Expectations and Frictions: Lessons from a Quantitative Model with Dispersed Information*

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Abstract

This paper examines the macroeconomic implications of information frictions within a quantitative business cycle model. We propose a solution method for linear DSGE models under dispersed information that allows the inclusion of endogenous state variables. Using Bayesian techniques, we estimate a medium-scale DSGE model embedding standard inertial frictions (such as habits in consumption, investment adjustment costs, and price indexation) and information frictions, disciplined by extensive macroeconomic and expectations data. Our model enables us to revisit crucial questions regarding business cycles, assess the relative importance of standard versus information frictions, and investigate the effects of learning about shocks and higher-order expectations. Expectations data require strong information frictions, which dampen general equilibrium effects and alter the relative importance of shocks driving economic fluctuations. Rather than substituting standard frictions, information frictions complement them: the former generates inertia in real macroeconomic variables, while the latter is key for inflation sluggishness.

Keywords: Business cycles, DSGE models, information frictions, higher-order beliefs.

JEL Codes: C72, D80, E30, E7.

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1 Introduction

Expectations play a central role in macroeconomics. Households' and firms' decisions in macroeconomic models are heavily influenced by expectations about future economic developments and economic policy.

A recent body of literature has made significant efforts to understand how expectations are formed. Coibion and Gorodnichenko (2015) (henceforth, CG) document deviations from full-information rational expectations using empirical measures of information frictions computed from survey data.[1] They show that forecasts errors and revisions co-move consistently with standard imperfect information theories such as sticky information (Mankiw and Reis; 2002) and noisy information (Woodford; 2002; Sims; 2003).

However, state-of-the-art DSGE models, such as those following the tradition of Christiano et al. (2005) and Smets and Wouters (2003, 2007) still rest on the full-information paradigm. These models are essential for understanding macroeconomic dynamics and economic policy, as they successfully fit the empirical properties of key macroeconomic aggregates. Nonetheless, estimates of those models typically do not incorporate expectations data to discipline the dynamics, despite the prominent role of expectations in these models (Milani; 2023).

This paper seeks to fill this gap by estimating a medium-scale DSGE model with heterogeneous and imperfect information using a broad set of macroeconomic aggregates and expectations data. We make two main contributions, one methodological and the other applied.

Solution method. Our methodological contribution is developing a novel method to solve DSGE models with dispersed and exogenous information. The computational challenge lies in the complexity of solving DSGE models when we relax the assumption of complete information: agents must form an infinite regress of expectations, as first pointed out by Townsend (1983). Then, state representations of DSGE models with such information structure become infinite-dimensional. To overcome this challenge, our method bridges Uhlig's (2001) undermined coefficient method for full-information models with the solution methods from Nimark (2008) and Melosi (2017) for dispersed information models that truncate the hierarchy of beliefs to a finite order.

Our method improves upon existing approaches by allowing the inclusion of endogenous state variables into the system of log-linearized equilibrium conditions. This enables the solution of larger-scale models with dispersed information. The method also uses a representation of the

[1] Mankiw et al. (2004) highlight the persistence of forecast errors and time-varying disagreement in expectations within major U.S. surveys of expectations. Coibion and Gorodnichenko (2012) explores the forecast error response to shocks to document deviations from full information. Bordalo et al. (2020) extends CG evidence for a broad set of macroeconomic data and individual expectations. Moreover, Kohlhas and Walther (2021) document extrapolation from recent events in expectations while Broer and Kohlhas (2024) provide evidence on overrevisions in macroeconomic expectations.

system of equations that depends only on the average expectation, instead of requiring writing equilibrium equations in terms of higher-order expectations. This substantially simplifies the characterization of equilibrium conditions and requires minimum additional effort compared to solving a full information model. Our solution algorithm is sufficiently fast to perform Bayesian estimation with standard techniques.

Application in a quantitative model. Our applied contribution is estimating a standard medium-scale DSGE model along the lines of Smets and Wouters (2007) (henceforth, SW) augmented with dispersed information. In the model, neither firms nor households can perfectly observe the underlying aggregate shocks. Instead, they form expectations based on exogenous and idiosyncratic signals about them.[2] Due to the limitations of previous solution methods, few attempts have been made to estimate macroeconomic models with incomplete information.

Our model embeds both standard (such as habits in consumption, investment adjustment costs, and price indexation) and information frictions and is estimated using a broad set of U.S. macroeconomic data and *average forecast revisions* of several real (output, consumption, and investment) and nominal variables (inflation and interest rate).

The model is used to revisit several important questions in the business cycle literature: i) which frictions are more relevant for matching the empirical properties of business cycle *and expectation* dynamics? ii) what are the key driving forces behind output and inflation fluctuations? iii) how do shocks propagate in an economy where agents are not fully informed?

We also explore two open questions of the literature on information frictions: i) Can information frictions replace standard frictions as the key source of the sluggishness in macroeconomic variables? ii) Do higher-order expectations quantitatively matter for the shocks' propagation?

To answer these questions, we compare the estimates of the baseline model with dispersed information (DI) with the nested model under full information (FI) using two different datasets: one including only U.S. macroeconomic data – as is common practice in the literature –, and another augmented with forecast revisions data. This clarifies the role of adding expectations data and information frictions on parameter estimates separately.

Estimated degree of information frictions. We measure the information friction of each shock using the (steady-state) Kalman gain: an object that measures how much expectation is revised after receiving new information about the shock (i.e., unpredictable change in agents' signals). Our simple information structure implies that the Kalman gain ranges from one (full

[2]This form of information friction is standard in the literature (Woodford; 2002; Angeletos and La'O; 2009; Melosi; 2014; Angeletos and Huo; 2021, for instance).

information) to zero ("no information").[3]

By including expectations data in the estimation, we document three key findings. First, the DI model has a substantially better fit of expectations data than the FI model, using a marginal likelihood criterion (roughly 200 log points higher).

Second, estimates of strong information frictions are pervasive. The posterior estimates of the Kalman gains of all shocks are tight distributions whose 95% credibility intervals range from 0.20 to 0.61, with one exception. The information friction is less severe for the monetary shock, ranging from 0.62 to 0.81 but still is far from the full information benchmark. Measuring information frictions as discussed above simplifies comparing those frictions between shocks and also to empirical measures of information frictions (Kalman gains extracted from the expectations data directly). We further validate our results by computing the empirical Kalman gains using a similar approach to CG in simulated data from the DI model and compare them to the same estimates using actual data. The model can match reasonably well those untargeted moments for output, consumption, investment, inflation, and interest rate.

Third, expectation data is key to identifying information frictions. For instance, preference, investment-specific technological, and price mark-up shocks had negligible information frictions when estimating the model using macroeconomic data only (95% posterior credibility intervals of the respective Kalman gains ranging from 0.73 to 0.98).

Finally, we examine the role of each information friction on the model's fit to the data. We remove each information friction one at a time and re-estimate the model, allowing the remaining frictions to compensate for the removed information friction. This exercise reveals that the information frictions related to preference and investment shocks are the most important for the model's fit to the data. Removing any of these frictions yields a reduction in the model's marginal likelihood of approximately 100 log points. Additionally, information frictions on price markup and government expenditure also play a significant role (about 40 log points lower for each), while those associated with other shocks have a minor impact on the model fit.

Business cycle drivers. Performing a forecast error variance decomposition (FEVD) exercise using the estimated DI model, we find that investment-specific technological and government expenditure shocks are the key drivers of fluctuations in output growth and labor hours. The former is more important for explaining short-run business cycle fluctuations and decreases in importance over time, whereas the latter does the opposite. Preference shocks are the primary drivers of consumption fluctuations, while investment is primarily influenced by investment-specific

[3]The Kalman gain is an endogenous object that arises in the optimal signal extraction from Gaussian shocks and signals. In the stationary equilibrium, agents update expectations based on the steady-state Kalman gain. It is a function of the shock persistence and volatility, and the signal's noise variance. Conditional on the shock's parameters, it has a one-to-one relationship to the signal's noise variance.

technological shocks. Price markup shocks account for most of the fluctuations in inflation and are increasingly significant in explaining nominal interest rates and real wage dynamics over time. Wage markup shocks only significantly affect real wage dynamics, while TFP shocks play a minor role in driving the dynamics of all macroeconomic variables.

Comparing the FEVD using only macroeconomic data and the complete dataset shows that incorporating expectation and information frictions increases the relative importance of shocks directly affecting each variable (e.g., preference shocks for consumption). Expectation data requires stronger information frictions which dampens the general equilibrium forces of the model, thereby increasing the relative importance of partial equilibrium (direct effects) in the model (Angeletos and Huo; 2021).

The role of information frictions on shocks' propagation. Under dispersed information, agents underestimate the magnitude of shocks in the short run and take time to fully understand their impact.

Responses to price markup shocks, the key driver of inflation, exhibit major differences between DI and FI models. The DI model leads to a stronger stagflationary stance with a hump-shaped response of inflation. The latter is explained by the high persistence of those shocks combined with strong information frictions, leading to hump-shaped responses of expectations about the shock with a peak after three quarters. In contrast, the macroeconomic responses to investment-specific technological shocks, the major driver of output, are similar in the DI and FI models. The combination of low persistence of the shock with strong information frictions dampens the response (which is offset by higher estimated volatility) but cannot generate additional inertia. Monetary policy shocks have a more significant impact on macroeconomic aggregates and inflation, primarily due to changes in parameter estimates rather than due to information frictions.

We also decompose the role of information frictions on IRFs into first-order uncertainty (gradual learning about shocks) and higher-order uncertainty (uncertainty about other agents' expectations). For investment shocks, imperfect learning (first-order uncertainty) is the key factor behind the dampening effect of information frictions. For cost-push shocks, both first- and higher-order expectations contribute similarly to the propagation of real variables, while first-order uncertainty explains most of the dynamics of nominal variables (inflation and nominal interest rate).

Information frictions: substitutes or complements for standard frictions? Our application provides a quantitative assessment of the role of information frictions as an alternative source of persistence and inertia in macroeconomic aggregates (Woodford; 2002; Mankiw and Reis; 2002). Even closer connected, Angeletos and Huo (2021) develop a sharp equivalence between the equilibrium effects of incomplete information and standard frictions that induce inertia such as habits

or investment adjustment costs.

Our model embeds both types of frictions with reasonably flat priors, leaving the macroeconomic and expectations data to pin down their relative importance in a fully-fledged quantitative model. To explore this point further, we re-estimate the model shutting down frictions that generate inertia and compare the marginal likelihoods. This exercise suggests that, rather than perfect substitution as proposed in previous literature, information and standard frictions exhibit a more complementary relationship.

Specifically, removing all information frictions (the full information case) leads to a loss of marginal likelihood that is nearly twice as large as the loss incurred by abstracting from standard frictions. Yet, the case without standard frictions leads to a decrease of approximately 115 log points in the same measure. The key standard frictions are habit consumption and investment adjustment costs. This aligns with point estimates of those parameters: the posterior modes of the consumption habits parameter increased from 0.87 to 0.97, and the investment adjustment costs parameter nearly doubled from 5.47 to 9.73 when expectations data is included. Other standard parameter estimates remain largely unchanged.

The intuition of this result is the following. Macroeconomic and expectations data require low persistence and strong informational frictions for preference and investment shocks, which are the key drivers of consumption and investment fluctuations, respectively. This parameter mix cannot generate strong persistence in expectations, limiting their role in explaining inertia in consumption and investment. While information frictions can generate persistence in expectations, the persistence in expectations about shocks also depends on the shock persistence.[4] As a result, intrinsic persistence from habits and investment costs plays a major in generating inertia.

Moreover, we document that information frictions on cost-push shocks are crucial for generating hump-shaped responses in the inflation response to the same shock, which is the key driver of inflation fluctuations. Our results suggest that the estimated FI model cannot generate this hump-shaped response.

Related literature. Our paper also relates to papers that develop solution methods for general DSGE models with dispersed information. Our setting extends methods from Nimark (2008) and Melosi (2017) that truncate the hierarchy of beliefs to a finite order to avoid the infinite regress of higher-order expectations. There are other alternatives using frequency domain techniques. For instance, Huo and Takayama (2024) use a combination of the Wiener-Hopf prediction formula and the Kalman filter to achieve a tractable finite-state representation for the equilibrium.[5] They

[4]For instance, expectations about i.i.d. shocks have no persistence independent of the degree of information friction.

[5]For other papers using frequency domain techniques for solving dispersed information models see Kasa (2000); Kasa et al. (2014); Rondina and Walker (2021) and references therein.

apply this methodology to a small-scale dynamic beauty contest model and a HANK-type model with incomplete information.

This paper also relates to a broad literature that seeks to explain expectations and business cycle fluctuations quantitatively by relaxing the full-information rational expectations (FIRE) paradigm. Information frictions within rational agents in terms of imperfect information (Collard et al.; 2009; Collard and Dellas; 2010; Levine et al.; 2012), sticky information Mankiw and Reis (2007), rational inattention (Maćkowiak and Wiederholt; 2015) and noisy information (Melosi; 2014, 2017) were considered a promising alternative to sticky prices as a source of slow adjustment. These papers use small-scale models and macroeconomic data only. Our application uses a medium-scale DSGE model and a broad set of macroeconomic and expectations data in a model that features learning about shocks and higher-order expectations in addition to standard frictions.

Finally, our work is complementary to a growing literature that relaxes the rational expectations assumption in business cycle models by incorporating various behavioral frictions. These frictions have been instrumental in explaining distinct features of business cycles. For instance, adaptive learning (Milani; 2017) and cognitive discounting (Gabaix; 2020) have been shown to induce persistence and inertia in macroeconomic models. In contrast, diagnostic expectations (Bianchi et al.; 2024; L'Huillier et al.; 2024) and misspecified forecasts (Hajdini; 2023) can generate overreaction and boom-bust cycles. Our contribution differs by focusing on the role of incomplete information in explaining the observed persistence and inertia in macroeconomic data and then examining its implications for the propagation of shocks and the drivers of business cycles.

Outline. The remainder of this paper is organized as follows. Section 2 presents the details of the model and its equilibrium relations. Section 3 explains the new solution method we developed. Section 4 details the dataset used, the estimation procedure, and its results. Section 5 performs different quantitative exercises to evaluate the ability of the DI model to explain the macroeconomic and expectations data. Section 6 concludes.

2 Model

The model is a standard medium-scale DSGE model along the lines of Christiano et al. (2005) and Smets and Wouters (2007) (henceforth, SW). It features sticky prices and wages, variable capital utilization, fixed cost in intermediate production, and five frictions that generate endogenous persistence in the model: (i) habit formation in consumption, (ii) adjustment cost in investment, (iii, iv) partial indexation of prices and wages to past inflation, and (v) smoothing in the monetary policy rule.

The economic fluctuations are driven by seven orthogonal exogenous shocks: total factor and

investment-specific productivity shocks, a preference shock, wage and price mark-up shocks, and government expenditure and monetary policy shocks.

The innovation of our application is the introduction of dispersed information (DI) to this environment. Specifically, households and firms do not observe perfectly the shocks hitting the economy. Instead, they receive noisy idiosyncratic signals about them and form expectations rationally conditional on this information.

Timing

Time is discrete and each period contains two stages. In the first stage, shocks and signals realize, intermediate goods firms choose optimal prices, and households choose consumption, investment, installed capital, and its utilization level, and set optimal wages, based on information from their signals. In the second stage, rental rates and wages of differentiated labor are uncovered. Competitive final good firms buy intermediate goods to sell the final good to households. Competitive labor packers use the supply of differentiated labor from households and sell a homogeneous labor bundle to intermediate firms. Intermediate firms rent capital and hire labor to produce the intermediate goods.

This timing protocol is standard in the literature[6] and ensures two features. First, all markets clear. Competitive final good firms ensure that the supply of final good matches consumption and investment demands. Given prices of intermediate goods, differentiated wages, and rental rate chosen at stage 1, firms allocate capital and labor to accommodate the demand from final good firms. Finally, given the wages set at the first stage, labor packers aggregate the differentiated labor services from households and supply homogeneous labor to intermediate firms such that the labor market clears.

Second, intermediate good firms do not use information from the production process to extract information from aggregate variables. This implies that both intermediate firms and households use only information from their signals to form expectations.

Final good firms

The homogeneous final good Y_t is a bundle of intermediate goods, $Y_{i,t}$, where the index $i \in [0,1]$ denotes the continuum of intermediate firms. The production function is given by

$$Y_t = \left(\int_0^1 (Y_{i,t})^{\frac{1}{1+\mu_t^p}} \, di \right)^{1+\mu_t^p}, \tag{1}$$

[6] See for instance Nimark (2008), Angeletos and La'O (2011).

where μ_t^p is the time-varying price mark-up of intermediate goods following the process $log(1+\mu_t^p) = log(1+\bar{\mu}^p) + x_t^p$, and $\bar{\mu}^p$ is the steady-state value of the price mark-up. The price mark-up shock x_t^p follows

$$x_t^p = \rho_p x_{t-1}^p + \varepsilon_t^p, \quad \varepsilon_t^p \sim \mathcal{N}(0, \sigma_p^2). \tag{2}$$

Final goods firms operate in a perfectly competitive market and sell the final goods to households for a price P_t. They solve a usual profit maximization problem

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{3}$$

subject to (1). From the first-order conditions and the zero-profit condition, we derive the demand for intermediate production

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{1+\mu_t^p}{\mu_t^p}} Y_t, \tag{4}$$

where $P_t = \left(\int_0^1 (P_{i,t})^{-\frac{1}{\mu_t^p}} di\right)^{-\mu_t^p}$ is the price level.

Intermediate good firms

Each intermediate producer is a monopolistic competitive firm $i \in [0,1]$ that uses labor and capital to produce their goods. Good i is produce using technology

$$Y_{i,t} = e^{x_i^a} (K_{i,t})^{\alpha} (\gamma^t L_{i,t})^{1-\alpha} - \gamma^t \Phi_p, \tag{5}$$

where γ represents the labor-augmenting deterministic growth rate of productivity and x_t^a is a total factor productivity shock. $K_{i,t}$ and $L_{i,t}$ denote the amount of capital and labor demanded by firm i at period t respectively, and Φ_p is a fixed cost included in the production function.[7] The productivity shock follows

$$x_t^a = \rho_a x_{t-1}^a + \varepsilon_t^a, \quad \varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2). \tag{6}$$

Intermediate firms are subject to a Calvo (1983)-like pricing friction: in every period, only a fraction $1 - \xi_p$ of firms can adjust prices. Firms not allowed to reset prices apply the following indexation rule

$$P_{i,t} = (\Pi_{t-1})^{\iota_p} \left(\bar{\Pi}\right)^{1-\iota_p} P_{i,t-1}, \tag{7}$$

where $\Pi_t \equiv P_t / P_{t-1}$ is gross inflation and $\bar{\Pi}$ is its steady-state value. Firms that are able to reoptimize prices choose the level $P_{i,t}^*$ that maximizes their expected discounted flow of profits

[7] Scaling the fixed cost by γ^t avoids it to decrease over time in terms of output.

given by

$$E_{it}\sum_{s=0}^{\infty}(\beta\xi_p)^s\Lambda_{t,t+s}\left[\left(X_{t,t+s}P_{i,t}^{*}-MC_{i,t+s}\right)Y_{i,t+s}\right], \tag{8}$$

subject to $Y_{i,t+s}=\left(\frac{X_{t,t+s}P_{i,t}^{*}}{P_{t+s}}\right)^{-\frac{1+\mu_{t+s}^{p}}{\mu_{t+s}^{p}}}Y_{t+s}$. $MC_{i,t}$ is the marginal cost of firm i at period t. $\Lambda_{t,t+s}$ is the household's stochastic discount factor between periods t and $t+s$ and $X_{t,t+s}$ is the indexation between the same periods. Consistently with the indexation rule (7), $X_{t,t+s}$ is given by

$$X_{t,t+s}=\begin{cases}\bar{\Pi}^{(1-\iota_p)s}\prod_{j=1}^{s}\left(\Pi_{t+j-1}^{\iota_p}\right) & \text{if} \quad s\geq 1\\ 1 & \text{if} \quad s=0.\end{cases} \tag{9}$$

Given the optimal prices and the indexation rule, the price level has the following law of motion

$$P_t=\left[\xi_p\left(\Pi_{t-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}P_{t-1}\right)^{-\frac{1}{\mu_t^p}}+(1-\xi_p)\left(\int_0^1 P_{i,t}^{*}di\right)^{-\frac{1}{\mu_t^p}}\right]^{-\mu_t^p}. \tag{10}$$

Labor packers

There is a continuum of households indexed by $h\in[0,1]$, each supplying differentiated labor services. Following Erceg et al. (2000), there are labor packers that hire labor from the households and aggregate them according to

$$L_t=\left(\int_0^1 L_{h,t}^{\frac{1}{1+\mu_t^w}}dh\right)^{1+\mu_t^w}, \tag{11}$$

where μ_t^w denote the agency's wage mark-up such that $log(1+\mu_t^w)=log(1+\bar{\mu}^w)+x_t^w$, and $\bar{\mu}^w$ is the steady-state value of the wage mark-up. The shock follows the process

$$x_t^w=\rho_w x_{t-1}^w+\varepsilon_t^w, \quad \varepsilon_t^w\sim\mathcal{N}(0,\sigma_w^2). \tag{12}$$

Labor packers pay the wage $W_{h,t}$ for each household h, and sell a homogeneous labor service to intermediate firms at a cost W_t. Agents maximize profits

$$W_tL_t-\int_0^1 W_{h,t}L_{h,t}dh \tag{13}$$

subject to (11). Thus, the labor demand for each household h's labor service is given by

$$L_{h,t}=\left(\frac{W_{h,t}}{W_t}\right)^{-\frac{1+\mu_t^w}{\mu_t^w}}L_t, \tag{14}$$

where $W_t = \left(\int_0^1 \left(W_{h,t} \right)^{-\frac{1}{\mu_t^w}} dh \right)^{-\mu_t^w}$ is the nominal wage index.

Households

Household $h \in [0,1]$ derive utility from consumption and leisure. In order to maximize their expected utility, they choose consumption ($C_{h,t}$), holdings of government bonds ($B_{h,t}$), installed capital level ($K_{h,t}$) and its utilization rate ($U_{h,t}$). The capital rented to firms, K_t^u, is determined by the installed capital and the utilization rate. The objective function that each household h optimizes is given by

$$U = E_{ht} \left[\sum_{s=0}^{\infty} \beta^s e^{x_{t+s}^c} \left(ln \left(C_{h,t+s} - \varphi C_{h,t+s-1} \right) - \frac{L_{h,t+s}^{1+\chi}}{1+\chi} \right) \right], \tag{15}$$

where $C_{h,t}$ is consumption and $L_{h,t}$ denotes the supply of differentiated labor services of household h at period t. Households' preferences display external habit persistence, captured by the parameter φ, while χ is the inverse of the Frisch elasticity of labor supply. $E_{ht}[\cdot]$ is the expectation operator conditional on household h's information set, and x_t^c is a preference shock that follows

$$x_t^c = \rho_c x_{t-1}^c + \varepsilon_t^c, \quad \varepsilon_t^c \sim \mathcal{N}(0, \sigma_c^2). \tag{16}$$

The capital stock $K_{h,t}$ owned by household h evolves according to

$$K_{h,t} = (1-\delta) K_{h,t-1} + e^{x_t^i} \left(1 - S(I_{h,t}/I_{h,t-1}) \right) I_{h,t}, \tag{17}$$

where $S(I_t/I_{t-1})$ is the adjustment investment cost function that denotes the share of investment which does not become new capital. As in Christiano et al. (2005), the cost function $S(\cdot)$ has the following properties: $S(\gamma) = S'(\gamma) = 0$, and $S''(\gamma) = s'' > 0$.

The investment-specific technological shock x_t^i follows

$$x_t^i = \rho_i x_{t-1}^i + \varepsilon_t^i, \quad \varepsilon_t^i \sim \mathcal{N}(0, \sigma_i^2). \tag{18}$$

Households rent to firms an effective amount of capital $K_{h,t}^u$ given by

$$K_{h,t}^u = U_{h,t} K_{h,t-1}, \tag{19}$$

where $U_{h,t}$ is the level of capital utilization. They receive $R_t^k K_{h,t}^u$ for renting capital but pay a cost $a(U_{h,t}) K_{h,t-1}$ in terms of the consumption good. Following Christiano et al. (2005), this function has the properties $a(\bar{U}) = 0$ and $a''(\bar{U}) = a''$, where $\bar{U}$ is the value of the capital utilization rate in

the steady-state.

The households' budget constraint is given by

$$\begin{aligned} &P_t C_{h,t} + P_t I_{h,t} + B_{h,t} + P_t a(U_{h,t}) K_{h,t-1} + Q_{t+1,t} A_{h,t} \leq \\ &R_{t-1} B_{h,t-1} + W_{h,t} L_{h,t} + R_t^k U_{h,t} K_{h,t-1} + P_t A_{h,t-1} + T_{h,t}, \end{aligned} \tag{20}$$

in each period. $T_{h,t}$ denote net transfers from the government, $A_{h,t}$ is a vector of one-period state-contingent securities and $Q_{t+1,t}$ is the price of such asset.[8]

Households supply labor in a market with monopolistic competition subject to a wage-setting friction. Following Erceg et al. (2000), in every period only a fraction $1 - \xi_w$ of households are able to optimize their wages. Those households choose their optimal wage by setting a mark-up over the marginal rate of substitution between consumption and labor. Households who are not able to optimize update their wage using a indexation rule given by

$$W_{h,t} = (\Pi_{t-1})^{\iota_w} \left(\bar{\Pi}\right)^{1-\iota_w} \gamma W_{h,t-1}, \tag{21}$$

which is a geometrically weighted average of steady-state wage growth ($\gamma\bar{\Pi}$) and last period wage growth ($\gamma\Pi_{t-1}$).

Each household minimizes their expected discounted labor disutility

$$E_{ht}\left[\sum_{s=0}^{\infty} (\beta\xi_w)^s \left(-\frac{L_{j,t+s}^{1+\chi}}{1+\chi}\right)\right] \tag{22}$$

subject to the budget constraint (20) at all periods $s \in [0, \infty)$ and to the period $t+s$ nominal wage $W_{h,t+s} = X_{t,t+s}^w W_{h,t}^*$ and

$$X_{t,t+s}^w = \begin{cases} (\gamma\bar{\Pi}^{1-\iota_w})^s \prod_{j=1}^{s} \left(\Pi_{t+j-1}^{\iota_w}\right) & \text{if} \quad s \geq 1 \\ 1 & \text{if} \quad s = 0. \end{cases} \tag{23}$$

Given optimal prices and the indexation rule, the aggregate wage level has the following law of motion

$$W_t = \left[\xi_w \left(\gamma\Pi_{t-1}^{\iota_w}\bar{\Pi}^{1-\iota_w} W_{t-1}\right)^{-\frac{1}{\mu_t^w}} + (1-\xi_w)\left(\int_0^1 W_{h,t}^* dh\right)^{-\frac{1}{\mu_t^w}}\right]^{-\mu_t^w}. \tag{24}$$

[8]The assumption of a complete set of state-contingent securities guarantees that all households h make the same consumption and saving choices. This is true despite the fact that they have differentiated wages and form expectations based on idiosyncratic signals. This assumption is for tractability such that standard techniques of log-linearization to solve DSGE models can be applied even with informational frictions in the households' decisions.

Government Policies

The central bank sets the nominal interest rate according to a standard Taylor rule with smoothing according to

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\phi_R} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{(1-\phi_R)} e^{x_t^r}, \tag{25}$$

where variables with a bar denote the steady-state values and x_t^r is the monetary policy shock, which follows

$$x_t^r = \rho_r x_{t-1}^r + \varepsilon_t^r, \quad \varepsilon_t^r \sim \mathcal{N}(0, \sigma_r^2). \tag{26}$$

For simplicity, we assume that the monetary authority responds to deviations of output to its steady-state value instead of deviations from the natural output.

The government budget constraint is the following

$$P_t G_t + R_{t-1} B_{t-1} = B_t + T_t \tag{27}$$

where T_t and B_t are total lump-sum taxes and bonds, respectively. Government expenditure follows a simple stochastic process given by $G/Y = g_y + x_t^g$, where $g_y \equiv \bar{G}/\bar{Y}$ is the steady-state ratio of government expenditure to output and x_t^g is a government expenditure shock with process

$$x_t^g = \rho_g x_{t-1}^g + \varepsilon_t^g, \quad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2). \tag{28}$$

Resource constraint and market clearing

The aggregate resource constraint

$$C_t + I_t + G_t + a(U_t) K_t = Y_t, \tag{29}$$

is derived by integrating households' budget constraint over h, and combining it with the zero profit condition of final goods firms and labor packers and the government budget constraint.

Market clearing in labor and capital markets holds

$$K_t^u = \int_0^1 K_{i,t} di, \quad L_t = \int_0^1 L_{i,t} di.$$

The bond supply and the transfers

$$B_t = \int_0^1 B_{h,t} dh, \quad T_t = \int_0^1 T_{h,t} dh,$$

are consistent with the government spending rule and the public budget constraint (27).

Information and signal extraction

Intermediate good firms and households do not observe perfectly the structural shocks driving the economy, but instead receive noisy idiosyncratic signals about them. Formally, the signal follows the process

$$s_{j,t}^{l} = x_t^l + v_{j,t}^l, \quad v_{j,t}^l \sim \mathcal{N}(0, \tau_l^2), \tag{30}$$

where $l \in \{a, c, i, g, p, w, r\}$ denote each type of shock and $j \in [0, 1]$ is a index that pools both intermediate good firms i and households h. This assumption implies that firms and household receive signals with the same properties and are subject to the same degree of informational frictions. Hence, an agent j's information set is described as

$$\mathcal{I}_t^j = \{s_{j,\tau}^a, s_{j,\tau}^c, s_{j,\tau}^i, s_{j,\tau}^g, s_{j,\tau}^p, s_{j,\tau}^w, s_{j,\tau}^r : \tau \leq t\}. \tag{31}$$

Given that households and firms have the same unit mass, their average expectation is the same and denoted by $\bar{E}_t[\cdot] \equiv \int_0^1 E_{j,t}[\cdot]dj$, where $E_{j,t}[\cdot] \equiv E[\cdot|\mathcal{I}_t^j]$ denotes the j's individual expectation. Given the AR(1) structure of the shocks and signal structure (30), j's rational expectation about the shock x_t^l is computed by using the Kalman filter such that

$$E_{j,t}[x_t^l] = E_{j,t-1}[x_t^l] + \bar{k}_l \left[s_{j,t}^l - E_{j,t-1}[s_{j,t}^l]\right] \tag{32}$$

where denotes $\bar{k}_l$ the steady-state Kalman gain of shock l given by

$$\bar{k}_l = \frac{\tilde{p}_l}{\tilde{p}_l + r_l^2}. \tag{33}$$

$\tilde{p}_l = \frac{\bar{p}_l}{\sigma_l^2}$ is defined as the ratio of the individual expectation steady-state variance and the shock variance and $r_l \equiv \tau_l/\sigma_l$ denotes the noise-to-signal ratio of signal l. Solving the standard Riccati equation for $\tilde{p}_l$, it is possible to show it is a function of the shock persistence and the noise-to-signal ratio, $\tilde{p}_l = \tilde{p}_l(\rho_l, r_l)$. Therefore, the steady-state Kalman gain $\bar{k}_l = k(\rho_l, r_l)$ for each shock is a function of the same parameters.

The Kalman gain is the key object that measures the degree of informational friction, *conditional on the shock's parameters*. In other words, conditional on (ρ_l, σ_l) there is a one-to-one relationship between $\bar{k}_l$ and the signal's noise ratio τ_l (via $r_l = \tau_l/\sigma_l$). We get back on this when discussing priors choices.

Moreover, the average expectation is given by

$$\bar{E}_t[x_t^l] = (1-\bar{k}_l)\bar{E}_{t-1}[x_t^l] + \bar{k}_l x_t^l = (1-\bar{k}_l)\rho_l \bar{E}_{t-1}[x_{t-1}^l] + \bar{k}_l x_t^l. \tag{34}$$

Thus, the current expectation is a weighted average between the past expectation and the actual shock, whose weight is given by the Kalman gain.

If $\bar{k}_l = 1$ ($\tau_l \to \infty$) the model boils down to the full information benchmark. If $\bar{k}_l = 0$ ($\tau_l \to 0$), the average expectation is zero (equals to the unconditional expectation).[9]

Equation (34) helps to understand why and when information frictions can generate inertia. Strong informational frictions (low $\bar{k}_l$) has two implications for average expectations: it dampens the response to the actual shock and increases the expectation persistence. The latter also depends on the persistence of the unobserved shock, ρ_l. For instance, the expectation has no persistence independently on the degree of information friction when $\rho_l = 0$.[10]

In section 4, the model estimation implies that some shocks have a combination of high persistence and high informational frictions, which generates dampening and inertia. However, many shocks combine low persistence with high informational frictions. This cannot generate inertia while preserving a weakened response to shocks. This distinction is key to understanding whether informational frictions substitutes or complement standard frictions discussed in section 5.4.

2.1 Log-linearized equilibrium

For brevity, the log-linearized equations and detrending procedure are detailed in Appendix A.2. The derivation of these equations can be found in the Online Appendix. Hereafter, lowercase variables represent log-deviations from the steady state of their uppercase counterparts.

To clarify how dispersed information affects equilibrium conditions, we highlight two key equilibrium conditions.

First, consider the Euler equation:

$$c_t = \frac{\varphi/\gamma}{1+\varphi/\gamma} c_{t-1} + \frac{1}{1+\varphi/\gamma}\bar{E}_t[c_{t+1}] - \frac{(1-\varphi/\gamma)}{1+\varphi/\gamma}\bar{E}_t[r_t - \pi_{t+1} - (x_{t+1}^c - x_t^c)].$$

This is the same Euler equation as in standard representative-agent models (Smets and Wouters; 2003) with two key differences due to information frictions.

First, expectations are heterogeneous, and hence average expectations replace the full-information expectation operator. Second, current endogenous variables and structural shocks are not observed.

[9] Zero is the solution for the equation $\bar{E}[x_t^l] = \rho_l \bar{E}_{t-1}[x_{t-1}^l]$.

[10] In that case, the Kalman gain equals to the well-known weight with static fundamentals and signals: $\bar{k}_l = \frac{1}{1+r_l^2} = \frac{1/\sigma_l^2}{1/\sigma_l^2+1/\tau_l^2}$.

Thus, their expectations arise in optimal choices. In the context of the households' intertemporal consumption decision, households must form expectations about the current nominal rate, r_t, and the current preference shock, x_t^c.

Now consider the New Keynesian Phillips (NKPC) curve given by

$$\pi_t = \xi_p \iota_p \pi_{t-1} + \kappa_p \xi_p \bar{E}_t \left[mc_t + x_t^p \right] + \psi_p \bar{E}_t \left[\pi_t \right] + (1-\xi_p)\beta \xi_p \int_0^1 E_{it}[p_{i,t+1}^*] di \tag{35}$$

where $\psi_p \equiv (1-\xi_p)(1-\iota_p \beta \xi_p)$.

An additional difference appears in the NKPC as the average expectations of firms' future *own optimal prices* matters for inflation. Note that $\int_0^1 E_{it}[p_{i,t+1}^*] di \neq \bar{E}_t \left[p_{t+1}^* \right]$, since the law of iterated expectations does not hold for average expectations (Morris and Shin; 2005). Thus, we cannot write the last term in terms of the average expected future *aggregate optimal price*, which is related directly to future inflation.

As pointed out by Nimark (2008), as inflation depends on its own average expectation, higher-order expectations matter for inflation dynamics. By taking the average expectations of this equation and substituting it iteratively, we obtain that

$$\pi_t = \frac{\xi_p \iota_p}{1-\psi_p} \pi_{t-1} + \kappa_p \xi_p \sum_{k=1}^{\infty} \psi_p^{k-1} \bar{E}_t^{(k)} [mc_t + x_t^p] + (1-\xi_p)\beta \xi_p \sum_{k=0}^{\infty} \psi_p^k \bar{E}_t^{(k)} \left[\int_0^1 E_{it}[p_{i,t+1}^*] di \right], \tag{36}$$

where $\bar{E}_s^{(k)}[z_t] \equiv \int_0^1 E_{is} \left[\bar{E}_s^{(k-1)}[z_t] \right] di$, for any variable z_t, all $k \geq 1$ and $s \leq t$, with the convention that $\bar{E}_t^{(0)}[z_t] \equiv z_t$ and $\bar{E}_t^{(1)}[z_t] \equiv \bar{E}_t[z_t]$.

Firms must form not only beliefs about marginal cost, price mark-up shock, and future own price but also higher-order expectations about these variables. $\psi_p \in (0,1)$ is a key parameter that determines how strong the dependence of inflation on higher-order expectations is. One key feature is that inflation dependence on higher-order expectations decreases with the order.

The latter differs from Nimark's (2008) NKPC in two key aspects. First, our model includes partial indexation to past prices, which generates a backward term. Interestingly, ψ_p is decreasing in the indexation parameter, ι_p, i.e., indexation decreases the relevance of higher-order expectations. The intuition is the following. As non-optimizing firms index their prices to past inflation, optimal prices are less responsive to expectations about inflation. Thus, inflation is less dependent on higher-order expectations.

Second, it explicitly considers that $\int_0^1 E_{it}[p_{i,t+1}^*] di \neq \bar{E}_t \left[p_{t+1}^* \right]$, which prevents representing inflation as a function of the future inflation as in the full-information benchmark.[11]

[11] As pointed out in Appendix D of Angeletos and Huo (2021), Nimark (2008) and Melosi (2017) abstract in their derivation that $\int_0^1 E_{it}[p_{i,t+1}^*] di \neq \bar{E}_t \left[p_{t+1}^* \right]$. While this may not be quantitatively relevant, it is important for correctly defining the general system of equilibrium conditions (37) and the guessed policy function (40).

One key feature of the solution method discussed in the next section is that we do not need to represent equilibrium conditions in terms of higher-order expectations as in equation (36) to solve the model. We can find a policy function that depends on higher-order expectations even defining equilibrium conditions in the system of equation in terms of average expectations as in equation (35).

3 Solution method

In this section, we present a novel solution method for DSGE models featuring imperfect and dispersed information.

The log-linearized equilibrium equations (65) can be written as the following general system of linear rational equations

$$\begin{aligned} F_1\bar{E}_t[Y_{t+1}] + F_2\int_0^1 E_{it}[Y_{i,t+1}]di + G_1Y_t + G_2\bar{E}_t[Y_t] + HY_{t-1} + \\ L\bar{E}_t[x_{t+1}] + M_1x_t + M_2\bar{E}_t[x_t] = 0_{m\times 1}, \end{aligned} \tag{37}$$

where $i \in [0,1]$ indexes an agent, $Y_{i,t}$ is a $m \times 1$ vector of individual choices, Y_t is a $m \times 1$ vector of aggregate endogenous variables and x_t is a $n \times 1$ vector of unobservable exogenous shocks.

The system of equations (37) generalizes the existing methods by allowing: i) average expectation about future own variables (the term post multiplying F_2) and ii) endogenous state variables (the term post multiplying H). The latter allows solving medium-scale DSGE models such as the one in section 2.

Structural shocks x_t follow a stationary stochastic process

$$x_t = A_1x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon) \tag{38}$$

where A_1 is a diagonal matrix storing the persistence of each shock, and Σ_ε is a diagonal covariance matrix. Idiosyncratic signals are given by

$$s_{i,t} = C_xx_t + Dv_{i,t}, \quad v_{i,t} \sim \mathcal{N}(0, \Sigma_v), \tag{39}$$

where Σ_v is a diagonal covariance matrix.[12]

Define the k-th order average expectation $E_t^{(k)}[\cdot]$ given information of period t about the vector of unobservable aggregate shocks x_t as $E_s^{(k)}[x_t] \equiv \int_0^1 E_{is}\left[E_s^{(k-1)}[x_t]\right]di$, for all $k \geq 1$ and s, t, with the convention that $E_t^{(0)}[x_t] \equiv x_t$ and $E_t^{(1)}[x_t] \equiv \bar{E}_t[x_t]$.

[12]The model from Section 2 implies a diagonal structure for A_1, Σ_ε and Σ_v. The solution method does not require these assumptions.

It is useful to denote the expectations hierarchy of x_t from order l to s as the vector

$$x_t^{(l:s)} \equiv \left[\begin{array}{cccc} E_t^{(l)}\left[x_t\right]' & E_t^{(l+1)}\left[x_t\right]' & \cdots & E_t^{(s)}\left[x_t\right]' \end{array} \right]',$$

for $s > l \geq 0$.

Building on the insights of Nimark (2008) and Melosi (2017), our solution method involves truncating the state space to address the issue of infinite regression of expectations, a problem first highlighted by Townsend (1983).[13] As noted in these previous studies, this approach not only provides an arbitrarily accurate approximation for sufficiently large truncations of higher-order beliefs but also offers a bounded rationality interpretation that relates to level-k thinking (Nagel; 1995).

Specifically, we guess that the individual and aggregate equilibrium law of motion and expectation hierarchy are given by

$$Y_{i,t} = \mathbf{R}Y_{i,t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_{i,t}\left[x_t^{(0:\bar{k})}\right], \tag{40}$$

$$Y_t = \mathbf{R}Y_{t-1} + \mathbf{Q}x_t^{(0:\bar{k})} \tag{41}$$

$$x_t^{(0:\bar{k})} = \mathbf{A}x_{t-1}^{(0:\bar{k})} + \mathbf{B}\varepsilon_t, \tag{42}$$

where $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1}, \mathbf{A}, \mathbf{B})$ are finite dimensional matrices to be determined in equilibrium. $\mathbf{Q} = \mathbf{Q_0}e_x + \mathbf{Q_1}T$, e_x is the selection matrix such that $x_t = e_x x_t^{(0:\bar{k})}$ and T is a order transformation matrix such that $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = Tx_t^{(0:\bar{k})}$[14].

Our guess and verify strategy extends Melosi (2017) in two aspects: First, we guess the policy function of individuals (40) and then aggregate to find the aggregate policy function (41). In contrast, Melosi (2017) abstracts from the average expectation of future own variables in the system of equations (specifically, the term multiplying F_2 in equation 37) and directly guesses the aggregate policy function.

Secondly, both the individual and aggregate policy functions include a lagged term with a coefficient $\mathbf{R}$. This is significant because our system incorporates endogenous state variables, such as Y_{t-1} in equation (37). By doing so, we allow endogenous variables to exhibit intrinsic persistence, in addition to the persistence arising from shocks and higher-order expectations.

Proposition 1 shows the dynamics for expectation hierarchy and the expressions for the fixed-point solution for $(\mathbf{A}, \mathbf{B})$.

Proposition 1. *Suppose that x_t is a stationary process, the expectation hierarchy follow equation*

[13] Nimark (2017) demonstrates that if the impact of higher-order expectations on equilibrium outcomes diminishes with their order, there exists a threshold $\bar{k}$ at which the approximation error becomes arbitrarily small, less than any $\epsilon > 0$.

[14] See details in Appendix D.

(42) and agents use information from signals (39). Assuming common knowledge of rationality, the individual and average expectations about the hierarchy are given by

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = (I_k - \mathbf{K}C)\,\mathbf{A}E_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \mathbf{K}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \mathbf{K}C\mathbf{B}\varepsilon_t + \mathbf{K}Dv_{it}$$
$$\bar{E}_t\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}\bar{E}_{t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{\mathbf{K}}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C\mathbf{B}\varepsilon_t.$$

where $C = C_x e_x$.

The expectation hierarchy consistent with the expectations above has coefficients $(\mathbf{A}, \mathbf{B})$ *that satisfy the matrix equations:*

$$\begin{aligned}\left(I_k - T'\bar{\mathbf{K}}C\right)\mathbf{A} &= e_x' A_1 e_x + T'\left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}T \\ \left(I_k - T'\bar{\mathbf{K}}C\right)\mathbf{B} &= e_x'\end{aligned} \tag{43}$$

such that $\bar{\mathbf{K}}$ *and* $\bar{\mathbf{P}}$ *solve the Riccati equation resulting from equations*

$$\begin{aligned}\bar{\mathbf{K}} &= \bar{\mathbf{P}}C'\left[C\bar{\mathbf{P}}C' + D\Sigma_v D'\right]^{-1} \\ \bar{\mathbf{P}} &= \mathbf{A}\left[\bar{\mathbf{P}} - \bar{\mathbf{K}}C\bar{\mathbf{P}}\right]\mathbf{A}' + \mathbf{B}\Sigma_\varepsilon\mathbf{B}'\end{aligned} \tag{44}$$

The Ricatti equation resulting from equations (44) solves the steady-state mean square error of the expectation hierarchy. The equations uses the standard Kalman Filter with hierarchy dynamics (42) as state equation and measurement equation as signals (39).

The key difference is that $(\mathbf{A}, \mathbf{B})$ is a result from agents signal extraction. Thus, the hierarchy persistence $(\mathbf{A})$ and response to shocks $(\mathbf{B})$ depend on the Kalman gain matrix $\bar{\mathbf{K}}$. As any signal extraction, the $\bar{\mathbf{K}}$ depends on the unobserved state dynamics $(\mathbf{A}, \mathbf{B})$.

Therefore, the equilibrium solution $(\mathbf{A}, \mathbf{B}, \mathbf{K}, \mathbf{P})$ is the fixed point from equations (43-44).

Since the signals (39) are exogenous, they do not depend on equilibrium conditions (37). This simplifies finding the guessed coefficients for the equilibrium $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1})$ and tremendously diminishes the computational costs.[15]

Before presenting the equilibrium dynamics in Proposition below, we want to emphasize one assumption that simplifies substantially this task. We assume that when forming expectations about Y_t using the guessed solutions (40-41), agents know the past value of endogenous variables, Y_{t-1}.

[15]For instance, Melosi (2017) explains in his replication files that: "Be aware that estimating the DIM may take several weeks or even a few months depending on the computer used for this task." In our application, which has a considerable higher scale model, takes rougly 72 (36) hours for 400k posterior draws using the RB-RWHM (RWMH) algorithm in a standard notebook.

This assumption contrasts with the expectation formation from Proposition 1 in which expectations depend only on exogenous signals. This avoid learning from (past) endogenous variables, which generates a feedback between the signal extraction and endogenous variable responses to the hierarchy.[16] Ribeiro (2017) considers the general case with endogenous signals.

This assumption is implicit in standard solution methods for imperfect under common information such as Blanchard et al. (2013) and Baxter et al. (2011). This is also a common assumption when studying the New Keynesian Phillips Curve under dispersed information (e.g., Angeletos and La'O; 2009; Angeletos and Huo; 2021).

Proposition 2. *For a given dynamics expectation hierarchy from equation (42), the system of equations (37) has a recursive equilibrium law of motion (41) whose matrix coefficients satisfy:*

1. $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1})$ *satisfy the matrix equations:*

$$F\mathbf{R}^2 + G\mathbf{R} + H = 0_{m\times m} \tag{45}$$

$$[F_1\mathbf{R} + G_1]\,\mathbf{Q_0} + M_1 = 0_{m\times n} \tag{46}$$

$$\begin{aligned}&[F_1\mathbf{R} + G_1]\,\mathbf{Q_1} + F_1\mathbf{Q_1}\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q_1}T + F_2\mathbf{Q_1}T\mathbf{A} + \\ &[(F_2\mathbf{R} + G_2)\mathbf{Q_0} + F\mathbf{Q_0}A_1 + (LA_1 + M_2)]\,e_x = 0_{m\times k}\end{aligned} \tag{47}$$

   where $F \equiv F_1 + F_2$, $G \equiv G_1 + G_2$ *and* $k = n(\bar{k} + 1)$.

2. *(Uhlig; 2001)* $\mathbf{R}$ *has a unique stable solution if all eigenvalues of* $\mathbf{R}$ *are smaller than unity in absolute value.*

3. *Given the solution of* $\mathbf{R}$*, denote the matrices* (V_0, V_1) *such that:*

$$V_0 = F_1\mathbf{R} + G_1 \tag{48}$$

$$V_1 = I_k \otimes (F_1\mathbf{R} + G_1) + T' \otimes (F_2\mathbf{R} + G_2) + \mathbf{A}' \otimes F_1 + T'\mathbf{A}' \otimes F_2 \tag{49}$$

   Provided that there exists a inverse for V_0 *and* V_1*, the equilibrium solution for* $(\mathbf{Q_0}, \mathbf{Q_1})$ *is given by*

$$\mathbf{Q_0} = -V_0^{-1}M_1 \tag{50}$$

$$vec(\mathbf{Q_1}) = -V_1^{-1}vec\left([(F_2\mathbf{R} + G_2)\mathbf{Q_0} + F\mathbf{Q_0}A_1 + (LA_1 + M_2)]\,e_x\right) \tag{51}$$

   where $vec(\cdot)$ *denotes columnwise vectorization.*

[16] One can think this assumption as a deviation from rationality as agents do not incorporate that their knowledge about Y_{t-1} can be used to learn about $x_{t-1}^{(0,\bar{k})}$, which would also change their beliefs about $x_t^{(0,\bar{k})}$.

Given the individual response to shocks ($\mathbf{Q_0}$) *and to the individual expectation about the hierarchy* ($\mathbf{Q_1}$)*, the aggregate response to the hierarchy is given by:* $\mathbf{Q} = \mathbf{Q_0}e_x + \mathbf{Q_1}T$

Proposition 2 connects the solution of imperfect common knowledge models with the standard undetermined coefficients solution for full information DSGE models.

Condition (45) is exactly the same as the usual "brute force" approach of Uhlig (2001). In other words, he equilibrium $\mathbf{R}$ is the same that would happen if agents had full information, as shown in Appendix D.[17] Note that when we abstract from state endogenous variables ($H = 0_{m\times m}$), $\mathbf{R} = 0_{m\times m}$, leading to a policy function without intrinsic persistence from endogenous variable as in Melosi (2017).

This has two key implications. First, $\mathbf{R}$ can be computed with standard techniques. Second, informational frictions does not affect how endogenous variables respond to state variables. This does not imply that information frictions are not important their persistence. Endogenous variables persistence also reflects the hierarchy persistence.

Our solution method is a natural extension of Uhlig's (2001) full information and Blanchard et al.'s (2013) imperfect information methods. It is also related to the dispersed information method from Melosi (2017). The key difference, besides the ones discussed above, is that he allows for endogenous signals.

Appendix D further explore the relation with those methods and show the details of the algorithm to find the fixed point solution from Proposition 1.

4 Estimation

In this section, we discuss the data, the prior choices, and the posterior results of the Bayesian estimation. We estimate the model featuring dispersed information ('DI model') using two different datasets: one containing macroeconomic data only and other including both macroeconomic and expectation data. For comparison, we also estimate the same model under full-information ('FI model') where we abstract from information frictions in the same datasets.

4.1 Data

We collect U.S. quarterly macroeconomic and expectation data from 4Q:1981 to 4Q:2007. The data on macroeconomic aggregates include the series of real GDP, real consumption, real investment, GDP deflator index, annualized Fed funds rate, hours worked index and nominal wages index, and come from the Federal Reserve Database (FRED).

[17] The key assumption for this result is that agents observe past endogenous variables at period t.

We also gather the average expectation about one-quarter ahead real GDP, real consumption and real investment, inflation and three-month Treasure bill rate from the US Survey of Professional Forecasters (SPF). A detailed description of the datasets can be found in Appendix C.

When estimating medium scale DSGE models, the inclusion of expectation data in the dataset is scarce. In recent years, few papers have tried to incorporate expectation data to discipline macroeconomic models with complete and incomplete information. For instance, Del Negro and Eusepi (2011) adds one-year ahead inflation expectations to estimate a conventional New Keynesian model. Melosi (2017) estimates a small scale dispersed information model using data on inflation forecasts one-quarter and one-year ahead. Del Negro et al. (2015) uses ten-year inflation expectations from the Blue Chip and SPF.

In this paper, we divert from previous literature in two ways. First, we use expectation data on a broader set of variables. Second, we use data on *current forecast revisions* (the nowcast minus last period forecast). The reasoning for the latter is the following.

By aggregating equation (32), one can see that

$$\overline{Rev}_{t|t-1}[x_t^l] \equiv \bar{E}_t[x_t^l] - \bar{E}_{t-1}[x_t^l] = \bar{k}_l \left[s_t^l - \bar{E}_{t-1}[s_t^l] \right] = \bar{k}_l \overline{Fe}_{t-1}[x_t^l] \tag{52}$$

where $s_t^l = \int_0^1 s_{it}^l di = x_t^l$ is the aggregate signal, which due to the simply signal structure (30) is equal to the shock l. $\overline{Rev}_{t|s}[x_t^l]$ denotes the average expectation revision about x_t^l at period t of a forecast made in period s and $\overline{Fe}_s[x_t^l] = x_t^l - \bar{E}_s[x_t^l]$ denotes the forecast error at period t made from the same forecast.

Note that the current forecast revision of shock l depends on the information frictions and signals only.

By iterating equation (34), one can see that forecast $\bar{E}_t[x_t^l]$ depends on the whole history of previous signals, the information frictions and shock's persistence.

Note that the current forecast revision of shock l depends on the information frictions and signals only. Longer forecast horizons ($\bar{E}_t[x_{t+h}^l]$) and revisions ($\bar{Rev}_{t|t-1}[x_{t+h}^l]$) are likely to be less informative as it depends also on the shocks persistence.[18]

The discussion above is true if we had data on the expectations about shocks. In practice, we have data on endogenous variables, which creates another layer of complexity.

Considering these issues, short-horizon forecast revisions are more appropriate for identifying the degree of informational friction, in our view.

[18]The h-step ahead forecast is given by $\bar{E}_t[x_{t+h}^l] = \rho_l^h \bar{E}_t[x_t^l]$ and analogous revision is $\overline{Rev}_{t|t-1}[x_{t+h}^l] = \rho_l^h \overline{Rev}_{t|t-1}[x_t^l]$. They also depend on shocks' persistence, ρ_l and the horizon h.

4.2 Measurement equations

The measurement equations that link data with the variables for both models are the following

$$
\begin{aligned}
dy_t^{obs} &= \bar{\gamma} + y_t - y_{t-1} \\
dc_t^{obs} &= \bar{\gamma} + c_t - c_{t-1} \\
di_t^{obs} &= \bar{\gamma} + i_t - i_{t-1} \\
dw_t^{obs} &= \bar{\gamma} + w_t - w_{t-1} \\
l_t^{obs} &= l_t \\
\pi_t^{obs} &= \bar{\pi} + 4\pi_t \\
r_t^{obs} &= \bar{r} + 4r_t \\
rev_{dy,t|t-1}^{obs} &= \bar{E}_t[\Delta y_t] - \bar{E}_{t-1}[\Delta y_t] + \varepsilon_{dy,t}^{me} \\
rev_{dc,t|t-1}^{obs} &= \bar{E}_t[\Delta c_t] - \bar{E}_{t-1}[\Delta c_t] + \varepsilon_{dc,t}^{me} \\
rev_{di,t|t-1}^{obs} &= \bar{E}_t[\Delta i_t] - \bar{E}_{t-1}[\Delta i_t] + \varepsilon_{di,t}^{me} \\
rev_{\pi,t|t-1}^{obs} &= 4\left(\bar{E}_t[\pi_t] - \bar{E}_{t-1}[\pi_t]\right) + \varepsilon_{\pi,t}^{me} \\
rev_{r,t|t-1}^{obs} &= 4\left(\bar{E}_t[r_t] - \bar{E}_{t-1}[r_t]\right) + \varepsilon_{r,t}^{me},
\end{aligned}
\tag{53}
$$

where $\bar{\gamma} \equiv 100(\gamma - 1)$ is the quarterly trend growth rate of productivity. The variables with superscript "*obs*" correspond to the variables in the database. The notation dx_t^{obs} denotes the quarterly growth rate of the variables $x \in \{y, c, i, w\}$ at period t. The data on hours worked (l_t^{obs}) is demeaned, while inflation (π_t^{obs}) and nominal interest rate (r_t^{obs}) are computed in annual terms. $\bar{\pi} \equiv 400(\bar{\Pi} - 1)$ and $\bar{r} \equiv 400(\bar{R} - 1)$ stand for the steady-state annualized inflation and nominal interest rates, respectively. The variables $rev_{\pi,t|t-1}^{obs}$, for $x \in \{dy, dc, di, \pi, r\}$, are the current forecast revision.

Measurement errors ε_x^{me} in the observational equations associated with forecast errors are introduced to avoid stochastic singularity.[19] They also capture the fact that the expectation data from SPF are not completely consistent with the average expectations of the agents populating our model. Measurement errors follow an i.i.d. Gaussian distribution with standard deviations denoted by σ_x^{me}, with $x \in \{dy, dc, di, \pi, r\}$.

4.3 Prior distributions

As standard in the literature, we fix the value of some parameters that are not easily identified. The quarterly depreciation rate δ is set to 2.5%. The ratio of government expenditures to output g_y is calibrated to its historical mean value of 18%. The steady-state wage markup $\bar{\mu}^w$ is fixed at

[19] This is required since the complete dataset includes 12 time series, while the model has only 7 shocks.

1.5 as in SW. Finally, the capital share of the production function α is fixed 0.3 to be consistent with the historical capital-output ratio.

The introduction of informational frictions is likely to affect the estimation of real and nominal frictions, so we do not rely completely on estimates from previous papers to choose priors. Instead, most of the priors are relatively loose. Table 1 shows the choices for priors on the left side. The priors are the same for the FI and DI models. By definition, the full information model does not include the informational friction parameters and estimates with macroeconomic data only do not include measurement errors for revision data.

The prior for parameters that have support between 0 and 1, such as the persistence of shocks, partial adjustment of the Taylor rule, indexation of prices and wages to past inflation and habit persistence in consumption, follow a beta distribution ($\mathcal{B}$) with a mean of 0.5 and standard deviation of 0.2. One important exception is the Calvo frictions on prices and wages. For those to parameters we use very tight prices around micro data estimates for prices from Nakamura and Steinsson (2008) and for wages from Grigsby et al. (2021). Those imply average duration of three quarters for prices and one year for wages.

The trend growth, inflation rate, and nominal interest rate steady-state parameters have priors with gamma distribution ($\mathcal{G}$) whose mean equals to their sample average. The standard deviations are 0.10 for the first prior and 0.50 for the others.

The priors on the adjustment cost of investment, capital utilization, and the inverse of Frisch elasticity are taken from Del Negro et al. (2007). The parameters specifying the Taylor rule have priors with gamma distribution whose mean values are $\phi_\pi = 1.5$ and $\phi_y = 0.2$ and the standard deviations are 0.5 and 0.1, respectively.

Priors for the standard deviation of the structural shocks are distributed as an inverse gamma ($\mathcal{IG}$) with means chosen to match standard deviations of observables in the pre-sample 1957Q1-1981Q1 as in Del Negro and Eusepi (2011). Standard deviations are set to 1.00, which implies fairly loose priors.

Priors for Kalman gains of each shock are set to follow a beta distribution with a mean of 0.5 and a standard deviation of 0.2. The value of 0.5 is similar to estimates from CG using one-year inflation expectations and Bordalo et al. (2020) for one-year data on output, consumption and investment growth rates, inflation, and nominal rates. Note that their estimates of frictions are using macroeconomic aggregates and not actual shocks. We discuss more on this in section 5.1.

We use the Kalman gain ($\bar{k}_l$) directly as a parameter instead of the noise-to-signal ratio ($r_l = \tau_l/\sigma_l$) as Del Negro and Eusepi (2011) or the private signal variances (τ_l) as Melosi (2017). As discussed by Del Negro and Eusepi (2011), forming priors on r_l of a shock implicitly forms a prior for the private signal's variance dependent on the shock's variance, σ_l. Analogously, when placing priors on the Kalman gains of each shock, it implicitly generates a prior for the NRS dependent

on the shock's persistence, ρ_l (see equation 33).[20]

Therefore, despite the shock processes (ρ_l, σ_l) having different priors for each shock l, when assuming the same prior for the Kalman gains of all shocks, there is an implicit prior for $\tau_l|\rho_l, \sigma_l$ that allows the same information friction for all shocks a priori.

Finally, priors for the standard deviation of measurement errors on the time series of forecast errors are distributed as inverse gamma with a mean that matches roughly 10% of the total variance of each revision data. The standard deviations are small to ensure very tight priors, which avoids measurement errors to explain more than 25% of the variance in the revision data.

4.4 Posterior distributions

Bayesian techniques are applied to estimate the models, by combining the prior densities described before with the likelihood function computed using the Kalman filter. We solve the model truncating the average higher-orders at the sixth order, i.e., $\bar{k} = 6$.[21] We use the Random Block Random-Walk Metropolis-Hastings (RB-RWMH) algorithm with five blocks to draw from the posterior distribution.[22] Table 1 reports the posterior estimates for both models (FI and DI models) for both datasets. For each model and dataset, the table shows the mode and the 5%-95% percentiles of the posterior density.

Several interesting results arise from our estimation exercise. First, we compare estimates with macroeconomic data only and the complete dataset including expectation data. The parameters estimates related to endogenous propagation are remarkably stable with two key exceptions. Investment adjustment costs has a three fold increase for the FI model and almost two fold under DI. Consumption habits slightly increases in the DI model from 0.87 to 0.97. The latter estimate is relatively higher than standard values around 0.7-0.8. However, the standard pratice is to impose a very tight prior around 0.7 whereas we use a loose prior centered on 0.5. Moreover, the response to inflation in the Taylor rule also increases reaching a quite strong response of 3.32.

Including data on forecast revisions changes substantially the estimated levels of information frictions. Investment, price mark-up, TFP and preference shocks have sharp decreases in their Kalman gain estimates (higher frictions) while those from monetary shock increases substantially (lower frictions). In the next section, we discuss the drivers of those changes.

[20] Del Negro and Eusepi (2011) has also a robustness check that they estimate the Kalman gain directly without taking into account its relationship with other parameters.

[21] Nimark (2017) shows that $\bar{k} = 6$ is a accurate approximation in a simple model. Which order is sufficiently high to provide good approximation to the policy function is model dependent. We find similar results $\bar{k} = 10$ and $\bar{k} = 15$ using the complete dataset. Truncating at lower orders speedy-up substantially the estimation.

[22] Estimates from RB-RWMH have substantially better chain convergence diagnostics than standard RWMH as they generate chains with lower autocorrelation. We use 400k draws and a 50% burn-in. The candidate distribution is a multivariate normal with covariance matrix based on the estimate of the Hessian matrix at posterior mode as in An and Schorfheide (2007).

Table 1: Prior and Posterior distributions

Parameters	Prior			Dataset: Macroeconomic data only						Dataset: Macroeconomic and expectations data					
				Posterior: FI model			Posterior: DI model			Posterior: FI model			Posterior: DI model		
	Dist.	Mean	Std. Dev.	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
Endogenous propagation parameters															
χ	$\mathcal{G}$	2.00	0.75	1.50	1.15	2.49	1.21	1.01	1.83	1.99	1.50	2.80	1.48	1.15	2.02
φ	$\mathcal{B}$	0.50	0.20	0.96	0.83	0.98	0.87	0.55	0.92	0.98	0.95	0.99	0.97	0.95	0.98
a''	$\mathcal{G}$	0.20	0.10	0.40	0.28	0.66	0.37	0.25	0.63	0.42	0.29	0.69	0.41	0.28	0.68
s''	$\mathcal{G}$	4.00	1.50	4.35	2.58	8.30	5.47	3.27	9.21	14.76	11.67	20.00	9.73	7.22	13.72
ϕ_y	$\mathcal{G}$	0.20	0.10	0.06	0.03	0.13	0.06	0.03	0.13	0.06	0.04	0.11	0.07	0.04	0.14
ϕ_π	$\mathcal{G}$	1.50	0.50	3.05	2.39	4.05	2.84	2.24	3.77	2.85	2.35	3.53	3.32	2.71	4.21
ϕ_r	$\mathcal{B}$	0.50	0.20	0.87	0.81	0.90	0.87	0.82	0.91	0.89	0.86	0.91	0.90	0.87	0.92
ι_p	$\mathcal{B}$	0.50	0.20	0.11	0.04	0.28	0.08	0.03	0.23	0.03	0.01	0.09	0.03	0.01	0.10
ι_w	$\mathcal{B}$	0.50	0.20	0.56	0.24	0.85	0.52	0.20	0.83	0.47	0.17	0.80	0.59	0.23	0.86
ξ_p	$\mathcal{B}$	0.67	0.05	0.83	0.80	0.80	0.80	0.75	0.83	0.85	0.83	0.87	0.85	0.82	0.88
ξ_w	$\mathcal{B}$	0.75	0.05	0.75	0.68	0.80	0.70	0.63	0.77	0.79	0.73	0.83	0.75	0.68	0.81
Steady-state parameters															
π_{ss}	$\mathcal{G}$	2.62	0.50	2.29	1.70	2.82	2.22	1.52	2.85	2.76	2.34	3.21	3.06	2.46	3.59
r_{ss}	$\mathcal{G}$	5.15	0.50	6.10	5.40	6.97	6.47	5.69	7.61	6.04	5.46	6.86	6.88	6.03	7.98
γ_{ss}	$\mathcal{G}$	0.50	0.10	0.33	0.28	0.37	0.33	0.27	0.36	0.35	0.32	0.37	0.32	0.29	0.36
Structural shocks parameters															
ρ_c	$\mathcal{B}$	0.50	0.20	0.33	0.18	0.70	0.77	0.19	0.83	0.24	0.16	0.34	0.13	0.07	0.21
ρ_i	$\mathcal{B}$	0.50	0.20	0.62	0.48	0.91	0.46	0.31	0.65	0.25	0.19	0.32	0.24	0.18	0.29
ρ_g	$\mathcal{B}$	0.50	0.20	0.98	0.96	0.99	0.99	0.98	1.00	0.97	0.96	0.98	0.99	0.98	1.00
ρ_p	$\mathcal{B}$	0.50	0.20	0.96	0.92	0.98	0.95	0.92	0.98	0.90	0.87	0.93	0.82	0.77	0.87
ρ_w	$\mathcal{B}$	0.50	0.20	0.22	0.11	0.39	0.22	0.13	0.34	0.24	0.12	0.38	0.32	0.22	0.42
ρ_r	$\mathcal{B}$	0.50	0.20	0.57	0.41	0.70	0.65	0.48	0.82	0.31	0.22	0.41	0.45	0.33	0.57
ρ_a	$\mathcal{B}$	0.50	0.20	0.92	0.86	0.97	0.93	0.86	0.97	0.88	0.84	0.92	0.91	0.86	0.96
σ_c	$\mathcal{IG}$	0.30	1.00	0.17	0.10	0.22	0.11	0.09	0.36	0.20	0.17	0.23	1.70	1.25	2.28
σ_i	$\mathcal{IG}$	1.50	1.00	0.90	0.74	1.13	1.18	0.98	1.92	1.27	1.11	1.44	4.70	3.93	5.57
σ_g	$\mathcal{IG}$	0.30	1.00	0.56	0.49	0.65	0.53	0.45	0.62	0.60	0.53	0.68	0.56	0.48	0.63
σ_p	$\mathcal{IG}$	0.10	1.00	0.05	0.04	0.06	0.06	0.05	0.09	0.04	0.04	0.05	0.17	0.12	0.23
σ_w	$\mathcal{IG}$	0.75	1.00	0.57	0.47	0.67	1.07	0.77	2.71	0.60	0.50	0.71	1.56	0.99	2.48
σ_r	$\mathcal{IG}$	0.10	1.00	0.13	0.11	0.14	0.12	0.11	0.14	0.13	0.11	0.14	0.13	0.11	0.15
σ_a	$\mathcal{IG}$	0.30	1.00	0.40	0.37	0.46	0.39	0.36	0.45	0.41	0.37	0.46	0.41	0.37	0.46
Information friction parameters[a]															
k_c	$\mathcal{B}$	0.50	0.20				0.92	0.73	0.97				0.26	0.20	0.32
k_i	$\mathcal{B}$	0.50	0.20				0.93	0.78	0.98				0.45	0.41	0.50
k_g	$\mathcal{B}$	0.50	0.20				0.58	0.44	0.75				0.57	0.53	0.61
k_p	$\mathcal{B}$	0.50	0.20				0.94	0.81	0.98				0.37	0.29	0.46
k_w	$\mathcal{B}$	0.50	0.20				0.44	0.23	0.72				0.41	0.25	0.57
k_r	$\mathcal{B}$	0.50	0.20				0.22	0.07	0.56				0.72	0.62	0.81
k_a	$\mathcal{B}$	0.50	0.20				0.64	0.32	0.88				0.33	0.08	0.57
Measurement errors standard deviation[b]															
σ^{me}_{dy}	$\mathcal{IG}$	0.10	0.02							0.46	0.41	0.53	0.28	0.25	0.32
σ^{me}_{dc}	$\mathcal{IG}$	0.10	0.02							0.42	0.38	0.47	0.21	0.19	0.23
σ^{me}_{di}	$\mathcal{IG}$	0.50	0.20							2.23	2.01	2.52	1.26	1.12	1.41
σ^{me}_{π}	$\mathcal{IG}$	0.15	0.05							0.74	0.67	0.84	0.54	0.48	0.61
σ^{me}_{r}	$\mathcal{IG}$	0.15	0.05							0.29	0.26	0.33	0.28	0.24	0.33
Marginal Likelihood				-925.46			-942.25			-1544.50			-1333.20		

Note: $\mathcal{G}$, $\mathcal{IG}$ and $\mathcal{B}$ stand for: Gamma, Inverse Gamma and Beta distributions, respectively.
[a]Applies to the dispersed information model.
[b]Applies to the dataset including macroeconomic and expectations data.

Expectation data also has important implications for parameter of exogenous processes for the DI model.[23] It changes substantially estimates for most shocks except government expenditure

[23]Interestingly, it does not have much impact on estimates of the FI model because they cannot be compensated

and TFP shocks. The estimated persistence of some shocks declines moderately for investment-specific technological, price mark-up, and monetary shocks. The decrease is more pronounced in preference shocks (from 0.77 to 0.13). Moreover, standard deviations also increases sharply for investment, price and wage mark-up, and preference shocks.

Overall, the estimates from the complete dataset exhibit stronger habits and adjustment costs, and robust informational frictions across all shocks. Other sources of inertia such as price and wage indexation and smoothing in Taylor rule are relatively stable across datasets and models.

Comparing parameter estimates for the FI and DI models under both datasets reveals a considerable decrease in the variance of measurement errors, a first indication that the DI model provides a better explanation for expectation data, which we confirm further when comparing the marginal likelihoods of the FI and DI models in the bottom of Table 1. For the complete dataset, the difference in favor of the DI model increases considerably to 211 log points.

5 Frictions, shocks and business cycles when accounting for informational frictions

This section is divided into two parts. The first (subsection 5.1) explores whether the model can explain expectations data using empirical measures of information frictions. In this part, we also explore the relative importance of standard frictions in explaining expectations data. In the second part, we leverage our dispersed information model, disciplined with expectations data, to reassess the relative importance of various shocks in driving business cycles (subsections 5.2 and 5.3).

5.1 Does the model match empirical measures of information frictions?

A growing body of literature following CG, explores a general feature of models with information frictions that relate forecast errors and revisions to provide simple measures of information frictions from survey data on expectations.

In this section, we follow a similar empirical strategy. For each variable z, consider the following regression

$$\overline{Rev}_{t|t-1}[z_t] = \beta \overline{FE}_{t-1}[z_t] + error_t, \tag{54}$$

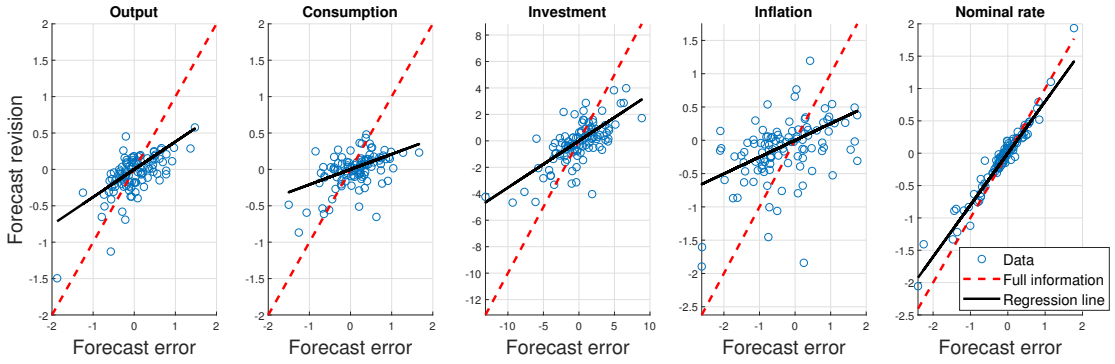
where $\overline{Rev}_{t|t-1}[z_t] \equiv \bar{E}_t[z_t] - \bar{E}_{t-1}[z_t]$ is the average forecast revision at time t and $\overline{FE}_{t-1}[z_t] \equiv z_t - \bar{E}_{t-1}[z_t]$ is the average one-step ahead forecast error at time t.

by informational frictions as we discuss below.

Following the same steps as equation (52), one can see that, if z follows an AR(1) and agents form rational expectations under dispersed information, then β is the Kalman gain, $\bar{k}_z$.

Figure 1 shows the scatter plot with the fitted line in solid black for each variable in the SPF used in the estimation of the model. Hereafter, we refer to those estimates as empirical Kalman gains (eKGs), i.e., Kalman gains directly estimated in the data. If agents had full information,

Figure 1: Empirical Kalman gains from SPF expectations data



Note: Empirical Kalman gains are the coefficients from regression (54), which relate revisions in one-quarter-ahead forecasts to past one-quarter-ahead forecast errors. Specifically, output, consumption, and investment refer to expectations regarding their growth rates, while inflation and interest rates pertain to expectations of their levels. Sample: 1981Q4-2007Q4.

all data points should be in the red dashed line (the 45º line). Thus, Figure 1 suggests a clear departure from the full-information benchmark in the SPF data, in line with previous studies such as CG and BGMS. It is also clear that variables differ in their information friction measure. For instance, the Fed funds rate has forecast revisions that are much more aligned with the forecast errors than any other series.

The AR(1) assumption holds for the dynamics of the exogenous shocks in our model. Thus, the regression (54) strictly applies only if we had data on shocks instead of endogenous variables. The equilibrium dynamics for endogenous variables (41) depend on their lags and the full hierarchy of expectations about all shocks. Therefore, the relationship between forecast errors and forecast revisions from endogenous variables does not have a closed-form representation as in equation (54).

In a dynamic beauty context model with one action and $AR(1)$ fundamental, Angeletos and Huo (2021) show analytically that eKGs are determined by the interaction of information frictions with other frictions in the model. Therefore, in our general model, estimates from those regressions capture a combination of i) information frictions of each shock and ii) standard frictions associated with endogenous persistence (e.g., habit formation and price stickiness, among others).[24] Our

[24]CG extends their empirical specification to more general data-generating processes such as $AR(p)$ for inflation or a $VAR(1)$ for some selected macroeconomic models. In our model, the persistence of endogenous variables,

estimated model can shed light on the relative importance of those frictions.

In the following, we assess whether the data generated by the DI model produces estimates for Kalman gains that align with those found in the SPF data. This exercise verifies the model's ability to match an important untargeted moment of the expectations data.

Let Θ be a vector that collects all parameters of the model and z_t a particular endogenous variable of the model. For a given Θ, we simulate n_{sim} samples of this variable and the respective average expectation $\{z_t, \bar{E}_t[z_t], \bar{E}_{t-1}[z_t]\}$ and measurement errors $\{\varepsilon^{me}_{z,t}\}$ for $t = 1, \cdots, T_{sim}$ and $z \in \{dy, dc, di, \pi, r\}$. Then, we construct n_{sim} time-series for average forecast revisions and forecast errors such that

$$\begin{aligned}\overline{Rev}^{sim}_{z,t|t-1} &= \bar{E}_t[z_t] - \bar{E}_{t-1}[z_t] + \varepsilon^{me}_{z,t} \\ \overline{Fe}^{sim}_{z,t-1} &= z_t - \bar{E}_{t-1}[z_t]\end{aligned}$$

for $t = 1, \cdots, T_{sim}$ and $z \in \{dy, dc, di, \pi, r\}$. Note that we add simulated measurement errors $\varepsilon^{me}_{z,t}$ to the series of forecast revisions so that it is consistent with the measurement equations (53) used in the estimation. Then, we compute the average coefficient and standard error from n_{sim} regressions (54).

Figure 2 shows the results for $T_{sim} = 105$, $n_{sim} = 100$ and simulations based on the the prior mean ($\Theta = \underline{\Theta}$) and the posterior mean ($\Theta = \bar{\Theta}$).[25] The black error bars (with $\circ$) represent the estimates from the SPF presented in Figure 1.

The eKGs from simulated data using the priors mean (red error bar with $\square$) are similar for all variables and close to the common prior mean for the Kalman gains of exogenous shocks. When comparing the same estimates from the posterior mean (blue error bar with $\diamond$), those are much more aligned with the direct estimates from the data.

Recall that we use data from forecast revisions at the same horizon in the estimation. Still, the Bayesian estimates do not attempt to match the empirical dependence between forecast revisions and forecast errors.

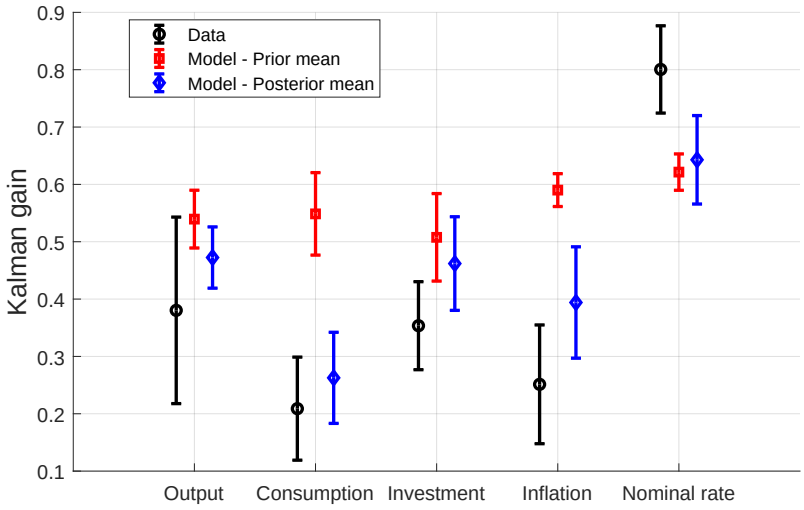
5.2 What are the main drivers of business cycle fluctuations?

We explore the drivers of business cycles in the DI and FI models by performing a forecast error variance decomposition (FEVD) exercise for selected variables at various horizons h. Figure 3 shows the results of this exercise for both models and datasets. We find that investment-specific technological shocks and government expenditure shocks are the key drivers of fluctuations in

represented by the matrix R in our solution (41), is a function of structural parameters.

[25] We use $T_{sim} = 105$ to match the sample size from SPF data. $n_{sim} = 100$ is sufficiently large to avoid sampling variability affecting the Kalman gain estimates meaningfully.

Figure 2: Empirical Kalman gains from simulated data from DI model



Note: Output, consumption and investment refer to the Kalman gains associated with the expectations about the *growth rate* of the corresponding variables. Inflation and interest rates refer to the Kalman gains associated with the expectations about the *level* of the corresponding variables. Data from the Survey of Professional Forecasters (SPF), covering the period from 1981Q4 to 2007Q4.

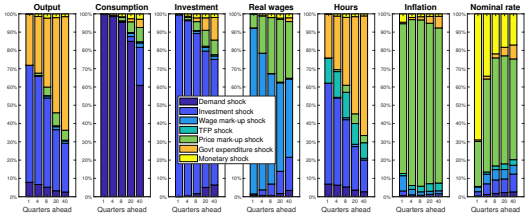
output growth and hours worked. Roughly 50-60% of short-run fluctuations in those variables are explained by the investment shock, whereas the government spending shock explains about 25-30%. In the long run, the relevance of these shocks inverts.

This pattern applies to both models and datasets. Our findings are in contrast with those of SW, who suggest that wage mark-up and TFP shocks play a significant role. Our results align with the conclusions of Justiniano et al. (2011), indicating that the difference can be attributed to including inventories in total investment, which SW does not account for. Auclert et al. (2020) also find that investment-specific technological shocks are key to output fluctuations using an estimated HANK model with sticky information.

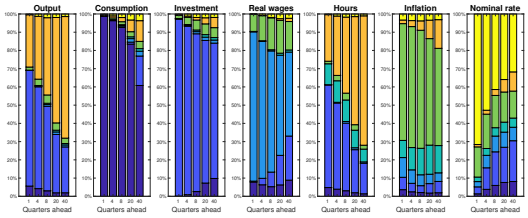
In our model, consumption and investment fluctuations are mainly driven by preference shocks and investment-specific technological shocks, respectively. Price mark-up shocks explain most of inflation's fluctuations and have an important role in explaining nominal rates and real wage dynamics, with increasing importance over time. Wage mark-up shocks play a major role in explaining fluctuations in real wages only. TFP shocks play a minor role in explaining fluctuations in all observables.

The FEVD exhibited in Panel (a) reveals that, in the baseline DI model, macroeconomic variables are primarily driven by the shock that directly affects them (e.g., price mark-up shocks

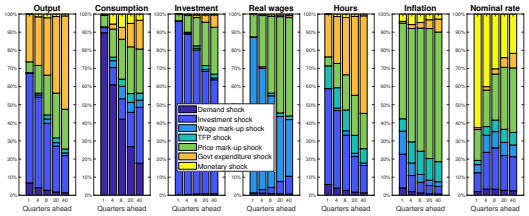
Figure 3: FEVD for macroeconomic variables by model and dataset



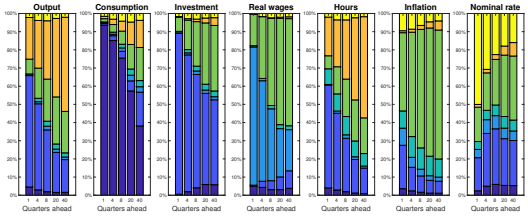
(a) DI model - Forecast revision dataset



(b) FI model - Forecast revision dataset



(c) DI model - Macroeconomic only dataset



(d) FI model - Macroeconomic only dataset

Note: Forecast error variance decomposition for macroeconomic variables (output, consumption, investment, real wages, hours worked, inflation, and nominal interest rates) across models and datasets. Panels (a) and (b) display results from the dispersed information and full information models, respectively, using the complete dataset. Panels (c) and (d) show results from the same models but using a dataset that includes only macroeconomic data.

explain the bulk of inflation fluctuations). Information frictions can explain the pattern discussed above. The mechanism is anchoring implied by dispersed information: it weakens the general equilibrium forces of the model, increasing the relative importance of partial equilibrium (direct effects) in the model (see Angeletos and Huo; 2021). As we show in the following, this is driven by the underreaction of average expectations and higher-order beliefs to shocks.

When comparing FEVDs from FI models across both datasets (panels b and d), we observe that the inclusion of data on forecast revisions generates the same pattern (variables being driven by the mostly related shock) to some degree. This pattern is reinforced when introducing information frictions (compare panels a and c). Interestingly, the DI and FI models have very similar FEVDs when using macroeconomic data only (panels c and d), which highlights the importance of disciplining the model with expectations data.

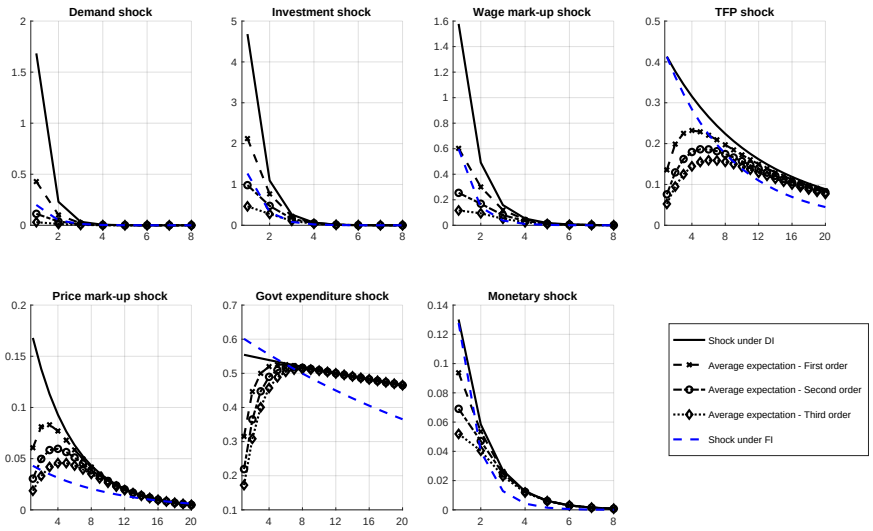
5.3 Propagation of shocks

In this subsection, we study how shocks propagate in different models by comparing the Bayesian impulse response functions (IRFs) of the DI and FI models estimated using the complete dataset.

Learning and higher-order average expectations about exogenous shocks. Figure 4 shows the IRFs of each shock and the average expectations about them up to the third order.

Under full-information rational expectations, all these impulse responses should coincide. The dynamics of shocks differ under full information (dashed blue line) and dispersed information (solid black line) due to distinct estimates for the exogenous processes in each model.

Figure 4: Estimated learning process for each shock



Note: The figure shows the posterior mean of impulse responses for each shock, triggered by a one standard deviation shock, alongside the estimated responses of higher-order beliefs about the shock.

When using the complete dataset, models suggest that preference, investment-specific technological, and wage mark-up shocks are the least persistent shocks.[26] The key difference is that shocks have a much larger variance under DI, but agents display a high degree of inattention to them. Agents underestimate the magnitude of shocks in the short run and are only fully aware of them after they have dissipated.

The dynamics of monetary policy shocks and the learning process associated with them are very similar for both models. In the DI model, agents take about three quarters to fully learn about them. TFP and government expenditure shocks have high persistence and similar variance in both models. Moreover, the high estimates of information frictions generate rich dynamics of higher-order expectations in the DI model. Interestingly, price mark-up shocks differ in terms of variance, but have high persistence for both models. This generates slow-moving higher-order expectations that are similar to the path of the actual shock under the FI model.

We find that shocks when both high persistence and significant departures from full information can accurately generate inertia expectations those shocks. In the following, we show that those

[26] When using macroeconomic data only, FI and DI models differ more regarding the shocks' persistence estimates.

shocks are the ones that induce strong inertia in macroeconomic variables. Usual quantitative explorations with small models typically calibrate models consistently with this. In contrast, a combination of low persistence and strong information frictions (present in half of the shocks) fails to generate sufficient inertia. In that case, information frictions only dampen the impact of shocks in the short run, without producing the sustained effects characteristic of high inertia.

This can be easily seen in equation (34). For each shock l, the persistence depends on both $1-\bar{k}_l$ and ρ_l and the responsiveness of the average expectation to the shock depends on $\bar{k}_l$ only. The distinction above underscores the importance of correctly specifying both persistence and information structure jointly when estimating macroeconomic models.

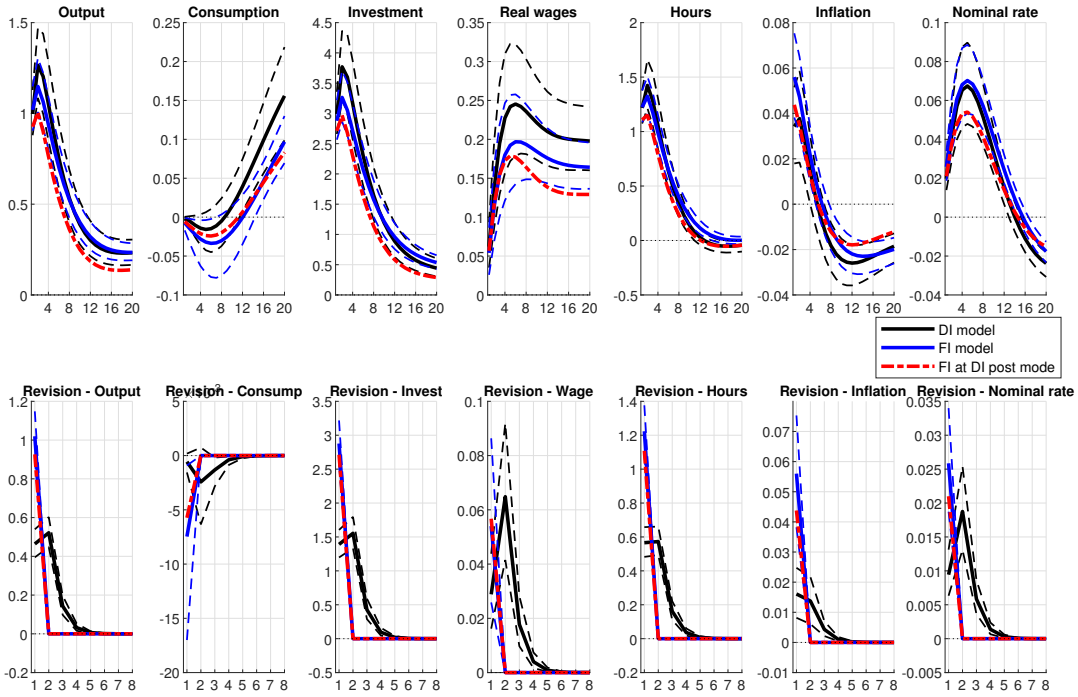
Endogenous variables. We now study the propagation of key structural shocks to relevant macroeconomic variables in the DI and FI models. The figures below compare the IRFs from the FI model (blue solid line) and the DI model (black solid line) with 90% Bayesian confidence bands. To highlight the effect of information frictions on the model's dynamic properties, we also compute the IRFs of a FI model solved at the posterior mean of the DI model (red dot-dashed lines)[27]. This comparison helps to determine whether the dynamics of the DI model differ from those of the FI model due to information frictions or differences in estimates from the remaining parameters. Indeed, the gap between the black line and the red line is entirely explained by information friction.

An investment-specific technological shock leads to an increase in output, hours, and investment, with the latter showing the largest percentage increase (Figure 5). Consumption has a weak negative response in the first year and increases in the following years. Inflation increases in the first year, but later decline. The nominal interest rate rise in response to higher inflation and output. The dynamics of the DI and FI models are quite similar with slightly stronger responses in the DI model. The key difference is the response of real wages, which is much more pronounced in the DI model. By comparing red with black lines, one can see that the differences (although small) between the DI and FI models result from the information frictions and not from the change in common parameter estimates.

Agents are not fully aware of the boom in output, hours and investment, so they only adjust their forecasts by about half of the actual changes in those variables. It takes them about a year to fully understand the impact of the shock on macroeconomic variables. After about a year, agents fully grasp the impact of the shock on macroeconomic variables. This pattern occurs because of the weak persistence and high information frictions for this shock implied in the estimated model. In contrast, the FI model generates a spike in the forecast revision on impact followed by an immediate return to zero. This pattern is a feature of any macroeconomic model with

[27]The impulse from the FI model with the mean of posterior estimates of the DI model is computed using the standard deviation from the FI model for generating comparable IRFs. Since the model is log-linearized, this normalization does not affect the shape of the IRF.

Figure 5: Impulse responses to an investment-specific technological shock



Note: The figure displays impulse response for a one standard deviation shock, based on the estimated DI and FI models. For comparison purposes, the IRF for the FI model using the DI model's posterior mode (red dot-dashed line) employs a one standard deviation shock from the FI model's posterior mode. The dashed lines represent 90% credible intervals.

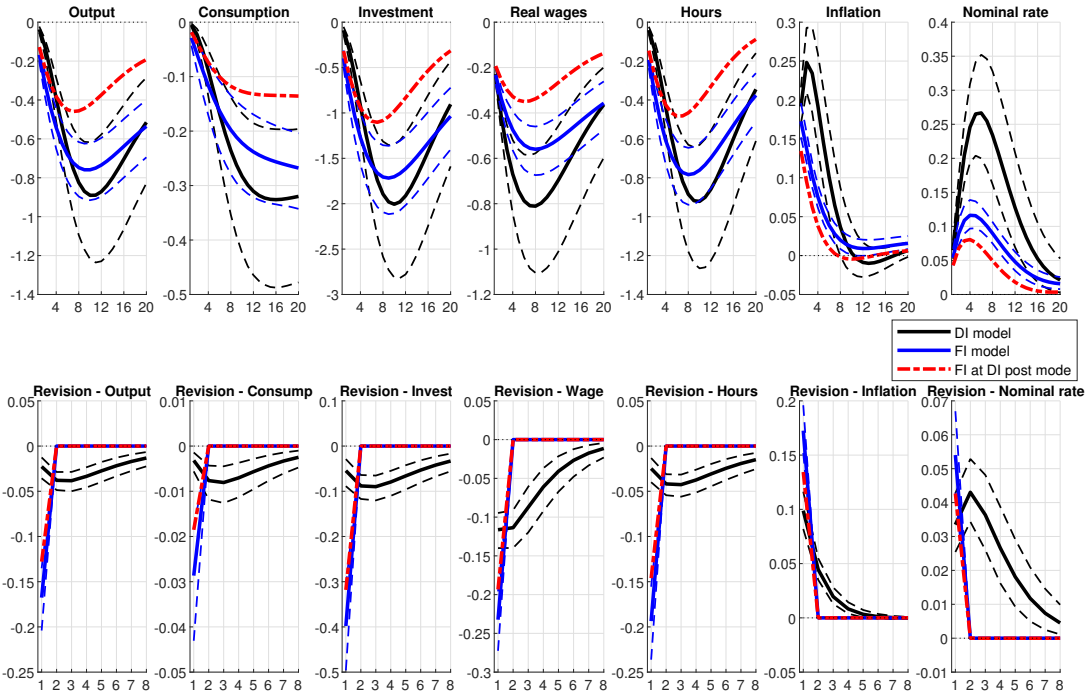
full-information rational expectations, and it is one of the key reasons why these models cannot provide a good fit to the expectations data.

There are noticeable differences in the responses of DI and FI dynamics to a cost-push (price markup) shock. These differences are primarily due to information frictions (Figure 6). The DI model generates a more pronounced and hump-shaped response of inflation, along with a slightly more significant decrease in real aggregates (output, consumption, investment, and hours) and a considerably larger decrease in real wages. The peak effect happens after 10 quarters for output, hours and investment, slightly longer than in the FI model. At the same time, average higher-order expectations converge to the path of the actual shock (Figure 4).

The DI model implies a stronger nominal rate response, with the peak response after 6 quarters roughly two times higher than the FI model. Overall, the DI model implies that a typical cost-push shock has a stronger impact on inflation and real variables, with a stronger response from the central bank than the FI model.

Interestingly, the combination of persistent shocks and strong information frictions – as discussed in Figure 4 – implies an attenuated and hump-shaped response of forecast revisions for real aggregates. After the shock, agents correctly anticipate the fall in real activity and the increase in inflation which are gradually revised quarter by quarter for more than two years.

Figure 6: Impulse responses to price markup shock

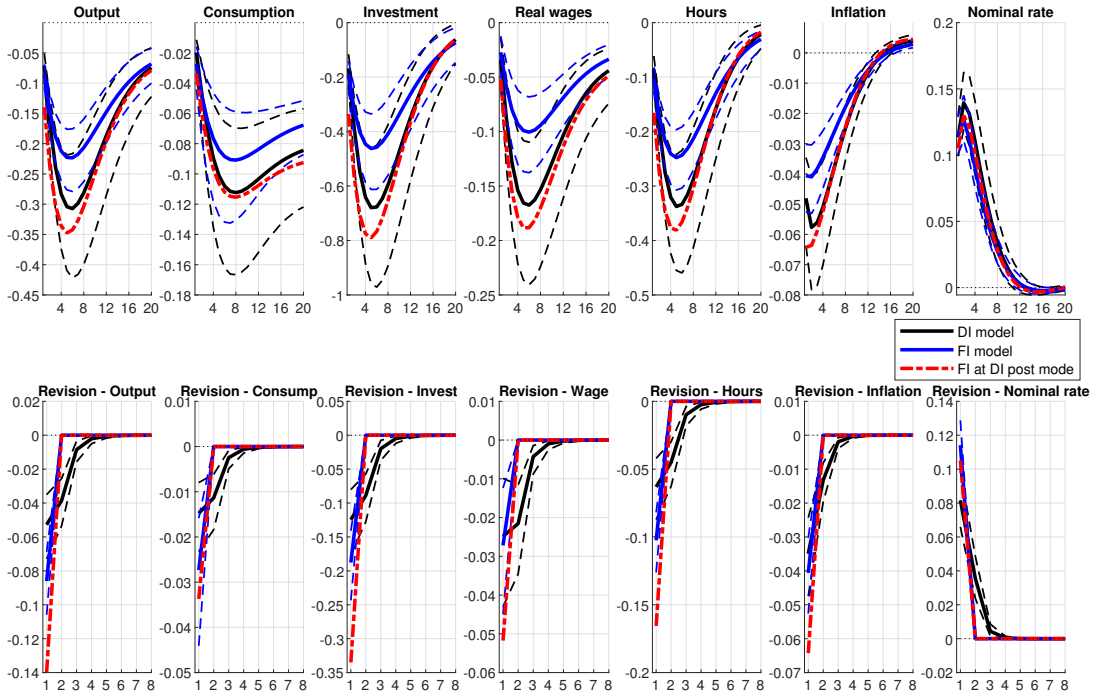


Note: The figure displays impulse response for a one standard deviation shock, based on the estimated DI and FI models. For comparison purposes, the IRF for the FI model using the DI model's posterior mode (red dot-dashed line) employs a one standard deviation shock from the FI model's posterior mode. The dashed lines represent 90% credible intervals.

Finally, a monetary policy shock leads to a decline in output and investment with peak response after six quarters and eight quarters for consumption. Inflation also decreases with peak response in the quarter ahead (Figure 7). The impulse responses of forecast revisions indicate that agents adjust their expectations about real variables and inflation downward after the shock hits, while revising their expectations about the nominal rate upward.

Comparing the FI and DI models, we find that monetary policy shocks have stronger effects on macroeconomic aggregates and inflation in the DI model. The difference is primarily due to changes in parameter estimates rather than information frictions, as the red dashed and black lines are very similar. This is consistent with the fact that the monetary shock has the lowest estimated level of information friction compared to all other shocks.

Figure 7: Impulse responses to a monetary policy shock



Note: The figure displays impulse response for a one standard deviation shock, based on the estimated DI and FI models. For comparison purposes, the IRF for the FI model using the DI model's posterior mode (red dot-dashed line) employs a one standard deviation shock from the FI model's posterior mode. The dashed lines represent 90% credible intervals.

Figures with impulse responses to the remaining shocks are available in Appendix E.

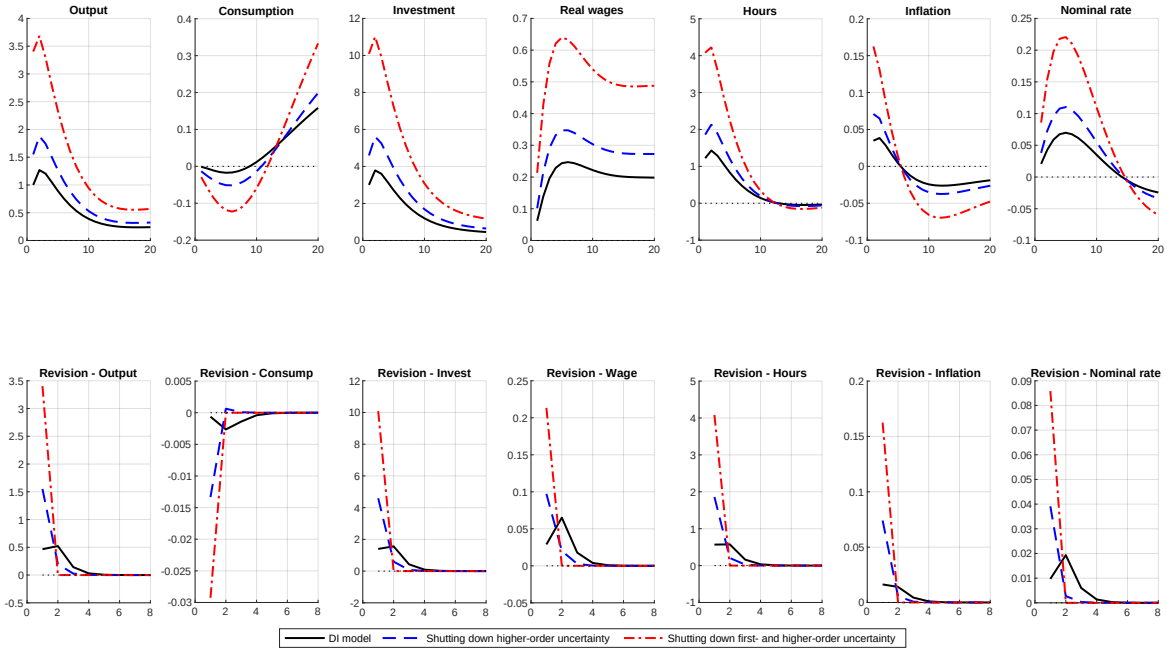
Decomposing the role of information frictions. The implications of incomplete information, in the form of dispersed information in our model, are twofold: i) it generates gradual learning about exogenous shocks (first-order uncertainty), and ii) uncertainty about other agents' expectations (higher-order uncertainty). In this section, we investigate the role of each force in shaping the effects of information frictions on the propagation of shocks. Here, we focus on investment-technological and price mark-up (cost-push) shocks, which information frictions play a role and are the main drivers of output and inflation, respectively.

We assess the relative importance of those forces by constructing two auxiliary IRFs. The first shuts down higher-order uncertainty by imposing that higher-order beliefs follow the same dynamics as first-order beliefs. This holds in any model with imperfect but common information (dashed blue lines in the following figures). To construct the second, we shut down both first- and higher-order uncertainty, i.e., the full hierarchy of expectations is equal to the true dynamics of

the shock (dot-dashed red lines). This corresponds to the full information benchmark, solved with the same parameter values of the DI model.[28]

Figure 8 shows the counterfactual exercise for the investment shock. For almost all variables, higher-order beliefs do not have a great bite, as evidenced by the small difference between the solid black and dashed blue lines. Imperfect learning (first-order uncertainty) is the key factor behind the dampening effect of information frictions, as evidenced by the large differences between the dot-dashed red and solid black lines.

Figure 8: Decomposition of impulse responses to an investment shock



Note: The figure shows the role of first-order and higher-order uncertainty for the propagation of an investment shock.

Figure 9 shows the same analysis to a cost-push (price markup) shock. We find that for real variables, first- and higher-order expectations contribute roughly the same to the shock propagation, both mitigating the dynamic effects of the shock. The learning contributes in postponing the peak effect on those variables in one quarter (from 7 to 8 quarters) and higher-order expectations in additional two quarters for output, investment and hours (from 8 to 10 quarters).

[28]IRFs named as 'FI at DI post mode' in Figures 5-6 depict the IRF in response of shock with the same size of a one-standard-deviation from the FI model for better comparison with the FI model. In Figure 8-9 the impulses from a one-standard deviation from the DI posterior.

For inflation, imperfect learning about the shock (first-order uncertainty) explains almost entirely the less inflationary effect of the cost-push shock in the DI model compared to the FI model. Learning is key to generating a hump-shape in the first year following the shock, with a peak in the second quarter.

Interestingly, first-order and higher-order uncertainty drive the nominal rate in different directions. The learning mitigates the nominal rate response in the first two years, while the higher-order expectations push nominal rates higher after the first year. The intuition is that learning about the shock mitigates the inflationary effect of the cost-push shock in the first year. Both first- and higher-order expectations do not play a major role in the inflation dynamics after the first year. In contrast, higher-order expectations have a major role in dampening the shock transmission to output. This drives a slightly tighter policy as the recessionary impact is attenuated. Despite altering largely the output response to the cost-push shock, higher-order uncertainty marginally affects the nominal rate because of the estimated high response to inflation ($\phi_\pi = 3.32$) and small response to output ($\phi_y = 0.07$).

Finally, both first- and higher-order expectations are key for expectations dynamics. The former attenuates the immediate response of expectations to the shock and generates persistence in forecast revisions. The latter weakens further the impact and creates a hump shape with a peak in the third quarter. Higher-order expectations do not generate the same pattern for inflation revision as its peak response is on impact.

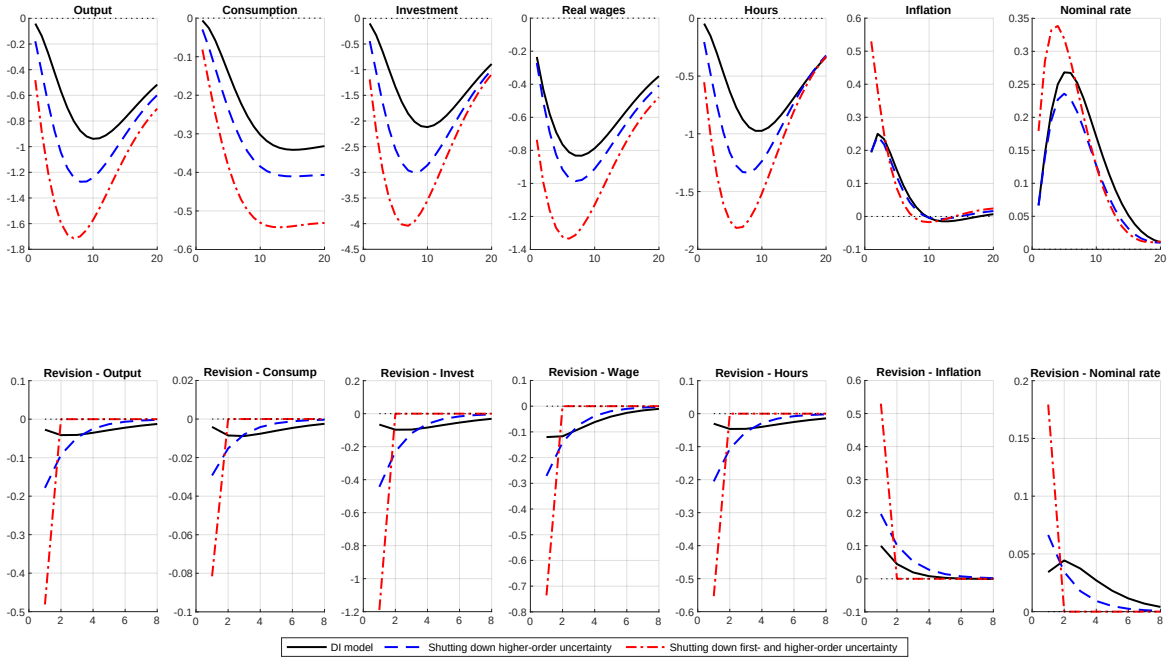
5.4 Information frictions: substitutes or complements for standard frictions?

As discussed before, a large body of literature has studied the potential of information frictions to substitute for standard frictions that generate inertia in macroeconomic models. In our quantitative exploration, standard and information frictions are both essential to fit the macroeconomic and expectations data.

In this section, we further investigate which frictions are most crucial for explaining the data by re-estimating the model shutting down frictions that generate inertia and comparing the marginal likelihood of the re-estimated models with our baseline specification. Table 2 shows the marginal likelihood of the models re-estimated with both datasets, the one incorporating only macroeconomic data and the one including expectations data.

When estimating the model with both standard and information frictions using macroeconomic data only, standard frictions remain at high levels as estimated in mainstream full-information macroeconomic models, while information frictions are relatively weaker. Moreover, the comparison of marginal likelihoods suggests that information frictions do not play such an important role

Figure 9: Decomposition of impulse responses to a price mark-up shock



Note: The figure shows the role of first-order and higher-order uncertainty for the propagation of a price mark-up shock.

in explaining macroeconomic data when standard frictions are present (the marginal likelihood of the full-information model is about 17 log points higher).

Interestingly, when using macroeconomic data only, abstracting from all standard frictions leads to a rather small loss in marginal likelihood, roughly 12 log points lower. Shutting down habit formation or investment adjustment costs leads to similar losses (roughly 8 log points less) while abstracting from price or wage indexation is marginally beneficial. When shutting down those frictions, the estimates from information frictions and shocks persistence increase. As discussed before, the combination of high persistence of shocks and strong information frictions can generate inertia, and those compensate for the absence of standard frictions in the model. This point is especially true for preference shocks and investment-specific technological shocks. [29] SW does the same exercise for the full information benchmark with macroeconomic data only. They point out the major role of habits and investment adjustment costs in fitting the data and the minor

[29] In the baseline estimation, $\rho_c = 0.77$, $k_c = 0.92$, $\rho_i = 0.46$ and $k_i = 0.93$. The case without habits estimates $\rho_c = 0.94$ and $k_c = 0.60$ while shutting down investment adjustment costs estimates $\rho_i = 0.91$, $k_i = 0.87$. See Table 5 in Appendix F for details.

importance of price and wage indexation. Using the same macroeconomic dataset and including information frictions, abstracting from habits and adjustment costs does not impact the model fit as much.

This is consistent with the hypothesis of substituting standard frictions with information frictions as proposed by previous literature. However, our baseline results suggest that information frictions are not well identified when abstracting from expectations data in the estimation. This comparison is better suited when we have data that can properly identify those deep parameters.

The same exercise using macroeconomic and expectations data leads toward a more complementary view of the frictions. When all frictions are removed, the log marginal likelihood is roughly 115 points lower. Eliminating only habits leads to a drop of similar magnitude. Adjustment costs in investment are important, but only contribute to half of the drop in log marginal likelihood compared to the former. Shutting down price or wage indexation leads to better marginal likelihoods than the baseline model. These results are consistent with SW's original findings regarding the relevance of habits and investment adjustment costs for fitting the data (now including expectations data) and the minor importance of price and wage indexation. In other words, the absence of standard frictions cannot be fully compensated by information frictions when disciplining the model with expectations data since there are fewer degrees of freedom.[30]

We also evaluate the relative importance of incomplete information for the model's fit to the complete dataset.[31] First, removing all information frictions (the full information case) leads to a reduction in the log marginal likelihood of about 211 points, almost twice as much as the case without standard frictions. This underscores the relevance of the information frictions for explaining the expectations data. Second, among all information frictions, the ones related to preference and investment shocks are the most crucial. Both lead to losses in the model's marginal likelihood of a similar magnitude to shutting down all standard frictions or habits only. Government expenditure and cost-push shocks also play an important role (roughly 46 and 35 drops, respectively), in a magnitude similar to the investment adjustment costs. Information frictions on other shocks play a minor role.

In summary, our results portray a more complementary view of information and standard frictions, compared to most of the previous literature. The intuition behind this key conclusion is the following. Macroeconomic and expectations data require the combination of low persistence and strong information frictions for preference and investment shocks, which are insufficient to create the necessary persistence in endogenous variables. Therefore, intrinsic persistence in en-

[30] In the baseline estimation, $\rho_c = 0.13$, $k_c = 0.26$, $\rho_i = 0.24$ and $k_i = 0.45$. The case without habits estimates $\rho_c = 0.90$ and $k_c = 0.66$ while shutting down investment adjustment costs estimates $\rho_i = 0.72$, $k_i = 0.58$. Thus, information frictions do not increase to compensate other frictions as in the estimation with macroeconomic dataset onyl. See Table 4 in Appendix F for details.

[31] We do not pursue the same exercise for the dataset with macroeconomic data only because our main results suggest that those frictions are not very well identified using this data.

Table 2: Marginal Likelihoods shutting down key frictions

Dataset:	Macroeconomic data only		Macroeconomic and expectations data	
Model	Marginal Likelihood	Difference from baseline	Marginal Likelihood	Difference from baseline
Baseline	-942.25	-	-1333.20	-
Shutting down:				
All standard frictions	-954.81	-12.56	-1448.70	-115.50
Habits	-951.22	-8.96	-1436.90	-103.70
Invest. adjustment costs*	-950.97	-8.71	-1377.70	-44.50
Price indexation	-938.74	3.51	-1327.60	5.60
Wage indexation	-937.72	4.54	-1322.90	10.30
Full information for:				
All shocks	-925.45	16.80	-1544.50	-211.30
Preference shock	-	-	-1438.50	-105.30
Inv. specific tech. shock	-	-	-1438.01	-104.81
Gov. expenditure shock	-	-	-1379.90	-46.70
Price markup shock	-	-	-1368.70	-35.50
Wage markup shock	-	-	-1348.60	-15.40
Monetary policy shock	-	-	-1337.50	-4.30
TFP shock	-	-	-1336.30	-3.10

Notes: Estimates from each model use 400k draws and 50% burn-in from the same algorithm as in the baseline model. Log-marginal likelihoods are computed using the modified harmonic mean estimator of Geweke (1999). *When shutting down investment adjustment costs, we substitute them with standard capital adjustment costs.

dogenous variables is needed, which traditionally comes from standard frictions such as habits and investment costs.

One crucial exception is inflation. The forecast error variance decomposition from the DI model shows that mark-up shocks play a major role in explaining inflation fluctuations (section 5.2). Figure 6 shows that the model with information frictions generates a hump-shaped response of inflation to a cost-push shock that the FI model cannot generate. This is due to the high estimates for information frictions and persistence for this particular shock, along with a low estimated price indexation. Angeletos and La'O (2009) shows in a Calvo model with incomplete information that higher-order expectations play an important role in inflation dynamics even in the presence of nominal rigidity and learning about shocks. Our results corroborate this view.[32]

[32]The key difference is that they study a nominal demand shock while we find price mark-up shocks as the main driver of inflation.

6 Conclusion

We develop a general solution method that allows enriching a standard medium-scale DSGE model with dispersed information and estimate using Bayesian techniques with comprehensive macroeconomic and expectation data. We draw important conclusions regarding the role information frictions play in business cycles. The degree of informational friction varies significantly across shocks and indicates important departures from perfect information. Relatedly, simulated data from the model can match standard empirical measures of informational frictions when using data on forecast revisions in the estimation.

Moreover, we contribute to the business cycle literature by showing that the inclusion of informational frictions and expectation data changes the relative importance of shocks, as the general equilibrium effects are dampened due to a weaker response to expectations about future conditions. In general, shocks that directly impact some particular variables are more relevant to explaining their fluctuations in the DI model. For instance, price mark-up shocks play a stronger role in explaining inflation dynamics in the DI model compared to the FI model.

We also contribute to the literature by re-assessing a long-standing view that information frictions could be an alternative source of sluggishness in macroeconomic data. Our results suggest a more complementary view of the standard and informational frictions. When disciplining our model with expectation data, standard frictions such as habits and investment costs increase substantially despite the pervasive informational frictions. Hence, both types of frictions are important to explain the data.

In our estimated model, shocks with stronger informational frictions also exhibit weak persistence. This combination helps to lessen the impact of shocks and expectations on endogenous variables, but it does not produce the necessary high inertia, as observed in the data. Strong standard frictions instead provide this inertia. One important exception is the price-mark-up shock, for which information frictions introduce additional sluggishness in inflation and nominal rates that cannot be produced by standard frictions alone.

A natural extension for our work is embedding a richer informational structure, including both public endogenous and exogenous signals, for instance. Additionally, our model provides a natural setting to study the role of additional shocks related to agents' information sets such as noise shocks (Lorenzoni; 2010; Blanchard et al.; 2013) or confidence shocks (Angeletos and La'O; 2013; Angeletos et al.; 2018). In conclusion, we see an open venue for exploring the quantitative potential of DSGE models with informational frictions.

References

An, S. and Schorfheide, F. (2007). Bayesian analysis of dsge models, *Econometric reviews* **26**(2-4): 113–172.

Angeletos, G.-M., Collard, F. and Dellas, H. (2018). Quantifying confidence, *Econometrica* **86**(5): 1689–1726.

Angeletos, G.-M. and Huo, Z. (2021). Myopia and Anchoring, *American Economic Review* **111**(4): 1166–1200.

Angeletos, G.-M. and La'O, J. (2009). Incomplete Information, Higher-order Beliefs and Price Inertia, *Journal of Monetary Economics* **56**: S19 – S37.

Angeletos, G.-M. and La'O, J. (2011). Optimal Monetary Policy with Informational Frictions, *Technical report*, National Bureau of Economic Research Working Paper Series.

Angeletos, G.-M. and La'O, J. (2013). Sentiments, *Econometrica* **81**(2): 739–779.

Auclert, A., Rognlie, M. and Straub, L. (2020). Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model, *NBER Working Papers 26647*, National Bureau of Economic Research, Inc.

Baxter, B., Graham, L. and Wright, S. (2011). Invertible and non-invertible information sets in linear rational expectations models, **35**(3): 295–311.

Bianchi, F., Ilut, C. and Saijo, H. (2024). Diagnostic business cycles, *The Review of Economic Studies* **91**(1): 129–162.

Blanchard, O. J., L'Huillier, J.-P. and Lorenzoni, G. (2013). News, noise, and fluctuations: An empirical exploration, *American Economic Review* **103**(7): 3045–70.

Bordalo, P., Gennaioli, N., Ma, Y. and Shleifer, A. (2020). Overreaction in macroeconomic expectations, *American Economic Review* **110**(9): 2748–82.

Broer, T. and Kohlhas, A. N. (2024). Forecaster (mis-)behavior, **106**(5): 1334–1351.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* **12**(3): 383–398.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of Political Economy* **113**(1): 1–45.

Coibion, O. and Gorodnichenko, Y. (2012). What can survey forecasts tell us about information rigidities?, *Journal of Political Economy* **120**(1): 116–159.

Coibion, O. and Gorodnichenko, Y. (2015). Information rigidity and the expectations formation process: A simple framework and new facts, *American Economic Review* **105**(8): 2644–2678.

Collard, F. and Dellas, H. (2010). Monetary misperceptions, output, and inflation dynamics, **42**(2/3): 483–502.

Collard, F., Dellas, H. and Smets, F. (2009). Imperfect information and the business cycle, *Journal of Monetary Economics* **56**: S38–S56.

Del Negro, M. and Eusepi, S. (2011). Fitting observed inflation expectations, *Journal of Economic Dynamics and Control* **35**(12): 2105–2131.

Del Negro, M., Giannoni, M. P. and Schorfheide, F. (2015). Inflation in the great recession and new keynesian models, *American Economic Journal: Macroeconomics* **7**(1): 168–96.

Del Negro, M., Schorfheide, F., Smets, F. and Wouters, R. (2007). On the fit of new keynesian models, *Journal of Business & Economic Statistics* **25**(2): 123–143.

Erceg, C. J., Henderson, D. W. and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics* **46**(2): 281–313.

Gabaix, X. (2020). A behavioral new keynesian model, *American Economic Review* **110**(8): 2271–2327.

Geweke, J. (1999). Using simulation methods for bayesian econometric models: inference, development,and communication, *Econometric Reviews* **18**(1): 1–73.

Grigsby, J., Hurst, E. and Yildirmaz, A. (2021). Aggregate nominal wage adjustments: New evidence from administrative payroll data, **111**(2): 428–71.

Hajdini, I. (2023). Mis-specified forecasts and myopia in an estimated new keynesian model, *Working Paper 22-03R*, Federal Reserve Bank of Cleveland.

Hamilton, J. D. (1995). *Time series analysis*, Cambridge Univ Press.

Huo, Z. and Takayama, N. (2024). Rational expectations models with higher-order beliefs, *The Review of Economic Studies* p. rdae096.

Justiniano, A., Primiceri, G. E. and Tambalotti, A. (2011). Investment shocks and the relative price of investment, *Review of Economic Dynamics* **14**(1): 102–121. Special issue: Sources of Business Cycles.

Kasa, K. (2000). Forecasting the forecasts of others in the frequency domain, *Review of Economic Dynamics* **3**(4): 726–756.

Kasa, K., Walker, T. B. and Whiteman, C. H. (2014). Heterogeneous beliefs and tests of present value models, **81**(3): 1137–1163.

Kohlhas, A. N. and Walther, A. (2021). Asymmetric attention, **111**(9): 2879–2925.

Levine, P., Pearlman, J., Perendia, G. and Yang, B. (2012). Endogenous persistence in an estimated dsge model under imperfect information, **122**(565): 1287–1312.

L'Huillier, J.-P., Singh, S. R. and Yoo, D. (2024). Incorporating diagnostic expectations into the new keynesian framework, *The Review of Economic Studies* **91**(5): 3013–3046.

Lorenzoni, G. (2010). Optimal monetary policy with uncertain fundamentals and dispersed information, **77**(1): 305–338.

Maćkowiak, B. and Wiederholt, M. (2015). Business cycle dynamics under rational inattention, *The Review of Economic Studies* **82**(4): 1502–1532.

Mankiw, N. G. and Reis, R. (2002). Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve, *The Quarterly Journal of Economics* **117**(4): 1295–1328.

Mankiw, N. G. and Reis, R. (2007). Sticky information in general equilibrium, *Journal of the European Economic Association* **5**(2-3): 603–613.

Mankiw, N. G., Reis, R. and Wolfers, J. (2004). *Disagreement about Inflation Expectations*, The MIT Press, pp. 209–270.

Melosi, L. (2014). Estimating models with dispersed information, *American Economic Journal: Macroeconomics* **6**(1): 1–31.

Melosi, L. (2017). Signalling effects of monetary policy, *The Review of Economic Studies* **84**(2): 853–884.

Milani, F. (2017). Sentiment and the u.s. business cycle, *Journal of Economic Dynamics and Control* **82**: 289–311.

Milani, F. (2023). Chapter 18 - expectational data in dsge models, *in* R. Bachmann, G. Topa and W. van der Klaauw (eds), *Handbook of Economic Expectations*, Academic Press, pp. 541–567.

Morris, S. and Shin, H. S. (2005). Central bank transparency and the signal value of prices, *Brookings Papers on Economic Activity* **2005**(2): 1–66.

Nagel, R. (1995). Unraveling in guessing games: An experimental study, *American Economic Review* **85**(5): 1313–1326.

Nakamura, E. and Steinsson, J. (2008). Five Facts about Prices: A Reevaluation of Menu Cost Models, *The Quarterly Journal of Economics* **123**(4): 1415–1464.

Nimark, K. (2008). Dynamic pricing and imperfect common knowledge, *Journal of Monetary Economics* **55**(2): 365–382.

Nimark, K. (2017). Dynamic higher order expectations, *Mimeo* .

Ribeiro, M. (2017). Solution of linear rational expectations model with imperfect common knowledge, *Mimeo* .

Ribeiro, M. (2018). *Essays on imperfect common knowledge in macroeconomics*, PhD thesis.

Rondina, G. and Walker, T. B. (2021). Confounding dynamics, **196**: 105251.

Sims, C. A. (2003). Implications of rational inattention, *Journal of Monetary Economics* **50**(3): 665–690.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area, *Journal of the European Economic Association* **1**(5): 1123–1175.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach, *American Economic Review* **97**(3): 586–606.

Townsend, R. M. (1983). Forecasting the forecasts of others, *Journal of Political Economy* **91**(4): 546–588.

Uhlig, H. (2001). 30A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily, *Computational Methods for the Study of Dynamic Economies*, Oxford University Press.

Woodford, M. (2002). Imperfect common knowledge and the effects of monetary policy, *Aghion, P., R. Frydman, J. Stiglitz, and M. Woodford (eds.) Knowledge, Information and Expectations in Modern Macroeconomics*, Princeton University Press.

Appendix

A Log-linear model and full derivation

A.1 Optimality conditions

Households

From the households' utility maximization problem, we get the following first-order conditions for consumption and bonds, respectively

$$E_{ht}[\Lambda_{h,t}P_t] = \frac{E_{ht}\left[e^{\eta_t^c}\right]}{C_{h,t} - \varphi C_{h,t-1}} \tag{55}$$

$$E_{ht}[\Lambda_{h,t}] = \beta E_{ht}\left[\Lambda_{h,t+1}R_t\right] \tag{56}$$

where $\Lambda_{h,t}$ is the Lagrange multiplier of households' budget constraint (20). The optimal conditions for capital and investment are given by, respectively:

$$\Phi_{h,t} = \beta E_{ht}\left[\Lambda_{h,t+1}\left(R_{t+1}^k U_{h,t+1} - P_{t+1}a(U_{h,t+1})\right)P_{t+1}\right] + (1-\delta)\beta E_{ht}\left[\Phi_{h,t+1}\right] \tag{57}$$

$$\begin{aligned} E_{ht}[\Lambda_{h,t}P_t] = \Phi_{h,t}E_{ht}\left[e^{\eta_t^i}\right]\left(1 - S\left(\frac{I_{h,t}}{I_{h,t-1}}\right) - S'\left(\frac{I_{h,t}}{I_{h,t-1}}\right)\frac{I_{h,t}}{I_{h,t-1}}\right) \\ +\beta E_{ht}\left[\Phi_{h,t+1}e^{\eta_{t+1}^i}S'\left(\frac{I_{h,t+1}}{I_{h,t}}\right)\left(\frac{I_{h,t+1}}{I_{h,t}}\right)^2\right] \end{aligned} \tag{58}$$

where $\Phi_{h,t}$ is the Lagrange multiplier associated with the law of motion of capital. The optimal value of the capital utilization rate solves

$$E_{ht}[R_t^k] = a'(U_{h,t})E_{ht}[P_t]. \tag{59}$$

Recall that households can buy state-contingent assets in terms of consumption (but not for leisure). Therefore, it must hold that for all households $h \in [0,1]$ that $C_{h,t} = C_t$, $I_{h,t} = I_t$, $U_{h,t} = U_t$, $\Lambda_{h,t} = \Lambda_t$ and $\Phi_{h,t} = \Phi_t$. However, households still have heterogeneous expectations, $E_{ht}[\cdot]$, wages $w_{h,t}$ and labor supply $L_{h,t}$.

The optimal condition of the wage setting problem is

$$E_{ht}\left[\sum_{s=0}^{\infty}(\beta\xi_w)^s\left(\Lambda_{t,t+s}\frac{X_{t,t+s}^w}{\mu_{t+s}^w} - \frac{1+\mu_{t+s}^w}{\mu_{t+s}^w}\frac{L_{h,t+s}^\chi}{W_{h,t}^*}\right)L_{h,t+s}\right] = 0 \tag{60}$$

where $\Lambda_{t,t+s} \equiv \frac{\Lambda_{t+s}}{\Lambda_t}$ is the stochastic discount factor from period t to period $t+s$. The marginal rate of substitution of labor and consumption is defined as

$$MRS_{h,t} \equiv -\frac{U_{L_{h,t}}}{U_{C_{h,t}}} = L_{h,t}^{\chi}\left(C_{h,t} - \varphi C_{h,t-1}\right). \tag{61}$$

Intermediate good firms

At stage 1, firms choose their optimal price based on the information from signals. The optimal price $P_{i,t}^*$ that maximize intermediate firm i's profit solve the following first-order condition

$$E_{it}\left[\sum_{s=0}^{\infty}(\beta\xi_p)^s\Lambda_{t,t+s}\left(\frac{X_{t,t+s}}{\mu_{t+s}^p} - \frac{1+\mu_{t+s}^p}{\mu_{t+s}^p}\frac{MC_{i,t+s}}{P_{i,t}^*}\right)Y_{i,t+s}\right] = 0. \tag{62}$$

At stage 2, they hire labor at the nominal wage W_t and rent capital at the rental rate R_t^k. Cost minimization subject to production function (5) implies that the capital-labor ratio is given by[33]

$$\frac{K_{i,t}}{L_{i,t}} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}. \tag{63}$$

We obtain that the marginal cost is given by

$$MC_{i,t} = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{\left(R_t^k\right)^\alpha\left(W_t\right)^{1-\alpha}}{a_t}. \tag{64}$$

Since the production function displays constant return of scale, capital-labor ratios and marginal costs are the same across firms in equilibrium.

A.2 Detrending and log-linearized model

Before showing the system of log-linearized equations that characterizes the model, we discuss the detrending procedure.

All real variables grow along with the productivity trend, so they are detrended as follows: $\hat{Z}_t = \frac{Z_t}{\gamma^t}$, for any real variable Z_t. Nominal variables grow along with the price level P_t, hence they are stationarized using the following procedure: $\hat{Z}_t = \frac{Z_t}{P_t}$.[34]

One exception is the optimal prices and wages are detrended by dividing them to the price level of last period: $\hat{P}_{i,t}^* = P_{i,t}^*/P_{t-1}$, while $\hat{W}_{h,t}^* = \hat{W}_{h,t}^*/\gamma^t P_{t-1}$.[35]

[33] Since factor prices are revealed at stage 2, firms does not need to form expectations for W_t and R_t^k.

[34] One exception is the Lagrange multiplier of the budget constraint, Λ_t, that must be normalized to $\hat{\Lambda}_t = \Lambda_t\gamma^t P_t$, since it reflects the marginal utility of an additional unit of money, which declines as consumption grows at rate γ and the price level rises.

[35] This simplifies the derivation as $E_{h,t}[\hat{W}_{h,t}^*] = \hat{W}_{h,t}^*$ and $E_{i,t}[\hat{P}_{i,t}^*] = \hat{P}_{i,t}^*$ since observe P_{t-1}. This would not be

Stationary variables are transformed by taking their log-deviation to steady-state value as follows: $z_t = log(Z_t/\bar{Z})$, for any stationary variable Z_t. Thus, lower case variables denote log-deviation from steady-state of the upper case variables.

The log-linearization of optimal conditions leads to the following system of equations

$$c_t = \frac{\varphi/\gamma}{1+\varphi/\gamma}c_{t-1} + \frac{1}{1+\varphi/\gamma}\bar{E}_t[c_{t+1}] - \frac{1-\varphi/\gamma}{1+\varphi/\gamma}\bar{E}_t[r_t - \pi_{t+1} - (x^c_{t+1} - x^c_t)] \tag{65.1}$$

$$i_t = \left(\frac{1}{1+\beta}\right) i_{t-1} + \left(\frac{\beta}{1+\beta}\right)\bar{E}_t[i_{t+1}] + \left(\frac{1}{s''(1+\beta)\gamma^2}\right)\left(q_t + \bar{E}_t\left[x^i_t\right]\right) \tag{65.2}$$

$$q_t = \left(\frac{\beta(1-\delta)}{\gamma}\right)\bar{E}_t[q_{t+1}] + \left(1 - \frac{\beta(1-\delta)}{\gamma}\right)\bar{E}_t\left[r^k_{t+1}\right] - \bar{E}_t[r_t - \pi_{t+1}] \tag{65.3}$$

$$k_t = \left(\frac{1-\delta}{\gamma}\right) k_{t-1} + \left(1 - \frac{1-\delta}{\gamma}\right)\left(i_t + x^i_t\right) \tag{65.4}$$

$$u_t = \left(\bar{R}^k/a''\right)\bar{E}_t[r^k_t] \tag{65.5}$$

$$k^u_t = k_{t-1} + u_t \tag{65.6}$$

$$mrs_t = \left(\frac{1}{1-\varphi/\gamma}\right)(c_t - \varphi/\gamma c_{t-1}) + \chi l_t \tag{65.7}$$

$$y_t = \left(1 + \frac{\Phi_p}{\bar{Y}}\right)(\alpha k^u_t + (1-\alpha)l_t + x^a_t) \tag{65.8}$$

$$k^u_t - l_t = w_t - r^k_t \tag{65.9}$$

$$mc_t = (1-\alpha)w_t + \alpha r^k_t - x^a_t \tag{65.10}$$

$$y_t = \frac{1}{1-g_y}\left(\frac{\bar{C}}{\bar{Y}}c_t + \frac{\bar{I}}{\bar{Y}}i_t + \frac{\bar{R}^k\bar{K}}{\bar{Y}}u_t + x^g_t\right) \tag{65.11}$$

$$r_t = \phi_r r_{t-1} + (1-\phi_r)(\phi_\pi \pi_t + \phi_y y_t) + x^r_t \tag{65.12}$$

$$\begin{aligned}\pi_t =& \xi_p\iota_p\pi_{t-1} + \kappa_p\xi_p\bar{E}_t[mc_t + x^p_t] + (1-\xi_p)(1-\iota_p\beta\xi_p)\bar{E}_t[\pi_t] \\ &+ (1-\xi_p)\beta\xi_p\int_0^1 E_{it}[p^*_{i,t+1}]di\end{aligned} \tag{65.13}$$

$$w_t + \pi_t = \xi_w(\iota_w\pi_{t-1} + w_{t-1}) + (1-\xi_w)w^*_t \tag{65.14}$$

$$w^*_t = \frac{(1-\beta\xi_w)}{1+\chi\theta_w}\bar{E}_t[mrs_t + w_t + x^w_t] + (1-\iota_w\beta\xi_w)\bar{E}_t[\pi_t] + \beta\xi_p\int_0^1 E_{ht}\left[w^*_{h,t+1}\right]dh \tag{65.15}$$

where $\kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}$, $\kappa_w = \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(1+\chi\theta_w)}$ and $\theta_w = \frac{1+\bar{\mu}^w}{\bar{\mu}^w}$. $\bar{E}_t[\cdot] \equiv \int_0^1 E_{it}[\cdot]di$ denotes average expectation, where $E_{it}[\cdot] \equiv E[\cdot \mid \mathcal{I}_{it}]$ is agent i's expectation based on her information set at period t, $\mathcal{I}_{it}$.

All equations are standard in macroeconomic models. The first equation is the Euler equation

true if we detrended them using P_t.

for consumption, which is derived by combining the log-linearized versions of equations (55) and (56). The second equation is the equilibrium condition for investment from (58) and the third determines the marginal real value of a unit of capital q_t that come from condition (57). The forth is log-linearized version of the usual law of motion of capital (17). The fifth is the log-linearized version of the optimal condition for capital utilization (59) and the sixth comes from the definition of utilized capital (19). The seventh and eighth are log-linearized versions of the aggregation of the marginal rate of substitution between consumption and labor from equation (61) and the production function (5), respectively. The ninth and tenth comes from aggregating firms' cost minimization condition (63) and marginal cost (63), respectively. The eleventh derives from the aggregate resource constraint (29), while twelfth is the log-linearized version of the Taylor rule (25). Finally, the thirteenth is the New Keynesian Phillips curve for prices that combines log-linearized versions of equations (62) and (10). The last equation is the New Keynesian Phillips curve for wages that combines log-linearized versions of equations (60) and (24).

The derivation of the model can be found in the Online Appendix.

B Signal extraction and steady-state Kalman gain

In section 4, we estimate the steady-state Kalman gain for each shock directly. While the Kalman gain is a endogenous object, our information structure implies that it has a one-to-one relationship to the private signal variance, conditional on the shock persistence and variance. In the following, we show that it is the case.

Recall that all shocks have the same AR(1) structure given by

$$x_t^l = \rho_l x_{t-1}^l + \varepsilon_t^l, \quad \varepsilon_t^l \sim \mathcal{N}(0, \sigma_l^2). \tag{66}$$

where $l \in \{a, c, i, g, p, w, r\}$.

Agents use information from their signals (30), restated here to covenience:

$$s_{j,t}^l = x_t^l + v_{j,t}^l, \quad v_{j,t}^l \sim \mathcal{N}(0, \tau_l^2). \tag{67}$$

Agents must form expectations about the whole hierarchy of expectations of all shocks and the solution method takes into account. Here we simply show how our measure of information friction – the steady-state Kalman gain from the first-order expectation of individual j – can be found as a function of deep parameters.

Since shocks and their respective signals are independent, their signal extraction can be written separately. The rational expectation of each shock l can be computed by the Kalman filter using (66) as state equation and (67) as measurement equation.

The update equation of the Kalman filter is given by

$$E_{j,t}[x_t^l] = E_{j,t-1}[x_t^l] + \bar{k}_l \left[s_{j,t}^l - E_{j,t-1}[s_{j,t}^l] \right] \tag{68}$$

where $\bar{k}_l$ is the steady-state Kalman gain, as discussed in Appendix D.1. The steady-state Kalman gain for each shock l can be computed by equations (76)-(77) applied to the state equation as (66) and measurement equation (67) and evaluated at the steady-state. This implies the following expressions:

$$\bar{k}_l = \frac{\bar{p}_l}{\bar{p}_l + \tau_l^2} \tag{69}$$

$$\bar{p}_l = \rho_l^2 (1 - \bar{k}_l) \bar{p}_l + \sigma_l^2 \tag{70}$$

where $\bar{k}_l$ and $\bar{p}_l$ are the Kalman gain and the mean squared error (MSE) of the one-period ahead prediction error both at steady-state.

Note that, the Kalman gain can be expressed alternatively in terms of the MSE relative to the shock variance such that

$$\bar{k}_l = \frac{\tilde{p}_l}{\tilde{p}_l + r_l^2}$$

$$\tilde{p}_l = \rho_l^2 (1 - \bar{k}_l) \tilde{p}_l + r_l$$

where $\tilde{p}_l = \bar{p}_l / \sigma_l^2$ is the relative MSE and $r_l = \tau_l / \sigma_l$ the noise-to-signal ratio. The first equation simply divides the denominator and numerator of equation (69) by σ_l^2 while second equation can be found by dividing equation (70) by σ_l^2 on both sides.

Both equations leads to the quadratic Riccati equation in terms of the noise-to-signal ratio:

$$\tilde{p}_l^2 - \left(1 - (1 - \rho_l^2) r_l^2 \right) \tilde{p}_l - r_l^2 = 0$$

This equation also has a unique positive solution, leading to $\tilde{p}_l = \tilde{p}_l(\rho_l, r_l)$.

This implies that the steady-state Kalman filter depends only on the persistence of the shock and the noise-to-signal ratio $\bar{k}_l = \bar{k}_l(\rho_l, r_l) \in (0, 1)$.

C Data

Data on macroeconomic aggregates are collected from the Federal Reserve Database (FRED) and expectation data come from the Survey of Professional Forecasters (SPF). The time series collected to construct the database are displayed in Table 3. It starts in 4Q81, the first quarters with data

for all time series on expectations that I use in this work, and ends in 4Q07, before the Great Recession.

Table 3: Database

Series	**Description**	**Measure**	**Source**
GDPC1	Real Gross Domestic Product	Billions of Chained 2012 Dollars	BEA
PCECC96	Real Personal Consumption Expenditures	Billions of Chained 2012 Dollars	BEA
GPDIC1	Real Gross Private Domestic Investment	Billions of Chained 2012 Dollars	BEA
GDPDEF	Gross Domestic Product: Implicit Price Deflator	Index 2012=100	BEA
FF	Effective Federal Funds Rate	Percentage points	FED Board
PRS85006063	Nonfarm Business Sector: Compensation	Index 2012=100	BEA
HOANBS	Nonfarm Business Sector: Hours of All Persons	Index 2012=100	BLS
CNP16OV	Civilian Noninstitutional Population	Thousands of People	BLS
RGDP	Forecast for the real GDP	Chained Dollars (base year varies)	SPF
RCONSUM	Forecast for the real PCE	Chained Dollars (base year varies)	SPF
RNRESINV	Forecast for the nonresidential fixed investment	Chained Dollars (base year varies)	SPF
PGDP	Forecast for the GDP price index	Index (base year varies)	SPF
TBILL	Forecast for the annual three-month Treasury bill rate	Percentage points	SPF

Note: BEA, BLS, Fed Board and SPF stand for, respectively: U.S. Bureau of Economic Analysis, U.S. Bureau of Labor Statistics, Board of Governors of the Federal Reserve and Survey of Professional Forecasters.

Real output (GDPC1), consumption (PCE) and investment (GDPIC1) are computed as per capita aggregates by dividing them to the Hodrick-Prescott filtered population index (CNP16OV). Real wage is constructed by dividing the compensation in the nonfarm business sector (PRS85006063) by the GDP deflator (GDPDEF). Per capita total hours is computed by dividing the hours of all persons out nonfarm business sector (HOANBS) from the smoothed population index. Inflation is defined as the annualized quarterly growth rate of GDP deflator (GDPDEF). The nominal interest rate is computed by the log of the gross federal funds rate (FF) to be consistent with the log-linearization procedure.

$$
\begin{aligned}
Y_t^{obs} &= GDPC1_t / HP(CNP16OV_t) \\
C_t^{obs} &= PCE_t / HP(CNP16OV_t) \\
I_t^{obs} &= GPDIC1_t / HP(CNP16OV_t) \\
W_t^{obs} &= PRS85006063_t / GDPDEF_t \\
dy_t^{obs} &= 100 \log(Y_t^{obs} / Y_{t-1}^{obs}) \\
dc_t^{obs} &= 100 \log(C_t^{obs} / C_{t-1}^{obs}) \\
di_t^{obs} &= 100 \log(I_t^{obs} / I_{t-1}^{obs}) \\
dw_t^{obs} &= 100 \log(W_t^{obs} / W_{t-1}^{obs}) \\
L_t^{obs} &= HOANBS_t / HP(CNP16OV_t) \\
\Pi_t^{obs} &= 400 \log(GDPDEF_t / GDPDEF_{t-1}) \\
R_t^{obs} &= 100 \log(1 + FF_t / 100)
\end{aligned}
$$

Expectation data are constructed as follows

$$
\begin{aligned}
\bar{E}_t[Y_{t+1}^{obs}] &= RGDP3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t[Y_t^{obs}] &= RGDP2_t/HP(CNP16OV_t) \\
\bar{E}_t[C_{t+1}^{obs}] &= RCONSUM3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t[C_t^{obs}] &= RCONSUM2_t/HP(CNP16OV_t) \\
\bar{E}_t[I_{t+1}^{obs}] &= RINVEST3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t[I_t^{obs}] &= RINVEST32_t/HP(CNP16OV_t) \\
\bar{E}_t[dy_t^{obs}] &= 100\log\left(\bar{E}_t[Y_{t+1}^{obs}]/\bar{E}_t[Y_t^{obs}]\right) \\
\bar{E}_t[dy_t^{obs}] &= 100\log\left(\bar{E}_t[Y_t^{obs}]/\bar{E}_t[Y_{t-1}^{obs}]\right) \\
\bar{E}_t[dc_t^{obs}] &= 100\log\left(\bar{E}_t[C_t^{obs}]/\bar{E}_t[C_{t-1}^{obs}]\right) \\
\bar{E}_t[di_t^{obs}] &= 100\log\left(\bar{E}_t[I_t^{obs}]/\bar{E}_t[I_{t-1}^{obs}]\right) \\
\bar{E}_t[\pi_t^{obs}] &= 400\log(PGDP3t/PGDP2t) \\
\bar{E}_t[R_t^{obs}] &= \log(1+TBILL2_t/100) \\
\bar{F}[dy_t] &= dy_t^{obs} - \bar{E}_{t-1}[dy_t^{obs}] \\
\bar{F}[dc_t] &= dc_t^{obs} - \bar{E}_{t-1}[dc_t^{obs}] \\
\bar{F}[di_t] &= di_t^{obs} - \bar{E}_{t-1}[di_t^{obs}] \\
\bar{F}[\pi_t] &= \pi_t^{obs} - \bar{E}_{t-1}[\pi_t^{obs}] \\
\bar{F}[R_t] &= R_t^{obs} - \bar{E}_{t-1}[R_t^{obs}]
\end{aligned}
$$

D Solution method and Algorithm

We first start with two Lemmas. The first writes the first-order expectation of the hierarchy as a linear transformation of the hierarchy.

Lemma 1. *The first-order expectation of the hierarchy of expectations satisfy*

$$
E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = Tx_t^{(0:\bar{k})} \tag{71}
$$

where $T = \begin{bmatrix} 0_{n\bar{k}\times n} & I_{n\bar{k}} \\ 0_{n\times n} & 0_{n\times n\bar{k}} \end{bmatrix}$ *is the order transformation matrix.*

Proof. By the definition of $x_t^{(0:\bar{k})}$ one can see that $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = x_t^{(1:\bar{k}+1)}$. By definition for $\bar{k}$, any order s such that $s > \bar{k}$ does not affect the equilibrium. Then, without loss of generality, one can

set $E_t^{(s)}[x_t] = 0$ if $s > \bar{k}$. Therefore, one can rewrite $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$ as

$$E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = \begin{bmatrix} x_t^{(1:\bar{k})} \\ E_t^{(\bar{k}+1)}[x_t] \end{bmatrix} = \begin{bmatrix} x_t^{(1:\bar{k})} \\ 0_{n\times 1} \end{bmatrix} = \begin{bmatrix} 0_{n\bar{k}\times n} & I_{n\bar{k}} \\ 0_{n\times n} & 0_{n\times n\bar{k}} \end{bmatrix} \begin{bmatrix} x_t \\ x_t^{(1:\bar{k})} \end{bmatrix} = T x_t^{(0:\bar{k})}, \tag{72}$$

where the first equality uses the definition of $x_t^{(1:\bar{k}+1)}$ and the second equality uses that $E_t^{(\bar{k}+1)}[x_t] = 0$. In the last equality, T is defined accordingly.

□

This is Lemma builds on Proposition 1 from Online Appendix of Melosi (2017). It explores the truncation of the hierarchy.

The second Lemma does the opposite: writes the hierarchy as a linear transformation of the first-order expectation of the hierarchy.

Lemma 2. *$x_t^{(0:\bar{k})}$ can be rewritten as a linear function of x_t and its average expectation such that*

$$x_t^{(0:\bar{k})} \equiv e_x' x_t + T' E_t^{(1)}\left[x_t^{(0:\bar{k})}\right],$$

Proof. By definition, $x_t^{(0:\bar{k})}$ can be decomposed as $x_t^{(0:\bar{k})} = \left[x_t' \quad (x_t^{(\bar{1}:\bar{k})})'\right]'$. Using this decomposition, the average expectation decomposed as $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = \left[(x_t^{(\bar{1}:\bar{k})})' \quad E_t^{(\bar{k}+1)}[x_t]'\right]'$. Therefore, one can use this two definitions such that

$$x_t^{(0:\bar{k})} \equiv \begin{bmatrix} x_t \\ x_t^{(\bar{1}:\bar{k})} \end{bmatrix} = \begin{bmatrix} I_n \\ 0_{n\bar{k}\times n} \end{bmatrix} x_t + \begin{bmatrix} 0_{n\times n\bar{k}} & 0_{n\times n} \\ I_{n\bar{k}} & 0_{n\bar{k}\times n} \end{bmatrix} \begin{bmatrix} x_t^{(1:\bar{k})} \\ E_t^{(\bar{k}+1)}[x_t] \end{bmatrix} = e_x' x_t + T' E_t^{(1)}\left[x_t^{(0:\bar{k})}\right],$$

where last equality uses the definitions of T in Lemma 1 and e_x. □

Lemmas are taken from Ribeiro (2018, chap. 3).

D.1 Proof of Proposition 1

Individual and average expectations about the hierarchy. The Kalman filter delivers the individual conditional expectation, $E_{it}[\cdot] \equiv E[\cdot|\mathcal{I}_t^i]$, where $\mathcal{I}_t^i = \{s_{i,\tau}, \tau \leq t\}$ is the information set of individual i in period t.

The state equation is the hierarchy of expectations that is given by the state equation (42), restated for convenience:

$$x_t^{(0:\bar{k})} = \mathbf{A} x_{t-1}^{(0:\bar{k})} + \mathbf{B}\varepsilon_t, \tag{73}$$

and the agent i with the observational equation (39), also restated:

$$s_{i,t} = C_x x_t + D v_{i,t}.$$

Note that $s_{i,t}$ is a signal about the shocks and not the whole hierarchy of expectations.

Let the selection matrix $e_x \equiv \begin{bmatrix} I_n & 0_{n \times n\bar{k}} \end{bmatrix}$ such that $x_t = e_x x_t^{(0:\bar{k})}$. Then, we can rewrite the signal in terms of the hierarchy as

$$s_{i,t} = C x_t^{(0:\bar{k})} + D v_{it}. \tag{74}$$

where $C = C_x e_x$.

Each agent i uses the Kalman filter and find the update equation given by

$$E_{i,t}\left[x_t^{(0:\bar{k})}\right] = E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] + \mathbf{K_t}\left[s_{i,t} - E_{i,t-1}\left[s_{i,t}\right]\right], \tag{75}$$

where $\mathbf{K_t}$ is the Kalman gain given by

$$\mathbf{K_t} = \mathbf{P_{t/t-1}} C' \left[C \mathbf{P_{t/t-1}} C' + D \Sigma_v D'\right]^{-1}. \tag{76}$$

As usual, the mean squared error (MSE) of the one-period ahead prediction error is given by

$$\mathbf{P_{t+1/t}} = \mathbf{A}\left[\mathbf{P_{t/t-1}} - \mathbf{K_t}\left[C \mathbf{P_{t/t-1}}\right]\right]\mathbf{A}' + \mathbf{B}\Sigma_\varepsilon \mathbf{B}'. \tag{77}$$

For details of this deviation, see for instance Hamilton (1995, chap. 13).

Using the observational equation, (74), taking expectations and inserting in (75) one can find:

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] + \mathbf{K_t}\left[C x_t^{(0:\bar{k})} + D v_{it} - C E_{i,t-1}\left[x_t^{(0:\bar{k})}\right]\right] \tag{78}$$

Therefore, one can rewrite the equation above as

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \mathbf{K_t} C\right) E_{it-1}\left[x_t^{(0:\bar{k})}\right] + \mathbf{K_t}\left[C x_t^{(0:\bar{k})} + D v_{it}\right] \tag{79}$$

where $k = n(\bar{k}+1)$. Using the fact that $E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] = \mathbf{A} E_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right]$ and substituting equation (73), one can find:

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \mathbf{K_t} C\right)\mathbf{A} E_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \mathbf{K_t} C \mathbf{A} x_{t-1}^{(0:\bar{k})} + \mathbf{K_t} C \mathbf{B} \varepsilon_t + \mathbf{K_t} D v_{it} \tag{80}$$

We follow the literature by focusing in the stationary equilibrium. Therefore, the expectation of each individual i in the stationary equilibrium is the one which the MSE is in steady-state, i.e.,

agents update their forecast based on the steady-state Kalman gain. In other words, the dynamics of expectations depends only in the properties of the process they are forecasting and signals, but do not depend in the period t.

Combining equations (76-77), one can find the Riccati equation

$$\mathbf{P_{t+1/t}} = \mathbf{A}\left[\mathbf{P_{t/t-1}} - \mathbf{P_{t/t-1}}C'\left[C\mathbf{P_{t/t-1}}C' + D\Sigma_v D'\right]^{-1} C\mathbf{P_{t/t-1}}\right]\mathbf{A}' + \mathbf{B}\Sigma_\varepsilon\mathbf{B}' \tag{81}$$

Therefore, one need to iterate this equation to find the steady-state MSE, $\bar{\mathbf{P}}$, and compute its counterpart Kalman gain, $\bar{\mathbf{K}}$. Nimark (2017) shows that if is x_t stationary process, then the expectations hierarchy about this process, $x_t^{(0:\bar{k})}$, is also stationary. This and the fact that Σ_ε is positive definite, then there exists a steady-state solution such that $\bar{\mathbf{P}} = \mathbf{P_{t+1|t}} = \mathbf{P_{t|t-1}}$ which implies the steady-state Kalman gain $\bar{\mathbf{K}} = \mathbf{K_t} = \mathbf{K_{t-1}}$ (see Hamilton; 1995, chap. 13).

Expressions (44) in the Proposition 1 are equations (76-77) using the steady-state Kalman gain, $\bar{\mathbf{K}}$, instead of $\bar{\mathbf{K}}_\mathbf{t}$.

Moreover, the average expectation is easily computed by

$$\bar{E}_t\left[x_t^{(0:\bar{k})}\right] \equiv \int_0^1 E_{it}\left[x_t^{(0:\bar{k})}\right]di = \left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}\bar{E}_{t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{\mathbf{K}}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C\mathbf{B}\varepsilon_t \tag{82}$$

Analogously, the individual and average expectation of Proposition 1 are equations (80) and (82) using the steady-state Kalman gain, $\bar{\mathbf{K}}$, instead of $\bar{\mathbf{K}}_\mathbf{t}$.

Verify guess for $x_t^{(0:\bar{k})}$. In the first part of the proof, we found the average expectation (82) for the guessed the dynamics of the hierarchy of expectations (73).

Now we verify the guessed hierarchy and find the coefficients $(\mathbf{A}, \mathbf{B})$ consistent with the average expectation.

Substituting the average expectation from equation (82) into the expression from Lemma 2 one can find that:

$$x_t^{(0:\bar{k})} = e_x' x_t + T'\left[\left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}E_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{\mathbf{K}}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C\mathbf{B}\varepsilon_t\right]$$

Then, using the shocks definition (38) and the fact that $x_t = e_x x_t^{(0:\bar{k})}$ one can rewrite equation above as

$$x_t^{(0:\bar{k})} = e_x'\left(A_1 e_x x_{t-1}^{(0:\bar{k})} + \varepsilon_t\right) + T'\left(I_k - \bar{\mathbf{K}}C\right)AE_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + T'\bar{\mathbf{K}}CAx_{t-1}^{(0:\bar{k})} + T'\bar{\mathbf{K}}C\mathbf{B}\varepsilon_t$$

Using Lemma 1 at period $t-1$ into equation above and rearranging:

$$x_t^{(0:\bar{k})} = \left[e_x' A_1 e_x + T'\left(I_k - \bar{\mathbf{K}}C\right)AT + T'\bar{\mathbf{K}}CA\right] x_{t-1}^{(0:\bar{k})} + \left[T'\bar{\mathbf{K}}C\mathbf{B} + e_x'\right]\varepsilon_t. \tag{83}$$

This expression shows that the expectations hierarchy is a function of its lag and structural shocks, as guessed in equation (73). Therefore, the expression above verifies that $x_t^{(0:\bar{k})}$ follows the guessed form and the square brackets terms provide identities for A and $\mathbf{B}$ in equations (43)

□

D.2 Proof of Proposition 2

This proof builds on techniques developed by Ribeiro (2018, chap. 3). The key difference is that we guess a law of motion for individual endogenous variables instead of guessing the aggregate. This allows recovering expressions for solving for $(\mathbf{Q_0}, \mathbf{Q_1})$ and then get the solution for $\mathbf{Q}$. We also consider the exogenous information case only.

The guessed law of motion for individual endogenous variables is given (40), restated for convenience:

$$Y_{i,t} = \mathbf{R}Y_{i,t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_{i,t}\left[x_t^{(0:\bar{k})}\right]. \tag{84}$$

Aggregating the individual law of motion (84):

$$Y_t = \mathbf{R}Y_{t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] \tag{85}$$

$$= \mathbf{R}Y_{t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}Tx_t^{(0:\bar{k})} \tag{86}$$

where the last equality uses Lemma 1.

By computing the individual expectation of the endogenous aggregate variables, one can find that

$$\begin{aligned} E_{it}\left[Y_t\right] &= \mathbf{R}Y_{t-1} + \mathbf{Q_0}E_{it}\left[x_t\right] + \mathbf{Q_1}TE_{it}\left[x_t^{(0:\bar{k})}\right] \\ &= \mathbf{R}Y_{t-1} + (\mathbf{Q_0}e_x + \mathbf{Q_1}T)E_{it}\left[x_t^{(0:\bar{k})}\right] \\ E_t^{(1)}\left[Y_t\right] &= \mathbf{R}Y_{t-1} + (\mathbf{Q_0}e_x + \mathbf{Q_1}T)E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] \end{aligned} \tag{87}$$

where the first equality uses that Y_{t-1} is known, second equality uses the definition of the selection matrix, e_x, and the third equality aggregates.

Similarly, the individual expectation for endogenous variables in $t+1$:

$$\begin{aligned} E_{it}\left[Y_{t+1}\right] &= \mathbf{R}E_{it}\left[Y_t\right] + \mathbf{Q_0}E_{it}\left[x_{t+1}\right] + \mathbf{Q_1}TE_{it}\left[x_{t+1}^{(0:\bar{k})}\right] \\ &= \mathbf{R}E_{it}\left[Y_t\right] + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})E_{it}\left[x_t^{(0:\bar{k})}\right] \\ &= \mathbf{R}^2Y_{t-1} + \left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})\right]E_{it}\left[x_t^{(0:\bar{k})}\right] \\ E_t^{(1)}\left[Y_{t+1}\right] &= \mathbf{R}^2Y_{t-1} + \left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})\right]E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] \end{aligned} \tag{88}$$

where the second equality uses equations (38) and (42), and the definition of e_x. Third equation uses equation (87) and the fourth aggregates.

Finally, taking the individual expectation of the law of motion (40) in period $t+1$ implies that

$$\begin{aligned} E_{it}[Y_{i,t+1}] &= \mathbf{R}Y_{i,t} + \mathbf{Q_0}E_{it}[x_{t+1}] + \mathbf{Q_1}E_{it}\left[E_{i,t+1}\left[x_{t+1}^{(0:\bar{k})}\right]\right] \\ &= \mathbf{R}Y_{i,t} + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})E_{it}\left[x_t^{(0:\bar{k})}\right] \end{aligned}$$

where the second equality uses the law of iterated expectations (which holds for i's expectation but not for the average expectation), equations (38) and (42) as before.

Aggregating the expectation above leads to

$$\begin{aligned} \int_0^1 E_{it}[Y_{i,t+1}]di &= \mathbf{R}Y_t + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})E_t^{(1)}[x_t^{(0:\bar{k})}] \\ &= \mathbf{R}\left[\mathbf{R}Y_{t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})E_t^{(1)}[x_t^{(0:\bar{k})}] \\ &= \mathbf{R}^2Y_{t-1} + \mathbf{R}\mathbf{Q_0}x_t + \left[\mathbf{R}\mathbf{Q_1} + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})\right]E_t^{(1)}[x_t^{(0:\bar{k})}] \end{aligned} \tag{89}$$

where the second equality uses equation (86). Note that $\int_0^1 E_{it}[Y_{i,t+1}]di \neq E_t^{(1)}\left[Y_{t+1}\right]$.

Substituting the guessed solution (86) and the expectations (87-89) into the system of equations (37) one can find:

$$\begin{aligned} &F_1\left[\mathbf{R}^2Y_{t-1} + \mathbf{R}\mathbf{Q_0}x_t + \left[\mathbf{R}\mathbf{Q_1} + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})\right]E_t^{(1)}[x_t^{(0:\bar{k})}]\right] + \\ &F_2\left[\mathbf{R}^2Y_{t-1} + \left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})\right]E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + \\ &G_1\left[\mathbf{R}Y_{t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + G_2\left[\mathbf{R}Y_{t-1} + (\mathbf{Q_0}e_x + \mathbf{Q_1}T)E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + \\ &HY_{t-1} + \left[(L_1A_1 + L_2)\right]e_xE_t^{(1)}\left[x_t^{(0:\bar{k})}\right] + M_1x_t = 0_{m\times 1} \end{aligned}$$

which can be rearranged to

$$\begin{aligned}
&\left[F\mathbf{R}^2 + G\mathbf{R} + H\right] Y_{t-1} + \left[\left(F_1\mathbf{R} + G_1\right)\mathbf{Q_0} + M_1\right] x_t \\
&\left[F_1\left[\mathbf{R}\mathbf{Q_1} + \left(\mathbf{Q_0}A_1 e_x + \mathbf{Q_1}\mathbf{A}\right)\right] + F_2\left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1 e_x + \mathbf{Q_1}T\mathbf{A})\right] + \right. \\
&\left. G_1\mathbf{Q_1} + G_2(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (L_1A_1 + L_2 + M)e_x\right] E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = 0_{m\times 1}
\end{aligned}$$

which can be simplified to

$$\begin{aligned}
&\left[F\mathbf{R}^2 + G\mathbf{R} + H\right] Y_{t-1} + \left[\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_0} + M_1\right] x_t + \\
&\left[\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_1} + F_1\mathbf{Q_1}\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q_1}T + F_2\mathbf{Q_1}T\mathbf{A}\right. \\
&\left.\left[(F_2\mathbf{R} + G_2)\mathbf{Q_0} + (F_1 + F_2)\mathbf{Q_0}A_1 + (L_1A_1 + L_2)\right] e_x\right] E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = 0_{m\times 1}
\end{aligned}$$

This condition must hold for all realizations of Y_{t-1}, x_t and $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$. Therefore, all coefficients between square brackets must be zero, which leads to

$$\begin{aligned}
&F\mathbf{R}^2 + G\mathbf{R} + H = 0_{m\times m} \\
&\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_0} + M_1 = 0_{m\times n} \\
&\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_1} + F_1\mathbf{Q_1}\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q_1}T + F_2\mathbf{Q_1}T\mathbf{A} \\
&\left[(F_2\mathbf{R} + G_2)\mathbf{Q_0} + F\mathbf{Q_0}A_1 + (LA_1 + M_2)\right] e_x = 0_{m\times k}
\end{aligned}$$

which leads to the same equations from Proposition 2. $\mathbf{R}$ can be solved using Uhlig (2001) method. For a given solution of $\mathbf{R}$, $\mathbf{Q_0}$ and $\mathbf{Q_1}$ can be solved by straightforward vectorization as discussed in the Proposition.

□

D.3 Algorithm

Algorithm. Set the initial values $(\mathbf{A}^{(0)}, \mathbf{B}^{(0)})$, a small tolerance $\epsilon > 0$ and set $i = 1$. Then, follow the steps:

1. Given $\mathbf{A} = \mathbf{A}^{(i-1)}$ and $\mathbf{B}^{(i-1)}$, compute $\bar{\mathbf{K}}$ and $\bar{\mathbf{P}}$ using equations (44) using standard solver for Ricatti equations. Set $\bar{\mathbf{K}}^{(i)} = \bar{\mathbf{K}}$ and $\bar{\mathbf{P}}^{(i)} = \bar{\mathbf{P}}$.

2. Given $\mathbf{A}^{(i-1)}$ and $\bar{\mathbf{K}}^{(i)}$, compute the right hand side of equations (43) and solve for $\mathbf{B}$ and $\mathbf{A}$ the equations by matrix inversion. Set $\mathbf{B}^{(i)} = \mathbf{B}$, $\mathbf{A}^{(i)} = \mathbf{A}$.

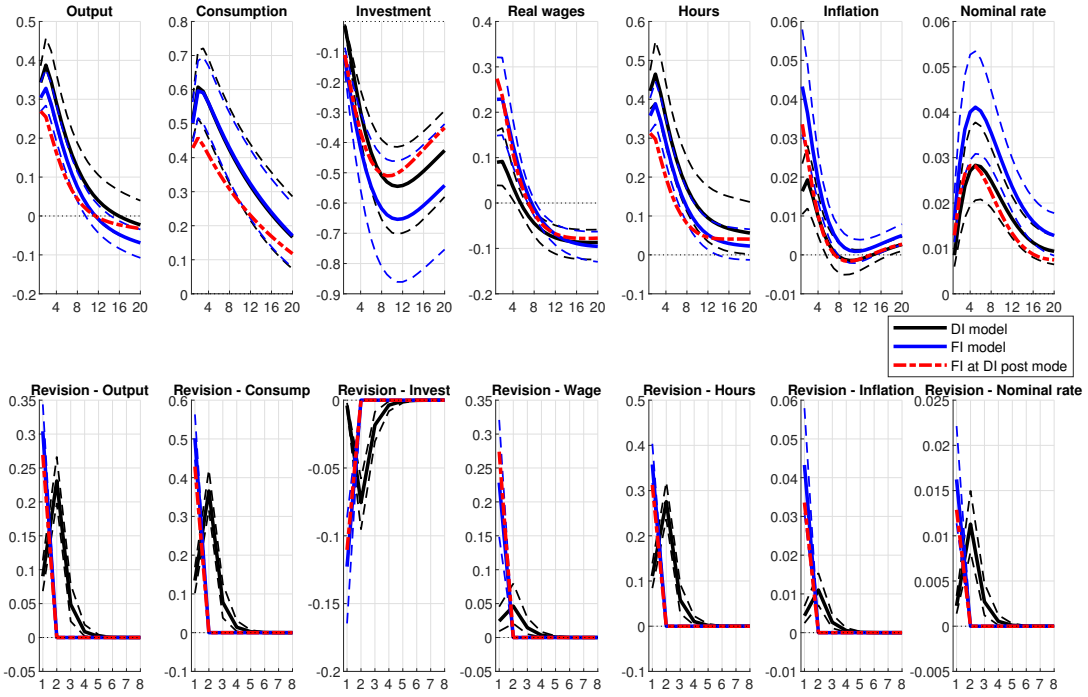
3. If $\max\left\{||\mathbf{B}^{(i)} - \mathbf{B}^{(i-1)}||, ||\mathbf{A}^{(i)} - \mathbf{A}^{(i-1)}||, ||\mathbf{P}^{(i)} - \mathbf{P}^{(i-1)}||\right\} < \epsilon$, stop iterating. Otherwise, set $i = i + 1$ and go back to step 1.

Given the solution for $\mathbf{A}$, one can use standard techniques for solving for $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1}, \mathbf{Q})$ using Proposition 2.

E Impulse response functions

In this section, the remaining impulse responses for the FI and DI models estimated with the complete dataset are displayed.

Figure 10: Impulse responses to preference shock



F Posterior estimates when shutting down frictions

Table 2 in section 5.4 shows the marginal likelihood of the models shutting down frictions that generate inertia re-estimated for both datasets. In this appendix, we show the details of the posterior estimates of each of those models. The priors are the same as the baseline for all unaffected parameters.

Figure 11: Impulse responses to TFP shock

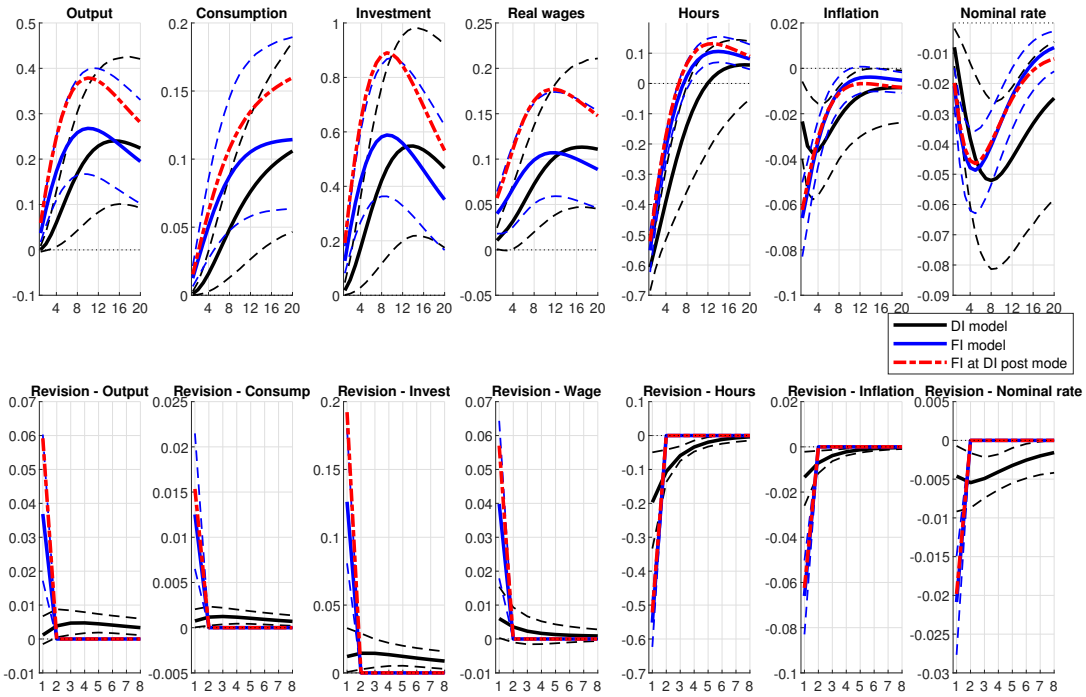


Figure 12: Impulse responses to wage mark-up shock

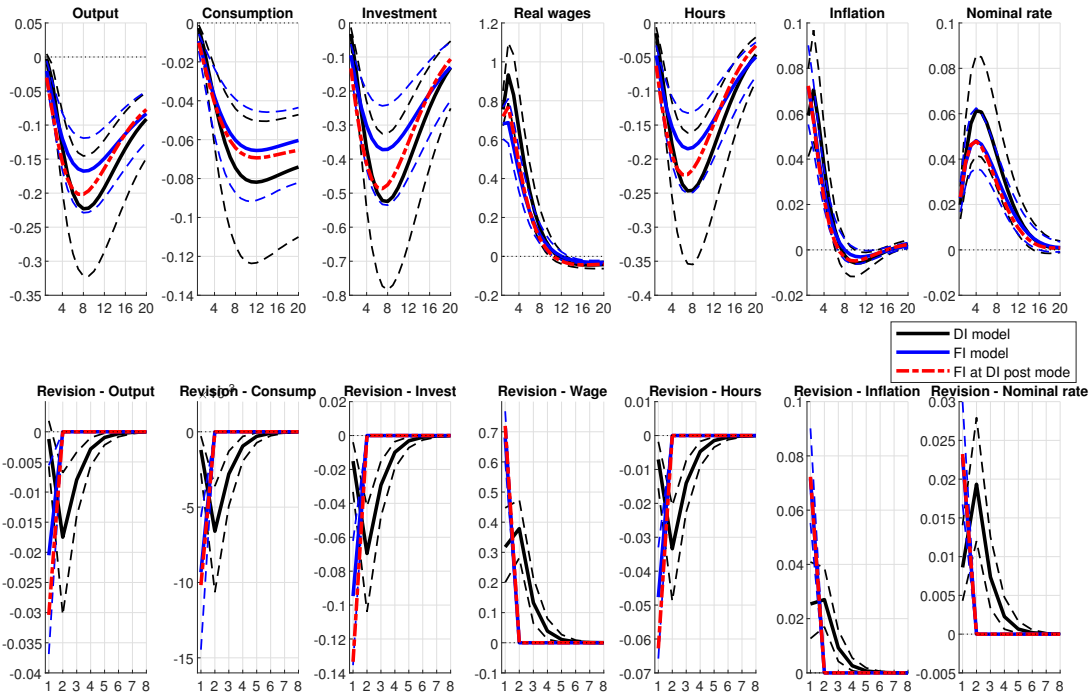


Figure 13: Impulse responses to price mark-up shock

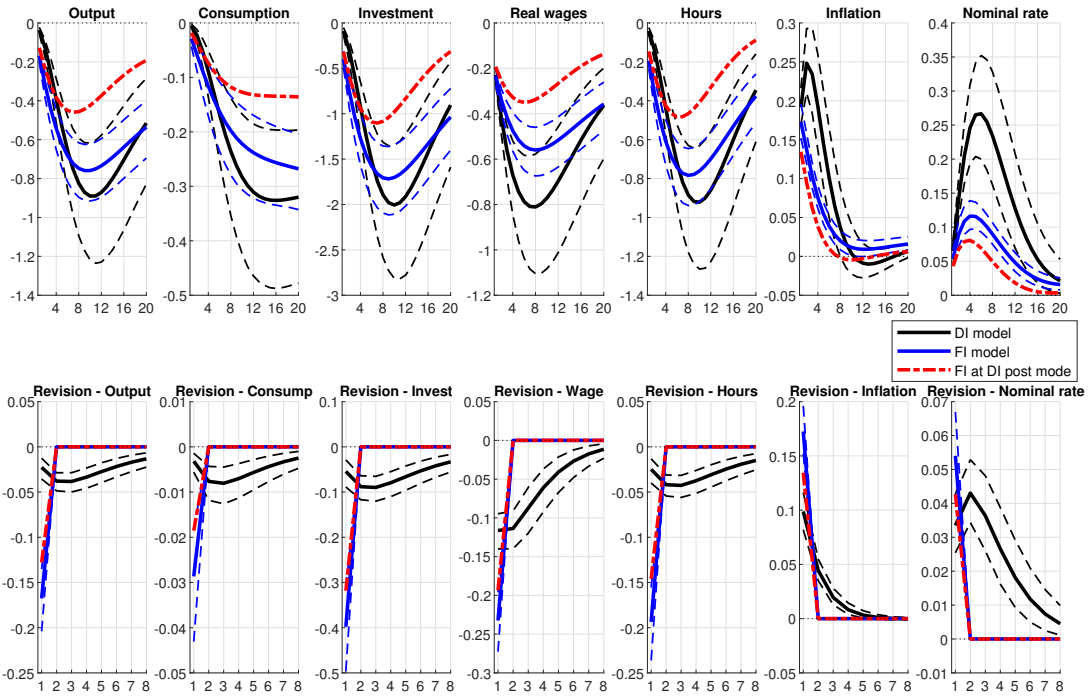


Figure 14: Impulse responses to government expenditure shock

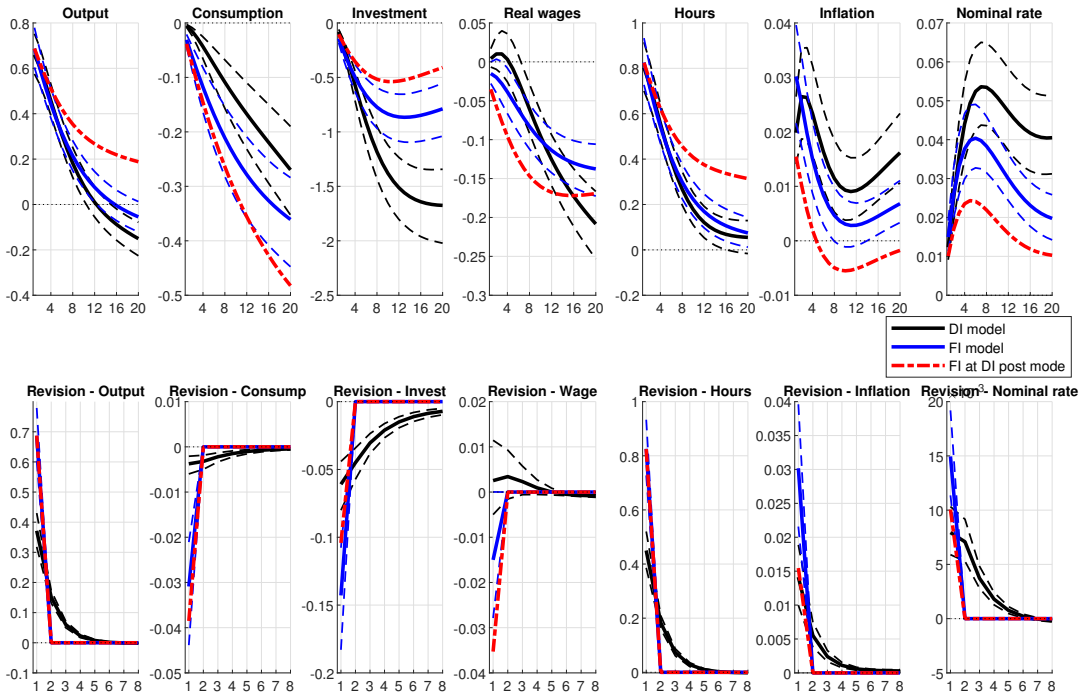


Table 4: Posterior distributions when shutting down key standard frictions: Complete dataset

	Baseline			No standard frictions			No habits			No inv. adjustment costs			No price indexation			No wage indexation		
Parameters	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
Endogenous propagation parameters																		
χ	1.48	1.15	2.02	0.90	0.77	1.05	1.43	1.09	1.87	0.58	0.41	0.92	1.53	1.15	2.03	1.45	1.15	1.95
φ	0.97	0.95	0.98	-	-	-	-	-	-	0.98	0.95	0.99	0.97	0.95	0.98	0.97	0.95	0.98
a''	0.41	0.28	0.68	0.32	0.18	0.50	0.35	0.19	0.55	0.29	0.20	0.53	0.45	0.28	0.67	0.42	0.28	0.66
s''	9.73	7.22	13.72	-	-	-	9.22	6.13	12.89	-	-	-	10.16	7.02	13.94	9.70	7.08	13.52
s''_k	-	-	-	1.04	0.72	1.42	-	-	-	1.28	1.01	1.87	-	-	-	-	-	-
ϕ_y	0.07	0.04	0.14	0.07	0.04	0.11	0.12	0.07	0.19	0.24	0.15	0.38	0.09	0.04	0.14	0.07	0.04	0.14
ϕ_π	3.32	2.71	4.21	2.67	1.86	3.54	2.88	2.17	3.67	3.22	2.67	4.23	3.42	2.72	4.24	3.27	2.77	4.30
ϕ_r	0.90	0.87	0.92	0.84	0.80	0.88	0.87	0.83	0.90	0.92	0.89	0.94	0.90	0.87	0.92	0.90	0.87	0.92
ι_p	0.03	0.01	0.10	-	-	-	0.05	0.01	0.11	0.05	0.02	0.16	-	-	-	0.03	0.01	0.09
ι_w	0.59	0.23	0.86	-	-	-	0.52	0.19	0.84	0.53	0.21	0.85	0.55	0.22	0.85	-	-	-
ξ_p	0.85	0.82	0.88	0.79	0.74	0.83	0.82	0.78	0.86	0.89	0.84	0.91	0.85	0.82	0.88	0.85	0.82	0.88
ξ_w	0.75	0.68	0.81	0.53	0.45	0.62	0.61	0.53	0.69	0.81	0.75	0.86	0.74	0.68	0.80	0.74	0.67	0.80
Steady-state parameters																		
π_{ss}	3.06	2.46	3.59	2.08	1.52	2.69	2.53	2.01	3.07	1.79	1.37	2.46	3.07	2.51	3.61	3.04	2.50	3.62
r_{ss}	6.88	6.03	7.98	6.44	5.25	7.64	6.05	5.28	6.96	6.86	6.05	7.80	7.03	6.10	8.05	6.97	6.08	8.07
γ_{ss}	0.32	0.29	0.36	0.24	0.19	0.29	0.36	0.32	0.40	0.40	0.33	0.43	0.33	0.30	0.36	0.33	0.29	0.36
Structural shocks parameters																		
ρ_c	0.13	0.07	0.21	0.91	0.76	0.98	0.90	0.82	0.97	0.12	0.07	0.20	0.14	0.07	0.21	0.13	0.07	0.21
ρ_i	0.24	0.18	0.29	0.89	0.83	0.94	0.30	0.23	0.36	0.72	0.65	0.77	0.23	0.18	0.29	0.23	0.17	0.29
ρ_g	0.99	0.98	0.997	0.98	0.96	0.99	0.96	0.94	0.98	1.00	0.99	0.998	0.99	0.98	1.00	0.99	0.98	0.997
ρ_p	0.82	0.77	0.87	0.74	0.64	0.81	0.82	0.76	0.87	0.67	0.57	0.75	0.82	0.77	0.87	0.82	0.76	0.86
ρ_w	0.32	0.22	0.42	0.34	0.23	0.45	0.34	0.24	0.45	0.33	0.23	0.43	0.32	0.22	0.42	0.32	0.22	0.43
ρ_r	0.45	0.33	0.57	0.67	0.49	0.81	0.50	0.34	0.66	0.37	0.26	0.55	0.45	0.33	0.57	0.44	0.33	0.57
ρ_a	0.91	0.86	0.96	0.94	0.89	0.99	0.91	0.86	0.95	1.00	0.99	1.00	0.91	0.86	0.96	0.91	0.86	0.96
σ_c	1.70	1.25	2.28	0.22	0.14	0.35	0.19	0.11	0.30	1.45	1.11	2.02	1.71	1.26	2.30	1.59	1.28	2.29
σ_i	4.70	3.93	5.57	6.48	4.67	8.61	4.68	3.89	5.60	7.53	5.55	10.08	4.72	3.93	5.62	4.64	3.93	5.57
σ_g	0.56	0.48	0.63	0.52	0.43	0.62	0.61	0.52	0.70	0.51	0.45	0.59	0.55	0.48	0.63	0.55	0.48	0.63
σ_p	0.17	0.12	0.23	0.36	0.20	0.50	0.20	0.12	0.35	0.23	0.17	0.36	0.16	0.12	0.22	0.16	0.12	0.24
σ_w	1.56	0.99	2.48	1.45	0.98	2.10	1.42	0.97	2.05	1.37	0.94	2.26	1.64	1.00	2.70	1.26	0.92	2.45
σ_r	0.13	0.11	0.15	0.13	0.11	0.14	0.12	0.11	0.14	0.13	0.11	0.14	0.13	0.11	0.15	0.13	0.11	0.15
σ_a	0.41	0.37	0.46	0.44	0.39	0.50	0.42	0.37	0.47	0.45	0.41	0.52	0.41	0.37	0.46	0.41	0.37	0.46
Information friction parameters																		
k_c	0.26	0.20	0.32	0.63	0.58	0.68	0.66	0.61	0.72	0.28	0.22	0.35	0.26	0.20	0.32	0.26	0.20	0.31
k_i	0.45	0.41	0.50	0.55	0.51	0.58	0.45	0.41	0.50	0.58	0.56	0.60	0.45	0.41	0.50	0.45	0.41	0.50
k_g	0.57	0.53	0.61	0.57	0.51	0.64	0.58	0.54	0.62	0.60	0.57	0.67	0.57	0.53	0.61	0.57	0.53	0.61
k_p	0.37	0.29	0.46	0.28	0.21	0.39	0.38	0.24	0.51	0.31	0.24	0.42	0.37	0.29	0.46	0.36	0.29	0.47
k_w	0.41	0.25	0.57	0.38	0.26	0.51	0.41	0.29	0.55	0.41	0.28	0.61	0.40	0.23	0.57	0.42	0.25	0.61
k_r	0.72	0.62	0.81	0.29	0.22	0.37	0.46	0.30	0.60	0.77	0.59	0.85	0.71	0.61	0.80	0.72	0.61	0.81
k_a	0.33	0.08	0.57	0.11	0.01	0.23	0.36	0.12	0.56	0.06	0.05	0.09	0.34	0.09	0.56	0.37	0.10	0.58
Measurement errors standard deviation																		
σ^{me}_{dy}	0.28	0.25	0.32	0.33	0.30	0.37	0.31	0.27	0.34	0.30	0.27	0.34	0.28	0.25	0.32	0.28	0.25	0.32
σ^{me}_{dc}	0.21	0.19	0.23	0.32	0.28	0.36	0.36	0.32	0.41	0.21	0.19	0.23	0.21	0.19	0.23	0.20	0.19	0.23
σ^{me}_{di}	1.26	1.12	1.41	1.42	1.26	1.60	1.25	1.11	1.40	1.40	1.26	1.58	1.26	1.12	1.41	1.26	1.12	1.41
σ^{me}_{π}	0.54	0.48	0.61	0.51	0.45	0.58	0.55	0.47	0.63	0.49	0.44	0.57	0.54	0.48	0.61	0.53	0.48	0.62
σ^{me}_{r}	0.28	0.24	0.33	0.39	0.33	0.45	0.36	0.30	0.42	0.28	0.24	0.33	0.28	0.24	0.33	0.28	0.25	0.33
Marginal Likelihood	-1333.20			- 1448.70			-1436.90			-1377.70			-1327.60			-1322.90		

Note: $\mathcal{G}$, $\mathcal{IG}$ and $\mathcal{B}$ stand for: Gamma, Inverse Gamma and Beta distributions, respectively.

Table 5: Posterior distributions when shutting down key standard frictions: Macroeconomic dataset

	Baseline			No standard frictions			No habits			No inv. adjustment costs			No price indexation			No wage indexation		
Parameters	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
Endogenous propagation parameters																		
χ	1.21	1.01	1.83	0.85	0.75	1.04	1.22	1.00	1.88	1.08	0.96	1.36	1.24	1.01	1.76	1.16	0.98	1.71
φ	0.87	0.55	0.92	-	-	-	-	-	-	0.82	0.69	0.88	0.87	0.52	0.92	0.87	0.52	0.92
a''	0.37	0.25	0.63	0.35	0.24	0.60	0.36	0.22	0.60	0.35	0.25	0.64	0.37	0.25	0.63	0.36	0.24	0.62
s''	5.47	3.27	9.21	-	-	-	4.47	2.59	8.02	-	-	-	5.79	3.38	9.35	5.35	3.36	9.07
s''_k	-	-	-	0.54	0.40	1.04	-	-	-	0.73	0.53	1.22	-	-	-	-	-	-
ϕ_y	0.06	0.03	0.13	0.04	0.02	0.07	0.08	0.06	0.14	0.04	0.02	0.08	0.07	0.03	0.13	0.07	0.03	0.13
ϕ_π	2.84	2.24	3.77	1.51	1.34	1.88	1.67	1.43	2.30	2.74	2.20	3.73	2.76	2.27	3.81	2.82	2.23	3.76
ϕ_r	0.87	0.82	0.91	0.70	0.61	0.76	0.78	0.70	0.84	0.85	0.79	0.89	0.87	0.82	0.90	0.87	0.82	0.90
ι_p	0.08	0.03	0.23	-	-	-	0.05	0.02	0.19	0.06	0.02	0.19	-	-	-	0.07	0.03	0.23
ι_w	0.52	0.20	0.83	-	-	-	0.45	0.16	0.79	0.45	0.17	0.82	0.57	0.19	0.83	-	-	-
ξ_p	0.80	0.75	0.83	0.69	0.63	0.73	0.79	0.72	0.83	0.73	0.68	0.78	0.80	0.76	0.83	0.79	0.75	0.83
ξ_w	0.70	0.63	0.77	0.59	0.52	0.66	0.67	0.59	0.74	0.65	0.59	0.71	0.70	0.63	0.77	0.70	0.63	0.77
Steady-state parameters																		
π_{ss}	2.22	1.52	2.85	1.53	1.14	2.14	1.63	1.20	2.27	2.27	1.68	2.92	2.19	1.52	2.84	2.15	1.54	2.86
r_{ss}	6.47	5.69	7.61	7.18	6.22	8.34	6.94	5.89	7.99	6.46	5.62	7.47	6.52	5.73	7.61	6.53	5.74	7.63
γ_{ss}	0.33	0.27	0.36	0.27	0.21	0.32	0.37	0.32	0.41	0.29	0.24	0.32	0.32	0.28	0.36	0.32	0.28	0.36
Structural shocks parameters																		
ρ_c	0.77	0.19	0.83	0.89	0.80	0.95	0.94	0.88	0.97	0.20	0.10	0.47	0.76	0.18	0.83	0.76	0.18	0.84
ρ_i	0.46	0.31	0.65	0.93	0.86	0.96	0.39	0.28	0.50	0.91	0.84	0.95	0.45	0.30	0.65	0.47	0.32	0.66
ρ_g	0.99	0.98	1.00	0.99	0.97	1.00	0.99	0.97	1.00	0.99	0.98	1.00	0.99	0.98	1.00	0.99	0.98	1.00
ρ_p	0.95	0.92	0.98	0.96	0.92	0.98	0.94	0.87	0.97	0.97	0.95	0.99	0.95	0.92	0.98	0.95	0.91	0.97
ρ_w	0.22	0.13	0.34	0.18	0.10	0.28	0.23	0.14	0.34	0.19	0.11	0.30	0.22	0.13	0.33	0.23	0.14	0.35
ρ_r	0.65	0.48	0.82	0.94	0.80	0.97	0.89	0.68	0.95	0.58	0.42	0.76	0.65	0.48	0.82	0.63	0.47	0.83
ρ_a	0.93	0.86	0.97	0.99	0.94	0.99	0.95	0.88	0.98	0.98	0.88	0.99	0.92	0.86	0.97	0.93	0.86	0.97
σ_c	0.11	0.09	0.36	0.11	0.08	0.17	0.10	0.08	0.23	0.26	0.19	0.52	0.12	0.09	0.43	0.11	0.09	0.38
σ_i	1.18	0.98	1.92	3.23	2.28	5.76	1.86	1.41	2.71	3.17	2.31	5.61	1.19	0.98	2.02	1.21	0.97	1.82
σ_g	0.53	0.45	0.62	0.46	0.40	0.55	0.50	0.43	0.60	0.53	0.46	0.62	0.52	0.46	0.62	0.52	0.46	0.62
σ_p	0.06	0.05	0.09	0.15	0.10	0.20	0.07	0.05	0.16	0.09	0.07	0.13	0.06	0.05	0.08	0.07	0.05	0.09
σ_w	1.07	0.77	2.71	1.96	1.33	4.51	1.47	1.01	3.35	1.26	0.89	2.87	1.06	0.80	2.97	1.23	0.78	3.02
σ_r	0.12	0.11	0.14	0.13	0.12	0.15	0.13	0.11	0.14	0.12	0.11	0.14	0.12	0.11	0.14	0.12	0.11	0.14
σ_a	0.39	0.36	0.45	0.43	0.38	0.49	0.41	0.37	0.46	0.42	0.38	0.48	0.40	0.36	0.45	0.40	0.36	0.46
Information friction parameters																		
k_c	0.92	0.73	0.97	0.92	0.70	0.97	0.60	0.48	0.75	0.88	0.59	0.96	0.93	0.66	0.97	0.93	0.70	0.97
k_i	0.93	0.78	0.98	0.61	0.46	0.82	0.79	0.65	0.91	0.87	0.65	0.96	0.93	0.75	0.98	0.94	0.79	0.98
k_g	0.58	0.44	0.75	0.38	0.31	0.55	0.49	0.39	0.64	0.58	0.47	0.68	0.59	0.43	0.75	0.59	0.44	0.74
k_p	0.94	0.81	0.98	0.56	0.49	0.68	0.65	0.48	0.87	0.84	0.73	0.94	0.93	0.78	0.97	0.94	0.79	0.97
k_w	0.44	0.23	0.72	0.25	0.13	0.41	0.33	0.18	0.56	0.35	0.21	0.62	0.39	0.21	0.69	0.41	0.20	0.69
k_r	0.22	0.07	0.56	0.07	0.03	0.15	0.10	0.04	0.22	0.17	0.07	0.58	0.20	0.08	0.57	0.19	0.07	0.55
k_a	0.64	0.32	0.88	0.38	0.29	0.53	0.61	0.27	0.80	0.56	0.38	0.74	0.69	0.35	0.88	0.66	0.33	0.89
Marginal Likelihood	-942.25			- 954.81			-951.22			-950.97			-938.74			-937.72		

Note: $\mathcal{G}$, $\mathcal{IG}$ and $\mathcal{B}$ stand for: Gamma, Inverse Gamma and Beta distributions, respectively.

Table 6: Posterior distributions when shutting down one of each information friction: Complete dataset

Parameters	Preference Shock			Investment Shock			Gov. expenditure shock			Price markup shock			Wage markup shock			Monetary shock			TFP shock		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
Endogenous propagation parameters																					
χ	1.35	1.11	1.86	2.25	1.79	3.01	2.62	1.92	3.71	1.14	0.83	1.52	1.63	1.29	2.29	1.63	1.29	2.29	1.57	1.21	2.17
φ	0.96	0.93	0.97	0.99	0.97	0.99	0.99	0.98	0.99	0.97	0.95	0.99	0.97	0.95	0.98	0.97	0.95	0.98	0.97	0.95	0.98
a''	0.37	0.25	0.64	0.43	0.29	0.69	0.51	0.36	0.80	0.43	0.30	0.71	0.42	0.30	0.69	0.42	0.30	0.69	0.46	0.31	0.72
s''	9.31	6.58	12.40	10.98	8.35	15.03	12.10	9.41	17.04	14.53	10.79	18.73	11.07	8.20	15.22	11.07	8.20	15.22	10.73	8.09	15.06
ϕ_y	0.09	0.05	0.15	0.07	0.04	0.13	0.07	0.04	0.14	0.09	0.05	0.18	0.05	0.03	0.11	0.05	0.03	0.11	0.06	0.03	0.12
ϕ_π	3.26	2.68	4.15	3.07	2.56	4.02	3.10	2.53	4.16	3.10	2.57	4.01	3.15	2.60	4.01	3.15	2.60	4.01	3.19	2.67	4.09
ϕ_r	0.90	0.87	0.93	0.90	0.87	0.92	0.90	0.86	0.93	0.90	0.87	0.93	0.90	0.87	0.92	0.90	0.87	0.92	0.90	0.87	0.92
ι_p	0.03	0.01	0.10	0.02	0.01	0.09	0.03	0.01	0.10	0.02	0.01	0.08	0.03	0.01	0.10	0.03	0.01	0.10	0.03	0.01	0.10
ι_w	0.64	0.29	0.88	0.55	0.19	0.83	0.55	0.21	0.84	0.70	0.29	0.89	0.42	0.15	0.79	0.42	0.15	0.79	0.54	0.19	0.82
ξ_p	0.85	0.82	0.88	0.85	0.82	0.88	0.85	0.81	0.88	0.82	0.80	0.85	0.90	0.87	0.92	0.90	0.87	0.92	0.88	0.86	0.91
ξ_w	0.79	0.74	0.83	0.75	0.68	0.80	0.74	0.67	0.79	0.73	0.66	0.79	0.74	0.66	0.79	0.74	0.66	0.79	0.76	0.69	0.81
Steady-state parameters																					
π_{ss}	2.94	2.38	3.43	2.97	2.43	3.47	3.04	2.34	3.56	2.76	2.16	3.31	3.07	2.52	3.57	3.07	2.52	3.57	3.14	2.60	3.64
r_{ss}	6.46	5.69	7.47	6.58	5.80	7.54	7.08	6.22	8.14	6.73	5.90	7.71	7.02	6.06	8.00	7.02	6.06	8.00	7.03	6.12	8.07
γ_{ss}	0.32	0.29	0.35	0.34	0.31	0.37	0.33	0.30	0.36	0.34	0.30	0.38	0.33	0.30	0.36	0.33	0.30	0.36	0.33	0.29	0.36
Structural shocks parameters																					
ρ_c	0.26	0.16	0.35	0.15	0.08	0.22	0.13	0.07	0.20	0.11	0.05	0.18	0.13	0.07	0.21	0.13	0.07	0.21	0.13	0.07	0.21
ρ_i	0.28	0.22	0.34	0.23	0.16	0.29	0.17	0.11	0.23	0.25	0.18	0.30	0.24	0.18	0.30	0.24	0.18	0.30	0.24	0.18	0.30
ρ_g	0.99	0.98	1.00	0.99	0.98	1.00	0.99	0.98	0.99	0.99	0.98	1.00	0.99	0.98	1.00	0.99	0.98	0.997	0.99	0.98	0.997
ρ_p	0.85	0.80	0.89	0.85	0.79	0.88	0.86	0.80	0.89	0.92	0.89	0.94	0.78	0.70	0.83	0.78	0.70	0.83	0.80	0.74	0.85
ρ_w	0.26	0.16	0.35	0.28	0.19	0.38	0.27	0.19	0.38	0.30	0.20	0.42	0.41	0.27	0.55	0.41	0.27	0.55	0.32	0.22	0.43
ρ_r	0.43	0.32	0.54	0.43	0.31	0.54	0.53	0.40	0.67	0.47	0.36	0.60	0.44	0.33	0.57	0.44	0.33	0.57	0.45	0.33	0.57
ρ_a	0.91	0.86	0.95	0.91	0.86	0.95	0.92	0.87	0.98	0.90	0.85	0.95	0.91	0.86	0.96	0.91	0.86	0.96	0.86	0.81	0.90
σ_c	0.20	0.17	0.24	1.52	1.17	2.14	1.65	1.28	2.37	1.49	1.16	2.20	1.64	1.26	2.26	1.64	1.26	2.26	1.57	1.26	2.34
σ_i	4.16	3.49	5.08	1.32	1.17	1.52	3.22	2.70	4.08	4.45	3.80	5.52	4.69	3.94	5.64	4.69	3.94	5.64	4.58	3.92	5.59
σ_g	0.55	0.49	0.64	0.57	0.50	0.65	0.54	0.47	0.62	0.54	0.48	0.63	0.54	0.48	0.63	0.54	0.48	0.63	0.54	0.48	0.63
σ_p	0.12	0.09	0.17	0.11	0.09	0.17	0.13	0.10	0.19	0.05	0.04	0.06	0.16	0.11	0.24	0.16	0.11	0.24	0.14	0.11	0.21
σ_w	1.38	0.99	2.59	1.49	1.06	3.09	1.43	1.06	2.76	1.02	0.77	1.90	0.52	0.45	0.66	0.52	0.45	0.66	1.23	0.90	2.72
σ_r	0.13	0.12	0.15	0.13	0.11	0.15	0.13	0.12	0.15	0.13	0.11	0.15	0.13	0.11	0.14	0.13	0.11	0.14	0.13	0.11	0.15
σ_a	0.40	0.36	0.46	0.40	0.36	0.45	0.40	0.37	0.46	0.41	0.37	0.47	0.41	0.37	0.46	0.41	0.37	0.46	0.42	0.38	0.48
Information friction parameters																					
k_c	-	-	-	0.27	0.21	0.33	0.25	0.20	0.32	0.27	0.20	0.33	0.25	0.20	0.32	0.25	0.20	0.32	0.25	0.20	0.32
k_i	0.48	0.43	0.53	-	-	-	0.59	0.52	0.66	0.46	0.41	0.51	0.45	0.41	0.50	0.45	0.41	0.50	0.45	0.41	0.50
k_g	0.60	0.56	0.65	0.90	0.81	0.96	-	-	-	0.58	0.53	0.62	0.56	0.53	0.60	0.56	0.53	0.60	0.56	0.53	0.60
k_p	0.45	0.35	0.56	0.43	0.35	0.55	0.40	0.32	0.51	-	-	-	0.37	0.29	0.47	0.37	0.29	0.47	0.38	0.30	0.50
k_w	0.43	0.25	0.60	0.34	0.20	0.55	0.37	0.23	0.55	0.52	0.32	0.70	-	-	-	0.69	0.56	0.77	0.43	0.23	0.62
k_r	0.77	0.68	0.84	0.76	0.65	0.84	0.74	0.63	0.81	0.75	0.65	0.84	0.69	0.56	0.77	-	-	-	0.73	0.62	0.80
k_a	0.38	0.15	0.61	0.32	0.14	0.60	0.34	0.16	0.52	0.59	0.22	0.80	0.36	0.12	0.63	0.36	0.12	0.63	-	-	-
Measurement errors standard deviations																					
σ_{dy}^{me}	0.35	0.32	0.39	0.45	0.41	0.51	0.41	0.36	0.47	0.31	0.28	0.35	0.28	0.25	0.32	0.28	0.25	0.32	0.29	0.26	0.33
σ_{dc}^{me}	0.42	0.38	0.47	0.21	0.19	0.23	0.20	0.18	0.23	0.21	0.19	0.24	0.20	0.19	0.23	0.20	0.19	0.23	0.21	0.19	0.23
σ_{di}^{me}	1.27	1.15	1.45	2.16	1.97	2.47	1.42	1.26	1.65	1.30	1.18	1.49	1.24	1.12	1.40	1.24	1.12	1.40	1.27	1.13	1.42
σ_{π}^{me}	0.56	0.50	0.64	0.56	0.50	0.65	0.55	0.49	0.63	0.72	0.65	0.82	0.54	0.49	0.63	0.54	0.49	0.63	0.54	0.48	0.62
σ_{r}^{me}	0.26	0.23	0.30	0.26	0.24	0.31	0.26	0.23	0.31	0.29	0.25	0.33	0.29	0.26	0.34	0.29	0.26	0.34	0.27	0.24	0.32
Marginal Likelihood	-1438.50			-1438.01			-1379.90			-1368.70			-1348.60			-1337.50			-1336.30		

Note: $\mathcal{G}$, $\mathcal{IG}$ and $\mathcal{B}$ stand for: Gamma, Inverse Gamma and Beta distributions, respectively.