When Corporate Cash Talks: Dividends and State-Dependent Business Cycles*

Miguel H. Ferreira † Timo Haber ‡ Hanbaek Lee §

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Abstract

This paper studies the macroeconomic implications of corporate savings over the business cycle. We develop a heterogeneous-firm business cycle model in which firms accumulate cash for precautionary motives, influencing the transmission of aggregate shocks. When firms hoard cash, they absorb part of the aggregate risk, changing the sensitivity of macroeconomic variables to business cycle fluctuations. In particular, higher cash holdings allow firms to smooth dividends, dampening the response of aggregate consumption to adverse TFP shocks. Empirical evidence supports the role of the dividend channel in shaping state-dependent consumption dynamics.

*All errors are our own.

†Queen Mary University of London. migueltchferreira@gmail.com

‡De Nederlandsche Bank. habertimo@gmail.com. Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank or the Eurosystem.

§University of Cambridge. hanbaeklee1@gmail.com

1 Introduction

Corporate cash holdings have been rising steadily across multiple developed economies (Chen et al., 2017). Figure 1 presents the time series of the aggregate corporate cash holdings-to-GDP ratio in the US, where liquid assets are sourced from the Flow of Funds data and nominal GDP from NIPA.[1] It is evident that corporate cash holdings have grown significantly faster than output over time. While the corporate finance literature has explored various factors contributing to this trend, its macroeconomic implications remain relatively understudied. Despite a growing empirical and theoretical understanding of rising corporate cash balances, limited research has examined their role in shaping business cycle dynamics. This paper addresses this gap by developing a heterogeneous firm business cycle model with financial frictions, where firms hoard cash for precautionary motives.

In the model, firms make decisions each period regarding labor hiring, dividend payouts, and cash accumulation. A key mechanism driving cash holdings is the presence of a convex cost of external financing (*negative dividends*), which creates a precautionary motive: firms hold cash to smooth dividend payments and avoid costly equity issuance. Additionally, we assume that cash is not tradable across firms, further reinforcing the role of internal liquidity buffers.

While firms' savings behavior partially mirrors that of households, a fundamental difference arises due to the structure of their objective function. Unlike the household optimization problem, where marginal utility is strictly concave, the firm's contemporaneous objective is only strictly concave over a limited domain and remains weakly concave globally. This feature arises because internal financing (*positive dividends*) does not trigger adjustment costs, leading to a satiation point in the firm's cash hoarding decision. As a result, the intertemporal savings policy exhibits a flat region, making firm-level cash accumulation strongly nonlinear. This nonlinearity is a defining characteristic that distin-

[1] The definition of aggregate corporate cash stock is provided in Appendix A.1.

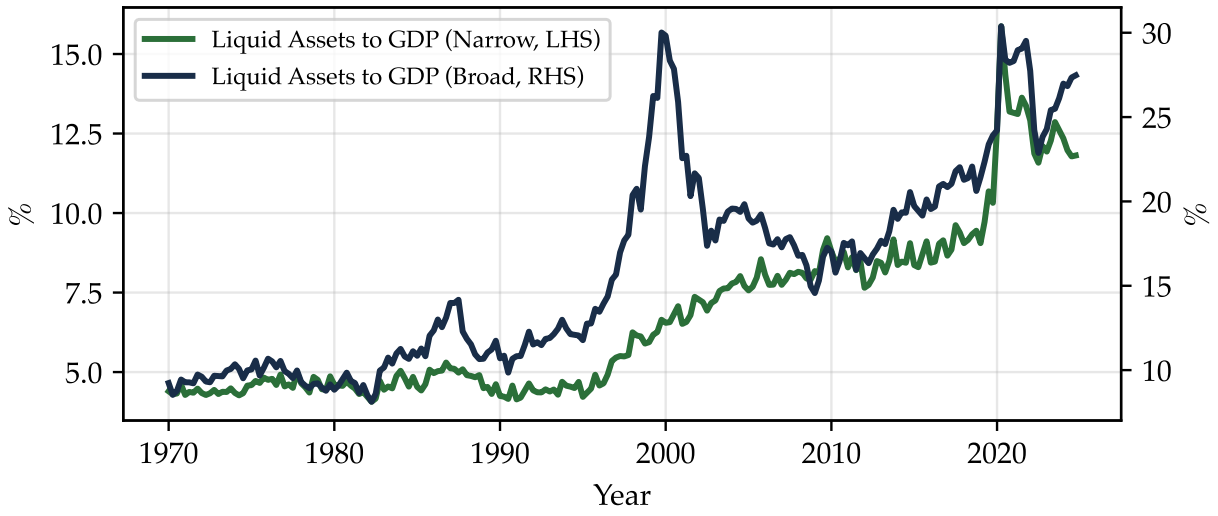


Figure 1: US nonfinancial corporate business liquid asset holdings to GDP

Note. The figure plots the time series of the liquid assets to GDP ratio. The liquid asset stock data is sourced from the Flow of Funds of the Federal Reserve Board, and the nominal GDP is from National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA). The definition of the broad and narrow measure can be found in Appendix A.1

guishes our model from standard heterogeneous household frameworks.

Based on this insight, we explore whether micro level nonlinearity survives aggregation and affects business cycle outcomes. In addition to the flat part in the cash holding policy function, one other factor is key to generate aggregate nonlinearity in cash holdings: cash is not tradable across firms, which prevents the price of cash from fluctuating over the business cycle. Due to the missing general equilibrium effect on the cash stocks, the firm-level nonlinearity is not washed out after aggregation, leading to a macro-level nonlinearity in the TFP-driven aggregate fluctuations.

Following a negative TFP shock, the responsiveness of firm-level dividends depends on the level of cash holdings. For example, when a firm is short of cash, a negative TFP shock causes a firm to reduce dividends further than it would do when it has abundant cash stocks. This is due to a firm with little cash needing not only to pay out dividends but save cash out of concern for the future. Due to this precautionary channel, household consumption responds more to negative TFP shocks when the levels of cash holdings are

low.

Using aggregate consumption, TFP and corporate cash holdings data, we show that this relation is present in the data: consumption is less responsive to negative TFP shocks in periods when corporate cash holdings are high. We find this relationship has strengthened since the 1980s, coinciding with the rapid rise in corporate cash holdings. This is in line with a model calibrated to the two different periods, which predicts the correlation between cash holdings and consumption responsiveness to TFP to be stronger when cash holdings are higher.

Additionally, we test whether the relationship between consumption responsiveness to TFP shocks and corporate cash holdings arises from the dividend channel. For this, we project consumption on aggregate dividends and then test the responsiveness of the part of consumption that co-varies with dividends. We find the dividend channel to display a significant role in the consumption state dependence.

Related Literature Our paper relates to three different branches of the literature.

First, to the literature studying risk sharing over the business cycle. Recent research has highlighted the role of risk sharing in financial accelerator models and its implications for macroeconomic fluctuations. Bernanke et al. (1999) introduced the financial accelerator mechanism, where credit market frictions amplify economic shocks due to entrepreneurs bearing all aggregate risk. However, Carlstrom et al. (2016) challenge this framework by showing that the assumed fixed lender return is not privately optimal. Instead, they derive an optimal contract that indexes debt repayments to aggregate conditions, significantly dampening fluctuations in leverage and the risk premium, thereby weakening the financial accelerator effect. Building on this, Jeenas (2024) explores how uninsurable idiosyncratic labor income risk influences risk-sharing dynamics, finding that when labor income risk is countercyclical, it further concentrates aggregate risk on entrepreneurs' balance sheets, amplifying business cycle fluctuations.

Furthermore, several papers such as Gertler and Kiyotaki (2010), Ottonello and Winberry (2020), Begenau and Salomao (2018), and Cloyne et al. (2023), Ferreira et al. (2023) explored how financial frictions amplify firm-level risk, and this increase in risk can spill over to aggregate consumption in response to negative shocks. These insights underscore that the concentration of risk in leverage firms/entrepreneurs is essential to the propagation of shocks and the response of aggregate variables.

We contribute to this literature by illustrating how firms cash hoarding, driven by precautionary motives, offload part of the consumption risk into equity prices and reduce consumption volatility over the business cycle.

Second, we contribute to the extensive literature that studies corporate cash hoarding as a function of firm-specific risk, financial constraints, and macroeconomic uncertainty. Opler et al. (1999) established that firms accumulate cash as a hedge against adverse financial conditions, according to the results found by Acharya et al. (2007), who emphasize the role of cash in reducing firms' reliance on costly external funding. Bates et al. (2009) highlighted the growing trend of cash holdings among US firms due to increased volatility in cash flows. Riddick and Whited (2009) argued that firms may accumulate cash even in the absence of financing frictions, as cash holdings optimize investment decisions over time. Chen et al. (2017) further demonstrated that the increase in profits together with dividend stickiness explains the increase in corporate savings. We contribute to this literature by illustrating the importance of financial frictions over the business cycle to explain firms' cash holdings, out of precautionary motives.

Third, we relate to the extensive literature on business cycle heterogeneous agent models. While heterogeneous agent models have been widely used to study macroeconomic fluctuations, early work by Aiyagari (1994) and Krusell and Smith (1998) showed that these models exhibit behavior that is largely linear at the aggregate level. Specifically, despite the presence of idiosyncratic shocks, the household savings policy function remains smooth and linear, leading to approximately linear aggregate dynamics. On the

firm side, Khan and Thomas (2008) demonstrated that investment policy functions are highly nonlinear due to fixed adjustment costs and capital irreversibility. However, their findings suggest that in a general equilibrium setting, the elasticity of the interest rate offsets firm-level nonlinearities, causing any potential nonlinearities at the micro-level to wash out in the aggregate. We contribute to this literature by showing that firms cash hoarding phenomenon can generate highly nonlinear dynamics over the business cycle.

Structure The rest of the paper is organized as follows: Section 2 introduces the model, in Section 3 we compare the nonlinearity of the cash holdings policy function to a standard Aiyagari (1994) model, Section 4 illustrates the aggregate implications of this micro level nonlinearity, **??** analyses the policy implications and Section 5 concludes.

2 Model

There are three sectors in our economy; a production sector, a household sector and a government. The production sector is populated by a continuum of measure one of ex-ante homogeneous firms that face idiosyncratic productivity shocks. These firms can insurance against future negative shocks that may require costly external finance using cash, in the spirit of Froot et al. (1993). The household sector features a representative household that consumes, prices the risk-free bond, owns the firms and supplies labor. The government collects taxes from firms and distributes them in a lump-sum fashion to the households. Idiosyncratic risk arises from firm-specific productivity shocks in the production sector, while aggregate risk stems from time-varying fluctuations in total factor productivity (TFP).

2.1 Technology

Firms use labor to produce the final good.[2] The production function has decreasing returns to scale and the following functional form:

$$y_{it} = A_t z_{it} l_{it}^{\gamma}, \tag{1}$$

where y_{it} denotes final good output, l_{it} labor demand, $\gamma < 1$ the span of control parameter, and z_{it} and A_t idiosyncratic and aggregate productivities, respectively. Regarding productivities, we assume that the log of idiosyncratic productivity follows an AR(1) process:

$$\log(z_{it}) = \rho_z \log(z_{it-1}) + \epsilon_{it}, \quad \epsilon_{it} \sim_{i.i.d} N(0, \sigma_z^2). \tag{2}$$

in which ϵ_{it} is an i.i.d standard normal shock with variance σ_z^2 and ρ_z denotes the persistence of the productivity process. The stochastic aggregate productivity process is assumed to follow a two-state Markov process.[3] Hence, the transition matrix and set of possible aggregate productivity states are:

$$\Gamma_A = \begin{bmatrix} p(A_{t+1} = A_B | A_t = A_B) & (1 - p(A_{t+1} = A_B | A_t = A_B)) \\ (1 - p(A_{t+1} = A_G | A_t = A_G)) & p(A_{t+1} = A_G | A_t = A_G) \end{bmatrix} \tag{3}$$

$$\text{and} \quad A_t \in \{A_B, A_G\}, \tag{4}$$

where $A_B := 1 - \Delta_A$ and $A_G := 1 + \Delta_A$, and we calibrate Δ_A to match the aggregate output volatility. Due to the presence of aggregate risk and incomplete markets, there

[2]The absence of capital in the model is equivalent to a setup where firms use both capital and labor, but optimal capital demand is frictionlessly embedded within the labor demand decision.

[3]This simplification does not affect our results significantly. In particular, the repeated transition method is able to handle finer discretization than two grid points. For example, Lee (2022) uses a finer discretization (five grid points). However, to simplify the interpretation of recessions and booms, we resort to two grid points only.

will be a time-varying distribution of firms over their idiosyncratic cash holdings n_{it} and productivity status z_{it}. Denote this distribution as Φ and its evolution over time as:

$$\Phi_{t+1}(z_{it+1}, n_{it+1}) = \mathbf{G}(\Phi_t(z_{it}, n_{it}), A_t). \tag{5}$$

where $\mathbf{G}$ denotes the transition operator that maps today's distribution and aggregate productivity into tomorrow's distribution. Finally, for ease of notation, denote the collection of aggregate state variables X_t as:

$$X_t \equiv \{\Phi_t(z_{it}, n_{it}), A_t\}. \tag{6}$$

2.2 Cash holdings and financial frictions

Cash Holdings Firms generate profits that equal revenues net of the wage bill and a fixed operation cost $\xi > 0$ in each period. They then decide how much to distribute as dividends (d_{it}) to their ultimate owners, the households. The portion of earnings retained after dividends (and any financing costs) is allocated to adjusting the firm's cash balance.[4] We assume that the evolution of cash holdings follows:

$$n_{it+1} = n_{it} + q_t^n h_{it}, \tag{7}$$

where n_{it} represents the firm's liquid asset holdings, q_t^n is the price of one unit of liquid asssets, and h_{it} denotes the net investment in liquid assets. Importantly, firms are subject to a no net borrowing constraint on cash, ensuring that cash balances remain non-negative, $n_{it+1} \geq 0$. This constraint mirrors the standard incomplete markets assumption with borrowing limits, as in Aiyagari (1994) or Huggett (1993).

Critically, we assume that the return on cash is lower than the risk free rate, closely

[4] In what follows we will, with a slight abuse of terminology, refer to liquid assets and cash holdings interchangeably.

following Alfaro et al. (2024) and Cooley and Quadrini (2001). Specifically, we set the price as a multiple κ of the risk-free bond price, such that $q_t^n = \kappa q_t$, where $\kappa > 1$ and q_t is the risk-free bond price, ensuring that cash is not an excessively attractive store of value.

On the aggregate level, the net supply of liquid assets is assumed to follow an exogenous process linked to the price of liquid assets q_t^n. In particular, we again follow Alfaro et al. (2024) and impose a constant elasticity of supply specification:

$$N_{t+1}^S = \mathcal{H}(q_t^n)^{\frac{1}{\zeta}}, \tag{8}$$

where ζ governs the elasticity of the liquid asset supply, and $\mathcal{H} > 0$ is a scaling constant. This assumption captures how changes in the risk free rate influence the aggregate availability of liquidity in the economy.

External finance cost Beyond the non-negativity constraint on cash holdings, we assume that firms face external financing costs when dividends are negative. In such cases, firms incur an additional cost $C(d_{it})$ in the spirit of Jermann and Quadrini (2012) and Riddick and Whited (2009). We assume that this costs takes the following functional form:

$$C(d_{it}) = \frac{\mu}{2}\mathbb{I}\{d_{it} < 0\}d_{it}^2. \tag{9}$$

where μ governs the magnitude of the financing cost, determining how costly it is for firms to raise external funds when dividends are negative. A higher μ implies that even small equity issuances result in significant costs, discouraging firms from relying on external financing and strengthening their precautionary savings motive. The net dividend is given by $d_{it} - \frac{\mu}{2}\mathbb{I}\{d_{it} < 0\}d_{it}^2$. This function is continuously differentiable ($\mathbb{C}^1$) and concave, ensuring smooth adjustment at $d_{it} = 0$ without kinks. Consequently, the standard theory of concave household utility applies seamlessly to the model.[5]

[5] Jermann and Quadrini (2012) highlight that the convexity assumption aligns with empirical findings by Altinkilic and Hansen (2000) and Hansen and Torregrosa (1992), who document that underwriting fees

The external financing cost captures firms' limited ability to adjust funding sources in response to financial conditions, as in Jermann and Quadrini (2012). This also aligns with the empirical literature on dividend smoothing where firms avoid drastic payout fluctuations due to managerial incentives and agency considerations (Leary and Michaely, 2011; Bliss et al., 2015). In particular, Leary and Michaely (2011) show that cash-rich firms smooth dividends significantly more than others, a pattern the model replicates.

Absent external financing costs, holding cash would not be optimal since cash earns a lower return than dividends. However, due to these costs, firms hoard cash as a precautionary measure, ensuring liquidity for adverse conditions (e.g., low z_t or low A_t). This precautionary motive leads firms to smooth dividend payouts in equilibrium, consistent with observed corporate behavior.

2.3 Recursive firm problem

We are now able to formulate the firm's problem in recursive form. At the beginning of each period, a firm i is identified by its idiosyncratic states n and z. Furthermore, firms have rational expectations and are aware of the full distribution of the firm-level state variables and its evolution. Using these pieces of the setup, the recursive formulation of

increase at a rising marginal cost as the offering size grows.

the firm's problem can be written as follows:

$$J(n,z;X) = \max_{n',d} \quad d - C(d) + \mathbb{E}\left[m(X;X')J(n',z';X')\right] \tag{10}$$

subject to:

$$d = \pi(z;X)(1-\tau^c) + n - q^n n',$$

$$n' \geq 0,$$

$$\pi(z;X) \equiv \max_l zAl^\gamma - w(X)l - \xi,$$

$$C(d) = \frac{\mu}{2}\mathbb{I}(d<0)d^2,$$

$$X \equiv \{\Phi, A\},$$

$$\text{and} \quad \Phi' = G(\Phi, A)$$

where J denotes the value function of a firm, d the dividend, m the stochastic discount factor used to price future payoffs, π the operational profits, τ^c the profit tax imposed by the government, w the wage and ξ the fixed production cost.

2.4 Households

The household sectors is populated by a representative household that chooses labor supply l^H, consumption c, bond holdings b and shares that households invest in firms, denoted by s. Thus, the household problem is given by:

$$V^H(b,s;X) = \max_{c,s',b',l^H} \left[\log(c) - \frac{\eta}{1+\frac{1}{\chi}}(l^H)^{1+\frac{1}{\chi}} + \beta\mathbb{E}\left[V^H(b',s';X')\right]\right] \tag{11}$$

subject to:

$$c + q(X)b' + \sum_{A'}\Gamma_{A,A'}\int m(X;X')s'd\Phi' = w(X)l^H + s + b + T(X)$$

$$X \equiv \{\Phi, A\}$$

where η is the labor disutility parameter, and χ is the Frisch elasticity parameter.

2.5 Government

The government can set a tax on profits τ^c and distribute its proceeds in a lump-sum fashion to the households. Consequently, the government budget constraint reads as follows:

$$\int \tau^c \pi(z; X) d\Phi = T(X) \tag{12}$$

2.6 Competitive equilibrium

Definition 1. *A recursive competitive equilibrium is a set of functions*

$$\left(q, q^n, m, w, T, J, N, D, L, V^H, C, L^H, S, B, N^S, \Phi\right)$$

that solve the firm problem, household problem, government budget constraint, and clear the markets for liquid assets, labor, output and household bond holdings, as described by the following conditions:

1. *Taking q^n, m and w as given, J solves the firm problem described in (10), and (N, D, L) are the associated policy functions for firms.*
2. *V^H solves (11) and (C, B, L^H, S) are the associated policy functions for households.*
3. *T solves the government budget constraint described in (12).*
4. *The goods market clears.*
5. *The market for shares clears $s(X) = \int [J(n, z; X) + C(D(n, z; X))] d\Phi$ as the external financing costs and aggregate firm values jointly determine the supply of equity.*
6. *The labor market clears $L^H(X) = \int L(n, z; X) d\Phi$*

7. *The liquid asset market clears* $(N^S(X))' = \int N(n,z;X)d\Phi$

8. *The evolution of the distribution is consistent with policy functions.*

9. *The bond market-clearing condition,* $B(X) = 0$ *is satisfied by Walras's law.*

3 The role of market incompleteness and the financial frictions

In this section, we analyze the individual firm's cash hoarding patterns in the stationary equilibrium. This analysis is essential to understand why the model can feature highly nonlinear dynamics under aggregate uncertainty. Due to the external financing cost, a firm uses cash as a precautionary savings instrument. However, there exists a target cash level, which the optimal cash holding does not go beyond. This is because after the target level of cash, holding cash becomes costlier: if a firm holds a large stock of cash, then the future risk of external financing is almost perfectly hedged while carrying additional cash bears only a smaller return than the household discount factor would imply. This intuition is reflected in Proposition 1 which states the existence of the target cash level.[6]

Proposition 1 (The existence of the target cash-holding level)**.**
Suppose policy functions are non-trivial: $n'(n,z) > 0$ *and* $d(n,z) > 0$ *for some* $n > 0$*, given* z*. Then, there exists* $\overline{n}(z) > 0$ *such that* $n'(n,z) \leq \overline{n}(z)$ *for* $\forall n \geq 0$*.*

Proof. See Online Appendix. ■

Therefore, the optimal future cash holding policy function is constant when the current cash stock reaches the target cash holding level.[7] Figure 2 shows the future cash

[6]It is worth noting that the implication of Proposition 1 is different from Proposition 4 of Aiyagari (1993). Proposition 4 of Aiyagari (1993) implies that a household with a excessively large wealth would gradually decrease the wealth. In contrast, Proposition 1 implies that a firm with a excessively large cash stock would immediately reduce the cash stock to the target level.

[7]The existence of the target cash holding level is similar to the prediction of the consumption buffer

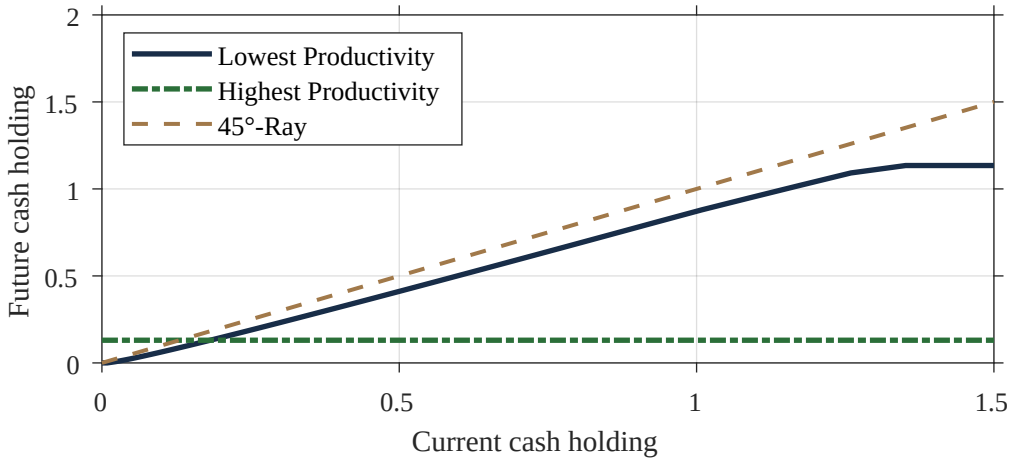


Figure 2: Cash holding policy function of individual firms

Note. The solid line plots the future cash-holding policy function of the lowest-productivity firms. The dash-dotted line plots the future cash-holding policy function of the highest-productivity firms.

holding policy function for the lowest and highest productivity firms. For the highest productivity firms, due to the persistence of the productivity shock, the target cash holding level is lowest: they are the least concerned firms about the future external financing cost. As can be seen from the figure, the policy function crosses the 45-degree ray. That is, the highest productivity firms with little cash increase the cash holding until they reach the target level. On the other hand, the lowest productivity firms' target cash holding level is the highest, but their low profit makes them decrease their future cash holding. Therefore, the lowest productivity firms' policy function does not cross the 45-degree ray.

The flat region in the cash-holding policy function is starkly contrasted with the wealth accumulation pattern of households in Aiyagari (1994). To make a direct comparison feasible, we define the liquidity on hand, which is a firm-side counterpart of the total resource in Aiyagari (1994):

$$\text{Liquidity on hand}_t := \underbrace{\pi_t}_{\text{Liquidity from operating profit}} + \underbrace{n_t}_{\text{Cash}}.$$

stock model (Carroll, 1997). Despite the flat region, there is no kink in the policy function, as the objective function is smooth over the entire domain ($\mathbb{C}^1$ class).

Figure 3 plots the saving and dividend policies in panel (a) and the future liquidity on hand in panel (b) as functions of the liquidity on hand. This figure is the firm-side counterpart of Figure I in Aiyagari (1994).[8] For a sharp illustration, we only plot the policy functions for a firm at the lowest productivity ($z = min\mathbb{Z}$). For both firms in my model and households in Aiyagari (1994), there are cases where the borrowing limit is bound, leaving the saving policy flat near the borrowing limit. However, that region spans only a tiny range for the firm side, so it is not visible in panel (a) of Figure 3.[9] If a firm does not hold enough liquidity on hand, an extra dollar increase in liquidity goes to both saving and dividends. Especially a greater amount goes to the saving than dividends. However, if a firm holds enough liquidity on hand, additional liquidity is solely paid out as a dividend, as can be seen from the parallel dividend curve to the 45-degree line for the high liquidity on hand region. In panel (b), the future liquidity on hand displays a flat region for the high level of the current liquidity on hand, similar to the dynamics of the cash policy function in Figure 2.

The nonlinear policy function at the firm level generates nonlinearity in the business cycle. Especially depending on the fraction of firms that are located over the flat policy region, the aggregate cash dynamics vary. Therefore, the aggregate fluctuations in this model crucially depend on the endogenous state Φ, the distribution of individual states.

[8] Dividend d is the counterpart of consumption c, and future cash holding n' is the counterpart of future wealth a_{t+1}.

[9] Households' saving pattern also displays a kink in Krusell and Smith (1998). However, in both Aiyagari (1994) and Krusell and Smith (1998), the fraction of these constrained households is negligibly small. Especially, this is one of the major reasons why the aggregate dynamics in Krusell and Smith (1998) do not feature a nonlinearity.

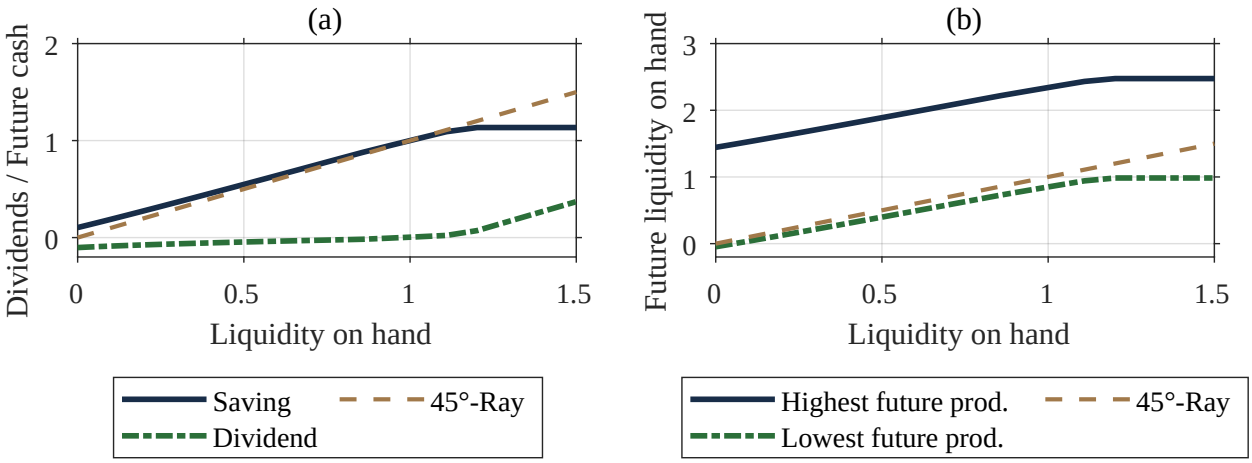


Figure 3: Dividends and savings and liquidity on hand policy functions (when $z = min\mathbb{Z}$)

Notes: This figure is the firm-side counterpart of Figure I in Aiyagari (1994).

4 Nonlinear business cycle and endogenous consumption Risk

In this section, we quantitatively analyze the recursive competitive equilibrium allocations computed from the global nonlinear solution method in the sequence space. For easier computation, we normalize the firm's value function by contemporaneous consumption c_t following Khan and Thomas (2008). Based on this, we define the marginal utility price of consumption as $p_t := 1/c_t$ and the normalized value function $\tilde{J}_t := p_t J_t$. From the intratemporal and intertemporal optimality conditions of households, we have $w_t = \eta / p_t$ and $q_t = \beta p_{t+1}/p_t$. Thus, p_t is the only price to characterize the equilibrium.

4.1 Calibration and Solution

We follow the standard approach of setting some parameters equal to values in the literature and calibrate key model parameters based on empirical data moments. The model period is a quarter, so we set the household discount factor β to 0.995, consistent with Chen et al. (2017), generating an annualized real rate of approximately 2% at the non-stochastic steady state. The Frisch labor supply elasticity is set equal to 1, consistent with

Table 1: Fixed Parameters

Parameter	Description	Value	Source
Households			
β	Discount factor	0.995	Chen et al. (2017)
η	Frisch labor supply elasticity	1.000	Kaplan et al. (2018)
Production			
γ	Span of control	0.930	Standard calibration
ρ_z	Idiosyncratic shock persistence	0.861	Bachmann et al. (2013)
σ_z	Idiosyncratic shock volatility	0.075	Bachmann et al. (2013)
Aggregate			
$p(A_B\|A_B)$	Persistence of low aggregate TFP	0.875	Krusell and Smith (1998)
$p(A_G\|A_G)$	Persistence of high aggregate TFP	0.875	Krusell and Smith (1998)

Note. This table presents the fixed parameters used in the model, along with their sources. For the idiosyncratic shock volatility σ_z we sum both the idiosyncratic and sector specific shock volatility from Bachmann et al. (2013).

previous literature, for example Kaplan et al. (2018).[10] On the firm side, we set the span of control parameter equal to 0.930,

The model's key parameters are the external financing cost parameter μ and the operating cost parameter ξ. The external financing cost μ is identified from the corporate cash-to-output ratio. In the moment calculation, the aggregate cash stock is obtained from the Flow of Funds of the Federal Reserve Board, and the aggregate output (GDP) is from the National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA).[11] In the model, as μ increases, the corporate cash-to-output ratio increases due to the heightened precautionary motivation. The identifying moment of the operating cost parameter ξ is the consumption-to-output ratio. The consumption data is from NIPA.[12] As operating cost increases, the ratio decreases due to the reduced dividends. The calibrated parameters and the corresponding moments are summarized in Table 2.

[10] Chetty et al. (2011) show that micro estimates imply a Frisch elasticity of 0.82. Our value is slightly above this.

[11] The detailed definition of aggregate cash holding is available in Appendix A.1

[12] Consumption includes both durable and non-durable consumptions.

Table 2: Calibrated Parameters and Target Moments

Parameters	Description	Data	Model	Deviation in (%)
μ	Corporate cash holdings relative to output (%)	10.00	10.12	0.08
ξ	Consumption relative to output (%)	66.00	62.85	0.15
η	Hours worked relative to time available (%)	33.00	33.00	13.50
Δ_A	Output volatility (% *p.q.*)	1.45	1.74	0.02

Note. This table presents the calibrated parameters along with the corresponding target moments, observed data values, model-implied values, and the level of precision achieved.

4.2 Nonlinear business cycle

Using the repeated transition method, we compute the recursive competitive equilibrium allocations over the simulated path of aggregate shocks. Using the aggregate cash stock, we take the sufficient statistic approach. The dynamics of the aggregate cash stocks are highly nonlinear for two reasons. First, the individual firm's cash holding policy function becomes flat for high levels of individual cash stocks, as described in Section 3. Second, the general equilibrium effect does not strongly affect each firm's cash holding demand. It is because the price of cash holding is exogenously fixed at r^n as the cash is not allowed to be traded across the firms.

Figure 4 plots a part of the simulated path of the price p_t (panel (a)) and aggregate corporate cash holding N_t (panel (b)) obtained from both the repeated transition method and the log-linear specification of the law of motion. The solid line plots the expected allocations (guess from the n^{th} iteration), and the dash-dotted line plots the realized allocations (simulation based on the policy in $(n+1)^{th}$ iteration) in the repeated transition method. The dashed line represents the dynamics of the allocations in the log-linear specification of the law of motion. To obtain the parameters in the log-linear specification, we fit the equilibrium allocations from the repeated transition method into the log-linear specifica-

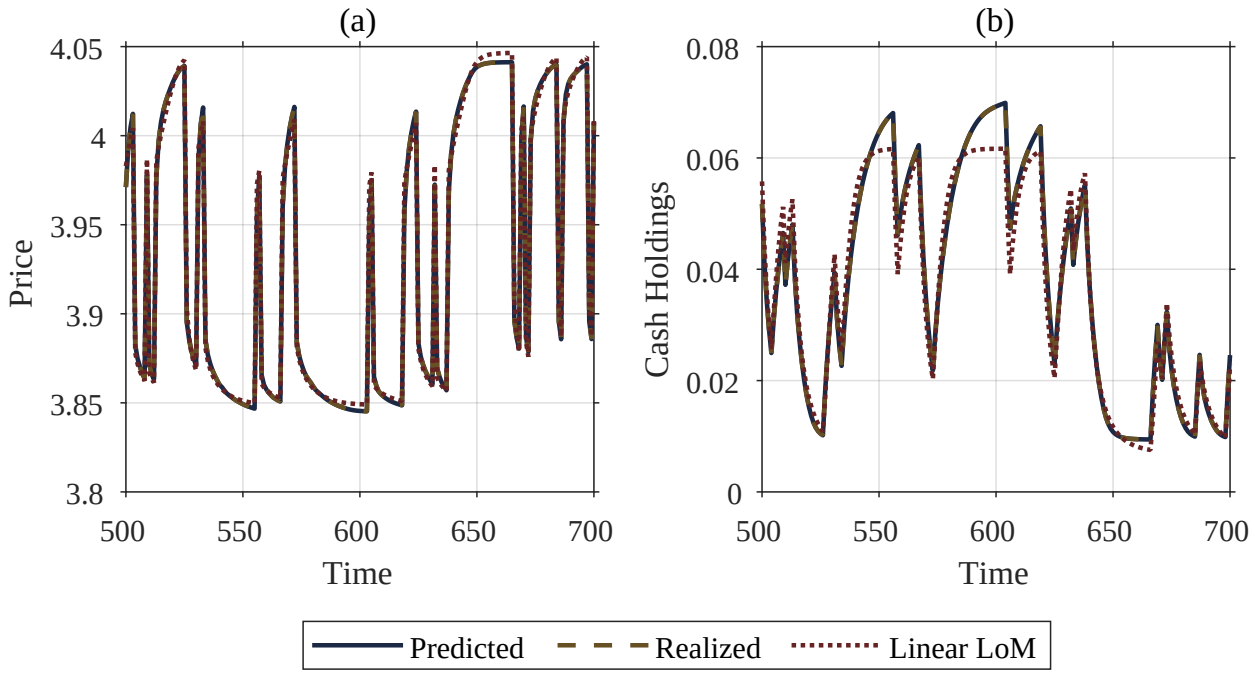


Figure 4: Aggregate fluctuations in the baseline model

Notes: The figure plots the time series of the price p_t the aggregate cash stock N_t in the baseline model. In both panels, the solid line is the predicted time series (n^{th} guess) $\{p_t^{(n)}, N_t^{(n)}\}_{t=400}^{500}$; the dash-dotted line is the realized time series $\{p_t^*, N_t^*\}_{t=400}^{500}$; the dashed line is the predicted time series implied by the linear law of motion.

tion, and the result is as follows:

$$log(N_{t+1}) = -0.5473 + 0.8899 * log(N_t), \quad \text{if } A_t = A_B, \text{ and } R^2 = 0.9977,\ MSE = 0.0006$$

$$log(N_{t+1}) = -0.7437 + 0.7331 * log(N_t), \quad \text{if } A_t = A_G, \text{ and } R^2 = 0.9914,\ MSE = 0.0011$$

$$log(p_t) = 1.3545 - 0.0093 * log(N_t), \quad \text{if } A_t = A_B, \text{ and } R^2 = 0.9332,\ MSE = 0.0000$$

$$log(p_t) = 1.3284 - 0.0073 * log(N_t), \quad \text{if } A_t = A_G, \text{ and } R^2 = 0.9573,\ MSE = 0.0000$$

In the repeated transition method, the expectation path and the realized path converge as can be seen from the figure. Therefore, R^2s are 1 and the mean squared errors are as small as 10^{-6} for both p_t and N_t. On the other hand, the log-linear fitting results in low R^2 and high mean squared errors. These results show that the aggregate fluctuations in this economy are highly nonlinear.

One important reason for the nonlinearity is the missing general equilibrium effect

on cash. If the price of cash is determined in the competitive market, the dynamics of aggregate cash stocks are smoothed. For example, when there is a surge of cash holding demand, the price of cash holding goes up to mitigate the surge and vice versa for the case of decreasing cash holding demand. In many of the models in the literature, this flattening force from the general equilibrium has been proven to be powerful enough to guarantee the log-linear specification as the true law of motion. One example is Khan and Thomas (2008), where the micro-level lumpiness is smoothed out by real interest rate dynamics. However, due to the missing general equilibrium effect, the log-linear prediction rule fails to capture the true law of motion in this paper.

On top of the nonlinearity, there is another complication in the model that the prototype method of Krusell and Smith (1998) cannot simply address: there is a non-trivial market-clearing condition with respect to price p_t. Krusell and Smith (1997) suggests an algorithm to solve this problem by considering an external loop in the algorithm that solves the market-clearing price p_t in each iteration. This algorithm is known to successfully solve the log-linear models with non-trivial market-clearing conditions, such as Khan and Thomas (2008). However, due to the extra loop in each iteration, the algorithm entails high computation costs. In contrast, the repeated transition method tracks the implied price instead of the market clearing price on the simulated path. Therefore, the method does not require an extra loop for computing the market-clearing price, so it saves a great amount of computation time.

4.3 The endogenous consumption risk and state-dependent consumption responsiveness

In this section, we analyze the role of corporate cash holdings on aggregate consumption fluctuations using the baseline model and support the model prediction from the empirical evidence.

In the model, the aggregate productivity A_t can take one of two values $\{A_B, A_G\}$,

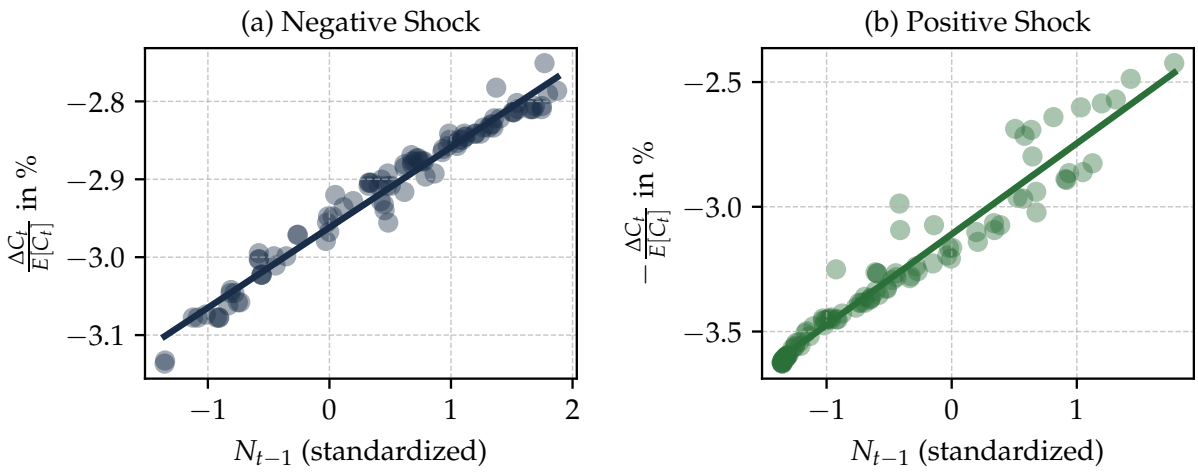


Figure 5: Endogenous state-dependence in the shock responses of consumption

Notes: The figure plots consumption responses (relative to average consumption, in percentages) to negative (panel (a)) and positive (panel (b)) aggregate TFP shocks as a function of lagged aggregate cash stocks on the horizontal axis. The positive shock responses are multiplied by -1.

and it follows a persistent process. We define the negative aggregate productivity shock as a TFP shift from A_G to A_B and the positive aggregate productivity shock as a TFP shift from A_B to A_G. In the baseline model, depending on the cash stock a firm holds, the responsiveness of the firm-level dividends to the exogenous aggregate TFP shock changes. For example, when a firm is short of cash, a negative TFP shock in productivity makes a firm reduce dividends further than it would do when it has abundant cash stocks. It is because the firm with little cash needs to not only pay out dividends but save cash out of concern for the future.

Then, due to this dividend channel, the responsiveness of household consumption becomes dependent on the aggregate cash stock. Figure 5 plots the relationship between the consumption responsiveness over the aggregate cash stocks separately for negative aggregate shock (panel (a)) and positive aggregate shock (panel (b)). In this model, the magnitude of aggregate shock is uniform at $|A_G - A_B|(= 2\%$ TFP shock$)$.[13] Therefore, If the consumption shock responses are different across the periods, it is due to the endoge-

[13]The aggregate shock is defined as a shift from one productivity to the other.

nous state dependence of the responsiveness rather than the shock magnitude variation. Then, we separately collect the periods of negative aggregate shock and positive aggregate shock and compute the consumption c_t and one-period-ahead aggregate cash stock N_{t-1} for each period. As can be seen from Figure 5, the consumption responsiveness decreases in the aggregate cash stock for the negative aggregate shock. From a similar intuition with the opposite direction, the consumption responsiveness increases in the aggregate cash stock for the positive aggregate shock.

Table 3: Endogenous state-dependence in consumption responses to negative and positive shocks

	Dep. Var.: $\lvert log(c_t) \rvert$ $(p.p.)$	
	Neg. (1)	Pos. (2)
$Cash_{t-1}(s.d.)$	-0.166	0.07
	(0.001)	(0.001)
Constant	Yes	Yes
Observations	83	84
R^2	0.996	0.994

Notes: The table reports the results of the regression of consumption responses to a negative and positive aggregate TFP shock on the lagged aggregate cash stocks using the simulated data. The first column is for the negative aggregate TFP shock, and the second is for the positive aggregate TFP shock. The numbers in the brackets are standard errors.

Table 3 reports the regression coefficient when the observations in Figure 5 are fitted into the linear regression. The numbers in the brackets are the standard errors. When the lagged aggregate cash stock increases by one standard deviation, the consumption responsiveness to the negative aggregate TFP shock (-2% TFP shock) significantly decreases by 0.17 percentage points. For the positive aggregate TFP shock (+2% TFP shock), the consumption responsiveness decreases by 0.07 percentage points when the lagged aggregate cash stock increases by one standard deviation.

Therefore, the aggregate cash holding gives a consumption buffer against a negative aggregate shock by smoothing the dividend stream in the simulated data. Also, the aggregate cash holding helps a positive productivity shock solely pass down to the consump-

tion. And the consumption buffer effect against the negative TFP shock is significantly stronger than the consumption boosting effect for the positive TFP shock in the model. We support this model prediction from the macro-level data. The data is the quarterly frequency and covers from 1951 to 2018. Consumption and the total dividend of the corporate sector are from NIPA; the aggregate cash holding and the total asset holding are obtained from the Flow of Funds.[14]

Table 4: State-dependent consumption responses to negative and positive shocks in data

	Dependent variables:			
	$\|log(c_t)\|$ $(p.p.)$ before 1980		$\|log(c_t)\|$ $(p.p.)$ after 1980	
	Neg. (1)	Pos. (2)	Neg. (3)	Pos. (4)
$Cash_{t-1}(s.d.)$	-0.108	0.036	-0.226	0.164
	(0.09)	(0.072)	(0.085)	(0.09)
Constant	Yes	Yes	Yes	Yes
Observations	63	49	77	79
R^2	0.023	0.005	0.086	0.041

Notes: The table reports the results of the regression of consumption responses to a negative and positive aggregate TFP shock on the lagged aggregate cash stocks using the macro-level data. The first column is for the negative aggregate TFP shock before 1980; the second is for the positive aggregate TFP shock before 1980; the third is for the negative aggregate TFP shock after 1980; the last is for the positive aggregate TFP shock after 1980. The numbers in the brackets are standard errors.

Table 4 reports the data counterpart of the results in Table 3, separately for the periods before 1980 (the first two columns) and after 1980 (the last two columns).[15] For this analysis, the consumption variations derived from the TFP variation are controlled by consumption residualization over the polynomials of TFP up to the fourth order.[16] As can be seen from Figure 1, since around 1980, the corporate cash stock has rapidly increased. This rising corporate cash holding has brought a change in the relationship between corporate cash stock and consumption. In the pre-1980 periods, consumption responsiveness was not dependent on the one-period-ahead corporate cash stock. In contrast, in the

[14] All time series are detrended using the HP filter with a frequency parameter at 1600.
[15] The result is not sensitive to the choice of the cutoff year.
[16] We use the utilization-adjusted TFP from Fernald (2014).

post-1980 periods, consumption responsiveness becomes significantly dependent on the corporate cash stock. When the lagged aggregate cash stock increases by one standard deviation, the consumption responsiveness to the negative aggregate shock significantly decreases by 0.23 percentage points. For the positive aggregate shock, the consumption responsiveness increases by 0.16 percentage points when the lagged aggregate cash stock increases by one standard deviation.[17]

4.4 Testing the dividend channel

In this section, we test whether the state dependence of the consumption responsiveness to a TFP is through the dividend channel or not in the data. For this, we first factor out the variation of consumption orthogonal to the contemporaneous dividend variation:[18]

$$C_t = C_t^D + \widetilde{C}_t, \quad \text{s.t. } \widetilde{C}_t \perp D_t.$$

Then, we check whether the responsiveness of C_t^D is dependent on the cash stocks. Table 5 shows the state dependence of C_t^D. The consumption that co-varies with the dividend still displays significant state-dependent shock responsiveness. Even in the data before 1980, this dividend channel displays a significant role in consumption state dependence. However, once the whole consumption is considered, this channel is muted down (Table 4), possibly due to the relatively low importance cash stocks before 1980.

5 Concluding remarks

Corporate cash holdings have been increasing globally over the past few decades. In this paper, we analyze the effects of this increase on the business cycle. Making use of a heterogeneous-firm business cycle model with financial frictions, in which firms hoard

[17] These two estimates are not statistically different from the model-side estimates in Table 3.
[18] We projected C_t onto the polynomials of D_t up to the sixth order.

Table 5: State-dependent consumption responses to negative and positive shocks through the dividend channel in data

	Dependent variables:			
	$\lvert log(c_t) \rvert$ $(p.p.)$ before 1980		$\lvert log(c_t) \rvert$ $(p.p.)$ after 1980	
	Neg. (1)	Pos. (2)	Neg. (3)	Pos. (4)
$Cash_{t-1}(s.d.)$	-0.045	0.036	-0.072	0.035
	(0.021)	(0.022)	(0.036)	(0.027)
Constant	Yes	Yes	Yes	Yes
Observations	63	49	77	79
R^2	0.068	0.055	0.05	0.021

Notes: The table reports the results of the regression of consumption responses to a negative and positive aggregate TFP shock on the lagged aggregate cash stocks using the consumption varying along with the dividend. The first column is for the negative aggregate TFP shock before 1980; the second is for the positive aggregate TFP shock before 1980; the third is for the negative aggregate TFP shock after 1980; the last is for the positive aggregate TFP shock after 1980. The numbers in the brackets are standard errors.

cash out of a precautionary motivation.

We find the cash hoarding by firms creates aggregate risk sharing between firms and households, leading to aggregate consumption sensitive to aggregate shocks becoming dependent on the level of corporate cash holdings. If cash holdings are higher, firms can better smooth their dividend, causing consumption to fall by less in response to a negative TFP shock. Using aggregate data for the US, we find empirical evidence in support of the dividend channel in consumption state dependence.

Despite the lower consumption volatility when cash hoarding is high, a trade-off arises, as output becomes more volatile in that case. This is driven by the dampened wealth effect that operates through the labor supply channel.

References

ACHARYA, V. V., H. ALMEIDA, AND M. CAMPELLO (2007): "Is cash negative debt? A hedging perspective on corporate financial policies," *Journal of financial intermediation*, 16, 515–554.

AIYAGARI, S. R. (1993): "Uninsured idiosyncratic risk and aggregate saving," Working Papers 502, Federal Reserve Bank of Minneapolis.

——— (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109, 659–684, publisher: Oxford University Press.

ALFARO, I., N. BLOOM, AND X. LIN (2024): "The Finance Uncertainty Multiplier," *Journal of Political Economy*, 132, 577–615.

ALTINKILIC, O. AND R. S. HANSEN (2000): "Are There Economies of Scale in Underwriting Fees? Evidence of Rising External Financing Costs," *The Review of Financial Studies*, 13, 191–218.

BACHMANN, R., R. J. CABALLERO, AND E. M. R. A. ENGEL (2013): "Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model," *American Economic Journal: Macroeconomics*, 5, 29–67.

BATES, T. W., K. M. KAHLE, AND R. M. STULZ (2009): "Why do US firms hold so much more cash than they used to?" *The journal of finance*, 64, 1985–2021.

BEGENAU, J. AND J. SALOMAO (2018): "Firm Financing over the Business Cycle," *The Review of Financial Studies*, 32, 1235–1274.

BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of macroeconomics*, 1, 1341–1393.

BLISS, B. A., Y. CHENG, AND D. J. DENIS (2015): "Corporate payout, cash retention, and the supply of credit: Evidence from the 2008–2009 credit crisis," *Journal of Financial Economics*, 115, 521–540.

CARLSTROM, C. T., T. S. FUERST, AND M. PAUSTIAN (2016): "Optimal contracts, aggregate risk, and the financial accelerator," *American Economic Journal: Macroeconomics*, 8, 119–147.

CARROLL, C. D. (1997): "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *The Quarterly Journal of Economics*, 112, 56.

CHEN, P., L. KARABARBOUNIS, AND B. NEIMAN (2017): "The global rise of corporate saving," *Journal of Monetary Economics*, 89, 1–19.

CHETTY, R., A. GUREN, D. MANOLI, AND A. WEBER (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," *American Economic Review*, 101, 471–75.

CLOYNE, J., C. FERREIRA, M. FROEMEL, AND P. SURICO (2023): "Monetary Policy, Corporate Finance, and Investment," *Journal of the European Economic Association*, jvad009.

COOLEY, T. F. AND V. QUADRINI (2001): "Financial Markets and Firm Dynamics," *American Economic Review*, 91, 1286–1310.

FERNALD, J. (2014): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," *Federal Reserve Bank of San Francisco, Working Paper Series*, 01–28.

FERREIRA, M. H., T. HABER, AND C. RÖRIG (2023): "Financial constraints and firm size: Micro-evidence and aggregate implications," Tech. rep., Working Paper.

FROOT, K. A., D. S. SCHARFSTEIN, AND J. C. STEIN (1993): "Risk Management: Coordinating Corporate Investment and Financing Policies," *Journal of Finance*, 48, 1629–1658.

GERTLER, M. AND N. KIYOTAKI (2010): "Financial intermediation and credit policy in business cycle analysis," in *Handbook of monetary economics*, Elsevier, vol. 3, 547–599.

HANSEN, R. S. AND P. TORREGROSA (1992): "Underwriter Compensation and Corporate Monitoring," *Journal of Finance*, 47, 1537–1555.

HUGGETT, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of Economic Dynamics and Control*, 17, 953–969.

JEENAS, P. (2024): "Idiosyncratic Labor Risk and Aggregate Risk Sharing with Financial Friction," Tech. rep., Working Paper.

JERMANN, U. AND V. QUADRINI (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102, 238–271.

KAPLAN, G., B. MOLL, AND G. L. VIOLANTE (2018): "Monetary Policy According to HANK," *American Economic Review*, 108, 697–743.

KHAN, A. AND J. K. THOMAS (2008): "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76, 395–436.

KRUSELL, P. AND A. A. SMITH, JR. (1997): "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, 1.

——— (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.

LEARY, M. T. AND R. MICHAELY (2011): "Determinants of Dividend Smoothing: Empirical Evidence," *Review of Financial Studies*, 24, 3197–3249.

LEE, H. (2022): "Striking While the Iron Is Cold: Fragility after a Surge of Lumpy Investments," *Working Paper*, 68.

OPLER, T., L. PINKOWITZ, R. STULZ, AND R. WILLIAMSON (1999): "The determinants and implications of corporate cash holdings," *Journal of financial economics*, 52, 3–46.

OTTONELLO, P. AND T. WINBERRY (2020): "Financial Heterogeneity and the Investment Channel of Monetary Policy," *Econometrica*, 88, 2473–2502.

RIDDICK, L. A. AND T. M. WHITED (2009): "The Corporate Propensity to Save," *The Journal of Finance*, 64, 1729–1766.

A Appendix

A.1 Aggregate liquid assets from the Flow of Funds

The nonfinancial corporate business holdings of liquid assets are defined both as a narrow measure and a broad measure. The narrow measure is defined as the sum of:

- private foreign deposits (`FL103091003.Q`)
- checkable deposits and currency (`FL103020005.Q`)
- total time and savings deposits (`FL103030003.Q`)
- money market fund shares (`FL103034000.Q`)

whereas the broad measure also includes:

- municipal securities (`FL103062003.Q`)
- commercial paper (`FL103069100.Q`)
- Treasury securities (`FL103061103.Q`)
- agency- and GSE-backed securities (`FL103061703.Q`)
- security repurchase agreements (`FL102051003.Q`)
- mutual fund shares (`FL103064203.Q`)
- corporate equities (`FL103064103.Q`)
- corporate and foreign bonds (`FL123063003.Q`)

A.2 Note on computation

Then, the equilibrium price p_t is determined from the following variant of the non-trivial market clearing condition:[19]

$$p = \arg_{\widetilde{p}} \left\{ c(\widetilde{p}) - \int \left[d(x; X, \widetilde{p}) + w(X, \widetilde{p}) l(x; X, \widetilde{p}) \right] d\Phi = 0 \right\}.$$

This is a fixed-point problem and computationally costly to solve. Instead, the repeated transition method uses the implied price p^*, which is obtained as follows:

$$\begin{aligned} p^* &= \arg_{\widetilde{p}} \left\{ c(\widetilde{p}) - \int \left[d(x; X, p^{(n)}) + w(X, p^{(n)}) l(x; X, p^{(n)}) \right] d\Phi = 0 \right\} \\ &= 1 / \int \left[d(x; X, p^{(n)}) + w(X, p^{(n)}) l(x; X, p^{(n)}) \right] d\Phi, \end{aligned}$$

where $p^{(n)}$ is the guessed price in the n^{th} iteration. We take the sufficient statistic approach, and the aggregate cash holdings N_t (the first moment of the distribution of cash holding) is the sufficient statistic.

[19] This condition is derived from combining the household's budget constraint and the equity market clearing condition. For a sharp illustration, the time subscript is lifted.

A.3 Recovering the true nonlinear law of motion

In this section, we recover the true law of motion from the converged equilibrium outcomes over the simulated path. Then, we test the validity of the true law of motion by fitting the law of motion into the out-of-sample simulated path.

Specifically, the following laws of motion are studied:

$$N_{t+1} = G_N(N_t, N_{t-1}, N_{t-2}, \ldots, N_{t-n}; A_t)$$
$$p_t = G_p(N_t, N_{t-1}, N_{t-2}, \ldots, N_{t-n}; A_t).$$

Table A.1 reports the goodness of fitness (R^2) of the different specifications. The first five rows report the fitness when only the contemporaneous aggregate cash stock N_t is considered up to different polynomial orders. When a single argument is considered without the higher-order polynomials, R^2 gets as low as 0.8956 for G_N if the contemporaneous productivity state is G.[20] As the more higher-order polynomials are included, the better the fitness becomes. However, the fitness of the specification of G_p stops improving after a certain threshold. This shows that the true law of motion can be recovered only by including further historical allocations.

The bottom seven rows of Table A.1 report the fitness of the law of motion when the additional lagged terms of the aggregate cash stock are considered. Up to the third order polynomials are included for each of lagged terms on top of the polynomial terms of the contemporaneous cash stocks up to the fifth order.[21] As more lagged terms are considered in the law of motion, the fitness improves, especially in G_p. However, only after the polynomials of the seven-period lagged aggregate cash stock are included in the law of motion, the accurate law of motion is recovered.

This exercise shows the substantial nonlinearity of the aggregate fluctuations in this model. In the repeated transition method, the contemporaneous aggregate cash stock is

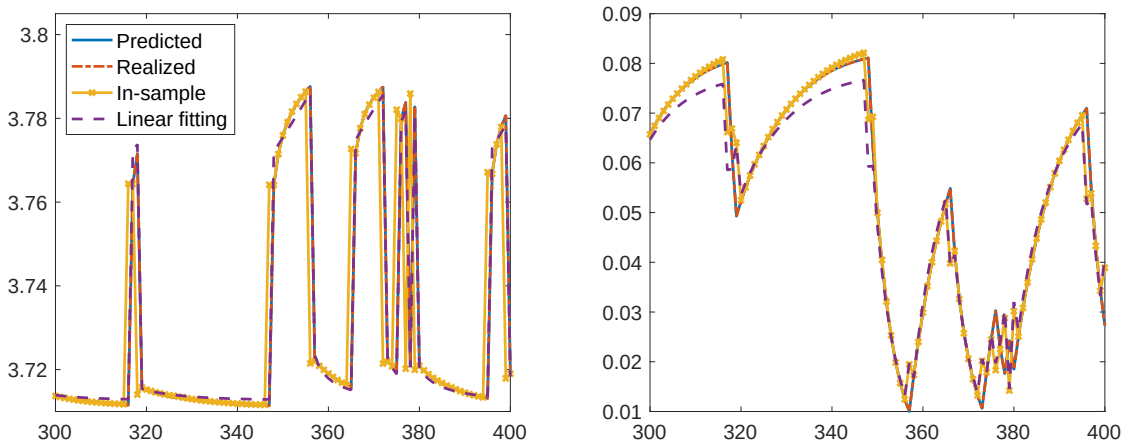
[20] This pure linear specification is different from the log-linear specification in Section 4.
[21] The results only negligibly change over different order choices.

Table A.1: The fitness of the law of motion across different specifications

			Goodness of fitness: R^2			
	# of lagged	order	CA_{t+1} : *Good*	CA_{t+1} : *Bad*	p_t : *Good*	p_t : *Bad*
Contemp.	0	1	0.8956	0.9452	0.9922	0.9966
	0	2	0.9839	0.9952	0.9927	0.9976
	0	3	0.9973	0.9995	0.9930	0.9976
	0	4	0.9993	0.9999	0.9932	0.9976
	0	5	0.9996	1.0000	0.9933	0.9976
Add. history	1	3	0.9999	1.0000	0.9987	0.9979
	2	3	0.9999	1.0000	0.9997	0.9984
	3	3	0.9999	1.0000	0.9998	0.9987
	4	3	0.9999	1.0000	0.9998	0.9991
	5	3	0.9999	1.0000	0.9998	0.9994
	6	3	0.9999	1.0000	0.9998	0.9996
	7	3	0.9999	1.0000	0.9998	0.9997

Notes: The first column (# of lagged) reports the number of lagged terms included in the specification. The second column (order) reports the highest order of polynomials considered. When lagged terms are included, contemporaneous terms are considered up to the fifth polynomial for all specifications. The third and fourth column reports the R^2 of the law of motion of N_{t+1} for Good and Bad shock realizations. The fifth and sixth column reports the R^2 of the law of motion of p_t for Good and Bad shock realizations.

used as a sufficient statistic of each period's cross-section. However, this does *not* imply that the true aggregate law of motion is a function of only the contemporaneous cash stock. The contemporaneous cash stock is rather a labeling of each period that correctly sorts the rankings of the value functions across the periods in the repeated transition method.



(a) Price p_t $(= 1/C_t)$ (b) Aggregate cash stocks N_t

Figure A.1: Fitting into the out-of-sample path

Notes: The figure plots the time series of the price p_t the aggregate cash stock N_t in the baseline model for an out-of-sample simulated path of the aggregate shocks. In both panels, the solid line is the predicted time series (n^{th} guess) $\{p_t^{(n)}, N_t^{(n)}\}_{t=300}^{400}$; the dash-dotted line is the realized time series $\{p_t^*, N_t^*\}_{t=300}^{400}$; the line with the star tick mark is the predicted time series implied by the exact law of motion recovered from the in-sample path; the dashed line is the predicted time series implied by the linear law of motion.

We validate the recovered law of motion by fitting it on the out-of-sample simulated path.[22] Specifically, we solve the model on another simulated path to obtain the converged equilibrium dynamics using the repeated transition method and compare the dynamics with the implied dynamics in the recovered true law of motion on the in-sample path. Figure A.1 plots p_t (panel (a)) and N_t (panel (b)) for 1) predicted time series (solid line), 2) realized time series (dot-dashed line), 3) time series implied by the recovered in-sample law of motion (solid line with ticks), and 4) time series implied by the linear law of motion (dashed line). The predicted time series and the realized time series are indistinguishably close to each other due to the convergence requirement of the repeated transition method. The time series implied by the recovered law of motion also closely tracks the converged equilibrium dynamics, validating the specification. The goodness

[22]The recovered true of motion refers to the specification that considers up to the seven-period lagged aggregate cash stocks, where R^2 is the highest in Table A.1.

of fitness (R^2) in the time series implied by the recovered law of motion is greater than 0.999 for both G_N and G_p in all shock realizations.

A.4 Proposition and proof of satiation point

Proposition 1 (The existence of the target cash-holding level)**.**
Suppose policy functions are non-trivial: $n'(n_1, z) > 0$ for some $n_1 > 0$ and $d(n_2, z) > 0$ for some $n_2 > 0$, given z. Then, there exists $\overline{n}(z) > 0$ such that $n'(n, z) \leq \overline{n}(z)$ for $\forall n \geq 0$.

Proof. To prove the proposition by contradiction, suppose there is no such $\overline{n}(z)$. That is, $n'(n, z) < n'(n + \epsilon, z)$ for $\forall(n, z)$ and $\forall \epsilon > 0$.

Define cash on hand as $m(n, z) = \pi(z) + n$. Then,

$$d(n, z) + q^n n'(n, z) = m(n, z).$$

$m(n, z)$ strictly increases in n. Due to the monotone preference on greater d and n' and strict monotonicity of m on n, d and n' weakly increase in n. Now consider $\widetilde{n}$ such that $n'(\widetilde{n}, z) > 0$ and $d(\widetilde{n}, z) > 0$. Such $\widetilde{n}$ exists as n' and d weakly increase in n. For example $\widetilde{n} = \max\{n_1, n_2\}$.

Then, for a marginal incremental ϵ in cash, the marginal cost of hoarding cash is 1 (forgone dividend), while the marginal benefit out of hoarding cash is $\frac{q}{q^n}$:[23]

$$\underbrace{1}_{\text{Marginal cost}} \quad > \quad \underbrace{\frac{q}{q^n}}_{\text{Marginal benefit}} .$$

where, $q = \beta$ at the non-stochastic steady state. This implies that for the extra cash, the firm does not have an incentive to hoard it in the cash reserve. Therefore, $d(n + \epsilon, z) =$

[23] In this argument, the non-negativity constraint does not matter, as $n'(\widetilde{n}, z) > 0$.

$d(n,z)+\epsilon$, if $d(n,z)>0$. Then, from a firm's budget constraint,

$$\begin{aligned} q^n n'(\widetilde{n}+\epsilon,z) &= \widetilde{n}\epsilon + \pi(z) - d(\widetilde{n}+\epsilon,z) \\ &= \widetilde{n} + \pi(z) - (d(\widetilde{n}+\epsilon,z)-\epsilon) \\ &= \widetilde{n} + \pi(z) - d(\widetilde{n},z) \\ &= q^n n'(\widetilde{n},z). \end{aligned}$$

Therefore, any extra increase in the current cash stock $\widetilde{n}$ does not change the future cash stock:

$$q^n n'(\widetilde{n},z) = q^n n'(\widetilde{n}+\epsilon,z),$$

which is a contradiction. Therefore, there exists the target cash stock $\overline{n}(z)$. ■