(Ir)responsible Takeovers*

Philip Bond† **Doron Levit**‡

March 20, 2025

Abstract

Takeovers change corporate policies, affecting both firm value and externalities imposed on various stakeholders. How, if at all, do shareholders, bidders and incumbents respond to externalities in takeovers? We study this question by introducing externalities and social preferences into a canonical model of takeovers (Bagnoli and Lipman 1988). Our analysis highlights the interplay between the holdout problem in takeovers (Grossman and Hart 1980) and free riding in public good provision. We show that the dual free-riding problems offset each other. Shareholders' preferences over externalities generated by firms they have divested from play a key role. If shareholders care about such externalities, then acquisitions succeed if and only if they are socially efficient, free-riding problems notwithstanding. Moreover, both incumbents and bidders have incentives to maximize the social value of the firm. In contrast, "warm-glow" shareholders accept some socially inefficient acquisitions while rejecting some socially efficient ones; and incumbents and bidders generally respond by adopting socially inefficient policies. Incumbents use corporate social responsibility as a takeover defense. Social responsibility by bidders is counterproductive if target shareholders care about divested externalities, but helps offset inefficiencies created by warm-glow shareholders. Overall, our analysis sheds light on the intricate interplay between shareholder behavior, social responsibility, and acquisition dynamics.

Keywords: Corporate Social Responsibility, Takeovers, M&A, ESG, Externalities, Divestment, Corporate Governance

JEL classifications: D74, D82, D83, G34, K22

*We are grateful for helpful comments from seminar participants at the Hebrew University; Georgia State University; the University of California, Los Angeles; and the University of Washington.

†University of Washington. Email: apbond@uw.edu.

‡University of Washington. Email: dlevit@uw.edu.

1 Introduction

Many shareholder profess to care about the externalities generated by their portfolio firms. The point at which shareholders enjoy their most direct influence over corporate decisions is during contests for corporate control. In this paper we ask: If shareholders indeed care about the externalities generated by their portfolio firms then what is the outcome of a corporate control contest? In particular, is the outcome socially desirable? In other words: Is a world with externality-minded shareholders something that we should wish for?

This analysis serves as a thought experiment, but one with immediate relevance to current social and legal debates. At its core is the question of whether investors should consider externalities generated by their portfolio companies, or instead focus exclusively on profit-maximization as advocated by the Friedman doctrine and common interpretations of fiduciary duty. Beyond its normative implications, our analysis generates novel predictions about takeover outcomes when externality considerations influence investor decisions.

Even absent considerations of externalities, takeovers are exposed to freerider problems (Grossman and Hart 1980, Bagnoli and Lipman 1988). Externalities add to these freeriding problems, because even a shareholder who internalizes externalities reasons (correctly) that he/she has little influence over takeover outcomes.

Our main results characterize the conditions under which the market for corporate control works well, in the sense of blocking socially inefficient takeovers and allowing socially efficient ones. A leading case in which the market works well is that in which target shareholders continue to care about externalities even after they sell their shares, while bidders follow the "Friedman doctrine" and focus entirely on shareholder value. At the opposite extreme, the market for corporate control *also* works well if target shareholders have "warm glow" preferences and cease to care about externalities after divestment, while bidders have similar warm glow preferences and care about externalities only in proportion to their ownership stake. More generally, we characterize the mapping from bidder and target-shareholder preferences to outcomes.

Private equity (PE) firms play an important but controversial role in the corporate-control market. Our analysis provides support for the arguments of both critics and defenders of PE. Leveraged acquisition offers can indeed lead to socially inefficient acquisitions, as critics charge.

But more positively, leveraged offers also increase the success probability of socially efficient acquisitions.

Many investors say that they incorporate environmental and social (ES) factors into investment decisions and thereby practice "responsible investing."[1] However, the effectiveness of responsible investment strategies in shaping corporate policies—whether through voting, engagement with the company's management, or divestment of the company's shares from the investor's portfolio—remains a topic of ongoing research and debate.[2] The market for corporate control is a natural domain for responsible investment to make an impact. Takeovers, which change the ownership and control of corporate assets, are a powerful mechanism for influencing corporate policies or preempting anticipated changes. Transactions such as mergers, acquisitions, and leveraged buyouts are typically driven by operational and financial synergies, or the capture of private benefits of control. These not only affect the shareholder value, but also generate externalities for various stakeholders of the firms. Examples include layoffs of employees (e.g., Dessaint, Golubov and Volpin 2017), improved workplace safety (Cohn, Nestoriak, and Wardlaw 2021), increased market power and lower consumer welfare (e.g., Eckbo 1983 and Borenstein 1990), increased innovation (Phillips and Zhdanov 2013), curbed innovation (Cunningham, Ederer, and Ma 2021), pollution (Bellon 2020), and broader societal impacts such as free speech (e.g., Elon Musk's buyout of Twitter), journalism (Ewens, Gupta, and Howell 2022), education (Eaton, Howell, and Yannelis 2019), and healthcare (Gupta et al. 2021; Liu 2022). Importantly, takeovers often require the approval of the target shareholders, and thus

[1] In 2006, the United Nations established the Principles for Responsible Investment (PRI), which was signed by 63 investors managing a total of $6.5 trillion. By the end of 2021, this had grown to 3,404 investors, representing $121 trillion. Beyond the UN PRI, many asset managers have signed up to national stewardship codes which also require incorporation of ESG factors into investment decisions. 1,500 asset owners, collectively managing $40 trillion, have publicly committed to divest from fossil fuels (see https://divestmentdatabase.org/).

[2] Teoh, Welch, and Wazzan (1999) found that the South Africa exclusion campaign had minimal impact on company valuations. Berk and van Binsbergen (2021) argue that exclusion driven by ES motivations has little effect on capital costs. Gibson et al. (2022) demonstrated that U.S. institutional investors adhering to the Principles for Responsible Investment do not enhance the ESG scores of their portfolio companies. Heath et al. (2023) showed that while ES funds target firms with strong ES performance, their investment does not improve it. In contrast, Zerbib (2022) reported a significant annual return premium from exclusion. Green and Vallee (2023) observed that bank divestment policies reduce total debt and assets for coal firms. Hartzmark and Shue (2023) found that higher financing costs for brown firms lead to significant negative changes in their impact. Gantchev, Giannetti, and Li (2022) showed that the threat of exit due to negative ES incidents motivates managers to improve ES performance. Several studies provide evidence that shareholder activism and engagement influence firms' ES policies: Dimson, Karakas, and Li, (2015); Hoepner et al., (2024); Naaraayanan, Sachdeva, and Sharma, (2021); Akey and Appel, (2020); and Chen, Dong, and Lin, (2020).

present a unique opportunity for investors to exercise their social responsibility. Empirical evidence suggests that externalities and social preferences do play a role in the market for corporate assets and control (e.g., Duchin, Gao, and Xu 2024; Li, Peng, and Yu 2023; Berg, Ma, and Streitz 2023), and recent surveys by PWC and Deloitte of M&A practitioners further highlight the growing importance of ES considerations in the dynamics of deal-making.[3]

Formally, we introduce externalities and social preferences into the canonical takeover model of Bagnoli and Lipman (1988). Bagnoli and Lipman study a tender offer model with a finite number of dispersed target shareholders. The bidder makes a cash offer to buy shares from target shareholders, and then shareholders individually decide whether to tender or retain their shares. The takeover is successful if and only if a majority of shares are tendered. If the takeover fails, no shares are acquired, and the incumbent retains control. If the takeover succeeds, all tendered shares are acquired, the bidder obtains control of the target, and non-tendering shareholders become minority shareholders in the acquired firm.

Crucially, the takeover affects not only the firm's value but also the externalities it generates, which can be either positive or negative. We label a takeover as privately efficient if it increases firm value, and socially efficient if it increases the sum of firm value and its externalities.[4] Our framework allows for situations where a takeover is socially inefficient but privately efficient, and vice versa. Importantly, target shareholders consider both the value of their shares and the firm's externalities. We explore cases in which shareholders' social preferences are invariant to their ownership in the firm and cases in which they are not (e.g., "warm-glow" preferences). In our baseline analysis the bidder is motivated solely by profit.

As noted, our analysis revolves around two fundamental free-rider problems. First, there is the "holdout" problem in takeovers (Grossman and Hart 1980): if shareholders expect the takeover to succeed, they can benefit from retaining their shares and enjoying the full post-takeover value. This free-riding problem stems from the non-excludability of the takeovers' pecuniary effects. Second, there is the classic free-riding problem in public goods (e.g., Baumol 1952): each shareholder can rely on others to contribute, in this case, by rejecting (accepting) high (low) premium offers when the takeover externality is negative (positive). This free-riding

[3] See https://www2.deloitte.com/content/dam/Deloitte/us/Documents/mergers-acqisitions/us-2024-ma-esg-survey.pdf and https://www.pwc.com/gx/en/services/sustainability/publications/private-equity-and-the-responsible-investment-survey.html

[4] Notice, a takeover can create negative (positive) externalities and still be socially efficient (inefficient).

problem stems from the non-excludability of a takeover's social externalities.

We show that, in equilibrium, these dual free-rider problems have the same force and offset each other. Specifically, when the social preferences of target shareholders are invariant to their ownership, efficiency is achieved—socially inefficient (efficient) takeovers fail (succeed) even if they are privately efficient (inefficient). Notably, this result holds no matter how dispersed shareholders are and even though the bidder's sole objective is maximizing profits from the takeover.

To see the intuition, suppose the takeover is privately efficient but generates externalities that are sufficiently negative that it is socially inefficient. One might expect the takeover bid to succeed because even a socially responsible shareholder may reason that individually he/she has little ability to prevent an inefficient outcome—and that it is consequently better to accept the bid premium and tender. This reasoning reflects the classic free-rider problem in public goods, now manifesting itself in the context of takeovers with externalities. However, there is another force at play. An individual shareholder might be tempted to reject the bid, retain his/her shares, and become a minority shareholder in the acquired firm. By doing so, he/she captures the full appreciation of the share value post-takeover, rather than settling for the smaller bid premium like others. This is the well-known holdout problem in takeovers—which in this case provides a safeguard against the public-good free-riding problem and prevents a socially inefficient takeover. If instead the takeover is both socially and privately efficient, then the takeover externalities cannot be too negative, and shareholders have even stronger incentives to tender. In this case, the public good free-riding prevails, and the takeover succeeds, which is the efficient outcome.

Now, consider the opposite case in which the takeover is privately inefficient but generates externalities that are sufficiently positive that it is socially efficient. With socially responsible shareholders, this should allow the bidder to lower the bid and acquire the firm while still making a profit (even though the takeover would decrease firm value). However, the public good free-riding implies that target shareholders don't fully account for the social benefits from the takeover, limiting the price discount that would induce them to tender—making it unprofitable for the bidder. Surprisingly, the takeover still succeeds. When the takeover is privately inefficient, the flip side of the holdout problem is the pressure on target shareholders to tender out of fear that if they don't tender, they will be left with a depreciated share value

under the bidder's control (Bebchuk 1987). This force facilitates a socially efficient outcome. If instead the takeover is both socially and privately inefficient, then the externalities are not sufficiently positive to induce the responsible shareholders to tender, the public good free-riding prevails, and the takeover fails.

The efficiency results described above change dramatically when target shareholders have warm-glow social preferences. With such preferences, shareholders do not internalize the firm's externalities if they no longer own its shares. Specifically, when the takeover externalities are negative, socially inefficient takeovers sometimes succeed in equilibrium, and when the takeover externalities are positive, socially efficient takeovers may sometimes fail.

Intuitively, with warm-glow preferences, shareholders dislike (like) holding a share of a firm that generates negative (positive) externalities—giving them additional incentives to tender (keep) their shares. If the takeover externalities are negative, the bidder can "threaten" target shareholders with these externalities if they don't tender their shares; the bidder lowers the premium and makes a strictly positive profit even when the target's ownership is widely dispersed. In this case, warm-glow preferences amplify public good free-riding and weaken the holdout problem that provided a safeguard against socially inefficient takeovers. By contrast, if the takeover externalities are positive, the bidder must "tempt" shareholders to tender by offering even a larger premium, leaving the bidder with insufficient profits. In this case, warm-glow preferences amplify the public good free-riding and weaken the pressure to tender phenomena that was needed in order to facilitate socially efficient takeovers.

Our framework is flexible enough to embed social preferences under which shareholders internalize the takeover externalities to a larger extent if their individual actions contributed to these externalities, that is, if they tender their shares and takeover succeeded. Intuitively, shareholders bear responsibility for their actions. These preferences are the opposite of warm-glow preferences, and we label them as "hyper-consequentialism." With such preferences, an analogous inefficiency result is obtained: when the takeover externalities are negative, socially efficient takeovers can fail, and when the takeover externalities are positive, socially inefficient takeovers might succeed.

In practice, the motivation of the bidding firm (or its own shareholders) for the takeover might go beyond profit. We therefore consider cases in which the bidder internalizes, at least partly, the externalities from the takeover, i.e., the bidder itself is socially responsible. We show

that the bidder's social responsibility "adds" to the target shareholders' social responsibility. Specifically, if target shareholders' social preferences are invariant to their ownership or exhibit hyper-consequentialism, then the bidder's social responsibility can be counterproductive and reduce efficiency of takeover outcomes. However, if target shareholders exhibit warm-glow preferences, then the bidder's social responsibility can increase efficiency of takeover outcomes. Combined, the efficiency of the market for corporate control depends on the social preferences of both counterparties, and more social responsibility is not always better.

Generally, the pecuniary value and the externalities created by a takeover are determined by the same production technology or corporate policies the firm employs. For example, the negative externalities to consumer welfare from horizontal mergers could be the byproduct of the acquiring firm's plan to realize operational synergies via increased market power. In the last part of the paper, we consider how the bidder and the incumbent balance pecuniary value and social externalities in their decisions on how best to run the firm.

Consider the bidder first. We show that if shareholders' social preferences are invariant to their ownership, the bidder will commit to producing the efficient level of externalities post-takeover. That is, the profit-maximizing bidding strategy is to maximize the social value from the takeover, reinforcing our first efficiency result. By contrast, if target shareholders have warm-glow (hyper-consequentialism) preferences, then the bidder has incentives to create more (less) value and produce (less) more negative externalities than is socially optimal. Intuitively, with such social preferences, the production technology can be used to threaten (tempt) shareholders to sell the firm, and in this respect, social irresponsibility is an effective bidding strategy.

Interestingly, the incumbent can also use similar policies pre-takeover to either increase the welfare of target shareholders or as an effective takeover defense (i.e., to limit the probability of a successful takeover). Specifically, we show that if shareholders' social preferences are invariant to their ownership, then the socially optimal production plan both maximizes target shareholders' welfare and reduces the probability of a takeover. Intuitively, with such preferences, the social optimum on one hand maximizes the shareholder welfare if the takeover fails, but it also forces the bidder to make a higher offer in order to convince them to tender, which in turn benefits shareholders and minimize the probability of a takeover. By contrast, when shareholders' social preferences exhibit warm-glow or hyper-consequentialism, maximiz-

ing their welfare might require the incumbent to deviate from the socially optimal production to limit the bidder's ability to exploit shareholders' social preferences. Moreover, in those cases, socially irresponsible production would achieve the lowest probability of a takeover. Combined, our analysis shows that social (ir)responsibility can serve as an effective takeover defense.

Overall, our analysis demonstrates that externalities in takeovers and social responsibility have significant positive and normative implications for the market for corporate control.

Related Literature

Our paper is related to two main strands of literature. First, we contribute to the theoretical literature on takeovers. Besides Bagnoli and Lipman (1988), Holmstrom and Nalebuff (1992), Gromb (1993), Cornelli and Li (2002), Marquez and Yılmaz (2008), Dalkır and Dalkır (2014), Dalkır (2015), Ekmekci and Kos (2016), Dalkır, Dalkır, and Levit (2019), and Voss and Kulms (2022) study variants of tender offer models with a finite number of shareholders. Unlike these studies, we examine the effects of takeover externalities and social preferences on the takeover dynamics. Baron (1983), Ofer and Thakor (1987), Harris and Raviv (1988), Bagnoli, Gordon, and Lipman (1989), Berkovitch and Khanna (1990), Hirshleifer and Titman (1990), and Levit (2017) studied various mechanisms through which managers and boards of target companies can resist and influence takeover outcomes. Relative to these papers, we study the circumstances under which social (ir)responsibility can be an effective takeover defense.[5]

Second, we contribute to the theoretical literature on the effects of responsible investment on corporate policies. A growing number of papers studies the effects of portfolio allocations and divestment strategies on corporate policies: Heinkel, Kraus, and Zechner (2001), Davies and Van Wesep (2018), Oehmke and Opp (2024), Edmans, Levit, and Schneemeier (2022), Landier and Lovo (2024), Green and Roth (2024), and Chowdhry, Davies, and Waters (2019), Huang and Kopytov (2022), Gupta, Kopytov and Starmans (2022), Piccolo, Schneemeier, and Bisceglia (2022), Pastor, Stambaugh, and Taylor (2021), Pedersen, Fitzgibbons, and Pomorski (2021), Baker, Hollifield, and Osambela (2022), and Goldstein et al. (2022). Broccado, Hart,

[5] A larger body of literature followed Grossman and Hart (1980) and studied various implications and variants of the holdout problem in takeovers when shareholders are infinitesimal: Yarrow (1985), Shleifer and Vishny (1986), Kyle and Vila (1991), Burkart, Gromb, and Panunzi (1998, 2000), Amihud, Kahan and Sundaram (2004), Mueller and Panunzi (2004), Marquez and Yilmaz (2008), Gomes (2012), At, Burkart and Lee (2011), Burkart and Lee (2015, 2022), Burkart, Lee and Petri (2023), and Burkart, Lee and Voss (2024).

and Zingales (2022) and Gollier and Pouget (2022) also study engagement and voting as alternative mechanisms to affect firm's externalities. Relative to this burgeoning literature, we study responsible investment in the context of takeovers. The decision of shareholders to tender their shares to the bidder can be viewed as combination of exit (selling the firm to the bidder) and voice (influencing who controls the target). Moreover, while the existing literature focuses on the classic free-rider problems in public goods, we highlight its interaction with another well-known free-rider problem, namely, the holdout problem of Grossman and Hart (1980).

2 Model

There are $N > 1$ shareholders indexed by i, each owns a single share in a target firm and each share carries one vote. A bidder is interested in taking over the firm, which is currently managed by the incumbent. The standalone value of the firm under the incumbent management is V_0. The value of the firm if the bidder successfully takes over is V_1. The firm also produces externalities. Under the incumbent, the externality is Φ_0, and under the bidder it is Φ_1. The change of control has implications for the externalities produced by the firm: If $\Phi_1 > \Phi_0$ ($\Phi_1 < \Phi_0$) then the takeover creates more (less) positive externalities or less (more) negative externalities, and in this respect, it is socially (ir)responsible. We label a takeover as socially efficient whenever $V_1 + \Phi_1 > V_0 + \Phi_0$, and socially inefficient if $V_1 + \Phi_1 < V_0 + \Phi_0$. Similarly, a takeover is privately efficient whenever $V_1 > V_0$, and privately inefficient if $V_1 < V_0$. Bagnoli and Lipman's (1988) model is a special case of our model when $\Phi_1 = \Phi_0 = 0$. We assume throughout that $V_1 + \Phi_1 \neq V_0 + \Phi_0$.

The bidder makes a tender offer at price of p per share. The bidder successfully gains control and takes over the firm if at least $K < N$ shares are tendered. We let $\kappa = \frac{K}{N} \in (0, 1)$ be the majority rule. We focus on conditional offers, that is, the bidder's offer is conditional on the success of the takeover: if less than K shares are tendered, the bidder buys no shares and the takeover fails. If K or more shares are tendered, then the bidder buys all tendered share, and the takeover succeeds. Conditional offers are the most common tender offers in practice.

Given the offer p, all shareholders decide simultaneously whether to tender their shares or reject the offer. Shareholder i can either tender or keep his/her share. A (mixed) strategy of shareholder i in a tender offer subgame is denoted by $\gamma_i \in [0, 1]$, where γ_i is the probability

that shareholder i tenders.

For use throughout, define $v_0 \equiv \frac{V_0}{N}$ and $v_1 \equiv \frac{V_1}{N}$, the per-share firm values under the incumbent and bidder. As discussed in the introduction, we want to understand the consequences of shareholders who take the externalities created by a firm seriously. Accordingly, we write $\phi_0 \equiv \frac{\Phi_0}{N}$ and $\phi_1 \equiv \frac{\Phi_1}{N}$, and assume that an individual's utility from holding a share if the incumbent retains control is $v_0 + \phi_0$, and is $v_1 + \phi_1$ if the bidder acquires control.

Since the offer is conditional on the takeover success, if the takeover fails, then shareholders' utilities are $v_0 + \phi_0$ regardless of whether or not they tender. If the takeover succeeds, the utilities of shareholders who retain their shares are $v_1 + \phi_1$. That is, if the takeover succeeds, non-tendering shareholders hold on to their shares and get the full post-takeover value,[6] as well as experience the externalities created by the takeover.

In contrast, shareholders who tender their shares when the takeover succeeds receive utility $p+\eta\phi_1$, where $\eta \geq 0$ captures the extent of consequentialism in shareholders' preferences. If $\eta \in [0, 1)$ then target shareholders internalize the post-takeover externalities to a larger extent when they keep their ownership in the firm, i.e., shareholders have *warm-glow* preferences.[7] If $\eta = 1$ then shareholders internalize the externalities to the same extent irrespective of their ownership in the firm; we label these as *ownership-invariant* preferences, and sometimes abbreviate this to simply *invariant* preferences. If $\eta > 1$ then shareholders internalize the externalities to a larger extent when selling shares, reflecting, for example, a sense of responsibility for outcomes they actively contributed to; we label this case as *hyper-consequentialist* preferences.

We focus on symmetric Nash equilibria, which is standard in the literature, and denote by (p^*, γ^*) the equilibrium offer and tendering strategy. If the bidder is indifferent between making a bid p and not making a bid at all, we assume the bidder does not make a bid, reflecting, for example, infinitesimal costs of bid preparation.

[6] In a freeze-out merger, minority shareholders are guaranteed to receive the original offer price regardless of whether they tender their shares. This might suggest that freeze-out mergers completely solve the holdout problem in takeovers (e.g., Amihud, Kahan, and Sundaram 2004). However, Dalkır, Dalkır, and Levit (2019) show that freeze-out mergers do not fully resolve the holdout problem as long as shareholders can be pivotal for the takeover, even if the probability of being pivotal is arbitrarily small. Mueller and Panunzi (2004) also highlight the limitations of freeze-out mergers, noting their vulnerability to legal challenges when shareholders are infinitesimal. Additionally, Bates, Becher, and Lemmon (2006) provide empirical evidence suggesting that minority shareholders retain some bargaining power in freeze-out mergers, further indicating that these mergers are not a complete solution to the holdout problem.

[7] The "warm-glow" label is most apt for positive externalities, $\phi_1 > 0$. For negative externalities $\phi_1 < 0$, "warm-glow" preferences are in fact "cold-prickle" preferences.

3 Analysis

3.1 Preliminaries

We start with notations and identities that will be useful in our analysis below. Suppose in equilibrium each shareholder is expected to tender with probability $\gamma \in [0,1]$. The probability of a successful takeover if $N-1$ shareholders follow tendering strategy γ while the remaining shareholder retains is

$$q \equiv \sum_{j=K}^{N-1} \binom{N-1}{j} \gamma^j (1-\gamma)^{N-1-j}. \tag{1}$$

Similarly, the probability that an individual shareholder's tendering decision is pivotal conditional on the other $N-1$ shareholders following tendering strategy γ is

$$\Delta \equiv \binom{N-1}{K-1} \gamma^{K-1} (1-\gamma)^{N-K}. \tag{2}$$

Hence $q + \Delta$ is the probability of a successful takeover if one shareholder tenders and all others follow tendering strategy γ. Moreover, the probability of a successful takeover if all shareholders follow strategy γ is

$$\Lambda \equiv (1-\gamma)\, q + \gamma\, (q + \Delta)\,. \tag{3}$$

Note that $\Lambda(\cdot)$, $q(\cdot)$, and $\Delta(\cdot)$, are all functions of γ. In order to ease notation, we omit the argument γ whenever possible.

3.2 Tendering decisions

In this section we analyze the tendering subgame. A shareholder's expected utility from retaining a share is

$$v_0 + \phi_0 + q\,(v_1 + \phi_1 - v_0 - \phi_0)\,, \tag{4}$$

that is, if the takeover nevertheless succeeds, the shareholder enjoys the full post-takeover value of the firm as a minority shareholder, as well as the externalities created by the takeover. A

shareholder's expected utility from tendering is

$$v_0 + \phi_0 + (q + \Delta)(p + \eta\phi_1 - v_0 - \phi_0). \tag{5}$$

That is, the likelihood the takeover succeeds increases by Δ, and if the takeover indeed succeeds, the shareholders gets the offer p and a utility $\eta\phi_1$ from the externalities created by the takeover. Thus, the marginal benefit of tendering is

$$\tau(\gamma; p) \equiv \underbrace{(p - v_1)\, q + (p - v_0)\, \Delta}_{pecuniary} + \underbrace{(\eta - 1)\, \phi_1 q + (\eta\phi_1 - \phi_0)\, \Delta}_{social}. \tag{6}$$

The first two terms of $\tau(\gamma; p)$ comprise the marginal pecuniary benefit from tendering, and the last two terms the marginal social benefit from tendering. Notice that we can rewrite

$$\tau(\gamma; p) = (q + \Delta)(p - \mu(\gamma)), \tag{7}$$

where

$$\mu(\gamma) \equiv v_0 + \phi_0 - \eta\phi_1 + \frac{q}{q + \Delta}(v_1 + \phi_1 - v_0 - \phi_0). \tag{8}$$

Since $\tau(0; p) = 0$, non-tendering ($\gamma^* = 0$) is always an equilibrium. Similarly, since $\tau(1; p) = p - v_1 - (1 - \eta)\phi_1$, tendering ($\gamma^* = 1$) is an equilibrium if and only if $p \geq v_1 + (1 - \eta)\phi_1$. Finally, a mixed strategy equilibrium with tendering probability $\gamma^* \in (0, 1)$ exists if and only if shareholders are indifferent between tendering and keeping their shares, that is, $p = \mu(\gamma^*)$.

Throughout the analysis we require the equilibrium to be stable: for some $\epsilon > 0$, $\tau(\gamma; p) > 0$ for all $\gamma \in (\gamma^* - \epsilon, \gamma^*)$ and $\tau(\gamma; p) < 0$ for all $\gamma \in (\gamma^*, \gamma^* + \epsilon)$. Intuitively, if other shareholders were to deviate and follow a slightly more (less) aggressive tendering strategy, the marginal benefit from tendering of each shareholder must be negative (positive), disincentivizing deviation and reinforcing the equilibrium.

The next result characterizes all stable equilibria of the tendering subgame.

Lemma 1. *Suppose the bidder makes a conditional tender offer p. Then:*

(*i*) *$\gamma^* = 0$ is an equilibrium if and only if either $p < v_0 + \phi_0 - \eta\phi_1$, or $p = v_0 + \phi_0 - \eta\phi_1$ and the takeover is socially efficient.*

(*ii*) $\gamma^* = 1$ *is an equilibrium if and only if either* $p > v_1 + (1-\eta)\,\phi_1$, *or* $p = v_1 + (1-\eta)\,\phi_1$ *and the takeover is socially efficient.*

(*iii*) $\gamma^* \in (0,1)$ *is an equilibrium if and only if* $p \in (v_0 + \phi_0 - \eta\phi_1, v_1 + (1-\eta)\,\phi_1)$. *A necessary condition is that the takeover is socially efficient. In this case,* γ^* *is the unique solution to*

$$\mu(\gamma^*) = p, \tag{9}$$

and satisfies

$$\gamma^* > \hat{\gamma}(p) \equiv \frac{1}{1 + \frac{v_1+\phi_1-v_0-\phi_0}{p+\eta\phi_1-v_0-\phi_0}\frac{N-K}{K-1}}. \tag{10}$$

Moreover, γ^* *increases continuously from* 0 *to* 1 *as* p *increases from* $v_0 + \phi_0 - \eta\phi_1$ *to* $v_1 + (1-\eta)\,\phi_1$*; decreases in* ϕ_0*; and increases in* ϕ_1 *if and only if* $\eta > \frac{q(\gamma^*)}{q(\gamma^*)+\Delta(\gamma^*)}$.

(*iv*) *Multiple equilibria exist if and only if* $p \in (v_1 + (1-\eta)\,\phi_1, v_0 + \phi_0 - \eta\phi_1)$. *A necessary condition is that the takeover is socially inefficient.*

Lemma 1 is somewhat intuitive: in the tendering subgame equilibrium, shareholders are more likely to tender their shares when the offer is higher, that is γ^* increases in p. Notice that if $\eta = 1$, then the tendering strategy in the subgame equilibrium depends only on $\phi_1 - \phi_0$, the change in the externalities produced by the firm due to the takeover, rather than their level.

The red curve in Figure 1 below depicts the marginal benefit from tendering under different scenarios. The blue points on left (right) column represent stable equilibria of the tendering subgame when the takeover is socially efficient (inefficient), and the yellow points represent

unstable equilibria.

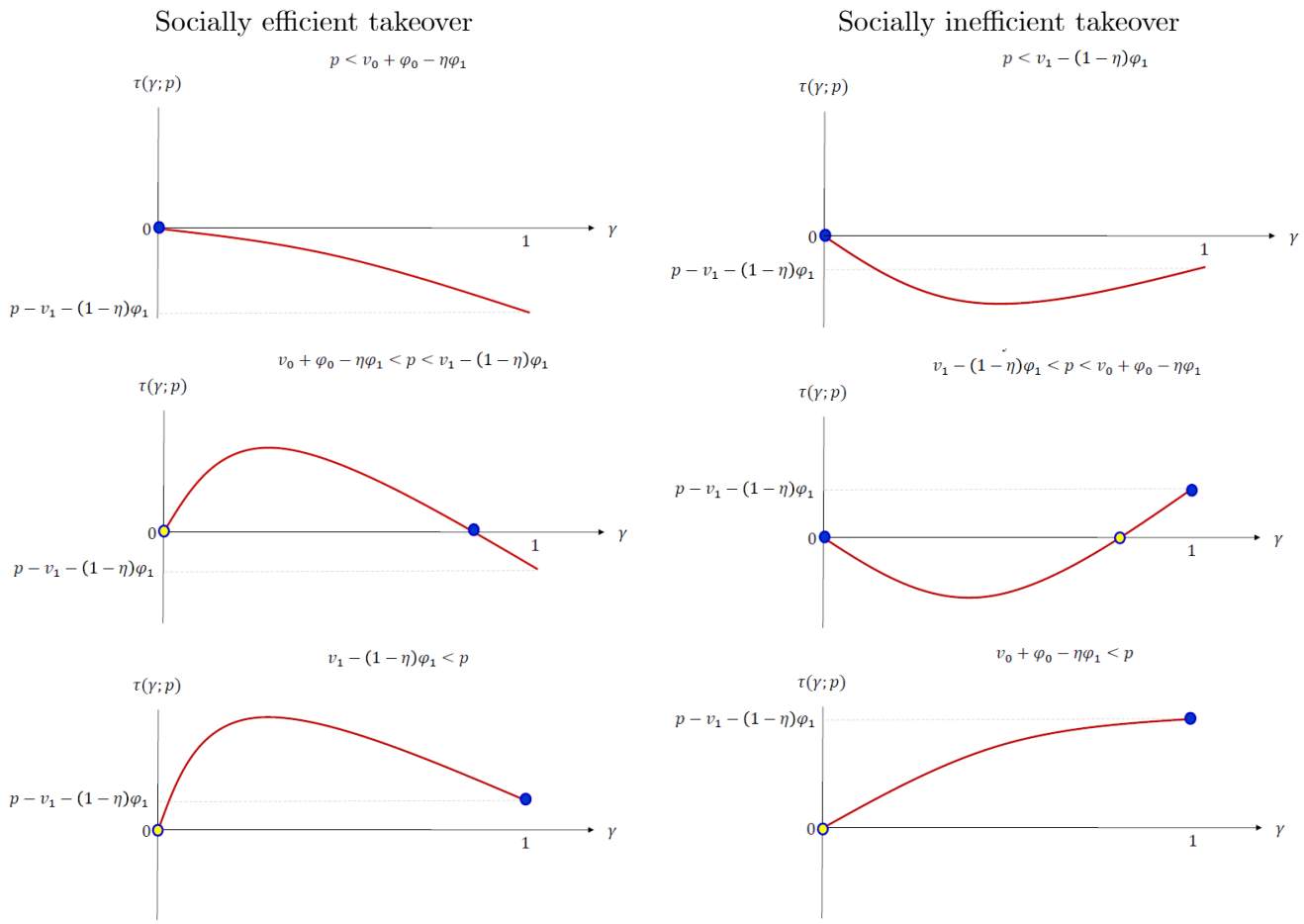


Figure 1

More generally, however, for a given p, the tendering probability γ^* increases in the post-takeover externalities ϕ_1 if and only if η is sufficiently large. To see the intuition, recall that the marginal social benefit from tendering, as stated in (6), can be rewritten as $\eta\left(q+\Delta\right)\phi_1 - q\phi_1 - \Delta\phi_0$. Thus, if $\eta < \frac{q^*}{q^*+\Delta^*}$, shareholders' warm-glow preferences are relatively strong, and they do not expect to experience the full extent of externalities if they tender shares and the takeover succeeds. Therefore, holding onto the shares, becoming a minority shareholder of the acquired firm, and essentially free-riding on other shareholders' tendering decisions, is relatively more valuable as ϕ_1 increases. If instead $\eta > \frac{q^*}{q^*+\Delta^*}$ then shareholders internalize enough of the externalities even if they no longer hold shares. Thus, larger ϕ_1 increases a shareholder's incentives to tender since doing so increases the likelihood of takeover-success and externalities

are realized. The comparative static of γ^* with respect to ϕ_0 is simpler: larger pre-takeover externalities implies a larger reservation utility for the shareholder, and hence, reduces the incentives to tender.

Interestingly, multiple equilibria potentially exist if the takeover is socially inefficient. By Lemma 1, multiple equilibria exist if and only if both tendering with probability one, and tendering with probability zero, are equilibria. Intuitively, if $v_1 + \phi_1 < v_0 + \phi_0$, the takeover is socially inefficient, and the benefit from keeping the share is decreasing with the likelihood the takeover succeeds. Thus, if shareholders expect a high (low) probability of tendering by other shareholders, then the benefit from retention is relatively low (high), and they might indeed choose to tender with a higher (lower) probability. In those cases, the tendering probability is self-fulfilling due to strategic complementarity among shareholders.[8]

Notice that multiple equilibria exist even if the equilibrium is privately efficient, that is, $v_1 - v_0 > 0$. Indeed, without externalities (i.e., $\phi_1 = \phi_0 = 0$), the equilibrium would have been unique in that case, and so, takeover externalities and shareholders' social concerns can result in unpredictable takeover outcomes.

3.3 Bidder's payoff

In this section we consider the optimal bid and the bidder's payoff. The bidder's expected profit from an offer to an individual shareholder, conditional on that shareholder tendering and all other shareholders playing the equilibrium strategy γ, is

$$(q + \Delta)(v_1 - p). \tag{11}$$

Indeed, since the offer is conditional on the takeover's success, if it fails, the bidder's payoff is zero, and if it succeeds, the bidder gets $v_1 - p$. In an equilibrium with $\gamma \in (0, 1)$, Lemma 1 implies $p = \mu(\gamma)$, where $\mu(\gamma)$ can be rewritten as

[8] In he knife edge case where $v_1 + \phi_1 = v_0 + \phi_0$ and $p = v_0 + \phi_0 - \eta\phi_1$ then $\gamma^* = [0, 1]$.

$$\mu(\gamma) = \underbrace{v_1 - (v_1 - v_0) - (\phi_1 - \phi_0)}_{\text{bid under full excludability}} + \underbrace{\frac{q}{q+\Delta}(v_1 - v_0)}_{\text{pecuniary non-excludability}} + \underbrace{\frac{q}{q+\Delta}(\phi_1 - \phi_0)}_{\text{social non-excludability}} \quad (12)$$

$$+ \underbrace{(1-\eta)\phi_1}_{\text{preference shift}} .$$

The quantity $\mu(\gamma)$ is the bid needed to induce a given tendering probability γ. The right hand side expresses this in terms of a discount relative to the highest bid that the bidder would offer, namely v_1.

The combined quantity $v_1 - (v_1 - v_0) - (\phi_1 - \phi_0)$ is the offer a bidder would make in a theoretical benchmark in which it is possible to exclude non-tendering shareholders from both the pecuniary and social effects of the takeover, and moreover, preferences are ownership invariant. In this case, the bidder is able to reduce its bid from v_1 by an amount equal to the sum of pecuniary and social value it creates for shareholders.

The second term is the effect of non-excludability of pecuniary benefits, relative to this benchmark. Similarly, the third term is the effect of non-excludability of social benefits, relative to the same benchmark.

The two non-excludability terms reflect, respectively, the holdout problem in takeovers, and the free-riding problem in public good provision. Importantly, these two problems are of exactly the same magnitude, as reflected by the common factor of $\frac{q}{q+\Delta}$ in both terms. In essence, the holdout problem is simply a specific manifestation of free-riding in public good provision.

The fourth term reflects how attitudes towards social externalities shift when shareholders sell their shares.

Substituting (8) into (11), the bidder's profit from an offer to an individual investor condi-

tional on him/her tendering, is

$$\underbrace{(q+\Delta)(v_1+\phi_1-v_0-\phi_0)}_{\text{value creation}} - \underbrace{q(v_1+\phi_1-v_0-\phi_0)}_{\text{cost to bidder of non-excludability}} - \underbrace{(q+\Delta)(1-\eta)\phi_1}_{\text{preference shift}} \quad (13)$$

$$= \Delta(v_1+\phi_1-v_0-\phi_0) - (q+\Delta)(1-\eta)\phi_1. \quad (14)$$

The first term in (13) is the bidder's profit in the benchmark described above, in which non-tendering shareholders can somehow be excluded from both the pecuniary and social effects of a takeover, and moreover shareholders' social preferences are ownership-invariant ($\eta = 1$).

The second term is the cost to the bidder of non-excludability. In particular, as the number of shareholders grows large, the pivotal probability grows small, and the cost of non-excludability approaches value creation. If a takeover has purely pecuniary effects, this statement is simply a recapitulation of the familiar holdout argument.

The third term reflects how attitudes towards social externalities shift when shareholders sell their shares. As is already clear from (13), if N is large then bidder profits are determined primarily by this preference shift. We explore the consequences in more detail below.

Since each shareholder tenders with probability $\gamma \in (0,1)$, and tendering decisions are independent across shareholders, a bidder's total profits are simply a multiple $N\gamma$ of (14), and are given by the function

$$\pi(\gamma) \equiv N\gamma\Delta(v_1-v_0+\phi_1-\phi_0) - N\gamma(q+\Delta)(1-\eta)\phi_1. \quad (15)$$

Combining the above observations

$$\Pi(\gamma) = \begin{cases} 0 & \text{if } \gamma = 0 \\ \pi(\gamma) & \text{if } \gamma \in (0,1) \\ N(v_1-p) & \text{if } \gamma = 1. \end{cases} \quad (16)$$

Notice $\pi(0) = \Pi(0)$. Also, $\pi(1) = -N(1-\eta)\phi_1$, and hence, $\pi(1) > \Pi(1) \Leftrightarrow p > v_1+\phi_1-\eta\phi_1$. Recall from Lemma 1 that $\gamma^* = 1$ is an equilibrium if and only if $p \geq v_1+\phi_1-\eta\phi_1$. Thus, $\Pi(\gamma)$

is continuous everywhere, possibly the with exception of $\gamma = 1$.[9] We let Π^* be the bidder's expected profit in equilibrium.

3.4 Equilibrium characterization

We obtain the following full characterization of equilibrium.

Proposition 1.

(*i*) *Suppose the takeover is socially inefficient,* $v_1 + \phi_1 < v_0 + \phi_0$*:*

- (*a*) *If* $v_1 + \phi_1 - v_0 - \phi_0 \leq (1-\eta)\phi_1$ *then there is an equilibrium in which the takeover fails with certainty.*
- (*b*) *If* $(1-\eta)\phi_1 < 0$ *then there is a continuum of equilibria, indexed by* $p^* \in (v_1 + (1-\eta)\phi_1, v_1]$*, in which the bidder offers* p^* *and all shareholders accept with certainty.*
- (*c*) *No other equilibrium exists.*

(*ii*) *Suppose the takeover is socially efficient,* $v_1 + \phi_1 > v_0 + \phi_0$*. In the unique equilibrium:*

- (*a*) *If* $(1-\eta)\phi_1 > 0$ *then* $\gamma^* \in [0, \kappa)$*. As* N *grows large the takeover success probability approaches* 0*, as does the bidder's payoff* Π^**.*
- (*b*) *If* $(1-\eta)\phi_1 = 0$ *then* $\gamma^* = \kappa$ *and* $p^* = \mu(\kappa)$*. The takeover success probability is bounded away from both* 0 *and* 1*. As* N *grows large the bidder's payoff* Π^* *approaches* 0*.*
- (*c*) *If* $(1-\eta)\phi_1 < 0$ *then* $\gamma^* \in (\kappa, 1)$*. As* N *grows large the takeover success probability approaches* 1 *and the bidder's payoff* Π^* *approaches* $-(1-\eta)\Phi_1$*.*

To develop intuitions, we consider several special cases of Proposition 1. We start with a benchmark case in which there are no externalities. In this case, the parameter η is irrelevant.

[9] If the bidder expects the takeover to succeeds with probability one then the optimal offer would be $p = v_1 + \phi_1 - \eta\phi_1$, and in that case, $\Pi(\gamma)$ is also continuous at $\gamma = 1$.

Corollary 1. *Suppose the firm does not produce any externalities, $\phi_1 = \phi_0 = 0$. Then, in the unique equilibrium:*

(*i*) *If a takeover is privately inefficient ($v_1 < v_0$) then it fails with probability one.*

(*ii*) *If a takeover is privately efficient ($v_1 > v_0$) then in equilibrium, $\gamma^* = \kappa$. For large N, the takeover's success is uncertain, and $\Pi^* \to 0$.*

The comparison between Proposition 1 and Corollary 1 has a couple of interesting takeaways. First, echoing the discussion that follows Lemma 1, takeover externalities and shareholders' social concerns result in unpredictable takeover outcomes when takeovers are socially inefficient. Second, if $(1-\eta)\,\phi_1 < 0$ then in spite of the holdout problem in takeovers, the bidder can make a strictly positive profit even when the target's ownership is widely dispersed (i.e., $N \to \infty$). Notice that $(1-\eta)\,\phi_1 < 0$ requires either negative post-takeover externalities and warm-glow preferences, or positive post-takeover externalities and $\eta > 1$. Intuitively, in the former cases the bidder exploits shareholders' disproportional disutility from becoming a minority shareholders in a firm that creates negative externalities, and in the latter case he exploits their disproportional utility from selling their shares to a firm that is creating a public good. Either way, the takeover externalities mitigate the holdout problem in takeovers.

3.4.1 Ownership-invariant preferences ($\eta = 1$)

We start by considering the special case of Proposition 1 in which shareholders have ownership-invariant preferences ($\eta = 1$). This case serves both as an important benchmark, and elucidates economic forces that shape outcomes when $\eta \neq 1$.

Corollary 2. *Suppose $\phi_1 \neq \phi_0$ and $\eta = 1$. Then, in the unique equilibrium:*

(*i*) *If a takeover is socially inefficient ($v_1 + \phi_1 < v_0 + \phi_0$) then it fails with probability one.*

(*ii*) *If a takeover is socially efficient ($v_1 + \phi > v_0 + \phi_0$) then $\gamma^* = \kappa$, $p^* = \mu(\kappa)$, and $p^* \in (v_0 - (\phi_1 - \phi_0), v_1)$ is decreasing in $\phi_1 - \phi_0$, and Π^* is increasing in $\phi_1 - \phi_0$.*

When shareholders have ownership-invariant preferences, the equilibrium outcome only depends on the difference between the post-takeover and pre-takeover externalities $\phi_1 - \phi_0$, rather than their individual levels.

Corollary 2(i) says that, no matter how dispersed shareholders are, a socially inefficient takeover is always blocked.

It is instructive to consider the case of a takeover that is privately efficient ($v_1 > v_0$) but sufficiently socially costly ($\phi_1 < \phi_0$) that the takeover is socially inefficient ($v_1 + \phi_1 < v_0 + \phi_0$).

One might have expected that successful bids would occur under these circumstances, on the grounds that shareholders suffer from a free-rider problem, with even a socially responsible shareholder reasoning that he/she has little ability to prevent the negative externality, and that, as such, it is individually better to accept the bid premium (recall, $p^* > v_0 - (\phi_1 - \phi_0) > v_0$). Indeed, decomposition (12) formalizes exactly this: with full excludability of social costs, the bidder increases its offer by $\phi_0 - \phi_1 > 0$ to compensate shareholders for the takeover's social costs. Free-riding arises from non-excludability, and reduces the compensation that the bidder must offer for social costs from $\phi_0 - \phi_1$ to $\phi_0 - \phi_1 - \frac{q}{q+\Delta}(\phi_0 - \phi_1)$.

However, tendering a share is subject to the well-known "holdout" problem—itself a free-rider problem—in which an individual shareholder is tempted to keep his/her share, become a minority shareholder in the acquired firm, and benefit from the increase in private value $v_1 - v_0$ rather than accept the smaller bid premium $p - v_0$. Again, decomposition (12) formalizes this: with full excludability of the pecuniary effects of a takeover, the bidder doesn't deliver any of the pecuniary benefits $v_1 - v_0$ to shareholders. The holdout problem arises from non-excludability, and forces the bidder to deliver $\frac{q}{q+\Delta}(v_1 - v_0)$ of the pecuniary benefits to shareholders.

Corollary 2(i) establishes that the "holdout" problem in takeovers safeguards against the free-rider problem in social externalities (public good provision). Specifically: social efficiency entails a comparison of the pecuniary benefit $v_1 - v_0$ with the social cost $\phi_0 - \phi_1$. The social free-riding problem implies that the bidder internalizes only a fraction of the social cost, specifically $\phi_0 - \phi_1 - \frac{q}{q+\Delta}(\phi_0 - \phi_1)$. But the holdout problem implies that the bidder internalizes only a fraction of the pecuniary benefit, specifically $v_1 - v_0 - \frac{q}{q+\Delta}(v_1 - v_0)$. As is clear from these expressions, the bidder's ordering of the internalized pecuniary benefit and internalized social cost precisely matches the ordering of the full pecuniary benefit and the full social cost.

One implication is that if a bidder were able to conduct an aggressive freeze-out merger, with non-tendering shareholders receiving just v_0 in a post-takeover freeze-out, then social costs would play only a limited role in takeover outcomes, precisely because of the free-riding problem. That is, a bidder would compare $v_1 - v_0$ with $\phi_0 - \phi_1 - \frac{q}{q+\Delta}(\phi_0 - \phi_1)$, where the

latter object is small when free-riding problems are acute (Δ small).

We next turn to Corollary 2(ii), on socially efficient takeovers. Here, it is instructive to consider the opposite case to that discussed above, viz., a privately inefficient takeover ($v_1 < v_0$) that generates sufficient social benefits ($\phi_1 > \phi_0$) that the takeover is socially efficient ($v_1 + \phi_1 > v_0 + \phi_0$).

Corollary 2 shows that takeovers succeed with positive probability in this case. At first sight this is surprising, again because of the free-rider problem in public good provision. That is: under excludability of social consequences, a bidder would be able to reduce its bid by the amount of the social benefits $\phi_1 - \phi_0$; but under non-excludability, a bidder can instead reduce the bid by only $\phi_1 - \phi_0 - \frac{q}{q+\Delta}(\phi_1 - \phi_0)$, which is a small quantity if there are many shareholders and the pivotal probability Δ is low.

The reason why takeovers nonetheless succeed is that in this case non-excludability of a takeover's pecuniary consequences facilitates a successful bid. To illustrate this point, consider briefly the case of no social effects. With full excludability of pecuniary consequences, a bidder would need to offer v_0 to shareholders. But with non-excludability, a bid p only marginally above $v_1 < v_0$ is sufficient to induce shareholders to tender. In detail: Each shareholder compares tendering—and getting p whenever the bid succeeds—with retention—and getting v_1 if the bid succeeds without his/her support, and getting v_0 if his/her retention decision is pivotal and prevents the bid's success. Because the pivotal probability is small, a bid p that is only marginally above $v_1 < v_0$ is enough to induce a shareholder to tender. This flip side of the holdout problem in takeovers is also known as the "pressure to tender" (Bebchuk 1987).

Combining the above observations: in this case, non-excludability of social benefits hampers a successful bid, but non-excludability of pecuniary costs facilitates one. Exactly as in Corollary 2, the two effects offset each other, so that a takeover occurs with positive probability precisely when it is socially efficient.

For this case of ownership-invariant preferences ($\eta = 1$), it turns out that there is an easy way to map the standard setting without social externalities to the case of social externalities. Consider two sets of parameters: $(\bar{v}_0, \bar{v}_1, \bar{\phi}_0, \bar{\phi}_1)$ and $(\tilde{v}_0, \tilde{v}_1, \tilde{\phi}_0, \tilde{\phi}_1)$ where $\bar{\phi}_0 = \bar{\phi}_1 = 0$, $\tilde{v}_0 + \tilde{\phi}_0 = \bar{v}_0$ and $\tilde{v}_1 + \tilde{\phi}_1 = \bar{v}_1$. That is: the "bar" parameters correspond to the standard case without social externalities, and the "tilde" parameters introduce social externalities while leaving the combination of pecuniary and social value unchanged. Consider an arbitrary offer by the

bidder, $\bar{p}$, made under the bar parameters. From (8), an offer $\tilde{p} = \bar{p} - \tilde{\phi}_1$ made under the the tilde parameters induces exactly the same tendering behavior as the offer $\bar{p}$ under the bar parameters. The bidder's payoff is also exactly the same in the two cases: if the offer is rejected, its payoff is 0 in both cases, while if the offer is accepted, the offer $\tilde{p}$ entails paying $\tilde{\phi}_1$ less for a firm that generates $\tilde{\phi}_1$ less of pecuniary value.

3.4.2 Warm-glow and cold-prickle preferences

Corollary 3. *Suppose $\eta < 1$. Then, in equilibrium:*

(*i*) *If $\phi_1 > 0$ then as N grows large the takeover success probability approaches 0.*

(*ii*) *If $\phi_1 < 0$ then there are equilibria such that, as N grows large, the takeover success probability approaches 1. If the takeover is socially efficient then the equilibrium is unique.*

Corollary 3 demonstrates that the equilibrium outcome changes dramatically when shareholders have warm-glow preferences. In particular, with warm-glow preferences, socially inefficient takeovers sometimes succeed, while some socially efficient takeovers always fail. Moreover, it highlights that the effects of the post-takeover and pre-takeover externalities on the takeover outcome are asymmetric.

Relative to the benchmark case with no externalities, the existence of negative post-takeover externalities ($\phi_1 < 0$) raises the probability of a privately efficient takeover. Indeed, as Corollary 3 establishes, when shareholders are sufficiently dispersed, negative post-takeover externalities enable a takeover to succeed independent of either the private ($v_1 - v_0$) or social ($v_1 - v_0 + \phi_1 - \phi_0$) value created.

To understand the economic forces that drive this conclusion, start from the case of ownership-invariant preferences ($\eta = 1$) analyzed immediately above. For that case, we showed that the holdout problem (non-excludability of pecuniary effects) means that the bidder derives very little profit from any pecuniary value created by the takeover. Similarly, the free-rider problem in public good provision (non-excludability of social externalities) means that the bidder derives very little profit/bears very little cost from any change in social externalities.[10]

[10] Individually, of course, both effects are familiar from prior research; the contribution of the analysis above is to show that both effects have the same magnitude, and hence offset each other.

Decomposition (12) shows that warm-glow preferences ($\eta < 1$) shift behavior relative to ownership-invariant preferences only via the preference shift term $(1-\eta)\,\phi_1$. In words, and for the case $\phi_1 < 0$, this term reflects that the fact that shareholders dislike holding a share of a firm that generates negative externalities—and that this dislike is alleviated by getting rid of the share. This effect gives shareholders a direct motive to tender—in turn allowing the bidder to reduce its bid, and thereby facilitating successful takeovers. In essence, the bidder is able to "threaten" shareholders with negative externalities if they don't tender.

Importantly, the direct motive to tender that arises from warm-glow preferences and negative post-takeover externalities operates regardless of whether or not an individual shareholder is pivotal. Hence the bidder can extract a discount from shareholders even when shareholders are arbitrarily dispersed, i.e., $N \to \infty$. At the same time, recall that the holdout and public-good free-rider problems prevent the bidder from benefiting significantly from any social value $(v_1 - v_0 + \phi_1 - \phi_0)$ created. Consequently, the discount that the bidder obtains from warm-glow shareholders' direct motive to tender is dominant when shareholders are sufficiently dispersed.

The case of positive post-takeover externalities ($\phi_1 > 0$) is directly analogous. In this case, warm-glow preferences induce a direct motive for shareholders to *retain* shares. To overcome this, a bidder would have to offer a premium. But when shareholders are dispersed, the holdout and public-goods free-riding problems imply that the bidder derives little benefit from the value created by the takeover. Because of this, the bidder is unable to afford the premium that would be required to overcome warm-glow shareholders' direct preference to retain shares in a firm that generates positive externalities.

The two halves of Corollary 3 imply that warm-glow preferences can lead to perverse consequences. This is especially clear in the case in which the status quo is associated with zero social externalities ($\phi_0 = 0$). In this case, takeovers that are socially destructive ($\phi_1 < 0$) occur, while takeovers that are socially beneficial ($\phi_1 > 0$) are blocked.

A specific prediction of Corollary 3 that is worth noting is that firms that produce negative externalities will be acquired, by private bidders, even absent any change in either pecuniary value creation or in social externalities (i.e., $v_1 = v_0$ and $\phi_1 = \phi_0 < 0$).

3.4.3 Hyper-consequentialism preferences

Corollary 4. *Suppose $\eta > 1$. Then, in equilibrium:*

(*i*) *If* $\phi_1 < 0$ *then as* N *grows large the takeover success probability approaches* 0.

(*ii*) *If* $\phi_1 > 0$ *then there are equilibria such that, as* N *grows large, the takeover success probability approaches* 1. *If the takeover is socially efficient then the equilibrium is unique.*

Corollary 4 is the analog of Corollary 3, with the sole difference that in the predictions the sign on the externality flips. In particular, relative to the benchmark case with no externalities, the existence of positive externalities raises the probability of a privately efficient takeover. The reason is that, absent externalities, the holdout problem reduces the takeover probability. But the combination of positive externalities and hyper-consequentialism preferences dampens the holdout problem, since a shareholder who tenders in expectation of a takeover succeeding knows that he has contributed to the creation of positive externalities, which he values a lot.

4 Exclusionary mechanisms

4.1 Freeze-out mergers

Once a bidder gains a controlling stake in a target firm, the bidder is often able to acquire the shares of all non-tendering shareholders via a post-acqusition *freeze-out* merger. For example, the bidder can use its control of the target to sell the target to a separate entity that it wholly owns. Consequently, freeze-out mergers limit the ability of target shareholders to retain a minority stake and to profit from the fruits of a value-increasing acquisition.

Here, we analyze the effect of freeze-out mergers in our setting. Perhaps surprisingly, the possibility of a post-acquisition freeze-out merger doesn't affect the main conclusions of our analysis so far, at least qualitatively.

Specifically, and as in Mueller and Panunzi (2004), suppose that following a successful takeover, the bidder is able to execute a successful freeze-out merger with (exogenous) probability $\theta \in [0, 1)$; our baseline model is the special case $\theta = 0$. Following the discussion in Amihud et al (2004) and Mueller and Panunzi (2004), we assume that legal restrictions ensure that minority shareholders receive at least the original offer p in a freeze-out; and since the bidder's profits are decreasing in the freeze-out payment, it follows that minority shareholders

receive exactly p. Consequently, a shareholder's expected utility from retaining a share is

$$v_0 + \phi_0 + q(1-\theta)(v_1 + \phi_1 - v_0 - \phi_0) + q\theta(p + \eta\phi_1 - v_0 - \phi_0). \tag{17}$$

That is: with probability $q(1-\theta)$ the takeover succeeds but the freeze-out fails, and a retaining-shareholder's payoff is exactly as in the non-freeze-out case; but with probability $q\theta$ the takeover succeeds and the freeze-out succeeds, and in this case a retaining shareholder receives p for the share and values the externalities associated with the takeover at $\eta\phi_1$.[11]

A shareholder's expected utility from tendering does not depend on the success of the freeze-out, and so is the same as in the no-freeze-out baseline, see (5). Hence the marginal benefit of tendering is

$$\begin{aligned} \tau_\theta(\gamma; p) &\equiv (q + \Delta - \theta q)(p + \eta\phi_1 - v_0 - \phi_0) - (1-\theta) q (v_1 + \phi_1 - v_0 - \phi_0) && (18) \\ &= (1-\theta) q (p - v_1 + (\eta - 1)\phi_1) + \Delta(p + \eta\phi_1 - v_0 - \phi_0). && (19) \end{aligned}$$

Importantly, and as (18) makes clear, a shareholder is indifferent between tendering and retention in the probability $q\theta$ event that a successful freeze-out occurs, because either way the shareholder receives a payment p and values externalities at $\eta\phi_1$. Accordingly, expression (19) takes the same form as its no-freeze-out analogue (6), with the sole difference being that the probability q of holding out and benefiting from a takeover is reduced to $(1-\theta)q$.

As in the no-freeze-out case (see (12)) it is useful to write the marginal benefit of tendering as

$$\tau_\theta(\gamma; p) = ((1-\theta) q + \Delta)(p - \mu_\theta(\gamma))$$

where

$$\mu_\theta(\gamma) \equiv v_1 - (v_1 - v_0) - (\phi_1 - \phi_0) + \frac{(1-\theta) q}{(1-\theta) q + \Delta}(v_1 - v_0) + \frac{(1-\theta) q}{(1-\theta) q + \Delta}(\phi_1 - \phi_0) + (1-\eta)\phi_1. \tag{20}$$

As before, $\mu_\theta(\gamma)$ is the bid required to induce a tendering probability of γ; and (20) decomposes

[11] See Dalkır at al. (2019) for the analysis of tender offers without externalities in which freeze-out mergers succeed if and only if the number of tendered shares is at least $F \in \{K+1, ..., N-1\}$. In this model, the analog of $q\theta$ is $\sum_{j=F}^{N-1} \binom{N-1}{j} \gamma^j (1-\gamma)^{N-1-j}$, and in particular, the probability of a successful freeze-out merger is endogenous.

this into the bid made under full-excludability, with adjustments for non-excludability of a takeover's pecuniary and social consequences, and also the preference shift associated with divesting shares.

Expression (20) highlights two key points. First, the possibility of a freeze-out ameliorates but does not eliminate non-excludability of the pecuniary and social consequences of a takeover. Second, a freeze-out affects the two non-excludability problems in the same proportion, so that—exactly as before—non-excludability means that the bidder adjusts the offer by some fraction of total surplus creation $v_1 - v_0 + \phi_1 - \phi_0$. Intuitively, the reason is that, as noted, a target shareholder is indifferent between tendering and retention in the event that a freeze-out succeeds.

The bidder's expected profits per shareholder are

$$(\gamma (q + \Delta) + (1 - \gamma) \theta q) (v_1 - p) . \tag{21}$$

That is: Fix a representative shareholder. With probability γ the shareholder tenders; conditional on this, the takeover succeeds with probability $q + \Delta$, and the bidder makes $v_1 - p$ from the tendered share. With probability $1 - \gamma$ the shareholder retains the share; conditional on this, the takeover succeeds with probability q; and conditional on this, a freeze-out succeeds with probability θ, and the bidder again makes $v_1 - p$ from the share acquired in the freeze-out.

Freeze-outs benefit the bidder in two ways. First, from (20), by ameliorating non-excludability, freeze-outs reduce the offer needed to induce a given tendering probability γ. Second, and more mechanically, for a given tendering probability γ the bidder's probability of acquiring a share is increased by a probability of $(1 - \gamma) \theta q$.

Our main result here is:

Proposition 2. *If a takeover is socially inefficient then the equilibrium success probability is invariant to the freeze-out probability θ.*
If a takeover is socially efficient, $\eta = 1$, and $K \geq \frac{N}{2}$, then the equilibrium success probability is strictly increasing in the freeze-out probability θ.

Proposition 2 first says that freeze-outs don't affect the success rate of socially inefficient takeovers. The reason is that shareholders respond to a socially inefficient takeover either by

rejecting the offer for sure ($\gamma = 0$) or by accepting it for sure ($\gamma = 1$). There is no room for freeze-outs to play a role in either case. Consequently, any equilibrium of the tendering subgame without freeze-outs remains an equilibrium when freeze-outs are possible. Moreover, and as established in the proof, the possibility of freeze-outs doesn't introduce new equilibria.

Second, Proposition 2 says that the possibility of freeze-outs increases the success rate of socially efficient takeovers in the leading case of ownership-invariant preferences ($\eta = 1$). Here, it is instructive to recall Corollary 2's finding that socially efficient takeovers are associated with a tendering probability of $\gamma = \kappa$ in the no-freeze-out case. As discussed, the free-riding forces associated with public-goods contributions and with the hold-out problem in takeovers balance each other, and so socially efficient takeovers occur with positive probability. From (20), these statements remain true when freeze-outs are possible. Freeze-outs raise the takeover-success probability associated with any given bid. In principle it is possible that the bidder responds by aggressively reducing the bid, so that the equilibrium success probability falls. But Proposition 2 establishes that this doesn't happen, and that the success probability rises; a rough intuition is that the value of freeze-outs rises in the probability of takeover success, which raises the value that the bidder attaches to takeover success.

4.2 Leveraged offers

Bidders often finance the acquisitions with debt collateralized against the target's assets, for example, private equity firms that specialize in leveraged buyouts (LBOs). Mueller and Panunzi (2004) showed that leverage enables the bidder to "tunnel" resources out of the target at the expense of minority shareholders, thereby ameliorating the holdout problem. In this section, we consider the effect of leverage on takeovers with externalities.

Suppose the bidder issues debt $Nd > 0$, owed by the target if the takeover succeeds. Since debt is a senior claim, the target share value post takeover is $\max\{v_1 - d, 0\}$. Legal protections for minority shareholders are typically interpreted to constrain

$$v_1 - d \geq v_0. \tag{22}$$

We therefore assume throughout that $v_1 > v_0$ and $d \in [0, v_1 - v_0]$. Thus, a shareholder's

expected utility from retaining is

$$v_0 + \phi_0 + q\left(v_1 - d + \phi_1 - v_0 - \phi_0\right), \tag{23}$$

and a shareholder's expected utility from tendering does not depend on leverage, and thus, is the same as in the baseline model and given by (5). Let $\tau\left(\gamma; p, v_1\right)$ and $\mu\left(\gamma; v_1\right)$ be the parameterization of (6) and (8) as a function of v_1, respectively, then the marginal benefit from tendering is

$$\tau\left(\gamma; p, v_1 - d\right) = \left(q + \Delta\right)\left(p - \mu\left(\gamma; v_1 - d\right)\right). \tag{24}$$

Thus, for a given (p, d), the equilibrium of the tendering subgame is fully characterized by Lemma 1, where v_1 is replaced everywhere by $v_1 - d$.

The bidder's expected profits per representative shareholder are

$$d\left(\gamma\left(q + \Delta\right) + \left(1 - \gamma\right)q\right) + \gamma\left(q + \Delta\right)\left(v_1 - d - p\right). \tag{25}$$

If the takeover succeeds, the bidder makes a profit of d whether or not the shareholder tenders his share, which is the first term above. Indeed, the bidder obtains the leverage proceeds d which is paid back by the target. If the shareholder tenders and the takeover succeeds, then as in the baseline model, the bidder makes a profit of $v_1 - d - p$; the difference being that the post-takeover equity value is diluted by a liability of d.

Rearranging terms, the bidder profits are

$$\Pi = N\gamma\left(q + \Delta\right)\left(v_1 - p\right) + Nd\left(1 - \gamma\right)q. \tag{26}$$

Hence leverage enables the bidder to "tunnel" resources out of the firm at the expense of minority shareholders: conditional on a successful takeover, the bidder obtains the entire leverage proceeds Nd, but incurs only part of the total liability, with the remaining $Nd\left(1 - \gamma\right)q$ falling on minority shareholders. In addition, since $\mu\left(\gamma; v_1 - d\right)$ decreases in d, leverage also ameliorates non-excludability, and thereby reduces the cost to the bidder of inducing any given tendering probability γ.

We next characterize the bidder's profit-maximizing bid p and leverage d .

Proposition 3. *Suppose $\eta = 1$. Then:*

- *If a takeover is socially efficient and $\phi_1 \geq \phi_0$ then $d^* = v_1 - v_0$ and $\gamma^* \in (\kappa, 1)$.*
- *If a takeover is socially efficient and $\phi_1 < \phi_0$ then for any $\beta \in (\frac{v_1 - v_0 + \phi_1 - \phi_0}{v_1 - v_0}, 1]$ there is an equilibrium in which the takeover succeeds with probability β.*
- *If a takeover is socially inefficient then for any $\beta \in [0, 1]$ there is an equilibrium in which the takeover succeeds with probability β.*

In the first case, leverage enhances efficiency. Intuitively, and similar to the intuition behind freeze-out mergers, it ameliorates the holdout problem in takeovers, and thereby, further facilitates a socially efficient takeover. Interestingly, if $v_1 < v_0$ then legal protections of minority shareholders prevent the bidder from taking on debt and thereby facilitating socially efficient takeovers that are privately inefficient. Our analysis therefore provides a normative argument in favor of relaxing legal protections and allow bidders that create positive externalities to take leverage up to $v_1 - v_0 + \phi_1 - \phi_0$.

In the second case, if $\frac{v_1 - v_0 + \phi_1 - \phi_0}{v_1 - v_0} < \kappa$, then leverage could in fact reduce the probability a socially efficient takeover succeeds in equilibrium. Intuitively, by taking more leverage, the bidder can more effectively pressure shareholders to tender, which increases his profits but also the risk that shareholders will reject the takeover altogether. Indeed, as noted in Section 3.2, when a takeover is socially inefficient (net of debt), the takeover success is self-fulfilling which entails a risk of failure. Essentially, the bidder trades off two alternatives: (i) low leverage ($d = v_1 - v_0 + \phi_1 - \phi_0$), high probability of takeover success, but lower profits if the takeover succeeds, with (ii) high leverage ($d = v_1 - v_0$), low probability of takeover success, but higher profits if the takeover succeeds. When the latter dominates, leveraged offers increase inefficiencies. Thus, then again, our analysis provides a normative argument in favor of expanding legal protections by constraining bidders that create negative externalities to take leverage up to $v_1 - v_0 + \phi_1 - \phi_0$.

In the third case, socially inefficient takeovers can succeed. The ability to take leverage and dilute shareholders enables the bidder to make profit even from socially inefficient takeovers, albeit, at the expense of target shareholders.

Finally, note that if $\phi_1 = \phi_0 = 0$ then $v_1 > v_0$ implies the takeover is socially efficient and leverage has only positive consequences (if $v_1 < v_0$ then leverage has no effect). In other words,

the combination of diluting leverage and negative takeover externalities (i.e., $\phi_1 < \phi_0$) increase the scope takeover inefficiencies.

4.3 Discussion: freeze-out mergers vs. leveraged offers

Our analysis of freeze-out mergers and leveraged offers suggests these two exclusionary mechanisms operate differently when takeovers create externalities. To understand the differences, suppose $\eta = 1$, and note that the diluted value of minority shareholders is given by $d \in [0, v_1 - v_0]$ under leveraged offers and by $f \equiv \theta (v_1 - p)$ under freeze-out mergers.

A couple of immediate observations. First, under freeze-out mergers, larger dilution requires offering a lower price. The former increases the probability of tendering while the later decreases it, creating a tension from the bidder's perspective. In contrast, under leveraged offers the bidder has more degrees of freedom which enable him to increase profits at the expense of social efficiency. Second, if a takeover is privately inefficient but socially efficient, then leveraged offers have no bite as a diluting mechanism, but freeze-out mergers do. Conversely, if a takeover is privately efficient but socially inefficient, then freeze-out mergers have no bite as a diluting mechanism, but leveraged offers do. If a takeover is both privately and socially inefficient, neither mechanism has any bite.

Suppose a takeover is both privately and socially efficient. If $p = \mu_\theta (\gamma)$ then

$$f = \frac{\theta}{(1-\theta)\frac{q}{\Delta} + 1} (v_1 - v_0 + \phi_1 - \phi_0). \tag{27}$$

Thus, the extent of dilution through freeze-out mergers depend on the takeover externalities, $\phi_1 - \phi_0$. Specifically, if $\phi_1 - \phi_0 < 0$ then $f < v_1 - v_0$ and the extent of dilution under freeze-out mergers is more limited than under leveraged offers. Intuitively, if $\phi_1 - \phi_0 < 0$ then shareholders internalize the reduction in externalities and hence require a lower price to tender, which reduces the scope of dilution via freezed-out. If $\phi_1 - \phi_0 > 0$ then $f > v_1 - v_0$ if θ is sufficiently large. Roughly, freeze-out mergers are more effective as diluting mechanism when the takeover creates positive externalities.

5 Socially responsible bidder

In this section we consider the possibility that the bidder itself (or the bidding firm's shareholders) internalizes the externalities the takeover is expected to create. Formally, suppose the bidder's payoff per share it acquires in the takeover is $v_1 + \delta\phi_1 - p$, where $\delta \geq 0$ is a parameter that captures the bidder's social responsibility. The baseline model is a special case where $\delta = 0$.[12]

The analysis of the tendering subgame, where the tender offer is exogenous, does not change and Lemma 1 continues to hold. What changes is the bidder's expected payoff. Specifically, the bidder's expected profit from an offer to an individual shareholder, conditional on that shareholder tendering and all other shareholders playing the equilibrium strategy γ, is

$$(q + \Delta)(v_1 + \delta\phi_1 - p) \tag{28}$$

Since $p = \mu(\gamma)$, the above expression can be written as

$$\Delta(v_1 - v_0 + \phi_1 - \phi_0) - (q + \Delta)(1 - \eta)\phi + (q + \Delta)\delta\phi_1. \tag{29}$$

Relative to (14), the socially responsible bidder's expected benefit from an offer to an individual shareholder is higher by $(q + \Delta)\delta\phi_1$. Overall, the expected payoff of a socially responsible bidder is

$$\Pi(\gamma; \delta) \equiv \Pi(\gamma) + N\gamma(q + \Delta)\delta\phi_1, \tag{30}$$

where $\Pi(\gamma)$ is given by (16).

Proposition 4. *Consider socially responsible bidder with parameter δ. The equilibrium characterization is identical to the one in Proposition 1, with the exception of replacing η by $\eta + \delta$.*

Proposition 4 says that the equilibrium characterization with a socially responsible bidder is the same as in the baseline model with the exception that warm-glow preferences of shareholders is $\eta + \delta$ instead of η. For example, if $\eta = 1 - \delta$ and the bidder is socially responsible with parameter δ, then the outcome is efficient as described in Corollary 2. This means that when

[12] Notice, the bidder's social preferences have a flavor of warm-glow preferences; internalizing the firm's externalities only if the takeover succeeds and only in proportion to the acquired shares.

shareholders' preferences are ownership invariant ($\eta = 1$), having also a socially responsible bidder leads to inefficient outcomes. And vice versa, when the bidder is socially responsible and shareholders have warm glow preferences, increasing shareholders' social responsibility (i.e., increasing η) can lead to inefficient outcomes. Clearly, there are cases where larger bidder's social responsibility leads to better outcomes.[13]

6 Externality production in takeovers

In this section, we extend the baseline model by allowing either the bidder or the incumbent to choose the level of externalities produced, with different choices associated with different pecuniary firm values. In Section 6.1 we allow the bidder to pledge a level of externalities as part of its takeover offer, and analyze what level it chooses. In Section 6.2 we allow the incumbent to change the status quo level of externalities. Throughout, we maintain the assumption from the baseline model that the bidder's motive is to maximize profit.

6.1 Social responsibility as a takeover offense

In this section, we consider the possibility that the bidder chooses social responsibility as a takeover offense tactic, to increase the takeover's profitability. Specifically, suppose that the bidder chooses both an offer p, and credibly pledges social externalities of ϕ_1. The associated pecuniary value of the firm under the bidder is $v_1(\phi_1)$, where the function $v_1(\cdot)$ captures potential trade-offs between social externalities and pecuniary firm value. Note that we allow for the possibility that $v_1(\phi_0) \neq v_0$, reflecting any operational or financial synergies that don't change the level of social externalities.

Define ϕ_1^{**} as the level that maximizes social value, i.e.,

$$\phi_1^{**} \equiv \arg\max_{\phi_1} \{v_1(\phi) + \phi_1\}, \tag{31}$$

where we assume that ϕ_1^{**} is well-defined, and also that $v_1(\cdot)$ is differentiable at ϕ_1^{**}. We further

[13] Similar to the discussion at the end of Section 3.4.1, when $\eta \neq 1$, the equilibrium outcome is markedly different form the outcome of a model in which there are no externalities and the post-takeover firm value is $v_1 + \delta\phi_1$ instead of v_1.

assume that

$$v_1\left(\phi_1^{**}\right) + \phi_1^{**} > v_0 + \phi_0. \tag{32}$$

That is: if the bidder chooses the socially efficient level of externalities then the takeover would be socially efficient.

Recall that multiple equilibria exist for some parameter configurations. We adopt the mild assumption that if ϕ_1 leads to a unique equilibrium in which the takeover fails, and $\tilde{\phi}_1$ generates multiple equilibria, at least one of which gives a strictly positive payoff to the bidder, then the bidder prefers $\tilde{\phi}_1$ to ϕ_1.

Proposition 5. *Let ϕ_1^* be the externality that maximizes the bidder's profit. Then,*

(*i*) *If shareholder preferences are ownership invariant ($\eta = 1$) then the bidder chooses $\phi_1^* = \phi_1^{**}$.*

(*ii*) *If shareholders have warm-glow preferences, $\eta < 1$ (hyper-consequentalist preferences, $\eta > 1$), then the bidder chooses $\phi_1^* < \phi_1^{**}$ ($\phi_1^* > \phi_1^{**}$) .*

We note that, regardless of the value of η, the takeover succeeds with positive probability—so the bidder's choice of ϕ_1 has material consequences.

Proposition 5(i) demonstrates that when shareholders have ownership-invariant preferences, the bidder pledges the efficient level of externalities. Intuitively, since shareholders fully internalize the social benefit from the takeover, the bidder maximizes its profit from the takeover by maximizing social value creation.

If instead shareholders have warm-glow or hyper-consequentialism preferences, then the bidder's pledge deviates from the social optimum. Specifically, if shareholders have warm-glow (hyper-consequentialism) preferences, then the bidder pledges smaller (larger) externalities than the socially optimum level, since with such social preferences, shareholders have lower (stronger) incentives to keep (tender) their shares when the externalities are more negative (positive). In other words, the bidder uses the production of externalities to threaten (tempt) shareholders to sell the firm, and in this respect, social responsibility is used as a takeover offense tactic.

6.2 Social responsibility as a takeover defense

In this section, we consider the possibility that the incumbent uses social responsibility as a takeover defense tactic—either with the aim of increasing the welfare of target shareholders, or to reduce the probability of a successful takeover.

Specifically, suppose that prior to any offer from the bidder, the incumbent chooses externalities ϕ_0. The associated pecuniary value of the firm is $v_0(\phi_0)$. Define ϕ_0^{**} as the level that maximizes social value, i.e.,

$$\phi_0^{**} \equiv \arg\max \{v_0(\phi_0) + \phi_0\},$$

where we assume that ϕ_0^{**} is well-defined, and also that $v_0(\cdot)$ is differentiable at ϕ_0^{**}. We further assume that

$$v_1 + \phi_1 > v_0(\phi_0^{**}) + \phi_0^{**}.$$

That is: even if the incumbent chooses the socially efficient level of externalities then the takeover would still be socially efficient.[14] This assumption focuses attention on the case in which the takeover succeeds with positive probability, and in which the role of the incumbent's choice of ϕ_0 is to affect that probability.

How does the incumbent's choice of ϕ_0 affect the takeover success probability? The answer depends on shareholders' preferences (η) and on whether the prospective takeover generates positive or negative social externalities:

Lemma 2 *The probability that shareholders tender (γ) is increasing in the social value of the target under the incumbent, $v_0(\phi_0) + \phi_0$, if $(1-\eta)\phi_1 < 0$; is independent of $v_0(\phi_0) + \phi_0$ if $(1-\eta)\phi_1 = 0$; and is decreasing in $v_0(\phi_0) + \phi_0$ if $(1-\eta)\phi_1 > 0$.*

To understand Lemma 2: Recall first that if shareholders have ownership-invariant preferences then the bidder focuses on maximizing the probability $\gamma\Delta$, because by doing so the bidder minimizes the effects of non-excludability of the benefits of a takeover. In this case, the bidder makes an offer p that delivers $\gamma^* = \kappa$. The bidder's payoff is proportional to the value created by the takeover.

[14] Notice, the incumbent's choice affects the private and social gains from the takeover without directly affecting the post-takeover value or externalities.

To illustrate the other parts of Lemma 2, we focus on the case in which shareholders have warm-glow preferences and the takeover yields negative externalities ($\phi_1 < 0$). In this case: the bidder additionally extracts the benefits that shareholders experience from their reduced interest in negative social externalities that accompanies selling their shares. Because of this, the bidder benefits from raising the tendering probability γ. From (15), the bidder sets γ to maximize the weighted average of the probabilities $\gamma\Delta$ and $q + \Delta$, where the weight on the former is the social surplus associated with the takeover.

Lemma 2 follows from the observation that as the social value of the firm under the incumbent increases, the surplus created by the takeover decreases, and so the bidder places more weight on $q + \Delta$—leading it to choose a higher value of γ.

We are now in a position to characterize the incumbent's choice of ϕ_0, using Lemma 2. We consider two possible objectives of the incumbent. First, the incumbent may be *shareholder-focused* and seek to maximize the payoff of its current shareholders. Because, in equilibrium, shareholders are indifferent between tendering and retaining their shares, a shareholder-focused incumbent seeks to maximize

$$W = N \times \begin{cases} (1-q)\left(v_0\left(\phi\right)+\phi\right) + q\left(v_1+\phi_1\right) & \text{if } \gamma \in [0,1) \\ p + \eta\phi_1 & \text{if } \gamma = 1. \end{cases} \tag{33}$$

Second, the incumbent may be *entrenched,* and seek to minimize the takeover probability.

Proposition 6.

(*i*) *If* $(1-\eta)\,\phi_1 < 0$ *then an entrenched incumbent minimizes* $v_0\left(\phi_0\right)+\phi_0$ *while a shareholder-focused incumbent chooses* ϕ_0^{**}.

(*ii*) *If* $(1-\eta)\,\phi_1 = 0$ *then an entrenched incumbent is indifferent while a shareholder-focused incumbent chooses* ϕ_0^{**}.

(*iii*) *If* $(1-\eta)\,\phi_1 > 0$ *then an entrenched incumbent chooses* ϕ_0^{**} *while a shareholder-focused incumbent faces a trade-off between increasing* $v_0\left(\phi_0\right)+\phi_0$ *and increasing the takeover probability.*

Proposition 6 is immediate from Lemma 2. It is again useful to consider the case of shareholders with warm-glow preferences and a prospective takeover with negative externalities. In this case, increasing the status quo value of the firm $v_0(\phi_0) + \phi_0$ increases the probability of a successful takeover—because by reducing the social surplus created by the takeover, doing so pushes the bidder to prioritize the portion of its profits stemming from the preference shift that accompanies a successful takeover.

Accordingly, an entrenched incumbent reduces the probability of a takeover by making the status quo value of the firm as low as possible. Conversely, a shareholder-focused incumbent seeks to make $v_0(\phi_0) + \phi_0$ as high as possible—both because doing so increases the probability that a value-increasing takeover occurs, and because if the takeover fails then shareholders benefit directly from a firm with more social value.

If instead a prospective takeover yields positive externalities, then exactly the opposite implications hold. In this case, and perhaps more intuitively, increasing the status quo social value of the firm reduces the probability of a takeover. Accordingly, an entrenched incumbent makes $v_0(\phi_0) + \phi_0$ as high as possible. In contrast, a shareholder-focused incumbent faces a potentially difficult trade-off—increasing $v_0(\phi_0) + \phi_0$ is desirable in the positive probability event that a takeover fails, but undesirable because it reduces the probability of a successful takeover.

7 Conclusion

In this paper, we have studied the effects of responsible investment on the market for corporate control and takeover dynamics. We introduced externalities and social preferences into the canonical takeover model of Bagnoli and Lipman (1988). Our analysis highlights the interplay between the holdout problem in takeovers (Grossman and Hart 1980) and free-riding in public good provision. We show that the dual free-riding problems offset each other. Shareholders' preferences over externalities generated by firms they have divested from play a key role. If shareholders care about such externalities, then acquisitions succeed if and only if they are socially efficient, free-riding problems notwithstanding. Moreover, both incumbents and bidders have incentives to maximize the social value of the firm. In contrast, "warm-glow" shareholders accept some socially inefficient acquisitions while rejecting some socially efficient ones; and

incumbents and bidders generally respond by adopting socially inefficient policies. Incumbents can use corporate social responsibility as an effective takeover defense. Social responsibility by bidders is counterproductive if target shareholders care about divested externalities, but helps offset inefficiencies created by warm-glow shareholders. Overall, our analysis sheds light on the intricate interplay between shareholder behavior, social responsibility, and acquisition dynamics.

References

[1] Akey, P., and I. Appel, 2020, Environmental externalities of hedge funds, *Working paper.*

[2] Amihud, Y., M. Kahan, and R. Sundaram. 2004. The foundations of freeze-out laws in takeovers. *Journal of Finance* 59:1325–44.

[3] At, C., M. Burkart, and S. Lee. 2011. Security-voting structure and bidder screening. *Journal of Financial Intermediation* 20:458–76.

[4] Bagnoli, M., Gordon, R., Lipman, B. L., 1989. Stock Repurchase as a Takeover Defense. *Review of Financial Studies* 2, 423-443.

[5] Bagnoli, M., Lipman, B. L., 1988. Successful Takeovers Without Exclusion. *Review of Financial Studies* 1, 89–110.

[6] Baker S. D., B. Hollifield, E. Osambela, 2022, Asset Prices and Portfolios with Externalities*, Review of Finance,* 26(6):1433–1468,

[7] Baron, D .P., 1983. Tender Offers and Management Resistance. J*ournal of Finance* 38, 331-343.

[8] Baumol, W. (1952). Welfare Economics and the Theory of the State. *Cambridge, Massachusetts: Harvard University Press.*

[9] Bebchuk, L. A., 1988, "The Pressure to Tender: An Analysis and a Proposed Remedy," *Delaware Journal of Corporate Law*, Vol. 12, pp. 911–949

[10] Bellon A. 2020. Does private equity ownership make firms cleaner? The role of environmental liability risks. *Working paper.*

[11] Berg, T., L. Ma, D. Streitz, 2023, Climate Risk and Strategic Asset Reallocation, *Working paper.*

[12] Berk, Jonathan and Jules H. van Binsbergen, 2021, The Impact of Impact Investing, *Working paper.*

[13] Berkovitch, E., Khanna, N., 1990. How target shareholders benefit from value-reducing defensive strategies in takeovers. *Journal of Finance* 45, 137-156.

[14] Broccado, E., O. Hart, and L. Zingales, 2022, Exit versus Voice, *Journal of Political Economy* 130, 3101–3145.

[15] Borenstein, S., 1990, Airline Mergers, Airport Dominance, and Market Power, *American Economic Review*, 80, 400-404

[16] Burkart, M., D. Gromb, and F. Panunzi. 1998. Why higher takeover premia protect minority shareholders. *Journal of Political Economy* 106:172–204.

[17] Burkart, M., D. Gromb, and F. Panunzi. 2000. Agency conflicts in public and negotiated transfers of corporate control. *Journal of Finance* 55, 647-677

[18] Burkart, M., D. Gromb, H. M. Mueller, and F. Panunzi (2014). Legal investor protection and takeovers. *Journal of Finance* 69 (3), 1129–1165

[19] Burkart, M. and S. Lee, 2015, Signaling to Dispersed Shareholders and Corporate Control, *Review of Economic Studies*, 82(3), 922-62.

[20] Burkart, M. and S. Lee, 2022, Activism and Takeovers, *Review of Financial Studies*, 35(4), 1868-96.

[21] Burkart, M. and S. Lee, and H. Petri, 2023, LBO Financing, *Working paper.*

[22] Burkart, M. and S. Lee, and P. Voss 2024, The Evolution of the Market for Corporate Control, *Working paper.*

[23] Chen, Tao, Hui Dong and Chen Lin, 2020, Institutional shareholders and corporate social responsibility, *Journal of Financial Economics* 135, 483–504.

[24] Chowdhry, B., S. W. Davies, and B. Waters, 2019, Investing for Impact. *Review of Financial Studies* 32 (3):864-904.

[25] Cohn, J., N. Nestoriak, and M. Wardlaw, 2021, Private Equity Buyouts and Workplace Safety, *Review of Financial Studies* ,34(10): 4832–4875,

[26] Cornelli, F., and D. Li. 2002. Risk arbitrage in takeovers. *Review of Financial Studies* 15:837–68.

[27] Cunningham C, Ederer F, Ma S. 2021. Killer acquisitions, *Journal of Political Economy* 129(3):649–702

[28] Dalkır, E. 2015. Shareholder information and partial tender offers. *Economics Letters* 136:64–66.

[29] Dalkır, E., and M. Dalkır. 2014. On the optimality of partial tender offers. *Journal of Economic Theory* 151:561–70.

[30] Dalkır, E., and M. Dalkır. D. Levit 2019. Freeze-out Mergers, *Review of Financial Studies*, 32(8), 3266-3297

[31] Davies, S. W. and E. D. Van Wesep, 2018, The Unintended Consequences of Divestment. *Journal of Financial Economics* 128, 558–575.

[32] Dessaint, O., Golubov, A., and Volpin, P. 2017. Employment protection and takeovers, *Journal of Financial Economics* 125(2), 369-388.

[33] Dimson, Elroy, Oguzhan Karakas, and Xi Li, 2015, Active ownership, *Review of Financial Studie*s 28, 3225–3268.

[34] Duchin, R., J. Gao, and Q. Xu, Sustainability or greenwashing: Evidence from the asset market for industrial pollution, forthcoming *Journal of finance*

[35] Eaton, C., S. Howell, and C. Yannelis, 2019, When investor incentives and consumer interests diverge: Private equity in higher education, *The Review of Financial Studies*, 22(9):4024–406.

[36] Eckbo, E., 1983, Horizontal mergers, collusion, and stockholder wealth, *Journal of Financial Economics*, 11

[37] Edmans, A„ D. Levit, and J. Schneemeier,2022, Socially Responsible Divestment. *Working Paper*

[38] Ekmekci, M., and N. Kos. 2016. Information in tender offers with a large shareholder. *Econometrica* 84:87–139.

[39] Ewens, M., A. Gupta, and S. Howell, 2022, Local journalism under private equity ownership. *Working paper.*

[40] Gantchev, N., M. Giannetti, and R. Li, 2022, Does Money Talk? Divestitures and Corporate Environmental and Social Policies. *Review of Finance* 26, 1469–1508.

[41] Gibson, R., S. Glossner, P. Krueger, P. Matos, and T. Steffen, 2022, Do Responsible Investors Invest Responsibly?" *Review of Finance* 26, 1389–1432.

[42] Goldstein, I., A. Kopytov, L. Shen, and H. Xiang, 2022 On ESG Investing: Heterogeneous Preferences, Information, and Asset Prices, *Working Paper*

[43] Gollier, C. and S. Pouget, 2022, Investment Strategies and Corporate Behaviour with Socially Responsible Investors: A Theory of Active Ownership. *Economica* 89, 997–1023.

[44] Gomes, A. 2001. Takeovers, freeze-outs, and risk arbitrage. *Working Paper.*

[45] Green, D. and B. Roth ,2024, The Allocation of Socially Responsible Capital, *Journal of Finance*, forthcoming.

[46] Green, D. and B. Vallee, 2023, Measurement and Effects of Bank Coal Exit Policies, *Working Paper*

[47] Gromb, D. 1993. Is one share—one vote optimal? *Working Paper.*

[48] Grossman, S., and O. Hart. 1980. Takeover bids, the free rider problem, and the theory of the corporation. *Bell Journal of Economics* 11:42–64.

[49] Gupta A, Howell S. T, Yannelis C, Gupta A, 2021, Owner Incentives and Performance in Healthcare: Private Equity Investment in Nursing Homes. *Working Paper*

[50] Gupta, D., A. Kopytov, and J. Starmans, 2022, The Pace of Change: Socially Responsible Investing in Private Markets, *Working paper.*

[51] Harris, M., Raviv, S., 1988. Corporate control contests and capital structure. Journal of Financial Economics 20, 55-86.

[52] Hartzmark, S. M. and K. Shue, 2023, Counterproductive Sustainable Investing: The Impact Elasticity of Brown and Green Firms, *Working paper.*

[53] Heath, D., D. Macciocchi, R. Michaely, and M. C. Ringgenberg, 2023, Does Socially Responsible Investing Change Firm Behavior? *Review of Finance* 27, 2057–2083.

[54] Heinkel, R., A. Kraus and J. Zechner,2001, The Effect of Green Investment on Corporate Behavior, *Journal of Financial and Quantitative Analysis* 36, 431–449.

[55] Hirshleifer, D., and S. Titman, 1990, Share tendering strategies and the success of hostile takeover bids. *Journal of Political Economy* 98:295–324.

[56] Hoepner, A. G.F., I. Oikonomou, Z. Sautner, L. T. Starks, and X. Y. Zhou, 2024, ESG shareholder engagement and downside risk, *Review of Finance*, Volume 28, Issue 2, March 2024, Pages 483–510

[57] Holmstrom, B., and B. Nalebuff. 1992. To the raider goes the surplus? A reexamination of the free-rider problem. *Journal of Economics and Management Strategy* 1:37–62.

[58] Huang, S. and A. Kopytov, 2022, Sustainable finance under regulation, *Working paper.*

[59] Kyle, A., and J. Vila. 1991. Noise trading and takeovers. *Rand Journal of Economics* 22: 54–71.

[60] Landier, A. and S. Lovo, 2024, ESG Investing: How to Optimize Impact? *Review of Financial Studies*, forthcoming.

[61] Levit, Doron. 2017, Advising Shareholders in Takeovers, *Journal of Financial Economics* 126 (3): 614–34.

[62] Li, T., Q. Peng, and L. Yu. 2023, ESG Considerations in Acquisitions and Divestitures: Corporate Responses to Mandatory ESG Disclosure, *Working paper.*

[63] Liu, T., 2022, Bargaining with private equity: implications for hospital prices and patient welfare, *Working paper.*

[64] Marquez, R., Yilmaz, B., 2008. Information and Efficiency in Tender Offers. *Econometrica* 76, 1075-1101.

[65] Marquez, R., Yilmaz, B., 2012. Takeover bidding and shareholder information. *Review of Corporate Finance Studies* 1, 1-27.

[66] Mueller, H. M., and F. Panunzi. 2004. Tender offers and leverage. *Quarterly Journal of Economics* 119:1217–48.

[67] Naaraayanan, S. L., K. Sachdeva, and V. Sharma, 2021, The real effects of environmental activist investing, *Working paper.*

[68] Oehmke, Martin and Marcus Opp (2024): "A Theory of Socially Responsible Investment." *Review of Economic Studies*, forthcoming.

[69] Ofer, A. R., Thakor, A. V., 1987. A theory of stock price response to alternative corporate disbursement methods: Stock repurchases and dividends. *Journal of Finance* 42, 365–394.

[70] Pastor, L., R. F. Stambaugh, and L. A. Taylor, 2021, Sustainable Investing in Equilibrium. *Journal of Financial Economics* 142, 550–571.

[71] Pedersen, L.H., S. Fitzgibbons, and L. Pomorski, 2021, Responsible Investing: The ESG-Efficient Frontier. *Journal of Financial Economics* 142, 572–597.

[72] Phillips GM, Zhdanov A. 2013. R&D and the incentives from merger and acquisition activity. *Review of Financial Studies.* 26(1):34–78

[73] Piccolo, A., J. Schneemeier, and M. Bisceglia, 2022), Externalities of Re sponsible Investments, *Working paper.*

[74] Rossi, S. , Volpin, P.F. , 2004. Cross-country determinants of mergers and acquisitions. *Journal of Financial Economics* 74, 277–304 .

[75] Shleifer, A., and R. W. Vishny. 1986. Large shareholders and corporate control. *Journal of Political Economy* 94:461–488.

[76] Voss, P. & Kulms, M. (2022), 'Separating ownership and information', *American Economic Review* 112(9), 3039–62.

[77] Yarrow, G. K. 1985. Shareholder protection, compulsory acquisition and the efficiency of the takeover process. *Journal of Industrial Economics* 34 (September):3–16.

[78] Zerbib, O. D., 2022, A Sustainable Capital Asset Pricing Model (S-CAPM): Evidence from Environmental Integration and Sin Stock Exclusion., *Review of Finance* 26, 1345–1388.

A Appendix

A.1 Proofs of main results

Proof of Lemma 1. Rearranging,

$$\begin{aligned}\tau(\gamma;p) &= (p-v_0+\eta\phi_1-\phi_0)(q+\Delta)-(p-v_0+\eta\phi_1-\phi_0+v_1-p+(1-\eta)\phi_1)q \\ &= (p-v_0+\eta\phi_1-\phi_0)(q+\Delta)-(v_1-v_0+\phi_1-\phi_0)q. \end{aligned} \tag{34}$$

Auxiliary Lemma 3 in Section A.2 implies

$$\frac{\partial\tau(\gamma;p)}{\partial\gamma}=\left[(p-v_0+\eta\phi_1-\phi_0)\frac{K-1}{\gamma}-(v_1-v_0+\phi_1-\phi_0)\frac{N-K}{1-\gamma}\right]\Delta. \tag{35}$$

Note that

$$\begin{aligned}\tau(0;p) &= 0 \\ \tau(1;p) &= p-v_1-(1-\eta)\phi_1.\end{aligned}$$

From (35), the shape of τ is determined by the following four cases:

(i) τ is increasing then decreasing if $p>v_0+\phi_0-\eta\phi_1$ and the takeover is socially efficient

(ii) τ is monotonically increasing if $p\geq v_0+\phi_0-\eta\phi_1$ and the takeover is socially inefficient

(iii) τ is decreasing then increasing if $p<v_0+\phi_0-\eta\phi_1$ and the takeover is socially inefficient

(iv) τ is monotonically decreasing if $p\leq v_0+\phi_0-\eta\phi_1$ and the takeover is socially efficient

Moreover, in the non-monotone cases (i) and (iii) the interior extremum occurs at $\hat{\gamma}(p)$, defined in (10).

Hence:

$\gamma^*=0$ is an equilibrium if and only if one of cases (iii) and (iv) holds.

$\gamma^*=1$ is an equilibrium if and only if $p>v_1+(1-\eta)\phi_1$, or if $p=v_1+(1-\eta)\phi_1$ and case (i) holds. Note that if $p=v_1+(1-\eta)\phi_1$ and the takeover is socially efficient then $p>v_0+\phi_0-\eta\phi_1$.

$\gamma^*\in(0,1)$ is a equilibrium if and only if both case (i) holds and $p<v_1+(1-\eta)\phi_1$. Note that social efficiency is implied by the combination of $p>v_0+\phi_0-\eta\phi_1$ and $p<v_1+(1-\eta)\phi_1$. In this case, γ^* is the unique solution to $\tau(\gamma^*;p)=0$, or equivalently, $\mu(\gamma^*)=p$. Note that $\gamma^*\in(\hat{\gamma}(p),1)$. Since τ is continuous and increasing in p it follows that γ^* is continuous and increasing in p. Moreover, $\gamma^*\to 0$ as $p\to v_0+\phi_0-\eta\phi_1$ and $\gamma^*\to 1$ as $p\to v_1+(1-\eta)\phi_1$.[15]

[15]Note from (34) that $\tau(\gamma;v_0+\phi_0-\eta\phi_1) = -(v_1-v_0+\phi_1-\phi_0)q$ and $\tau(\gamma;v_1+(1-\eta)\phi_1) = (v_1-v_0+\phi_1-\phi_0)\Delta$.

Similarly, γ^* decreases in ϕ_0 because τ decreases in ϕ_0. Finally, γ^* is locally increasing in ϕ_1 if and only if $\tau(\gamma^*; p)$ is increasing in ϕ_1 (holding γ^* fixed), i.e., if and only if $\eta(q+\Delta) - q > 0$.

From the above characterization: if $\gamma^* \in (0,1)$ is an equilibrium then it is unique. Hence the only case in which multiple equilibria exist is if both $\gamma^* = 0$ and $\gamma^* = 1$ are equilibria. Note that if $p = v_0 + \phi_0 - \eta\phi_1$ and the takeover is socially efficient then $p < v_1 + (1-\eta)\phi_1$ and $\gamma^* = 1$ isn't an equilibrium; and similarly, if $p = v_1 + (1-\eta)\phi_1$ and the takeover is socially efficient then $p > v_0 + \phi_0 - \eta\phi_1$ and $\gamma^* = 0$ isn't an equilibrium. Hence $\gamma^* = 0$ and $\gamma^* = 1$ coexist as equilibria if and only if $p \in (v_1 + (1-\eta)\phi_1, v_0 + \phi_0 - \eta\phi_1)$. ■

Proof of Proposition 1. We prove Proposition 1 for a general case with a socially responsible bidder with parameter $\delta \geq 0$, as described in Section 5. Proposition 1 is obtained when $\delta = 0$.

First, consider the case $v_1 + \phi_1 < v_0 + \phi_0$. From Lemma 1, shareholders' response is given by the correspondence

$$\gamma^* = \begin{cases} 0 & \text{if } p \leq v_1 + (1-\eta)\phi_1 \\ \{0,1\} & \text{if } p \in (v_1 + (1-\eta)\phi_1, v_0 + \phi_0 - \eta\phi_1) \\ 1 & \text{if } p \geq v_0 + \phi_0 - \eta\phi_1, \end{cases} \tag{36}$$

From (16) and (30), $\Pi(0) = 0$ and $\Pi(1) = N(v_1 + \delta\phi_1 - p)$. Hence if $v_1 + (1-\eta)\phi_1 \geq v_1 + \delta\phi_1$, or equivalently $(1-\eta-\delta)\phi_1 \geq 0$, then the bidder's payoff is strictly negative in any equilibrium with $\gamma^* = 1$. Conversely, if $(1-\eta-\delta)\phi_1 < 0$ then for any $p \in (v_1 + (1-\eta)\phi_1, v_1 + \delta\phi_1]$ there is an equilibrium in which the bidder offers p and shareholders accept with $\gamma^* = 1$.

The bid $p = v_0 + \phi_0 - \eta\phi_1$ guarantees both $\gamma^* = 1$ and a strictly positive payoff for the bidder if $v_1 + \delta\phi_1 > v_0 + \phi_0 - \eta\phi_1$. Hence an equilibrium with $\gamma^* = 0$ exists if and only if $v_1 + \phi_1 - v_0 - \phi_0 \leq (1-\eta-\delta)\phi_1$.

Second, consider the case $v_1 + \phi_1 > v_0 + \phi_0$. From Lemma 1, shareholders' response is given by the function

$$\gamma^* = \begin{cases} 0 & \text{if } p \leq v_0 + \phi_0 - \eta\phi_1 \\ \mu^{-1}(p) \in (\hat{\gamma}(p), 1) & \text{if } p \in (v_0 + \phi_0 - \eta\phi_1, v_1 + (1-\eta)\phi_1) \\ 1 & \text{if } p \geq v_1 + (1-\eta)\phi_1. \end{cases}$$

Offers $(v_0 + \phi_0 - \eta\phi_1, v_1 + (1-\eta)\phi_1)$ deliver shareholder acceptance probabilities γ satisfying $\mu(\gamma) = p$ and (from (15)) associated bidder payoffs of

$$\pi(\gamma; \delta) \equiv N\gamma\Delta(v_1 + \phi_1 - v_0 - \phi_0) - N\gamma(q+\Delta)(1-\eta-\delta)\phi_1.$$

The offer $p = v_1 + (1-\eta)\,\phi_1$ delivers a shareholder acceptance probability of $\gamma = 1$ and a bidder payoff of

$$N\left(v_1 + \delta\phi_1 - p\right) = -N\left(1 - \eta - \delta\right)\phi = \pi\left(1;\delta\right).$$

Recall from Lemma 1 that as p increases over the interval $(v_0 + \phi_0 - \eta\phi_1, v_1 + (1-\eta)\,\phi_1)$ the shareholder acceptance probability increases continuously from 0 to 1. Hence the bidder effectively picks γ (via choice of offer p) to solve

$$\max_{\gamma\in[0,1]} \pi\left(\gamma;\delta\right).$$

Rearranging,

$$\frac{\pi\left(\gamma;\delta\right)}{N} = \gamma\left(q+\Delta\right)\left(v_1 + \phi_1 - v_0 - \phi_0 - \left(1-\eta-\delta\right)\phi_1\right) - \gamma q\left(v_1 + \phi_1 - v_0 - \phi_0\right).$$

From Lemma 3,

$$\begin{aligned}
\frac{\partial}{\partial\gamma}\left(\gamma\left(q+\Delta\right)\right) &= q + K\Delta \\
\frac{\partial}{\partial\gamma}\left(\gamma q\right) &= q + \frac{N-K}{1-\gamma}\gamma\Delta
\end{aligned}$$

and hence

$$\begin{aligned}
\frac{1}{N}\frac{\partial\pi\left(\gamma;\delta\right)}{\partial\gamma} &= \left(q+K\Delta\right)\left(v_1 + \phi_1 - v_0 - \phi_0 - \left(1-\eta-\delta\right)\phi_1\right) \\
&- \left(q + \frac{N-K}{1-\gamma}\gamma\Delta\right)\left(v_1 + \phi_1 - v_0 - \phi_0\right) \\
&= -\left(q+K\Delta\right)\left(1-\eta-\delta\right)\phi_1 + \frac{K-\gamma N}{1-\gamma}\Delta\left(v_1 + \phi_1 - v_0 - \phi_0\right). \qquad (37)
\end{aligned}$$

Hence

$$\frac{\partial\pi\left(\gamma;\delta\right)}{\partial\gamma} > 0 \Leftrightarrow \frac{\kappa-\gamma}{1-\gamma}\left(v_1 - v_0 + \phi_1 - \phi_0\right) > \left(\frac{q}{N\Delta} + \kappa\right)\left(1-\eta-\delta\right)\phi_1.$$

From Lemma 3 and L'HÃŽpital's rule, $\frac{\Delta}{q} \to \infty$ as $\gamma \to 0$. Hence $\lim_{\gamma\to 0}\frac{\partial\pi(\gamma;\delta)}{\partial\gamma} > 0$ if and only if

$$v_1 - v_0 + \phi_1 - \phi_0 > \left(1-\eta-\delta\right)\phi_1. \qquad (38)$$

If instead (38) is violated,

$$\frac{\kappa-\gamma}{1-\gamma}\left(v_1 - v_0 + \phi_1 - \phi_0\right) \leq \kappa\left(v_1 - v_0 + \phi_1 - \phi_0\right) \leq \kappa\left(1-\eta-\delta\right)\phi_1,$$

and so $\frac{\partial \pi(\gamma;\delta)}{\partial \gamma} < 0$ for all $\gamma > 0$. It follows that if (38) holds the bidder chooses $\gamma^* > 0$, while if it is violated the bidder chooses $\gamma^* = 0$.

For profit calculations below: Recall that $Nv_1 = V_1$ is independent of N, etc. Moreover, by Stirling's approximation $\Delta \to 0$, regardless of the limiting behavior of γ^*. Consequently, the bidder's payoff approaches $-(1-\eta-\delta)\Phi_1 \lim_{N\to\infty} \gamma^* q$.

There are three subcases:

Subcase $(1-\eta-\delta)\phi_1 > 0$: There exists $\epsilon > 0$ (independent of N) such that $\frac{\partial \pi(\gamma;\delta)}{\partial \gamma} < 0$ if $\gamma \geq \kappa - \epsilon$. So the bidder chooses $\gamma^* \in (0, \kappa - \epsilon)$ if (38) holds and $\gamma^* = 0$ otherwise. In either case, the success probability approaches 0 as N grows large.

Subcase $(1-\eta-\delta)\phi_1 = 0$: The bidder chooses $\gamma = \kappa$.

Subcase $(1-\eta-\delta)\phi_1 < 0$: There exists $\epsilon > 0$ (independent of N) such that $\frac{\partial \pi(\gamma;\delta)}{\partial \gamma} > 0$ if $\gamma \leq \kappa + \epsilon$. So the bidder chooses $\gamma^* \in (\kappa + \epsilon, 1)$. The success probability approaches 1 as N grows large, while $\Delta \to 0$. From Lemma 1, if the bidder offers $p = v_1 + (1-\eta)\phi_1$ then all shareholders tender with probability 1, and so the bidder's payoff is $N(v_1 + \delta - p) = -(1-\eta-\delta)\Phi_1$. This offer is suboptimal, and so $-(1-\eta-\delta)\Phi_1$ is a lower bound for the bidder's payoff. Hence the bidder's payoff approaches $-(1-\eta-\delta)\Phi_1$ as N grows large (which also establishes that $\gamma^* \to 1$). ■

Proof of Corollary 2. The result follows directly from Proposition 1; it is only left to establish the comparative statics stated in part (ii). Notice that if $\eta = 1$ then

$$\mu(\kappa) = v_0 + \frac{(v_1 - v_0)q - (\phi_0 - \phi_1)\Delta}{q + \Delta},$$

where both q and Δ are independent of $\phi_1 - \phi_0$. Therefore, $\mu(\kappa)$ is decreasing in $\phi_0 - \phi_1$. Also, it can be verified that $\mu(\kappa) < v_1$ in this case. To prove Π^* is increasing in $\phi_0 - \phi_1$ notice

$$\Pi^* = N\kappa\Delta(v_1 - v_0 + \phi_1 - \phi_0).$$

■

Proof of Proposition 2. It is straightforward to replace τ with τ_θ in the proof of Lemma 1, where $\theta \in [0, 1)$. For socially inefficient takeovers the equilibrium of the tendering subgame exactly coincides with the no-freeze-out baseline ($\theta = 0$). Moreover, in this case the only possible equilibria are $\gamma = 0$ and $\gamma = 1$, and freeze-outs don't affect the bidder's profits in these cases. Consequently, in this case both the bidder's equilibrium offer and shareholders' equilibrium response coincide with the no-freeze-out baseline.

The remainder of the proof deals with the case of socially efficient takeovers. The bidder's

profit $\pi(\gamma)$ in a mixed-strategy equilibrium is N times

$$\begin{aligned}
& (\gamma(q+\Delta)+(1-\gamma)q\theta)(v_1-\mu_\theta(\gamma)) \\
= \; & (\gamma(q(1-\theta)+\Delta)+q\theta)\frac{\frac{\Delta}{1-\theta}}{q+\frac{\Delta}{1-\theta}}(v_1+\phi_1-v_0-\phi_0)-(\gamma(q+\Delta)+(1-\gamma)q\theta)(1-\eta)\phi_1 \\
= \; & \left(\Delta\gamma+\frac{\Delta\theta q}{q(1-\theta)+\Delta}\right)(v_1+\phi_1-v_0-\phi_0)-(\gamma(q+\Delta)+\theta q(1-\gamma))(1-\eta)\phi_1. \qquad (39)
\end{aligned}$$

Moreover, the minimum offer that generates $\gamma = 1$ as an equilibrium for the tendering subgame is $p = v_1 + (1-\eta)$, which gives profits of $-N(1-\eta)$, coinciding with the expression above as $\gamma \to 1$. Consequently the bidder effectively chooses $\gamma \in [0,1]$ to maximize (39).

For preferences $\eta = 1$, the bidder's problem reduces to choosing γ to maximize

$$\Delta\gamma + \theta\frac{\Delta q}{q(1-\theta)+\Delta}. \qquad (40)$$

The term $\Delta\gamma$ is single-peaked and obtains its maximum at $\gamma = \frac{K}{N}$. Differentiating the second term in (40) gives

$$\begin{aligned}
\frac{\partial}{\partial\gamma}\left(\frac{\Delta q}{q(1-\theta)+\Delta}\right) &= \frac{((1-\theta)q+\Delta)\left(q\frac{\partial\Delta}{\partial\gamma}+\Delta\frac{\partial q}{\partial\gamma}\right)-\Delta q\left((1-\theta)\frac{\partial q}{\partial\gamma}+\frac{\partial\Delta}{\partial\gamma}\right)}{((1-\theta)q+\Delta)^2} \\
&= \frac{(1-\theta)q^2\frac{\partial\Delta}{\partial\gamma}+(1-\theta)q\Delta\frac{\partial q}{\partial\gamma}+\Delta q\frac{\partial\Delta}{\partial\gamma}+\Delta^2\frac{\partial q}{\partial\gamma}-\Delta q(1-\theta)\frac{\partial q}{\partial\gamma}-\Delta q\frac{\partial\Delta}{\partial\gamma}}{((1-\theta)q+\Delta)^2} \\
&= \frac{(1-\theta)q^2\frac{\partial\Delta}{\partial\gamma}+\Delta^2\frac{\partial q}{\partial\gamma}}{((1-\theta)q+\Delta)^2}. \qquad (41)
\end{aligned}$$

To establish that the bidder selects $\gamma > \frac{K}{N}$ we show that (41) is strictly positive for all $\theta > 0$ and $\gamma \leq \frac{K}{N}$. Note that $\frac{\partial\Delta}{\partial\gamma} > 0$ if and only $\gamma < \frac{K-1}{N-1}$,[16] while $\frac{\partial q}{\partial\gamma} > 0$ for all $\gamma \in (0,1)$. Hence (41) is strictly positive for all $\gamma \leq \frac{K-1}{N-1}$. So it remains to show that (41) is strictly positive for $\gamma \in \left(\frac{K-1}{N-1}, \frac{K}{N}\right]$; and it suffices to establish this statement at $\theta = 0$.

Expanding (using Lemma 3), we must show

$$\left(\frac{K-1}{\gamma}-\frac{N-K}{1-\gamma}\right)\Delta q^2+\Delta^2\frac{N-K}{1-\gamma}\Delta > 0 \text{ for } \gamma \in \left(\frac{K-1}{N-1}, \frac{K}{N}\right],$$

[16] $\frac{\partial\Delta}{\partial\gamma} > 0$ is equivalent to $\frac{K-1}{\gamma}-\frac{N-K}{1-\gamma} > 0$, i.e., to $(1-\gamma)K-(1-\gamma)-\gamma N+\gamma K > 0$, and hence to $K-1 > \gamma(N-1)$.

or equivalently,

$$\frac{1-\gamma}{\gamma}(K-1)-(N-K)+\left(\frac{\Delta}{q}\right)^2(N-K)>0 \text{ for } \gamma\in\left(\frac{K-1}{N-1},\frac{K}{N}\right].$$

From Lemma 4, the ratio $\frac{\Delta}{q}$ is decreasing in γ, and so it suffices to establish the inequality at $\gamma=\frac{K}{N}$. By straightforward manipulation, this is equivalent to

$$\sqrt{K}\Delta>q \text{ at } \gamma=\frac{K}{N}. \tag{42}$$

We establish inequality (42) in two steps. First, we fix $K\geq 2$, and establish the inequality for $N=2K$. Second, we show that if (42) holds for $N=2K$ then it also holds for any $N<2K$.

For the first step, consider $N=2K$. Note that there are K binomial terms from $\binom{N-1}{0}$ to $\binom{N-1}{K-1}$, and likewise $N-K=K$ binomial terms from $\binom{N-1}{K}$ to $\binom{N-1}{N-1}$. So by symmetry, it follows that if $\gamma=\frac{K}{N}=\frac{1}{2}$ then $q=\frac{1}{2}$. Hence we need to show

$$\sqrt{K}\tbinom{2K-1}{K-1}\left(\frac{1}{2}\right)^{2K-1}>\frac{1}{2}.$$

We establish this by induction in K. At $K=1$ the LHS evaluates to $\frac{1}{2}$. Hence it suffices to show that for any $K\geq 1$,

$$\sqrt{K+1}\tbinom{2(K+1)-1}{(K+1)-1}\left(\frac{1}{2}\right)^{2(K+1)-1}>\sqrt{K}\tbinom{2K-1}{K-1}\left(\frac{1}{2}\right)^{2K-1},$$

i.e.,

$$\sqrt{\frac{K+1}{K}}\left(\frac{1}{2}\right)^2>\frac{(2K-1)!}{(2K+1)!}\frac{K!}{(K-1)!}\frac{(K+1)!}{K!}=\frac{(K+1)\,K}{(2K+1)\,2K},$$

i.e.,

$$K+\frac{1}{2}>\sqrt{K}\sqrt{K+1},$$

which indeed holds by the concavity of the log function.

For the second step, we show that if (42) holds for $N=2K$ then it also holds for any $N<2K$. It suffices to show that $\frac{q(\frac{K}{N})}{\Delta(\frac{K}{N})}$ is increasing in N (holding K fixed). Note

$$\frac{q}{\Delta}=\frac{\sum_{j=K}^{N-1}\binom{N-1}{j}\gamma^j(1-\gamma)^{N-1-j}}{\binom{N-1}{K-1}\gamma^{K-1}(1-\gamma)^{N-1-(K-1)}}=\sum_{j=K}^{N-1}\frac{(K-1)!\,(N-K)!}{j!\,(N-1-j)!}\gamma^{j-K+1}(1-\gamma)^{K-1-j}.$$

Defining $\tilde{j} = j - K + 1$ and substituting in $\gamma = \frac{K}{N}$,

$$\frac{q}{\Delta} = \sum_{\tilde{j}=1}^{N-K} \frac{(K-1)!\,(N-K)!}{\left(\tilde{j}+K-1\right)!\left(N-K-\tilde{j}\right)!} \left(\frac{K}{N-K}\right)^{\tilde{j}}.$$

Expanding,

$$\frac{q}{\Delta} = \sum_{j=1}^{N-K} \frac{(N-K)\cdot\ldots\cdot(N-K-j+1)}{K\cdot\ldots\cdot(j+K-1)} \left(\frac{K}{N-K}\right)^{j} = \sum_{j=1}^{N-K} \frac{1\cdot\left(1-\frac{1}{N-K}\right)\cdot\ldots\cdot\left(1-\frac{j-1}{N-K}\right)}{1\cdot\left(1+\frac{1}{K}\right)\cdot\ldots\cdot\left(1+\frac{j-1}{K}\right)},$$

which is indeed increasing in N, thereby establishing (42).

Finally, we establish that the bidder's profit-maximizing choice of γ is strictly increasing in the freeze-out probability θ. Recall that the bidder sets γ to

$$\arg\max_{\gamma\in[0,1]} \left(\Delta\gamma + \theta\frac{\Delta q}{q\,(1-\theta)+\Delta}\right). \tag{43}$$

Note that both $\gamma = 0, 1$ give zero bidder profits, and so certainly the bidder's choice of γ is interior. From (41) we know that if

$$\frac{\partial}{\partial\gamma}\left(\frac{\Delta q}{q\,(1-\theta)+\Delta}\right) \geq 0$$

for some γ and θ, then this inequality holds strictly for any $\tilde{\theta} > \theta$: this follows trivially if $\frac{\partial\Delta}{\partial\gamma} \geq 0$, and follows easily if $\frac{\partial\Delta}{\partial\gamma} < 0$. Consequently, (43) is strictly increasing in θ over any neighborhood of θ-values in which it is unique. Finally, suppose there is some θ at which both γ_1 and $\gamma_2 > \gamma_1$ maximize the bidder's objective (40). Let $q_1, \Delta_1, q_2, \Delta_2$ denote q and Δ evaluated at γ_1 and γ_2. Note that

$$\Delta_1\gamma_1 + \theta\frac{\Delta_1 q_1}{q_1\,(1-\theta)+\Delta_1} = \Delta_2\gamma_2 + \theta\frac{\Delta_2 q_2}{q_2\,(1-\theta)+\Delta_2}.$$

We know $\gamma_2 > \gamma_1 > \kappa$ and hence both $\Delta_1 > \Delta_2$ and $\Delta_1\gamma_1 > \Delta_2\gamma_2$, and hence

$$\frac{\Delta_1 q_1}{q_1\,(1-\theta)+\Delta_1} < \frac{\Delta_2 q_2}{q_2\,(1-\theta)+\Delta_2}.$$

Note that

$$\frac{\partial}{\partial\theta}\frac{\Delta q}{q\,(1-\theta)+\Delta} = \frac{1}{\Delta}\left(\frac{\Delta q}{q\,(1-\theta)+\Delta}\right)^2,$$

implying that for $\tilde{\theta} > \theta$

$$\Delta_1\gamma_1 + \tilde{\theta}\frac{\Delta_1 q_1}{q_1\left(1-\tilde{\theta}\right)+\Delta_1} < \Delta_2\gamma_2 + \tilde{\theta}\frac{\Delta_2 q_2}{q_2\left(1-\tilde{\theta}\right)+\Delta_2}.$$

It again follows that the bidder's profit-maximizing choice γ is increasing in θ. ■

Proof of Proposition 3. Suppose $\eta = 1$. We first characterize the bidder's profit-maximizing behavior conditional on some choice of d; and then optimize over d. There are three cases to consider:

1. First, suppose $v_1 - d + \phi_1 > v_0 + \phi_0$. From Lemma 1, $p = \mu(\gamma; v_1 - d) \in (v_0 + \phi_0 - \phi_1, v_1 - d)$. The bidder profits are

$$\Pi = N\gamma\Delta(v_1 + \phi_1 - v_0 - \phi_0) + Nqd.$$

   If $p = v_1 - d + (1-\eta)\phi_1$ then $\gamma = 1$ in equilbirum, and bidder profits are Nd. Hence the bidder effectively chooses $\gamma \in [0,1]$ to maximize the term above.

2. Second, suppose $v_1 - d + \phi_1 < v_0 + \phi_0$. From Lemma 1, $\gamma = 1$ is an equilibrium if $p > v_1 - d$. Moreover, $\gamma = 0$ is also an equilibrium if $p < v_0 + \phi_0 - \phi_1$.

3. Suppose $v_1 - d + \phi_1 = v_0 + \phi_0$. If $p > v_1 - d$ then shareholders always accept; while if $p = v_1 - d$ then any γ is a tendering probability in the subgame, and bidder profits are $Nd(q + \gamma\Delta)$, which is increasing in γ.

Next,

1. Suppose $v_1 + \phi_1 > v_0 + \phi_0$ and $\phi_1 \geq \phi_0$. The bidder sets $d = v_1 - v_0$ and chooses γ to maximize

$$N\gamma\Delta(v_1 + \phi_1 - v_0 - \phi_0) + Nq(v_1 - v_0).$$

   Hence $\gamma \in (\kappa, 1)$. In this case the bidder would like to further raise leverage, but cannot do so. Leverage raises bidder profits, though they fall short of full surplus extraction.

2. Suppose $v_1 + \phi_1 > v_0 + \phi_0$ and $\phi_1 < \phi_0$. If the bidder sets $d = v_1 + \phi_1 - v_0 - \phi_0 - \epsilon$, profits are

$$\max_{\gamma\in[0,1]} N(q + \gamma\Delta)(v_1 + \phi_1 - v_0 - \phi_0) - N(1-\gamma)q\epsilon,$$

   which can be made arbitarily close to the full deal surplus $N(v_1 + \phi_1 - v_0 - \phi_0)$. By further increasing d to $d = v_1 - v_0$ the bidder moves to a situation in which $v_1 - d + \phi_1 <$

$v_0 + \phi_0$, and so multiple equilibria may arise. In particular, if $d = v_1 - v_0$ and $p = v_0 + \epsilon$ there is an equilibrium in which $\gamma = 1$ and profits are $N(v_1 - v_0 - \epsilon)$—though there is also an equilibrium of the tendering subgame in which $\gamma = 0$.

If the former equilibrium is selected with probability β when $d = v_1 - v_0$, then the bidder chooses $d = v_1 + \phi_1 - v_0 - \phi_0 - \epsilon$ if and only if

$$N(v_1 + \phi_1 - v_0 - \phi_0) > \beta N(v_1 - v_0) \Leftrightarrow \beta < 1 + \frac{\phi_1 - \phi_0}{v_1 - v_0}.$$

Thus, in equilibrium, the takeover succeeds with probability not smaller than $1 + \frac{\phi_1 - \phi_0}{v_1 - v_0}$. There is an equilibrium in which the takeover fails with probability $-\frac{\phi_1 - \phi_0}{v_1 - v_0}$ or smaller.

3. Suppose $v_1 + \phi_1 < v_0 + \phi_0$. Recall that we are assuming $v_1 > v_0$. Hence $\phi_1 < \phi_0$. The bidder sets $d = v_1 - v_0$. Bidding $p \geq v_0 + \phi_0 - \phi_1 > v_1$ would result in negative profits. Hence $p < v_0 + \phi_0 - \phi_1$, and $\gamma = 0$ is an equilibrium. Setting $p = v_0 + \epsilon$ gives a second equilibrium in which $\gamma = 1$.

■

Proof of Proposition 5. First consider the case $\eta = 1$. By Corollary 2, if the bidder chooses ϕ_1 such that $v_1(\phi_1) + \phi_1 < v_0 + \phi_0$ then the bidder's payoff is 0. If instead the bidder chooses ϕ_1 such that $v_1(\phi_1) + \phi_1 > v_0 + \phi_0$ then the bidder's payoff is $N\kappa\Delta(\kappa)(v_1(\phi) + \phi_1 - v_0 - \phi_0)$. Hence the bidder chooses $\phi_1 = \phi_1^{**}$.

Next consider the case $\eta < 1$. From Proposition 1, if the bidder pledges ϕ_1^{**} then shareholders tender with probability $\gamma^{**} \in [0, 1)$, and the bidder's payoff is (writing $\Delta^{**} = \Delta(\gamma^{**})$ and $q^{**} = q(\gamma^{**})$)

$$N\gamma^{**}\Delta^{**}(v_1(\phi_1^{**}) + \phi_1^{**} - v_0 - \phi_0) - N\gamma^{**}(q^{**} + \Delta^{**})(1 - \eta)\phi_1^{**}.$$

First note that there is a $\phi_1^* < \phi_1^{**}$ yielding a higher payoff for the bidder. If $\gamma^{**} > 0$ this follows by envelope arguments: because $\frac{\partial}{\partial \phi_1}(v_1(\phi_1) + \phi_1) = 0$, one can find a ϕ_1^* marginally below ϕ_1^{**} such that, holding the acceptance probability unchanged at γ^{**}, the bidder's payoff is strictly higher. If instead $\gamma^{**} = 0$ then the bidder's profit from ϕ_1^{**} is zero; moreover, by (38) a necessary condition for this case is $\phi_1^{**} > 0$. If the bidder instead chooses $\phi_1^* < 0$, then either the takeover is socially efficient, (38) holds, and the bidder's payoff is strictly positive; or else the takeover is socially inefficient, and there is an equilibrium in which the bidder's payoff is strictly positive.

Conversely, consider any pledge $\tilde{\phi}_1 > \phi_1^{**}$. If this pledge yields a zero payoff for the bidder then it is dominated by ϕ_1^* above. Otherwise, let $\tilde{\gamma}, \tilde{\Delta}, \tilde{q}$ be the associated probabilities. Note that regardless of whether $\tilde{\gamma} \in (0,1)$ or $\tilde{\gamma} = 1$, the bidder's payoff is bounded above by

$$\begin{aligned} & N\tilde{\gamma}\tilde{\Delta}\left(v_1\left(\tilde{\phi}_1\right) + \tilde{\phi}_1 - v_0 - \phi_0\right) - N\tilde{\gamma}\left(\tilde{q} + \tilde{\Delta}\right)(1-\eta)\,\tilde{\phi}_1 \\ < \;& N\tilde{\gamma}\tilde{\Delta}\left(v_1\left(\phi_1^{**}\right) + \phi_1^{**} - v_0 - \phi_0\right) - N\tilde{\gamma}\left(\tilde{q} + \tilde{\Delta}\right)(1-\eta)\,\phi^{**} \\ \leq \;& N\gamma^{**}\Delta^{**}\left(v_1\left(\phi_1^{**}\right) + \phi_1^{**} - v_0 - \phi_0\right) - N\gamma^{**}\left(q^{**} + \Delta^{**}\right)(1-\eta)\,\phi^{**}, \end{aligned}$$

so that $\tilde{\phi}_1$ is dominated by ϕ_1^{**}, which is in turn dominated by ϕ_1^*.

Finally, the case $\eta > 1$ follows from parallel arguments to the case of $\eta < 1$. ■

Proof of Lemma 2. The proof builds on Proposition 1 and its proof. Since the takeover is socially efficient, part (ii) of Proposition 1 applies. Recall that the bidder effectively selects γ to maximize $\pi(\gamma)$. From (37), $\frac{\partial \pi(\gamma;\delta)}{\partial \gamma}$ is decreasing in $v_0(\phi_0) + \phi_0$ if $\gamma < \kappa$, is independent of $v_0(\phi_0) + \phi_0$ if $\gamma = \kappa$, and is increasing in $v_0(\phi_0) + \phi_0$ if $\gamma > \kappa$.

If $(1-\eta)\,\phi_1 > 0$ then, from Proposition 1, $\gamma^* < \kappa$ independent of the specific value of $v_0(\phi_0) + \phi_0$. It follows from the above characterization of $\frac{\partial \pi(\gamma;\delta)}{\partial \gamma}$ that γ^* is decreasing in $v_0(\phi_0) + \phi_0$.

The cases of $(1-\eta)\,\phi_1 = 0$ and $(1-\eta)\,\phi_1 < 0$ follow similarly. ■

A.2 Supplemental results

Lemma 3. *The following identities hold:*

$$\frac{\partial \Lambda}{\partial \gamma} = N\Delta \tag{44}$$

$$\frac{\partial \Delta}{\partial \gamma} = \left(\frac{K-1}{\gamma} - \frac{N-K}{1-\gamma}\right)\Delta \tag{45}$$

$$\frac{\partial q}{\partial \gamma} = \frac{N-K}{1-\gamma}\Delta \tag{46}$$

Proof. Here, adopt the convention that if $j > N$ then $\binom{N}{j} = 0$. We prove identity (45):

$$\begin{aligned}
\frac{\partial \Delta}{\partial \gamma} &= \frac{\partial}{\partial \gamma}\tbinom{N-1}{K-1}\gamma^{K-1}(1-\gamma)^{N-K} \\
&= \left(\frac{K-1}{\gamma} - \frac{N-K}{1-\gamma}\right)\tbinom{N-1}{K-1}\gamma^{K-1}(1-\gamma)^{N-K} \\
&= \left(\frac{K-1}{\gamma} - \frac{N-K}{1-\gamma}\right)\Delta.
\end{aligned}$$

We prove identity (46):

$$\begin{aligned}
\frac{\partial q}{\partial \gamma} &= \frac{\partial}{\partial \gamma}\sum_{j=K}^{N-1}\tbinom{N-1}{j}\gamma^{j}(1-\gamma)^{N-1-j} \\
&= \sum_{j=K}^{N-1}\tbinom{N-1}{j} j\gamma^{j-1}(1-\gamma)^{N-1-j} - \sum_{j=K}^{N-1}\tbinom{N-1}{j}(N-1-j)\gamma^{j}(1-\gamma)^{N-2-j} \\
&= (N-1)\sum_{j=K}^{N-1}\tbinom{N-2}{j-1}\gamma^{j-1}(1-\gamma)^{N-1-j} - (N-1)\sum_{j=K}^{N-2}\tbinom{N-2}{j}\gamma^{j}(1-\gamma)^{N-2-j} \\
&= (N-1)\sum_{j=K-1}^{N-2}\tbinom{N-2}{j}\gamma^{j}(1-\gamma)^{N-2-j} - (N-1)\sum_{j=K}^{N-2}\tbinom{N-2}{j}\gamma^{j}(1-\gamma)^{N-2-j} \\
&= (N-1)\tbinom{N-2}{K-1}\gamma^{K-1}(1-\gamma)^{N-1-K} \\
&= \frac{N-K}{1-\gamma}\tbinom{N-1}{K-1}\gamma^{K-1}(1-\gamma)^{N-K} \\
&= \frac{N-K}{1-\gamma}\Delta.
\end{aligned}$$

We prove identity (44). From (3), (45), and (46), we have:

$$\begin{aligned}
\frac{\partial \Lambda}{\partial \gamma} &= \frac{\partial}{\partial \gamma}\left[q + \gamma\Delta\right] \\
&= \frac{\partial q}{\partial \gamma} + \Delta + \gamma\frac{\partial \Delta}{\partial \gamma} \\
&= \frac{N-K}{1-\gamma}\Delta + \Delta + \gamma\left(\frac{K-1}{\gamma} - \frac{N-K}{1-\gamma}\right)\Delta \\
&= N\Delta.
\end{aligned}$$

■

Lemma 4. *The ratio $\frac{q}{\Delta}$ is strictly increasing in $\gamma \in (0,1)$.*

Proof. We need to show

$$\frac{\partial q}{\partial \gamma}\Delta > q\frac{\partial \Delta}{\partial \gamma}.$$

From Lemma 3, this inequality is equivalent to

$$\frac{N-K}{1-\gamma}\Delta > \left(\frac{K-1}{\gamma} - \frac{N-K}{1-\gamma}\right) q,$$

which is in turn equivalent to

$$\begin{aligned}
\frac{N-K}{1-\gamma}(q+\Delta) &> \frac{K-1}{\gamma}q \Leftrightarrow \\
\frac{N-K}{1-\gamma}\sum_{j=K-1}^{N-1}\binom{N-1}{j}\gamma^j(1-\gamma)^{N-1-j} &> \frac{K-1}{\gamma}\sum_{j=K}^{N-1}\binom{N-1}{j}\gamma^j(1-\gamma)^{N-1-j} \Leftrightarrow \\
(N-K)\sum_{j=K-1}^{N-1}\binom{N-1}{j}\gamma^j(1-\gamma)^{N-1-(j+1)} &> (K-1)\sum_{j=K}^{N-1}\binom{N-1}{j}\gamma^{j-1}(1-\gamma)^{N-1-j} \Leftrightarrow \\
(N-K)\sum_{j=K-1}^{N-1}\binom{N-1}{j}\gamma^j(1-\gamma)^{N-1-(j+1)} &> (K-1)\sum_{j=K-1}^{N-2}\binom{N-1}{j+1}\gamma^j(1-\gamma)^{N-1-(j+1)}.
\end{aligned}$$

Hence it is sufficient to establish that, for any $j = K-1, \ldots, N-2$,

$$(N-K)\binom{N-1}{j} > (K-1)\binom{N-1}{j+1},$$

i.e.,

$$\frac{N-K}{K-1} > \frac{N-1-j}{j+1}.$$

At $j = K-1$ this inequality is equivalent to $\frac{1}{K-1} > \frac{1}{K}$, which indeed holds. Since the RHS is decreasing in j, this completes the proof. ■