Robinson Meets Roy: Monopsony Power and Comparative Advantage*

Preliminary Draft

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Abstract

We provide a number of insights into the nature and consequences of monopsony power through the lens of comparative advantage, where employers' power in wage setting stems from match-specific rents. Chief among them is that employers will apply larger wage markdowns to workers with greater comparative advantage at their firm. This leads to stronger monopsony power over more productive workers, provided the workers' comparative advantage aligns with their absolute advantage. Using Brazilian administrative data, we confirm this prediction: monopsony disproportionately affects high-wage workers within firms and workers at high-paying firms. Because monopsony power is skewed against high-wage workers, monopsony acts as a force for wage equality and minimum wage policies are unlikely panaceas for labor market power.

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1 Introduction

Research on labor markets is increasingly focused on the role of monopsony power, examining its impact on income inequality and labor's allocation, as well as its implications for policy interventions, specifically minimum legal wages. While traditional approaches associate monopsony power with firm characteristics—especially a firm's employment share—this paper proposes a novel perspective by attributing monopsony power to comparative advantage at the level of a worker-firm match.

Our study integrates Robinson's (1933) model of non-competitive firm wage setting with Roy's (1951) model of worker comparative advantage. The reasoning is as follows. Monopsony power emerges when a firm faces an imperfectly elastic labor supply, implying the firm has workers it could employ at lower wages. The differential between wage received and wage required represents a rent for those inframarginal workers. The Roy model associates rents with a worker's comparative advantage in their job, i.e., their productivity at their chosen job relative to alternatives. As a result, we can associate monopsony power in wage setting with workers who have a comparative advantage at their firm.

This synthesis directs attention to match characteristics in looking for monopsony power in the data. In particular, our model predicts that employers will discriminate in wage setting, exercising larger wage markdowns on workers who exhibit greater comparative advantage at their firm. When worker comparative advantage aligns with their absolute advantage, employers apply larger markdowns to their most productive workers. Hence, monopsony power promotes wage *equality* within firms. Intuitively, monopsony taxes worker rents; so, presuming higher-wage workers command larger employer rents, they will face steeper markdowns.

We study an oligopsony equilibrium where employers compete for workers, initially focusing on symmetric firms to analytically characterize employers' wage strategies over more and less productive workers. The allocation of workers across firms is efficient because firms' markdowns are symmetric. Firms offer wages that reflect a worker's productivity and the distribution of second-best productivities for workers of that productivity. In turn, markdowns at each productivity reflect a weighted average of workers' comparative advantage with their employer—greater comparative advantage means steeper markdown. We also show how comparative advantage, and hence markdowns, are affected by the number of firms competing for a worker and the correlation of that worker's productivity across firms. Finally, we show that a minimum wage, through the strategic interaction of firms in oligopsony, acts to raise equilibrium wages throughout the wage distribution.

We next examine an asymmetric duopsony where one firm exhibits higher productivity. The more productive firm pays higher wages, is larger, but consequently has more inframarginal

workers with rents, leading it to impose larger wage markdowns. These predictions align with previous models that associate monopsony power with large employers and predict that monopsony reduces aggregate productivity by distorting labor away from the largest, most productive firms (Berger, Herkenhoff and Mongey, 2022).

We also consider duopsony where one firm systematically exhibits comparative advantage for skilled workers (e.g., its technology is more skill-biased). This firm is high-wage, because it predominantly employs high-skilled workers, but especially marks down wages for its most-productive workers whose comparative advantage is greatest. Conversely, low-wage firms apply larger markdowns to their least-skilled workers. Two key implications emerge. First, monopsony impairs skill sorting across employers by diverting workers away from their comparative-advantage employer, beyond its distortion to relative firm sizes. Second, while existing literature emphasizes monopsony's effects on lower-wage workers, particularly in minimum wage analyses,[1] our model suggests the most substantial markdowns occur for highest-wage workers at highest-wage firms, thereby promoting wage equality.

We test our predictions using data from Brazil's formal labor market through the Relação Anual de Informações Sociais (RAIS). The RAIS tracks establishment-worker matches, providing information on location, industry, employment, earnings, hours, and occupations. We segment workers into markets based on their metropolitan area and broad occupation. We then further stratify workers by their relative wages at their employer to see how an employer's rate of employment growth affects both wages and employment throughout its wage distribution. The inverse elasticity of an employer's labor supply curve — which determines wage markdowns — is revealed through the relative responses of wages versus employment at different points in the wage distribution.

Our key finding is that employment growth generates larger wage changes for an employer's high-wage workers while employment responds more strongly at the bottom of its wage distribution. This indicates less elastic labor supply for employers' higher-wage workers. More exactly, our estimates imply markdowns greater than 20% for those in the top wage quartile at an employer, while showing minimal monopsony power for employers' low-wage workers. Combining all workers our estimates imply an average markdown of about 13%.

Moreover, an employer's market share does not predict larger wage responses to its employment growth, implying that markdowns do not systematically differ between larger and smaller Brazilian employers. By contrast, high-wage employers face less elastic labor supply schedules, resulting in larger markdowns for their workers, except at the bottom of the wage distribution where markdowns are generally smaller.

We end by examining the impact of an informal labor market and a minimum wage—both

[1] References include Dube, Giuliano and Leonard (2019), Godoey and Reich (2021), and Jardim et al. (2022).

important features of the Brazilian labor market—in a model calibrated to match our markup estimates. While the literature suggests minimum wages can reduce monopsony power by turning employers into wage-takers at the legal minimum, our model indicates this approach may not be so effective. Since monopsony power is concentrated among higher-wage workers, a minimum wage high enough to address these workers would displace substantial numbers of lower-wage workers from the formal sector. Furthermore, while a higher legal minimum raises wages in our model throughout the wage distribution, for a reasonable calibration, we find it actually increases the average markdown across workers by displacing less-productive workers where markdowns are already low. Those displaced workers get reallocated to the informal sector where they are less productive, and, hence, earn even lower wages.

After discussing some of the most relevant literature, Section 2 presents our model and its predictions. Sections 3 and 4 describe our data, empirical specifications, and findings. Section 5 examines minimum-wage policy in our model extended to include a competitive informal sector where minimum wages go unenforced. Section 6 concludes.

Related Literature. Our paper unites two separate streams of economic theory — Roy's 1951 treatment of worker sorting and Robinson's 1933 analysis of employer market power. The Roy model is traditionally used to address comparative advantage and selection across industries or occupations under competitive wage setting. Our paper offers a novel application of the Roy Model by shifting its focus from broad occupational or industry sorting to firm-level labor allocation. Framing the Roy model at the firm level naturally leads to monopsonistic wage setting where employers' market power varies across its workers.

Like much of the recent literature, our treatment of employers' market power follows in Robinson's footsteps by using heterogeneous worker rents to trace out firms' labor supply curves, e.g., Card et al. (2018), Berger, Herkenhoff and Mongey (2022), Azkarate-Askasua and Zerecero (2022), Lamadon, Mogstad and Setzler (2022), Chan et al. (2024), and Volpe (2024).[2] However, these rents are typically modeled via differences in workers' outside options in wages or job amenities, unknown to the firm.[3] We allow worker productivity to vary within and across firms, enabling employers to discriminate their wage markdowns based on

[2]Search frictions provide another way to motivate rents, generating monopsony power or bargaining, depending on model assumptions. Examples include Burdett and Mortensen (1998), Manning (2003), Hurst et al. (2022), and Jarosch, Nimczik and Sorkin (2024). These models can relate wage setting to an employer's market share. Their implications are less clear for how monopsony power should vary across workers at a given employer, which is our primary emphasis and derives naturally from the Roy model setting.

[3]Volpe (2024) allows that workers differ in their marginal rates of substitution (MRS) of wages versus employer amenities. This generates a novel implication that high-wage firms systematically attract workers who care less about amenities, implying high-wage firms face a more elastic labor supply. Volpe also allows for the MRS of wages versus amenities to correlate with certain observable worker characteristics, e.g., schooling. As a result, a firm will vary the markdown across its workers if they differ in those observables.

worker productivity. Importantly, we model oligopsony in a strategic general-equilibrium setting, with each employer recognizing the wage strategies of competing employers as in Berger, Herkenhoff and Mongey (2022) and Chan et al. (2024).

Empirically, our estimated markdown for Brazil of 13%, *not* stratifying workers by their relative wages, roughly aligns with a number of estimates for the U.S. and elsewhere. Felix (2022) and Lobel (2024), like us, estimate labor supply elasticities for firms in Brazil's formal sector. Lobel, based on payroll tax changes as an instrument, finds an implied markdown of 19%, whereas Felix, instrumenting with tariff changes, reports a much higher markdown of 50%. Among empirical studies on the U.S. labor market, Staiger, Spetz and Phibbs (2010) estimate 10% markdowns in nursing; Dube, Giuliano and Leonard (2019) estimate markdowns of 20% in retailing; and Lamadon, Mogstad and Setzler (2022) and Berger, Herkenhoff and Mongey (2022) respectively estimate 15% and 25% markdowns economy-wide.

Central to our contribution, we provide new evidence that markdowns vary considerably by the wages of workers at an employer, with 20-25% markdowns for higher-wage workers but near zero for lower-wage. These results, and our model predictions, align with recent papers that find higher earnings responses within firms to tax changes, Carbonnier et al. (2022) and Lobel (2024), and to new patents, Kline et al. (2019).

A number of papers have examined minimum wages in light of monopsony power. Engbom and Moser (2022), Parente (2023), and Luduvice, Martinez and Sollaci (2024), like us, consider minimum wages in the presence of both market power and an informal sector. Hurst et al. (2022) and Berger, Herkenhoff and Mongey (2024) point out that minimum wages have limited impact on monopsony if most workers' wages are well above a plausible legal minimum. Our analysis highlights new limitations on the effectiveness of minimum wage policies in reducing monopsony power, given its concentration among higher-wage workers.

2 Model

We analyze wage setting and employment allocations for a symmetric oligopsony and for an asymmetric duopsony. We first present the general features of the economy and its equilibrium.

2.1 Economic Environment and Preliminaries

Consider a labor market consisting of K firms and a unit mass of heterogeneous workers. Each worker is endowed with a set of skills that translate to $Y_k \geq 0$ units of labor efficiency at firm $k \in \{1, \cdots, K\}$. Workers' efficiency levels can be correlated across firms. Let $G(y_1, \cdots, y_K) = P(Y_1 \leq y_1, \cdots, Y_K \leq y_K)$ denote the joint distribution of efficiency units in the market with marginal distributions denoted by $F_k(y_k) = P(Y_k \leq y_k)$. We assume that F_k are strictly in-

creasing and continuous. Under these conditions, G admits a unique copula representation: $G(y_1, \cdots, y_K) = C(u_1, ..., u_K)$, where C is a copula and $u_k = F_k(y_k)$ (Sklar, 1959).

We define a worker's comparative advantage in production across firms relative to the distribution of efficiency levels in the population.

Definition 1 *A worker with efficiency levels y_k and y_j at firms k and j has a comparative advantage at firm k over firm j if $\frac{y_j}{y_k} \leq \frac{\mathbb{E}[Y_j]}{\mathbb{E}[Y_k]}$.*

The definition above is ordinal. The (cardinal) strength of comparative advantage is determined by how much better a worker is at firm k versus j relative to an average worker. Accordingly, we quantify that strength by the distance between the ratio of a worker's efficiency levels at the two firms and the corresponding ratio of population means:

$$R(y_j/y_k) = \frac{\mathbb{E}[Y_j]}{\mathbb{E}[Y_k]} - \frac{y_j}{y_k},$$

for $y_j \leq y_k$. Note that when $\mathbb{E}[Y_j] = \mathbb{E}[Y_k]$, the strength of the comparative advantage varies between 0 and 1.

Each worker can be hired by a single employer. An allocation (of workers to firms) is therefore a K-dimensional partition $\Pi = \{\pi_1, \cdots, \pi_K\}$ of $\mathbb{R}^K_+$. Employment at firm k is given by the measure of workers allocated to that firm: $n_k(\pi_k) = P(\{Y_1, \cdots, Y_K\} \in \pi_k)$. Naturally, $\sum_k n_k(\pi_k) = 1$. We consider an allocation to be efficient if workers are allocated where they are most productive. Formally,

Definition 2 *An allocation Π is efficient if $\mathbf{y} = \{y_1, \cdots, y_K\} \in \pi_k$ if and only if $y_k \geq y_j$ for all $j, k \in \{1, \cdots, K\}$ and for all $\mathbf{y} \in \mathbb{R}^K_+$.*

2.2 Oligopsonistic Competition

Firms compete over workers by simultaneously posting wage offers that depend on a worker's productivity at their firm: $w_k(y) : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that firm k observes worker's productivity y_k, but not its productivity at other firms. Worker's see their wage offers across firms. Based on those offers, a worker chooses the employer that pays them the highest wage:

$$\max_{k \in \{1, \cdots, K\}} w_k(y_k). \tag{1}$$

Thus a firm attracts a worker only if its wage offer tops all other firms. In turn, labor supplied to firm k at wage w by workers with efficiency $Y_k = y$ are those whose efficiency levels elsewhere do not garner a better wage:

$$n_k(w;y) = P(w_j(y_j) \leq w, \forall j \neq k | Y_k = y) f_k(y), \tag{2}$$

where $f_k(y) = F_k'(y)$ is the density of workers with $Y_k = y$. The probability that w is the worker's highest wage offer is conditional on $Y_k = y$, because workers' productivities are generally correlated across firms.

The labor supply function faced by firm k is non-decreasing in w since the probability of w being the highest offer is increasing in w by definition. Recognizing that, oligopsonistic firms set their wage schedules to maximize profits:

$$\max_{0 \leq w_k(y) \leq y} \int (y - w_k(y)) n_k(w_k(y); y) dy \tag{3}$$

The following defines the equilibrium in this labor market:

Definition 3 *An oligopsonistic labor market equilibrium is a set of wage schedules $w_k(y)$ and an employment allocation Π such that:*

1. *For each $k = \{1, \cdots, K\}$, $w_k(y)$ maximizes firm k's profits given the labor supply function $n_k(w;y)$ and wage schedules of other firms $\{w_j(y)\}_{j \neq k}$.*
2. *Workers choose the highest wage offer given the set of wage schedules, $\{w_k(y)\}_{k=1}^K$.*
3. *The labor market clears $n_k(\pi_k) = n_k(w_k(y);y)$ for all $k = \{1, \cdots, K\}$.*

Oligopsonistic equilibrium in our setting is similar to that in first-price, sealed-bid auctions. The existence of an equilibrium with monotone bidding strategies in that context is addressed in Athey (2001) and Reny and Zamir (2004). To characterize the equilibrium, note that firm k's profit-maximizing wage offer for workers with efficiency level y satisfies:

$$(y - w) \frac{\partial n_k(w;y)}{\partial w} = n_k(w;y). \tag{4}$$

By marginally raising the wage, firm k raises employment by $\partial n_k(w;y)/\partial w$. Each additional employee contributes $y - w$ to profits. The optimal wage offer balances this gain against the transfers it entails to inframarginal workers, those for whom firm k's wage offer would already be their best. That cost, per dollar higher wage, is $n_k(w;y)$. Rearranging the optimality condition gives the well-known representation in monopsonistic labor markets:

$$\frac{w}{y} = \frac{\eta_k(w;y)}{1 + \eta_k(w;y)},$$

where $\eta(w;y) = \partial n_k(w;y)/\partial w \times w/n_k(w;y)$ is the elasticity of labor supplied by workers of efficiency level y to firm k at wage level w. More elastic labor supply schedules, higher $\eta_k(w;y)$, call for higher w/y and, therefore, lower wage markdowns $1-w/y$.

2.3 Symmetric Equilibria

We begin our analysis with a symmetric market, where firm productivities are identically drawn. This allows us to analytically characterize the oligopsonistic equilibrium with symmetric wage strategies. Formally,

Definition 4 *A market is symmetric if C is exchangeable and has common marginals $F_k(y) = F(y)$.*

For any arbitrary distribution function $\tilde{F}(x|y)$ over the interval $x \in [0, y]$, define $\mathbb{E}_{\tilde{F}}[x|x \leq y] = \int_0^y x d\tilde{F}(x|y)$ as the weighted upper-truncated mean of x with weights given by $\tilde{F}'(x|y)$.

Theorem 1 (Symmetric Equilibrium) *Consider a symmetric market with K firms. The symmetric oligopsony equilibrium is characterized by the following wage offer policy and employment allocation.*

1. *Wage offer policy*
$$w(y) = \int_0^y x d\tilde{F}(x|y) = \mathbb{E}_{\tilde{F}}[x|x \leq y],$$
where $\tilde{F}(x|y) = \exp(-\int_x^y \sum_{j\neq k} \frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))} F'(z)dz)$ and C_{kj} is the cross-partial derivative of the C w.r.t. $u_k = F_k(y)$ and $u_j = F_j(y)$ for all $j \neq k \in \{1,\ldots,K\}$.

2. *For any $\mathbf{y} = \{y_1,\ldots,y_K\} \in \mathbb{R}_+^K$, $\mathbf{y} \in \pi_k$ if and only if $k = argmax_k\{y_1,\ldots,y_K\}$, and employment at firm k is given by:*
$$n_k(\pi_k) = C_k(F(y),\cdots,F(y))F'(y) \quad \forall k = \{1,\ldots,K\}.$$

Theorem 1 characterizes firm k's optimal wage offer as a weighted average of efficiency levels that are inferior to the worker's productivity at k. The weight assigned to each efficiency level, $x < y$, reflects the probability that x represents the worker's second-best productivity. To see this, note the following (heuristic) identity.

$$\frac{\sum_{j\neq k} C_{jk}(F(x),\ldots,F(x))}{C_k(F(x),\ldots,F(x))} \approx \frac{P[\max_{j\neq k} Y_j \simeq x \mid Y_k = x]}{P[\max_{j\neq k} Y_j \leq x \mid Y_k = x]},$$

where $Y_j \simeq x$ means $Y_j \in [x-\epsilon, x+\epsilon]$ for some small $\epsilon > 0$. The denominator represents the fraction of workers, with efficiency level $Y_k = x$ at firm k, who are less productive elsewhere. All such workers are employed at firm k by Theorem 1. The numerator is the relative density of workers with $Y_k = x$ who are (almost) equally productive elsewhere. Therefore the above ratio can be viewed as the probability that a worker at k is equally productive elsewhere.[4] If that probability is higher, then $\tilde{F}(x|y)$ is lower given any $x < y$, effectively raising the wage offer. Because $C_k(F(x), ..., F(x))F'(x) = n_k(x)$, the ratio can also be interpreted as the density of marginal employees to total employment at firm k of workers with efficiency level $Y_k = x$. Two factors that could naturally raise that probability are if the market has a larger number of competitors, K, or exhibits stronger (positive) correlation of worker productivity across firms. We discuss these further below.

Because the wage offer is effectively a truncated mean, it follows that $0 < w(y) < y$ whenever $y > 0$.[5] This implies markdowns (or profits) are always positive. Furthermore, $w(y)$ is (strictly) increasing in y.[6] The strict monotonicity of the wage function delivers the following two results:

Theorem 2 *The symmetric oligopsonistic equilibrium is efficient.*

A worker with efficiency levels y_k and y_j at firms k and j works for k if and only if $w(y_k) > w(y_j)$, which only happens if $y_k > y_j$ since $w(y)$ is strictly increasing. Because firms are identical, their offers exhibit the same wage markdown at a given y. Therefore, a worker can only receive a better offer elsewhere if they are more efficient at that firm. This result does not extend to equilibria with asymmetric firms as we illustrate in the next subsection.

Lemma 1 *The joint distribution of wage offers $w(y_k)$ is described by the copula $C^w = C$, and the common marginal distribution $F^w(x) = F(w^{-1}(x))$.*

Lemma 1 states that the joint distribution of wage offers inherits the dependency structure of the productivity distribution, but with different marginal distributions. The result follows from the stability of copulas under (strict) monotonic transformations (Nelsen, 2006).[7] Because workers take their highest wage offer, the equilibrium distribution of wages is defined by the distribution of $\max\{w(y_1), \cdots, w(y_k)\}$, which is given by the diagonal section of C copula that describes the wage offer distribution:

[4] We mean this heuristically. Since the distribution of $\max_{j \neq k} Y_j$ is continuous, the density to distribution ratio may generally exceed one.

[5] Because $\tilde{F}'(x|y) > 0$ for all $x \leq y$, non-zero weights are given to efficiency levels that are strictly below y.

[6] See the proof for Theorem 1 in the Appendix.

[7] Because the stability of copulas to monotonic transformations does not require the transformations be identical for all y_k, this result extends to asymmetric equilibria with strictly increasing wage-offer strategies.

Theorem 3 *The distribution of wages in symmetric oligopsony equilibrium, is:*

$$P(w^* \leq \omega) = C(F^w(\omega), \cdots, F^w(\omega)). \tag{5}$$

Markdowns in Oligopsony Equilibrium

Oligopsonistic wage policy relates equilibrium markdowns closely to comparative advantage. For any worker with productivity y, the markdown is:

$$1 - \frac{w(y)}{y} = \mathbb{E}_{\tilde{F}}\left[1 - \frac{x}{y}\middle| x < y\right] = \mathbb{E}_{\tilde{F}}[R(x/y)], \tag{6}$$

where the last equality follows from $\mathbb{E}[Y_j] = \mathbb{E}[Y_j]$ in a symmetric market. Equation (6) makes the role of comparative advantage in wage setting explicit. For worker with efficiency y, the markdown is a weighted average of their comparative advantage at their firm, relative to others, with larger markdowns if comparative advantage is stronger. Recall that $\tilde{F}$ weights each $x < y$ according to the density of marginally employed workers conditional on employment: $\delta'(x|Y_k = x)/\delta(x|Y_k = x)$. So, equivalently, the markdown is determined by weighting comparative advantage at firm k versus all competitors. If the joint distribution of productivity at k and best productivity elsewhere is dense along its diagonal (where equal), then markdowns are small. This could reflect either productivities that are highly correlated across firms, with little room for comparative advantage, or many competitors, raising a worker's outside option.

How markdowns vary by worker productivity will generally reflect the marginal distribution of efficiencies, as well as the dependence structure of workers' efficiencies across employers. The following proposition gives the necessary and sufficient condition for markdowns to increase with productivity.

Theorem 4 *Let* $G(y) = \ln \int_0^y \tilde{F}(x|y)dx$. *Then for any* $y > 0$, $1 - w(y)/y$ *is increasing in* y *if and only if*

$$-\frac{G''(y)}{G'(y)}y > 1.$$

For markdowns to increase with productivity, $G(y)$ has to be sufficiently concave. Concavity alone implies $dw/dy < 1$, which ensures profits, $y - w(y)$, increase with productivity. Because wages also increase with productivity, rents from comparative advantage are shared between the firm and the worker. For markdowns to increase with productivity, firm's share of the rents has to increase disproportionately more, which requires G to be strongly concave. Conversely, if $G(y)$ is convex ($G'' > 0$), then markdowns necessarily decrease with productivity.

At this level of generality, markdowns can be increasing, decreasing, or non-monotonic in productivity, depending on the joint distribution of efficiencies across employers. We illustrate

this next via specific examples chosen to highlight the roles of competition and correlation in wage setting.

Examples

We begin with a market where a worker's efficiency levels are independent across employers and continue with three cases of co-dependence. Worker efficiencies correlate differently between employers in these cases, leading to diverse implications for markdowns.

Example 1 (Independent Copula) *If Y_k are independent across employers, then the optimal wage offer policy is $w(y) = \int_0^y x d\tilde{F}^I(x|y)$ with*

$$\tilde{F}^I(x|y) = \left[\frac{F(x)}{F(y)}\right]^{K-1}.$$

Consider the simplest case first: a duopsony where Y_1 and Y_2 are independent. Then, $\tilde{F}(x|y) = F(x)/F(y)$, which implies $w(y) = \mathbb{E}[x|x < y]$. The equilibrium wage offer for employees with productivity y reflects their average efficiency at the competing firm conditional on that efficiency being inferior to y. Equivalently, the markdown reflects the average comparative advantage of workers with efficiency level y: $1 - w(y)/y = \mathbb{E}[1 - x/y|x < y]$.

More generally, when $K > 2$, $\tilde{F}^I(x|y)$ is the conditional probability that the highest productivity across $K-1$ competitors is inferior to y. Because the equilibrium wage is the conditional expectation of the second-best productivity level, it monotonically rises with K, matching productivity in the limit, reflecting that second-best productivity stochastically increases with the number of competitors.[8]

Next, consider a duopsony where efficiencies at the two firms are co-dependent. The literature typically treats dependence via the correlation coefficient while assuming joint log-normality of efficiencies.

Example 2 (Gaussian Copula) *Let $K = 2$ and $C = C^G(\rho)$ be the Gaussian Copula with correlation $\rho \in [-1, 1]$ and normal marginals $F_k(\ln x) = \Phi(\frac{\ln x - \mu}{\sigma})$ for $k \in \{1, 2\}$. The wage offer policy in symmetric duopsony equilibrium is $w^G(y, \rho) = \int_0^y x d\tilde{F}^G(x|y; \rho)$ with*

$$\tilde{F}(x|y;\rho) = \left[\frac{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln x - \mu)}{\sigma}\right)}{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln y - \mu)}{\sigma}\right)}\right]^{\frac{1}{1-\rho}}.$$

[8] Formally, $\tilde{F}^I(x|y)$ declines with K for all $x < y$ with $\tilde{F}^I(y|y) = 1$, creating a first-order stochastic ranking with respect to K. In the extreme, $\lim_{K\to\infty} \tilde{F}^I(x|y) = 0$ for all $x < y$, implying $w(y) = y$.

Example 2 captures distinct roles of co-dependence in wage setting. In the definition for $\tilde{F}(x|y;\rho)$ here, ρ appears twice inside the bracketed term and once in the exponent. By raising the exponent, a higher ρ lowers $\tilde{F}(x|y;\rho)$ for all $x < y$, which raises wage offers for all productivity levels. This mechanism is similar to the role of K above. Intuitively, competition gets stiffer either when the number of competitors increase or when workers are similarly productive at all firms. Wages equal productivity in either limit, as $K \to \infty$ or as $\rho \to 1$. But this need not happen symmetrically for all workers, because the bracketed term, which captures the dependence between productivity and outside option, is not monotonic in ρ. To see this, note that when $\rho = 0$, all workers have the same expected productivity at the competing firm. A positive ρ better aligns the competing offer with productivity, which is good news for high-productivity workers, but not so for low-productivity workers. As a result, wage markdowns may increase or decrease with correlation depending on the worker's productivity, y, even though the wage does converge to productivity as $\rho \to 1$.

The Gaussian copula imposes a specific structure on the distribution of comparative advantage across efficiency levels, in particular, that the degree of correlation is common at all levels. We next consider Archimedean copulas, which allow for richer co-dependence:

Proposition 1 (Archimedean Copulas) *Let $C^A = \psi^{-1}(\sum_j \psi(u_j))$ be an Archimedean Copula with the generator function $\psi : [0,1] \to [0,1]$. Then the optimal wage offer policy is $w^A(y) = \int_0^y x d\tilde{F}(x|y)$ with*

$$\tilde{F}(x|y) = \left[\frac{\psi'(C^A(F(x),...,F(x))}{\psi'(C^A(F(y),...,F(y))}\right]^{-\frac{K-1}{K}}.$$

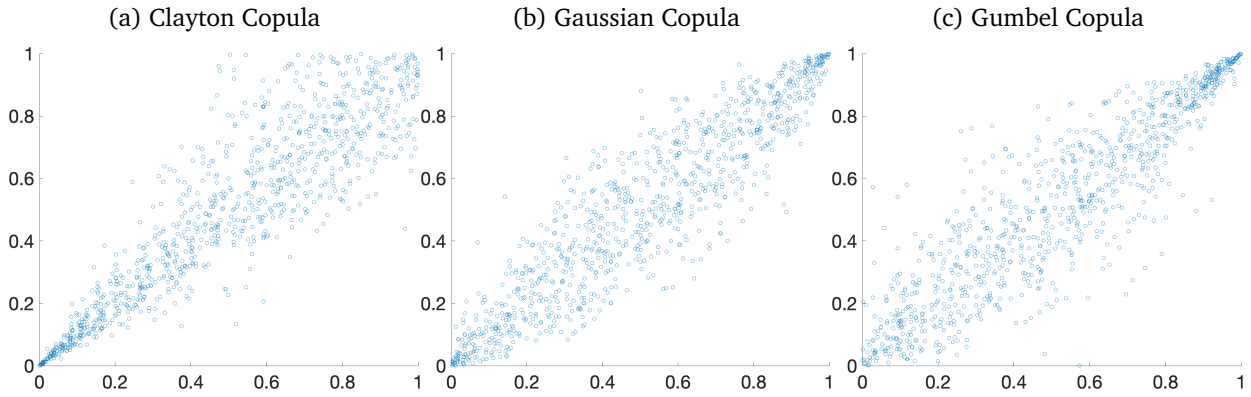
Two Archimedean copulas are of particular interest: Clayton copula, which features weaker dependency at higher productivity levels, and Gumbel copula, which features the reverse pattern. Clayton copula sets $\psi(u) = (u^{-\theta} - 1)/\theta$, where $\theta \geq -1$ measures the strength of dependence. Higher θ values imply more positive dependence, with $\theta < 0$ implying negative dependence, $\theta > 0$ positive dependence, and $\theta = 0$ independence.[9] Gumbel copula uses the generator function $\psi(u) = -(\ln(u))^\alpha$ for $\alpha \geq 1$. Unlike Gaussian or Clayton copulas, Gumbel is limited to positive dependencies with $\alpha = 1$ representing the independent copula. The corresponding wage policies in oligopsony equilibrium are as follows:

Example 3 (Clayton Copula) *Let $C^{AC}(\theta) = \left(\sum_k (u_k^{-\theta} - 1)\right)^{-1/\theta}$ where $\theta \in (-1, \infty)$ and $u_k = F(y)$ for all $k \in \{1,...,K\}$. Then the optimal wage-offer policy is $w^{AC}(y,\theta) = \int_0^y x d\tilde{F}(x|y;\theta)$ with*

$$\tilde{F}(x|y;\theta) = \left[\frac{F^{-\theta}(x) - 1 + 1/K}{F^{-\theta}(y) - 1 + 1/K}\right]^{-\frac{K-1}{K}\frac{(1+\theta)}{\theta}} = \left[\frac{C^C(F(x),...,F(x))}{C^C(F(y),...,F(y))}\right]^{\frac{K-1}{K}(1+\theta)}.$$

[9] The independent copula ($\psi(u) = \ln u$) is obtained in the limit as $\theta \to 0$.

Figure 1: Dependence Structures with Copulas



Notes: Figure shows the joint distribution for three different bivariate copulas with a Kendall's τ of 0.75.

Example 4 (Gumbel Copula) *Let* $C^{AG}(\theta) = \exp\left(-\left(\sum_k(-(\ln(u_k))^\alpha)\right)^{1/\alpha}\right)$ *where* $\alpha \in [1,\infty)$ *and* $u_k = F(y)$ *for all* $k \in \{1,...,K\}$. *Then the optimal wage offer policy is* $w^{AG}(y,\theta) = \int_0^y x d\tilde{F}(x|y;\theta)$ *with*

$$\tilde{F}(x|y;\alpha) = \left[\left(\frac{F(x)}{F(y)}\right)^{K^{1/\alpha}}\left(\frac{\log F(y)}{\log F(x)}\right)^{\alpha-1}\right]^{\frac{K-1}{K}}.$$

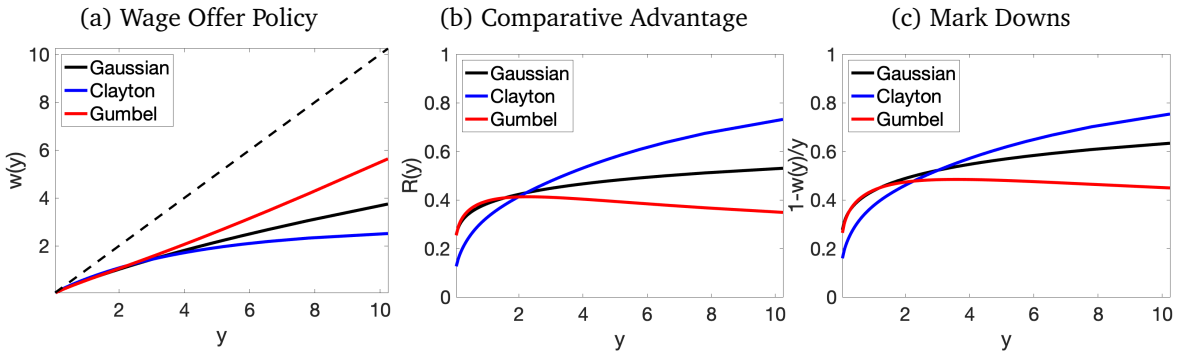
Figure 1 compares the dependence structure in Gumbel and Clayton copulas with the standard Gaussian copula in a bivariate setting. Each panel shows the joint distribution of $F(y_1)$ and $F(y_2)$. The dependency parameter of each copula is calibrated to the same degree of dependency as measured by Kendall's $\tau = 0.75$.[10] Recall that Gaussian copula (panel b) features a constant correlation across efficiency levels. In panel (a), the correlation between efficiencies is relatively weaker at low levels and rises with productivity. The reverse is true in Panel (c) for the Clayton copula.

The corresponding wage offer policies in a symmetric duopsony equilibrium are shown in Figure 2-a. While always increasing in productivity, wages are quite different across the cases reflecting how they differ in the dependence structure between efficiencies at the two firms. Relative to the Gaussian copula, wages track productivity more closely if dependence of efficiencies across firms strengthens with productivity (Gumbel) than if it weakens (Clayton).

Panel b shows how the strength of comparative advantage, $R(y_j) = 1 - \mathbb{E}[y_i/y_j|y_i < y_j]$, varies by productivity. This directly dictates how the wage markdowns respond along the efficiency distribution. Markdowns increase with productivity under the Gaussian copula, especially at lower levels. The Clayton copula displays a steeper markup trajectory with productivity, reflecting the weakening in cross-firm correlation in efficiencies. Interestingly, markdowns

[10] The corresponding values are $\rho = \sin(\tau\rho/2) = 0.92$ for Gaussian copula, $\theta = 2\tau/(1-\tau) = 6$ for Clayton copula and $\alpha = 1/(1-\tau) = 4$ for Gumbel copula.

Figure 2: Wage Policies in Duopsony Equilibria



Notes: Figure shows comparative advantage, wage policies, and wage markdowns in duopsony equilibria. Each copula is calibrated to a Kendall's τ of 0.5; marginal productivity distributions are log-normal with $\mathbb{E}[\ln y_i] = 0$ and $\mathbb{V}[\ln y_i] = 1$ for $i \in \{1, 2\}$. Comparative advantage is measured by $R(y_j) = 1 - \mathbb{E}[y_i | y_i < y_j]/y_j$.

follow a hump-shaped pattern with the Gumbel copula. That reflects the strong correlation of efficiencies at high levels, which makes the market highly competitive for productive workers. Which scenario best describes an economy is essentially an empirical question.

Figure 3 shows how correlation alters the markdown distribution. Generally, a higher correlation of productivity across firms brings pay closer to productivity. But that can affect high- versus low productivity workers differently. With higher correlation, productive workers at one firm are also productive at competing firms, raising their outside option. Similarly, higher correlation lowers outside options of less productive workers. As a result, firms mark down wages of high-productivity workers less, while marking down those of low-productivity workers more. This is visible for Gaussian and Gumbel copulas among least productive workers when correlation is low, e.g., independent.[11] By contrast, with Clayton copula (panel b), higher correlation raises wages uniformly, reducing markdowns at all dependency levels:

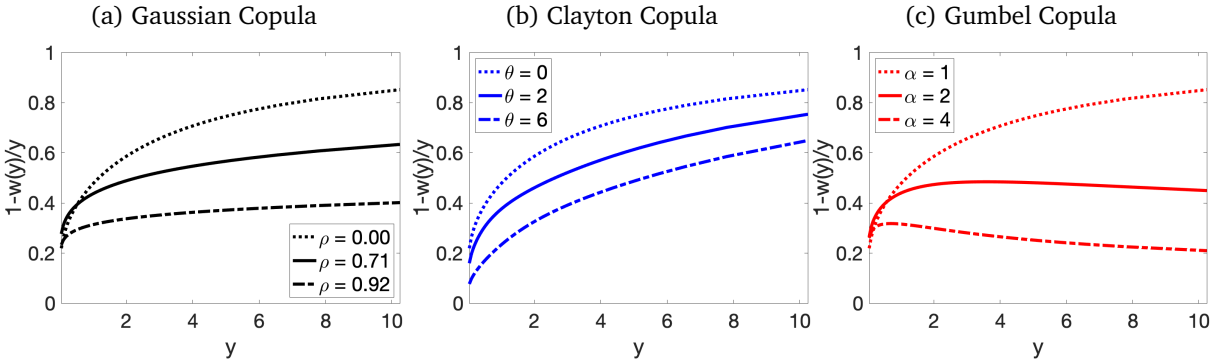
Remark 1 *For all $y > 0$, $w^{AC}(y;\theta)$ is increasing in θ.*

This implies $1 - w^{AC}(y)/y$ is decreasing in θ. Because correlation is always relatively stronger among low-productivity workers, higher correlation via θ mainly affects productive workers on the margin.

Figure 4 shows how more competitors affects markdowns. A higher K reduces markdowns at all productivities. This is necessarily so for the copulas we display in the figure, Clayton and Gumbel, as well as for independence:

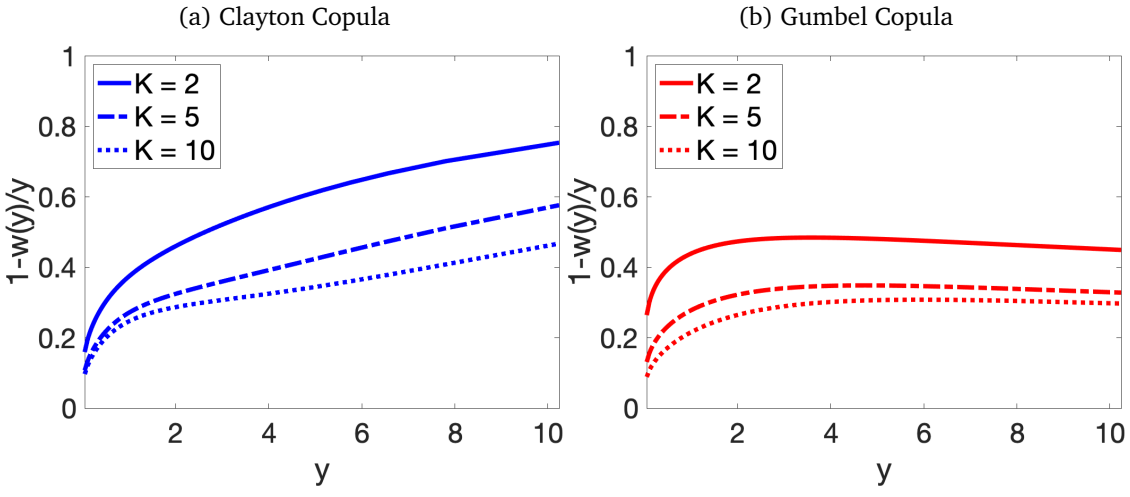
[11] But those economies imply much larger wage markdowns than typically estimated. Independent copula, for instance, implies an average markdown of about a half, which is two to four times most estimates. At empirically relevant dependency levels, markdowns decline for both copulas as correlation increases. Raising Kendall's τ from 0.5 to 0.75 in Figure 2, markdowns decline uniformly in all panels.

Figure 3: Markdowns in Symmetric Equilibria



Notes: Figure shows wage markdowns in duopsony equilibria by levels of co-dependence. In each panel, copula parameters respectively correspond to Kendall's τ of 0 (independence), 0.5 and 0.75. Marginal productivity distribution is log-normal with $\mathbb{E}[\ln y_i] = 0$ and $\mathbb{V}[\ln y_i] = 1$ for $i \in \{1, 2\}$.

Figure 4: Markdowns in Symmetric Equilibria



Notes: Figure shows wage markdowns in oligopsony equilibria by number of competitors for Clayton Copula (left) and for Gumbel Copula (right) with $\tau = 0.5$. Solid lines in each panel replicate the corresponding markdowns in Figure 2. Marginal productivity distribution is log-normal with $\mathbb{E}[\ln y_i] = 0$ and $\mathbb{V}[\ln y_i] = 1$ for $i \in \{1, K\}$.

Remark 2 *Let $w(y) \in \{w^I(y), w^{AG}(y), w^{AC}(y)\}$. For all $y > 0$, $w(y)$ is increasing in K and $1 - w(y)/y$ is decreasing in K.*

With more competitors, the odds that a worker is highly productive elsewhere increases, reducing expected match-specific rents and, thereby, markdowns.

Overall, the simulations show that markdowns within a firm may increase or decrease with wages depending on the underlying distribution of efficiencies across firms. Pay is better aligned with productivity in economies with a larger number of competitors or those where worker productivity is highly correlated across firms.

2.4 Asymmetric Equilibria

We make two additional assumptions to study asymmetric equilibria. First, we assume that worker productivity is bounded: $Y_k \leq \bar{y} < \infty$ for all k. Second, because asymmetric equilibria with K firms is inherently complex, we simplify the problem by setting $K = 2$.[12] Generality is not entirely lost by these assumptions because $\bar{y}$ can be arbitrarily large. Furthermore, because the effect of the number of competitors on wage setting is similar to that of co-dependence, the degree of competition can also be captured by raising the correlation of workers' efficiencies between the two firms.

The following proposition characterizes the equilibrium.

Proposition 2 (Asymmetric Duopsony Equilibrium) *Let $K = 2$ and $C(u, v)$ denote the joint distribution of worker efficiencies at the two firms with marginal productivity distributions $u = F_1(y)$ and $v = F_2(y)$. The asymmetric duopsony equilibrium is characterized by wage offer policies $\{w_k(y)\})_{k\in\{1,2\}}$ and an employment allocation such that:*

1. *Inverse wage offer policies solve the following system of differential equations:*

$$\begin{aligned}(y_1(w) - w)C_{uv}(u, v)F_2'(y_2(w)))dy_2 - C_v(u, v)dw &= 0\\ (y_2(w) - w)C_{vu}(u, v)F_1'(y_1(w)))dy_1 - C_u(u, v)dw &= 0\end{aligned}$$

   where $y_k(w) = w_k^{-1}(y)$.

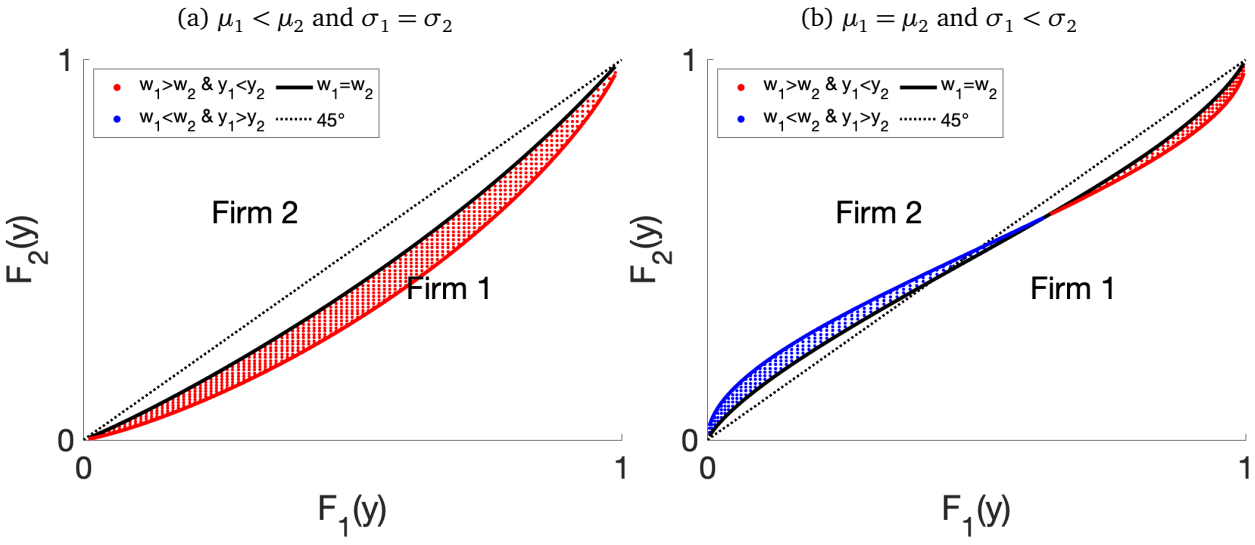
2. *Employment levels are:*

$$n_1(y) = C_u(F_1(y), F_2(y_2(w_1(y))))F_1'(y) \quad (7)$$

$$n_2(y) = C_v(F_1(y_1(w_2(y))), F_2(y))F_2'(y) \quad (8)$$

[12] For this reason, much of the auction-theory literature assumes symmetry. Reny and Zamir (2004) and Jackson and Swinkels (2005) analyze first-price auctions with multiple asymmetric bidders, focusing on existence of equilibrium.

Figure 5: Employment Allocations in Asymmetric Equilibria



Notes: Figure shows the allocation of workers in duopsony equilibria. Workers below the black curve choose Firm 1. Dashed 45-degree line indicates the allocation in symmetric equilibrium. Areas colored in red and blue show inefficiencies in asymmetric equilibria, where the firm-worker allocation deviates from comparative advantage. Productivity levels are log-normal at the firm level with parameters μ_i, σ_i with $i \in \{1, 2\}$ and the joint distribution is given by a Clayton Copula with positive dependence: $\tau = 0.5$.

We consider two cases of asymmetry. In the first we allow one firm to be generally more productive than the other. We view this as a firm-specific component of productivity and formalize it with the first-order stochastic dominance relationship between the marginal productivity distributions: $F_1(y) > F_2(y)$ for all $y > 0$.[13] In this scenario, because Firm 2 is generally more productive, it offers higher wages as the following proposition shows assuming Archimedean copulas $C(u, v) \in C^A(u, v)$.

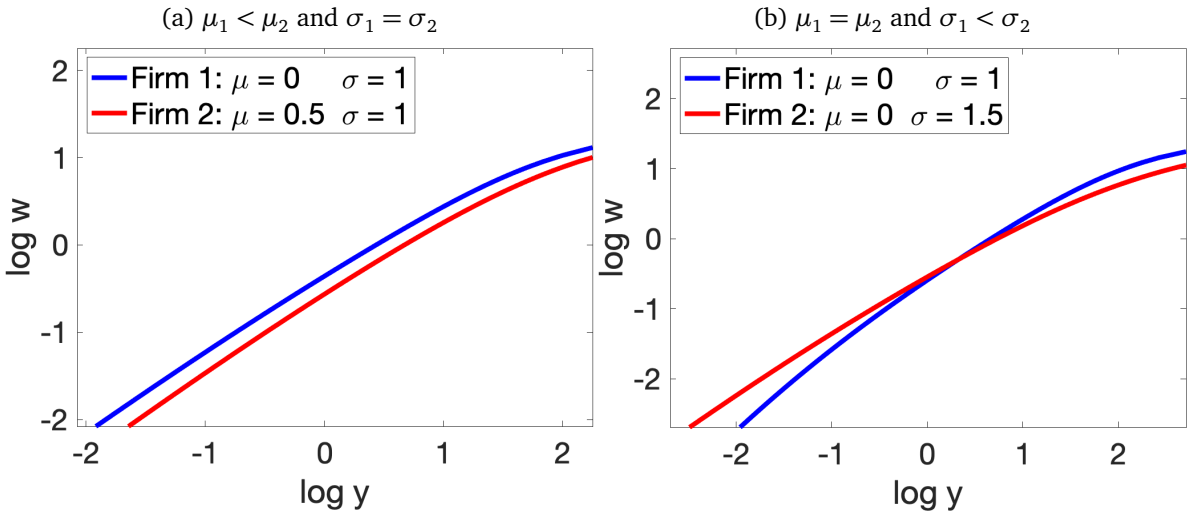
Theorem 5 *Suppose $C(u, v) \in C^A(u, v)$ and $F_1(y) > F_2(y)$ for all $y > 0$. Then, for all $w > 0$: $F_1(y_1(w)) > F_2(y_2(w))$.*

Theorem 5 says that if the productivity distribution at Firm 2 stochastically dominates Firm 1's, a larger fraction of the workforce is offered a wage below w at Firm 1 than at Firm 2. Equivalently, to merit the same wage offer w, a worker has to rank higher in Firm 1's productivity distribution than they would in Firm 2's. Consequently, because Firm 2 generally pays better, in a stochastic sense, it employs a larger share of the workforce.

Figure 5-a illustrates a typical equilibrium in this scenario. In the chart, $\ln Y_1$ and $\ln Y_2$ are normally distributed with common standard deviations, $\sigma_1 = \sigma_2$, but different means: $\mu_2 > \mu_1$. The joint distribution reflects a Clayton copula with positive dependence: $\theta > 0$. The axes

[13] A common example is when $\ln Y_j$ are normal with common variance but different means.

Figure 6: Wage Offer Policies in Asymmetric Equilibria



Notes: Figure shows wage offers in asymmetric duopsony equilibria. Productivity levels are log-normal at each firm with parameters μ_i, σ_i, $i \in \{1,2\}$, and a joint distribution that reflects a Clayton Copula with positive dependence: $\tau = 0.5$.

show the productivity quantiles at each firm. The solid curve is the locus of worker indifference in asymmetric equilibrium when Firm 2 is more productive. Workers with efficiency bundles above the curve are offered, and accept, higher wages at Firm 2. The indifference locus lying below the 45-degree line implies Firm 2 is larger.[14]

The red line shows the locus of equal productivity: $y_1 = y_2 = y$. Given $F_1(y) > F_2(y)$, this line is strictly below the 45-degree line where $F_1(y_1) = F_2(y_2)$. It is also below the locus of equal pay because pay-to-productivity is higher at Firm 1: $w/y_1(w) > w/y_2(w)$.[15] Therefore, the red shaded area represents workers who are more productive at Firm 2, but work at Firm 1—those offered a higher wage at Firm 1 due only to its lower markdown than Firm 2's.

The second asymmetric case we consider is when the firms' marginal productivity distributions cross once. This scenario would arise, for instance, if one firm values labor quality more than the other. The following theorem characterizes sorting of workers between firms in that case, proving a single crossing result in the interior of the (u, v) space if any crossing exists.[16]

Theorem 6 *If $C(u,v) \in C^A(u,v)$ and $\exists \tilde{y} > 0$ such that $F_1(y) > F_2(y)$ for all $y > \tilde{y}$ and $F_1(y) < F_2(y)$ for all $y < \tilde{y}$, then there is a unique $w^* \geq 0$ such that $F_1(y_1(w)) \geq F_2(y_2(w))$ if and only*

[14] By definition, the distributions of $u = F_1(y_1)$ and $v = F_2(y_2)$ are uniform and symmetric around the 45-degree line, which therefore represents equal division of employment in Figure 5.

[15] Because $y_2(w) > y_1(w)$, for all $w > 0$, in equilibrium, we have $F_2(y_2(w))/F_1(y_1(w)) > F_2(y_1(w))/F_1(y_1(w))$, where the left term traces the equal-pay locus and right term traces the equal-productivity locus.

[16] Single-crossing may not exist when $\tilde{y}$ is sufficiently large, e.g., if $\ln Y_2$ is more dispersed *and* it has a larger mean. Without a crossing, the equilibrium sorting is similar to that characterized by Theorem 5

if $w \geq w^*$ in equilibrium.

An example is shown in Figure 5-b, where $\ln Y_j$ are normal with $\mu_1 = \mu_2$ but $\sigma_2 > \sigma_1$. The joint distribution is described by a Clayton copula as in 5-a. The black curve shows the locus of points with equal wage offers, $F_1(y_1(w)) = F_2(y_2(w))$, and the colored curve shows the locus of points with equal productivity: $F_1(y) = F_2(y)$.

When there is a crossing, firms attract different segments of the workforce. Firm 2 employs a larger share of the more productive workers: the separation curve in Figure 5 lies below the 45-degree line when y is high. Among highly paid workers, where the black equal-pay curve lies above the red equal-productivity curve, $y_2(w) > y_1(w)$. Consequently, both wage offers conditional on productivity and markdowns conditional on the wage rate are larger at Firm 2. Equilibrium in this segment of the market is much like in Panel a. The situation is reversed at low productivity levels. Firm 1 employs a larger share of the workforce, offers better pay conditional on productivity, but marks wages down by more at any wage level.

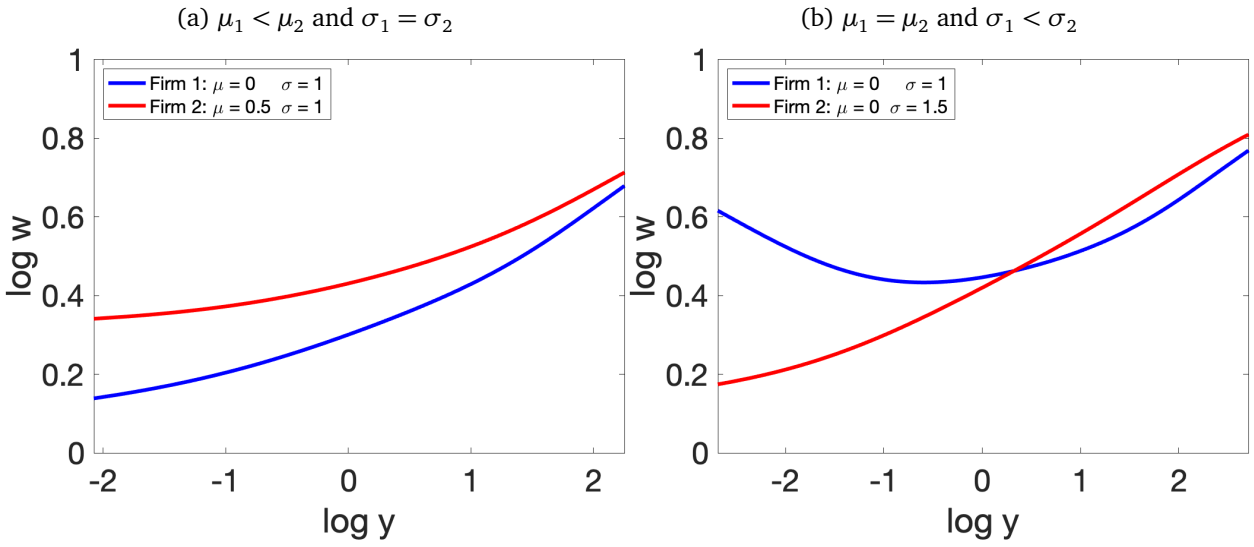
In this scenario, Firm 1 has a comparative advantage among low-productivity (low-wage) workers and Firm 2 has a comparative advantage among high-productivity (high-wage) workers. Yet, because the firm at a comparative disadvantage competes more aggressively by paying closer to productivity, neither firm can fully exploit their comparative advantage. This results in an inefficient employment allocation as depicted by shaded zones in Figure 5. Workers in the blue zone are more productive at Firm 2, but are employed by Firm 1. The reverse is true for workers in the red zone. The overall employment shares of the two firms depends on the densities of comparative advantage zones, and is, therefore, generally ambiguous.

Figures 6 and 7 show firms' wage offers and the corresponding markdowns by productivity for our two asymmetric cases. In each figure firm productivities are marginally distributed $\ln Y_j \sim N(\mu_j, \sigma_j)$ for $j \in \{1, 2\}$, with joint distribution described by a Clayton copula. The left panels show case 1: $\sigma_1 = \sigma_2$ and $\mu_2 > \mu_1$, i.e., firm 1's productivity distribution stochastically dominates. The right panels assume $\sigma_2 > \sigma_1$ and $\mu_2 = \mu_1$. This can be interpreted either as skills specific to Firm 2 are more dispersed, or, equivalently, skills are more valuable at Firm 2.

In the first case Firm 2 offers a higher wage conditional on productivity (Figure 6-a)) consistent with Theorem 5. Markdowns (Figure 7-a) increase with productivity in both firms, but Firm 1, having a productivity disadvantage, competes by marking wages down relatively less.

In the second case, $\sigma_2 > \sigma_1$ and $\mu_2 = \mu_1$, Firm 1 has a comparative advantage among low-productivity workers and Firm 2 among high-productivity workers. Consistent with Theorem 6, Firm 2 has a steeper pay-productivity profile (Figure 6-b), with Firm 1's wage offer beating that of Firm 2 for low-productivity workers and vice versa for high-productivity workers. In turn, markdowns (7-b) track comparative advantage: Firm 1 marks down wages of low-productivity workers by more than Firm 2, while the reverse is true for high-productivity workers.

Figure 7: Markdowns in Asymmetric Equilibria



Notes: Figure shows the distribution of markdowns in duopsony equilibria. Productivity levels are log-normal at each firm with parameters μ_i, σ_i with $i \in \{1,2\}$; the joint distribution reflects a Clayton Copula with positive dependence: $\tau = 0.5$.

Interestingly, we see that wage markdowns display different patterns within the two firms. Whereas markdowns monotonically increase with productivity at Firm 2, they display a U-shaped pattern at Firm 1. Recall that Clayton copula always generates increasing markdown-productivity profiles in symmetric equlibria. The declining profile among low-productivity workers at Firm 1 (Figure 7-b) stems from the asymmetry between the two firms. Because $\sigma_1 < \sigma_2$, workers have greater productivity differences at Firm 2, Firm 1's competitor. From the perspective of Firm 1, marginally better-skilled workers command disproportionately better pay at Firm 2 (in expectation), forcing Firm 1 to offer a larger share of the rent to stay competitive for those workers.

We can summarize the asymmetric cases above. i) If an employer is generally more productive, then it pays higher wages, is therefore larger, and at the same time marks down wages more at any productivity level. ii) If efficiency units for an employer are especially responsive to skill, then it disproportionately attracts high-productivity workers—so pays a higher average wage—while exhibiting a steeper profile of markdowns with respect to productivity. But this higher-wage employer is not necessarily larger in terms of employment.

In the empirical work these predictions naturally lead us to stratify employers both by size and average wage. If employer size differences are driven by productivity differences, then we should find larger employers exhibiting bigger markdowns. Comparing employers by their median wage, high-wage employers should unambiguously display higher markups at the top of

their wage distributions. Employers could be high-wage either because they are generally more productive or because they are skill elastic. Both forces predict larger markdowns for higher-wage workers at high-wage employers. But the two forces have countervailing predictions for their lower-wage workers: If employers are high wage due to generally high productivity they should show bigger markdowns even for their lower-wage workers; but if being high-wage reflects being skill-elastic, then this prediction is reversed.

2.5 Informal Sector and the Legal Minimum Wage

We now consider two prevalent features of the Brazilian labor market: a legal minimum wage and an informal sector. A minimum wage implies that firms do not hire workers whose productivity is below the minimum wage. Workers with productivity levels that equal the minimum wage are offered the minimum wage. Those that are more productive see their wages marked down relative to their productivity, albeit less than they would have been in the absence of a legal minimum wage because the minimum wage raises wage offers all along the productivity distribution in our model. The following theorem gives the equilibrium wage offer policy with a minimum wage in symmetric equilibrium:

Theorem 7 *Consider a symmetric market with K firms and a legal minimum wage $\bar{w}$. For all $y \geq \bar{w}$, the wage offer policy in oligopsony equilibrium is given by:*

$$w_m(y;\bar{w}) = w(y) + \bar{w} - w(\bar{w}),$$

where $w(y)$ is the wage offer in the absence of minimum wage defined in Theorem 1.

With a minimum wage, all workers receive a wage bump because the minimum wage shifts the initial condition for the differential equation that describes the equilibrium wage policy in oligopsony equilibrium. That additional pay is $\bar{w} - w(\bar{w}) > 0$, which is the firms' profits in the absence of a legal minimum wage associated with a worker whose productivity equals $\bar{w}$. Minimum wage regulation redistributes that portion from the firm to the worker, regardless of their productivity level, so long as it exceeds $\bar{w}$, thereby reducing wage markdowns for all workers. Because this is a fixed and additive pay bump, the mark down reduction is most significant for low-wage workers, relative to their productivity or pay. This result is consistent with a number of studies that estimate a positive impact of minimum wage legislation on wages well-above the legal minimum, e.g., Lee (1999) and Engbom and Moser (2022).[17]

[17] Engbom and Moser (2022) estimate a significant minimum-wage impact for most of the wage distribution for Brazil's formal market. To try to capture this, they consider a model with a sequence of Burdett and Mortensen (1998) economies separated by workers' skill-types.

A legal minimum wage necessarily lowers employment because workers with below-minimum-wage productivity at *all* firms, $\max_k\{y_k\} < \bar{w}$, are not employed while all workers are employed in the absence of the minimum wage. Formally, total employment is $1 - C(F(\bar{w}), ..., F(\bar{w}))$, which is decreasing in $\bar{w}$ since both F and C are strictly increasing in their arguments. Furthermore, there is no reallocation between firms from legal minimum wage in symmetric equilibrium, implying firms have equal employment shares although each has fewer employees.

Next, we add an informal economy. Specifically, we assume workers have the option of working outside the formal sector with pay w_0, which is distributed i.i.d. according to distribution $F_I(w_0)$. We interpret this option as the informal sector in Brazil. More generally, $F_I(w_0)$ represents the distribution of the reservation wage for participating in the (formal) labor market. In this economy, firms have to beat not only their competitors' offers but also the informal sector's offer to attract a worker. The labor supply to a firm is now given as:

$$n_k(w; y) = P(w_j(y_j) \leq w, \forall j \neq k | Y_k = y) \cdot P(w_0 \leq w) \cdot f_k(y), \tag{9}$$

where $f_k(y) = F_k'(y)$ is the density of workers with $Y_k = y$ and $P(w_0 \leq w) = F_I(w)$. Because $F_I(w) < 1$, an informal sector option necessarily reduces total employment in the formal sector.

Because the informal sector intensifies competition for workers, it also raises wages in the formal sector for all workers and, thereby, reduces wage markdowns. An explicit analytical expression for the wage offer policy in oligopsony equilibrium is not feasible in this case. The following theorem characterizes the wage offer policy as a solution to a differential equation.

Theorem 8 *Consider a symmetric market with K formal firms, an informal sector and a legal minimum wage $\bar{w}$. The wage offer policy in oligopsony equilibrium solves the following differential equation.*

$$\frac{dw}{dy} = (y - w)\left[\frac{\sum_{j\neq k} C_{kj}(F(y), \cdots, F(y))F'(y)}{C_k(F(y), \cdots, F(y))} + \frac{F_I'(w)}{F_I(w)}\right], \tag{10}$$

with the initial condition $w(\bar{w}) = \bar{w}$.

Relative to our benchmark economy, the addition of the informal sector introduces the last term in brackets in equation (10). $F_I'(w)/F_I(w) > 0$ for all w; so dw/dy, the wage productivity slope, is steeper everywhere. Given the common initial condition $w(\bar{w}) = \bar{w}$, a steeper profile implies that wage offers are strictly greater at all productivity levels $y > \bar{w}$, and, hence, markdowns are lower. While the minimum wage provides a common pay bump, the wage reactions to informal sector competition vary with productivity in a manner dictated by the distribution of informal wages. Empirically informal sector pay is lower, on average, in the formal sector; so the wage gains from informal competition will skew toward low-wage workers. In that sense,

introduction of the informal sector reinforces our prediction that wage markdowns are largest among high productivity workers.

These effects are common to all firms in symmetric equilibria. In asymmetric equilibria, the effects of competition from informal sector is disproportionate across firms, affecting more severely firms that have a comparative advantage among low-wage workers.

3 Data

Our model predicts monopsony power is most severe for workers in matches that exhibit strong comparative advantage, whom we anticipate to be disproportionately high-wage workers. To test these predictions, in Section 4 we estimate the elasticity of labor supply facing employers in the Brazilian formal labor marker. But first we discuss our data, how we stratify workers by relative wages at an employer, and how we measure employment and wage changes.

3.1 RAIS Data

Brazil's Annual Social Information Report, RAIS for short, collects establishment-level data on each employees' compensation as well as other characteristics. The RAIS provides the basis for government statistics on Brazil's formal labor market. It is also the basis for government monitoring of employment laws and required payments for benefits.[18]

The RAIS data have important advantages for our purposes. For one, we clearly need micro data at the employer-worker level, as in the RAIS, to establish how both employment and wages change at different segments of an employer's wage distribution. The RAIS panel structure allows us to calculate wage changes for continuing workers at the employer, controlling for compositional changes. The RAIS has several advantages relative to many matched-data for other countries. For one, in additional to annual information, the RAIS provides earnings and hours data specifically for the month of December, allowing us to calculate, not just worker earnings, but also a wage rate. The RAIS also provides information on worker characteristics including age, education, and occupation. Information on employer characteristics is relatively limited; it provides the establishment-level location and industry, but no production information beyond labor (e.g., revenue, capital, inventory, etc.). While the RAIS has comprehensive coverage of Brazil's formal labor market, its primary shortcoming is its omission of Brazil's large informal labor market. We return to this issue in Section 5.

[18] See http://www.rais.gov.br/sitio/index.jsf. Benefits include social security benefits, severance pay, and the legislative one-twelfth annual pay bonus.

3.2 Sample Construction

Our estimates are based on years 2006-2018 of the RAIS.[19] While employers are required to report information on all workers employed during the year, they specifically report how many workers were employed at the end of the year and workers' earnings for the month of December. These variables underlie our measures of employment and wage growth to estimate supply elasticities. Therefore, we begin with a sample of RAIS workers who are still employed at year's end with reported earnings for December. Combining years 2006-2018, this yields 553,266,008 worker-year observations.

Table 1 presents restrictions imposed in constructing our sample. Panel A lists restrictions that apply to all statistics (e.g., worker's wage rank, employer's employment, market share, etc.), while Panel B gives further restrictions imposed for estimating wage responses to an employer's employment growth. We restrict our sample to workers aged 18 to 64 whose reported December pay is at least 95% of the legal minimum. We exclude military and government workers because wage settings in these sectors presumably do not reflect profit maximization. We exclude certain occupations: workers on apprentice contracts, upper management (directors/managers), and science or art professionals. We view the management and professional positions as sufficiently idiosyncratic that wage setting could well reflect individual bargaining. The excluded sectors and occupations are 29% of observations. Below we will define a worker's labor market largely by their metropolitan area. Therefore, we restrict our sample to establishments in one of 74 metropolitan areas. Excluding rural areas drops 28.3% of observations. The union of the above restrictions excludes 50.9% of the worker-year observations.

Panel B further reports the sample restrictions for our main regressions. Our wage measure is based on monthly pay for the month of December. That pay reflects overtime, bonuses, commissions, and tips, but not the annual bonus, the "thirteenth-month" pay, requried in Brazil. In order that pay corresponds to a wage, we restrict our regression sample to those paid monthly, were employed the entirety of December, and who report working 40 to 44 hours per week. Respectively, 90.8%, 96.2%, and 91.2% of observations meet these constraints. December wages are deflated by the December IPCA price index.[20]

We examine year-over-year changes in employment and wages at each employer. For instance, for year t we compare an employer's employment at end of year t to its December 31st employment of year $t-1$. For wage responses, we compare a worker's wage for December of

[19] 2006 is the first year that reports establishments' 7-digit industry, i.e., the second edition of Brazil's National Classification of Economic Activities (CNAE 2.0). Industry employment forms one of our instrumental variables for employment changes. We exploit years of the RAIS back to 1985 to benchmark the age of establishments, which forms another of our instruments.

[20] The IPCA is constructed by The Brazilian Institute of Geography and Statistics (IBGE) to tracks prices for goods and services with weights reflecting household expenditure shares.

Table 1: Sample Selection Criteria

Panel A: Cross-Section Sample Restrictions

Restrictions	% meeting each[†]
$18 \leq$ age ≤ 64	98.2
Paid ≥ 0.95 times legal minimum	97.5
Not military, government, apprentice, director/manager, or science/art prof.	71.0
In one of 74 metropolitan areas	71.7
Meeting all Panel A restrictions	49.1

Panel B: Regression Sample Restrictions

Restrictions	% meeting each[‡]
Paid monthly	90.8
At establishment all of December	96.2
$40 \leq$ weekly hours ≤ 44	91.2
Employer ≥ 10 workers with stayers above & below median	61.0
Stayer with consecutive wages	52.6
Meeting all Panel B restrictions	28.9
Resulting sample size	
worker-year	78,582,409
employer-year	3,308,773

[†]Expressed in terms of sample of workers employed at year's end with a December wage.

[‡]Expressed in terms of sample of workers who met all restrictions in Panel A.

Table 2: Rates of Hiring from Same Metro, Occupation, Industry

	% of hires
Same metro area	79.9
Same occupation	77.8
Same 7-digit industry	22.9

Notes: Sample is restricted to employers with 10 or more employees at end of year. Shares calculated based on those hired from formal sector. Sample size is 13,410,198.

year t to the same worker's wage in December of $t-1$. We focus on wage changes for stayers at an employer to avoid any impact on the wage change from compositional effects.

We stratify workers at an employer into quartiles based on their relative wages.[21] To make meaningful distinctions across an employer's wage distribution, we restrict the sample to employers (establishments by occupation) that average at least 10 employers between the year-ends of t and $t+1$ and who have at least one stayer at both above and below the median wage. This cuts 39% of observations from our sample. The requirement of having a wage at the same employer for consecutive Decembers, necessary only for our wage-change regression, excludes 47.4% of observations. The union of the regression-sample restrictions in Panel B excludes 71.1% of workers. The final sample for wage-change regressions is comprised of 78,582,409 worker-year-level observations at 3,308,773 employer-years.

We judge an employer's employment growth relative to its labor market by including a full set of year-market fixed effects. We define a labor market by two dimensions: (i) the worker's metropolitan area, and (ii) the worker's broad occupation. We distinguish three occupations defined in the RAIS as technicians/repairman, service workers, and production workers. We define an employer by each of the three occupations at an establishment. Thus our estimates ask when an establishment expands employment of an occupation, say service workers, how much does it affect the wage they pay to service workers.

Our definition of a market as a metropolitan/area appears roughly consistent with transitions in the RAIS data. The first row of Table 2 reports that 80% of hires in our sample reflect workers previously employed in the same metro area.[22] From Row 2, nearly the same fraction, 78%, are hired from within the same occupation. By contrast, we see from Row 3 that only 23% are hired from the same 7-digit industry. We highlight the low inter-industry hiring rate because one instrument we consider for an employer's growth is the growth rate for its industry nationally, relative to that for other employers in its market.

[21] We base stayers' relative wage on the average of their real wages in December of years t and $t-1$. For those departing during t, we average their real wage in December of $t-1$ with an imputed wage for December t, imputed based on the employers' average wage change for stayers. Similarly, for those arriving in t, we average their real wage in December of t and an imputed wage for December of $t-1$. We then weight the exciters and entrants by 0.5 and group the workers according to the weighted quartiles: a worker is assigned to group q if its weighted cumulated density (i.e., the weighted share of workers with smaller or equal wages) in the employer is strictly greater than $0.25(q-1)$ and weakly smaller than $0.25q$. Because of ties, the quartiles are not equal-sized in our sample; the first to fourth quartiles account respectively for 14%, 23%, 26%, and 37% of workers.

[22] Note the fractions in Table 2 are calculated based only on hires who were recently employed in the formal market. More exactly, for each hire the sample reflects the formal sector employment contract that ended most recently prior to the hire. We require that it ended in the same or prior year as the hire.

4 Empirical Results

4.1 Empirical Specifications and Estimating Strategy

Identifying the employer-level supply elasticities requires measuring the relative responses of employment and wages to a shift in an employer's labor demand, holding its supply curve fixed. Consider a set of workers whose productivity is y at employer j, but then shifts to $y' = z \cdot y$. The shifter z could reflect a productivity change or a shift in demand for the employer's output. The elasticity of supply for workers of productivity y at employer j is:

$$\eta_j(w; y) = \frac{d \ln n_j(w; y)/d \ln z_j(y)}{d \ln w_j(y)/d \ln z_j(y)}. \tag{11}$$

Note that, while the elasticity $\eta_j(w; y)$ is specific to y, the labor demand shifter, $z_j(y)$, can be specific to y-workers or more employer-wide, i.e., $z_j(y) = z_j$ for all y. The key is that the shift in labor demand for workers of productivity y be orthogonal to any shifts in outside opportunities for those workers. With that in mind, in estimating $\eta_j(w; y)$ we will consider a variety of instruments for an employer's employment growth.

Specifically, we estimate wage and employment responses at each quartile of an employer's wage distribution to year-over-year growth in its total employment. For worker i at employer j in year t, let q be the worker's wage quartile. We consider the specifications:

$$\Delta \ln \text{wage}_{it} = \beta_q \Delta \ln \text{emp}_{jt} + \boldsymbol{x}_{it}\boldsymbol{\mu} + \alpha_{mqt} + \varepsilon_{it}, \tag{12}$$

$$\Delta \ln \text{emp}_{jqt} = \gamma_q \Delta \ln \text{emp}_{jt} + \kappa_{mqt} + \xi_{jqt}. \tag{13}$$

The first equation is estimated at the worker-year level, just for stayers at their employer, reflecting the 78,582,409 worker-year observations. The operator Δ denotes the backward-looking one-year difference: $\Delta \ln \text{wage}_{it}$ denotes stayer i's rate of wage growth from December $t-1$ to December t, while $\Delta \ln \text{emp}_{jt}$ denotes employer j's rate of employment growth between year ends $t-1$ and t. The row vector $\boldsymbol{x}_{it}$ controls for quadratics in the worker's years of schooling and age. The subscript m denotes the market (metropolitan area and occupation), with α_{mqt} market-by-quartile-by-year fixed effects in stayers' wage changes. ε_{it} is residual wage growth for stayer i. The coefficients of interest are the β_q's; differences in β_q's across quartiles capture how wages respond for an employer's higher- versus lower-wage workers.

The second equation is estimated at the employer-quartile-year level. The sample reflects 2,234,769 employer-year observations. For each employer-year, $\Delta \ln \text{emp}_{jqt}$ denotes the year-end rate of employment change from year $t-1$ to t for wage quartile q within the employer. κ_{mqt} is a market-by-quartile-by-year fixed effect for rate of employment growth; ξ_{jqt} is residual

employment growth. We weight each observation by the average of the employer's number of employees at year ends $t-1$ and t. The coefficients of interest are the γ_q's; its cross-quartile differences capture whether, when employment grows for an employer, that growth occurs more at the top or bottom of the employer's wage distribution.

The estimates from equations (12) and (13) jointly yield estimates of quartile-specific labor-supply elasticities, $\widehat{\eta}_q = \widehat{\gamma}_q/\widehat{\beta}_q$, with implied quartile-specific markdowns, $1/(1+\widehat{\eta}_q)$. Of course, OLS parameter estimates are biased unless employment changes are unrelated to potential shifts in the employer's labor supply—in particular, the wage response and implied markdown are biased downward if employment is partly responding to supply shocks.

To address this we consider three sets of instruments. We first exploit that younger and smaller establishments predictably grow faster. More exactly, the instrumental variables are quadratics in the employer's age and its log employment in $t-2$.[23] The identifying assumption is that faster growth for these employers primarily reflects productivity gains or product development that expands labor demand, not systematic shifts in their labor supply functions. To highlight this possible challenge to identification, we note that Kim (2024) predicts an employer's labor supply will grow over its life-cycle because, by learning, workers resolve their uncertainty about its productivity. In this scenario, our instrument would yield an upward-biased estimate of employer labor supply elasticity, implying a downward-biased markdown.

As an alternative strategy we treat national employment growth for an industry as an instrument for employment growth for employers in that industry, *holding fixed employment in each employer's labor market*. This strategy exploits that each labor market—defined by metropolitan area and occupation—contains workers from many industries. More exactly, the instrument is the annual growth rate in an employer's 7-digit industry across all labor markets.[24]

The identifying assumptions behind this instrument are two-fold. First, it assumes that an industry's nation-wide employment growth reflects shifts to its labor demand. Second, the instrument assumes an employer's labor supply is dictated by its location and occupation of its workers, not its 7-digit industry.[25] For robustness with respect to the first assumption, in Appendix B.2 we replace industry employment growth as an instrument with an interaction of the Brazilian business cycle with an industry's employment cyclicality, defined by its employment elasticity to aggregate employment. This weakens the assumption to require only that demand factors (e.g., producing durable goods) dictate why some industries are more cycli-

[23] The panel data reveal establishment births back to 1985. Given our regression sample begins in 2006, for all years we can construct exact age only up to 21 years. For this reason, our exact instruments are a dummy variable indicating age 21 or below plus its interaction with a quadratic in establishment age.

[24] Industries reflect Brazil's National Classification of Economic Activities (CNAE 2.0). Our sample has 1,393 industries. Across our 222 markets (3 occupations times 74 metropolitan areas) the average number of industries is 847. See Appendix B.1 for detailed summary statistics on the concentration of industries within labor markets.

[25] Table 2 showed that most hires are from the same metro area and occupation, but not the same industry.

cal than others. For robustness to the second assumption, in Appendix B.2 we exclude those industries for which more than 10% of hires, on average, come from the same industry.

Lastly, we combine and interact both sets of instruments. This approach exploits not only the labor demand changes rooted in employers' life-cycle and industry; the interaction term allows, for instance, that employment volatility in response to industry demand shocks changes over an employer's life-cycle (see recent evidence in Crouzet and Mehrotra, 2020; Clymo and Rozsypal, 2023). The identifying assumption, once again, requires these three sources of employment variation to be uncorrelated with changes in labor supply.

4.2 Within Employers: Less elastic Labor Supply for Higher-Wage Workers

We present wage and employment responses, β_q's and γ_q's, to an employer's employment growth, together with the implied labor supply elasticity and markdown. Standard errors are clustered at the employer-by-year level.[26] Table 3 presents the OLS results. The first row reports the supply elasticity without distinguishing wage quartiles.[27] Stayers' wage growth exhibits an elasticity of 0.026 to its employer's employment growth. The associated standard error is only 0.001—so the wage response is clearly statistically different from zero. Nevertheless, it implies an extremely high labor supply elasticity, η, of 37.8 with standard error 1.3. That maps to a markdown of only 2.6%, which is quite low compared to estimates in the literature.

The rest of Table 3 presents the OLS results for (12) and (13), i.e., how both wages and employment at each quartile of an employer's wage distribution respond to its overall employment growth.[28] Wage responses monotonically increase in size as we move up an employer's wage quartiles—the top quartile responds about two times more than the bottom. By sharp contrast, the employment responses systematically decline moving up the wage distribution—the top quartile responds two-thirds as much as the bottom. This indicates a less elastic labor supply for higher-wage workers. The supply elasticity declines from 81.6 to 23.3 from the bottom to the top quartile, with the implied markdown going from 1.2% to 4.1%.

The markdown implied by the OLS estimates is likely biased downward unless employment changes are unrelated to potential shifts in supply. To address this, Table 4 reports estimates under each set of instruments for employment growth described in Section 4.1. Each set passes the weak-instrument test.[29] The first panel instruments with the employer's life-cycle via quadratics in its log size and age; both being smaller and younger increases expected employ-

[26] Standard errors for labor supply elasticities and markdowns are derived by Delta Method.

[27] When not distinguishing across quartiles, the fixed effect α_{mqt} in equation (12) simplifies to market by year, α_{mt}. This also applies to first-row regressions of each panel in Tables 4 and A2.

[28] Average wages across the quartiles, expressed relative to the lowest quartile, are 1, 1.18, 1.53, and 2.27.

[29] We report the Kleibergen and Paap (2006) Wald rk F-statistic for weak identification because standard errors are clustered.

Table 3: OLS Estimates: Wage and Employment Responses by Wage Quartile

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.026 (0.001)	1.00 (—)	37.8 (1.3)	2.6 (0.1)
1st	0.015 (0.001)	1.21 (0.01)	81.6 (4.7)	1.2 (0.1)
2nd	0.021 (0.001)	0.97 (0.01)	45.5 (1.8)	2.2 (0.1)
3rd	0.028 (0.001)	0.86 (0.01)	30.4 (1.4)	3.2 (0.1)
4th	0.033 (0.001)	0.77 (0.01)	23.3 (0.9)	4.1 (0.2)

Notes: Wage responses reflect 78,582,409 worker-year observations; employment response reflect 3,308,773 employer-year observations, with observations weighted by employer's employment. Standard errors (in parentheses) for wage and employment responses are clustered at employer-by-year level. Those for labor supply elasticity and markdown are derived by Delta Method. Not every employer is represented in all four quartiles.

ment growth, with a first-stage F-statistic of 3479. The second panel instruments with industry employment growth; its first-stage F-statistic equals 39. The third panel, which combines and interacts the previous two instrument sets, yields a first-stage F-statistic of 9089.

We begin again, in the first row of each panel, with the employer-level supply elasticity without distinguishing across quartiles. As expected, the IV-estimated wage responses are much higher than OLS. Across the first two sets of instruments, stayers' wages respond with respective elasticities of 0.20 and 0.12, with corresponding implied markdowns of 16.6 and 10.9%. Combining the instruments, Panel 3, yields estimates that are intermediate, with an elasticity of wage response of 0.15 (s.e. 0.02) and an implied markdown of 13.1% (s.e. 1.4%). As discussed in the literature review, this markup estimate is comparable to Lobel's (2024) 19% estimate for Brazil's formal labor market, instrumenting with payroll-tax changes, and comparable to many estimates for the U.S. labor market.

The remaining rows in each panel of Table 4 present the IV estimates by workers' wage quartiles. Instrumenting yields the same relative pattern as OLS, but with much larger wage responses and implied markdowns. Furthermore, these patterns are remarkably consistent across the choice of instruments. In particular, all three panels show wage responses that increase monotonically across wage quartiles, with a top-quartile response of about four times that at the bottom, while at the same time showing employment responses that monotonically decline, with a top-quartile response that is half that of the bottom's. As a result, regardless of instrument set, the estimates imply a supply elasticity that is nearly an order of magnitude larger for the bottom quartile as the top. The upshot is that we estimate that monopsony power is far greater at the top of employers' wage distributions. Focusing on Panel 3 combining instruments, the estimated markup for top quartiles is 23.4% (s.e. 2.3%), whereas for bottom quartiles it is extremely low at 3.2% (s.e. 1.3%).

These findings are consistent with the prediction of Proposition 4 in Section 2. Through the lens of our model this implies that, for Brazil's formal labor market, workers' comparative and absolute advantages typically align at their employers, with the higher-wage workers exhibiting higher match rents and larger monopsony markdowns on those rents. Moreover, our empirical findings are consistent with estimates by Kline et al. (2019), Carbonnier et al. (2022), and Lobel (2024) that firm labor-demand shocks have a larger impact on wages of firms' higher-wage workers.

In Appendix B.2, Table A2, we report two robustness regressions for the industry growth instrument. First, we exclude industries if more than 10% of its hires are from within the industry. This is to avoid conflating the demand effect from industry-wide growth with more competition to find workers (a backward supply shift to employers). Second, to avoid confounding changes in industry labor supply, we replace industry employment growth as instrument with the industry's exposure to business cycles, predicted by the industry's average cyclicality. In both cases, the estimation results are quite similar; but standard errors are somewhat larger.[30]

4.3 Across Employers: Less Elastic Labor Supply for Higher-Wage Employers

We next explore how labor supply elasticities vary across employers in light of Section 2.4's predictions under asymmetric duopsony: i) bigger markdowns at larger employers; ii) steeper profiles of markdowns with respect to productivity for higher-wage employers. Thus we group employers by relative size and relative average wage.

We first divide employers into large versus small based on their relative size in their market. To be comparable with the common definition in the literature (see, e.g., Berger, Herkenhoff and Mongey, 2022), we define large employers as the top 5% in each market in number of workers. These employers account for 44% of workers in our sample, with small employers accounting for 56%.

We next group employers according to wages. To be specific, we first rank employers in each market by median wage, then label high-wage employers as the minimal upper part of that distribution required to account for 50% of workers in each market. High-wage employers are 47% of all employers in our sample. The average median wage among our high-wage employers is 91% higher than that among the low-wage employers.

Denote the employer group as $g \in \{\text{large}, \text{small}\}$ when grouping by size, and $g \in \{\text{high-wage}, \text{low-wage}\}$ when grouping by wages. We modify the regression specifications (12) and (13) to

[30] We report two additional robustness tests for our IV regressions in Appendix B.2, Table A3. The first restricts the sample to workers with wages at least 120% of the legal minimum. The second constructs wage quartiles for each employer after first removing the impact of observable worker characteristics (e.g., age, education, gender, etc.). In both cases, our findings are qualitatively unaffected. That is, within-employer wage dispersion remains large and the estimated elasticity remains much lower for employers' higher-wage workers.

Table 4: IV Estimates: Wage and Employment Responses by Wage Quartile

IV1: employer's life-cycle (quadratics in log size and log age)

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.20 (0.02)	1.00 (—)	5.0 (0.6)	16.6 (1.6)
1st	0.06 (0.03)	1.21 (0.07)	19.6 (9.4)	4.8 (2.2)
2nd	0.15 (0.02)	1.06 (0.03)	7.2 (1.0)	12.2 (1.6)
3rd	0.23 (0.03)	0.69 (0.03)	3.1 (0.4)	24.6 (2.1)
4th	0.25 (0.02)	0.58 (0.03)	2.4 (0.2)	29.7 (2.1)

IV2: industry's employment growth

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.12 (0.02)	1.00 (—)	8.2 (1.6)	10.9 (1.9)
1st	0.04 (0.03)	1.44 (0.13)	35.7 (25.3)	2.7 (1.9)
2nd	0.09 (0.03)	0.89 (0.10)	10.3 (3.3)	8.8 (2.5)
3rd	0.14 (0.03)	0.83 (0.06)	5.8 (1.2)	14.6 (2.6)
4th	0.17 (0.03)	0.65 (0.05)	3.8 (0.7)	20.7 (3.0)

IV3: employer's life-cycle × industry's employment growth

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.15 (0.02)	1.00 (—)	6.7 (0.8)	13.1 (1.4)
1st	0.04 (0.02)	1.24 (0.13)	30.7 (13.0)	3.2 (1.3)
2nd	0.11 (0.02)	0.96 (0.09)	8.8 (1.6)	10.2 (1.6)
3rd	0.17 (0.02)	0.79 (0.05)	4.5 (0.6)	18.2 (2.1)
4th	0.20 (0.02)	0.64 (0.04)	3.3 (0.4)	23.4 (2.3)

Notes: The IV1 instruments include quadratics for log establishment size and age, where establishment size is measured by 2-period lagged employment; in addition, we interact log age quadratics with an indicator for age below 21 because the birth year of establishment is truncated at 1985 by construction. IV2 instrument is the establishment's 7-digit CNAE industry employment growth. IV3 instruments are those for IV1 and IV2 and their interactions. The pooled regressions (labeled as "all") control for market-by-year fixed effects; others control for markety-by-year-by-quartile fixed effects. Standard errors in parentheses, clustered at the employer-by-year level. Standard errors in Columns 3 and 4 are calculated by the Delta Method.

allow for differing wage and employment responses across employer groups (β_q now β_{gq} and γ_q now γ_{gq}) and fixed effects that span g as well as m, q, and t. Because the three instrument sets delivered similar findings in the previous subsection, here we only report estimates using the last set of instruments, i.e., the combination and interaction of life-cycle and industry growth instruments. Standard errors are, again, clustered at the employer-by-year level.

Figure 8 depicts the results: Panel (a) groups employers by relative size, while Panel (b) does so by relative wage. In each panel the top figure plots the estimated stayers' wage responses, $\widehat{\beta}_{gq}$, by within-employer wage quartile q; the middle plots the estimates for employment responses, $\widehat{\gamma}_{gq}$, by q; the bottom plots the implied markdown, $1/(1+\widehat{\gamma}_{gq}/\widehat{\beta}_{gq})$. For each Panel (a) figure, the red and blue lines depict respective estimates for large and small employers; for Panel (b), the red and blue lines depict respective estimates for high-wage and low-wage employers. The shaded areas represent the estimates' 95% confidence intervals.[31]

We highlight three findings. First, the *within-employer* pattern of markdowns by wage quartile, documented above, holds for each employer group: compared to lower-wage workers, higher-wage workers' wages are more responsive to labor demand while their employment is less responsive, indicating their less elastic labor supply and, hence, larger markdown.

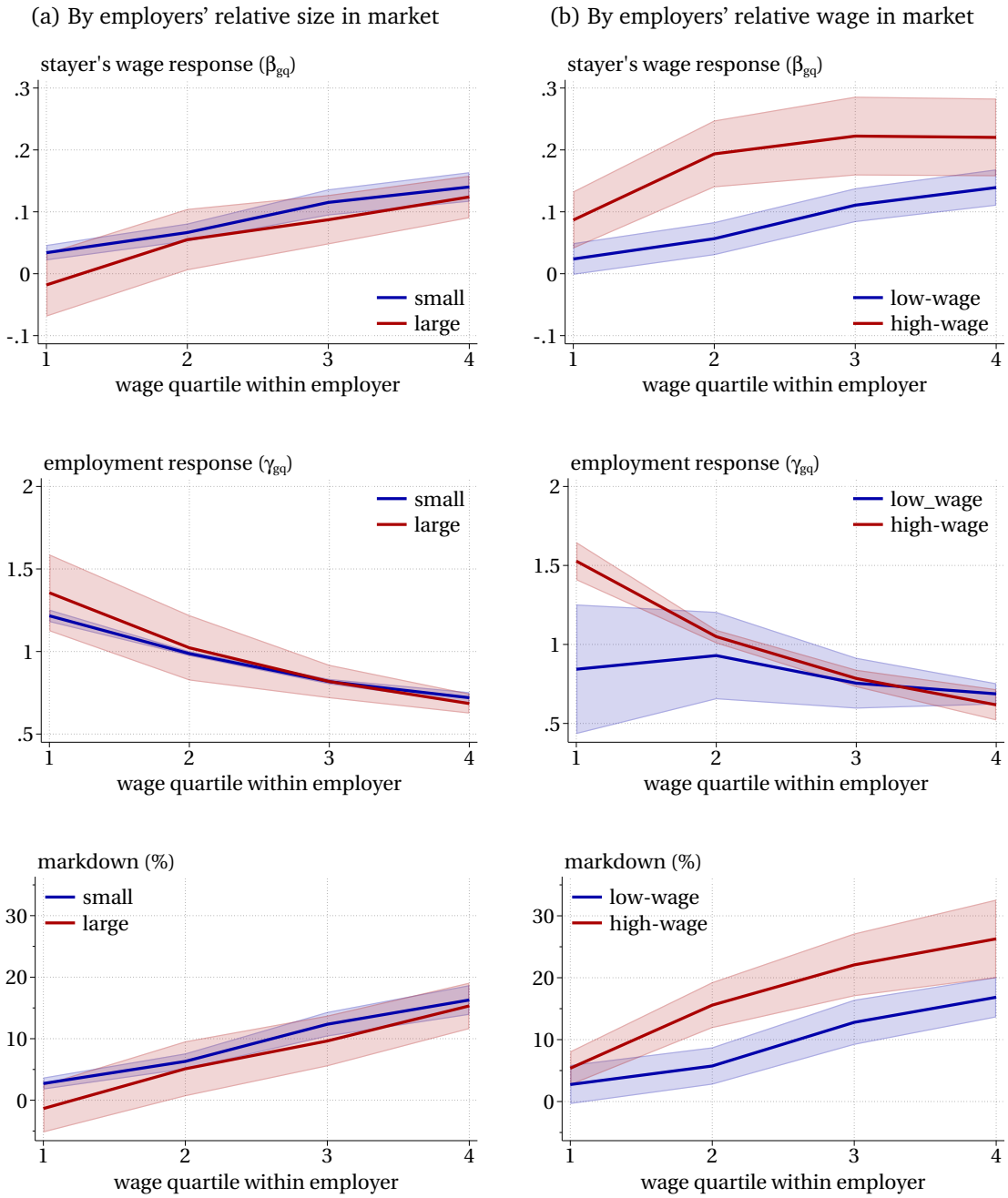
Second, wage responses, and hence implied markdowns, do not differ economically or statistically between large and small employers. Through the lens of our model, if large firms are generally more productive, they should exhibit a smaller supply elasticity, so larger wage responses. Thus, one interpretation of our findings in Panel (a) is that size differences across employers in the Brazilian labor market are not primarily driven by productivity. Two stylized facts on the Brazilian formal labor market are worth noting in this regard. First, employment is quite de-concentrated; so size variations are relatively small. In particular, the average market employment share of the employers we classify as large is only 0.3%. Second, the elasticity of median employer wage with respect to employment is quite low in Brazil's formal sector, equaling 0.04 (correlation of 0.08).[32] As a result, average wages for our large employers are only 11% higher than for small employers. Again through the model's lens, if size differences reflect general productivity differences, then relative size and wage should be tightly related across employers. In turn this suggests that wage differences across employers may primarily reflect how higher- versus less-skilled workers are selected across employers, as in Panel (b) of Figures 5 - 7, rather than general productivity differences in Panel (a).

The third finding is that high-wage employers face less elastic labor supply, showing greater wage responses to demand, indicating a bigger markdown. Compared to the case of asymmet-

[31] Standard errors for labor supply elasticities and markdowns are derived by Delta Method.

[32] The elasticity reflects projecting an employer's log median wage on its log employment controlling for market-by-year fixed effects.

Figure 8: Markdown Heterogeneity across Employers



Notes: For each Panel (a) figure, the red and blue lines depict respective estimates for large and small employers; for Panel (b), the red and blue lines depict respective estimates for high-wage and low-wage employers. The shaded areas represent the estimates' 95% confidence intervals.

ric duopsony in Section 2 with a single-crossing in productivities (Figure 7-b), we do not see a crossing of markdowns, i.e., estimated markdowns are bigger for high-wage employers for all within-employer wage quartiles. But, high-wage employers do show a steeper profile of markups with respect to wage quartile. This suggests that skill-dependent sorting into high- and low-wage employers, such as in Panel (b) of Figures 5 - 7 from differences in skill elasticity of productivity, plays a role in explaining the cross-employer patterns in markdowns.

5 Implications of Estimated Markdowns through the Lens of Model

Section 2 showed how the elasticity of a firm's labor supply reflects the cross-firm dependence of productivities. Our estimates in Section 4 reveal a sharply decreasing labor supply elasticity moving up an employer's wage distribution. We now return to the model to see what discipline our estimates impose in describing monopsony power in the Brazilian labor market. To focus on the within-firm variation of markdowns, we first look at our symmetric duopsony model and examine what our estimates tell us about the dependence structure of worker productivities across firms. We then turn to our extended model allowing for an informal labor market (or non-employment) as well as a binding legal minimum wage. After calibrating the model to our estimates, as well as a few salient features of the Brazilian labor market, we then gauge the extent one should expect minimum wages to counter monopsony power in Brazil.

5.1 Disciplining the Cross-Firm Dependence of Productivities

We first consider a symmetric duopsony without an informal sector nor a minimum wage. We assume the marginal productivity distribution is log-normal and calibrate three model versions corresponding to joint productivities reflecting a Gausian, Clayton, or Gumbel copula.[33] Normalizing the the marginal distribution's mean to zero, two parameters remain to calibrate: The standard deviation (σ) of the marginal distribution and the dependence parameter (ρ, θ, or α) conditional on each copula. We calibrate these to match the average within-employer standard deviation of wages in our RAIS sample of 0.31 (weighting employers by their employments) and an average markdown of 16.2%, reflecting the average of markdown estimates for the four quartiles in Table 4 using the third set of instruments.[34]

The calibrated parameters are given in the top panel of Table 5. The model, under each

[33] Given the parameters, we first compute the equilibrium wage function using the formulas in Examples 2, 3, and 4 of Section 2. We then simulate workers from the given joint productivity distribution, calculate wages and job choices for each worker using the wage function, and use the simulated moments as a numerical approximation to the model moments.

[34] Repeating from above, because of ties the quartiles differ in size, with respective shares of 14%, 23%, 26%, and 37% of workers.

Table 5: Calibrated Parameters and Implied Markdowns across Copulas

Calibrated parameters

	Gaussian	Clayton	Gumbel
Marginal distribution σ	0.36	0.40	0.35
Copula parameter	$\rho = 0.90$	$\theta = 4.12$	$\alpha = 3.06$

Markdowns (%) by within-firm wage quartile

quartile (q)	Data	Gaussian	Clayton	Gumbel
1st	3.2 (1.3)	11.4	7.8	13.3
2nd	10.2 (1.6)	12.7	12.2	13.9
3rd	18.2 (2.1)	13.5	16.9	13.5
4th	23.4 (2.3)	15.2	26.3	11.6

Notes: For each copula, parameters are calibrated to match (i) a 0.31 within-employer standard deviation of log wages and (ii) a 16.2 average wage markdown across workers. The first number is calculated by the authors using the RAIS data with a sample that satisfies Table 1 restrictions except for being stayers (i.e., the last restriction in Panel B). The second number and the data markdowns by quartiles are previously reported as IV3 in Table 4.

copula, is able to match the two calibrated moments quite precisely. All three models require high overall dependence to hit an average wage markdown of 16.2%. In particular, the calibrated correlation coefficient for the Gaussian copula is 0.90. Kendall's tau for the calibrated Gaussian, Clayton, and Gumbel copulas are respectively 0.71, 0.67, and 0.67.

The pattern for markdowns by wage quartile are presented in Table 5. Starting with the Gaussian copula, we see wage markdowns increase moving up the wage distribution, as anticipated by Figure 2. But the model fails to match the sharpness at which the estimated markdowns increase across wage quartiles, shown in Column 1. The model matches the estimates much more closely under the Clayton copula, which allows for higher correlation at low productivities. In particular, the model captures most of the 20 percentage point differential in markdowns between the 4^{th} and 1^{st} quartiles. The model under a Gumbel is directly at odds with the estimates—the markdown is relatively flat across the wage quartiles, in fact decreasing moving from the 2^{nd} to the 4^{th} quartile. We therefore focus on Clayton copula in our simulations below.[35]

[35]Given the parameters, we first compute formal firms' equilibrium wage function using a reverse shooting algorithm to solve the differential equation specified in Theorem 8 of Section 2.5. We then simulate workers from the given productivity and informal wage distributions, calculate wages and job choices for each worker using the wage function, and approximate the model moments using the simulated moments.

Table 6: Targeted Moments for Calibration with Minimum Wage and Informal Sector

Targeted Moments	Data Source	Data	Model
Within-formal-firm std of log wage	Authors' calculation	0.31	0.33
Formal sector average markdown (%)	Authors' estimation	16.2	16.2
Formal sector employment share (%)	Engbom, Gonzaga and Olivieri (2022)	67	68
Formal-to-minimum wage ratio	Authors' calculation	2.04	2.04
Formal-to-informal wage ratio	Engbom, Gonzaga and Olivieri (2022)	1.82	1.73
Untargeted Moments	**Data Source**	**Data**	**Model**
Wage markdown (%)			
1st quartile	see Table 4	3.2	5.4
2nd quartile	"	10.2	9.9
3rd quartile	"	18.2	14.8
4th quartile	"	23.4	25.2

Notes: The within-employer std of log wage and the formal-to-minimum wage ratio are calculated by the authors using the RAIS data with a sample that satisfies Table 1 restrictions except for being stayers (i.e., the last restriction in Panel B). The formal sector average markdown are previously reported as IV3 in Table 4.

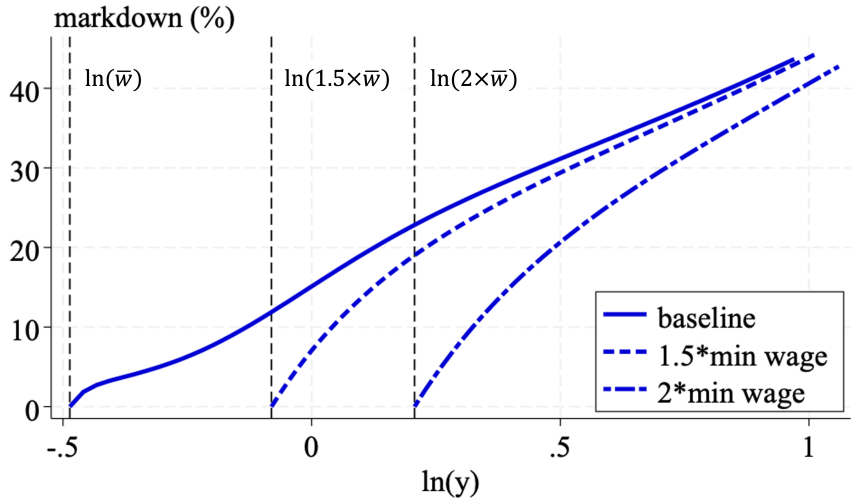
5.2 The Effect of Minimum Wage Policies

We now extend the model calibration to allow for a legal minimum wage, $\bar{w}$, and a competitive informal labor market. (See Section 2.5.) We assume the informal-market wage, w_0, is distributed log-normally with mean μ_0 and standard deviation σ_0, independent of formal-sector productivities, y_1 and y_2. This requires calibrating three additional parameters: μ_0, σ_0, and the minimum wage $\bar{w}$. To do so, we additionally target an informal sector that is one-third of total employment with an average wage that is 55% that in the formal sector. We base these two moments on Engbom, Gonzaga and Olivieri (2022)'s comparison of informal and formal employments and earnings from survey data for Brazil's six largest cities.[36] We target a minimum-wage that is 49% of the average formal-sector wage, with this figure calculated from comparing a time series for the Brazilian minimum wage to wages in our RAIS sample. Table 6 summarizes the calibration targets and presents the model moments.

Model implied markdowns by wage quartiles are presented in the second column of Table 7. Relative to markdowns in Table 5, the bottom three quartiles display lower markdowns, consistent with the model's prediction, Section 2.5, that both an informal sector and a minimum wage reduce markdowns especially for lower-wage workers. As a result, the markdown markdown profile rises even more steeply along the wage distribution, closely mirroring what we estimate from the RAIS.

[36]Informal-to-formal wage ratio is computed from the difference between the weekly earnings gap and the weekly hours gap between the two sectors as reported in Table 2 of Engbom, Gonzaga and Olivieri (2022).

Figure 9: Markdown by Log Productivity: Baseline vs. Counterfactuals



Notes: The baseline model is calibrated to match moments reported in Table 6. The two counterfactual models raise the minimum wage, $\bar{w}$, from the baseline model by 1.5 and 2 times its calibrated value respectively, holding the other parameters the same as the baseline.

Larger markdowns for higher wage workers imply that monopsony power tends to reduce wage inequality. In our calibrated model, the standard deviation of log-productivity in the formal sector is 0.43 whereas that of log-wages is 0.33. Therefore, compared to competitive wage setting, where wages equals productivity, monopsony power reduces (within firm) wage dispersion in the formal sector by 23 percent, holding fixed the worker allocation between informal and formal sectors.

To better illustrate the effects of a change in the minimum wage on wage markdowns and employment, we simulate counterfactual equilibria by raising $\bar{w}$ by 50% and 100%. Figure 9 shows the markdowns by productivity in each economy. Recall from Section 2.5 that a minimum wage brings a pay bump to all workers with productivity levels above the minimum wage. As a result, higher minimum wages reduce markdowns in Figure 9 at all productivities, but the decline is particularly significant for less productive workers.

At the same time, workers whose productivities are eclipsed by the minimum wage are forced to the informal sector. Table 7, last row, reports the formal-sector share shrinks from 68%, to 57%, to 40% as the minimum wage is increased by 50%, then 100%. Workers displaced by higher minimum wages are precisely those whose wages are marked down less. As a result, even though a higher minimum reduces markdowns at each productivity, it actually acts to raise the average markdown by skewing formal employment to high-wage, high-markdown workers. From our calibration (Table 7, third row) the average markdown actually rises from 16.2%, to 17.3%, to 18.6% as the minimum wage is increased by 50%, then 100%.

Table 7: Model Implications: Baseline vs. Counterfactuals

		Counterfactuals	
Model moments	Baseline	$1.5 \times \bar{w}$	$2 \times \bar{w}$
Within-formal-firm std of log wage	0.33	0.27	0.19
Within-formal-firm std of $\ln(y)$	0.43	0.38	0.31
Formal sector average markdown (%)	16.2	17.3	18.6
Formal sector employment share (%)	68	57	40

Notes: The baseline model is calibrated to match moments reported in Table 6. The two counterfactual models raise the minimum wage, $\bar{w}$, from the baseline model by 1.5 and 2 times its calibrated value respectively, holding the other parameters the same as the baseline.

6 Concluding Remarks

We show that monopsony power can be attributed to comparative advantage in worker-employer matches—an employer marks down the wage to tax what it perceives as a workers' comparative-advantage rents. As a result, if a firm's more-productive workers have greater comparative advantage, monopsony will flatten a firm's profile of wages with respect to worker productivity.

We first characterize a symmetric oligopsony, i.e. all employers draw from the same productivity distribution, showing how the markup maps to comparative advantage. That advantage reflects the number of competing firms, the dependence of worker productivity across firms, and a worker's relative productivity at their best match. We also show that, via the interaction of firms' wage polices, a minimum wage reduces markdowns throughout the wage distribution.

We then drop symmetry to first consider a duopsony equilibrium where one firm is generally more productive. The more-productive firm is both larger and pays higher wages, but also employs an inefficiently low share of workers. Finally, we consider an asymmetric duopsony where efficiency dictates one firm disproportionately employ more-skilled workers. This gives each employer distinct monopsony power vis a vis the workers where comparative advantage is greatest: the highest-skilled workers at the high-wage firm and the lowest-skilled at the low-wage firm. As a consequence, there is suboptimal sorting of workers by skill.

We illustrate our model's predictions for the Brazilian formal labor market. Our key empirical finding is that employment growth generates larger wage changes for an employer's higher-wage workers, while employment responds more at the bottom of its wage distribution. Thus employers face less elastic supply, implying bigger markdowns, for their higher-wage workers, consistent with match comparative advantage being greatest for these workers. In fact, we estimate that monopsony power has little impact on wages for employers' lower-wage workers. We also estimate that high-wage employers face less elastic supply, implying

bigger markdowns for their workers. Together these results, greater markdowns for higher-wage employers and for higher-paid workers within employers, imply that monopsony acts to reduce wage inequality relative that in worker productivities.

Lastly, we show that our extended model, allowing for an informal labor market and a minimum-wage, fits well the within-firm pattern of estimated markups provided productivity for lower-wage workers is highly correlated across firms. The literature highlights that a minimum wage can reduce monopsony power by making firms wage-takers at the legal minimum. Our results suggest little room for this policy, at least for Brazil. The low degree of monopsony estimated for low-wage formal-sector workers implies that a minimum wage modestly above the monopsony wage would divert a large number of workers to unemployment or to the informal sector.

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Appendix

A Mathematical Appendix

Theorem 1 (Symmetric Equilibrium) *Consider a symmetric market with K firms. The symmetric oligopsony equilibrium is characterized by the following wage offer policy and employment allocation.*

1. *Wage offer policy*
$$w(y) = \int_0^y x d\tilde{F}(x|y) = \mathbb{E}_{\tilde{F}}[x|x \leq y],$$
where $\tilde{F}(x|y) = \exp(-\int_x^y \sum_{j\neq k} \frac{C_{kj}(F(z),...,F(z)}{C_k(F(z),...,F(z))} F'(z)dz)$ *and* C_{kj} *is the cross-partial derivative of the C w.r.t.* $u_k = F_k(y)$ *and* $u_j = F_j(y)$ *for all* $j \neq k \in \{1,...,K\}$.

2. *For any* $\mathbf{y} = \{y_1,...,y_K\} \in \mathbb{R}_+^K$, $\mathbf{y} \in \pi_k$ *if and only if* $k = argmax_k\{y_1,...,y_K\}$, *and employment at firm k is given by:*
$$n_k(\pi_k) = C_k(F(y),\cdots,F(y))F'(y) \quad \forall k = \{1,...,K\}.$$

Proof. Suppose that the wage strategy is strictly increasing in productivity: $w_k'(y_k) > 0$. We will verify that this is the case at the end. Define the inverse wage function $a_k(w) = w_k^{-1}(w)$ for each firm k.

Step 1 – Firm k's problem: The probability that firm k's wage offer beats competition for a worker is equivalent to that worker not being sufficiently efficient to warrant a better wage offer at other firms: $\forall j \neq k,\ w_k(y_k) > w_j(y_j) \Leftrightarrow y_j < a_j(w_k(y_k))$. Using that fact, the conditional probability in equation 2 can be represented as:
$$P(w_j(y_j) \leq w, \forall j \neq k | Y_k = y) = P(y_j \leq a_j(w); \forall j \neq k | Y_k = y)$$
Using the copula notation to represent the probability above, the labor supply faced by firm k for workers with efficiency $Y_k = y$ is:
$$n_k(w; y) = C_k(u_1,\cdots,F_k(y),\cdots,u_K)F_k'(y),$$
where $u_j = F_j(a_j(w))$ for all $j \neq k$ and $C_k()$ is the partial derivative of C copula with respect to u_k. The slope of the labor supply curve is:
$$n_w(w; y) = \sum_{j\neq k} C_{kj}(F_1(a_1(w)),\cdots,F_k(y),\cdots,F_K(a_K(w)))F_j'(a_j(w))a_j'(w)F_k'(y),$$
where C_{kj} denotes the cross-partial derivative of C w.r.t. u_k and u_j. Substituting into the optimality condition for wages (equation (4)) and canceling $F_k'(y)$ gives:

$$(y-w)\sum_{j\neq k} C_{kj}(u_1,\cdots,F_k(y),\cdots,u_K)F_j'(a_j(w))a_j'(w) = C_k(u_1,\cdots,F_k(y),\cdots,u_K), \tag{A1}$$

with $u_j = F_j(a_j(w))$ for all $j \neq k$. Equation (A1) defines a K-dimensional system of first-order, non-linear differential equations. Note that, the solution to each equation gives $w_k(y)$, or, equivalently, $y = a_k(w)$. Substituting $F_k(y) = a_k(w)$, and then imposing symmetry, by setting $a_j(w) = a(w)$ and $F_j(y) = F(y)$ for all j, reduces it to a single differential equation, stated in standard notation (which sets $a'(w) = da/dw$) below:

$$(a-w)\sum_{j\neq k} C_{kj}(F(a),\cdots,F(a))F'(a)da - C_k(F(a),\cdots,F(a))dw = 0, \tag{A2}$$

Step 2 – Solving the Differential Equation:

To solve the differential equation above, consider the integrating factor below.

$$I(a) = \exp\left(-\int_0^a F'(x)\frac{C_{kk}(F(x),..F(x))}{C_k(F(x),..F(x))}dx\right) \equiv e^{-\gamma(a)},$$

where we introduce $\gamma(a)$ for notational convenience. Multiplying equation (A2) by $I(a)$ gives an exact ODE of the form $\Psi_a(a,w)da + \Psi_w(a,w)dw = 0$ with a generic solution $\Psi(a,w) = d_0$ for any constant of integration $d_0 \in \mathbb{R}$:

$$\underbrace{e^{-\gamma(a)}(a-w)\sum_{j\neq k} C_{kj}(F(a),\cdots,F(a))F'(a)}_{\Psi_a(a,w)}da \overbrace{-e^{-\gamma(a)}C_k(F(a),\cdots,F(a))}^{\Psi_w(a,w)}dw = 0,$$

Exactness can be established by verifying $\Psi_{wa}(a,w) = \Psi_{aw}(a,w)$:

$$\begin{aligned}\Psi_{wa}(a,w) &= e^{-\gamma(a)}\left[\gamma'(a)C_k(F(a),...,F(a)) - F'(a)\sum_{j=1}^{K} C_{kj}(F(a),...,F(a))\right] \\ &= -e^{-\gamma(a)}F'(a)\sum_{j\neq k} C_{kj}(F(a),...,F(a)) = \Psi_{aw}(a,w),\end{aligned}$$

where the second equality sets $\gamma'(a) = F'(a)C_{kk}(F(a),...,F(a))/C_k(F(a),...,F(a))$ using the definition of $\gamma(a)$ above.

The solution can be obtained in two steps. First, integrate $\Psi_w(a,w)$ along w:

$$\Psi(a,w) = \int \Psi_w(a,w)dw = -e^{-\gamma(a)}C_k(F(a),...,F(a))w + d(a), \tag{A3}$$

where $d(a)$ is constant of integration, which may depend on a. To determine $d(a)$, differentiate the

term above w.r.t. a and equate to $\Psi_a(a,w)$ from the ODE:

$$\Psi_a(a,w) = -we^{-\gamma(a)}F'(a)\sum_{j\neq k} C_{kj}(.) + d'(a) = e^{-\gamma(a)}(a-w)\sum_{j\neq k} C_{kj}(.)F'(a)$$

implying:

$$d'(a) = ae^{-\gamma(a)}\sum_{j\neq k} C_{kj}(.)F'(a)$$

Integrating $d'(a)$ and substituting back in equation (A3) gives the solution for $\Psi(a,w)$:

$$\Psi(a,w) = -we^{-\gamma(a)}C_k(F(a),...,F(a)) + \int_0^a xe^{-\gamma(x)}\sum_{j\neq k} C_{kj}(F(x),...,F(x))F'(x)dx = d_0. \tag{A4}$$

To determine d_0, note that it is optimal to offer $w(0) = 0$ when productivity is nil. Therefore, $d_0 = \Psi(0,0) = 0$. Finally, solving for w and setting $a = y$ gives the optimal wage offer policy:

$$w(y) = \int_0^y \frac{\sum_{j\neq k} e^{-\gamma(x)}C_{kj}(F(x),...,F(x))F'(x)}{e^{-\gamma(y)}C_k(F(y),...,F(y))}xdx \tag{A5}$$

Notice that the term in the numerator is the derivative of the term in the denominator, implying:

$$\int_0^y \sum_{j\neq k} e^{-\gamma(x)}C_{kj}(F(x),...,F(x))F'(x)dx = e^{-\gamma(y)}C_k(F(y),...,F(y)).$$

Therefore setting

$$\tilde{F}(x) = \frac{e^{-\gamma(x)}C_k(F(x),...,F(x))}{e^{-\gamma(y)}C_k(F(y),...,F(y))}$$

gives the following representation for the optimal wage function:

$$w(y) = \int_0^y x d\tilde{F}(x) = \mathbb{E}_{\tilde{F}}[x|x \leq y], \tag{A6}$$

where $\tilde{F}'(x)$ are weights assigned to $x \leq y$ since $\tilde{F}'(x) \geq 0$ for all $x \in [0,y]$ and $\int_0^y \tilde{F}'(x) = 1$.

For the alternative representation of $\tilde{F}(x)$ stated in the proposition, consider the following:

$$
\begin{aligned}
\ln C_k = \int \frac{d}{dy} \ln C_k &= \int \frac{C_{kk}}{C_k} F' + \int \sum_{j \neq k} \frac{C_{kj}}{C_k} F' \\
-\int \frac{C_{kk}}{C_k} F' &= -\ln C_k + \int \sum_{j \neq k} \frac{C_{kj}}{C_k} F' \\
\underbrace{e^{-\int \frac{C_{kk}}{C_k} F'}}_{e^{-\gamma(y)}} &= \frac{e^{\int \sum_{j \neq k} \frac{C_{kj}}{C_k} F'}}{C_k} \\
e^{-\gamma(y)} C_k &= e^{\int \sum_{j \neq k} \frac{C_{kj}}{C_k} F'}
\end{aligned}
$$

Substituting this back in the definition of $\tilde{F}$ gives:

$$
\tilde{F}(x) = \frac{e^{\int_0^x \sum_{j \neq k} \frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))} F'(z) dz}}{e^{\int_0^y \sum_{j \neq k} \frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))} F'(z) dz}} = e^{-\int_x^y \sum_{j \neq k} \frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))} F'(z) dz} \tag{A7}
$$

Step 3 – Verify $w'(y) > 0$: To conclude the proof, we next verify that $w'(y) > 0$, which ensures the existence of $y = a(w) = w^{-1}(w)$ with $a'(w) > 0$ (continuity of $w(y)$ is ensured by the continuity of the density function $h(y_1, ..., y_k)$.

In equilibrium, $a'(w(y)) = 1/w'(y)$. From the optimality condition, we have:

$$
w'(y) = (y - w) \left(\sum_{j \neq k} \frac{C_{kj}(F(y), \cdots, F(y))}{C_k(F(y), \cdots, F(y))} F'(y) \right).
$$

From equation (A6), $y - w \geq 0$, with strict inequality when $y > 0$. $F'(y) > 0$ by assumption. Also recall that for any $k \in \{1, ..., K\}$, $C_k(F(y), \cdots, F(y)) = P(Y_j \leq y, \forall j \neq k | Y_k = y) > 0$ and, for any $j, k \in \{1, ..., K\}$ with $j \neq k$, $C_{kj}(F(y), \cdots, F(y)) = P(Y_j = y, Y_l \leq y, \ \forall l \notin \{j, k\} \ \text{and} \ j \neq k | Y_k = y) > 0$, which are both positive. Therefore the derivative, $w'(y)$, is strictly positive for $y > 0$ (and $w'(0) = 0$).

■

Lemma 1 *The joint distribution of wage offers $w(y_k)$ is described by the copula $C^w = C$, and the common marginal distribution $F^w(x) = F(w^{-1}(x))$.*

Proof. Proof follows from the stability of copulas under monotonic transformations and the strict monotonicity of the wage offer function. Formally, suppose X_k are K random variables whose joint distribution is described by copula C^x and marginals $F_k(X_k)$. Then for any set of continuous and strictly monotonic functions $H_k : \mathbb{R} \to \mathbb{R}$,

$$
P(H_k(X_k) \leq h_k) = P(x_k \leq H_k^{-1}(h_k)) = C^x(F_k(H_k^{-1}(h_k)), \forall k = 1, ..., K).
$$

Noting that the equilibrium wage offer function $w(y)$ is continuous and strictly increasing implies the

lemma. ■

Theorem 3 *The distribution of wages in symmetric oligopsony equilibrium, is:*

$$P(w^* \leq \omega) = C(F^w(\omega), \cdots, F^w(\omega)). \tag{5}$$

Proof. For any K dimensional copula C that describes the joint distribution of random variables X_k for $k = 1, ..., K$, the distribution of $\max\{X_k : k = 1, ..., K\}$ is given by its diagonal section. Hence:

$$P(\max_j\{w(y_j)\} \leq \omega) = P(w(y_1) \leq \omega, ...w(y_k) \leq \omega) = C(F^w(\omega), ..., F^w(\omega))$$

■

Example 1 (Independent Copula) *If Y_k are independent across employers, then the optimal wage offer policy is $w(y) = \int_0^y x d\tilde{F}^I(x|y)$ with*

$$\tilde{F}^I(x|y) = \left[\frac{F(x)}{F(y)}\right]^{K-1}.$$

Proof. The independence Copula is $C^I = \Pi_k u_k$. At $Y_k = z$ for all k, $u_k = F(z)$ for all k, implying, $C_k = F^{K-1}(z)$ and $C_{kj} = F^{K-2}(z)$ for all k. But then:

$$\int_x^y \sum_{j \neq k} \frac{C_{kj}(F(z), ..., F(z)}{C_k(F(z), ..., F(z))} F'(z) dz = (K-1) \int_x^y \frac{F'(z)}{F(z)} dz = (K-1)[\ln F(y) - \ln F(x)]$$

Substituting this in the definition of $\tilde{F}$ in Theorem 1 gives the result:

$$\tilde{F}(x) = e^{-(K-1)[\ln F(y) - \ln F(x)]} = \frac{F^{K-1}(x)}{F^{K-1}(y)} \tag{A8}$$

■

Proposition 1 (Archimedean Copulas) *Let $C^A = \psi^{-1}(\sum_j \psi(u_j))$ be an Archimedean Copula with the generator function $\psi : [0,1] \to [0,1]$. Then the optimal wage offer policy is $w^A(y) = \int_0^y x d\tilde{F}(x|y)$ with*

$$\tilde{F}(x|y) = \left[\frac{\psi'(C^A(F(x), ..., F(x))}{\psi'(C^A(F(y), ..., F(y))}\right]^{-\frac{K-1}{K}}.$$

Proof. Taking the appropriate derivatives of the equation $\psi(C^A) = \sum_j \psi(u_j)$, the following can be established for Archimedean copulas for any $j \neq k$:

$$\frac{C^A_{kj}}{C^A_k} = \frac{\psi''(C^A)}{\psi'(C)} C^A_j,$$

with $C_j^A = \psi'(u_j)/\psi'(C)$. Substituting back in the definition of $\tilde{F}$ gives the result:

$$\begin{aligned}
\tilde{F}(x) &= \exp\left(-\int_x^y \sum_{j\neq k} \frac{C_{jk}^A(F(z),...,F(z))}{C_k^A(F(z),...,F(z))} F'(z)dz\right) \\
&= \exp\left(-\int_x^y (K-1)\frac{\psi''(C^A(.)}{[\psi'(C^A(.)]^2}\psi'(F(z))dz)\right) \\
&= \exp\left(-\frac{K-1}{K}\ln\psi'(C^A(F(z),...,F(z))|_x^y\right) \\
&= \left[\frac{\psi'(C^A(F(y),...,F(y))}{\psi'(C^A(F(x),...,F(x))}\right]^{\frac{K-1}{K}} \\
&= \left[\frac{\psi'(\psi^{-1}(K\psi(F(y))))}{\psi'(\psi^{-1}(K\psi(F(x))))}\right]^{\frac{K-1}{K}}.
\end{aligned}$$

The last equality follows from the definition of C^A under symmetry. ■

Example 2 (Gaussian Copula) *Let $K = 2$ and $C = C^G(\rho)$ be the Gaussian Copula with correlation $\rho \in [-1,1]$ and normal marginals $F_k(\ln x) = \Phi(\frac{\ln x - \mu}{\sigma})$ for $k \in \{1,2\}$. The wage offer policy in symmetric duopsony equilibrium is $w^G(y,\rho) = \int_0^y x d\tilde{F}^G(x|y;\rho)$ with*

$$\tilde{F}(x|y;\rho) = \left[\frac{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln x-\mu)}{\sigma}\right)}{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln y-\mu)}{\sigma}\right)}\right]^{\frac{1}{1-\rho}}.$$

Proof. ■

Theorem 4 *Let $G(y) = \ln\int_0^y \tilde{F}(x|y)dx$. Then for any $y > 0$, $1 - w(y)/y$ is increasing in y if and only if*

$$-\frac{G''(y)}{G'(y)}y > 1.$$

Proof. Applying integration by parts to the definition of equilibrium wage policy gives:

$$w(y) = \int_0^y x d\tilde{F}(x) = y - \int_0^y \tilde{F}(x)dx,$$

since $\tilde{F}(y) = 1$. Solving for markdown, we have:

$$1 - \frac{w(y)}{y} = \frac{\int_0^y \tilde{F}(x)dx}{y}$$

Differentiating w.r.t. y gives;

$$\frac{d}{dy}\left(1-\frac{w(y)}{y}\right)=\left[y\cdot\frac{d}{dy}\ln\int_0^y\tilde{F}(x)dx\right]^{-1}=[yG'(y)]^{-1}.$$

Monotonicity of markdowns is inversely related to the monotonicity of term inside the brackets. By differentiation:

$$\frac{dyG'(y)}{dy}=G'(y)\left[1+\frac{yG''(y)}{G'(y)}\right]<0 \qquad \Leftrightarrow \qquad -\frac{yG''(y)}{G'(y)}>1$$

■

Theorem 5 *Suppose $C(u,v)\in C^A(u,v)$ and $F_1(y)>F_2(y)$ for all $y>0$. Then, for all $w>0$: $F_1(y_1(w))>F_2(y_2(w))$.*

Proof. The proof is established by contradiction. Denoting $u(w)=F_1(y_1(w))$ and $v(w)=F_2(y_2(w))$, the ratio of the optimality conditions for the two firms gives the following:

$$\frac{\psi'(v)}{\psi'(u)}\frac{dv}{du}=\frac{y_2(w)-w}{y_1(w)-w},$$

This is a differential system with boundary values $v(0)=u(0)=0$ and $v(\bar{w})=u(\bar{w})=1$ at some $\bar{w}>0$ (Lebrun, 1999).

Suppose $u(w)\leq v(w)$ were true for some $w\in(0,\bar{w})$ in equilibrium. This implies $y_1(w)<y_2(w)$, because $F_1(y)>F_2(y)$ and $F_1'(y)>0$ by assumption. Furthermore, by concavity of ψ, and the fact that $\psi'<0$, we have $0>\psi'(v)\geq\psi'(u)$, implying $0<\psi'(v)/\psi'(u)\leq 1$.

The relations above together imply $\frac{dv}{du}=\frac{y_2(w)-w}{y_1(w)-w}\cdot\frac{\psi'(u)}{\psi'(v)}>1$ whenever $v(w)\geq u(w)$ from the equation above. But that reinforces $v>u$ for all wage levels above w, which contradicts $v=u=1$ at $\bar{w}$. Therefore, $u(w)>v(w)$ must hold in equilibrium for all $w\in(0,\bar{w})$. This implies employment at Firm 2 is larger than that at Firm 1 at any $w\in(0,\bar{w})$. Visually, this is implied by the equilibrium locus lying below the 45-degree line and the symmetry of Archimedean copulas.

■

Theorem 6 *If $C(u,v)\in C^A(u,v)$ and $\exists\tilde{y}>0$ such that $F_1(y)>F_2(y)$ for all $y>\tilde{y}$ and $F_1(y)<F_2(y)$ for all $y<\tilde{y}$, then there is a unique $w^*\geq 0$ such that $F_1(y_1(w))\geq F_2(y_2(w))$ if and only if $w\geq w^*$ in equilibrium.*

Proof. Define $\tilde{w}$ where $y_2(\tilde{w})>\tilde{y}$ in equilibrium. Following the same steps as in the proof or Proposition 5, it can be shown that $u(w)>v(w)$ for all $w\in(\tilde{w},\bar{w})$, because $F_1(y)>F_2(y)$ for all $y>\tilde{y}$. Therefore, if there is a w^* such that $u(w^*)=v(w^*)$, $w^*\leq\tilde{w}$ must hold.

It is possible that $w^*=0$ is the only crossing point with $u(w)>v(w)$ for all $w>0$. In that case, the equilibrium is similar to that described by Theorem 5. Next, we show that there is a single *interior* crossing point: that if there is a $w^*>0$, it must be the only such point. We will do so by showing that

for all $w \in [w^*, \tilde{w}]$, $dv/du < 1$. That is, the slope at the crossing point is less than one and for all wage levels above that, the differential system reinforces $v(w) < u(w)$ until $\tilde{w}$. Since we already established $u(w) > v(w)$ for all $w \in (\tilde{w}, \bar{w})$, w^* must be the only interior crossing point.

For any $w < \tilde{w}$, suppose $u(w) \geq v(w)$. Then $0 > \psi'(u) > \psi'(v)$ by strict convexity of ψ. Because $y_2(w) < \tilde{y}$ when $w < \tilde{w}$ by definition, $F_2(y) > F_1(y)$ when $y < \tilde{y}$ by assumption, and $u(w) \geq v(w)$ by supposition, $F_1(y_1(w)) > F_2(y_2(w)) > F_1(y_2(w))$ must hold for all $w < \tilde{w}$, which implies $y_1(w) > y_2(w)$. These inequalities imply:

$$\frac{dv}{du} = \underbrace{\frac{\psi'(u)}{\psi'(v)}}_{<1} \underbrace{\frac{y_2(w)-w}{y_1(w)-w}}_{<1} < 1.$$

The inequality above implies that $v(w)$ remains below $u(w)$ in equilibrium, for all wage levels $w \geq w^* > 0$. Equivalently, if the equilibrium equal-pay locus $(v(w), u(w))$ ever touches the 45-degree line, it remains strictly below that line and does not cross it again (that is, of course, until the terminal point: $v(\bar{w}) = u(\bar{w}) = 1$). ■

Theorem 7 *Consider a symmetric market with K firms and a legal minimum wage $\bar{w}$. For all $y \geq \bar{w}$, the wage offer policy in oligopsony equilibrium is given by:*

$$w_m(y; \bar{w}) = w(y) + \bar{w} - w(\bar{w}),$$

where $w(y)$ is the wage offer in the absence of minimum wage defined in Theorem 1.

Proof. Recall from equation (A4) that the differential equation that describes wage offers in symmetric oligopsony equilibrium is:

$$\Psi(a, w) = -we^{-\gamma(a)} C_k(F(a), ..., F(a)) + \int_0^a xe^{-\gamma(x)} \sum_{j \neq k} C_{kj}(F(x), ..., F(x)) F'(x) dx = d_0.$$

with the initial condition $w(\bar{w}) = \bar{w}$ when there is a minimum wage.[A1] Using that condition to solve the constant of integration, d_0, we have:

[A1] There is no bunching at $\bar{w}$ in equilibrium, because competitors can bid away infra-marginal workers, those with $y > \bar{w}$ and $w(y) = \bar{w}$, by offering marginally more than $\bar{w}$ if a firm offers

$$\begin{aligned}
\Psi(\bar{w},\bar{w}) = -\bar{w}e^{-\gamma(\bar{w})}C_k(F(\bar{w}),...,F(\bar{w})) + \int_0^{\bar{w}} xe^{-\gamma(x)}\sum_{j\neq k} C_{kj}(F(x),...,F(x))F'(x)dx &= d_0 \\
e^{-\gamma(\bar{w})}C_k(F(\bar{w}),...,F(\bar{w}))\left[-\bar{w} + \frac{\int_0^{\bar{w}} xe^{-\gamma(x)}\sum_{j\neq k} C_{kj}(F(x),...,F(x))F'(x)dx}{e^{-\gamma(\bar{w})}C_k(F(\bar{w}),...,F(\bar{w}))}\right] &= d_0 \\
e^{-\gamma(\bar{w})}C_k(F(\bar{w}),...,F(\bar{w}))\left[-\bar{w} + \mathbb{E}_{\tilde{F}}[x|x\leq\bar{w}]\right] &= d_0.
\end{aligned}$$

Solving for the wage function from equation (A4), now denoted $w(y,\bar{w})$ and substituting for d_0 from above gives the solution.

$$\begin{aligned}
w(y,\bar{w}) &= \mathbb{E}_{\tilde{F}}[x|x\leq\bar{w}] - \frac{d_0}{e^{-\gamma(\bar{w})}C_k(F(\bar{w}),...,F(\bar{w}))} \\
w(y,\bar{w}) &= \mathbb{E}_{\tilde{F}}[x|x\leq y] + \left[\bar{w} - \mathbb{E}_{\tilde{F}}[x|x\leq\bar{w}]\right] \\
w(y,\bar{w}) &= \bar{w} + \mathbb{E}_{\tilde{F}}[x|x\leq y] - \mathbb{E}_{\tilde{F}}[x|x\leq\bar{w}] \\
w(y,\bar{w}) &= \bar{w} + w(y) - w(\bar{w}),
\end{aligned}$$

where $w(y)$ is the equilibrium wage offer in the absence of minimum wage, i.e. when $\bar{w}=0$, from Theorem 1. ■

Theorem 8 *Consider a symmetric market with K formal firms, an informal sector and a legal minimum wage $\bar{w}$. The wage offer policy in oligopsony equilibrium solves the following differential equation.*

$$\frac{dw}{dy} = (y-w)\left[\frac{\sum_{j\neq k} C_{kj}(F(y),\cdots,F(y))F'(y)}{C_k(F(y),\cdots,F(y))} + \frac{F_I'(w)}{F_I(w)}\right], \tag{10}$$

with the initial condition $w(\bar{w})=\bar{w}$.

Proof. Recall from equation (4) that the optimality condition for firm k's wage offer is

$$(y-w)\frac{\partial n_k(w;y)}{\partial w} = n_k(w;y),$$

and from equation (9) that the labor supply to a firm is given by:

$$\begin{aligned}
[n_k(w;y) &= P(w_j(y_j)\leq w, \forall j\neq k|Y_k=y)\cdot P(w_0\leq w)\cdot f_k(y) \\
&= C_k(u_1(w),\cdots,F_k(y),\cdots,u_K(w))\cdot F_I(w)\cdot F_k'(y),
\end{aligned}$$

where $u_j(w)=F_j(y_j(w))$ and $y_j(w)$ areninverse wage offer functions for $j\neq k$. Differentiating with respect to w and dividing by labor supply gives:

$$\frac{\partial n_k(w;y)}{\partial w}\frac{1}{n_k(w;y)} = \frac{\sum_{j\neq k} C_{kj}(F(y),\cdots,F(y))F'(y)}{C_k(F(y),\cdots,F(y))}\frac{dy_j(w)}{dw} + \frac{F_I'(w)}{F_I(w)}$$

Setting $y_j(w) = y(w)$ by symmetry, substituting in the optimality condition and solving for dw/dy above gives:

$$\frac{dw}{dy} = (y-w)\left[\frac{\sum_{j\neq k} C_{kj}(F(y),\cdots,F(y))F'(y)}{C_k(F(y),\cdots,F(y))} + \frac{F_I'(w)}{F_I(w)}\right].$$

■

B Empirical Appendix

B.1 Concentration of Industries per Market

Table A1: Concentration of Industries per Market

percentile	number of industries	employer-based ENI	worker-based ENI
1st	210.6	2.1	1.0
25th	738.1	53.9	5.5
50th	898.9	68.6	7.2
75th	1003.3	72.6	9.5
99th	1053.8	112.2	21.7
Mean	847.3	61.8	8.0

Notes: The effective number of industries (ENI) is the hypothetical number of equal-sized industries that yields the same Herfindahl–Hirschman index as the data distribution of size across industries, where industry size is captured by the number of employers in Column 3 and as the number of workers in Column 4. Summary statistics across markets are weighted by employment.

B.2 Regression Robustness

Table A2: Robustness check for instrumenting with industry growth

IV: industry's growth, excluding industries with $>10\%$ internal hires

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}^s_q$)	markdown (%)
1st	0.04 (0.03)	1.44 (0.13)	35.7 (25.3)	2.7 (1.9)
2nd	0.09 (0.03)	0.89 (0.10)	10.3 (3.3)	8.8 (2.5)
3rd	0.14 (0.03)	0.83 (0.06)	5.8 (1.2)	14.6 (2.6)
4th	0.17 (0.03)	0.65 (0.05)	3.8 (0.7)	20.7 (3.0)

IV: industries exposure to business cycle due to cyclicality

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}^s_q$)	markdown (%)
1st	0.06 (0.08)	1.72 (0.34)	26.9 (33.4)	3.6 (4.3)
2nd	0.20 (0.10)	1.15 (0.30)	5.7 (3.1)	14.9 (6.9)
3rd	0.17 (0.09)	0.71 (0.14)	4.1 (2.2)	19.6 (8.6)
4th	0.19 (0.10)	0.47 (0.27)	2.4 (1.9)	29.1 (15.9)

Notes: Standard errors in parentheses, clustered at the employer-by-year level.

Table A3: Robustness check for sample constructions

Restricting to workers with wage $\geq 1.2\times$ minimum wage

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}^s_q$)	markdown (%)
1st	-0.01 (0.02)	1.73 (0.06)	-182.2 (384.9)	-0.6 (1.2)
2nd	0.07 (0.02)	0.95 (0.01)	13.7 (3.3)	6.8 (1.5)
3rd	0.13 (0.02)	0.63 (0.03)	4.8 (0.8)	17.1 (2.2)
4th	0.16 (0.02)	0.56 (0.03)	3.5 (0.5)	22.2 (2.4)

Residualizing wages by workers' characteristics

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}^s_q$)	markdown (%)
1st	0.09 (0.02)	1.34 (0.03)	15.3 (2.8)	6.1 (1.0)
2nd	0.14 (0.02)	1.03 (0.01)	7.7 (1.0)	11.5 (1.3)
3rd	0.16 (0.02)	0.88 (0.01)	5.4 (0.7)	15.6 (1.6)
4th	0.17 (0.02)	0.75 (0.02)	4.3 (0.6)	18.8 (2.0)

Notes: Standard errors in parentheses, clustered at the employer-by-year level.

Figure A1: Markdown Heterogeneity across Employers

