The Economic Geography of American Slavery

Treb Allen
Dartmouth and NBER

Winston Chen
Yale

Suresh Naidu
Columbia and NBER

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Abstract

We study the economic structure of U.S. slavery on the eve of the Civil War and quantify the welfare gains from emancipation. We consider three facets of the institution of slavery: first, enslaved persons were *coerced* into supplying more labor than they would have chosen; second, slaverholders acted like *monopsonists* when setting enslaved persons' consumption and labor; and third, slaveholders *misallocated* enslaved persons across occupations and locations to maximize output rather than utility. We develop a novel quantitative general equilibrium spatial model of slavery featuring each of these facets and use detailed spatial microdata to document a number of stylized facts consistent with the model predictions. We then combine the data with the framework to estimate how dismantling the institution of slavery affected the spatial economy. We find that the (equivalent variation) welfare gains from emancipation for enslaved persons exceeded 1,100%, with coercion (196%), monopsony (160%) and mis-allocation (56%) all contributing importantly to the gains. Emancipation also increased U.S. GDP (16.8%), although the welfare of free persons fell by 0.8%.

1 Introduction

American racial slavery was a distinctive economic institution, with trade-able, legally enforced restrictions to Black labor and life in widespread economic use until the Civil War. The 4 million slaves in 1860 were a primary axis of political economy throughout the early republic. Southern slaveowners were among the wealthiest individual in the United States, and output per capita was also high, driven by cash-crop agriculture and cotton in particular. Scholars have debated whether property rights in people, and in particular removing workers' choices of where and how much to work, fundamentally altered the distribution of economic activities across space and sector, and the relationship between slavery, racial inequality, and regional patterns of economic development and structural change. While universally condemned, the effects and legacy of historical enslavement remains a source of perpetual historical and political debate, and have been a core topic of economic analysis at least since Fogel and Engerman's "Time on the Cross" (Fogel and Engerman, 1995).

In economics and economic history, researchers have classically focused on the economic productivity and profitability of slavery, and more recently on the large negative effects of the system on the well-being of enslaved workers and families. Other researchers have looked at differences in economic structure induced by unfree labor: scholars have studied whether slavery altered the composition of economic activity between agriculture and manufacturing(Bateman and Weiss, 2002), as well as whether the slave economy was overly specialized in plantation cotton (Wright, 2022)[1].

All of these debates have either implicitly or explicitly specified counterfactuals to the slave economy. Recent analyses have either focused on spatial discontinuities between slave and free states (Bleakley and Rhode, 2022), or differences between the pre-Civil War South and the postbellum South, with a few focused on within-slavery differences. However, the local spatial comparisons, and indeed most "local" comparisons that attempt to isolate the causal effect of slavery are intrinsically limited in their ability to recover the general equilibrium effects of emancipating the third of the Southern U.S. population that was enslaved. A natural aggregate counterfactual also exists in the pre-post Civil War comparison. But emancipation after 1860 is confounded by both the destruction of the Civil War(Feigenbaum, Lee, and Mezzanotti, 2022) as well as the rapid imposition of both military Reconstruction (Frieden, Grossman, and Lowery, 2024) and incipient Jim Crow(Althoff and Reichardt, 2024). Constructing a plausible counterfactual to the slave economy requires detailed modeling of the ways in which coercion was used, and how costly it was.

[1]Other recent literatures include studies of the informativeness of slave pricesCalomiris and Pritchett (2016) and the availability of slave-backed financial instruments Narayanan and Pritchett (2025); González, Marshall, and Naidu (2017).

In this paper, we analyze the general equilibrium effect of slavery on aggregate productivity, welfare, and the incidence of forced labor on enslaved workers. Our basic insight is that slavery induces mis-allocation: across space, between sectors, and between labor and leisure/home production. Enslaved labor can be thus modeled as labor where slaveowners, not the workers themselves, choose what enslaved workers do, as well as how much and where they do it. We integrate this idea into a multi-sector, multi-location quantitative general equilibrium spatial equilibrium model of the United States economy in 1860. We use data on quantities of free and enslaved labor by county and occupation, novel data on wages and slave rents from the census social statistics and military forts, along with exogenous variation in cotton productivity, malaria, and ease of slave flight, to estimate and quantify the model.

We begin by showing a number of stylized facts consistent with the misallocation hypothesis. The intuition of our model is that enslaved workers' preferences are ignored by slaveholders, who make decisions to maximize their own profits instead of the enslaved's welfare. In spatial equilibrium, this means that enslaved workers will be coerced into oversupplying their own labor and will be allocated to high productivity, low amenity locations, while free workers will choose based on both high productivity and amenities. The geographic comparative advantage of enslaved labor is to be located in productive places where free workers do not want to live or work. Across sectors with a location, free workers will select out of high disamenity occupations, while there will be no such pattern for enslaved workers. Third, on the extensive margin, free workers will supply more labor in high productivity, high amenity locations, but the availability of enslaved labor allows slaveowners to choose tasks that are relatively attractive, blunting the disamenities of geography. Finally, free workers will have higher wages in high productivity, low amenity counties, as in standard spatial equilibrium models, but (proxies for) enslaved workers consumption will be marked-down relative to free persons within the same location-sector, holding labor supply constant.

We next develop a quantitative general equilibrium spatial model of the institution of slavery. Consistent with the stylized facts, we show how the institution of slavery reduces the welfare of the enslaved through three channels: first, enslaved persons are coerced to provide greater labor supply than they would have themselves chosen (as in Acemoglu and Wolitzky (2011)); second, slaveholders act as monopsonists, marking-down enslaved persons wages in order to maximize their own profits; third, slaveholders assign the enslaved to locations and occupations that have lower amenities but higher productivity than they would have chosen on their own. We derive straightforward (partial equilibrium) expressions for the welfare costs of each of these margins as functions of the distribution of productivities and amenities across locations.

In general equilibrium, however, the sectoral and geographic reallocation of newly emancipated workers will alter the prices for the outputs of different places, induce reallocation of the already free population, and these can have complex effects on both the welfare of the newly emancipated as well as the already free. For example, if enslaved workers are (as we find) disproportionately located in remote, disease-ridden locations with high agricultural productivity, then emancipation will result in them leaving both those locations as well as the agricultural sector. But this will drive up wages for free workers in those sectors and locations, as well as the price of the products produced in those locations. The incidence of these wage and price changes will be propagated throughout the economy (depending on e.g. the transportation costs between slave-intensive areas and other population centers). As another example, enslaved workers will be disproportionately located in agriculture far from free states (to deter fugitives as in Allen (2015)) in order to more easily coerce enslaved labor. With emancipation those workers will be free to move to more productive manufacturing or agricultural sectors in locations closer to the North. The incidence of the resulting wage and price changes will again be propagated throughout the economy. Quantifying these complex effects is the task we set our model to.

In order to do so, we begin by using the detailed spatial data on labor allocations and output to "invert" the model to recover the underlying productivities and amenities of each location and occupation. The crucial identifying assumption necessary is that were slavery to not exist, enslaved persons would act the same as free persons. For example, if we observe many more free people working in one occupation than another even if they both earn the same wage, we can infer that the more popular occupation offers greater amenities. We show that the recovered productivities and amenities correlate well with the proxies we employed in the stylized facts.

We then simulate a series of counterfactuals that dismantle the institution of slavery to estimate the welfare and spatial economic impacts for both free and enslaved workers. Perhaps unsurprisingly, we find that removing the institution of slavery leads to substantial welfare gains for enslaved workers. In equivalent variation terms, an enslaved person would have to have their earnings under slavery increased by 1,100% to be indifferent between slavery and emancipation. Each of the three sources of misallocation play an important role: allowing enslaved individuals to choose their own labor supply while remaining in slavery yields a welfare gain equivalent to a 196% increase in wages; allowing enslaved workers to retain all of their earnings (i.e. remove the monopsonist slaveholder) yields a welfare gain equivalent to a further 160% wage increase; and allowing enslaved workers to choose any occupation in any location in the U.S. (so that they act identically to free persons) leads to a welfare gain equivalent to a 56% wage increase.

Emancipation also has large general equilibrium impacts on the economy. U.S. GDP increases by 16.8%, an effect which is driven by an expansion of the formerly-enslaved labor supply in manufacturing and a move toward more productive locations in the U.S. North. Welfare of free persons declines by a modest 0.8% as they face increased competition in high amenity locations.

Our paper is related most closely to the recent economics of slavery. Hornbeck and Logan (2023), perhaps closest to us, similarly calculate welfare gains from emancipation, analogizing slavery to an externality imposed on the enslaved. They rely on a Hulten (1978)-type intuition: starting from a competitive slave economy with no other distortions, marginal economic benefit from emancipating enslaved worker is equal to the uncompensated externality. They then calculate bounds on missing "compensating differential" using both value of statistical life estimates as well as data on wages for gang-labor after emancipation, and find welfare gain to enslaved of emancipation on the order of the industrial revolution increase in GDP. In a simple, partial equilibrium description, Naidu (2020) suggests that free labor is monopsonistically competitive, with idiosyncratic tastes for employers, while the enslaved labor market, being and is competitive: less allocative inefficiency in slave economy (ignoring amenities). As mentioned above, Bleakley and Rhode (2022), analyze discontinuities at the Mason-Dixon line. They present evidence that slavery itself was a disamenity for free workers, with the slave side of the border characterized by low land values, high free wages, and few free whites on slave side of border. As inflows of free white workers were reduced by slave property rights, a lot of human capital and potential agglomeration was lost, contributing to underdevelopment.[2]These recent papers contribute to a large economics literature on slavery going back to Conrad and Meyer (1958), and we review the relevant works in the next section. In contrast to these papers, we estimate a full model of the economy, allowing heterogeneous tastes across many margins of labor supply, and recovering the dispersion of tastes from free-worker elasticities across the same margins. [3]

Our paper also contributes to a recent literature on spatial general equilibrium models, and their application to economic history. Studying the 20th century Black migration, both Takahashi (2022)and Yang (2024) build quantitative spatial models to quantify the contribution of the Great Migration of African-Americans to economic growth. Heblich, Redding, and Voth (2022)use a quantitative trade model to study the contribution of the

[2]Francis (2024), provides a critical discussion of the paper along with evidence that slavery facilitated westward expansion.

[3]Ager, Boustan, and Eriksson (2021) study persistent effects of emancipation on slaveowners, finding that slaveowner descendents had largely converged to non-slaveowner wealth levels by 1900. Althoff and Reichardt (2024) look at individual intergenerational persistence of slavery, and find that much of the persistent legacy of slavery (compared to African Americans emancipated before the civil war) can be explained by differential exposure to more intensive Jim Crow laws in the deep South.

slave trade to industrialization in England, emphasizing the role of frictional capital markets. Donaldson and Hornbeck (2016) study the effect effects of the 19th century US railroad through the lens of a spatial equilibrium model, while Allen and Donaldson (2020) study multiple equilibria in a quantitative dynamic spatial model of the United States back to 1790.

The theoretical contribution of our paper is to integrate a static spatial general equilibrium model with the key insight of coercive labor markets highlighted by Acemoglu and Wolitzky (2011). In addition to the standard location specific productivities and amenities, we allow each location to a have a specific "coercion" parameter that determines the extent to which monopsonistic slaveowners can extract labor *above* the level implied by the worker's labor supply curve alone. Our tractable formulation of the extensive margin of enslaved labor supply allows us to estimate these parameters and compare it with the productivity and amenity parameters. Our location-specific coercion parameters embody heterogeneity in the public and private threats of violence exercised by slave owners, and we show they are correlated with other measures of slave compulsion. While our application is location-specific coercion, the incorporation of location-specific extensive-labor margin "wedges" into an otherwise standard spatial equilibrium model may be of use for studying other policies (e.g. unemployment insurance), and we return to this in the conclusion.

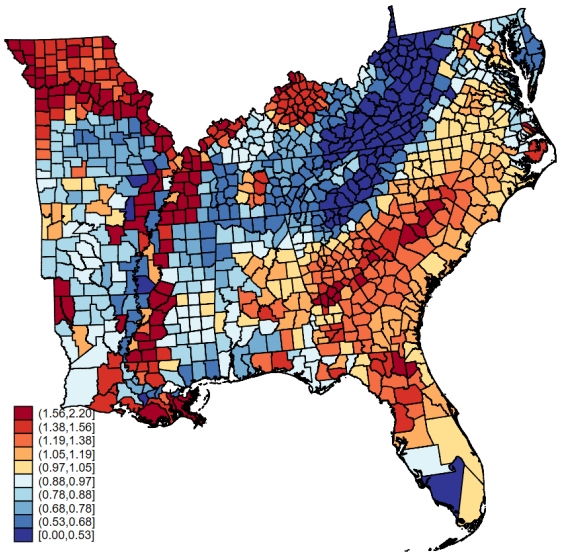
2 Data and Stylized Facts

We motivate our model by documenting a series of stylized facts about the U.S. slave economy. We begin by describing the data sources we use to document these facts, some of which are new.

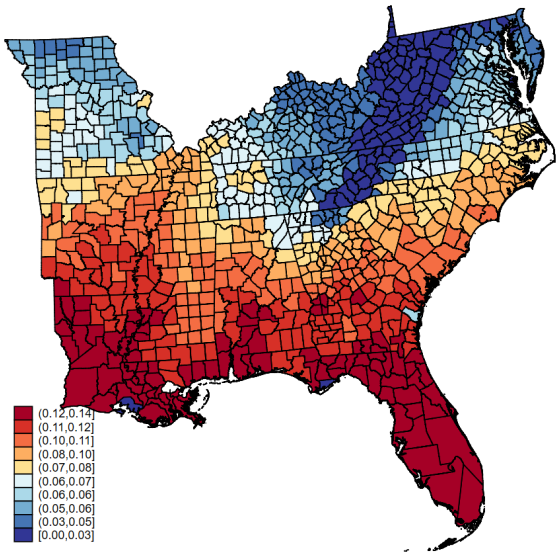
Geography: We use a variety of geographic datasets. We use malaria suitability from (Kiszewski et al. 2004), cotton and other crop suitabilities from the FAO-GAEZ database (we use rain-intensive, irrigation). We also measure distance of each county centroid to the nearest Northern (free) county, using Vintner distances. Figure 1 shows maps of the raw geographic variables, which have expected patterns, and suggest numerous potential confounds (e.g. latitude and state). However, once we absorb state fixed effects and condition each variable on the other two, we get much more variation across counties, as seen in Figure 2,and this will be the variation we use in our empirical exercises below.

Figure 1: Maps of Main Geographic Variables

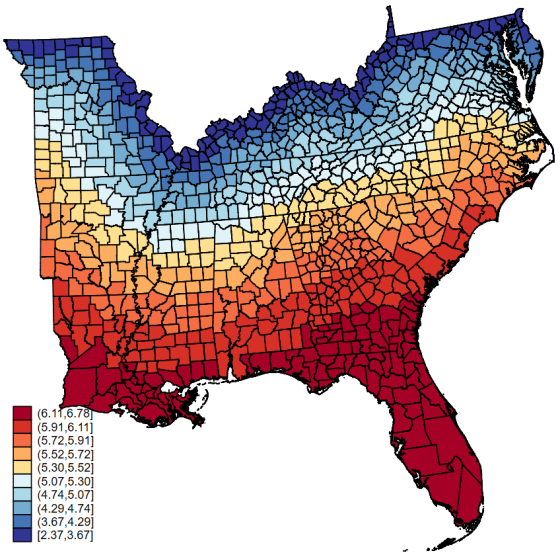
(a) Cotton suitability



(b) Malaria suitability



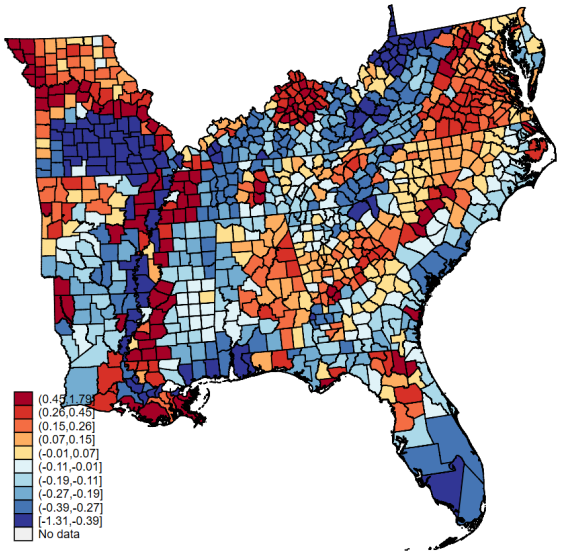
(c) Distance to North



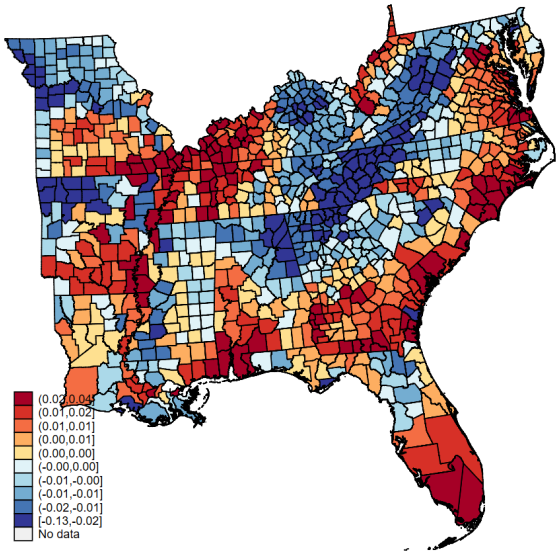
Notes: County-level maps of raw data using variables described in text.

Figure 2: Maps of Main Geographic Variables (Residualized)

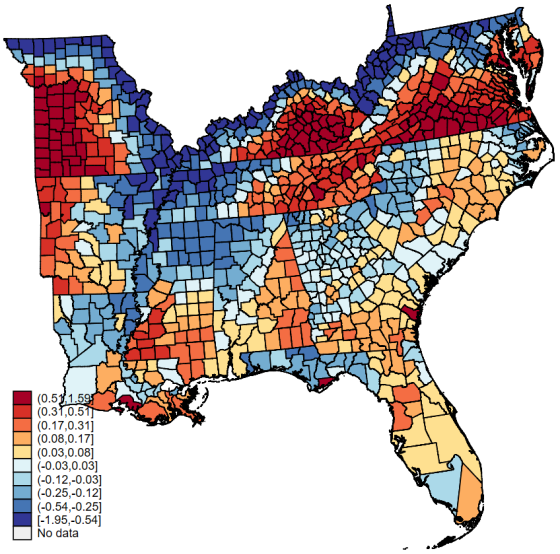
(a) Cotton suitability



(b) Malaria suitability



(c) Log distance to North



Notes: County-level maps of residualized data using variables described in text. Values shown control for state fixed effects and the values of each of the other two geographic variables

Workforce: We use the 100% Count 1860 Free Census from IPUMS, and consider workers between 15-50 as in the labor force. We use the industry variable to code households as agriculture or non-agriculture. We use the IPUMS 5% count of the enslaved schedules, assume all slaves are in the labor force, and impute agriculture/non-agriculture from the size of the slaveholding and the location (rural slaveholdings with more than 5 slaves are considered agricultural).

Economic Activity: We use the 1860 Census County Level Social Statistics, digitized by Paul Rhode and John Clegg, for wages of free workers by occupation and county,[4] We merge these data to the 1860 county-level agricultural and manufacturing censuses compiled by Michael Haines and available on ICPSR.

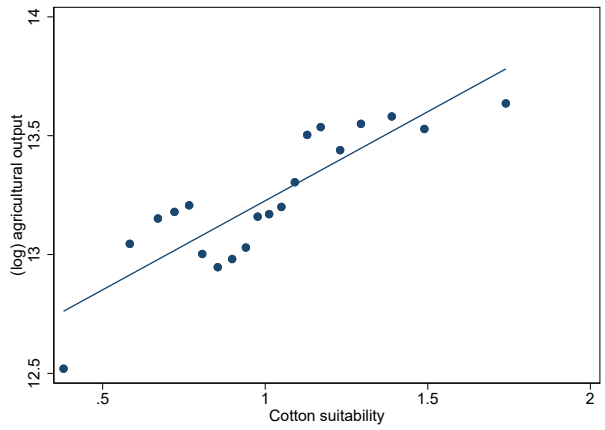
To validate that our exogenous geographic, county-specific proxies for agricultural productivity (cotton suitability) disamenities (malaria suitability), and coercion (distance to freedom) make sense, we check that they are correlated with direct measures of economic activity, health, and ease of slave escape. Figure 3 shows the within-state binned scatterplot of cotton suitability with the nominal value of agricultural output in the first panel, and reassuringly shows that agricultural output is indeed differentially higher in high cotton-suitability counties. The second panel shows the within-state correlation of malaria intensity with the share of old people among free persons. Owing to either differential mortality or out-migration, elderly individuals are significantly under-represented in high malaria areas, as would be expected. The third panel shows the within-state correlation of the share of enslaved workers reported as fugitives in the 5% sample with the within-state distance to the North. Reassuringly, these correlations remain significant and of the same sign when both proxies are included as regressors.

Fort data: A novel source of data we bring to bear on the study of slavery is the individual level U.S. Army Fort quartermaster records (Margo, 2009). Slaves were rented for employment in a variety of occupations on U.S. army forts prior to 1865, doing many tasks alongside free labor. It is not clear whether the recorded payments from the military are paid to the enslaved or their owner; however, in the case of the latter they are likely an upper bound on slave consumption. The virtue of the fort data is that it allows us to control for occupation, and thus compare wage rates to enslaved and free workers doing the same job in the same fort. Summary statistics are shown in the Appendix.

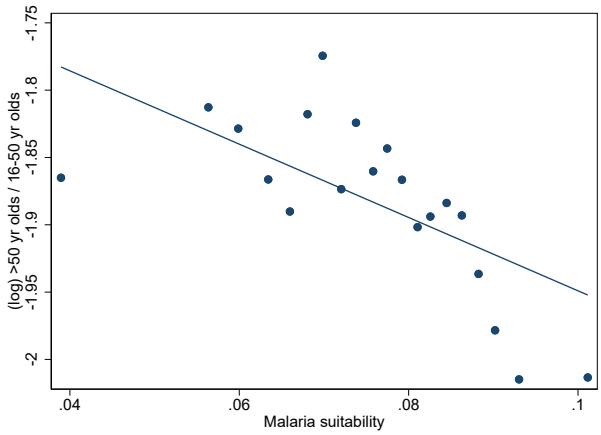
[4]We thank both of these authors for sharing these with us.

Figure 3: Geographic Variables Predict Productivity, Mortality, and Runaways

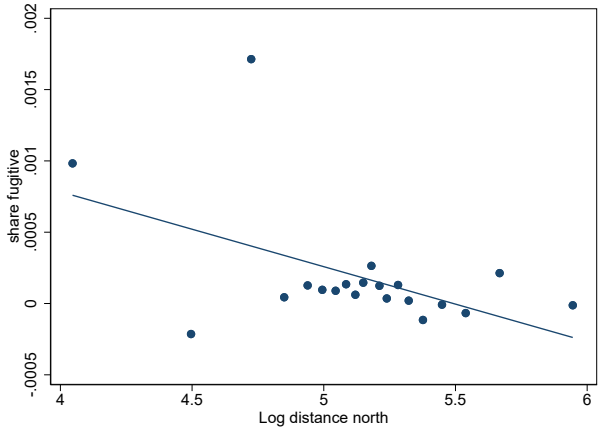
(a) Agricultural output is increasing with cotton suitability



(b) The share of old persons is decreasing with malaria suitability



(c) The share of fugitive slaves is decreasing with distance to the North



Notes: County-level binscatters showing that cotton suitability is correlated with agricultural productivity, malaria suitability is correlated with share of elderly free people, and distance to the North is correlated with share of enslaved workers that are reported as runaways. Each of these of binscatters controls for state fixed effects and the other two geographic variables.

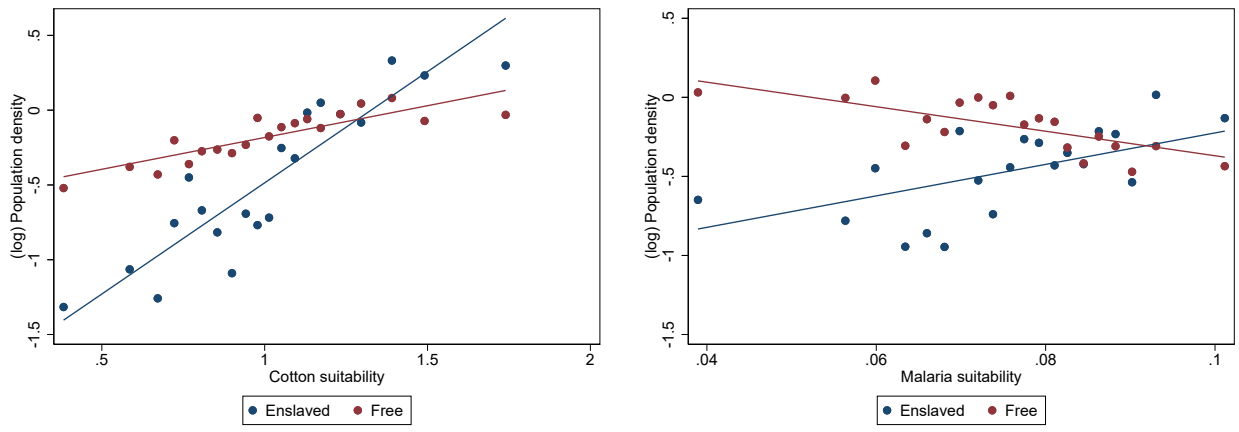
2.1 Stylized Facts

Stylized Fact #1: Free and enslaved sorting across locations. We begin by documenting that slavery allowed an allocation of labor to remote, disease-intensive locations difficult for free farmers to cultivate, building on Esposito (2022), and that enslaved labor was disproportionately allocated towards high malaria, high cotton productivity areas that were far from freedom. Further, enslaved labor should be disproportionately allocated towards places where coercion was easier, proxied in our data by distance to the North, while no such correlation should exist for free labor. Figure 4 shows a series of binned scatterplots showing that all these facts hold. Each of the binned scatterplots controls for both of the other variables as well as state-fixed effects, and shows that while both enslaved and free worker density increases with cotton suitability, the slope of enslaved population density is steeper, showing that enslaved labor is much more responsive, at least in the cross-section, to agricultural productivity. The flip side of the enslaved labor allocation's additional responsiveness to productivity is that free labor should be much less likely to live in unpleasant locations, in our data proxied by malaria suitability, compared to enslaved labor, as shown in the second panel. Indeed enslaved population density is increasing in within-state malaria suitability, while free labor is decreasing. Finally, the enslaved worker population density is increasing with the (within-state) distance to the North, as the costs of coercion decline (as running away becomes more difficult), while there is no such relationship (indeed it is opposite signed) for free workers.

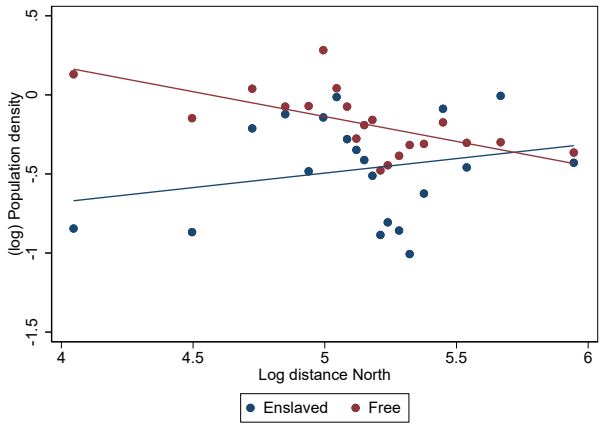
Figure 4: Free and Enslaved Sorting Across Locations

(a) Cotton suitability

(b) Malaria suitability



(c) Distance to North



Notes: County-level binned scatterplots showing how free and enslaved workers allocate across counties by geographic characteristics. All plots control for state fixed-effects and the other two geographic variables.

Table 1: Stylized Fact #1: Sorting across locations

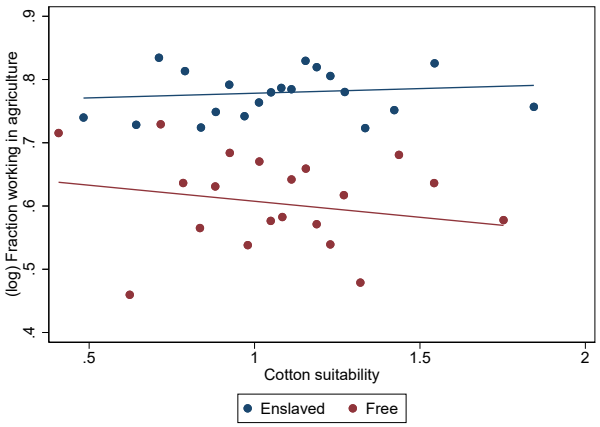
	Enslaved	Free	Enslaved/Free ratio
	(1)	(2)	(3)
Malaria suitability	9.954***	-7.753***	10.604***
	(3.081)	(1.773)	(1.952)
Cotton suitability	1.487***	0.425***	0.719***
	(0.134)	(0.076)	(0.080)
Log distance north	0.183*	-0.314***	0.307***
	(0.110)	(0.057)	(0.075)
State FE	Yes	Yes	Yes
R-squared	0.381	0.448	0.449
Observations	921	921	921

Notes: Ordinary least squares. The dependent variable is the (log) population density. Each observation is an 1860 county in the U.S. South. Cotton suitability is the predicted cotton yield given agro-climatic conditions in absence of irrigration and fertilizer. Malaria suitability is the malaria ecology index developed by Kiszewski et al. (2004). Standard errors reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

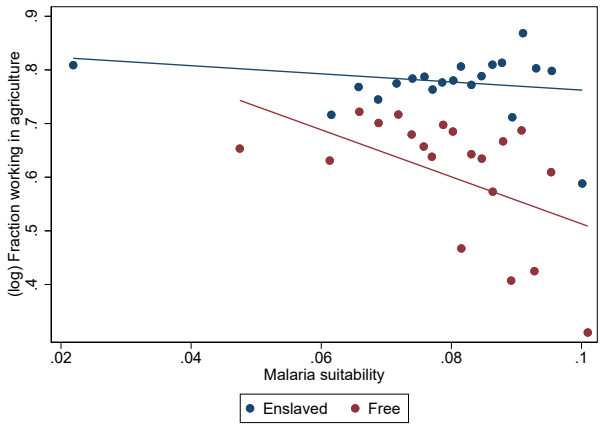
Stylized Fact #2: Free and enslaved sorting across sectors A common view is that enslaved labor organization had a comparative advantage in agriculture. Slavery allowed "gang labor system" that could be "driven" in large teams, increasing productivity(Fogel and Engerman, 1974). Slavery solves recruitment and retention problems that plague crops in with long tending periods and demanding harvests (Fleisig, 1976; Wright, 2006). (Hanes, 1996) argues that costly turnover sectors were disproportionally enslaved. However, in our data, while we do not see enslaved workers disproportionately in agriculture as a function of cotton suitability, we do see that free workers are slightly less likely to be working in agriculture as a function of cotton suitability, so there is a difference in slopes. Similarly, enslaved workers are, however, much more likely to be in agriculture in high malaria counties, when compared to free workers. Finally, there is only a small difference in the share employed in agriculture as a function of distance to the North.

Figure 5: Free and Enslaved Sorting Across Sectors

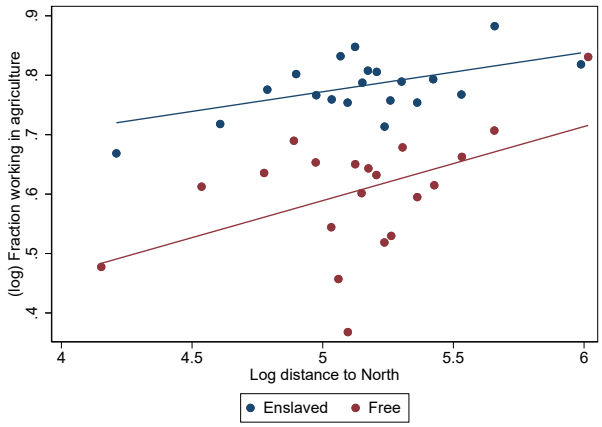
(a) Cotton suitability



(b) Malaria suitability



(c) Log distance to North



Notes: Individual binned scatterplots showing the shares of free and enslaved workers allocated to agriculture by county-level geographic characteristics. All plots control for state fixed-effects and the other two geographic variables.

Table 2: Stylized Fact #2: Sorting across tasks (within location)

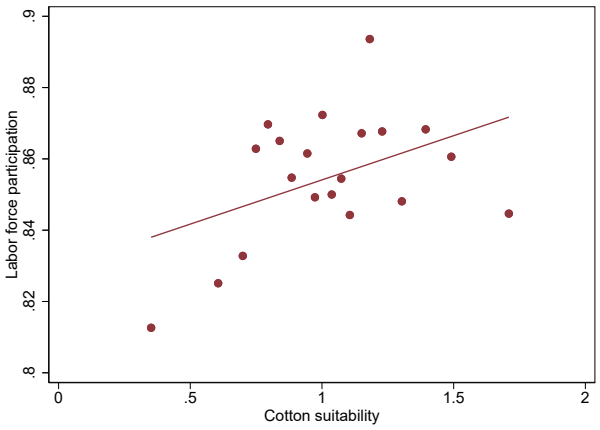
	(1)	(2)	(3)	(4)
Log distance north			0.112***	
			(0.034)	
Log distance north * Enslaved			0.011	0.014*
			(0.012)	(0.007)
Malaria suitability	-2.615**		-3.551***	
	(1.090)		(1.202)	
Malaria * Enslaved	2.291***	1.925***	2.264***	1.291***
	(0.711)	(0.332)	(0.852)	(0.471)
Cotton suitability	-0.035		0.008	
	(0.063)		(0.056)	
Cotton * Enslaved	0.030	-0.007	-0.027	-0.023
	(0.050)	(0.023)	(0.046)	(0.025)
State FE	Yes	Yes	Yes	Yes
County FE	No	Yes	No	Yes
R-squared	0.111	0.267	0.121	0.267
Number of counties	974	974	974	974
Number of workers	3399905	3399905	3399905	3399905

Notes: Ordinary least squares. The dependent variable is a dummy variable equal to one if a worker aged 15-50 is engaged in agriculture. Each observation is an individual worker residing in the U.S. South. Cotton suitability is the predicted cotton yield given agro-climatic conditions in absence of irrigration and fertilizer. Malaria suitability is the malaria ecology index developed by Kiszewski et al. (2004). Log distance north is the Vincenty centroid distance from each county to the U.S. North. Standard errors clustered at the county level reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

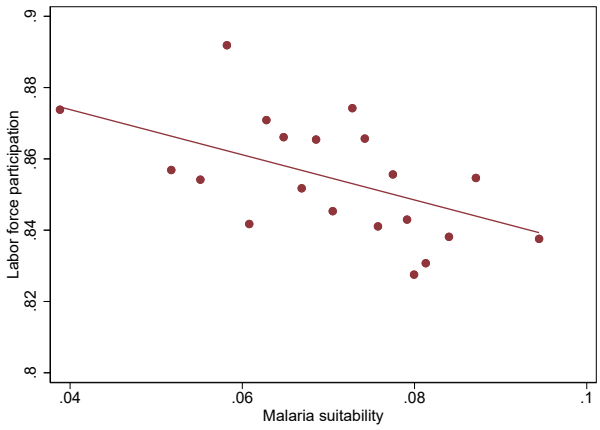
Stylized Fact #3: Labor force participation of free workers Free workers can choose how much to work, and thus we should expect their labor supply to vary positively with our productivity proxy, and negatively with the disamenity proxy. Figure 6 and Table 3 shows this pattern holds in our data.

Figure 6: Free Labor Supply-Extensive Margin

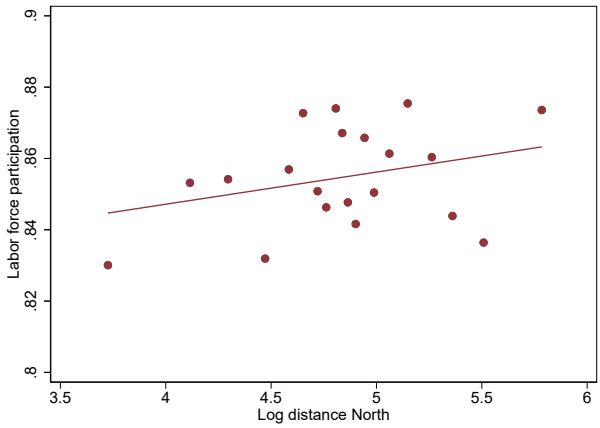
(a) Cotton suitability



(b) Malaria suitability



(c) Log distance to North



Notes: Individual binned scatterplots showing the share of prime-age free workers in the labor force by county-level geographic characteristics. All plots control for state fixed-effects and the other two geographic variables.

Table 3: Stylized Fact #3: Labor force participation

	(1)	(2)	(3)
Malaria suitability	-0.634**	-0.682**	-0.705**
	(0.271)	(0.272)	(0.279)
Cotton suitability	0.025*	0.021	0.023
	(0.013)	(0.013)	(0.014)
Log distance north	0.009	0.007	0.006
	(0.008)	(0.008)	(0.009)
Slaveholder		0.034***	-0.006
		(0.003)	(0.032)
Malaria * slaveholder			0.014
			(0.172)
Cotton * slaveholder			-0.014
			(0.010)
Log distance North * slaveholder			0.011
			(0.006)
State FE	Yes	Yes	Yes
R-squared	0.004	0.005	0.006
Number of counties	973	973	973
Number of adults	1067915	1067915	1067915

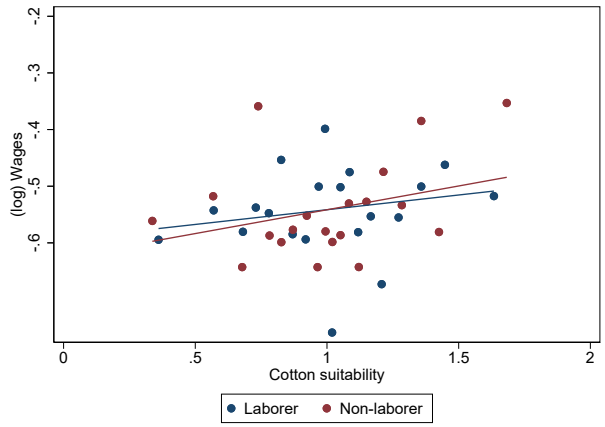
Notes: Ordinary least squares. The dependent variable is a dummy variable equal to one if an individual aged 15-50 of a given type is in the labor force. Each observation is a free white head of household residing in the U.S. South. Cotton suitability is the predicted cotton yield given agro-climatic conditions in absence of irrigration and fertilizer. Malaria suitability is the malaria ecology index developed by Kiszewski et al. (2004). Standard errors clustered at the county level reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

Stylized Fact #4: Free and enslaved wages Fact#1 has a corresponding implication for free worker wages: wages ought to be higher in high productivity places, as well as in high disamenity places. Figure **??** shows free worker wages from the 1860 census social statistics, separately for laborers (farm workers) and non-laborers (e.g. carpenters). Wages for both types of free workers are increasing in cotton suitability, increasing in malaria suitability, and decreasing with distance to the North.

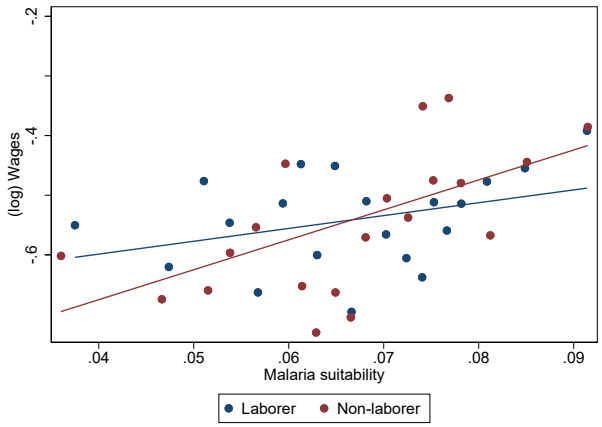
We can also use the U.S. army fort data to look at wage gaps between free and enslaved workers doing the same occupation. Table 5 below shows regressions that control for fort fixed effects, year-by-month fixed effects, and occupation fixed effects. Most importantly for our purposes, the table shows there is a level gap, of around 25%, between free and enslaved wages, within fort and occupation. This suggests that doing the same job, at the same intensity, enslaved workers are paid less.

Figure 7: Free Wages by Occupation

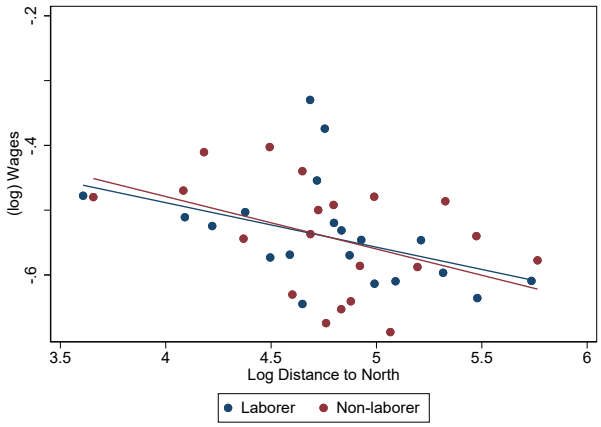
(a) Cotton suitability



(b) Malaria suitability



(c) Log distance to North



Notes: County-level binned scatterplots showing how free worker wages vary by geographic characteristics and sector. All plots control for state fixed-effects and the other two geographic variables.

Table 4: Stylized Fact #5: Free wages

	(1)	(2)	(3)
Cotton suitability	0.062**	-0.036	
	(0.028)	(0.052)	
Malaria suitability	3.194***	7.255***	
	(0.788)	(1.176)	
Log distance north	-0.073***	-0.153***	
	(0.016)	(0.028)	
Cotton * laborer		0.154***	0.132**
		(0.056)	(0.053)
Malaria * laborer		-6.415***	-6.333***
		(1.149)	(1.096)
Log distance north * laborer		0.124***	0.140***
		(0.034)	(0.034)
Occupation FE	Yes	Yes	Yes
State FE	Yes	Yes	Yes
County FE	No	No	Yes
R-squared	0.692	0.697	0.782
Observations	3945	3945	3931

Notes: Ordinary least squares. The dependent variable is the (log) wage. Each observation is an occupation-county pair in the U.S. South in the year 1860. There are five occupations: farmhand, laborer without board, laborer with board, carpenter, and women domestic servant. We classify farm hands and laborers as laborers. Cotton suitability is the predicted cotton yield given agro-climatic conditions in absence of irrigration and fertilizer. Malaria suitability is the malaria ecology index developed by Kiszewski et al. (2004). Log of distance to north calculated using Vincenty distance between county centroids. All interactions are demeaned, so effects are at sample averages. Observations weighted by number of workers with that occupation in a county. Standard errors clustered at the county reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

Table 5: Stylized Fact #4: Free and enslaved wages (from U.S. forts)

	(1)	(2)	(3)	(4)
Enslaved	-0.239***	-0.241***	-0.071	-0.029
	(0.035)	(0.038)	(0.157)	(0.220)
Malaria suitability	8.258*		8.224*	
	(4.722)		(4.804)	
Cotton suitability	-0.001		-0.000	
	(0.002)		(0.002)	
Log distance North	0.158		0.180*	
	(0.096)		(0.092)	
Malaria * Enslaved			1.645	1.830
			(1.774)	(2.218)
Cotton * Enslaved			-0.002	-0.001
			(0.002)	(0.002)
Log distance North * Enslaved			-0.170	-0.218
			(0.178)	(0.248)
State FE	Yes	Yes	Yes	Yes
YearXMonth FE	Yes	Yes	Yes	Yes
Occupation FE	Yes	Yes	Yes	Yes
Fort FE	No	Yes	No	Yes
R-squared	0.687	0.707	0.687	0.707
Number of forts	41	38	41	38
Number of workers	18248	18245	18248	18245

Notes: Ordinary least squares. The dependent variable is the (log) wage. Each observation is a worker contract; observations weighted by number of workers on a contract. Cotton suitability is the predicted cotton yield given agro-climatic conditions in absence of irrigation and fertilizer. Malaria suitability is the malaria ecology index developed by Kiszewski et al. (2004). Log of distance to north calculated using Vincenty distance between county centroids. All interactions are demeaned, so effects are at sample averages. Standard errors clustered at the fort reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

Together, these stylized facts show that the allocation of free and enslaved population across countries, and across sectors, reflect the intuition that enslaved workers were allocated ignoring their own preferences, at least under the (very plausible) assumption that these preferences for higher wages, more leisure/homeproduction, and healthier locations, are similar to those of free workers.

3 A general equilibrium model of the slave economy

We now present a general equilibrium spatial equilibrium model of slavery that matches the stylized patterns of labor allocation and pricing documented in the previous section. The model allows us to disentangle slavery's three fundamental distortions, coercion, market power, and misallocation, and show how each of these different aspects of slavery shaped the American economy. Further, our model allows us to calculate both the productivity and welfare impacts of large scale counterfactual emancipations, and compare them to the actual experience of the post-bellum U.S. South.

3.1 Setup

We build a standard quantitative economic geography model with two key ingredients. We have a standard free labor population, that chooses labor, location, and sector to maximize welfare, inclusive of both commonly valued amenities as well as idiosyncratic tastes. More novel is the inclusion of enslaved labor, whose labor supply is *coerced*, and which is *assigned* to locations and sectors to maximize *profits*.

Formally, we have N locations (U.S. counties), $O = 2$ sectors (agriculture and non-agriculture), and perfectly substitutable enslaved and free labor. Each location is characterized by a sector specific productivity A_{io}, and a sector-specific amenity U_{io}, and a "coercion" parameter $c_{io,s}$ indicating the relative cost of coercing enslaved labor in that county. Thus, each county is characterized by the vector $((A_{io}, c_{io,s}, u_{io})_{o=A,NA})$. As is standard, each location-pair is endowed with (iceberg) trade cost $\tau_{ij} \geq 1$.

Each location produces a differentiated variety by combining a constant elasticity of substitution (CES) combination of different occupations: $Q_i = \left(\sum_{o \in O} q_{io}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$, where q_{io} is the efficiency units of labor assigned to occupation o in location i and $\rho > 0$ is the elasticity of substitution across sectors. Final consumption is a CES aggregate of these location-specific outputs, discounted by trade costs. Workers, indexed by ω, receive an idiosyncratic location-sector productivity shock given by $\varepsilon_{io}(\omega) = \varepsilon_i(\omega)\,\varepsilon_o(\omega)$, where $\varepsilon_i(\omega)$ and $\varepsilon_o(\omega)$ follow independent Frechét distributions with shape parameters θ and η respectively.

3.2 The Allocation of Free Labor

Free laborers choose their labor supply, location, and sector, accounting for consumption, amenities, and idiosyncratic shocks. The opportunity cost of work (either leisure or home-production) is a constant elasticity function such that the overall utility for free workers in location-occupation io is given by:

$$W_{io}(Q_i, L, (\omega)) = u_{io}\left(\sum_i^N Q_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \frac{l^{\phi+1}}{\phi+1} = \frac{w_{io}u_{io}}{P_i}A_{io}\varepsilon_{io}(\omega)l - \frac{\phi}{1+\phi}l^{\frac{1+\phi}{\phi}},$$

where $\sigma > 1$[5] is the elasticity of substitution across aggregate location goods, ϕ is the elasticity of labor supply, and the CES price index $P_i = \left(\sum_j \tau_{ji}^{1-\sigma}p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is a function of trade costs τ_{ji} and location-variety prices p_j. Profit maximization implies that, for all locations and occupations, wages w_{io} are defined by the marginal product of labor:

$$w_{io} = p_i \times \left(\frac{q_{io}}{Q_i}\right)^{-\frac{1}{\rho}}.$$

Given a location-occupation, free workers choose a degree of labor supply l, so that

$$l_{io}(\omega) = \left(\frac{w_{io}u_{io}}{P_i}A_{io}\varepsilon_{io}(\omega)\right)^{\phi}. \tag{1}$$

Substituting back into utility, we get that the indirect utility from choosing location i and occupation o is

$$W_{io}(\omega) = \left(\frac{1}{1+\phi}\right)\left(\frac{u_{io}}{P_i}w_{io}A_{io}\varepsilon_{io}(\omega)\right)^{1+\phi}.$$

Free-workers thus choose a location-occupation to maximize W_{io}. Then, the occupational shares within a location are given by[6]

$$\pi_{io,f|i} = \frac{(w_{io}A_{io}u_{io})^{\eta}}{\sum_{o'}(w_{io}A_{io}u_{io})^{\eta}}.$$

[5]We specify $\sigma > 1$ to guarantee the existence of a unique mapping from parameters and observed data to unobserved location characteristics (see Appendix 8.1).

[6]Note that $\varepsilon_o(\omega)^{1+\phi} \sim Frechet(\frac{\eta}{1+\phi})$ since $\Pr\{x \leq \bar{x}\} = e^{-\bar{x}^{-\eta}}$ so that $\Pr\left\{x^{1+\phi} \leq \bar{x}\right\} = \Pr\left\{x \leq \bar{x}^{\frac{1}{1+\phi}}\right\} = e^{-\bar{x}^{-\frac{\eta}{1+\phi}}}$

Using the nested structure of the location-specific occupation utility, the expected utility from location i is given by:

$$\Pi_{i,f} = \left(\frac{1}{1+\phi}\right)\Gamma\left(\tfrac{\eta-1-\phi}{\eta}\right)\left(\sum_o \left(\tfrac{u_{io}}{P_i} w_{io} A_{io}\right)^{\eta}\right)^{\frac{1+\phi}{\eta}}.$$

Lastly, the share of free workers employed in location i is given by

$$\pi_{i,f} = \frac{\Pi_{i,f}^{\theta}}{\sum_{i'} \Pi_{i',f}^{\theta}}.$$

3.3 The Allocation of Enslaved Labor

Enslaved labor differs from free labor in three ways. First, their labor supply is chosen by the slave owner, who behaves as a perfect monopsonist able to extract all surplus from the enslaved. Second, their labor supply is *coerced*, following Acemoglu and Wolitzky (2011), distorting the labor-supply curve faced by a monopsonistic owner towards more labor at a given wage. Finally, the enslaved worker is *misallocated*, in that the amenities of their location and occupation are discounted relative to free workers. Formally, the slaveholder chooses a location and an occupation, along with labor supply l and an enslaved wage w_s to maximize the net surplus (difference between productivity and enslaved wage per unit of effective labor, times amount of effective labor:

$$\max_{i,o,l,w_s} \Big(w_{io} - w_s\Big) A_{io} \varepsilon_{io}(\omega) l, \tag{2}$$

subject to a participation (or rebellion/fugitive) constraint:

$$\frac{w_s u_{io}}{P_i} A \varepsilon_{io,s}(\omega) l - \frac{\phi}{1+\phi} l^{\frac{1+\phi}{\phi}} \geq \widetilde{c_{io,s}} \frac{w_s}{P_i} A_{io} \varepsilon_{io}(\omega) l \tag{3}$$

$$\iff \frac{w_s u_{io}}{P_i} A \varepsilon_{io}(\omega) l - \frac{\phi}{1+\phi} \frac{l^{\frac{1+\phi}{\phi}}}{c_{io,s}} \geq 0.$$

Where $c_{io,s} = 1 - \widetilde{c_{io,s}}/u_{io} \geq 1$. The term $c_{io,s}$ is a "coercion parameter" that enables slaveowners to compel additional labor supply, holding the wage fixed. If $c_{io,s} = 1$, the participation constraint is equivalent to holding the enslaved utility equal to zero. With regards to the slaveowners' problem (but not the enslaved utility), it is isomorphic to a productivity differential for enslaved labor relative to free labor in the same occupation and location. Acemoglu and Wolitzky (2011) present a principal-agent model where coercive employers can (individually or collectively) invest in "guns" that reduce workers outside

options. They show that this allows employers to elicit more effort, as it slackens the limited liability constraint that normally binds a free worker. To embed this idea in our model, we allow there to be location-sector-specific coercion parameters, and suppose that coercion is proportional to an enslaved worker's wage (including amenities). Taken literally, this implies that more productive workers, as well as workers who get more utility from a given location-occupation, will be coerced more by the local authorities.

The slaveholder sets the constraint to bind in equilibrium, and substituting it into the slaveowner objective function yields:

$$\max_{l,w_s} \left(w_{io} - w_s\right) A_{io}\varepsilon_{io}\left(\omega\right) l \iff$$
$$\max_{l} w_{io}A_{io}\varepsilon_{io}\left(\omega\right) l - \frac{\phi}{1+\phi}\frac{l^{\frac{1+\phi}{\phi}}}{c_{io,s}}\left(\frac{u_{io}}{P_i}\right)^{-1},$$

leading to an enslaved labor supply of:

$$l_{io,s} = \left(\frac{u_{io}}{P_i} w_{io}\left(A_{io}c_{io,s}\right)\varepsilon_{io}\left(\omega\right)\right)^{\phi} \tag{4}$$

From the first-order conditions, enslaved labor supply is implying enslaved workers exert more effort than free labor if $c_{io,s} > 1$. Their utility (net of forced effort) can be negative, reflecting the cost of coercion. The payoff to the slaveowner for assigning an enslaved worker to (i, o) is

$$W_{io}^{sh} = \Big(w_{io} - w_s\Big)A_{io}\varepsilon_{io}(\omega)l_{io,s} = \frac{1}{1+\phi}\Big(w_{io}A_{io}\varepsilon_{io}(\omega)\Big)^{1+\phi}\Big(\frac{u_{io}}{P_i}c_{io,s}\Big)^{\phi}.$$

Thus, the occupational share in each location i is

$$\pi_{io,s|i} = \frac{\left(u_{io}^{\frac{\phi}{1+\phi}} c_{io,s}^{\frac{\phi}{1+\phi}} w_{io}A_{io}\right)^{\eta}}{\sum_{o'}\left(u_{io'}^{\frac{\phi}{1+\phi}} c_{io',s}^{\frac{\phi}{1+\phi}} w_{io'}A_{io'}\right)^{\eta}}.$$

The expected profits a slaveowner gets from allocating a slave to location i is:

$$\Pi_{i,sh} = \Gamma\Big(\tfrac{\eta-1-\phi}{\eta}\Big) \underbrace{\tfrac{1}{1+\phi}}_{Monopsony} \left(\sum_{o}\Big(\underbrace{\big(\tfrac{u_{io}}{P_i}\big)^{-\frac{1}{\phi}}}_{Misallocation}\underbrace{\big(c_{io,s}\big)^{\frac{\phi}{1+\phi}}}_{Coercion}\tfrac{u_{io}}{P_i}w_{io}A_{io}\Big)^{\eta}\right)^{\frac{1+\phi}{\eta}}.$$

The highlighted terms show where $\Pi_{i,sh}$ differs from $\Pi_{i,f}$. There is a monopsony component, that is the share of surplus appropriated by the slave owner, net of what is paid the enslaved worker. This is a pure transfer, with no distortion of quantities. The misallocation term shows that enslaved workers are employed in sectors and locations with lower amenities than free workers, because owners do not internalize any of the disamenities of places of living or content of work. Finally, the coercion term captures the additional labor supply that enslavers elicit from enslaved workers. Then, the allocation of enslaved labor across locations satisfies

$$\pi_{i,s} = \frac{\Pi_{i,sh}^{\theta}}{\sum_{i'} \Pi_{i',sh}^{\theta}}.$$

3.4 Equilibrium

We now close the model with standard general equilibrium conditions. First, given CES demand, we have the typical gravity equation:

$$X_{ij} = \tau_{ij}^{1-\sigma} p_i^{1-\sigma} P_j^{\sigma-1} E_j,$$

where total expenditure in location j is equal to total income paid to free labor, enslaved labor, and slaveowners:

$$E_j = \sum_o w_{jo} L_{jo,f} + \frac{\phi}{1+\phi} \sum_o w_{jo} L_{jo,s} + \lambda_j \frac{1}{1+\phi} \sum_i \sum_o w_{io} L_{io,s}.$$

Here, λ_j is the share of slaveholder profits going to location j. We can then formally define an equilibrium: For any geography$\{(A_{io}, c_{io}, u_{io}, \lambda_i)\}_{i \in N}$, aggregate labor endowments $\bar{L}_f$ and $\bar{L}_s$, trade costs $\{\tau_{ij}\}_{(i,j) \in N \times N}$, and model parameters $\{\sigma, \rho, \theta, \eta, \phi\}$, equilibrium is a set of prices (p_i, w_{io}) and labor allocations (π_{io}^s, π_{io}^f) such that:

Goods markets clear:

$$p_i Q_i = \sum_j \tau_{ij}^{1-\sigma} p_i^{1-\sigma} P_j^{\sigma-1} E_j;$$

Trade is balanced:

$$P_i^{\sigma-1} = \sum_j \tau_{ji}^{1-\sigma} p_j^{1-\sigma};$$

Wages are equal to the marginal product:

$$w_{io} = p_i \times \left(\frac{L_{io,f} + L_{io,s}}{Q_i} \right)^{-\frac{1}{\rho}},$$

where the quantities of effective labor are given by:

$$L_{io,f} = \Gamma\left(\frac{\eta-\phi}{\eta}\right)\Gamma\left(\frac{\theta-1}{\theta}\right)A_{io}\left(\frac{u_{io}}{P_i}w_{io}A_{io}\right)^{\phi}\left(\pi_{io,f|i}\right)^{\frac{\eta-\phi}{\eta}}\left(\pi_{i,f}\right)^{\frac{\theta-1}{\theta}}\bar{F}$$
$$L_{io,s} = \Gamma\left(\frac{\eta-\phi}{\eta}\right)\Gamma\left(\frac{\theta-1}{\theta}\right)A_{io}\left(\frac{u_{io}}{P_i}w_{io}A_{io}c_{io,s}\right)^{\phi}\left(\pi_{io,s|i}\right)^{\frac{\eta-\phi}{\eta}}\left(\pi_{i,s}\right)^{\frac{\theta-1}{\theta}}\bar{S};$$

and labor allocations satisfy equations (1) and (4) for free and enslaved labor respectively.

3.5 Identification of Unobserved Location Characteristics

We now show results on the identification of unobserved location characteristics. For exposition, let us define the following objects:

$$A_{io} \equiv A_i \times a_{io}$$
$$u_{io} \equiv U_i \times \tilde{u}_{io}$$

Here, A_i and U_i capture the across-location relative productivities and amenities respectively, while a_{io} and $\tilde{u}_{io}$ capture the within-location, across-occupation relative productivities and amenities. We define the set $\{\mathbf{a} = [a_{io}], \tilde{\mathbf{u}} = [\tilde{u}_{io}], \mathbf{c} = [c_{io}]\}$ as the unobserved within-location occupation characteristics, which differ across occupations within a location. Similarly, we define the set $\{\mathbf{A} = [A_i], \mathrm{U} = [\mathbf{U}_i], P = [P_i]\}$ as the unobserved across-location characteristics.

Proposition 1. *Given parameter values $\{\sigma, \theta, \eta, \phi, \rho\}$, observed data $\{\mathbf{L}_{persons}, \pi, P_{enslaved}, \mathbf{Y}_f\}$ and trade costs $\{\tau\}$, there exists a unique (up to scale) set of within-location occupation characteristics $\{\mathbf{a}, \tilde{\mathbf{u}}, \mathbf{c}\}$ and across-location characteristics $\{\mathbf{A}, \mathbf{U}, \mathbf{P}\}$ that are consistent with model equilibrium.*

Proof. See Appendix 8.1. □

This result shows that, given parameters and a set of observables, we can solve for a unique up-to-scale set of unobservable location characteristics. We first discuss the intuition for the within-location results. First, free location-occupation output $\mathbf{Y}_f$ and labor allocations $\mathbf{L}_{persons,f}$ allow us to back out wages for each location-occupation. Adjusting for the labor supply, the wages and free workers' choices of occupations allow us to identify the relative amenity of occupations within a location, $\tilde{\mathbf{u}}$. If a free worker works in a location-occupation with low wages, the amenity value of that location-occupation must be relatively high. On the other hand, since enslaved workers are allocated to maximize profits

for slaveholders, their labor allocation (again conditional on adjusted wages) identifies the relative coercion parameters, $\mathbf{c}$. The national average price of an enslaved worker $P_{enslaved}$ pins down the overall scale of the coercion parameters. Next, given the adjusted wages, labor allocations, and coercion parameters, we can identify the output across occupations. Then, the relative output across occupations (conditional on wages and amenities) identifies the relative productivity of occupations in each location, $\mathbf{a}$.

We now move to the across-location intuition. Similarly to the within-location amenities, free worker location choices, conditional on within-location characteristics, wages, and labor shares, identify the across-location amenities relative to prices, $\frac{\mathbf{U}}{\mathbf{P}}$. This term captures the non-wage benefits of living in a particular location. For the across-location productivities, $\mathbf{A}$, identification comes from the CES consumption specification and the relative expenditures of each location on goods from other locations, conditional on wages, within-location characteristics, across-location amenities relative to prices, labor allocations, and trade costs. Specifically, we express $\mathbf{A}$ as an argument in an excess-demand system for each location aggregate good, for which we show there exists a unique (up to scale) solution. Lastly, price indices $\mathbf{P}$ are identified by substituting previously identified location characteristics, adjusted wages, labor allocations, and trade costs into the balanced trade condition. Note that identifying $\frac{\mathbf{U}}{\mathbf{P}}$ and $\mathbf{P}$ implicitly identifies $\mathbf{U}$ as well.

3.6 Estimating the Model

We observe labor shares $\pi_{io|i,s}$ (enslaved) and $\pi_{io|i,f}$ (free), total output Y_{io} by sector and location, and aggregate labor stocks $\bar{S}, \bar{F}$. Model parameters $(\sigma, \theta, \eta, \phi, \rho)$ are known or calibrated. We seek to identify:

$$\{A_{io}\}, \quad \{u_{io}\}, \quad \{c_{io,s}\}.$$

Below are the main equations used for recovering these objects.

3.7 Core Equations for Recovery

Define an auxiliary variable $w'_{io} \equiv w_{io}\, A_{io} \left(\frac{u_{io}}{P_i}\, w_{io}\, A_{io}\right)^{\phi}$, which is observable up to parameters and occupation shares. Then:

1. *Relative Amenity (Free Labor):*

$$\frac{u_{io}}{\sum_{o'} u_{io'}} \;=\; \frac{\left(w'_{io}\right)^{-1} \left(\pi_{io,f|i}\right)^{\frac{1+\phi}{\eta}}}{\sum_{o'} \left(w'_{io'}\right)^{-1} \left(\pi_{io',f|i}\right)^{\frac{1+\phi}{\eta}}}. \tag{5}$$

2. *Coercion Parameter:* from enslaved occupation shares,

$$c_{io,s} = \left[\left(\pi_{io,s|i}\right)^{\frac{1}{\eta}} \left(w'_{io}\right)^{-\frac{1}{1+\phi}} \left(\frac{\left(\pi_{i,s} \sum_{i'} \Pi^{\theta}_{i',sh}\right)^{\frac{1}{\theta}}}{\Gamma\left(\frac{\eta-1-\phi}{\eta}\right)^{\frac{1}{1+\phi}}} \right)^{\frac{1}{1+\phi}} \right]^{\frac{1+\phi}{\phi}}.$$

3. *Free and Enslaved Output:* (sector o, location i)

$$\begin{aligned} Y_{io,f} &= \Gamma\left(\tfrac{\eta-\phi}{\eta}\right) w'_{io} \left(\pi_{io,f|i}\right)^{\frac{\eta-\phi}{\eta}} L_{i,f}, \\ Y_{io,s} &= \Gamma\left(\tfrac{\eta-\phi}{\eta}\right) w'_{io} \left(c_{io,s}\right)^{\phi} \left(\pi_{io,s|i}\right)^{\frac{\eta-\phi}{\eta}} L_{i,s}. \end{aligned} \tag{6}$$

4. *Relative Productivity Within Location:*

$$\frac{A_{io}}{\sum_{o'} A_{io'}} = \frac{\left(Y_{io,f} + Y_{io,s}\right)^{\frac{1}{\rho-1}} \left(w'_{io}\right)^{\frac{1}{1+\phi}} \left(u_{io}\right)^{-\frac{\phi}{1+\phi}}}{\sum_{o'} \left(Y_{io',f} + Y_{io',s}\right)^{\frac{1}{\rho-1}} \left(w'_{io'}\right)^{\frac{1}{1+\phi}} \left(u_{io'}\right)^{-\frac{\phi}{1+\phi}}}.$$

3.8 Across-Location Productivities and Amenities

4 Estimation Algorithm (Sketch)

1. *Within Each Location:* Use free-labor occupation shares (5) to recover *relative* amenities u_{io}. Use enslaved occupation shares to recover coercion parameters $c_{io,s}$, then compute $Y_{io,f}, Y_{io,s}$.

2. *Relative Productivities (Within Location):* Compute A_{io} up to a normalization using the ratio form of sectoral outputs.

3. *Across Locations:* Solve the trade/price index system for A_i and U_i using location-level market clearing.

4. *Iteration:* Update until consistency with observed labor shares, outputs, and the national price of enslaved labor is achieved.

This procedure identifies $\{A_{io}, u_{io}, c_{io,s}\}$ from the data on allocations, wages, and outputs, given structural parameters $(\sigma, \theta, \eta, \phi, \rho)$.

5 The Welfare Consequences of Misallocation

5.1 Baseline: Enslaved Welfare and Participation Constraint

We assume that the slaveholder chooses enslaved workers' labor supply so that the enslaved worker's participation constraint is always binding. Hence, the enslaved worker's utility under slavery is effectively normalized to zero. Once emancipation occurs, however, formerly enslaved workers can gain significantly. In fact, once freed, their welfare gain from *not* being at the subsistence (zero utility) level is infinite.

Because such infinite gains are not very informative from a *comparative statics* standpoint, we focus on the *additional*, finite welfare gains from being able to *choose* allocations (sector, location) rather than being assigned.

5.2 Misallocation Across Sectors

Consider a partial-equilibrium thought experiment in which we fix prices and wages. Suppose an enslaved individual is now freed but *cannot* change her location; however, she can choose which sector to work in and can retain the full produce of her labor. The expected increase in welfare from this freedom to choose occupation is given by:

$$\frac{\Pi_{i,s}^{\text{chooseocc}}}{\Pi_{i,s}^{\text{assignedocc}}} = \frac{\left(\sum_o \pi_{io,s|i} \left(\frac{u_{io}}{P_i}\right)^{\frac{\eta}{1+\phi}}\right)^{\frac{1+\phi}{\eta}}}{\sum_o \pi_{io,s|i} \left(\frac{u_{io}}{P_i}\right)}, \tag{7}$$

where:

- $\pi_{io,s|i}$ denotes the observed sector share (under slavery) for enslaved workers in sector o and location i;
- u_{io} is the amenity in location i and sector o;
- P_i is the local price index (cost of living) in location i;
- η and ϕ are elasticity parameters governing preferences over goods and leisure.

Interpretation of Equation (7)**.** This ratio measures how much higher the worker's utility can be if she chooses the sector with the highest amenity-to-cost-of-living ratio (weighted appropriately) rather than being assigned to the sector that the slaveholder selected. In other words, if $\left(\frac{u_{io}}{P_i}\right)$ varies widely across sectors, the gains to free choice can be large.

5.3 Misallocation Across Sectors and Locations

Next, consider the partial equilibrium scenario where a formerly enslaved person:

- Receives the produce of her labor,
- Can choose both sector *and* location,
- Faces the same fixed prices and wages as under slavery (no general equilibrium feedback yet).

Define $W_s^{\text{chooseall}}$ as the welfare of the formerly enslaved population when they choose their location and sector, and $W_s^{\text{choosenone}}$ as their welfare when both are assigned. Then from a lemma in the paper, we have:

$$\frac{W_s^{\text{chooseall}}}{W_s^{\text{choosenone}}} = \left[\sum_i \pi_{i,s} \left(\frac{\Pi_{i,s}^{\text{chooseocc}}}{\Pi_{i,s}^{\text{assignedocc}}} \times \frac{\Pi_{i,s}^{\text{chooseloc}}}{\Pi_{i,s}^{\text{assignedloc}}} \right)^{\theta} \right]^{\frac{1}{\theta}}, \tag{8}$$

where:

$$\frac{\Pi_{i,s}^{\text{chooseloc}}}{\Pi_{i,s}^{\text{assignedloc}}} \equiv \frac{\sum_o \pi_{io|i,s} \left(\frac{u_{io}}{P_i}\right)}{\sum_i \pi_{i,s} \left(\sum_o \pi_{io,s|i} \left(\frac{u_{io}}{P_i}\right)\right)}.$$

Interpretation of Equation (8). The term $\frac{\Pi_{i,s}^{\text{chooseocc}}}{\Pi_{i,s}^{\text{assignedocc}}}$ captures the welfare gain from choosing a *sector* within a fixed location. The term $\frac{\Pi_{i,s}^{\text{chooseloc}}}{\Pi_{i,s}^{\text{assignedloc}}}$ captures the welfare gain from being able to *move* to a more attractive location. The overall expression sums over all initial locations i, weighted by the enslaved share $\pi_{i,s}$, and then aggregates them with an elasticity parameter θ. Thus, Equation (8) shows how total welfare changes when enslaved workers are free to choose both location and occupation, relative to being fully assigned.

6 Counterfactuals: A Preview

We identify the model's key parameters (productivities $\{A_{io,t}\}$, amenities $\{u_{io}\}$, and elasticities $\{\sigma, \rho, \theta, \eta, \phi\}$) so that the model matches observed allocations and economic output in the historical data. We then recompute equilibria under alternative assumptions regarding slavery and compare counterfactual outcomes.

- **Baseline:** Slavery as observed.
- **Dismantling slavery:** We ask:

– What would happen if enslaved workers could choose their labor supply (but slaveholders still acted as monopsonists and assigned their location and occupation)?

– What would happen if enslaved workers could choose their labor supply and retained the fruits of their labors (but slaveholders still assigned their location and occupation)?

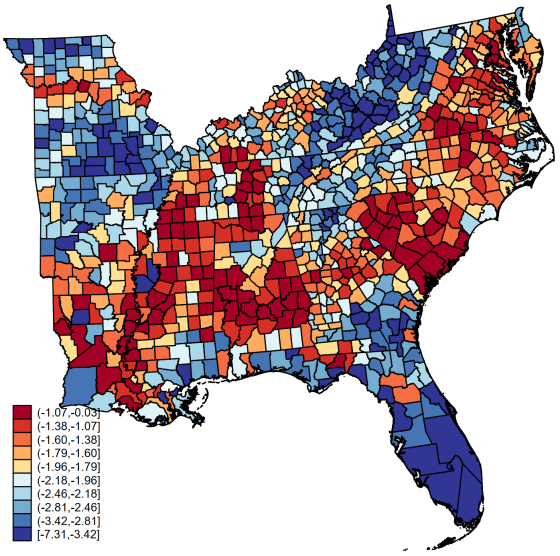
– What would happen if enslaved workers could choose their labor supply, retained the fruits of their labors, and chose their location and occupation? We refer to this case as emancipation, as the enslaved workers act identically to the free workers, although we also consider a variant where the "emancipated" enslaved workers can only reside in the U.S. South.

6.1 Estimated Productivity and Amenities

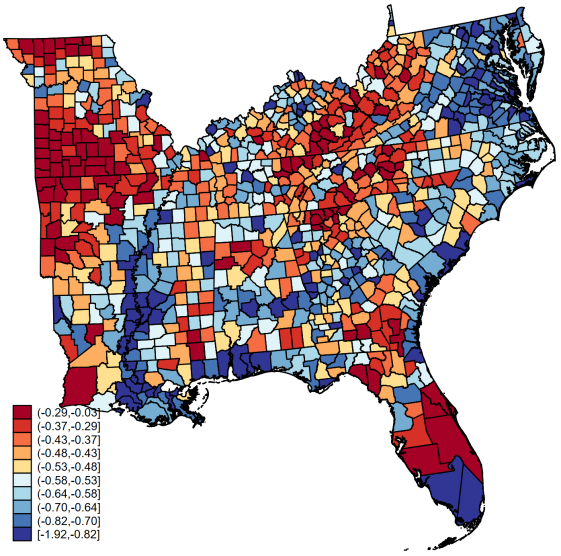
Figure 8 illustrates the estimated productivity and amenity parameters. Figure 9 shows that these are strongly correlated with our geographic proxies, as expected.

Figure 8: Estimated agricultural productivity and amenities across locations.

(a) Agricultural productivity



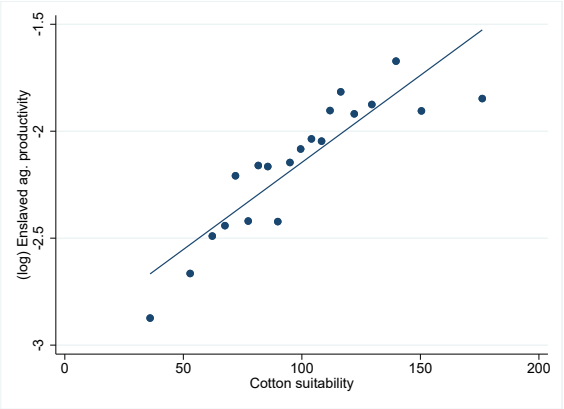
(b) Disamenity



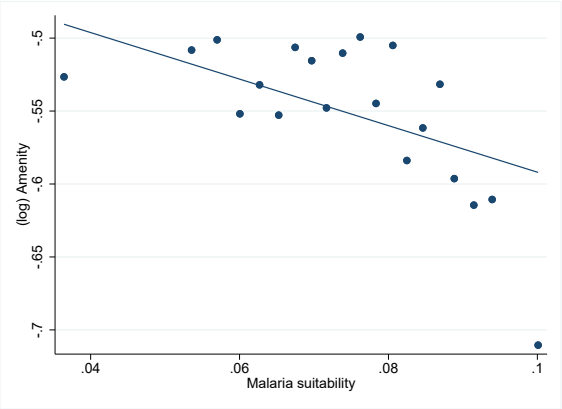
Notes: Maps of estimated model parameters.

Figure 9: Agricultural productivity and amenity estimates correlate with agricultural productivity and disease factors.

(a) Agricultural productivity vs. cotton suitability



(b) Agricultural amenity vs. malaria intensity



Notes: County-level binscatters showing correlation of estimated model parameters with underlying geographical characteristics.

6.2 The Welfare and Economic Impacts of Emancipation

In the preliminary results that follow, we choose the structural parameters are $\theta = \eta = \rho = \sigma = 3$ and $\phi = 0.3$, which allows for modest supply and demand elasticities across locations and sectors and a modest labor supply elasticity.

Table 6: Welfare and economic impacts of emancipation

				Welfare impacts			Economic impacts		
				(1)	(2)	(3)	(4)	(5)	(6)
	Coercion?	Monopsony?	Misallocation?	Enslaved persons	Free persons	Slaveholders	Ag. output	Man. output	Total GDP
Slavery	✓	✓	✓	-$140.66	$658.45	$150.00	$108.89	$203.54	$312.44
No coercion		✓	✓	$0.00	$664.98	$340.57	$106.59	$234.52	$341.11
No monopsony			✓	$339.95	$665.25	$0.00	$106.50	$234.71	$341.21
Emancipation (South only)				$408.86	$657.13	$0.00	$110.08	$227.13	$337.20
Emancipation				$651.95	$651.95	$0.00	$115.17	$249.81	$364.98

Notes: This table reports the aggregate welfare and economic impacts of removing the institution of slavery. Both welfare and economic impacts are measured in dollars per capita per year. No coercion means that enslaved people can choose their own labor supply, No monopsony means that enslaved people also receive the proceeds of their labor, emancipation (South only) means that enslaved people can also choose their occupation and location but must remain in the U.S. south, and emancipation means they can choose occupations and locations in the U.S. North as well, i.e. they act as free persons.

Table 6 presents the counterfactual results. We report both the welfare impacts and the economic impacts in dollars per person per year. For welfare, we also consider an equivalent variation measure which is the percentage change in wages from the slavery baseline that would yield an equivalent change in welfare. For enslaved persons, this equivalent variation measure holds their labor supply fixed (as it is beyond their control), whereas for free persons, their labor supply is allowed to respond (as is standard).

The first row presents our slavery baseline where the model is calibrated to exactly match the observed data. The baseline welfare of the enslaved is negative. Recall that in the absence of coercion, slaveholders act as monopsonists by setting a marked down wage for enslaved that ensures their welfare is zero. Because coercion increases the enslaved's labor supply above their preferrred choice, this causes their welfare to fall below zero. Note that the slaveholders per-slave profits are \$150, a value that we have calibrated.

The second row presents what would occur if there were no coercion. This results in a rise of enslaved welfare to \$0, which is a welfare gain equivalent to a 196.5% increase in enslaved wages from the baseline scenario. Somewhat surprisingly, slaveholder profits per slave actually increase: this is because we estimate coercion parameters less than one in the non-agricultural sector in most locations. Intuitively, coercion is difficult in the non-agricultural sector (as evinced by the low share of enslaved workers in that sector despite the high observed wages), so that the enslaved supply less labor in that sector than they would if they could freely choose.

The third row presents what would occur if there were no coercion and enslaved persons were able to retain all the fruits of their labor. This transfers the slaveholders profits to the enslaved persons (and has a small general equilibrium impact on prices, as slaveholder's profits are assumed to be collected nationally before being redistributed). The gains from receiving the value of all of one's labors (or equivalently, removing the monopsonist mark down) is large, equal to a 671.3% increase in wages in the baseline case, or an additional 160.1% increase over and above removing coercion on its own.

The fourth row presents what would occur if enslaved persons were free but confined to remain in the U.S. South. The overall gains relative to the baseline case are equivalent to a 867.5% in wages, or an additional 12.5% increase over and above removing coercion and monopsony. Not surprisingly, allowing enslaved persons to choose their location and occupation to maximize welfare rather than maximizing profits does come at a cost of reducing aggregate GDP per capita, although this effect is modest (a decline from \$341.21 to \$337.20).

Finally, the fifth row presents what would occur if enslaved persons were free and could choose to reside anywhere in the U.S. In this case, they act identically to the free persons, so the welfare impacts for the two groups are identical. All told, the gains are equivalent

to an 1207% increase in baseline wages, or a further 56.5% increase over removing coercion and monopsony on their own. Because the U.S. North is estimated to be more attractive in both its amenities and productivities than the U.S. South, the welfare gains from being able to move anywhere in the U.S. are substantially larger than the gains from just reallocating within the U.S. South. Moreover, the ability of the formerly enslaved to move to the more productive North enables emancipation to substantially increase U.S. per capita GDP to $364.98, a 16.8% increase over the baseline case. On the other hand, free person welfare declines from $658.45 in the baseline case to $651.95, equivalent to a 0.8% reduction in wages.

All told, these results suggest that the institution of slavery had large impacts on the U.S. spatial economy and enormous welfare implications for the enslaved.

7 Conclusion

We view our model as a first attempt to capture the complexity of the antebellum slave economy in the United States. We have built a rich model of the allocation of free and enslaved workers in a model with agricultural and non-agricultural sectors, each with location specific productivity and amenity parameters, as in quantitative, state-of-the art, static models of spatial equilibrium. The key idea in our model is that enslaved workers do not get to choose neither how much they work, nor what sector they are working in, nor where to live and work, but free workers do. Using the observed allocations of free and enslaved workers, together with new data on wages, we quantify the parameters of the model. We then use the model to simulate emancipation, accounting for the general equilibrium response, and quantitatively recover the welfare gains to emancipation from a variety of emancipation scenarios.

We have omitted a number of key dimensions of both American slavery and the larger American economy. We have neglected a very important gender dimension of the household allocation of labor between home production and market labor, as well as variation in the efficacy of home production in the slave economy (e.g. the task vs gang system of provisioning). We have also ignored the important credit and capital market dimensions of slavery, where enslaved people enabled a form of collateral that enabled complex financial contracts. Finally, we have ignored market and production network linkages between the North and the South, including international trade dimensions, for example where Southern exports facilitated Northern importation of capital equipment and foreign investment. We leave exploration of these potentially important economic forces for future work.

Our results suggest that observed allocation of workers in 1870 reflects more than eman-

cipation itself. Our model predicts that enslaved workers would have left their 1860 locations more than we see in the data. As discussed in the introduction, actual emancipation came alongside an extremely destructive war, a military occupation, and political attempts to restore something close to the slave system, and perhaps these forces can rationalize the persistence of slavery and its effects well into the present. Again we think exploring these empirical facts through the lens of (perhaps dynamic and political) models like the one we have constructed could be a direction for future research.

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8 Appendix

8.1 Identifying Location Characteristics from Parameters and Observed Data

Proposition 2. *Given parameter values* $\{\sigma, \theta, \eta, \phi, \rho\}$*, observed data* $\{\mathbf{L}_{persons}, \pi, P_{enslaved}, \mathbf{Y}_f\}$ *and trade costs* $\{\tau\}$*, there exists a unique (up to scale) set of within-location occupation characteristics* $\{\mathbf{a}, \tilde{\mathbf{u}}, \mathbf{c}\}$ *and across-location characteristics* $\{\mathbf{A}, \mathbf{U}, \mathbf{P}\}$ *that are consistent with model equilibrium.*

Proof. To ease notation, let us first define a few objects that will be useful for the proof. First, let us define $\tilde{w}_{io}$ as the wage that justifies the observed location-occupation output, given the population:

$$\tilde{w}_{io} \equiv \frac{Y_{io,f}}{L_{persons,io,f}}.$$

It will be useful for us to define the labor supply-adjusted wage (from now on known as the "adjusted wage"), w'_{io}, as a function of $\tilde{w}_{io}$:

$$\begin{aligned} w'_{io} &\equiv w_{io}A_{io} \times \left(\frac{u_{io}}{P_i} w_{io}A_{io}\right)^{\phi} \\ &= \left(\Gamma\left(\frac{\theta-1}{\theta}\right)\Gamma\left(\frac{\eta-\phi}{\eta}\right)\right)^{-1} \tilde{w}_{io} \times \left(\pi_{io,f|i}\right)^{\frac{\phi}{\eta}} \times \left(\pi_{i,f}\right)^{\frac{1}{\theta}} \end{aligned} \tag{9}$$

Note that w'_{io} is a function of parameters and known data.

We can split the proof into two parts: within location and across location. First, we summarize the steps for identification within location:

1. Given parameters $\{\phi, \eta\}$ and the adjusted wages $\{w'_{io}\}$, the relative within-location amenities $\{\tilde{\mathbf{u}}\}$ are identified.

2. Given parameters $\{\phi, \eta, \theta\}$, the adjusted wages $\{w'_{io}\}$, and enslaved person occupation and location shares $\left\{\pi_{io,s|i}, \pi_{i,s}\right\}$, the location-occupation coercion parameters $\{\mathbf{c}\}$ are identified.

3. Given parameters $\{\phi, \eta, \rho, \theta\}$, the adjusted wages $\{w'_{io}\}$, within-location amenities $\{\tilde{\mathbf{u}}\}$, and free and enslaved populations and shares $\{\mathbf{L}_{persons}, \pi\}$, the relative within-location productivities $\{\mathbf{a}\}$ are identified.

Relative within-location amenities

First, we can rewrite the adjusted wage equation (9) to recover the compensation per unit of labor:

$$w_{io}A_{io} = (w'_{io})^{\frac{1}{1+\phi}} \left(\frac{P_i}{u_{io}}\right)^{\frac{\phi}{1+\phi}}. \tag{10}$$

Then, we can substitute this expression into the within-location free occupation share expression to solve for the location-occupation amenities:

$$\begin{aligned}
\pi_{io,f|i} &= \frac{\left(\frac{u_{io}}{P_i} w_{io} A_{io}\right)^{\eta}}{\sum_{o'} \left(\frac{u_{io'}}{P_i} w_{io'} A_{io'}\right)^{\eta}} \iff \\
u_{io} &= \left(\frac{P_i}{(w'_{io})^{\frac{1}{1+\phi}} \left(\frac{P_i}{u_{io}}\right)^{\frac{\phi}{1+\phi}}}\right) \times \left(\pi_{io,f|i}\right)^{\frac{1}{\eta}} \times \left(\sum_{o'} \left(\frac{u_{io'}}{P_i} (w'_{io'})^{\frac{1}{1+\phi}} \left(\frac{P_i}{u_{io'}}\right)^{\frac{\phi}{1+\phi}}\right)^{\eta}\right)^{\frac{1}{\eta}} \iff \\
u_{io} &= \left[\left(\frac{P_i^{\frac{1}{1+\phi}}}{(w'_{io})^{\frac{1}{1+\phi}}}\right) \times \left(\pi_{io,f|i}\right)^{\frac{1}{\eta}} \times \left(\sum_{o'} \left(\left(\frac{u_{io'}}{P_i}\right)^{\frac{1}{1+\phi}} (w'_{io'})^{\frac{1}{1+\phi}}\right)^{\eta}\right)^{\frac{1}{\eta}}\right]^{1+\phi}
\end{aligned}$$

The relative amenities in location i are then

$$\frac{\tilde{u}_{io}}{\sum_{o'} \tilde{u}_{io'}} = \frac{(w'_{io})^{-1} \times \left(\pi_{io,f|i}\right)^{\frac{1+\phi}{\eta}}}{\sum_{o'} (w'_{io'})^{-1} \times \left(\pi_{io',f|i}\right)^{\frac{1+\phi}{\eta}}}. \tag{11}$$

Coercion parameters

Similarly to amenities, we can substitute equation (10) into the within-location enslaved occupation share to solve for the coercion parameters:

$$\pi_{io,s|i} = \frac{\left(u_{io}^{\frac{\phi}{1+\phi}} c_{io,s}^{\frac{\phi}{1+\phi}} w_{io} A_{io}\right)^{\eta}}{\sum_{o'}\left(u_{io'}^{\frac{\phi}{1+\phi}} c_{io,s}^{\frac{\phi}{1+\phi}} w_{io'} A_{io'}\right)^{\eta}} \iff$$

$$\left(\pi_{io,s|i}\right)^{\frac{1}{\eta}} = \frac{u_{io}^{\frac{\phi}{1+\phi}} c_{io,s}^{\frac{\phi}{1+\phi}} \left(w'_{io}\right)^{\frac{1}{1+\phi}} \left(\frac{P_i}{u_{io}}\right)^{\frac{\phi}{1+\phi}}}{\left(\sum_{o'}\left(u_{io'}^{\frac{\phi}{1+\phi}} c_{io',s}^{\frac{\phi}{1+\phi}} \left(w'_{io'}\right)^{\frac{1}{1+\phi}} \left(\frac{P_i}{u_{io'}}\right)^{\frac{\phi}{1+\phi}}\right)^{\eta}\right)^{\frac{1}{\eta}}} \iff$$

$$\left(\pi_{io,s|i}\right)^{\frac{1}{\eta}} = \frac{c_{io,s}^{\frac{\phi}{1+\phi}} \left(w'_{io}\right)^{\frac{1}{1+\phi}}}{\left(\sum_{o'}\left(c_{io',s}^{\frac{\phi}{1+\phi}} \left(w'_{io'}\right)^{\frac{1}{1+\phi}}\right)^{\eta}\right)^{\frac{1}{\eta}}},$$

which gives us

$$c_{io,s\cdot} = \left[\left(\pi_{io,s|i}\right)^{\frac{1}{\eta}} \times \left(w'_{io}\right)^{-\frac{1}{1+\phi}} \times \left(\sum_{o'}\left(\left(c_{io',s}\right)^{\frac{\phi}{1+\phi}} \left(w'_{io'}\right)^{\frac{1}{1+\phi}}\right)^{\eta}\right)^{\frac{1}{\eta}}\right]^{\frac{1+\phi}{\phi}}. \tag{12}$$

As an intermediate step, note that we can rewrite the expected slaveowner utility equation as

$$\left(\frac{\left(\pi_{i,s} \times \sum_{i'} \Pi_{i',s}^{\theta}\right)^{\frac{1}{\theta}}}{\Gamma\left(\frac{\eta-1-\phi}{\eta}\right)\left(\frac{1}{1+\phi}\right)}\right)^{\frac{1}{1+\phi}} = \left(\sum_{o'}\left(\left(c_{io',s}\right)^{\frac{\phi}{1+\phi}} \left(w'_{io'}\right)^{\frac{1}{1+\phi}}\right)^{\eta}\right)^{\frac{1}{\eta}}. \tag{13}$$

Note the right hand side is the same expression as in equation (12). Moreover, we can express the term $\sum_{i'} \Pi_{i',s}^{\theta}$ as a function of the average national price of an enslaved worker:

$$\sum_{i'} \Pi_{i',s}^{\theta} = \left[\frac{P_{enslaved}}{\Gamma\left(\frac{\theta-1}{\theta}\right)}\right]^{\theta}. \tag{14}$$

Finally, substituting equations (14) and (13) into equation (12) gives us

$$c_{io,s} = \left[\left(\pi_{io,s|i}\right)^{\frac{1}{\eta}} \times \left(w'_{io}\right)^{-\frac{1}{1+\phi}} \times \left(\frac{\left(\pi_{i,s} \times \sum_{i'} \Pi_{i',s}^{\theta}\right)^{\frac{1}{\theta}}}{\Gamma\left(\frac{\eta-1-\phi}{\eta}\right)\left(\frac{1}{1+\phi}\right)}\right)^{\frac{1}{1+\phi}}\right]^{\frac{1+\phi}{\phi}}, \tag{15}$$

which gives us the coercion parameters in terms of parameters and known data.

Relative within-location productivities

The last within-location step is to identify the relative productivities. First, note that the location-level labor efficiency units are functions of parameters, observed population shares, and aggregate populations:

$$L_{i,f} = (\pi_{i,f})^{\frac{\theta-1}{\theta}} \times \Gamma\left(\frac{\theta-1}{\theta}\right) \times \bar{F},$$
$$L_{i,s} = (\pi_{i,s})^{\frac{\theta-1}{\theta}} \times \Gamma\left(\frac{\theta-1}{\theta}\right) \times \bar{S}.$$

Next, we can substitute adjusted wages into the expression for location-occupation free and enslaved output using equation (10):

$$Y_{io,f} = \Gamma\left(\frac{\eta-\phi}{\eta}\right) w'_{io} \left(\pi_{io,f|i}\right)^{\frac{\eta-\phi}{\eta}} L_{i,f},$$
$$Y_{io,s} = \Gamma\left(\frac{\eta-\phi}{\eta}\right) w'_{io} \left(c_{io,s}\right)^{\phi} \left(\pi_{io,s|i}\right)^{\frac{\eta-\phi}{\eta}} L_{i,s}.$$

We have now expressed location-occupation output in terms of parameters, observed data, and the now-identified coercion parameters. To find the relative productivities, first note that the firms' first order condition gives us an expression for the relative occupation output:

$$\begin{aligned}
p_i \left(\frac{q_{io}}{Q_i}\right)^{-\frac{1}{\rho}} &= w_{io} \iff \\
p_i Q_i^{\frac{1}{\rho}} &= w_{io} q_{io}^{\frac{1}{\rho}} \iff \\
Y_i Q_i^{\frac{1}{\rho}-1} &= Y_{io} q_{io}^{\frac{1}{\rho}-1} \iff \\
Y_i^{\frac{\rho}{1-\rho}} \left(\sum_{o' \in O} q_{io'}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} &= Y_{io}^{\frac{\rho}{1-\rho}} q_{io} \iff \\
\frac{Y_{io}}{Y_i} &= \frac{\left(\frac{1}{w_{io}}\left(Y_{io,s} + Y_{io,f}\right)\right)^{\frac{\rho-1}{\rho}}}{\sum_{o' \in O}\left(\frac{1}{w_{io'}}\left(Y_{io',s} + Y_{io',f}\right)\right)^{\frac{\rho-1}{\rho}}}.
\end{aligned}$$

We can substitute for w_{io} using equation (10), which gives us an expression for productivities:

$$A_{io} = \left(\frac{Y_{io}}{Y_i}\right)^{\frac{\rho}{\rho-1}} \times \frac{\left[\sum_{o' \in O} \left((A_{io'})^{-1} \left(w'_{io'}\right)^{-\frac{1}{1+\phi}} \left(u_{io'}\right)^{\frac{\phi}{1+\phi}} \left(Y_{io',s} + Y_{io',f}\right)\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}}{\left(w'_{io}\right)^{-\frac{1}{1+\phi}} \left(u_{io}\right)^{\frac{\phi}{1+\phi}} \left(Y_{io,s} + Y_{io,f}\right)}. \tag{16}$$

Relative productivities within a location are thus given by

$$\frac{a_{io}}{\sum_{o'} a_{io}} = \frac{\left(Y_{io}\right)^{\frac{1}{\rho-1}} \left(w'_{io}\right)^{\frac{1}{1+\phi}} \left(u_{io}\right)^{-\frac{\phi}{1+\phi}}}{\sum_{o'} \left(Y_{io'}\right)^{\frac{1}{\rho-1}} \left(w'_{io'}\right)^{\frac{1}{1+\phi}} \left(u_{io'}\right)^{-\frac{\phi}{1+\phi}}}, \tag{17}$$

where the right-hand-side consists of parameters, identified characteristics, and observed data.

Across location identification

We now summarize the steps for identification across locations:

1. Given parameters $\{\phi, \eta, \theta\}$, the adjusted wages $\{w'_{io}\}$, identified within-location characteristics $\{\mathbf{a}, \tilde{\mathbf{u}}, \mathbf{c}\}$, and free and enslaved populations and shares $\{\mathbf{L}_{persons}, \pi\}$, the relative across-location amenities relative to price indices $\left\{\frac{\mathbf{U}}{\mathbf{P}}\right\}$ can be expressed in terms of parameters, identified characteristics, and observed data.

2. Given parameters $\{\phi, \eta, \rho, \sigma\}$, the adjusted wages $\{w'_{io}\}$, identified within-location characteristics $\{\mathbf{a}, \tilde{\mathbf{u}}, \mathbf{c}\}$, free and enslaved populations and shares $\{\mathbf{L}_{persons}, \pi\}$, and trade costs τ, the relative across-location productivities $\{\mathbf{A}\}$ can be expressed as a function of other relative location-level productivities $\{\mathbf{A}\}$ and the across-location amenities relative to price indices $\left\{\frac{\mathbf{U}}{\mathbf{P}}\right\}$. Additionally, if there is a solution to the equation, it is unique up to scale, Lyapunov stable, and globally attractive.

3. Given parameters $\{\phi, \eta, \rho, \sigma\}$, the adjusted wages $\{w'_{io}\}$, identified within-location characteristics $\{\mathbf{a}, \tilde{\mathbf{u}}, \mathbf{c}\}$, free and enslaved populations and shares $\{\mathbf{L}_{persons}, \pi\}$, and trade costs τ, the relative across-location price indices $\{\mathbf{P}\}$ can be expressed as a function of relative location-level productivities and the across-location amenities relative to price indices, $\left\{\mathbf{A}, \frac{\mathbf{U}}{\mathbf{P}}\right\}$.

Relative across-location amenities

Using the expression for free person location shares, $\pi_{i,f} = \frac{\Pi^{\theta}_{i,f}}{\sum_{i'} \Pi^{\theta}_{i',f}}$, we can solve for across-location amenities:

$$\pi_{i,f} = \frac{\Pi_{i,f}^{\theta}}{\sum_{i'} \Pi_{i',f}^{\theta}} \iff$$

$$\pi_{i,f} = \frac{\left(\left(\left(\sum_o \left(\frac{u_{io}}{P_i} w_{io} A_{io}\right)^{\eta}\right)^{\frac{1}{\eta}}\right)^{1+\phi}\right)^{\theta}}{\sum_{i'}\left(\left(\left(\sum_o \left(\frac{u_{i'o}}{P_{i'}} w_{i'o} A_{i'o}\right)^{\eta}\right)^{\frac{1}{\eta}}\right)^{1+\phi}\right)^{\theta}} \iff$$

$$\pi_{i,f} = \frac{\left(\frac{U_i}{P_i}\right)^{\theta(1+\phi)} \left(\sum_o \left(\tilde{u}_{io} w_{io} A_{io}\right)^{\eta}\right)^{\frac{\theta(1+\phi)}{\eta}}}{\sum_{i'}\left(\frac{U_{i'}}{P_{i'}}\right)^{\theta(1+\phi)} \left(\sum_o \left(\tilde{u}_{i'o} w_{i'o} A_{i'o}\right)^{\eta}\right)^{\frac{\theta(1+\phi)}{\eta}}} \iff$$

$$U_i = P_i \times \left(\pi_{i,f}\right)^{\frac{1}{\theta(1+\phi)}} \times \left[\sum_o \left(\tilde{u}_{io} w_{io} A_{io}\right)^{\eta}\right]^{-\frac{1}{\eta}} \times \left[\sum_{i'} \left(\frac{U_{i'}}{P_{i'}}\right)^{\theta(1+\theta)} \left(\sum_o \left(\tilde{u}_{i'o} w_{i'o} A_{i'o}\right)^{\eta}\right)^{\frac{\theta(1+\phi)}{\eta}}\right]^{\frac{1}{\theta(1+\phi)}} \iff$$

$$U_i = P_i \times \left(\pi_{i,f}\right)^{\frac{1}{\theta(1+\phi)}} \times \left[\sum_o \left(\tilde{u}_{io} \times \left(w'_{io}\right)^{\frac{1}{1+\phi}} \left(\frac{P_i}{u_{io}}\right)^{\frac{\phi}{1+\phi}}\right)^{\eta}\right]^{-\frac{1}{\eta}} \times \left[\sum_{i'} \left(\frac{U_{i'}}{P_{i'}}\right)^{\theta(1+\phi)} \left(\sum_o \left(\tilde{u}_{i'o} w_{i'o} A_{i'o,f}\right)^{\eta}\right)^{\frac{\theta(1+\phi)}{\eta}}\right]^{\frac{1}{\theta(1+\phi)}} \iff$$

$$U_i^{\frac{1}{1+\phi}} = P_i^{\frac{1}{1+\phi}} \times \left(\pi_{i,f}\right)^{\frac{1}{\theta(1+\phi)}} \times \left[\sum_o \left(\tilde{u}_{io} w'_{io}\right)^{\frac{\eta}{1+\phi}}\right]^{-\frac{1}{\eta}} \times \left[\sum_{i'} \left(\frac{U_{i'}}{P_{i'}}\right)^{\theta(1+\phi)} \left(\sum_o \left(\tilde{u}_{i'o} w_{i'o} A_{i'o,f}\right)^{\eta}\right)^{\frac{\theta(1+\phi)}{\eta}}\right]^{\frac{1}{\theta(1+\phi)}} \iff$$

$$U_i = P_i \times \left(\pi_{i,f}\right)^{\frac{1}{\theta}} \times \left[\sum_o \left(\tilde{u}_{io} w'_{io}\right)^{\frac{\eta}{1+\phi}}\right]^{-\frac{1+\phi}{\eta}} \times \left[\sum_{i'} \left(\left(\frac{U_{i'}}{P_{i'}}\right)^{\theta(1+\theta)} \sum_o \left(\tilde{u}_{i'o} w_{i'o} A_{i'o}\right)^{\eta}\right)^{\frac{\theta(1+\phi)}{\eta}}\right]^{\frac{1}{\theta}}.$$

Then, the relative across-location amenities can be written as

$$\frac{U_i}{\sum_{i'} U_i} = \frac{P_i \times \left(\pi_{i,f}\right)^{\frac{1}{\theta}} \times \left[\sum_o \left(\tilde{u}_{io} w'_{io}\right)^{\frac{\eta}{1+\phi}}\right]^{-\frac{1+\phi}{\eta}}}{\sum_{i'} P_{i'} \times \left(\pi_{i',f}\right)^{\frac{1}{\theta}} \times \left[\sum_o \left(\tilde{u}_{i'o} w'_{i'o}\right)^{\frac{\eta}{1+\phi}}\right]^{-\frac{1+\phi}{\eta}}}. \tag{18}$$

Similarly, the across-location amenities relative to price indices can be written as

$$\frac{\frac{U_i}{P_i}}{\sum_{i'}\left(\frac{U_{i'}}{P_{i'}}\right)} = \frac{\left(\pi_{i,f}\right)^{\frac{1}{\theta}} \times \left[\sum_o \left(\tilde{u}_{io} w'_{io}\right)^{\frac{\eta}{1+\phi}}\right]^{-\frac{1+\phi}{\eta}}}{\sum_{i'} \left(\pi_{i',f}\right)^{\frac{1}{\theta}} \times \left[\sum_o \left(\tilde{u}_{i'o} w'_{i'o}\right)^{\frac{\eta}{1+\phi}}\right]^{-\frac{1+\phi}{\eta}}}. \tag{19}$$

In this case, the right hand side is exclusively a function of parameters, identified characteristics, and observed data. Thus, $\frac{\mathbf{U}}{\mathbf{P}}$ is identified to scale.

Relative across-location productivities

First, we can rewrite the CES location production function as

$$Q_i = \left(\sum_{o \in O} q_{io}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

$$Q_i = \left(\sum_{o \in O} \left(\frac{1}{w_{io}} \left(Y_{io',s} + Y_{io',f} \right) \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \iff$$

$$Q_i = \left(\sum_{o \in O} \left(A_{io} \left(w'_{io} \right)^{-\frac{1}{1+\phi}} \left(\frac{P_i}{u_{io}} \right)^{-\frac{\phi}{1+\phi}} \left(Y_{io,s} + Y_{io,f} \right) \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \iff$$

$$Q_i = A_i \left(\frac{U_i}{P_i} \right)^{\frac{\phi}{1+\phi}} \tilde{Q}_i, \tag{20}$$

where

$$\tilde{Q}_i = \left(\sum_{o \in O} \left(a_{io} \left(w'_{io} \right)^{-\frac{1}{1+\phi}} \left(\tilde{u}_{io} \right)^{\frac{\phi}{1+\phi}} \left(Y_{io,s} + Y_{io,f} \right) \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}.$$

$\tilde{Q}_i$ is a function of parameters, identified within-location characteristics, and observed data. We can substitute equation (20) into the market clearing condition:

$$Y_i = \sum_j \frac{\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} Q_i^{\sigma-1}}{\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} Q_k^{\sigma-1}} E_j \iff$$

$$Y_i = \sum_j \frac{\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i} \right)^{\frac{\phi}{1+\phi}} \right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1}}{\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k} \right)^{\frac{\phi}{1+\phi}} \right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}} E_j. \tag{21}$$

We can write this equation in the style of an excess demand equation (with productivities instead of wages):

$$D_i(\mathbf{A}) = Y_i - \sum_j \frac{\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i} \right)^{\frac{\phi}{1+\phi}} \right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1}}{\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k} \right)^{\frac{\phi}{1+\phi}} \right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}} E_j.$$

Lemma S.7 of Redding, Ahfeldt, Sturm and Wolf (2015) shows that if certain properties of $D_i(\mathbf{A})$ are satisfied, there is a unique (up to scale) vector $\mathbf{A}^*$ such that $D_i(\mathbf{A}^*) = 0$. Said properties are:

1. $D_i(\mathbf{A})$ is continuous.

2. $D_i(\mathbf{A})$ is homogeneous of degree zero.

3. $\sum_i D_i(\mathbf{A}) = 0$ for all $\mathbf{A} \in \mathbb{R}^N_{++}$.

4. $D_i(\mathbf{A})$ exhibits gross substitution: that is, $\frac{\partial D_i(\mathbf{A})}{\partial D_h(\mathbf{A})} > 0$ for all $\mathbf{A}$, for all i, h such that $i \neq h$, and $\frac{\partial D_i(\mathbf{A})}{\partial D_i(\mathbf{A})} < 0$ for all $\mathbf{A}$, for all i.

We now show that these properties apply to $D_i(\mathbf{A})$. Continuity is clear from the fact that $\tau_{ij}, Y_i, A_i, \frac{U_i}{P_i}, Q_i > 0$ for all i and j. For homogeneity of degree zero, note

$$D_i(c\mathbf{A}) = Y_i - \sum_j \frac{\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times cA_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1}}{\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times cA_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}} E_j$$

$$D_i(c\mathbf{A}) = Y_i - \sum_j \frac{\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1}}{\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}} E_j$$

$$D_i(c\mathbf{A}) = D_i(\mathbf{A}).$$

For the third condition, we have

$$\begin{aligned}
\sum_i D_i(\mathbf{A}) &= \sum_i \left(Y_i - \sum_j \frac{\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1}}{\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}} E_j \right) \\
&= \sum_i Y_i - \sum_j \frac{\sum_i \tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1}}{\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}} E_j \\
&= \sum_i Y_i - \sum_j E_j \\
&= 0.
\end{aligned}$$

Finally, for gross substitution, we have

$$\frac{\partial D_i(\mathbf{A})}{\partial A_h} = \sum_j \frac{(\sigma-1)\left[\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1} \times E_j\right] \times \left[\tau_{hj}^{1-\sigma} Y_h^{1-\sigma} \times A_h^{\sigma-2} \times \left(\left(\frac{U_h}{P_h}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_h^{\sigma-1}\right]}{\left[\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}\right]^2}$$

$$> 0,$$

as $\sigma > 1$. Similarly,

$$\begin{aligned}
\frac{\partial D_i(\mathbf{A})}{\partial A_i} &= \sum_j \frac{(\sigma-1)\left[\sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}\right] \times \left[\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-2} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1} \times E_j\right]}{\left[\sum_k \tau_{kh}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}\right]^2} \\
&\quad - \sum_j \frac{(\sigma-1)\left[\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1}\right] \times \left[\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-2} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1} \times E_j\right]}{\left[\sum_k \tau_{kh}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}\right]^2} \\
&= (\sigma-1) \sum_j \frac{\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-2} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1} \times E_j}{\left[\sum_k \tau_{kh}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-1} \times \left(\left(\frac{U_k}{P_k}\right)^{\frac{\phi}{1+\phi}}\right)^{1-\sigma} \times \tilde{Q}_k^{\sigma-1}\right]^2} \left(\tau_{ij}^{1-\sigma} Y_i^{1-\sigma} \times A_i^{\sigma-1} \times \left(\left(\frac{U_i}{P_i}\right)^{\frac{\phi}{1+\phi}}\right)^{\sigma-1} \times \tilde{Q}_i^{\sigma-1} - \sum_k \tau_{kj}^{1-\sigma} Y_k^{1-\sigma} \times A_k^{\sigma-} \right. \\
&< 0.
\end{aligned}$$

Therefore, there exists a unique solution to $D_i(\mathbf{A})$, and $\mathbf{A}$ is identified up to scale.

Relative price indices

Using equation (20), we can rewrite the price index as

$$\begin{aligned}
P_i &= \left(\sum_j \tau_{ji}^{1-\sigma} p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \iff \\
P_i &= \left(\sum_j \tau_{ji}^{1-\sigma} Y_j^{1-\sigma} Q_j^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \iff \\
P_i &= \left(\sum_j \tau_{ji}^{1-\sigma} Y_j^{1-\sigma} \left(A_j \left(\frac{U_j}{P_j}\right)^{\frac{\phi}{1+\phi}} \tilde{Q}_j\right)^{\sigma-1}\right)^{\frac{1}{1-\sigma}}. \qquad (22)
\end{aligned}$$

Prices are thus expressed as a function of parameters, data, and previously identified terms.

Since our solution is up-to-scale, we can choose an arbitrary normalization for two out of three characteristics within and across location respectively. Within location, we normalize **a** and **u**to have mean one for each location. Across location, we normalize $\mathbf{A}$ and $\mathbf{U}$ to have mean one. To justify this, we show that this is equivalent to a normalization of $\mathbf{P}$ and $\frac{\mathbf{U}}{\mathbf{P}}$: □

Lemma 3. *Any scaling of* $\mathbf{A}$ *and* $\boldsymbol{U}$ *that is consistent with model equilibrium can be represented by a model consistent scaling of* $\mathbf{P}$ *and* $\frac{\mathbf{U}}{\mathbf{P}}$.

Proof. Note that equations (19) and (21) are unaffected by the scaling of any variables, so it suffices to consider equation (22). Suppose we have some scale c_A for $\mathbf{A}$ and c_U for $\mathbf{U}$. We can then write

$$P_i = \left(\sum_j \tau_{ji}^{1-\sigma} Y_j^{1-\sigma} \left(c_A A_j \left(\frac{c_U U_j}{P_j} \right)^{\frac{\phi}{1+\phi}} \tilde{Q}_j \right)^{\sigma-1} \right)^{\frac{1}{1-\sigma}} \iff$$

$$\frac{1}{c_A} P_i = \left(\sum_j \tau_{ji}^{1-\sigma} Y_j^{1-\sigma} \left(A_j \left(c_U \frac{U_j}{P_j} \right)^{\frac{\phi}{1+\phi}} \tilde{Q}_j \right)^{\sigma-1} \right)^{\frac{1}{1-\sigma}}.$$

Thus, an equivalent and consistent scaling can be generated by scaling $\mathbf{P}$ by $\frac{1}{c_A}$ and $\frac{\mathbf{U}}{\mathbf{P}}$ by c_U. □