An "Orthodox" Theory of Dual Labor Markets

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Abstract

We study the equilibrium segmentation of the labor market into a primary segment with stable employment relationships and high wages, a secondary segment with unstable matches with low wages, and a tertiary segment consisting of those who do not participate. This segmentation emerges endogenously in a standard search and matching model with a limited set of additional assumptions. Duality is possible in our framework because we assume that there are two production technologies into which workers self-select based on heterogeneous costs of supplying labor. Thus, duality in the labor market is a natural outcome and there is no need for any institutional barriers to explain it. We use our model to examine how the structure of the labor market reacts to productivity shocks and find that the secondary segment is more cyclically sensitive than the primary. We then estimate a Hidden Markov Model (HMM) on SIPP data using our model structure and estimate each respondent's labor market segment. We examine how segment membership relates to (i) workers' demographic characteristics; (ii) job characteristics, and (iii) workers' unobserved types.

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1 Introduction

We study the equilibrium segmentation of the labor market into a primary segment with stable employment relationships and high wages, a secondary segment with unstable matches and low wages, and a tertiary segment consisting of those who do not participate. We show that this segmentation emerges endogenously in a Mortensen and Pissarides (1994) model with a limited set of additional assumptions. Thus, duality among labor market participants is a natural outcome in the sense that no institutional barriers between the primary and secondary segment are needed to sustain the existence of both in equilibrium. This duality together with a nonparticipating segment does a good job of capturing U.S. labor market dynamics.

The study of duality in the labor market goes back, at least, to Doeringer and Piore (1970). Their Dual Labor Market (DLM) Hypothesis argued that the segmentation and the resulting lack of opportunities for low-skilled workers that we see in the labor market is a result of many different types of barriers facing the less educated, minorities, immigrants, and women (Reich *et al.* , 1973; Harrison and Sum, 1979; Berger and Piore, 1980; Dickens and Lang, 1985).

Several recent studies provide evidence that duality is a useful stylized way to think about the U.S. labor market. They show that labor market dynamics are well captured as driven by a limited number of subgroups, the members of which have markedly different labor market outcomes in terms of the stability of their jobs, the frequency with which they are unemployed, and their attachment to the labor force (e.g. Hall and Kudlyak, 2019; Gregory *et al.* , 2021; Ahn *et al.* , 2023). Among those who are attached to the labor force, most have relatively stable jobs as characterized in the primary segment in the DLM Hypothesis. A small group has less stable jobs, is frequently unemployed, and often temporarily gives up looking for a job. This latter group corresponds to the secondary segment in the DLM Hypothesis and accounts for most of the fluctuations in the aggregate unemployment rate.

These studies also find that only a small fraction of the variation in segment membership across individuals can be explained by the observable characteristics such as education, gender, and race, which were the main focus of the DLM Hypothesis (Gregory *et al.* , 2021; Ahn *et al.* , 2023). Despite reductions in many of the barriers the DLM Hypothesis emphasized, the share of the population in the secondary segment in the U.S. labor market has not declined much in recent decades (Ahn *et al.* , 2023). In fact, despite differences in labor-market institutions, this share is similar to that estimated for many European countries (Bentolila *et al.* , 2019).

This indicates that the causes of labor-market duality are related to factors that have been relatively constant over time and are common across labor markets in advanced economies. Recent models that have been used to illustrate the importance of market segmentation for

aggregate dynamics assume this segmentation reflects exogenously given and immutable worker types (e.g. Ferraro, 2018; Hall and Kudlyak, 2019; Gregory *et al.* , 2021). Thus, they do not look further for the causes of labor market segmentation. There are only a few papers in which duality emerges as an endogenous equilibrium outcome (Albrecht and Vroman, 1992; Bulow and Summers, 1986; Saint-Paul, 1997). In all of these models, the segmentation is due to imperfect information about worker effort and an associated efficiency wage mechanism that drives a wedge between primary and secondary segment jobs.

The main point of our paper is that duality is an equilibrium outcome in a relatively standard search and matching model of the labor market in the spirit of Mortensen and Pissarides (1994) with a small number of additional assumptions. The first core assumption that makes duality possible in our framework is that there are two production technologies. One is a baseline technology, not unlike that in Mortensen and Pissarides (1994), associated with the secondary market. The other, used in the primary market, requires an additional flow cost from the worker which, when incurred, results in a boost in productivity. Firms choose a technology when they post a vacancy, and job seekers decide in which labor market segment to search.

We deliberately describe these two technologies in a very generic way because our assumption fits many different specific interpretations. One is that the primary segment technology uses capital which requires additional effort by the worker to operate. Another is that the firm monitors workers in the primary segment to ensure they put in additional effort required for the productivity boost. We take a similar approach with the other assumptions we add in order to emphasize the broad applicability of our results.

The second core assumption that drives duality in our model is that workers differ in their cost of effort. This makes some workers more willing to incur the extra effort associated with primary-segment work. This heterogeneity leads to endogenous sorting of workers across the two available production technologies. We assume a continuous distribution of effort costs, which means that as labor market conditions change, workers can reallocate themselves across segments. The assumption of heterogeneity in effort costs is also open to multiple interpretations. The most straightforward is to interpret the cost of effort as the disutility from working or as the foregone value of home production. However, one can also think about effort cost as inversely related to skill and thus the ease with which workers perform tasks.

These two assumptions alone are enough to generate duality in the labor market and, when interpreted in terms of monitoring and effort costs, are similar to those in Albrecht and Vroman (1992). But the type of duality they generate does not necessarily capture the main dynamic characteristics of the labor market segments in the data because many of these characteristics have to do with turnover rates that these assumptions do not touch upon. With this in mind,

we make two additional assumptions to capture the fact that jobs in the primary segment are more stable than those in the secondary segment and to generate movements into and out of participation as well as between the secondary and primary segments.

The first additional assumption is related to the distribution of match-specific productivity shocks. As in the standard Mortensen and Pissarides (1994) model, our model features endogenous separations due to declines in idiosyncratic match-specific productivity. In order for the endogenous separation rate to be substantially higher in the secondary segment than in the primary one, we assume that the variance of productivity shocks is greater for the secondary-segment technology than for the high-productivity technology used in the primary segment.[1]

The second additional assumption is that workers' cost of effort varies over time. This variation is driven by frequent small shocks. These small shocks reflect minor changes in the disutility of working, possibly even driven by things such as their mood and the weather. In addition, we assume that a worker can be either "attached" or "unattached" with movements out of these two states at Poisson rates λ_a, where $a \in \{att, unatt\}$. There are two differences between the two states. First, the stochastic processes driving the change in the cost of effort over time differ between the two states. On average, attached workers have a lower cost of effort than unattached workers. Second, we assume that an unattached worker is subject to a death risk. When an unattached worker exits the model, we assume that he or she is replaced by a new, attached worker with the departing worker's cost of effort. The attachment status of workers in our model adds a life-cycle component to the evolution of a worker's cost of effort. Together, that life-cycle component and the small shocks to a worker's cost of effort generate flows between participation and non-participation and across segments over time.

The four assumptions we add to the standard Mortensen and Pissarides (1994) model yield an equilibrium in which the labor market splits into three segments, the sizes of which are endogenously determined. The primary segment consists of high-productivity matches involving workers with low cost of effort, who are willing to incur the additional cost necessary to benefit from the boost in productivity. These matches, on average, have a high surplus resulting in high wages and a lower likelihood that the match ends. The secondary segment consists of low-productivity matches in which workers with high cost of effort use the baseline technology. These matches tend to generate very little surplus and pay low wages. The result of this low surplus is that workers in the secondary segment are similar to workers in the Hagedorn and Manovskii (2008) model in the sense that their matches are more likely to terminate as a result of negative productivity shocks. Those in the secondary segment are only marginally attached

[1] This is consistent with empirical evidence that secondary segment jobs are more prevalent in sectors and occupations with higher seasonal fluctuations (Dickens and Lang, 1985; Ahn *et al.* , 2023).

to the labor force and likely to move into and out of the tertiary segment that consists of those who are out of the labor force and not searching.

The marked bifurcation of labor market participants that emerges in this equilibrium is due to differences in the types of matches in terms of productivity and worker's cost of effort that are sustained in the two segments in equilibrium. These differences are possible because of the adoption of different technologies in the two segments of the labor market. Thus, duality is not about worker or firm type but about equilibrium sorting of workers and firms between the two segments in response to the availability of different technologies. Indeed, firms are identical before choosing segments and workers with the same cost of effort may be in different segments in equilibrium.

This reason for duality contrasts sharply with the reasons given in the DLM Hypothesis literature. Those reasons had a much more normative flavor in that duality was interpreted as undesirable because those in the secondary segment are stuck in a low-wage low-opportunity cycle of unstable jobs due to specific barriers that prevent them from pursuing careers in the primary segment. Contrary to many of the earlier explanations of duality, such as those in Doeringer and Piore (1970) and Reich *et al.* (1973), which were often very U.S.-specific, the generality of the mechanism that we emphasize in our model might explain why labor-market duality is a phenomenon we observe in other advanced labor markets as well.

Because the main point of this paper is the generality of conditions under which duality can occur in equilibrium, the rest of the paper has the following structure. In the next section, we introduce the four main assumptions we add to the standard Mortensen and Pissarides (1994) model and discuss their implications for the equilibrium structure of the labor market. In Section 3, we solve our model and illustrate the characteristics of our equilibrium in more detail. We also show which aspects are most important for qualitatively matching the average differences between the market segments. In Section 4, we then discuss which of the possible interpretations of the generic model specification are most relevant in the data. Conclusions are presented in the final section.

2 An "Orthodox" model of a dual labor market

In this section, we introduce an "orthodox" search and matching model that has a dual labor market equilibrium outcome. The "orthodoxy" of our model refers to the fact that, at its core, our model is a standard DMP model with endogenous separations in the spirit of Mortensen and Pissarides (1994). We add four assumptions and a participation margin to this basic core. In the following subsections, we discuss what our assumptions add to the Mortensen and Pissarides

(1994) model and why we think they are important. Just as in the basic DMP model, we solve our model in continuous time.

2.1 Main assumptions

Labor market states: Addition of non-participation Most labor market search and matching models only focus on the labor force and ignore the participation margin.[2] Several recent papers, however, have emphasized that an important aspect of labor market heterogeneity is that the participation decisions of those in the secondary segment differ substantially from those in the primary segment.[3] In order for our model to capture these differences, we include a participation margin. Those who do not participate in the labor market are involved in home production.

Thus, as in the data, individuals are in one of three labor market states. They are either not participating in the labor market and instead involved in home production (N), searching for work and unemployed (U), or they are employed (E).

Three production technologies The key assumption in our paper is that, in each of the segments of the labor market, output is produced using a different technology. Those who don't participate, and are in the tertiary labor-market segment, use a home production technology which yields a flow value of b.

The "baseline" market production technology is used by workers and firms in the secondary segment of the labor market. It is similar to the one in Mortensen and Pissarides (1994) in that the match of the worker and firm yields a stochastic flow of output. We denote this output flow by Ae^z. A is the common component of productivity and is the source of business cycle fluctuations that we consider below. The other term, e^z, is the idiosyncratic match-specific level of productivity, the log of which we model as an Ornstein-Uhlenbeck process

$$dz = k_z^S(\mu_z^S - z)dt + \sigma_z^S dW_z, \tag{1}$$

where μ_z^S is the mean, k_z^S is the mean-reversion parameter, W_z is a standard Brownian Motion, and the variance of the process is determined by σ_z^S.

The "advanced" market technology used in the primary segment requires, at every instant,

[2]Several papers add a participation margin to labor search models, for example, Chapter 8 of Pissarides (2000), Garibaldi and Wasmer (2005), Pries and Rogerson (2009) and Albrecht *et al.* (2010). In these papers, workers are heterogeneous with respect to the flow value of nonparticipation, the cost of search, and/or match productivity. None of these papers allows for a dual labor market.

[3]See Ahn *et al.* (2023) for an analysis of participation in the context of the type of duality we consider here. The worker types in Hall and Kudlyak (2019) also exhibit very different participation dynamics.

additional effort from workers to operate. Incurring this flow cost yields two benefits; it increases the average productivity of the match, and it reduces the volatility of the idiosyncratic productivity shocks. As in the baseline technology in the secondary segment, z follows an Ornstein-Uhlenbeck process. But we assume that such a match has a higher mean productivity, i.e. $\mu_z^P > \mu_z^S$. The volatility of the productivity shocks for this advanced technology is lower than that in the secondary segment for two reasons. First, the variance of the process is lower, i.e. $\sigma_z^P < \sigma_z^S$; and second, the mean reversion of the process is weaker, i.e. $k_z^P < k_z^S$.

Heterogeneity in workers' cost of labor supply One of the main facts documented by empirical studies of the labor market is that unobserved heterogeneity is important for segment membership of workers.[4] With this in mind, we include heterogeneity in the cost of supplying labor. In particular, for a unit of effort needed to operate the baseline technology in the secondary segment, workers incur a flow utility cost that scales linearly with θ.

The parameter θ can be interpreted in several ways: (i) the disutility of labor, (ii) the value of foregone home production when working, or (iii) the effective effort needed to supply the labor necessary to operate the technology (inverse of ability). Thus, θ represents cross-individual variation in preferences and abilities, the specifics of which we do not model in detail and which are only partially explained by observables in the data. The sources of variation in θ vary over time across individuals.

We assume that θ evolves according to a regime-switching Ornstein-Uhlenbeck process with the regime indicated by attachment status, $a \in \{att, unatt\}$:

$$d\theta = k_a(\mu_a - \theta)dt + \sigma_a dW_\theta. \tag{2}$$

In the state-dependent processes determining movements in θ, k_a governs the reversion to the process mean μ_a, σ_a determines the process variance, and W_θ is a standard Brownian motion.

The differences and variation in the cost of supplying labor are an important driver of the selection of workers into and transitions between segments. Those in the tertiary segment supply no labor and thus do not incur the cost; those in the secondary segment, who use the baseline technology, incur the cost which scales with θ; and those in the primary segment incur a cost which scales faster with θ than for secondary workers due to the additional effort required to operate the advanced technology.

Labor market frictions: Segment-specific search Search and matching frictions in this model can be best summarized as "segment-specific search" in that both non-employed workers

[4] Observable characteristics only explain a small part of cross-individual variation in segment membership identified to be important for aggregate labor market dynamics. (Gregory *et al.* , 2021; Ahn *et al.* , 2023)

who decide to search as well as firms that post vacancies decide in which labor market segment to do so. In each segment the number of matches, m, is determined by the segment-specific tightness ratio, η_j for $j \in \{P, S\}$.

Search is costly for both workers and firms. Non-employed workers who search have a flow value less than what they would obtain if they were involved in home production. The reduction in this flow value is segment-specific and given by s_j for $j \in \{P, S\}$. It is costly for these workers to enter the labor market. There is a fixed cost, ν, associated with movement from nonparticipation to search.[5] Flow vacancy-posting costs for firms also vary by segment and are given by c_j for $j \in \{P, S\}$. Firms are *ex ante* identical, in that match-specific productivity is only realized after a match occurs, and they can choose to post vacancies in either the primary or the secondary segment. Just as in most search and matching models, firms enter and exit freely.

As is typical in DMP models, once a worker and firm meet, there is full information. In our context, this means that both the worker and firm know the realized match-specific productivity, z, the value of the worker's cost of labor supply, θ, and the worker's state, $a \in \{att, unatt\}$.

2.2 Labor supply, labor demand, and wage setting

Market production takes place in heterogeneous matches between workers and firms. This heterogeneity is captured by the quadruple $(z, \theta, a; j)$, where $j \in \{P, S\}$, and is reflected in the wage paid to the workers, i.e. $w_j(z, \theta, a)$. In this subsection, we discuss the labor supply decisions of the workers and the labor demand decisions of the firms taking this wage schedule as given. We then describe how these wages are set at the end of this subsection.

Labor supply and job search Non-employed workers, who vary in terms of θ and a have the choice not to participate in the labor market with value $N(\theta, a)$, search for a job in the primary segment with value $U^P(\theta, a)$, or search in the secondary segment with value $U^S(\theta, a)$.

If they decide to search for a job, then the segment they search in is the one that gives them the maximum value. That is, those for whom $U^P(\theta, a) \geq U^S(\theta, a)$ search in the primary segment, and otherwise they search in the secondary segment. The decision whether or not to search depends on the fixed cost of entering the labor force, i.e. ν. For a non-participant in state a with cost of effort θ to start searching for a job, it must be the case that

$$\max\left\{U^P(\theta, a), U^S(\theta, a)\right\} - \nu \geq N(\theta, a). \tag{3}$$

[5] The fixed cost ν is not essential for the existence of duality in equilibrium. It reduces the cyclical flow across the participation margin in case of a shock to A.

Similarly, for an unemployed worker in state a whose cost of effort is θ to drop out of the labor force, it must be the case that

$$N(\theta, a) \geq \max\left\{U^P(\theta, a), U^S(\theta, a)\right\}. \tag{4}$$

If the labor-force entry cost were zero, then unemployed workers and non-participants with the same a and θ would make the same labor supply decisions. The labor-force entry cost induces inertia, however, in the sense that, for some θ's, searchers continue to search and non-participants stay out of the labor force.

Workers are risk-neutral and discount the future at rate r. The present discounted values, $N(\theta, a)$, $U^P(\theta, a)$, and $U^S(\theta, a)$ are determined by three continuous-time Hamilton-Jacobi-Bellman (HJB) equations.

Non-participants of type θ in state a receive home-production flow value b and are still non-participants in the next instant. Their type, θ, evolves, however, and its changes are driven by three forces: (i) It has a tendency to revert to its mean, μ_θ, (ii) it is subject to small stochastic changes at any instant, and (iii) the process changes when the worker switches attachment state. The HJB for $N(\theta, a)$ reflects these three forces. It is

$$\begin{aligned} rN(\theta,a) = b + N_\theta(\theta,a)k_a(\mu_a - \theta) + \frac{1}{2}N_{\theta\theta}(\theta,a)\sigma_a^2 + \\ \lambda_a\left(\max\left\{N(\theta,a^c), U_S(\theta,a^c) - \nu, U_P(\theta,a^c) - \nu\right\} - N(\theta,a)\right) + \\ \mathcal{I}[a = unatt]\zeta(-N(\theta,a)). \end{aligned}$$

Here $\mathcal{I}[.]$ is the indicator function.

The terms on the first line are due to the mean reversion and small stochastic changes in θ at each instant. Here $N_\theta(\theta, a)$ and $N_{\theta\theta}(\theta, a)$ are the first- and second-order derivatives of $N(\theta)$ respectively. The second line accounts for the fact that the worker's state can change from attached to unattached or vice versa. In this line, a^c is the complement of the worker's current state. Finally, the third line reflects the death risk faced by the unattached.

An unemployed worker receives a flow value $b - s_j$, where the search cost s_j depends on the segment that the worker searches in, $j \in \{P, S\}$. At each instant, one of four things can happen. First, there can be a small change in the worker's θ. Second, the worker's state can switch from a to a^c with a corresponding change in value. Third, the worker can meet a firm in segment j. This occurs at the Poisson rate $m(\eta_j)$, where η_j is the ratio of vacancies to unemployment in segment j. Once the worker and firm meet, the worker learns the match-specific productivity, z, associated with the job and the firm observes both the worker's effort cost, θ and his or

her state, a. The worker and firm jointly decide whether to match and, if not, the worker and firm continue to search in segment j. Finally, a worker who is is unattached faces the death risk ζ. These four distinct possibilities are reflected in the associated Hamilton-Jacobi-Bellman equation (HJB). For $j \in \{P, S\}$:

$$\begin{aligned} rU_j(\theta,a) = b - s_j + U_{j,\theta}(\theta,a)k_a(\mu_a - \theta) + \frac{1}{2}U_{j,\theta\theta}(\theta,a)\sigma_a^2 + \\ \lambda_a\left(\max\left\{N(\theta,a^c), U_S(\theta,a^c), U_P(\theta,a^c)\right\} - U_j(\theta,a)\right) + \\ m(\eta_j)\left(\left[\int \max\{U_j(\theta,a), E_j(z,\theta,a)\}g_j(z)dz\right] - U_j(\theta,a)\right) + \\ \mathcal{I}[a = unatt]\zeta(-U_j(\theta,a)) \end{aligned}$$

The structure of the HJB is the same for both the primary and secondary segments. The only things that are different across the segments are the level of the search cost s_j, the rate at which matches arrive $m(\eta_j)$, the distribution of match specific productivity in case of a match, $g_j(z)$, and the worker's value of a successful match, $E_j(z,\theta,a)$.

The worker value of a match of type (z,θ,a) in segment j is determined by the following HJB:

$$\begin{aligned} rE_j(z,\theta,a) = w_j(z,\theta,a) - \theta e_j + E_{j,z}(z,\theta,a)k_z(\mu_z - z) + \frac{1}{2}E_{j,zz}(z,\theta,a)\sigma_z^2 + \\ E_{j,\theta}(z,\theta,a)k_a(\mu_a - \theta) + \frac{1}{2}E_{j,\theta\theta}(z,\theta,a)\sigma_a^2 + \\ \lambda_a\big(\max\{N(\theta,a^c), U_S(\theta,a^c), U_P(\theta,a^c), E_j(z,\theta,a^c)\} - E_j(z,\theta,a)\big) + \\ \mathcal{I}[a = unatt]\zeta(-E_j(z,\theta,a)) \end{aligned}$$

This equation reflects that at every instant, an employed worker in segment j receives a flow value equal to the wage minus the effort cost and that, subsequently, the match type evolves because of gradual movements in z and θ, changes in the worker's state, and the death risk suffered by the unattached.

Labor demand, vacancy posting, and dissolution of matches Firms that employ a worker in attachment state a who is of type θ at match-specific productivity z in segment j receive the discounted value of the match, $J_j(z,\theta,a)$. At any point the firm and the worker have the option to costlessly dissolve the match and then the firm receives a value of 0, which is implied by the free entry of vacancies in both segments. Thus, matches continue so long as $J_j(z,\theta,a) \geq 0$.

At each instant, the firm receives the flow value of output minus the wage paid. Moreover,

at any point in time, the match value may evolve for several reasons. First, the match-specific productivity, z, can change. Second, the worker's cost of labor supply, θ, can shift slightly. Third, the worker's state can change. Finally, if the worker's state is unattached, the worker can die, thus ending the match. The impact of these changes on the value of the match is captured by the following HJB: For $j \in \{P, S\}$:

$$\begin{aligned} rJ_j(z,\theta,a) = Ae^z - w_j(z,\theta,a) + J_{j,z}(z,\theta,a)k_z(\mu_z - z) + \frac{1}{2}J_{j,zz}(z,\theta,a)\sigma_z^2 + \\ J_{j,\theta}(z,\theta,a)k_a(\mu_a - \theta) + \frac{1}{2}J_{j,\theta\theta}(z,\theta,a)\sigma_a^2 + \\ \lambda_a\left(\max(J_j(z,\theta,a^c), V_S, V_P, 0) - J_j(z,\theta,a)\right) + \\ \mathcal{I}[a = unatt]\zeta(\max(V_S, V_P, 0) - J_j(z,\theta,a)). \end{aligned}$$

A firm obtains the match value $J_j(z,\theta,a)$ if it meets an unemployed worker of type θ who is in state a and the match-specific productivity z is realized. This requires posting a vacancy in segment j at the flow cost c_j. This is a segment-specific search environment in the sense that vacancies are posted in either the primary or secondary segment, but within each segment, firms cannot focus their recruitment efforts on workers of particular types θ or a.

A vacancy in segment j meets an unemployed worker at Poisson rate $m(\eta_j)/\eta_j$. The worker type associated with such a meeting is a draw from the distribution of unemployed workers in segment j while the match-specific productivity is drawn from a segment-specific productivity distribution. If the surplus generated by the potential match is positive, then a match forms. Otherwise, the worker retains the value $U_j(\theta,a)$ and the firm can continue to post a vacancy or exit the labor market. The HJB determining the value associated with posting a vacancy in segment j is:

$$rV_j = -c_j + \frac{m(\eta_j)}{\eta_j}\sum_a P(a \mid U_j)\left(\int\int \max(J_j(z,\theta,a), V_S, V_P, 0) f(\theta \mid U_j, a) g_j(z) d\theta dz - V_j\right)$$

where $f(\theta \mid U_j, a)$ is the endogenously determined distribution of worker types θ in attachment state a, who search in segment j and $g_j(z)$ is the distribution from which the match-specific productivity is drawn upon meeting.

Free entry of vacancies in each of the segments implies that vacancies are posted up to the point at which the level of labor market tightness, η_j, is such that $V_j = 0$ for $j \in \{P, S\}$.

Wage setting, match surplus, and separations Just as in Mortensen and Pissarides (1994), wages in both segments are set through Nash bargaining. These wages are continuously adjusted in response to changes in z, θ and a. Nash bargaining implies that workers obtain a share α of

the match surplus while firms receive the rest. This surplus is given by

$$S_j(z,\theta,a) = [E_j(z,\theta,a) - \max\{N(\theta,a), U_P(\theta,a), U_S(\theta,a)\}] + J_j(z,\theta,a). \tag{5}$$

Wages are then set such that

$$J_j(z,\theta,a) = (1-\alpha)S_j(z,\theta,a) \text{ and} \tag{6}$$

$$E_j(z,\theta,a) - \max\{N(\theta,a), U_P(\theta,a), U_S(\theta,a)\} = \alpha S_j(z,\theta,a) \text{ for } j \in \{P,S\}\ . \tag{7}$$

If match-specific productivity declines so much or the worker's effort cost increases so much that the surplus of the match is negative, the firm and worker agree to dissolve the match and separate. The match can also dissolve in reaction to a change in the worker's attachment state.

2.3 Structure of equilibrium with duality

In Mortensen and Pissarides (1994), equilibrium is represented by (*i*) the fraction of the labor force that is unemployed, (*ii*) a match-specific productivity cutoff above which matches continue, and (*iii*) labor market tightness, i.e. the ratio of vacancies to unemployed. The core equilibrium objects in our model are similar, but, because of the addition of worker heterogeneity, a participation margin, and the technology choice which leads to segmentation, the representation of equilibrium is somewhat more complicated. Here we discuss its main properties, which will be useful for understanding the results we present in the rest of the paper.

Distribution of the population across five labor market states In equilibrium, the population is distributed across five separate states. First, there are those who are engaged in home production with value $N(\theta,a)$ who are not participating in the labor force. Second, the unemployed are split between those searching in the primary and secondary segments with values $U^P(\theta,a)$ and $U^S(\theta,a)$, respectively. Finally, the employed are also divided between the primary and secondary segments with values, $E^P(z,\theta,a)$ and $E^S(z,\theta,a)$, respectively.[6] The distributions of θ among the unemployed in the primary and secondary segments, $f(\theta \mid U_j)$ for $j \in \{P,S\}$, determine the value of posting vacancies in the two segments.

This means that, in equilibrium, the three groups normally analyzed in macroeconomic analyses of the labor market, i.e. non-participants, unemployed, and employed, are a mixture of unobserved underlying types. These types are defined by the individual's cost of effort, θ, and, if they are in the labor force, by the segment they are in, $j \in \{P,S\}$, and, if they

[6] The evolution of the distribution of the population follows a set of flow equations implied by the discussion above.

are employed, by the match-specific productivity, z.[7] As a result, consistent with the facts emphasized in Kudlyak and Lange (2018), the observed labor market transitions between the three observed labor market states are Non-Markovian.

Choice sets Given an attachment status a, workers' labor supply decisions are characterized by five cutoff levels. The first, $\theta^{P,S}(a)$, is the level above which searchers in the primary segment start searching in the secondary segment and below which the opposite happens. The other four cutoff levels, $\theta^{N,j}(a)$ and $\theta^{j,N}(a)$ for $j \in \{P, S\}$, are the levels of θ below which non-participants start searching in segment j and the levels above which searchers in segment j stop searching and exit the labor force. As a result of the fixed cost of entering the labor force, ν, $\theta^{N,j}(a) \neq \theta^{j,N}(a)$ for $j \in \{P, S\}$. The difference between these two values of θ in both segments reflects the *participation inertia* induced by the fixed cost.

Whereas in the conventional Mortensen and Pissarides (1994) model, there is a cutoff level of z above which matches continue, in our model, whether a match continues depends on both z and θ as well as on the worker's state a. Given a, the (z, θ) combinations at which matches continue in the primary and secondary segments are given by the sets $\mathcal{C}_j(a)$ for $j \in \{P, S\}$. The continuation set in the primary segment, $\mathcal{C}_P(a)$, has the property that if a primary-segment match of type (z, θ) continues, then all matches, (z', θ'), with higher or equal match-specific productivity, $z' \geq z$, and lower or equal cost of effort, $\theta' \leq \theta$, will also continue. This is not the case in the secondary segment. The continuation set in the secondary segment, $\mathcal{C}_S(a)$, is such that if a secondary-segment match of type (z, θ) continues then all matches, (z', θ) with higher or equal match-specific productivity, $z' \geq z$, will also continue. However, it can be the case that matches (z', θ') in which $\theta' < \theta$ dissolve. The reason is that, at a lower θ, the worker's option value of searching in the primary segment increases and there can be a point at which θ' is so low that the worker is better off pursuing the *probability of upward movement* by searching for a job in the primary segment instead continuing in the secondary-segment match.

Two-dimensional labor-market tightness Since the labor market is segmented, two levels of labor market tightness, η_S and η_P, determine the level of labor demand at which firms make zero expected profits when posting vacancies. The firms' values of a job match depend on the expected duration of the match and that duration depends on the opportunities workers have in the other segment. As a result, η_P and η_S are both determined not only by what happens in their own segment but also in the other segment of the labor market.

[7] In our model, these transitions follow a Coupled Hidden Markov Process (Brand *et al.* , 1997).

3 Quantitative analysis

3.1 Calibration

We calibrate our model by targeting various moments in the 2014 panel of the SIPP.[8] There are two sets of parameters. The first govern the search and matching process and the second set of parameters capture the stochastic processes that drive productivity and cost of labor supply. Parameter values are tabulated in Tables A.5 and A.6.

Targets We target 44 moments in the data. Given the interactions in the model it is hard to pinpoint the value of a certain parameter by targeting a specific moment. That is why we minimize the distance between targeted and model-implied moments. The list of moments we target are: the aggregate unemployment and participation rates (2); the unemployment and labor force participation rates by age (6); labor force flow rates (6); job-finding and separation rates by age (6); and the distribution of time spent unemployed, employed and not in the labor force over the 48 months individuals spend in the SIPP (24); which gives us a total of 44 moments.

Age-dependent moments in the model Age is not a state in our model. However, the attachment status of individuals and the death risk for unattached individuals introduce a life-cycle component to the model. We utilize this to compute model-implied, age-dependent unemployment and participation rates as well as labor force flow rates. In particular, we suppose that when an unattached individual exits the model, the attached individual who replaces them is a nonparticipant and 16 years old. Using the distribution of 16-year-olds over the state space as a starting point, we simulate forward to extract age-group-specific distributions of individuals over the state space $(z, \theta, a; j)$ for each age group of interest. We then simulate 4 years of labor force histories from these distributions for consistency with the SIPP.

Model fit There are clearly more moment conditions than the number of parameters that we calibrate, even though some conditions are implied by a combination of others such as the aggregate unemployment and participation rates. Table 1 and Figure 1 show the fit of the model. The calibrated model captures the statistics well especially the age-specific patterns even though we do not model age specifically. Moreover, the distribution of time spent in each labor market state is well captured. The employment and nonparticipation distribution is U shaped while the unemployment distribution is right skewed.

[8] See Appendix B for details on the model solution method.

	Data	Model
LFPR	65.8	66.9
u	4.9	5.7
EE	98.6	99.3
EU	0.4	0.6
EN	0.9	0.2
UE	10.7	12.0
UU	83.2	87.1
UN	6.1	0.9
NE	1.7	0.1
NU	0.7	0.4
NN	97.5	99.5

	Age Group	Data	Model
LFPR	*16-24*	66.3	67.8
	25-54	81.7	75.2
	55-64	59.3	66.5
u	*16-24*	8.9	11.4
	25-54	4.3	4.9
	55-64	3.3	3.2
Job-Finding	*16-25*	10.5	12.7
	25-54	10.4	11.2
	55-64	9.9	12.7
Separation	*16-24*	2.9	1.1
	25-54	1.0	0.7
	55-64	1.4	0.6

Table 1: Calibration targets in the 2014 SIPP panel and model-equivalent values for aggregate moments (left) and age-specific targets (right).

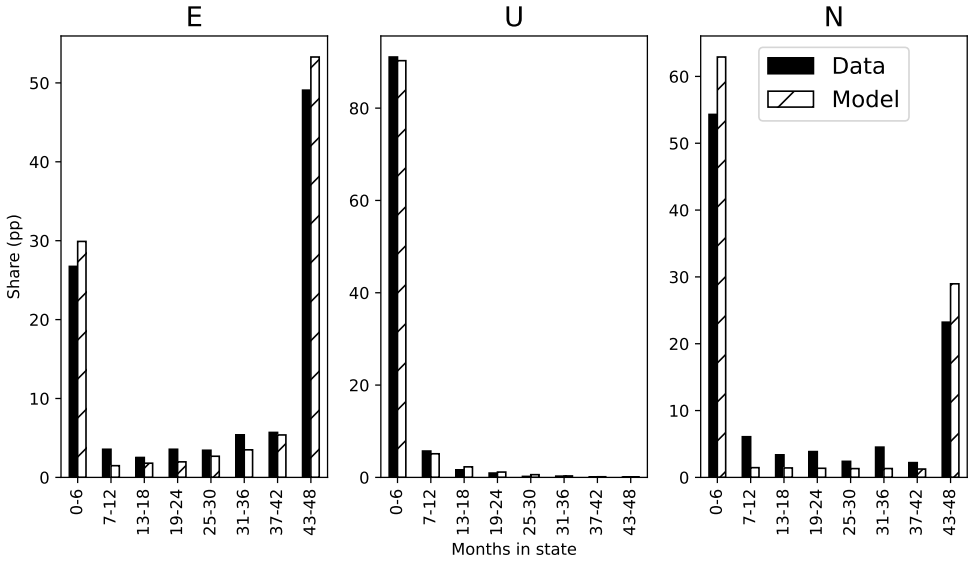


Figure 1: Distribution of time spent in each labor market state over a 4-year span in both the SIPP and the model.

3.2 Steady-state properties of the model

The steady-state of the model exhibits three separate segments: primary, secondary and tertiary. These segments have starkly different properties as Table 2 shows. The primary segment has low unemployment, high wages and high wage dispersion and constitutes 50% of the population. The secondary segment is 17.3% of the population and it has significantly higher unemployment rate and lower wages. The tertiary segment captures nonparticipation. Table 3

shows that there is a clear age component to segment allocations. Younger and older workers are more likely to be out of the labor force and less likely to be in the primary segment compared to workers in the 25-54 year old category.

	Primary	Secondary	Tertiary
Population share	49.6%	17.3%	33.1%
Unemployment rate	4.8%	8.2%	NA
Mean wage	7.3	3.8	NA
Variance of wages	0.5	0.2	NA

Table 2: Aggregate statistics by segment.

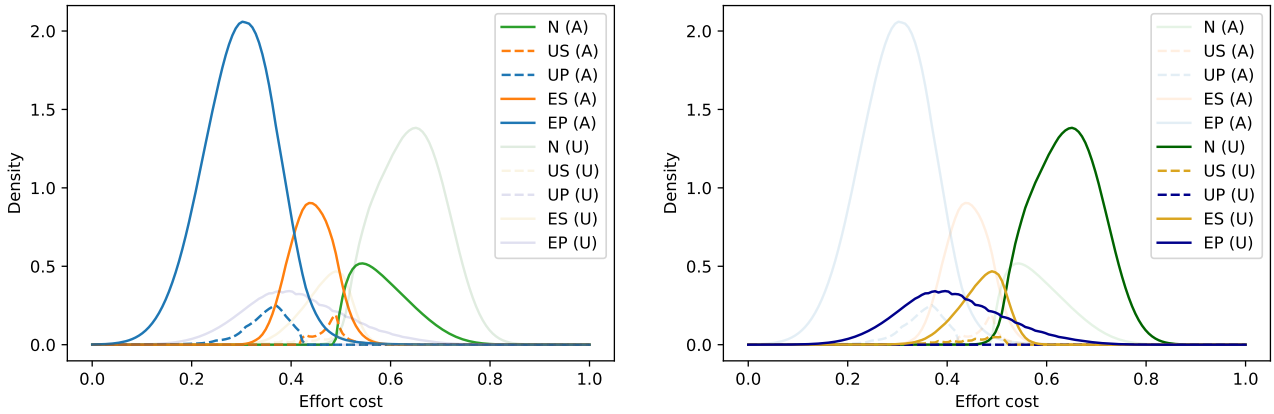
	Total	16-24	25-54	55-64
Share in P	49.6	27.1	58.1	56.2
Share in S	17.3	40.6	17.1	10.3
Share in N	33.1	32.2	24.8	33.5
u	5.7%	11.4%	4.9%	3.2%

Table 3: Aggregate statistics by segment and age.

3.3 Labor-market segmentation as an equilibrium

The steady-state equilibrium features three different segments and five separate states as we have described in Section 2.3. First, there are those who are engaged in home production with value N who are not participating in the labor force. Second, the unemployed are split between those searching in the primary and secondary segments with values U_P and U_S. Finally, the employed are also divided between the primary and secondary segments with values E_P and E_S. Membership in these segments and states is determined by an individual's cost of effort θ. The left panel of Figure 2 shows the distribution of attached workers across different labor market segments and effort costs θ in our steady state. The right panel of the workers shows the distribution of unattached workers. Workers who have a higher effort cost are more likely to be nonparticipants and even more so if they are unattached. Workers with low values of θ participate in the primary segment and the ones in between are likely to be in the secondary segment.

Recall that in our model, whether a match continues depends on both z and θ as well as on the worker's state a. The worker enters the secondary segment when their θ crosses the participation threshold. As θ gets lower, the worker's option value of searching in the primary segment increases and there can be a point at which θ' is so low that the worker is better off



(a) Attached workers (b) Unattached workers

Figure 2: The steady-state distribution of individuals over labor market states and θ.

pursuing the *probability of upward movement* by searching for a job in the primary segment instead continuing in the secondary-segment match.

3.4 Cyclical sensitivity of segments

Given the selection into primary and secondary segments, the surplus distribution of matches in the two segments look very different. Figure 3 shows that matches in the secondary segment have small surplus while primary segment matches, on average, have higher surplus. Moreover, there is more dispersion in the surplus of jobs in the primary segment. This is due to the differences in productivity process and the cost of effort differentials. Jobs in the secondary segment are similar to the Hagedorn and Manovskii (2008) model in the sense that their matches are more likely to terminate as a result of negative productivity shocks.

We consider the economy's response to a 2% aggregate productivity shock to examine our model's implications for aggregate shocks. The rise in the unemployment rate in the secondary segment is about 0.6 ppts while primary segment unemployment rate increases by only 0.2 ppts. The participation rate also declines by 0.6 ppts.

4 From macro to micro analysis

We have intentionally left the significance of individual types and segment-specific production technologies in our model open to interpretation to accommodate multiple explanations for the emergence of the dual labor market. But in this section, we investigate the extent to which our modeling approach can be used to explicitly relate observable characteristics of individuals

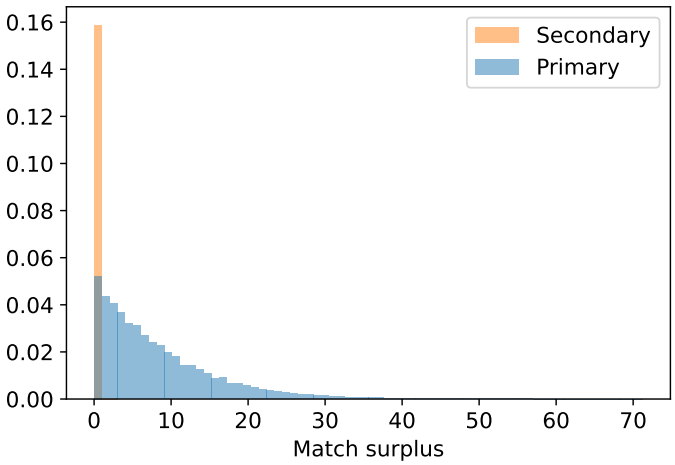


Figure 3: Distribution of match surplus by labor market segment

to membership in the primary, secondary, or tertiary labor market segments. We do this by conducting a classification exercise in the SIPP. Specifically, we use the structure of our model to assign *model-implied probabilities* of segment membership to each respondent in the SIPP micro data. After assigning these probabilities, we are able to examine the relationship between the probability of primary or secondary segment membership with observables that the existing literature on dual labor markets has linked to segmentation.

4.1 Classifying individuals in the data using the model

Our approach relies on the fact that in the SIPP, we observe a month-by-month path of labor market states $\hat{x} := \{x_1, x_2, \ldots, x_{48}\}$, where $x_t \in \{E, U, N\}$. Assuming that our model was the data generating process for the SIPP, then this path $\hat{x}$ would be the result of individuals self-selecting into segments of the labor market based on the path of their state variables $\hat{s} := \{s_1, s_2, \ldots, s_{48}\}$, where s_t includes an individual's θ (and z if they are employed), the segment they currently belong to, and their labor force status at each month t.

Under this conceptual framework, which is represented visually in Figure 4, the SIPP can be considered a hidden Markov model (HMM). In an HMM, there are *emissions*, which are observable to the econometrician but non-Markovian, and *hidden states*, which are unobservable but Markovian. At a given month t, an individual i's hidden state is their vector of model state variables $s_{i,t}$, since effort cost, match-specific productivity, and segment membership are unobservable in the data. Meanwhile, individual i's emission will be their labor market status $x_{i,t}$, since this is the only state variable in the model which is observable in the SIPP.

Conditional on a respondent's emissions $\hat{x}$, one can compute an implied probability that

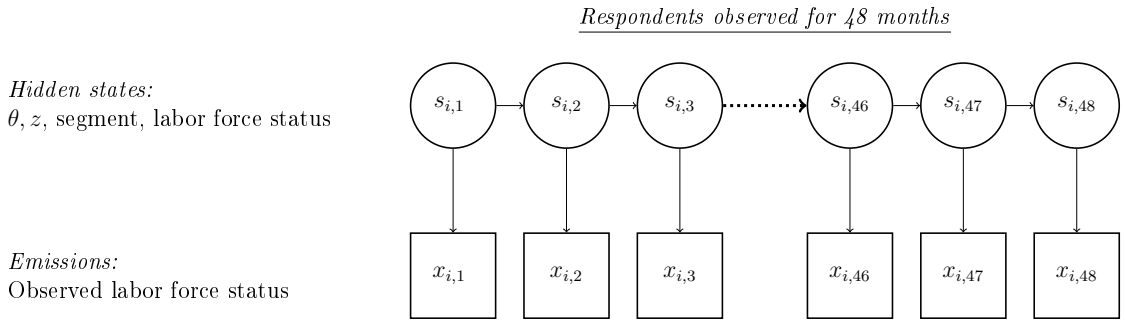


Figure 4: Representing the SIPP as a hidden Markov model based on the hidden states in our model.

they are in a particular hidden state s at the end of their survey tenure:

$$P(S_{48} = s \mid \hat{x}) = \frac{\alpha_s(\hat{x})}{\sum_{s'} \alpha_{s'}(\hat{x})}$$

where α_s is the joint probability of the respondent's emissions and the ending state s:

$$\alpha_s(\hat{x}) = P(X_1 = x_1, \ldots, X_{48} = x_{48}, S_{48} = s).$$

These probabilities α_s are computed using the forward step of the Baum-Welch algorithm used in estimating HMMs.[9] Taken together, these probabilities α_s imply a path-specific conditional probability distribution over the model's state space for every possible path $\hat{x}$ of 48 labor market states. Therefore, by summing over relevant s values—for example, all states s where an individual is in the primary segment—we can compute the model-implied probability that a SIPP respondent is in the primary segment. This holds true for other relevant moments as well, like a respondent's model-implied expected effort cost or match productivity.

4.2 Segmentation and observed heterogeneity

Table 4 computes the fraction of workers in each labor market segment in the SIPP. We find that men and more educated workers are more likely to be in the primary segment while older workers and women are more likely to be in the tertiary segment. Women and workers who identify as Black and Hispanic are underrepresented in the primary segment. We also find that primary segment workers are on average older and slightly more highly educated than

[9]The Baum-Welch algorithm is a forward-backward algorithm usually used to estimate the parameters of an HMM. In our case, we only need to use the forward step because our model parameters are calibrated based on other moments in the SIPP.

secondary segment workers. The relationship between age, education, and segment membership is presented in more detail in Appendix C.1.

	Primary	Secondary	Tertiary
Mean age	42.1	36.1	55.5
Mean years schooling	14.2	13.8	12.8
Male	43.5	28.3	28.2
Female	32.4	27.3	40.3
White	38.4	27.2	34.4
Black	33.0	30.8	36.2
Asian	40.2	28.5	31.3
Hispanic	36.9	33.2	29.9

Table 4: Model-implied segment membership by demographic characteristics in the SIPP. Rows other than means are presented such that they sum to 100 percent of individuals who belong to the given demographic group.

4.3 Segmentation and job characteristics

The market segmentation in our model is due to differences in the types of jobs in terms of productivity and worker's cost of effort. These differences could reflect different production technologies and/or tasks in the two segments of the labor market. Given that heterogeneity in the types of jobs is also an important driver of the segmentation, we estimate each SIPP respondent's probability of primary segment membership conditional on their final reported occupation and industry. We report these probabilities in a heat map in Figure 5. While occupation and industry alone do not capture primary segment membership well, the combination of the two has more explanatory power. We plan to examine these patterns in more detail with a regression framework.

4.4 Segmentation and unobserved heterogeneity

While the observables correlate with segments, observables are not enough to account for segment membership, consistent with the earlier findings in the literature. Our estimation based on HMM in the SIPP data provides estimates of each individual's hidden state, which includes their cost of effort, represented by θ. The left panel of Figure 6 shows that the cost of effort follows a U-shaped pattern with age. It decreases as workers age until they reach their prime working years, then remains stable between ages 30 and 50. This is followed by a steep increase as workers get older. Women are estimated to have a higher cost of effort at any age, as the

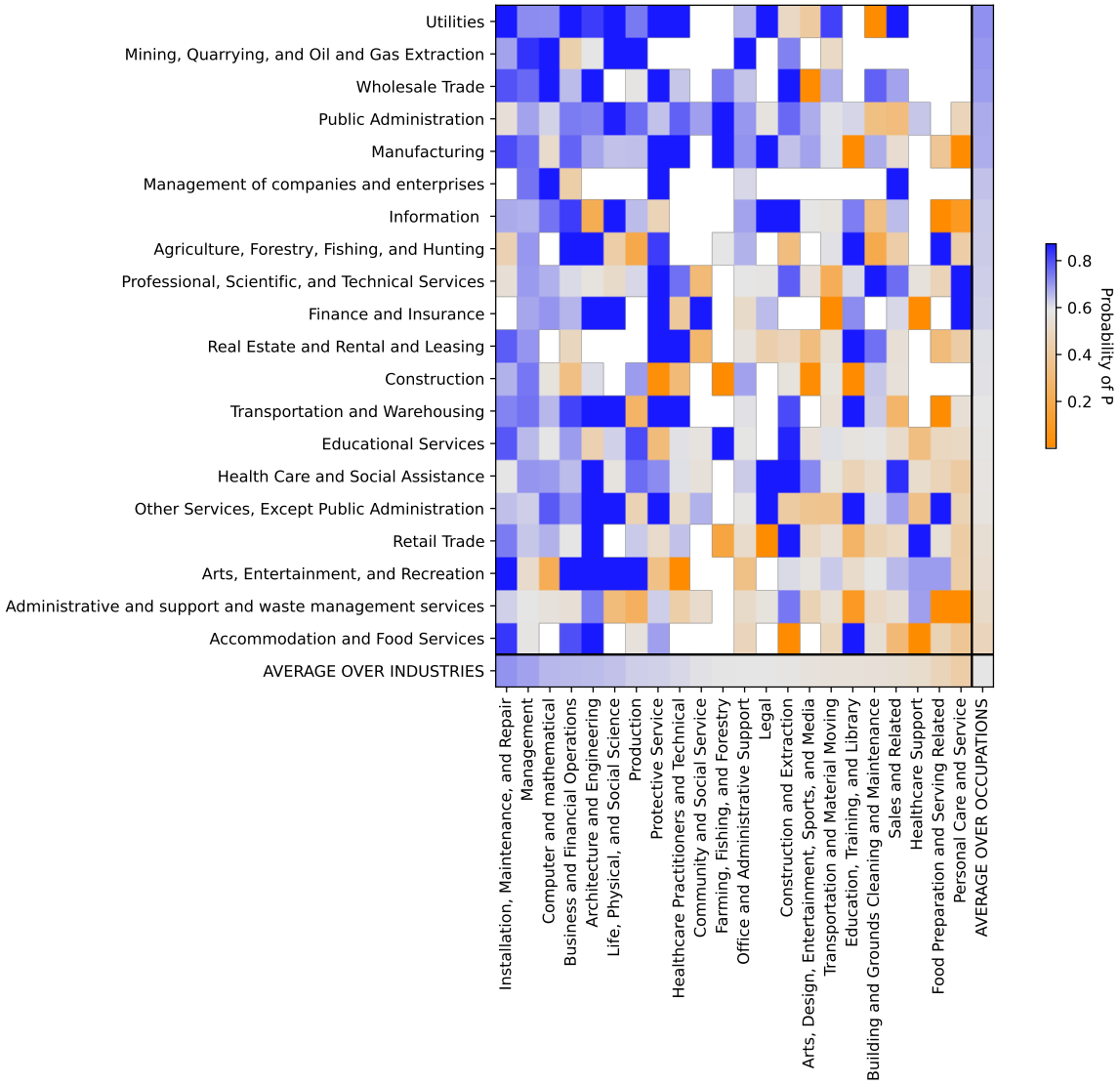


Figure 5: Model-implied probability of primary segment membership for individuals employed at the end of the SIPP sample period, conditional on each respondent's final reported occupation and industry. Industries and occupations are ordered by the probability of primary membership when averaging over occupations or industries, respectively.

right panel of Figure 6 shows. The differences in cost of effort are the highest at child-bearing ages and tend to be lower for younger and older workers.

5 Future work and conclusion

We extend the standard Mortensen and Pissarides (1994) model and show that in equilibrium the labor market splits into three segments, the sizes of which are endogenously determined. The primary segment consists of high-productivity matches involving workers with low cost

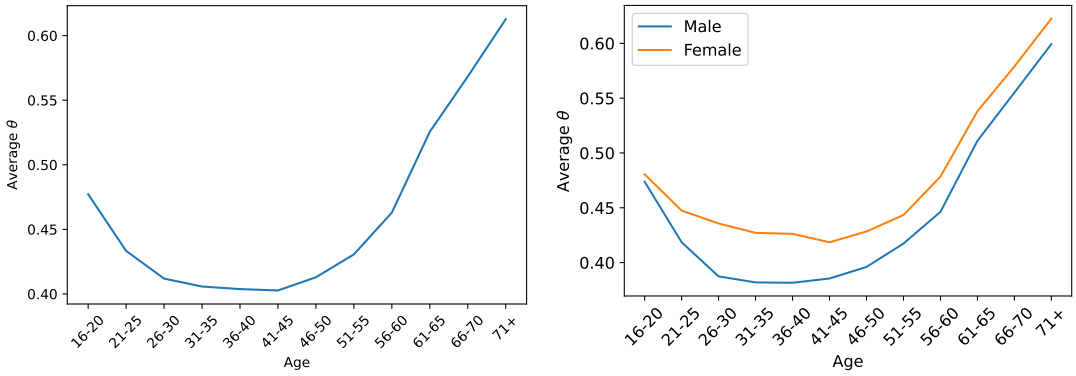


Figure 6: Expected effort cost θ by age in the SIPP (left panel) and expected effort cost θ by age and gender (right panel).

of effort, who are willing to incur the additional cost necessary to benefit from the boost in productivity. These matches, on average, have a high surplus resulting in high wages and a lower likelihood that the match ends. The secondary segment consists of low-productivity matches in which workers with high cost of effort use the baseline technology. These matches tend to generate very little surplus and pay low wages.

Our paper contributes to the recent and growing literature that focuses on estimating worker types using statistical models in three important ways. The first is that market segmentation is an equilibrium outcome and is endogenous to market conditions. The second important finding is that segmentation is an outcome of both worker and job heterogeneity and it is not a fixed characteristic of the worker. The third is since we can estimate an HMM within our model for the SIPP, we are able to examine the relative importance of different factors to segmentation; which we plan to do next.

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A Calibration details

	Parameter	Value
r	Discount rate	0.1
n	Flow value, nonparticipation	2.1
s_S	Search cost, S	0.2
s_P	Search cost, P	1.9
e_S	Effort, S	3.8
e_P	Effort, P	12
c_S	Vacancy cost, S	0.14
c_P	Vacancy cost, P	0.2
ν	Participation cost	0.5
α	Worker bargaining power	0.5
γ	Elasticity of v in $m(u,v)$	0.35

Table A.5: Parameter values related to search and matching process.

	Parameter	Value
k_{att}	Mean reversion, att.	0.09
σ^2_{att}	OU variance, att.	0.001
μ_{att}	OU mean, att.	0.275
λ_{att}	Exit rate, att.	0.04
k_{unatt}	Mean reversion, unatt.	0.09
σ^2_{unatt}	OU variance, unatt.	0.0005
μ_{unatt}	OU mean, unatt.	0.7
λ_{unatt}	Exit rate, unatt.	0.02
$k_{z,S}$	Mean reversion, z, S	1.2
σ^2_S	OU variance, z, S	0.0065
$k_{z,P}$	Mean reversion, z, P	0.003852
σ^2_P	OU variance, z, P	0.0004
μ_S	OU mean, z, S	1.4
μ_P	OU mean, z, P	1.7

Table A.6: Parameter values related to the stochastic processes for productivity and cost of labor supply.

B Mathematical and computational details

B.1 Description of solution method

We solve the model using a finite-element method in which workers and firms' continuous time problems are approximated with discrete time problems with small time increments of size dt.

The continuous state space for θ and z is represented with a discrete grid of points with a high resolution. We approximate the continuous-time Ornstein-Uhlenbeck processes for these variables with a finite-state Markov chain using the method in Lefebvre and Guilbault (2009). We use 75 grid points for match productivity z and 150 grid points for θ. Our time increment dt is $\frac{1}{500}$ of a year. Simulated examples of the approximations to the continuous time processes appear in Figures B.7 and B.8.

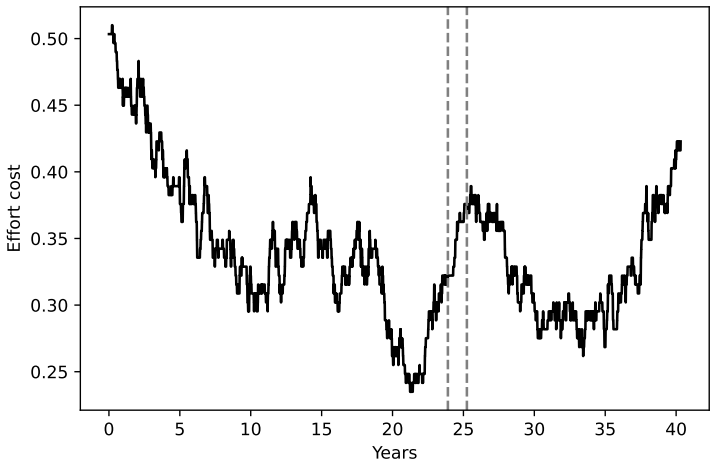


Figure B.7: Example path of θ over 40 years, starting with an initial θ of 0.5 in the attached state. Dashed lines indicate a switch from attached to unattached and then a switch back to attached.

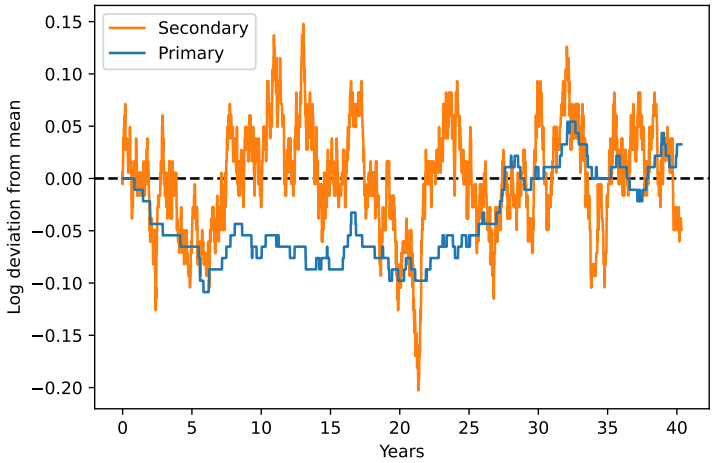


Figure B.8: Example paths of z in each segment over 40 years, starting with initial z's at the mean of the Ornstein-Uhlenbeck process in each segment.

To solve our model in general equilibrium, three steps are required. First, we use value function iteration to solve for the values N, U_j, and E_j for $j \in \{P, S\}$ based on an initial guess η_j for the ratio of vacancies to unemployed in each segment. Second, we update the

distribution of individuals over the state quadruple $(z, \theta, a; j)$ until convergence, given the cutoff points $\theta^{P,S}(a), \theta^{N,j}(a), \theta^{j,N}(a)$ and match continuation sets $\mathcal{C}_j(a)$ implied by the value functions. Third, we update the ratios η_j of vacancies to unemployed such that the free entry condition is satisfied given the distribution of individuals computed in the previous step. We repeat these three steps until the ratios η_j converge.

B.2 Discrete time approximation used in solution method

Here we present the discrete time approximation to our continuous time model. Following the method in Lefebvre and Guilbault (2009) to represent the Ornstein-Uhlenbeck processes for θ and z as finite-state Markov chains, we obtain parameters p and ρ which translate the variance and mean reversion of an Ornstein-Uhlenbeck process into a probability of upward or downward movement at each grid point for these states.

Note that the solution method is more efficient if instead of solving for the values J and E of employing a worker and being employed separately, we simply solve for the value of the surplus S in each segment. To allow for this, we also write the continuation values in the value functions for non-employment and posting vacancies in terms of the shares of the surplus that workers and firms receive rather than J or E.

B.2.1 Individuals

A non-employed individual has the following value functions. Starting with N:

$$
\begin{aligned}
N_t(\theta, a) \;=\; & b dt \\
& + (1 - r dt - 2p_a dt - \lambda_a dt - \mathcal{I}[a = unatt]\zeta dt) \\
& \qquad \max(N_{t+dt}(\theta, a), U_{S,t+dt}(\theta, a) - \nu, U_{P,t+dt}(\theta, a) - \nu) \\
& + (p_a - \rho_a(\theta - \mu_a))\, dt \max(N_{t+dt}(\theta + d\theta, a), U_{S,t+dt}(\theta + d\theta, a) - \nu, \\
& \qquad U_{P,t+dt}(\theta + d\theta, a) - \nu) \\
& + (p_a + \rho_a(\theta - \mu_a))\, dt \max(N_{t+dt}(\theta - d\theta, a), U_{S,t+dt}(\theta - d\theta, a) - \nu, \\
& \qquad U_{P,t+dt}(\theta - d\theta, a) - \nu) \\
& + \lambda_a dt \max(N_{t+dt}(\theta, a^c), U_{S,t+dt}(\theta, a^c) - \nu, U_{P,t+dt}(\theta, a^c) - \nu) \\
& + \mathcal{I}[a = unatt]\zeta dt \cdot 0.
\end{aligned}
$$

Unemployed individuals have the following value functions for $j \in \{S, P\}$:

$$\begin{aligned}
U_{j,t}(\theta, a) &= (b - s_j)dt \\
&\quad + (1 - rdt - 2p_a dt - \lambda_a dt - \mathcal{I}[a = unatt]\zeta dt - m_j(\eta_j)dt) \\
&\qquad \max(N_{t+dt}(\theta, a), U_{S,t+dt}(\theta, a), U_{P,t+dt}(\theta, a)) \\
&\quad + (p_a - \rho_a(\theta - \mu_a))\, dt \max(N_{t+dt}(\theta + d\theta, a), U_{S,t+dt}(\theta + d\theta, a), \\
&\qquad U_{P,t+dt}(\theta + d\theta, a)) \\
&\quad + (p_a + \rho_a(\theta - \mu_a))\, dt \max(N_{t+dt}(\theta - d\theta, a), U_{S,t+dt}(\theta - d\theta, a), \\
&\qquad U_{P,t+dt}(\theta - d\theta, a)) \\
&\quad + \lambda_a dt \max(N_{t+dt}(\theta, a^c), U_{S,t+dt}(\theta, a^c), U_{P,t+dt}(\theta, a^c)) \\
&\quad + m_j(\eta_j)dt \int \Big[\max(\alpha S_{j,t+dt}(z, \theta, a), 0) + \\
&\qquad \max(N_{t+dt}(\theta, a), U_{S,t+dt}(\theta, a), U_{P,t+dt}(\theta, a)) \Big] g(z)dz \\
&\quad + \mathcal{I}[a = unatt]\zeta dt \cdot 0.
\end{aligned}$$

B.2.2 Firms

A firm in segment $j \in \{S, P\}$ has the following value function for posting vacancies:

$$\begin{aligned}
V_{j,t} &= -c_j dt \\
&\quad + \left(1 - rdt - \frac{m_j(\eta_j)}{\eta_j} dt\right) \max(V_{S,t+dt}, V_{P,t+dt}, 0) \\
&\quad + \frac{m_j(\eta_j)}{\eta_j} dt \sum_a P(a \mid U_j) \int \int \Big[\max((1 - \alpha) S_{j,t+dt}(z, \theta, a), 0) + \\
&\qquad \max(V_{S,t+dt}, V_{P,t+dt}, 0) \Big] f_t(\theta \mid U_j, a) g(z) d\theta dz.
\end{aligned}$$

B.2.3 Matches

Once a match in segment $j \in \{S, P\}$ begins, the surplus has the following value function:

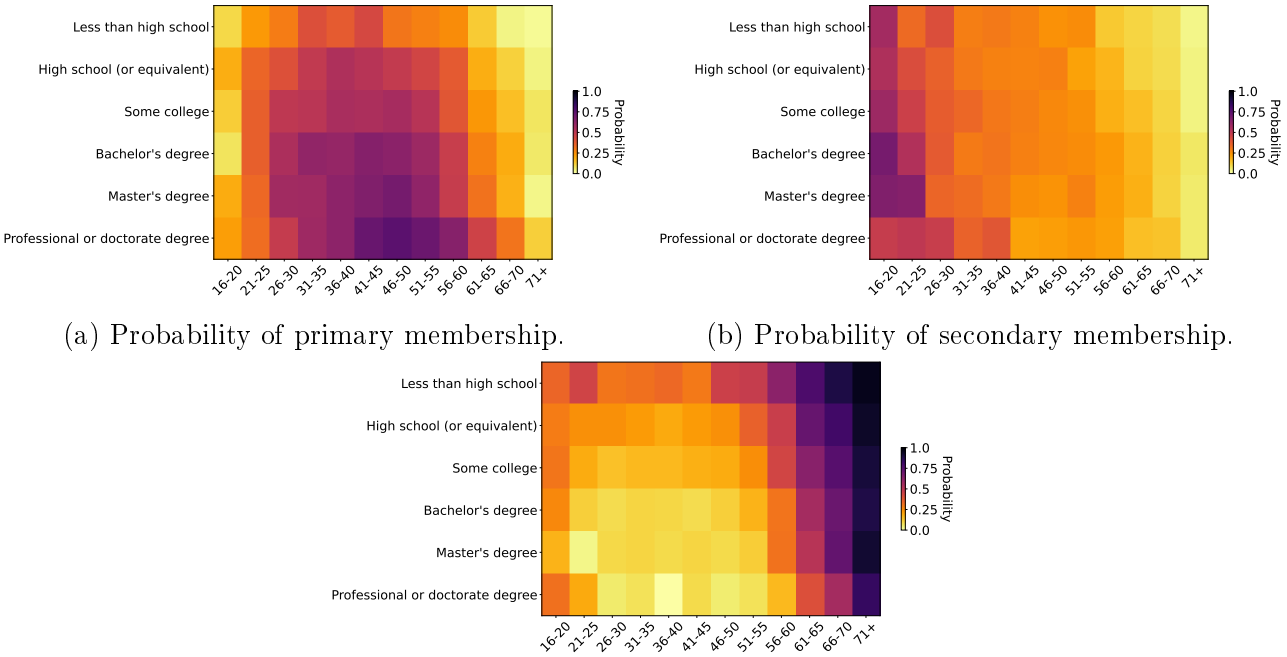
$$\begin{aligned}
S_{j,t}(z,\theta,a) &= (A_t e^z - \theta e_j)dt + \\
&\quad + (1 - rdt - 2p_a dt - 2p_z dt - \lambda_a dt - \mathcal{I}[a = unatt]\zeta dt)\, S^{cont}_{j,t+dt}(z,\theta,a) \\
&\quad + (p_a - \rho_a(\theta - \mu_a))\, dt S^{cont}_{j,t+dt}(z,\theta + d\theta,a) \\
&\quad + (p_a + \rho_a(\theta - \mu_a))\, dt S^{cont}_{j,t+dt}(z,\theta - d\theta,a) \\
&\quad + (p_{z,j} - \rho_{z,j} z)\, dt S^{cont}_{j,t+dt}(z + dz,\theta,a) \\
&\quad + (p_{z,j} + \rho_{z,j} z)\, dt S^{cont}_{j,t+dt}(z - dz,\theta,a) \\
&\quad + \lambda_a dt S^{cont}_{j,t+dt}(z,\theta,a^c) \\
&\quad + \mathcal{I}[a = unatt]\zeta dt \max(V_{S,t+dt}, V_{P,t+dt}, 0) \\
&\quad - \max(N_t(\theta,a), U_{S,t}(\theta,a), U_{P,t}(\theta,a)) \\
&\quad - \max(V_{S,t}, V_{P,t}, 0)
\end{aligned}$$

$$\begin{aligned}
S^{cont}_{j,t+dt}(z,\theta,a) &= \max(S_{j,t+dt}(z,\theta,a), 0) + \max(N_{t+dt}(\theta,a), U_{S,t+dt}(\theta,a), U_{P,t+dt}(\theta,a)) \\
&\quad + \max(V_{S,t+dt}, V_{P,t+dt}, 0).
\end{aligned}$$

C Additional results

C.1 Segment membership, education, and age

In Figure C.9, we present a SIPP respondent's model-implied probability of segment membership given their education and age in three heatmaps. In each panel, the value being plotted in the heatmap is the probability that a respondent in a given education and age cell belongs to a particular segment. Primary membership is concentrated among middle-aged and highly educated individuals, while secondary membership is concentrated among younger and less educated individuals. Tertiary membership is heavily concentrated among older individuals, with some representation among young individuals as well.



(a) Probability of primary membership.

(b) Probability of secondary membership.

(c) Probability of tertiary membership.

Figure C.9: The model-implied probability of segment membership in each age and education cell. Age bins are defined by a respondent's initial age when they enter the SIPP.