Not-So-Cleansing Recessions*

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Abstract

Recessions are periods in which the least productive firms in the economy exit, and as the economy recovers, they are replaced by new and more productive entrants. These *cleansing effects* imply that business cycles generate improvements in the average firm productivity. We argue that this is not sufficient to induce long-run gains in GDP and welfare. We show that these are driven by the intensity of *love-of-variety* in aggregate production. If industry output is aggregated with the standard CES aggregator, recessions do not bring about any improvement in GDP and welfare. If the economy features more *love-of-variety* than CES, the social planner finds it optimal to subsidize economic activity in recessions to avoid firm exit. We use the model and quasi-exogenous variation in demand to estimate love-of-variety. We find it to be significantly higher than implied by CES aggregation, suggesting that even the long-run effects of recessions are likely negative. Finally, we characterize the optimal policy response both along the transition and in the steady state.

Keywords: cleansing effects of recessions, business cycle, love-of-variety.
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This is really at the bottom of the recurrent troubles of capitalist society. They are but temporary. They are the means to reconstruct each time the economic system on a more efficient plan. But they inflict losses while they last, drive firms into the bankruptcy court, throw people out of employment, before the ground is clear and the way paved for new achievement of the kind which has created modern civilization and made the greatness of this country.

— Joseph A. Schumpeter *on the Great Depression, 1934*

1 Introduction

Recessions are often periods of increased reallocation of economic activity. The *liquidationist* view of business cycles has long posited that during downturns unproductive economic units exit, and, as the economy recovers, they are replaced by new and more productive firms. Stiglitz (1993) refers to this effect as the *"silver lining of economic recessions"*. The intellectual roots of this argument go back to Schumpeter (1934) who viewed recessions as a necessary short-term pain through which the economy becomes more efficient and improves its long-run outcomes.

In this paper, we revisit the role of recessions as moments of *cleansing* and creative destruction and, in particular, their long-run effects on GDP and welfare. While the costs of business cycles have been the subject of an extensive literature following Lucas (1987), we are interested in isolating their potential to improve steady-state outcomes. We consider a firm dynamics economy with heterogeneous firms à la Hopenhayn (1992)-Melitz (2003) and study the cleansing effect of business cycles driven by fluctuations in the fixed costs of production. In a partial equilibrium version of our model, we show that fixed cost recessions have cleansing effects through Schumpeterian forces: after a recession, fewer firms operate, and they are, on average, more productive than before the downturn. However, these gains need not translate into higher GDP and welfare. When products are differentiated, both households and firms might value the availability of varieties per se, independently of the elasticity of substitution. If this is the case, they are not willing to trade off higher average firm productivity for fewer available products. In fact, the cleansing effects of recessions can result in higher average productivity of surviving firms, yet lower GDP and welfare in steady state. In our model, temporary recession can permanently change the firm size distribution, generating history-dependence.

We start by showing that when industry output is generated through a CES aggregator, pre-cycle GDP, and long-run post-cycle GDP and welfare are identical despite the composition of the economy being different: the economy features fewer but more productive firms. Whenever the aggregator values varieties more than implied by CES, a fixed cost cycle induces long-run GDP losses, despite its cleansing effect. We show that, in economies characterized by the more general aggregators in Dixit and Stiglitz (1975) where *love-of-variety* (LoV) and the elasticity of substitution are governed by different parameters (see also Ethier, 1982; Benassy, 1996), the effect of recessions on long-run output can be decomposed into two terms: i) the effect on the output level of the counterfactual economy with CES aggregation; and ii) the effect

on the number of available varieties, weighed by how much higher or lower is love-of-variety than in the CES case. In partial equilibrium, the first effect is always zero, as the free entry equilibrium implies history-independent output levels under CES. However, since recessions trade off varieties for average productivity, an economy can be strictly worse or better off depending on the extent of love-of-variety. This element makes output in non-CES economies path-dependent since the composition of firms matters for welfare. This result is our point of departure to characterize the behavior of similar economies in general equilibrium, empirically estimate the long-run effects of recessions and characterize the socially optimal policy both along the cycle and in steady state.

First, we show that this intuition extends to general equilibrium with a minor caveat: in GE, the cleansing effect frees up labor resources. After a recession the economy features fewer, more productive firms. As a consequence, less of the finite labor endowment is used to pay the fixed costs of production. This implies that fixed cost cycles induce higher long-run welfare even in CES economies. Nonetheless, we show that for any fixed cost cycle there exists a unique level of LoV, such that long-run GDP and welfare are identical to their pre-cycle levels.

We conclude our positive analysis by studying the role of recessions depth in Section 3.2. We start by showing that, ranking economies by their LoV, it is possible to partition them based on how cleansing recessions are. There are high LoV economies for which recessions are always welfare-reducing, independently of their magnitude. Similarly, sufficiently low LoV economies always show cleansing effects. However, in economies with intermediate levels of love of variety, some recessions are welfare-improving, while others reduce long-run output and welfare. The intuition behind this result is that while the variety effect on welfare is constant in logs, the cleansing forces exhibit diminishing returns. As a consequence, while some recessions might increase welfare, others might reduce it. We conclude that for these economies, it is not possible to generally say whether recessions improve long-run outcomes or not, and such a statement has to be empirically evaluated recession by recession.

In Section 4, we set out to estimate the key parameters of the model, separately governing the elasticity of substitution in production σ and love-of-variety q. These parameters are the fundamental forces to determine whether recessions can generate long-run gains. In our model, σ uniquely determines the profit rate of the economy. We use the WIOD data (Timmer et al., 2015) to obtain the distribution of profit rates for the 56 industries in the sample across 43 countries.[1] We find a median elasticity of substitution of 5.3, associated with a profit rate of 18.7%. To estimate q, we make use of another structural relation in our model: love-of-variety governs the extent of external returns in our economy. We show that, in our setting, the elasticity of output to an increase in downstream expenditure is equal to $1+q$, where the "1" obtains from our constant technological returns to scale assumption. We use the quasi-exogenous shifts in foreign income from the instrumental variable in Ferrari (2024). This instrument combines shifts in destination markets for WIOD industries via their network exposure shares. We obtain

[1] WIOD reports capital expenses together with economic profits, we, therefore, make the conservative assumption that all capital expenses are, in fact, profits, leading to an upward bias of the profit rate and a downward bias in σ.

an estimate of love-for-variety q between .51 and .63, depending on the specification. Comparing these estimates with the love-of-variety in CES implied by our distribution of σ, we conclude that all but 4 out of the 56 industries in our sample have a stronger love-of-variety than implied by CES. The direct consequence of our empirical finding is that, for most of these industries, recessions are costly even in the long run.

Given our empirical findings, we conclude in Section 5 by studying the normative implications of our model both analytically and quantitatively. First, we extend results in Dixit and Stiglitz (1977) and Dhingra and Morrow (2019) on the constrained efficiency of CES economies with monopolistically competitive firms and free entry. We show that such results extend to economies that also consider the presence of incumbent firms. We then show that when the economy values varieties more or less than implied by CES, the market equilibrium features either too few or too many firms. The planner can correct this steady-state inefficiency with a combination of entry and fixed costs subsidies.

Importantly, we show that in economies in which output is aggregated with stronger love-for-variety than in the CES benchmark, under mild assumptions on the productivity distribution, the social planner finds it optimal to increase fixed costs subsidies during recessions. This policy is motivated by inducing a smaller amount of exit during bad times than implied by the market equilibrium. This result speaks to frequent efforts by governments to reduce churn during bad times, as seen during the Covid-19 recession.[2] Using the empirical estimates of Section 4, we fully characterize the optimal policy along the entire transition, in a quantitative version of our model. [*TO BE COMPLETED*]

Finally, we consider a number of extensions of our framework. First, we show that business cycles induced by movements in aggregate TFP do not carry any long-run effects. In fact, aggregate TFP movements do not induce any entry or exit since they do not change relative productivities. As a consequence, the economy reverts to the initial steady state as soon as the temporary shocks subside. Next, we consider a setting where firms produce multiple products. In such an economy, when fixed costs increase, some firms exit, while the ones that stay expand their product portfolios. Since varieties made by the same firms are more substitutable than those made by different firms, our main insights hold true. Finally, we study a setting where firms are subject to idiosyncratic shocks to their productivity, as in Hopenhayn (1992). We show that if shocks are bounded, then our steady-state results remain unchanged; If instead shocks are not bounded or firm exit is purely random, recessions have persistent effects in the form of "echoes" (Caballero and Hammour, 1994) during the transition after a cycle.

Related Literature This paper draws insights from the literature on the optimal number of varieties (Dixit and Stiglitz, 1977; Dhingra and Morrow, 2019)[3] to discuss the cleansing effects of recessions. We build on Hopenhayn (1992), Grossman and Helpman (1993), and Melitz (2003) to study the effect of recessions in economies with love-of-variety and firm dynamics.

The notion that recessions can generate long-run benefits thanks to an acceleration of cre-

[2] See Kozeniauskas et al. (2022) for an empirical evaluation of such programs.
[3] See Brakman and Heijdra (2001) for a comprehensive review.

ative destruction dates back to Schumpeter (1934, 1939) and has been formalized in Caballero and Hammour (1994). They study an embedded capital model in which recessions induce firm exit, thereby accelerating the speed of modernization of installed capital. We study an economy in which there is no capital, but producers are heterogeneous. During recessions, the least productive firms exit and are subsequently replaced by entrants with higher average productivity. Importantly, we show that this is not enough to infer the behavior of output or welfare, as the equilibrium number of firms drops after a recession. The patterns predicted by our model are consistent with the large empirical evidence on the procyclical properties of entry and exit, both at the firm and product level (Dunne et al., 1989; Davis and Haltiwanger, 1992; Broda and Weinstein, 2010; Argente et al., 2018; Tian, 2018; Argente et al., 2024).

Theoretically, our work related to the literature on the aggregate consequences of entry, exit and firm heterogeneity, such as Chatterjee and Cooper (1993); Clementi and Palazzo (2016); Carvalho and Grassi (2019); Gouin-Bonenfant (2022); Ferrari and Queirós (2024), and the literature on external effects in macroeconomics following Baxter and King (1991). Related to our conclusion, Hamano and Zanetti (2017, 2022) highlight the existence, in a CES economy, of the welfare tradeoff between higher average productivity and fewer available varieties. We show that for a CES economy, the two effects perfectly offset each other in PE. In GE, the only welfare effect is the labor savings and the income windfall that this generates. The tradeoff between better firm selection and loss of variety becomes welfare-relevant only away from CES. In particular, when love-of-variety is larger than implied by the CES benchmark. Barlevy (2002) and Ouyang (2009) suggest that the cleansing effects may be lower than expected. Barlevy (2002) argues that job match quality is procyclical and that this effect quantitatively dominates the cleansing role of recessions. Ouyang (2009) considers a setting in which recessions may halt the entry of very productive firms and, thereby, reducing long-run growth. Our point is related but distinct: even when measured productivity improves during recessions, this may induce a loss of varieties that manifests in lower GDP and welfare.

Finally, our paper is closely related to Baqaee et al. (2023) and Ardelean (2006), who provide estimates for love-of-variety in production and in demand, respectively. Baqaee et al. (2023) find an elasticity of -.3% for marginal costs of firms adding suppliers. Ardelean (2006), using import data, finds a love-of-variety 42% lower than the one implied by CES. Our estimates based on foreign demand shocks suggest that the aggregate love-of-variety effect is approximately .5-.6, and significantly larger than implied by a CES aggregator.

2 An Industry Model of Entry and Exit

We begin by laying out the structure of production and consumption in the economy and then proceed to study the effects of business cycles.

2.1 Setup

Production We consider an industry in monopolistic competition, in which firms hire labor to produce their variety with constant returns to scale in labor l and heterogeneous productivity z: $y(z) = zl$. Productivity is distributed according to some density function m. To produce, firms must pay a fixed cost of f^c labor units. In this partial equilibrium context, we consider a small industry in isolation. Labor can be hired at a market-clearing wage w, which is not affected by the industry. We set the wage as the numeraire, $w = 1$. Firms maximize their profits $\pi(z) = y(z)(p(y) - 1/z) - f^c$ by choosing their price.

Industry Output A perfectly competitive intermediary combines varieties and sells the final composite good to households. The intermediary aggregates with a generalized CES production function à la Dixit and Stiglitz (1975), Ethier (1982), Benassy (1996)

$$Y = M^{q-\frac{1}{\sigma-1}} \left[\int y(z)^{\frac{\sigma-1}{\sigma}} m(z)dz\right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

We follow Hopenhayn (1992) and define $m(z)$ as a density function. The total number of varieties M is given by $\int m(z)\, \mathrm{d}z$. We stress that in the paper we use $m(z)$ when referring to density functions, and $\mu(z)$ when referring to probability density functions that integrate to 1, i.e. $\mu(z) := m(z)/M$.

Two parameters govern the aggregator: the elasticity of substitution, σ, which measures product similarity, and love-of-variety (LoV). Following Benassy (1996), we define LoV as the "gain derived from spreading a certain amount of production between M differentiated products instead of concentrating it on a single variety". We highlight in Remark 1 that in our aggregator, LoV is exactly equal to q. LoV measures the additional productivity the intermediary derives from having a larger number of available varieties. This effect constitutes an aggregate production externality.[4] The classic Dixit and Stiglitz (1977) CES-aggregator is a special case where $q = q^{CES} := 1/(\sigma - 1)$. Hence, $q > 1/(\sigma - 1)$ implies stronger LoV than in the CES case and vice versa.[5]

Remark 1. *The aggregator in eq.(1) exhibits love-of-variety equal to q.*

Demand There is a representative household with utility $\mathcal{U}(Y)$, $\mathcal{U}' > 0$, and exogenous total income $\mathcal{I}$. We relax exogeneity when we consider a general equilibrium economy in Section 3. Households spend all their income on the composite good.

[4] Our setting can be interpreted as having an intermediary downstream firm, with production function Y, purchasing its inputs from the upstream variety producers. Therefore, LoV in our model closely aligns with the LoV estimated in Baqaee et al. (2023). In an equivalent reading without intermediaries, the aggregator sits in the household utility function. The extra utility from having a larger number of available varieties is then an aggregate demand externality as in Blanchard and Kiyotaki (1987).

[5] Readers more familiar with the Melitz (2003) model aggregation can think of our aggregator as being $Y = M^{q+1}\left[\int y(z)^{\frac{\sigma-1}{\sigma}}\mu(z)dz\right]^{\frac{\sigma}{\sigma-1}}$, with $\mu(z) = m(z)/M$ being a probability density function.

Entry and Exit Entry of ex-ante homogeneous firms grows the mass of available varieties. Entrants pay a fixed cost of entry f^e, also in terms of labor, to enter and draw a productivity level z from a baseline probability distribution μ^E.[6] Potential entrants make entry decisions based on their expected post-entry profits, and firms cannot borrow against future profits. As a consequence, firms with negative current profits exit the economy, even if the present discounted value of the firm is positive.

2.2 Equilibrium

The intermediary's problem implies a demand schedule for each variety, which is taken into account by the corresponding monopolist. The monopolistic competition solution is independent of the aggregate production externality and implies the usual constant-markup pricing $p(z) = \frac{\sigma}{\sigma-1}\frac{1}{z}$. At the optimum, firm profits, π, depend on the firm's productivity z and on competition, m. Notably, m only matters through an aggregate statistic: the index $\int z^{\sigma-1}m(z)\,\mathrm{d}z$, which we call *market intensity*. Profits are given by

$$\pi(z,m) = \frac{\mathcal{I}}{\sigma}\frac{z^{\sigma-1}}{\int z^{\sigma-1}m(z)\,\mathrm{d}z} - f^c. \tag{2}$$

To characterize entry and exit, suppose that, at the onset, there are firms in the economy whose fixed cost of entry has been paid in the past and is thus sunk. These firms, which we call incumbents, can be arranged according to their productivity, giving rise to the incumbent distribution $z \sim m^I$. New firms enter if they expect a profit net of the fixed cost of entry, i.e., if $\mathbb{E}_{\mu^E}[\max\{\pi(z,m),0\}] - f^e > 0$, where the expectation is taken over the baseline probability distribution of productivity, μ^E. Firms understand that their entry decision is simultaneous to that of other potential entrants. As a consequence, they do not just consider the competition given by current incumbents but rather what the market will look like once all potential entrants have entered. We call this ex-post distribution m. If a firm chooses to pay the entry cost and draw a productivity level, it may still find it unprofitable to produce due to the fixed production cost f^c. This fixed cost implies the existence of a threshold productivity $\underline{z}$ such that $\pi(\underline{z},m) = 0$. Entrants and incumbents alike produce if their productivity z lies above $\underline{z}$ and leave otherwise.

Denote the mass of entrants drawing from μ^E by $E \geq 0$. Then, after entry, the distribution of firms, m, is given by the sum of surviving entrants and incumbents. It is spelled out in eq. (5).[7] Equilibrium is reached if no additional entrepreneurs wish to enter (eq. 3) and no additional incumbents or recent entrants wish to quit (eq. 4); the two equilibrium objects—

[6] We subsume any discounting into f^e. Hence, this can be understood as the $f^e = (1-\beta)\widetilde{f}^e$ so that a firm discounting time at rate $\beta \in (0,1)$ compares the present discounted value of profits to the entry cost $\widetilde{f}^e$.

[7] This is the law of motion of $m(z)$, which is our key equilibrium object controlling both the mass of available varieties and the probability distribution of productivities in the industry at any time. The law of motion has been defined recursively: the post-entry distribution $m(z)$ characterizes the new incumbents in the market. An equivalent formulation of the law of motion is: $m_\tau(z) = (m_{\tau-1}(z) + E_\tau \mu^E(z))\mathbb{I}_{\{z \geq \underline{z}_\tau\}}$ $(\forall z \geq 0)$.

cutoff, $\underline{z}$ and mass of entrants, E—adjust to jointly satisfy eqs. (3) and (4):

$$f^e \geq \mathbb{E}_{\mu^E}[\max\{\pi(z,m),0\}], \tag{3}$$

$$0 = \pi(\underline{z},m), \tag{4}$$

$$m(z) = (m^I(z) + \underbrace{E\mu^E(z)\mathbb{I}_{\{z\geq\underline{z}\}}}_{m^E(z)}) \ (\forall z \geq 0). \tag{5}$$

If there is scope for entry, then $E > 0$ and eq. (3) holds with equality. Combining (2), (4) and (3), we obtain the following equilibrium conditions:

$$E = \frac{\mathcal{I}}{\sigma f^c}\frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}} z^{\sigma-1}\mu^E(z)\,\mathrm{d}z} - \frac{\int_{\underline{z}} z^{\sigma-1}m^I(z)\,\mathrm{d}z}{\int_{\underline{z}} z^{\sigma-1}\mu^E(z)\,\mathrm{d}z}, \tag{6}$$

$$\frac{f^e}{f^c} = \int_{\underline{z}}^{\infty}\left[\left(\frac{z}{\underline{z}}\right)^{\sigma-1} - 1\right]\mu^E(z)dz. \tag{7}$$

Notably, the equilibrium conditions do not depend on q. LoV only affects the production of the downstream intermediary firm. This change in production does not translate into higher demand for the upstream firms, because total expenditure on the composite good is fixed to $\mathcal{I}$. The price of the final good adjusts to counterbalance the change in its quantity such that the revenue of the intermediary remains fixed at $\mathcal{I}$.

Examining eq. (7), we note the following result.

Lemma 1 (Cutoff determination)**.** *Whenever there is entry into the economy ($E > 0$), the productivity cutoff $\underline{z}$ is independent of the incumbent distribution m^I.*

To build intuition for Lemma 1, we discuss the sufficiency of market intensity. Note that the labor share $\frac{\sigma-1}{\sigma}$ of total income spent on the final product, $\mathcal{I}$, constitutes expenditures for production labor (as opposed to fixed cost payments). Therefore, the total number of hours spent on production is $\mathcal{I}\frac{\sigma-1}{\sigma}$, where w is normalized to 1. There are two ways in which firms decrease labor employed by other firms. First, new entrants draw labor from existing firms. This leads to decreasing expected profits, hence decreasing marginal returns to entry, for the potential entrant and to decreasing returns to production for any given existing firm. Second, an increase in the productivity of competing firms decreases the expected profits of a potential entrant or any given incumbent, because more productive firms take up more labor. We learn that, for any given z, profits are affected at essentially two aggregate margins: number of competitors and productivity of competitors. This also implies that we can manipulate one of the margins (e.g., increase the number of firms by 1%) and adjust the other (e.g., decrease productivity of all firms by x_z%) to keep $\pi(z,m)$ unchanged. Essentially, in CES monopolistic competition, every firm's equally differentiated product is competing with the *same* aggregate of all other products, which suggests that the change in this product at one margin (the 1% increase) is undone by the *same* change at the other, such that $x_z\% = x\%$ for all z. These insights suggest the existence of an aggregate statistic $s(m)$, which allows us to write $\pi(z,m) = \widetilde{\pi}(z,s(m))$;

this statistic is precisely given by market intensity. Given that firm profits only depend on the competitive environment through the level of market intensity, $\int z^{\sigma-1}m(z)dz$, the exact composition of m is not payoff-relevant for firm production or entry choice. As a consequence, whether the competitors are other current entrants or incumbents is also irrelevant to firm decisions, and there is only one level of *market intensity* that solves eq. (4), which directly implies the result of the Lemma.[8] In contrast, if $E = 0$, then the cutoff relies heavily on the distribution of incumbents because if there is exit, then the marginal exiting firm is an incumbent. Equation (3) is slack and eq. (4) rearranges to $0 = \pi(\underline{z}, m^I)$.

We conclude the equilibrium characterization by noting that our results hold more generally than we have stated so far. We have defined the entry process as an instantaneous equilibrium in which all market participants anticipate the firm distribution after entry, m. In an alternative equilibrium characterization without anticipation of m, firms iteratively enter the market, only considering the ex-ante distribution of firms in each iteration. Given a distribution of incumbents, initially, a small mass of firms enters the market, and the low productivity subset folds immediately. This play repeats until no additional firm wishes to enter and no active firm wishes to quit. Remark 2 states that the two entry processes are equivalent.

Remark 2 (Entry game equivalence)**.** *The instantaneous equilibrium has a unique solution* $(E, \underline{z}, m)$*, which coincides with the limit point of the iterative entry game.*

This completes the characterization of the partial equilibrium of this economy. Next, we study the effect of business cycles.

2.3 Business Cycles

We define business cycles as one-time, unexpected shocks to the fixed costs of production, f^c. Such cycles could, for example, be driven by varying financing conditions in working capital constraints.[9]

Since we are interested in isolating the long-run consequences of temporary fluctuations, we focus on steady states. Following Lucas (1987), we want to answer the following question: how much would consumers pay to move to a world that has gone through a recession in the past? Formally, let the economy run through three phases, capturing the dynamics of crises. In phase $\tau = 1$, fixed costs equal f_l^c, then unexpectedly increase to $f_h^c > f_l^c$ in $\tau = 2$, and subsequently revert to f_l^c in $\tau = 3$; subscripts l and h refer to low and high, respectively.[10] In phase 1, there is scope for entry, and the industry reaches the equilibrium determined by eqs. (6) and (7). In phase 2, the rise of fixed costs leaves no scope for entry and forces some

[8] This result relies heavily on the monopolistic competition assumption as all firms have infinitesimal size. In an oligopolistic context, it would be payoff-relevant for each individual firm whether, fixing a level of aggregate productivity, the competitive environment is composed of few productive or many less productive firms. See Ferrari and Queirós (2024) for the effect of firm heterogeneity on aggregate outcomes in oligopoly models.

[9] An alternative interpretation of these shocks is that the fixed costs are produced in a different sector c from labour and that downturns are triggered by reductions in the TFP of sector c.

[10] Note that considering the phase 3 reversion to f_l^c is equivalent to studying the limit of a slow-moving process of mean-reversion of the fixed cost to the long-run mean of f_l^c.

of the incumbents—the firms that entered in phase 1, so, $m_2^I \equiv E_1 \mu^E(z)\, \mathbb{I}_{\{z \geq \underline{z}_1\}}$—to exit. The productivity cutoff, determined by the zero-profit condition alone (eq. 4), increases to $\underline{z}_2 > \underline{z}_1$, generating the new distribution $m_2 = m_2^I(z) \mathbb{I}_{\{z \geq \underline{z}_2\}}$. We show this process in panels A and B of Figure 1. In phase 3, the fixed costs revert to f_l^c. Survivors of the crisis are now incumbents, so $m_3^I \equiv m_2$. Additionally, new firms enter. Equations (6) and (7) hold, and $E_3 > 0$. The post-crisis distribution of firms m_3 is given by eq. (5), with $m^I = m_3^I$ and $E = E_3$. Panel C of Figure 1 displays the post-recession distribution m_3. Note that the productivity of the least productive active firm is the same before and after the cycle, since by Lemma 1, the cutoff $\underline{z}_3$ equals the pre-crisis cutoff $\underline{z}_1$. Yet, the distribution of firms differs and $m_1 \neq m_3$ due to the presence of incumbents.

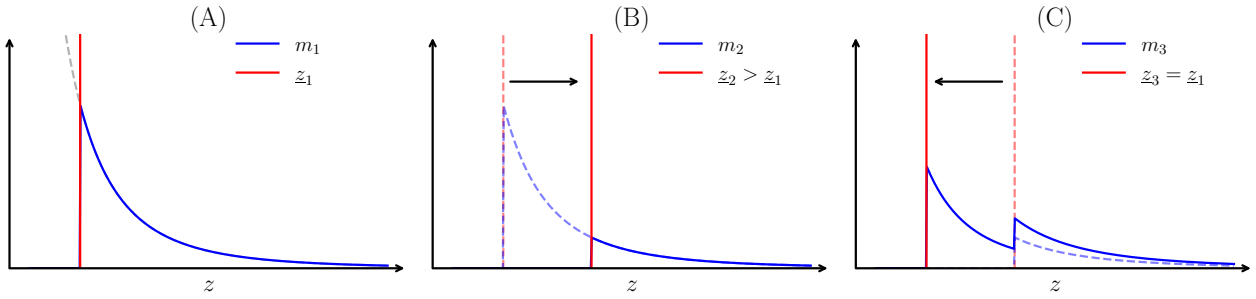


Figure 1: The figure shows the entry and exit dynamics over the business cycle. Panel (A) shows the distribution m_1 before the shock hits. Upon impact, the left tail of firms with productivity less than $\underline{z}_2$ leave, creating distribution m_2 (B). Finally, after fixed costs return to pre-shock levels and new firms drawn from the baseline distribution (μ^E) enter, m_3 becomes the distribution of productivities in the market (C). The dashed light-blue line refers to $m_{\tau-1}$.

By substituting equilibrium production of each firm into the aggregator in eq. (1), we can write aggregate output (GDP) in equilibrium as

$$Y_\tau = M_\tau^{q-\frac{1}{\sigma-1}} L_\tau^d \left(\int z^{\sigma-1} m_\tau(z)\, \mathrm{d}z \right)^{\frac{1}{\sigma-1}}. \tag{8}$$

Here, L_τ^d is the amount of labor demanded for production, and $\left(\int z^{\sigma-1} m_\tau(z)\, \mathrm{d}z \right)^{\frac{1}{\sigma-1}}$ is aggregate TFP. Proposition 1 characterizes how business cycles affect aggregate output relative to pre-cycle values.

Proposition 1 (Cleansing Effects of Cycles)**.** *First, the aggregate demand for production labor by the industry, L^d, is independent of m, f^c and f^e, and thus does not change through the cycle. Second, the ratio of post-crisis output to pre-crisis output is given by*

$$\frac{Y_3}{Y_1} = \left[\frac{M_3}{M_1} \right]^{q-\frac{1}{\sigma-1}} \left[\frac{\int z^{\sigma-1} m_3(z)\, \mathrm{d}z}{\int z^{\sigma-1} m_1(z)\, \mathrm{d}z} \right]^{\frac{1}{\sigma-1}}, \tag{9}$$

where

$$\frac{M_3}{M_1} < 1 \text{ and } \frac{\int z^{\sigma-1} m_3(z)\, \mathrm{d}z}{\int z^{\sigma-1} m_1(z)\, \mathrm{d}z} = 1.$$

Furthermore,

$$\frac{Y_3}{Y_1} \gtreqless 1 \Leftrightarrow q \lesseqgtr q^{CES}. \tag{10}$$

Proposition 1 is our first main result: whether GDP and welfare are larger than before the recession depends uniquely on the LoV parameter q. It comes with several additional important observations. First, our result refers to long-run effects: any potential welfare loss is not driven by the transition in phase 2. When $q < q^{CES}$, the economy might still be worse off once transition costs are accounted for. What we show is that if $q > q^{CES}$, long-run welfare is lower even without the transition costs. Second, all the classic Schumpeterian forces of recessions are present: new entrants are, on average, more productive than exiters during the cycle. Recessions induce a *selection effect*. Nonetheless, they may still result in long-run welfare losses. Third, we remark that the CES case is a knife-edge parametric restriction:

Remark 3 (CES Aggregation)**.** *The special case of CES aggregation* $\left(q = q^{CES}\right)$ *implies that* $Y_1 = Y_3$*: long-run output is unchanged pre- and post-recession.*

To build intuition for the above proposition and remark, it helps to look at the production of an individual firm with productivity z, which is given by

$$y_\tau(z) = z \cdot \underbrace{\mathcal{I}\frac{\sigma-1}{\sigma}\frac{z^{\sigma-1}}{\int z^{\sigma-1} m_\tau(z)\, \mathrm{d}z}}_{:= l^d(z),\ \text{production labor demand}}. \tag{11}$$

We recall that firms produce as if they were living in a CES economy, which implies that any deviation of Y_τ from its CES counterpart is driven by the aggregation, not by individual production decisions. Therefore, we first discuss the mechanics of the business cycle on y_τ for a CES economy and then explain deviations from the CES aggregate as driven by the externality alone. Consider a firm with productivity z large enough to survive throughout the cycle. In phase 2, the presence of fewer competitors in the market lowers market intensity; hence, the firm expands its output: $y_2(z) > y_1(z)$. In phase 3, entrants start producing: facing an initially low competition, they drive up market intensity until the free entry condition holds. Since all costs have reverted back to pre-crisis levels, the only way to make potential entrants indifferent between entering and staying out is by adjusting aggregate productivity to pre-crisis levels.[11]

In a CES economy, output depends uniquely on aggregate TFP and aggregate demand for production labor, L^d_τ. The latter, however, does not depend on the fixed cost cycle but exclusively on market size $\mathcal{I}$ and σ, as already argued in Section 2.2. Since aggregate TFP and

[11] Note that, in our model, there is no strategic advantage from being an incumbent. In a different strategic setting, e.g., if incumbents were Stackelberg leaders and entrants followers, this may no longer be true.

aggregate labor used in production are unchanged, so is output and, therefore, welfare.

This effect is best understood considering eq. (8). Taking logs and differencing pre- and post-crisis values yields

$$\Delta \log Y = q\Delta \log M + \Delta \log L^d + \Delta \log \bar{z}, \tag{12}$$

where we call $\bar{z}_\tau := \left(\int z^{\sigma-1}\mu_\tau(z)\,\mathrm{d}z\right)^{\frac{1}{\sigma-1}}$ *average productivity*. The first term, $q\Delta \log M$, is the *variety effect*, the second term, $\Delta \log L^d$, the *labor cleansing effect*, and the third term the *selection effect* of the crisis. Given that production labor is constant throughout the cycle, there is no labor cleansing effect in PE. The variety effect is driven by changes in M and amplified through LoV, q. The selection effect is channeled through changes in average productivity as exiting firms, on average, are less productive than entrants. In PE, the variety effect dominates the selection effect for $q > q^{CES}$, while for CES economies, the two cancel out exactly.

As a final discussion point, we consider output during the recession. If $q > q^{CES}$, it is immediate that Y_2 drops relative to Y_1 because both the number of varieties and market intensity in eq. (1) decrease. However, in the limit case of $q = 0$, we can write $Y_\tau = L^d \cdot \bar{z}_\tau$. In this extreme case, the intermediary is indifferent to the number of varieties and only cares about average productivity among active firms, $\bar{z}_\tau$, which *increases during the crisis*. The selection effect is positive, while the variety effect is zero. What may be perceived as a crisis within the industry turns out to be beneficial for aggregate output.[12]

Proposition 1 highlights another important aspect of our model, formalized in Remark 4.

Remark 4 (Path Dependence)**.** *The stationary steady-state equilibrium is path-dependent.*

We considered economies that, in phases 1 and 3, feature identical parameters. Nonetheless, they are characterized by different equilibrium allocations. This property is fully driven by the presence of incumbents. Our business cycles involve changes in the fixed costs of production, and the productivity cutoff is a direct function of these costs. Starting from a steady state with a certain level of fixed cost, f^c, and distribution of active firms, m, the distribution of entrants $m^E(z) := E\mu^E(z)\mathbb{I}_{\{z\geq \underline{z}\}}$ in m is truncated at a new cutoff during the transition phase, i.e., phase 2. This alone already implies that the composition of firms in the new steady state, where fixed costs have reverted to f^c, cannot possibly be the same, as the new and old entrant's distributions have different cutoffs. Additionally, the mass of entrants (for $E > 0$), and the cutoff (for $E = 0$) are both functions of the incumbents' distribution, creating another source of path dependency.

2.4 Discussion

We conclude this section with a brief discussion of the most salient model primitives and their interpretations. We provide a formal treatment of all these extensions in Appendix A.

[12]Such a scenario is even more likely in general equilibrium when the exit of low-productivity firms relaxes the resource constraint. For the rest of the paper, we abstract from such extremes and use the terms crisis and recession interchangeably.

Entry and Exit Selection In our model, the path dependence of steady states is driven by an asymmetry embedded in the process of entry and exit. Firms enter under the veil of ignorance and draw their productivity. Selection occurs upon the draw, when firms with productivity $z < \underline{z}$ exit. As a consequence, as the economy is recovering from a recession, firms populate the entire support of the distribution above $\underline{z}$. Exit instead occurs in a strongly selected way. At the onset of a recession, firms leave the economy from the bottom of the distribution ($z < \underline{z}_2$). In this sense, recessions generate *selected* exit followed by *unselected* entry during recoveries. This property follows from our assumption that firms do not know their productivity upon paying the entry cost.

This modeling assumption provides us with a sharp and analytically tractable result which qualitatively carries through in less extreme versions of our model. Consider, for example, a setting in which potential entrants have a belief about their productivity which is positively correlated with their true type yet noisy. Provided that some noise is present, our results carry through qualitatively. These results would also arise in a context in which potential entrants know their productivity exactly but face uncertainty about the entry cost.

Alternative Exit Conditions We have modeled exit as a forced event, triggered by $\widetilde{\pi}(z, m) < f^c$, where $\widetilde{\pi}$ are profits before fixed cost. This model primitive may be interpreted as firm default due to liquidity constraints in a world with prohibitively costly borrowing, but this is not its exclusive interpretation. Alternatively, the firm may compare the NPV of fixed cost $f^c/(1-\beta)$ to the NPV of profits $\widetilde{\pi}/(1-\beta)$. Under this reading, firm exit is not driven by a constraint, but by a rentability consideration. Equivalence to the first interpretation obtains because firm owners do not expect parameter changes (shocks are 'MIT shocks'). In similar vein, f^c may be construed as a measure of opportunity cost faced by the firm owner. In Appendix A, we relax the assumption that firm owners do not expect any future changes in fixed cost, allowing them to forecast the reversion of fixed cost to pre-cycle levels. This extension is consistent with our propositions, and does not alter results beyond a change in notation.

Unit of Fixed Cost Payments and Shocks to Productivity Our primitive that fixed costs are paid in labor units can be easily disposed of. Fixed cost may also be paid in terms of any other good whose price is determined outside the industry. More generally, the model can be restated in terms of many inputs instead of just labor. Under constant returns to scale (CRS) at the firm level, one can swap labor units and the wage for the optimal input bundle and corresponding price. None of our results would be affected by this interpretation.

If fixed costs are paid in units other than labor, a natural link between fixed cost cycles and TFP shocks arises. Consider two industries, and suppose firms of industry A must pay their fixed cost in terms of industry B's output. A negative TFP shock to industry B increases the price of B's goods, raising the fixed cost of firms in industry A and causing extensive margin adjustments in A. TFP shocks may also travel via the consumer under standard assumptions on preferences. If households consume goods from A and B, and both goods are normal goods, an increase in the price of B reduces total expenditure on goods in A, $\mathcal{I}^A$. From eq. (2), we

note that a decline in income has equivalent effects on firm exit to a surge in fixed cost.

Changing the unit of fixed cost does not alter the model, except in the special case where fixed costs are to be paid in units of the final good. Under this reading, in which the upstream firms acquire some output of the downstream intermediary, a few details change. Take the empirically relevant case in which $\sigma \geq 2$ (see Section 4.5). Like above, $q = q^{CES}$ is the knife edge parametrization at which pre- and post-cycle output are the same (see Proposition A.1). However, different to our baseline model q^{CES} also is the value at which pre- and post cycle output are equal in general equilibrium (see Section 3), and the decentralized economy is no longer constrained efficient (Barro and Sala-i Martin (2004)).

Another reason to focus on fixed cost cycles is, besides expositional convenience, that productivity shocks cause dynamics at the extensive margin only if they are idiosyncratic. Proposition A.2 states that with constant downstream expenditure, a proportional shock to each firm's productivity causes output to drop, but has no other long-run effects. The intuitive reason is that given fixed $\mathcal{I}$, profits are unchanged on the aggregate. Because market shares are unaffected, profits also do not change at the micro level, and no entry/exit dynamics are set off. This changes in the presence of idiosyncratic shocks. A firm receiving a large enough adverse idiosyncratic shock will die. In the extreme case of random death shocks, all effects described in this paper vanish in the very long run and would be visible through what Caballero and Hammour (1996) refer to as "echo" effects. For a more precise characterization, see Proposition A.3 and the surrounding discussion.

Varieties and the Number of Firms In our model, varieties and firms are, by assumption, the same thing. However, our results readily generalize to economies in which entry costs and the elasticity of substitution are symmetric across products. In an asymmetric environment, in which varieties produced by the same firm are more substitutable than varieties across firms, we can distinguish between LoV for products within and across firms. We analyse this extension and how our main results change in Proposition A.4. Our main conclusions continue to hold.

3 General Equilibrium

In GE, we consider the whole economy, and not one small industry in isolation. This new setting entails two fundamental changes. Firstly, labor demanded by firms now affects the market-clearing wage. Labor market clearing dictates:

$$L^d + Mf^c + Ef^e = \bar{L}. \tag{13}$$

Secondly, revenues generated by operating firms, $R = PY$, constitute the entirety of household's income $\mathcal{I} = \bar{L} + \Pi$. Therefore, firm profits become:

$$\pi(z, m) = \frac{R}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} m(z)\, \mathrm{d}z} - f^c. \tag{14}$$

Entry and exit are still pinned down by the *free-entry condition* and *zero-profit condition* (eqs. 3 and 4). When entry is strictly positive, eqs. (3) and (4) can be rearranged into a GE analog of eq. (6)

$$E = \frac{R}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} - \frac{\int_{\underline{z}}^{\infty} z^{\sigma-1} m^I(z) \, \mathrm{d}z}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z}, \tag{15}$$

and eq. (7), which is unchanged from PE. The equilibrium is fully characterized by eqs. (7), (13), and (15).

When there is no scope for entry, then $E = 0$, the *free-entry condition* is slack, and the *zero-profit condition*

$$\frac{R}{\sigma} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} m^I(z) \, \mathrm{d}z} - f^c = 0 \tag{16}$$

pins down the mass of active firms in the economy. The equilibrium is fully characterized by eqs. (13) and (16), and $E = 0$.

3.1 Business Cycles

In GE, entry and fixed costs divert labor resources away from production. However, entry costs are only temporary, and free up labor once paid. The income effect from these labor savings induces subsequential entry. Therefore, while we let the economy run through the same three phases as in PE, we cannot think of each phase as corresponding to one period anymore. In GE, each phase τ consists of an infinite number of periods $t \in \mathbb{N}$, with the labor savings effect vanishing as $t \to \infty$. The total number of entrants along this process in phase τ is called *steady state mass of entrants* and given by $E_\tau^{ss} \equiv \sum_{t=1}^{\infty} E_{\tau,t}$. For a formal treatment of the steady-state equilibrium, see Definition 1 in the appendix. The following result extends Proposition 1 to the GE setting:

Proposition 2 (Cleansing Effects of Cycles)**.** *The ratio of post-cycle steady-state output to pre-cycle steady-state output is given by*

$$\frac{Y_3}{Y_1} = \left[\frac{M_3}{M_1}\right]^{q-\frac{1}{\sigma-1}} \left[\frac{L_3^d}{L_1^d}\right] \left[\frac{\int z^{\sigma-1} m_3 \, \mathrm{d}z}{\int z^{\sigma-1} m_1 \, \mathrm{d}z}\right]^{\frac{1}{\sigma-1}}, \tag{17}$$

where

$$\frac{M_3}{M_1} < 1, \quad \frac{L_3^d}{L_1^d} > 1, \quad \frac{\int z^{\sigma-1} m_3 \, \mathrm{d}z}{\int z^{\sigma-1} m_1 \, \mathrm{d}z} > 1. \tag{18}$$

There exists a unique $q^\star > q^{CES}$ (which generally depends on the fixed cost cycle values f_l^c, f_h^c)

for which $\frac{Y_3}{Y_1} = 1$. *Furthermore,*

$$\frac{Y_3}{Y_1} < 1 \iff q > q^\star. \tag{19}$$

General equilibrium introduces an additional effect of cleansing via recessions, the *labor cleansing effect*: as the long-run economy features fewer operating firms, fewer labor resources are used for fixed costs as opposed to production. As a consequence, the income obtained by the household, both in terms of labor payments and profits, increases. To see this, note that the fixed supply of labor $\bar{L}$ is used in production L^d and for fixed costs payments Mf^c (recall that we are looking at steady-states and, thus, contemporary entry is zero). Furthermore, by the constant markup, total income is given by $\frac{\sigma}{\sigma-1}L^d$. Combining these two, it is immediate that the income obtained by the household is $\frac{\sigma}{\sigma-1}(\bar{L} - Mf^c)$. This GE income effect expands market size and creates additional entry in phase 3 relative to PE, and thus, the economy loses fewer varieties. Put differently, the labor cleansing effect works directly though a relaxation of the resource constraint by increasing L^d, and indirectly as an income effect on market size.

The direct consequence of the combination of these forces is an output gain, even in the CES case, which is highlighted in Remark 5.

Remark 5 (CES Aggregation in General Equilibrium)**.** *The special case of CES aggregation* $\left(q = q^{CES}\right)$ *implies that* $\frac{Y_3}{Y_1}\left(q^{CES}\right) > 1$.

Based on the above intuition and remark, it is immediate that when $q > q^{CES}$, the economy values the loss of variety more than in the CES case. As a consequence, there exists a level of love-of-variety such that the household is indifferent between pre- and post-cycle outcomes as the three effects exactly cancel.

3.2 Depth of Crises

Until now, we have considered a fixed business cycle characterized by (f_l^c, f_h^c). Here, we study crises of varying intensity of the fixed cost increase, f_h^c. The following discussion examines the conclusions highlighted in panel (C) of fig. 2 in detail: (i) economies with large q relative to q^{CES} always experience a long-run output loss induced by recessions, (ii) the reverse holds for low-q economies, and (iii) intermediate-q economies elicit ambiguous long-run responses.

In a given economy, the effect of a fixed cost increase on long-run output is governed by the elasticity:

$$\frac{\partial \log Y_3}{\partial \log f_h^c} = \frac{\partial \log M_3}{\partial \log f_h^c} \times \left[\frac{\partial \log Y_3^{CES}}{\partial \log M_3} + (q - q^{CES})\right], \tag{20}$$

where $Y_\tau^{CES} := L_\tau^d(\int z^{\sigma-1} m_\tau(z)\ \mathrm{d}z)^{1/(\sigma-1)}$ is the phase τ output in the CES economy with $q = q^{CES}$. We study each element in turn. First, we analyze the effect of a marginal change in the depth of the crisis on the number of active firms in the long run, M_3. This is the first term

of eq. (20). Second, we analyze the effect of marginal changes in the number of firms in the economy on long-run output. This is the term in the bracket of eq. (20).

Effects of f_h^c on the mass of active firms M_3. We have argued above that the long-run number of firms necessarily declines after a crisis. This effect is driven by selection, as the average entrant is strictly more productive than the average exiter. In particular, the successful entrants have an average productivity of $\mathbb{E}_{\mu^E}[z|z \geq \underline{z}_1]$, whereas the exiters have an average productivity of $\mathbb{E}_{\mu^E}[z|\underline{z}_1 \leq z \leq \underline{z}_2]$. As a consequence, the average firm in the economy becomes more productive, and all else equal, there is space for fewer firms, i.e., less entry is needed to drive market intensity to its equilibrium level. This is true for all recession intensities f_c^h and independently of LoV, q.

However, larger crises do not always imply marginally fewer firms in the long run: the sign of $\frac{\partial \log M_3}{\partial \log f_h^c}$ is ambiguous. This is driven by the selection effect: the magnitude of the selection force depends on how large the rise in the fixed cost f_c^h is. To understand this, compare the average successful entrant with the marginal exiter. The productivity of the marginal exiter is $\underline{z}_2$. When $\underline{z}_2$ is very low, $\underline{z}_2 < \mathbb{E}_{\mu^E}[z|z \geq \underline{z}_1]$, the selection effect is strong, and less than one average entrant is needed to fill the gap left by the exiter: $\frac{\partial \log M_3}{\partial \log f_h^c} < 0$. However, when $\underline{z}_2$ is high, $\underline{z}_2 > \mathbb{E}_{\mu^E}[z|z \geq \underline{z}_1]$, selection is weak, and the survivor lost at the margin creates space for more than one firm to enter the market: $\frac{\partial \log M_3}{\partial \log f_h^c} > 0$. We formalize this insight following in Lemma 2.

Lemma 2 (On $M_3(f_h^c)$)**.** *The long run number of firms active in the economy $M_3(f_h^c) \leq M_1$ attains a unique minimum M_3^{min} at some crisis level, $f_h^{c,*} \in (f_l^c, \infty)$. For each crisis driven by $f_h^c \in (f_l^c, f_h^{c,*})$ there exists some crisis $f_h^{c\prime} \in (f_h^{c,*}, \infty)$, such that $M_3(f_h^c) = M_3(f_h^{c\prime})$.*

The mechanism underlying this result is fully driven by the presence of incumbents. The marginal effect of the magnitude of the crisis on the long-run number of firms is given by

$$\begin{aligned}\frac{\partial \log M_3}{\partial \log f_h^c} &= \frac{f_h^c}{M_3} \cdot \frac{\partial}{\partial f_h^c} \left[\underbrace{M_1 p_E(\underline{z}_2)}_{\text{Surviving Firms in Phase 2}} + \underbrace{E_3 p_E(\underline{z}_1)}_{\text{Entrants in Phase 3}} \right] \\ &= \frac{f_h^c}{M_3} \cdot \left[\underbrace{M_1 \frac{\partial p_E(\underline{z}_2)}{\partial \underline{z}_2}}_{<0} + \underbrace{p_E(\underline{z}_1) \frac{\partial E_3}{\partial \underline{z}_2}}_{>0} \right] \cdot \underbrace{\frac{\partial \underline{z}_2}{\partial f_h^c}}_{>0}, \end{aligned} \tag{21}$$

where $p_E(\underline{z}) = \int_{\underline{z}}^{\infty} \mu^E(z) dz$. As f_h^c increases, the crisis cutoff $\underline{z}_2$ shifts up. At low levels of f_h^c, the first term in the bracket of eq. (21) dominates: marginal exiters are less productive than entrants, and there is a marginal decline in the number of firms. At high levels of f_h^c, the marginal exiter is more productive than the average entrant, and, therefore, any additional increase in the fixed cost is associated with marginally more firms in the long run.

Effect of M_3 on long-run output, Y_3. Fix LoV at some arbitrary level, q. In this q-economy, as highlighted by eq. (20), the elasticity of output Y_3 with respect to M_3 decomposes into (i) the elasticity of Y_3^{CES}—output if the economy had LoV of level q^{CES}—and (ii) the elasticity of the variety effect in the q-economy relative to a CES economy. The latter is constant and its sign depends on whether LoV in the economy is higher or lower than in the CES benchmark. The elasticity of Y^{CES} with respect to M_3 is always negative. A higher number of firms M_3 affects Y^{CES} in two different ways: i) more firms directly imply more fixed cost payments and, therefore, less labor used in production, and ii) more firms in the economy are associated with weaker selection. Both of these effects reduce Y_3^{CES}.

To complete the characterization, note that $\frac{\partial \log Y_3^{CES}}{\partial \log M_3}$ is globally negative but not constant across different crisis intensities, f_h^c. Instead, it is decreasing in absolute terms. Consider $\frac{\partial Y_3}{\partial M_3}\frac{M_3}{Y_3}$ if $f_h^c = f_l^c$, such that no cycle occurred and $M_3 = M_1$. To decrease the number of firms by 1%, one needs to eliminate substantially more than 1% of firms during the crisis, otherwise, entrants in phase 3 would be too numerous. The reason is as follows: killing the least productive percent of firms leads to large savings in fixed costs relative to losses in production. Therefore, income increases in phase 3, which expands equilibrium market intensity and the number of entrants, largely undoing the removal of firms during the crisis. Thus, one requires large labor cleansing and income effects in the crisis, begetting a large relative increase in output and, therefore, a high elasticity. Alternatively, if we started from a somewhat smaller M_3, Y_3 has already reaped the benefits of labor cleansing and selection and is large compared to Y_1. To decrease M_3 by an additional 1% now only requires removing a few more firms during the crisis since now the marginal survivor is much closer in productivity to the marginal entrant. This, in turn, implies a smaller increase through labor cleansing over (an already large) Y_3. In effect, the elasticity is low.

We can now return to the total effect of having more firms in the post-cycle steady state. On the one hand, additional firms generate one-to-one gains from varieties, provided that $q > q^{CES}$. On the other hand, additional firms generate more-than-linear reductions in the labor-saving and selection gains from the crisis, and thus in Y_3^{CES}. We conclude that reducing firm and hence product variety, M_3, has always a negative effect on long-run output if q is large and always a positive effect if q is small. In between these two extremes, reducing firm and hence product variety M_3 has a negative effect on long-run output for small values of M_3 (where the variety effect is stronger than cleansing and selection) and positive for large values of M_3. Next, we discuss this region of ambiguity and its implication for the effect of crises of varying magnitude, f_h^c, on total output.

Effect of f_h^c on Y_3. Taking stock, we know that as the crisis intensifies, the mass of varieties available in phase 3 first decreases then increases, and that this mass of varieties can have unambiguously positive, negative, or indeterminate effects on total output. We assemble these insights about the effect of f_h^c on Y_3 in Proposition 3:

Proposition 3 (Interaction of cycle depth and LoV)**.** *Index economies with otherwise equal*

parameters by their love-of-variety, q ('q-economies'). Then, there exists a unique, nonempty interval $(q_\circ, q^\circ)$ with $q_\circ > q^{CES}$ such that:

1. $q \geq q^\circ \implies Y_3/Y_1 \leq 1$ *for all* f_h^c.
2. $q \leq q_\circ \implies Y_3/Y_1 \geq 1$ *for all* f_h^c.
3. *for all* $q \in (q_\circ, q^\circ)$*, larger crises can be welfare improving or welfare reducing depending on their intensity* f_h^c.

Proposition 3 provides three results. First, there exists a level of LoV $q\circ$ such that any economy with $q \geq q^\circ$ faces GDP and welfare drops for any crisis intensity f_h^c. These are economies that value the presence of varieties so much that they experience long-run welfare losses, even for the smallest crises. Symmetrically, there is a subset of q-economies that do not value varieties as much and find any recession *cleansing*, independently of its intensity. Finally, the set of economies indexed by $q \in (q_\circ, q^\circ)$ is such that small and extremely large recessions can be cleansing while medium recessions reduce welfare. This follows by the decreasing returns to labor cleansing and selection effects as the number of varieties shrinks. We sketch these behaviors out in Figure 2.

Panel (A) shows how CES output, Y^{CES}, and the total mass of firms, M_3, respond to different increases in the fixed costs, f_h^c. Both quantities are independent of q. Relative to before the crisis, the mass M_3 drops, reaching a minimum at depth $f_h^{c,*}$, where all the incumbents with productivity less than the average successful entrant have been replaced. CES output always increases relative to before the cycle and reaches its highest level when no further returns from labor cleansing and selection effects can be extracted, i.e., at $f_h^{c,*}$. As the fixed cost rises toward eliminating every firm during phase 2, all quantities return to their pre-crisis levels.

Next, panel (B) shows the evolution of the love-of-variety effect for different levels of q. When $q > q^{CES}$, this effect tracks the path of the number of firms in panel (A). When $q < q^{CES}$, the LOV externality is such that the economy is better off when fewer firms are active, so the effect mirrors that of M_3/M_1.

Finally, panel (C) shows the behaviour of output for different $q-$economies. When q is large, recessions are unambiguously welfare reducing and, vice versa, when q is small they are always cleansing. In the intermediate range, some recessions can induce welfare gains while others generate welfare losses depending on their depth.

This set of intermediate $q-$economies is characterized by an inference problem. In fact, past data on recessions cannot inform on whether future recessions will have cleansing effect or not.

We conclude by noting that in our economy, recessions are path-dependent and that the depth of the current crisis has implications for output and welfare in subsequent downturns. This is formalized in Remark 6.

Remark 6 (Path Dependence of Recessions)**.** *Let a q-economy experience two cycles of the same intensity f_c^h. Then: a) the post-crisis distribution, output, and welfare are different across*

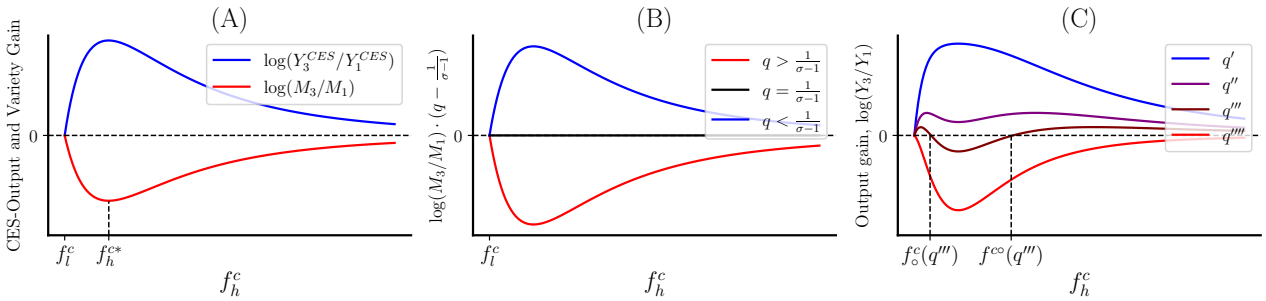


Figure 2: Decomposition of the log-output ratio for varying levels of LoV, q and crisis, f_h^c. All curves start in $(f_l^c, 0)$ and are simulated from the model. Panel (A) shows the how $\ln(Y_3^{CES}/Y_1^{CES})$ and $\ln(M_3/M_1)$ vary with f_h^c. Panel (B) shows how LOV contributes to the output ratio through the factor $\ln((M_3/M_1)^{q-q^{CES}})$. Panel (C) sketches the curve of $\ln(Y_3/Y_1)$ for values of q, $q' < q'' < q''' < q''''$, whereby only $q''' \in (q_\circ, q^\circ)$ holds (cf. Proposition 3). Correspondingly, at q''', the effect of crises is ambiguous, and there are two crises $f_\circ^c(q''') < f^{c\circ}(q''')$ at which the output ratio returns equals 1. A proof of the 'wiggly' shape when LoV is q''' is provided in the proof of Proposition 3.

the two cycles; b) the recessions generate different degrees of cleansing, captured by $\underline{z}_2^1 \leq \underline{z}_2^2$, *where* $i \in \{1, 2\}$ *denote the cycle.*

Having experienced the first recession, the q-economy starts the second crisis with a different incumbent population. During downturns, the composition of incumbents determines the strength of selection effects for a given increase in the fixed cost f_c^h (see eq. 16). This directly implies a different long-run distribution of firms and, therefore, output and welfare.

4 Estimation

In order to quantify the forces of interest, we have to estimate our two key parameters: q and σ, governing love-for-variety and substitutability across varieties. We estimate q and σ in a cross-industry version of our model for all the countries available in the World Input-Output Database (Timmer et al., 2015), henceforth WIOD. Because we consider a large number of different industries, $i \in \mathbf{I}$, over time, $t \in \mathbf{T}$, and across countries, $c \in \mathbf{C}$, we consider the partial equilibrium framework of Section 2.3. We subdivide the economy by industry, assuming that the whole economy absorbs the labor cleansing effects from shocks to any given industry. Throughout, we assume that entrants and incumbents are distributed according to some entry measure, μ^E, which is truncated by some cutoff, $\underline{z}$, for entrants, and z^* for incumbents. We discuss the identification of q and σ in turn.

4.1 Identification of q

To estimate the intensity of love-for-variety, we use a structural relation of the model, linking changes in income to changes in output through q. We start by taking logs of the aggregate

production function in eq. (8), and differentiating with respect to $\log \mathcal{I}$ to obtain

$$\frac{\partial \log Y}{\partial \log \mathcal{I}} = q \frac{\partial \log M}{\partial \log \mathcal{I}} + 1 + \frac{\partial \log \bar{z}}{\partial \log \mathcal{I}}. \tag{22}$$

A *positive* change in income, $\mathcal{I}$, which is nominal expenditure of downstream customers of the industry, induces a 1-for-1 direct increase in output as well as two indirect effects. First, higher income induces entry, which in turn increases M. Second, by inducing entry, higher income changes the average productivity of firms, $\log \bar{z}$, provided entrants do not have the same average productivity as incumbents. Since in equilibrium factor demand for labor L^d equals expenditure $\mathcal{I}$, the same equations and all following results hold for changes in L^d, too. The following Lemma characterizes the elasticities given *small positive changes in* $\mathcal{I}$. We do so for two cases: (a) the economy has not undergone any recent recession such that the productivity cutoffs of entrants and incumbents (z^*) are equal, i.e., $\underline{z} = z^*$; and (b) where selection through fixed cost cycles has left more productive incumbents, such that $\underline{z} < z^*$[13].

Lemma 3 (Elasticities In Output Equation). *Suppose, entrants and incumbents are distributed according to* $\mu^E(\cdot \mid z \geq \underline{z})$ *and* $\mu^E(\cdot \mid z \geq z^*)$*, respectively. Then, the right-derivatives* $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}$ *and* $\frac{\partial_+ \log \bar{z}}{\partial_+ \log \mathcal{I}}$ *are given by:*

$$\frac{\partial_+ \log \bar{z}}{\partial_+ \log \mathcal{I}} = \frac{1}{\sigma - 1}\left(1 - \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}\right), \quad \textit{and} \qquad \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} = \frac{E_{|I=0} p_0(\underline{z})}{E_{|I=0} p_0(\underline{z}) + I\, k(\underline{z}, z^*)}, \tag{23}$$

where $E_{|I=0}$ *is the number of entrants if there are no incumbents, and* k *is a function satisfying* $k(\underline{z}, z^*) \lesseqgtr 0$ *if* $\underline{z} \lesseqgtr z^*$.

For point-identification, we make Assumption 1, which states that the latent entrant distribution and the incumbent distribution have the same cutoff. This has three interpretations. Either (i) the industry has not undergone a sequence of selection-inducing fixed-cost recessions yet. Or (ii) the industry is in a long-run equilibrium with exogenous death shocks (cf. Proposition A.3), such that the cleansing effects since the last recession have fully subsided. Or, finally, (iii) entrants 'learn' from incumbents, such that their technologies, conditional on clearing the cutoff, $\underline{z}$, are no worse than those of existing businesses.

Assumption 1 (Entrant and Incumbent Distribution). $\underline{z} = z^*$.

Under Assumption 1, entry induced by shocks such as income expansions that leave f^c/f^e and hence the cutoff unchanged do not affect average productivity, $\bar{z}$. As a consequence, we have the following identification result.

Proposition 4 (Point Identification of q). *Under Assumption 1,*

$$\frac{\partial_+ \log Y}{\partial_+ \log X} = 1 + q, \quad X \in \{\mathcal{I}, L^d\}, \tag{24}$$

[13] The lemma also characterizes the case of $\underline{z} > z^*$. However, this case is inconsistent with an expansion in $\log \mathcal{I}$.

and in the regression equation

$$\Delta \log Y_{i,t} = \beta \Delta \log X_{i,t} + \gamma \Delta \log \bar{z}_{i,t} + \epsilon_{i,t}, \quad X_{i,t} \in \{\mathcal{I}_{i,t}, L^d_{i,t}\}, \tag{25}$$

we have

$$\Delta \log X_{i,t} > 0 \implies \Delta \log \bar{z}_{i,t} = 0 \quad \textit{and} \quad \beta = 1 + q. \tag{26}$$

This result allows us to identify q as the determinant of the elasticity of output to income changes. Underlying this result is that, by the free entry condition, an increase in income $\mathcal{I}$ increases the number of firms one-for-one. The change in the number of firms induces a direct change in output mediated by q. In this sense, we estimate q as governing the intensity of external returns to scale over and above the technological returns, which in our model are equal to 1 by the constant returns to scale assumption. Importantly, this only holds for positive changes in income and demanded labor since any negative change necessarily induces exit from the bottom of the distribution and, therefore, a response in $\log \bar{z}$. We conclude that we can estimate love of variety q provided some exogenous increase in employment L^d or income $\mathcal{I}$ via the regression

$$\Delta \log Y_{i,t} = \beta \Delta \log X_{i,t} + \epsilon_{i,t}, \quad X_{i,t} \in \{\mathcal{I}_{i,t}, L^d_{i,t}\}. \tag{27}$$

Proposition 4 enables us to estimate eq. (27) with an instrument inducing *positive variation* in L^d or $\mathcal{I}$, while disregarding the average productivity term. We use this identification strategy as our benchmark and discuss possible threats to identification in the next section. In particular, how eq. (25) directly extends to a setting with multiple factor inputs, constant returns to scale, and growth. Additionally, we discuss how failure of Assumption 1 still allows us to identify the sign of $q - q^{CES}$, given that we can identify $q^{CES} = \frac{1}{\sigma - 1}$, or a lower bound thereof.

4.2 Discussion of Identification

Multiple Factor Inputs Part of our identification result in Proposition 4 relies on the assumption that labor is the only factor of production and that it enters the production function linearly. This is what drives the "1" in our estimated coefficient of $\beta = 1 + q$. Clearly, these assumptions are counterfactuals in empirical applications. Our identification strategy readily extends to settings in which firms have multiple inputs combined under constant returns to scale (CRS). The "1" we identify is our regression should be interpreted as the returns to the optimal input bundle (in our model, trivially made only of labor). Deviations from constant returns to scale would induce an upward bias in our estimated q if the true data-generating process features decreasing returns, and a downward bias if it features increasing returns to scale.[14] Recent evidence estimating returns to scale points to small deviations from CRS and,

[14] In such a case our estimated coefficient would be $\widehat{\beta} = RTS + q$, where RTS is the returns to scale of the production function. As a consequence, our implied $\widehat{q} = \widehat{\beta} - 1$ would be higher than the true q only if $RTS < 1$

if anything, increasing returns (see, for example, Chiavari, 2024, who finds returns to scale of at most 1.05). Importantly, absent relative price changes of factor inputs, CRS implies a constant input mix for the industry. Hence, all other inputs would be perfectly collinear with L^d and are, through the inclusion of L^d, already accounted for. One can think of an input bundle of constant composition used by firms. Since positive idiosyncratic shocks to a single industry are unlikely to distort factor prices in the economy, the absence of relative price changes in our regression setting is plausible.

Time Trends, Technological Growth, and Controls We allow for productivity growth in a given industry at variable rates, $1 + \lambda_t$. This tacitly assumes that entrants and existing businesses experience the same growth rates. Since such growth does not lead to entry/exit dynamics per se, it is absorbed in a time fixed effect, following from $1+\lambda = \Delta \log \bar{z}$ in a state of entry. Other trended variables in the data, like inflation, can likewise be accounted for by the inclusion of time fixed effects and control variables, $H_{i,t}$. Importantly, since we are identifying the effect of variety on output indirectly by using the unit-coefficient pass-through from labor and income via M to output, control variables on the RHS are not allowed to confound this mediation relation. Other contemporaneous regressors are *bad controls* since they might respond to the exogenous shifts, thereby hindering the identification of our coefficient of interest. To sidestep this problem, include control variables are lagged to increase the precision of our estimates without biasing the estimation.

Weighting of Country-Industry-Year Observations Since we are estimating a homogeneous global q, we treat all country-industry-year observations symmetrically. If our primary goal were instead to estimate an average q under a hypothesis of heterogeneity, we would want to weight observations appropriately. We demonstrate in the appendix that weighting by industry size does not substantially alter our results.

Estimates without Assumption 1 Our identification strategy requires that there is at most negligible selection in incumbents. However, even if Assumption 1 fails entirely and $\underline{z} < z^*$ holds, we are still able to identify the sign of $q - q^{CES}$ and determine whether recessions (in partial equilibrium) are welfare-improving. This is stated in the following proposition.

Proposition 5 (Identification of $\text{sgn}(q - q^{CES})$)**.** *Suppose that $q^{CES} = \frac{1}{\sigma-1}$ is identified. Let β be the regression coefficient on $\Delta \log \mathcal{I}_{i,t}$ in eq. (27). Then, the bias of $\widehat{q} = \beta - 1$ is given by*

$$bias_+ = \widehat{q} - q = \left(\frac{1}{\sigma - 1} - q\right)\left(1 - \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}\right), \tag{28}$$

assuming an expansion in $\mathcal{I}$. Furthermore, for $\underline{z} < z^*$,

$$\text{sgn}\left(\widehat{q} - \frac{1}{\sigma - 1}\right) = \text{sgn}\left(q - \frac{1}{\sigma - 1}\right), \tag{29}$$

and lower if $RTS > 1$.

Which implies that even without Assumption 1, we can identify the sign of $q - q^{CES}$.

Firms, Establishments, and Varieties As a final remark, note that our approach circumvents a fundamental problem in mapping the model to the data: the definition of a variety. In principle, if we assumed that establishments/firms only produced a single variety, we could use the number of firms or establishments as a direct measure of M. This mapping is fragile as establishments/firms may produce multiple products (as we highlighted in the extension section), or new establishments of firms, such as factories, stores, or offices, may indicate larger quantities of existing rather than new product varieties. In such a case, we would need to observe both the number of firms and the number of products per firm. Our empirical strategy sidesteps this issue since it relies on the observation that q governs the intensity of external aggregate returns to scale.

4.3 Data

Input-Output Data The primary data source is the World Input-Output Database (WIOD) 2016 release, see Timmer et al. (2015). It contains the Input-Output structure of sector-to-sector flows for $J = 44$ countries from 2000 to 2014 yearly. The data is available at the 2-digit ISIC rev-4 level. The number of sectors in WIOD is $S = 56$, which amounts to 6,071,296 industry-to-industry flows and 108,416 industry-to-country flows for every year in the sample.

The World Input-Output Table is an $(S \times J)$ by $(S \times J)$ matrix whose entries. Each element Z_{ij}^{rs} denotes the value of output of industry r in country i sold to industry s in country j for intermediate input use. Additionally, the table WIOT includes an $(S \times J)$ by J matrix of final use. Each element F_{ij}^{r} is the value of output of industry r in country i sold to and consumed by households, government, and non-profit organizations in country j. Denote $F_{i}^{r} = \sum_j F_{ij}^{r}$, namely the value of output of sector r in country i consumed in any country in the world. The total value of sold output is $Y_i^r = F_i^r + \sum_s \sum_j Z_{ij}^{rs}$.

Instrumental Variable For each of the WIOD industries, we would like a plausibly exogenous shift in expenditure. We build this as in Ferrari (2024) through a shift-share, aggregating destination-specific shifters through destination shares. Formally, an industry i in country r at time t is assigned a change in demand equal to

$$\sum_j \xi_{ij}^r \Delta \log F_{jt}, \tag{30}$$

where ξ_{ij}^r represents the fraction of the value of output of industry r in country i consumed directly or indirectly in destination j in the first sample period and $\Delta \log F_{jt}$ is the growth rate of final consumption expenditure in country j at time t. The exposure share ξ_{ij}^r, assumed to be time-invariant, includes direct sales from the industry to consumers in j and output sold to other industries, which eventually sell to consumers in j.

The input-output structure of the data allows a full account of these indirect linkages when

analyzing sales composition. Formally, define the share of sales of industry r in country i that is consumed by country j as

$$\xi_{ij}^{r} = \frac{F_{ij}^{r} + \sum_s \sum_k a_{ik}^{rs} F_{kj}^{s} + \sum_s \sum_k \sum_q \sum_m a_{ik}^{rs} a_{km}^{sq} F_{mj}^{q} + ...}{Y_i^r}, \tag{31}$$

where a_{ij}^{rs} is dollar amount of output of sector r from country i needed to produce one dollar of output of sector s in country j, defined as $a_{ij}^{rs} = Z_{ij}^{rs}/Y_j^s$. The first term in the numerator represents sales from sector r in country i directly consumed by j; the second term accounts for the fraction of sales of sector r in i sold to any producer s in country k that uses i, r as input and then sells to country j for consumption. The same logic applies to higher-order terms. By definition $\sum_j \xi_{ij}^r = 1$.

For the shifters, we estimate the common component of all industries selling to a given destination j at time t. For each industry, we iteratively exclude flows from the same country and in the same sector

$$\Delta \log F_{kjt}^{s} = \eta_{jt}(i, r) + \nu_{kjt}^{s} \quad k \neq i, s \neq r. \tag{32}$$

This boils down to identifying our effect of interest through changes in foreign demand on other products, we refer to Ferrari (2024) for an extended discussion of the identification. Therefore, our instrument is given by

$$\eta_{it}^{r} = \sum_j \xi_{ij}^{r} \widehat{\eta}_{jt}(i, r). \tag{33}$$

Following Proposition 4, we estimate

$$\Delta \log Y_{it}^{r} = \beta \Delta \log \widehat{X}_{it}^{r} + \epsilon_{it}^{+r}, \tag{34}$$

Where $\Delta \log \widehat{X}_{it}^{r}$ is the instrumented variable. Practically, WIOD only provides information on sales. We detail in appendix E how we combine sales data with common deflators to obtain output.

4.4 Identification of σ

In our model, σ can be identified directly from the profit share since $\pi/R = 1/\sigma$. However, the WIOD data is built such that the sum of material, labor, and capital expenditure is equal to gross output. This implies lumping together economic profits with the cost of capital. For the purpose of identifying σ we make the conservative assumption that all capital expenses are, in fact, profits. Hence, we compute $\sigma = CAP/K$ for all industry-country-year triplets. Clearly, this implies that we are overstating the profit share in our measurement. This implies that our estimated σ is a lower bound for the true elasticity of substitution and, therefore, the implied measured q^{CES} is larger than the true q^{CES}.

4.5 Results

Estimates of q^{CES} We start with a description of our estimated σ. Our estimation relies on the conservative assumption that all capital remuneration represents profits through the lens of our model. Based on the WIOD data on capital returns, we find a distribution of profit rates across industries which is summarized in Table 1. The profit rate ranges between 4.4% and 57.9% with a median of 18.7%. The corresponding σ distribution is between 1.7 and 22.6, with a median of 5.3, in line with the estimates in the literature (see Anderson and Van Wincoop, 2004, for a review). We are interested in the q^{CES} associated with this distribution of elasticities. We find a distribution ranging between 0.05 and 1.37, with a median of 0.23, which is our chosen measure of q^{CES} for the quantitative application in 5.3.

Table 1: Distribution of Profit Rate, σ, and q^{CES} across Industries

Percentiles	**Profit Rate**	$\boldsymbol{\sigma}$	$\mathbf{q^{CES}}$
1	4.4%	22.6	0.05
5	6.5%	15.3	0.07
10	8.1%	12.4	0.09
25	13.2%	7.5	0.15
50	18.7%	5.3	0.23
75	27.0%	3.7	0.37
90	32.8%	3.1	0.49
95	42.1%	2.4	0.73
99	57.9%	1.7	1.37

Estimates of q We report different estimations of our object of interest q in Table 2. Given that positive demand shocks could induce firm entry with a delay, we use both contemporary and lagged shocks as instruments.[15] In panels (A) and (B), we report the results of estimating eq. (34) using both income and labor as the endogenous variables. The first and second columns in each panel show the first- and second-stage estimation results, respectively. In all our estimations, we find a significant first stage and βs larger than 1. Recall that $q = \beta - 1$, which implies that all our estimates of q are strictly positive. Depending on the specification, we find estimates of q between .512 and .634. For comparison, using a different identification and firm-level data, Baqaee et al. (2023) estimate a love-for-variety elasticity of .3. We interpret our larger estimate as a sign of even stronger aggregate increasing returns than implied by individual firms' production functions.

Panel (C) in Table 2 reports the results of an empirical model treating eq. (34) for labor and income as not independent. We estimate a 3SLS model explicitly accounting for the correlation between the error terms across the system of equations. Additionally, we impose common coefficients, as we fail to reject the null hypothesis that the coefficients are the same at 1, 5, and 10% significance levels. The joint estimation helps us increase the precision of our estimates.

[15] In Appendix E.2, we show that our results are robust to this choice. Furthermore, we show that the findings hold when weighting observations by industry size.

In Figure 3, we plot the estimate of q from panel (C), which we choose as our q for the quantitative application in 5.3, against the distribution of q^{CES} at the industry level. Our estimate implies that all but 4 industries out of 56 in our sample have $q < q^{CES}$ and for the vast majority of industries, q is substantially larger than q^{CES}. Furthermore, the aggregated q^{CES} lies below the 95% confidence interval of our estimated q. Therefore, when conducting a one-sided t-test, we can reject the hypothesis that $q \leq q^{CES}$ at the 5% significance level as per Table 2.

This result suggests that recessions have a significant potential to reduce long-run welfare as agents are sensitive to the loss of varieties.

Table 2: Estimation Results

	(A)		(B)		(C)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^r_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log L^{d,r}_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log Y^r_{i,t}$
$\eta^{+,r}_{i,t}$	0.0854***		0.103***		
	(0.0215)		(0.0204)		
$\eta^{+,r}_{i,t-1}$	0.0941***		0.0646***		
	(0.0231)		(0.0204)		
$\Delta \log \mathcal{I}^r_{i,t}$		1.512***			1.568***
		(0.191)			(0.151)
$\Delta \log L^{d,r}_{i,t}$				1.634***	1.568***
				(0.245)	(0.151)
$P(q \leq q^{CES})$		.07		.049	.013
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	10067	10067	10083	10083	10067
F-Stat	22	63	23	45	
R^2	0.0315	0.0860	0.0413	0.0880	
F-Stat eq.(1)					2843
F-Stat eq.(2)					131
R^2 eq.(1)					0.0870
R^2 eq.(2)					0.0880

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

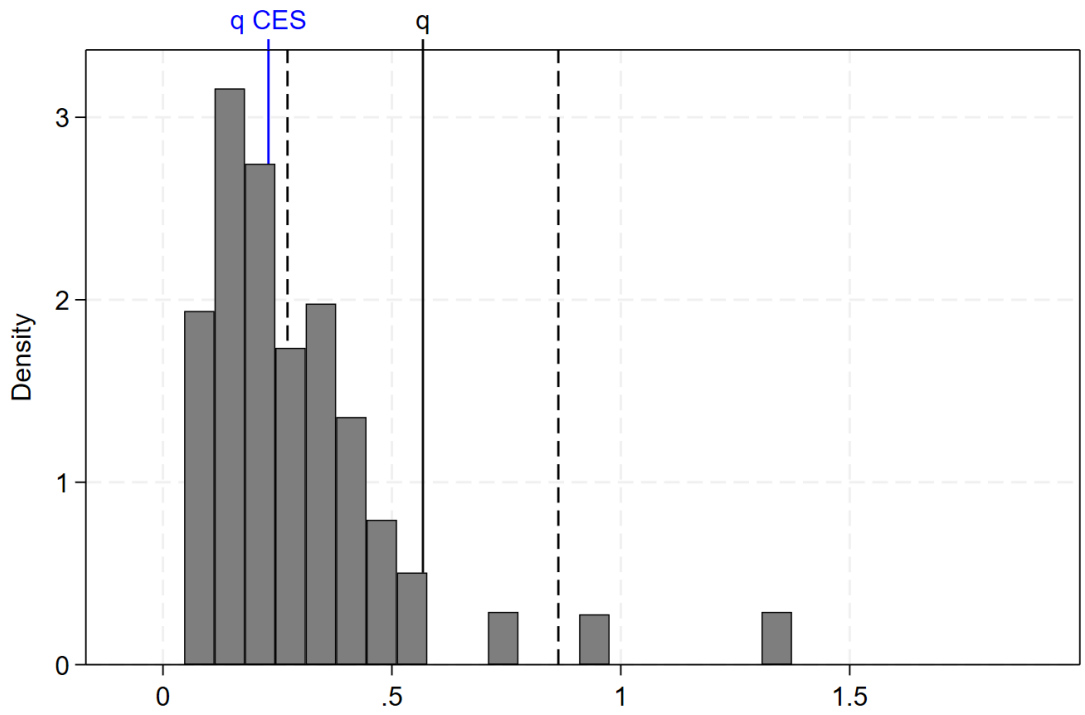


Figure 3: Estimated q and q^{CES}

5 Policy

In this section, we first study a social planner problem for economies with incumbents. The planner chooses the allocation of labor between entry costs, fixed costs, and each individual firm's production, controlling the entry mass E^{SP}, the cutoff $\underline{z}^{SP}$. Since taste-for-variety and incumbencies do not distort the firms' decision problem, a result by Dhingra and Morrow (2019) applies, which says that any given amount of production labor is split efficiently between firms and therefore E^{SP} and $\underline{z}^{SP}$ are sufficient tools for the social planner to steer the economy.[16] The social planner makes decisions sequentially, in period t and, possibly, phase of cycle, τ. Wherever it matters, we therefore write $E^{SP}_{\tau,t}$ and $\underline{z}^{SP}_{\tau,t}$.

We show that the government can use subsidies to achieve the first-best allocation even when $q \neq 1/(\sigma-1)$. The easiest way to achieve this is to subsidize or tax f^e and f^c in eq. (7) and (15); when firms and entrants face costs $\delta^c_{\tau,t} f^c_\tau$ and $\delta^e_{\tau,t} f^e$ instead, the government can shift $E_{\tau,t}$ and $\underline{z}_{\tau,t}$ to efficient levels in general equilibrium, by solving the corresponding equations for $(\delta^e_{\tau,t}, \delta^c_{\tau,t})$, while financing the intervention using a lump-sum tax paid by the households. In the knife-edge case of CES, the economy is constrained-efficient, and no intervention is necessary.

We end this section with quantitative characterization of fixed cost cycles. We simulate firm exit and entry in a mean-reverting fixed cost cycle, and characterize output losses along the transition. Our final objective is to compute the elasticity of total welfare to f^c given the empirical estimates of Section 4, and to solve for an optimal output policy. [*To be completed.*]

[16] This observation uses the fact that if the social planner wanted to keep unprofitable firms in the economy, she would always choose the most productive unprofitable firms.

5.1 Planner Entry and Exit Choices

We build the optimal behavior of the planner over the cycle by studying in isolation the cases of pure entry and pure exit. We combine these insights to study the socially optimal policy over the cycle. The social planner chooses $(E^{SP}, \underline{z}^{SP})$ to maximize the objective

$$\begin{aligned} \max_{E^{SP},\underline{z}^{SP}} \quad & Y^{SP} = (M^{SP})^{q-\frac{1}{\sigma-1}} L^{d,SP} \left[\int z^{\sigma-1} m^{SP}(z)\, \mathrm{d}z\right]^{\frac{1}{\sigma-1}} \qquad \text{s.t.} \qquad (35) \\ & \bar{L} \geq L^{d,SP} + (p_E(\underline{z}^{SP}) f^c + f^e) E^{SP} + f^c I, \\ & m^{SP}(z) = (m^I(z) + E^{SP}\mu^E(z)) \mathbb{I}_{\{z \geq \underline{z}^{SP}\}}(z), \\ & \underline{z}^{SP}, E^{SP} \geq 0. \end{aligned}$$

where $I := \int m^I(z) dz$.

The objective, Y^{SP}, is current output, which a myopic social planner would want to maximize. One can expect the solution to this tractable static problem to be a reasonable approximation to that of a dynamic Ramsey problem—not only for $\beta = 0$ but also for strictly positive discount factors. The reason is that the trade-off that a social planner makes is not as stark as a classical consumption-savings decision. This follows from the dual role of allocating labor to entry. Not only does a unit of labor allocated to entry today rather than tomorrow free up f^e resources tomorrow, but also entry today may increase today's output through the presence of an additional firm. If the economy lacks firms, there is no trade-off between entry today versus entry tomorrow, and it pays off to rush establishments into the market.

For characterizations of social planner solutions through the cycle, one thinks of (τ, t)-subscripts being attached to the choice variables of (35), and a law-of-motion of the incumbent measure $m^I_{\tau,t} = (m^I_{\tau,t-1} + E_{\tau,t}\mu^E)\mathbf{1}_{\{\bar{z} \geq \underline{z}^{SP}_{\tau,t}\}}$, given some $m^I_{\tau,0}$. From the first order conditions on E^{SP} and $\underline{z}^{SP}$, we characterize how entrants, cutoff, and steady-state mass of entrants $E^{ss,SP}$ change through the presence of the aggregation externality ($q \neq q^{CES}$) and incumbents.

We characterize optimal policy in Proposition 6, where we distinguish two cases. In case [A], the social planner faces an economy with only a few incumbents all of which are highly productive. For example, this is the state of an economy treated in Section 3, which stands at the beginning of phase 3. Such an economy is in a state of entry, and the cutoff chosen by the social planner, $\underline{z}^{SP}$, forces unproductive entrants but none of the previous incumbents to leave. Second, in [B], we treat the case of an economy that features *too many* incumbents, like the economy in phase 2 of the cycle. Specifically, we assume that incumbents were drawn from the baseline distribution, such that $m^I(z) \propto \mu^E(z)\mathbb{I}_{\{z \geq \underline{z}\}}(z)$ holds for some $\underline{z}$.

Additionally, we make the distributional assumption (A1) that $\mathbb{E}_{\mu^E}\left[(z/\underline{z})^{\sigma-1} \,\middle|\, z \geq \underline{z}\right]$ is weakly decreasing in the cutoff $\underline{z}$ throughout, which guarantees that points [A]-4 and [B] of Proposition 6 (monotonicity of $\underline{z}$ in q) hold. Distributions satisfying the assumption are, for example, Pareto, Exponential, Weibull (for shape parameter $k > 1$), Burr, and Fréchet (for shape parameter $\xi \leq 1$). It can be interpreted as requiring that as the condition for survival

becomes more stringent, the productivity of marginal exiter and new entrant become more similar on expectation.[17]

Proposition 6 (Social Planner Solutions)**.** *The interior solutions $E^{SP}, \underline{z}^{SP}$ of the social planner problem given by eq. (35) have the following properties:*
[A] Case with entry: $E^{SP} > 0$ (assuming $m^I(z) = 0 \; \forall z \leq \underline{z}^{SP}$).

1. *$E^{SP}, \underline{z}^{SP}, E^{ss,SP}$ coincide with general equilibrium allocations $E, \underline{z}, E^{ss}$ if and only if $q = q^{CES}$.*

2. *Number of entrants E^{SP} and total mass of entrants $E^{ss,SP}$ are both strictly increasing in q, and $(E^{SP}, M^{SP}) = (E, M)$ if and only if $q = q^{CES}$.*

3. *The presence of incumbents m^I with higher average productivity than entrants in the market decreases all $E^{SP}, E^{SP,ss}, E$ and E^{ss}. It also decreases the final number of firm $M^{SP,ss} = E^{SP,ss} + I$.*

4. *Incumbents do not affect the social planner steady-state cutoff.*

5. *Under (A1): Without incumbents before entry, the cutoff $\underline{z}^{SP}$ is decreasing in q, i.e., the social planner discards fewer of the entering firms.*

[B] Under (A1): Case with exit, $m^I(z) \propto \mu^E(z)\mathbb{I}_{\{z \geq \underline{z}\}}(z)$ holds for some $\underline{z}$
In this case, $E^{SP} = 0$ and $\underline{z}^{SP}$ depends again negatively on q, furthermore, $\underline{z}^{SP} = \underline{z}^{GE}$ if and only if $q = q^{CES}$.

The result in Proposition 6 extends previous insights from Spence (1976); Dixit and Stiglitz (1977); Mankiw and Whinston (1986); Parenti et al. (2017) and Dhingra and Morrow (2019) to economies with firm heterogeneity and incumbents. When the planner is constrained by the presence of already existing incumbent firms m^I, it optimally chooses the same cutoff and mass of entrants as in the market equilibrium if and only if the intensity of love-of-variety is that of CES preferences. If LoV is larger, there are inefficiently few firms operating in the market. These potential inefficiencies represent steady-state distortions. For any LoV, the optimal number of entrants is decreasing in the incumbent mass. In our setting, where incumbents are, on average, more productive than successful entrants, the planned economy eventually contains fewer firms the larger the incumbent mass, I, when approaching a steady state. Intuitively, the social planner internalizes the trade-off between allocating more labor to the most productive firms versus producing more varieties through a less productive entry margin.

Without incumbents, LoV affects the cutoff in the expected fashion: a higher q leads the social planner to decrease $\underline{z}$, e.g., by subsidizing fixed costs, such that lower-productivity firms can survive. While having ambiguous effects in the short run, a general distribution of incumbents, m^I, does *not* affect the cutoff as the economy approaches its steady state. The reason

[17] A sufficient condition for $\mathbb{E}_{\mu^E}\left[(z/\underline{z})^{\sigma-1} \,|\, z \geq \underline{z}\right]$ to be decreasing in the cutoff $\underline{z}$ is that the elasticity of the anti-cumulative corresponding to μ^E with respect to the truncation $\underline{z}$, $\varepsilon_{1-F}(\underline{z})$, is globally larger than -1, i.e. $\varepsilon_{1-F}(\underline{z}) \geq -1, \; \forall \underline{z} \geq 0$.

is that the social planner problem is, just like the equilibrium conditions, fundamentally agnostic regarding the precise firm heterogeneity at hand and only balances average productivity post-entry with the number of available varieties.

Next, we characterize the optimal policy intervention during business cycles.

Remark 7 (Ramsey Planner)**.** *One can compare the short-run planner solution to the steady state solution of a Ramsey planner problem facing a discount factor β in eq. (36):*

$$E^{ss,Ramsey}(\underline{z};q) = \bar{L}\frac{q}{q+1}\frac{1}{f^c p_E(\underline{z}) + f^e \frac{1}{(1+q)(1+\beta)}}. \tag{36}$$

Starting from an empty economy, the Ramsey solution approaches the cumulative sum of entrants of the social planner as $\beta \to 0$. Notably, the role of β is rather small. Even for values of β close to 1, the effect on total entry in the long run is moderate as long as f^c/f^e is large. We present the Ramsey problem in more detail in the appendix.

5.2 Optimal Policy through the Cycle

To contrast planner solutions and market allocations, consider a cycle like in Section 2.3, starting from an empty economy. We show that the social planner solution dominates the equilibrium allocation throughout the cycle with phases $\tau \in \{1,2,3\}$, even if the social planner cannot anticipate changes in fixed cost. For sake of this analysis, assume that $q >> q^{CES}$. Note that $\underline{z}_{1,t}^{SP} = \underline{z}_1^{SP}$ and $\underline{z}_{2,t}^{SP} = \underline{z}_2^{SP}$ are constant throughout, and that $\underline{z}_{3,t}^{SP}$ converges to some $\underline{z}_3^{SP}$ (this is shown in the proof of Proposition 6). Figure 4 sketches the evolution of the distribution of active firms throughout the cycle.

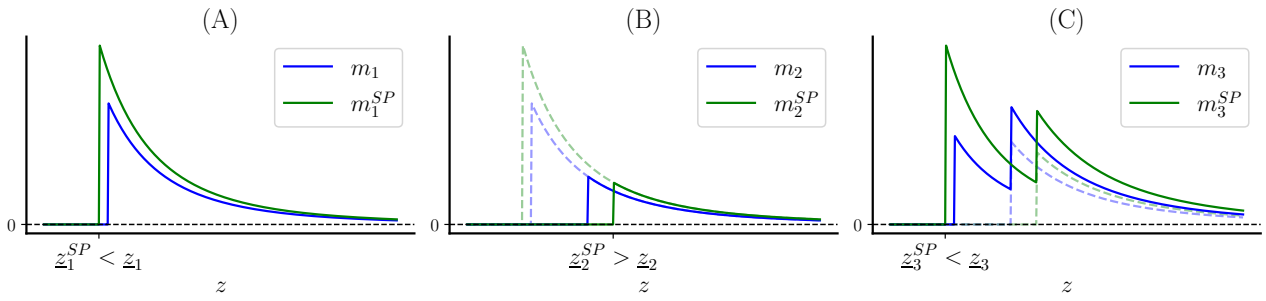


Figure 4: Evolution of m through the business cycle for the market and social planner allocations, assuming that $q >> q^{CES}$.

Panel (A) displays the pre-crisis steady-state allocation ($\tau = 1, t \to \infty$). Because q is large, the social planner sets $\underline{z}_1^{SP}$ small and adds entrants to the economy. It holds that $\underline{z}_1^{SP} < \underline{z}_1$ and $M_1^{SP} > M_1$. Once fixed costs go up, the social planner chooses $\underline{z}_2^{SP}$. For large q, $\underline{z}_2^{SP} > \underline{z}_2$ must also hold, since the large entrant mass $E_1^{ss,SP}$ chosen beforehand now requires extensive dropout so that fixed costs can still be paid with positive production. It is true that the crisis in the social planner economy kills more firms than in the GE economy. This, however, is exclusively caused by the fact that the social planner economy carries a lot more firms into

the crisis than the GE economy. Were the social planner to suddenly leave the SP economy to market forces in phase 2, even more firms would perish.

In Panel (B), we see the firm distribution during the crisis ($\tau = 2$). The social planner solution pre-cycle has caused a deepening of the crisis: an economy that sustains a larger number of varieties becomes more vulnerable to an increase in fixed cost. The relative decrease in the number of varieties and total productivity (as given by the productivity integral) is much larger in the centrally planned economy than in the equilibrium case. Note, however, that welfare is still larger for the social planner economy: the large stock of firms caused by a large $E_1^{ss,SP}$ allows the social planner to feasibly choose some $\underline{z}'$ which equates total number of firms to the number of firms in the GE distribution m_2, while creating a distribution which dominates m_2. Hence, $Y_2^{ss,SP} \geq Y_2^{ss}$.

Finally, in panel (C), the distribution has settled on the post-crisis firm distribution. The market cutoff level has reverted back to its original level $\underline{z}_1$, whereas $\underline{z}_3^{SP}$ may lie right of $\underline{z}_1^{SP}$ due to the presence of incumbents. Overall, the social planner solution clearly dominates the general equilibrium distribution in terms of output since the social planner inherits a distribution of incumbents with larger mass and higher cutoff. We summarize that the short-run social planner achieves consistently higher output but with exacerbated volatility through the cycle.

When there is entry, the optimal values for $\delta_{\tau,t}^c, \delta_{\tau,t}^e$ change with the state of the economy and converge to values δ_τ^c and δ_τ^e as the entry dynamics subside. For the sake of this discussion, we compare these steady-state policies when considering phases of the cycle. Additionally, an equilibrium with $\underline{z} = 0$ is not actually implementable as it would require an infinite ratio $\frac{\delta^e f^e}{\delta^c f^c}$. We thus consider a situation in which $\frac{\delta_t^e f^e}{\delta_t^c f_t^c}$ is large yet finite. For a given small $\underline{z}_1^{SP}$ in the above discussion, firms must face a small fixed cost of production $\delta_1^c f_1^c$ (by eq. 15) and a much larger fixed cost of entry, $\delta_1^e f_1^e$ (potentially a tax). The value of $\delta_{\tau,t}^c$ is determined jointly by the zero-profit condition and the cutoff equation (eqs. (4), (7)); however, once the steady-state has been reached, δ_τ^c satisfies eq. (4) with zero entry and the steady-state entrants as incumbents, while fixing $\underline{z}$ at the social planner value. By evaluating eq. (4) at the $\underline{z}^{SP}$ which satisfies the first-order-condition of the social planner, we immediately obtain the following proposition:

Proposition 7 (Optimal Policy)**.** *The optimal subsidy for production fixed-cost when the social planner economy attains its steady state after entry or when the social planner raises $\underline{z}^{SP}$ to provoke exit, i.e., whenever $E^{SP} = 0$, is given by*

$$\delta^c(\underline{z}^{SP}) = \left[[q(\sigma-1)-1] \left(\mathbb{E}_{\mu^E} \left[\left(\frac{z}{\underline{z}^{SP}} \right)^{\sigma-1} \mid z \geq \underline{z}^{SP} \right] \right)^{-1} + 1 \right]^{-1}. \tag{37}$$

Furthermore, under (A1), fixed-cost subsidies are counter-cyclical, and in the cycle discussed in Section 5.2, we have $\delta_1^c \geq \delta_2^c$ and $\delta_3^c \geq \delta_2^c$.

Recall that $\underline{z}_1^{SP} < \underline{z}_2^{SP}$. Using the first part of Proposition 7, we see that $\delta_1^c \geq \delta_2^c$, i.e., the government should increase subsidies precisely if (A1) holds, which yields the second part.[18]

[18] In the special case of a Pareto baseline distribution, it turns out that $\delta_1^c = \delta_2^c$.

Since the cutoff reverts back below $\underline{z}_2^{SP}$ at the end of the cycle, we deduce that, under the monotonicity assumption made above, subsidies decrease again, and thus $\delta_3^c \geq \delta_2^c$ holds, too.

We conclude that the social planner in an economy with $q > q^{CES}$ finds it optimal to increase the fixed cost subsidy during recessions. This policy change is motivated by having fewer firms exit the market.

5.3 Quantitative Model

[*TO BE COMPLETED*]

6 Conclusions

We study the cleansing effect of recessions in an economy where households value the consumption of multiple differentiated varieties. We show that recessions driven by rising fixed costs of production generate *cleansing* in the sense that exiting firms are replaced by more productive entrants. Nonetheless, we show that this need not translate into long-run gains in GDP and welfare. Whether the household is better off in the long run fundamentally depends on the extent of *love-of-variety* in the downstream aggregation technology for industry output. We show that when *love-of-variety* is stronger than in the benchmark CES economy, a social planner finds it optimal to increase fixed-cost subsidies during recessions to reduce the extent of firm exit.

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Online Appendix

Not-so-Cleansing Recessions

A Extensions

Forward-Looking Exit A straightforward extension of the model is one in which firms anticipate the mean-reverting path of an adverse fixed cost shock, allowing them to see and discount $f^c_{t+1}, f^c_{t+2}, ...$ However, they do not foresee the path of future entry, forecasting future profits using $\widetilde{\pi}(z, m_t)$. The decision to exit then hinges on

$$NPV(\{f^c_{t+k}\}_k) \gtreqless \widetilde{\pi}(z, m_t)/(1-\beta). \tag{38}$$

Accordingly, the cutoff jumps up less during a recession, while the dynamics of the model are unaffected.

Fixed costs in units of the final good To preserve efficiency in the decentralized economy with $q = q^{CES}$, we have assumed that fixed costs are paid in units of labor (Barro and Sala-i Martin, 2004). Proposition A.1 extends Propositions 1 and 2 in the case in which fixed costs are paid in units of the final good.

Proposition A.1 (Cleansing with Fixed Costs in Final Good's units)**.** *The difference between post-cycle and pre-cycle output is given by*

$$Y_3 - Y_1 = \left(\int z^{\sigma-1} m_3 \, dz - \int z^{\sigma-1} m_1 \, dz \right) \frac{\sigma f^c}{\underline{z}_1^{\sigma-1}} \tag{39}$$

where

$$q = q^{CES} \implies Y_3 = Y_1.$$

Furthermore:

$$\begin{cases} \sigma \in (1,2): & a)\ Y_3 > Y_1\ \forall\, q > q^{CES}; \\ & b)\ \exists\ \underline{q}\ s.t. \\ & \quad q \in [0, \underline{q}) \implies Y_3 > Y_1 \\ & \quad q \in (\underline{q}, q^{CES}) \implies Y_3 < Y_1 \\ & \quad q = \underline{q} \implies Y_3 = Y_1. \\ \sigma > 2: & a)\ Y_3 > Y_1\ \forall\, q < q^{CES}; \\ & b)\ \exists\ \bar{q}\ s.t. \\ & \quad q \in (\bar{q}, \infty) \implies Y_3 > Y_1 \\ & \quad q \in (q^{CES}, \bar{q}) \implies Y_3 < Y_1 \\ & \quad q = \bar{q} \implies Y_3 = Y_1. \end{cases}$$

If fixed costs are paid in units of the final good, entry becomes a function of q through the fixed costs whose price, P, is directly affected by the LoV externality. Therefore, aggregate productivity and the mass of available varieties are directly affected by q. In common with our main results, $q = q^{CES}$ is a knife-edge case in which pre and post output and welfare are identical. Contrary to our main results, for $q \neq q^{CES}$, the cleansing effects of cycles are more nuanced and depend on the elasticity of substitution σ.

Aggregate TFP Cycles So far, we have considered business cycles driven by fluctuations in the fixed cost of production. We now study how our conclusions would change if recessions were caused by changes in aggregate TFP. Consider a once and for all decrease in aggregate productivity A, such that the productivity of all firms goes from z to $Az < z$. We characterize the behavior of our economy in Proposition A.2.

Proposition A.2 (Aggregate TFP Cycles)**.** *Recessions driven by temporary decreases in aggregate TFP have no long-run effects.*

To understand this surprising result, note that, in our economy, temporary shocks have long-run effects only if they affect entry and exit decisions. An aggregate TFP shock moves the effective productivity of all firms equally. As a consequence, the relative productivity is unchanged. Entry/exit decisions might still be affected if the size of the economy, R, changes (see eq. 14). However, since the labor supply is perfectly inelastic, a drop in real wages exactly offsets the reduced productivity, and firms' marginal costs are unchanged. The negative TFP shock implies lower output as long as A remains lower than 1. Yet, as entry and exit are not affected, when A returns to its initial value, the economy returns to its original steady state.

Importantly, this conclusion relies on our assumption that total labor supply is fixed. If we had assumed an elastic labor supply, households would optimally choose more leisure since the real wage drops. As a consequence, firms' entry and exit choices would respond to the change in total income and generate long-run effects of temporary TFP shocks.

Stochastic Idiosyncratic Productivity Our baseline model posits that firms draw their productivity once and for all. Here, we study a setting in which they experience fluctuations in idiosyncratic productivity even after entry. Suppose firms are subject to individual, transitory productivity shocks as in Hopenhayn (1992). Productivity shocks are i.i.d. across firms and periods. Firms decide at the beginning of the period, before learning their idiosyncratic shock, whether or not to produce. Effective productivity $\widetilde{z}$ consists of a fixed component, z, drawn from μ^E on entry, and a shock ε, such that $\widetilde{z} = z + \varepsilon$ and $\varepsilon \sim \psi$.[19] Consider GE, so $\mathcal{I}$ is endogenous (although this does not matter here). Firm profits now read

$$\pi(\widetilde{z}, m) = \frac{\mathcal{I}}{\sigma} \frac{(z+\varepsilon)^{\sigma-1}}{\mathcal{Z}(m,\psi)} - f^c, \tag{40}$$

where

$$\mathcal{Z}(m,\psi) := \int \int (z+\varepsilon)^{\sigma-1} m(z) \psi(\varepsilon) \, \mathrm{d}z \, \mathrm{d}\varepsilon \tag{41}$$

is market intensity, which nests our baseline model for $\psi = \delta_0$ (the Dirac distribution centered about 0). Given the timing assumption, the zero profit condition holds with equality at the cutoff for productivity z. The equilibrium conditions describing firm entry and exit with idiosyncratic risk are given by

$$E_\varepsilon(\mathbb{E}_{\mu^E}[\max\{\mathbb{E}_\psi \pi, 0\}] - f^e) \leq 0 \tag{42}$$

$$E_\varepsilon \geq 0 \tag{43}$$

$$\mathbb{E}_\psi \pi(\underline{z}_\varepsilon + \varepsilon, m) = 0 \tag{44}$$

$$m(z) = (m^I(z) + E_\varepsilon \mu^E(z)) \mathbb{I}(z \geq \underline{z}_\varepsilon), \tag{45}$$

and all other equilibrium equations are as in the baseline model. We note the following properties relative to our baseline model:

Proposition A.3 (Equilibrium with idiosyncratic risk)**.** *The following holds*

1. *As in the baseline model, the cutoff, $\underline{z}_\varepsilon$ is independent of m^I in a state of entry, and equilibrium is established by adjusting $\mathcal{Z}$ through entry. Thus, any fixed cost cycle as in Section 3.1 and thereafter features exactly two levels of $\underline{z}_\varepsilon$.*

2. *In any given phase, τ, of the cycle, at time $t = 0$ an incumbent with permanent productivity component z is part of the steady-state if and only if*

$$\mathbb{P}_\psi(z + \varepsilon < \underline{z}_\varepsilon) = 0, \tag{46}$$

[19] For ease of notation, we assume that $z + \varepsilon \geq 0$ always holds. Similar results can be achieved if one parametrizes the distribution of ε by z to ensure $\varepsilon + z \geq 0$.

and the steady states of phases 1 and 3 differ if and only if

$$\int \mathbb{P}_\psi(z+\varepsilon < \underline{z}_\varepsilon)\mu^E(z)\ \mathrm{d}z > 0. \tag{47}$$

An immediate consequence of Proposition A.3 is that whenever eq. (47) holds, Propositions 1–3, which all exploit the path dependency of our model, still hold true.

An alternative scenario is one in which *every* firm exits in some future, and incumbency plays no role in the steady state. For example, this would be the case whenever there are random death shocks (Melitz, 2003; Bilbiie et al., 2012). Once we introduce such shocks, no matter how unlikely, the path-dependency driven by incumbency collapses, and the only steady state is the one reached from an empty economy. All the fixed cost cycle effects we discussed above become strictly transitory. Nonetheless, the dynamics we described throughout would be visible through what Caballero and Hammour (1996) refer to as "echo" effects. Namely persistent fluctuations caused by past recessions that only vanish in the very long run.

Multiproduct Firms An important assumption of our model so far is that firms produce a single variety. As a consequence, firm exit necessarily implies the loss of a variety. Here we study an extension of our baseline model where firms can produce multiple varieties. Consider an economy in which individual firms pay a fixed cost $f^p > 0$ per variety they produce, and firm productivity stays the same across varieties. Then, a firm with productivity z produces a total number of varieties $N(z)$. To aggregate varieties $j \in [0, N(z)]$ within a firm, we use a CES-aggregator with an elasticity of substitution $\eta > 1$:

$$y(z) = \left(\int_0^{N(z)} y_j(z)^{\frac{\eta-1}{\eta}}\ \mathrm{d}j \right)^{\frac{\eta}{\eta-1}}. \tag{48}$$

Varieties across firms are aggregated by a generalized CES aggregator with elasticity of substitution $\sigma \in (1, \eta)$. This aggregator features, in addition to the aggregate externality running through LoV for different firms, q_F, an externality term for LoV through products, q_V:

$$Y = M^{q_F - \frac{1}{\sigma-1}} \widetilde{N}^{q_V - \frac{1}{\eta-1}} \left[\int y(z)^{\frac{\sigma}{\sigma-1}} m(z)\ \mathrm{d}z \right]^{\frac{\sigma-1}{\sigma}}. \tag{49}$$

Here, the number of firms is $M := \int m\ \mathrm{d}z$, and $\widetilde{N}$ is the number of products per firm: $\widetilde{N} = N/M = \int N(z)\mu(z)\ \mathrm{d}z$. Since the cost to produce an extra variety is linear while the gains are concave, each firm chooses to operate an optimal mass $N(z) > 0$ of varieties:

$$N(z) = \frac{L^d}{(\eta-1)f^p} \frac{z^{\phi-1}}{\int z^{\phi-1} m(z)\ \mathrm{d}z}, \tag{50}$$

where $\phi - 1 := \frac{(\sigma-1)(\eta-1)}{\eta-\sigma}$. Like individual output in our baseline model, the number of varieties of each firm is increasing in its relative productivity $z^{\phi-1} / \left(\int z^{\phi-1} m(z)\ \mathrm{d}z \right)^{\frac{1}{\phi-1}}$ and decreasing

in the fixed cost f^p. By aggregation, the total number of varieties is given by

$$N = \frac{L^d}{(\eta - 1) f^p}, \tag{51}$$

and therefore varieties scale linearly in the amount of available production labor. Equivalently, the ratio of labor expended on product fixed cost to that used for production, Nf^p/L^d is constant and given by $1/(\eta - 1)$. Intuitively, firms trade off allocating labor to producing more varieties versus more output of existing varieties. Since firms behave as if they lived in a world with CES aggregation, this trade-off is independent of scale, and Nf^p/L^d is a constant fraction. To solve for L^d, we use the labor market clearing condition:

$$L^d + Mf^c + Ef^e + Nf^p = \bar{L}, \tag{52}$$

from which we conclude

$$L^d = \frac{\eta - 1}{\eta}(\bar{L} - Mf^c) \qquad \text{and} \qquad N = \frac{(\bar{L} - Mf^c)}{\eta f^p}. \tag{53}$$

As before, we consider the steady state, i.e., $E = 0$. By substituting the equilibrium production of each firm into the aggregator in eq. (49), we can write aggregate output in equilibrium as

$$Y \propto \overbrace{M^{q_F - q_V - \frac{1}{\phi - 1}}}^{= M^{\left(q_F - \frac{1}{\sigma - 1}\right) - \left(q_V - \frac{1}{\eta - 1}\right)}} (\bar{L} - Mf^c)^{1 + q_V} \overbrace{\left(\int z^{\phi - 1} m(z) \, \mathrm{d}z \right)^{\frac{1}{\phi - 1}}}^{=: Z}, \tag{54}$$

suppressing positive constants depending on η and f^p only. Contrary to eq. (8), both the mass of firms and the mass of varieties matter for aggregate output, and we allow for two separate parameters capturing how much each is valued: q_F and q_V, respectively. However, we also note the high similarity to our baseline formulation. By inspection of eq. (54), it is clear that cleansing effects are qualitatively the same, with a few remarks. First, for $q_V > 0$, there are increasing returns to scale to *uncommitted labor*, $\bar{L} - Mf^c$, because more uncommitted labor increases both the number of varieties per firm $\widetilde{N}$, and firm output $y(z)$. Second, an increase in the number of firms M is valued through two channels: a mechanical contraction of varieties per firm $\widetilde{N}$ with elasticity $-(q_V - \frac{1}{1-\eta})$ and an expansion of firm-variety with elasticity $(q_F - \frac{1}{\sigma - 1})$.

With appropriately redefined boundaries for the inequalities in Propositions 1–3, all our results on cleansing through fixed-cost cycles hold true if dynamics of the extensive margin are unchanged relative to the baseline model. This is indeed true since firm-level profits are given by

$$\pi(z, m) = \frac{L^d}{\phi - 1} \frac{z^{\phi - 1}}{\int z^{\phi - 1} m(z) \, \mathrm{d}z} - f^c, \tag{55}$$

eqs. (3), (4) and (5) yield exactly the same equilibrium conditions and entry/exit dynamics with the 'compound' elasticity of substitution ϕ in place of σ. We formalize the long-run effects

of business cycles, driven by f^c, in part (i) of Proposition A.4.

Proposition A.4 (Fixed-cost Cycles with Multi-Product Firms)**.** *In the presence of multi-product firms, the following holds true.*

(i) For recessions driven by temporary increases in f^c, where $q_F^{CES} := \frac{1}{\sigma-1}$ and $q_V^{CES} := \frac{1}{\eta-1}$:

(a) an analogue of Proposition 1 holds with

$$M_3 < M_1,\ L_3^d = L_1^d,\ N_3 = N_1,\ Z_3 = Z_1$$

and

$$\frac{Y_3}{Y_1} \gtreqless 1 \Leftrightarrow q_F - q_F^{CES} \lesseqgtr q_V - q_V^{CES};$$

(b) an analogue of Proposition 2 holds with

$$M_3 < M_1,\ L_3^d > L_1^d,\ N_3 > N_1,\ Z_3 > Z_1$$

and there exists a unique $q_F^\star$ with

$$q_F^\star - q_F^{CES} > q_V - q_V^{CES},$$

such that

$$\frac{Y_3}{Y_1} \gtreqless 1 \iff q_F \lesseqgtr q_F^\star;$$

(c) an analogue of Proposition 3 holds with

$$\exists!\ (q_\circ, q^\circ)\ \ \textit{with}\ \ q_\circ - q_F^{CES} > q_V - q_V^{CES}$$

such that long-run effects depend on f_h^c intensity.

(ii) Recessions driven by temporary increases in f^p have no long-run effects.

The setup of this extension also allows us to examine a second kind of fixed cost cycle: a cycle in f^p. Part (ii) states that f_p cycles are akin to aggregate TFP cycles in that they have no long-run effects. The intuition is as follows. First, note that, differently from firm entry, product creation does not require a sunk investment, hence there is no notion of 'product-incumbency'. Our cycles always require that the parameters of our economy revert back to their initial level. Hence, if f_p cycles do not induce change in the firm distribution, the available products will be the same before and after the crisis. Firms reduce their product set while the recession occurs and create new products as f_p falls to the original level. Furthermore, note that in our economy, the presence of multiple products operates like a shifter to the productivity of each individual firm: if $z' > z$, the effective productivity difference is even larger since the z' firm

chooses to operate more product varieties than the z firm. However, the elasticity of $N(z)$ to f_p is independent of z. Hence, when f_p increases, all firms reduce the product set of the same percentage. This operates like a drop in aggregate productivity and does not affect the relative productivity of firms. Like in our previous case, this shift does not induce any exit and, therefore, no change in the firm distribution before and after the recession. As a consequence, the economy reverts to the initial steady state.

B Definitions

Definition 1 (Steady-State Equilibrium After Entry)**.** *The allocation* $(L^{d,ss}, M^{ss})$ *and measure of active firms* m^{ss} *constitute a steady-state equilibrium after entry, given an initial measure of incumbents* m^I*, if*

1. *the cutoff* $\underline{z}$ *is determined by eq. (7),*
2. *the steady-state distribution of firms is given by* $m^{ss} = (m^I + E^{ss}\mu^E)\mathbf{1}_{\{z \geq \underline{z}\}}$*, where* $E^{ss} \equiv \sum_{t=1}^{\infty} E_t$ *denotes the total entry mass across time,*
3. *eq. (15) holds with* $E = 0$*, and* m^{ss} *as incumbent measure,*
4. *the mass of active firms is given by* $M^{ss} = \int_{\underline{z}}^{\infty} m^I \, \mathrm{d}z + E^{ss} p_E(\underline{z})$*,*
5. *labor markets clear* $\bar{L} = L^{d,ss} + M^{ss} f^c$*,*

and if the economy before entry (given m^I*) is in a state of positive entry.*

C Proofs

Proof of Remark 1. Suppose wlog that z_i is increasing in the firm index, $i \in [0, M]$. We can then perform the change of measure:

$$V_M(\{y_i\}_{i\in[0,M]}) := M^{q-\frac{1}{\sigma-1}} \left(\int_0^M y_i^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}} = M^{q-\frac{1}{\sigma-1}} \left(\int_{z\in\mathcal{Z}} y(z)^{\frac{\sigma-1}{\sigma}} m(z) \, \mathrm{d}z \right)^{\frac{\sigma}{\sigma-1}},$$

where the right-hand side is our aggregator. As in Benassy (1996), we define

$$v(M) := \frac{V_M(1)}{M} = \frac{M^{q-\frac{1}{\sigma-1}} M^{1+\frac{1}{\sigma-1}}}{M} = M^q.$$

The function $v(M)$ characterizes the relative "gain derived from spreading a certain amount of production between M differentiated products instead of concentrating it on a single variety" (Benassy, 1996). The elasticity of $v(M)$ with respect to M is defined as LoV, which indeed equals q. ■

Proof of Remark 2. Let μ^E be the baseline probability density function from which productivity is drawn. We then have the following iterative entry procedure:

1. At iteration $k \geq 0$, a mass of firms, $M^{(k)}$, enter initially until:

$$\begin{aligned} 0 &= \mathbb{E}_{\mu^E}[\max\{\pi(z, m^I, m^{(k-1)}, M^{(k)}), 0\}] - f^e \\ &= \int_{\underline{z}^{(k)}}^{\infty} \left[\frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} m' dz + \int z^{\sigma-1} m^I dz} - f^c\right] \mu^E dz - f^e \end{aligned} \tag{56}$$

where $m' := M^{(0)}\mu^E$ in the first iteration and $m' := m^{(k-1)} + M^{(k)}\mu^E$ in the kth iteration.

2. Currently active firms check their profitability, and all firms with productivity less than $\underline{z}^{(k)}$ leave the market. The cut-off $\underline{z}^{(k)}$ is determined by:

$$\begin{aligned} \pi(\underline{z}^{(k)}, m^I, m^{(k-1)}, M^{(k)}) = \frac{\mathcal{I}}{\sigma} \frac{(\underline{z}^{(k)})^{\sigma-1}}{\int z^{\sigma-1} m' dz + \int z^{\sigma-1} m^I dz} - f^c = 0 \\ (\underline{z}^{(k)})^{\sigma-1} = f^c \frac{\sigma}{\mathcal{I}} \left[\int z^{\sigma-1} m' dz + \int z^{\sigma-1} m^I dz\right] \end{aligned}$$

3. After exit, the density function corresponding to the firms left in the market is relabeled as the new incumbent density function and is given by:

$$m^I \leftarrow m^I + m'(z)\mathbb{I}_{\{z \geq \underline{z}^{(k)}\}} = m^I + m^{(k)}$$

where m^I is not affected by the truncation since scope for entry implies that the threshold is weakly lower than the one previously incurred by the incumbents.

Steps 1 to 3 are iterated until convergence.

Equivalence of threshold $\underline{z}$

By step 1, we have that at iteration k the following holds:

$$f^e = \int_{\underline{z}^{(k)}}^{\infty} \left[\frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} m^{(k-1)} dz + M^{(k)} \int z^{\sigma-1} \mu^E dz + \int z^{\sigma-1} m^I dz} - f_c\right] \mu^E dz \tag{57}$$

$$= \frac{\mathcal{I}}{\sigma} \frac{p_E(\underline{z}^{(k)})\, \mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{\int z^{\sigma-1} m^{(k-1)} dz + M^{(k)} \mathbb{E}_{\mu^E}[z^{\sigma-1}] + \int z^{\sigma-1} m^I dz} - p_E(\underline{z}^{(k)}) f_c \tag{58}$$

where $p_E(\underline{z}^{(k)}) = \int_{\underline{z}^{(k)}}^{\infty} \mu^E(z) dz$.
Solving for $M^{(k)}$, we obtain:

$$M^{(k)} = \frac{1}{\mathbb{E}_{\mu^E}[z^{\sigma-1}]} \left[\frac{\mathcal{I}}{\sigma} \frac{p_E(\underline{z}^{(k)})\, \mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{f^e + f^c p_E(\underline{z}^{(k)})} - \int z^{\sigma-1} m^{(k-1)} dz - \int z^{\sigma-1} m^I dz\right] \tag{59}$$

Substituting the derived $M^{(k)}$ in the cut-off equation of step 2, we obtain:

$$(\underline{z}^{(k)})^{\sigma-1} = f^c \frac{p_E(\underline{z}^{(k)})\ \mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{f^e + f^c\ p_E(\underline{z}^{(k)})} \tag{60}$$

$$\frac{f^e}{f^c} = p_E(\underline{z}^{(k)}) \left[\frac{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{(\underline{z}^{(k)})^{\sigma-1}} - 1 \right] \tag{61}$$

First, note that clearly none of this is dependent on having started at iteration k. We would have obtained eq. (61) also by having derived $M^{(0)}$ in eq. (59). Second, note that $\underline{z}^{(k)}$ is a constant sequence as it depends only on f^c and f^e, which do not change with k. Hence, we can write $\underline{z}_k = \underline{z}\ \forall k \in \mathbb{N}$. Third, note that the equation pinning down the threshold in the iterative entry procedure is the same as in the simultaneous entry game eq. (7).

Equivalence of entry mass E

With $\underline{z}^{(k)} = \underline{z}\ \forall k \in \mathbb{N}$, it is clear that all we are doing is stacking scaled versions of the baseline density truncated at $\underline{z}$. Therefore, eq. (59) can be written as:

$$M^{(k)} = \frac{1}{\mathbb{E}_{\mu^E}[z^{\sigma-1}]} \left[\frac{\mathcal{I}}{\sigma} \frac{\underline{z}^{\sigma-1}}{f^c} - (M^{(0)} + ... + M^{(k-1)}) \int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E dz - \int z^{\sigma-1} m^I dz \right] \tag{62}$$

where we used eq. (60) to substitute in $\frac{\underline{z}^{\sigma-1}}{f^c}$.
This shows that $\{M^{(k)}\}$ is a decreasing sequence. Since $M^{(k)} \geq 0$ holds $\forall k$, $\{M^{(k)}\}$ has a real limit point, and this limit point must be 0. Hence, taking the limit as $k \to \infty$, we obtain:

$$0 = \frac{\mathcal{I}}{\sigma} \frac{\underline{z}^{\sigma-1}}{f^c} - \sum_{i=0}^{\infty} M^{(i)} \int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E dz - \int z^{\sigma-1} m^I dz \tag{63}$$

$$\sum_{i=0}^{\infty} M^{(i)} = E = \frac{\mathcal{I}}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E dz} - \frac{\int z^{\sigma-1} m^I dz}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E dz} \tag{64}$$

which is clearly the same entry mass as in the simultaneous entry game (see eq. 6).

Equivalence of measure μ

Assuming that μ^E is a bounded density function, it is clear that $m^{(k)} := \sum_{i=0}^{k} M^{(i)} \mu^E(z) \mathbb{I}_{\{z \geq \underline{z}\}}$ converges to $m^E := E\mu^E(z)\mathbb{I}_{\{z \geq \underline{z}\}}$ as $k \to \infty$, given that the partial sums of the entry mass series form a Cauchy sequence. The limiting density function $m = m^I + E\mu^E(z)\mathbb{I}_{\{z \geq \underline{z}\}}$ is the same density function as in the simultaneous entry game equilibrium. ■

Proof of Lemma 1. We note that the existence of the incumbent density function m^I does not

matter for the determination of the cutoff. To see this, let

$$f^e = \mathbb{E}_{\mu^E}[\max\{\pi(z,m),0\}] \tag{65}$$

$$= \int_{\{z\geq \underline{z}\}} \frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{E\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz + \int_{\{z\geq \underline{z}\}} z^{\sigma-1} m^I dz} - f^c \mu^E dz \tag{66}$$

$$= \frac{\mathcal{I}}{\sigma} \frac{\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz}{E\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz + \int_{\{z\geq \underline{z}\}} z^{\sigma-1} m^I dz} - f^c p_E(\underline{z}) \tag{67}$$

$$= \frac{\mathcal{I}}{\sigma} \frac{\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz}{\left(\frac{\mathcal{I}}{\sigma f^c}\frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^E dz} - \frac{\int z^{\sigma-1} m^I dz}{\int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^E dz}\right)\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz + \int_{\{z\geq \underline{z}\}} z^{\sigma-1} m^I dz} - f^c p_E(\underline{z}) \tag{68}$$

Simplifying terms, and using the fact that $\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz = \mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}] p_E(\underline{z})$ we obtain

$$\frac{f^e}{f^c} = p_E(\underline{z})\left(\frac{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}]}{\underline{z}^{\sigma-1}} - 1\right). \tag{69}$$

■

Proof of Proposition 1. Start by noticing that

$$\int z^{\sigma-1} m_3 \, dz = \int z^{\sigma-1}\left(E_1 \mu^E \mathbf{1}_{\{z\geq \underline{z}_2\}} + E_3 \mu^E \mathbf{1}_{\{z\geq \underline{z}_3\}}\right) dz \tag{70}$$

$$= E_1 \int_{\{z\geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz + E_3 \int_{\{z\geq \underline{z}_3\}} z^{\sigma-1}\mu^E dz \tag{71}$$

$$= E_1 \int_{\{z\geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz + E_1 \left(1 - \frac{\int_{\{z\geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z\geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_{\{z\geq \underline{z}_3\}} z^{\sigma-1}\mu^E dz \tag{72}$$

$$= E_1 \int_{\{z\geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz \tag{73}$$

$$= \int z^{\sigma-1} m_1 \, dz. \tag{74}$$

Therefore, the factor $\int z^{\sigma-1} m_\tau \, dz$ remains unchanged in before and after the cycle. Next,

consider some $a \geq \underline{z}_2 > \underline{z}_1$, then

$$\frac{\int_a^\infty m_3(z)dz}{\int m_3(z)dz} = \frac{E_1 \int_a^\infty \mu^E(z)dz + E_3 \int_a^\infty \mu^E(z)dz}{\int m_3(z)dz} \tag{75}$$

$$= \frac{E_1 \int_a^\infty \mu^E(z)dz + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_a^\infty \mu^E(z)dz}{\int m_3(z)dz} \tag{76}$$

$$= \frac{E_1 \int_a^\infty \mu^E(z)dz + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_a^\infty \mu^E(z)dz}{E_1 \int_{\underline{z}_2}^\infty \mu^E(z)dz + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_{\underline{z}_1}^\infty \mu^E(z)dz} \tag{77}$$

$$> \frac{E_1 \int_a^\infty \mu^E(z)dz + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_a^\infty \mu^E(z)dz}{E_1 \int_{\underline{z}_1}^\infty \mu^E(z)dz + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_{\underline{z}_1}^\infty \mu^E(z)dz} \tag{78}$$

$$= \frac{\int_a^\infty \mu^E(z)dz}{\int_{\underline{z}_1}^\infty \mu^E(z)dz} \tag{79}$$

$$= \frac{\int_a^\infty m_1(z)dz}{\int m_1(z)dz}. \tag{80}$$

Likewise, for $\underline{z}_1 \leq a < \underline{z}_2$

$$1 - \frac{\int_a^\infty m_3(z)dz}{\int m_3(z)dz} = \frac{\int_0^a m_3(z)dz}{\int m_3(z)dz} \tag{81}$$

$$= \frac{E_1 \int_a^a \mu^E(z)dz + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_{\underline{z}_1}^a \mu^E(z)dz}{E_1 \int_{\underline{z}_2}^\infty \mu^E(z)dz + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_{\underline{z}_1}^\infty \mu^E(z)dz} \tag{82}$$

$$\leq \frac{E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_{\underline{z}_1}^a \mu^E(z)dz}{E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) \int_{\underline{z}_1}^\infty \mu^E(z)dz} \tag{83}$$

$$= \frac{\int_0^a m_1(z)dz}{\int m_1(z)dz} = 1 - \frac{\int_a^\infty m_1(z)dz}{\int m_1(z)dz}, \tag{84}$$

so that again $\frac{\int_a^\infty m_3(z)dz}{\int m_3(z)dz} \geq \frac{\int_a^\infty m_1(z)dz}{\int m_1(z)dz}$. Thus, the probability density function $\frac{m_3(z)}{\int m_3(z)dz}$ strictly first-order dominates the probability density function $\frac{m_1(z)}{\int m_1(z)dz}$. Therefore, their respective expectations, which by the Layer-Cake-Representation Theorem can be calculated as follows, are related by

$$\int \frac{\int_a^\infty m_1(z)dz}{\int m_1(z)dz} da < \int \frac{\int_a^\infty m_3(z)dz}{\int m_3(z)dz} da.$$

However, this is equivalent to

$$\frac{\int\int_a^\infty m_1(z)dzda}{\int m_1(z)dz} = \frac{\int z^{\sigma-1}m_1(z)dz}{\int m_1(z)dz} < \frac{\int z^{\sigma-1}m_3(z)dz}{\int m_3(z)dz} = \frac{\int\int_a^\infty m_3(z)dzda}{\int m_3(z)dz}.$$

and combined with the first part of this proof, we obtain $\int m_1(z)dz > \int m_3(z)dz$.

■

Proof Proposition 2. Note first that in the absence of fixed cost of entry, f^e, Equation 15 determines E^{ss} directly: there is no additional labor freed up from lower entry expenses. Hence, the entry process terminates immediately and jumps to E^{ss}. Rewrite Equation 15 now as

$$E_3 = \frac{R_3}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz} - E_1\frac{\int_{\{z\geq\underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz} \tag{85}$$

$$= \frac{R_1}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz} - E_1\frac{\int_{\{z\geq\underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz} + (R_3-R_1)\frac{1}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz} \tag{86}$$

$$= E_1\left(1 - \frac{\int_{\{z\geq\underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right) + (R_3-R_1)\frac{1}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz} \tag{87}$$

where R_τ is steady-state GE income (i.e., without fixed cost of entry). Note that

$$(R_3 - R_1) = \frac{\sigma}{\sigma-1}f^c(E_1 p_E(\underline{z}_1) - (E_1 p_E(\underline{z}_2) + E_3 p_E(\underline{z}_1))) \tag{88}$$

$$= \frac{\sigma}{\sigma-1}f^c(M_1 - M_3). \tag{89}$$

Suppose that $(R_3 - R_1) \leq 0$, then $M_3 \geq M_1$. Since $E_3 \leq E_1\left(1 - \frac{\int_{\{z\geq\underline{z}_2\}} z^{\sigma-1}\mu^E dz}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz}\right)$, following the steps of the proof of Proposition 1, we know that $\int z^{\sigma-1}m_3 dz \leq \int z^{\sigma-1}m_1 dz$. But then, it also holds that $\int z^{\sigma-1}\mu_3 dz \leq \int z^{\sigma-1}\mu_1 dz$, i.e., the average productivity of a firm active in the market is lower post cycle. By construction of the exit and entry process (clipping the left tail of the productivity distribution in period 2 and allowing entry of average firms in period 3) this cannot be. Hence, $(R_3 - R_1) > 0$ and thus

$$M_3 < M_1, \text{ and } \int z^{\sigma-1}m_3 dz > \int z^{\sigma-1}m_1 dz. \tag{90}$$

Consequently, $L_3^d > L_1^d$ holds, too. The output ratio between pre- and post-crisis steady-state depending on q is then given by

$$\frac{Y_3}{Y_1}(q) = \left[\frac{M_3}{M_1}\right]^{q-\frac{1}{\sigma-1}}\left[\frac{L_3^d}{L_1^d}\right]\frac{(\int z^{\sigma-1}m_3 dz)^{1/(\sigma-1)}}{(\int z^{\sigma-1}m_1 dz)^{1/(\sigma-1)}}, \tag{91}$$

and none of the equilibrium quantities on the right-hand-side depend on q. It follows that $\frac{Y_3}{Y_1}\left(\frac{1}{\sigma-1}\right) > 1$ for the CES case, and there exists a unique $q^* > \frac{1}{\sigma-1}$ for which $\frac{Y_3}{Y_1}(q^*) = 1$. ■

Proof of Proposition 3 and Figure 2. We prove the characteristics of Figure 2, panels A and B,

first. To this end, note that $\underline{z}_2$ is a strictly increasing, monotonic function of f_h^c defined by the zero profit condition (4), with $\underline{z}_2 \to \infty$ as $f_h^c \to \infty$, and any analysis in terms of $\underline{z}_2$ carries over in terms of f_h^c. Now note that

$$\begin{aligned} M_3 &= E_1 p_E(\underline{z}_2) + E_3 p_E(\underline{z}_1) \text{ and} \\ E_3 &= \left(1 + \frac{\underline{z}_1}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}_1](\sigma-1)}\right)^{-1} \\ & \quad E_1 \left[1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E dz}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E dz} + (p_E(\underline{z}_1) - p_E(\underline{z}_2)) \frac{\underline{z}_1}{\left(\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E dz\right)(\sigma-1)}\right], \end{aligned} \tag{92}$$

which follows from substituting eq. (89) into eq. (87) and rearranging for E_3. Using eq. (92), we calculate

$$\frac{\partial}{\partial \underline{z}_2} M_3 = E_1 \mu^E(\underline{z}_2) \left(\frac{\frac{\underline{z}_2 + \frac{1}{\sigma-1}\underline{z}_1}{\mathbb{E}_{\mu^E}[z^{\sigma-1} | z \geq \underline{z}_1]}}{1 + \frac{1}{\sigma-1} \frac{\underline{z}_1}{\mathbb{E}_{\mu^E}[z^{\sigma-1} | z \geq \underline{z}_1]}} - 1 \right). \tag{93}$$

If $\underline{z}_2 = \underline{z}_1$, one verifies that the term in parentheses is negative, while for large enough $\underline{z}_2$, it is positive. By continuity, there exists a $\underline{z}_2^*$, such that

$$\frac{\partial M_3}{\partial \underline{z}_2} \begin{cases} < 0 \text{ if } \underline{z}_2 < \underline{z}_2^* \\ = 0 \text{ if } \underline{z}_2 = \underline{z}_2^* \\ > 0 \text{ if } \underline{z}_2 > \underline{z}_2^*. \end{cases} \tag{94}$$

Now, partial derivatives are

$$\frac{\partial [M_3/M_1]^{q-q^{CES}}}{\partial \underline{z}_2} = (q - q^{CES}) \left[\frac{M_3}{M_1}\right]^{q-q^{CES}-1} \frac{1}{M_1} \frac{\partial M_3}{\partial \underline{z}_2}, \tag{95}$$

$$\frac{\partial (L_3^d / L_1^d)}{\partial \underline{z}_2} = \frac{-f_l^c}{L_1^d} \frac{\partial M_3}{\partial \underline{z}_2}, \tag{96}$$

$$\frac{\partial}{\partial \underline{z}_2} \left[\frac{\int z^{\sigma-1} m_3 dz}{\int z^{\sigma-1} m_1 dz}\right]^{1/(\sigma-1)} = \left[\frac{\int z^{\sigma-1} m_3 dz}{\int z^{\sigma-1} m_1 dz}\right]^{1/(\sigma-1)-1} \frac{1}{\sigma-1} \frac{\underline{z}_1}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E dz} \frac{-1}{E_1(\sigma-1)} \frac{\partial M_3}{\partial \underline{z}_2}, \tag{97}$$

from which we see that eq. (96, 97) have the reverse sign order of (94), while eq. (95) has the same sign order, iff $q > q^{CES}$. Combined with the facts that each factor is 1 at f_l^c (no crisis in phase 2) and at $f_h^c \to \infty$, the graphs in panels A and B of Figure 2 follow. Furthermore, if $q \leq q^{CES}$, then the graph for q' in panel C follows, and $(Y_3/Y_1)(q, f_h^c) \geq 1$ for all $f_h^c \geq f_l^c$.

Assume now that $q > q^{CES}$. For reference, note that

$$\frac{\int z^{\sigma-1} m_1 dz}{\int z^{\sigma-1} m_3 dz} = 1 + \frac{M_1 - M_3}{E_1(\sigma-1)} \frac{\underline{z}_1}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E dz} \tag{98}$$

is the ratio of productivity integrals. The output ratio only depends on f_h^c through $\underline{z}_2$, and it depends on $\underline{z}_2$ only through M_3. Consider the output ratio now as a function of M_3 given q, i.e. $\frac{Y_3}{Y_1}(M_3;q)$, and note that the domain of $M_3(\underline{z}_2)$ is $[M_{min}, M_1]$ for some M_{min}, and that M_3 runs from M_1 monotonically down to $M_{min} := M_3(\underline{z}_2^*)$ and back up to M_1 in the limit. We write

$$\frac{Y_3}{Y_1}(M_3;q) = F_1(M_3,q)F_2(M_3)F_3(M_3),$$

corresponding to its three contributing factors, and take the derivative

$$\frac{\partial}{\partial M_3}\frac{Y_3}{Y_1} = (F_1'(M_3,q)/F_1(M_3,q) + F_2'(M_3)/F_2(M_3) + F_3'(M_3)/F_3(M_3))\frac{Y_3}{Y_1}(M_3;q) \tag{99}$$

$$=: g(M_3,q)\frac{Y_3}{Y_1}(M_3;q). \tag{100}$$

The derivative vanishes iff (substituting in)

$$0 = (F_1'(M_3,q)/F_1(M_3,q) + F_2'(M_3)/F_2(M_3) + F_3'(M_3)/F_3(M_3)) \tag{101}$$

$$= \frac{(q-q^{CES})}{M_3} \underbrace{-\frac{f_l^c}{L_3^d(M_3)} - \frac{\int z^{\sigma-1}m_1dz}{\int z^{\sigma-1}m_3dz}\frac{1}{\sigma-1}\frac{\underline{z}_1}{\int_{\{z\geq\underline{z}_1\}} z^{\sigma-1}\mu^E dz}\frac{1}{E_1(\sigma-1)}}_{:=-s(M_3)/M_3} \tag{102}$$

$$= v(M_3,q), \tag{103}$$

which is strictly decreasing in M_3 (to see that the third summand is decreasing cf. eq. 98). Therefore, if M_3^* satisfying eq. (101) exists, it is unique. Since $M_{min} \leq M_3 \leq M_1$, it can only exist if $v(M_{min},q) \geq 0 \geq v(M_1,q)$. Clearly, if q is large enough, then v must be globally positive and if q gets close to q^{CES}, then v is negative for some, and eventually all $M_3 \in [M_{min}, M_1]$. One checks that an interior critical point exists iff $q \in (\underline{q}, \overline{q})$, where $\underline{q} = q^{CES} + s(M_1)$ and $\overline{q} = q^{CES} + s(M_{min})$. On the one hand, if $q \leq \underline{q}$, even though there is LOV relative to CES, the output ratio is decreasing in the number of post-crisis varieties, and any crisis is cleansing. On the other hand, if $q \geq \overline{q}$, then the output ratio is globally increasing in the number of varieties, and all crises come at an output loss. For the region in between, $q \in (\underline{q}, \overline{q})$, there is an extremum at some M_3^*. This is a maximum because Y_3/Y_1 satisfies second-order conditions at M_3^* for such q. See this as follows. As argued, $g(M_3;q)$ is decreasing, so $g' < 0$. The second derivative of Y_1/Y_3 evaluated at M_3^* is

$$g'(M_3^*;q)\frac{Y_3}{Y_1}(M_3^*;q) + g(M_3^*;q)\frac{Y_3}{Y_1}'(M_3^*;q) = g'(M_3^*;q)\frac{Y_3}{Y_1}(M_3^*;q) + g(M_3^*;q) \times 0 < 0. \tag{104}$$

Consequently, there is an optimal number of firms in phase 3, $M_3^* \in (M_{min}, M_1)$, created by an "optimally cleansing business cycle". Because the mapping between f_h^c and M_3 is parabola-shaped, its inverse image always has two elements, and any optimal post-crisis number of firms can be created by two different values of f_h^c. This implies the graph displayed in panel C for q''. It is easy to check that the output ratio uniformly decreases in q, which explains why the

graph for q''' lies below. Therefore, as the curve of Y_3/Y_1 shifts down, there must be an interval $(q_\circ, q^\circ) \subseteq (\underline{q}, \overline{q})$ where the output ratio crosses 1 multiple times, creating two nonempty, disjunct intervals in which crises are cleansing.

The partial equilibrium result follows immediately since F_2 and F_3 are independent of f_h^c and of q, by Proposition 1. ■

Proof Proposition 6. We start by characterizing the optimal policy in the case of entry and then study the one of pure exit.

Entry: The interior first-order condition on entry is

$$E^{SP}(\underline{z}; m^I, q) = \frac{\bar{L} - If^c}{p_E(\underline{z})f^c + f^e}\frac{1}{\sigma} - \frac{\int_{\{z\geq\underline{z}\}} z^{\sigma-1}m^I dz}{\int_{\{z\geq\underline{z}\}} z^{\sigma-1}\mu^E dz}\frac{\sigma-1}{\sigma} + \Delta E^{SP}, \tag{105}$$

which equals the CES GE-entry plus a difference term, ΔE^{SP}. The latter is given by

$$\Delta E^{SP} = \left(q - \frac{1}{\sigma-1}\right) L^d \frac{\sigma}{\sigma-1}(p_E(\underline{z})f^c + f^e)^{-1} Q(m^I, \underline{z}), \tag{106}$$

where $Q(m^I, \underline{z})$ is the ratio between the average productivity of firms post-entry and the average productivity of (successful) entrants. It is immediate that if the household has CES preferences ($q = 1/(\sigma-1)$), then $\Delta E^{SP} = 0$, and the market outcome is constrained efficient. Over time, a stock of firms builds up (see discussion in Section 3.1), so $\sum_{t\geq 0} E_t^{SP} \to E^{SP,ss}$, where we call $E^{SP,ss}$ the (steady-state) total mass of entrants for the social planner problem. As E_t^{SP} tends to 0, Q tends to 1, and the incumbent mass building up is, eventually, $\widetilde{I} = I + E^{SP,ss}$. (Similarly, the measure of incumbents is stacked up.) Substituting these into eq. (105) and rearranging, we find

$$E^{SP,ss} = \frac{(\bar{L} - If^c)q}{f^c p_E(\underline{z})(1+q) + f^e} - \frac{\int_{\{z\geq\underline{z}\}} z^{\sigma-1}m^I dz}{\int_{\{z\geq\underline{z}\}} z^{\sigma-1}\mu^E dz}\frac{f^c p_E(\underline{z}) + f^e}{f^c p_E(\underline{z})(1+q) + f^e}. \tag{107}$$

The socially optimal number of entrants is increasing in taste-for-varieties, q, and decreasing in the mass and productivity of active incumbents. Fixed cost of entry, f^e, stop to matter if and only if $q \to \infty$. Comparing this to eq. (15), it so follows that the general equilibrium steady-state mass converges to $E^{SP,ss}$ if and only if $q = \frac{1}{\sigma-1}$. It also follows that in an economy with high $q > \frac{1}{\sigma-1}$, the equilibrium number of entrants is too small compared to the short-run optimal number, so $E^{GE} < E^{SP}$. Conversely, for small q, the equilibrium number of varieties is larger than the short-run efficient number.

Using the first order condition on E^{SP}, the interior first order condition on $\underline{z}^{SP} = \underline{z}^{SP}(E; q, m^I)$

yields:

$$
\begin{aligned}
&(\bar{L} - If^c)\left(\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz\right) \\
&\left(\frac{f^e}{f^c p_E(\underline{z}) + f^e}\frac{1}{\sigma}\frac{J}{K} - \left[1 - \frac{\underline{z}^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right]\frac{1}{\sigma}\frac{\sigma - \frac{J}{K}}{\sigma - 1}\right) \\
&= \left(\int_{\{z\geq \underline{z}\}} z^{\sigma-1} m^I dz\right)(f^c p_E(\underline{z}) + f^e) \\
&\left(\frac{f^e}{f^c p_E(\underline{z}) + f^e}\left(1 - \frac{\sigma - 1}{\sigma}\frac{1}{K}\right) + \frac{1}{\sigma}\frac{1}{K}\left[1 - \frac{\underline{z}^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right]\right).
\end{aligned} \tag{108}
$$

The fraction J/K and J and K individually take value 1 precisely when $q = \frac{1}{\sigma-1}$. Only in this case, are the terms in the large parentheses both equivalent to $\frac{f^e \int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz}{\delta_f} - \left(\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz - \underline{z}^{\sigma-1} p_E(\underline{z})\right)$. Eq. (108) is then equivalent to eq. (7), the GE cutoff.

In general, we need to unpack J and K to see how the cutoff is determined in a non-CES world. Here, note that $J = 1 + [(\sigma-1)q - 1]Q$ and $K = 1 + \frac{1}{\sigma}[(\sigma-1)q - 1]Q$. If we approach a steady state (or drop the incumbents), $Q \to 1$. Then eq. (108) simplifies:

$$
\frac{f^e}{f^c p_E(\underline{z}) + f^e}(\sigma - 1)q - \left[1 - \frac{\underline{z}^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right] = 0, \tag{109}
$$

and under mild regularity conditions on μ^E, we can conclude that $\frac{\partial \underline{z}}{\partial q} < 0$. Intuitively, if the economy has a strong preference for varieties, the entry barriers for new establishments should be lower. Eq. (109) also holds with entrants, if all entrants were drawn from the same distribution, i.e., if $m^I \propto \mu^E \mathbb{I}(z \geq \underline{z})$.

Exit: Suppose incumbent firms are distributed according to $m^I = M_0 \mu^E \mathbb{I}(z \geq \underline{z}_0)$, and that fixed costs f^c are such that the optimal cutoff $\underline{z}$ is larger than $\underline{z}_0$, and that optimal entry, therefore, is nil. Hence, the relevant production function to maximize is

$$
Y = (M_0)^q \, p_E(\underline{z})^{q - \frac{1}{\sigma-1}} \left(\bar{L} - f^c(M_0 p_E(\underline{z}))\right)\left(\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz\right)^{\frac{1}{\sigma-1}}, \tag{110}
$$

where taking first order conditions yields

$$
f^c M_0 p_E(\underline{z}^{SP}) = \frac{\bar{L} - f^c M_0 p_E(\underline{z}^{SP})}{\sigma - 1}\left([q(\sigma-1) - 1] + \frac{(\underline{z}^{SP})^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{SP}]}\right), \tag{111}
$$

which is equal to the market equilibrium outcome only if $q = \frac{1}{\sigma-1}$. For larger q, the cutoff is again optimally set lower than in the equilibrium allocation. ■

Proof of Proposition 7. In the steady-state targeted by the social planner, eq. (111) must hold once entry has subsided, i.e., with $E^{SP} = 0$ and some $\underline{z}^{SP}$. We now evaluate eq. (4) at this

$\underline{z}^{SP}$, whereby we have swapped f^c for $\delta^c f^c$ beforehand. Solving for δ^c yields the result. ■

Proof of Proposition A.2. We write the equilibrium conditions for an economy with aggregate productivity shock A as below and index all endogenous quantities by A:

$$f^e \geq \int_{\underline{z}_A} \left[\frac{R_A}{\sigma} \frac{(Az)^{\sigma-1}}{\int_{\underline{z}_A} (Az)^{\sigma-1} m_A(z)\, \mathrm{d}z} - f^c \right] \mu^E(z)\, \mathrm{d}z \tag{112}$$

$$0 = \frac{R_A}{\sigma} \frac{(A\underline{z}_A)^{\sigma-1}}{\int_{\underline{z}_A} (Az)^{\sigma-1} m_A(z)\, \mathrm{d}z} \tag{113}$$

Note that A cancels in both equations above, such that the system is equivalent to the equilibrium equations of our baseline model if and only if $R_A = R$. To establish this equality, note

$$\begin{aligned} R_A &= \bar{L} + \Pi_A \\ &= \bar{L} + R_A/\sigma - \left(\int m_A(z) dz \right) f^c \quad \text{(integrate } \pi_A(z,m)\text{)}. \end{aligned} \tag{114}$$

Eqs. (112–114) are then exactly the same as in the baseline equilibrium, and thus any $(m, \underline{z}, R)$ satisfying the baseline equilibrium also satisfy (112–114). We can give more detail. In the case with entry, use $m_A = (E_A \mu^E + m^I)\mathbb{I}_{z \geq \underline{z}_A}$ in eqs. (112–113) to derive eq. (7), so $\underline{z}_A = \underline{z}$. Use eq. (114) in (113) and the definition of m_A to see that $E_A = E$, hence $m_A = m$. Clearly, then also $R_A = R$. In the case without entry, $E_A = 0 = E$ and eq. (113) alone pins down $\underline{z}_A$, after substituting for m_A. Since this equation is again independent of A, $\underline{z}_A = \underline{z}$.

■

Proof of Proposition A.4. For (ii), a very similar argument to Proposition A.2 applies. Note that individual firms' profits in eq. (55) are independent of f^p. To see this, note that the only element in eq. (55) that could depend on f^p is L^d. Using eq. (52) in steady state, i.e. with $E = 0$, we can write L^d as:

$$L^d = \bar{L} - M f^c - N f^p \tag{115}$$

Integrating eq. (50), and substituting it in the above, we obtain:

$$L^d = \bar{L} - M f^c - \frac{L^d}{(\eta - 1) f^p} f^p. \tag{116}$$

which gives us that:

$$L^d = \frac{\eta - 1}{\eta} (\bar{L} - M f^c) \tag{117}$$

As firm profits are independent of f^p, entry and exit decisions are unaffected by f^p. Therefore, movements in f^p do not change $(m, \underline{z})$, and there is no path-dependency and, thus, no long-run effects.

For (i) (a), firstly, note that in PE $\mathcal{I}$ is exogenously fixed. Therefore, $\mathcal{I}_3 = \mathcal{I}_1$, and given that $L_\tau^d = \frac{\sigma-1}{\sigma}\mathcal{I}_\tau$, also $L_3^d = L_1^d$. Secondly, note that the proof of Proposition 1 applies with $z^{\phi-1}$ instead of $z^{\sigma-1}$, which has no material effect. Therefore, following the steps of Proposition 1, we obtain that $Z_3 = Z_1$ and $M_3 < M_1$. In addition, given that by integrating eq. (50) we have $N = \frac{L^d}{(\eta-1)f^p}$, we obtain that $N_3 = N_1$. The condition on the q's immediately follows.

For (i) (b), note that the proof of Proposition 2 applies with $z^{\phi-1}$ instead of $z^{\sigma-1}$, which has no material effect, and with:

$$R_3 - R_1 = \frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}f^c(M_1 - M_3) \tag{118}$$

where $R_3 - R_1$ is obtained by using that $R_\tau = \frac{\sigma}{\sigma-1}L_\tau^d$. Using the argumentation in the proof of Proposition 2, we obtain that $M_3 < M_1$ and $Z_3 > Z_1$. For the multi-product setting, we have that:

$$L^d = \frac{\eta-1}{\eta}(\bar{L} - Mf^c) \tag{119}$$

$$N = \frac{\bar{L} - Mf^c}{\eta f^p} \tag{120}$$

where to obtain N, we substitute L^d in $N = \frac{L^d}{(\eta-1)f^p}$. Therefore, we can conclude that: $L_3^d > L_1^d$, $N_3 > N_1$. The condition on the q's immediately follows.

For (i) (c), firstly note that: $Y_\tau^{CES} := N_\tau^{q_V} L_\tau^d Z_\tau$ and thus:

$$Y_\tau^{CES} \propto (\bar{L} - Mf^c)^{1+q_V} Z_\tau.$$

Therefore, the only changes to eq. (20) are that within the parenthesis:

1. instead of $(q - q^{CES})$ we have $(q_F - \frac{1}{\sigma-1}) - (q_V - \frac{1}{\eta-1})$
2. for $\frac{\partial \log Y_3^{CES}}{\partial \log M_3}$ we just have that the labor cleansing effect is amplified. Recall that in the single-product firm case, we had $Y_\tau^{CES} = (\bar{L} - Mf^c)Z_\tau$. However, the elasticity is still increasing (in absolute terms) in M_3.

Therefore, the proof of Proposition 3 applies, with the only difference that the q-interval in which the long-run effects are dependent on f_h^c needs to have that $q_\circ - \frac{1}{\sigma-1} > q_V - \frac{1}{\eta-1}$, as stated in the proposition.

■

Proof of Proposition A.3. In a state of entry, we obtain

$$\frac{f^e}{f^c} = \int_{\underline{z}}^{\infty}\left[\int\left(\frac{z+\varepsilon}{\underline{z}}\right)^{\sigma-1}\psi(\varepsilon)d\varepsilon - 1\right]\mu^E(z)dz = \left[\frac{\mathcal{Z}(\mu,\psi)}{\underline{z}^{\sigma-1}} - p_E(\underline{z})\right] \tag{121}$$

from which we see immediately that cutoffs are independent of incumbents. Like in the baseline, equilibrium is established through entry E_ε which shifts $\mathcal{Z}$. For the second part of the

proposition, note that a firm which enters with productivity z in period k of a cycle phase $\tau \in \{1,2,3\}$ is still alive in period $t > k$ with probability $\mathbb{P}_\psi(z+\varepsilon < \underline{z}_\varepsilon)^{t-k}$, which converges to 0 if and only if $\mathbb{P}_\psi(z+\varepsilon < \underline{z}_\varepsilon) > 0$. Therefore, all incumbents at the beginning of the period for whom $\mathbb{P}_\psi(z+\varepsilon < \underline{z}_\varepsilon) > 0$ holds true will also exit eventually, and only if there is a positive measure of incumbents who never exit, path dependency holds. ■

Proof of Proposition A.1. With fixed costs paid in units of the final good, the zero profit condition characterizing entry becomes:

$$\sigma f^c \int z^{\sigma-1} m_\tau dz = \frac{R}{P_\tau}\, \underline{z}_\tau^{\sigma-1} \tag{122}$$

where we use that revenue is constant: $R = \frac{\sigma}{\sigma-1}\bar{L}\ \forall\tau$, because $L_\tau^d = \bar{L}\ \forall\tau$ when fixed costs are paid in units of the final good.

For $q = q^{CES}$, starting from eq. (122) we obtain:

$$\begin{aligned}\int z^{\sigma-1} m_\tau \, \mathrm{d}z &= \frac{R}{\sigma P_\tau^{CES} f^c}\, \underline{z}_\tau^{\sigma-1} \\ &= \frac{\bar{L}}{(\sigma-1) f^c}\, \underline{z}_\tau^{\sigma-1} \left(\int p(z)^{1-\sigma} m_\tau dz\right)^{\frac{1}{\sigma-1}} \\ &= \frac{\bar{L}}{\sigma f^c}\, \underline{z}_\tau^{\sigma-1} \left(\int z^{\sigma-1} m_\tau dz\right)^{\frac{1}{\sigma-1}}\end{aligned}$$

Therefore, we have that:

$$\int z^{\sigma-1} m_\tau \, \mathrm{d}z = \left(\frac{\bar{L}}{\sigma f^c}\, \underline{z}_\tau^{\sigma-1}\right)^{\frac{\sigma-1}{\sigma-2}}$$

By Lemma 1, which also applies here, $\underline{z}_3 = \underline{z}_1$. Therefore, $\int z^{\sigma-1} m_3 dz = \int z^{\sigma-1} m_1 dz$. By eq. (39), we obtain $Y_3 = Y_1$.

For $q \neq q^{CES}$, we just need to determine whether E_3 is increasing or decreasing in q given that $\int z^{\sigma-1} m_3\, dz$ is increasing in E_3. Starting again from eq. (122), with $\tau = 3$, we obtain:

$$E_3 = \frac{R}{\sigma\, P_3\, f^c} \frac{\underline{z}_1^{\sigma-1}}{\int_{\underline{z}_1}^\infty z^{\sigma-1} \mu^E(z) dz} - \frac{E_1 \int_{\underline{z}_2}^\infty z^{\sigma-1} \mu^E(z) dz}{\int_{\underline{z}_1}^\infty z^{\sigma-1} \mu^E(z) dz}$$

Adding and subtracting E_1, this becomes:

$$E_3 = \left(\frac{1}{P_3} - \frac{1}{P_1}\right)\left[\frac{R}{\sigma f^c} \frac{\underline{z}_1^{\sigma-1}}{\int_{\underline{z}_1}^\infty z^{\sigma-1} \mu^E(z) dz}\right] + E_1 \left[1 - \frac{\int_{\underline{z}_2}^\infty z^{\sigma-1} \mu^E(z) dz}{\int_{\underline{z}_1}^\infty z^{\sigma-1} \mu^E(z) dz}\right]$$

with $P_\tau = M_\tau^{-\left(q - \frac{1}{\sigma-1}\right)} \underbrace{\left(\int p(z)^{1-\sigma} m_\tau(z) dz\right)^{\frac{1}{1-\sigma}}}_{P^{CES}}$, and $p(z) = \frac{\sigma}{\sigma-1}\frac{1}{z}$.

Substituting for P_τ and M_τ, we obtain:

$$E_3 = \left[\frac{(E_1 p_E(\underline{z}_2) + E_3 p_E(\underline{z}_1))^{q-\frac{1}{\sigma-1}}}{\left(E_1 \int_{\underline{z}_2}^{\infty} p(z)^{1-\sigma}\mu^E(z)dz + E_3 \int_{\underline{z}_1}^{\infty} p(z)^{1-\sigma}\mu^E(z)dz\right)^{\frac{1}{1-\sigma}}} - \frac{(E_1 p_E(\underline{z}_1))^{q-\frac{1}{\sigma-1}}}{\left(E_1 \int_{\underline{z}_1}^{\infty} p(z)^{1-\sigma}\mu^E(z)dz\right)^{\frac{1}{1-\sigma}}}\right] \left[\frac{R}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\underline{z}_1}^{\infty} z^{\sigma-1}\mu^E(z)dz}\right] + E_1\left[1 - \frac{\int_{\underline{z}_2}^{\infty} z^{\sigma-1}\mu^E(z)dz}{\int_{\underline{z}_1}^{\infty} z^{\sigma-1}\mu^E(z)dz}\right]$$

Let

$$F = \left[\frac{(E_1 p_E(\underline{z}_2) + E_3 p_E(\underline{z}_1))^{q-\frac{1}{\sigma-1}}}{\left(E_1 \int_{\underline{z}_2}^{\infty} p(z)^{1-\sigma}\mu^E(z)dz + E_3 \int_{\underline{z}_1}^{\infty} p(z)^{1-\sigma}\mu^E(z)dz\right)^{\frac{1}{1-\sigma}}} - \frac{(E_1 p_E(\underline{z}_1))^{q-\frac{1}{\sigma-1}}}{\left(E_1 \int_{\underline{z}_1}^{\infty} p(z)^{1-\sigma}\mu^E(z)dz\right)^{\frac{1}{1-\sigma}}}\right] \left[\frac{R}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\underline{z}_1}^{\infty} z^{\sigma-1}\mu^E(z)dz}\right] + E_1\left[1 - \frac{\int_{\underline{z}_2}^{\infty} z^{\sigma-1}\mu^E(z)dz}{\int_{\underline{z}_1}^{\infty} z^{\sigma-1}\mu^E(z)dz}\right] - E_3.$$

Using the implicit function theorem, we have that:

$$\frac{dE_3}{dq} = \frac{\left[\frac{ln(M_1)M_1^{q-\frac{1}{\sigma-1}}}{P_1^{CES}} - \frac{ln(M_3)M_3^{q-\frac{1}{\sigma-1}}}{P_3^{CES}}\right]\left[\frac{R}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\underline{z}_1}^{\infty} z^{\sigma-1}\mu^E(z)dz}\right]}{\left[\frac{(q-\frac{1}{\sigma-1})M_3^{q-\frac{1}{\sigma-1}-1}p_E(\underline{z}_1)P_3^{CES} + \frac{1}{\sigma-1}P_3^{CES,\sigma}\int_{\underline{z}_1}^{\infty} p(z)^{1-\sigma}\mu^E(z)dz M_3^{q-\frac{1}{\sigma-1}}}{P_3^{CES,2}}\right]\left[\frac{R}{\sigma f^c}\frac{\underline{z}_1^{\sigma-1}}{\int_{\underline{z}_1}^{\infty} z^{\sigma-1}\mu^E(z)dz}\right] - 1}.$$

Simplifying the expression, we obtain:

$$\frac{dE_3}{dq} = \frac{E_1\left[ln(M_1) - ln(M_3)\frac{Y_3}{Y_1}\right]}{(q-\frac{1}{\sigma-1})\frac{\int z^{\sigma-1}\mu_3 dz}{\int z^{\sigma-1}\mu_1 dz} - \frac{\sigma-2}{\sigma-1}} \tag{123}$$

Let $\sigma \in (1,2)$.

Let $q > q^{CES}$. Suppose $Y_3 \leq Y_1$. By eq. (39), $\int z^{\sigma-1}m_3 dz \leq \int z^{\sigma-1}m_1 dz$. Then, $M_3 < M_1$, because $\int z^{\sigma-1}\mu_3 dz > \int z^{\sigma-1}\mu_1 dz$. Let $M_\tau > 1$, then the numerator of eq. (123) is positive. Given that $q > q^{CES}$, the denominator is also positive. But then $E_3 > E_3^{q=q^{CES}}$, thus $Y_3 > Y_1$. For $q << q^{CES}$, repeating the same steps, we obtain $\frac{dE_3}{dq} < 0$, so again $E_3 > E_3^{q=q^{CES}}$ and $Y_3 > Y_1$. Thus, there exists an interval a $\underline{q}$, s.t. for $q \in [0, \underline{q})$, output and welfare are higher post cycle, and for $q \in (\underline{q}, q^{CES})$ they are not, with equality holding for $q = \underline{q}$.

Let $\sigma > 2$.

Using the same logic as above, for $q < q^{CES}$, recessions are always cleansing, i.e. $Y_3 > Y_1$. In the case of $q > q^{CES}$, there exists a $\bar{q}$, s.t. for $q \in (\bar{q}, \infty)$, output and welfare are higher post cycle, and for $q \in (q^{CES}, \bar{q})$ they are not, with equality holding for $q = \bar{q}$.

■

Proof of Lemma 3. Since we are using the entry equation for positive entry, all derivatives are right-derivatives. By definition, $M = Ep_0(\underline{z}) + Ip_0(z^*)$. The entry condition determines

$E = E(\mathcal{I})$ such that

$$Ep_0(\underline{z}) + Ip_0(z^*) = \frac{\mathcal{I}}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}]} - Ip_0(z^*) \left(1 - \frac{\mathbb{E}[z^{\sigma-1} \mid z \geq z^*]}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right). \tag{124}$$

Letting

$$k(\underline{z}, z^*) = p_0(z^*) \left(1 - \frac{\mathbb{E}[z^{\sigma-1} \mid z \geq z^*]}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right), \tag{125}$$

we obtain

$$k(\underline{z}, z^*) \begin{cases} > 0 & \text{for } \underline{z} > z^*, \\ = 0 & \text{for } \underline{z} = z^*, \\ < 0 & \text{for } \underline{z} < z^*. \end{cases} \tag{126}$$

One now takes elasticities with respect to $\mathcal{I}$ to obtain the expression for $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}$. Next, substitute the distribution post-entry

$$\frac{Ep_0(\underline{z})\mu^E(z \mid z \geq \underline{z}) + Ip_0(z^*)\mu^E(z \mid z \geq z^*)}{Ep_0(\underline{z}) + Ip_0(z^*)} \tag{127}$$

into the integral of the average productivity, $\log \bar{z}$. After some algebra to write integrals as expectations, substitute out $E(\mathcal{I})$ to obtain

$$\log \bar{z} = \frac{1}{\sigma - 1} \left(\log \mathcal{I} - \log \sigma f^c + \log \underline{z} - \log[p_0(\underline{z}) E_{|I=0} + I\, k(\underline{z}, z^*)]\right) \tag{128}$$

$$= \frac{1}{\sigma - 1} \left(\log \mathcal{I} - \log \sigma f^c + \log \underline{z} - \log M\right) \tag{129}$$

with

$$E_{|I=0} = \frac{\mathcal{I}}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}] p_0(\underline{z})}. \tag{130}$$

Differentiating with respect to $\log \mathcal{I}$ yields the result. ■

Proof of Proposition 4. Using Lemma 3, note that $k(\underline{z}, \underline{z}) = 0$. Hence, $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} = 1$ and $\frac{\partial_+ \log \bar{z}}{\partial_+ \log \mathcal{I}} = 0$. ■

Proof of Proposition 5. Using Lemma 3, note that

$$\frac{\partial_+ \log Y}{\partial_+ \log \mathcal{I}} = q \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} + 1 + \frac{1}{\sigma - 1} \left(1 - \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}\right) \tag{131}$$

The LHS is the estimated coefficient from our regression. Define the implied $\widehat{q} = \beta - 1$. Then,

we have

$$\widehat{q} = \beta - 1 = \frac{1}{\sigma - 1} + \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}(q - q^{CES}) \tag{132}$$

If we have a measure of $\frac{1}{\sigma-1}$ we can write

$$\widehat{q} - \frac{1}{\sigma - 1} = \beta - 1 - \frac{1}{\sigma - 1} = \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}(q - q^{CES}). \tag{133}$$

Since $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} > 1$, we conclude that

$$\operatorname{sgn}\left(\widehat{q} - \frac{1}{\sigma - 1}\right) = \operatorname{sgn}\left(q - \frac{1}{\sigma - 1}\right).$$

■

D Ramsey Planner

To further motivate working with the short-run social planner, we take a look at an economy starting with no incumbents through the eyes of a Ramsey planner. She takes a (household) discount factor β and utility function u as given, and maximizes $\sum_{t\geq 0} \beta^t u(Y_t)$ under the constraint that $Y_t = Y(M, E)$. The term $Y(M, E)$ describes total output if a measure of $M\mu^E\mathbf{I}(z \geq \underline{z})$ former entrants, now incumbents, are present and a mass of E entrepreneurs newly flows into the market. She splits labor between new entrants and production and allocates production labor to each variety. Since the split of production labor remains efficient in equilibrium even in the presence of the aggregate externality (the proof in Dhingra and Morrow (2019) still applies), it suffices to consider a planner who chooses E_t and $\underline{z}_t$ to steer the entire economy while respecting the resource constraints. For tractability, we consider a social planner who takes the cutoff as given and constant. Taking the cutoff to be a parameter allows us to analyze the recursive formulation of the social planner problem given by

$$V(M; \underline{z}) = \max_{E\geq 0} u\big(Y(M, E)\big) + \beta V(M + E; \underline{z}), \text{ where} \tag{134}$$

$$Y(M, E) = L^d(E)\left(\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\mu^E dz\right)^{\frac{1}{\sigma-1}} (M + E)^q p_E(\underline{z})^{q-\frac{1}{\sigma-1}} \tag{135}$$

and $L^d = \bar{L} - (p_E(\underline{z})f^c + f^e)E - (Mp_E(\underline{z}))f^c \geq 0$ is production labor.

We differentiate with respect to E, use the envelope theorem, and impose that in the steady state, $E = E' = 0$ holds. The steady-state cumulative sum of entrants is given by eq. (36).

E Appendix: Estimation

E.1 Data treatment

The WIOD data contains information on gross output at current prices for all industries in a given year. It also provides a version of the same information at previous year prices (PYP). Define $S_t = P_t Y_t$ and $S_t^{t-1} = P_{t-1} Y_t$. We can write

$$\log \frac{S_t^{t-1}}{S_{t-1}^{t-1}} = \log P_{t-1} + \log Y_t - \log P_{t-1} - \log Y_{t-1} = \\ = \log Y_t - \log Y_{t-1} = \Delta \log Y_t. \tag{136}$$

E.2 Robustness analysis

Weighting by industry We now estimate q using weights based on the industry median log total sales across countries and years. This approach gives greater influence to industries with higher median total sales. As it can be seen from Table 3 and Figure 5, the point estimates are very similar to section 4, and q is significantly higher than q^{CES}.

Table 3: Estimation Results with Industry Weights

	(D)		(E)		(F)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^r_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log L^{d,r}_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log Y^r_{i,t}$
$\eta^{+,r}_{i,t}$	0.0890***		0.101***		
	(0.0209)		(0.0192)		
$\eta^{+,r}_{i,t-1}$	0.0967***		0.0646***		
	(0.0225)		(0.0191)		
$\Delta \log \mathcal{I}^r_{i,t}$		1.460***			1.535***
		(0.174)			(0.141)
$\Delta \log L^{d,r}_{i,t}$				1.648***	1.535***
				(0.236)	(0.141)
$P(q \leq q^{CES})$		.094		.039	.015
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	10067	10067	10083	10083	10067
F-Stat	25	70	25	49	
R^2	0.0323	0.0860	0.0426	0.0880	
F-Stat eq.(1)					1300
F-Stat eq.(2)					127
R^2 eq.(1)					0.0870
R^2 eq.(2)					0.0870

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

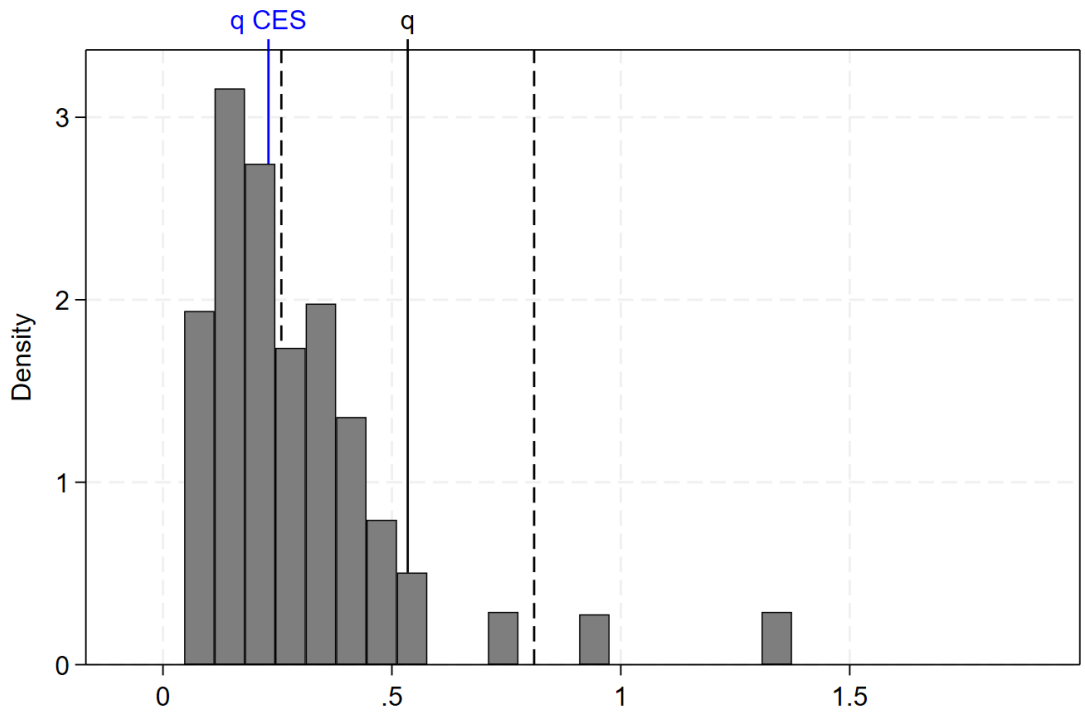


Figure 5: Estimated q (weighted by industry size) and q^{CES}

Single instrument In our main analysis, we estimate q using both lagged and contemporary positive demand shocks as instruments for income. As shown in Tables 4 and 5, and Figures 6 and 7, our results are robust to the choice of instruments and do not depend on whether we use them together or individually.

Table 4: Estimation Results with only lagged positive demand shocks

	(G)		(H)		(I)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^r_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log L^{d,r}_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log Y^r_{i,t}$
$\eta^{+,r}_{i,t-1}$	0.104***		0.112***		
	(0.0188)		(0.0154)		
$\Delta \log \mathcal{I}^r_{i,t}$		1.680***			1.622***
		(0.255)			(0.165)
$\Delta \log L^{d,r}_{i,t}$				1.568***	1.622***
				(0.216)	(0.165)
$P(q \leq q^{CES})$		.039		.059	.009
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	15507	15507	15526	15526	15505
F-Stat	31	44	53	53	
R^2	0.0254	0.124	0.0353	0.124	
F-Stat eq.(1)					7
F-Stat eq.(2)					10
R^2 eq.(1)					0.124
R^2 eq.(2)					0.124

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

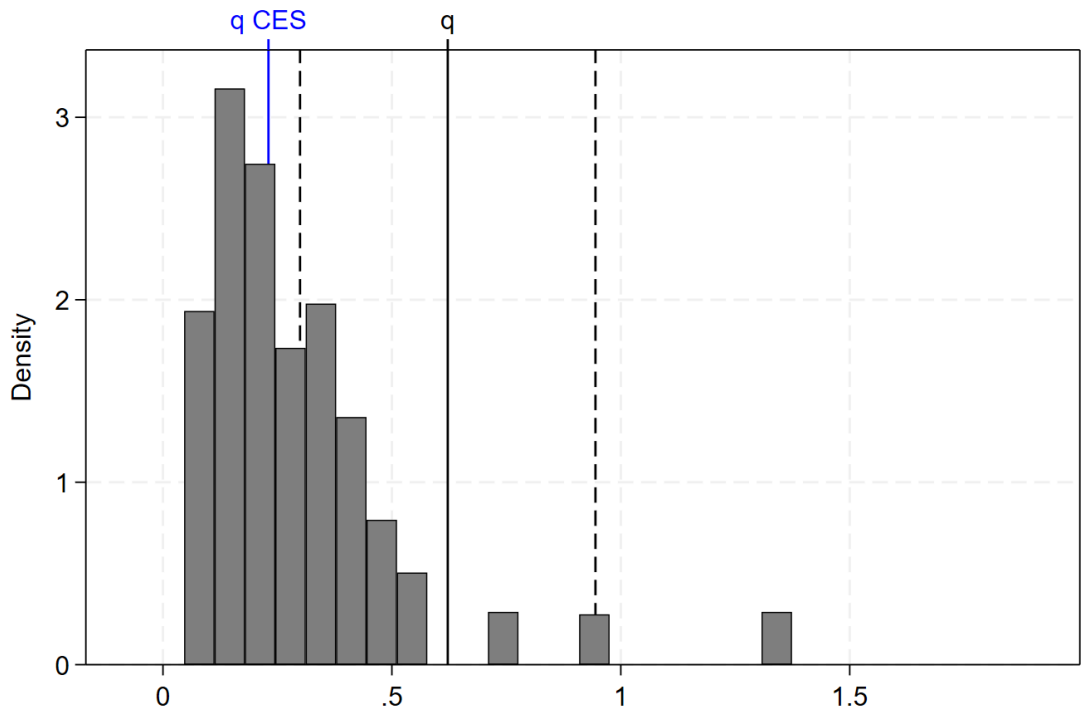


Figure 6: Estimated q (using only the lagged instrument) and q^{CES}

Table 5: Estimation Results with only contemporary positive demand shocks

	(J)		(K)		(L)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^r_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log L^{d,r}_{i,t}$	$\Delta \log Y^r_{i,t}$	$\Delta \log Y^r_{i,t}$
$\eta^{+,r}_{i,t}$	0.114***		0.104***		
	(0.0179)		(0.0169)		
$\Delta \log \mathcal{I}^r_{i,t}$		1.745***			1.816***
		(0.235)			(0.186)
$\Delta \log L^{d,r}_{i,t}$				1.919***	1.816***
				(0.316)	(0.186)
$P(q \leq q^{CES})$		.014		.015	.001
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	15029	15029	15048	15048	15028
F-Stat	41	55	38	37	
R^2	0.0264	0.0690	0.0317	0.0690	
F-Stat eq.(1)					6
F-Stat eq.(2)					5
R^2 eq.(1)					0.0690
R^2 eq.(2)					0.0690

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

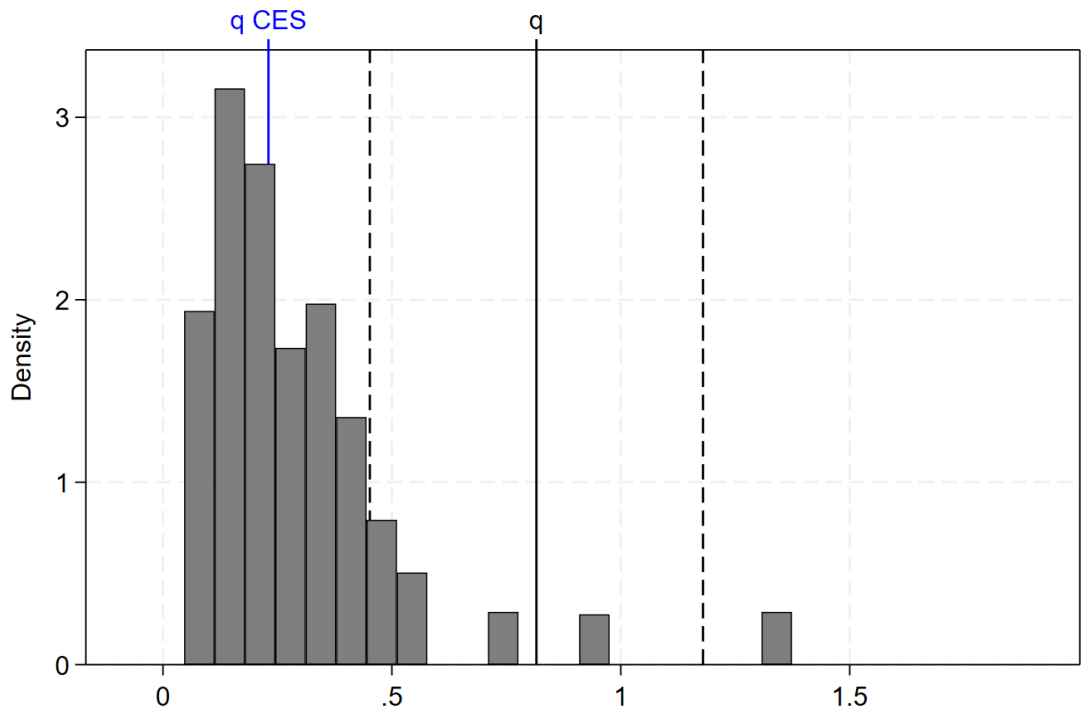


Figure 7: Estimated q (using only the contemporary instrument) and q^{CES}