The Dynamics of Firm-level Pay: Theory and Evidence from Portugal.*

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Abstract

We document that Portugal's earnings inequality has been steeply decreasing since the mid-1990s, mostly due to a compression in firm pay. We investigate the determinants of this evolution, by deploying a tractable model of firm-pay heterogeneity featuring labor market's monopsonistic power in a Hopenhayn-style firm dynamics framework. Our analysis suggests that a substantial increase in minimum wage was responsible for the wages of low-productivity firms to rise with respect to the mean. Our quantitative exercise hints that the wage policies of more productive firms are unlikely to have changed much as a result of the minimum wage hike.

Keywords: Firm Dynamics, Earnings Inequality, Minimum Wage, Monopsony.
JEL Codes: D33, E24, E25, L11, L25.

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1 Introduction

Recent empirical work made possible by the availability of employer-employee matched datasets has shown that firms may have played an important role in shaping the dynamics of earnings inequality in both developed and developing countries.[1] This paper shows that firms have also played a major role in shaping the evolution of Portugal's earnings inequality.

Using high-quality administrative data, we document that, after a fast-paced increase between the mid-80s and the mid-90s, earnings inequality has been steeply declining, driven exclusively by a compression in firm pay. In fact, other components such as cross worker-specific inequality and assortative matching between workers and firms have pushed inequality in the opposite direction.

We did not find any appreciable decline in the heterogeneity of observable firm–level characteristics, such as productivity. The lower firm pay dispersion was instead associated to a decline in the pass-through from firm-level productivity onto wages.

These empirical findings are consistent with contemporaneous and independent research by Silva et al. (2022). The fundamental different between ours and their work lies in the methodological approach that is adopted to rationalize the evidence.

To investigate the ultimate causes of the dynamics illustrated above, here we build a tractable theory of firm-level pay heterogeneity, by introducing labor market's monopsonistic power in the familiar firm dynamics model due to Hopenhayn (1992). The model features ex-ante worker heterogeneity and a minimum wage. These modeling choices are motivated by two key developments that affected Portugal's economy over the last quarter of a century: Sizeable increases in the real minimum wage and in the fraction of workers with college degree, respectively.

Figure 1a illustrates the evolution of the general (non-agricultural) real minimum wage since the mid-1980s. In 2019, the minimum wage was 40% higher than in the mid-1990s. Figure 1b shows that the share of college graduates among working individuals has also increased dramatically, from just under 5% in the mid-1990s to 19% in 2019.



(a) Real minimum wage

(b) Share of college graduates

Figure 1: Minimum wage and share of college graduates among workers

[1] See for example Abowd et al. (1999); Card et al. (2013); Song et al. (2018); Alvarez et al. (2018).

According to our theory, the minimum wage constraint binds at the bottom of the firms' productivity distribution. It follows that a minimum wage hike forces low-productivity firms to pay higher wages. By itself, this contributes to a decrease in the variance of firm pay.

Turning to higher productivity firms, which offer higher wages, we find that raising the minimum wage has pro- and anti-competitive effects on their pay practices. Everything else equal, higher-productivity firms need to pay higher wages in order to keep attracting workers when constrained firms post more attractive offers. This means that a minimum wage hike spills over to unconstrained firms, leading their wage markdowns to decline, as in Engbom and Moser (2022) and Berger et al. (2022). However, the higher minimum wage also leads low-productivity firms to reduce hiring, leading to softer competition for unconstrained firms.

Finally, a higher minimum wage reduces labor market tightness via a reduction in the mass of operating firms. This is the case because, all else equal, a greater minimum wage reduces the value of incumbency. In turn, this implies that firm selection at entry must improve: Only a smaller mass of relatively more productive candidates for entry will find optimal to start producing. Every operating firm will see its contact rate with job applicants increase, driving wage offers by unconstrained firms down.

Our quantitative exercise suggests that the increase in the minimum wage experienced by Portugal over the last three decades is likely to have had a sizeable effect on the variance of firm pay and overall earnings inequality. This was due mostly to the direct effect on constrained firms. The minimum wage hike had a limited net effect on the wage practices of relatively more productive firms, as the two mechanism outlined above canceled each other out.

This said, the minimum wage hike falls short of accounting for the observed compression in firm pay. About half of the decline in earnings inequality still needs to be accounted for.

Our paper contributes to a recent literature whose goal is to quantitatively assess the determinants of inequality dynamics by means of structural equilibrium models. Our conclusion regarding the relevance of the minimum wage in Portugal's experience is reminiscent of Engbom and Moser (2022), who find that 45% of the compression in Brazil's labor earnings is attributable to the large increase in the real minimum wage experienced by that country over a period of 22 years.

Our theoretical framework, however, is different. While Engbom and Moser (2022) envision labor markets as search and matching environments, we assume monopsony à la Robinson. In that, our theory is closer to Berger et al. (2022) and Hurst et al. (2022).

In common with the latter, we assume a full measure of firms and monopsonistic competition. Differently from theirs, however, our focus is on firm-level heterogeneity and on the long-run adjustment of the industry to policy changes.

A key difference from Berger et al. (2022) lies in the thrust of the analysis. Theirs is normative in nature, as they seek to identify the optimal level of the minimum wage. Ours is positive. Since Portugal experienced a large increase in the minimum wage, we want to understand its role in shaping inequality dynamics.

From the modeling standpoint, we note that while Berger et al. (2022) assume a finite number of firms that competes à la Cournot, our taxonomy of minimum wage effects is very close to theirs. While their framework is more detailed along several dimensions, we strive to model carefully firms' entry and exit. We do so to account for the long-run response of the distribution of firms to a variety of factors, among which a minimum wage increase. In

general, such a policy leads to a reduction in the mass of operating firms and to an increase in average productivity, with non-trivial effects on markdowns and on production efficiency.

The remainder of this paper is structured as follows. In Section 2, we carefully document the evolution of earnings inequality in Portugal and highlight the role played by firms. In Section 3 we introduce a simple static model of the labor market with monopsonistic competition. We show how labor market power shapes the level and slope of firms' wage profiles. In Section 4 we embed the model of the labor market in a more general framework, better suited for quantitative analysis. In Section 5, we address the key question of the paper. Section 6 concludes.

2 Earnings inequality in Portugal: Evidence

2.1 Quadros de Pessoal

The earnings data is from Quadros de Pessoal (QP), an administrative data set covering all private-sector firms employing at least one salaried worker, including those owned by the government. The data is collected annually by the Ministry of Labor to ensure compliance with labor laws. The reporting is for the month of March until 1993 and for October since. The data is unavailable for 1990 and 2001.

QP covers all workers employed in Portugal, except civil servants and independent contractors, as well as temporary rural and domestic workers. It is a matched employer-employee data set, providing comprehensive information on workers and some basic information on firms. We focus on workers aged 18–65, working full-time (at least 140 normal monthly hours), and earning at least the minimum wage as base pay.[2] We exclude firms in farming and fishing, since earnings in these sectors are often not reported on a monthly basis, and focus on for-profit non-financial market-oriented sectors.

Our favored measure of compensation is total monthly earnings, i.e. base salary plus overtime and regular monthly supplements such as productivity and seniority premia, as well as compensating differentials.[3] To avoid outliers, we trim the top 1% earnings observations per year. Further details on the data are in Appendix A.

2.2 The dynamics of earnings inequality

Figure 2 plots the variance of log earnings since 1986, the year Portugal joined the European Union, then known as the European Economic Community.

Inequality has increased dramatically from 1986 until the mid-1990s. The variance grew by about 50% over just a decade. From the mid-1990s onwards, however, inequality has first plateaued and then briskly declined since the financial and sovereign debt crisis of 2008-9. The variance of earnings dropped by more than one fifth between the mid-1990s' peak and 2019. An yearly rate of decline of about 1% is economically significant and comparable to

[2]The data on the (general) minimum wage and on the (private consumption, base year 2016) CPI are from https://www.pordata.pt/en/Home

[3]When running our analysis using hourly wages instead, the magnitudes change slightly, but the message does not.

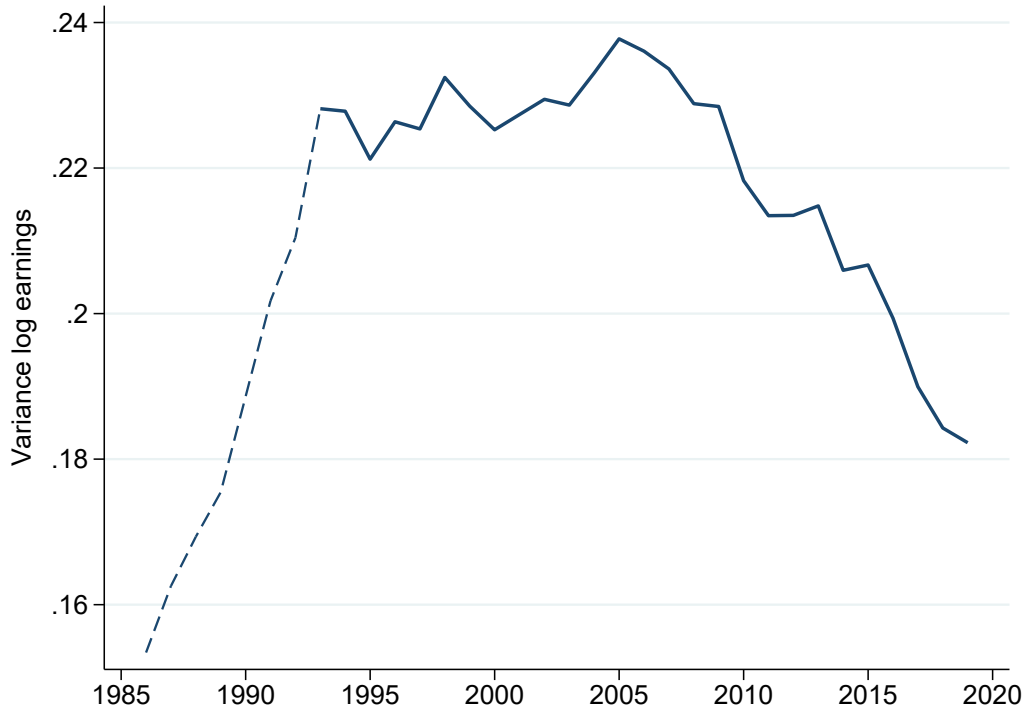


Figure 2: Quadros de Pessoal, 1986-2019

the rate of increase in the same measure of inequality reported by Song et al. (2018) for the United States between 1985 and 2009.

As far as we know, this non-monotonic dynamics is unique in the developed world. The sharp increase in inequality from the mid-1980s until the mid-1990s was also documented by Cardoso (1999), using the same data. This paper focuses on inequality past the mid-90s, a period during which, thanks to the reduction in informality, Quadros de Pessoal has significantly increased worker coverage.

Inspired by the recent empirical literature highlighting the key role that firms have played in shaping inequality in Germany (Card et al., 2013), the U.S. (Song et al., 2018) and Brazil (Alvarez et al., 2018), we ask whether employers have played a substantial role in Portugal as well.

2.2.1 The role of firms: A first look

Figure 3 displays a decomposition of the variance of log earnings into the variance of average earnings between firms and the employment-weighted average of the within-firm earnings variances.

The decomposition gives a first indication of the importance of firms in the Portuguese context. The between-firm component accounts for about 60% of the level. Most importantly for our purpose, it is also the main component behind the dynamics, accounting about 2/3 of the increase until the mid-1990s and for about 90% of the decline since the mid-1990s.

A natural question is whether the evolution of the between-firm component is driven by

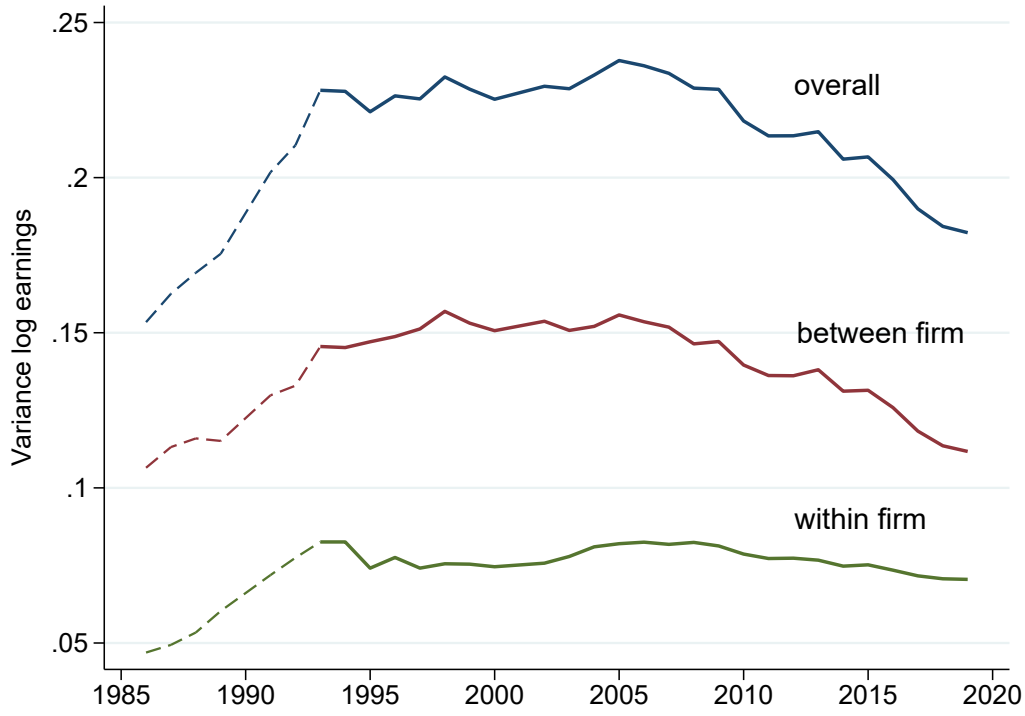


Figure 3: Between-within firm decomposition

changes over time in the composition of the workforce at the firm level or by variation in intrinsic firm–level characteristics.

2.2.2 AKM regression

To isolate the role played by intrinsic firm–level characteristics, we now resort to the regression framework due to Abowd et al. (1999). The AKM approach extends the prototypical Mincer wage regression specification by adding firm fixed effects to control for observed and unobserved systematic firm-level characteristics, in addition to worker fixed effects and a set of time-varying observables. Our regression equation for the two-way fixed effect (AKM) model is

$$y_{ijt} = \text{WFE}_i + \text{FFE}_j + \mathbf{X}'_{ijt}\beta + \varepsilon_{ijt}. \tag{1}$$

For the time-varying observables we follow Card et al. (2013) and include a full set of year dummies and a worker age cubic polynomial, both fully interacted with schooling attendance dummies.[4] As is standard in the literature, we estimate (1) over six non-overlapping subperiods p of four/five years each. This is a simple way to capture variation over time in the estimated coefficients.[5] We therefore estimate cross-sectional distributions of WFE_i^p and

[4] As in Card et al. (2013), age is normalized (by deviations from the age of 40) so that the polynomial implies flat experience profiles at age 40. The linear effect of age is dropped as it is captured by the worker fixed effects together with time. Likewise, the independent effect of schooling is also captured by the worker fixed effects.

[5] Engbom et al. (2021) consider an alternative approach by extending the original AKM's identification

FFE_j^p per subperiod p, plus the coefficients in β^p.

For estimation purposes, we restrict our sample in a number or ways. First, we focus on the largest connected set of all firms (and their workers) for which every worker in the set has ever worked for. Second, we discard singletons (unique firm-worker observations). Appendix C describes further sample restrictions, such as dropping very small firms to mitigate the limited mobility bias when estimating the cross-sectional variances, and restricting to workers sufficiently away from the minimum wage. Our main results are not very sensitive to excluding these firms.

2.2.3 Basic variance decomposition

Ignoring the time-varying observables for now, the AKM regression delivers the following basic variance decomposition by subperiod p:

$$\text{var}\left(y_{ijt}\right) = \underbrace{\underbrace{\underbrace{\text{var}\left(\text{WFE}_i\right)}_{\text{fixed}} + \underbrace{\text{var}\left(\varepsilon_{ijt}\right)}_{\text{random}}}_{\text{workers}} + \underbrace{\underbrace{\text{var}\left(\text{FFE}_j\right)}_{\text{firm pay}} + \underbrace{2\text{cov}\left(\text{WFE}_i, \text{FFE}_j\right)}_{\text{assortative matching}}}_{\text{firms}}}. \tag{2}$$

The first two components are attributed to workers. $\text{var}(\text{WFE}_i)$ is the dispersion in the systematic component that captures observable features such as schooling and gender, as well as unobservable characteristics such as motivation and inter-personal traits. $\text{var}(\varepsilon_{ijt})$ is the dispersion in the random component capturing permanent or transitory earnings changes due to shocks to, say, health or training. The two remaining components are attributed to firms. The first is the dispersion in the systematic firm-level component capturing characteristics such as productivity, size, or management quality. This is the firm pay component, enjoyed by any worker employed by a specific firm. The second is an assortative matching component conveying the extent to which high-wage workers are employed by high-wage firms.

The dispersion in worker fixed effects could vary across subperiods due to either persistent changes in the systematic components of continuing workers, or to cohort effects induced by the characteristics of either labor force entrants or exiters. In turn, the latter could stem from the significant increases in college graduation and female labor force participation which have taken place since the mid-1980s. The dispersion in firm pay may have changed due to cross-period changes in incumbents' pay policies, or may be due to cohort effects induced by the specific characteristics of entrants and exiters within each subperiod.

The results of this decomposition exercise are in Table 1. We emphasize two main trends across three subperiods: The increase in inequality from the mid-1980s until the mid-1990s and the decline since the mid-1990s. The table contains also the time-varying observables (Xb) that were omitted in (2).

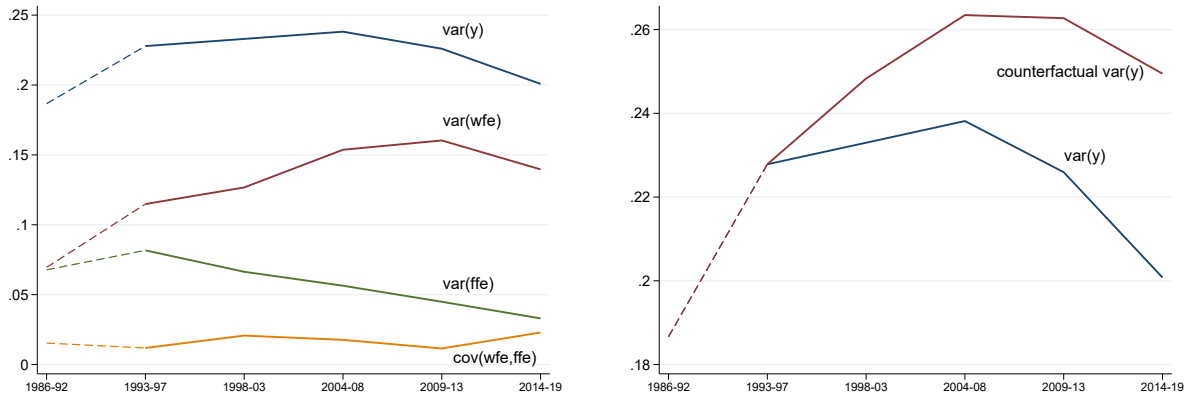
Figure 4a shows the period-by-period dynamics of the most significant components from Table 1. The variance of firm pay is a significant driver of the increase in overall earnings inequality until the mid-1990s and it's by far the key force behind the decline since then. In fact, it is the only major force pushing for earnings compression. It accounts for 180% of

requirement. They show that firm-specific time trends are identified within a set of workers which are mobile not just across firms, but across firm-years.

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.187	–	.228	–	.201	–	.041	–	-.027	–
Var(WFE)	.07	37.3	.115	50.4	.14	69.5	.045	110	.025	-91.8
Var(FFE)	.068	36.3	.082	35.9	.033	16.4	.014	33.8	-.049	180
Var(Xb)	.012	6.7	.004	1.9	.008	3.9	-.008	-19.7	.003	-12.9
Var(ϵ)	.015	7.9	.014	6.3	.011	5.3	0	-1	-.004	13.4
2Cov(WFE, FFE)	.015	8.2	.012	5.2	.023	11.4	-.003	-8.4	.011	-40.9
2Cov(WFE, Xb)	.002	1	-.001	-.5	-.013	-6.2	-.003	-7.2	-.011	42.4
2Cov(FFE, Xb)	.005	2.6	.002	.8	-.001	-.4	-.003	-7.5	-.003	9.7

Table 1: Basic Decomposition

the overall decline in inequality and outmatches the impact of greater worker heterogeneity, responsible for a negative contribution of 92%.



(a) Main components

(b) Counterfactual earnings variance

Figure 4: Basic variance decomposition

An alternative way to appreciate the importance of the compression in firm pay for the overall inequality decline is to evaluate (2) while counterfactually holding the variance of firm fixed effects constant at the mid-1990s level. Figure 4b shows that earnings inequality would have increased rather than declined.

Such outsize role of firm pay as a driver of earnings inequality is in common with the experiences of Brazil and Ecuador, where inequality has steadily declined, and with those of U.S. and Germany, where inequality has steadily increased.[6]

2.2.4 Detailed variance decomposition

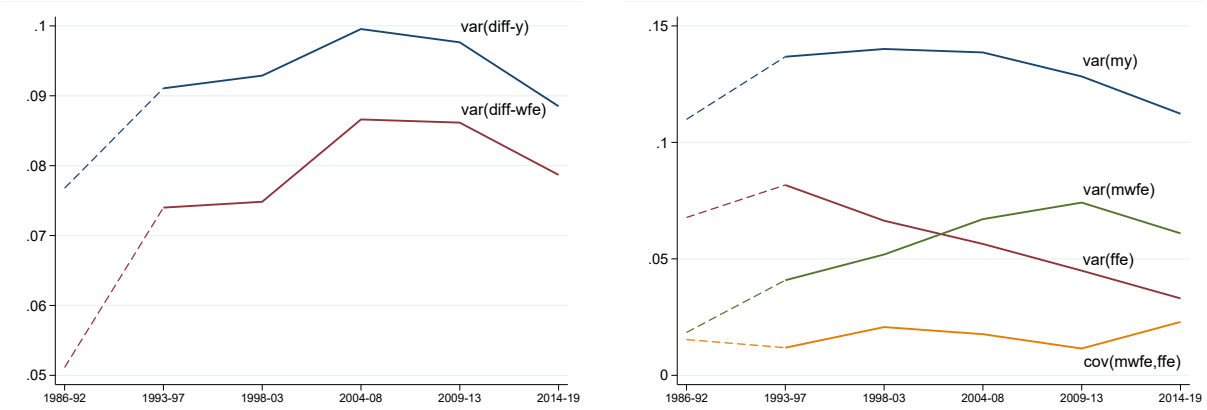
A more detailed decomposition, along the lines of Song et al. (2018), splits the overall variance into a within-firm and a between-firm component. Once again abstracting from the time-varying observables:

$$\text{var}\left(y_{ijt}\right) = \underbrace{\text{var}\left(\text{WFE}_i - \overline{\text{WFE}}_j\right) + \text{var}\left(\varepsilon_{ijt}\right)}_{\text{within-firm}} + \underbrace{\underbrace{\text{var}\left(\text{FFE}_j\right)}_{\text{firm pay}} + \underbrace{2\text{cov}\left(\overline{\text{WFE}}_j, \text{FFE}_j\right)}_{\text{sorting}} + \underbrace{\text{var}\left(\overline{\text{WFE}}_j\right)}_{\text{segregation}}}_{\text{between-firm}}. \quad (3)$$

The various components add up to the between and within firm components of Figure 3. The payoff consists in gaining further insight into the worker-firm matching patterns. We now have a sorting component reflecting the tendency for high-wage firms to attract high-wage workers and a segregation component reflecting the tendency for firms to employ similar workers.

[6] See Alvarez et al. (2018) for Brazil, Messina and Silva (2019) for Ecuador, Card et al. (2013) for Germany, and Song et al. (2018) for the United States

Table 2 provides a comprehensive account of the results, while Figure 5 zeroes in on the main components. Figure 5a suggests that the overall increase in the variance of worker fixed effects is mirrored within Portuguese firms. Within-firm inequality rose simply because the dispersion in relevant worker-level characteristics rose in Portugal. Figure 5b confirms that the compression in firm pay since the 1990s has been the chief driver of earnings inequality. Whereas segregation exerts a relatively small force towards an increase in inequality (-17.3% of the total variance since the mid-1990s), sorting between Portuguese workers and firms has increased significantly, a powerful force leading to higher inequality (-40.9%). The bottom-line is that compression in firm pay has counteracted both of these forces and has single-handedly brought earnings inequality down.



(a) Within-firm

(b) Between-firm

Figure 5: Detailed Decomposition

2.3 Determinants of compression in firm pay since the mid-1990s

What are the economic forces primarily responsible for the greater compression in firm pay? The literature has suggested that firm-level heterogeneity in pay may be intimately tied to the variation in firm-level productivity in a non-competitive labor market. This motivates us to investigate the association between firm pay and firm-level productivity. We postulate the following log-linear specification for firm j's pay policy in subperiod p:

$$\text{FFE}_{jp} = \alpha_p + \gamma_p \log z_{jp}, \tag{4}$$

where z_{jp} is the firm's within-period systematic productivity level, and γ_p is the pass-through from productivity to the firm-level component of wages. From the firm pay policy (4) we obtain

$$\text{var}\left(\text{FFE}_{jp}\right) = \gamma_p^2 \text{var}\left(\log z_{jp}\right), \tag{5}$$

which indicates two possible sources for the compression in firm pay: (i) A compression in the distribution of firm–level productivity and (ii) a decline in the pass-through.

We first ask whether the Portuguese data is consistent with a specification such as (4). We do this by regressing the estimated AKM firm fixed effects on measures of firm–level

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.187	–	.228	–	.201	–	.041	–	-.027	–
Var(m_y)	.11	58.9	.137	60	.112	55.9	.027	65.3	-.024	90.5
Var(m_WFE)	.018	9.9	.041	17.9	.061	30.4	.022	54.4	.02	-74.5
Var(FFE)	.068	36.3	.082	35.9	.033	16.4	.014	33.8	-.049	180
Var(m_Xb)	.002	1	.001	.3	.001	.5	-.001	-2.8	0	-1.6
2Cov(m_WFE, FFE)	.015	8.2	.012	5.2	.023	11.4	-.003	-8.4	.011	-40.9
2Cov(m_WFE, m_Xb)	.002	.9	0	0	-.005	-2.4	-.002	-4.2	-.005	17.7
2Cov(FFE, m_Xb)	.005	2.6	.002	.8	-.001	-.4	-.003	-7.5	-.003	9.7
Var(diff_y)	.077	41.1	.091	40	.089	44.1	.014	34.7	-.003	9.5
Var(diff_WFE)	.051	27.4	.074	32.5	.079	39.2	.023	55.5	.005	-17.3
Var(diff_Xb)	.011	5.7	.004	1.6	.007	3.4	-.007	-16.8	.003	-11.4
Var(ϵ)	.015	7.9	.014	6.3	.011	5.3	0	-1	-.004	13.4
2Cov(diff_WFE, diff_Xb)	0	.1	-.001	-.4	-.008	-3.8	-.001	-3	-.007	24.7

Table 2: Detailed Decomposition

productivity, following Alvarez et al. (2018) very closely.

2.3.1 Sistema de Contas Integradas das Empresas

We link the Quadros de Pessoal with the Sistema de Contas Integradas das Empresas (SCIE), which allows us to compute measures of firm-level productivity. The SCIE provides a broad coverage of private-sector non-financial and for-profit firms operating in Portugal. However, it is only available since 1996.

Between 1996 and 2003, the information is drawn from the Inquérito à Empresa Harmonizado (IEH), a survey compulsory for all large private-sector corporations and a random sample of small corporations (less than 100 employees, or sales less than 5 million euros).

Starting in 2004, it is based upon administrative information, from tax records on a broader set of firms in 2004-5, and since 2006 from Informação Empresarial Simplificada (IES) on the universe of private-sector non-financial and for-profit firms operating in Portugal, drawn in particular from balance-sheet data. This is significantly higher quality information, used by the Public Administration and the Bank of Portugal. The number of available variables was increased with the IES, and SCIE coverage was also extended to include entrepreneurs and independent contractors, although we restrict our sample to corporations. In 2010 there was a major overhaul of the data reporting, for harmonization purposes with the rest of the E.U., which was applied retroactively to some of the variables starting in 2004. Some of the variables are therefore only available since 2004, and some others since 2010. We construct population weights to make the pre and post-IES information comparable. Further details can be found in Appendix B.

2.3.2 Firm–level productivity and firm pay

We compute two alternative measures of firm-level productivity. Our favorite is firm–level TFP, computed for firm j in sector s and year t as

$$z_{jt} = \frac{y_{jt}}{n_{jt}^{\alpha_n^s} m_{jt}^{\alpha_m^s} k_{jt}^{1-\alpha_n^s-\alpha_m^s}},$$

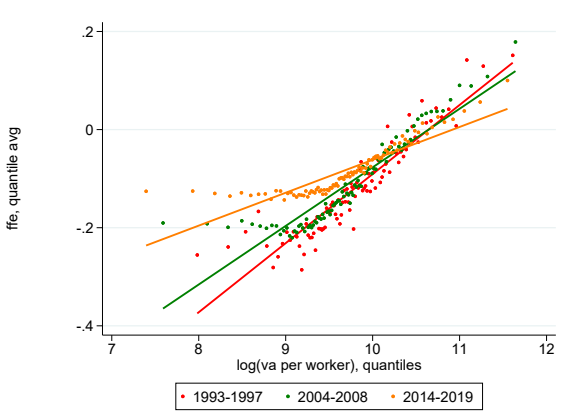
where y is total operating revenue (CPI deflated), n is the number of employees, m is the real value of intermediates, k is the real book value of capital, and the α's are the sector-specific averages of the firm–level revenue shares of total expenditures in labor and materials and services.[7] Unfortunately information on the book value of capital is only available since 2010, and so is firm-level TFP. We therefore rely mostly on real value-added per worker (CPI deflated), which is available since 1996 and hence covers the whole period of firm pay compression. We average each measure of productivity over each subperiod. To avoid outliers, we trim the top and bottom 1% observations by subperiod.

Figure 6 displays the evolution of the two potential determinants of firm pay compression (firm fixed effects are normalized by the subperiod employment-weighted average). Figure 6a illustrates the relationship between firm pay and value-added per worker across different

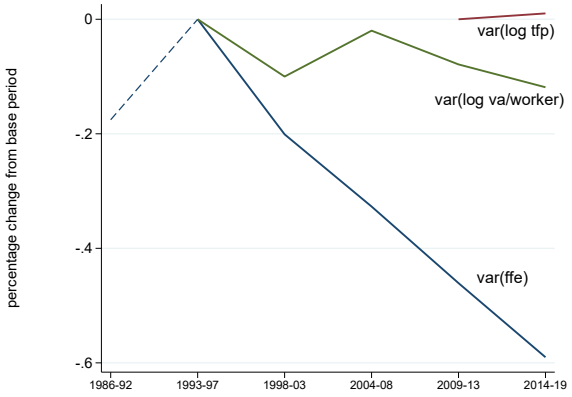
[7] We average over observations with strictly positive value added.

subperiods. It plots the average log value-added per worker quantiles against the corresponding firm fixed effects (scaled by the period average firm fixed effect), averaged over each quantile. Quite clearly, high-productivity firms pay more. However, it is also quite clear that the pass-through has weakened over time, especially from the mid-1990s to the mid-2000s.

Figure 6b displays the evolution of firm–level productivity dispersion against the dispersion in firm fixed effects which we wish to explain. The dispersion in value-added per worker declined only very slightly over time, whereas the dispersion in firm–level TFP remained essentially flat, although our sample is unfortunately very short as explained above. The evidence therefore suggests that the compression of firm pay is entirely due to the weakening of the pass-through from productivity to wages.



(a) Firm pay and value added per worker



(b) Variance of firm pay, value-added per worker, and TFP

Figure 6: Determinants of Firm Pay Compression

To provide a quantitative assessment of the contribution of the pass-through dynamics, in each subperiod we regress the firm fixed effects on log value-added per worker (regression weighted by worker-years, for comparability with the variance of firm fixed effects computed out of the AKM regression). We also consider sector and firm size controls, since these are likely co-determinants of firm pay. The results, reported in Table 3, show a reduction in the pass-through. The baseline regression without controls displays a decline from 0.186 to 0.113. These figures are somewhat lower but yet comparable to those estimated by Alvarez et al. (2018) for Brazil – a decline from 0.26 to 0.14 between 1996-2000 and 2008-2012.[8]

3 A labor market with monopsonistic competition

In this section we introduce a static model of the labor market along the lines of Card et al. (2018) and show how firm-level wage policies vary with competition in labor markets. We

[8]Our figures are also very much in line with the 0.1-0.15 range reported by Card et al. (2018) for Portugal, although they restrict to male workers in the late 2000s and employ a different data set for firm–level characteristics.

	1993-97			1998-03			2004-08			2009-13			2014-19		
log va per worker	.186	.156	.128	.18	.152	.138	.153	.132	.123	.123	.117	.112	.113	.109	.101
log size	–	.043	.039	–	.037	.033	–	.032	.03	–	.018	.016	–	.017	.017
sector dummies	no	no	yes	no	no	yes	no	no	yes	no	no	yes	no	no	yes
R^2	.22	.316	.373	.234	.329	.364	.207	.271	.289	.166	.205	.216	.178	.232	.243

Notes: All coefficients are significant at 1%. Weighted by worker-years and population weights.

Table 3: Regression of firm pay on firm characteristics

depart from Card et al. (2018) by considering a continuum of firms and solving for equilibrium wages. In Section 4 we will nest this framework in a firm-dynamics model with endogeneous entry and exit, better suited for quantitative analysis.

A mass N_f of firms produce an homogeneous good by means of the production function $y = zl^\gamma$, where l denotes labor and $0 < \gamma \leq 1$. The variable z is a firm-level (idiosyncratic) productivity index, distributed over a discrete finite set according to the probability density μ. Product demand is infinitely elastic at the unitary price.

There is a constant mass $N_w > 0$ of individuals that sell labor services to firms. The value to a worker of a wage offer w is

$$u(w) = w + \varepsilon,$$

where ε is a match-specific amenity shock distributed according to the Gumbel distribution with location parameter 0 and scale parameter $\sigma > 0$. The shock proxies for non-pecuniary aspects of the job offer that the worker may value, such as proximity, colleagues' characteristics, and required effort.

Each worker is matched with two firms, randomly drawn from the population. Confronted with wage offers $\{w_i, w_j\}$, her optimization problem is

$$\max \{w_i + \varepsilon_i, w_j + \varepsilon_j\}.$$

Workers' optimization, jointly with firms' wage offers, induces a firm-level labor supply curve $\bar{m}\Phi(w)$, where $\bar{m} \equiv \frac{2N_w}{N_f}$ are the offers extended by each firm and $\Phi(w)$ is the probability that a wage offer w is accepted. The firm's optimization problem is

$$\begin{aligned} \max_w \; & z_i[l(w)]^\gamma - wl(w) \\ \text{s.t.} \; & l(w) = \bar{m}\Phi(w). \end{aligned} \tag{6}$$

Necessary condition for optimality is

$$z_i \gamma l^{\gamma-1} - w = \frac{\Phi(w)}{\Phi'(w)}. \tag{7}$$

Since the elasticity of labor supply is $\epsilon_s(w) = \frac{\Phi'(w)w}{\Phi(w)}$, equation (7) can be rewritten as the

familiar condition relating a monopsonistic firm's wage markdown to labor supply elasticity:

$$\frac{z_i \gamma l^{\gamma-1} - w}{w} = \frac{1}{\epsilon_s(w)}.$$

The wage is pushed up to the point where the marginal gain deriving from adding to labor equals the marginal cost from adding to the wage bill of infra-marginal workers.

Definition. A labor market equilibrium consists of wage policies $w_i \equiv w(z_i)$ and a map $\Phi(w)$ such that $\forall\, i$, (1) w_i solves problem (6) given $\Phi(w)$, (2) $\Phi(w)$ is the probability that offering w leads to a hire, i.e.

$$\Phi(w) = \sum_{j=1}^{N} \frac{1}{1 + e^{\frac{w_j - w}{\sigma}}} \mu_j,$$

and (3) workers choose job offer i over j if and only if $w_j + \sigma\varepsilon_i > \sigma\varepsilon_i$ for all i, j.

3.1 Equilibrium firm-level pay

We now turn to describing salient features of the equilibrium allocation. When suitable, we illustrate such features by means of the numerical approximation of equilibrium in a parameterized version of the model.

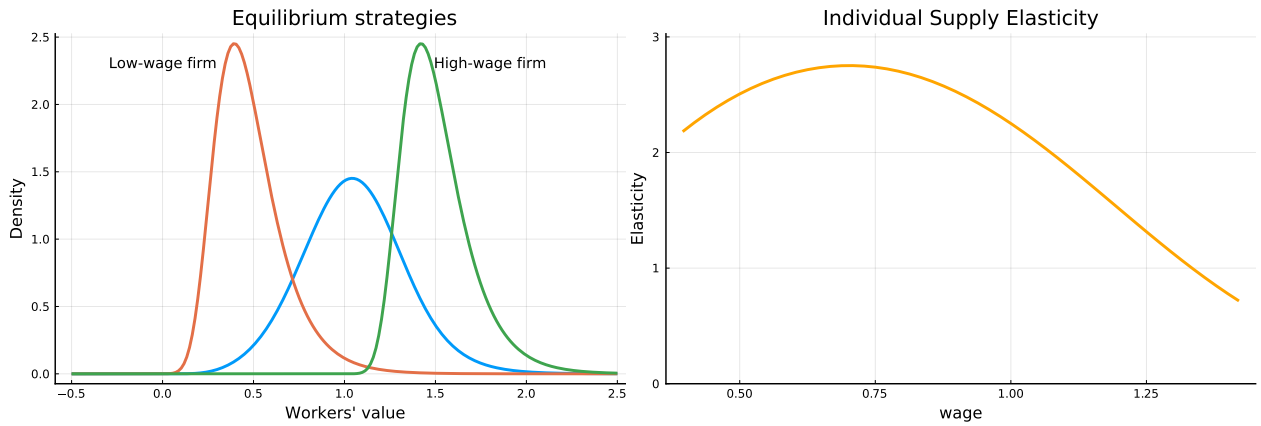
Consistent with the empirical evidence, our theory of the labor market predicts that more productive firms are larger and pay higher wages. The reason is straightforward: Higher productivity means higher marginal product of labor and calls for more labor input. Since the mass of contacts is given at $\bar{m}$, hiring more workers requires paying higher wages.

The assumption that all firms have the same mass of contacts with workers is made for simplicity. One could think of relaxing it my allowing firms to expend resources in order to contact more job candidates. In such scenario, increasing the contact rate would lead to greater hiring for given wage offer. As long as the cost of increasing the contact rate is increasing and convex, equilibrium would still feature more productive firms paying higher wages. Our assumption calls for that marginal cost to be zero until $\bar{m}$ and infinitely high thereafter.

Figure 7a depicts the distribution of the value to workers of wage offers by a low- and high- productivity firms, respectively. Because of the Gumbel-distributed amenity value, the probability of recruiting a worker is always positive, no matter the wage offered.

When making an offer to a job candidate, recruiters don't know which firm they are competing with. Consistent with rational expectations, we assume that they can divine the unconditional equilibrium distribution of workers' values associated with the competing offer, also depicted at the center of Figure 7a.

The elasticity of the labor supply schedule faced by firms is also an equilibrium object. Importantly, it is not constant. We know that it will be lowest for high wage, since the firm distribution of productivity has a thin right tail. For high-productivity firms, raising their wage offer will have a negligible effect on their yield. It follows that high-productivity firms will be able to extract larger wage markdowns. In general however, as shown in the right panel of Figure **??**, the elasticity will not be monotone decreasing.



(a) Equilibrium Strategies

(b) Supply elasticity

Figure 7: Illustration of equilibrium

Both the level and the slope of the map between firm productivity and wage, an element at the center of our analysis, depend upon the parameter σ. This is the case because σ shapes firms' labor market power by disciplining the importance of job amenities relative to wages in workers' choice. For low σ, workers will simply choose the job featuring the highest salary. This is the scenario where firms enjoy the least labor market power. Markdowns will be relatively small. The slope tends to decline as σ gets smaller, since increasing minimally the wage leads to larger gains in hiring. The slope converges to zero, akin to the perfectly competitive outcome, when both σ converges to zero and $\bar{m}$ converges to N_w, so that neither amenities nor matching frictions play a role.

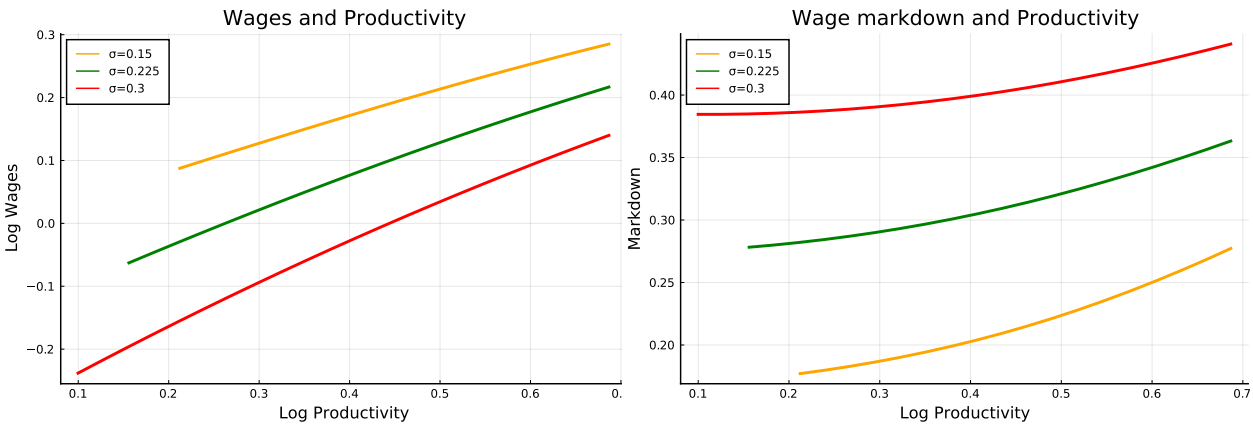
For σ arbitrary large, wages essentially don't matter for workers' choices. In the limit, the probability of the worker accepting a job is $1/2$ no matter her wage offers. This corresponds to the least competitive scenario. Markdowns increase with σ. The slope of the wage-productivity map also increases with σ, since the marginal hiring yield of raising the wage is small when amenities are very relevant.

Figure 8a illustrates the equilibrium pay policy for three different levels of σ. As expected, a decline in σ is associated with higher wages and a decline in the slope of the map between productivity and wages.

A lower σ means a more elastic labor supply function. The illustration also confirms that the effect of a smaller σ on wages is proportionally larger for low-productivity firms, leading to a flatter wage-productivity profile. Accordingly, the wage markdown declines at a faster rate for low-productivity firms. See Figure 8b.

4 Firm dynamics and minimum wage

We embed our model of the labor market in a more general framework that features firm entry and exit, a minimum wage, and unemployment. Time is discrete and goes from $t = 0$ to infinity. Since our analysis will focus on stationary equilibria, we suppress the time notation.



(a) Wages (b) Markdown

Figure 8: Wages, Markdown, and Productivity

4.1 Workers

Workers have no access to either storage or financial markets and therefore are hand-to-mouth. Their value of non-employment is denoted as $b + \varepsilon_b$, where ε_b is an amenity shock also conveniently distributed according to a Gumbel distribution with location parameter 0 and scale parameter $\sigma > 0$. When confronting job offers $\{w_i, w_j\}$, a worker's optimization problem is now

$$\max \left\{ w_i + \varepsilon_i, w_j + \varepsilon_j, b + \varepsilon_b \right\}.$$

The probability of accepting job i when facing alternative j is $\frac{e^{w_i/\sigma}}{e^{w_i/\sigma}+e^{w_j/\sigma}+e^{b/\sigma}}$.

4.2 Incumbent firms

Incumbent firms are distributed over the ordered discrete set $\{z_i\}_{i=1}^{N}$ according to the (endogenously-determined) density μ. The dynamics of individual productivity are determined by the Markov chain Π, with $Prob(z_{t+1} = z_j | z_t = z_i) = \pi_{ij} > 0$. Firms discount the future at the rate $\beta > 0$, incur a random per-period operating cost $\kappa > 0$, expressed in unit of the numeraire good, and exit when their expected discounted value is negative. We assume that κ is distributed according to the cdf Ψ, defined over the positive orthant and allowing a density ψ.

Every period, each worker is matched with two randomly drawn firms. In turn, this means that each firm is matched with a mass $\bar{m} \equiv 2N_w/N_f$ of workers, where N_f is now the endogenously determined mass of operating firms.

Firms are not compelled to make an offer to each worker they are matched with. We anticipate that, when the minimum wage is low enough with respect to their productivity, they will find optimal to only extend $m < \bar{m}$ offers.

The optimization problem of an incumbent of productivity z_i can be conveniently stated

in a recursive fashion as

$$v(z_i,\Phi)=\int_0^{+\infty}\max\left\{0,\max_{\{m,w\}}\ z_i l(w)^{\gamma}-wl(w)-\kappa+\beta\sum_{j=1}^{N}\pi_{ij}v(z_j,\Phi)\right\}d\psi(\kappa) \tag{8}$$

$$\text{s.t. } l(w)=m\Phi(w),$$

$$w\geq \underline{w}, \tag{9}$$

$$m\leq \bar{m}. \tag{10}$$

The constraint (9) ensures that firms comply with minimum wage legislation. The constraint (10) imposes that firms can extend offers only to the workers they are matched with.

The notation $\Phi(w)$ still denotes the individual labor supply function, i.e. the probability of hiring a worker when offering a given wage. We have that

$$\Phi(w)=\sum_{j=1}^{N}\left[\left(1-\frac{m_j}{\bar{m}}\right)\frac{1}{1+e^{\frac{b-w}{\sigma}}}+\frac{m_j}{\bar{m}}\frac{1}{1+e^{\frac{b-w}{\sigma}}+e^{\frac{w_j-w}{\sigma}}}\right]\mu_j,\quad \mu_j=\frac{\hat{\mu}_j}{N_f}. \tag{11}$$

Condition (11) reflects the fact that with probability $\sum_{j=1}^{N}\left(1-\frac{m_{jh}}{\bar{m}_h}\right)$ a firm will face no competition when trying to hire.

The solution of problem (8) will feature a non-decreasing set of thresholds for the operating cost $\{\kappa_i\}_{i=1}^{N}$ such that firms with productivity z_i will exit if and only if $\kappa>\kappa_i$. In turn, this means that $1-\Psi(\kappa_i)$ is the conditional probability of exit for firms of productivity z_i.

4.3 Entry

Entry is modeled along the lines of Clementi and Palazzo (2016). Every period, there is a mass of potential entrants $M>0$ that draw signals $\eta>0$ about their initial productivity from the continuous distribution H. We assume that $\Pr(z\leq z_i|\eta)=G_i(\eta)$.

For every η, the collection $\{G_i(\eta)\}_{i=1,N}$ is a cumulative distribution function. Crucially, G_i is strictly decreasing in η for all i.

Firms must pay a cost $c_e>0$, expressed in units of numeraire good, in order to learn their initial productivity. Therefore, a potential entrant will pay c_e and learn its initial productivity if and only if

$$\sum_{i=1}^{N}v(z_i,\Phi)g_i(\eta)\geq c_e,$$

where g_i is the pdf associated to G_i. It follows that the measure of entrants characterized by initial productivity z_i will be

$$\hat{\mu}_e(z_i)=\Psi(\kappa_i)M\int_{\eta^*}g_i(\eta)dH(\eta).$$

4.4 Equilibrium

Definition. A recursive stationary equilibrium consists of

- Policies for wages and job offers $\{w_i, m_i\}_{i=1}^N$
- Thresholds η^*, $\{\kappa_i\}_{i=1}^N$
- A firm-level labor supply functions $\Phi(w)$,
- Measure of active firms $\hat{\mu}(z)$, and associated mass $N_f \equiv \sum_{i=1}^N \hat{\mu}_i$,

such that

- Wages and job offers solve problem (8) given $\Phi(w)$,
- $\hat{\mu}$ is consistent with η^* and $\{\kappa_i\}_{i=1}^N$, i.e. for all i,

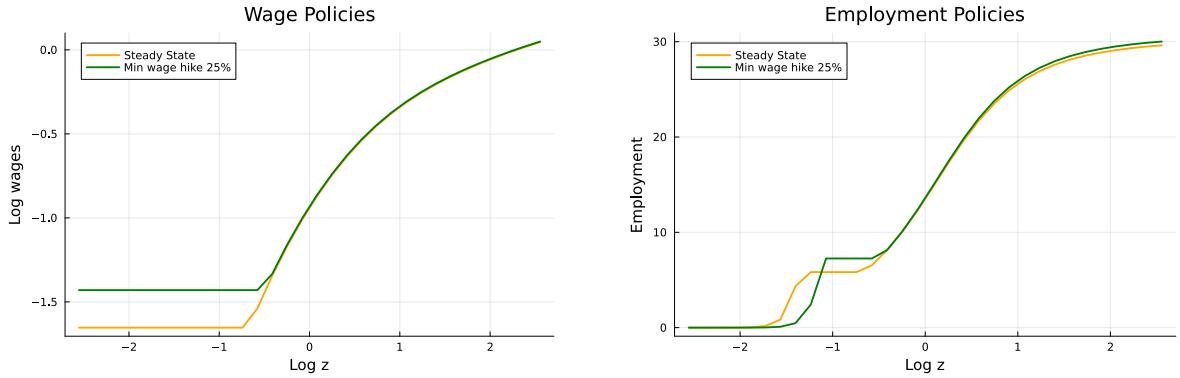
$$\hat{\mu}_i = \Psi(\kappa_i) \left[M \int_{\eta^*} g_i(\eta) dH(\eta) + \sum_{j=1}^N \pi_{ji} \hat{\mu}_j \right]$$

- $\Phi(w)$ is the probability of hiring a worker, i.e.

$$\Phi(w) = \sum_{j=1}^N \left[\left(1 - \frac{m_j}{\bar{m}}\right) \frac{1}{1 + e^{\frac{b-w}{\sigma}}} + \frac{m_j}{\bar{m}} \frac{1}{1 + e^{\frac{b-w}{\sigma}} + e^{\frac{w_j - w}{\sigma}}} \right] \mu_j, \quad \mu_j = \frac{\hat{\mu}_j}{N_f}.$$

4.5 A raise of the minimum wage

We now turn to illustrating the effect of a minimum wage hike on the equilibrium allocation, by means of a simple comparative statics exercise. We parameterize the model in order to reproduce key features of the Portuguese economy in the period 1993-97, along the lines of the quantitative exercise described in Section 5. We then compute the impact on such allocation of a 25% increase in minimum wage – the same magnitude of the increase in Portugal's minimum wage between 1993 and 2019. This exercise will also allow us to illustrate key features of the model.



(a) Wages (b) Employment

Figure 9: Comparative statics with respect to the minimum wage

Refer to Figure 9. In equilibrium, firms can be conveniently partitioned in three connected and ordered subsets of the productivity support. We will refer to them as regions 1, 2, and 3, respectively. The fraction of firms closer to the productivity lower bound (region 1) are constrained by the minimum wage. Their productivity is so low that they do not extend offers to all workers they are matched with. If they did so, their marginal product of labor would fall short of the minimum wage. These firms are price takers at the minimum wage level and hire up the level for which the marginal product of labor equals the minimum wage.

Moving to the right along the productivity support, we find firms whose productivity is large enough for them to make offers to all workers they are matched with. This is region 2. Extending offers to all workers they are matched with and paying the minimum wage leads to a marginal product of labor which is greater than the minimum wage and lower than the marginal cost. It follows that their wage markdown falls short of the monopsonistic optimum. In other words, equating marginal revenue to marginal cost as a monopsonist would do, would lead to wages lower than the minimum allowed. This is why these firms find it optimal to offer $\underline{w}$ to a mass $\bar{m}$ of job candidates.

Further to the right is region 3, where firms' productivity is so large that wage setting results in equating the marginal product of labor to the marginal cost. These firms are akin to textbook monopsonists.

An increase in the minimum wage has obvious effects on the firms in region 1. Their wages increase and their employment declines. Furthermore, region 1 now extends further to the right. Firms in region 2 also see their wages increase. Those with relatively lower productivity now fall in region 1. Their employment decreases. On the contrary, those with relatively higher productivity see their employment increase.

Firms in region 3 are not directly affected by the policy change. However, the equilibrium distribution of firms, their mass, and the wages they charge are now different. In turn, this means that firms in region 3 face different individual labor supply functions.

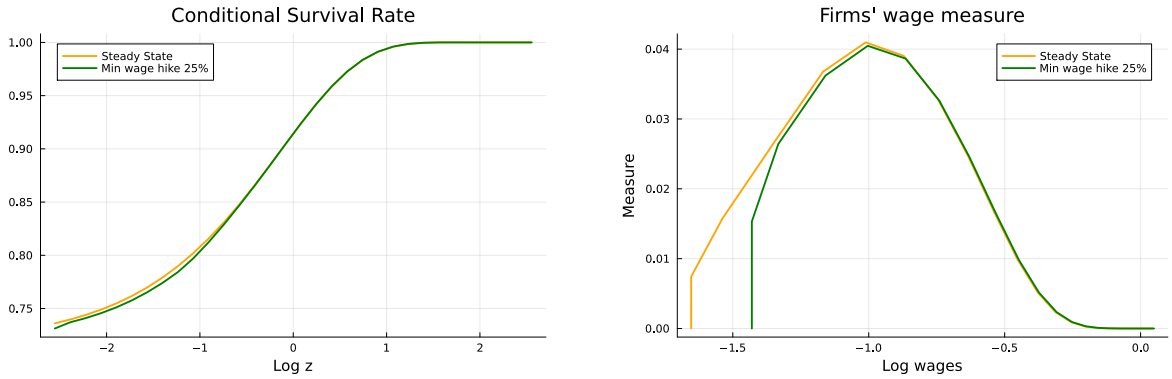
Firms in region 1 and 2 offer higher wages. Everything else equal, this causes the elasticity of labor supply for region-3 firms to rise. This is often referred to as the spillover effect of a minimum wage hike, that in principle may lead to a propagation of the wage rise to unconstrained firms.

Let's now turn to the equilibrium effects on the measure of operating firms. An increase in the minimum wage lowers the expected value of entry. The free-entry condition calls for an increase in the entry threshold η^* and a lower mass of entrants. In turn, this results in a smaller equilibrium mass of firms N_f. Every incumbent meets a larger mass of workers. Everything else equal, this allows for lower wages.

Figure 10a illustrates the equilibrium change in survival rates. With a higher minimum wage, low-productivity firms exit at a higher rate. The decline in the equilibrium mass of firms is confined to the bottom of the productivity distribution. This is consistent with Figure 10b, which reproduces the measure of firms over the wage space. Thanks to the minimum wage hike, the lower bound of the wage distribution rises. However, the measure of firms offering those wages declines.

In net, the effect on the wage policies by firms in region 3 is negligible.

Spillover effects from a minimum wage increase have been found to be quite significant in Brazil according to Engbom and Moser (2022), however the minimum wage in Brazil increased four times as much as in Portugal over a similar period of time. Spillover effects



(a) Conditional survival probability

(b) Firms' measure over wage space

Figure 10: Equilibrium effects of a minimum wage hike

are more muted in our setting, just like in Berger et al. (2022) when they consider a moderate minimum wage increase.

5 Accounting for the recent decline in Portugal's earnings inequality

We are now ready to address the main question of this paper: What accounts for the recent decline in Portugal's earnings inequality? In particular, we are interested in gauging the contribution of the increase in minimum wage to the reduction in the variance of firm-level pay, which we found to be largely responsible for the decline in overall earnings inequality.

In Section 1, we documented that the share of college graduates has increased dramatically in Portugal. We therefore deem important to take such dynamics into account.

We assume that production is a CES aggregate of the services of two types of labor, which we call skilled and unskilled, respectively. The production function is

$$y = z_i \left[\chi l_s^{\frac{\nu-1}{\nu}} + (1-\chi) l_u^{\frac{\nu-1}{\nu}} \right]^{\frac{\gamma}{\nu-1}},$$

with $\chi \in (0,1)$, $\nu > 0$ and $0 < \gamma \leq \nu$.

We assume that the two labor markets are segmented. Each firm receives a mass $2N_f/N_i$ of applications in the market for skill $i = s, u$, with N_i the mass of workers of type i. For the sake of brevity, we do not lay out an amended definition of equilibrium. The interested reader is referred to the Appendix.

Our methodological approach calls for identifying structural parameters' values under which the steady-state of our model is consistent with a set of salient statistics for the Portuguese economy in the mid-1990s. We will then ask what must have happened, beyond the documented increases in minimum wage and educational attainment, for our model to be consistent with the evolution of inequality up to 2019. We focus on a steady-state comparison.

By simulating our artificial economy for four periods, we obtain an employer-employee

matched panel dataset over which we estimate the same AKM regression we ran on our actual data. In particular, we can compute the variance of worker and firm fixed effect, respectively.

Parameter	Meaning	Initial eq.	Final eq.
ν	Elasticity of substitution	1.4	Same
γ	Returns to scale	1.26	Same
χ	Skill bias	0.71	0.58
σ	Amenities	0.07	Same
A	Mean of log TFP	0.0	0.1222
ρ_z	TFP persistence	0.89	Same
σ_z	TFP cond. var.	0.22	Same
N_s	Fraction skilled	0.05	0.16
$\underline{w}$	Minimum wage	0.115	0.145

Table 4: Parameter Values

The first column in Table 4 reports the parameters associated with the initial steady-state equilibrium. Some of them are set externally. According to Acemoglu and Autor (2011), the summary of the evidence for the U.S. suggests an elasticity of substitution between skilled and unskilled labor in the 1.4-2.0 range. Centeno and Novo (2014) provide similar estimates for Portugal, in the 1.5-2.6 range. We set $\nu = 1.4$, which is the estimate in the original Katz and Murphy (1992) study. We also set $\gamma = 1.26$, which implies returns to scale of 0.9.

The Markov chain Π is set in such a way that the implied stochastic process approximates an AR(1) with autocorrelation 0.89 and conditional standard deviation $\sigma_z = 0.22$. These are the values we recovered when estimating an AR(1) process for total factor productivity in our sample of firms (available from 2012 onwards)

We normalize the total mass of workers to 1. We then set $N_s = 0.05$ to reflect the observation that the average fraction of workers with a college degree was 5% between 1993 and 1997.

Finally, we set the mass of potential entrants M, the minimum wage $\underline{w}$, as well as the parameters σ e χ, so that in stationary recursive equilibrium the average size of firms, the ratio of minimum wage to median wage, the variance of the worker fixed effect and the variance of the firm fixed effects are as close as possible to the average values recorded in Portugal over the period 1993-97. The targets are listed in Table 5.

Description	Initial Steady State (1993-97)	Final Steady State (2014-19)
Minimum wage / median wage	0.57	0.64
Variance of WFE	0.116	0.139
Avg firm size (in workers)	14.0	12.0

Table 5: Calibration Targets.

Next, we turn to the period 2014-2019. In such period, the average real minimum wage was 25% higher than in the mid-90s. The fraction of college-educated workers was 16%, 11 percentage points higher than in the mid-90s. We update the values of $\underline{w}$, N_u and N_s accordingly.

We accommodate aggregate productivity growth, by allowing the support of the productivity distribution to shift at the rate A. Then we ask how A and χ must have changed for the model to be consistent with the observed time-variation in the ratio of minimum wage to median wage, the variance of worker fixed effect, and the average firm size. Finally, we ask what is the implied change in the variance of firm fixed effect.

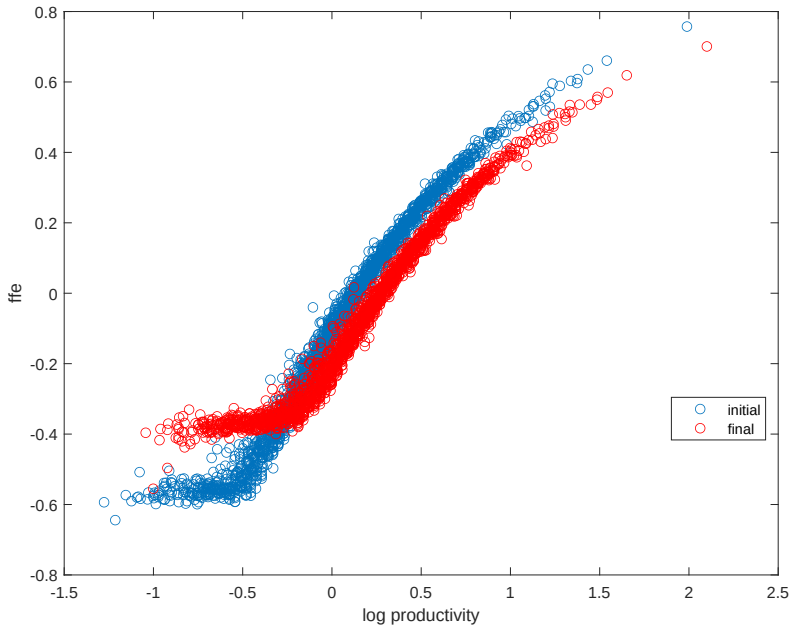
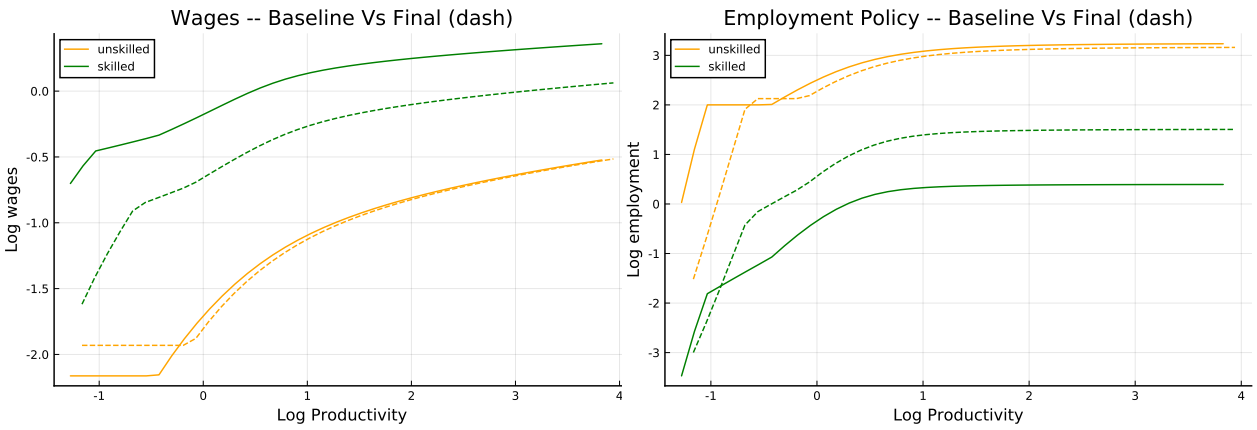


Figure 11: Distribution of model-generated firm fixed effects

Figure 11 illustrates the distribution of the estimated firm fixed effects in the initial and final steady state equilibrium, respectively. One can notice the wage compression caused by the mechanical effect of a greater minimum wage on compensation by relatively unproductive firms. For higher productivity levels, the relation between productivity and fixed effects becomes flatter. In other words, the pass-through declines. This is the case mostly because of composition.

Figure 12, which describes firms' equilibrium policies for wages and employment in the initial and final steady states, respectively, helps us understand why. Overall population is constant. However, the fraction of high-skill workers increase. The estimate pass-through is essentially a weighted average of the pass-throughs for high- and low-skilled workers. The increase in educational attainment was such that the weights on the flatter wage profiles of skilled workers increased.



(a) Wages (b) Employment

Figure 12: Initial (solid) Vs Final (dashed)

The exercise also suggests that the minimum wage increase can account for about half of the increase in the variance of firm fixed effect. The impact of the rise in minimum wage alone was not large enough to generate the reduction in the variance of firm-level pay that we measured for Portugal. Other forces must have been at work.

6 Conclusion

TBA

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Appendix

A Quadros de Pessoal

We rely upon the following variables:

- Total monthly earnings (*rganho*) is the sum of base earnings, plus overtime pay, plus regular monthly supplements such as productivity and seniority premia, as well as compensating differentials.
- Monthly hours (*hnormais*) are normal working-time paid hours.
- Worker age (*idade_Cod*) is the actual worker age at the time of the survey (bottom-coded at 17 and top-coded at 68). To avoid measurement error, age is recomputed as the difference between the current year and the mode of the year of birth implied by reported age.
- Schooling (*habil*) is the highest degree attended. Based upon this variable, we construct three schooling dummies, for the equivalent of 9th grade or less, secondary or post-secondary non-college, and college and over (including Bachelor). We then take the mode of each indicator for each worker.
- Firm–level sectoral codes are at two–digit level, based upon the *C.A.E. - Classificação Portuguesa das Atividades Económicas* classification system. They have been harmonized over time (C.A.E. was revised in 1995, 2003, and 2007). Within an estimation subperiod, the firm–level code is taken to be mode across years.
- Number of employees (*pemp*) equals all persons that have worked for the firm in October, either remunerated by the firm or not (e.g. owner-managers and family members), and including short-term (less than one month) absentees. This variable is used to select our AKM estimation sample (dropping small firms), our primary measure for the number of employees in the rest of the paper is the analogously-defined variable drawn from SCIE.

Our focus on for-profit non-financial market-oriented sectors entails dropping firms in the financial sector, public administration, education, health, social services, families with domestic workers, and international organizations.

For workers reported to be working at different firms in a given year, we kept a single firm observation based upon the following criteria: (i) the firm where the individual works the highest number of normal hours, otherwise (ii) the firm where the individual earns the highest (straight-time) earnings, otherwise (iii) the firm where the individual has the longest tenure.

B Sistema the Contas Integradas das Empresas (SCIE)

We rely upon the following variables:

- Gross income (*produção*) equals sales revenue plus the change in inventories, plus some additional operating revenue.
- Gross value added (*valor acrescentado bruto a preços de mercado*) equals gross income minus the expenditure on materials and services.
- Materials and services (*consumos intermédios*) equals the cost of materials and services external to the firm.
- Capital (*ativos fixos tangívies + intangíveis + biológicos e propriedades de investimento*) equals the book value of fixed intangible assets, plus intangible assets, plus investment property.
- Number of employees (*pessoal ao serviço*) equals all persons that have worked for the firm during the year, either remunerated by the firm or not (e.g. owner-managers and family members), and including short-term (less than one month) absentees. This is our primary variable for the number of employees, and is used to compute value-added per worker and firm size (whenever missing, the number of employees from Quadros de Pessoal is used instead).
- Total labor cost (*gastos com o pessoal* equals total (fixed plus transitory) payments to employees, inclusive of social insurance, pension, and other mandatory contributions such as private insurance, plus hiring and firing costs.

The basic firm identifier we use in our empirical analysis is the one from Quadros de Pessoal. Firms in Quadros de Pessoal come also with the same fictitious fiscal number that identifies firms in the SCIE. We are therefore able to match each basic firm identifier from Quadros de Pessoal with a SCIE fictitious fiscal number in any given year. We restrict to SCIE firms which are also in Quadros de Pessoal, which for the most part implies dropping independent contractors and firms without paid employees.

SCIE does not provide population weights for the years prior to 2006, i.e. prior to broad coverage associated with IES. As mentioned in the main text, smaller firms are underrepresented in the earlier years. We infer population weights by comparing the firm counts in SCIE and in Quadros de Pessoal (which contains the universe of firms for our purpose), according to a stratification of firms based upon the number of employees and sales. All the tables, figures, and regressions in our paper are weighted by these relative counts.

C Robustness

We show that our bottom-line results from Section 2 are not very sensitive to alternative sample selection criteria. First, Table 6 displays the basic decomposition without restricting to firms in the market sector. Second, Table 7 contains the basic decomposition when our

baseline sample is further restricted to firms with at least 10 workers on average over their entire life-cycle.[9]

Third, Table 8 restricts the baseline sample to worker observations with base earnings above 1.2 of the minimum wage. The wage compression since the mid-1990s becomes muted. However, we still observe the basic dynamics we highlight in the paper, of significant compression in firm pay while inequality in worker characteristics increases. Figure 13 plots the dynamics of the main components of the basic AKM decomposition.

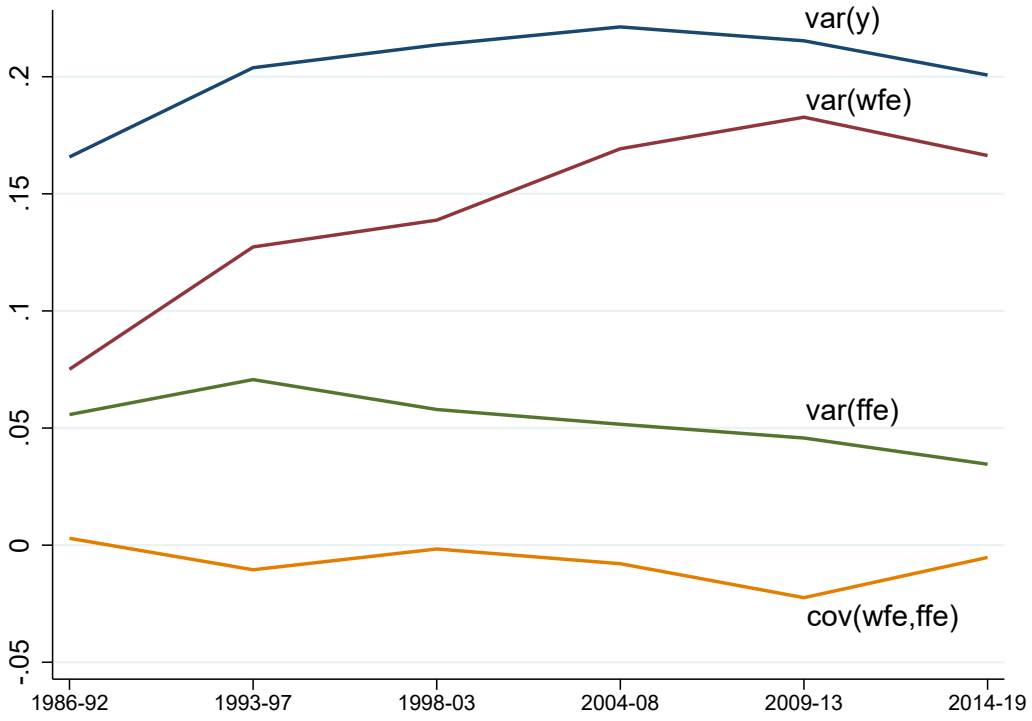


Figure 13: Basic Decomposition (base wages over 1.2 of min wage)

D Numerical approximation of the stationary equilibrium

In stationary equilibrium, the measure of firms over productivity is an invariant vector $\hat{\mu}$ of size N, where N is the number of points in the grid for productivity. Let $N_f = \sum_{i=1}^{N} \hat{\mu}_i$ the mass of incumbent firms and $\mu_i \equiv \hat{\mu}_i / N_f$.

[9] Our motivation is to act conservatively in order to avoid biased estimates of the cross-sectional variances, what the literature refers to as "limited mobility bias" (Bonhomme et al., 2020; Engbom et al., 2021). This problem is magnified by the presence of small firms, whose workers tend to exhibit lower mobility. Our simple approach is therefore to eliminate small firms in a long-run/life-cycle sense.

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.2	–	.256	–	.217	–	.056	–	-.039	–
Var(WFE)	.07	35.1	.121	47.4	.151	69.6	.051	91.3	.03	-75.7
Var(FFE)	.077	38.4	.094	36.9	.033	15.4	.018	31.4	-.061	155.8
Var(Xb)	.012	6.2	.004	1.5	.007	3.2	-.009	-15.6	.003	-8
Var(ϵ)	.015	7.4	.014	5.7	.01	4.7	0	-.6	-.004	11.2
2Cov(WFE, FFE)	.017	8.5	.021	8.1	.029	13.2	.004	6.8	.008	-20.1
2Cov(WFE, Xb)	.002	1.2	0	-.1	-.012	-5.7	-.003	-4.9	-.012	30.8
2Cov(FFE, Xb)	.006	3.2	.002	.6	-.001	-.3	-.005	-8.4	-.002	6

Table 6: Basic Decomposition (baseline sample plus non-market firms)

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.205	–	.262	–	.232	–	.057	–	-.03	–
Var(WFE)	.073	35.6	.126	48.3	.168	72.3	.053	93.7	.041	-138.1
Var(FFE)	.073	35.4	.086	32.9	.028	12	.014	23.7	-.058	195.1
Var(Xb)	.013	6.2	.004	1.4	.007	3.2	-.009	-15.6	.004	-12.4
Var(ϵ)	.015	7.4	.015	5.8	.01	4.4	0	0	-.005	17
2Cov(WFE, FFE)	.022	10.8	.029	11.1	.033	14.3	.007	12.2	.004	-13.7
2Cov(WFE, Xb)	.003	1.3	0	-.1	-.014	-5.9	-.003	-4.9	-.014	45.4
2Cov(FFE, Xb)	.007	3.2	.001	.5	-.001	-.3	-.005	-9	-.002	6.7

Table 7: Basic Decomposition (baseline sample minus small firms)

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.166	–	.204	–	.201	–	.038	–	-.003	–
Var(WFE)	.075	45.3	.127	62.5	.166	82.9	.052	137.2	.039	-1245
Var(FFE)	.056	33.6	.071	34.7	.035	17.2	.015	39.2	-.036	1153.6
Var(Xb)	.013	7.9	.004	1.8	.011	5.6	-.009	-24.8	.007	-239.1
Var(ϵ)	.015	9.3	.015	7.2	.011	5.3	-.001	-1.9	-.004	128.9
2Cov(WFE, FFE)	.003	1.8	-.011	-5.2	-.005	-2.6	-.013	-35.3	.005	-168.2
2Cov(WFE, Xb)	.001	.5	-.002	-1.1	-.017	-8.4	-.003	-8	-.015	466.8
2Cov(FFE, Xb)	.003	1.5	0	.1	0	0	-.002	-6.3	0	3

Table 8: Basic Decomposition (base wages over 1.2 of min wage)

D.1 Solving for the optimal firm policies, given μ

Given μ, the firm policies $\{m_i, w_i\}_{i=1}^N$ are such that for all i,

$$\begin{aligned} m_i, w_i = \arg\max_{\{m,w\}} \; & z_i l(w)^\gamma - w l(w) \\ \text{s.t. } & l(w) = m\Phi(w) \\ & w \geq \underline{w} \\ & m \leq \bar{m}, \end{aligned}$$

where $\bar{m} = \frac{2N_w}{N_f}$ and

$$\Phi(w) = \sum_{j=1}^{N} \left[\left(1 - \frac{m_j}{\bar{m}}\right) \frac{1}{1 + e^{\frac{b-w}{\sigma}}} + \frac{m_j}{\bar{m}} \frac{1}{1 + e^{\frac{b-w}{\sigma}} + e^{\frac{w_j - w}{\sigma}}} \right] \mu_j.$$

The support of productivity levels can be partitioned into three connected subsets. In region 1, $w_i = \underline{w}$ and $m_i < \bar{m}$. The mass of offers m_i satisfies

$$z_i \gamma [m_i \Phi(\underline{w})]^{\gamma - 1} = \underline{w}.$$

For firms in region 2, $w_i = \underline{w}$ and $m_i = \bar{m}$. In such region,

$$\underline{w} < z_i \gamma [\bar{m} \Phi(\underline{w})]^{\gamma - 1} < \underline{w} + \frac{\Phi(\underline{w})}{\Phi'(\underline{w})}.$$

For firms in region 3, $m_i = \bar{m}$ and $w_i > \underline{w}$. The wage w_i is pinned down by the condition

$$z_i \gamma [\bar{m} \Phi(w_i)]^{\gamma - 1} = w_i + \frac{\Phi(w_i)}{\Phi'(w_i)}.$$

D.2 A fixed point for μ

For every i, denote by $R(z_i, \mu)$ the vector of operating profits conditional on productivity z_i and distribution μ, computed as described above. The incumbent's Bellman equation is

$$V(z_i, \mu) = \int \max \left\{ 0, R(z_i, \mu) - \kappa + \beta \sum_j \pi_{ij} V(z_j, \mu) \right\} \psi(\kappa) d\kappa.$$

As long as the function $R(z_i, \mu) + \beta \sum_j \pi_{ij} V(z_j, \mu)$ is strictly monotone, there exists $\hat{\kappa}(z_i)$ such that

$$\hat{\kappa}(z_i) = R(z_i, \mu) + \beta \sum_j \pi_{ij} V(z_j, \mu). \tag{12}$$

It follows that one can rewrite

$$V(z_i, \mu) = \int^{\hat{\kappa}(z_i)} \left\{ R(z_i) - \kappa + \beta \sum_j \pi_{ij} V(z_j, \mu) \right\} \psi(\kappa) d\kappa.$$

In turn, using (12), one can write

$$\begin{aligned} V(z_i,\mu) &= \int^{\hat{\kappa}(z_i)} \{\hat{\kappa}(z_i) - \kappa\}\, \psi(\kappa) d\kappa \\ &= \Psi[\hat{\kappa}(z_i)]\hat{\kappa}(z_i) - \int^{\hat{\kappa}(z_i)} \kappa\psi(\kappa) d\kappa \\ &= \Psi[\hat{\kappa}(z_i)]\left[\hat{\kappa}(z_i) - E(\kappa|\kappa \leq \hat{\kappa}(z_i)\right]. \end{aligned}$$

Substituting back in (12), one obtains the condition

$$\hat{\kappa}(z_i) - \beta \sum_j \pi_{ij} \Psi[\hat{\kappa}(z_j)]\left[\hat{\kappa}(z_j) - E(\kappa|\kappa \leq \hat{\kappa}(z_j)\right] = R(z_i,\mu). \tag{13}$$

For $i = 1, ..., N$, the system of equations (13) identifies the vector of cost thresholds $\hat{\kappa}(z_i)$ consistent with the distribution μ.

Recall that $\Psi[\hat{\kappa}(z_i)]$ is the probability of survival, conditional on z_i. Invoking the law of large number, $\Psi[\hat{\kappa}(z_i)]$ is also the fraction of firms that survive, conditional on a productivity draw z_i.

Let $\mathcal{E}(z_i)$ denote the mass of entrants that obtained a draw z_i. Conditional a signal threshold η^*, we have

$$\mathcal{E}(z_i) = M \int_{\eta^*} g_i(\eta) h(\eta) d\eta.$$

In turn, the threshold is pinned down by the free-entry condition

$$\sum_i g_i(\eta^*) \Psi[\hat{\kappa}(z_i)]\left[\hat{\kappa}(z_i) - E(\kappa|\kappa \leq \hat{\kappa}(z_i)\right] = c_e.$$

Stationarity requires that

$$\mu(z_i) = \Psi[\hat{\kappa}(z_i)] \left(\sum_j \pi_{ji} \mu(z_j) + \mathcal{E}(z_i) \right).$$

Let $\hat{\mathbf{\Psi}}$ denote a $N \times N$ diagonal matrix such that $\hat{\mathbf{\Psi}}_{ii} = \Psi[\hat{\kappa}(z_i)]$. We can rewrite the above condition as

$$\mu = (\Pi \hat{\mathbf{\Psi}})^T \mu + \hat{\mathbf{\Psi}} \mathcal{E}$$

It follows that the stationary measure must satisfy

$$\mu = \left[I - (\Pi \hat{\mathbf{\Psi}})^T \right]^{-1} \hat{\mathbf{\Psi}} \mathcal{E},$$

where $\hat{\mathbf{\Psi}}$ depends on μ by (13) and $\mathcal{E}$ depends itself on $\hat{\mathbf{\Psi}}$.

D.3 Non-parametric specification of the operating cost distribution

Instead of assuming a parametric form for the distribution Ψ, we retrieve it non-parametrically by imposing that in the initial steady state, the model be consistent with the vector of conditional exit rates X_i. The conditional exit rate (i.e. the probability of exit conditional on a given productivity level) is simply

$$X_i = \sum_i \pi_{ij} \left\{1 - \Psi[\hat{\kappa}(z_j)]\right\}.$$

The unconditional exit rate is

$$\sum_i \mu_i \sum_j \pi_{ij} \left\{1 - \Psi[\hat{\kappa}(z_j)]\right\}$$

We select values for Ψ_j such that

$$\sum_i \pi_{ij} \left\{1 - \Psi_j\right\} = \mathrm{X}_i$$

for all i.

The vector of probabilities Ψ_j can then be used to back out μ and solve for the equilibrium $R(z)$. Finally, the thresholds for the fixed cost $\hat{\kappa}_i$ can be backed out from the system of equations

$$\hat{\kappa}_i - \beta \sum_j \pi_{ij} \Psi_j \left[\hat{\kappa}_j - E(\kappa | \kappa \leq \hat{\kappa}_j\right] = R(z_i).$$

Notice that the above conditions yield no restrictions regarding the distribution $\Psi(\kappa)$ for $\kappa < \hat{\kappa}_1$. We will assume that $\kappa \in \Re^+$ and that the distribution features an atom for $\kappa = 0$. The execution will reveal that none of our conclusions depend upon this assumption.

D.4 Algorithm – The initial steady state

1. Choose a vector of conditional exit rates $X_{(N\times 1)}$
2. Given X, compute the vector $\hat{\Psi}_{(N\times 1)}$ using $\hat{\Psi} = \mathbf{1} - \Pi^{-1} X$. For every j, the element $\hat{\Psi}_j$ is the cdf of Ψ evaluated at the cost threshold $\hat{\kappa}_j$
3. Choose the threshold for the signal η^*
4. The choice of η^* pins down the vector of entrants $\mathcal{E}$
5. Denote as $\hat{\mathbf{\Psi}}_{(N\times N)}$ the diagonal matrix with the elements of $\hat{\Psi}_{N\times 1}$ on the diagonal. Then we can determine μ by computing

$$\mu = \left[I - (\Pi \hat{\mathbf{\Psi}})^T\right]^{-1} \hat{\mathbf{\Psi}}^T \mathcal{E},$$

6. Once we have μ, we can determine the vector of revenues R

7. Once we have the vector R, we can compute the thresholds for exit on the cost space using the system of linear equations

$$\hat{\kappa}_i - \beta \sum_j \pi_{ij} \hat{\Psi}_j \left[\hat{\kappa}_j - E(\kappa | \kappa \leq \hat{\kappa}_j) \right] = R_i.$$

Denote as ψ the pdf associated to the cdf Ψ. Rewrite the latter, which is equation (13) recovered above, as

$$\hat{\kappa}_i - \beta \sum_j \pi_{ij} \hat{\Psi}_j \hat{\kappa}_j + \beta \sum_{j=1} \pi_{ij} \int_{\hat{\kappa}_0}^{\hat{\kappa}_j} \kappa \psi(\kappa) d\kappa = R_i, \tag{14}$$

where $\hat{\kappa}_0$ is arbitrary.

For the purpose of computing the integral in (14), we need to make assumptions about the density ψ. We will assume that it is piece-wise uniform.

D.4.1 Piece-wise uniform representation of the density ψ

We posit that

$$\psi(\kappa) = \psi_s \quad \forall\, \kappa \in [\hat{\kappa}_{s-1}, \hat{\kappa}_s], \quad s = 1, ..., N.$$

We also assume that the lower bound of the support is $\kappa_0 = 0$, and we allow for an atom at $\kappa_0 = 0$, i.e. $\Psi_0 \geq 0$. Then, the sequence of constants ψ_s are pinned down by the condition

$$\int_{\hat{\kappa}_{s-1}}^{\hat{\kappa}_s} \psi_s d\kappa = \Psi_s - \Psi_{s-1}, \quad \forall s > 0 \tag{15}$$

It follows that

$$\psi_s = \frac{\Psi_s - \Psi_{s-1}}{\hat{\kappa}_s - \hat{\kappa}_{s-1}}$$

and that

$$\int_{\hat{\kappa}_{s-1}}^{\hat{\kappa}_s} \psi_s \kappa d\kappa = \frac{1}{2} \left(\Psi_s - \Psi_{s-1} \right) \left(\hat{\kappa}_s + \hat{\kappa}_{s-1} \right)$$

$$\hat{\kappa}_i - \beta \sum_j \pi_{ij} \hat{\Psi}_j \hat{\kappa}_j + \frac{\beta}{2} \sum_{j=1} \pi_{ij} \sum_{s=1}^{j} \left(\Psi_s - \Psi_{s-1} \right) \left(\hat{\kappa}_s + \hat{\kappa}_{s-1} \right) = R_i,$$

Let $\mathcal{A}$ denote the diagonal matrix $(N \times N)$ with generic element $\mathcal{A}_{ss} = \Psi_s - \Psi_{s-1}$. Then, $\mathcal{A}\kappa$ is a vector $(N \times 1)$ with generic element s equal to $(\Psi_s - \Psi_{s-1})\hat{\kappa}_s$.

$$\mathcal{A} = \begin{pmatrix} \Psi_1 - \Psi_0 & 0 & \dots & 0 \\ 0 & \Psi_2 - \Psi_1 & \dots & 0 \\ \vdots & & & \vdots \\ 0 & \dots & \dots & \Psi_N - \Psi_{N-1} \end{pmatrix}.$$

Let $\mathcal{B}$ denote the matrix $(N \times N)$ with element $\mathcal{B}_{s+1,s} = \Psi_s - \Psi_{s-1}$ and all other elements

equal to 0. Then, $\mathcal{B}\kappa$ is a vector $(N \times 1)$ with generic element s equal to $(\Psi_s - \Psi_{s-1})\hat{\kappa}_{s-1}$.

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & \dots & \dots & 0 \\ \Psi_2 - \Psi_1 & 0 & \dots & \dots & 0 \\ 0 & \Psi_3 - \Psi_2 & & & \vdots \\ \vdots & & & & \vdots \\ 0 & \dots & \dots & \Psi_N - \Psi_{N-1} & 0 \end{pmatrix}.$$

Let $\mathcal{T}$ denote a lower-triangular matrix were all the non-zero elements are 1. That is, for every s, $\mathcal{T}_{sr} = 1$ if $r \leq s$ and $\mathcal{T}_{sr} = 0$ otherwise.

$$\mathcal{T} = \begin{pmatrix} 1 & 0 & \dots & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & 1 & 1 & & \vdots \\ \vdots & 1 & & 1 & 0 \\ 1 & 1 & \dots & 1 & 1 \end{pmatrix}.$$

Then, the system of equations above is linear and can be written in matrix form as

$$\kappa - \beta\Pi\hat{\mathbf{\Psi}}\kappa + \frac{\beta}{2}\Pi\mathcal{T}(\mathcal{A} + \mathcal{B})\kappa = R$$

It follows that

$$\kappa = \left[I - \beta\Pi\hat{\mathbf{\Psi}} + \frac{\beta}{2}\Pi\mathcal{T}(\mathcal{A} + \mathcal{B})\right]^{-1} R.$$

The cost of entry c_e is given by free-entry condition:

$$\sum_i g_i(\eta^*)\Psi_i \left[\hat{\kappa}_i - E(\kappa|\kappa \leq \hat{\kappa}_i)\right] = c_e.$$

The latter can be rewritten as

$$\sum_i g_i(\eta^*)\Psi_i\hat{\kappa}_i - \sum_i g_i(\eta^*) \int_{\hat{\kappa}_0}^{\hat{\kappa}_i} \kappa\psi(\kappa)d\kappa = c_e.$$

The algorithm also yields the discrete approximation for the cdf $\Psi(\kappa)$ and the associated density $\psi(\kappa)$, defined over the support $[\hat{\kappa}_0, +\infty)$, which can be used for comparative statics.

D.5 Comparative Statics

1. Guess η^*.

   (a) Make an initial guess for the vector of cost thresholds $\tilde{\kappa}_0$. Associated with it, there will be a cdf $\widetilde{\Psi}_0$.

(b) Figure out the vector of entrants $\mathcal{E}_0$ and the stationary distribution of firms is

$$\mu_0 = \left[I - (\Pi \widetilde{\mathbf{\Psi}}_0)^T \right]^{-1} \widetilde{\mathbf{\Psi}}_0^T \mathcal{E}_0,$$

(c) Compute the revised guess of the vector of cost thresholds $\tilde{\kappa}_1$ by solving

$$\tilde{\kappa}_{1i} = R_i + \beta \sum_j \pi_{ij} \Psi(\tilde{\kappa}_{0j}) \left[\tilde{\kappa}_j - E(\kappa | \kappa \leq \tilde{\kappa}_j) \right],$$

(d) Repeat the process until convergence, i.e. $||\tilde{\kappa}_T - \tilde{\kappa}_{T-1}||$ is small enough

2. Check that the free-entry condition holds

$$\sum_i g_i(\eta^*) \Psi(\tilde{\kappa}_{Ti}) \left[\tilde{\kappa}_{Ti} - E(\kappa | \kappa \leq \tilde{\kappa}_{Ti}) \right] = c_e$$

3. If not, then revise the guess for η^* and repeat