiv \left(\mu_t^E\left(j,\omega,k\right), \mu_t^U(j,\omega,d)\right)$ where $\mu_t^E\left(j,\omega,k\right)$ is the distribution for an employed workers of age j, skill type ω, in sector k and $\mu_t^U\left(j,\omega,d\right)$ is the distribution for unemployed workers of duration $d = U, D$. As is standard of directed search models (Menzio and Shi, 2011), the equilibrium features a block recursive structure.

Definition 1. Given a speed of transition η and an initial distribution of worker ages, skills and employment states $\boldsymbol{\mu_0}(j,\omega)$, a *block-recursive equilibrium (BRE)* consists of value functions for employed workers $W_t^E(j,\omega,k)$ and unemployed workers $W_t^U(j,\omega,d)$, firm values $V_t(j,\omega,k)$, as well as policy functions $k_t^{E,\star}(j,\omega,k)$ and $k_t^{U,\star}\left(j,\omega,d\right)$, and distributions $\boldsymbol{\mu}_t(j,\omega)$. These functions satisfy the following conditions:

1. The value of employed workers in sector k, $W_t^E(j,\omega,k)$, satisfies the Bellman equation 2.6 with boundary condition $W_t^E(J,\omega,k) = 0\ \forall t,\omega,k$,
2. The value of unemployed workers, $W_t^U(j,\omega,d)$, satisfies the Bellman equation (2.8) with boundary condition $W_t^U(J,\omega,d) = 0\ \forall t,\omega,d$,
3. Workers' search behavior $k_t^{E,\star}(j,\omega,k)$ satisfies expression (2.7) for employed workers, and $k_t^{U,\star}\left(j,\omega,d\right)$ is defined similarly for unemployed workers,
4. The value of a filled job in sector k, $V_t(j,\omega,k;\eta)$, satisfies Bellman equation (2.9) with boundary condition $V_t(J,\omega,k) = 0\ \forall t,\omega,k$,
5. Vacancy posting satisfies the free entry condition (2.5) in every period t and market k, with equality if a positive measure of vacancies are posted, and
6. The evolution of the distribution of workers across employment states $\boldsymbol{\mu_t}(j,\omega)$ is consistent with the flows implied by the value functions.

The block recursive structure of the equilibrium allows efficient computation of both distributions and value functions, as detailed in Appendix B. We initialize the economy in period 0 at a steady state assuming only the old sector exists.

3 Two Useful Limiting Economies

To elucidate the determinants of the welfare effects of fast and slow technological transitions, this section considers two informative limits of the model. We first consider a Walrasian limit of the model without unemployment risk to focus on the the role of worker heterogeneity. Then, we consider an economy in which workers have homogeneous skills but labor markets are imperfect.

3.1 A Walrasian Economy

The Walrasian limit occurs when workers are free to switch across occupations $\lambda^U, \lambda^E \to \infty$ and there are no vacancy-posting costs $\nu = 0$ so markdowns disappear $\pi = 0$. In this limit, workers are only indexed by their birth period τ and their skill transferability ω. We reintroduce labor market imperfections from Section 3.2 onwards.

3.1.1 Labor Supply and Consumption Profiles

We start by inspepecting the time path of consumption for an individual worker. The marginal product of a worker of skill type ω is $A_t(k;\eta)\psi(\omega,k)$ in sector k and period t. This is also their income in each sector since labor markets are competitive. Workers will therefore choose to work in the new sector if and only if

$$\hat{\psi}(\omega) \equiv \frac{\psi(\omega,\text{new})}{\psi(\omega,\text{old})} \geq \frac{A_t(\text{old};\eta)}{A_t(\text{new};\eta)} \equiv \hat{A}_t(\omega)^{-1} \tag{3.1}$$

as they can reallocate freely between the two. The left-hand side corresponds to the worker's relative productivity in the new sector. In turn, the right-hand side is the inverse of the relative TFP in the new sector $\hat{A}_t(\omega)$ and hence captures the relative wage cut that a worker experiences upon switching. The new sector is growing faster than the old sector, which implies a cutoff strategy. Workers will work in the old sector until a stopping time $t^\star\left(\hat{\psi}(\omega);\eta\right)$ when the new sector becomes sufficiently productive to justify switching. The stopping time depends on the worker's relative productivity in the new sector $\hat{\psi}(\omega)$ and the transition speed η. It ensures that expression (3.1) holds with equality if that equation has a solution or $t^\star\left(\hat{\psi}(\omega);\eta\right) \equiv \infty$ otherwise. A worker switches sector whenever this

stopping time occurs during their life span. We formalize this discussion below.

Lemma 1. *The set of workers* (ω, τ) *can be partitioned into:*

1. ***Always switchers*** *with* $\hat{\psi}(\omega) \geq \hat{A}_0^{-1}$ *move immediately to the new sector as* $t^\star\left(\hat{\psi}(\omega);\eta\right) = 0$ *for any* η*,*

2. ***Never switchers*** *with* $\hat{\psi}(\omega) \leq \hat{A}_\infty^{-1}$ *never move to the new sector as* $t^\star\left(\hat{\psi}(\omega);\eta\right) = \infty$ *for any* η*,*

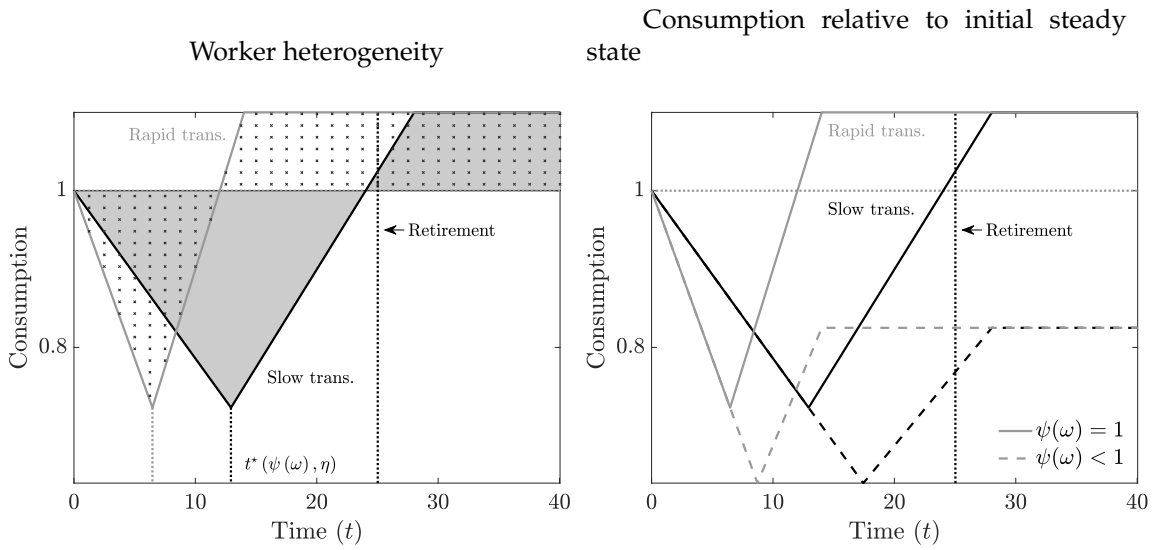
3. ***Occasional switchers*** *with* $\hat{\psi}(\omega) \in \left(\hat{A}_\infty^{-1}, \hat{A}_0^{-1}\right)$ *potentially move to the new sector as* $0 < t^\star\left(\hat{\psi}(\omega);\eta\right) < \infty$*. For these workers,* $t^\star\left(\hat{\psi}(\omega);\eta\right)$ *strictly decreases with the relative productivity in the new sector* $\hat{\psi}(\omega)$ *and the transition speed* η *and satisfies* $t^\star\left(\hat{\psi}(\omega);\eta\right) \to 0$ *as* $\eta \to \infty$*. Furthermore, these workers switch sectors during their lifetime if they are sufficiently young and the transition is sufficiently fast that* $\tau \geq t^\star\left(\hat{\psi}(\omega);\eta\right) - J$*.*

To illustrate this result, consider a transition where a new sector replaces the old sector without becoming substantially more productive in the long-run. We view this as approximating the transition to clean technologies, which are unlikely to be much more productive than their dirty counterparts but may come with other societal benefits. Specifically, suppose that sectoral productivities evolve according to (2.4) with $A_{\text{old},0} = A_{\text{new},\infty} = b = 0.4$, $A_{\text{old},0} = 1$ and $A_{\text{new},\infty} = 1.1$.[12]

The left panel of Figure 1 illustrates the effect that a faster transition has on workers' income and consumption by plotting the consumption dynamics for two transition speeds η for a worker with perfectly transferable skills ($\hat{\psi}(\omega) = 1$). The shaded of hashed areas correspond to consumption changes relative to steady state. Workers stay in the old sector until a stopping time $t^\star(\omega;\eta)$. At that point, the difference in productivity across sectors makes up exactly for the worker's lower productivity in the new sector. Workers' consumption declines over time until they switch and then rises until either the transition is complete or the worker retires. A faster transition reduces the stopping time $t^\star(\omega;\eta)$ and moves the trough of the consumption profile closer to $t = 0$. Older workers who retire sooner are less likely to switch to the new sector and enjoy consumption gains there. Therefore

[12] Since no workers are unemployed in the Walrasian model, b represents the uniform outside option for workers. Our qualitative insights carry through even when $A_{\text{new},\infty} \leq A_{\text{old},0}$, i.e., when the new sector is somewhat less productive in the long-run than the old sector originally was.

Figure 1: Workers' consumption over the transition



Notes: Figure plots workers' consumption profile in the Walrasian limit of our model. The left panel highlights workers' consumption changes relative to the initial steady state as the transition speed changes for a w worker with perfectly transferable skills ($\hat{\psi}(\omega) = 1$). The shaded region corresponds to a slow transition (low η) while the crossed region corresponds to a rapid transition (higher η). The right panel shows the consumption paths for workers of different skill types. The solid lines correspond to a versatile worker with $\hat{\psi}(\omega) = 1$, i.e., no loss of skills between old and new sectors, while the dashed lines correspond to a less versatile worker with $\hat{\psi}(\omega) = 0.75$. The black lines correspond to a slow transition (low η) and the lines to a rapid transition (higher η). The transition is linear as in (2.4) with $A_{\text{new},0} = A_{\text{old},\infty} = 0.4$ and $A_{\text{new},\infty} = 1.1$, while $A_{\text{old},0} = 1$. Time is measured in years on the horizontal axis.

speeding up the transition has two opposing effects. On the one hand, a faster transition accelerates the decline in consumption while the worker remains in the old sector. On the other hand, it accelerates the increase in consumption after switching.

The right panel of Figure 1 repeats this exercise for workers with two different levels of skill specificity ω. The black lines correspond to a slow transition whereas the grey lines correspond to a rapid one. In turn, the solid lines correspond to a versatile worker with $\hat{\psi}(\omega) = 1$ that is equally productive in both sectors while the dashed lines correspond to a less versatile worker with $\hat{\psi}(\omega) = 0.75$ who experiences a 25% loss in human capital when switching to the new sector. Less versatile workers switch to the new sector later and therefore experience a deeper, more prolonged decline in consumption.

3.1.2 Individual Welfare

How does a faster transition affect workers' welfare? To answer this question, we start by characterizing the continuation welfare of an individual worker born in period $\tau \geq -J$ with skill transferability ω. In the following, we let $t_0(\tau) \equiv \max\{\tau, 0\}$ denote the first period where the worker is present during the transition. The welfare of such a worker is given by

$$W(\tau, \omega; \eta) = \psi(\omega, \text{old}) \int_{t_0(\tau)}^{t^\star(\psi(\omega);\eta)} e^{-\rho(t-t_0(\tau))} u\left(A_t(\text{old}; \eta)\, \psi(\omega, \text{old})\right) dt + \psi(\omega, \text{new}) \int_{t^\star(\psi(\omega);\eta)}^{J+\tau} e^{-\rho(t-t_0(\tau))} u\left(A_t(\text{new}; \eta)\, \psi(\omega, \text{new})\right) dt \quad (3.2)$$

where the integrals are understood to be equal to zero if the upper limit of the integral is lower than the lower limit.[13]

One can understand the effect of a faster transition on worker welfare by inspecting Figure 1. Speeding up the transition leads to a steeper consumption decline while the worker is in the old sector, a steeper increase in the new sector, and an earlier switching time. The total effect on welfare for an individual is given by the size of the shaded triangle below 1 relative to the shaded trapezoid above 1, appropriately weighted by workers' utility function u and discount rate ρ. Formally, the marginal change of worker welfare from a faster transition is

$$\frac{dW(\tau, \omega; \eta)}{d\eta} \propto \int_{t_0(\tau)}^{t^\star\left(\hat{\psi}(\omega);\eta\right)} e^{-\rho(t-t_0(\tau))} u'\left(A_t(\text{old}; \eta)\right) \frac{\partial A_t(\text{old}; \eta)}{\partial \eta} dt + \hat{\psi}(\omega) \int_{t^\star\left(\hat{\psi}(\omega);\eta\right)}^{J+\tau} e^{-\rho(t-t_0(\tau))} u'\left(A_t(\text{new}; \eta)\, \hat{\psi}(\omega)\right) \frac{\partial A_t(\text{new}; \eta)}{\partial \eta} dt, \quad (3.3)$$

where the proportionality factor is $\psi(\omega, \text{old})^{1-\sigma} > 0$ and where we have used the fact that the stopping time $t^\star\left(\hat{\psi}(\omega); \eta\right)$ is chosen optimally so its response has no first order effect on welfare. The first term in expression (3.3) is negative for everyone except always-switchers who have $t^\star\left(\hat{\psi}(\omega); \eta\right) \equiv 0$. The second term is positive for everyone except never-switchers who have $t^\star\left(\hat{\psi}(\omega); \eta\right) \equiv \infty$. As this expression makes clear, the welfare of always-switchers strictly increases with the transition speed η whereas the welfare of never-switchers strictly decreases with

[13] Periods preceeding the transition, i.e., $t < 0$, are irrelevant for welfare.

η.

Whether the net effect of a faster transition is positive or negative for occasional switchers depends on two key features. First, the worker's relative productivity in the two sectors $\hat{\psi}(\omega)$ determines both their stopping time $t^\star\left(\hat{\psi}\left(\omega\right);\eta\right)$ and the size of consumption gains after switching. Second, worker's age determines their horizon $J+\tau$, which in turn determines whether the switch occurs during their lifetime and how much of the faster consumption growth in the new sector they will enjoy after switching.[14] When the transition is slow, workers retire before switching but suffer only modest income losses. As the transition accelerates, these losses increase. Eventually, the transition is sufficiently fast that occasional switchers move to the new sector. Their welfare reaches a minimum when the (weighted) area of the shaded region below 1 in Figure 1 is maximized. Beyond that point, occasional switchers prefer a faster transition so they benefit from a steeper income increase in the new sector. In summary, the welfare of these workers is U-shaped in the transition speed. They are most harmed by *medium-speed* transitions. We formalize this discussion below.

Proposition 1. *Individual welfare $\mathcal{W}\left(\tau,\omega;\eta\right)$ is: (i)*

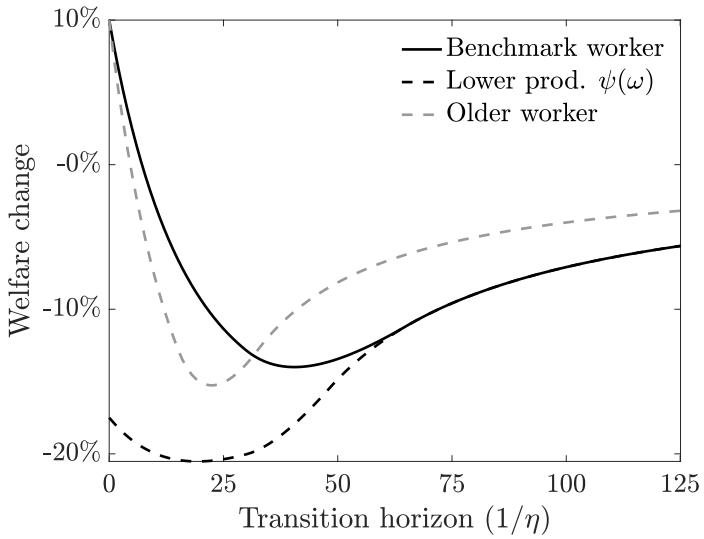
1. ***Increasing** in η for the always-switchers with $\hat{\psi}(\omega)\geq\hat{A}_0^{-1}$,*
2. ***Decreasing** in η for the never-switchers with $\hat{\psi}(\omega)\leq\hat{A}_\infty^{-1}$,*
3. ***U-shaped** in η for the occasional switchers with $\hat{\psi}(\omega)\in\left(\hat{A}_\infty^{-1},\hat{A}_0^{-1}\right)$. Formally, for each skill ω and birth date τ, there exists a critical speed $\eta^\star\left(\omega,\tau\right)$ with $0<\eta^\star\left(\omega,\tau\right)<\infty$ such that $\mathcal{W}\left(\tau,\omega;\eta\right)$ is decreasing in η for $\eta<\eta^\star\left(\omega,\tau\right)$ and increasing in η for $\eta>\eta^\star\left(\omega,\tau\right)$.*

Proof. See Appendix A. □

Figure 2 illustrates this result when the relative risk aversion is $\sigma=2$ and the skill in the old sector is normalized to $\psi\left(\omega,\text{old}\right)=1$ for all ω. We focus in this plot on workers who are occasional switchers. All welfare changes are expressed in consumption-equivalent terms. The horizontal axis considers the horizon of the transition $1/\eta$: a longer horizon corresponds to a slower transition. The solid

[14] Workers' subjective discount rate ρ plays a similar role by shortening workers' horizon.

Figure 2: Individual welfare changes



Notes: Figure plots a worker's consumption-equivalent welfare change (3.3) from the transition as a function of the horizon of the transition $1/\eta$ (measured in years). The solid black curve corresponds to a benchmark worker who has $\hat{\psi}(\omega) = 1$ and a remaining working life of 40 years. The dashed black line corresponds to a worker with a more specific skills ($\hat{\psi}(\omega) = 0.75$), while the gray dashed line corresponds to an older worker with a remaining working life of 20 years.

black line plots welfare changes as a function of the transition horizon $1/\eta$ for a worker with 40 years left to work with $\hat{\psi}(\omega) = 1$. The figure confirms that individual welfare changes are U-shaped in the speed of transition. That is, the labor market costs of the transition are the largest for medium-speed transitions for which workers experience a long decline in income while they stay in the old sector, without being sufficiently long-lived to enjoy the steeper increase in income after they switch sectors.

Next consider the dashed black line, which corresponds to a less versatile worker with $\hat{\psi}(\omega) = 0.75$. The welfare changes are identical to the previous worker with $\hat{\psi}(\omega) = 1$ if the transition is sufficiently slow that neither of them end up switching sectors during their lifetime. As the transition accelerates, workers with less transferable skills are stuck in their sector and experience large welfare losses whereas workers with more transferable skills benefit from a faster transition. Finally, the dashed gray line considers a worker with transferable skills but only 20 years of working life remaining. For relatively slow transitions, this older worker is less harmed than its younger counterpart as they have a shorter horizon over which they experience income losses in the old sector. However, the older worker is more

sensitive to medium-speed transitions since they retire before realizing the benefits the transition.

Note that this broader conceptual point does not depend on the assumption that workers are hand-to-mouth consumers.[15] One can perform the same analysis assuming risk-neutral workers, who only value the present value of their income. Changes in this present value are still generally non-monotone in the speed of transition through exactly the same mechanisms described above.

3.1.3 Social Welfare

Having characterized individual welfare, we are now ready to investigate how social welfare changes as the transition accelerates. Social welfare is defined as

$$\overline{\mathcal{W}}(\eta) \equiv \int_{\Omega} \int_{-J}^{\infty} \Lambda(\tau, \omega) e^{\rho \min\{\tau, 0\}} W(\tau, \omega; \eta) \hat{\mu}(\omega, \tau) d\tau d\omega \tag{3.4}$$

where $\Lambda(\tau, \omega)$ are Pareto weights, $W(\tau, \omega; \eta)$ denotes the continuation values (3.2), $\hat{\mu}(\omega, \tau)$ still denotes the distribution of skill ω and birth date (or cohort) τ and the term $e^{\rho \min\{\tau, 0\}}$ captures the fact that utilities are evaluated at the time of birth.[16] In the rest of this section, we suppose that the government is utilitarian and discounts future generations using the subjective discount rate ρ.[17] As the

[15] In practice, about 41% of US households are hand-to-mouth (Kaplan and Violante, 2022). More generally, median wealth excluding housing is quite low among US workers (i.e., their precautionary savings buffer is small).

[16] Expressing utilities at the time of birth is standard in the literature. Indeed, it is necessary to ensures that social preferences are time consistent (Calvo and Obstfeld, 1988). There are no profits in the Walrasian limit so the welfare of entrepreneurs is zero.

[17] Both assumptions are common in the literature. See Itskhoki and Moll (2019) and Guerreiro et al. (2022) among many others. Of course, the conclusions that we reach below about the role of the skill distribution $\hat{\mu}(\tau, \omega)$ apply just as well to the Pareto weights $\Lambda(\tau, \omega)$ since they are isomorphic in the definition of social welfare (3.4).

transition speed η increases, the change in social welfare is

$$\begin{aligned}\frac{d\overline{\mathcal{W}}(\eta)}{d\eta} = &\int_{\left\{\omega:\hat{\psi}(\omega)\leq\hat{A}_{\infty}^{-1}\right\}}\int_{-J}^{\infty}\Lambda(\tau,\omega)\frac{\partial W(\tau,\omega;\eta)}{\partial\eta}\hat{\mu}(\omega,\tau)\,d\tau d\omega \\ &+\int_{\left\{\omega:\hat{\psi}(\omega)\geq\hat{A}_{0}^{-1}\right\}}\int_{-J}^{\infty}\Lambda(\tau,\omega)\frac{\partial W(\tau,\omega;\eta)}{\partial\eta}\hat{\mu}(\omega,\tau)\,d\tau d\omega \\ &+\int_{\left\{\omega:\hat{\psi}(\omega)\in\left(\hat{A}_{\infty}^{-1},\hat{A}_{0}^{-1}\right)\right\}}\int_{-J}^{\infty}\Lambda(\tau,\omega)\frac{\partial W(\tau,\omega;\eta)}{\partial\eta}\hat{\mu}(\omega,\tau)\,d\tau d\omega \qquad (3.5)\end{aligned}$$

where individual welfare changes $\partial W/\partial\eta$ are still given by (3.3) and where we have split the integral over ω into the contribution of never-switchers, always-switchers, and occasional switchers (Lemma 1). As discussed earlier, these three groups respond very differently to a faster transition. As a result, the shape of the social welfare function $\overline{\mathcal{W}}(\eta)$ may be almost arbitrary depending on the skill distribution $\hat{\mu}(\omega,\tau)$. In the following, we will first consider an age-invariant distributions of skill $\hat{\mu}_0(\omega,\tau)\equiv\hat{\mu}^{\star}(\omega)$. We will then turn attention to a set of correlated distributions $\hat{\mu}_1(\omega,\tau)$ with the same marginal distribution of skills

$$\int_{-J}^{0}\hat{\mu}_1(\omega,d\tau)=\int_{-J}^{0}\hat{\mu}^{\star}(\omega)\,d\tau \quad\text{and}\quad \hat{\mu}_1(d\omega,\tau)=\delta(g(\tau)) \qquad (3.6)$$

for incumbent generations, where δ is the Dirac delta function and g is some monotonic function of the birth date τ. In other words, the distribution $\hat{\mu}_1(\omega,\tau)$ assigns a weight 1 to a worker with $\omega=g(\tau)$ so age $-\tau$ and skill ω are perfectly correlated. When this function is decreasing, older workers with lower τ have a higher productivity in the new sector (i.e., age and productivity are positively correlated). The opposite happens when the function is decreasing.

Proposition 2. *There exist age-invariant distributions of skill $\hat{\mu}_0(\tau,\omega)\equiv\hat{\mu}^{\star}(\omega)$ such that aggregate social welfare $\overline{\mathcal{W}}(\eta)$ is either:*

1. ***Increasing** or **decreasing** in η*
2. ***U-shaped** in η, or*
3. ***Inverted U-shaped** over some region for η provided that the final productivity in the new sector is not too large relative to the one in the old sector $A_{\text{new},\infty}/A_{\text{old},\infty}<\hat{A}$ for some threshold $\hat{A}>1$.*

Furthermore, fix some transition speed η and a correlated distribution $\hat{\mu}_1(\omega, \tau)$ that satisfies (3.6) for some monotonic g. Then, the slope of the aggregate welfare function $\overline{\mathcal{W}}(\eta)$ can ***reverse*** *depending on g (i.e., positive or negative correlation between age and skill).*

Proof. See Appendix A. □

This proposition states that workers' net marginal cost from a faster transition can be positive, negative, flat and extremely non-linear depending on the skill distribution $\hat{\mu}(\omega, \tau)$. The first two cases are a corollary to Proposition 1: social welfare is monotonic or U-shaped when the age-invariant distribution $\mu^\star(\omega)$ puts all the mass in the regions corresponding to always-switchers, never-switchers or occasional switchers. To understand the logic between the potential inverse U-shape, consider a distribution that places a small mass $\varepsilon > 0$ on a group of incumbent workers who are never-switchers and suffer from a faster transition. The remaining mass is placed on always-switchers who have a productivity $\bar{\psi} = \hat{A}_0^{-1}$ in the new sector such that they are indifferent between the two sectors at first and then benefit from a faster transition.[18] After combining expressions (3.3) and (3.5) and assuming that the government is utilitarian, the change in social welfare is

$$\begin{aligned}\frac{d\overline{\mathcal{W}}(\eta)}{d\eta} =& \bar{\psi}\int_0^{+\infty} e^{-\rho t}\left(1-\varepsilon h_t(J)\right)u'\left(\bar{\psi}A_t(\text{new};\eta)\right)\frac{\partial A_t(\text{new};\eta)}{\partial \eta}dt \\ &+\varepsilon\int_0^{J} e^{-\rho t}h_t(J)\,u'\left(A_t(\text{old};\eta)\right)\frac{\partial A_t(\text{old};\eta)}{\partial \eta}dt \end{aligned} \tag{3.7}$$

where $h_t(J) \equiv 1 - \min\{t, J\}/J$ is the mass of incumbent workers who survive until period t. Starting from a very slow transition with $\eta \to 0$, the marginal utilities are roughly equalized in (3.7) since the always-switchers are indifferent between the two sectors, by assumption. Therefore, social welfare increases if the mass ε is sufficiently small. Instead, when starting from a very rapid transition with $\eta \to \infty$, the marginal utilities are constant over time since the economy jumps to its new steady state. The marginal utility of never-switchers can become arbitrarily large when $\Delta_{\text{new}} < 0$ is very negative. Furthermore, the transition occurs instantly: that is, when h_t is the largest. As a result, the second term in (3.7) can dominate and

[18] In this example, new generations are also born with productivity $\bar{\psi}$ in the new sector.

welfare declines in η.[19] Taken together, these observations imply that the social welfare function must feature some inverse U-shaped region under the skill distribution that we considered.

The final part of Proposition 2 shows that it is crucial to discipline not only the distribution of skills ω but also its *correlation* with age $-\tau$. Indeed, the welfare implications of a faster transition can be fully reversed depending on this correlation. To understand why, consider the previous example where the marginal distribution $\mu^{\star}(\omega)$ puts a small weight on workers with low skill transferability ω. Suppose that distribution of skills $\hat{\mu}(\omega, \tau)$ is now perfectly correlated with age. For instance, suppose that old workers tend to be less versatile (i.e., a negative correlation between skill ω and age $-\tau$). For the sake of illustration, suppose that the skill distribution (3.6) takes the following form

$$g(\tau) = \begin{cases} g_0 & \text{if } \tau < J(\varepsilon - 1) < 0 \\ \bar{\psi} & \text{otherwise} \end{cases}, \tag{3.8}$$

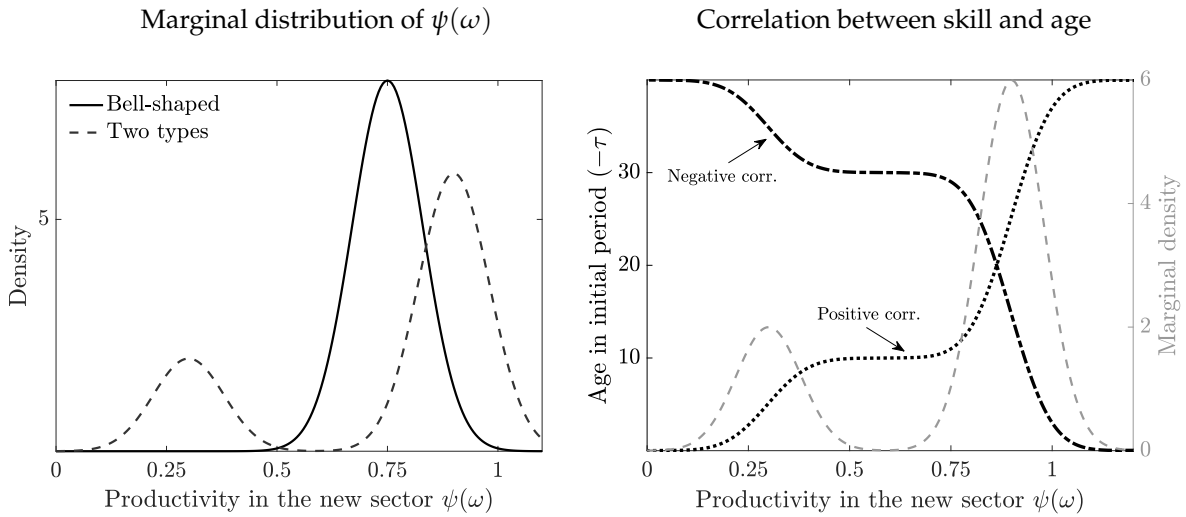
where g_0 is small and $\bar{\psi}$ is defined as before, i.e., old generations who only live for a small period of time εJ are never-switchers whereas younger and new generations are always-switchers. In this case,

$$\begin{aligned} \frac{d\overline{\mathcal{W}}(\eta)}{d\eta} =& \bar{\psi} \int_0^{+\infty} e^{-\rho t} \left(1 - \varepsilon \phi_t(\varepsilon)\right) u'\left(\bar{\psi} A_t(\text{new}; \eta)\right) \frac{\partial A_t(\text{new}; \eta)}{\partial \eta} dt d\tau \\ &+ \varepsilon \int_0^{\varepsilon} e^{-\rho t} \phi_t(\varepsilon) u'\left(A_t(\text{old}; \eta)\right) \frac{\partial A_t(\text{old}; \eta)}{\partial \eta} dt d\tau \end{aligned} \tag{3.9}$$

where $\phi_t(\varepsilon) \equiv 1 - \min\{t, \varepsilon\} / \varepsilon$. Comparing (3.7) and (3.9) reveals the role that the correlation between skill and age can plays for welfare change. The two expressions are identical but for the integration bound for never-switchers and the associated weights ϕ_t. While the mass of workers who are never switchers in $t = 0$ is the same in both expressions (i.e., ε), the *horizon* over which income changes are integrated is very different. When skill and age are correlated, never-switchers make up for a small share of total income changes precisely because they are short-lived. Therefore, the contribution of the negative term in expression (3.9) is much

[19] We show numerically below that this case can easily arise for empirically reasonable parametrizations.

Figure 3: Distribution of skills and birth dates $\hat{\mu}(\omega, \tau)$



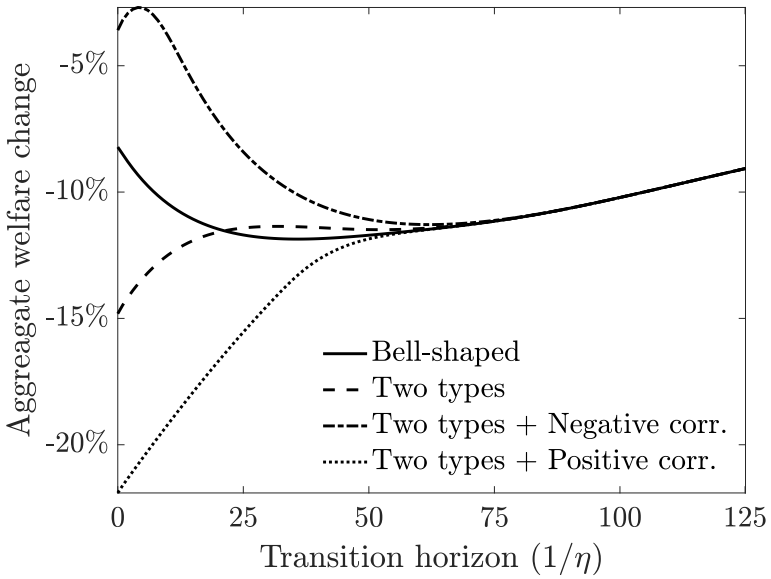
Notes: The left panel plots two skill distributions with mean 0.75. The solid curve is a uni-model "bell-shaped" distribution. The dashed curve is a bi-modal distribution with "two types" where some households have much less versatile skills in the new sector. The right panel illustrates cases when skill and age are perfectly correlated. The gray dashed line plots the density of the assumed two-types distribution. The black dotted and dash-dotted curves plot the age of workers who have a particular level of new sector skill when age and skill are positively and negatively perfectly correlated, respectively.

smaller than it is in (3.7) which can reverse the sign of $\overline{W}'(\eta)$.

Figures 3 and 4 illustrate these results. The left panel of Figure 3 plots two distributions of productivities in the new sector $\psi(\omega, \text{new})$ with a common mean of 0.75. The solid line corresponds to a bell-shaped distribution, while the dashed line corresponds to a "two-type" distribution where many workers have very transferable skills (high ψ) and some workers have much less transferable skills (low ψ). We continue to normalize $\psi(\omega, \text{old}) = 1$ in the old sector for all workers. Throughout, we assume that workers live for 40 years and that new cohorts of workers have perfectly transferable skills to the new sector ($\psi = 1$). The right panel plots workers' skill $\psi(\omega)$ as a function of their age $-\tau$ in the initial period. We consider three cases: an age-invariant skill distribution, one where age and skill are positively correlated (dotted) and one where they are negatively correlated (dash-dotted). In both cases, the marginal distribution coincides with the "two-type" distribution discussed previously.

Figure 4 plots social welfare changes (in consumption-equivalent terms) as a function of the transition horizon $1/\eta$. A longer horizon indicates a slower tran-

Figure 4: Social welfare changes



Notes: This figure plots social welfare changes (in consumption-equivalent terms) as a function of the transition horizon $1/\eta$ (measured in years). The four curves correspond to the distributions plotted in Figure 3. The solid and dashed lines plot age-invariant distributions from the left panel of Figure 3. The dotted line plots welfare changes from a two-type marginal distribution of skills with perfect positive correlation between skill and age as illustrated by the dotted line in the right panel of Figure 3, while the dash-dotted line considers perfect negative correlation.

sition. The four curves correspond to the four distributions of skill and age in Figure 3. First, the solid line reports the social welfare changes when skills are independent of age and distributed according to the bell-shaped distribution. Social welfare features a smooth U-shape in this case, which reflects the micro welfare effects studied in Figure 2 since most incumbent workers have similar skills under this distribution. Medium-speed transitions are still the most painful. In turn, the dashed line corresponds to social welfare under the two-type distribution, maintaining the assumption of independence between skill type and age. In contrast to the bell-shaped distribution, social welfare is now declining in the speed of transition (increasing in transition horizon) and features a relatively long segment where the marginal cost of a faster transition is close to zero. The presence of less versatile workers with low $\psi(\omega)$ reduce social welfare as the transition gets faster. Introducing a negative correlation between age $-\tau$ and skill ω reverses that conclusion as low $\psi(\omega)$ workers are short-lived and contribute less to social welfare. In contrast, introducing a positive correlation completely once again flips the pic-

ture completely and social welfare declines quickly with the speed of transition. Overall, these findings show that the joint distribution of age and skill is a crucial determinant how social welfare changes as technological transitions accelerate. Accordingly, this distribution will be a key input in our quantitative analysis.

3.1.4 Other Technological Transitions

Thus far, we have studied the case in which the old sector declines while a new sector rises. We believe that this scenario is a good approximation of the transition from polluting to clean technologies or, more broadly, of Schumpeterian dynamics where the marginal revenue product of an aging technology declines while a new technology takes over. While this section has largely focused on the non-monotonicities that can arise in this scenario, the broader point is that welfare can be strongly *non-linear* in the speed of transition.

To illustrate this, consider the case where the marginal revenue product in the old sector does not decline, i.e., $\Delta_{old} = 0$, despite being taken over by the new technology. In this case, Figure 1 would feature a flat line at 1 followed by a linear increase and leveling out when the new sector reaches its final steady state. Then, the integral of consumption changes corresponds to the area of the trapezoid above 1 (or triangle if the worker retires earlier before the transition is complete). As the transition accelerates, the point at which the worker switches to the new sector shifts to the left, therefore expanding the area of this trapezoid as in the scenario studied above. As a result, a worker's total income grows *quadratically* with η once the transition is sufficiently fast that they switch to the new sector.[20] In others words, individual welfare is flat at first and then strongly non-linear in the speed of transition η. In this case, estimating the joint distribution of skill and age is crucial to identify the *inflection point(s)* in social welfare as opposed to (local) maximum or minimum.

3.2 The Role of Structural Unemployment

The previous section assumed that labor markets were frictionless. We now show that structural unemployment along the transition can also generate non-linearities

[20] As above, curvature in utility and discounting also play a role in these non-linearities.

and even non-monotonicities.[21] This section illustrates the forces at play qualitatively, while we present our quantitative analysis in Section 4 onwards.

To isolate the role of labor market imperfections, we suppose in this section that all incumbent workers are ex ante homogeneous and share the same degree of skill transferability with productivities $\psi(\text{old}) \equiv 1$ and $\psi(\text{new}) > 0$. We continue to assume that new generations have one unit of human capital in both sectors.

The emergence of structural unemployment along the transition depends on firms' propensity to fire and hire workers as their productivity changes. We suppose that the separation probability conditional on incurring an operating cost is

$$\mathcal{F}(V;\phi) = 1 - \Phi\left(\frac{\log(V) - \phi}{\gamma}\right) \tag{3.10}$$

where V denotes the firm's value, Φ is the standard normal cumulative distribution function (CDF), $\phi > 0$ controls the baseline separation rate and $\gamma > 0$ determines the elasticity of separations with respect to firm value. Section 4 microfounds this functional form and elaborates on our calibration approach. We set γ to match an elasticity of separations to firm value of -2, following evidence from Chan et al. (2023), and we choose ϕ to match a steady state unemployment rate of 5%. The frequency of operating costs is not identified separately from the average cost ϕ. We normalize $\chi_0 \equiv 4$ so operating costs are incurred every quarter on average, reflecting quarterly reviews. In turn, hirings depend on both the matching technology and the value of a successful match. Following Den Haan et al. (2000), we assume that the matching function is $f(\theta) = (1+\theta^{-\alpha})^{-1/\alpha}$ and $q(\theta) = (1+\theta^{\alpha})^{-1/\alpha}$ and we set $\alpha = 1.6$ as in Schaal (2017). The job-finding rate as a function of the value of a filled job V is thus

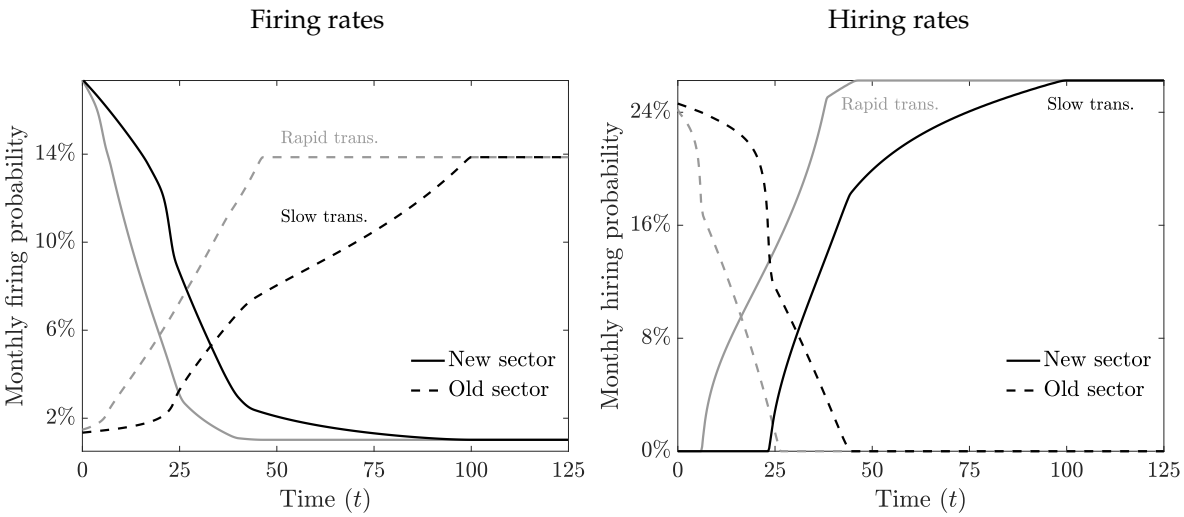
$$\tilde{f}(V) = \left(1 + \left[(\nu/V)^{-\alpha} - 1\right]^{-1}\right)^{-1/\alpha} \tag{3.11}$$

where ν captures the cost of posting a vacancy. We set $\nu = 1/5$, which lies between the estimates of Gavazza et al. (2018) and Muehlemann and Strupler Leiser (2018).[22] Finally, the value of a successful match depends on the markdown π

[21] In our directed search model, these non-linearities arise even absent congestion externalities that are typically present in random search models.

[22] In this baseline model with exogenous markdowns, the vacancy posting cost ν is not identified separately from the workers' search intensity λ_U when unemployed. We endogeneize mark-

Figure 5: Hiring and firing rates



Notes: The left panel plots the monthly firing rate in each sector, for two transition speeds. The solid curves correspond to the new sector and the dashed curves to the old sector. The black curves correspond to a slow transition and the gray curves to a rapid transition. Firing rates are measured as the probability of experiencing an EU transition over the next month when starting in a given sector, averaged using the age distribution among employed workers the initial steady state. The right panel plots the monthly hiring rates, which are the probability of experiencing an UE transition when searching in a given sector, averaged using the age distribution among unemployed workers at the initial steady state. Time is measured in years on the horizontal axis. Incumbent workers assumed to have $\hat{\psi}(\omega) = 0.65$ while new cohorts have $\hat{\psi}(\omega) = 1$. All workers have $\psi(old) = 1$.

which we set to 25% reflecting the estimates of Berger et al. (2022).

We pick λ_U to match an average job-finding rate of 25% per month. We set the replacement rate for the short-term unemployed to $b^U = 40\%$ and the one for long-term unemployment to $b^D = 0.5 \times b^U$. We feed in the same path for sectoral productivities $A_t(k;\eta)$ as in the Walrasian model. Finally, we experiment with different values for ψ (new) for incumbent workers.

3.2.1 Structural Unemployment

Figure 5 plots the evolution of the firing and hiring rates over time, in each sector. The solid curves correspond to the new sector and the dashed curves to the new sector. The black curves correspond to a slow transition and the gray curves to a rapid one. We measure firing rates as the probability that a worker employed

downs in our quantitative model, which allows us to pin down ν.

in a given sector experiences an EU transition over the next month.[23] Similarly, we measure the hiring rates as the probability that an unemployed worker who might choose to search in a given sector experiences an UE transition over the next month.[24] We weight the age-specific EU and UE rates using the age distribution that prevails at the initial steady state among employed and unemployed workers, respectively. This allows us to tease firms' behavior apart from general equilibrium changes in the age composition of the pool of employed or unemployed workers. The plots here correspond to the case where incumbent workers have $\hat{\psi}(\omega) = 0.65$.

As the marginal revenue product falls in the old sector, the firing rate rises and the hiring rate declines. The opposite happens in the new sector. Note, however, that the hiring rate remains persistently low (and even zero) in the new sector when the marginal revenue product is so low that firms do not find it profitable to post vacancies. As a result, workers displaced from the old sector struggle to find work. Workers in the old sector thus face an unpleasant choice: they may either remain in their declining sector, realizing higher layoff risk and falling wages, or seek a near-unattainable job in the growing sector. This dynamic generates structural unemployment and potentially large welfare losses as workers may not find employment for a long time. Accelerating the transition leads hiring and firing rates to reach their new steady state faster.[25]

Note that there is a period where separation rates are high in both new and old sectors, while hiring rates are low relative to steady state values. Peak unemployment over the transition is largely determined by the amount of time the economy spends in this intermediate period in which both sectors have very elevated separations and depressed hiring.

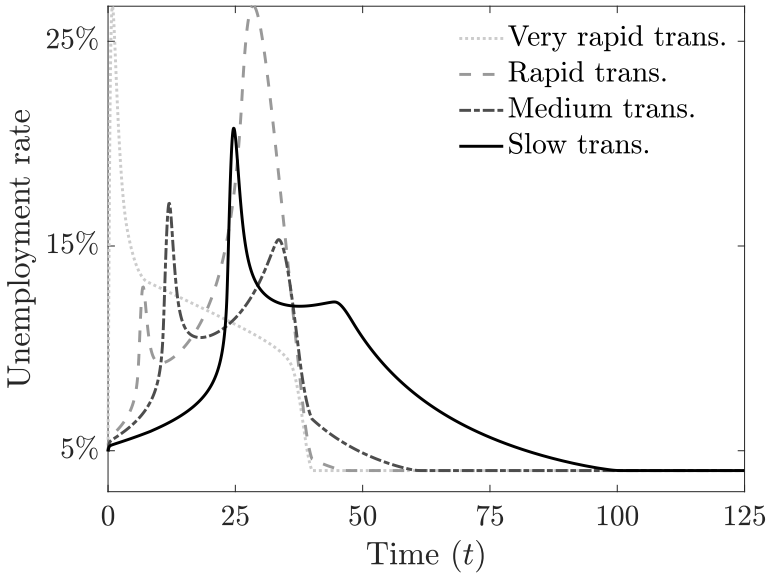
The transition speed can have a very non-linear effect on the unemployment dynamics. Figure 6 plots the evolution of the stock of total unemployment (both short- and long-term) over time. We consider the same two transitions as in Figure

[23] To be consistent with the data, we define an EU transition as a event where a worker loses their job at some point during the month without being rehired in the same month. Accounting for this time aggregation ensures, for instance, that the steady state hiring rate coincides with the UE rate that we target in our calibration. We provide more details in section 4.2.

[24] These are hypothetical rates. We can compute the firing rate in a sector even when no worker is employed in that sector in equilibrium; and we can compute the hiring rate even when no worker searches in that sector in equilibrium.

[25] An additional force comes from anticipation effects: when the transition is rapid, firms anticipate rapid changes in marginal revenue products, which amplifies their hiring and separation responses relative to what would be implied by static productivity movements.

Figure 6: Unemployment rate



Notes: This figure plots the evolution of the unemployment stock over time for four transition speeds. Time is measured in years on the horizontal axis. Incumbent workers assumed to have $\hat{\psi}(\omega) = 0.65$ while new cohorts have $\hat{\psi}(\omega) = 1$. All workers have $\psi(old) = 1$.

5, as well as an intermediate one that lies between the two and a very fast one. The path of unemployment shifts dramatically and non-monotonically as the transition accelerates. Note first that peak unemployment is non-monotone in the transition speed. This is because medium-speed transitions spend a great deal of time in the regime with many separations and few hirings.

A few additional features stand out. First, the path of unemployment has two peaks for all but the very rapid transition. Second, and perhaps surprisingly, unemployment can peak *later* when the transition becomes *faster*. The reason behind these two phenomena lies in changes in the composition of the unemployment pool over time. A higher firing rate drives up unemployment initially. Then, new generations with versatile skills find jobs as the hiring rate increases in the new sector, which leads the unemployment rate to temporarily fall. In the meantime, old workers are more and more likely to be laid off from the old sector. Firms in the new sector do not find vacancy creation profitable for these old workers with less versatile skills and shorter horizon. Old workers are therefore structurally unemployed and the unemployment rate resumes its ascent.[26] Eventually, old workers

[26] Most of this unemployment should be considered "structural" rather than "cyclical," as our

retire and unemployment converges to its long-run steady state.[27] Overall, the second peak can occur later for rapid transitions as old workers are displaced faster from the old sector than they are absorbed by the new sector. The two peaks only coincide when the transition is so rapid that firms in the new sector find it profitable to post vacancies right away even for old workers.

3.2.2 Social Welfare

Social welfare inherits these strong non-linearities. The left panel in Figure 7 plots social welfare changes (in consumption-equivalent terms) as a function of the transition horizon $1/\eta$.[28] The black curves correspond to the model with unemployment and the gray ones to its Walrasian counterpart with no unemployment. The solid curves correspond to a low skill versatility $\hat{\psi}(\omega) = 0.65$ and the dashed curves to a higher one $\hat{\psi}(\omega) = 0.75$. Starting with the dashed curves for workers with relatively transferable skills, social welfare is U-shaped in the Walrasian model for the reasons that we discussed in Section 3.1. Social welfare changes in the unemployment model partly reflects the forces that are at play in the Walrasian benchmark: workers still choose optimally when to switch to the new sector and their income declines while they stay in the old sector. However, the model with unemployment generates much larger declines in welfare and a more pronounced U-shape in welfare. This is because unemployment – and particularly long-term unemployment – is costly in itself since unemployed workers have lower consumption than employed workers. Therefore, the difference between the Walrasian and unemployment models is driven by the extent that structural unemployment rises and persists, and the gap in values between employment and unemployment. Medium-speed transitions are particularly painful for workers in the unemployment model as they generate a large and persistent surge in structural unemployment during which long-term unemployed workers lose access to generous unemployment benefits.[29]

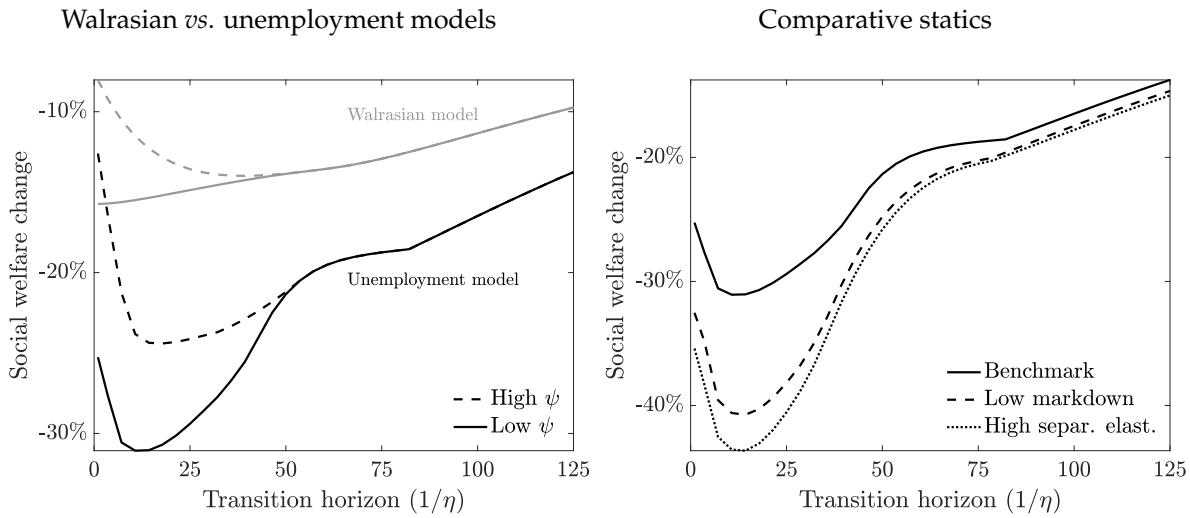
The non-linearity in welfare is so pronounced in the unemployment model that it can introduce non-monotonicities in and of itself. The black curves illustrate this

model inherits the well-known fast convergence properties of labor search models (Shimer, 2005).

[27] The unemployment rate is lower at the final steady state since $A_{\text{new},\infty} > A_{\text{old},0}$.

[28] We focus on workers' welfare. Profits (and hence workers' welfare) are zero in the Walrasian model. In the model with unemployment, profits are zero in steady state by free entry, but not necessarily over the transition. We account for entrepreneurs' welfare in our quantitative

Figure 7: Social welfare changes



Notes: The left panel plots social welfare changes as a function of the transition horizon $1/\eta$ (measured in years). The black curves correspond to the model with unemployment and the gray curves to the Walrasian model with no unemployment. Welfare changes are denominated in consumption-equivalent terms relative to each model's steady state. The solid curves correspond to a low skill versatility $\psi = 0.65$ and the dashed curves to a higher one $\psi = 0.75$. The right panel considers the effect of a lower markdown ($\pi = 0.2$ instead of 0.25) and more responsive separations (an elasticity of -2.5 instead of -2), both in the model with unemployment and $\psi = 0.65$.

scenario when $\psi = 0.65$. In this region of the parameter space, the Walrasian model features a monotonic decline in welfare as a function of the transition speed (increase in transition horizon). In contrast, the unemployment model predicts a clear U-shape since medium-speed transitions are particularly painful.

Medium-speed transitions are so costly within the unemployment model for two key reasons. First, as discussed above, medium-speed transitions lead incumbent workers to spend a great deal of time in the region where separations are high and job-finding rates are low in both sectors, leading to high peak unemployment. Second, the types of workers that are laid off during a fast and slow transition are different. Early layoffs are old workers with relatively specific skills, while late layoffs are a mix of old workers and younger workers with transferable skills. Such early layoffs are more likely to generate long-term structural unemployment, as firms are less likely to post vacancies for older workers who both have more spe-

analysis.

[29] Figure 10 in Appendix A.3 reports short- and long-term unemployment separately, confirming that long-term unemployed workers account for most of the increase in the unemployment stock.

cific skills and a shorter horizon over which to recover the vacancy posting cost. Since fast transitions generate many such early layoffs, they can be quite costly in aggregate. Very fast transitions, however, reduce the harm of these early layoffs, because they also lead to early increases in hiring rates, which makes it less likely these laid-off workers become long-term unemployed. Slower transitions ensure that most of the workers subject to layoffs are new cohorts with transferable skills, who may have an easier time finding employment in the new sector; this force pushes towards aggregate welfare to be decreasing in the transition speed.[30]

The right panel of Figure 7 shows that the size of this non-monotonicity is largely driven by two quantities: firms' markdowns, and the sensitivity of layoffs and hires to firm productivity. The solid curve is the same as on the left panel. A lower markdown on wages (dashed curve) reduces the profitability of each job, therefore reducing the number of vacancies and aggravating structural unemployment. Similarly, a higher separation elasticity (dotted) results in more job displacement and structural unemployment. Indeed, the elasticities of hiring and firing determine job creation and destruction over the transition, and hence the extent of the surge in structural unemployment.

3.3 Discussion

This section characterize the shape of costs borne by workers as a technological transition accelerates. In practice, a government might also take into account the cost of implementing a particular transition speed or some other objectives such as reduced carbon emissions. Suppose that *total* welfare is $\overline{\mathcal{W}}(\eta) + \mathcal{W}^{\text{other}}(\eta)$, where $\overline{\mathcal{W}}(\eta)$ is workers' welfare (3.4) and $\mathcal{W}^{\text{other}}(\eta)$ reflects all welfare effects of a faster transition outside of our model, such as reduced carbon emissions or the costs of implementing a transition speed η with distortive policies. A government that chooses the transition speed optimally would set $\eta^\star$ so that the marginal benefit of speeding up a transition for other reasons is equated with the marginal cost

[30] An additional effect is generated if separations carry an additional cost per se through unemployment scarring effects, as is the case in the quantitative model. Here, a prolonged increase in separations carry their own welfare losses, so the non-linear response of separations introduces a new non-linearity.

imposed on workers:

$$\frac{d}{d\eta}\overline{\mathcal{W}}\left(\eta^{\star}\right)+\frac{d}{d\eta}\mathcal{W}^{\text{other}}\left(\eta^{\star}\right)=0.$$

The shape of $\mathcal{W}^{\text{other}}$ and hence its slope are relatively well understood (e.g., using an climate integrated assessment model). We have shown in this section that worker welfare $\overline{\mathcal{W}}$ can be strongly non-linear and even non-monotonic in the speed of transition. Focusing solely on the labor market, the government may be particularly worried about medium-speed transitions that impose large reallocation costs on workers without the benefits from improvements in the new sector. Extremely slow transitions give the economy time to adjust smoothly to the new technology, while very fast transitions allow workers to benefit from the increased productivity in the rising sector. Depending on the speed η that the economy starts at, a government that seeks to accelerate the transition might, for instance, encounter a very flat region for $\overline{\mathcal{W}}$ before hitting a very steep region soon after. Two features need to be disciplined carefully to quantify this non-linearity: the joint distribution of skill transferability and age, and sensitivity of layoffs and hires to firms' productivity which governs the response of structural unemployment. In the next section, we enrich our model and bring it to the data.

Before proceeding, a few caveats are warranted. First, we do not consider transitions in which the endpoints of the transition are endogenous to the speed the transition occurs. This may rule out settings with, for instance, learning-by-doing. Second, we abstract from human capital accumulation in our analysis. One should therefore interpret our skill types as reflecting present-values of human capital supplies. Finally, we only consider transitions in which the speed of decline in the old sector is tied to the speed of growth in the new sector. Settings in which these speeds can be independently altered would trivially lead welfare to be maximized when the decline of the old sector is as slow as possible and the growth in the new sector is as rapid as possible.

4 Quantitative Model

We now enrich our model and discipline it empirically. Unless otherwise stated, the model is the same as in Section 2 including the preferences, life cycle, skills

and technologies. Our quantitative model extends this baseline setup in five key directions. First, we allow for on-the-job search with a job ladder to microfound the distribution of markdowns. Second, we introduce random operating costs to microfound the probability of involuntary layoffs. Third, we model the scarring costs associated with unemployment. Fourth, we allow for idiosyncratic amenity values in the search decision to replicate the substantial share of wage cuts observed after a job switch. Finally, we introduce a third sector which neither grows nor declines relative to trend and which provides displaced workers with a better outside option than unemployment. Appendix B describes the model and its numerical solution in more details.

4.1 Environment

Outside sector. Our goal remains to quantify the welfare changes experienced by workers initially employed in the old (or dirty) sector. These are the workers who risk being displaced and are the subject of the ongoing policy debate. In addition to the new (or clean) sector, we assume that these workers can be employed in a third, outside sector which does not undergo a technological transition. We index this sector by $k =$ "out" and assume that $A_t(\text{out}; \eta) \equiv 1$ for all periods t and transitions speeds η.[31] This outside sector captures jobs that displaced workers can secure more easily but that come with a lower wage. We suppose that incumbent cohorts have a common productivity

$$\psi^{\text{comp}}(\omega, \text{out}) = \psi^{\text{comp}}_{\text{out}} < \psi^{\text{comp}}(\omega, \text{old}) \equiv 1$$

in that third sector.[32] We allow the productivity of new cohorts in the third sector $\psi^{\text{comp},\star}_{\text{out}}$ to be different from $\psi^{\text{comp}}_{\text{out}}$.[33] This allows us to capture the idea that new cohorts can secure high paying jobs in activities that are neither inheritently clean or dirty (e.g., doctors) but which are inaccessible to incumbent cohorts. The distribution of ω by age and hence productivities $\psi^{\text{comp}}(\omega, \text{new})$ in the new sector are unrestricted at this point — we will estimate them in the data.

[31] As before, this productivity should be understood as relative to a common trend among the three sectors with an exogenous growth rate g.

[32] Assuming that all workers have the same skill in this outside sector ensures that our state space remains manageable.

[33] Formally, new cohorts have a different skill ω_0 such that $\psi^{\text{comp}}(\omega_0, \text{out}) \neq \psi^{\text{comp}}_{\text{out}}$.

Job separations. Firms incur fixed costs to continue operations, which generate endogeneous and involuntary separations for workers.[34] Firms incur these costs at a constant rate $\chi_0 > 0$. These fixed costs are random and log-normally distributed with location parameter ϕ and scale parameter γ.[35] A firm lays off its worker when the operating cost exceeds its continuation value. This specification microfounds the firing hazard (3.10) that we introduced in Section 2.

Unemployment scarring. Workers who become unemployed face a higher separation risk after reemployment (Jarosch, 2023). To capture this idea, we suppose that workers transition from a safe state $(z = \bar{z})$ to a risky state $(z = \underline{z})$ when losing their job. Workers in the safe state face endogenous separation, as discussed above. Those in the risky state face *additional* exogeneous separations at rate $\chi_1 > 0$. Unemployment scarring results from this extended period of elevated separation risk. Workers transition from the risky state to the safe state at rate $\kappa > 0$ when employed.

Competitive search. Workers search for work both on- and off-the-job and direct their search within and across sectors. They search at rate $\lambda^E > 0$ when employed and at rate $\lambda^U > 0$ when unemployed. Firms pay workers a constant wage $w \geq 0$ during their tenure for each unit of effective labor $\psi(j, \omega, k)$ that they supply.[36] We abstract from wage renegotiation.[37] There is a continuum of submarkets in each occupation, indexed by worker's age j, their skill ω, their productivity z, their occupation k, and their wage w. As in the baseline model, firms incur a cost $\nu > 0$ when posting a vacancy. There is free entry into each submarket.

Job amenities. Workers choose which submarket $s \equiv (w, k)$ to search in. To rational-

[34] An alternative approach is to introduce idiosyncratic firm productivity shocks as in Hopenhayn (1992). Our approach economizes one state variable.

[35] Random fixed costs of operation or production are commonly used in the literatures on firm dynamics (Clementi and Palazzo (2016)), trade (Atkeson and Burstein, 2010), and macroeconomics more generally (Guerrieri et al., 2022).

[36] Workers' earnings rises *on-the-job* as they get older (as is the case in practice) since $\psi(j, \omega, k)$ features a life-cycle component in (2.2).

[37] As discused earlier, firms are often reluctant to renegociate wages downward (Grigsby et al., 2021).

ize the fact that job switches often entail wage cuts (Sorkin, 2018), we assume that workers experience some idiosyncratic amenity shocks ζ_s when searching in each market s, which we assume are distributed i.i.d. according to a Type-1 Extreme Value distribution with mean 0 and scale parameter γ_ζ. For instance, a worker with age j, skill ω and safety z who is currently employed in submarket s solves

$$s_t^\star\left(j,\omega,z,s\right) \equiv \arg\max_{s'\equiv(w',k')} \left\{ f\left(\theta_t\left(j,\omega,z,s'\right)\right) \times \left[W_t^E\left(j,\omega,z,s'\right) - W_t^E\left(j,\omega,z,s\right)\right] + \zeta_{s'} \right\} \tag{4.1}$$

Earnings and profits. An employed worker earns $w\psi\left(j,\omega,k\right)$ where $\psi\left(j,\omega,k\right)$ still denotes their effective labor supply (2.2). The flow profits for the firm employing them is $(A_t(k;\eta) - w)\psi(j,\omega,k)$ before operating costs.

4.2 Calibration

We start by calibrating externally some standard parameters. We discipline internally the remaining ones. Appendix B provides details about our numerical implementation. We discuss the calibration in more details in Appendix B.6, including how we account for time aggregation when computing moments over discrete periods of time (a month, a year, etc.) in our continuous time model.[38]

External calibration. Table 1 summarizes the external calibration. The preferences and life span are the same as in our baseline model (Section 3.2). Our matching function still follows Den Haan et al. (2000) with $\alpha = 1.599$ as in Schaal (2017). Operating costs still arrive every quarter on average so $\chi_0 = 4$.

Internal calibration. We discipline 10 parameters internally by matching moments at the initial steady state. Table 2 summarizes the internal calibration. All targets are matches to numerical precision.

We first describe how we choose the parameters that govern workers' search behavior. We calibrate the search intensity for unemployed workers λ^U to target a monthly UE rate of 25% (Shimer, 2005). Similarly, we pick the search intensity for

[38] In short, we discretize our continuous time model into short periods of 2 weeks which allows us to capture high-frequency movements in and out of employment.

employed workers λ^E to match the monthly EE rate of 2.5% (Fujita et al., 2024). We set the scale of job amenity shocks γ_ζ to replicate the fact that 37% of job-switchers experience a wage cut (Grigsby et al., 2021). We obtain a relatively small standard deviation of 0.036. The short-term unemployment benefit b^U is chosen so that workers who get laid off have an average replacement rate of 40% as a share of their previous wage.[39] We obtain $b^U = 0.291$ which reflects the fact that workers' wages are less than 1 in equilibrium but also the selection among those who lose their job.[40]

We set the vacancy posting cost ν to match an average markdown of 75%, which is in the middle of the range surveyed by Berger et al. (2022).[41] This is a natural moment to match since firms' ex ante values increase with the markdown and are equal to the hiring cost by free entry. We obtain a vacancy posting cost of 0.329, which lies in the range spanned by Shimer (2005) and Hagedorn and Manovskii (2008).

Turning to separations, the average log operating cost ϕ governs the rate at which matches are endogenously destroyed. We pick ϕ to match an average steady state unemployment rate of 5%.[42] The scale of the log operating costs γ_ϵ controls the elasticity with which firms destroy matches when experiencing a negative productivity shock. To understand why, note that the elasticity of separation to productivity shocks goes to zero as $\gamma_\epsilon \to +\infty$; in contrast, the elasticity goes to infinity as $\gamma_\epsilon \to 0$.[43] This elasticity is an important moment to match in our context, as we have shown in Section 3.2.2. We target an elasticity of separations of –2 following a 15% productivity shock, based on the evidence from Chan et al. (2023).[44] We obtain a scale parameter (or standard deviation) of 0.207, which is slightly smaller than the flow-profit share of 25%. The parameters governing the job-safety state

[39] We currently set the long-term unemployment benefit to $b^D = b^U$. We are in the process of adding the long-run employment state in our quantitative model.

[40] For instance, workers in the low safety state are more likely to be laid off and are paid lower wages on average. On the contrary, older workers are more likely to be laid off since they have a shorter active life but have higher average wages.

[41] That is, workers are paid 25% less than their marginal product on average.

[42] This is equivalent to matching the EU rate directly, since our calibration already targets the UE and EE rates.

[43] Of course, ϕ must also adjust in the limit $\gamma_\epsilon \to +\infty$. See Beraja and Zorzi (2024a) for an application to durable spending.

[44] This elasticity is inline with the estimates of Carlsson et al. (2021). The 15% shock lies between the standard deviations of productivity shocks reported in these two papers. Separations include EU and EE transitions, both in the model and in the data.

Table 1: External calibration

Parameter	**Description**	**Value**	**Source**
Preferences			
σ	Inverse IES	2	–
J	Lifespan	40	Career length
ρ	Discount rate	0.05	Real interest rate
Technology			
α	Elast. of matching function	1.599	Schaal (2017)
χ_0	Operating cost (arrival rate)	4	Quarterly review of workers

are chosen to replicate the elevated separation rates following an involuntary job loss documented by Jarosch (2023). We set the exogenous separation rate for risky workers χ_1 to match a 13% higher separation rate in the first year following a separation (Figure 11 in Jarosch, 2023). We then pick the transition rate to the "safe" state κ to fit the slope of the excess separations rate over the 5 first years post separation. This is again a natural moment to match since a faster transition rate κ implies a steeper decline in excess separations.

Finally, we approximate the life-cycle productivity $\psi^{\text{age}}(j)$ using linear splines with 11 knots. We choose these nots to match the average life cycle real wage profile from the Current Population Survey Outgoing Rotation Group (CPS ORG) over the period 1980–2019 (Figure 12 in Appendix D). In our current calibration, we assume away the outside sector by setting $\bar{A}_N = 0$ *(calibration with third sector under construction)*.

4.3 Estimating Sectoral Productivities and the Skill Distribution

Two objects still have to be estimated: the distribution of skill specificity $\hat{\mu}(\omega, t)$; and the time path of sectoral productivities $A_t(k; \eta)$ for the new and old sectors.[45] Our calibration from Section 4.2 could be performed with no knowledge of these

[45] In addition, we will estimate a time-invariant productivity for the outside sector once we have added it to our quantitative model.

Table 2: Internal calibration

Param.	Description	Value	Moment	Source	Target	Model
Search						
λ^U	Search intensity (U)	7.297	UE rate (M)	Shimer (2005)	25%	25%
λ^E	Search intensity (E)	0.691	EE rate (M)	Fujita et al. (2024)	2.5%	2.5%
γ_ζ	Amenity (scale)	0.036	Share of wage cuts	Grigsby et al. (2021)	37%	37%
b^U	U benef. (short-term)	0.291	Average UI replac.	US UI Rules	40%	40%
Vacancies						
ν	Vacancy post. cost	0.329	Average markdown	Berger et al. (2022)	75%	75%
Separations						
ϕ	Oper. costs (loc.)	-3.490	Unempl. rate	US Average	5%	5%
γ_ϵ	Oper. costs (scale)	1.410	Elasticity of separ.	Chan et al. (2023)	-2.0	-2.0
χ_1	Exogenous separ.	0.334	Excess separ. (interc.)	Jarosch (2023)	13%	13%
κ	Safety rate	0.445	Excess separ. (slope)	Jarosch (2023)	-0.031	-0.031
Life-cycle						
ψ^{age}	Life cycle prod.	–	Life cycle wage	CPS ORG	–	–

parameters.[46] We now explain how these parameters can be inferred using available data on job switches and sectoral wages over the early stages of the climate transition. Our estimation strategy leverages the block recursivity of competitive search equilibria (Menzio and Shi, 2011).

For estimation purposes, we parameterize the marginal distribution of new-sector skills for incumbent workers to be a mixture of two log-normal distributions to allow for the possibility of a bi-modal skill distribution. That is, the marginal distribution of skill has five parameters: the location and scale parameters of each log-normal distribution $\mu_1^\omega, \mu_2^\omega, \sigma_1^\omega, \sigma_2^\omega$, and weight π^ω placed on the first distribution in the mixture. To parameterize the correlation between age and skill, we

[46] All the moments were matched at the initial steady state before the transition occurs. Workers could only be employed in the "old" and "outside" sectors — the "new" sectors hadn't emerged yet — so the distribution $\hat{\mu}(\omega, t)$ was irrelevant.

assume that the joint distribution of skill and cohort is given by the combination of their marginal distributions and the Clayton copula with parameter l. That is, if the CDF of the marginal distribution of new sector skill ψ_{new} is u and the CDF of the marginal distribution of cohort τ is v, the CDF of the joint distribution is given by

$$C(u,v) = \left[\max\left\{u^{-l} + v^{-l} - 1, 0\right\}\right]^{1/l},$$

where the parameter $l \in [-1,\infty)\setminus\{0\}$ governs the strength of dependence between age and skill. If $l > 0$, then cohort and skill are positively correlated so that old workers (early cohorts) have specific skills (low ψ_{new}), while the opposite is true if $l < 0$. We collect the six parameters of the joint distribution of skill and age in a vector $\Theta^{\tau,\omega} = (\mu_1^\omega, \mu_2^\omega, \sigma_1^\omega, \sigma_2^\omega, \pi^\omega, l)$.

Without loss of generality, we normalize the transition speed in the data to $\bar{\eta} = 1$.[47] Furthermore, we suppose that the paths of labor productivities $A_t(k;\bar{\eta})$ follow S-curves parameterized as

$$A_t(k;\bar{\eta}) = A_{k0} + \Delta_k \bar{\Phi}(t|\tilde{\mu},\tilde{\sigma},T), \tag{4.2}$$

where $\bar{\Phi}(t|\tilde{\mu},\tilde{\sigma},T)$ is the CDF of a truncated normal distribution between period 0 and period T, with location parameter $\tilde{\mu}$ and scale parameter $\tilde{\sigma}$, so that

$$\bar{\Phi}(t|\tilde{\mu},\tilde{\sigma},T) = \min\left\{\frac{\Phi\left(\frac{t-\tilde{\mu}}{\tilde{\sigma}}\right) - \Phi\left(\frac{0-\tilde{\mu}}{\tilde{\sigma}}\right)}{\Phi\left(\frac{T-\tilde{\mu}}{\tilde{\sigma}}\right) - \Phi\left(\frac{0-\tilde{\mu}}{\tilde{\sigma}}\right)}, 1\right\} \qquad \text{for } t \geq 0,$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. In other words, $\bar{\Phi}(\cdot)$ represents the share of the transition that is complete as of period t, with the transition fully completing in period T. The parameter $\tilde{\mu}$ determines where the steepest part of the S-curve lies, while the $\tilde{\sigma}$ controls the slope at this point. We normalize $A_{\text{old},0} = 1$ and suppose that the new sector first emerges in $t = 0$ with a low marginal revenue product $A_{\text{new},0} = b_0^U$.

The structural paths of labor productivity $A_t(k;1)$ are not observed in the data. The reason is that measured labor productivity depends on the composition of un-

[47] Starting with an arbitrary η', one can always rescale the sectoral labor productivities to obtain the same transition with $\eta = 1$.

observed worker quality which is itself endogenous over the transition.[48] We must therefore *jointly* estimate the parameters of the transition process $\Delta_k, \tilde{\mu}, \tilde{\sigma}$ and those governing the distribution of skills and age $\Theta^{\tau,\omega}$. To do so, we adopt the iterative approach described below.

Algorithm (estimation)**.** Fix some guess for the transition process $\Delta_k, \tilde{\mu}, \tilde{\sigma}$ for labor productivities $A_t\left(k;\bar{\eta}\right)$.

Step 1 (skill distribution). The model is block-recursive so workers' optimal policies depend exclusively on $\Delta_k, \tilde{\mu}, \tilde{\sigma}$ and *not* on the distribution $\Theta^{\tau,\omega}$. For each worker type (τ,ω), we can compute moments $\boldsymbol{x}_t^{MODEL}(\tau,\omega)$ at horizon t, including the collection of monthly flow rates from any sector k to another sector k' over the next month. Different distributions $\Theta^{\tau,\omega}$ will imply different averages

$$\overline{\boldsymbol{x}}_t^{MODEL}\left(\tau;\Theta^{\tau,\omega}\right) \equiv \int \mu\left(\omega,\tau;\Theta^{\tau,\omega}\right) \boldsymbol{x}_t^{MODEL}\left(\tau,\omega\right) d\omega \tag{4.3}$$

for each cohort τ and period t. In particular, the timing with which a given cohort τ moves from the old to the new sector is informative about the shape of the distribution $\hat{\mu}\left(\cdot,\tau\right)$. Suppose for instance that the reallocation is slow initially, then accelerates 10 years into the transition. This means that few workers have a strong comparative advantage in the new sector and most of them have a lower $\psi\left(\omega\right)$. By matching the collection of moments (4.3), we are therefore able to trace the full distribution of skills. Specifically, the distribution $\Theta^{\tau,\omega}$ can be estimated by GMM to minimize the distance between the set of moments $\hat{\boldsymbol{x}}_t^{MODEL}\left(\tau;\Theta^{\tau,\omega}\right)$ and their empirical counterpart $\overline{\boldsymbol{x}}_t^{MODEL}\left(\tau;\Theta^{\tau,\omega}\right)$. We then proceed to Step 2.

Step 2 (update the transition process). The previous step yields a distribution $\Theta^{\tau,\omega}$ conditional on a transition process $\Delta_k, \tilde{\mu}, \tilde{\sigma}$. Given this distribution, we can simulate the model forward and compute average wages $\overline{w}_{k,t}^{\text{MODEL}}$ in each sector k and period t which reflects workers' selection into labor reallocation. We then adjust the transition process $\Delta_k, \tilde{\mu}, \tilde{\sigma}$ so the paths of average wages match their empirical counterparts $\overline{w}_{k,t}^{\text{DATA}}$.[49] We then repeat Step 1 until convergence.

[48] See Grigsby (2024) for instance.

[49] This approach allows an estimation of the transition and skill distribution that best fits the observed data on labor flows and wages. Of course, projections into the future are subject to large uncertainty. We compare the results of model-implied employment projections with those elsewhere in the literature in section 5.2 (under construction).

In summary, labor flows from the old to the new sector discipline the joint distribution of skill and age, while aggregate wage dynamics in old and new sectors discipline the parameters for the transition.

5 Application to the Climate Transition

We apply our model to ask how accelerating the transition away from polluting technologies towards "green" technologies would affect U.S. workers.

5.1 Institutional Background and Data

Institutional background. Although climate change was first recorded by the scientific community during the 1970s, international efforts to coordinate policy addressing climate change began in 1992 with the establishment of the United Nations Framework Convention on Climate Change (UNFCCC). This group has coordinated international agreements on climate policy ever since. The most important early agreement was the Kyoto Protocol, which established binding reduction targets for greenhouse gas (GHG) emissions for the 192 countries that ratified it. The Kyoto protocol was adopted in 1997 and entered into force in 2005. We therefore choose our period 0 to be the year 2003, a couple of years before the enforcement of the protocol. Since the first Kyoto protocol was adopted, many additional agreements have been entered into, such as the notable Paris Climate Agreement of 2015 which called to keep global warming to no more than 1.5^oC. In order to achieve this target, it is understood that most economies need to have net-zero carbon emissions by the year 2050. The year 2050 has thus been adopted as a common target to achieve net-zero emissions by many developed countries.[50] The US, however, is one country which does not currently maintain such a target. We therefore assume that the data reflect a transition which began in 2005 and will end in 2070.

Defining polluting and green sectors. In order to study the labor market effects of the transition with our model, knowledge of the productivity of the old and new sectors is required. A first challenge is to appropriately define polluting and green

[50] See Naser and Pearce (2022) for more on the history of international climate accords.

sectors: the k in the data. There have been many approaches to produce such definitions.[51] To define polluting industries, we follow the approach of the 2024 OECD employment report report (Bassanini et al., 2024). We first obtain information on emissions intensity – measured as the kilograms of CO2-equivalent gas emissions per dollar of output – by 6-digit NAICS industry codes from the Supply Chain Greenhouse Gas Emission Factors published by the Environmental Protection Agency (EPA).[52] This measure accounts for emissions produced throughout the supply chain of production of each commodity. An industry is deemed to be "high-emissions" if it is in the top decile of emissions intensities, weighted by employment. Examples of industries classified as dirty in this way include "Petroleum refineries," "Beef cattle ranching and farming" and "Industrial gas manufacturing."[53] We then handcode a few sectors for which emissions data are unavailable as dirty; for instance, the emissions data do not provide information on "Fossil fuel electric power generation," while "Gasoline stations" are not themselves polluting since the act of selling gasoline does not produce many emissions, but demand for gas stations is likely to fall as electric cars become more common. We detail these handcodings and all other such decisions in Appendix C.

To define "green" sectors, we combine two approaches. First, we follow the output based approach of Bontadini and Vona (2023), who find that 95% of the production of "green goods"[54] is concentrated in just 13 out of 119 4-digit European NACE industries and that green and polluting production does not occur in the same sector. We adopt their list of green sectors by merging these 4-digit NACE industries to 6-digit NAICS codes. Examples of the 54 sectors that are green by this definition include "Automatic environmental control manufacturing," "Electric power and specialty transformer manufacturing," and "Turbine and turbine generator set units manufacturing." While a useful starting point, this

[51] See Merino-Saum et al. (2020) for a meta-analysis of 140 definitions of "green" sectors, and Granholm (2024) and Bassanini et al. (2024) for additional conceptual discussion. Appendix C (under construction) provides additional details on our approach.

[52] Specifically, we use version 1.3 which is available here (accessed March 7, 2025).

[53] Appendix Figure 11 plots emissions intensity by NACE2 sector as produced by the 2024 OECD Employment report (Bassanini et al. (2024)), which are less granular than our 6-digit data but offer a similar ranking.

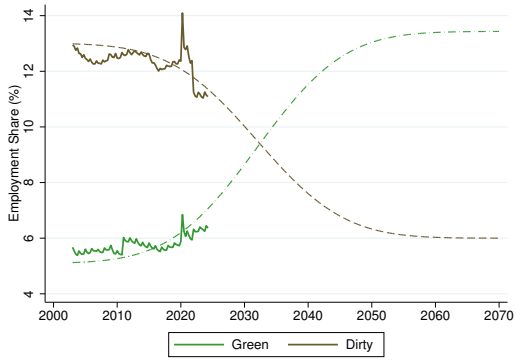
[54] The list of green goods employed by Bontadini and Vona (2023) is based on the Combined List of Environmental Goods (CLEG) of the OECD and considers goods which are useful for the green transition. For instance, solar panels and wind turbines are classified as "green goods" even if their production is emission-intensive, since the goods themselves are green by nature.

approach only permits goods-producing sectors to be green. Therefore, we supplement this output-based approach with the task-based approach employed by the US Energy Employment Report (Granholm (2024)). This approach uses data from the Green Goods and Services (GGS) survey produced by US Bureau of Labor Statistics (BLS) to study the share of workers employed in each industry that are engaged in green tasks. Such green tasks can be conservation efforts, clean energy generation, or energy efficiency efforts, among others. We consider sectors to be green if they employ workers engaged in green tasks. This adds an additional 59 sectors, such as "Solar electric power generation" and power generation from other renewable sources, "Materials recovery facilities" and "Primary battery manufacturing." When there are sectors classified both as green and polluting, we classify them as neither.

All other sectors form the basis of the "not-transitioning" category. However, our focus is on workers who might be affected by the climate transition. Many jobs – such as doctors, nurses and consultants – are unlikely to be directly affected by this transition.. To better reflect displaced workers outside options such as jobs in food/accommodation, agriculture, other manufacturing, construction, real estate, and custodial services, we therefore drop the education, healthcare, information, finance, management of companies, and professional services sectors – which we term the "skilled service sectors" – from the "not-transitioning category."

Data. We merge our definitions for clean and dirty sectors with the Quarterly Census of Employment and Wages (QCEW), which provides administrative quarterly data on total employment and average weekly wages for private establishments every 6-digit NAICS sector in the U.S.. Figure 8 plots the path of employment of green (green lines) and polluting sectors (brown lines) in the U.S. from 2003 onward, measured as a share of total employment excluding the skilled service sectors. As a visual aid, an S-curve transition is fitted following the functional form of equation 4.2, assuming the transition ends in 2070. By the definitions above, green sector employment accounted for 5.5% of employment as of the beginning of 2003, while polluting industries accounted for around 13% of employment, or 9.5 million jobs. By the second quarter of 2024, the dirty sector had declined to 11% of employment and the clean sector had grown to around 6.5% of employment, an increase from 4.1 million to 5.5 million jobs. Since 5.5% of employment

Figure 8: Employment Projections: Green and Dirty Sectors



Notes: Figure plots the share of employment in green and dirty sectors over time. Employment shares are relative to total employment excluding skilled service sectors, which includes education, healthcare, information, finance, management of companies, and professional service sectors. Dirty sectors are those 6-digit NAICS sectors in the top 10% of emissions intensity, while green sectors are those with either green output or which employ workers engaged in green tasks according to the BLS Green Goods and Services (GGS) survey. Employment data come from the QCEW. Dashed lines report a fitted transition assuming the transition path follows that of a truncated normal CDF beginning in 2003 and ending in 2070.

is in green sectors even at the beginning of our transition, we assume that 5.5% of the population is comprised of "always-switchers" who have $\psi_{new} = 1/A_{new,0}$.[55]

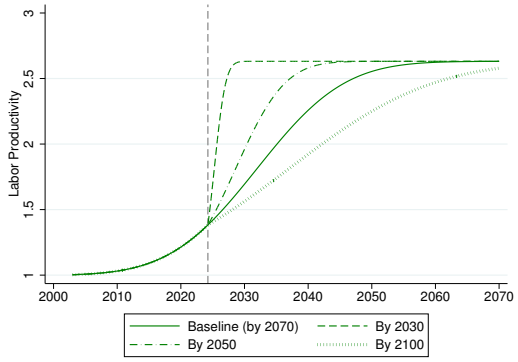
We construct flows between green and dirty sectors using data from the Current Population Survey (CPS). The CPS is a monthly rotating panel designed to be representative of the US population. Households remain in the sample for four consecutive months, before rotating out for 8 months and back in for an additional four month spell. This structure permits the construction of monthly, quarterly and annual flows between sectors. We construct quarterly flows between our three sectors, including both E-E and E-U-E flows, for households between 25 and 65 years old over the period 2003 through 2019.

5.2 Estimation Results and Welfare Effects of Baseline Transition Speed

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[55] Percentages are reported as a share of aggregate private sector employment excluding skilled service sectors.

Figure 9: Transition Path under Different Transition Speeds η



Notes: Figure plots how changing the transition speed η changes the path of labor productivity, holding fixed the path observed in the data through the first quarter of 2025. The four lines correspond to different horizons for the transition to be complete by.

5.3 Counterfactual Speeds of Transition

We study the effect of speeding up the transition following beginning in 2025. Letting t_{2024} be the value of t associated with the fourth quarter of 2024, we suppose that the value of labor productivity in period t is given under a transition speed η by

$$A_t(k;\eta) = A_{k0} + \Delta_k \begin{cases} \bar{\Phi}\left(t|\tilde{\mu},\tilde{\sigma},T\right) & \text{if } t \leq t_{2024} \\ \bar{\Phi}\left(t_{2024} + \eta\left(t - t_{2024}\right)|\tilde{\mu},\tilde{\sigma},T\right) & \text{if } t > t_{2024} \end{cases}, \tag{5.1}$$

Increasing η compresses the transition by ensuring a period t is at the same point as a future period under the baseline transition. Figure 9 illustrates how varying η changes the path of labor productivity into the future, holding fixed the path up through the first quarter of 2025.

Under Construction

6 Conclusion

This paper studies how the speed of technological transitions affects workers' welfare. We develop an overlapping generations model in which workers with heterogeneous skills direct their job search across a declining old sector and a grow-

ing new sector. The speed of transition generally has non-linear or even non-monotonic effects on workers' welfare. We show that two key features affect the shape of this relation: the joint distribution of age and skill, and the sensitivity of layoffs and hires to firms' productivity which determines the response of structural unemployment.

We bring the model to the data and use it to study the welfare effects of accelerating the transition away from polluting jobs and towards "green" jobs. These results are under construction.

The conceptual points raised here inform the policy debate surrounding net-zero emissions targets. While the social benefits of curbing carbon emissions faster are well understood and quantified, we know much less about the costs imposed on workers as the transition gets faster. Quantifying these costs is a crucial step to decide how ambitious climate goals should be. Future work should seek to understand optimal policy when the transition speed is endogenously determined by market forces and to quantify appropriate subsidies to technology adoption when labor market reallocation is costly.

References

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous, "The environment and directed technical change," 2 2012.

_ , _ , Lint Barrage, and David Hémous, "Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution," 2023.

_ , Ufuk Akcigit, Douglas Hanley, and William Kerr, "Transition to Clean Technology," *Journal of Political Economy*, 2 2016, *124*.

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach [On the Existence and Uniqueness of Stationary Equilibrium in Bewley Economies with Production]," *The Review of Economic Studies*, 2022, *89* (1), 45–86.

Adão, Rodrigo, Martin Beraja, and Nitya Pandalai-Nayar, "Fast and Slow Technological Transitions," *Journal of Political Economy Macroeconomics*, 6 2024, *2*, 183–227.

Aghion, Philippe and Peter Howitt, "A Model of Growth Through Creative Destruction," *Econometrica*, 1992, *60* (2), 323–351.

Arkolakis, Costas and Conor Walsh, "Clean Growth," 2023.

_ **and** _ , "The Economic Impacts of Clean Energy," *Brookings Papers on Economic Activity*, 9 2024.

Atkeson, Andrew and Ariel Tomas Burstein, "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, June 2010, *118* (3), 433–484.

_ **and Patrick J Kehoe**, "Social Insurance and Transition," *International Economic Review*, May 1996, *37* (2), 377–401.

Barrage, Lind and William D Nordhaus, "Policies, Projections, and the Social Cost of Carbon: Results from the DICE-2023 Model," 2023.

Bassanini, Andrea, Andrea Garnero, Agnes Puymoyen, Cesar Barreto, Jonas Fluchtmann, Alexander Hijzen, Dzana Topalovic, Michele Tuccio, Assia Elgouacem, Herwig Immervoll, Anasuya Raj, Jules Linden, Cathal O'Donoghue, Denisa Sologon, Sandrine Cazes, Sebastien Martin, and Andrea Salvatori, *OECD Employment Outlook 2024*, OECD, 7 2024.

Beraja, Martin and Nathan Zorzi, "Durables and Size-Dependence in the Marginal Propensity to Spend," Working Paper 32080, National Bureau of Economic Research January 2024.

_ **and** _ , "Inefficient Automation," *The Review of Economic Studies*, 02 2024, *92* (1), 69–96.

Berger, David, Kyle Herkenhoff, and Simon Mongey, "Labor Market Powe," *American Economic Review*, 4 2022, *112*, 1147–1193.

Bilal, Adrien and Diego R Känzig, "The Macroeconomic Impact of Climate Change: Global vs. Local Temperature *," 2024.

_ **and Esteban Rossi-Hansberg**, "Anticipating Climate Change Across the United States," 2023.

_ **and James H Stock**, "Macroeconomics and Climate Change," Working Paper 33567, National Bureau of Economic Research March 2025.

Blanchard, Olivier J., "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 1985, *93* (2), 223–247.

Blanco, Andrés, Andrés Drenik, Christian Moser, and Emilio Zaratiegui, "A Theory of Labor Markets with Inefficient Turnover *," Technical Report 2024.

Bontadini, Filippo and Francesco Vona, "Anatomy of Green Specialisation: Evidence from EU Production Data, 1995–2015," *Environmental and Resource Eco-*

nomics, 8 2023, *85*, 707–740.

Braxton, J. Carter and Bledi Taska, "Technological Change and the Consequences of Job Loss," *American Economic Review*, 2 2023, *113*, 279–316.

Braxton, J Carter and Bledi Taska, "Technological Change and Insuring Job Loss," 2024.

Calvo, Guillermo A and Maurice Obstfeld, "Optimal Time-Consistent Fiscal Policy with Finite Lifetimes," *Econometrica*, March 1988, *56* (2), 411–432.

Carlsson, Mikael, Julián Messina, and Oskar Nordström Skans, "Firm-Level Shocks and Labour Flows," *The Economic Journal*, 2021, *131* (634), 598–623.

Chan, Mons, Sergio Salgado, and Ming Xu, "Heterogeneous Passthrough from TFP to Wages," 2023.

Clementi, Gian Luca and Berardino Palazzo, "Entry, exit, firm dynamics, and aggregate fluctuations," *American Economic Journal: Macroeconomics*, 2016, *8*, 1–41.

Costinot, Arnaud and Iván Werning, "Robots, Trade, and Luddism: A Sufficient Statistic Approach to Optimal Technology Regulation," *Review of Economic Studies*, 10 2023, *90*, 2261–2291.

Curtis, E. Mark and Ioana Marinescu, "Green Energy Jobs in the United States: What Are They, and Where Are They?," *Environmental and Energy Policy and the Economy*, 1 2023, *4*, 202–237.

_ , Layla O'Kane, and R. Jisung Park, "Workers and the Green-Energy Transition: Evidence from 300 Million Job Transitions," *NBER Working Paper*, 2023, *31539*.

Davis, Steven J. and Pawel M. Krolikowski, "Sticky Wages on the Layoff Margin," *American Economic Review*, 2 2025, *115*, 491–524.

Dix-Carneiro, Rafael, "Trade Liberalization and Labor Market Dynamics," *Econometrica*, 2014, *82*, 825–885.

Elsby, Michael W L and Axel Gottfries, "Firm Dynamics, On-the-Job Search, and Labor Market Fluctuations [Optimal Investment with Costly Reversibility]," *The Review of Economic Studies*, 2022, *89* (3), 1370–1419.

Fujita, Shigeru, Giuseppe Moscarini, and Fabien Postel-Vinay, "Measuring Employer-to-Employer Reallocation," *American Economic Journal: Macroeconomics*, 7 2024, *16*, 1–51.

Gavazza, Alessandro, Simon Mongey, and Giovanni L. Violante, "Aggregate Recruiting Intensity," *American Economic Review*, August 2018, *108* (8), 2088–2127.

Granholm, Jennifer M, "United States Energy & Employment Report 2024," 2024.

Grigsby, John, "Skill Heterogeneity and Aggregate Labor Market Dynamics," NBER Working Papers 30052, National Bureau of Economic Research, Inc May 2024.

_ , Erik Hurst, and Ahu Yildirmaz, "Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data," *American Economic Review*, 2 2021, *111*, 428–471.

Guerreiro, Joao, Sergio Rebelo, and Pedro Teles, "Should Robots be Taxed?," *Review of Economic Studies*, 2021, *89*, 279–311.

_ , _ , and _ , "Should Robots Be Taxed?," *Review of Economic Studies*, 2022, *89* (1), 279–311.

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning, "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?," *American Economic Review*, May 2022, *112* (5), 1437–1474.

Haan, Wouter J Den, Garey Ramey, and Joel Watson, "Job Destruction and Propagation of Shocks," *American Economic Review*, 6 2000, *90*.

Hagedorn, Marcus and Iourii Manovskii, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 2008, *98*, 1692–1706.

Hanson, Gordon H, "Local Labor Market Impacts of the Energy Transition: Prospects and Policies," 2023.

Hopenhayn, Hugo A., "Entry, Exit, and firm Dynamics in Long Run Equilibrium," *Econometrica*, 1992, *60* (5), 1127–1150.

Itskhoki, Oleg and Benjamin Moll, "Optimal Development Policies With Financial Frictions," *Econometrica*, January 2019, *87* (1), 139–173.

Jacobson, Louis S, Robert J Lalonde, and Daniel G Sullivan, "Earnings Losses of Displaced Workers," *American Economic Review*, 1993, *83*, 685–709.

Jäger, Simon, Benjamin Schoefer, and Josef Zweimüller, "Marginal Jobs and Job Surplus: A Test of the Efficiency of Separations," *Review of Economic Studies*, 5 2023, *90*, 1265–1303.

Jarosch, Gregor, "Searching for Job Security and the Consequences of Job Loss," *Econometrica*, 2023, *91*, 903–942.

Känzig, Diego R., "The Macroeconomic Effects of Oil Supply News: Evidence from OPEC Announcements," *American Economic Review*, April 2021, *111* (4), 1092–1125.

_ , "The Unequal Economic Consequences of Carbon Pricing," NBER Working Papers 31221, National Bureau of Economic Research, Inc May 2023.

Kaplan, Greg and Giovanni L. Violante, "The Marginal Propensity to Consume in Heterogeneous Agent Models," *Annual Review of Economics*, 2022, *14* (1), 747–775.

Lehr, Nils H and Pascual Restrepo, "Optimal Gradualism," *NBER Working Paper*, 2023, *30755*.

Menzio, Guido and Shouyong Shi, "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy*, 2011, *119* (3), 468–510.

Merino-Saum, Albert, Jessica Clement, Romano Wyss, and Marta Giulia Baldi, "Unpacking the Green Economy concept: A quantitative analysis of 140 definitions," 1 2020.

Mortensen, Dale T and Christopher A Pissarides, "Technological Progress, Job Creation, and Job Destruction," *Review of Economic Dynamics*, 1998, *1*, 733–753.

Muehlemann, Samuel and Mirjam Strupler Leiser, "Hiring costs and labor market tightness," *Labour Economics*, 2018, *52* (C), 122–131.

Naser, Mostafa Mahmud and Prafula Pearce, "Evolution of the International Climate Change Policy and Processes: UNFCCC to Paris Agreement," 11 2022.

Nath, Ishan B, Valerie A Ramey, and Peter J Klenow, "How Much Will Global Warming Cool Global Growth?," 2024.

Postel-Vinay, Fabien, "The Dynamics of Technological Unemployment," *International Economic Review*, 2002, *43*, 737–760.

Schaal, Edouard, "Uncertainty and Unemployment," *Econometrica*, 2017, *85*, 1675–1721.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 3 2005, *95*, 25–49.

Sorkin, Isaac, "Ranking firms using revealed preference," *Quarterly Journal of Economics*, 2018, *133*, 1331–1393.

Thuemmel, Uwe, "Optimal Taxation of Robots," *Journal of the European Economic Association*, 6 2023, *21*, 1154–1190.

Traiberman, Sharon, "Occupations and Import Competition: Evidence from Denmark," *American Economic Review*, 12 2020, *109*, 4260–4301.

Violante, Giovanni L, "Technological Acceleration, Skill Transferability, and the Rise in Residual Inequality," *The Quarterly Journal of Economics*, 2002, *117*, 297–338.

Vona, Francesco, Giovanni Marin, Davide Consoli, and David Popp, "Environmental regulation and green skills: An empirical exploration," *Journal of the Association of Environmental and Resource Economists*, 10 2018, *5*, 713–753.

Walker, W. Reed, "The Transitional Costs of Sectoral Reallocation: Evidence From the Clean Air Act and the Workforce," *The Quarterly Journal of Economics*, 2013, *128* (4), 1787–1835.

Yaari, Menahem E., "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, 1965, *32* (2), 137–150.

A Baseline Model Appendix

This section provides additional information and proofs about the baseline model studied in sections 2 and 3.

A.1 An Economy with Constant Growth Rates

This section studies a version of our model with long-run growth. Specifically, suppose that $A_t(k,\eta) = \tilde{A}_t(k;\eta)e^{gt}$ where $\tilde{A}_t(k,\eta)$ is the deviation from the long-run trend growth brought about by the transition and g is the long-run trend growth rate.[56] We assume this trend growth rate is the same for both sectors beginning in period 0, when the new sector is unexpectedly introduced. $\tilde{A}_t(k;\eta)$ is the piece subject to the transition and takes the linear form of equation 2.4.

The model set up is the same as that presented in Section 2 with a few key exceptions. First, we specialize the utility function to take the CRRA form from the outset: $u(c) = c^{1-\sigma}/(1-\sigma)$. Second, the cost of vacancy posting ν_t and unemployment benefit b_{Dt} grow at the same rate as productivity: $\nu_t = \nu e^{gt}, b_{Dt} = b_D e^{gt}$. Third, suppose that the separation function is now $\mathcal{F}(V_t e^{-gt})$, so that its input is the value of the firm scaled by the exogenous growth rate. Finally, we assume entrepreneurs are risk-neutral and die stochastically at rate σg. They have access to a market for annuities that insures them against the risk of death as in Blanchard (1985) and Yaari (1965). Therefore, their effective discount rate becomes $\rho + \sigma g$.[57]

With these assumptions, the free entry condition becomes

$$q\left(\theta_t(j,\omega,k)\right) V_t(j,\omega,k) \leq \nu e^{gt} \tag{A.1}$$

in every period t and market k, with equality if a positive measure of vacancies are posted. This condition implies that the probability that a worker searching in sector k is able to find a job is given by a time-varying function $\tilde{f}_t(V_t) \equiv$

[56] Workers earn a constant piece rate in the baseline model introduced in Section 2. Their earnings therefore growth a rate g. In the quantitative model of Section 4, workers' wages are fixed in a given job for each effective units of labor supplied. We suppose that it is precisely this number of effective units of labor that grows at rate g so workers' earnings also grow at this rate in this case and the economy can be detrended accordingly.

[57] This discount rate is naturally arises in Schumpeterian growth models. For instance, the equilibrium interest rate in Aghion and Howitt (1992)'s classic paper is equal to $\rho + \sigma g$, for g the equilibrium growth rate of the economy.

$f\left(q^{-1}\left(e^{gt}\nu/V_t\right)\right)$. We can write the value of an unemployed worker after the introduction of the new sector in period 0 as follows

$$\rho W_t^U(j,\omega,D) = \frac{b_D^{1-\sigma}e^{(1-\sigma)gt}}{1-\sigma} + \lambda^U \max_k \left\{\tilde{f}_t\left(V_t(j,\omega,k)\right)\left[W_t^E(j,\omega,k) - W_t^U(j,\omega,D)\right]\right\} + \frac{d}{d\epsilon}W_{t+\epsilon}^U(j+\epsilon,\omega,D)\Big|_{\epsilon=0}, \tag{A.2}$$

the value of an employed worker in sector k is given by

$$\begin{aligned}\rho W_t^E(j,\omega,k) = &\frac{\left(\left(1-\pi\right)\psi\left(j,\omega,k\right)\tilde{A}_t(k,\eta)\right)^{1-\sigma}e^{(1-\sigma)gt}}{1-\sigma} \\ &+ \lambda^E \cdot \max_{k'}\left\{\tilde{f}_t\left(V_t(j,\omega,k')\right)\left[W_t^E(j,\omega,k') - W_t^E(j,\omega,k)\right]\right\} \\ &+ \chi_0\mathcal{F}\left(V_t(j,\omega,k)e^{-gt}\right)\left[(1-\xi)^{1-\sigma}W_t^U(j,\omega,0) - W_t^E(j,\omega,k)\right] \\ &+ \frac{d}{d\epsilon}W_{t+\epsilon}^E(j+\epsilon,\omega,k)\Big|_{\epsilon=0},\end{aligned} \tag{A.3}$$

and the value of a filled job to a firm in sector k is given by

$$\begin{aligned}(\rho+\sigma g)V_t(j,\omega,k) = &\pi\psi\left(j,\omega,k\right)\tilde{A}_t(k,\eta)e^{gt} \\ &- \left[\chi_0\mathcal{F}\left(V_t(j,\omega,k)e^{-gt}\right) + \lambda^E\tilde{f}_t\left(V_t\left(j,\omega,k_t^\star(j,\omega,k)\right)\right)\right]V_t(j,\omega,k) \\ &+ \frac{d}{d\epsilon}V_{t+\epsilon}(j+\epsilon,\omega,k)\Big|_{\epsilon=0}.\end{aligned} \tag{A.4}$$

Now define the following normalized value functions:

$$\tilde{W}_t^E(j,\omega) \equiv W_t^E(j,\omega,k)e^{-(1-\sigma)gt} \quad , \quad \tilde{W}_t^U(j,\omega,D) \equiv W_t^U(j,\omega,D)e^{-(1-\sigma)gt} \quad ,$$

$$\tilde{V}_t(j,\omega,k) \equiv V_t(j,\omega,k)e^{-gt}$$

Note that we can then write the job-finding rate as a function only of $\tilde{V}_t$ which does not change over time: $\tilde{f}\left(\tilde{V}_t\right) = f\left(q^{-1}\left(\nu/\tilde{V}\right)\right)$. Substituting these normalized value functions into equations A.2-A.4 yields an expression for the normalized

value of unemployed workers

$$\rho \tilde{W}_t^U(j,\omega,D)e^{(1-\sigma)gt} = \frac{b_D^{1-\sigma}e^{(1-\sigma)gt}}{1-\sigma} + \lambda^U \max_k \left\{ \tilde{f}\left(\tilde{V}_t(j,\omega,k)\right)\left[\tilde{W}_t^E(j,\omega,k) - \tilde{W}_t^U(j,\omega,D)\right] e^{(1-\sigma)gt} \right\} \tag{A.5}$$

$$+ e^{(1-\sigma)gt}\left.\frac{d}{d\epsilon}\tilde{W}_{t+\epsilon}^U(j+\epsilon,\omega,D)\right|_{\epsilon=0} + (1-\sigma)g\tilde{W}_t^U(j,\omega,D)e^{(1-\sigma)gt}, \tag{A.6}$$

for normalized values of employed workers

$$\rho \tilde{W}_t^E(j,\omega,k)e^{(1-\sigma)gt} = \frac{\left((1-\pi)\,\psi\left(j,\omega,k\right)\tilde{A}_t(k,\eta)\right)^{1-\sigma}e^{(1-\sigma)gt}}{1-\sigma} + \lambda^E \cdot \max_{k'}\left\{ \tilde{f}\left(\tilde{V}_t(j,\omega,k')\right)\left[\tilde{W}_t^E(j,\omega,k') - \tilde{W}_t^E(j,\omega,k)\right] e^{(1-\sigma)gt} \right\} + \chi_0 \mathcal{F}\left(\tilde{V}_t(j,\omega,k)\right)\left[(1-\xi)^{1-\sigma}\tilde{W}_t^U(j,\omega,0) - \tilde{W}_t^E(j,\omega,k)\right] e^{(1-\sigma)gt} + e^{(1-\sigma)gt}\left.\frac{d}{d\epsilon}\tilde{W}_{t+\epsilon}^E(j+\epsilon,\omega,k)\right|_{\epsilon=0} + (1-\sigma)g\tilde{W}_t^E(j,\omega,k)e^{(1-\sigma)gt}, \tag{A.7}$$

and for firm values of a filled job

$$(\rho+\sigma g)\tilde{V}_t(j,\omega,k)e^{gt} = \pi\psi\left(j,\omega,k\right)\tilde{A}_t(k,\eta)e^{gt} - \left[\chi_0\mathcal{F}\left(\tilde{V}_t(j,\omega,k)\right) + \lambda^E\tilde{f}\left(\tilde{V}_t\left(j,\omega,k_t^\star(j,\omega,k)\right)\right)\right]\tilde{V}(j,\omega,k)e^{gt} \tag{A.8}$$

$$+ e^{gt}\left.\frac{d}{d\epsilon}\tilde{V}_{t+\epsilon}(j+\epsilon,\omega,k)\right|_{\epsilon=0} + g\tilde{V}_t(j,\omega,k)e^{gt}. \tag{A.9}$$

Dividing equations A.6 and A.7 by $e^{(1-\sigma)gt}$, dividing equation A.9 by e^{gt}, and

rearranging yields expressions for the normalized value of unemployed workers

$$(\rho + (\sigma-1)g)\,\tilde{W}_t^U(j,\omega,D) = u(b_D) + \lambda^U \max_k \left\{ \tilde{f}\left(\tilde{V}_t(j,\omega,k)\right)\left[\tilde{W}_t^E(j,\omega,k) - \tilde{W}_t^U(j,\omega,D)\right]\right\} + \frac{d}{d\epsilon}\tilde{W}_{t+\epsilon}^U(j+\epsilon,\omega,D)\Big|_{\epsilon=0},$$

normalized values of employed workers

$$\begin{aligned}(\rho + (\sigma-1)g)\,\tilde{W}(j,\omega,k) = {} & u\left((1-\pi)\,\psi\left(j,\omega,k\right)\tilde{A}_t(k,\eta)\right) \\ & + \lambda^E \cdot \max_{k'}\left\{\tilde{f}\left(\tilde{V}_t(j,\omega,k')\right)\left[\tilde{W}_t^E(j,\omega,k') - \tilde{W}_t^E(j,\omega,k)\right]\right\} \\ & + \chi_0 \mathcal{F}\left(\tilde{V}_t(j,\omega,k)\right)\left[(1-\xi)^{1-\sigma}\tilde{W}_t^U(j,\omega) - \tilde{W}_t^E(j,\omega,k)\right] \\ & + \frac{d}{d\epsilon}\tilde{W}_{t+\epsilon}^E(j+\epsilon,\omega,k)\Big|_{\epsilon=0},\end{aligned}$$

and for firm values of a filled job

$$\begin{aligned}(\rho + (\sigma-1)g)\,\tilde{V}_t(j,\omega,k) = {} & \pi\psi\left(j,\omega,k\right)\tilde{A}_t(k,\eta) \\ & - \left[\chi_0\mathcal{F}\left(\tilde{V}_t(j,\omega,k)\right) + \lambda^E \tilde{f}\left(\tilde{V}_t\left(j,\omega,k_t^\star(j,\omega,k)\right)\right)\right]\tilde{V}(j,\omega,k) \\ & + \frac{d}{d\epsilon}\tilde{V}_{t+\epsilon}(j+\epsilon,\omega,k)\Big|_{\epsilon=0}\end{aligned}$$

that are nearly identical to those presented in the main text with $g = 0$ (equations 2.8, 2.6 and 2.9). The only difference is that agents' effective discount rate is now $\rho + (\sigma-1)g$. Therefore, the positive implications for deviations from trend growth are identical in this model as in the model with zero growth studied in the main text calibrated to a different discount rate.

The normative implications of the model with growth are also identical to a model without growth which places different Pareto weights on different cohorts. Recall that social welfare as of period 0 is given by

$$\bar{\mathcal{W}}(\eta) = \int_\Omega \int_{-J}^{\infty} \Lambda(\tau,\omega) W(\tau,\omega;\eta)\hat{\mu}(\tau,\omega)\,d\tau\,d\omega$$

In the economy with exogenous growth at rate g, our work above shows that the value of workers is equal to $e^{(1-\sigma)gt}$ times the value of workers in a world with zero trend growth, while the value of firms is equal to e^{gt} times the value of firms in a zero growth world. Since calendar time and age are colinear, we can substitute this into the above equation to arrive at

$$\bar{\mathcal{W}}(\eta) = \int_{\Omega} \int_{-J}^{\infty} \Lambda(\tau, \omega) e^{-(1-\sigma)g\tau} \tilde{W}(\tau, \omega; \eta) \hat{\mu}(\tau, \omega) d\tau d\omega$$

where $\tilde{W}(\tau, \omega; \eta)$ is the value of a worker of type (τ, ω) given a transition speed η in the world where $g = 0$. Therefore, one can define alternative Pareto weights $\tilde{\Lambda}(\tau, \omega) \equiv \Lambda(\tau, \omega) e^{-(1-\sigma)g\tau}$ and show that aggregate social welfare is takes the same form as in the zero growth model:

$$\bar{\mathcal{W}}(\eta) = \int_{\Omega} \int_{-\infty}^{J} \tilde{\Lambda}(\tau, \omega) \tilde{W}(\tau, \omega; \eta) \hat{\mu}(\tau, \omega) d\tau d\omega.$$

Therefore, our focus on the zero trend-growth model is without loss of generality both normatively and positively in the baseline framework. One can pursue the same steps for the quantitative model as well so long as the random component of operating costs grow at rate g and the variance of the search amenity shocks grows at rate $(1 - \sigma)g$.

A.2 Proofs in Walrasian Benchmark with Ex Ante Heterogeneity

This section proves the claims made in the Walrasian benchmark under weaker conditions on the transition. Throughout this section, we break from the linear transition assumed in the main text, and instead maintain the more general Assumption 1.

Assumption 1. *Assume:*

1. *$A_t(k; \eta)$ and $\hat{A}_t(\eta)$ is twice differentiable in t and η at $t = 0$ and almost everywhere,*
2. *For all t and η, $\partial_t A_{old}(t; \eta) \leq 0 \leq \partial_t A_{new}(t; \eta)$, $\partial_\eta \partial_t A_t(new, \eta) \geq 0$, $\partial_\eta \partial_t A_t(old, \eta) \leq 0$,*
3. *$\lim_{\eta \to \infty} \hat{A}(t; \eta) = \dfrac{A_{new,\infty}}{A_{old,\infty}}$ for all t and $\partial_t A_{old}(t; 0) = \partial_t A_{new}(t; 0) = 0$, and*
4. *There exists some $n > 0$ such that the n^{th} partial derivative of $A_t(old; \eta)$ with*

respect to η *is bounded at* $t = 0$ *in the limit as* $\eta \to \infty$*:* $\left| \lim_{\eta \to \infty} \frac{\partial^n A_0(old;\eta)}{\partial \eta^n} \right| < \infty$.

The first assumption assumes smoothness of the transition almost everywhere. The second assumption defines the kinds of transitions we can consider – the old sector declines (relative to trend), the new sector grows, and η speeds up the transition in the sense that a higher η translates into a faster growth in the new sector's productivity and a faster decline in the old sector's. The third states that, in the limit as η goes to infinity, the ratio of new-to-old productivity instantly jumps to its long run level at all t, while there is no transition at all if $\eta = 0$. The fourth is a technical condition to ensure that as the transition grows infinitely fast, some rate of change of the old sector's technology path is bounded. Many standard functional forms for technological paths satisfy these assumptions, such as exponential, linear, or logistic transitions.[58] Therefore, this set of assumptions is strictly weaker than those presented in the main text – our assertions of non-monotonicity are not dependent on the linear transition we assumed for expositional clarity. We now prove Proposition 1 under this assumption.

A.2.1 Proof of Proposition 1

First, consider the always-switchers. These individuals switch as soon as the new sector appears, regardless of the transition speed. Therefore, they are characterized by $t^\star(\omega;\eta) = 0$ for all η, which is obtained if $\hat{A}_0 \geq \hat{\psi}(\omega)^{-1}$: their skills are sufficiently high in the new sector relative to the old sector to span the productivity gap between the sectors even in period 0. Since such workers are always in the new sector, whose growth rate is increasing in η, they are trivially made better off by a faster transition.

Never-switchers are the reverse. They are defined as $t^\star(\omega;\eta) = \infty$ for all η, which is obtained if $\hat{A}_\infty \leq \hat{\psi}(\omega)^{-1}$: relative skills are insufficient to overcome

[58] Consider an example with an exponential transition: $A_t(old;\eta) = A_{old0} + \Delta_{old}\{1 - \exp(-\eta t)\}$. Differentiating with respect to η yields

$$\frac{\partial A_t(old;\eta)}{\partial \eta} = \Delta_{old} t \exp(-\eta t)$$

which is bounded (indeed, equals 0) at $t = 0$ as desired. If the transition is linear as in equation 2.4, the partial derivative of $A_t(old;\eta)$ with respect to η is equal to t, which is trivially 0 when evaluated at $t = 0$.

the productivity differential even when the transition is complete. These workers are made monotonically worse by a faster transition because they only realize the faster declines in the old sector.

Now consider the sometimes-switchers, for whom $\hat{\psi}(\omega)^{-1} \in (\hat{A}_0, \hat{A}_\infty)$. For these workers, there exists some η for which $0 < t^\star(\omega;\eta) < \infty$. For these workers, we claim that there exists some η_h for which $\partial_\eta \mathcal{W}(\tau,\omega;\eta) > 0$ for all $\eta \geq \eta_h$ and $\eta_l < \eta_h$ such that $\partial_\eta \mathcal{W}(\tau,\omega;\eta) < 0$ for all $\eta \leq \eta_l$: that is, the worker's welfare is increasing in η for some values of η and decreasing for others.

Proof: First note that $t^\star(\omega;\eta)$ is continuous and decreasing in η since $\hat{A}_t(\eta)$ is continuous and increasing in η. First consider workers who are not born as of period 0; that is $\tau > 0$. Under Assumption 1, since there exists some η for which $0 < t^\star(\omega;\eta) < \infty$ and $\tau > 0$, the Intermediate Value Theorem implies that there must be some η_h for which $t^\star(\omega;\eta_h) = \tau$ (the worker switches immediately) and some η_l for which $t^\star(\omega;\eta_l) = J + \tau$ (the worker would switch after retirement). Then, for all $\eta < \eta_l$, the worker remains in the old sector for their whole career and are made worse off by a faster transition, while for all $\eta > \eta_h$ they remain in the new sector their whole career and are made better off by a fast transition. Thus welfare is non-monotone in η for cohorts not yet born.

Now consider an incumbent worker $\tau < 0$ for whom there exists some η for which $0 < t^\star(\omega;\eta) < \infty$. First, note that one can derive η_l in the same way as above: for slow enough transitions such that the worker does not want to switch before retirement, the worker acts like a never-switcher. Now we prove the existence of an η_h. Consider an η with $0 \leq t^\star(\omega;\eta) \leq J + \tau$: that is, an η such that the

worker would like to switch at some point before retirement. We have

$$\begin{aligned}
\frac{d\mathcal{W}(\tau,\omega;\eta)}{d\eta} &= \psi(\omega,old)\int_0^{t^\star(\omega;\eta)} e^{-\rho t}u'\left(A_t(old;\eta)\psi(\omega,old)\right)\frac{\partial A_t(old;\eta)}{\partial\eta}dt \\
&\quad + \psi(\omega,new)\int_{t^\star(\omega;\eta)}^{J+\tau} e^{-\rho t}u'\left(A_t(new;\eta)\psi(\omega,new)\right)\frac{\partial A_t(new;\eta)}{\partial\eta}dt \\
&\geq \psi(\omega,old)\left(u'(b)\int_0^{t^\star(\omega;\eta)} e^{-\rho t}\frac{\partial A_t(old;\eta)}{\partial\eta}dt\right. \\
&\quad \left. + \hat{\psi}(\omega)\int_{t^\star(\omega;\eta)}^{J+\tau} e^{-\rho t}u'\left(A_t(new;\eta)\psi(\omega,new)\right)\frac{\partial A_t(new;\eta)}{\partial\eta}dt\right) \\
&\geq \psi(\omega,old)\left(u'(b)\int_0^{t^\star(\omega;\eta)} e^{-\rho t}\frac{\partial A_t(old;\eta)}{\partial\eta}dt\right. \\
&\quad \left. + u'\left(A_{new,\infty}\psi(\omega,new)\right)\hat{\psi}(\omega)\int_{t^\star(\omega;\eta)}^{J+\tau} e^{-\rho t}\frac{\partial A_t(new;\eta)}{\partial\eta}dt\right)
\end{aligned}$$

The first inequality uses the fact that the household always maintains the option of becoming unemployed and receiving b, so marginal utilities are bounded above by $u'(b)$, and that $\partial A_t(old;\eta)/\partial\eta \leq 0$. The second inequality uses the fact that $\partial A_t(new;\eta)/\partial\eta \geq 0$ and that $A_{new,\infty} \geq A_t(new;\eta)$ for all t so that the lower bound on marginal utilities is $u'\left(A_{new,\infty}\psi(\omega,new)\right)$. Finally, factor out the smallest possible (or most negative) value of $\dfrac{\partial A_t(old;\eta)}{\partial\eta}$ and $\dfrac{\partial A_t(new;\eta)}{\partial\eta}$ and evaluate the remaining integral to arrive at:

$$\begin{aligned}
\frac{d\mathcal{W}(\tau,\omega;\eta)}{d\eta} \geq \psi(\omega,old)\left(\frac{u'(b)}{\rho}\inf_{0\leq t\leq t^\star(\omega;\eta)}\left(\frac{\partial A_t(old;\eta)}{\partial\eta}\right)\left(1-e^{-\rho t^\star(\omega;\eta)}\right)\right. \\
\left. + \frac{u'\left(A_{new,\infty}\psi(\omega,new)\right)}{\rho}\hat{\psi}(\omega)\inf_{t^\star\leq t\leq J+\tau}\left(\frac{\partial A_t(new;\eta)}{\partial\eta}\right)\left(e^{-\rho t^\star(\omega;\eta)}-e^{-\rho(J+\tau)}\right)\right).
\end{aligned}$$

We wish to claim that this is larger than 0 for some value of η. Observe that the second line of the above inequality is larger than 0, as all objects in it are weakly positive. We therefore must bound only the first line for some value of η. Note that by the definition of sometimes-switchers and since the transition is instantaneous as $\eta \to \infty$, there exists a value of η such that $t^\star(\omega;\eta) = 0$ and workers switch

immediately. Define η_h to be the smallest value of η such that $t^\star(\omega;\eta) = 0$. Then for $\eta > \eta_h$, $\left(1 - e^{-\rho t^\star(\omega;\eta)}\right) = 0$. The first line therefore equals zero for $\eta > \eta_h$ so long as the absolute value of $\dfrac{\partial A_t(old;\eta)}{\partial \eta}$ grows slower than $1 - e^{-\rho t^\star}$ goes to zero. If $t^\star(\omega;\eta) = 0$, then $\inf_{0 \leq t \leq t^\star(\omega;\eta)} \partial A_t(old;\eta)/\partial\eta = \partial A_0(old;\eta)/\partial\eta$. Sending $\eta \to \infty$ we have that the operative part of the above equation becomes

$$\lim_{\eta \to \infty} \frac{\left(\dfrac{\partial A_0(old;\eta)}{\partial \eta}\right)}{\left(1 - e^{-\rho t^\star(\omega;\eta)}\right)^{-1}}.$$

Note that $e^{-\rho t^\star(\omega;\eta)} \to 1$ as $\eta \to \infty$. Applying L'Hopital's rule, we thus have

$$\lim_{\eta \to \infty} \frac{\left(\dfrac{\partial A_0(old;\eta)}{\partial \eta}\right)}{\left(1 - e^{-\rho t^\star(\omega;\eta)}\right)^{-1}} = \lim_{\eta \to \infty} -\frac{\dfrac{\partial_2 A_0(old;\eta)}{\partial \eta^2}}{\left(1 - e^{-\rho t^\star(\omega;\eta)}\right)^{-2} e^{-\rho t^\star(\omega;\eta)}}$$

We can continually apply L'Hopital's rule to have these limits involving progressively higher order derivatives of $A_0(t;\eta)$ with respect to η in the numerator, and something going to infinity on the denominator. Therefore if $\lim_{\eta\to\infty} \dfrac{\partial_n A_0(old;\eta)}{\partial \eta^n}$ is bounded for some n (item 4 of Assumption 1), this limit will equal 0, and $\dfrac{d\mathcal{W}(\tau,\omega;\eta)}{d\eta} \geq 0$. Therefore, there exists some η_h such that for all $\eta > \eta_h$, incumbent sometimes-switchers are made better off by a faster transition. Therefore, we have non-monotone micro welfare effects of a faster transition for any worker for whom there exists some finite $t^\star$ after which the worker would switch to the new sector.

A.2.2 Proof of Proposition 2

First note that the first two possibilities, i.e., monotonicity or U-shaped behavior of social welfare $\overline{\mathcal{W}}(\eta)$, are direct corollaries of Proposition 2. One need simply place all the mass on sets of points (ω, τ) such that the corresponding workers are always-stayers (for monotone decreasing), always-switchers (for monotone increasing) or sometimes-switchers (for U-shaped). In this case, aggregate social

welfare function inherits the shape of the corresponding micro welfare effects.

We therefore need only prove that there exist distributions such that social welfare $\overline{\mathcal{W}}(\eta)$ features inverted-U-shaped regions if $A_{\text{old},\infty}$ is sufficiently small. In the following, we will construct a two-point, age-invariant distribution $\hat{\mu}(\omega, \tau) = \hat{\mu}(\omega)$ such that

$$d\overline{\mathcal{W}}(\eta^\star)/d\eta = 0 \quad \text{and} \quad \overline{\mathcal{W}}(\eta^\star) > \max\left\{\overline{\mathcal{W}}(\eta_0), \overline{\mathcal{W}}(\eta_1)\right\}$$

for some transition speed $\eta^\star$ and all transitions speeds $\eta_0 < \eta^\star < \eta_1$ in a small neighborhood around $\eta^\star$. We let $\omega_0 < 1$ and $\omega_1 \equiv 1$ denote the two points of this distribution and $1 - \hat{\mu}_1$ and $\hat{\mu}_1$ denote the associated weights. We consider the limit $\omega_0 \to 0$ so these workers decide to stay in the old sector and earn $A_t(old; \eta)$. We will consider the limit $A_{old,\infty} \to 0$ in the following. To streamline the exposition, we focus on existing generations.[59]

Note that there is some $\bar{\eta}$ such that $d\mathcal{W}(\omega_1; \eta)/d\eta$ is U-shaped in η for all $\eta \geq \bar{\eta}$ for the reasons discussed above. Let $\eta^\star$ be any transition speed above $\bar{\eta}$. By inspecting expressions (3.3)–(3.5), it is clear that there exists a weight $\hat{\mu}_1$ such that

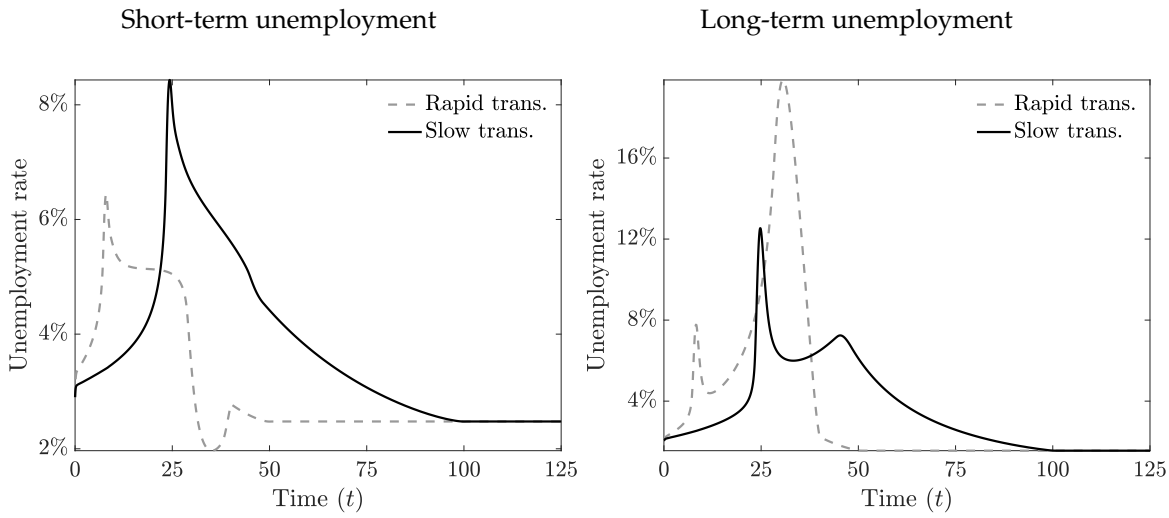
$$(1 - \hat{\mu}_1)\frac{d\mathcal{W}(\omega_0; \eta^\star)}{d\eta} + \hat{\mu}_1 \frac{d\mathcal{W}(\omega_1; \eta^\star)}{d\eta} = 0$$

and hence $d\overline{\mathcal{W}}(\eta^\star)/d\eta = 0$ since marginal utility is unbounded as $A_{old,\infty} \to 0$. One can easily verify that $\overline{\mathcal{W}}(\eta_0), \overline{\mathcal{W}}(\eta_1) < \overline{\mathcal{W}}(\eta^\star)$ for $\eta_0 < \eta^\star < \eta_1$ since utility is concave.

[59] The presence of new generations does not change the proof since they benefit from a faster transition and their utility is bounded.

A.3 Additional Results

Figure 10: Decomposing the unemployment rate



Notes: The left panel plots two skill distributions with mean 0.75. The solid curve is a uni-model "bell-shaped" distribution. The dashed curve is a bi-modal distribution with "two types" where some households have much less versatile skills in the new sector. The right panel illustrates the skill distribution () dotted and dotted-dashed curves maintain the two-types distribution but assume that age is positive and negatively correlated with new sector skill, respectively.

B Quantitative Model

B.1 Workers' and Firm's Values

In this appendix, we formulate the workers' and firms' values recursively.

States. Workers are indexed by their age j; their skill ω; their safety state z; their employment status $e = E, U$; and, if employed, their occupation k; and their wage w. The last three states can be combined into a single state s which corresponds to a market (k, w) when the worker is employed, or to U when the worker is unemployed.[60] In the following, we let $W_t^E(j, \omega, z, s)$ denote the value of an employed worker and $W_t^U(j, \omega)$ denote the value of an unemployed worker. Firm's values

[60] Combining these states allows to reduce the size of the state space by not keeping track of the wage w, for instance, for workers who are unemployed. Similarly, the safety state z can be included in s. Doing so requires introducing additional notation, e.g., by allowing workers to search only in markets s associated to the safety z that they currently enjoy. For clarity, we keep these states separate in this appendix.

are denoted by $V_t(j, \omega, z, s)$.

Workers. The value of an employed worker satisfies

$$\begin{aligned}\rho W_t^E(j,\omega,z,s) = u\left(w(s)\psi(j,\omega,k(s))\right) \\ + \left\{\chi_0 \mathcal{F}_t(j,\omega,z,s) + \mathbf{1}_{\{z=\underline{z}\}}\chi_1\right\} \times \left\{W_t^U(j,\omega,0) - W_t^E(j,\omega,z,s)\right\} \\ + \lambda^E \left\{\max_{s'\equiv(k',\pi')} f\left(\theta_t(j,\omega,z,s')\right)\left\{W_t^E(j,\omega,z,s') - W_t^{E,\text{stay}}(j,\omega,z,s)\right\} + \zeta_{s'}\right\} \\ + \kappa\left\{W_t^E(j,\omega,\bar{z},s) - W_t^E(j,\omega,z,s)\right\} \\ + \frac{d}{d\epsilon} W_{t+\epsilon}^E(j+\epsilon,\omega,z,s)\Big|_{\epsilon=0} \end{aligned} \tag{B.1}$$

for all ages before retirement $j \in [0, J)$, skills ω, safeties z and markets $s \equiv (k, w)$.[61] The match can be dissolved for three reasons. First, the firm incurs an operating cost with intensity χ_0 and decides to dissolve the match and fire the worker with an endogenous probability $\mathcal{F}_t(j, \omega, z, s)$, defined below. When workers separate from their job, they become short-term unemployed. Second, employed workers search on-the-job with intensity λ^E. The probability that they find a job depends on the market that they search in, which solves

$$s_t^\star(j,\omega,z,s|\boldsymbol{\zeta}) \equiv \arg\max_{s'\equiv(k',\pi')} f\left(\theta_t(j,\omega,z,s')\right)\left\{W_t^E(j,\omega,z,s') - W_t^{E,\text{stay}}(j,\omega,z,s)\right\} + \zeta_{s'} \tag{B.2}$$

where $\zeta_{s'}$ is an iid preference shock for searching in sector s' which is distributed according to a type 1 extreme value distribution with variance γ_ζ. The match is

[61] Our approach bears some resemblance with the penalty methods often used in the literature on optimal stopping time (see Elsby and Gottfries, 2022 for an application). Our approach constructs a matrix of transition flows that allow us to conveniently derive the law of motion for workers' states (Appendix B.3).

dissolved when this search is successful.[62] Let

$$p_t^E\left(s';j,\omega,z,s\right) \equiv \frac{\exp\left(\frac{f(\theta_t(j,\omega,z,s'))\left\{W_t^E(j,\omega,z,s')-W_t^{E,\text{stay}}(j,\omega,z,s)\right\}}{\gamma_\zeta}\right)}{\sum_{\tilde{s}}\exp\left(\frac{f(\theta_t(j,\omega,z,\tilde{s}))\left\{W_t^E(j,\omega,z,\tilde{s})-W_t^{E,\text{stay}}(j,\omega,z,s)\right\}}{\gamma_\zeta}\right)} \tag{B.3}$$

denote the probability that a type (j,ω,z) worker searches in market s' when they receive a search opportunity and are employed in sector s. Further, let

$$Q_t(j,\omega,z,s) \equiv \sum_{s'} \underbrace{p_t^E\left(s';j,\omega,z,s\right)}_{Pr\{\text{Search in market } s'|j,\omega,z,s\}} \times \underbrace{f\left(\theta_t\left(j,\omega,z,s'\right)\right)}_{\text{Job-Finding Rate in } s'} \tag{B.4}$$

denote the probability that a worker who receives an opportunity to search successfully finds a job and thus quits their current employer. Third, the match is dissolved exogeneously at rate $\chi_1 > 0$ if workers' safety is low $z = \underline{z}$. The safety state z switches to $\bar{z} \equiv 1$ with flow rate $\kappa > 0$ when they are employed.

In turn, the value of an unemployed worker satisfies

$$\begin{aligned}\rho W_t^U\left(j,\omega,d\right) = {} & u\left(b_d\psi^{\text{age}}\left(j\right)\right) + \kappa_d\left[W_t^U\left(j,\omega,1\right) - W_t^U\left(j,\omega,0\right)\right] \\ & + \lambda^U\left\{\max_{s'\equiv(k',\pi')} f\left(\theta_t\left(j,\omega,\underline{z},s'\right)\right)\left\{W_t^E\left(j,\omega,\underline{z},s'\right) - W_t^U\left(j,\omega,D\right)\right\} + \zeta_{s'}\right\} \\ & + \frac{d}{d\epsilon}W_{t+\epsilon}^U\left(j+\epsilon,\omega,D\right)\Big|_{\epsilon=0},\end{aligned} \tag{B.5}$$

for all ages before retirement $j \in [0,J)$ and each unemployment status $d = U,D$. An unemployed worker searches off-the-job with intensity λ^U and workers have a low safety $z = \underline{z}$ upon leaving unemployment. At rate κ_d they become long-term unemployed with $d = D$. The boundary conditions associated with (B.1) and (B.5)

[62] Workers never search in their own market in models where separation is exogenous, i.e., they keep searching along a monotonically increasing job ladder. In our model with endogenous firings, workers trade-off a higher wage w against a higher probability of firing $\mathcal{F}_t$. Depending on the parametrization, there might be a "preferred" market that becomes an absorbing state. However, a simple variational argument shows that no worker searches in this market directly. Instead, workers move closer and closer to this market as they progress along the job ladder.

are

$$W_t^E(J,\omega,z,s) = W_t^U(J,\omega,D) = 0 \tag{B.6}$$

for all workers (ω,z,s) when they retire at age $j = J$. It is worth noting that employed and unemployed workers typically direct their search towards different markets since they have different outside options.

Firm. The value of a firm determines the firing probability $\mathcal{F}_t(j,\omega,z,s)$ and the equilibrium tightnesses $\theta_t(j,\omega,z,s)$. The value of a firm, excluding vacancy-posting and current operating costs, satisfies

$$\begin{aligned} rV_t(j,\omega,z,s) = {} & y_t(a,\omega,s) - w(s)\psi(j,\omega,k(s)) + \chi_0\{\mathcal{O}_t(j,\omega,z,s) - V_t(j,\omega,z,s)\} \\ & - \left\{\lambda^E Q_t(j,\omega,z,s) + \mathbf{1}_{\{z=\underline{z}\}}\chi_1\right\} V_t(j,\omega,z,s) \\ & + \kappa\{V_t(j,\omega,\bar{z},s) - V_t(j,\omega,z,s)\} \\ & + \frac{d}{d\epsilon}V_{t+\epsilon}(j+\epsilon,\omega,z,s)\Big|_{\epsilon=0}, \end{aligned} \tag{B.7}$$

for all ages before retirement $j \in [0,J)$, skills ω, safeties z and markets $s \equiv (k,w)$, where $r \equiv \rho$ is the interest rate. The firm incurs a fixed cost of production with flow probability χ and continues operating when this cost is lower than the firm's continuation value of keeping the worker. The expected value of the resulting discrete choice is

$$\begin{aligned} \mathcal{O}_t(j,\omega,z,s) \equiv {} & \Phi\left(\frac{\log\left(V_t^{\text{keep}}(j,\omega,z,s)/\psi(j,\omega,k)\right) - \phi}{\gamma_\epsilon}\right) V_t^{\text{keep}}(j,\omega,z,s) \\ & - \psi(j,\omega,k)\exp\left(\phi + \frac{1}{2}\gamma_\epsilon^2\right)\Phi\left(\frac{\log\left(V_t^{\text{keep}}(j,\omega,z,s)/\psi(j,\omega,k)\right) - \phi - \gamma^2}{\gamma_\epsilon}\right) \end{aligned} \tag{B.8}$$

where $V_t^{\text{keep}}(j,\omega,z,s)$ is the value of keeping the worker (defined below) and Φ is the cumulative distribution function of a standard normal distribution and ϕ and γ_ϵ are the location and scale parameters of the distribution of operating costs. The firm continues operation and pays the operating cost whenever this cost is less than the value of the firm, i.e., $\exp(\phi+\gamma\epsilon) \leq V_t^{\text{keep}}(j,\omega,z,s)$ where $\epsilon \sim \Phi$. The first line in (B.8) corresponds to expected continuation value and the second

line corresponds to the expected operating cost paid by the firm when it continues operation.[63] Accordingly, the firm dissolves the match and fires the worker with probability

$$\mathcal{F}_t(j,\omega,z,s) = 1 - \Phi\left(\frac{\log\left(V_t^{\text{keep}}(j,\omega,z,s)\right) - \phi}{\gamma_\epsilon}\right) \tag{B.9}$$

The match may dissolve for two additional reasons. Workers search on-the-job at rate λ^E and separate from the firm when this search is successful with probability $Q(j,\omega,z,s)$. The match is also dissolved exogeneously at rate $\chi_1 > 0$ if workers' safety is low $z = \underline{z}$. Workers' safety state z switches to $\bar{z} \equiv 1$ with flow rate $\kappa > 0$ when they are employed. The boundary condition associated with (B.7) is

$$V_t(J,\omega,z,s) = 0 \tag{B.10}$$

for all workers (ω,z,s) when they retire at age $j = J$.

Free entry. The equilibrium tightnesses $\theta(j,\omega,z,s)$ are pinned down by the firms' free entry condition

$$q\left(\theta_t(j,\omega,z,s)\right) V_t(j,\omega,z,s) = \nu \tag{B.11}$$

B.2 Solving for the Workers' and Firms' Values

Discretized system. We discretize the differential equations of Appendix B.1 using a finite difference scheme that is implicit in the safety z and employment status s and explicit in age j and time t.[64]

After discretization, the value of being employed becomes

[63] Formally, the second line is equal to $\int_{-\infty}^{\bar{\epsilon}} \exp(\gamma\epsilon + \phi) \exp\left(-\frac{\epsilon^2}{2}\right) / \sqrt{2\pi} d\epsilon$ with $\bar{\epsilon} \equiv (\log(V_t(j,\omega,s)) - \phi)/\gamma$.

[64] Workers' skill ω is permanent so workers' and firms' values can be solved separately for different ω.

$$
\begin{aligned}
\rho W_t^E(j,\omega,z,s) = {} & u\left(w(s)\,\psi(j,\omega,k(s))\right) \\
& + \left\{\chi_0 \mathcal{F}_{t+\Delta}(j+\Delta,\omega,z,s) + \mathbf{1}_{\{z=\underline{z}\}}\chi_1\right\} \\
& \qquad \times \left\{W_{t+\Delta}^U(j+\Delta,\omega,0) - W_{t+\Delta}^E(j+\Delta,\omega,z,s)\right\} \\
& + \lambda^E \gamma_\zeta \exp\left(\sum_{s'\neq U} f\left(\theta_{t+\Delta}(j+\Delta,\omega,z,s')\right)\right. \\
& \qquad \left. \times \left\{W_{t+\Delta}^E(j+\Delta,\omega,z,s') - W_{t+\Delta}^E(j+\Delta,\omega,z,s)\right\}/\gamma_\zeta\right) \\
& + \kappa\left\{W_{t+\Delta}^E(j+\Delta,\omega,\bar{z},s) - W_{t+\Delta}^E(j+\Delta,\omega,z,s)\right\} \\
& + \frac{W_{t+\Delta}^E(j+\Delta,\omega,z,s) - W_t^E(j,\omega,z,s)}{\Delta}
\end{aligned}
\tag{B.12}
$$

for all markets $s \equiv (k,w)$ excluding unemployment, where $\Delta > 0$ denotes the time step. The terms $p_{t+\Delta}^E(s';j+\Delta,\omega,z,s)$ correspond the probability that a worker of age $j+\Delta$, skill ω, productivity z and that is employed in market s searches in the market s' in period $t+\Delta$, while $\mathcal{F}_{t+\Delta}$ captures the layoff decisions (B.9).[65] These terms are inheritently non-linear in workers' and firms' values. For this reason, these terms are evaluated using the values in the *following* period (as usual) so the remaining system is linear in the values in period t.

In turn, the value of being unemployed is

$$
\begin{aligned}
\rho W_t^U(j,\omega,D) = {} & u\left(b_D \psi^{\text{age}}(j)\right) \\
& + \lambda^U \gamma_\zeta \exp\left(\sum_{s'\neq U} f\left(\theta_{t+\Delta}(j+\Delta,\omega,\underline{z},s')\right) \times\right. \\
& \qquad \left. \times \left\{W_{t+\Delta}^E(j+\Delta,\omega,\underline{z},s') - W_{t+\Delta}^U(j+\Delta,\omega,1)\right\}/\gamma_\zeta\right) \\
& + \kappa_D\left\{W_{t+\Delta}^U(j+\Delta,\omega,1) - W_{t+\Delta}^U(j+\Delta,\omega,0)\right\} \\
& + \frac{W_{t+\Delta}^U(j+\Delta,\omega) - W_t^U(j,\omega)}{\Delta}
\end{aligned}
\tag{B.13}
$$

where $p_{t+\Delta}^U(s';j+\Delta,\omega)$ is the probability that a worker of age $j+\Delta$, skill ω and

[65] Note that employed workers can only search in markets corresponding to their current level of safety z. In turn, unemployed workers can only search in markets corresponding to the lower safety $\underline{z}$. We set the probabilities of searching in other markets to zero.

that is unemployed searches in the market s' in period $t+\Delta$, defined analogously to (B.3). The boundary conditions associated to (B.12) and (B.13) are still given by (B.6).

Finally, the value of a firm becomes

$$\begin{aligned}\rho V_t(j,\omega,z,s) = {} & y_t(j,\omega,s) - w(s)\psi(j,\omega,k(s)) + \chi_0\{\mathcal{O}_{t+\Delta}(j,\omega,z,s) - V_t(j,\omega,z,s)\} \\ & - \left(\lambda^E Q_{t+\Delta}(j+\Delta,\omega,z,s) + \mathbf{1}_{\{z=\underline{z}\}}\chi_1\right) V_{t+\Delta}(j+\Delta,\omega,z,s) \\ & + \kappa\{V_{t+\Delta}(j+\Delta,\omega,\bar{z},s) - V_{t+\Delta}(j+\Delta,\omega,z,s)\} \\ & + \frac{V_{t+\Delta}(j+\Delta,\omega,z,s) - V_t(j,\omega,z,s)}{\Delta}\end{aligned} \tag{B.14}$$

with $\mathcal{O}_{t+\Delta}(j+\Delta,\omega,z,s)$ still defined by (B.8).

Workers' and firms' values are characterized by the system (B.12)–(B.14) together with the definition of $\mathcal{O}_t$ (B.8), the definition of $\mathcal{F}_t$ (B.9), and the free entry condition (B.11) that pins down θ_t. Note that this system is independent of the distribution of workers' states — this is the block-recursive property of directed search model (Menzio and Shi, 2011).

Numerical solution. We explain below how we solve for workers' and firms' values.

Algorithm 1. Solve for workers' and firms' values separately for each value of ω.

Step 0. Let the terminal values for workers $W_T^E(J,\omega,z,s)$, $W_T^U(J,\omega,D)$ and firms $V_T(j,\omega,z,s)$ satisfy the boundary conditions (B.6) and (B.10) at retirement at age J. Similarly, fix the the search decisions $p_T^E(s';J,\omega,z,s) \equiv 0$ and $p_T^U(s';J,\omega) \equiv 0$ for all markets s'. Let the value $\mathcal{O}_T(J,\omega,z,s) \equiv 0$ and the hazard $\mathcal{F}_T(J,\omega,z,s) \equiv 1$. Initiate the recursion by setting the time index to $t = T - \Delta$ and the age index to $j = J - \Delta$.

Step 1. Solve for workers' values $W_t^E(j,\omega,z,s)$ and $W_t^U(j,\omega,D)$:

Solve for the values

$$\mathbf{W}_t(j,\omega) = \left(\rho + \frac{1}{\Delta}\right)^{-1}\{\mathbf{B}_t(j,\omega) - \mathbf{M}_{t+\Delta}(j+\Delta,\omega)\mathbf{W}_{t+\Delta}(j,\omega)\}, \tag{B.15}$$

where the column vector

$$\mathbf{W}_t(j,\omega)_{[(z,s),1]} \equiv W_t^E(j,\omega,z,s) \qquad \mathbf{W}_t(j,\omega)_{[\text{end},1]} \equiv W_t^U(j,\omega) \tag{B.16}$$

with the index (z,s) denoting vectors over markets s stacked by productivities z and

$$\mathbf{M}_t(j,\omega) \equiv \begin{bmatrix} \mathbf{M}_{0,t}(j,\omega) + \mathbf{M}_{1,t}(j,\omega) & \mathbf{M}_{2,t}(j,\omega) \\ \mathbf{M}_{3,t}(j,\omega) & \mathrm{M}_{4,t}(j,\omega) \end{bmatrix} \quad \mathbf{B}_t(j,\omega) \equiv \begin{bmatrix} \mathbf{B}_{1,t}(j,\omega) \\ B_{2,t}(j,\omega) \end{bmatrix} \tag{B.17}$$

with[66]

$$\mathbf{M}_{0,t}(j,\omega) \equiv \mathbf{M}_{0,t}^A(j,\omega) + \mathbf{M}_{0,t}^B(j,\omega), \tag{B.18}$$

with the diagonal matrix[67]

$$\mathbf{M}_{0,t}^A(j,\omega)_{[(z,s),(z',s')]} \equiv \begin{cases} \chi_0 \mathcal{F}_{t+\Delta}(j+\Delta,\omega,z,s) + \mathbf{1}_{\{z=\underline{z}\}}(\chi_1+\kappa) & \text{if } (z',s') = (z,s) \\ 0 & \text{otherwise} \end{cases} \tag{B.19}$$

for markets s and s' excluding unemployment, the square matrix

$$\mathbf{M}_{0,t}^B(j,\omega)_{[(z,s),(z',s')]} \equiv \begin{cases} -\kappa & \text{if } z = \underline{z}, z' = \bar{z}, s' = s \\ 0 & \text{otherwise} \end{cases} \tag{B.20}$$

the square matrix

$$\mathbf{M}_{1,t}(j,\omega)_{[(z,s),(z,s')]} \equiv 0, \tag{B.21}$$

the column vector

$$\mathbf{M}_{2,t}(j,\omega)_{[(z,s),1]} \equiv -\left\{\chi_0 \mathcal{F}_{t+\Delta}(j+\Delta,\omega,z,s) + \mathbf{1}_{\{z=\underline{z}\}}\chi_1\right\} \tag{B.22}$$

[66] We use the same subscript for $\mathbf{M}_{0,t}^A(j,\omega)$ and $\mathbf{M}_{0,t}^B(j,\omega)$ since they both apply to values at the *same* job. Distinguishing between flows that occur within or across jobs is important in our calibration (Appendix B.6).

[67] Note that imposing $z = \underline{z}$ is not required for the flow κ. However, it allows us to ensure that $\mathbf{M}_{0,t}^A(j,\omega)$ and $\mathbf{M}_{0,t}^B(j,\omega)$ below actually capture outflows and inflows relative the current state, which will turn out to be convenient when iterating on the distribution (Appendix B.3). Similarly, introducing the matrix $\mathbf{M}_{1,t}(j,\omega)$ below will be useful when iterating on the distribution.

the row vector

$$\mathbf{M}_{3,t}\left(j,\omega\right)_{[1,(z,s')]} \equiv 0, \tag{B.23}$$

the scalar

$$\mathrm{M}_{4,t}\left(j,\omega\right) \equiv 0, \tag{B.24}$$

the column vector

$$\begin{aligned}\mathbf{B}_{1,t}\left(j,\omega\right)_{[(z,s),1]} \equiv & u\left(w\left(s\right)\psi\left(j,\omega,k\left(s\right)\right)\right) \\ & + \lambda^{E}\gamma_{\zeta}\exp\left(\sum_{s'\neq U} f\left(\theta_{t+\Delta}\left(j+\Delta,\omega,z,s'\right)\right)\right. \\ & \qquad \left.\times\left\{W^{E}_{t+\Delta}\left(j+\Delta,\omega,z,s'\right)-W^{E}_{t+\Delta}\left(j+\Delta,\omega,z,s\right)\right\}/\gamma_{\zeta}\right) \\ & + \frac{W^{E}_{t+\Delta}\left(j+\Delta,\omega,z,s\right)}{\Delta}\end{aligned} \tag{B.25}$$

and the scalar

$$\begin{aligned}B_{2,t}\left(j,\omega\right) \equiv & u\left(b_{D}\psi^{\mathrm{age}}\left(j\right)\right) \\ & + \lambda^{U}\gamma_{\zeta}\exp\left(\sum_{s'\neq U} f\left(\theta_{t+\Delta}\left(j+\Delta,\omega,\underline{z},s'\right)\right)\times\right. \\ & \qquad \left.\times\left\{W^{E}_{t+\Delta}\left(j+\Delta,\omega,\underline{z},s'\right)-W^{U}_{t+\Delta}\left(j+\Delta,\omega,1\right)\right\}/\gamma_{\zeta}\right) \\ & + \frac{W^{U}_{t+\Delta}\left(j+\Delta,\omega\right)}{\Delta}\end{aligned} \tag{B.26}$$

Step 2. Solve for the firms' values $V_t\left(j,\omega,z,s\right)$:

Solve for the values

$$\mathbf{V}_{t}\left(j,\omega\right)=\left(\rho+\frac{1}{\Delta}\right)^{-1}\left\{\mathbf{C}_{t}\left(j,\omega\right)-\mathbf{P}_{t+\Delta}\left(j+\Delta,\omega\right)\mathbf{V}_{t+\Delta}\left(j+\Delta,\omega\right)\right\} \tag{B.27}$$

where the column vector

$$\mathbf{V}_{t}\left(j,\omega\right)_{[(z,s),1]} \equiv V_{t}\left(j,\omega,z,s\right) \tag{B.28}$$

with the index (z, s) denoting vectors over markets s stacked by productivities z and

$$\mathbf{P}_t(j,\omega) \equiv \mathbf{P}_t^A(j,\omega) + \mathbf{P}_t^B(j,\omega), \tag{B.29}$$

with the diagonal matrix

$$\mathbf{P}_t^A(j,\omega)_{[(z,s),(z',s')]} \equiv \begin{cases} \lambda^E Q_{t+\Delta}(j+\Delta,\omega,z,s) & \\ +\chi_0 + \mathbf{1}_{\{z=\underline{z}\}}(\chi_1+\kappa) & \text{if } (z',s') = (z,s), \\ 0 & \text{otherwise} \end{cases} \tag{B.30}$$

the square matrix

$$\mathbf{P}_t^B(j,\omega)_{[(z,s),(z',s')]} \equiv \begin{cases} -\kappa & \text{if } z = \underline{z}, z' = \bar{z}, s' = s \\ 0 & \text{otherwise} \end{cases}, \tag{B.31}$$

and the column vector

$$\begin{aligned} \mathbf{C}_t(j,\omega)_{[z,s,1]} = {} & y_t(j,\omega,s) - w(s)\psi(j,\omega,k(s)) + \chi_0 \mathcal{O}_{t+\Delta}(j+\Delta,\omega,z,s) \\ & + \frac{V_{t+\Delta}(j+\Delta,\omega,z,s)}{\Delta} \end{aligned} \tag{B.32}$$

Step 3. Compute the probabilities $p_t^E(s';j,\omega,z,s)$ and $p_t^U(s';j,\omega)$ using (B.3). Compute the value $\mathcal{O}_t(j,\omega,z,s)$ using (B.8), the hazard $\mathcal{F}_t(j,\omega,z,s)$ using (B.9), and the tightnesses $\theta_t(j,\omega,z,s)$ using (B.11).

Step 4. Set the age $j' = j - \Delta$, repeat Steps 1–3, and continue until $j = 0$.

Step 5. When solving for transitions, set $t' = t - \Delta$, repeat Steps 1–4 in period t', and continue until $t = 0$. This last step is redundant when solving for the steady state.

B.3 Distribution of Workers' States and Free Entry

We denote the density of wokers' states by $\mu_t(\mathbf{x})$. This density is a continuous function of age j and skill ω, and a discrete function of safety z and the sector k. While the wage w is a continuous variable, only a subset of markets are active in equilibrium. Therefore, the distribution admits mass points along this continuous

dimension. Accordingly, we define an integral of the form $\int f(s, \mathbf{x}_{-1}) \mu_t(ds, \mathbf{x}_{-1})$ as a discrete sum over markets $s \equiv (k, w)$ with positive mass, where $\mathbf{x}_{-1}$ denotes the other states.

As usual, the law of motion of the distribution and the recursive formulation of workers' values are closely related. In fact, the matrices $\mathbf{M}_t(j, \omega)$ (Appendix B.2) encodes the information required to iterate on the distribution (Achdou et al., 2022). This is intuitive: the matrices $\mathbf{M}_t(j, \omega)$ capture the outflows from the current state and the inflows into subsequent ones. The only flows that are not captured by $\mathbf{M}_t(j, \omega)$ are those associated to the workers' job search, which are directly captured by the terms (B.25)–(B.26). To ensure that the transition matrix used to iterate for the distribution accounts for these flows, we define a new matrix $\widetilde{\mathbf{M}}_t(j, \omega)$ similarly to $\mathbf{M}_t(j, \omega)$ in Appendix B.2, but with the matrices

$$\mathbf{M}^A_{0,t}(j, \omega)_{[(z,s),(z',s')]} \equiv \begin{cases} \lambda^E Q_{t+\Delta}(j+\Delta, \omega, z, s) \\ \quad + \chi_0 \mathcal{F}_{t+\Delta}(j+\Delta, \omega, z, s) + \mathbf{1}_{\{z=\underline{z}\}}(\chi_1 + \kappa) & \text{if } (z', s') = (z, s) \\ 0 & \text{otherwise} \end{cases} \tag{B.33}$$

with Q_t defined by (B.4),

$$\widetilde{\mathbf{M}}_{1,t}(j, \omega)_{[(z,s),(z,s')]} \equiv -\lambda^E p^E_{t+\Delta}(s'; j+\Delta, \omega, z, s) f\left(\theta_{t+\Delta}(j+\Delta, \omega, z, s')\right), \tag{B.34}$$

and

$$\widetilde{\mathbf{M}}_{3,t}(j, \omega)_{[1,(z,s')]} \equiv -\lambda^U p^U_{t+\Delta}(s'; j+\Delta, \omega) f\left(\theta_{t+\Delta}(j+\Delta, \omega, \underline{z}, s')\right), \tag{B.35}$$

and the scalar

$$\widetilde{\mathrm{M}}_{4,t}(j, \omega) \equiv \lambda^U \sum_{s' \neq U} p^U_{t+\Delta}(s'; j+\Delta, \omega) f\left(\theta_{t+\Delta}(j+\Delta, \omega, \underline{z}, s')\right), \tag{B.36}$$

One can easily verify that the rows of $\widetilde{\mathbf{M}}_t(j, \omega)$ sum up to zero.

In the limit $\Delta \to 0$, the law of motion of the workers' distribution is

$$\frac{\boldsymbol{\mu}_{t+\Delta}(j, \omega) - \boldsymbol{\mu}_t(j-\Delta, \omega)}{\Delta} = -\widetilde{\mathbf{M}}_t(j, \omega)^T \boldsymbol{\mu}_t(j, \omega), \tag{B.37}$$

for all ages $j \in [0, J-\Delta)$, where

$$\boldsymbol{\mu}_t(j,\omega)_{[(z,s),1]} \equiv \mu_t^E(j,\omega,z,s) \qquad \boldsymbol{\mu}_t(j,\omega)_{[\text{end},1]} \equiv \mu_t^U(j,\omega) \tag{B.38}$$

with the index (z,s) denoting vectors over markets s stacked by safety states z. Equivalently,

$$\boldsymbol{\mu}_{t+\Delta}(j,\omega) = \left\{\mathbb{I} - \widetilde{\mathbf{M}}_t(j,\omega)^T \Delta\right\} \boldsymbol{\mu}_t(j-\Delta,\omega) \tag{B.39}$$

Finally, the boundary conditions associated with the law of motion (B.39) are

$$\mu_t^E(0,\omega,z,s) = 0 \quad \text{and} \quad \mu_t^U(0,\omega) = \begin{cases} \mu_t^R & \text{if } \omega = 0 \\ 0 & \text{otherwise} \end{cases} \tag{B.40}$$

for all (ω,z,s) at age $j = 0$, as new generations are born with $\omega = 0$ and are unemployed $s = U$ initially. The mass of newborns is

$$\mu_t^R = \int\int\int \mu_{t-\Delta}^E(dJ, d\omega, dz, ds) + \int \mu_{t-\Delta}^U(J, d\omega)\, \mathrm{d}\omega \tag{B.41}$$

Stationary distribution. The economy starts from a stationary distribution where all workers are active in the "old" sector, i.e., $A^{\text{old}} \equiv 1$ and $A^{\text{new}} \equiv 0$. We let μ^E and μ^U denote the stationary distribution in this case. We normalize this distribution so it has a unit mass.

Note that (B.40) assumes that $\omega = 0$ for *all* new generations so the ergodic distribution described above in fact has a mass point at $\omega = 0$. While this is true for those born in $t \geq 0$ (after the shock), this is not the case for those born in $t < 0$ (before the shock) as they have not acquired the right skills before the aggregate shock. Instead of introducing separate laws of motions for the distribution before and after $t = 0$, we simply initialize the distribution in $t = 0$ with

$$\int_\Omega \mu^{E,\prime}(j, d\omega, z, s) = \mu^E(j,\omega_0,z,s)\Big|_{\omega_0=0} \times \int_\Omega \hat{\mu}(j, d\omega) \tag{B.42}$$

and

$$\int_\Omega \mu^{U,\prime}(j, d\omega) = \mu^U(j,\omega_0)\Big|_{\omega_0=0} \times \int_\Omega \hat{\mu}(j, d\omega)\,, \tag{B.43}$$

for all skill sets Ω, where $\hat{\mu}(j,\omega)$ is the same distribution of skill conditional on age that we use in the Walrasian model with ex ante heterogeneity only (Section 3.1).

B.4 Vacancy Posting and Firm Entry

The flow of vacancies created in a given market $N_t(j,\omega,z,s)$ satisfies the market clearing condition

$$q(\theta_t(j,\omega,z,s))\, N_t(j,\omega,z,s) \tag{B.44}$$

$$= \left[-\widetilde{\mathbf{M}}_t^{\text{EEUE}}(j,\omega)^T \mu_t(j,\omega)\right]_{(z,s)}, \tag{B.45}$$

which stipulates that the number of vacancies posted $N_t(j,\omega,z,s)$ and subsequently filled with probability $q(\theta_t(j,\omega,z,s))$ is equal to the number of succesful job searches $-\widetilde{\mathbf{M}}_t^{\text{EEUE}}(j,\omega)^T \mu_t(j,\omega)$.[68] The matrix $\widetilde{\mathbf{M}}_t^{\text{EEUE}}(j,\omega)$ is defined similarly to $\mathbf{M}_t(j,\omega)$ in Appendix B.3 except that the *inflows* corresponding to EE or UE transitions are set to zero.[69]

B.5 Social Welfare

Workers. Workers' social welfare is given by

$$\begin{aligned}\text{Welf}^{\text{work}}(\eta) \equiv &\int_{-J}^{0}\int\int\int \Lambda^E(\tau,\omega,z,s)\exp(\rho\tau)\, W_0^E(-\tau,\omega,z,s)\,\mu^E(-\tau,\omega,z,s)\,\mathrm{d}s dz \mathrm{d}\omega d\tau \\ &+\int_{-J}^{0}\int\int \Lambda^U(\tau,\omega)\exp(\rho\tau)\, W_0^U(-\tau,\omega,D)\,\mu^U(-\tau,\omega)\,\mathrm{d}\mathrm{D}\mathrm{d}\omega d\tau, \\ &+1/J\times\int_0^{+\infty}\Lambda^{\text{new}}(\tau)\, W_\tau^U(0,\omega,0)\Big|_{\omega=0} d\tau,\end{aligned} \tag{B.46}$$

where $1/J$ is the rate at which new generations are born, and λ^{E}, λ^{U} and λ^{new} are the Pareto weights on existing workers that either employed or unemployed in $t=0$ and those that are not yet born. The values for existing and new workers are given by (B.1) and (B.5). All values are evaluated at birth.[70] New workers are born with $\omega=0$ and are unemployed initially. These values depend implicitly on the transition speed η, which we omit for notation. The stationary distributions μ^E

[68] The flow of vacancies created can be equivalently backed out from the definition of the market tightnesses, using the fact that $q(\theta_t(j,\omega,z,s)) \equiv \theta_t(j,\omega,z,s)\, f(\theta_t(j,\omega,z,s))$.

[69] Therefore, $\mathbb{I} - \widetilde{\mathbf{M}}_t^{\text{EEUE}}(j,\omega)^T \Delta$ captures the outflow of workers who succesfully search for a job. See Appendix B.6 for additional details.

[70] This explains why the values are discounted for existing generations but not new generations in social welfare (B.46). The same comment applies to expression (B.47) below.

and μ^U are described in Appendix B.3.

Entrepreneurs. Entrepreneur's social welfare is given by

$$\text{Welf}^{\text{entrepr}}(\eta) \equiv \int_{-J}^{0} \lambda^{\text{invest}}(\tau) \exp(\rho\tau) W_\tau^{\text{entrepr}} d\tau \tag{B.47}$$

where λ^{invest} is the investors' Pareto weights. Entrepreneurs born in period t claim the profits of the matches created in that period, net of vacancy posting costs. the *continuation* welfare of existing generations $\tau < 0$ of entrepreneurs is

$$W_\tau^{\text{entrepr}} = \int\int\int\int \mathcal{M}(j,\omega,z,s;\tau) V_0(j,\omega,z,s) \, dsdzd\omega dj \tag{B.48}$$

where $\mathcal{M}(j,\omega,z,s;\tau)$ is the distribution of matches created in $t = \tau < 0$ that survived until period $t = 0$.[71] In turn, the welfare of new generations $\tau \geq 0$ of entrepreneurs is 0 by the free entry condition.

Pareto weights. We assume that the welfare of different generations is discounted using the subjective discount rate ρ. This ensures that social preferences are time consistent (Calvo and Obstfeld, 1988). We focus on the case where the social welfare function is utilitarian. That is, $\Lambda^E(\tau,\omega,s) = \Lambda^U(\tau,\omega) = \Lambda^{\text{new}}(\tau) = \Lambda^{\text{invest}}(\tau) = \exp(-\rho\tau)$ for all agents.

Social welfare. Social welfare is the sum of workers' and investors' welfare

$$\text{Welf}^{\text{social}}(\eta) = \text{Welf}^{\text{work}}(\eta) + \text{Welf}^{\text{entrepr}}(\eta) \tag{B.49}$$

We express changes in workers' welfare $\mathcal{W}^{\text{work}}(\eta)$ or social welfare $\mathcal{W}^{\text{social}}(\eta)$ in

[71] The initial distribution is $\mathcal{M}(j,\omega,z,s;0) \equiv q(\theta_t(j,\omega,z,s)) N_t(j,\omega,z,s)$. The subsequent distributions $\mathcal{M}(j,\omega,z,s;\tau)$ for $\tau < 0$ are obtained by iterating on that initial distribution over τ periods, setting the inflows in the law of motion (B.39) to zero for EE and EU transitions (see Appendix B.6 for more details). Finally, note that the continuation welfare of existing entrepreneurs is typically positive since the vacancy posting cost ν is sunk for them.

consumption equivalent terms. For instance,

$$\mathcal{W}^{\text{work}}(\eta) \equiv \frac{u^{-1}\left(\text{Welf}^{\text{work}}(\eta)\right)}{u^{-1}\left(\text{Welf}^{\text{work}}(0)\right)} - 1 \tag{B.50}$$

for workers.

B.6 Calibration

We discretize our continuous time model into short periods corresponding to 2 weeks. This allows us to capture high-frequency movements in and out of employment. We calibrate our benchmark model by targeting six moments: (i) the EE probability, (ii) the EU probability, (iii) the UE probability, (iv) the elasticity of the separation rate over one year w.r.t. a permanent change in a firm's output, (v) the excess separation rate during year 1 after a prior separation and (vi) the excess separation rate during year 5 after a prior separation. We first explain how we compute the relevant employment flows. We then explain how we compute the last three moments based on these measures.

Employment flows. We compute the EE, EU and UE probabilities over one quarter. We compute separation probabilities over one year ($T \equiv 1$) to be consistent with the empirical targets that we match. We focus below on the separation probabilities since their computation is more involved. We assume below that a "separation" occurs when a worker experiences at least one EU transition during a year.[72] We discuss later how to adjust our approach when separations include EE transitions as well.

We first explain how we compute the steady-state separation probability. We focus on workers who are sufficiently young that they do not retire over the following year, i.e., $j \in J^y \equiv \{j' | j' \leq J - T\}$. Let $\mu^\star$ be some distribution of workers' states conditional on being employed. The probability that a worker does *not* experience any separation over the next year can be computed by adjusting the law

[72] An alternative way to measure EU flows is to count only workers who were employed initially and are unemployed a year later. The two measures can differ if a worker looses their job and finds a new one in the same year. The measure that we use is consistent with our empirical targets.

of motion (B.39). Specifically, let $\widetilde{\mathbf{M}}^0(j,\omega)$ be defined as the steady state transition matrix $\mathbf{M}(j,\omega)$ (Appendix B.3) but set the *inflows* to zero for the events where the workers becomes voluntarily unemployed, is fired from the firm or experiences an exogeneous separation. This correction allows us to keep track exclusively of "surviving" workers who do not experience a separation. We then use the law of motion

$$\tilde{\boldsymbol{\mu}}_{t+\Delta}(j,\omega) = \left\{\mathbb{I} - \widetilde{\mathbf{M}}^0(j,\omega)^T \Delta\right\} \tilde{\boldsymbol{\mu}}_t(j-\Delta,\omega) \tag{B.51}$$

for all ages j, skills ω and periods $t = 0, \Delta, \ldots, T-\Delta$ with initial condition $\tilde{\boldsymbol{\mu}}_0(j-\Delta,\omega) = \mu^\star$, where the vector $\tilde{\boldsymbol{\mu}}_t$ is defined as in expression (B.38). The probability of experiencing a separation over a time interval T is then given by

$$\begin{aligned}\text{Separation}(\mu^\star) \equiv 1 &- \int\int\int\int \tilde{\mu}_T^E(j,\omega,z,s)\, ds\mathrm{d}zd\omega dj \\ &- \int\int \tilde{\mu}_T^U(j,\omega)\, d\omega dj,\end{aligned} \tag{B.52}$$

When defining a separation as any transition away from the current job (i.e., EU or EE transitions), we also set the off-diagonal elements to zero for the event where the workers moves between jobs. The EE, EU and UE probabilities over one month are computed anagolously by setting $\mu^\star$ to be the steady state distribution conditional on employment or unemployment, choosing the appropriate matrices $\widetilde{\mathbf{M}}^0(j,\omega)$ and setting $T \equiv \Delta$ which corresponds to one month.

Elasticity of separation rate w.r.t. output change. We compute the elasticity of the separation rate following a permanent and unanticipated decrease $\alpha \in (0,1)$ in a firm's output. We compute this elasticity over the year following the shock to be consistent with the empirical target that we match. This is a partial equilibrium exercise that only affects one firm and its worker. All aggregate quantities remain unchanged and so are workers' outside options. We then compute the average response across such firms and workers subject to this change.

We start by computing alternative values for the workers $\widehat{W}_t(j,\omega,z,s)$ and firms $\widehat{V}_t(j,\omega,z,s)$ that are subject to this shock. These new values satisfy recursions similar to (B.1)–(B.5) and (B.7)–(B.9) with a few differences. First, output is now $(1-\alpha)\, y_t(\omega,s)$. Second, the *continuation* values $W_t^E(j,\omega,z,s')$ and $W_t^U(j,\omega)$ are treated as fixed in (B.12) and (B.13) and evaluated at steady state since the outside

options are unchanged.[73] The worker's job finding probabilities $f\left(\theta_t\left(j,\omega,z,s\right)\right)$ are also unchanged. In constrast, the labor supply decisions of employed workers $q_t^w\left(j,\omega,z,s\right)$ and $p_t^E\left(s';j,\omega,z,s\right)$ and firms' firing decisions $q_t^f\left(j,\omega,z,s\right)$ and $\mathcal{F}_t\left(j,\omega,z,s\right)$ are endogenous.

Algorithm 2. Solve for workers' and firms' values after the shock.

Steps 0 and 1. These steps are the same as in Algorithm 1 in Appendix B.2.

Step 2. Solve for workers' values $\widehat{W}_t^E\left(j,\omega,z,s\right)$:

for each age $j = J_w, J_w - \Delta, \ldots, 0$

↳ Solve for the values

$$\widehat{\mathbf{W}}_t\left(j,\omega\right) = \left[\left(\rho + \frac{1}{\Delta}\right)\mathbb{I} + \widehat{\mathbf{M}}_t\left(j,\omega\right)\right]^{-1}\widehat{\mathbf{B}}_t\left(j,\omega\right), \tag{B.53}$$

where $\widehat{\mathbf{W}}_t\left(j,\omega\right)$, $\widehat{\mathbf{M}}_t\left(j,\omega\right)$ and $\widehat{\mathbf{B}}_t\left(j,\omega\right)$ take a form similar to (B.16)–(B.18). The matrices $\widehat{\mathbf{M}}_{0,t}^A\left(j,\omega\right)$, $\widehat{\mathbf{M}}_{0,t}^B\left(j,\omega\right)$ and $\widehat{\mathbf{M}}_{2,t}\left(j,\omega\right)$ are defined as (B.19)–(B.20) and (B.22) and are evaluated at the equilibrium job finding rates $f\left(\theta_t\left(\cdot\right)\right)$ but the new policies q_t^w, q_t^f, p_t^E and $\mathcal{F}_t$ relevant for a workers and firm affected by the shock. The other matrices are $\widehat{\mathbf{M}}_{1,t}\left(j,\omega\right) = \widehat{\mathbf{M}}_{2,t}\left(j,\omega\right) = \widehat{\mathbf{M}}_{3,t}\left(j,\omega\right) = \widehat{\mathrm{M}}_{4,t}\left(j,\omega\right) = \mathbf{0}$ since the outside options and the value of unemployment are treated as exogenous in this exercise. These continuation values are now included in

$$\begin{aligned}\widehat{\mathbf{B}}_{1,t}\left(j,\omega\right)_{[(z,s),1]} \equiv & u\left(w\left(s\right)\psi\left(j,\omega,k\left(s\right)\right)\right) \\ & + \lambda^E\gamma_\zeta \exp\left(\sum_{s'\neq U} p_{t+\Delta}^E\left(s';j+\Delta,\omega,z,s\right) f\left(\theta_{t+\Delta}\left(j+\Delta,\omega,z,s'\right)\right)\right. \\ & \left.\times\left\{W_{t+\Delta}^E\left(j+\Delta,\omega,z,s'\right) - \widehat{W}_{t+\Delta}^E\left(j+\Delta,\omega,z,s\right)\right\}/\gamma_\zeta\right) \\ & + \frac{\widehat{W}_{t+\Delta}^E\left(j+\Delta,\omega,z,s\right)}{\Delta}\end{aligned} \tag{B.54}$$

where $\mathbf{W}_t\left(j,\omega\right)$ are the equilibrium outside values (i.e., without the shock)

[73] The continuation values $W_t^E\left(j,\omega,\bar{z},s\right)$ are treated as endogenous in (B.1), however, since they capture improvement in the safety state conditional on remaining employed in the same job.

and $f(\theta_t(\cdot))$ are the equilibrium job finding rates. Finally, the value of unemployment is unchanged so we impose the identity

$$\widehat{B}_{2,t}(j,\omega) \equiv \left(\rho + \frac{1}{\Delta}\right) W_t^U(j,\omega) \tag{B.55}$$

The terminal values (B.6) and (B.10) are also unchanged.

end

Step 3. Solve for firms' values $\widehat{V}_t(j,\omega,z,s)$:

for each age $j = J_w, J_w - \Delta, \ldots, 0$

↳ Solve the firms' values $\widehat{\mathbf{V}}_t(j,\omega)$ which still satisfy (B.27)–(B.32) excep that output is now given by $(1-\alpha)\, y_t(\omega,s)$.

end

Step 4. This step is the same as in Algorithm 1 in Appendix B.2.

We then compute the matrices $\widetilde{\mathbf{M}}_t(j,\omega)$ (Appendix B.3) and $\widetilde{\mathbf{M}}_t^0(j,\omega)$ (see above) using the new probabilities q_t^w, q_t^f and p_t^E and the new hazard $\mathcal{F}_t$ for the workers and firms affected by the shock. We then use $\widetilde{\mathbf{M}}_t^0(j,\omega)$ to compute the separation probability (B.52) over one year for the workers and firms affected by the shock. When computing this moment, we focus on any transition away from the current job (i.e., EU or EE transitions) to be consistent with the literature and we set $\mu_t^\star(j,\omega,z,s)$ to be equal to the stationary distribution conditional on being employed. We consider a permanent productivity change of $\alpha = 0.1$. We then compare the new separation probability to the one that prevails in steady state. We express this change as an elasticity in terms of $1-\alpha$.

Excess separation probability after a prior separation. We compute the probability of a subsequent separation at various horizons for the average worker who becomes unemployed in a given period. We compute the same probability for the average worker who remains employed in the same period. The excess separation probability at a given horizon is the difference between the two. We assume the following timing which is consistent with our empirical target (Jarosch (2023)). A worker experiences a separation in year $= 0$ if that separation occurred in $t = -3\Delta, -2\Delta, -\Delta, 0$. We then measure the separation rates over year $= 1, 2, \ldots$

after that.

Algorithm 3. Compute the excess separation probability.

Step 1. Initialize $\boldsymbol{\mu}_{-\Delta}(j,\omega)$ with the stationary distribution *conditional* on employment. Iterate forward on the stationary distribution for one period using the law of motion (B.39) to obtain an initial distribution $\boldsymbol{\mu}_0(j,\omega)$.[74] Let $\boldsymbol{\mu}_0^{\text{empl}}(j,\omega)$ and $\boldsymbol{\mu}_0^{\text{unempl}}(j,\omega)$ denote the same distribution conditional on being employed or unemployed, respectively.

Step 2. Construct the separation probabilities:

for each group $f = \text{empl}, \text{unempl}$

1. **for each** $t = 0, \Delta, 2\Delta, \ldots$
    1. Compute Separation_t^f over one year using (B.52) evaluated at $\mu_t^\star(j,\omega) \equiv \mu_t^f(j,\omega)$.
    2. Iterate forward on the distribution to obtain $\boldsymbol{\mu}_{t+\Delta}^f(j,\omega)$.

    end
2. Average out Separation_t^f by year, i.e., over intervals $\text{year} + [0, \Delta, \ldots, 1-\Delta]$ for each $\text{year} = 1, 2, \ldots$ since we assume that workers experience their original separation uniformly during year $= 0$. Let $\text{Separation}_{\text{year}}^f(T)$ denote this yearly separation rate.

end

Step 3. Construct the excess separation probabilities as

$$\text{Excess}_{\text{year}} \equiv \text{Separation}_{\text{year}}^{\text{unempl}}(T) - \text{Separation}_{\text{year}}^{\text{empl}}(T) \tag{B.56}$$

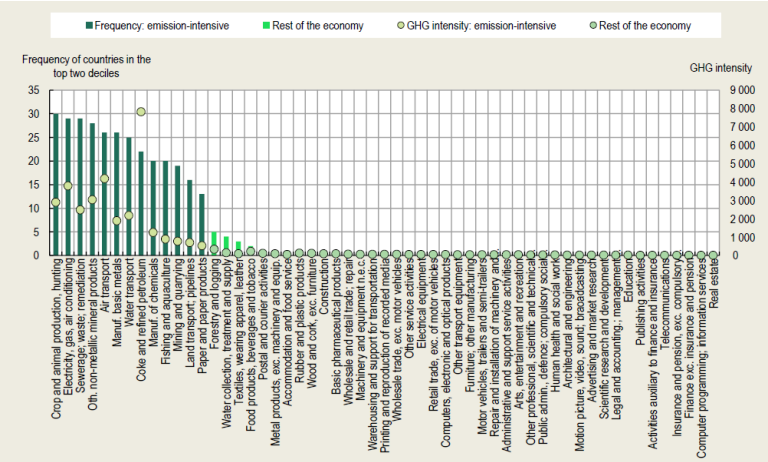
for each $\text{year} = 1, 2, \ldots$

C Empirical Appendix

Details under construction.

[74] Note that this distribution is different from the stationary distribution. The reason is that workers who *become* unemployed can be different from workers who *remain* unemployed, i.e., there is a selection into who stays in the unemployment pool (by age for instance). Step 1 allows us to focus on workers who just got laid off, which is consistent with the event study in Jarosch.

Figure 11: GHG intensity within 2-digit NACE rev.2 sectors and frequency in top 2 deciles across 31 OECD countries, 2009-20 average



Note: GHG intensity is expressed as CO2- equivalent emissions in tonnes per million EUR of gross value added.
Source: Eurostat – Air emissions accounts by NACE Rev. 2 activity. OECD Employment Outlook 2024 (Bassanini et al. (2024))

D Additional Results

Figure 12 presents the quantitative model's fit of the data's life cycle wage profile in steady state.

Figure 12: Life Cycle Wage Profile in Model vs Data

