Linking Lifecycle and Cross-sectional Inequality: Cohort Dynamics and the Role of Technological Change

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Abstract

This paper examines the determinants of cross-sectional and lifecycle inequality using a lifecycle earnings process model that incorporates earnings mobility and non-employment risks across birth cohorts and over time. Unobserved skill prices and the variance of individual fixed effects across cohorts are the primary drivers, with unemployment risk being a key determinant of life cycle inequality. To explain the increase in the variance of fixed effects, we interpret these within a Roy model as realized productivity influenced by both ability and task choice. We provide evidence that technological change can amplify inequality by widening the variation in individual productivity, in addition to the standard mechanism that operates through price changes.

Keywords: Wage Inequality, Lifecycle Earnings, Skill-biased Technological Change

JEL codes: J23, J24, J31, D31

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1. Introduction

Most research on wage and earnings inequality focuses on cross-sectional inequality (e.g., Katz and Murphy, 1992; Card and DiNardo, 2002; Lemieux, 2006; Goldin and Katz, 2007; Dustmann et al., 2009; Moretti, 2013). However, welfare implications depend on the underlying earnings dynamics, as high cross-sectional inequality could reflect rapid earnings growth and temporary wage fluctuations or stem from persistent disparities starting at labor market entry, resulting in low or high lifecycle inequality, respectively. Although income process models have been widely used to study earnings dynamics, research linking and understanding the determinants of cross-sectional and lifecycle inequality is limited. While research incorporating non-stationary shocks into earnings dynamics often focuses on age- or year-dependent variations (e.g., Haider, 2001; Baker and Solon, 2003; Moffitt and Gottschalk, 2012; Debacker et al., 2013; Blundell et al., 2015), it rarely considers heteroskedasticity in permanent wage shocks across cohorts, thus limiting our understanding of how lifecycle inequality evolves.[2] Moreover, cross-sectional wage measures often overlook the elevated non-employment risks low-wage workers face.

This paper addresses this gap by analyzing cross-sectional and lifecycle inequality across labor market entry cohorts in Germany, estimating a lifecycle earnings process model that incorporates multiple sources of non-stationarity in wage shocks and non-employment risks across birth cohorts and over time. Specifically, the distribution of individual fixed effects depends on birth cohorts, while skill prices and the distribution of other shocks vary by calendar year. This framework enables us to decompose inequality into cohort-specific lifecycle and year-specific cross-sectional components, facilitating a direct comparison. We also estimate a process of employment-non-employment transitions to capture the correlation between wages and job insecurity, which we show is crucial for understanding lifecycle earnings inequality but matters less for cross-sectional inequality.

Our wage process separates the cohort-specific individual fixed effects from the

[2] Another strand of research relies on models with stationary shocks (e.g., Lillard and Willis, 1978; Lillard and Weiss, 1979; MaCurdy, 1982; Meghir and Pistaferri, 2004; Guvenen, 2009).

time-varying skill price component. Our analysis reveals that the overall increase in cross-sectional inequality is equally attributable to changes in price and variance of individual fixed effects. A key contribution of our paper is to rationalize this within an extended Roy model, where technological change enhances returns to complex tasks and incentivizes individuals to sort across sectors, leading to an increase in the cohort-specific variance of the unobserved productivity component of individual fixed effects.

Our paper begins with a descriptive analysis using German administrative data from 1975 to 2019, showing that cross-sectional inequality in Germany expanded from the late 1980s but plateaued after the Great Recession (see also Dustmann et al., 2024). We also find that wage inequality at age 25 follows a similar trend to cross-sectional inequality, and that wage mobility decreases for later cohorts. These results suggest that significant inequality arises early in workers' careers and tends to persist throughout their working lives. Furthermore, life-cycle earnings inequality for birth cohorts from 1950 to 1985, defined as total labor earnings between ages 25 and 34, increased by more than 25 percent up to the 1975 cohort but declined by about 5 percent for cohorts born between 1975 and 1985. Consequently, while cross-sectional inequality has remained stable over the past decade, life-cycle inequality has decreased among more recent cohorts.

We then turn to our model of the wage and employment process to uncover the mechanisms driving the evolution of cross-sectional and lifecycle inequality. Similar to findings in the U.S. (Guvenen et al., 2022), we identify a substantial increase in the variance of individual fixed effects. Comparing the 1955 and 1994 birth cohorts, this variance nearly doubled. Additionally, we demonstrate that the price of unobserved skills rose until the Great Recession but declined slightly thereafter. The rise in cross-sectional inequality is primarily explained by changes in the variance of individual fixed effects and skill prices. Our model also replicates the recent plateau in cross-sectional inequality, which is attributed to a reduction in the magnitude of persistent wage shocks. Meanwhile, transitory wage shocks and non-employment play minor roles in shaping cross-sectional inequality, although the magnitude of transitory shocks temporarily increased around the Great Recession.

We further investigate the effect of non-employment risk on cross-sectional inequality. While the non-employed are excluded from such analysis, selection could still

lead to non-employment influencing cross-sectional inequality. We illustrate that while such selection is indeed present, its magnitude is too small to affect cross-sectional inequality meaningfully. The employment margin is, however, crucial for lifecycle earnings, given that low-wage earners experience greater job instability and non-employment is persistent over time. Our findings reveal that the decline in lifecycle inequality after the 1975 birth cohort is primarily driven by shorter and less frequent non-employment spells in recent years.[3]

Earnings mobility links cross-sectional and lifecycle inequality. Studying its evolution, we find a decline in earnings mobility starting with the 1966 birth cohort, which stabilizes after the 1975 cohort. This decline was mainly driven by the stark increase in the variance of individual fixed effects and the price of unobserved skills. Moreover, changes in returns to and composition of observables had a small positive impact on mobility after the 1975 cohort, partly offsetting the negative effect of the variance of individual fixed effects and the price of unobserved skill.

A key finding of our analysis is that the increase in the variance of unobserved wage determinants is driven not only by an increase in the variance of unobserved skill prices—commonly cited as the primary explanation in earlier literature (see, e.g., Juhn et al. 1993)—but also by a rise in the variance of the individual fixed effects. While the individual fixed effects are commonly interpreted as ability or talent, it seems implausible that subsequent generations experience such substantial changes in talent distribution. To provide an economic interpretation of the rising variance in individual fixed effects, we develop a Roy model of within-occupational task choice and skill-biased technological change. In this model, the individual fixed effect measures realized productivity, shaped by *both* individual unobserved innate ability and task choice, rather than being a direct measure of ability only. Thus, the variance of individual productivity *within* an existing talent pool can increase due to technological change.

Specifically, workers choose to perform routine or complex tasks. Since productivity depends more heavily on ability in complex tasks, high-ability workers have

[3] This is consistent with Boenke et al. (2015a), who highlight the significance of unemployment in shaping lifecycle inequality through Gini coefficient analysis.

a comparative advantage in complex tasks while low-ability workers have a comparative advantage in routine tasks. Technological change draws a growing share of low-ability workers into complex tasks. Because ability translates more directly into productivity in complex tasks than in routine tasks, this shift in the composition of workers across task sectors amplifies inequality. This mechanism complements the increase in the relative price of complex tasks in the canonical model of technological change, as described by e.g. Acemoglu and Autor (2011). Our framework accounts for the rise in the variance of workers' unobserved productivity without requiring changes in the underlying distribution of ability.

To test our model, we focus on occupations as labor market units with a complex and routine subsector, where the size of the complex sector varies. This is motivated by the observation that three-quarters of the increase in the variance of unobserved individual heterogeneity occurs *within* occupations. Our model predicts that the effect of worker sorting on inequality is higher in occupations with low or high initial complexity and smaller in occupations with moderate initial complexity. In line with these predictions, we show that occupations with the highest and lowest share of initial complexity experienced the largest increase in entry-level wage inequality, while those with moderate complexity experienced the lowest increase.

The contribution of our paper is three-fold. First, this is the first study that examines cross-sectional and lifecycle inequality through the lens of a lifecycle earnings model to quantify the relative importance of wage determinants and non-employment risk. Building on the longstanding literature that uses income process models to disentangle sources of lifecycle inequality (MaCurdy, 1982; Abowd and Card 1989; Moffitt and Gottschalk 1995), we add by incorporating cohort-specific non-stationarity, which proves essential to account for lifecycle inequality across cohorts. Moreover, much of the literature does not consider employment-non-employment transitions, either by focusing on workers for whom non-employment is less important (Baker and Solon, 2003; Guvenen, 2009) or by analyzing earnings, which implicitly account for non-employment as a part of earnings shocks (Haider, 2001; DeBacker et al, 2013; Blundell et al, 2015; Boenke et al., 2015b). An exception is Guvenen et al. (2021), who developed a rich model that explicitly accounts for non-employment risk but does not model inequality across

cohorts. We add to that by quantifying the contribution of the different sources on the evolution of cross-section and lifecycle inequality and wage mobility.

Our second contribution is to provide an interpretation of the role of the individual fixed effect in the wage process model. Individual fixed effects are usually interpreted as individual ability with a time-invariant distribution. This interpretation contrasts with our finding that the variance of the individual fixed effects changes across cohorts. We provide a new interpretation where individual fixed effects are regarded as realized productivity due to a worker's task choice within occupations. Incentivized through technological change, task changes increase the variance of realized productivity, holding the ability distribution constant. Our model thus also explains findings in the literature that the expansion of wage inequality is due to assortative matching between workers and firms (Card et al., 2013, 2018; Song et. al. 2013). Since task specialization may occur at the firm level and high-productive firms specialize in complex tasks, skill-biased technological change incentivizes high-productive workers to sort into high-productive firms via higher wage premiums for complex tasks. Consequently, their ability is better utilized and reflected in wages, increasing inequality.

Lastly, our study contributes to the literature on German wage inequality by providing new evidence on the sources of the evolution of lifecycle inequality across cohorts. Most existing research focuses on cross-sectional inequality (e.g., Dustmann et al., 2009; Card et al., 2013; Goldschmidt and Schmieder, 2017; Biewen et al. 2018; Biewen et al., 2019; Drechsel-Grau et al., 2022; Bossler and Schank, 2023; Dustmann et al., 2024). A few exceptions focus on Gini coefficients but do not quantify the contribution of stochastic wage determinants to lifecycle inequality (Boenke et al. 2015a) or estimate a lifecycle wage model but do not consider employment-non-employment transitions (Boenke et al. 2015b). Our study jointly models the stochastic wage process and employment-non-employment transitions to disentangle the sources of the evolution of lifecycle inequality in the German context.

The remainder of this paper is structured as follows: Section 2 describes the data, and Section 3 shows our wage and employment process model. Section 4 discusses the estimation results and counterfactual simulation results. Section 5 demonstrates the economic model to interpret the estimation results. Section 6 concludes this study.

2. Data and descriptive analysis

2.1. Data

Our analysis uses administrative data from the Sample of Integrated Labour Market Biographies (SIAB) from 1975 to 2019. [4] The SIAB is a two percent random sample of the Integrated Employment Biographies (IEB), containing detailed employment records for individuals covered by social security and records of unemployment benefit receipts. This dataset enables us to track individual employment histories across the entire 1975–2019 period. In addition to daily wages and employment duration, the data provides rich information on workers and jobs, including age, gender, educational attainment, occupation, industry, and workplace characteristics (see Frodermann et al., 2021, for further details). Dating back to 1975, it enables us to analyze wage inequality over a span of 45 years. Since the SIAB is based on administrative records, daily wages are measured with high precision. This is crucial because measurement errors in wages can complicate the estimation of wage process models (see Bound and Krueger, 1991; Meghir and Pistaferri, 2011).

While the SIAB does not cover self-employed individuals, family workers, or civil servants, it represents about 85 percent of employment in 2019 (DESTATIS, 2023). Wages in the SIAB are top-coded at the upper limit of social security contributions. To address this, we apply a two-step imputation method, following the approaches of Gartner (2005), Dustmann et al. (2009), Card et al. (2013), and Dauth and Eppelsheimer (2020).

When estimating the wage process model, we focus on males to avoid complications arising from voluntary labor force participation choices and variations in hours worked. We also exclude East Germans, as records are unavailable prior to 1991.[5]

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[4] Data access was provided via a Scientific Use File supplied by the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB)

[5] Since birthplace information is unavailable in our data, we follow Boelmann et al. (forthcoming) and classify workers as West German if their first recorded workplace in the dataset is located in West Germany.

We restrict the sample to individuals aged 25 to 59 to focus on prime-age workers.[6] Since the SIAB does not contain detailed data on hours worked—only indicators for full-time, part-time, or marginal employment—we limit the sample to full-time employment records to ensure that daily wages reflect full-time work. Specifically, we exclude individuals who held part-time or marginal jobs between the ages of 25 and 59. However, since most prime-age men work full-time, our results are not sensitive to this restriction and remain largely unchanged when part-time workers are included in the analysis sample. Additionally, we exclude individuals who had not completed their apprenticeship or university studies or were not employed by age 25 (the starting point of our lifecycle definition). To address data quality concerns, we also drop the top two percent of wages to eliminate unrealistic imputed values and the bottom two percent to account for the potential misclassification of part-time employment as full-time (Fitzenberger and Seidlitz, 2020).[7]

Our analysis uses two measures of labor income: wages and lifecycle earnings. Wages are calculated as the total wages earned in full-time employment within a calendar year, divided by the total number of days spent in full-time employment during that year.[8] While wages are useful for analyzing the wage distribution within a given year, such as cross-sectional wage inequality, they provide limited insight into earnings over the lifecycle, which is more relevant from a welfare perspective. Therefore, we measure lifecycle earnings as the total labor earnings accumulated between ages 25 and 34,

$$Lifecycle\ Earnings = \sum_{Age=25}^{34} Yearly\ Earnings_{Age}$$

Although our definition of "lifecycle earnings" is not full lifecycle earnings between ages 25 and 59, we focus on earnings during these ten years to extend our analysis to recent

[6] Our target population is similar to Baker and Solon (2003), who analyze men between 25 and 58.

[7] The wage here refers to the annual average wage defined below.

[8] In calculating wages, we assign zero weight to unemployment spells. Following Stueber et al. (2023), we allocate separately recorded one-time payments to the corresponding regular employment spells with the same employer within the same calendar year. If a worker holds multiple jobs simultaneously, we consider only the job with the highest daily wage.

cohorts. As Guvenen (2022) emphasizes, most wage growth occurs during the first ten years (see also Topel and Ward, 1992), and in fact, we show that this is a valid proxy for lifecycle earnings in Section 2.2.

This approach allows us to compute lifecycle inequality for cohorts born between 1950 and 1985. To ensure comparability, we calculate cross-sectional inequality for workers aged 25 to 34 unless otherwise specified. For the lifecycle inequality analysis, we focus on workers with a total employment duration of at least three years over the ten-year period. All wages and earnings are adjusted to 2015 values using the consumer price index provided by the German Federal Statistical Office.

In addition to the SIAB, we use the BIBB/IAB Career and Qualification Survey, which asks workers about the frequency of various tasks performed in their jobs. Task content from this survey has been used in the German context, for example, by Spitz-Oener (2006) and Black and Spitz-Oener (2010). We use this information to characterize occupations by their tasks, similar to O*NET data in the U.S. Specifically, we rely on the 1991/1992 survey to define a measure of each occupation's complexity and merge this information with the SIAB at the occupation level. See Section 5 for further details.

2.2. Evolution of Cross-sectional and Lifecycle Inequality

Figure 1 Panel A shows the trend in cross-sectional inequality for our sample, which began rising in the early 1990s and accelerated during the 2000s, driven by declining earnings at the lower percentiles. After the Great Recession, earnings grew sharply across the distribution, with no further expansion in inequality during this period. This pattern is similar when considering the entire population of West German male workers (see Figure 1 Panel B).[9]

Figure 1 Panel C shows that the evolution of wage inequality at age 25 closely mirrors overall cross-sectional inequality, suggesting that cross-sectional inequality is driven by inequality in entry-level wages, similar to patterns observed in the U.S. (see Guvenen et al., 2022). Due to the lag before entry-level wage inequality is fully reflected

[9] We do not show the 90th percentile, because wages at this level are censored in the overall distribution due to the social security contribution limit.

in cross-sectional inequality, the rise in entry-level inequality occurred throughout the 1990s and mid-2000s, driven by declines in the lower percentiles. After the Great Recession, entry-level wages increased across the entire distribution, stabilizing inequality. We show that entry-level wages are the primary driver of wage inequality more formally when estimating a wage process model in Section 3, where we decompose wage inequality into key determinants, such as individual fixed effects, skill price, persistent wage shocks, and transitory wage shocks.

Figure 1 Panel D illustrates the trend in lifecycle earnings across different birth cohorts, measured as the sum of earnings between ages 25 and 34. The median and lower percentiles declined for cohorts born up to the late 1970s, though the 10th percentile saw a slight increase among early 1970s birth cohorts that entered the market from 1995 onwards. In contrast, the 90th percentile was relatively flat until the 1970 cohort and exhibited steady growth from the 1970 cohort onward. Lifecycle earnings increased substantially for more recent cohorts, especially at the lower end of the distribution. As a result, the 90-10 percentile log difference follows a bell-shaped pattern, peaking with the 1975 birth cohort (see Figure A1). Lifecycle earnings inequality increased by approximately 25 percent from 1960 to the 1975 birth cohort and decreased by about 5 percent from the 1975 birth cohort to the 1985 birth cohort.

As inequality in entry-level wages expanded among early cohorts, the increase in lifecycle inequality indicates persistence of initial inequality over the lifecycle. We study this in more detail in our wage process model in Section 3. It should be noticed, however, that the evolution of entry-wage inequality cannot account for the shrinking lifecycle inequality observed in recent cohorts. While wage inequality at age 25 (measured as the p90p10 log wage differential) plateaued in the mid-2000s, it did not decline afterward, in contrast to life cycle inequality. To understand this difference, the employment margin—often overlooked in cross-sectional inequality studies—plays a crucial role.

During the period we studied, the German unemployment rate experienced significant fluctuations (Figure 2 Panel A). Starting at a very low level of below 3 percent in 1980, unemployment rate rose sharply in the early 1980s before moderately decreasing until the German reunification in 1990. In the 1990s, unemployment rate climbed to 10 percent and peaked at over 11 percent in 2005. Since then, it has steadily declined,

reaching a level slightly above 3 percent by 2019.[10]

We provide descriptive evidence on how unemployment rates can influence lifecycle earnings inequality in Figure 2 Panel B, which illustrates the correlation between wages at age 25 and total employment duration (in years) from age 25 to 34. The figure plots the mean and median total employment duration against each vintile (20 bins) of the wage distribution at age 25.[11] The figure reveals that low-wage workers are particularly vulnerable to non-employment risk: workers in the lowest 5% of the wage distribution have a median employment duration of less than 8.8 years, whereas those at the top of the initial wage distribution are far more likely to remain employed for the full 10 years.

As unemployment rates have decreased since 2005, this aligns with the sharp rise in the 10th percentile of lifecycle inequality among recent cohorts (Figure 1 Panel D), which should have mostly benefited low-wage earners. Section 4 quantifies the contribution of the employment margin to the observed decline in lifecycle inequality.

Our primary analysis focuses on workers' lifecycles between 25 and 34. To validate that examining the first ten years of workers' careers—when most wage growth typically occurs—is a reliable proxy for lifecycle earnings, we follow the approach of Guvenen et al. (2022) and plot the median wage across ages and years in Figure 3 Panel A. In this figure, the solid lines depict median wages at each age across the calendar year, while the dotted lines illustrate the age profile within each cohort from the point of their labor market entry at age 25. For example, the first dotted line from the left-hand side allows us to track the evolution of the median log wage of the cohort born in 1960 at age 25 in 1985, age 30 in 1990, and so on. Among cohorts born in 1960, wage growth between ages 25 and 55 amounts to approximately 45 percent, with more than 60 percent occurring by age 34. A similar pattern emerges for other cohorts, reinforcing that total earnings during the first ten years provide a reasonable approximation of entire lifecycle earnings.

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[10] The Great Recession caused only a small and temporary increase in the unemployment rates.

[11] In this sample, the maximum possible employment duration is 10 years. To ensure we only consider individuals with a strong attachment to the labor market, we set the minimum employment duration at 3 years, as described above.

2.3. Evolution of Earnings Mobility

A critical link between cross-sectional inequality and lifecycle inequality is earnings mobility. For instance, in the absence of mobility, individuals at the bottom of the earnings distribution remain at the bottom in subsequent years, while those at the top consistently stay at the top. In such a scenario, cross-sectional inequality directly corresponds to lifecycle inequality. Conversely, in the case of high earnings mobility, where all earnings shocks are transitory, substantial cross-sectional inequality may exist without resulting in any lifecycle inequality. Thus, earnings mobility serves as an (inverse) indicator of the extent to which cross-sectional inequality reflects lifecycle inequality.

To assess earnings mobility, we compute the Shorrocks mobility index (Shorrocks, 1978) across cohorts as in Kopczuk et al. (2010). This mobility index is defined as the ratio of an inequality measure (we use the variance) of average long-term earnings over the average of that inequality measure of short-term earnings:

$$Shorrocks_c = 1 - \frac{Var\left(\frac{1}{10}\sum_{a=25}^{34} Earnings_{a,c}\right)}{\frac{1}{10}\sum_{a=25}^{34} Var\left(Earnings_{a,c}\right)}$$

The Shorrocks index ranges from 0 to 1, with higher values indicating greater earnings mobility. When the index is close to 1, the variance of long-term earnings is much smaller than short-term earnings. This implies that earnings variance within a given year is mainly transitory, and earnings mobility across the lifecycle is very high. Conversely, when the Shorrocks index is close to 0, the variance of long-term earnings closely resembles the variance of short-term earnings. In this case, earnings shocks in a given year are almost permanent, resulting in low earnings mobility.

Figure 3 Panel B depicts the evolution of the Shorrocks index across cohorts. We note that earnings mobility is almost flat from birth cohort 1960 to 1966, ranging between 0.35 and 0.40, and thus, the magnitude of the long-term variance is 60-65% of the short-term variance. It decreased to 0.3 for birth cohorts 1966 to 1975. After the 1975 birth cohort, earnings mobility appears to be stable. The decline in earnings mobility is consistent with our argument that entry-level wage inequality has become increasingly

important and persistent among recent cohorts. In Section 4, we quantify the effect of changes in the structure of wages and employment on the evolution of mobility.

3. Wage and Employment Process Model

We model the residual wage using a linear factor model that builds on Baker and Solon (2003) and Guvenen (2009) but incorporates non-stationarity with respect to cohort and time. Specifically, the model includes individual fixed effects α_{ic} (representing time-invariant productivity) and their associated prices p_t, along with an AR(1) persistent shock η_{it} and an idiosyncratic transitory shock ϵ_{it}. We allow the distribution of individual fixed effects to vary across entry cohorts c, while the price of unobserved skill and the distributions of other shocks can differ across calendar years t.

Thus, the log wage of worker i from cohort c, with education level e, and age a in year t is modelled as:

$$\ln w_{iceat} = \mu_{ceat} + p_t\alpha_{ic} + z_{iat} + \epsilon_{it}, \quad (1)$$

$$z_{iat} = \rho_t z_{i,a-1,t-1} + \eta_{it}, \quad (2)$$

where μ_{ceat} is the mean log wage given cohort, education, and age, and z_{iat} is a non-stationary AR(1) process. In this specification, μ_{ceat} collects the observed determinants of wages such as age and education, while $p_t\alpha_{ic} + z_{iat} + \epsilon_{it}$ corresponds to the unobserved wage determinants (residual wage). The mean and variance of each component are specified as follows:

$$E[\alpha_{ic}|c] = E[\eta_{it}|t] = E[\epsilon_{it} \mid t] = 0,$$
$$Var\,(\alpha_{ic} \mid c) = \sigma^2_{\alpha c}, Var\,(\eta_{it} \mid t) = \sigma^2_{\eta t}, Var\,(\epsilon_{it} \mid t) = \sigma^2_{\epsilon t}.$$

Notably, we allow the distribution of the individual fixed effects α_{ic} to vary across cohorts. We thus extend the variance decomposition proposed by Lemieux (2006), which assumes that the within-group variance of unobserved skills remains constant to identify changes in skill prices. Both the price of skill across time and the cohort-specific variance of individual fixed effects are identified, as the autocorrelation of wages provides

additional identifying variation for estimating the sources of the wage variance. Specifically, the variance of the individual fixed effects $\sigma_{\alpha c}^2$ is the baseline intercept of the autocovariance profile of each cohort, and the price p_t shifts this intercept in the same way across cohorts but differently across time. Therefore, with multiple cohorts each year, we can identify the skill price p_t across time and the cohort-specific variance of the individual fixed effects $\sigma_{\alpha c}^2$ using the auto-covariance profiles.

The model parameters are identified and estimated by using the autocovariance of the residualized wage, $\hat{y}_{iceat} = \ln w_{iceat} - \mu_{ceat}$:

$$Cov\left(\hat{y}_{iceat}, \hat{y}_{ice,a+d,t+d}\right) = p_t p_{t+d}\, \sigma_{\alpha c}^2 + \left(\prod_{j=!}^{d} \rho_{t+j}\right) \sum_{k=0}^{a} \left(\prod_{s=0}^{k-1} \rho_{t-s}^2\right) \sigma_{\eta, t-k}^2 + 1\{d = 0\}\sigma_{\eta t}, \quad (3)$$

where a is normalized to be 0 at age 25, and the skill price p_t is normalized to be one in the initial year. We compute the empirical autocovariances for all cohorts and collect them in a single vector $\hat{\pi}$ to minimize

$$\min_{\theta \in \Theta} [\hat{\pi} - h(\theta)]' W\, [\hat{\pi} - h(\theta)],$$

where θ is the set of parameters in equations 1 and 2, $h(\theta)$ is a vector-value function represented by equation 3, and W is a diagonal weighting matrix whose element equals the number of observations for each component.[12] To manage the large number of parameters in our model, we group three consecutive calendar years and cohorts. This approach reduces the parameter space and enhances the precision of our estimates.[13] We estimate the model on all available yearly wage observations of individuals between age

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[12] Although the inverse of the variance-covariance matrix of the reduced-form estimator $\hat{\pi}$ is considered the asymptotically optimal weighting matrix, Altonji and Segal (1996) argue that it performs poorly in finite samples. Therefore, following the approach of Baker and Solon (2003) and Guvenen (2009), we use the number of observations contributing to each autocovariance as weights.

[13] For example, we specify the price and individual FE as $p_{1975} = p_{1976} = p_{1977} \neq p_{1978} = p_{1979} = p_{1980} \neq p_{1981} = \cdots$, and $\sigma_{\alpha 1950}^2 = \sigma_{\alpha 1951}^2 = \sigma_{\alpha 1952}^2 \neq \sigma_{\alpha 1953}^2 = \sigma_{\alpha 1954}^2 = \sigma_{\alpha 1955}^2 \neq \sigma_{\alpha 1956}^2 = \cdots$, and so on.

25 and 59.

We specify the employment-non-employment transition and non-employment duration as follows:

$$\text{Non-employment risk: } \Pr(U_{iat} = 1 \mid a, w_{i,t-1}, U_{i,t-1}) = q(a, t, w_{it-1}, U_{it-1}), \quad (4)$$

$$\text{Non-employment duration: } \ddot{v}_{iat} = \begin{cases} 0 \text{ if } U_{iat} = 0, \\ \min\{1, v_{iat}\} \text{ if } U_{iat} = 1, \end{cases} \quad (5)$$

where v_{iat} follows an exponential distribution as in Guvenen et al. (2021), and the distribution is characterized by calendar year, age, and the wage and employment status in the previous year with $v_{iat} \mid a, t, w_{t-1}, U_{t-1} \sim \exp\big(\lambda(a, t, w_{t-1}, U_{t-1})\big)$. The non-employment probability is directly and nonparametrically estimated as the non-employment rate in a year t given age, the wage quartile group, and the non-employment status of the previous year.[14] Since the non-employment duration is modeled as a truncated exponential distribution, the expected non-employment duration is:

$$E[\ddot{v}_{iat} \mid a, t, w_{i,t-1}, U_{i,t-1} = 1] = \frac{1 - \exp\big(\lambda(a, t, w_{t-1}, U_{t-1})\big)}{\lambda(a, t, w_{t-1}, U_{t-1})}. \quad (6)$$

The parameter $\lambda(a, t, w_{t-1}, U_{t-1})$ is estimated by solving the non-linear equation (6), where the left-hand side is replaced by the corresponding empirical expression.

While our model accounts for the correlation between earnings capability and non-employment risk through equations (4)–(6), it does not explicitly capture job-displacement effects on wages. Prior studies such as Jacobson et al. (1993) or Arulampalam (2001) show that job displacement effects tend to be persistent, which would be absorbed in persistent shocks in our model. As a result, we may understate the role of non-employment risk while overstating the impact of persistent shocks. Despite

[14] In practice, we construct five groups based on the wage and employment status from the previous year. Four groups correspond to employed individuals, categorized by wage quartiles, while the fifth group consists of those who were non-employed.

this limitation, our model fits the data well, as we show in Section 4. Furthermore, given the direction of the potential bias—that we might understate the role of non-employment risk and overstate the impact of persistent shocks—this caveat does not undermine our argument that the employment margin plays a crucial role in explaining the evolution of the lower tail of the lifecycle earnings distribution.

Our model follows a restricted income process (RIP). An alternative, a heterogeneous income process (HIP) model, allows the age profile to vary across individuals through random coefficients, while the RIP model absorbs this heterogeneity in wage growth in the persistent shock z_{iat}. We do not adopt the HIP model for three reasons. First, as Guvenen (2009) emphasizes, the identification of HIP heavily depends on higher-order autocovariances. Since few individuals contribute to these higher-order autocovariances, the resulting estimates can be challenging to interpret when using the full sample. Furthermore, as noted by Guvenen (2009), ensuring internal validity would require restricting the analysis to individuals with long-term wage observations (20 years in his case). Such a restriction poses significant challenges for analyzing wage inequality, and even more so for lifecycle earnings inequality, as it would necessitate focusing solely on workers with continuous employment. Additionally, identifying heterogeneous profiles for recent cohorts becomes difficult due to the lack of observable higher-order autocovariances. Second, the evolution of cross-sectional wage inequality closely mirrors the trajectory of entry-level wages (see Section 2), suggesting that individual fixed effects and their associated pricing play a more critical role than changes in wage growth. As such, the sources of wage variance throughout a career—whether RIP or HIP—are not central to our argument. Third, our current model already involves estimating a relatively large number of parameters. Incorporating a non-stationary HIP across cohorts would substantially increase computational complexity.

4. Results

4.1. Parameter Estimates

Panels (A) – (E) of Figure 4 present estimates of the parameters characterizing equations (1) and (2). Panel A illustrates the variance of the individual fixed effects (FE) $\sigma_{\alpha c}^2$, revealing a clear trend across cohorts. For the 1950 cohort, the variance is approximately 0.015, nearly doubling over the subsequent 45 years. As a result, the standard deviation of individual FE rises from 0.12 to 0.17, representing a 40 percent increase. This rise accounts for about half of the growth in the standard deviation of log wages during the same period (from 0.24 to 0.34). Since individual FE represent the permanent component of wages, these results underscore our descriptive evidence that inequality at market entry is a key driver of wage inequality, persisting over the lifecycle.[15]

The skill price, p_t, increased by approximately 50 percent between 1975 and 2011, remained stable until 2015, and declined slightly thereafter (Panel B). From 2002 onward, the estimates are statistically different from one at the five percent level. This overall trend in the (unobserved) skill price aligns broadly with the findings of Dustmann et al. (2024), who document a trend reversal in the evolution of the skill premium in Germany following the Great Recession. While Dustmann et al. (2024) analyzed prices for observed skills, our analysis complements their findings by demonstrating that the price for unobserved skills also ceased to increase after the Great Recession.

Panel C shows estimates of the parameter ρ_t that characterizes the persistency of the AR(1) process. The parameter is very close to one, suggesting that persistent shocks to wages are almost permanent for most of our sample period. Panel D depicts the estimates of the variance of the persistent shock $\sigma_{\eta t}^2$ of the AR(1) process. This parameter fluctuates around 0.005 in the first half of the period and then declines after 2000. This implies that the variance of accumulated persistent shocks over s years can be approximated by $s\sigma_{\eta t}^2$. Given the average estimated magnitude of the persistent shock

[15] Our findings for Germany are similar to those reported by Guvenen et al. (2022) for the United States, who show that the standard deviation of the initial wage at age 25, regarded as a proxy of individual fixed effect dispersion, increased by about 50 percent between 1968 and 2011.

$\sigma_{\eta t}^2$ of about 0.005, the accumulated variance $s\sigma_{\eta t}^2$ amounts to 0.05 ten years after labor market entry, which is comparable to the order of magnitude of the composite variance of the FE and the skill price ($p_t^2\sigma_{\alpha c}^2$). Therefore, the persistent shock η_{it} reflects sizable wage dispersion over the career. However, its change over time is smaller than in the FE variance, though we observe a non-negligible decrease in persistent shocks in recent years.

Finally, Panel E presents the estimates for the transitory shock ϵ_{it}, which remains relatively stable over time. For most of the period, the estimated variance $\sigma_{\epsilon t}^2$ ranges between 0.008 and 0.010, but it rises to almost 0.012 between 2005 and 2010—coinciding with the Great Recession.[16] This indicates that our model captures the macroeconomic shocks influencing wage volatility. The order of magnitude of the shock is comparable to the variance of the yearly innovation of the persistent shock $\sigma_{\eta t}^2$. However, as the transitory shock does not accumulate over time, our wage process model suggests that these temporary shocks are less influential than permanent or persistent shocks in driving wage dynamics over the long run. We confirm this in our counterfactual simulation in Section 4.

As shown in Figure 4, the parameters for persistent and transitory shocks are estimated with low precision for 1975–1977. This arises from weak identification, as our sample includes only individuals born in 1950 or later. Consequently, we observed just one cohort in 1975, two cohorts in 1976, and three in 1977. Distinguishing between persistent and transitory shocks relies on the autocovariance of wages; however, for these cohorts, the autocovariance matrix contains only four off-diagonal elements. Since estimating the AR(1) process requires two parameters, identification is barely achieved, resulting in large standard errors for these estimates. For this reason, we do not use these years as the baseline in the counterfactual simulations presented below.

Figure 5 summarizes the estimation result of the employment part of the model. The two histograms, in red and blue, show the distribution of the non-employment duration parameter λ among those non-employed (blue) and employed (red) in the previous period on the left y-axis. The black line depicts the relation between λ and the

[16] Note that the parameters are constrained to be constant within the intervals 2005–2007 and 2008–2010.

expected non-employment duration on the right y-axis, suggesting that a value of λ of 0 corresponds to full non-employment over the entire year (0 days of employment), while a value of 5 corresponds to a non-employment length in a given year of 20 percent (i.e., 80 percent of the days in a given year the person is employed).

Focusing first on those who were non-employed (blue histogram), the location of the distribution shows that those who were previously non-employed are likely to remain non-employed for the entire year ($\ddot{\nu}_{iat} = 1$) if they do not get reemployed, suggesting that non-employment is very persistent. In contrast, the distribution of λ for those who became non-employed this year but were employed last year (red) shows a much larger dispersion. For example, a worker with $\lambda = 1.5$ experienced roughly 6 months (50 percent of the year; see black solid line) of non-employment, whereas a worker with $\lambda = 0.75$ experienced 8.4 months (70 percent) of non-employment.

4.2. Counterfactual Simulation

We perform counterfactual simulations using the estimated wage process and non-employment risk parameters to disentangle and quantify the relative importance of the different sources of cross-sectional and lifecycle inequality. The steps of our simulation procedure are as follows:

1. Replicate the demographic composition (educational distribution and cohort size) at age 25.
2. Simulate log daily wages $\ln w_{iceat}$ between the ages 25 and 34 for all individuals according to equations (1) and (2), where all shocks are drawn from a normal distribution with mean zero and respective estimated variance for year t or cohort c.
3. Using the estimated (potential) wages from step 2, simulate the employment status U_{iat} and non-employment duration $\ddot{\nu}_{it}$ according to equations (4) and (5), where all individuals are assumed to be employed at age 25 (as in our sample restriction).
4. Calculate earnings, where those workers who are non-employed in step 3 receive only part of their yearly wages according to their simulated employment duration $1 - \ddot{\nu}_{it}$, computed as $\hat{e}_{it} = (1 - \ddot{\nu}_{it}) \exp(\mu_{ceat} + p_t \alpha_{ic} + z_{iat} + \epsilon_{it})$.

Appendix A provides a detailed description of our simulation procedure.

4.2.1. Counterfactual Simulation Scenarios

To perform the counterfactual simulation, we set the variance of the individual fixed effects (FE) $\sigma_{\alpha t}^2$ to its estimated value for the 1955 birth cohort while keeping all other parameters at their estimated values for 1980, the entry year of the 1955 cohort. We then simulate the wage distribution under various scenarios, sequentially allowing additional parameters to change over time. In scenario CF1, only the demographic and educational characteristics and their returns μ_{eat} are allowed to change. Scenario CF2 adds the variance of the individual FE $\sigma_{\alpha t}^2$. Scenario CF3 instead allows the skill price p_t (but not $\sigma_{\alpha t}^2$) to change, and in scenario CF4, both $\sigma_{\alpha c}^2$ and p_t are allowed to vary over time. Scenario CF5 adds changes in the estimated values of the persistent shock parameters ρ_t and $\sigma_{\eta t}^2$, and scenario CF6 additionally allows for changes in the transitory shock variance $\sigma_{\epsilon t}^2$. Scenario CF7 corresponds to our full model allowing also the non-employment risk and length to change over time.

4.2.2. Cross-Sectional Inequality

We present the counterfactual simulations for cross-sectional inequality in Figure 6.[17] As discussed in Section 2, the p90-p10 log difference of cross-sectional wage inequality—shown in solid black—was approximately 0.57 in 1985 and increased to around 0.83 by the onset of the Great Recession.

Scenario CF1 (dotted black line) fixes all parameters of the stochastic component of the wage process and non-employment probabilities at their estimated levels for 1980 and the 1955 cohort, allowing only the composition and returns to observables to vary over time. The level of cross-sectional inequality in this scenario is relatively close to the observed data (solid black line) but increases only marginally from about 0.53 to 0.58. Thus, changes in the composition and average returns to observables—specifically age, experience, and education—account for a significant portion of the *level* of inequality but

[17] In the simulation, cross-sectional inequality is calculated for individuals employed for at least one day within a given year, while lifecycle inequality is measured for individuals with at least three years of employment between ages 25 and 34, ensuring alignment with our sample restrictions.

do not explain the observed trend over time. This leaves the largest share of the trend in wage inequality to be explained by the stochastic component of wages or the employment margin.

Scenario CF2 (solid blue with small dots) additionally allows the variance of the individual FE $\sigma_{\alpha c}^2$ to vary across cohorts. This addition explains roughly one-third of the increase in wage inequality. Scenario CF3 (dotted blue line) instead allows observables and the skill price p_t (but not $\sigma_{\alpha c}^2$) to vary across time. Comparing the change from CF1 and CF3 with the change from CF1 to CF2, we see that the changes in the variance of the individual fixed effects and the skill price are roughly equally important in explaining the increase in cross-sectional inequality over our sample period. Scenario CF4 (solid red with symbol markers "v") allows both the variance $\sigma_{\alpha c}^2$ and the price p_t to vary together with observables. The combined effect explains almost the entire increase in cross-sectional inequality.

Next, to examine the role of persistent shocks, we additionally allow both the persistency ρ_t and the variance of the AR(1) innovation $\sigma_{\eta t}^2$ to vary over time (CF5, dotted red line). As suggested by the stable parameter estimates (Figure 4), the contribution of the AR(1) process, visible in the change from CF4 to CF5, is relatively modest. Although it slightly raises the p90-p10 ratio, this is primarily due to the low persistence parameter ρ_t, estimated to be around 0.9 in 1980. However, the decline in the variance of persistent shocks after 2000 (Figure 4 Panel D) helped mitigate the rise in inequality. Since the persistent shock η_{it} accumulates over time, the impact of its reduced variance was not immediately apparent in the early 2000s but became more significant during the 2010s. The stabilization of cross-sectional inequality during the 2010s can thus be attributed to this decreasing variance in persistent shocks, combined with stagnant growth of the skill price and the variance of the individual FE. Without it, our model would have predicted a slight increase in inequality over this period.

Scenario CF6 also allows for changes in the transitory shock (solid green line with "x" markers). While the variance of the transitory shock increased by 20 percent during the Great Recession (Figure 4 Panel E), its contribution to the rise in inequality is much smaller than that of the combined contribution of the variance of individual fixed effects and skill price, both in terms of magnitude and change over time.

The final scenario (CF7, dotted green line) corresponds to our full model, where all parameters, including the employment margin, can vary. The additional effect of changes in the employment margin (comparing CF6 to CF7) is negligible. Although cross-sectional wage inequality, by definition, only accounts for workers employed in a given year, non-employment can still influence inequality if selection into non-employment alters the wage distribution. However, our results suggest that changes in selection into non-employment had little impact on the observed trends in inequality.

To further examine the role of compositional changes through changes in selection into non-employment, we compute two additional counterfactual scenarios, one an extreme case where non-employment happens only at the bottom of the wage distribution and one where non-employment risk is uncorrelated with wages (*No Selection,* see Appendix B). In the first scenario, cross-sectional inequality would have been, on average, lower and continued to rise after 2010 as unemployment rates declined, with the lowest wage earners transitioning from non-employment into the wage distribution. However, our full model closely aligns with a *No Selection* scenario, suggesting that differences in non-employment probabilities across the wage distribution do not meaningfully impact the cross-sectional wage distribution. However, even if the employment margin is negligible in shaping cross-sectional inequality, the extensive margin of employment may play an important role in shaping lifecycle inequality, as discussed below.

4.2.3. Life-Cycle Inequality

Figure 7 presents the counterfactual simulation analysis of lifecycle inequality. The solid black line illustrates the evolution of lifecycle inequality across birth cohorts, as discussed in Section 2. Lifecycle inequality increased by approximately 25 percent from the 1960 birth cohort, which entered the labor market at age 25 in 1985, to the 1975 birth cohort, which entered in 2000. It then decreased by about 5 percent by the 1985 birth cohort.

Unsurprisingly, the decompositions reveal several similarities with those of cross-sectional inequality. Notably, the combined rise in the variance of individual fixed effects (FE) $\sigma_{\alpha c}^2$ and skill prices p_t—captured in scenario CF4—drove the steady increase in

lifecycle inequality throughout the sample period.[18] For instance, compared with the 1960 cohort, lifecycle inequality among the 1975 cohort is almost 25 log points higher. Changes in the variance of individual FE and skill prices alone contributed more than 60 percent to this increase but about 40 percent of the increase remain unexplained unlike in the cross-sectional inequality. Moreover, the role of the persistent shock was limited. Finally, the contributions of demographic composition, its returns, and the transitory shock were negligible.

A distinctive feature of lifecycle inequality is the role of non-employment risk. While studies on cross-sectional inequality often overlook this dimension—and, as shown above, it has only a minor impact on cross-sectional inequality—it significantly influences both the level and trend of lifecycle inequality. For instance, for the 1975 birth cohort, the log of the p90-p10 ratio of lifecycle earnings was approximately 0.85 when non-employment risk was excluded (fixed at the low unemployment level of 1980, Scenario CF6) but exceeded 1.0 when employment dynamics were incorporated (Scenario CF7). Furthermore, the wage process alone fails to account for the inverted U-shaped trend in lifecycle inequality. The employment margin is crucial for explaining the decline in lifecycle inequality observed after the 1975 cohort. Ignoring it not only distorts the level of inequality but also introduces a misleading bias in its trend.[19]

Non-employment risk influences lifecycle inequality through multiple channels. It is correlated with wages, as low-wage workers are more exposed to non-employment risk than their high-wage counterparts (see Figure 2 Panel B), amplifying earnings inequality. Panels A and B in Figure 8 illustrate that the 10th percentile of lifecycle earnings is far more sensitive to non-employment risk than the 90th percentile. Additionally, non-employment can be considered as a persistent shock since non-employment in a given year increases the probability and duration of non-employment in the following year (see Figure 5). Thus, while the effect of non-employment on cross-

[18] The counterfactuals CF2 and CF3 reveal that, similar to the cross-sectional analysis in Figure 6, both factors had a roughly equally significant impact.

[19] This finding aligns with Boenke et al. (2015a), who highlight the critical role of unemployment in shaping lifecycle earnings through their analysis of the Gini coefficient.

sectional inequality is negligible, it has a substantial impact on lifecycle inequality.

4.2.4. Earnings Mobility

We now use our counterfactual simulation to study the evolution of earnings mobility. Figure 9 displays the Shorrocks index across the various counterfactual scenarios. Scenario CF1 indicates that changes in observables and their returns (μ_{ceat}) had a small positive effect on mobility but cannot account for the observed decrease in earnings mobility. The decreasing variance of the individual FE (σ_{ac}^2) and the skill price (p_t) contributed over the entire period to a decline in mobility, starting with the 1960 birth cohort (see CF2-CF4). Comparing scenarios CF1 and CF4 reveals that the negative impact of these decreases outweighed the positive effect of the observables (μ_{ceat}) in CF1, leading to a stabilization of mobility after the 1975 cohort. Scenarios CF5 and CF6 show that permanent and transitory wage shocks have only a minor impact on the evolution of mobility. Meanwhile, the employment margin affects the level of mobility but has minimal influence on its trend over time. A comparison with scenario CF7 indicates that our full model accurately captures the level and the evolution of earnings mobility.

5. Mechanisms for an Increase in the Variance of Individual Fixed Effects

The estimation results in Section 4 indicate that a key factor driving the evolution of cross-sectional wage and lifecycle earnings inequality is the increase in the variance of individual fixed effects. This implies that inequality at the beginning of careers has widened and become more persistent over recent decades. While individual fixed effects are often interpreted as reflecting individual ability, it is unlikely that this is the primary driver in our case. Interpreting our findings as evidence of a near doubling in the dispersion of underlying unobserved ability seems implausible.

An alternative interpretation is that the increase in the variance of individual fixed effects represents a shift in how individual ability is translated into productivity and wages. More precisely, it could reflect changes in how workers' ability is utilized, influenced by

evolving technologies, labor market frictions, and institutional factors.[20] As a result, wage inequality may be shaped not only by the distribution of ability but also by how ability is allocated across sectors of the economy. To illustrate the key idea of this argument, consider two types of tasks, routine and complex. When the demand for routine tasks is high, high-ability workers may perform routine tasks where their ability has little impact on productivity, resulting in a small difference in productivity between low-ability and high-ability workers. In contrast, if there is a strong demand for complex tasks, more high-ability workers will perform complex tasks where their ability impacts productivity, resulting in a large difference in productivity (and thus wages) between low-ability and high-ability workers. Such a scenario is compatible with an increase in the skill price (see Figure 4), as the relative price between complex and routine tasks incentivizes high-ability workers to sort into complex tasks. We next formalize this intuition and show that technological change is a potential mechanism that strengthens the link between ability and productivity, thereby expanding wage inequality—even without changes in the underlying ability distribution or educational attainment.

5.1. A Roy Model of Skill-biased Technological Change within Occupations

5.1.1. Model

Since the individual fixed effects are time-invariant, we consider a static model, building on the canonical model of Acemoglu and Autor (2011) that endogenizes workers' selection into sectors. On the demand side, consider a firm that produces output using two types of labor, complex tasks C and routine tasks R. Consider the following profit maximization problem of the firm:

$$\max_{(C,R)\in\mathbb{R}_+^2} [\omega C^{\varphi} + (1-\omega)R^{\varphi}]^{\frac{1}{\varphi}} - w_C C - w_R R\,, \tag{7}$$

[20] In line with this, Hsieh et al. (2019) suggest that a significant portion of U.S. economic growth can be attributed to improved talent allocation.

where $\omega \in (0,1)$ is factor productivity of the complex (or non-routine) tasks, $\varphi \leq 1$ and the elasticity of substitution is $\frac{1}{1-\varphi}$. This gives rise to the relative demand function,

$$\frac{C^D}{R^D} = \left(\frac{\omega}{1-\omega}\frac{w_R}{w_C}\right)^{\frac{1}{1-\varphi}}, \tag{8}$$

indicating that relative demand for complex tasks increases in the factor productivity and decreases in the relative wage $\frac{w_C}{w_R}$.

On the supply side, suppose there is a continuum of workers with population one. Each worker draws ability $S \sim \mathcal{N}(0, \sigma^2)$, which affects their effective unit of labor. The effective unit of labor of the worker is determined by their ability and the type of tasks they perform, with $\exp(S)$ for complex tasks and $\exp(\delta S)$ for routine tasks, where $\delta \in (0,1)$. Thus, the parameter δ indicates the importance of ability when working in the routine vs the complex sector. If δ is close to 0, the required tasks are routinized so that low-ability workers are almost as productive as high-ability workers. The worker chooses the sector (either routine or complex) that maximizes their earnings. In consequence, they select the complex sector (and provide complex task input) if and only if

$$\exp(S)\, w_C \geq \exp(\delta S)\, w_R, \qquad \therefore\ S \geq S^* = \frac{1}{1-\delta}\ln\left(\frac{w_R}{w_C}\right). \tag{9}$$

Then, the aggregate labor supply is

$$C^S = \int_{S^*}^{\infty} \exp(s)\ f_S(s)ds = \exp\left(\frac{\sigma^2}{2}\right)\left[1 - \Phi\left(\frac{S^*}{\sigma} - \sigma\right)\right], \tag{10}$$

$$R^S = \int_{-\infty}^{S^*} \exp(\delta s)\ f_S(s)ds = \exp\left(\frac{\delta^2\sigma^2}{2}\right)\ \Phi\left(\frac{S^*}{\sigma} - \delta\sigma\right), \tag{11}$$

where $f_S(\cdot)$ is the probability density function of S and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Workers' sectoral choice is based on comparative advantage. As illustrated in

Figure 10, the effective unit of labor is larger (smaller) for complex tasks than routine tasks if $S > 0$ ($S < 0$). Moreover, the threshold ability level S^* is decreasing in the relative wage $\frac{w_C}{w_R}$, suggesting that the share of workers providing complex tasks increases when the relative wage increases.

Through the lens of our wage process model, the *effective* unit of labor corresponds to the individual fixed effects α_{ic}, whereas the relative wage of complex and routine tasks $\frac{w_C}{w_R}$ corresponds to the skill price p_t. Therefore, our comparative statics below focus on how skill-biased technological change affects the distribution of the realized effective unit of labor. Moreover, our assumptions on the ability distribution and the functional form of the effective unit of labor lead the earnings distribution to be log-normal (conditional on the type of tasks), which is a good approximation of the real-world distribution.

The equilibrium that satisfies equations (8)–(11) and the market clearing conditions, $C^D = C^S$ and $R^D = R^S$ is $\left(C^D, R^D, C^S, R^S, S^*, \frac{w_C}{w_R}\right)$. It is easy to see that this equilibrium is uniquely determined. From equations (8) and (9) and market clearing conditions, we observe

$$(1-\delta)\,\mu + \frac{(1-\delta^2)\sigma^2}{2} + \ln\frac{1-\Phi\left(\frac{S^*}{\sigma}-\sigma\right)}{\Phi\left(\frac{S^*}{\sigma}-\delta\sigma\right)} = \frac{1}{1-\varphi}\left[\ln\frac{\omega}{1-\omega} + (1-\delta)S^*\right]. \quad (12)$$

Since the left-hand side of this equation is decreasing and the right-hand side is increasing in S^*, the ability threshold S^* is uniquely determined. Equation (9) then determines the relative wage, and equations (10) and (11) show the equilibrium level of labor supply.

5.1.2. Model Implications and Comparative Statics

One testable implication from the model is that workers in the complex sector earn more than workers in the routine sector. This is intuitive because high-ability workers sort into the complex sector, whereas low-ability workers sort into the routine sector.

Proposition 1.

For any $S_0 \le S^* \le S_1$, $\exp(\delta S_0)\, w_R \le \exp(S_1)\, w_C$.

Proof

See Appendix C. ■

Next, to understand the role of skill-biased technological change, which increases the factor productivity ω in our model, we conduct comparative statics.

Proposition 2.

1. The relative wage is increasing in ω: $\frac{\partial}{\partial \omega}\frac{w_C}{w_R} > 0$.
2. The share of workers providing complex tasks is increasing in ω: $\frac{\partial}{\partial \omega}S^* < 0$.

Proof

See Appendix C. ■

This is the standard result shown, for example, by Acemoglu and Autor (2011). Intuitively, when complex task input becomes more productive than routine task input, the relative demand for complex task input rises, driving up the relative wage to attract more workers. The higher relative wage, in turn, incentivizes marginal workers to transition from routine to complex tasks.

Proposition 3.

Let $\xi(S;S^*)$ denote the (realized) effective unit of labor of a worker with ability S, given the threshold value S^*.

1. For $S^* > 0$, $Var\left(\xi(S;S^*)\right)$ is inverted U-shaped. For $S^* > 0$, there exists $\bar{S} > 0$ such that for $S^* > \bar{S}$, $\frac{d}{d\,S^*} Var\left(\xi(S;S^*)\right) < 0$, for $S^* = \bar{S}$, $\frac{d}{d\,S^*} Var\left(\xi(S;S^*)\right) = 0$ and for $S^* < \bar{S}$, $\frac{d}{d\,S^*} Var\left(\xi(S;S^*)\right) > 0$.
2. For $S^* = 0$, $\frac{d}{d\,S^*} Var\left(\xi(S;S^*)\right) = 0$.
3. For $S^* < 0$, $\frac{d}{d\,S^*} Var\left(\xi(S;S^*)\right) < 0$.

Proof

See Appendix C. ■

Figure 11, Panels A-C illustrate the intuition of Proposition 3. Panel A shows a situation where the level of technology ω is low, the threshold S^* is high, few workers work in the complex sector, and the mean of the effective unit of labor across all workers (dashed line) is low. A *decrease* in S^* due to technological change incentivizes the marginal worker with ability S^* to switch from the routine sector to the complex sector

(illustrated by the upward shift of the orange circle in the figure). In this situation, relatively high-ability workers are switching to the complex sector and become more productive than before because their ability is now better reflected in their productivity. Due to this shift, their *effective* unit of labor now deviates more substantially from the mean (dashed line) than it did when working in the routine sector (the distance of the orange circle from the dashed line has increased). Consequently, this switch *increases* the variance of the effective unit of labor, $Var\left(\xi(S;S^*)\right)$.

When S^* decreases further due to technical change, the marginal worker's effective unit of complex tasks declines, while the average effective unit of labor increases because more and more workers with comparative advantage in complex tasks have switched to that sector. At a certain value of S^*, the deviation from the mean of the effective unit of labor (dashed line) becomes smaller when working in the complex than in the routine sector (see Figure 11 Panel B). A switch of the marginal worker to the complex sector in response to a decrease in S^* will now lead to a *decrease* in the variance. In this situation, the marginal worker still increases their productivity by job switching but moves *closer* to the average productivity. Note that this job switch still increases the variance of productivity in the complex sector but decreases the variance of productivity in the overall economy due to this composition effect. For the special case where $S^* = 0$, the effective unit of labor of the marginal worker is the same across sectors ($\exp(S^*) = \exp(\delta S^*) = 1$), and thus, the variance is unaffected by the job switching.

Next, consider the case where $S^* < 0$, which corresponds to a high level of technology where most workers already work in the complex sector (Figure 11 Panel C). When $S^* < 0$, the marginal worker´s ability is so low that they are more productive in the routine sector than in the complex sector. However, a decrease in S^* will still induce low-ability workers to shift to the complex sector due to a higher price in that sector. Thus, if the mean is above their effective unit of routine tasks, the deviation from the mean (dashed line) is larger in the complex sector than in the routine sector, and switching from the routine sector to the complex sector increases the variance.

To illustrate how the variance of the effective unit of labor depends on S^* we present the results of a simulation exercise in Figure A5. Starting from the right-hand side, where S^* is high, corresponding to a low level of technology ω, we observe that the

variance initially increases when S^* decreases. This upward trend continues until $S^* = \bar{S}$, which aligns with the threshold defined in Proposition 3.1.[21] Beyond this point, as S^* decreases to moderate levels, the variance declines until $S^* = 0$, consistent with the result stated Proposition 3.1 and 3.2 for moderate levels of technology ω. After this, the variance increases again when S^* decreases further in line with Proposition 3.3 for high levels of technology ω.

To summarize, how does the variance of effective units of labor respond to a change in S^* depends on the relative locations of effective units, $\exp(S^*)$ and $\exp(\delta S^*)$, to the mean.

Our result of the non-monotonicity of the variance of the effective unit of labor holds even if we relax the distributional assumption on workers' ability and the functional form assumption on the effective unit of tasks (See Appendix D).

Combining Propositions 2 and 3, we obtain the following implications:

Corollary 1.

1. Skill-biased technological change increases the variance of the effective unit of labor in the sector with low or high technology
2. Skill-biased technological change does not affect or decrease the variance of the effective unit of labor in the sector with moderate technology.

Proof

See Appendix C. ■

Corollary 1 implies a heterogeneous impact of skill-biased technological change on the variance of the effective unit of labor, increasing the variance in a situation of high or low technology levels. This effect becomes smaller (and can even become negative) in situations with moderate technology where the threshold S^* is close to 0.

Our model extends Acemoglu and Autor's (2011) canonical framework by introducing workers' sector choice. In their model, labor supply to each sector is fixed, so changes in inequality are solely driven by variations in the skill premium. In contrast, our model accounts for workers' selection into sectors driven by increases in the skill

[21] In our numerical example $\bar{S}$ is just below 1.0.

premium.

This extension of the model is supported by estimates of our statistical model, where the evolution of inequality is driven by *both* the skill price p_t (corresponding to the relative wage or skill premium w_C/w_R), and the variance of the individual fixed effects $\sigma_{\alpha c}^2$ (corresponding to the variance of the effective unit of labor $Var\,(\xi)$). By endogenizing the workers' sector choice, our model implies that the variance of fixed effects in our statistical model can increase due to skill-biased technological change even without changing the underlying ability distribution of workers.

5.2. Testing the Implications of the Model

To test the model's predictions, we assume that the abovementioned dynamics characterize different occupations. This is motivated by the observation that the increase in entry-level wage inequality occurs mainly *within* rather than between occupations. We illustrate that in Figure 12, where we decompose the variance of log wages at age 25 by birth cohorts into changes between and within occupations.[22] The figure shows that the variance in entry-level wages almost tripled from 0.035 for the birth cohort 1950 to 0.09 for the birth cohort 1992.[23] About 75% of this increase is due to an increase in the variance of wages *within* occupations.

As a measure for factor productivity of complex task input (parameter ω in our model), we compute the level of complexity (non-routineness) for each occupation by averaging responses across five questions asked in the BIBB/IAB Career and Qualification Survey described in Section 2:[24] the degree of task stipulation, the

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[22] We regress log wages at age 25 on 120 3-digit occupational dummy variables separately for each calendar year. The R^2 of these regressions are then the shares of the variance that are explained by wage differences across occupations, while the $1 - R^2$ are the shares of the variance explained by differences within occupations.

[23] The occupational classification in the data follows the German Classification of Occupations (Klassifikation der Berufe 1988). The original classification includes 330 occupations at the 3-digit level, which have been aggregated into 120 occupations in our dataset. This aggregation primarily involved merging occupations with very few observations into closely related occupations.

[24] The Items are similar to O*NET measures of occupational requirements: Importance of Repeating

repetitiveness of tasks, the frequency of performing new tasks, the frequency of improving tasks or procedures, and the frequency of working in teams. For each item, workers indicate how often they engage in these activities, with response options ranging from "always" to "never," assigned values from 1 to 5, respectively. The use of these measures is motivated by recent findings of Deming (2021) on how technological change affects tasks. He associates the growth in employment within occupations requiring problem-solving and decision-making with technological change and higher life-cycle earnings.

Proposition 1 of our model predicts that within occupations, workers in the complex sector have higher wages than workers in the routine sector. Table A1 presents the results from regressions of monthly log wages in the 1991/1992 BIBB survey on our standardized measure of task complexity. Column (1) of Table A1 shows that a one standard deviation increase in complexity predicts a substantial 0.198 log-point increase in the monthly wage. Controlling for occupational fixed effects, estimates in Column (2) show that a one standard deviation increase in complexity predicts a 0.1 log-point increase in wages. This effect remains largely unchanged when additionally controlling for age and supervisory status. These findings align with Proposition 1 of our model.

Corollary 1 of our model suggests that the variance of the effective unit of labor increases more with technological change in occupations that exhibit a low or high level of technology (ω) in a base period. We thus classify occupations by task complexity using the occupational complexity index derived from the 1991/1992 survey. This information is merged with the SIAB at the occupation level, and occupations are grouped into four quartiles based on their complexity scores, as illustrated in Figure 13. Using the entry-level wage as a proxy for the effective unit of labor, we calculate the evolution of its variance for each occupational group using our administrative data.[25] Our model predicts

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Same Task, Structured vs Unstructured Work, Variety of Tasks Required, Making Decisions and Solving Problems, Work With Work Group or Team.

[25] In our wage process model, the residualized entry-level wage is expressed as $p\alpha + \eta + \epsilon$ with variance $p^2\sigma_\alpha^2 + \sigma_\eta^2 + \sigma_\epsilon^2$, which is well proxied by the individual fixed effects as the variances of the persistent and transitory shocks are far smaller than those of the individual fixed effects.

that the variance of the effective unit of labor increases over time, particularly in occupations that are either highly routine or highly complex (Proposition 3 and Corollary 1). Conversely, occupations with intermediate complexity are expected to experience a more moderate increase in the variance of the effective unit of labor.

Figure 14 shows the residual log wage standard deviations across quartiles of the complexity score. For occupations with the highest task complexity (Q4), the wage variance increased by more than 50% over the sample period. A similar rise is observed for occupations with the lowest initial complexity (Q1), particularly from the mid-1990s onward. In contrast, occupations in Q2 and Q3 experienced more moderate increases in wage variance, with Q2 showing the smallest growth. This empirical evidence supports our model's theoretical prediction: technological advancements amplify wage variance in occupations with high or low initial complexity, while the effect is more moderate for occupations with medium levels of initial complexity.

Table 1 presents the changes in the p90p10, p90p50, and p50p10 log differences of the residual log wage by complexity score quartiles across the two periods 1975-1991 and 1991-2009, distinguishing effects on upper- and lower-tail inequality. Column 2 suggests that for occupations with the highest measured complexity index in 1991/92 (Q4), upper tail inequality grew strongly in the period 1975-1991, but less so in the second period 1991-2009. Conversely, for the less complex occupation groups Q1, Q2 and Q3, upper-tail inequality increased little in the period up to 1991. In the period from 1991-2009 however, the increase in upper-tail inequality of groups Q1, Q2 and Q3 outpaced those of the group Q4 with Q1 growing by far the strongest by 0.188 log-points. Our model rationalizes that pattern. In occupations with initial low complexity, technological change should mainly affect the upper tail of the distribution as there are relatively few workers in complex tasks, and the marginal worker can be found in the upper tail of the wage distribution (Figure 11 Panel A). In contrast, in occupations with high complexity, upper tail inequality should increase less as the marginal worker is located in the lower tail of the distribution, although inequality should still increase by the price effect. The figures show indeed that from 1991 onwards, upper tail inequality in the most complex occupations grew the slowest. Moreover, the stark increase in upper-tail inequality in those highly complex occupations between 1975 and 1992 may suggest that those

occupations were initially also relatively non-complex and transformed into relatively highly-complex during that period.

Column 3 shows that growth in lower-tail inequality was relatively low for all occupational groups in the first period from 1975-1991. From 1991-2009, the change is now highest in the initially most complex occupations Q4. This also can be rationalized in our model since, for very complex occupations, the marginal worker is in the lower tail of the ability distribution (Figure 11 Panel C), increasing inequality through both the sorting and the skill price effect. However, the other occupation groups Q1, Q2, and Q3 also experienced stark increases in 1991-2009 due to the skill price effect.

Our findings are consistent with those of Card et al. (2013), who also examine the rise in wage inequality in Germany. Using a model with additive fixed effects (AKM effects), they decompose the increase in inequality from 1985 to 2009 into three components: individual-specific, firm-specific, and job-match effects. They find that while the job match effect remains small and stable over time, inequality during this period primarily increased due to rising variances in individual-specific and firm-specific effects. Notably, they conclude that a significant portion of this rise in inequality can be attributed to increasing assortative matching between workers and firms.

Assortative matching plays a similar role in our framework by amplifying underlying ability disparities. Thus, assortative matching across firms described in Card et al. (2013) is compatible with our model where workers change into the complex sector (through mobility across firms), incentivized by increasing skill prices, and where inequality increases because workers' abilities are more directly reflected in their productivity and wages. Within the same occupation, wage inequality arises through firm-to-firm mobility due to an increase in the relative price paid to complex over routine tasks $\left(\frac{w_C}{w_R}\right)$, and a sorting effect where workers are allocated to tasks in which their ability is better utilized and more accurately reflected in their wages. This mechanism not only aligns with the findings of Card et al. (2013, 2018) or Song et al. (2019) but also resonates with the concept of increasing superstar firms, as discussed by Autor et al. (2020). Sorting by task complexity across firms seems particularly plausible, as the degree of task specialization is likely to be shaped by the production processes determined at the firm level. Some firms specialize in producing goods or services that require more complex

tasks, while others focus on less complex, routine tasks.

6. Conclusion

This paper investigates the determinants of cross-sectional and lifecycle earnings inequality using a lifecycle earnings process model that incorporates earnings mobility and non-employment risks across birth cohorts and over time. The estimated wage process accounts for multiple sources of non-stationarity, where the distribution of individual fixed effects is cohort-dependent, while skill prices and the distribution of other shocks vary by calendar year. Additionally, we estimate employment-non-employment transitions to capture the correlation between wages and job insecurity, which is crucial for understanding lifecycle earnings and may also influence cross-sectional inequality through selection effects. Using our statistical model, we decompose the evolution of inequality into its underlying determinants, enabling a direct comparison between lifecycle (cohort-specific) and cross-sectional (year-specific) inequality.

A key finding of our analysis is that the dominant determinant in the rise of both cross-sectional and lifecycle inequality is the change in the price and variance of individual fixed effects. These factors equally explain the trends in both measures of inequality. Specifically, the increase in the variance of individual fixed effects across cohorts and the rise in the price of fixed effects explain 138% and 82% of the increase of cross-sectional and lifecycle inequality, respectively. Our analysis also highlights the crucial role of non-employment, where job instability has significantly impacted lifecycle earnings among low-wage workers. Conversely, selection into employment—which could potentially influence the measurement of cross-sectional inequality—turns out to be unimportant. Consequently, failing to account for employment risk can lead to misleading welfare implications when investigating lifecycle inequality, as it plays a significant role in shaping long-term earnings disparities.

To explain how the variance of individual productivity can increase without changing the underlying talent distribution, we develop a Roy model incorporating skill-biased technological change, which amplifies wage inequality by strengthening the relationship between ability and realized productivity. This leads to greater sorting of

workers into tasks where their abilities are most productive, contributing to rising inequality without changes in the underlying distribution of ability.

We empirically test key predictions of our model and find evidence supporting the proposed mechanisms. Our findings contribute to the literature on wage inequality by providing new insights into the link between cross-sectional and lifecycle inequality and its interaction with technological change. This explanation also aligns with the literature on assortative matching between workers and firms (Card et al., 2013). Specifically, high-productivity firms—often specializing in complex tasks—attract high-ability workers by offering higher wage premiums for complex tasks. As a result, technological change enhances the sorting of workers into firms that better utilize their skills, further reinforcing inequality.

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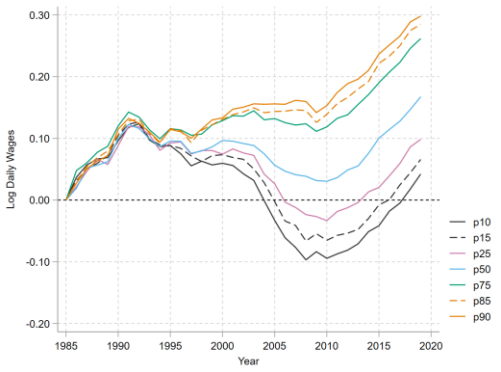
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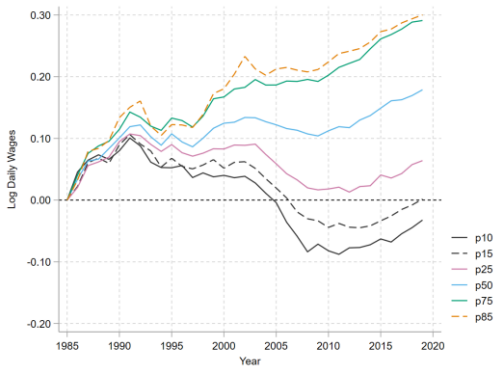
Figures

Figure 1 Distribution of wages and lifecycle earnings



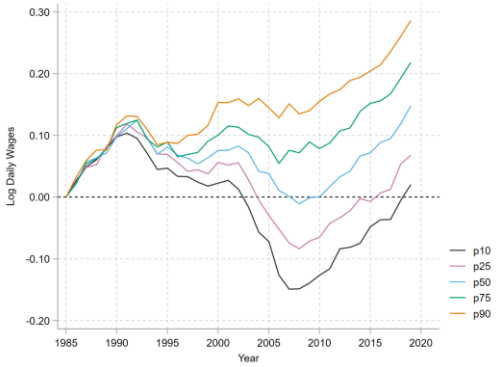
(a)

Distribution of cross-sectional wage in our sample



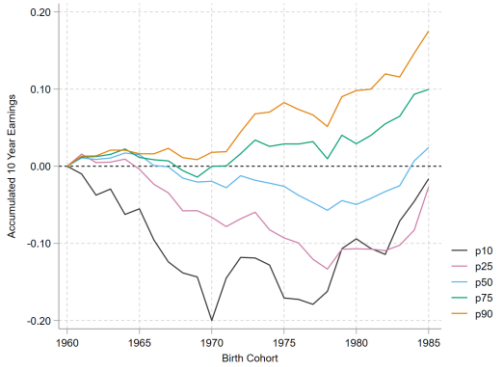
(b)

Distribution of cross-sectional wage for West German males



(c)

Distribution of wage at age 25

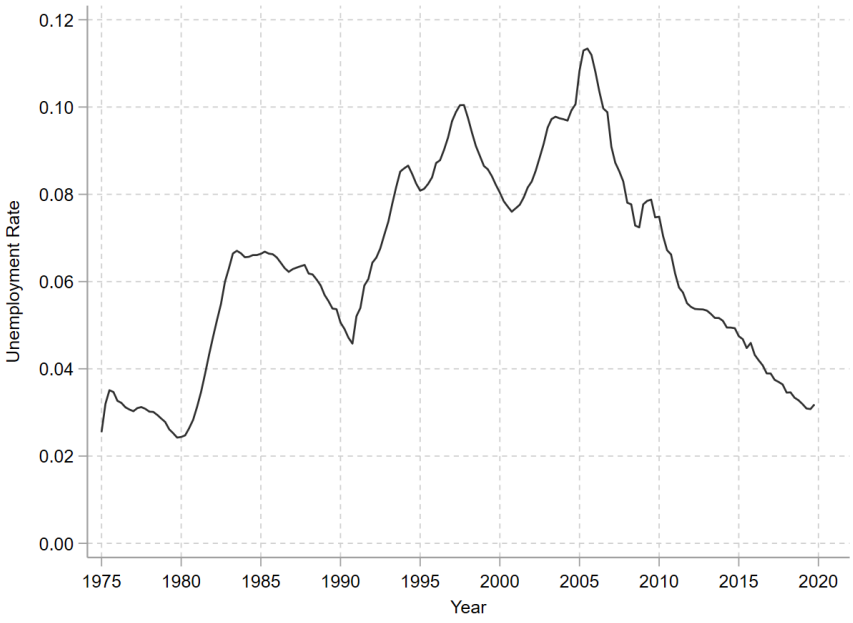


(d)

Distribution of lifecycle earnings across cohorts

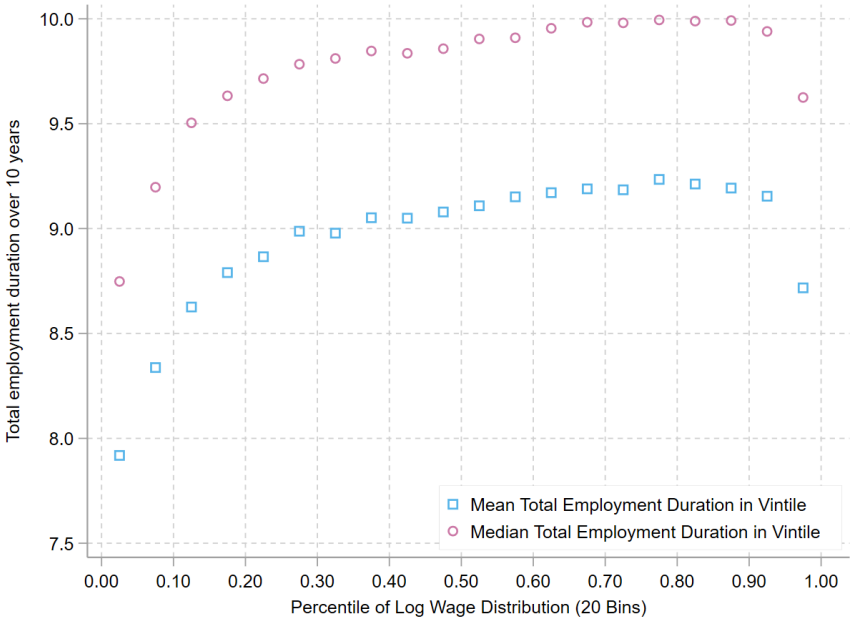
Note: Panel (a) shows the distribution of log wages of workers across calendar years in our sample aged 25 to 34. Panel (b) shows the distribution of log wages for West German males. Panel (c) shows the distribution of log wages of workers at age 25 in our sample. Panel (d) shows the distribution of our measure of lifecycle earnings calculated as the total earnings from age 25 to 34.
Source: Own calculations based on IAB data

Figure 2 Unemployment



(a)

Unemployment Rate (15 Years or over)



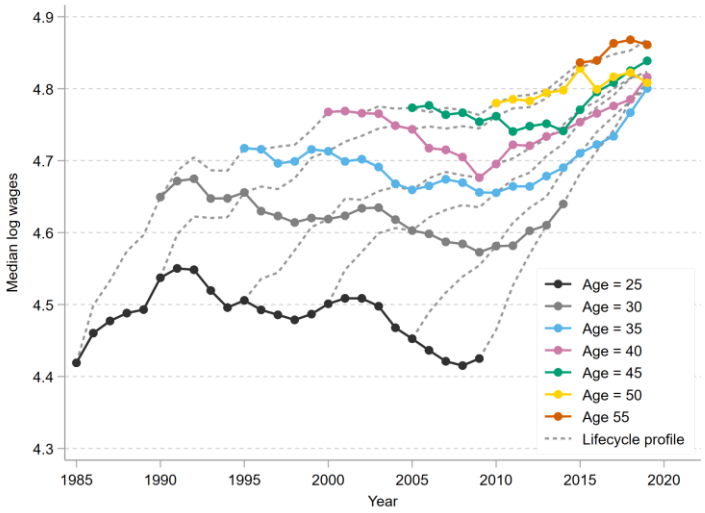
(b)

Total duration of employment over wage distribution at age 25

Note: Panel (a) shows seasonally adjusted quarterly unemployment rate for 15 years or over for Germany. Panel (b) shows the mean and median of total duration of non-employment in each Vintile (20 bins) over the wage distribution at age 25. For example, the first bin corresponds to the total duration of employment for the earners below the 5% percentile of wage at age 25

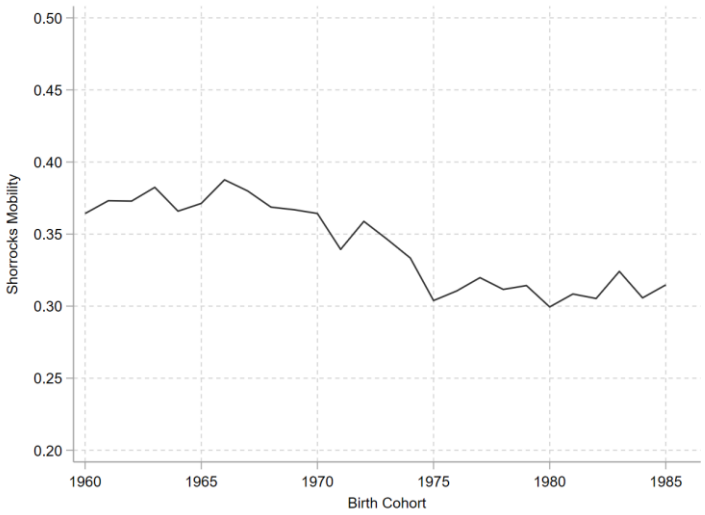
Source: Panel (a): OECD (2024). Panel (b): Own calculations based on IAB data

Figure 3 Median log wage across cohorts and lifecycle and earnings mobility across cohorts



(a)

Median log wage across cohorts and lifecycle

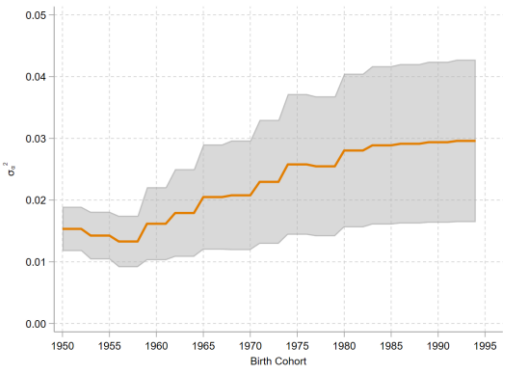


(b)

Shorrocks Index of Variance in Earnings

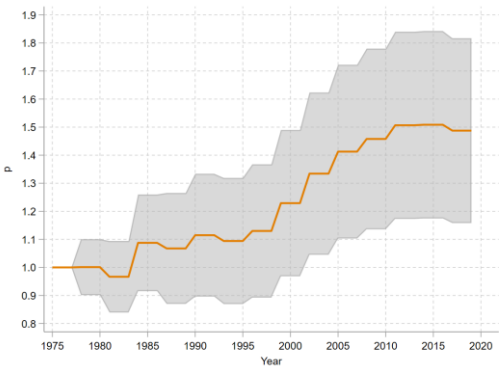
Note: Panel (a) shows the log median wage for different labor market entry cohorts at different ages. The solid lines with points allow us to follow the median log wage for a given age across calendar year. For example, the black dotted line follows the median wages of workers at age 25 across years. The dotted lines allow to follow lifecycle profiles of the median log wage of a given cohort. For example, the first dotted line from the left hand allows to track the log wage of the cohort born in 1960 at age 25 in year 1985, age 30 in year 1990 and so on. Panel (b) shows the Shorrocks index of the variance of earnings across birth cohort.
Source: Own calculations based on IAB data

Figure 4 Parameter estimates



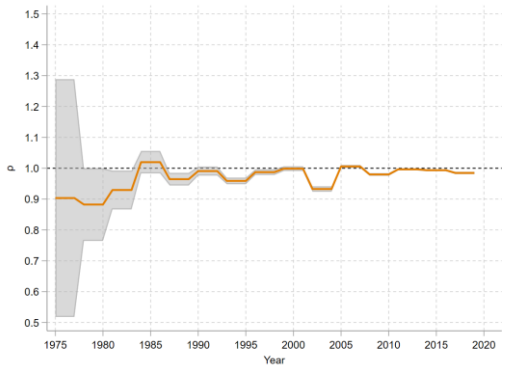
(a) Variance of individual FE

σ^2_{ac}



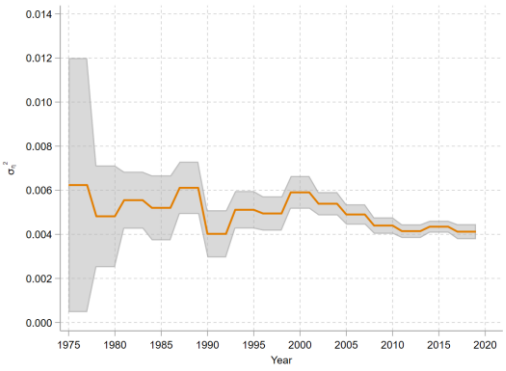
(b) Skill price

p_t



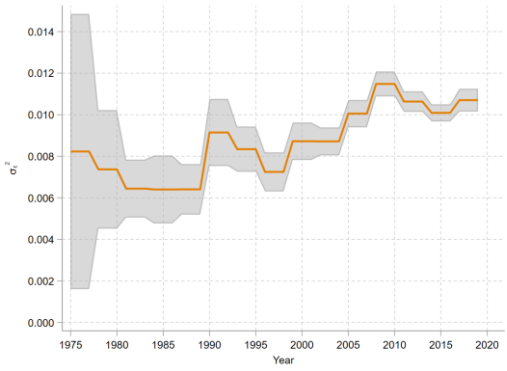
(c) Persistency of AR(1) process

ρ_t



(d) Variance of persistent shock (AR(1) innovation)

$\sigma^2_{\eta t}$

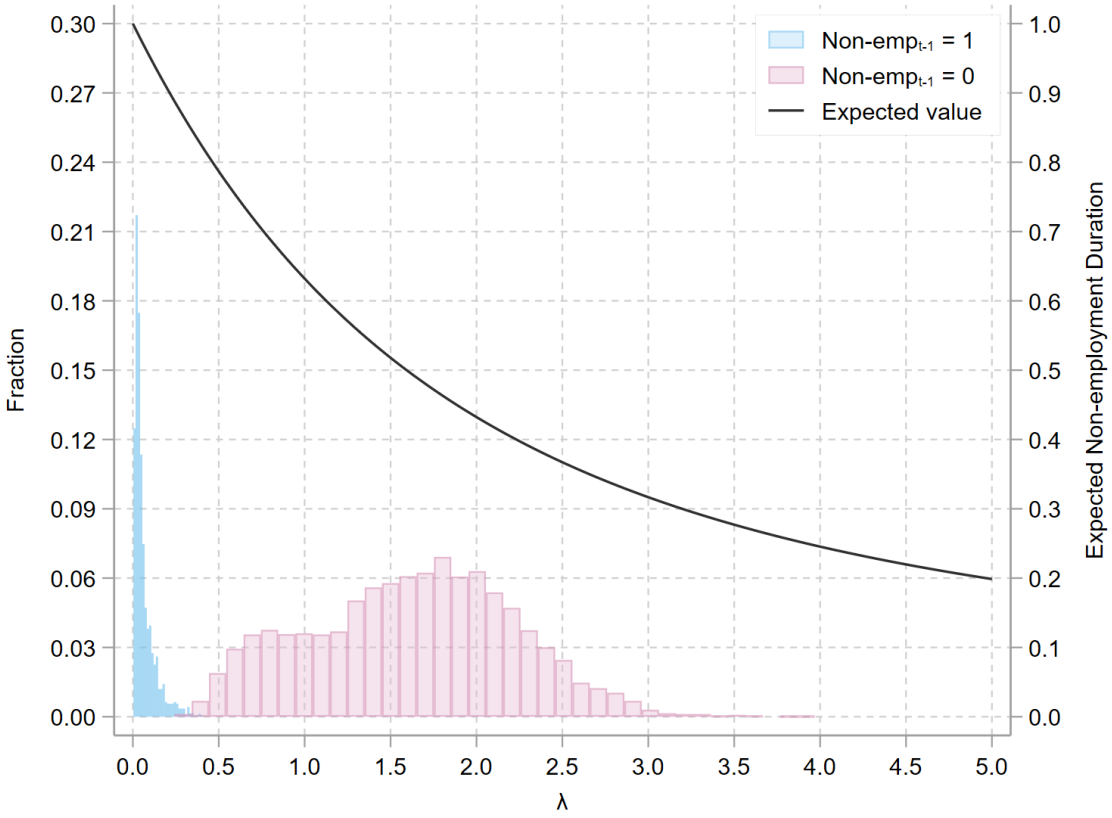


(e) Variance of transitory shock

$\sigma^2_{\epsilon t}$

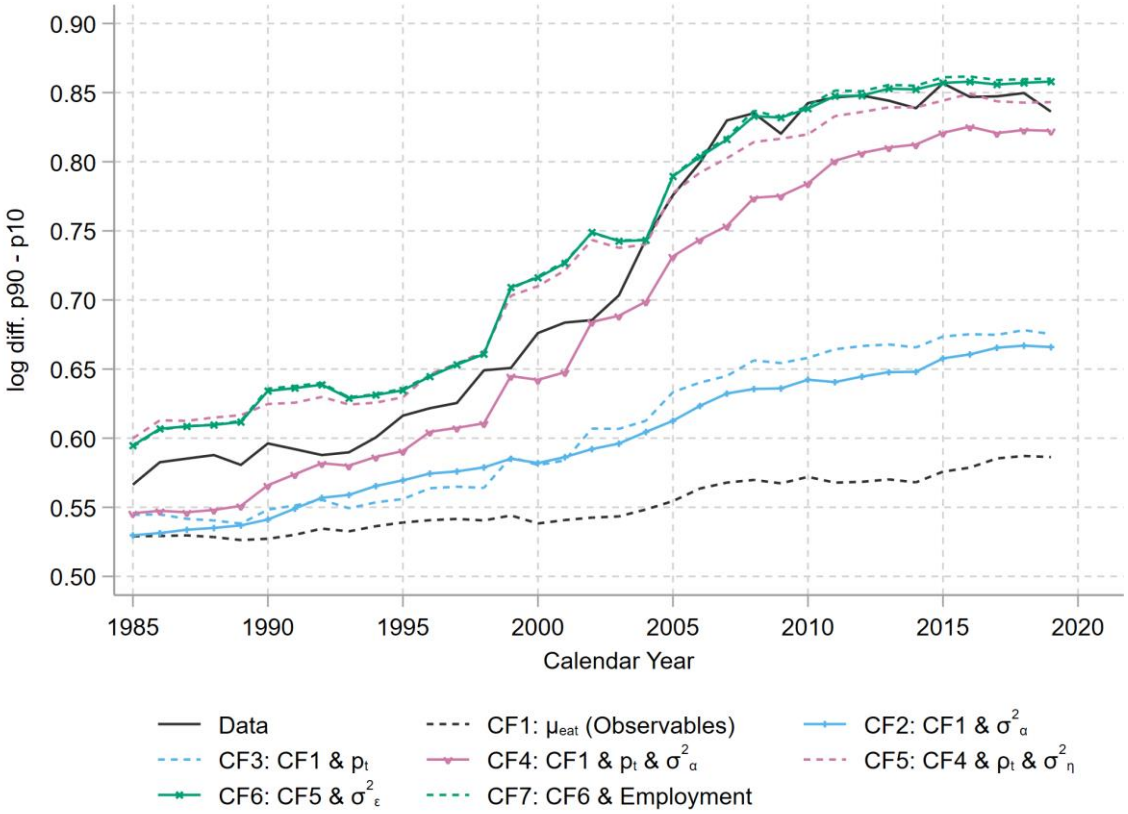
Note: The figure shows the estimated parameters of the stochastic wage growth model according to Equations 1 and 2 for West German workers born between 1950-1994. All parameters are indexed by calendar year, expect the variance of individual fixed effects σ^2_{ac} which varies by cohort. For estimation three cohorts and calendar years are group together respectively. Estimation was performed by minimization of the Euklidian norm of the covariance matrix of residual wages according to Equation 3. Shaded areas represent 90% confidence intervals.
Source: Own calculations based on IAB data

Figure 5 Parameter estimates: Non-employment duration



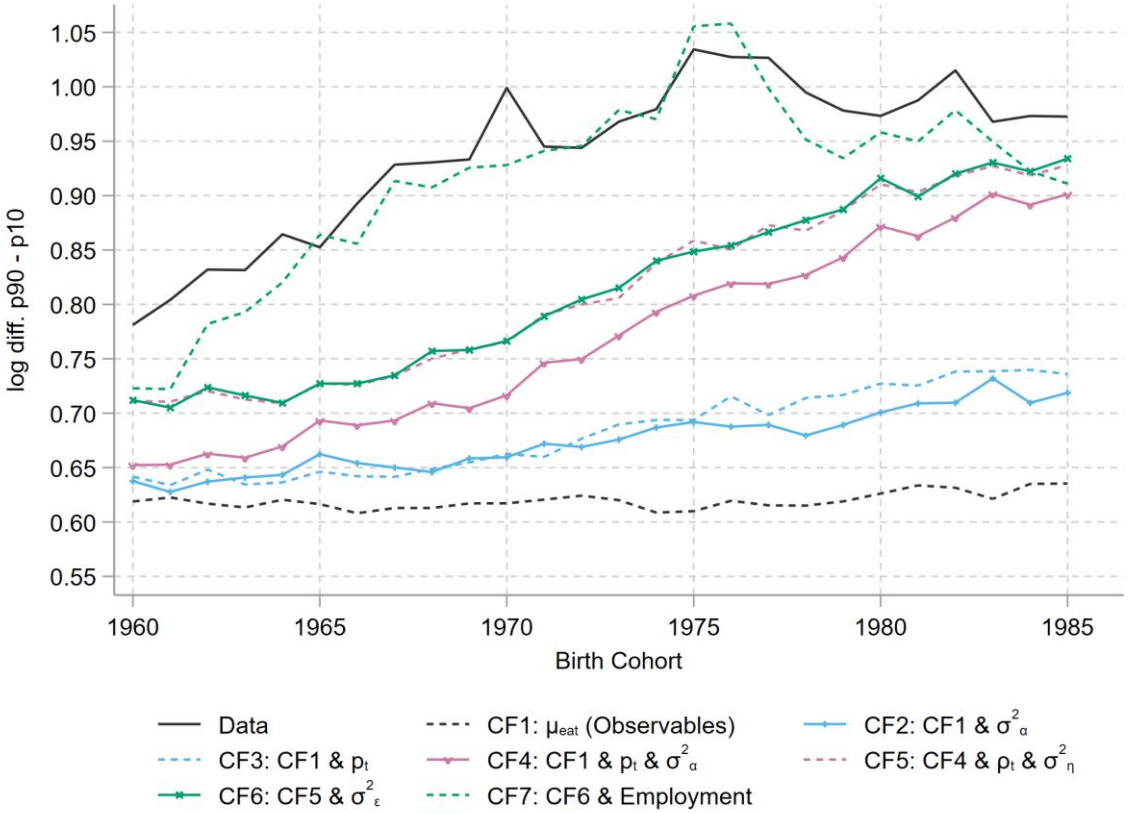
Note: This figure shows the distribution of the estimated value of the non-employment duration parameters λ (left scale) for unemployed workers in period t according to Equation 5. The estimate the in blue shows the distribution for those who were unemployed in the previous period t-1 (blue) and for those who were employed in the previous period t-1 (red). The solid line (right scale) shows the length of non-employment as a fraction of a year corresponding to the estimated value of the non-employment duration parameter λ.
Source: Own calculations based on IAB data

Figure 6 Counterfactual simulation: Cross-sectional inequality (p90-p10 ratio)



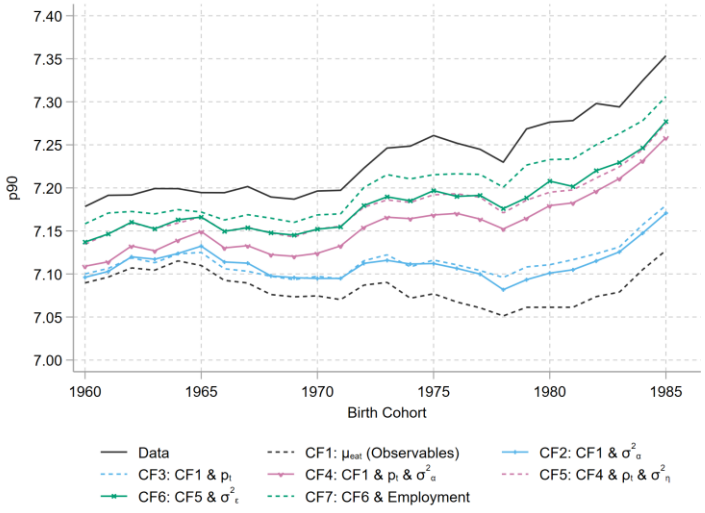
Note: The figure shows the p90-p10-ratio of the distribution of the cross-sectional wages across calendar years for different simulated counterfactual scenarios described in section 4.2 The Cross-sectional wage distribution in calendar year t consists of a rolling sample of the workers in our sample between age 25 and 34 at the given year. Data is calculated using the empirical distribution from our sample. Wages for CF1 to CF7 are simulated from a normal distribution with mean zero and variances according to our estimated parameters and employment/non-employment probabilities according to Equations (4) and (5). CF1 fixes all parameters at base year 1985 except the observable wage component μ_{ceat}. CF2 allows the observables and additionally the variance of skill $\sigma^2_{\alpha c}$ vary across cohort and time. CF3 allows the observables and the price for skill p_t vary across time (but not $\sigma^2_{\alpha c}$). CF4 allows observables and both variance of skill $\sigma^2_{\alpha c}$and price for skill p_t vary. CF5 allows observables, variance and price of skill as well as the components of the AR(1) process (the persistent shock), ρ_t and $\sigma^2_{\eta t}$, vary. In CF6 observables, variance and price of skill, the components of the AR(1) process (the persistent shock) and additionally the variance of the transitory shock $\sigma^2_{\epsilon t}$ are allowed to vary. CF7 additionally lets the parameters governing the employment process (non-employment probabilities and duration) to vary and corresponds to our full model.
Source: Own calculations based on IAB data

Figure 7 Counterfactual simulation: Lifecycle inequality (p90-p10 ratio)

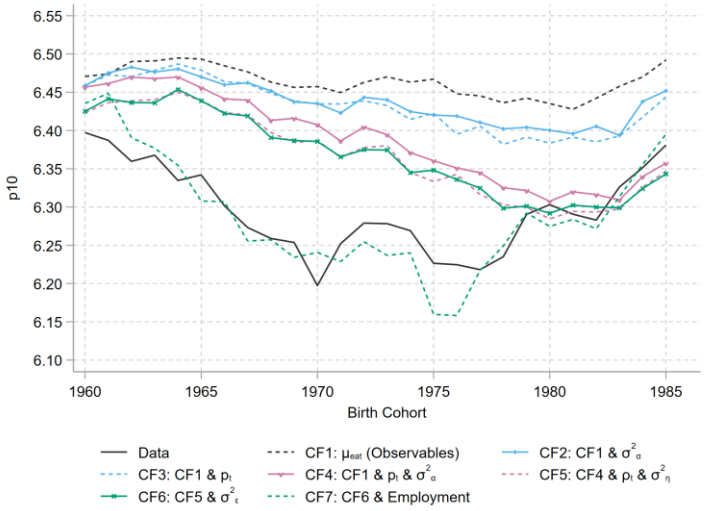


Note: The figure shows the p90-p10-ratio of the distribution of the lifecycle earnings across birth cohort for different simulated counterfactual scenarios described in section 4.2 The Lifecycle Earnings for workers in cohort c are calculated as the total earnings from age 25 to 34. The sample is restricted to individuals with at least 3 years of positive earnings between age 25 to age 34. Earnings are set as yearly wages in the case of full-time employment and 0 otherwise. Data is calculated using the empirical distribution from our sample. Wages and Earnings for CF1 to CF7 are simulated from a normal distribution with mean zero and variances according to our estimated parameters and employment/non-employment probabilities according to Equations (4) and (5). CF1 fixes all parameters at base year 1985 except the observable wage component μ_{ceat}. CF2 allows the observables and additionally the variance of skill $\sigma^2_{\alpha c}$ vary across cohort and time. CF3 allows the observables and the price for skill p_t vary across time (but not $\sigma^2_{\alpha c}$). CF4 allows observables and both variance of skill $\sigma^2_{\alpha c}$and price for skill p_t vary. CF5 allows observables, variance and price of skill as well as the components of the AR(1) process (the persistent shock), ρ_t and $\sigma^2_{\eta t}$, vary. In CF6 observables, variance and price of skill, the components of the AR(1) process (the persistent shock) and additionally the variance of the transitory shock $\sigma^2_{\epsilon t}$ are allowed to vary. CF7 additionally lets the parameters governing the employment process (non-employment probabilities and duration) to vary and corresponds to our full model.
Source: Own calculations based on IAB data

Figure 8 Counterfactual simulation: p10 and p90 of lifecycle earnings



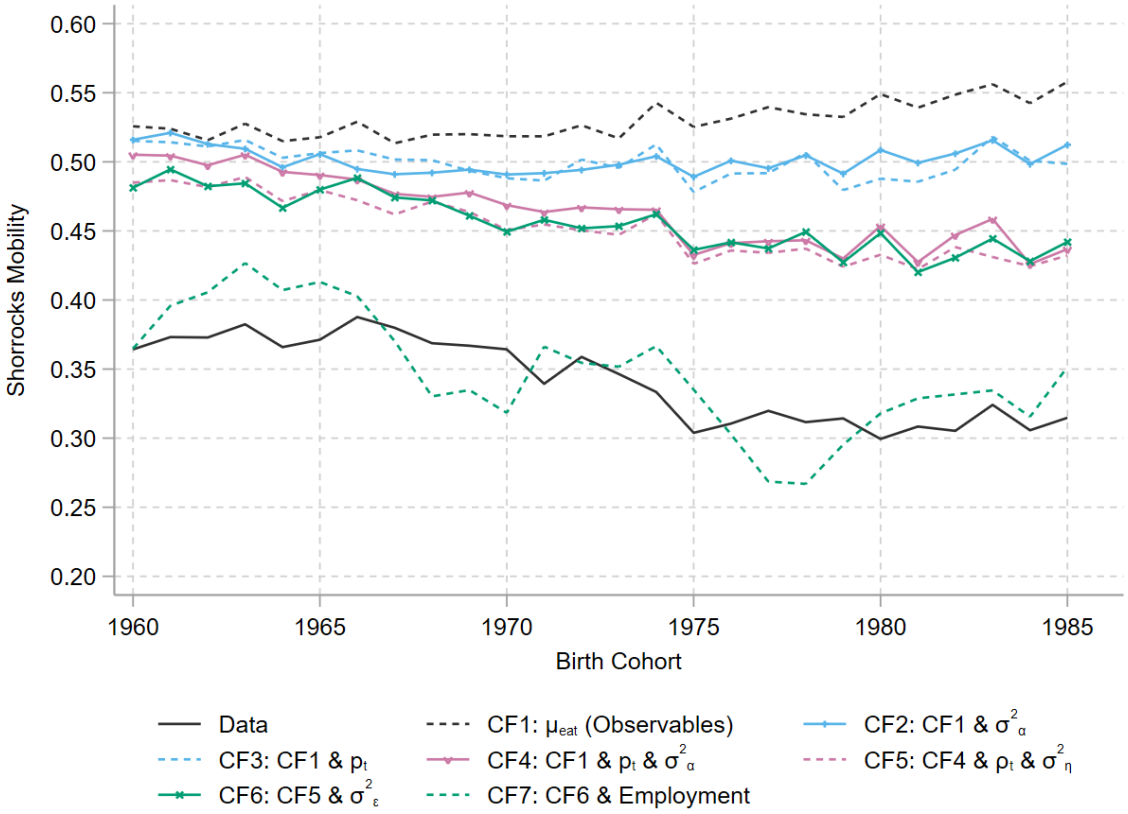
(a) p90



(b) p10

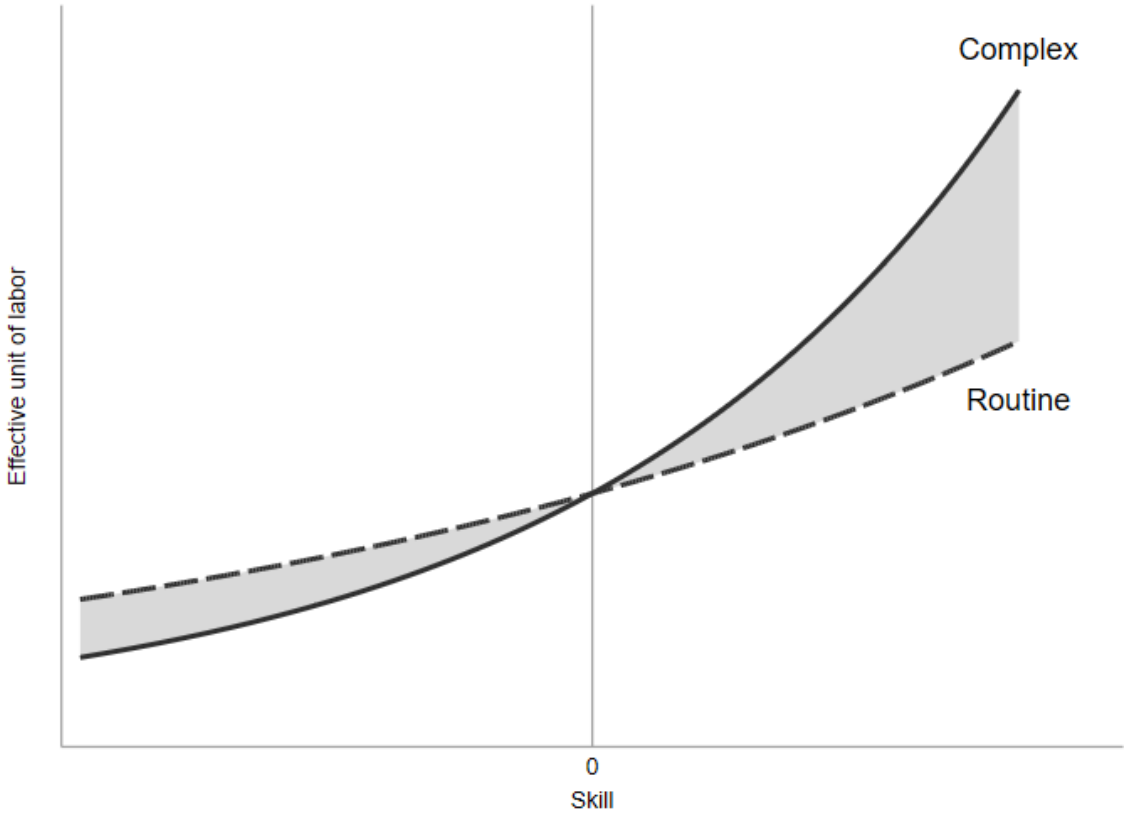
Note: The figure shows the 10th and 90th percentile of the distribution of the lifecycle earnings across birth cohort for different simulated counterfactual scenarios described in section 4.2 The Lifecycle Earnings for workers in cohort c are calculated as the total earnings from age 25 to 34. The sample is restricted to individuals with at least 3 years of positive earnings between age 25 to age 34. Earnings are set as yearly wages in the case of full-time employment and 0 otherwise. Data is calculated using the empirical distribution from our sample. Wages and Earnings for CF1 to CF7 are simulated from a normal distribution with mean zero and variances according to our estimated parameters and employment/non-employment probabilities according to Equations (4) and (5). CF1 fixes all parameters at base year 1985 except the observable wage component μ_{ceat}. CF2 allows the observables and additionally the variance of skill $\sigma^2_{\alpha c}$ vary across cohort and time. CF3 allows the observables and the price for skill p_t vary across time (but not $\sigma^2_{\alpha c}$). CF4 allows observables and both variance of skill $\sigma^2_{\alpha c}$and price for skill p_t vary. CF5 allows observables, variance and price of skill as well as the components of the AR(1) process (the persistent shock), ρ_t and $\sigma^2_{\eta t}$, vary. In CF6 observables, variance and price of skill, the components of the AR(1) process (the persistent shock) and additionally the variance of the transitory shock $\sigma^2_{\epsilon t}$ are allowed to vary. CF7 additionally lets the parameters governing the employment process (non-employment probabilities and duration) to vary and corresponds to our full model.
Source: Own calculations based on IAB data

Figure 9: Counterfactual simulation: Shorrocks Index of Variance in Earnings



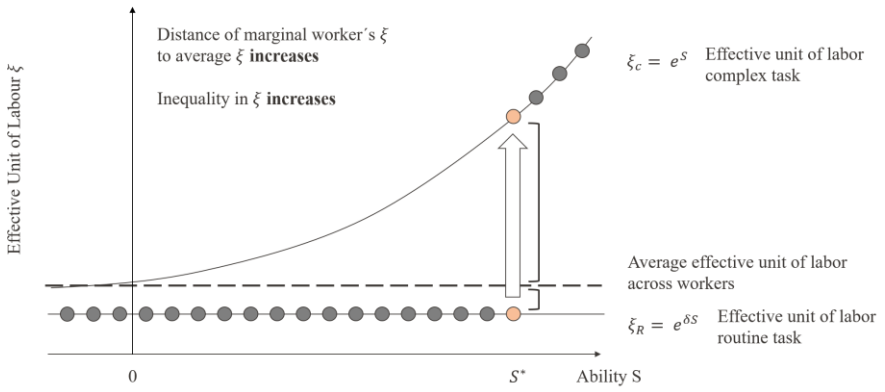
Note: The figure shows the Shorrocks index of the variance of earnings across birth cohort for different simulated counterfactual scenarios described in section 4.2. The sample is restricted to individuals with at least 3 years of positive earnings between age 25 to age 34. Earnings are set as yearly wages in the case of full-time employment and 0 otherwise. Data is calculated using the empirical distribution from our sample. Wages and Earnings for CF1 to CF7 are simulated from a normal distribution with mean zero and variances according to our estimated parameters and employment/non-employment probabilities according to Equations (4) and (5). CF1 fixes all parameters at base year 1985 except the observable wage component μ_{ceat}. CF2 allows the observables and additionally the variance of skill $\sigma^2_{\alpha c}$ vary across cohort and time. CF3 allows the observables and the price for skill p_t vary across time (but not $\sigma^2_{\alpha c}$). CF4 allows observables and both variance of skill $\sigma^2_{\alpha c}$and price for skill p_t vary. CF5 allows observables, variance and price of skill as well as the components of the AR(1) process (the persistent shock), ρ_t and $\sigma^2_{\eta t}$, vary. In CF6 observables, variance and price of skill, the components of the AR(1) process (the persistent shock) and additionally the variance of the transitory shock $\sigma^2_{\epsilon t}$ are allowed to vary. CF7 additionally lets the parameters governing the employment process (non-employment probabilities and duration) to vary and corresponds to our full model.
Source: Own calculations based on IAB data

Figure 10 Comparative advantage in complex and routine sectors by skill

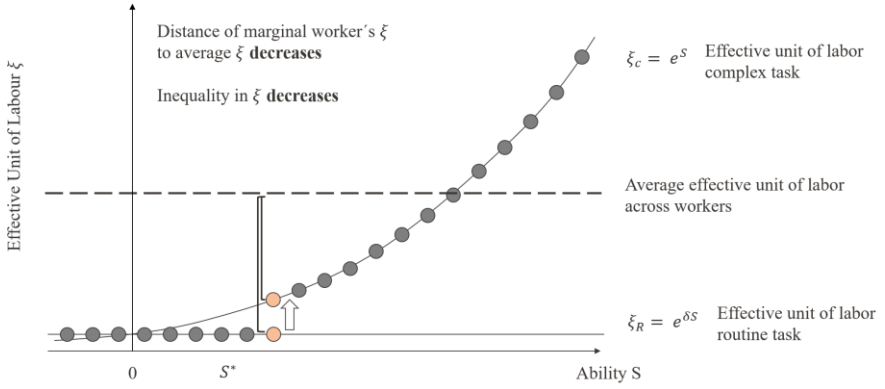


Note: The figure shows the effective unit of labor for complex and routine tasks by skill level.
Source: Own illustration

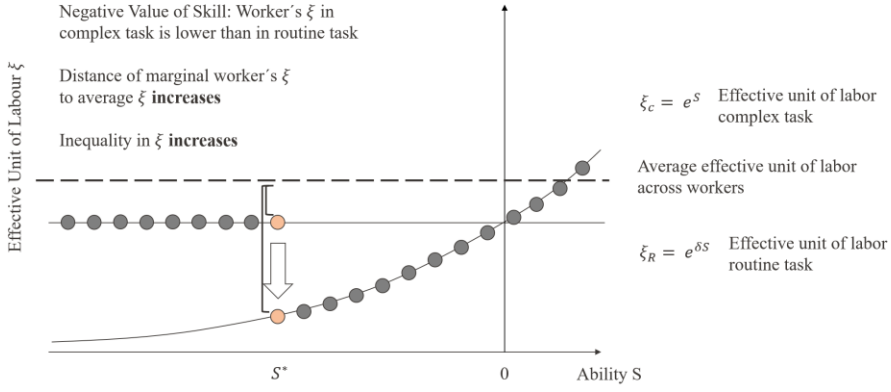
Figure 11 Job switching of the marginal worker under different levels of technology



(a) Low level of technology



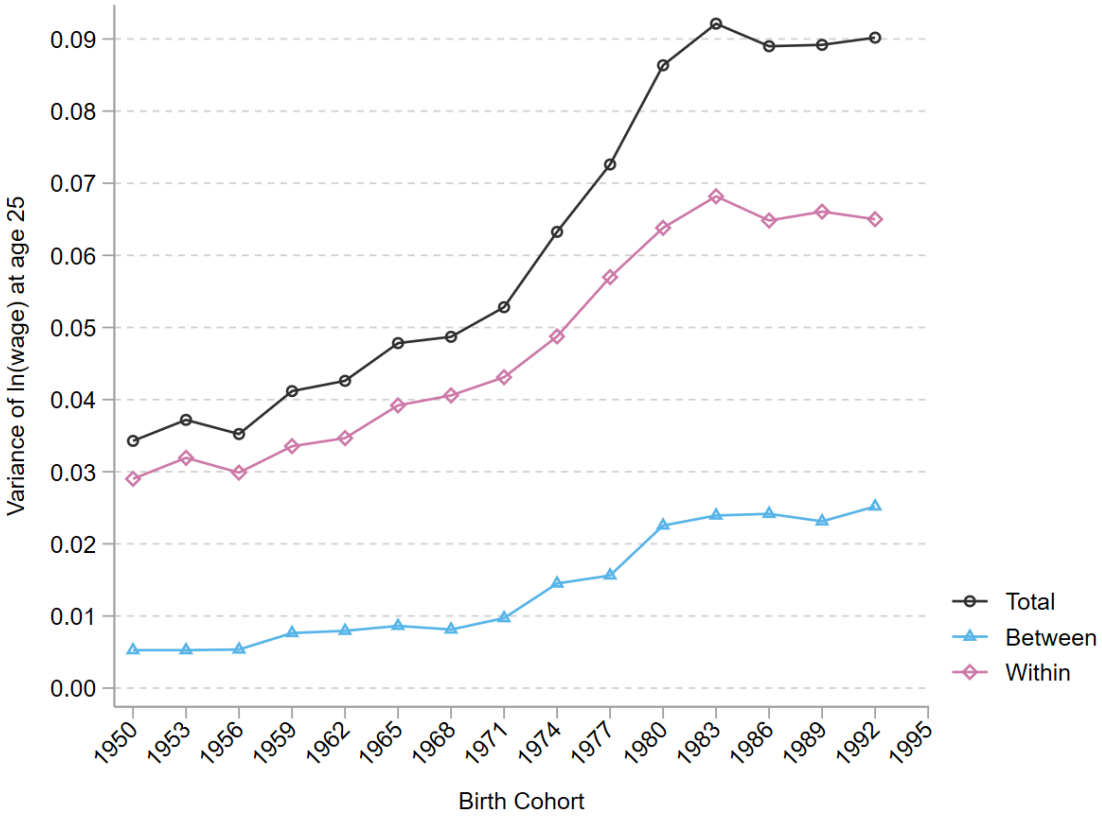
(b) Moderate level of technology



(c) High level of technology

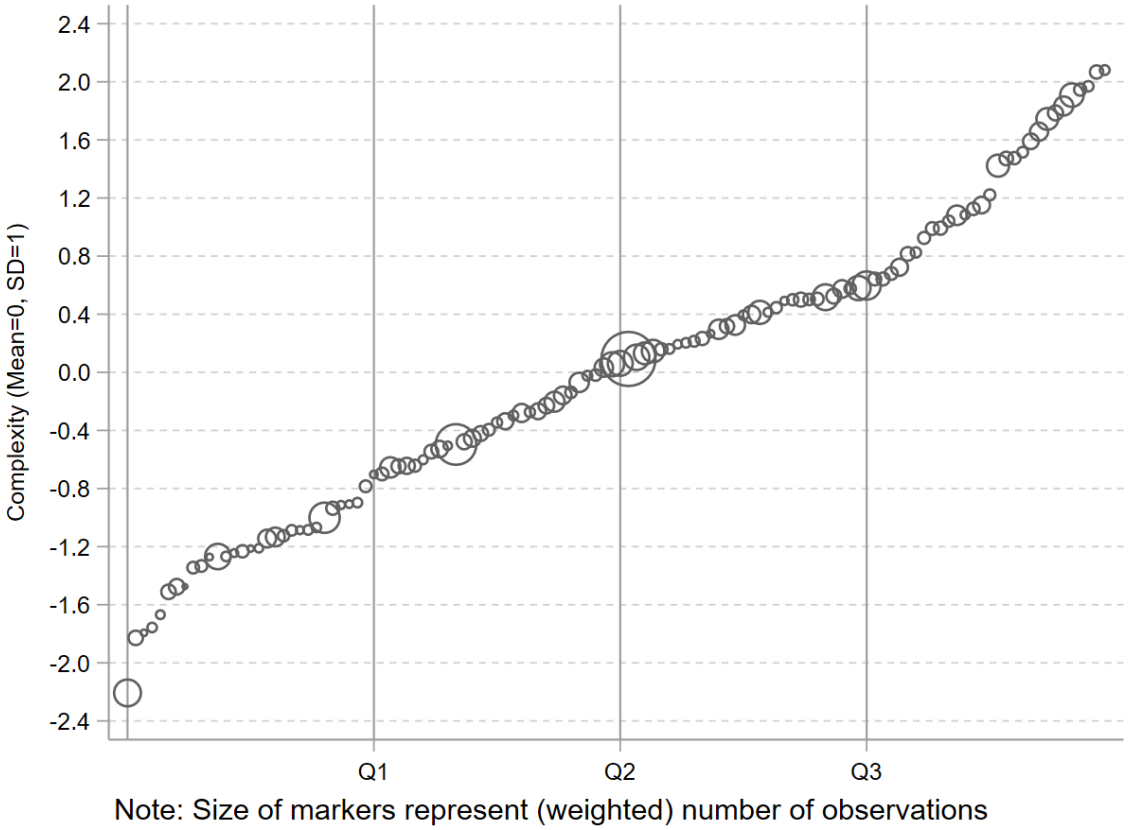
Note: The figure shows the effective unit of labor across skill leave. Each dot represents one worker. The orange dots represent the effective unit of labor of the marginal worker with skill S^* depending on whether he works in the complex or routine task sector. Note that the complex workers payoff working in complex tasks is equal working in routine tasks but his effective unit of labor differs.
Source: Own illustration

Figure 12 Between- and within-occupation variance of wage at age 25



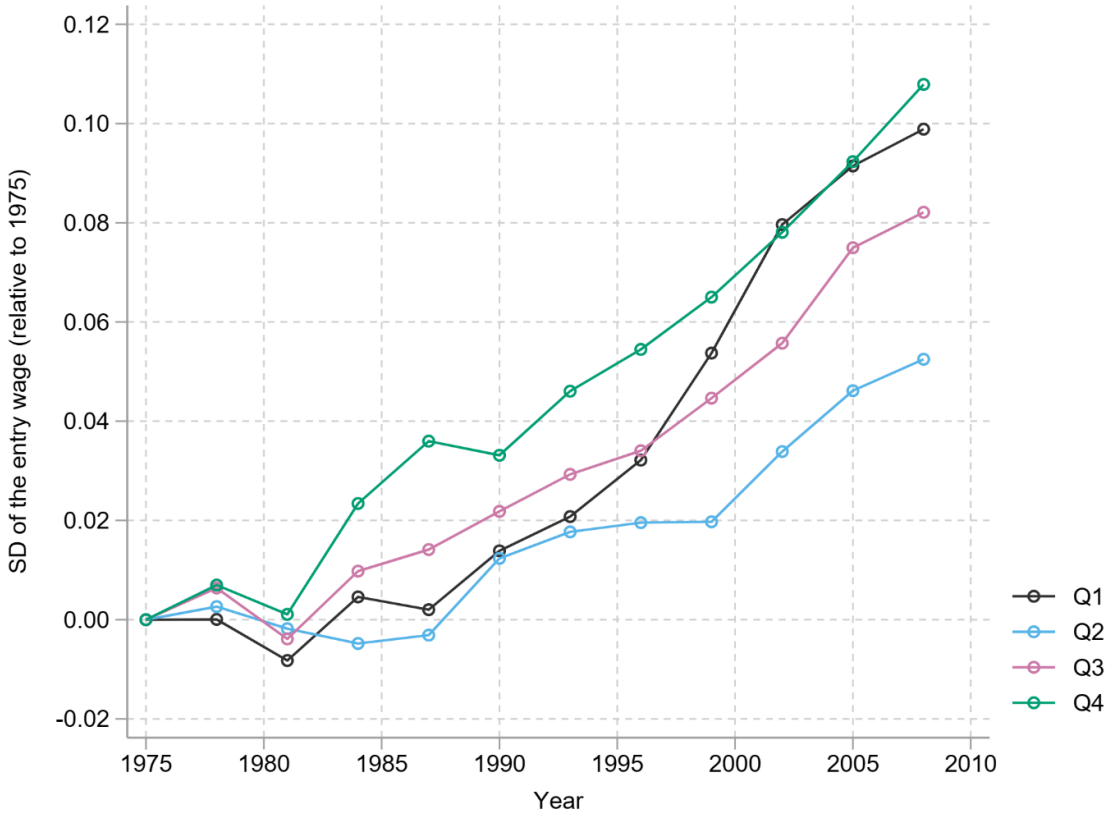
Note: The figure shows the evolution of the variance in log wages of workers at age 25 across birth year decomposed into changes between and within 120 occupations.
Source: Own calculations based on IAB data

Figure 13 Distribution of complexity measure across occupations



Note: The figure shows the distribution of our complexity measure across occupations.
Source: Own calculations based on IAB and BIBB data

Figure 14 Standard deviation of residualized log wage by occupation groups based on complexity



Note: This figure shows the standard deviation of residualized log wage by occupation groups based on complexity. Q1 refers to the occupations with the lowest level of complexity in 1991/1992 and Q4 to the occupations with the highest level of complexity.
Source: Own calculations based on IAB and BIBB data

Tables

Table 1 Changes in upper- and lower-tail inequality

Years	Complexity Group	p90p10	p90p50	p50p10
1975-1991	Q1	+0.064	+0.027	+0.037
	Q2	+0.070	+0.049	+0.021
	Q3	+0.059	+0.038	+0.022
	Q4	+0.120	+0.115	+0.005
1991-2009	Q1	+0.347	+0.188	+0.159
	Q2	+0.156	+0.081	+0.076
	Q3	+0.194	+0.113	+0.081
	Q4	+0.203	+0.041	+0.162

Note: This table shows the change in the p90p10, p90p50 and p50p10 log difference of residualized log wage by occupation groups based on complexity in the years 19975-1991 and 1991-2009. Q1 refers to the occupations with the lowest level of complexity in 1991/1992 and Q4 to the occupations with the highest level of complexity.
Source: Own calculations based on BIBB data

Appendix

A Simulation

To test how well our model fits the data and how the components of our both our earnings model contribute to the evolution of inequality we simulate our data. Our simulation is performed in 4 steps.

1. In the first step, we replicate the demographic composition, in our case the educational distribution and the cohort size at age 25. The number of observations in the simulation to be ten times larger than the original cohort size to minimize simulation errors.
2. In the second step we simulate for all workers the 10 wage observations $\ln w_{iceat}$ from age 25 to 34 (we can call this potential wage). To do this we simulate separately the three components of which residual wages are composed according to Equation (1) and (2): the FE $p_t\alpha_{ic}$, the persistent shock z_{iat} and the transitory shock ϵ_{it}. To simulate the three wage components, we draw the components from a normal distribution with mean zero (because the components are residual wages) and the variance depending on the estimated parameters of our model (Figure 4). The simulated FE $p_t\alpha_{ic}$ depends on the variance of the FE in the workers birth cohort $\sigma^2_{\alpha c}$ as well as the skill price p_t for the respective year the wage should be generated. For example, if we compute the fixed effect for a worker born in 1950 at age 30, we use the skill price in year 1980 and the variance of cohort 1950: $p_{1980}\alpha_{i1950}$. The variance of the transitory shock $\sigma^2_{\epsilon t}$ depends on the year in which the wage of the worker should be generated. The permanent shock is computed sequentially: first we compute the wage innovation at age 25. Then at age 26, the depreciated wage innovation from age 25 is added to the new wage innovation at age 26 which is based on the variance estimate of the new year. For example, for a worker aged 26 in year 1981 we compute: $z_{i,26,1981} = \rho_{1981} z_{i,25,1980} + \eta_{i1981}$. Finally, we also add the non-stochastic component of wages μ_{ceat} which is based on the average wage in each interacted cell of the observables age, year and education. After this procedure, every worker now has 10 simulated observations of a potential wage from age 25 to 34.

3. In the second step, we generate for each worker the employment status based on his potential wage from step 2. According to equation (4) each worker faces in every of his 10 periods a different probability to become unemployed $\Pr(U_{iat} = 1 \mid a, w_{i,t-1}, U_{i,t-1})$ which depends on a) the calendar year t b) his age a c) the wage quartile of his potential wage in the last period $w_{i,t-1}$ and d) whether he was employed in the last period or not $U_{i,t-1}$. Using these probabilities and the information of the potential wage from step 2 (which gives us the information in which wage quartile the worker is), we simulate sequentially for each worker if he will be unemployed from age 25 to 34.[26] If the worker is simulated to be unemployed in a given year, then we also draw their length of non-employment $\ddot{v}_{iat}$ from the truncated exponential distribution according to Equation (6), The estimate for Equation (6) was computed from the data. In some cases, an unemployed worker will be unemployed for the full year. In that case we exchange his potential earnings (based on his potential wage we computed in step 1) with 0. The worker could however also be simulated to be 75% of the year unemployed. We then exchange his potential earnings from step 1 for the fraction of the year with 0 according to $(1 - \ddot{v}_{it}) \exp(\mu_{ceat} + p_t \alpha_{ic} + z_{iat} + \epsilon_{it})$. For example, a worker with potential wage that equals 20.000 EUR earnings in a year who is 75% unemployed he is simulated to receive earnings of 5000 EUR. What happens to workers who are unemployed in the following period? We draw again from the probability distribution to simulate if he is employed or unemployed in the next year. Non-employment in our model can be persistent, because a worker faces a different (higher) probability to be unemployed if they were unemployed already in the previous period as can be seen in Figure 5. If the unemployed worker becomes employed again, he goes back to his original wage path which we calculated in step 2.
4. After this procedure we have for each individual 10 simulated values (for age 25-34) of a potential log wages $\ln(\widehat{w_{iceat}})$ for which some could be missing if the person

[26] We simulate this sequentially, as the probability to become unemployed depends on the employment status in the previous period.

is unemployed[27]. and 10 simulated values of annual earnings $\hat{e}_{it}$ for which some could be 0 if the person is unemployed for the full year. We then sum up the 10 years of earnings to arrive at the total accumulated earnings over the period. Here we also condition on workers whose accumulated working days are at least the equivalent of 3 out of 10 years to ensure we only consider worker with a minimum of attachment to the labor market.

B Selection into Non-employment

In principle, changing non-employment rates could affect the cross-sectional distribution of wages if selection into non-employment is sufficiently based on the wage and we explicitly consider this in our employment process. In this section we perform two additional decompositions to explain why in our counterfactual decomposition in Section 4.2 the employment margin - despite large changes in the non-employment rate and positive correlation between non-employment and wages - does not affect the evolution of cross-sectional wage inequality. Looking at the non-employment probability across the potential wage decile in Figure A2 we note that the probability to become unemployed is considerably higher for low wage earners than for high wage earners. We also note that the slope of this relationship becomes more negative in years of high unemployment. However, the question is, if this difference in non-employment probability across wages is strong enough to influence the distribution of wages when the unemployment varies across years.

To understand this, we perform two additional counterfactual scenarios of our employment process. Our employment process estimates for each simulated individual the employment status stochastically in each period based on his age, year and importantly his potential wage decile. This means workers with lower simulated wage having a higher probability of non-employment than simulated individuals with higher simulated wage.

We now change this scenario to account for the two possible extremes which are

[27] In that case we do not consider it in the cross-section.

illustrated in Figure A3 for an unemployment rate of 10%:

The first scenario is a scenario of full or perfect selection into non-employment. In this scenario the non-employment in a given year (determined by the aggregate unemployment rate in the data) is fully faced by the lowest earners (the individuals in our simulation which have the lowest potential wage in our first step estimation). Those individuals are assigned 0 earnings and missing wage and they are not considered in the cross-sectional wage inequality. In this scenario an increasing aggregate unemployment rate across years should ceteris paribus decrease the cross-sectional wage inequality because with more unemployment more low-wage earners leave the wage distribution.

In the second scenario, The *No Selection* Scenario, non-employment does not depend on the wage and changes in the unemployment rate should not affect the cross-sectional wage distribution at all.

Figure A4 shows the result of this counterfactual decomposition. We see that if the labor market operated under a system of perfect selection, where the lowest earners were systematically sorted into non-employment, cross-sectional inequality would (i) have been lower on average during the period we examine, and (ii) would have continued to rise after 2010 as unemployment rates declined and the lowest wage earners re-entered the wage distribution. Furthermore, our analysis shows that both the full model and the data closely align with a *No Selection* scenario, suggesting that the differences in non-employment probabilities along the wage distribution as we saw in Figure B1 are too small to significantly impact the cross-sectional distribution. Consequently, the employment margin has a minimal effect on shaping cross-sectional inequality during the period we study.

C Proof of Propositions

To show Proposition 1, note that the maximum earnings among workers in the routine sector are $\exp(\delta S^*)\ w_R$, whereas the minimum earnings among workers in the complex sector are $\exp(S^*)\ w_C$, since the effective unit of labor is increasing in ability S and workers engage in complex tasks if and only if $S \geq S^*$. Considering that $\exp(\delta S^*)\ w_R = \exp(S^*)\ w_C$ by the definition of S^*, we observe that for any $S_0 \leq S^* \leq S_1$,

$$\exp(\delta S_0)\, w_R \leq \exp(\delta S^*)\; w_R = \exp(S^*)\; w_C \leq \exp(S_1)\; w_C. \qquad \blacksquare$$

To show Proposition 2, note that $S^* = \frac{1}{1-\delta}\ln\frac{w_R}{w_C}$, and thus, $\frac{\partial S^*}{\partial \omega} < 0 \Leftrightarrow \frac{\partial}{\partial \omega}\frac{w_C}{w_R} > 0$. From equation (12), we have

$$\left[\frac{\Phi(z-\delta\sigma)}{1-\Phi(z-\sigma)}\cdot\frac{\partial}{\partial z}\frac{1-\Phi(z-\sigma)}{\Phi(z-\delta\sigma)}\cdot\frac{1}{\sigma}-\frac{1-\delta}{1-\varphi}\right]\frac{\partial S^*}{\partial \omega} = \frac{1}{1-\varphi}\frac{\partial}{\partial \omega}\ln\frac{\omega}{1-\omega} > 0,$$

where $z = \frac{S^*-\mu}{\sigma}$. Since $\frac{\partial}{\partial z}\frac{1-\Phi(z-\sigma)}{\Phi(z-\delta\sigma)} < 0$ and $\frac{\Phi(z-\delta\sigma)}{1-\Phi(z-\sigma)}\cdot\frac{\partial}{\partial z}\frac{1-\Phi(z-\sigma)}{\Phi(z-\delta\sigma)}\cdot\frac{1}{\sigma}-\frac{1-\delta}{1-\varphi} < 0$, it follows that $\frac{\partial S^*}{\partial \omega} < 0$. $\blacksquare$

To show Proposition 3, we first show that the expectation of the effective unit of labor is inverted U-shaped and peaked when $S^* = 0$. In fact,

$$E[\xi(S;S^*)] = \int_{S^*}^{\infty}\exp(s)\; f_S(s)ds + \int_{-\infty}^{S^*}\exp(\delta s)\; f_S(s)ds,$$

and hence,

$$\frac{\partial E[\xi(S;S^*)]}{\partial S^*} = f_S(S^*)[\exp(\delta S^*) - \exp(S^*)] < 0 \Leftrightarrow S^* > 0. \qquad (C1)$$

In addition, it follows that

$$E[\xi(S;0)] > \lim_{S^*\to-\infty} E[\xi(S;S^*)] = E[\exp(S^*)] = \exp\left(\frac{\sigma^2}{2}\right) \geq 1. \qquad (C2)$$

Note also that $Var\big(\xi(S;S^*)\big) = E[\xi^2(S;S^*)] - E[\xi(S;S^*)]^2$, and $E[\xi^2(S;S^*)] = \int_{S^*}^{\infty}\exp^2(s)\; f_S(s)ds + \int_{-\infty}^{S^*}\exp^2(\delta s)\; f_S(s)ds$. Thus, we observe

$$\frac{d}{dS^*}Var\big(\xi(S;S^*)\big) = f_S(S^*)[\exp(\delta S^*) - \exp(S^*)]\big[\exp(\delta S^*) + \exp(S^*) - 2\,E[\xi(S;S^*)]\big].$$

Proposition 3.2 immediately follows from this equation since if $S^* = 0$, we have $\exp(\delta S^*) - \exp(S^*) = 0$. $\blacksquare$

We next show Proposition 3.1. To this end, define $g(S^*) = \exp(\delta S^*) + \exp(S^*) - 2\,E[\xi(S;S^*)]$. For $S^* > 0$, since $\exp(\delta S^*) - \exp(S^*) < 0$, and equation (B1) implies that $g(S^*)$ is monotonically increasing, $\frac{d}{dS^*}Var\big(\xi(S;S^*)\big)$ is monotonically decreasing. Hence, Proposition 3.1 holds if we show that there exists $S_0 > 0$ such that $g(S_0) = 0$, or equivalently, $\frac{\exp(\delta S^*)+\exp(S^*)}{2} = E[\xi(S;S^*)]$. Since the left-hand side is continuous and monotonically increasing from 1 to ∞ for

$S^* > 0$, and since the right-hand side is continuous and monotonically decreasing for $S^* > 0$ with $1 < E[\xi(S;0)] < \infty$ from equation (C2), these two lines cross at one point S_0. ■

For Proposition 3.3, we next consider the case with $S^* < 0$, and since $\exp(\delta S^*) - \exp(S^*) > 0$ in this case, it is sufficient to show $g(S^*) < 0$. Given that $0 < \frac{\exp(\delta S^*) + \exp(S^*)}{2} < 1$ for $S^* < 0$, equations (C1) and (C2) imply that

$$\frac{\exp(\delta S^*) + \exp(S^*)}{2} < 1 \leq \lim_{S^* \to -\infty} E[\xi(S;S^*)] < E[\xi(S;S^*)],$$

and therefore, $g(S^*) < 0$ for $S^* < 0$. ■

Finally, we show Corollary 1. Since

$$\frac{\partial}{\partial \omega} Var\left(\xi(S;S^*)\right) = \frac{d}{d\,S^*} Var\left(\xi(S;S^*)\right) \cdot \frac{\partial}{\partial \omega} S^*,$$

and $\frac{\partial}{\partial \omega} S^* < 0$ from Proposition 2, the sign of $\frac{\partial}{\partial \omega} Var\left(\xi(S;S^*)\right)$ is opposite to $\frac{d}{d\,S^*} Var\left(\xi(S;S^*)\right)$. Therefore, given Proposition 3, it is sufficient to show that the equilibrium threshold $S^* = S^*(\omega, \varphi, \mu, \sigma, \delta)$ ranges $(-\infty, \infty)$ for $\omega \in (0,1)$. To this end. we first show that $S^*(\omega, \varphi, \mu, \sigma, \delta)$ is continuous and monotonically decreasing in ω. Rewriting equation (12) as

$$\Xi(S^*, \varphi, \mu, \sigma, \delta) = (1-\varphi)\left[(1-\delta)\,\mu + \frac{(1-\delta^2)\sigma^2}{2} + \ln\frac{1 - \Phi\left(\frac{S^*-\mu}{\sigma} - \sigma\right)}{\Phi\left(\frac{S^*-\mu}{\sigma} - \delta\sigma\right)} - \frac{(1-\delta)}{1-\varphi} S^*\right]$$
$$= \ln\frac{\omega}{1-\omega}, \qquad (C3)$$

we note that $\Xi(S^*, \varphi, \mu, \sigma, \delta)$ is continuous and monotonically decreasing in S^*. Hence, there exists a continuous and monotonically decreasing inverse function $\Xi^{-1}(\omega, \varphi, \mu, \sigma, \delta) = S^*(\omega, \varphi, \mu, \sigma, \delta)$. In addition, it is evident from equation (C3) that $\lim_{\omega \to 0} S^*(\omega, \varphi, \mu, \sigma, \delta) = \infty$ and $\lim_{\omega \to 1} S^*(\omega, \varphi, \mu, \sigma, \delta) = -\infty$. Therefore, given the continuity and monotonicity, $S^*(\omega, \varphi, \mu, \sigma, \delta)$ ranges $(-\infty, \infty)$ for $\omega \in (0,1)$. ■

D Model extension

This section generalizes our Roy model. In particular, we relax the distributional assumption on workers' skill and the functional form assumption on the effective unit of

labor. For skill distribution, we assume that S has the cumulative distribution function $F_S(\cdot)$ with support $\mathcal{S}$. We assume that $\inf \mathcal{S} < 0 < \sup \mathcal{S}$, so that there are workers who are more productive in routine (complex) tasks. We denote the effective unit of labor of a worker with ability S by $\psi(S)$ for complex tasks and $\psi(\delta S)$ for routine tasks, where $\psi: \mathcal{S} \to \mathbb{R}^2_{++}$ is continuous and monotonically increasing on $\mathcal{S}$. In addition, we assume that the first and second moments of $\psi(S)$ exist, and $h(S;\delta) = \frac{\psi(S)}{\psi(\delta S)}$ is monotonically increasing for all $S \in \mathcal{S}$, which serves as the single crossing condition. In fact, the worker chooses routine tasks if and only if

$$\psi(S)w_C \geq \psi(\delta S)w_R, \qquad \therefore S \geq S^* = h^{-1}\left(\frac{w_R}{w_C}; \delta\right), \tag{D1}$$

where $h^{-1}(\cdot\,;\delta)$ exists due to the monotonicity of $h(\cdot\,;\delta)$. Then, the effective unit of aggregate labor supply to each sector is

$$C^S = \int_{S^*}^{\sup \mathcal{S}} \psi(S) dF_S\,, \qquad R^S = \int_{\inf S}^{S^*} \psi(\delta S) dF_S\,. \tag{D2}$$

The equilibrium is $\left(C^D, R^D, C^S, R^S, S^*, \frac{w_C}{w_R}\right)$ that satisfies equations (8), (D1) and (D2) and the market clearing conditions, $C^D = C^S$ and $R^D = R^S$. The existence and uniqueness of the equilibrium are shown similarly to our baseline model in Section 5.1. Furthermore, it is obvious that Proposition 2 holds with almost identical proof. That is, we observe that $\frac{\partial}{\partial \omega}\frac{w_C}{w_R} > 0$ and $\frac{\partial}{\partial \omega}S^* < 0$. We now show the non-monotonic nature of the variance of effective unit of labor:

Proposition 4

Suppose that $\inf_{S \in \mathcal{S}} \psi(\delta S) < E[\psi(S)]$ and $\xi(0) \neq E[\psi(S;0)]$. Then, the variance of the realized effective unit of $Var\big(\xi(S;S^*)\big)$ is non-monotonic in S^*.

Proof

Note first that with similar argument in our baseline model, we observe that $E[\xi(S;S^*)]$ is inverted U-shaped with peak at $S^* = 0$, and the derivative of the variance is given by

$$\frac{d}{dS^*}Var\big(\xi(S;S^*)\big) = f_S(S^*)[\psi(\delta S^*) - \psi(S^*)]\big[\psi(\delta S^*) + \psi(S^*) - 2\,E[\xi(S;S^*)]\big].$$

We first show that the variance is decreasing when S^* is sufficiently small or large. Define $g(S^*) = \psi(\delta S^*) + \psi(S^*) - 2\,E[\xi(S;S^*)]$. Since $\psi(S)$ is increasing in S, we

have

$$g(\inf \mathcal{S}) = \frac{\psi(\delta \inf \mathcal{S}) + \psi(\inf \mathcal{S})}{2} - E[\xi(S; \inf \mathcal{S})] < \psi(\delta \inf \mathcal{S}) - E[\psi(S)] < 0,$$

where $E[\xi(S; \inf \mathcal{S})] = E[\psi(S)]$ by the definition of $\xi(\cdot)$ and the last inequality follows from the assumption of Proposition 3. As $g(\cdot)$ is continuous, there exists $\underline{S} \in \mathcal{S} \cap \mathbb{R}_-$ such that for all $S^* \leq \underline{S}$, $g(S^*) < 0$, and thereby, $\frac{d}{dS^*} Var\big(\xi(S; S^*)\big) < 0$. On the other hand, since $\psi(S)$ is increasing, we observe $E[\psi(S; \sup \mathcal{S})] = E[\psi(\delta S)] < \sup_{S^* \in \mathcal{S}} \psi(\delta S^*) \leq \sup_{S^* \in \mathcal{S}} \frac{\psi(\delta S^*) + \psi(S^*)}{2}$, implying $g(\sup \mathcal{S}) > 0$. So, there exists $\bar{S} \in \mathcal{S} \cap \mathbb{R}_+$ such that for all $S^* \geq \bar{S}$, $g(S^*) > 0$, and $\frac{d}{dS^*} Var\big(\xi(S; S^*)\big) < 0$.

Therefore, to show the non-monotonicity of the variance, it is sufficient to show $\frac{d}{dS^*} Var\big(\xi(S; S^*)\big) > 0$ for some S^*. If $\psi(0) < E[\xi(S; 0)]$, we have $g(0) < 0$. Since $g(\cdot)$ is continuous, there exists $\ddot{S}_+ \in \mathcal{S} \cap \mathbb{R}_+$ such that for all $S^* \in \big(0, \ddot{S}_+\big)$, $g(S^*) < 0$, and $\frac{d}{dS^*} Var\big(\xi(S; S^*)\big) > 0$ since $\psi(\delta S^*) - \psi(S^*) < 0$ for $S^* > 0$. Similarly, if $\psi(0) > E[\xi(S; 0)]$, we have $g(0) > 0$, and there exists $\ddot{S}_- \in \mathcal{S} \cap \mathbb{R}_-$ such that for $S^* \in \big(\ddot{S}_-, 0\big)$, $g(S^*) > 0$, and $\frac{d}{dS^*} Var\big(\xi(S; S^*)\big) > 0$ since $\psi(\delta S^*) - \psi(S^*) > 0$ for $S^* < 0$. ■

Note that with assumption $\psi(0) \leq E[\psi(S)]$, we can generalize Proposition 3:

Proposition 5

Let $\xi(S; S^*)$ be the (realized) effective unit of labor of a worker with skill S, given the threshold value S^*. Suppose that $\psi(0) \leq E[\psi(S)]$.[28] Then,

1. For $S^* > 0$, $Var\left(\xi(S; S^*)\right)$ is inverted U-shaped.
2. For $S^* = 0$, $\frac{d}{d\,S^*} Var\left(\xi(S; S^*)\right) = 0$.
3. For $S^* < 0$, $\frac{d}{d\,S^*} Var\left(\xi(S; S^*)\right) < 0$.

Proof

[28] This assumption holds when ability S has mean-zero distribution and $\psi(\cdot)$ is convex by Jensen's inequality.

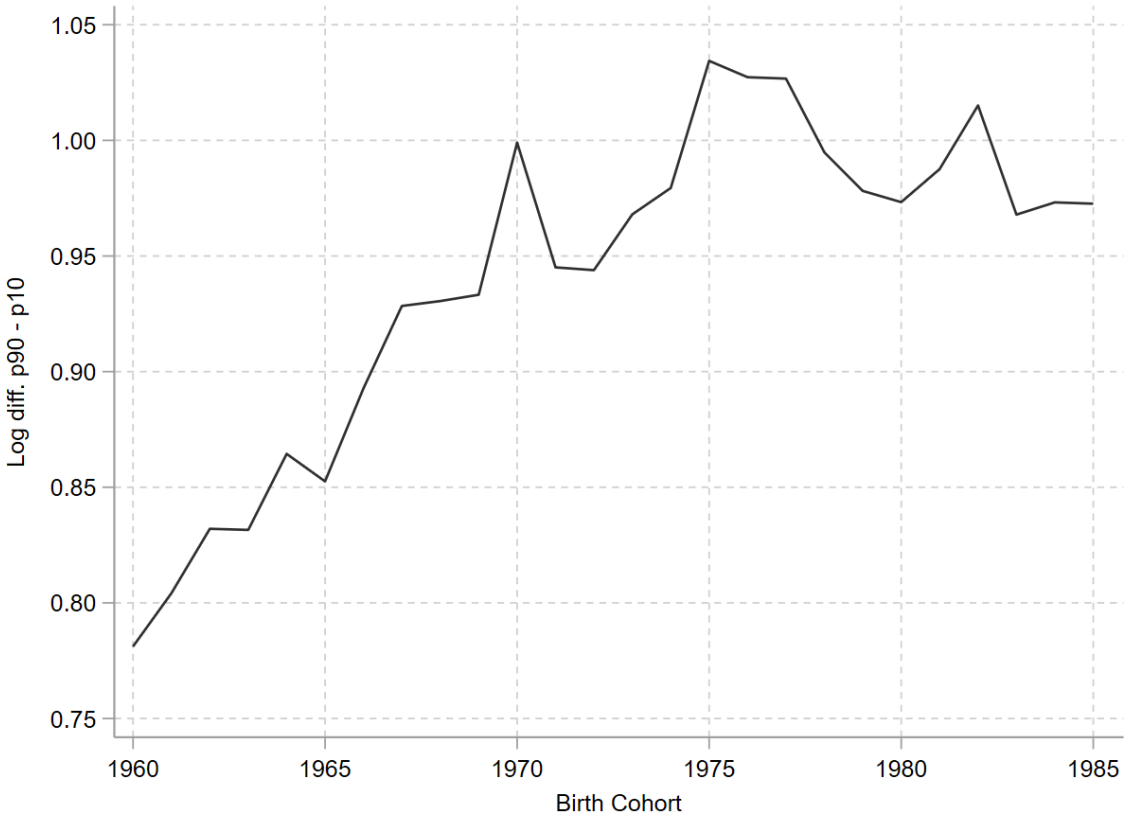
First, Proposition 5.2 is trivial. To show Proposition 5.3, note that by the definition of $\xi(\cdot)$, we have $E[\xi(S;\inf\mathcal{S})] = E[\psi(S)]$, and so, the assumption of the proposition implies that for $S^* < 0$,

$$\frac{\psi(\delta S^*) + \psi(S^*)}{2} < \psi(0) \leq E[\psi(S)] = E[\xi(S;\inf\mathcal{S})] < E[\xi(S;S^*)],$$

where the last inequality follows since $E[\xi(S;S^*)]$ is inverted U-shaped with peak $S^* = 0$. Therefore, for $S^* < 0$, we have $g(S^*) < 0$, and thus, $\frac{d}{dS^*}Var\big(\xi(S;S^*)\big) < 0$. To show Proposition 5.1, consider the case with $S^* > 0$. By the definition of $\xi(\cdot)$, $E[\xi(S;\inf\mathcal{S})] = E[\psi(S)]$ and $E[\xi(S;\sup\mathcal{S})] = E[\psi(\delta S)]$. The assumption implies that $\psi(0) \leq E[\psi(S)] = E[\xi(S;\inf\mathcal{S})] < E[\xi(S;0)]$, so that $g(0) < 0$. Furthermore, since $\psi(S)$ is monotonically increasing, we have $E[\xi(S;\sup\mathcal{S})] = E[\psi(\delta S)] < \sup_{S^*\in\mathcal{S}} \psi(\delta S^*) \leq \sup_{S^*\in\mathcal{S}} \frac{\psi(\delta S^*)+\psi(S^*)}{2}$, so that $g(\sup\mathcal{S}) > 0$. Hence, $g(0) < 0 < g(\sup\mathcal{S})$. Since $g(S^*)$ is continuous and monotonically increasing for $S^* > 0$, there exists $\tilde{S} \in \mathcal{S} \cap \mathbb{R}_+$ such that $g(S^*) < 0$ for $0 < S^* < \tilde{S}$, $g(S^*) = 0$ for $S^* = 0$, and $g(S^*) > 0$ for $S^* > 0$. Therefore, given that $\psi(\delta S^*) - \psi(S^*) < 0$ for $S^* > 0$, we observe that $Var\big(\xi(S;S^*)\big)$ is inverted U-shaped for $S^* > 0$ with peak $S^* = \tilde{S}$. ■

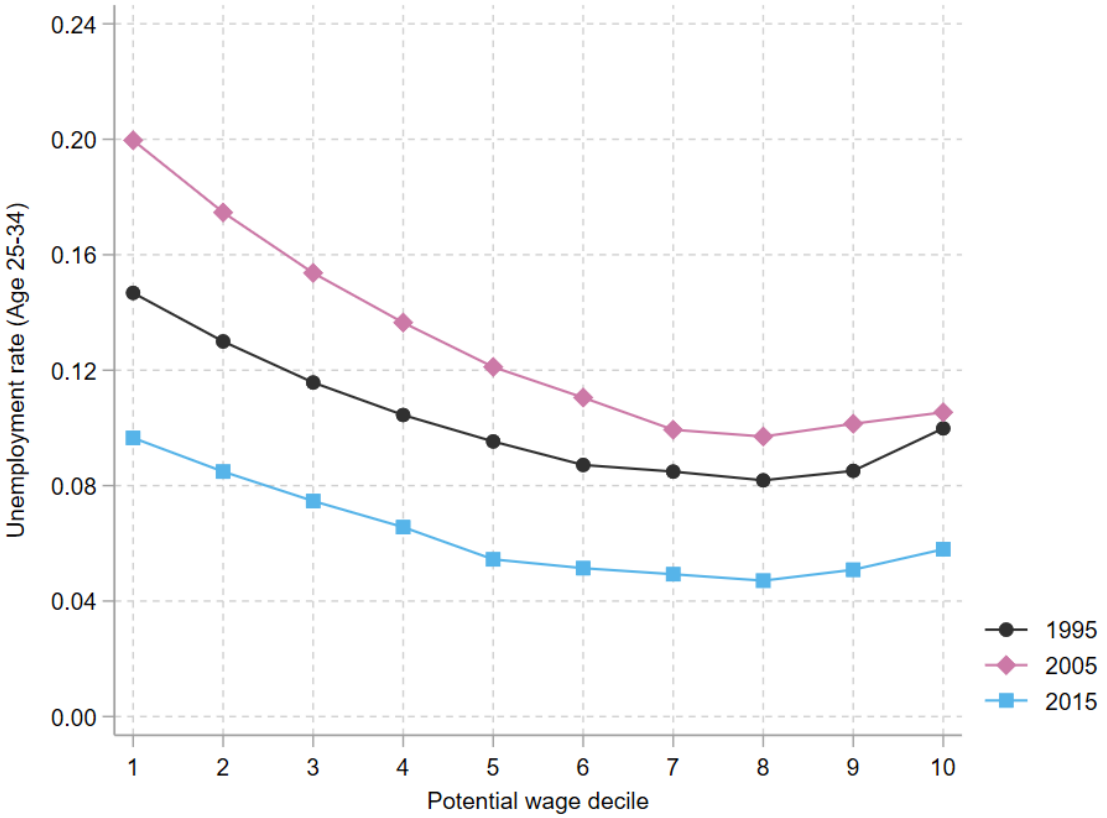
E Figures

Figure A1 p90-p10 ratio of lifecycle earnings



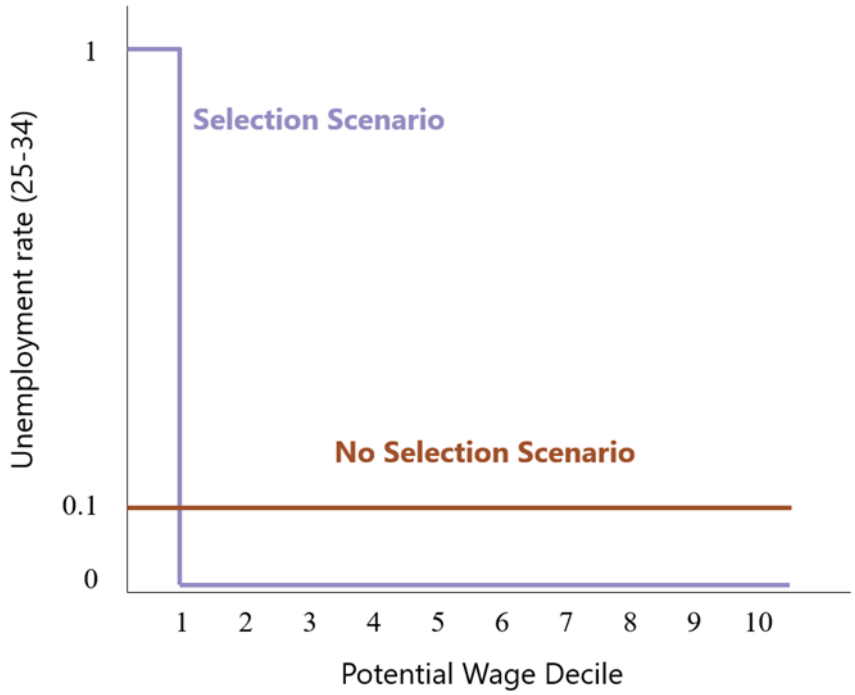
Note: This graph shows the p90p10 log ratio of our measure of lifecycle earnings calculated as the total earnings from age 25 to 34.
Source: Own calculations based on IAB data

Figure A2 Non-employment rate across the potential wage distribution



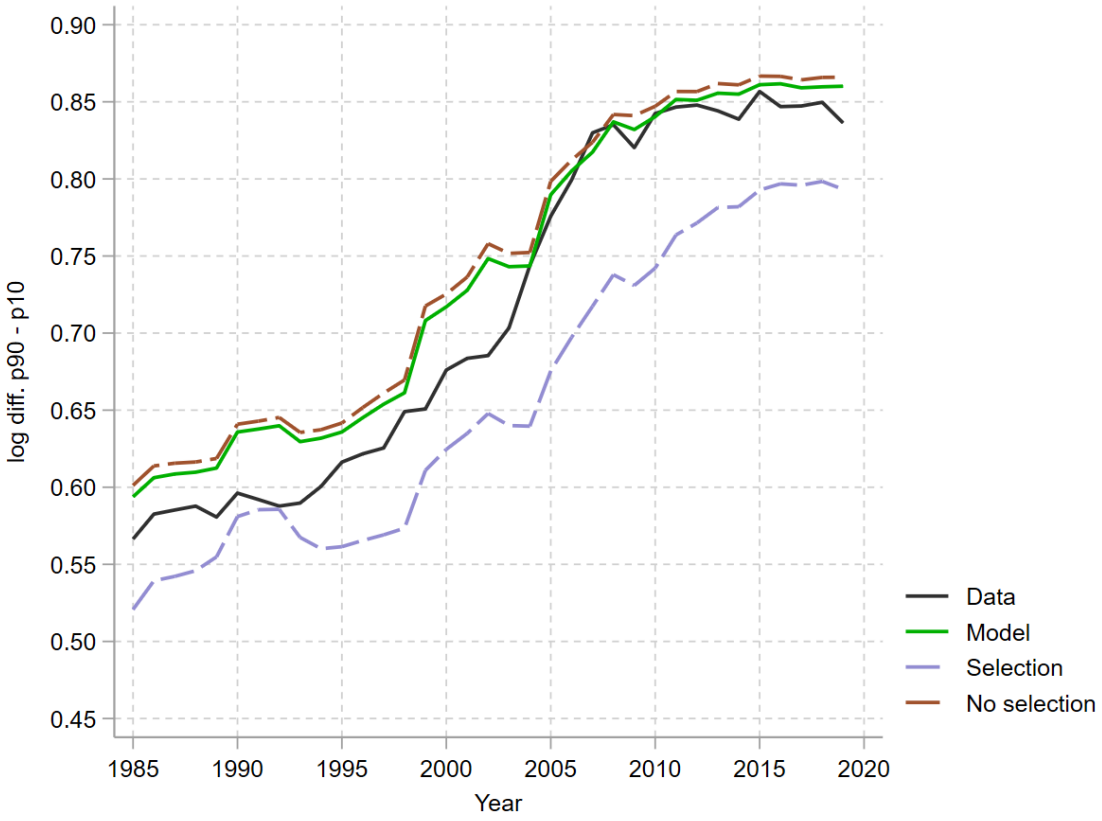
Note: The figure shows the rate of non-employment across potential wage in our simulated data across certain calendar years.
Source: Own calculations based on IAB data

Figure A3 Selection and No Selection Counterfactual Scenario



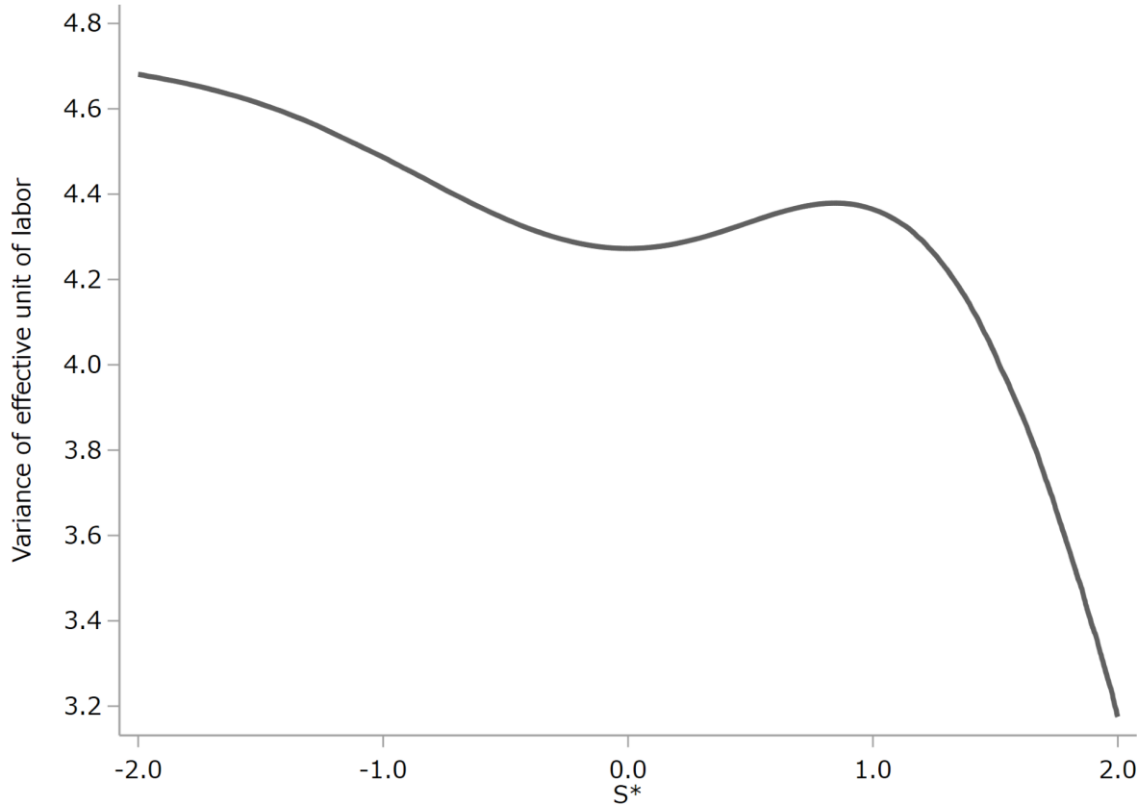
Note: The figure shows the probability of non-employment for an aggregate 10% unemployment rate under the counterfactual Selection and No Selection Scenario across the Potential Wage Distribution. In the Selection Scenario, the bottom wage decile has a 100% probability of being unemployed while the rest of the wage distribution has a 0% probability of being unemployed. In the No Selection Scenario, the probability of unemployment is 10% across the distribution.
Source: Own illustration

Figure A4 Selection and No Selection Counterfactual Simulation



Note: The figure shows the results of the counterfactual decomposition under the full model, the Selection scenario and the No Selection Scenario.
Source: Own calculations based on IAB data

Figure A5 Simulation of the Roy Model – Variance of Effective Unit of Labor



Note: To simulate the model we draw skill $S \sim N(0,1)$ for 100,000 individuals and set $\delta = 0.1$. Given the value of S^*, the worker with skill S is assigned the effective unit of labor $\xi = exp(S)$ if $S > S^*$ and $\xi = exp(\delta S)$ otherwise. We then compute $Var(\xi)$ for different levels of S^*.
Source: Own illustration

F Tables

Table A1 Regression results wage on complexity

Log Wage 1991/1992	(1)	(2)	(3)	(4)
Complexity	0.198***	0.100***	0.102***	0.079***
	(0.004)	(0.004)	(0.004)	(0.004)
Age			0.004***	0.003***
			(0.000)	(0.000)
Supervisor				0.215***
				(0.008)
Occupation Controls		x	x	x
R^2	0.127	0.385	0.392	0.415
N Obs	17,331	17,331	17,331	17,331

Note: This table shows the results of OLS regressions of monthly log wages of West Germans in 1991/1992 on our measure of task complexity (standardized), age and supervisor status and 3-digit occupation controls. Monthly log wages are calculated from a categorical variable representing wage ranges. The midpoint of each wage range is used to represent the wage for that category. For example, for the range [2000, 2500), the midpoint is 2250. The log transformation of the midpoint is then taken to compute the monthly log wage. Significance levels are denoted by *10%, **5% and ***1%.
Source: Own calculations based on BIBB data