On the Importance of Manufacturing Order Books*

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Abstract

A large part of manufacturing has sizable backlog of unfilled orders. This paper explores several implications of order backlog. First, backlog leads to *time to fill*, the time wedge between order arrival and fulfillment. Using a novel administrative micro dataset on order books in Germany, we document that time to fill is half a year on average and features a pronounced right tail across establishments. Second, backlog makes deflation challenging. We propose a theoretically coherent deflation scheme that avoids high informational requirements. Third, quantity and price of sales are predetermined by quantity and price of new orders. Fourth, the rigidity of sales prices invalidates conventional strategies to identify demand and supply shocks. Instead, variation in quantity and price of new orders can identify such shocks. Fifth, backlog constitutes an adjustment margin to shocks, which we analyze theoretically and for which we provide empirical evidence.

Keywords: Order books, backlog, time to fill, prices, supply shock, demand shock.
JEL Codes: D21, D22, D25, E23, E31.

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1 Introduction

A large part of manufacturing output is produced in response to customer orders, involving a non-trivial time wedge between order arrival and fulfillment, which we refer to as *time to fill*. Between order arrival and fulfillment, unfilled orders enter the order backlog. Despite the prevalence of order backlog in the manufacturing sector, order books are widely neglected in theoretical and empirical analyses of firms.

This paper argues that neglecting order books is not innocuous, for both theoretical and empirical reasons. In the presence of order backlog, the prices of new orders and sales differ contemporaneously but are dynamically linked. Deflation requires observing prices for all order book variables, an informational requirement typically not met in the data. Instead, we propose a scheme to construct theoretically coherent order book prices when only sales prices are observed. Since sales prices are predetermined by past order prices, conventional strategies to identify demand and supply shocks using variation in sales price and quantity are invalid. Instead, variation in price and quantity of new orders can be used to identify demand and supply shocks. Order backlog further matters for the propagation of these shocks. We propose a theory of the firm in which order backlog acts as a margin of adjustment to shocks, next to prices and production. On the empirical side, we introduce a novel micro dataset on order books in Germany. We document that order backlog is sizable, leading to a time to fill of six months on average across establishments with a pronounced right tail. Applying the deflation scheme, we document that sales prices differ strongly from contemporaneous new order prices and are predetermined by past new order prices. We further identify establishment-level demand and supply shocks and empirically characterize the response of order books to demand and supply shocks. Our estimates align with our theory of the firm.

We introduce administrative, granular, high-frequency data on order books in the manufacturing sector in Germany. The data are collected by the Federal Statistical Office of Germany (Destatis) and cover the universe of establishments with at least 50 employees. We observe order backlog, new orders, and sales (all nominal) at the monthly frequency for all within-establishment business segments with output in distinct four-digit manufacturing industries, which we refer to as establishments^{4d}, from 2014 through 2022. Order backlog and new orders are reported by all establishments in a subset of sectors for which order books are deemed important. These sectors account for 69-76% of sales, employment, and value-added in the manufacturing sector. Order backlog, new orders, and sales define a stock-flow equation that holds up to a residual which may capture cancellations, ex-post price changes, and measurement error. In the data, the residual is zero for the majority of observations

and small for most of the non-zero observations.

The nominal values in the order book data mask movements in price and quantity which we are ultimately interested in. Absent order backlog, the price of new orders equals the sales price. The presence of heterogeneous changes in idiosyncratic prices means we need to deflate by establishment^{4d}-month-specific prices. The presence of order backlog further raises the informational requirement. We need separate prices for order backlog, new orders, and sales for a given establishment^{4d}-month. To illustrate why, suppose new orders turn into sales two months later. The correct price deflator of new orders corresponds to the price deflator of sales two months later. We propose a deflation scheme, which under suitable assumptions constructs theoretically coherent order book prices based on observed sales prices. Empirically, we construct establishment^{4d}-level sales prices based on another Destatis dataset that records monthly product-establishment^{4d}-level sales values and quantities of production. We show that establishment^{4d}-month order prices can differ widely from sales prices in the same establishment^{4d}-month which further highlights the need to correctly deflate new orders and sales.

We document the distribution of time to fill in the data. The arithmetic average of time to fill across establishment^{4d}-months is between 5 and 7 months depending on whether or not we weight observations by sales and how we measure time to fill. We consider two measures of time to fill: an ex-post measure of the time it takes for a period t order to be filled based on realized future sales and an ex-ante measure based on expected future sales. The distribution of time to fill displays a pronounced right tail with time to fill exceeding 12 months for 8–11% of observations and 24 months for 1-3% of observations. Most of the variation in time to fill is not across narrowly-defined sectors, but within sectors across establishments, and within establishments across time. Time to fill implies that prices and quantity of sales are predictable by past prices and quantity of new orders. We provide evidence for predictability both at the micro level, but also at the aggregate level. The aggregate sales price lags the aggregate price of new orders by about 3 months.

We further develop a theory to analyze how order backlog shapes firm responses to demand and supply shocks. Backlog is a dynamic control variable that constitutes a third margin of firm adjustment to shocks, next to price and production. We characterize the optimal responses of the firm to demand and supply shocks. Expansionary demand shocks unambiguously raise backlog. In the presence of decreasing returns to scale, accumulating an order backlog allows the firm to lower production costs as production increases more gradually in response to an increase in new orders. The response of order backlog to expansionary supply shocks is ambiguous and depends on the persistence of the shock. A transitory supply shock raises production but has little impact on new orders, leading to lower backlog. Instead, a

sufficiently persistent supply shock raises new orders by more than production, leading to higher backlog.

Guided by the theory, we identify idiosyncratic demand and supply shocks and empirically characterize the effects of these shocks in our data. Our identification strategy estimates demand shocks as the residual when regressing (real) new orders on prices of new orders, using a Bartik shift-share instrumental variable that leverages exogenous shifts in production costs arising from imported input prices. Using the demand shock allows us to identify the supply shock as fluctuations in sales that are not explained by past order book information and that is orthogonal to the demand shock. We then characterize the dynamic response of order backlog, sales, new orders, and prices of new orders to demand and supply shocks. Our findings are qualitatively consistent with our theory. In response to expansionary demand shocks, order prices increase weakly and new orders display a short-lived sharp increase. Sales also increases, but less than new orders, leading to persistently higher order backlog. In response to expansionary supply shocks, order prices fall and new orders increases. Sales exhibit a transitory but strong increase, leading to a persistent decline in order backlog. We also implement a theoretically invalid identification strategy based on regressing (real) sales on sales prices using the same Bartik instrument. We obtain qualitatively different responses, which underlines the importance of accounting for order backlog in the analysis of the firm.

Our paper contributes to an older literature studying order backlogs, dating back to Zarnowitz (1962), Childs (1967), Belsley (1969), and Zarnowitz (1973). These studies provide conceptual discussions why some firms accumulate order backlogs and they document empirical regularities using aggregate or sectoral data on order books. To the best of our knowledge, we are the first to study order books using highly granular data. Some conceptual aspects that we address in this paper have already been pointed out in the previous literature. Foss et al. (1981) point out that different deflators are necessary for sales, new orders, and order backlogs and propose to extend the data collection. Haltiwanger and Maccini (1989) use the average (ex-ante) delivery lag to deflate the aggregate backlog with an (ad-hoc) weighted sum of producer price indices. In contrast, we propose a deflation procedure delivering theoretically coherent micro-level deflators when just observing a single price series. Second, the accounting literature (e.g., Rajgopal et al., 2003; Chang et al., 2018) shows that order backlogs predict future sales growth. We extend these findings by showing that the forecasting performance can be significantly increased by accounting for heterogeneity in time to fill at the micro level. Third, Belsley (1969), West (1989) and Haltiwanger and Maccini (1989) incorporate order backlogs in linear-quadratic models of inventories to study their role for shock propagation. Fourth, Odle et al. (1981) show that order backlogs partially absorb

demand fluctuations and that higher levels of backlogs are associated with lower sales price fluctuations at the sectoral level. In contrast, we estimate the response of order backlogs to demand and supply shocks that we identify relying on variation in order prices.

More broadly, we also relate to other strands of the literature considering time gaps between the order, production, and delivery of goods. These include the literature on capital goods with varying time to build (e.g., Kalouptsidi, 2014; Meier, 2020), the literature on inventories (e.g., Cooper and Haltiwanger, 1990; Ramey and West, 1999; Fisher and Hornstein, 2000; Bils and Kahn, 2000; Khan and Thomas, 2007; Kryvtsov and Midrigan, 2012; Auernheimer and Trupkin, 2014; Sarte et al., 2015; Görtz et al., 2024), and more recently the average time of production in general (Antràs and Tubdenov, 2025).

The remainder of the paper is organized as follows. Section 2 describes our data and documents the prevalence of order backlogs. Section 3 introduces theoretically coherent prices for all order book variables. Section 4 presents empirical evidence on time to fill and discusses its implications for forecasting real economic activity and prices. In Section 5, we introduce a theory of the firm with order backlog. Section 6 discusses the empirical identification and propagation of demand and supply shocks. Finally, Section 7 concludes.

2 Microdata on manufacturing order books

In this section, we describe the data, the prevalence of backlog across sectors, the sample selection, and provide descriptive statistics on order books in the manufacturing sector.

New data. We introduce a new dataset which so far has not been available to researchers, the *Monthly Reports of Manufacturing Establishments* (briefly: reports).[1] The data are administrative, high-frequency, highly granular, and include order book variables.

The reports are collected by the Federal Statistical Office of Germany (Destatis). Submitting a monthly report is legally mandatory for all manufacturing establishments in Germany with at least 50 employees in a given year. The reports include the nominal value of sales, new orders, and order backlog.[2] New orders are the total nominal value of all firmly accepted orders for delivery of products in a given month. The order backlog is the total nominal value of all firmly accepted orders (present and past) that have not yet passed through the sales account and that have not been canceled.

[1] A related dataset that has been available to researchers, *AFiD*, provides yearly aggregates of the monthly reports excluding all order book variables and not distinguishing sub-units within establishments.

[2] Destatis uses the reports in the production of the National Income and Product Accounts, and aggregate indices of sales, new orders, and order backlog that are closely monitored as business cycle indicators.

Establishments with output in multiple four-digit manufacturing sectors have to submit monthly reports separately by four-digit manufacturing sectors. Throughout the paper, we will refer to these sub-units as establishments[4d]. We have access to the universe of anonymized reports from 2014 through 2022. The reports go much further back in time, but only in 2014 was the order backlog added to the report. Initial order backlog data was frequently missing or implausible (Linz et al., 2016). We therefore omit 2014 data on order backlog throughout the paper. We can link the reports to other granular administrative datasets on unit prices and costs of production in the manufacturing sector, as we will do in subsequent sections. For further details on the data, see Appendix A.1.

Prevalence of backlog across sectors. The monthly reports include data on new orders and order backlog only for a subset of manufacturing sectors. These sectors, which we refer to as backlog sectors, are textiles, cloths, paper, chemicals, pharamceuticals, primary metals, fabricated metals, electronics, electrics, machinery, automotives, and other vehicles. Destatis deems the order backlog negligible in the other manufacturing sectors (e.g., food, wood, and petroleum manufacturing).[3] The backlog sectors cover an important segment of the manufacturing sector. Together, they account for 72% of total sales, 69% of employment, and 76% of value added in the manufacturing sector, see Table 1. The two backlog subsectors with the largest value added are the machinery and automobile sectors. In addition, backlog sectors represent a volatile segment of manufacturing. Year-over-year sales growth has a standard deviation of 0.76 for the backlog sectors and 0.62 for the non-backlog sectors in our sample period.

One way to think about the distinction between backlog and non-backlog sectors is that the former employs a produce-to-order approach, the latter a produce-to-stock approach. Produce-to-order is a favorable mode of operation if the costs of storage are high and demand for individual products is highly specific and hard to predict (Zarnowitz, 1962). In fact, our backlog sectors overlap relatively well with the sectors classified as produce-to-order in Belsley (1969). Some subsectors, however, may rather be in between produce-to-order and produce-to-stock. We therefore document the value of order backlog relative to the value of inventories. Ideally, we could observe the value of finished good inventories, but data for manufacturing in Germany only measure a composite of finished and semi-finished good inventories. With that caveat in mind, we find that order backlog is more than three times larger than (composite) inventory in the backlog sectors, see Table 1. In all broad subsectors, order backlog exceeds inventory, and inventory roughly equals one month of

[3]The selection of sectors was originally based on Eurostat guidelines (Linz et al., 2016) and has been unchanged for our sample. Appendix A.1 provides further details on the sectors. Note that the U.S. Census M3 Survey only collects order book data for a similar subset of manufacturing sectors.

Table 1: Sectoral prevalence of order backlog

	Sales (% of total)	Employment (% of total)	Value added (% of total)	Backlog /Inventory
Backlog Sectors	72.49	69.02	76.33	3.38
– *Textiles, Cloths, and Paper*	2.93	3.61	3.02	2.38
– *Chemicals and Pharmaceuticals*	10.66	7.22	10.87	1.44
– *Primary and Fabricated Metals*	10.52	14.41	11.84	3.29
– *Electronics and Electrics*	9.72	12.10	13.39	3.94
– *Machinery*	13.02	16.50	16.93	4.16
– *Automotives*	23.34	13.14	17.68	3.38
– *Other Vehicles*	2.29	2.04	2.61	3.99
Non-Backlog Sectors	27.51	30.98	23.67	–

Note: "Backlog Sectors" are the collection of subsectors listed in italics below. "Non-Backlog Sectors" are all other sectors in manufacturing. All variables are deflated by the PPI. The first three columns show the average annual contribution of a sector to total manufacturing from 2015 through 2022. The last column shows the average annual ratio of sector-level order backlog divided by sector-level inventory of finished and semi-finished goods for the same sample.

sales. The evidence suggests that finished good inventory is of relatively little importance for backlog sectors.

Stock-flow equation. Sales and new orders, respectively, constitute outflows and inflows to the stock of order backlog. Denoting the end-of-month t nominal value of order backlog of establishment[4d] i by B_{it}, the nominal value of new orders during month t by O_{it}, and the nominal value of sales by S_{it}, the following (nominal) stock-flow equation must hold,

$$B_{it} = B_{it-1} + O_{it} - S_{it} - R_{it}, \tag{2.1}$$

where R_{it} denotes an order book residual. The residual measures changes in the stock ($B_{it} - B_{it-1}$) that are not explained by O_{it} and S_{it}. Sources of the residual are order cancellations, post-order price changes (price discounts or raises, e.g., through price escalation clauses), and measurement error (e.g., an order not recorded in new orders and backlog). The residual R_{it} is not part of the manufacturing reports but is implicitly defined by the above identity.

Sample selection. The quality of the data is high. Important aspects in this regard are the legal mandate to submit the reports and a diligent data collection process with multiple

plausibility checks. Nonetheless, a small fraction of observations exhibits an implausible zigzag pattern in the order book residual, which we smooth out. We further exclude observations for which order backlog appears negligible in size or misreported. For example, some establishments[4d] report order backlog corresponding to less than half a month of sales in most months. Overall, we drop 25% of observations. The remaining sample still contains more than 50% of manufacturing sales and employment. Excluded observations are similar in sales and employment but have a substantially smaller order backlog compared to the remaining sample. Appendix A.1 presents further details.

Descriptive statistics. Table 2 provides descriptive statistics of key variables used in the paper. Our sample consists of 1.3 million establishment[4d]-month observations from January 2015 through December 2022.[4] On average, we observe sales of 4.6 million Euro, new orders of 5.1 million Euro, and an order backlog of 35.2 million Euro (all in 2019 prices). The averages mask vast heterogeneity. For example, sales ranges from 0.1 million to 7.4 million between the 10th and 90th percentile. Table 2 shows that the order book residual fluctuates around zero. That suggests that R_{it} is not dominated by order cancellations, because cancellations generate positive innovations to the residual. We also observe that R_{it} is close to zero in absolute terms. The ratio of the absolute residual to order backlog has a median of zero, a mean of 7%, and is 21% at the 90th percentile. Overall, R_{it} is a minor source of fluctuations in order backlog. In other words, order cancellations, post-order price changes, and measurement error in the order backlog do not appear to be quantitatively important features of the data.

Table 2: Descriptive statistics

	Mean	Sd	P10	P50	P90	Obs
Sales (S_{it})	4.64	36.17	0.10	1.07	7.38	1,315,354
New orders (O_{it})	5.10	45.31	0.07	1.06	7.74	1,315,354
Order backlog (B_{it})	35.16	451.12	0.16	3.74	42.19	1,315,354
Order book residual (R_{it})	0.24	22.20	-0.26	0.00	0.36	1,315,354
Relative residual ($\lvert R_{it}\rvert / B_{it-1}$)	0.07	0.19	0.00	0.00	0.21	1,315,354

Note: The table provides descriptive statistics at the establishment[4d]-month level from 2015 through 2022. All variables are deflated by the PPI normalized to the year 2019 and expressed in EUR million. The last row refers to the absolute order book residual normalized by the order backlog in the previous month.

[4]The average establishment has 1.3 establishments[4d], the average firm has 1.4 establishments[4d], and we observe an establishment[4d] for 68 months on average, see Table A.2 in the Appendix.

Table A.3 in the Appendix provides descriptive statistics across broadly-defined backlog sectors. In terms of average employment and sales, vehicle-producing establishments^{4d} tend to be substantially larger than establishments^{4d} in other manufacturing sectors. In contrast, the two sectors with the largest number of establishments^{4d} and the largest contribution to total employment and sales are metal and machinery manufacturing. More than 50% of the aggregate order backlog is recorded in the machinery and other vehicle manufacturing sectors. The latter includes manufacturing of ships, trains, and planes, which are all well known to operate long backlogs.

3 Order book prices

In this section, we discuss and address the non-trivial challenge of deflating order book variables that arises even if establishment^{4d}-level sales prices are available.

3.1 Deflation challenge

The generic shortcoming of nominal values is that they conflate real quantities and prices. Separating real quantities and prices is key, e.g., for the comparability of order book variables and the identification of shocks.

Deflating order book variables imposes high informational requirements. For example, consider an establishment that receives an order in period t, which passes into sales in $t+2$. If the price of new orders changes over the two periods, the price of sales in t will differ from the price of new orders in t, which is given by the price of sales in $t+2$. More generally, as new orders become sales in future periods, the contemporaneous prices of new orders and sales differ; that is, unless new order prices remain constant. Similarly, the order backlog generally contains a distribution of yet unfilled orders across multiple past periods, meaning the backlog price differs from the contemporaneous prices of new orders and sales. Thus, a single price for a given establishment^{4d} and month (i, t) will not allow us to correctly deflate all order book variables if prices vary over time at the establishment^{4d}-level. Deflating all order book variables with an aggregate PPI index (as in Table 2) is even more problematic because it fails to capture heterogeneous price fluctuations across establishments^{4d}.

Although the prices of backlog, new orders, sales, and order book residual generally differ from each other in any given period, they are dynamically linked to each other through a

real stock-flow equation, given by

$$\underbrace{b_{it}}_{B_{it}/p_{it}^B} = b_{t-1} + \underbrace{o_{it}}_{O_{it}/p_{it}^O} - \underbrace{s_{it}}_{S_{it}/p_{it}^S} - \underbrace{r_{it}}_{R_{it}/p_{it}^R}, \tag{3.1}$$

where the lowercase variables denote real order book variables, as opposed to the nominal uppercase variables. The price indices of all order book variables are implicitly defined. To reinforce the point that the different order book variables require different deflators, suppose the order price p_{it}^O has a positive time trend over some period, e.g., the recent inflation surge. Since sales prices are past new order prices, we obtain $p_{it}^O > p_{it}^B > p_{it}^S$. Naively deflating order backlog by sales prices would overstate the true size of real new orders and real order backlog during the period of rising new order prices.
The ideal order backlog data would include information on all four price indices. However, that is not the case for the manufacturing reports.

3.2 A coherent deflation scheme

We next provide a scheme to construct all order book price indices in a theoretically consistent way conditional on three assumptions:

Assumption 1: A sales price index (p_{it}^S) is observable.

As we explain further below, we have in fact data on establishment^{4d}-month-level sales prices. However, we lack direct information on any other order book price.

Assumption 2: The order backlog (b_{it}) follows a FIFO (First In, First Out) principle.

The assumption means the next order to be filled, i.e., passed into sales, is always the oldest unfilled order in the order backlog. Similarly, reductions (cancellations) or extensions of orders captured by the order book residual always apply to the oldest unfilled order in the order backlog.[5] An assumption on the filling order will be necessary to construct prices of new orders. Given the lack of direct information on the filling order—we do not have order-level but establishment^{4d}-level data—FIFO is the most plausible assumption we can impose.

Assumption 3: The prices of new orders (p_{it}^O) do not change in the months after the order. New orders (O_{it}) are not fully canceled in subsequent months.

[5] We assign the oldest unfilled orders proportionally between sales and the residual.

Finally, we need to impose some additional structure on the order book residual. The first part of the assumption means the price agreed upon when signing an order contract does not change later, which rules out ex-post price discounts and price escalation clauses. Ultimately, it means we interpret nominal order book residuals as reflecting real flows. Given that the residual is typically small (Table 2), we consider the assumption rather innocuous.[6] The second part rules out full cancellation of orders. This guarantees the existence of a sales price that includes information on the original new order price. In case the assumption is violated, we will impute the new order price using sales prices associated to new orders in subsequent periods.

We next describe how prices p_{it}^O, p_{it}^R, and p_{it}^B can be constructed based on Assumptions 1–3:

Step 1: Price of new orders (p_{it}^O)

$$
\begin{aligned}
p_{it}^O &= \sum_{j=\underline{\tau}_{it}}^{\overline{\tau}_{it}} w_{it+j} p_{it+j}^S \\
\underline{\tau}_{it} &= \min\left\{\tau \in \mathbb{N}^0 \,\middle|\, \sum_{k=0}^{\tau}(S_{it+k} + R_{it+k}) > B_{it-1}\right\} \\
\overline{\tau}_{it} &= \min\left\{\tau \in \mathbb{N}^0 \,\middle|\, \sum_{k=0}^{\tau}(S_{it+k} + R_{it+k}) \geq B_{it-1} + O_{it}\right\}
\end{aligned}
$$

Under Assumptions 2–3, $\underline{\tau}_{it}$ identifies the first period ($t + \underline{\tau}_{it}$) in which i starts depleting period t orders O_{it}. Before this period, the firm is still using up the backlog of orders from previous periods (B_{it-1}). This requires the cumulative net outflows ($S_{it+k} + R_{it+k}$) to exceed the backlog of orders at the beginning of period t, i.e., B_{it-1}. Similarly, $\overline{\tau}_{it}$ identifies the last period in which i still fills period t orders. In period $t + \overline{\tau}_{it}$, cumulative net outflows exceed $B_{it-1} + O_{it}$ for the first time.[7] Finally, we compute p_{it}^O as a sales-weighted average of sales prices between $t + \underline{\tau}_{it}$ and $t + \overline{\tau}_{it}$. We define sales weights as

$$
w_{it+j} = \frac{\tilde{S}_{it+j}}{\sum_{k=\underline{\tau}_{it}}^{\overline{\tau}_{it}} \tilde{S}_{it+k}},
$$

where $\tilde{S}_{it+k}$ takes into account the possibility that only a fraction of sales in $t+\underline{\tau}_{it}$ and $t+\overline{\tau}_{it}$ are attributable to the period t order. Appendix B.1 details the construction of $\tilde{S}_{it+k}$.

[6] Alternatively, we could construct order book deflators assuming the order book residual reflects only price adjustments or any convex combination between price and real adjustments.

[7] If an entire order O_{it} is fully filled in a single month, then $\overline{\tau}_{it} = \underline{\tau}_{it}$.

Step 2: Price of order book residual (p_{it}^R)

$$p_{it}^R = p_{it}^S$$

Assumptions 2–3 imply that the order book residual follows the same time pattern as sales and includes only real flows. Hence, the price index of the order book residual must equal the sales price.

Step 3: Price of order backlog (p_{it}^B)

$$p_{it}^B = \frac{B_{it}}{b_{it}}, \qquad b_{it} = b_{it-1} + o_{it} - s_{it} - r_{it}$$

Given Assumption 1 and following Steps 1 and 2, we can construct prices p_{it}^S, p_{it}^O, and p_{it}^R. Hence, we can construct real flow variables s_{it}, o_{it}, and r_{it}. The real stock-flow equation (3.1) implies that we can compute b_{it} for all periods given an initial level of the real backlog. To obtain the initial real backlog, we exploit that the real and the nominal stock-flow equation both have to hold simultaneously. Thus, the time to deplete the initial real backlog and the initial nominal backlog has to be the same. That condition uniquely pins down the initial real backlog, see Appendix B.2 for details.[8]

Discussion. Time aggregation creates measurement error in our deflation scheme. If we were able to observe the order book variables and the sales prices at any instant of time, and if the arrival of new orders, sales, and residual was discrete, then the assumptions above would be sufficient to correctly identify the prices p_{it}^O, p_{it}^R, and p_{it}^B. However, the available data are aggregated to a monthly frequency and our deflation scheme makes implicit intra-period timing assumptions. For illustration purposes, consider a scenario in which sales S_{it} and the residual R_{it} include parts of O_{it-2} and parts of O_{it-1}. First, this case requires an assumption on the intra-period timing of sales and the residual to assign them to the two orders when identifying the τ's. In the deflation scheme above we implicitly assign the oldest unfilled orders proportionally between sales and the residual. We regard this as the most agnostic assumption, given the available information. Second, the scenario above requires us to use the information contained in one sales price, p_{it}^S, for two different orders. Again, we make the most agnostic assumption and assume that fluctuations in p_{it}^S are *ex ante* equally likely to be associated with any of the two orders. In Step 1 we consistently assign a lower weight to p_{it}^S in the construction of p_{it-1}^O, the smaller the share of (net) sales associated with period $t-1$ orders.

[8] In Appendix B.3, we propose and implement an alternative step 3 that yields highly similar results.

3.3 Empirical implementation

Our deflation scheme builds on observed sales prices. The manufacturing reports do not contain sales prices. Instead, we leverage a second Destatis dataset, the *Production Census* of manufacturing in Germany (briefly: census). It records the quantity of production and the associated sales value at the level of nine-digit products and per establishment^{4d}-month. The coverage of establishment^{4d}-months almost perfectly overlaps with the manufacturing reports. We compute unit prices as value divided by quantity and aggregate to the establishment^{4d}-month level via Törnqvist indices with sales values acting as weights (cf., Gagliardone et al., 2023). This procedure allows us to construct a sales price index that covers 77.8% of establishment^{4d}-month observations. The primary reason why we do not obtain a sales price index for some observations is that the census does not elicit production quantities for some products. Appendix A.3 provides further detail on the census and the construction of the sales price index.

Following steps 1–3, the sales price index allows us to compute price indices for all other order book variables. However, naively applying the steps would run into two issues, which we next discuss and address. The first issue is selection on order book length. Computing the ex-post time gaps $\underline{\tau}_{it}$ and $\overline{\tau}_{it}$ requires data on future order book variables, which become increasingly unavailable towards the end of the sample. Not correcting for that fact would induce a selection bias in our sample of order book deflators and hence real order book variables: The longer the ex-post time gap of a given establishment^{4d}, the earlier real observations for that establishment^{4d} would be missing. To avoid such selection bias, we assume the price of new orders remains constant following the last period in which it can be computed. We consider the assumption acceptable in order to avoid selection bias. In addition, the data provides some support for the assumption. In our sample, sales-price inflation averages at close to zero between 2014 and 2019, and by the end of 2022, producer price inflation had nearly returned to zero.

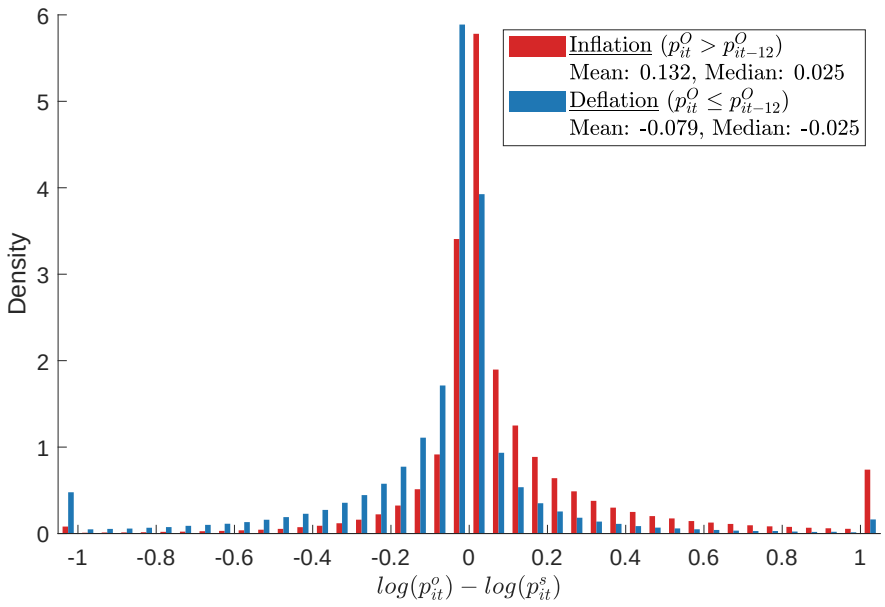
The second issue is that the real backlog series obtained from iterating the real stock flow equation in step 3 become negative occasionally. To avoid this issue, we impose a restriction on the real order backlog that is consistent with our assumptions. We impose that the real backlog b_{it} can not be smaller than the respective nominal backlog B_{it} divided by the maximum sales price of a given establishment^{4d}. We dynamically enforce that condition by adjusting the real order book residual whenever that condition is first violated. We then adjust the subsequent real backlog series according to the real stock-flow equation and identify the second instance (if any) in which the condition is violated. Notably, major adjustments of the residual (the adjusted residual differs from the original one by more than 1% of backlog) are only necessary for 2.1% of observations. The resulting real backlog series

is always weakly positive and zero if and only if the nominal backlog equals zero.[9]

3.4 Some evidence on prices and real quantities

We construct real order book variables and order book prices based on steps 1–3 with the two refinements discussed above. Our final real sample, that is the sample of non-missing data on prices and real quantities, has 1.00 million establishment^{4d}-time observations. The primary reason why we have fewer observations compared to the nominal sample (Table 2) is that production quantities are missing for some products in the census. For these products, we cannot compute unit prices. If that applies to all products an establishment^{4d} is producing, we have no information on the sales price of the establishment^{4d}.[10]

Figure 1: Sales prices and new order prices



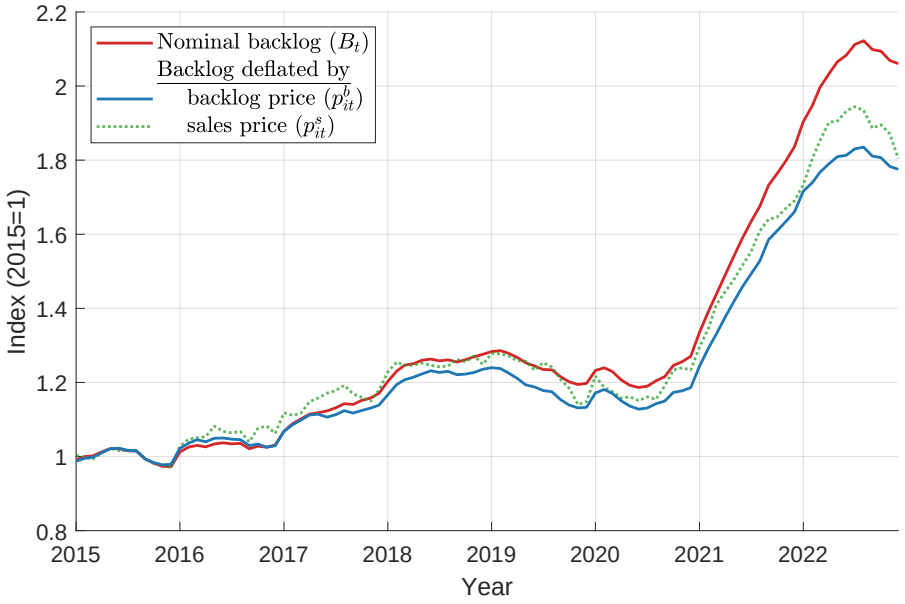
Note: This figure shows the distribution of log difference between the order price p_{it}^O and the sales price p_{it}^S across all observations with (relevant) real order book data. The sample spans 2015-2022. The distribution is winsorized at ± 1. The distribution is shown separately for "inflation" establishment^{4d}-months (in red) defined as observations in which $p_{it}^O > p_{it-12}^O$ and "deflation" establishment^{4d}-months (in blue) defined as observations in which $p_{it}^O \leq p_{it-12}^O$.

[9] In Appendix B.3, we present and implement an alternative step 3. The alternative procedure to obtain real order backlogs is equivalent to step 3 if our our assumptions hold but differs if our assumptions are violated in the data. Reassuringly, the real order backlog is highly similar between the two approaches, see Figure B.1.

[10] We cannot construct order book prices for an additional small fraction of observations because of data treatments dealing with measurement error in the census data, see Appendix A.3 for details.

We next provide evidence highlighting the quantitative importance of taking into account that prices of sales, new orders, and order backlog generally differ. Figure 1 shows that the price of sales p_{it}^S and the price of new orders p_{it}^O differ strongly from each other at the micro-level. For 10% of establishment^{4d}-months, the order price exceeds the sales price by at least 20%, for another 10%, the sales price exceed the order price by at least 30%. Hence, naively deflating the nominal value of new orders by the sales prices would create large errors in the imputed real quantities. These errors are not random but related to the evolution of new order prices. Today's sales prices are prices of past new orders. Hence, if new order prices grow, sales prices in period t will be lower than new order prices in t. In fact, we clearly observe this pattern in the data. The red bars show the log difference between new order price and sales conditional on positive year-over-year price growth in new order prices (inflation), the blue bars condition on negative new order price growth (deflation). In periods of inflation, new order prices are more likely to exceed sales prices than conversely, while the opposite holds in periods of deflation.[11]

Figure 2: Aggregate real vs. nominal backlog



Note: This figure shows three aggregate backlog series from 2015 to 2022. The series are normalized to one in 2015. The series are obtained by Törnqvist aggregation with (nominal) backlog weights similar to the procedure outlined in Appendix A.3. The micro-level growth rates are trimmed at 1% and 99% before aggregation. The red line corresponds to the nominal backlog, the blue line to the real backlog, and the green dotted line to nominal backlog deflated by establishment^{4d}-level sales prices.

[11]The main reason why we the distributions of $\log(p_{it}^O) - \log(p_{it}^S)$ overlap across the inflation and deflation scenarios is that these scenarios are constructed using year-over-year growth rates instead of the establishment^{4d}-month specific time for an order to turn into sales.

Building on that, Figure 2 provides evidence for the aggregate importance of these differences. The red and the blue line depict the aggregate nominal and real backlog, respectively. In 2022, the nominal backlog was more than 2.1 times higher than it was in 2015. In contrast, the real backlog only increased by up to 80% relative to the beginning of the sample. If the nominal order backlogs were instead naively deflated by (individual) sales prices, this increase would be overstated by 10 percentage points. This is in line with the evidence shown in Figure 1: In times of high inflation, order prices are (on average) higher than sales prices. Consequently, the real backlog should be (on average) lower compared to the naively deflated backlog. Especially in times of large price changes, our deflation method prevents parts of the nominal change in backlogs from being falsely reported as a real change.

4 Time to fill

In this section, we provide empirical evidence on the time to fill, i.e., the time wedge between new orders and sales resulting from the presence of order backlogs.

4.1 Measuring time to fill

We propose two measures of the time to fill, an *ex-ante* and an *ex-post* measure. The ex-post measure of time to fill builds directly on the deflation scheme developed in Section 3. In Step 1 of the deflation scheme, we determined $(t + \underline{\tau}_{it})$, the first period in which period t orders o_{it} start depleting, and $(t + \overline{\tau}_{it})$ as the last period in which parts of o_{it} are still filled. We use those objects to compute the ex-post time to fill as

$$\tau_{it}^{EP} = \frac{\underline{\tau}_{it} + \overline{\tau}_{it}}{2}. \tag{4.1}$$

Under Assumptions 2–3 in Section 3, τ_{it}^{EP} captures the average time for a new order in period t to be filled.[12] By construction, this measure reflects the *realized* time to fill and therefore relies on data beyond period t. As an alternative, we also compute the ex-ante time to fill of establishment[4d] i in month t as the real backlog-sales ratio

$$\tau_{it}^{EA} = \frac{b_{it}}{s_{it}^{12}}, \quad \text{where } s_{it}^{12} = \frac{1}{12}\sum_{j=0}^{11} s_{it-j}. \tag{4.2}$$

[12] In principle, we could further refine τ_{it}^{EP} computing it as weighted average across $\underline{\tau}_{it}$ and $\overline{\tau}_{it}$ taking into account how much of o_{it} is filled across the period from $\underline{\tau}_{it}$ through $\overline{\tau}_{it}$. However, that distribution can only be approximated. In practice, we expect this to make little difference, also because in 80% of the observations, $\underline{\tau}_{it}$ and $\overline{\tau}_{it}$ differ by one month or less. We therefore prefer the simple unweighted arithmetic average as our ex-post measure of the time to fill.

We refer to τ_{it}^{EA} as ex-ante (EA) time to fill because it requires only real order book data up to month t. The unit of τ_{it}^{EA} is months of (moving-average) sales. The ex-ante time to fill equals the expected number of months it takes for the end-of-period real order backlog b_{it} to be depleted by future real sales conditional on three assumptions. First, future real sales are expected to equal the moving-average real sales s_{it}^{12}. Second, the order book residual is expected to be zero in all future periods. Third, the order backlog follows a FIFO principle (as in Assumption 2 in Section 3). Under these assumptions, we can equivalently interpret τ_{it}^{EA} as the expected number of months it takes for a period t order o_{it} to be filled. The ex-ante time to fill is also commonly used in previous studies of order backlogs (see, e.g., Zarnowitz, 1973).[13] Note that τ_{it}^{EA} is not restricted to be an integer, e.g., it may equal 5.5 months if that is when the order can be expected to be filled. Throughout the remainder of the paper, we winsorize τ_{it}^{EA} and τ_{it}^{EP} at 60 months to reduce the impact of statistical outliers possibly related to measurement error.

4.2 Distribution of time to fill

We next provide descriptive empirical evidence on the distribution of the time to fill in our data. Panel (a) of Figure 3 shows the histogram of the ex-ante time to fill τ_{it}^{EA} across establishment^{4d}-months. The distribution has its mode at 0-1 months but features a long right tail. Panel (c) shows that the mean is 5.45 months, above the median of 3 months, which reflects the long right tail. In fact, we document a time to fill exceeding 12 months for more than 10% of observations, and a time to fill exceeding 24 months for more than 3% of observations. Weighting the distribution of τ_{it}^{EA} by sales (s_{it}^{12}) yields broadly similar moments. The weighted mean increases to 6.92 months meaning firms with higher sales tend to have a higher ex-ante time to fill.

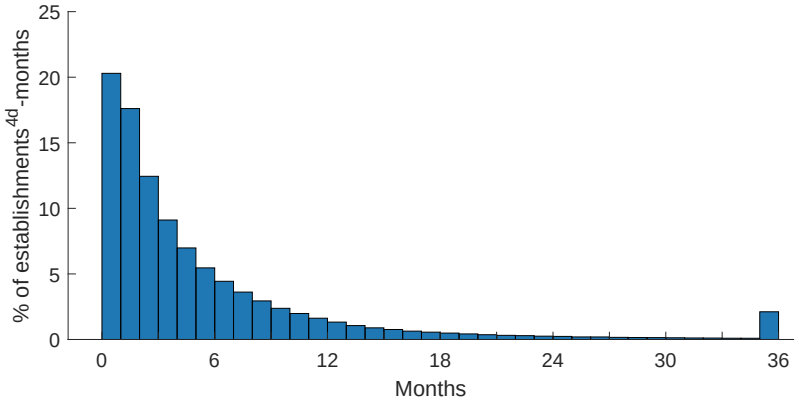
Panel (b) of Figure 3 shows the histogram of the ex-post time to fill τ_{it}^{EP}. Compared to panel (a), the histogram has more mass to the left and a less pronounced right tail. Overall, however, the two histograms are quite similar. Panel (c) reconfirms the pattern. The unweighted mean of the ex-post time to fill is 4.97 and the sales-weighted mean is 6.63, both somewhat below the corresponding means of the ex-ante time to fill.[14] The ex-ante time to fill tends to exceed the ex-post time to fill for two reasons. The first one is that fluctuations in past sales mechanically raise ex-ante to fill but not ex-post to fill. The second reason are trend increases in sales over time. Still, the two measures exhibit a high correlation of around 0.73. Taking stock, we find a mean time to fill between 5 and 7 months, and we find a long right tail with the time to fill exceeding 12 months for 8-11% of observations.

[13] Destatis publishes aggregate time series of a comparable backlog-sales ratio.

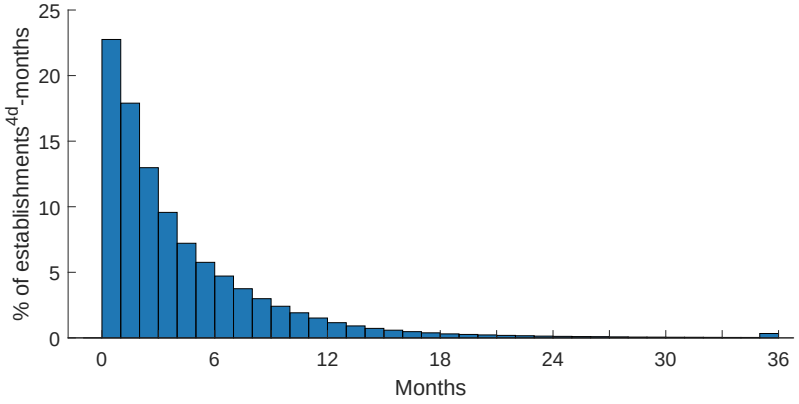
[14] Figure C.1 in Appendix C shows the sales weighted histograms for the two measures.

Figure 3: Distribution of time to fill

(a) Ex-ante time to fill (τ_{it}^{EA})



(b) Ex-post time to fill (τ_{it}^{EP})



(c) Moments of time to fill distributions

			Cumulative distribution function			
Time wedge	Mean	Std. Dev.	3 months	6 months	12 months	24 months
Ex-ante, unweighted	5.45	7.02	50.35	71.90	88.88	96.26
Ex-ante, weighted	6.92	8.51	47.33	72.55	90.81	97.00
Ex-post, unweighted	4.97	6.05	50.93	73.59	92.07	98.41
Ex-post, weighted	6.63	10.05	46.88	70.04	88.87	95.09

Note: Panels (a) and (b) show the (unweighted) histogram of the ex-ante time to fill (backlog-sales ratio) defined in equation (4.2) and the ex-post time to fill defined in equation (4.1), respectively. The histograms are winsorized at 36 months. Panel (c) shows additional moments of the distribution of the two measures. The second and fourth row refer to the sales-weighted distributions. The fifth to eighth column display the share of observations in percent for which the time to fill is at most X months. All panels include all establishment^{4d}-time observations between 2015 and 2022 for which real order backlog data are available.

Table 3: Variance decomposition of time to fill

Decomposition (4.3)	% of variance		Decomposition (4.4)	% of variance	
	τ_{it}^{EA}	τ_{it}^{EP}		τ_{it}^{EA}	τ_{it}^{EP}
Within establishment^{4d}	37.25%	28.55%	Within sector-month	91.38%	87.14%
Between establishments^{4d}	55.71%	61.26%	Between months	1.58%	2.08%
Between sectors	7.04%	10.19%	Between sectors	7.04%	10.78%

Note: The table shows the results of the variance decompositions defined in equations (4.3) and (4.4).

We next investigate to what extent differences in the time to fill across establishments^{4d} and months can be attributed to establishment^{4d}-, time-, and sector-specific components. We propose two within-between variance decompositions for which it will be useful to denote the time to fill by τ_{ist} where s denotes a four-digit sector. The first decomposition is defined by

$$Var\left(\tau_{ist}\right) = \underbrace{Var\left(\tau_{ist} - \overline{\tau}_{is}\right)}_{\text{within establishment}^{4d}} + \underbrace{Var\left(\overline{\tau}_{is} - \overline{\tau}_{s}\right)}_{\substack{\text{between establishments}^{4d} \\ \text{(within sector)}}} + \underbrace{Var\left(\overline{\tau}_{s} - \overline{\tau}\right)}_{\text{between sectors}}, \tag{4.3}$$

where $\overline{\tau}_{is}$ denotes the mean of τ_{ist} across t, $\overline{\tau}_{s}$ denotes the mean across t and all i within s, and $\overline{\tau}$ is the unconditional sample mean.[15] The second decomposition is defined by

$$Var\left(\tau_{ist}\right) = \underbrace{Var\left(\tau_{ist} - \overline{\tau}_{st}\right)}_{\text{within sector-month}} + \underbrace{Var\left(\overline{\tau}_{st} - \overline{\tau}_{s}\right)}_{\text{between months}} + \underbrace{Var\left(\overline{\tau}_{s} - \overline{\tau}\right)}_{\text{between sectors}}. \tag{4.4}$$

Table 3 provides the results when applying both variance decompositions to our sample. The largest share of the variance in the ex-post time to fill is accounted for by permanent differences in the time to fill across establishments^{4d} within a sector (61%), followed by within-establishment^{4d} variation over time (29%). Permanent differences across sectors explain the remaining 10% of the variance. Note that there are still sizable differences in the time to fill across sectors. Table C.1 in the Appendix shows that in particular the backlog sub-sectors "Other vehicles" and "Machinery" include many observations with long time to fill. However, the variance decomposition shows that differences within sectors are much larger than differences across sectors. The findings for ex-ante time to fill are of similar magnitude. The second decomposition shows that variation between establishments^{4d} within the same sector-month pair accounts for most of the variation (87-91%) in the time to fill.

[15] It can easily be verified that all covariances have to be zero.

That is precisely the variation that is not publicly available in the aggregate sectoral statistics published by Destatis. This finding therefore highlights the importance of using micro data to study the time to fill induced by order backlogs.[16]

4.3 Time to fill and aggregate inflation rates

Figure 3 shows that the average time to fill new orders across establishment^{4d}-months is between 5 and 7 months. Hence, for an average establishment^{4d}, we should also expect the sales price to lag behind the order price by a similar magnitude. We next investigate to what extent this holds true at the level of aggregate price indices.

We construct the aggregate sales price index and the aggregate new order price index via Törnqvist aggregation of the establishment^{4d}-level year-over-year price growth rates, using nominal sales as weights.[17] Figure 1 shows the aggregate year-over-year growth of both prices. We clearly observe the inflation surge starting in 2021. More importantly, we further observe a clear lead-lag pattern with new order prices leading sales prices. Such a lead-lag pattern is mechanically true at the micro-level, but does not necessarily arise at the aggregate level given that the composition of sales and new orders across establishments^{4d} changes over time. The highest correlation between the two series is reached at a lag of 3 months. Yet, note that the lag between the two series is not necessarily constant over time since the time to fill new orders also exhibits substantial time variation (c.f. Table 3).

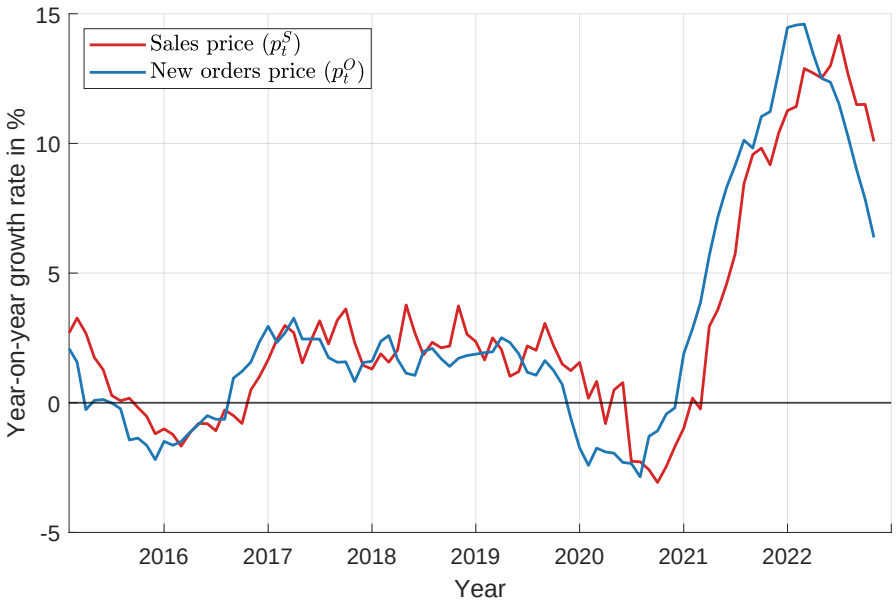
4.4 Forecasting real economic activity

We finally investigate to what extent information on the time to fill can help to predict future real economic activity. Abstracting from the order book residual, new orders mechanically turn into sales in the future. Hence, we should expect new orders to be a good predictor for future sales. However, new orders should be an even better predictor for future sales if we take into account the periods it takes for new orders to turn into sales, that is, the time to fill.

[16] Another consequence of the large heterogeneity within sector-time cells is that using (publicly available) sectoral sales price indices (e.g., PPI) to deflate order books may perform poorly.

[17] We proceed analogously when aggregating product-level prices to an establishment^{4d}-level price index, see Appendix A.3 for details.

Figure 4: Sales price vs. new orders price



Note: This figure shows the aggregate year-on-year price growth rate (measured in log-differences) for sales prices (red) and order prices (blue). The aggregate changes are computed using Törnqvist aggregation analogously to the procedure discussed in Appendix A.3.

We explore to what extent we can improve the forecasting of future sales by accounting for the establishment^{4d}-specific average time to fill. In particular, we estimate regressions defined by

$$s_{it+h} = \alpha_i + \alpha_{st} + \boldsymbol{\beta}'\mathbf{X}_{it} + u_{it}, \tag{4.5}$$

where s_{it} denotes real sales and h periods denotes the forecast horizon. The benchmark regressors in $\mathbf{X}_{it}$ are real sales in period $t, t-1, t-2, t-3, t-6, t-9, 1-12$. In two extensions, $\mathbf{X}_{it}$ further includes new orders in period t or new orders in periods $t, t-1, t-2$. The regression further includes α_i and α_{st} which denote establishment^{4d} and sector-time fixed effects. We consider the regression either specified to include sales and new orders in levels or in logs. The level specification is natural in the sense that it can be derived from the stock-flow equation (3.1). However, the level regression suffers from the typical problem that the regression estimates implicitly overweight observations with higher level realizations. We therefore also consider a specification in which sales and new orders are expressed in logs. Importantly, we consider as forecast horizon h both a variety of fixed horizons common across establishments^{4d}, and an establishment^{4d}-specific h that is set to

the rounded establishment^{4d}-specific median ex-ante or ex-post time to fill.

Table 4 summarizes the results of our forecasting regressions. In the first row (of both panels), we show the adjusted R^2 (within fixed effects) for the benchmark regressors across various forecast horizons h. The two rows below show the change in the adjusted R^2 when expanding the set of regressors by contemporaneous new orders, or additionally by two lags of new orders. The adjusted R^2 increases by 0.01 to 0.05 when including contemporaneous new orders as an explanatory variable and up to 0.09 when further including lags of new orders. The last two columns show the results when setting the forecast horizon h to the establishment^{4d}-specific median ex-ante and ex-post time to fill. Importantly, the increase in adjusted R^2 is largest in these two columns. In other words, the forecasting gain when adding new orders to the sales prediction is largest when the forecasting horizon corresponds to the establishment^{4d}-specific average time to fill new orders. The ex-post time to fill performs slightly better than the ex-ante measure. This is consistent with the ex-post measure reflecting the *realized* time to fill.[18]

Table 4: Adjusted R^2 of sales forecasting regressions

	s_{it+h}						
Horizon (h)	1	2	3	6	12	$\overline{\tau}_i^{EP}$	$\overline{\tau}_i^{EA}$
				(a) All variables in levels			
Benchmark regressors							
$(s_{it}, s_{it-1}, s_{it-2}, s_{it-3}, \ldots)$	0.245	0.186	0.217	0.162	0.127	0.131	0.132
& new orders (o_{it})	+0.013	+0.018	+0.021	+0.022	+0.017	+0.046	+0.035
& new orders (o_{it-1}, o_{it-2})	+0.024	+0.034	+0.038	+0.039	+0.025	+0.074	+0.058
				(b) All variables in logs			
Benchmark regressors							
$(s_{it}, s_{it-1}, s_{it-2}, s_{it-3}, \ldots)$	0.271	0.223	0.222	0.165	0.116	0.138	0.145
& new orders (o_{it})	+0.014	+0.020	+0.023	+0.027	+0.025	+0.056	+0.035
& new orders (o_{it-1}, o_{it-2})	+0.027	+0.037	+0.039	+0.047	+0.039	+0.090	+0.060

Note: This table shows the adjusted R^2 within fixed effects of multiple forecasting regressions. Each entry corresponds to one regression specification as defined in equation (4.5). Across columns, we vary the forecast horizon. $\bar{\tau}_{it}^{EA}$ and $\bar{\tau}_{it}^{EP}$ denote the establishment^{4d}-specific median of the ex-ante and the ex-post time to fill, respectively. Across the rows, we vary the control variables in the forecasting regression. The second, third, fifth, and sixth row report the change in R^2 compared to the Benchmark Sales-AR (first and third row).

[18] With the exception of $h = 12$, the adjusted R^2 for the benchmark regressors is lowest when the forecasting horizon is set to $\overline{\tau}_i^{EP}$ or $\overline{\tau}_i^{EA}$. This is not surprising insofar as the distribution of $\overline{\tau}_i$ has a long right tail. Forecasting sales far into the future is naturally harder because long forecast horizons provide more scope for disturbances to affect sales.

5 A theory of the firm with order backlog

In this section we introduce a theory of the firm with a non-trivial role for order backlog. Backlog is a dynamic control variable that constitutes a third margin of firm adjustment to shocks, next to the price and production. We characterize the optimal responses of the firm to such shocks. The theory motivates and guides the empirical identification of shocks in Section 6.

5.1 Problem of the firm

Time is discrete and runs forever. Consider a single firm that seeks to maximize its expected discounted profits by choosing real new orders o_t, their price p_t, real output q_t, and the real backlog b_t.[19] In doing so, the firm is subject to a real stock-flow equation. The firm's revenue in period t is given by real quantity of new orders it accepts, o_t, times the price of new orders in that period p_t. The firm is a (local) monopolist and internalizes that its new orders depend negatively on the price it sets. Moreover, real new orders are subject to an exogenous demand shifter x_t^d. The cost of the firm consists of two terms. First, the nominal production cost of the firm is given by $C(q_t)(x_t^s)^{-1}$, where x_t^s is an exogenous shifter lowering the cost of the firm. Second, the firm pays a monetary cost of starting into the period with backlog b_{t-1} and producing q_t given by the function $BP(q_t/b_{t-1})$. Formally, the problem of the firm is given by

$$\max_{\{o_t,p_t^o,q_t,b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[p_t o_t - C(q_t)(x_t^s)^{-1} - BP(q_t/b_{t-1})\right] \tag{5.1}$$

$$s.t. \quad o_t = x_t^d \left(p_t\right)^{-\sigma}; \tag{5.2}$$

$$b_t = b_{t-1} + o_t - q_t \tag{5.3}$$

$$\log(x_t^d) = (1-\rho_d)\mu_d + \rho_d \log(x_{t-1}^d) + \varepsilon_t^d$$

$$\log(x_t^s) = (1-\rho_s)\mu_s + \rho_s \log(x_{t-1}^s) + \varepsilon_t^s$$

$$\sigma > 1, \quad \beta, \rho_d, \rho_s \in (0,1)$$

where the innovations to the demand and supply shifter, ε_t^d and ε_t^d, are i.i.d. over time and mutually uncorrelated. We now introduce additional assumptions on the functional form of the two cost functions and discuss their economic rationales.

Assumption 1: The cost function $C(\cdot)$ is twice continuously differentiable with $C'(q) > 0, C''(q) > 0$ for all $q > 0$

[19] To lighten the notation we denote the price of new orders as p_t instead of p_t^O as in previous sections

In other words, we assume that the production cost is strictly convex, reflecting decreasing returns to scale of the underlying production function. This assumption gives rise to a production smoothing motive (as in West, 1989) since the increasing marginal cost of production makes fluctuations around the optimal scale of production costly.

> *Assumption 2: The backlog penalty function $BP(\cdot)$ is twice continuously differentiable, convex, and has the following property:*
> $\exists m \in (0, \infty) : (BP'(x) < 0,\ \forall x < m)\ \wedge\ (BP'(x) > 0,\ \forall x > m)$

As discussed above, the backlog penalty function $BP(\cdot)$ depends on production in period t relative to the level of backlog in the beginning of the period q_t/b_{t-1}, which is an inverse measure of the ex-ante time to fill, similar to the one introduced in section 4. The function $BP(\cdot)$, which depends on the inverse time to fill, reflects all economic costs associated with such a time wedge.[20]

First, it captures the costs of letting customers wait longer as the time to fill increases. *Ceteris paribus* we may expect the firm's customer to prefer a short waiting time between order and delivery. For tractability we model this motive in reduced form by including it in the cost term. Second, it captures a motive for the firm to keep a sufficiently positive amount of order backlog. Producing output in the same period that the order entered may be more expensive than producing it in a later period or stretching production over multiple periods. The underlying reasons may be technological, e.g., a higher capacity utilization has increasing marginal costs, or long planning horizons due to highly individualized outputs.

Assumption 2 disciplines this trade-off: If production is sufficiently low relative to backlog (i.e., if $q_t/b_{t-1} < m$), then the time to fill is large and the first motive dominates. Speeding up production relative to the backlog would lower the waiting time for customers and the associated penalty cost. If production is sufficiently high relative to backlog (i.e., if $q_t/b_{t-1} > m$), then the time to fill is small and the second motive dominates. In that case, decreasing production relative to the backlog would lower the penalty cost since it allows the firm to take more time for production. Note that Assumption 2 implies that the penalty function has a unique minimizer m, at which the marginal costs of the two motives balance out.

5.2 Optimal policies

In this subsection we characterize the firm's optimal choices up to first order around the deterministic steady state of the model. For tractability we let $\beta \to 1$ in the derivations.

[20] See Belsley (1969) and Zarnowitz (1973) for a more extensive discussion of these motives.

Under that assumption, it can be shown that the steady state satisfies:[21]

$$p = \frac{\sigma}{\sigma - 1} C'(q), \quad \frac{q}{b} = m.$$

In words, the firm sets the statically optimal CES markup over its marginal cost and accumulates a level of backlog that minimizes the backlog penalty function. We log-linearize the model around this steady state to understand the propagation patterns of demand and supply shocks. We start by characterizing the backlog policy function. We denote log deviations from the steady state by hatted variables.

Proposition 1 (**Backlog Dynamics**). *Up to a log-linear approximation around the deterministic steady state with $\beta \to 1$, the optimal order backlog policy satisfies*

$$\hat{b}_t = c_1 \hat{b}_{t-1} + c_2 \hat{x}_t^d + c_3 \hat{x}_t^s.$$

If $b/q = 1/m > 1$ in the steady state, the firm problem has a unique solution for c_1 with $c_1 \in (0, 1)$ such that $\hat{b}_t$ follows a stable process.

Proof: See Appendix D.1

Throughout the remaining analysis, we assume that $b/q > 1$ is satisfied.[22] In the histograms in section 4 we demonstrated that this condition holds for the vast majority of observations. It is immediate from Proposition 1 that the policy functions for prices, output and new orders are also linear functions of the same state variables. Formally, they are given by[23]

$$\begin{aligned}
\hat{p}_t &= d_1 \hat{b}_{t-1} + d_2 \hat{x}_t^d + d_3 \hat{x}_t^s \\
\hat{q}_t &= e_1 \hat{b}_{t-1} + e_2 \hat{x}_t^d + e_3 \hat{x}_t^s \\
\hat{o}_t &= f_1 \hat{b}_{t-1} + f_2 \hat{x}_t^d + f_3 \hat{x}_t^s.
\end{aligned}$$

The following proposition characterizes the response of the model's four endogenous variables to demand and supply shocks.

Proposition 2 (**Shock Propagation**).

- *An expansionary demand shock ($\varepsilon_t^d > 0$) increases the backlog, prices, output, and new orders ($c_2 > 0, d_2 > 0, e_2 > 0, f_2 > 0$).*

[21] It is straightforward to verify that the models has a unique steady state

[22] $b/q = 1/m > 1$ is only a sufficient condition for a unique solution with $c_1 \in (0, 1)$. In Appendix D.1 we provide a weaker (necessary and sufficient) condition in terms of the linearization coefficients of the model.

[23] See Appendix D.2 for a formal derivation

- *An expansionary supply shock ($\varepsilon_t^s > 0$) lowers prices ($d_3 < 0$) but raises output and new orders ($e_3 > 0, f_3 > 0$). There exists a persistency threshold $\bar{\rho}^s \geq 0$, such that an expansionary supply shock ($\varepsilon_t^s > 0$) lowers the backlog ($c_3 < 0$) if $0 \leq \rho_s < \bar{\rho}_s$ and (weakly) raises the backlog ($c_3 \geq 0$) if $\rho_s \geq \bar{\rho}_s$.*[24]

Proof: See Appendix D.2

The intuition for the first result is straightforward: If the firm faces an expansionary demand shock, it increases its price but new orders still increase. However, it doesn't increase production one-to-one due to the convex cost function. As a consequence, its backlog increases.
The case of a supply shock is more involved. For the sake of the argument, we discuss the intuition for the case of an *expansionary* supply shock, which induces the firm to temporarily increases its output and decreases its price. If these supply shocks are sufficiently transitory, the former effect dominates the latter, i.e., output increases by more than new orders and backlog decreases in response to an expansionary supply shock. Intuitively, if a supply shock in a given period t is sufficiently transitory, it will already have faded out once the firm actually starts producing the orders received in t. Therefore, the firm temporarily brings forward production of existing orders and only modestly decreases its price.
The more persistent supply shocks are, the more such a policy would decrease backlog over time. To avoid the associated penalty cost, a stronger price response is required to reduce the gap between new orders and output. If supply shocks are sufficiently persistent, the price response even dominates the output response, so that optimal backlog increases after an expansionary supply shock. Intuitively, if production is persistently cheaper, the firm has an incentive to increase production for multiple periods. To that end, it increases its order backlog by increasing new orders even more than production.

6 Identification and evidence on firm-level shocks

In this section, we discuss the challenge of identifying firm-level shocks to demand and supply in the presence of order backlog, highlighting that a class of conventional identification strategies become invalid in the presence of order backlog. We then propose a novel valid identification strategy and apply it to our sample. We estimate the empirical impulse response functions to the identified shocks. They are qualitatively consistent with the theoretical predictions in Section 5. Finally, we contrast our findings with comparable estimates

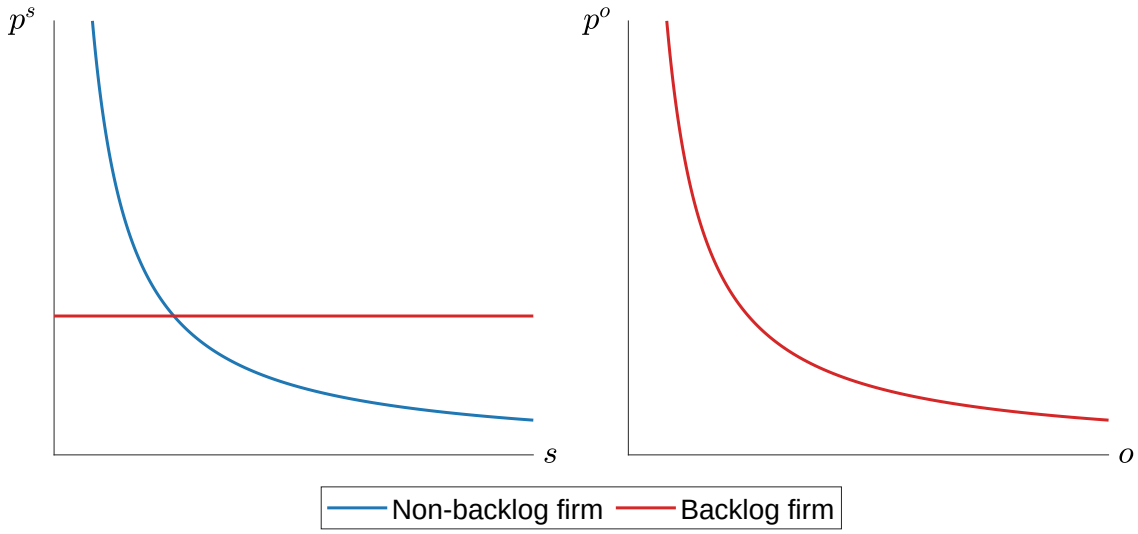
[24] We provide a formal characterization of $\bar{\rho}^s$ in terms of the linearization coefficients in Appendix D.2.

obtained when applying an invalid conventional identification strategy.

6.1 Identification of firm-level shocks

The identification of demand and supply shocks faces two key challenges. The first identification challenge is that backlog renders standard identification strategies for supply and demand shocks invalid – a fact that has been largely neglected in the literature on demand-curve estimation. In the presence of order backlog, a demand shock changes the order price for a given order quantity. Instead, the sales price remains unchanged on impact. We illustrate the fundamental impact of order backlog in Figure 5. On the left panel, for non-backlog firms, the sales price (p^s) and quantity (s) relationship follows a downward-sloping demand curve (in blue). This is not the case for a backlog firm where orders predetermine the firm's sales price (in red). In contrast, the relevant downward-sloping demand schedule for a backlog firm relates the order price (p^o) to order quantities (o) as depicted on the right panel. Consequently, estimating demand shocks for backlog firms based on sales-price and sales-quantity variation will not isolate the contemporaneous demand shock, but instead be contaminated by past demand shocks as well as past and present supply shocks.[25]
The second identification challenge is standard and based on the problem that variation in price and quantity data alone do not reveal whether observed variation stems from shifts in

Figure 5: Illustration of the identification problem



[25]In Subsection 6.3, we consider an exercise with wrongly identified supply and demand shock and find that these wrongly identified shocks do not generate firm responses that are in line with our model presented in Section 5.

demand or supply. Consistent with our model, we approach these challenges by estimating demand curves for backlog establishments[4d] in the order-price and order-quantity nexus while using a Bartik shift-share instrumental variable to exploit exogenous shifts in production costs. For an establishment[4d] i that is active in the two-digit sector $s(i)$, we regress the log of real orders during month t on the log of its order price and control for establishment[4d] and time fixed effects. To address serial correlation in the level of an establishment[4d]'s orders, we additionally control for the lag of real orders from the previous period:

$$\log(o_{it}) = \alpha_i + \alpha_{s(i)t} + \beta_{s(i)} \log(p_{it}^O) + \gamma_{s(i)} \log(o_{it-1}) + \varepsilon_{it}^d. \tag{6.1}$$

To allow for demand-curve slopes to vary by sector, we estimate (6.1) separately for each two-digit sector. We then use the estimated residuals $\hat{\varepsilon}_{it}^d$ as the identified demand shock. Note that this specification is consistent with the way how demand shocks enter the model introduced in Section 5.

Since the order price also responds endogenously to demand shocks, we need an instrument to rely on the exogenous part of variation in order prices. We build a Bartik shift-share instrumental variable that is a weighted aggregate of the prices charged by foreign producers, measured by sectoral import price indices, where the foreign input prices are the shifts and input cost shares are the shares of the instrument. We construct our instrument as follows:

$$SSIV_{it} = \sum_j \phi_{ij} p_{jt}^{imp}, \tag{6.2}$$

where ϕ_{ij} are time-invariant variable cost shares of establishment[4d] i on imported material or energy inputs from an input industry j. These cost shares are calculated based on combining two data sources. We first match the establishments[4d] from the reports data to the *Cost Structure Survey* of German manufacturing firms. This allows us to calculate the cost shares that a firm spends on material inputs and energy inputs.[26] In a second step, we narrow these costs shares down to foreign input industries j by linking the establishments[4d] to the direct requirements input-output table from Destatis which delivers foreign input expenditures measured at the level of two-digit input and output industries. To construct our shifter variable p_{jt}^{imp}, we rely on industry-level import price indices from Destatis. While these price indices originally vary at monthly frequency and measure month-on-month price changes, we recalculate the price indices to a monthly series based on year-on-year variation such that

[26] As opposed to the reports and censuses, the *Cost Structure Survey* is collected at the yearly frequency, covers firms and not individual establishments[4d], and covers changing random subsamples of firms. Most firms' cost structure is surveyed just once. If a firm is surveyed more than once in the *Cost Structure Survey*, we calculate average cost shares based on the available information.

the shifter p_{jt}^{imp} tracks year-on-year changes in input prices over the last 12 months before t relative to the average input prices during the 12 months in the base period 2015.[27]
Following Borusyak et al. (2022), our identification assumption is that of a shift-level idealized experiment, where we assume that long-run changes in foreign input prices are assigned as good as randomly across the input sectors j, conditional on observables. The relevance of the instrument stems from the impact of input prices on firms' production costs and ultimately order prices. We calculate the intra-class correlation (ICC) of the p_{jt}^{imp} across j after netting out the time fixed effects. The estimated ICC of these demeaned shift series is negative which implies that correlation of shocks across input sectors j is small or even negative and that the variation within input sectors over time exceeds the variation between input sectors. Borusyak et al. (2022) show that each shifter jt carries an importance weight that is proportional to the exposure share of that shifter for an average sample observation. The inverse of the Herfindahl-Hirschman concentration index of these importance weights is informative, as it indicates the effective number of shifts that are used in the regression to identify shocks to order prices. We consider 22 input industries j and our sample spans 96 months such that there are 2,112 observations for p_{jt}^{imp}. Given the concentration of exposure shares, the effective number of price shocks used for identification – measured by the inverse HHI – is fairly high and given by 550.14.

We first use our instrument to estimate an aggregate order demand curve spanning all backlog sectors. The slope of the demand curve is around one (-0.96) and significant at all usual significance levels.[28] The first-stage coefficient of input prices on order prices is 0.13 and thus has the expected sign – higher input prices are partially passed through on establishments[4d]' order prices. The Kleibergen-Paap first-stage F statistic is 195 when correcting for clustering of standard errors at the four-digit sector by month level. The reduced-form coefficient of order surprises on input prices is -0.13.
To obtain series of demand shocks $\hat{\varepsilon}_{it}^{d}$, we estimate (6.1) for each two-digit sector s. Table E.1 in Appendix E shows the estimated demand curves by sector. The slope of these sectoral order demand curves is negative throughout all sectors but one.[29] In a last step, we obtain supply shocks that are orthogonal to the demand shocks by running the following regression

[27] We can safely ignore the time wedge between order and sales prices in the instrument, as long as the autocorrelation in the input prices is sufficiently large to identify changes in the production costs of the establishments[4d]. Furthermore, input industries j span all manufacturing industries and energy-supplying industries such that order-price wedges are likely to play a smaller role, here.

[28] When estimating this aggregate demand curve, we follow Borusyak et al. (2022) and additionally control for the sum of cost shares $\sum_j \phi_{ij} \leq 1$ interacted with monthly fixed effects.

[29] Sector 14 (clothing) has a positive demand slope, presumably due to weak identification ($F < 5$). However, its share in the sample is fairly small (0.7% of observations).

for each two-digit sector:

$$\log(s_{it}) = \alpha_i + \alpha_{s(i)t} + \beta_{s(i)}\hat{\varepsilon}^d_{it} + \boldsymbol{\gamma}'_{s(i)}\mathbf{X}_{it} + \varepsilon^s_{it}, \tag{6.3}$$

where $\mathbf{X}_{it} = \left(\log(b_{it-1})\ \log(s_{it-1})\ \log(o_{it-1})\ \log(p^O_{it-1})\right)'$ is a vector of lagged control variables. The rationale of this regression is to mimic the policy function for sales obtained in Section 5. Conditional on lagged backlog and the demand shocks, all fluctuations in sales should be driven by the supply shock.

6.2 Propagation of firm-level shocks

Equipped with identified demand and supply shocks, we now estimate the impulse response functions (IRFs) of various order book variables to these shocks using panel local projections. For comparability with the theoretical predictions from Section 5, our main empirical analysis focuses on real order backlog, real sales, real new orders, and the price of new orders as the outcome variables of interest.[30] Formally, for each variable of interest x we estimate the following sequence of regressions:

$$\log(x_{it+h}) - \log(x_{it-1}) = \alpha^h_i + \alpha^h_{s(i)t} + \beta^h shock_{it} + v^h_{it}, \quad h = 0, ..., H$$

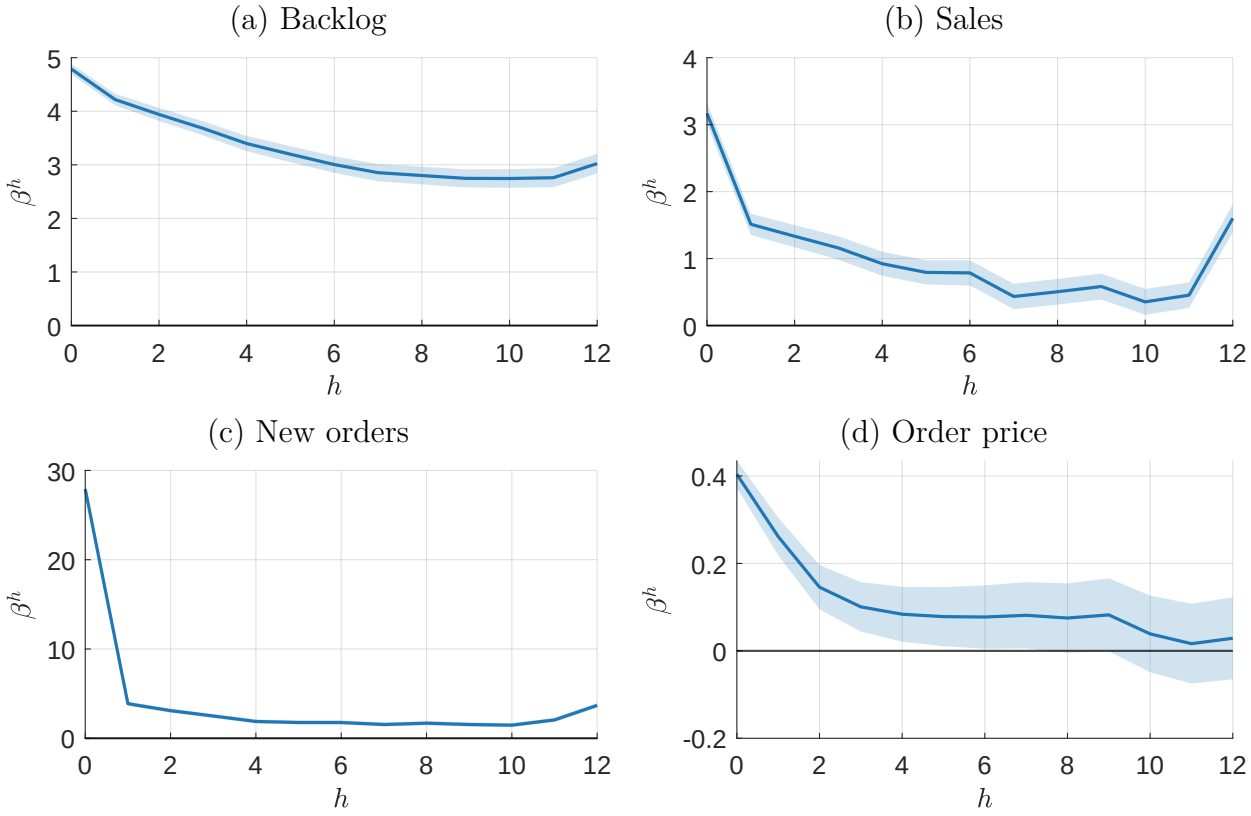
Here, α^h_i is an establishment[4d] fixed effect and α^h_{st} is a sector-time fixed effect.[31] The sequence of coefficients $\{\beta^h\}_{h=0}^H$ can be interpreted as the cumulative response of variable x at horizon h to a shock at horizon 0.

Figures 6 and 7 show the estimated IRFs to demand and supply shocks, respectively. The IRFs are normalized to one-standard deviation expansionary shocks and expressed in percent. In line with the theoretical predictions of Proposition 2 in Section 5, real order backlog, real sales, real new orders, and the price of new orders all increase in response to an expansionary demand shock. Quantitatively, order prices exhibit the weakest response and increase by only 0.4% on impact. That increase is also relatively short-lived: 8 months after the shock, the response is not significantly different from zero anymore. As rising prices barely counteract the demand shock's direct effect, new orders increase sharply by more than 25% on impact. Yet, the impact effect is very transitory and new orders only increase by around 4% at later horizons. Quantitatively, the sales response is weaker than the response of new orders, with a peak response of around 3% on impact. Consistently, order backlog increases by around 5% on impact. Even though the identified demand shocks have a rather transitory impact

[30] Appendix E provides additional results for sales prices (Figure E.1) and the real residual (Figure E.2)

[31] As in the identification of shocks, we define sectors by two-digit industry codes

Figure 6: Responses to a demand shock

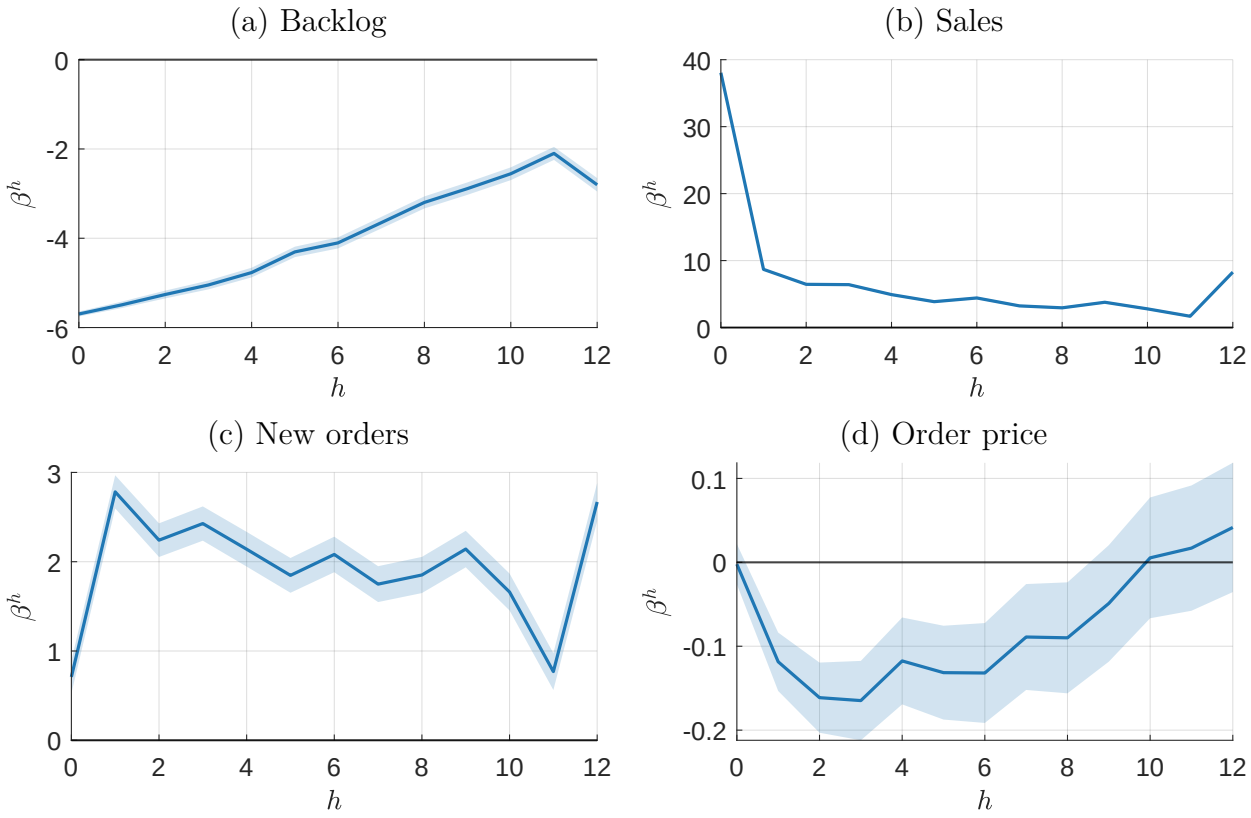


Note: This figure shows the IRFs to an expansionary demand shock of one standard deviation expressed in percent. The 95% confidence bands are computed using robust standard errors. The observations were trimmed at 1% and 99% for each horizon.

on the flow variables, the backlog response is more persistent and still at around 3% one year after the shock.

The empirical findings for supply shocks (Figure 7) align with our theory as well. Order prices do not react on impact but then fall in subsequent periods, up to a minimum of around -0.15% three months after the shock. The response of new orders is just the mirror image of that: The response is close to zero on impact but builds up in subsequent periods. This observation is in line with the demand curve (6.1) specified to identify the shocks: Supply shocks only affect new order indirectly through the price response. Hence, qualitatively, the order response and the price response should mirror each other. In contrast to these two responses, sales increase sharply on impact by more than 35%. Since sales react more strongly than new orders, the order backlog falls and only gradually approaches its pre-shock level again. Proposition 2 in Section 5 shows that order backlog increases in response to sufficiently persistent expansionary supply shocks but falls in response to transitory supply shocks. The empirical response shown in this section aligns well with the latter case.

Figure 7: Responses to a supply shock



Note: This figure shows the IRFs to an expansionary supply shock of one standard deviation expressed in percent. The 95% confidence bands are computed using robust standard errors. The observations were trimmed at 1% and 99% for each horizon.

6.3 Order prices vs. sales prices

In this subsection we provide additional evidence on the discrepancy between order prices and sales prices for the identification and propagation of demand and supply shocks.

We start by constructing an alternative invalid series of demand and supply shocks, which would be valid only for establishments^{4d} without order backlog. To see the intuition, consider Figure 5 again. In the previous subsections we estimated the demand curve in terms of orders and order prices depicted in the right panel. Now, we estimate the demand curve in terms of sales and sales prices depicted in the left panel, which is only valid in the absence of order backlog. Formally, the alternative specification is given by

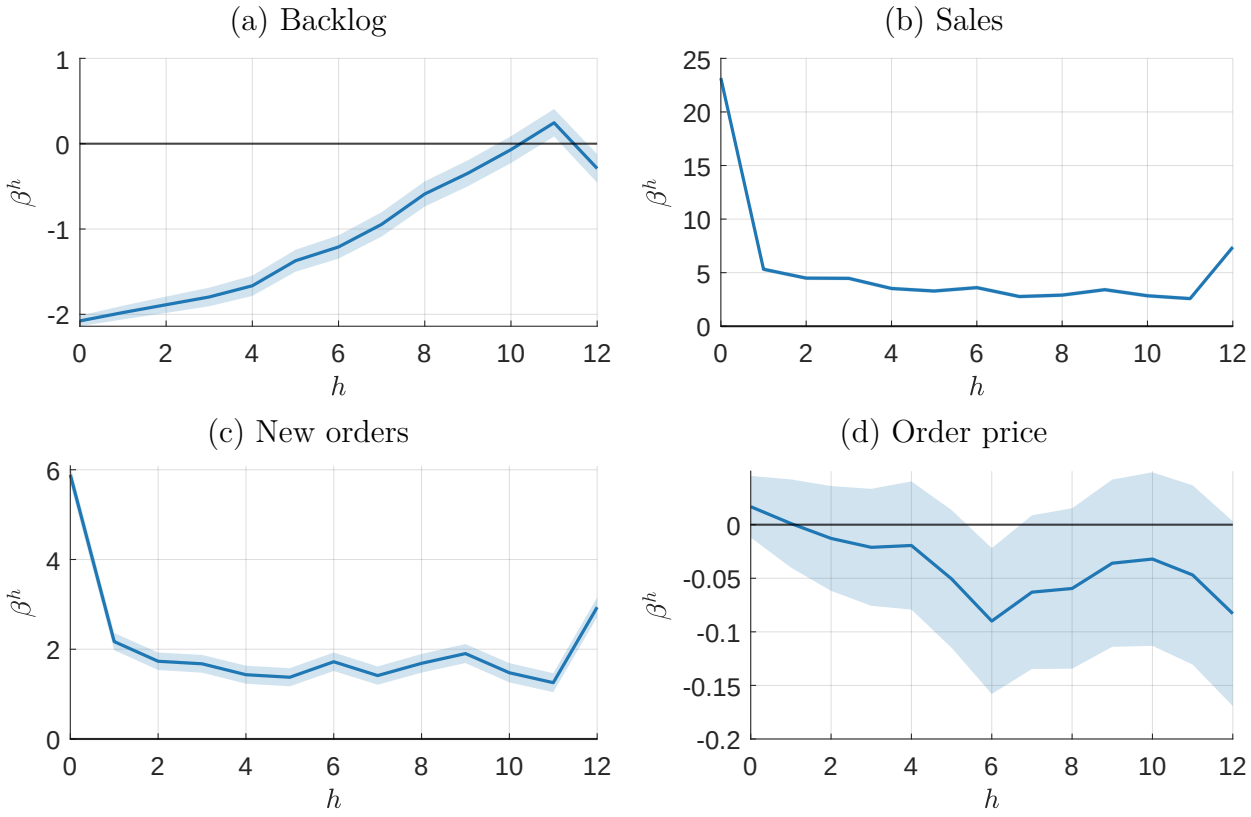
$$\log(s_{it}) = \alpha_i + \alpha_{s(i)t} + \beta_{s(i)} \log(p_{it}^S) + \gamma_{s(i)} \log(s_{it-1}) + \epsilon_{it} \tag{6.4}$$

We estimate equation (6.4) as outlined in section 6.1, i.e., we use the same IV as for our baseline. Moreover, we also identify the corresponding supply shocks given the demand

shocks as in the baseline.

At the identification stage, we find that 4 out of 12 sectors exhibit either a positive estimated demand slope or an F statistics less than 10 – even though the first stage only delivers a wrongly signed coefficient for a single sector.

Figure 8: Responses to incorrectly identified demand shocks

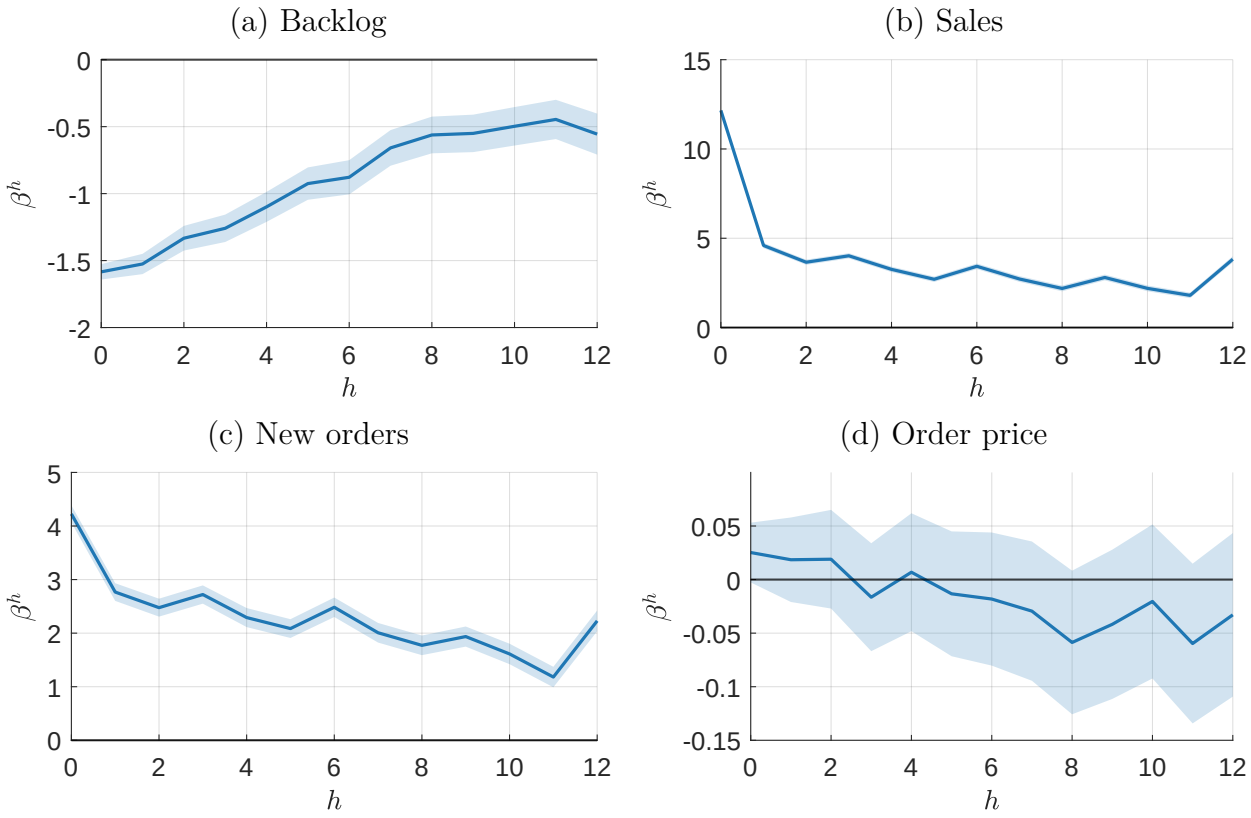


Note: This figure shows the IRFs to an expansionary (invalid) demand shock of one standard deviation expressed in percent. The 95% confidence bands are computed using robust standard errors. The observations were trimmed at 1% and 99% for each horizon.

The IRFs corresponding to the invalid demand and supply shocks are depicted in Figures 8 and 9, respectively. Two differences relative to the valid shocks stand out. First, the order backlog decreases in response to an expansionary demand shock. Note that the theory laid out in section 5 only implies an ambiguous response of backlog to a supply shock – the response to an expansionary demand shock is unambiguously positive contradicting the evidence in Figure 8. Second, the responses of order prices to demand and supply shocks are not significantly different from zero for most horizons. Surprisingly, even the response of sales prices to a demand shock (Panel (c) in Figure E.1 in Appendix E) is only positive on impact and essentially zero after that.

The discrepancy between order and sales prices does not only matter for the identification

Figure 9: Responses to incorrectly identified supply shocks



Note: This figure shows the IRFs to an expansionary (invalid) supply shock of one standard deviation expressed in percent. The 95% confidence bands are computed using robust standard errors. The observations were trimmed at 1% and 99% for each horizon.

but also for the propagation of shocks. In panels (a) and (b) of Figure E.1 in Appendix E we show the estimated IRFs of sales prices to the (correctly) identified demand and supply shocks. Remember that the sales price in period t of an establishment^{4d} with order backlog is predetermined. Hence, at any given horizon $h > 0$, only a subset of sales prices can react to a shock at horizon 0 – namely the sales prices of those establishments^{4d} with a time to fill $\leq h$. This also reflects in the IRFs: The response of sales prices to a demand shock is more sluggish than that of order prices. Summing up, both the identification of shocks and the propagation patterns significantly differ depending on which price is used.

7 Conclusion

This paper highlights the importance of order books in the manufacturing sector. We introduce a novel administrative micro dataset on order books in Germany, which includes order book data for two thirds of the manufacturing sector. We observe the order backlog, the

inflow of new orders, and the order book outflow, sales. The presence of order backlog induces a time wedge between new orders and sales—time to fill—, which averages between 5 and 7 months across establishments in our sample, and which features a pronounced right tail. Our analysis requires prices for all order book variables, which is an informational requirement not met in the data. Instead, we propose and apply a scheme to construct theoretically coherent establishment-level prices for all order book variables for which it suffices to observe establishment-level sales prices. The scheme exploits the fact that all prices are dynamically linked through the order backlog. In line with time to fill, we document that new orders predict future sales, especially at horizons that correspond to establishment-specific time to fill. Similarly, sales prices are predetermined by past prices of new orders, creating a novel source of nominal rigidity. The rigidity of sales prices invalidates conventional strategies to identify demand and supply shocks. Instead, variation in quantity and price of new orders can identify such shocks and we propose and apply a valid identification strategy. Finally, we provide a theory of the firm to analyze the propagation of demand and supply shocks when backlog constitutes an adjustment margin. Using the identified shocks, we test the theory in the data. We empirically characterize shock propagation and find responses that are qualitatively in line with the model.

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Appendix

Appendix A Data

A.1 Monthly Reports of Manufacturing Establishments

Data set. The *Monthly Reports of Manufacturing Establishments* (official name: *Monatsbericht für Betriebe des Verarbeitenden Gewerbes*) are collected by the (State) Statistical Offices of Germany. Submitting a monthly report is legally mandatory for all manufacturing establishments in Germany with at least 50 employees in a given year. The legal mandate is based on federal law (*BStatG* and *ProdGewStatG*) and EU regulations (1893/2006 and 2019/2152), and the EU commission implementing regulation 2020/1197. Further details on the data collection, data quality, and a sample questionnaire can be found in the quality reports published by Destatis (official name: *Qualitätsberichte der Monatsbericht für Betriebe des Verarbeitenden Gewerbes*).[32] Sector are classified according to the German classification system *WZ 2008*, which is comparable to the international NACE Rev. 2 standard.

Backlog sectors. Data on new orders and order backlog are only collected in a selected set of two-digit manufacturing sectors. These sectors are textiles (*WZ 2008* code 13), cloths (14), paper (17), chemicals (20), pharmaceuticals (21), primary metals (24), fabricated metals (25), electronics (26), electrics (27), machinery (28), automotives (29), and other vehicles (30). Excluded are the following two-digit manufacturing sectors: food (10), beverages (11), tobacco (12), leather (15), lumber products (16), print (18), petroleum and coal (19), rubber and plastics (22), other non-metallic minerals (23), furniture (31), other manufacturing (32), repair and installation (33).

Zigzag order book residual. The order book residual R_{it} occasionally displays a zigzag pattern. The residual is large (relative to other order book variables) in one month followed by a residual of similar magnitude but opposite sign in the subsequent month. We consider it likely that these zigzags reflect reporting mistakes in the order backlog. For example, if a purchase agreement of value Z in month t is recorded in new orders in t but is mistakenly not recorded in B_{it} but only starting from B_{it+1}, this will lead to a negative order book residual $R_{it} = -Z$ and a positive residual $R_{it+1} = Z$ in the next month. We smooth out such zigzags in the order book residual and adjust the order backlog to satisfy the stock-flow equation. Formally, we define a zigzag as two adjacent residuals (1) of opposite sign, $R_{it}R_{it+1} < 0$, (2) that are large relative to other order book variables, $0.5(|R_{it}| + |R_{it+1}|) > \min\{B_{it}, O_{it}, S_{it}\}$, and (3) that approximately offset each other, $10|R_{it} + R_{it+1}| < 0.5(|R_{it}| + |R_{it+1}|)$. For

[32] https://www.destatis.de/DE/Methoden/Qualitaet/Qualitaetsberichte/Industrie-Verarbeitendes-Gewerbe/einfuehrung.html

observations that satisfy the three criteria, we adjust the residuals and the backlog by

$$X_{it} = \min(|R_{it}|, |R_{it+1}|) \tag{A.1}$$

to obtain the corrected residual and backlog:

$$R_{it}^{corrected} = R_{it} - sign(R_{it})X_{it}, \tag{A.2}$$
$$R_{it+1}^{corrected} = R_{it+1} - sign(R_{it})X_{it}, \tag{A.3}$$
$$B_{it}^{corrected} = B_{it} + sign(R_{it})X_{it}. \tag{A.4}$$

The zigzag correction in the order book residual is consistent with the stock-flow equation and requires adjusting the order backlog only in a single period.[33] We identify and correct a zigzag pattern in the order book residual in less than 0.5% observations for the sample of backlog sectors.

Observations with negligible or implausible order data. We exclude observations for which the order data appear either implausible or negligible. For an establishment[4d] i that we observe for periods $t = 1, \ldots, T$, we consider all subsamples $\mathscr{T} = \{1, \ldots, t\}$ and determine whether they satisfy one of the following three criteria:

(1) $O_{it} < 0.2 \cdot S_{it}$, in at least 90% of the periods in $\mathscr{T}$
(2) $B_{it} \leq 0.5 \cdot S_{it}$, in at least 90% of the periods in $\mathscr{T}$
(3) $|S_{it} - O_{it}| < 0.005 \cdot S_{it}$, in at least 90% of the periods in $\mathscr{T}$

Criterion (1) means an establishment[4d] commonly reports far fewer new orders than sales. Rationalizing such a pattern requires an implausibly large negative growth rate of new orders. It seems plausible that a large fraction of orders is not recorded in this case. (2) means the order backlog is commonly reported as corresponding to less than half a month of sales. In this case, the backlog is either of negligible importance or under-reported. (3) states that sales closely tracks new orders in most periods, meaning the order backlog will vary by little. This case either reflects misreporting or a negligible order backlog.
For all establishment[4d] and subsamples $\mathscr{T}$, we first assess whether at least one of the criteria (1)–(3) is satisfied. In a second step we drop all subsamples (i.e., all observations t in $\mathscr{T}$ for a given i) for which at least one of the criteria is satisfied.

[33] To ensure a non-negative corrected backlog, we define a lower bound for the corrected backlog as $\underline{B}_{it} = \min\{B_{it}, 0.1 \cdot \frac{1}{12}\sum_{j=0}^{11} S_{it-j}\}$. If $sign(R_{it}) < 0$ and $B_{it} + sign(R_{it})X_{it} < \underline{B}_{it}$, we redefine X_{it} as $X_{it} = B_{it} - \underline{B}_{it}$ and compute $R_{it}^{corrected}$ and $B_{it}^{corrected}$ as described above.

Descriptive statistics on sample selection. Table A.1 provides descriptive statistics comparing the full sample for the entire manufacturing sector in panel (a) with the sample for all observations in backlog sectors after correcting order book residual zigzags in panel (b), and the final sample that excludes negligible and implausible order book data in panel (c). The full sample includes more than 3.0 million observations across establishments^{4d} and from 2015 through 2022. This number falls to 1.8 million in the full sample of backlog sectors. Establishments^{4d} in the backlog sectors are 20% larger in terms of employment and sales. In the final sample, we further exclude observations according to criteria (1)–(3). The number of observations falls by 27% to 1.3 million. The first criterion is the least important affecting only 0.4% of the observations. The second and third criteria apply to 22% and 25% of observations in the sample of backlog sectors, respectively. Given the diligent data collection process of Destatis, we consider it likely that most cases in which the second and third criteria are satisfied reflect a negligible order backlog rather than misreported data. Consistent with this view, our final sample covers 98% of the aggregate order backlog. We further observe that the firm size distribution in terms of employment and sales in our final sample is almost identical to the full sample of backlog sectors.

Table A.1: Sample selection

	Mean	Sd	P10	P50	P90	Obs
	(a) Entire manufacturing sector					
Employment	162.71	660.61	6.00	76.00	313.00	3,022,069
Sales	3.80	28.01	0.07	0.91	6.63	3,022,069
	(b) Full sample of backlog sectors					
Employment	193.59	837.18	9.00	81.00	359.00	1,805,063
Sales	4.60	33.31	0.09	1.03	7.59	1,805,063
New orders	4.80	39.67	0.06	1.01	7.79	1,805,063
Order backlog	26.12	385.79	0.00	1.61	29.66	1,805,063
Order book residual	0.05	21.35	-0.18	0.00	0.23	1,805,063
	(c) Final sample of backlog sectors (excl. negligible and implausible order book data)					
Employment	197.76	903.06	11.00	84.00	362.00	1,315,354
Sales	4.64	36.17	0.10	1.07	7.38	1,315,354
New orders	5.10	45.31	0.07	1.06	7.74	1,315,354
Order backlog	35.16	451.12	0.16	3.74	42.19	1,315,354
Order book residual	0.24	22.20	-0.26	0.00	0.36	1,315,354

Note: The table provides descriptive statistics at the establishment^{4d}-month level from 2015 through 2022. Employment is measured in number of employees. All other variables are deflated by the PPI normalized to the year 2019 and expressed in EUR million.

A.2 Additional descriptive statistics

Table A.2: Disaggregation of the manufacturing sector

	Mean	Sd	P10	P50	P90	P99	Obs
Establishments^{4d}/Establishment							
– non-weighted	1.40	1.31	1	1	2	5	937,112
– sales-weighted	1.64	2.26	1	1	3	8	937,112
Establishments^{4d}/Firm							
– non-weighted	1.30	0.79	1	1	2	4	1,013,805
– sales-weighted	1.20	0.80	1	1	2	4	1,013,805
Months/Establishments^{4d}							
– non-weighted	68.30	32.96	12	89	96	96	19,259
– sales-weighted	73.89	31.07	21	96	96	96	19,259

Note: The table provides descriptive statistics on the composition of establishment^{4d}-month observations in the final sample of backlog sectors from 2015 through 2022.

Table A.3: Descriptive statistics across backlog sub-sectors

	Mean	Sd	P10	P50	P90	Obs	Total
	Textiles, Cloths, and Paper						
Sales	3.12	5.24	0.20	1.54	6.66	86,292	4.4%
New orders	3.30	5.98	0.18	1.52	7.04	86,292	4.2%
Order backlog	8.47	26.28	0.14	2.41	18.00	86,292	1.6%
Order book residual	0.12	1.82	-0.20	0.00	0.36	86,292	
	Chemicals and Pharmaceuticals						
Sales	6.27	21.07	0.08	1.78	12.07	95,366	9.8%
New orders	6.49	22.46	0.05	1.77	12.46	95,366	9.2%
Order backlog	15.57	60.26	0.03	2.64	30.90	95,366	3.2%
Order book residual	0.09	7.14	-0.38	0.00	0.58	95,366	
	Primary and Fabricated Metals						
Sales	2.71	9.98	0.09	0.85	5.32	388,482	17.2%
New orders	2.80	11.16	0.07	0.82	5.54	388,482	16.2%
Order backlog	13.57	53.41	0.13	2.68	27.19	388,482	11.4%
Order book residual	0.00	3.65	-0.23	0.00	0.25	388,482	
	Electronics and Electrics						
Sales	3.57	13.66	0.10	1.04	6.54	260,719	15.2%
New orders	3.92	16.30	0.08	1.04	7.13	260,719	15.2%
Order backlog	22.05	100.93	0.18	3.51	35.93	260,719	12.4%
Order book residual	0.12	6.94	-0.20	0.00	0.33	260,719	
	Machinery						
Sales	3.51	10.84	0.10	1.07	7.29	401,846	23.1%
New orders	3.80	15.03	0.07	1.08	7.78	401,846	22.7%
Order backlog	33.29	222.76	0.26	5.70	56.77	401,846	28.9%
Order book residual	0.09	13.16	-0.27	0.00	0.40	401,846	
	Automotives						
Sales	24.77	153.44	0.25	1.96	18.20	61,068	24.8%
New orders	29.74	189.94	0.12	1.87	19.92	61,068	27.1%
Order backlog	125.72	752.15	0.29	7.81	124.31	61,068	16.6%
Order book residual	3.67	80.60	-0.80	0.00	1.07	61,068	
	Other Vehicles						
Sales	15.61	58.36	0.10	1.64	31.54	21,581	5.5%
New orders	16.76	94.86	0.04	1.26	26.40	21,581	5.4%
Order backlog	554.19	3062.41	0.35	14.87	850.04	21,581	25.9%
Order book residual	0.20	85.68	-0.33	0.00	0.64	21,581	

Note: The table provides descriptive statistics at the establishment^{4d}-month level from 2015 through 2022 across backlog sub-sectors. All variables are deflated by the PPI normalized to the year 2019 and expressed in EUR million.

A.3 Monthly Production Census

To construct establishment^{4d}-level sales price indices, we merge our main data, the *Monthly Report of Manufacturing Establishments*, with the *Monthly Production Census* of the German manufacturing sector, which records the sales value of production V_{ijt} and the quantity of production Q_{ijt} for (nine-digit) product j produced by establishment^{4d} i in month t.[34] Yearly aggregates of this data set have been used in prior research (e.g., Mertens, 2022; Bighelli et al., 2022). The census adheres to the PRODCOM standard of the European Union and is comparable to similar data sets in other EU countries (e.g., those used in Amiti et al., 2019; Gagliardone et al., 2023). It collects monthly data for all nine-digit products produced by establishments with at least 50 employees.[35] Given the same size threshold, it is not surprising that the census and reports almost perfectly overlap. The nine-digit product codes in the census are quite detailed. For example, product code `289317703` includes machines and devices for the beverage industry (excluding presses, mills, etc.) and `245310100` includes light metal foundry products used in the production of road vehicles. The quantity measure of production is time-consistent, e.g., pieces for `289317703` and tons for `245310100`.
We are interested in the product-level unit prices, which we compute as

$$P^S_{ijt} = \frac{V_{ijt}}{Q_{ijt}}. \tag{A.5}$$

The quantity measure is missing from the census for 24.9% of products meaning we do not observe the unit price. The raw data records in 1.3% of observations a nominal production value of 0 € or 1 € and in 1.2% of observations a production quantity of zero. Because these entries likely describe missing information, we set the respective entries to missing and linearly interpolate over the resulting gaps if they span only a single month. In a few cases, the unit price exhibits irregular spikes in a single month. We define these as months t in which the unit price 1) grows by a factor of more than two or shrinks by a factor of less than $1/2$ between $t-1$ and t and between t and $t+1$, 2) the product of the growth rate between $t-1$ and t and the growth rate between t and $t+1$ is negative, and 3) the two-month growth factor between $t-1$ and $t+1$ is less than half the average one-month growth factors in t and

[34] The official name of the dataset is *Monatliche Produktionserhebung*.

[35] The underlying product classification is the so-called GP (*Güterverzeichnis für Produktionsstatistiken*). Within our sample, the classification changes from "GP 2009" to "GP 2019." We use the official concordance tables to match 2009 codes to the 2019 classification. For the few unmatched 2009 and 2019 codes, we use our establishment^{4d}-level data to identify additional matches. For example, if an establishment^{4d} with a single unmatched 2009/2019 product, produces a similar value of that product before and after the classification changed, we assume the 2009 product code equals the 2019 product code. Whenever this procedure does not yield a clear match, we treat the unmatched codes as new products being introduced in 2019. This matching procedure is similar to other papers using PRODCOM micro data (Amiti et al., 2019; Mertens, 2022).

$t+1$. The latter criterion is to make sure that the spike is relatively more important than the two-month trend in the respective variable. These spikes occur in 0.9% of observations and we smooth them out via linear interpolation.

Finally, we aggregate the product-level unit prices P_{ijt}^S in the census to the establishment^{4d}-level and merge it with our main dataset, the manufacturing reports. We aggregate all nine-digit products attributed to an establishment^{4d} to four-digit product codes. Four-digit product codes are equivalent to the four-digit sector codes which define establishments^{4d}. The aggregation involves three steps:

1) Let F_{it} denote the set of products j produced by establishment^{4d} i in month t for which the unit price P_{ijt} is non-missing. We compute the establishment^{4d}-level sales value of production as

$$V_{it} = \sum_{j \in F_{it}} V_{ijt}. \tag{A.6}$$

2) We construct an establishment^{4d}-level sales price growth factors using Törnqvist aggregation (as in Eslava et al., 2004). Formally, we compute price growth factors by

$$\frac{P_{it}^S}{P_{it-1}^S} = \prod_{j \in F_{it}} \left(\frac{P_{ijt}}{P_{ijt-1}} \right)^{0.5 s_{ijt} + 0.5 s_{ijt-1}}, \quad s_{ijt} = \frac{V_{ijt}}{V_{it}}. \tag{A.7}$$

We set product-level price growth factors P_{ijt}/P_{ijt-1} as missing if the log growth factor is larger than one in absolute value, as in Gagliardone et al. (2023).

3) We create a price index from the growth rates by normalizing $P_{i0}^S = 1$ with $t = 0$ the first month in our data. If P_{it}^S/P_{it-1}^S is missing, we impute it with the value-weighted average of observed period t price growth rates across establishments^{4d} in the same four-digit industry (again following Eslava et al., 2004). Having constructed the price index series, we set individual observations P_{it}^S to missing if P_{it}^S/P_{it-1}^S was imputed.

Appendix B Deflation

In this appendix, we provide additional details on the deflation procedure and additional descriptive evidence for the real sample.

B.1 Sales weights for price of new orders

Section 3 defines the price of new orders as sales-weighted average of (future) sales prices

$$p_{it}^{O} = \sum_{j=\underline{\tau}_{it}}^{\overline{\tau}_{it}} w_{it+j} p_{it+j}^{S}, \quad w_{it+j} = \frac{\tilde{S}_{it+j}}{\sum_{k=\underline{\tau}_{it}}^{\overline{\tau}_{it}} \tilde{S}_{it+k}}.$$

where $\tilde{S}_{it+k}$ takes into account the case that only a fraction of sales in $t+\underline{\tau}_{it}$ and $t+\overline{\tau}_{it}$ are attributable to the period t order. Formally, we define $\tilde{S}_{it+k}$ as follows:

$$\begin{aligned}
\tilde{S}_{it+k} &= S_{it+k}, \quad \text{if } \underline{\tau}_{it} < k < \overline{\tau}_{it} \\
\tilde{S}_{it+\underline{\tau}_{it}} &= \underline{\zeta}_{it+\underline{\tau}_{it}} \cdot S_{it+\underline{\tau}_{it}} \\
\tilde{S}_{it+\overline{\tau}_{it}} &= \overline{\zeta}_{it+\overline{\tau}_{it}} \cdot S_{it+\overline{\tau}_{it}}
\end{aligned}$$

We define $\underline{\zeta}_{it+\underline{\tau}_{it}}$ as the share of net sales in period $t+\underline{\tau}_{it}$ that we attribute to period t orders:

$$\underline{\zeta}_{it+\underline{\tau}_{it}}: \quad B_{it-1} = \sum_{k=0}^{\underline{\tau}_{it}-1} (S_{it+k} + R_{it+k}) + (1 - \underline{\zeta}_{it+\underline{\tau}_{it}})(S_{it+\underline{\tau}_{it}} + R_{it+\underline{\tau}_{it}})$$

The first term is zero if the establishment[4d] starts depleting O_{it} in period t (i.e., $\underline{\tau}_{it} = 0$) and otherwise captures the cumulative net sales since period t until the beginning of period $t+\underline{\tau}_{it}$. The second term captures the remaining net sales in period $t+\underline{\tau}_{it}$ that is required to deplete B_{it-1}. The fraction $(1 - \underline{\zeta}_{it+\underline{\tau}_{it}})$ of net sales in $t+\underline{\tau}_{it}$ is implicitly defined by the equation. The remaining share $\underline{\zeta}_{t+\underline{\tau}_{it}}$ can be attributed to O_{it}.[36] Similarly, $\overline{\zeta}_{it+\overline{\tau}_{it}}$ is defined as the share of net output in period $t+\overline{\tau}_{it}$ that can be attributed to period t orders. Formally:

$$\overline{\zeta}_{it+\overline{\tau}_{it}}: \quad B_{it-1} + O_{it} = \sum_{k=0}^{\overline{\tau}_{it}-1} (S_{it+k} + R_{it+k}) + \overline{\zeta}_{it+\overline{\tau}_{it}}(S_{it+\overline{\tau}_{it}} + R_{it+\overline{\tau}_{it}})$$

The rationale for adjusting sales by the shares ζ is as follows: The closer ζ is to zero, the less informative is the sales price for the price of new orders.

[36] An alternative way to think about this definition is the following: In continuous time, $t+\underline{\tau}_{it}+\zeta_{it+\underline{\tau}_{it}}$ would be the exact instant of time in which the firm starts delivering period t orders.

B.2 Initial real backlog

Section 3 describes how real backlog can be computed given an initial real backlog. We next discuss how we can uniquely determine the initial real backlog by exploiting that the real and the nominal stock-flow equations have to be satisfied simultaneously. Let t_{i0} be the first period in which an establishment[4d] is observed, then $b_{it_{i0}-1}$ denotes the initial real backlog and $B_{it_{i0}-1}$ the initial nominal backlog.
We first define the time span to fully deplete the initial backlog by

$$\tau_{i0} = \min\left\{\tau \in \mathbb{N}^0 \,\middle|\, \sum_{k=0}^{\tau}(S_{it_{i0}+k} + R_{it_{i0}+k}) \geq B_{it_{i0}-1}\right\}. \tag{B.1}$$

Some net sales in period $t_{i0} + \tau_{i0}$ may fill orders not contained in $B_{it_{i0}-1}$. Let $\zeta_{it_{i0}+\tau_{i0}}$ be the share of net sales $(S_{t_{i0}+\tau_{i0}} + R_{t_{i0}+\tau_{i0}})$ that can be attributed to the initial backlog, and $1 - \zeta_{it_{i0}+\tau_{i0}}$ pertains to new orders subsequent to t_{i0}. Formally, $\zeta_{it_{i0}+\tau_{i0}}$ is defined by:

$$\zeta_{it_{i0}+\tau_{i0}}: \quad B_{it_{i0}-1} = \sum_{k=0}^{\tau_{i0}-1}(S_{it_{t0}+k} + R_{it_{i0}+k}) + \zeta_{it_{i0}+\tau_{i0}}(S_{it_{i0}+\tau_{i0}} + R_{it_{i0}+\tau_{i0}}) \tag{B.2}$$

Enforcing the same condition in real terms uniquely pins down the initial real backlog:

$$b_{it_{i0}-1} = \sum_{k=0}^{\tau_{i0}-1}(s_{it_{t0}+k} + r_{it_{i0}+k}) + \zeta_{it_{i0}+\tau_{i0}}(s_{t_{i0}+\tau_{i0}} + r_{t_{i0}+\tau_{i0}}) \tag{B.3}$$

B.3 Alternative backlog price index (alternative step 3)

We propose an alternative procedure to construct the order backlog price index. The procedure is equivalent to step 3 under our assumptions but may differ if our assumptions do not hold in the data. In that sense, the alternative step 3 allows us to assess the sensitivity of our results with respect to the assumptions that our deflation scheme rests on.
The alternative backlog price index departs from equation (B.2) above, which we rewrite as

$$p^B_{it} b_{it-1} = \sum_{k=0}^{\tau_{it}-1} p^S_{it}(s_{it+k} + r_{it+k}) + \zeta_{it+\tau_{it}} p^S_{it}(s_{it+\tau_{it}} + r_{it+\tau_{it}}) \tag{B.4}$$

where we used $p^S_{it} = p^R_{it}$ from step 2 in Section 3. Since the nominal and the real stock-flow equation have to hold simultaneously, (B.4) also has to hold in real terms:

$$b_{it-1} = \sum_{k=0}^{\tau_{it}-1}(s_{it+k} + r_{it+k}) + \zeta_{it+\tau_{it}}(s_{it+\tau_{it}} + r_{it+\tau_{it}}) \tag{B.5}$$

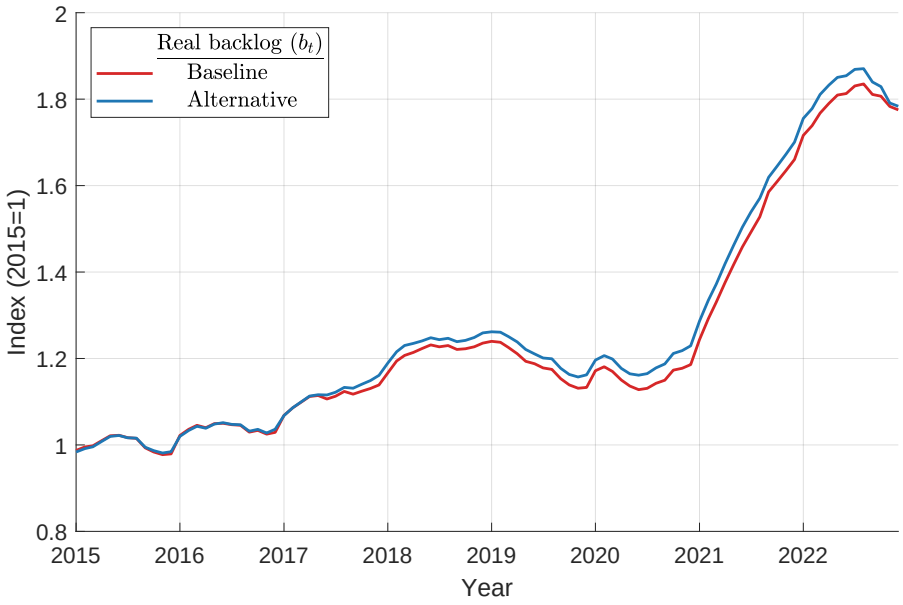
Dividing (B.4) by b_{it-1} and using (B.5) yields the order backlog price

$$p_{it}^B = \sum_{k=0}^{\tau_{it}-1} p_{it}^S \frac{(s_{it+k} + r_{it+k})}{\sum_{k=0}^{\tau_{it}-1}(s_{it+k} + r_{it+k}) + \zeta_{it+\tau_{it}}(s_{it+\tau_{it}} + r_{it+\tau_{it}})} + p_{it}^S \frac{\zeta_{it+\tau_{it}}(s_{it+\tau_{it}} + r_{it+\tau_{it}})}{\sum_{k=0}^{\tau_{it}-1}(s_{it+k} + r_{it+k}) + \zeta_{it+\tau_{it}}(s_{it+\tau_{it}} + r_{it+\tau_{it}})}. \tag{B.6}$$

In other words, p_{it}^B is a weighted average of different sales prices.

To compare our alternative backlog price index with the baseline, we compute the real order backlogs $b_{it} = B_{it}/p_{it}^B$. If all our assumptions were met, the alternative real backlog series should satisfy the real stock-flow equation (3.1). In practice, however, this is not guaranteed. Quantitatively, the deviations from the real stock-flow equation are larger than 1% of backlog in absolute terms for 12% of the observations. In one quarter of these cases, our baseline method would have resulted in a negative value for real backlog before adjustment. At the micro-level, the two series align more closely, the smaller the absolute residual of the stock-flow equation is. This aligns with our discussion on potential measurement issues in Section 3. Figure B.1 compares the aggregate real order backlog series across the two approaches. We find that the alternative real backlog series are qualitatively and quantitatively similar.

Figure B.1: Comparison of real backlog series

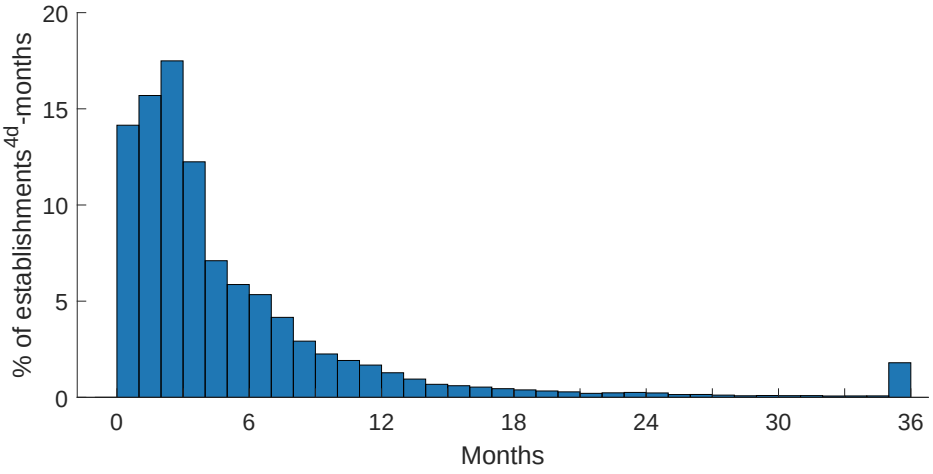


Note: This figure shows two aggregate backlog series from 2015 to 2022. The series are normalized to one in 2015. The series are obtained by Törnqvist aggregation with (nominal) backlog weights similar to the procedure outlined in Appendix A.3. The micro-level growth rates are trimmed at 1% and 99% before aggregation. The red line corresponds to the baseline real backlog as in Figure 2. The blue line corresponds to the real backlog series obtained with the alternative step 3.

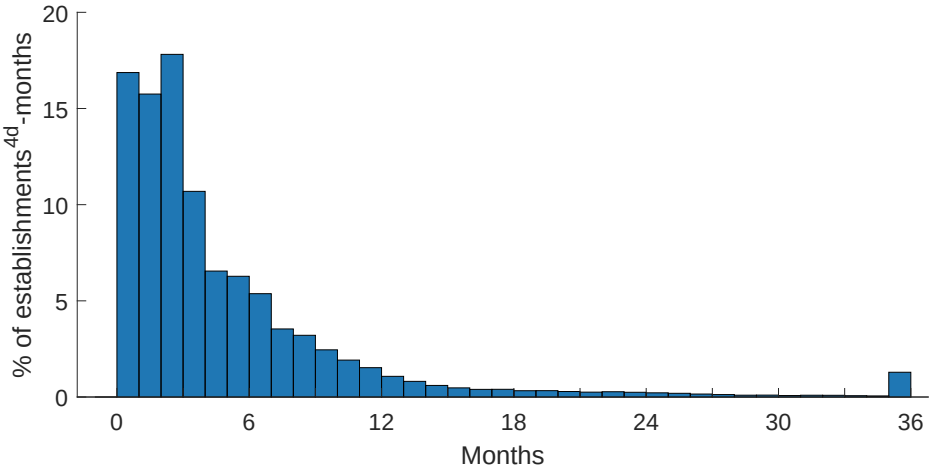
Appendix C Time to fill

Figure C.1: Sales-weighted distribution of ex-ante and ex-post to fill

(a) Ex-ante to fill



(b) Ex-post to fill



Note: The figure shows the sales-weighted histogram of the ex-ante time to fill (backlog-sales ratio) defined in equation (4.2) and the ex-post time to fill defined in equation (4.1). The histograms are winsorized at 36 months. Both panels include all establishment^{4d}-time observations between 2015 and 2022 for which real order backlog data are available.

Table C.1: Ex-post time to fill across industries

	(a) Non-weighted					
			Cumulative Distribution Function			
	Mean	Std. Dev.	3 months	6 months	12 months	24 months
All	5.0	6.1	50.9%	73.6%	92.1%	98.4%
Textiles, Cloths, Paper	2.4	3.0	77.1%	93.4%	98.4%	99.7%
Chemical, Pharma	2.4	3.5	78.2%	92.2%	97.9%	99.6%
Primary and Fabricated Metals	4.5	5.3	52.8%	75.8%	93.7%	99.1%
Electronics, Electrics	4.8	5.7	52.3%	75.7%	92.8%	98.5%
Machinery	6.2	6.5	38.6%	63.9%	88.6%	97.9%
Automobiles	5.4	5.8	43.1%	66.7%	92.4%	99.0%
Other Vehicles	12.7	14.6	28.5%	46.5%	66.4%	83.2%
	(b) Sales-weighted					
			Cumulative Distribution Function			
	Mean	Std. Dev.	3 months	6 months	12 months	24 months
All	6.6	10.0	46.9%	70.0%	88.9%	95.1%
Textiles, Cloths, Paper	2.4	2.7	75.7%	93.2%	98.8%	99.9%
Chemical, Pharma	2.4	2.9	79.9%	93.6%	98.4%	99.7%
Primary and Fabricated Metals	4.6	5.3	49.0%	76.1%	93.6%	99.1%
Electronics, Electrics	5.4	5.8	47.7%	69.8%	91.2%	97.9%
Machinery	8.5	10.0	29.7%	55.2%	81.6%	93.2%
Automobiles	4.2	3.3	50.3%	77.7%	98.6%	99.8%
Other Vehicles	31.0	22.4	14.6%	19.2%	28.0%	48.1%

Note: This table provides selected values of the cumulative distribution functions for the ex-ante time to fill by broad industry groups. The ex-ante time to fill is defined in equation (4.2). The sample spans 2015-2022 and includes all establishment^{4d}-month observations with real order book data. Textiles, Cloths, Paper: WZ 13, 14, 17 Chemical, Pharma: WZ 20, 21 Primary and Fabricated Metals: WZ 24, 25 Electronics, Electrics: WZ 26, 27 Machinery: WZ 28 Automobiles: WZ 29 Other Vehicles: WZ 30

Appendix D Theory

In this section of the appendix, we provide the proofs all propositions in Section 5.

D.1 Proof of Proposition 1

Non-linear FOCs. We first rewrite the problem of the firm in (5.1) - (5.3) by substituting the constraints in the objective function such that only the two choice variables b_t and p_t remain.

$$\max_{\{p_t,b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t^d p_t^{1-\sigma} - C\left(b_{t-1} + x_t^d p_t^{-\sigma} - b_t\right)(x_t^s)^{-1} - BP\left(\frac{b_{t-1} + x_t^d p_t^{-\sigma} - b_t}{b_{t-1}}\right)\right]$$

The first order condition for p_t can be express as

$$0 = (1-\sigma)p_t + C'\left(b_{t-1} + x_t^d p_t^{-\sigma} - b_t\right)\sigma(x_t^s)^{-1} + BP'\left(\frac{b_{t-1} + x_t^d p_t^{-\sigma} - b_t}{b_{t-1}}\right)\frac{\sigma}{b_{t-1}} \quad \text{(D.1)}$$

The first order condition for b_t can be express as

$$\begin{aligned} 0 =& C'\left(b_{t-1} + x_t^d p_t^{-\sigma} - b_t\right)(x_t^s)^{-1} + BP'\left(\frac{b_{t-1} + x_t^d p_t^{-\sigma} - b_t}{b_{t-1}}\right)\frac{1}{b_{t-1}} \\ &+ \beta E_t\left[-C'\left(b_t + x_{t+1}^d p_{t+1}^{-\sigma} - b_{t+1}\right)(x_{t+1}^s)^{-1} - BP'\left(\frac{b_t + x_{t+1}^d p_{t+1}^{-\sigma} - b_{t+1}}{b_t}\right)\frac{-x_{t+1}^d p_{t+1}^{-\sigma} + b_{t+1}}{b_t^2}\right] \end{aligned} \quad \text{(D.2)}$$

Steady state characterization. In the steady state, $b_t = b_{t-1} = b$ and $o = q$ has to hold. As $\beta \to 1$, it can be shown that the FOC for b_t, evaluated at that steady state, is given by

$$BP'\left(\frac{q}{b}\right) = 0$$

By the assumptions placed on the function $BP(\cdot)$, it must be that $\frac{q}{b} = m$, i.e. $\frac{q}{b}$ is the unique minimizer of the backlog penalty function. Using that result, the FOC for p_t, evaluated at the steady state, results in the standard CES markup equation:

$$p = \frac{\sigma}{\sigma - 1} C'(q)$$

Log-linearization. Define

$$a_1 = C''(q)(x^s)^{-1}b\frac{\sigma}{(\sigma-1)pa_3},\ a_2 = BP''(q/b)\frac{1}{b}\frac{\sigma}{(\sigma-1)pa_3},\ a_3 = \frac{b}{q} = \frac{1}{m}$$

Note that $a_1, a_3 > 0$; $a_2 \geq 0$ under our assumptions. We normalize the variance of $\hat{x}_t^s$ by $C'(q) \cdot (x^s)^{-1} \cdot \frac{\sigma}{(\sigma-1)a_3 p}$ (since the log-linear model features certainty equivalence, this is WLOG). Define $\gamma_1 = a_1a_3 - a_2(1-a_3), \gamma_2 = a_1 + a_2$ and note that $\gamma_1 = a_3\gamma_2 - a_2$. Note that our assumption $a_3 = b/q = 1/m > 1$ implies that $\gamma_1 > 0$. For the remaining derivations, the weaker assumption $\gamma_1 > 0$ would also be sufficient. Using these definitions, the log-linear approximation of the FOC for p (equation D.1) can be expressed as

$$\begin{aligned} & 0 = \hat{b}_{t-1}\gamma_1 - \hat{b}_t a_3\gamma_2 - \hat{p}_t(1+\sigma\gamma_2) + \hat{x}_t^d\gamma_2 - \hat{x}_t^s \\ \iff\quad & \hat{p}_t = \hat{b}_{t-1}\frac{\gamma_1}{1+\sigma\gamma_2} + \hat{b}_t\frac{-a_3\gamma_2}{1+\sigma\gamma_2} + \hat{x}_t^d\frac{\gamma_2}{1+\sigma\gamma_2} + \hat{x}_t^s\frac{-1}{1+\sigma\gamma_2} \end{aligned} \tag{D.3}$$

Similarly, we can log-linearize the FOC for b_t (equation D.2) to obtain:

$$\begin{aligned} & \hat{b}_{t-1}\gamma_1 a_3 + \hat{b}_t\left(-a_3^2\gamma_2 - \beta a_1 a_3^2 - \beta a_2(1-a_3)^2\right) + E_t\hat{b}_{t+1}\beta\gamma_1 a_3 + \hat{p}_t(-\sigma\gamma_2 a_3) \\ & E_t\hat{p}_{t+1}\beta\sigma\gamma_1 + \hat{x}_t^d(a_3\gamma_2 - \beta\gamma_1\rho_d) - \hat{x}_t^s a_3(1-\beta\rho_s) = 0 \end{aligned} \tag{D.4}$$

where we used $E_t\hat{x}_{t+1}^d = \rho_d\hat{x}_t^d$ and $E_t\hat{x}_{t+1}^s = \rho_s\hat{x}_t^s$. Combining equations (D.3) and (D.4), we obtain

$$0 = \tau_1\hat{b}_{t-1} + \tau_2\hat{b}_t + \tau_3 E_t\hat{b}_{t+1} + \tau_4 x_t^d + \tau_5 x_t^s, \tag{D.5}$$

with

$$\begin{aligned} \tau_1 &= \frac{a_3\gamma_1}{1+\sigma\gamma_2}, \quad \tau_2 = -\frac{a_1a_3^2(1+\beta) + \beta a_1 a_2\sigma + a_2(a_3^2 + \beta(1-a_3)^2)}{1+\sigma\gamma_2} \\ \tau_3 &= \frac{a_3\beta\gamma_1}{1+\sigma\gamma_2}, \quad \tau_4 = \frac{a_3\gamma_2 - \beta\rho_d\gamma_1}{1+\sigma\gamma_2}, \quad \tau_5 = \frac{-a_3(1-\beta\rho_s) + \beta\rho_s\sigma(a_3\gamma_2 - \gamma_1)}{1+\sigma\gamma_2} \end{aligned}$$

Functional form. Using equation (D.5), it is straightforward to verify that the policy function satisfying (D.5) must have the following functional form

$$\hat{b}_t = c_1\hat{b}_{t-1} + c_2 x_t^d + c_3 x_t^s.$$

We use the method of undetermined coefficients to pin down the coefficients. Since $a_1a_3 \neq$

$a_2(1-a_3)$, and thus $\gamma_1, \tau_3 \neq 0$, the solution for the coefficients is

$$c_1^{\pm} = -\frac{\tau_2}{2\tau_3} \pm \sqrt{\left(\frac{\tau_2}{2\tau_3}\right)^2 - \frac{\tau_1}{\tau_3}}, \tag{D.6}$$

$$c_2 = -\frac{\tau_4}{\tau_2 + \tau_3 c_1 + \tau_3 \rho_d}, \tag{D.7}$$

$$c_3 = -\frac{\tau_5}{\tau_2 + \tau_3 c_1 + \tau_3 \rho_s}. \tag{D.8}$$

Properties of c_1. In the following, we prove the properties of c_1 stated in Proposition 1 by showing that c_1^- is the unique, stable, and real solution for c_1. To this end, note that

$$-\frac{\tau_2}{2\tau_3} = \frac{a_1 a_3^2(1+\beta) + \beta a_1 a_2 \sigma + a_2(a_3^2 + \beta(1-a_3)^2)}{2\beta a_3 \gamma_1}, \quad \frac{\tau_1}{\tau_3} = \frac{1}{\beta}.$$

Claim 1: If $a_1 a_3 > a_2(1-a_3)$, then (i) $c_1^{\pm}$ is real, (ii) $c_1^+ > 1$, and (iii) $c_1^- \in (0,1)$.
Proof: (i) Note that $\tau_3 \neq 0$ under the assumption in the claim. Hence, to show that $c_1^{\pm}$ is real, it is sufficient to show

$$\left(-\frac{\tau_2}{2\tau_3}\right)^2 > \frac{1}{\beta}.$$

Since $-\frac{\tau_2}{2\tau_3} > 0$, this requires $-\frac{\tau_2}{2\tau_3} > \beta^{-0.5}$ or equivalently

$$a_1 a_3^2(1-\beta^{0.5})^2 + \beta a_1 a_2 \sigma + a_2(a_3 + \beta^{0.5}(1-a_3))^2 > 0,$$

which holds true given the assumptions on the parameters specified in (**??**).

(ii) The result $c_1^+ > 1$ immediately follows from (i) since

$$-\frac{\tau_2}{2\tau_3} > \left(\frac{1}{\beta}\right)^{0.5} > 1.$$

(iii) Finally, we show that $c_1^- \in (0,1)$. To obtain $c_1^- < 1$ requires

$$\begin{aligned} & 2\left(\frac{-\tau_2}{2\tau_3}\right) > \frac{\beta+1}{\beta} \\ \iff \quad & \beta a_1 a_2 \sigma + a_2(\beta + (1-\beta)a_3) > 0 \end{aligned}$$

which holds true. To obtain $c_1^- > 0$ requires equivalently

$$\frac{1}{\beta} > 0,$$

which also holds true. □

D.2 Proof of Proposition 2

Functional form of the remaining policy rules. Note that the log-linearized versions of the demand equation and the stock flow equation are given by

$$\hat{o}_t = \hat{x}_t^d - \sigma \hat{p}_t \tag{D.9}$$

$$\hat{b}_t = \hat{b}_{t-1} + \frac{1}{a_3}\left(\hat{o}_t - \hat{q}_t\right) \tag{D.10}$$

Using the first order condition for prices (D.3), the log-linearized stock-flow equation (D.10), and the log-linearized demand curve (D.9) we can characterize the solution for the price, output, and orders as:

$$\hat{p}_t = \underbrace{\frac{\gamma_1 - a_3\gamma_2 c_1}{1+\sigma\gamma_2}}_{\equiv d_1}\hat{b}_{t-1} + \underbrace{\frac{\gamma_2(1-a_3c_2)}{1+\sigma\gamma_2}}_{\equiv d_2}\hat{x}_t^d + \underbrace{\frac{-1-a_3\gamma_2 c_3}{1+\sigma\gamma_2}}_{\equiv d_3}\hat{x}_t^s$$

$$\hat{q}_t = \underbrace{\left(a_3 - a_3c_1 - \sigma d_1\right)}_{\equiv e_1}\hat{b}_{t-1} + \underbrace{\left(1 - a_3c_2 - \sigma d_2\right)}_{\equiv e_2}\hat{x}_t^d + \underbrace{\left(-a_3c_3 - \sigma d_3\right)}_{\equiv e_3}\hat{x}_t^s$$

$$\hat{o}_t = \underbrace{-\sigma d_1}_{\equiv f_1}\hat{b}_{t-1} + \underbrace{\left(1 - \sigma d_2\right)}_{\equiv f_2}\hat{x}_t^d + \underbrace{\left(-\sigma d_3\right)}_{\equiv f_3}\hat{x}_t^s$$

Signs of the policy coefficients. We prove the properties of the policy coefficients stated in Proposition 2 by proving a sequence of claims.

Claim 2: It holds $\forall \rho \in [0,1)$ that $\tau_2 + \tau_3 c_1 + \tau_3 \rho < 0$.

Proof: Since $a_1 > a_2 a_3(1-a_3)$, $\tau_2 + \tau_3 c_1 + \tau_3\rho < 0$ is equivalent to

$$\rho < \frac{-\tau_2 + \sqrt{\tau_2^2 - 4\tau_1\tau_3}}{2\tau_3} = \frac{-\tau_2}{2\tau_3} + \sqrt{\left(\frac{\tau_2}{2\tau_3}\right)^2 - \frac{\tau_1}{\tau_3}}.$$

Note that the right-hand side equals c_1^+, and we have shown in Claim 1 that $c_1^+ > 1$ if $a_1 a_3 > a_2(1-a_3)$. □

Claim 3: It holds $\forall \rho_d \in [0,1)$ that $c_2 > 0$.
Proof: As shown in Claim 2, the denominator of c_2 is strictly negative. Hence, $c_2 > 0$ requires $\tau_4 > 0$. Rewriting τ_4 as $\tau_4 = \frac{(1-\beta\rho_d)a_3\gamma_2 + \beta\rho_d a_2}{1+\sigma\gamma_2}$ proves that $\tau_4 > 0$ since $\gamma_2 > 0$. □

Claim 4: If $0 \leq \rho_s < \frac{a_3}{\beta(a_3+a_2\sigma)} \equiv \bar{\rho}_s > 0$, it holds that $c_3 < 0$. If $\rho_s \geq \bar{\rho}_s$, it holds that $c_3 \geq 0$.
Proof: By Claim 3, the denominator of c_3 is negative. Hence, $c_3 < 0$ iff $\tau_5 < 0$. Rearranging τ_5 yields the threshold $\bar{\rho}_s$. □

Claim 5: It holds $\forall \rho_d \in [0,1)$ that $e_2 > 0$.
Proof: Using the definitions of e_2, d_2, it follows that $e_2 > 0$ holds iff $c_2 < \frac{1}{a_3}$. Note that we can rewrite c_2 as

$$\begin{aligned} c_2 &= \frac{a_2 + \gamma_1(1-\beta\rho_d)}{a_1 a_3^2(1+\beta) + \beta a_1 a_2 \sigma + a_2(a_3^2 + \beta(1-a_3)^2) - \beta\gamma_1 a_3(c_1+\rho_d)} \\ &= \frac{a_2 + \gamma_1(1-\beta\rho_d)}{a_3(a_2+\gamma_1(1-\beta\rho_d)) + \underbrace{\beta a_1 a_3^2 + \beta a_1 a_2\sigma + \beta a_2(1-a_3)^2 - \beta a_3\gamma_1 c_1}_{\equiv \Psi_1}}. \end{aligned}$$

Hence, we only need to show that the remaining terms in the denominator (Ψ_1) are strictly positive. To satisfy $\Psi_1 > 0$, a sufficient condition is

$$\Psi_2 \equiv a_1 a_3^2 + a_2(1-a_3)^2 - a_3\gamma_1 c_1 > 0.$$

Rewriting yields

$$c_1 < \bar{c}_1 = \frac{a_1 a_3^2 + a_2(1-a_3)^2}{a_3\gamma_1} = \frac{a_3\gamma_1 + a_2(1-a_3)}{a_3\gamma_1}, \quad \gamma_1 = a_1 a_3 - a_2(1-a_3) > 0 \tag{D.11}$$

Case 1: $a_3 \in (0,1]$
This implies $\bar{c}_1 \geq 1$ and because $c_1 \in (0,1)$, it holds that $\Psi_2 > 0$.
Case 2: $a_3 > 1$
This implies $\bar{c}_1 \in (0,1)$ and so we need to verify $c_1 < \bar{c}_1$ for this case. Define $x = -\frac{\tau_2}{2\tau_3}$, then equation (D.11) writes as

$$x - \bar{c}_1 < \sqrt{x^2 - \frac{1}{\beta}},$$

and because $x > 1$ and $\bar{c}_1 \in (0,1)$, both sides of the inequality are positive. Thus, we can

square both sides to obtain

$$x > \frac{1}{\beta}\frac{\beta\bar{c}_1^2+1}{2\bar{c}_1}.$$

Plugging in for x and $\bar{c}_1$ yields

$$\begin{aligned} & \frac{a_1a_3^2(1+\beta)+\beta a_1a_2\sigma+a_2(a_3^2+\beta(1-a_3)^2)}{2\beta a_3\gamma_1} > \frac{\beta[a_1a_3^2+a_2(1-a_3)^2]^2+a_3^2\gamma_1^2}{2\beta a_3\gamma_1[a_1a_3^2+a_2(1-a_3)^2]}, \\ \iff \quad & \Psi_3 \equiv a_1a_3^2+\beta a_1a_2\sigma+a_2a_3^2-\frac{a_3^2\gamma_1^2}{a_1a_3^2+a_2(1-a_3)^2} > 0, \end{aligned}$$

A sufficient condition for $\Psi_3 > 0$ is

$$\Psi_4 \equiv a_1+a_2-\frac{\gamma_1^2}{a_1a_3^2+a_2(1-a_3)^2} > 0 \Leftrightarrow a_1a_2 > 0,$$

which is satisfied. □

Claim 6: It holds $\forall\rho_s \in [0,1)$ that $e_3 > 0$.
Proof: Using the definitions of e_3, d_3 it follows that $e_3 > 0$ iff $c_3 < \frac{\sigma}{a_3}$, which can be rewritten as

$$\begin{aligned} & c_3 = \frac{-a_3(1-\beta\rho_s)+\beta\sigma\rho_s a_2}{a_1a_3^2(1+\beta)+\beta a_1a_2\sigma+a_2(a_3^2+\beta(1-a_3)^2)-\beta\gamma_1a_3(c_1+\rho_d)} < \frac{\sigma}{a_3} \\ \iff \quad & 0 < \underbrace{(1-\beta\rho_s)a_3^2(1+\sigma a_1+\sigma a_2)+\beta\sigma\left(a_1a_3^2+a_1a_2\sigma+a_2(1-a_3)^2-a_3\gamma_1c_1\right)}_{\equiv\Phi_1} \end{aligned}$$

To shows $\Phi_1 > 0$, it is sufficient to show that

$$\Phi_2 \equiv (1-\beta)a_3^2(\sigma a_1+\sigma a_2)+\beta\sigma\left(a_1a_3^2+a_2(1-a_3)^2-a_3\gamma_1c_1\right) > 0.$$

Using the definitions of γ_1, one can simplify the condition as

$$\gamma_1a_3(1-\beta c_1)+a_2((1-\beta)a_3+\beta(1-a_3)) > 0.$$

If $a_3 < 1$ the statement is always true (remember $c_1 < 1$). Suppose instead $a_3 > 1$. To show $\Phi_2 > 0$, it is sufficient to show that

$$\Phi_3 \equiv a_3\gamma_1(1-c_1)+a_2(1-a_3) > 0,$$

which can be rearranged as

$$c_1 < \frac{a_3\gamma_1 + a_2(1-a_3)}{a_3\gamma_1}.$$

This was statement was already proven in Claim 5. □

Claim 7: It holds $\forall \rho_d \in [0,1)$ that $d_2 > 0$.
Proof: $d_2 > 0$ holds iff $c_2 < \frac{1}{a_3}$, which was shown in Claim 5

Claim 8: It holds $\forall \rho_s \in [0,1)$ that $d_3 < 0$.
Proof: $d_3 < 0$ holds iff $c_3 > -\frac{1}{a_3\gamma_2}$. Note that

$$\begin{aligned} c_3 &= \frac{-a_3(1-\beta\rho_s) + \beta\sigma\rho_s a_2}{a_1 a_3^2(1+\beta) + \beta a_1 a_2 \sigma + a_2(a_3^2 + \beta(1-a_3)^2) - \beta\gamma_1 a_3(c_1+\rho_d)} \\ &\geq \frac{-a_3}{a_1 a_3^2(1+\beta) + \beta a_1 a_2\sigma + a_2(a_3^2+\beta(1-a_3)^2)} \end{aligned}$$

Hence, it suffices to show

$$\begin{aligned} & \frac{-a_3}{a_1 a_3^2(1+\beta) + \beta a_1 a_2\sigma + a_2(a_3^2+\beta(1-a_3)^2)} > -\frac{1}{a_3\gamma_2} \\ \iff \quad & \beta a_1 a_3^2 + \beta a_1 a_2 \sigma + \beta a_2(1-a_3)^2 > 0, \end{aligned}$$

which is satisfied. □

Claim 9: It holds $\forall \rho_d \in [0,1)$ that $f_2 > 0$.
Proof: $f_2 > 0$ holds iff $d_2 < \frac{1}{\sigma}$. Note that

$$d_2 = \frac{\gamma_2(1-a_3c_2)}{1+\sigma\gamma_2} < \frac{1}{\sigma} \Leftrightarrow -c_2 a_3 \gamma_2 \sigma < 1.$$

Since $c_2 > 0$ (see Claim 3), the above inequality is satisfied. □

Claim 10: It holds $\forall \rho_s \in [0,1)$ that $f_3 > 0$.
Proof: $f_3 > 0$ holds iff $d_3 < 0$, which we have shown in claim 8. □

Appendix E Firm-level shocks

E.1 Identification

Table E.1: TSLS estimates of the demand curve

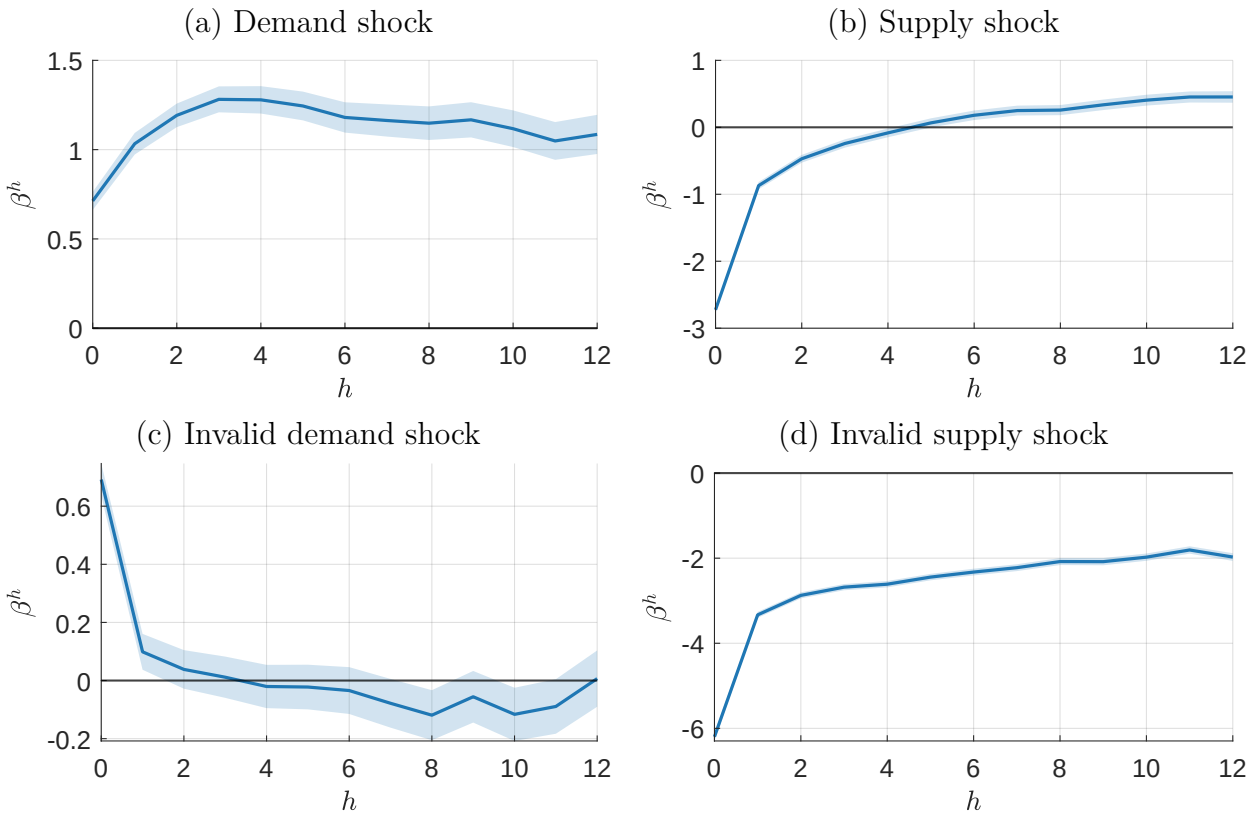
Sector	Aggregate	13	14	17	20	21	24
$\log(p_{it}^O)$	-0.9561	-0.0980	2.4111	-0.3929	-0.0008	-0.4944	-0.1818
	(0.1120)	(0.2485)	(1.9524)	(0.0584)	(0.1193)	(0.2457)	(0.1114)
R^2	0.3294	0.2282	-0.2914	0.2842	0.2728	0.2890	0.1903
Kleibergen-Paap F	194.15	153.92	4.26	388.98	260.45	22.95	191.51
N	826,391	23,499	5,854	40,368	61,874	1,682	68,931

Sector	25	26	27	28	29	30
$\log(p_{it}^O)$	-0.4761	-7.9110	-1.0851	-0.1174	-0.2307	-0.0578
	(0.1747)	(47.1985)	(0.5504)	(0.2824)	(0.3444)	(0.5897)
R^2	0.2710	-11.9235	0.3383	0.3417	0.2352	0.3876
Kleibergen-Paap F	166.50	0.03	15.40	30.45	34.39	13.52
N	233,488	78,620	103,462	250,143	27,754	2,852

Note: This table shows the estimated slopes of the demand curves for the aggregate of backlog sectors as well as for all (2-digit) sectors separately. All regressions contain an establishment^{4d} fixed effect and control for $\log(o_{it-1})$. The aggregate regression contains a time fixed effect interacted with the sum of shares $\sum_j \phi_{ij}$. The sectoral regressions contain a time fixed effect. The standard errors reported in parentheses are clustered at the (4-digit) industry-time level.

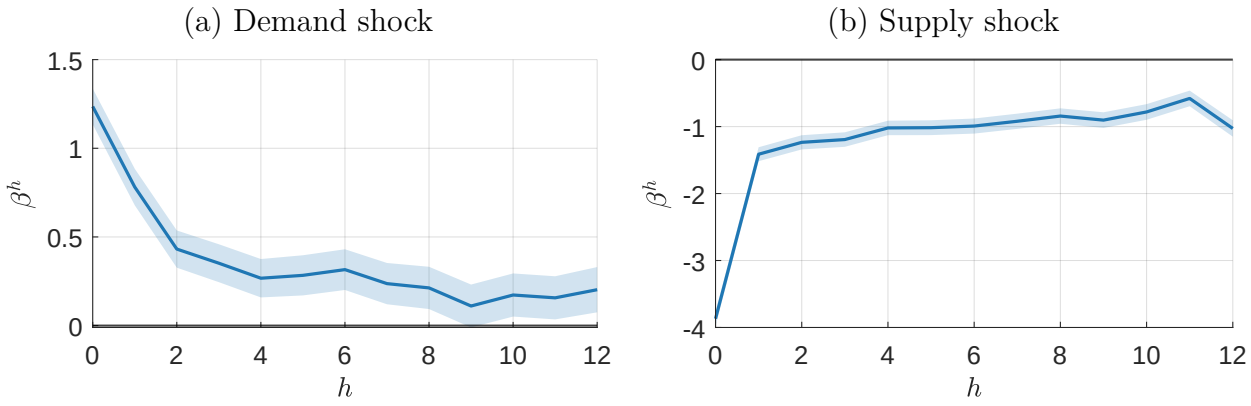
E.2 Additional evidence

Figure E.1: Sales price responses



Note: This figure shows the IRFs to expansionary demand and supply shocks of one standard deviation expressed in percent. The 95% confidence bands are computed using robust standard errors. The observations were trimmed at 1% and 99% for each horizon.

Figure E.2: Response of order book residual



Note: This figure shows the IRFs to an expansionary demand shock of one standard deviation expressed in percent of sales. The 95% confidence bands are computed using robust standard errors. The observations were trimmed at 1% and 99% for each horizon. As opposed to the other local projections, the left hand side of the regression is $(r_{it+1} - r_{it-1})/s_{it}^{12}$ to account for the fact that the residual can also be negative.