Robots and Inequality

Between and Within Occupations

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Abstract

We examine the impact of industrial robots on wage inequality both between and within occupations. Analyzing two decades of U.S. data, we find that moving from the 25th to the 75th percentile of industry-level robot adoption reduces the *within*-occupation wage inequality by 16% and increases the *between*-occupation inequality by 7% of their respective standard deviations. These results are robust to confounding factors, different inequality measures, and instrumental variables. Our evidence shows that robots compress within-occupation wage inequality by lowering wages at the top of the distribution while increasing between-occupation inequality by impacting the middle of the distribution. We show that these findings are consistent with a standard wage-posting model that incorporates endogenous robot adoption. Overall, our results suggest that robots exacerbate wage polarization while reducing the rents of the highest-paid workers in the occupations most vulnerable to automation.

Keywords: Robots, inequality, labor market frictions, technology adoption, labor search, labor occupations.

JEL Codes: D63, E22, E24, J23, J31, O14, O33.

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1 Introduction

The adoption of industrial robots has been identified as a key driver of rising wage inequality over recent decades. Previous literature has focused on the effects of robots on inequality across occupations.[1] This paper empirically shows that robot adoption increases inequality between occupations while reducing it within occupations. We show that these findings align with a standard search wage-posting model, augmented to allow for robot adoption.

Wage inequality has risen significantly in developed economies, with the spread of industrial robots recognized as a major contributing factor. Due to their superior capacity for automating routine tasks compared to non-routine tasks, researchers have adopted a task-based approach to asses their robots' distributional effects. However, although robots may displace workers in roles suited to automation—such as welders or assemblers—this does not mean they replace all workers in these occupations. This highlights the importance of analyzing the impact of robots on both between-occupation and within-occupation wage inequality, the latter of which has received less attention.

Our paper contributes to the literature by studying how robots affect wage inequality both within and between occupations. This focus is relevant since, as we document, the wage dispersion within occupations accounts for around 75% of total wage inequality from 2000 to 2018. Figure 1 shows that during this period, while robot penetration increased, the share of within-occupation wage inequality in overall inequality declined.

To study these relationships in depth, we use data from the American Community Survey (ACS), which provides information on employment and wages and other worker characteristics across 23 occupation groups in 21 industries from 2000 to 2018. To measure inequality, we compute the *Gini* coefficient for both between- and within-occupation wage inequality for the nearly two decades in our sample. We combine this data with robot adoption data

[1] See Autor (2015), who argues that automation disproportionately affects routine-based jobs leading to wage polarization, and Acemoglu and Restrepo (2022) who document that 50% to 70% of changes in the U.S. wage structure between 1980 and 2016 are accounted for by relative wage declines of worker groups specialized in routine tasks in industries experiencing rapid robot adoption.

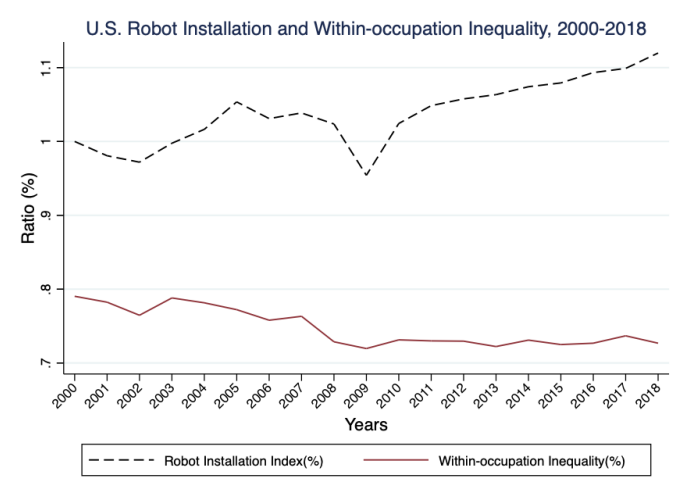


Figure 1: Robots and Within-Occupation Inequality

Note: The red line is the within-occupation inequality as a share of the total wage inequality, measured by the additive decomposable Theil-T index. The blue line is the value of robot installations in year t divided by the value in the year 2000.

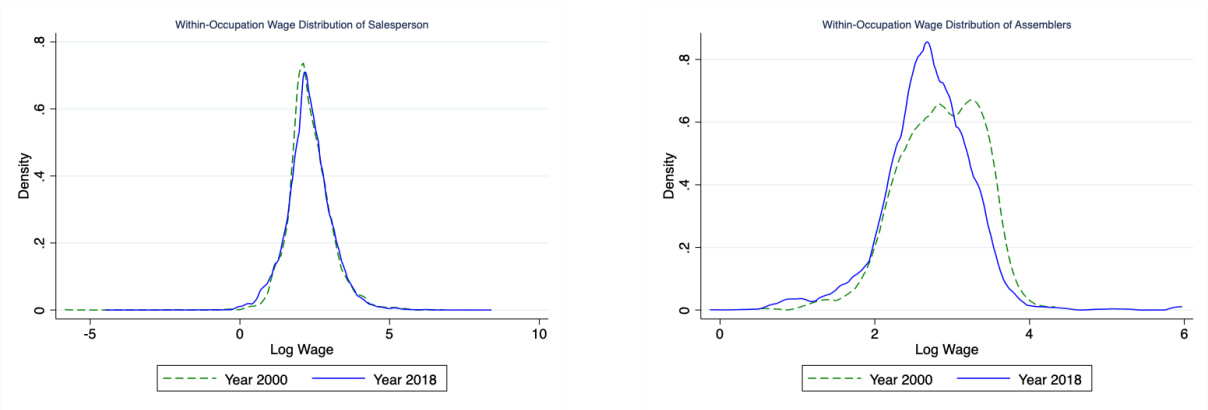
from the International Federation of Robotics (IFR).

We find that industry-level robot adoption is associated with reduced wage inequality within occupations and increased wage inequality between occupations.[2] These results are robust to controlling for contemporaneous shocks like import competition from China, or IT adoption, as well as other possible confounding factors. We also validate these outcomes using five alternative inequality measures, and the results stand firmly. Moreover, our results are robust to estimation with instrumental variables.

To illustrate the opposing directions of within- and between-occupation inequality, consider two occupation groups with different risks of automation: (i) salespersons *SOC-code 412* a non-routine occupation unlikely to be automated by industrial robots at the current state of development; and (ii) automotive assemblers *SOC-code 512* a routine occupation intensive in one of the tasks most performed by robots[3]. Figure 2a and 2b show the wage distribution for salespersons and assemblers in 2000 and 2018. The wage distribution for salespersons remains stable or with minor changes in the within-occupation wage inequality over time. In contrast, assemblers experienced two notable changes: a leftward shift in 2018,

[2]This last result is in line with the findings of Acemoglu and Restrepo (2019).
[3]Handling, welding, and painting are the other tasks most performed by robots according to the IFR.

indicating a decrease in average wages, and a more leptokurtic distribution concentrated around its new mean, suggesting reduced within-wage inequality for assembler workers.



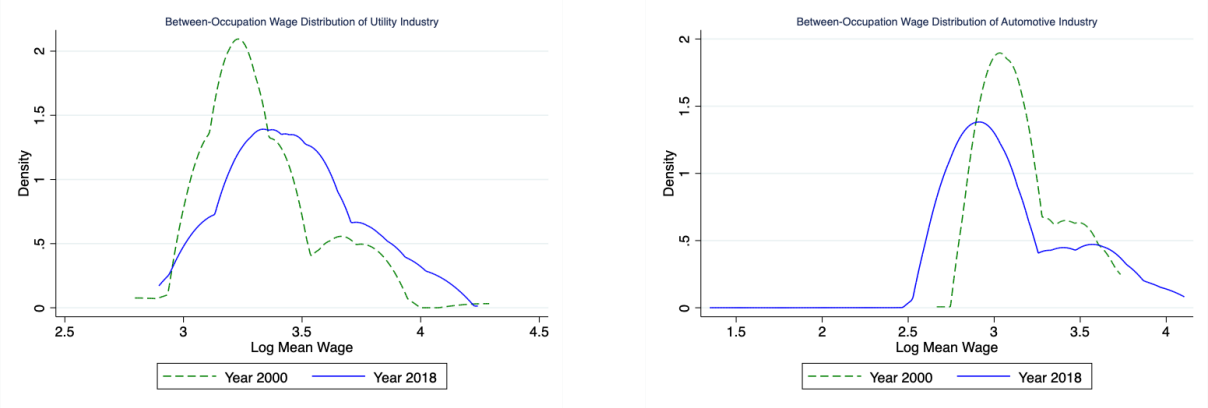
(a) Salesperson (b) Assembler

Figure 2: Within-Occupation Wage Distribution

To depict the effects of robots on between-occupation inequality, consider two industries: (i) the automotive industry *ISIC code 29*, which has the highest robot penetration; and (ii) the utility industry *ISIC code E* with one of the lowest levels of robot adoption. Figure 3b exhibits a notable increase in wage dispersion across occupations in the automotive industry from 2000 to 2018, driven largely by a widening lower tail. Figure 3a shows increased wage dispersion across occupations in the utility sector over the same period, due to an upward shift in the upper tail. This raises an important question: What role do robots play in shaping both within- and between-occupations inequality across sectors? This paper aims to provide an empirical answer to this question.

To further understand our results, we develop a model of occupational wage inequality, where occupations are defined as segmented labor markets, requiring specific expertise. Extending Burdett and Mortensen (1998) wage-posting model, we allow firms heterogeneous in their productivity to decide whether occupations will be performed by workers or robots. In equilibrium, the most productive firms offer higher wages to otherwise identical workers and are more willing to automate.

Analytically, we show that within-occupation inequality, measured by the *Gini* coefficient,



(a) Utility (b) Automotive

Figure 3: Between-Occupations Wage Distribution

decreases with robot adoption, given any non-degenerate productivity distribution of hiring firms. For between-occupation inequality, we provide conditions under which robotization increases inequality across occupations.

Literature. Our study contributes to the literature analyzing the relationship between technological change and rising inequality. Seminal work by Acemoglu (2002) and Autor et al. (2003) provides a foundational framework for understanding how technological change, can drive wage disparities. Goos and Manning (2007) and Autor and Dorn (2013) focus on job polarization in Europe and the US respectively, arguing that technological change has led to the growth of high-skill, high-wage jobs and low-skill, low-wage jobs while hollowing out middle-skill occupations. These studies focus on the between-occupation component of inequality by comparing workers in different occupations.

Our main contribution is to decompose and analyze the effects of industrial robots on both within and between-occupation wage inequality–with the former being unexplored by the literature. Van der Velde (2020) relates the task content of jobs to inequality within and between occupations in Europe. We go further and build a model that predicts when these two components of inequality might diverge due to robotization and empirically test these predictions.

This paper also contributes to the literature on the impacts of robots on labor markets.

Specifically, it builds on seminal work by Graetz and Michaels (2018) and Acemoglu and Restrepo (2020), which examine the adverse effects of robots on employment and wages, particularly in routine-manual occupations. Our work also aligns with studies on the distributional impacts of robots across occupations, such as Dauth et al. (2021), Humlum (2022), Rodrigo (2022), and Mukoyama et al. (2023). Unlike these studies, we abstract from labor reallocation across occupations, which is beyond the scope of this paper.

A closely related study is Acemoglu and Restrepo (2024), which similarly to ours, argues that automation targets tasks with higher rents, thereby reducing inequality within worker groups (defined by age, education, etc.) while increasing it across groups. Our paper is complementary to their work as we examine the consequences specifically of robot technology on occupational groups rather than worker groups defined by demographics. That said, we do study whether the impact of automation on inequality varies by age, gender, and other worker characteristics. Since they pool occupations, there is a parallelism between their analysis within groups and our between-occupation analysis.[4]

Other papers that study the impact of robots or automation in the context of labor search models include Arnoud (2018) Cords and Prettner (2022) and Leduc and Liu (2024). These papers either focus on the impact of robots on different skill levels or worker bargaining power and rely on the Mortensen and Pissarides (1994) random matching framework. In contrast, we employ the wage-posting framework of Burdett and Mortensen (1998), as it has a stable notion of employer productivity, a dimension of heterogeneity that the literature shows is key to the decision to adopt robots.

The rest of the paper is organized as follows. Section 2 outlines our empirical strategy and Section 3 describes our data. Section 4 reports our empirical findings, and Section 5 shows that these findings are consistent with a standard search model extended to allow for robot adoption. Section 6 concludes with suggestions for future work.

[4]On the theory side, the heterogeneity in our model is due to search frictions rather than wedges.

2 Empirical strategy

To analyze how robot adoption is related to within- and between-occupation inequality, we define the following empirical specification:

$$Inequality_{it} = \beta \times Robot_Adoption_{it-1} + X_{it} + \alpha_t + \delta_i + \epsilon_{it} \tag{1}$$

where the dependent variable $Inequality_{it}$ is the measure of inequality within-occupation $Gini^w_{it}$ or between-occupations $Gini^b_{it}$ both at the industry level for comparability as detailed in the next section. Our variable of interest is $Robot_Adoption_{it}$ measured as follows,

$$Robot_Adoption_{it} = \frac{Robots^{US}_{it}}{L^{US}_{i,t}} \tag{2}$$

that is the stock of robots at year t per worker in the same year for each of the 21 industries. We include the variable in our regressions lagged one period to account for the time gap between shipped, installed, and used.[5] X_{it} are demographic controls at the industry level that include the share of females, the share of high-skilled workers (Bachelor's degree and above), the share of young workers (age 16-24), the share of workers that moved across states, the share of self-employed and unemployed workers. α_t are year fixed effects, δ_i industry fixed effects, and ϵ_{it} is the error term.

2.1 Inequality measures

Our main inequality measure is the Gini Index, a widely known measure of inequality, commonly used to assess the distribution of wages or income in a population. This metric captures the area between the equality line and the Lorenz curve, which is a graphical representation of the cumulative distribution of income or wealth. The Gini ranges from 0 to 1, with 0 for perfect equality and 1 for maximal inequality. The Gini coefficient for

[5] In Appendix C.1 we show that our results are robust to using robot adoption contemporaneously or changing the unit of analysis to the occupation level for the within-occupation component.

workers in occupation o, at industry i, $Gini_{io}$ is computed as follows,

$$Gini_{io} = \frac{\sum_{j=1}^{n_{io}} \sum_{k=1}^{n_{io}} |w_{ioj} - w_{iok}|}{2n_{io}^2 \bar{w}_{io}} \tag{3}$$

where w_{ioj} and w_{iok} are the wages of individuals j and k within that specific industry-occupation group, $\bar{w}_{io}$ is the mean wage of that industry-occupation group, and n_{io} is the total number of individuals in that industry-occupation group.

The *Within-occupation* inequality measure captures the disparity in wages among individuals in the same occupation, while *between-occupation* inequality measure captures the disparity in average wage across different occupations. We compute these measures following Radaelli (2010), at the industry level to get comparable estimates for both components. Nevertheless, in Appendix C.1 we show that the effects are similar if within-occupation inequality is computed at the occupation level with a shift-share approach.

We compute the *within-occupation* inequality at the industry level $Gini_i^w$ by calculating the weighted average of the Gini indices $Gini_{io}$ for each occupation o within industry i. The weights assigned to each occupation o are its share of the industry wage income $IncShare_{io}$, multiplied by its share of the total industry workforce $PopShare_{io}$ as follows,

$$Gini_i^w = \frac{\sum_o IncShare_{io} * PopShare_{io} * Gini_{io}}{\sum_o IncShare_{io} * PopShare_{io}} \tag{4}$$

Between-occupation inequality within industries is determined by calculating the Gini coefficient for the average wage of each occupation in that industry $\bar{w}_o^i$. This effectively eliminates within-occupation wage variation within industries, allowing the Gini coefficient to solely reflect inequality stemming from differences in average wages across occupations. The *between-occupation* Gini index for industry i is,

$$Gini_i^b = \frac{\sum_{o=1}^{n} \sum_{o'=1}^{n} |\bar{w}_{io} - \bar{w}_{io'}|}{2n^2 \bar{w}} \tag{5}$$

where $\bar{w}_{io}$ and $\bar{w}_{io'}$ are the mean wages of occupation groups o and o' in industry i; n is

the number of occupation groups; and $\bar{w}$ is the overall mean wage for the industry.

2.2 Identification strategy

Equation 1 can be estimated using ordinary least squares (OLS). However some factors could cause robot adoption to be correlated with the error term ϵ_{it}, leading to biased estimates. Specifically, some industries may be adopting robots in response to contemporaneous shocks impacting their demand for different occupations, or a labor supply shock could be affecting firms' decisions like the adoption of robots.

To address these concerns, we include in our regressions two major contemporaneous shocks the Exposure to IT Capital and the Exposure to Chinese Imports, and we use two instrumental variables to identify the component of robot adoption driven by exogenous changes in technology that could replace certain occupations more prevalent in a given industry. First, following Graetz and Michaels (2018) we use the interaction between employment replaceability in U.S. industries and the average robot prices lagged 5 years:

$$Replaceable_{it} = Robot_Price_{t-5} \times Replaceability_i \tag{6}$$

This instrument captures two crucial factors influencing U.S. producers' robot adoption: the differences in replaceability across industries based on their occupations in 1970, before widespread robot use, and the inverse relationship between robot prices and subsequent demand. The exclusion restriction assumption relies on U.S. replaceability and robot prices[6] influencing robot adoption in the U.S. without direct effects on our inequality measures.

Our second instrument follows Acemoglu and Restrepo (2020), and is the robot intensity measure, using the adoption in five European countries with an advanced level of automation –Denmark, Finland, France, Italy, and Sweden– as follows,

[6]Data Source: https://www.ark-invest.com/articles/analyst-research/industrial-robot-cost-declines

$$Robot_Adoption_{it}^{EU} = \frac{1}{5} \sum_{k \in EU5} \frac{Robots_{kit}^{EU}}{L_{kit}^{EU}} \tag{7}$$

The exclusion restriction relies on the fact that European countries started the automation process earlier than the U.S. keeping a leading role in density and robotics advancements. Therefore, European adoption would influence U.S. robot adoption by pushing the technological frontier without directly impacting the U.S. labor markets.

While we acknowledge that neither of these instruments solves all potential problems of omitted variables and reverse causality, they deliver rigorous checks to the robustness of our empirical results.

3 Data

3.1 International Federation of Robotics Data

The IFR[7] data presents statistics on the installations or shipments of industrial robots by country and industry as defined by ISO 8373:2012: an automatically controlled, reprogrammable, multipurpose manipulator, programmable in three or more axes, which can be fixed in place or mobile for use in industrial automation applications. IFR data has been widely used for research on the effects of automation and robotics (e.g. Acemoglu and Restrepo (2019), Graetz and Michaels (2018), and Kugler et al. (2020)).

IFR uses the words "shipments" and "installations" of industrial robots synonymously. This is a compromise between the actual installation and use of the robot at the customer's site and the shipment, we lag our robot measure to account for this gap. The operational stock of industrial robots, representing the number in use, is the sum of installations over 12 years, assuming a 12-year service life with immediate withdrawal afterward.

[7]IFR data is primarily derived from a robot suppliers survey covering 90% of the global robot market. North American data includes reports from the Robotic Industries Association members, and from 2004 also from global suppliers and the Japan Robot Association enhancing this dataset.

The robot data are categorized into industries based on the International Standard Industrial Classification (ISIC) revision 4. Since in the early years, many robots are unclassified, we distribute unclassified robots to industries in the same proportions as the classified data. We use the US and European operational robot stock and group it into 21 industries.[8]

3.2 American Comunity Survey Data

Individual occupational employment information is sourced from IPUMS USA, which includes demographic (age, gender, race) and economic data (occupation, industry, income, work status, education, migration, location). We use high-precision samples from the ACS (2000-2019). Following standard ACS practices, hourly wage is computed as wage income divided by the product of weeks worked and hours worked per week; where reported weeks worked, originally categorical for 14-26 weeks or 50-52 weeks, is made numerical by assigning the midpoint of each interval. Observations of individuals not in the labor force are excluded.

ACS data reports worker occupation using the 2018 Standard Occupational Classification (SOC) system, and industry using the North American Industry Classification System (NAICS). To analyze the wage distribution within occupations, we define occupational groups using 2-digit codes. We use the NAICS-ISIC concordance table from the US Census, to match it with the IFR 21 industries. With these two elements, we compute our *RobotAdoption* measure according to Equation 2. Similarly, we combine the European employment from the Eurostat metadata[9] with the IFR data to compute Equation 7.

We complement this data using industry-level data on Information Technology (IT) capital sourced from the U.S. Bureau of Labor Statistics (BLS) productivity tables that detail capital inputs across major sectors and industries; and imports from China, from the U.S. Bureau of Economic Analysis (BEA). [10]

[8] Agricultural, Mining, Construction, Utility, Education, Food & Beverages, Paper, Textiles, Wood & Furniture, Pharmaceuticals, Rubber & Plastics, Other Chemicals, Basic Metals, Minerals, Metal Products, Electronics, Industrial Machinery, Automotive, Other Vehicles, Other Manufacturing, Other Non-Manufacturing.

[9] Data source: https://ec.europa.eu/eurostat/web/main/home

[10] Import Matrices After Redefinitions (97-22) dataset provides information on imports by industry.

3.3 Descriptive Statistics

Table 1: Summary Stats by Quartiles of The Change in Robot Adoption

	(1) All	(2) Q1	(3) Q2	(4) Q3	(5) Q4
	Panel A: Outcomes				
Within-occupation Gini	0.363	0.427	0.352	0.332	0.326
	(0.075)	(0.092)	(0.035)	(0.045)	(0.054)
Between-occupation Gini	0.174	0.155	0.179	0.181	0.185
	(0.050)	(0.054)	(0.042)	(0.025)	(0.062)
	Panel B: Shocks				
Robot Adoption US	0.007	0.00002	0.0002	0.004	0.027
	(0.018)	(0.000)	(0.000)	(0.004)	(0.028)
Robot Adoption EU	0.0111	0.0005	0.004	0.008	0.033
	(0.021)	(0.001)	(0.003)	(0.006)	(0.033)
Replaceability	20.812	4.183	28.411	22.598	31.382
	(15.671)	(3.327)	(11.278)	(13.764)	(14.148)
IT Capital Adoption	0.012	0.017	0.009	0.006	0.014
	(0.015)	(0.021)	(0.012)	(0.004)	(0.011)
Chinese Imports	0.014	0.0004	0.027	0.011	0.020
	(0.025)	(0.001)	(0.038)	(0.009)	(0.028)
	Panel C: Covariates				
Share of Female	0.289	0.313	0.322	0.271	0.246
	(0.142)	(0.213)	(0.122)	(0.081)	(0.073)
Share of High-skilled	0.245	0.279	0.206	0.236	0.250
	(0.133)	(0.179)	(0.115)	(0.089)	(0.110)
Share of Young	0.089	0.113	0.079	0.093	0.068
	(0.034)	(0.042)	(0.021)	(0.027)	(0.018)
Share of Movers	0.024	0.027	0.021	0.024	0.023
	(0.006)	(0.007)	(0.005)	(0.005)	(0.006)
Share of Self-employed	0.029	0.055	0.026	0.019	0.009
	(0.034)	(0.053)	(0.011)	(0.009)	(0.003)
Share of Unemployed	0.062	0.059	0.067	0.064	0.060
	(0.028)	(0.031)	(0.030)	(0.024)	(0.028)
N	378	108	90	90	90

Note: The table reports the mean values and standard deviations in parentheses for outcome variables and covariates used in our estimations. Columns 2 - 5 are the moments by quartiles of the U.S. Robot Adoption measure.

Table 1 provides basic descriptive statistics for the variables used in our analysis of the overall sample (Column 1) and by quartiles of robot adoption. Panel A, presents our main outcome variables, suggesting already two patterns: *(i) within-occupation* Gini is lower for industries in higher quartiles, *(ii) between-occupation* Gini is higher for the fourth quartile of

the robot adoption distribution. Although these patterns are not entirely monotone across quartiles, they hint at the possibility of a dichotomous result.

Panel B shows the technology and trade shocks across quartiles. It highlights the positive correlation between US robot adoption and our instrumental variables: European robot adoption and replaceability. This Panel also underlines a correlation with the import competition from China, emphasizing the importance of accounting for it in our estimations.

Panel C shows other labor market characteristics by different exposure to robots without a clear pattern for the share of female workers, the share of high-skill workers, and the share of workers moving across states. On the other hand, the share of young workers (ages 16-24) and the share of self-employed are decreasing for quartiles with higher robot adoption.

4 Empirical Results

This section presents the empirical results of the impacts of robot adoption on *within* and *between* occupation inequality, their robustness to confounding factors and alternative inequality measures, and explores the heterogeneous effects of robots for individuals with different income, skills, gender, age, and race. Finally, we present our IV estimates and discuss their quantitative implications.

4.1 *Effects of Robots on Inequality*

Table 2 reports the ordinary least squares results of estimating the effects of robots on the *within-occupation* (Panel A) and *between-occupation* (Panel B) Gini measures, computed at the industry-year level from 2001 to 2018, according to equation 1. Column 1 shows the unweighted results of our most basic specification, controlling for education, age, gender, self-employment status, worker mobility, as well as year and industry fixed effects. The rest of the regressions are weighted by the total labor force in each industry. Column 2 presents the results of weighting the specification from Column 1. Columns 3 and 4 control for two

major labor market shocks contemporaneous to automation: Chinese import competition and the adoption of information technologies (IT).[11] All regressions include standard errors robust to heteroskedasticity and account for correlations across industries.

Table 2: Within-occupation Inequality and Robots, Baseline

	A. Within-occupation			
	(1)	(2)	(3)	(4)
Robot Adoption	-1.656^{**}	-1.624^{*}	-1.554^{***}	-1.615^{**}
	(0.310)	(0.409)	(0.528)	(0.253)
Adj.Rsquare	0.773	0.891	0.892	0.892
	B. Between-occupation			
	(1)	(2)	(3)	(4)
Robot Adoption	0.706^{**}	0.622^{*}	0.741^{**}	0.773^{***}
	(0.077)	(0.213)	(0.285)	(0.059)
Adj.Rsquare	0.439	0.886	0.889	0.889
Demographic covariates	✓	✓	✓	✓
Year dummies	✓	✓	✓	✓
Industry dummies	✓	✓	✓	✓
Weighted		✓	✓	✓
IT capital			✓	✓
Chinese Imports				✓
Num of Obs	378	378	378	378

All regressions include demographic controls at the industry-year level, as well as year and industry-fixed effects. Column (1) presents unweighted regressions. Column (2)-(4) regressions are weighted by total industry employment. Column (3) adds controls for IT capital and Column (4) additionally includes Chinese imports, also at the industry level. Robust standard errors clustered at the industry level are reported in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Panel A reports negative and significant coefficients in all specifications relating robots and within-occupation wage inequality. Conversely, Panel B reports positive and significant coefficients associating robots and between-occupation inequality. In both cases, the estimated coefficients are statistically the same across specifications. Although the magnitudes slightly decrease (2.5-6% with respect to the unweighted) for the relation of robots and within-occupation inequality, while slightly increase (4.9-9.5%) for those relating robots and between-occupation inequality when weighting and controlling for contemporaneous shocks.

[11] IT adoption is measured as industry-level IT capital stock per worker, which considers software, databases, and other digital infrastructure, usually a required complementary investment to machine robots.

Column 4, weighting for industry labor force and factoring the effects of Chinese import competition and IT adoption both at the industry level, shows an estimated significant coefficient of -1.615 for the relation between robot adoption and within-occupation inequality, and a positive coefficient of 0.773 for the relation with between-occupation inequality. We take this specification as our baseline for the rest of the paper. These findings suggest that robots compress wages in more exposed occupations while exacerbating disparities between different occupation groups.

Table 6 in the Appendix C.1 shows that the estimated coefficients are fairly similar for both within- and between-occupation inequality if instead of lagging the robots variable it is concluded contemporaneously, although with smaller magnitudes. Moreover, Table 7 also in the Appendix C.1 shows that the negative and significant relation between robots and within-occupation inequality is also prevalent if the Gini Index is computed at the occupation level and robot adoption is measured as a shift-share variable instead.[12]

4.2 *Robustness*

Table 3 details the effects of robots on within- and between-occupation inequality, focusing on wage workers, accounting for other contemporaneous shocks that could bias our estimates.

Column 1 reports our baseline regression, that is Column 4 of Table 2, and that will be our benchmark. Aware of the measurement error induced by the unclassified robots in IFR data before 2004, we ran our baseline regression only for years after that year. Column 2 shows that the estimated coefficient for within-occupation inequality remains negative and significant but increases in magnitude around 20%, while the positive and significant coefficient for between-occupation inequality decreases around 30%.

Column 3, focusing on manufacturing industries –primarily affected by the robot shock–, shows 25% larger negative coefficients for within-occupation inequality (-2.022), indicating that in these industries the equalizing effect is more pronounced. The positive effects on

[12]Notice that a shift-share approach only works for the within inequality, breaking the comparability of our estimates, for that reason we believe that equation 1 is the best strategy in our case.

between-occupation inequality increase in magnitude by almost 20%.

Table 3: Effects of Robots on Inequality, Robustness

	A. Within-occupation					
	(1)	(2)	(3)	(4)	(5)	(6)
Robot Adoption	-1.615^{**}	-1.928^{***}	-2.022^{***}	-1.612^{**}	-1.652^{**}	-0.092
	(0.253)	(0.147)	(0.426)	(0.220)	(0.248)	(0.544)
Adj.Rsquare	0.892	0.918	0.394	0.749	0.889	0.882
	B. Between-occupation					
	(1)	(2)	(3)	(4)	(5)	(6)
Robot Adoption	0.773^{***}	0.512^{***}	0.921^{***}	0.624^{**}	0.790^{***}	0.414
	(0.059)	(0.037)	(0.221)	(0.073)	(0.068)	(0.448)
Adj.Rsquare	0.889	0.897	0.253	0.880	0.889	0.885
Baseline covariates	✓	✓	✓	✓	✓	✓
Year dummies	✓	✓	✓	✓	✓	✓
Industry dummies	✓	✓	✓	✓	✓	✓
After 2004		✓				
Manufaturing			✓			
Wage workers only				✓		
Drop most robotized					✓	
Future robot						✓
Num of Obs	378	315	270	378	360	378

All regressions control for demographics, IT adoption, and Chinese import competition, as well as year and industry fixed effects while weighting by industry-level employment. Column (1) presents the baseline regression, in Table 2 Column(4). Column (2) is restricted to years after 2004. Column (3) includes only manufacturing industries. Column (4) excludes self-employed workers. Column (5) excludes the most robotized industry. Column(6) shows a falsification test regressing on future robots. Robust standard errors clustered at the industry level are in parenthesis. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Column 4 excluding self-employed workers reports comparable coefficients for within-occupation inequality (-1.612) and the between component (0.624) to the baseline. Similarly, Column 5, confirms that these estimates are not driven by the industry adopting robots at a faster speed, reporting coefficients almost identical in magnitude and significance to the baseline for both outcome variables. This result underlies that robots are a pervasive technology since the effects remain after excluding the most robotized industries.

Finally, to discard the possibility that our estimates are capturing industry trends, we run a falsification test using future robot adoption data against current outcomes. Column 6 displays non-significant effects for the two components of the occupation inequality, in both cases, decreasing the magnitude a lot with respect to the baseline. This result indicates that anticipation effects are unlikely to explain the observed effects on wage inequality.

Additionally, to verify that our results are not driven by an *ad-hoc* selection of the inequality measure, we test five alternative measures commonly used in the inequality literature, capturing different aspects of the wage distribution. The results shown in Table 8 on Appendix C.2 are consistent with our baseline results.

4.3 *Effects of Robots by Occupation Type*

This section analyzes the differential impact of robot adoption on wage inequality across various occupation groups, categorized by their vulnerability to robot adoption such as *routineness, replaceability, offshorability,* and *wage levels*. Using occupation classifications from Graetz and Michaels (2018) and Autor and Dorn (2013), we examine how robot adoption influences within- and between-occupations wage disparities for these groups.

Routine vs. Non-routine Jobs

Panel A of Table 4 compares the relation of robot adoption on wage inequality for routine, Column 1, and non-routine jobs, Column 2. Using the task content of occupations (Abstract, Routine, Manual, and Offshoreability) from Autor and Dorn (2013), we compute routine at the industry-occupation level by averaging the Routine score across occupations in the 1970 Census with individual weights. Occupations in the top 35th percentile of routine are classified as routine jobs, while those below are non-routine.

Regarding within-occupation inequality, the significant negative coefficient of -2.128 for routine occupations contrasts with the non-significant coefficient of -0.099 for non-routine jobs, reflecting the difficulty of robots performing complex, non-routine tasks. For between-occupation inequality, both routine and non-routine occupations show a non-significant relationship with a coefficient of 0.158 and 0.162 respectively suggesting that the robot shock affects similarly the occupations within each of these two groups.

Replaceable vs. Non-replaceable Jobs

Panel B analyzes the differential impact of robots on replaceable versus non-replaceable occupations. A replaceable occupation is defined using the Graetz and Michaels (2018)

Table 4: Effects of Robot on Inequality, by Occupation Type

	Panel A: Routine		**Panel B: Replaceable**		**Panel C: Offshorable**		**Panel D: High-paid**	
	Within-occupation							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Robot Adoption	-2.128^{***}	-0.099	-1.766^{**}	-0.296	-0.379^{*}	-1.516^{**}	-1.911^{***}	-0.335
	(0.138)	(0.235)	(0.231)	(0.178)	(0.106)	(0.235)	(0.157)	(0.286)
Adj.Rsquare	0.504	0.747	0.670	0.714	0.811	0.706	0.788	0.600
	Between-occupation							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Robot Adoption	0.158	0.162	-0.003	0.042^{**}	0.315^{***}	0.843^{***}	-0.888^{**}	0.351^{*}
	(0.183)	(0.060)	(0.066)	(0.006)	(0.021)	(0.049)	(0.110)	(0.088)
Adj.Rsquare	0.856	0.928	0.562	0.827	0.712	0.948	0.960	0.720
Num of Obs	378	378	378	378	378	378	378	378

All regressions control for demographics, IT adoption, and Chinese import competition, and include year and industry FE, while weighting by industry-level employment. All columns show results for different groups based on occupation characteristics: Columns (1) and (2) show results for routine and non-routine occupations respectively; Columns (3) and (4) for replaceable and non-replaceable occupations respectively; Column (5) and (6) for offshorable and non-offshorable occupations. Column(7) and (8) present results for low-paid and high-paid occupations. Robust standard errors clustered at the industry level are in parenthesis. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

rubric that identifies occupations more likely to be replaced by robots depending on their task composition. We calculate replaceability at the industry-occupation level, those with positive values are defined as replaceable, otherwise are non-replaceable.

Columns 3 and 4 display the relationship between robots and inequality for replaceable and non-replaceable occupations respectively. For within-occupation inequality, replaceable occupations show a significant negative coefficient (-1.766), six times larger than the non-significant coefficient for non-replaceable occupations (-0.296) in Column 4. This aligns with results from routine occupations. For between-occupation inequality, replaceable occupations exhibit a non-significant negative coefficient, while the non-replaceable occupation coefficient is significant and positive. Although these results might seem counterintuitive, they are logical. They tell us that replaceable occupations are directly impacted by robots within occupations, while non-replaceable occupations are affected by robots between occupations.

Offshoreable vs. Non-Offshoreable Jobs

Panel C examines the relationship between robot adoption and offshoreable occupations versus non-offshoreable ones. We calculate offshorability using occupational task variables from Autor and Dorn (2013). This measure evaluates the potential for tasks within an occupation to be performed remotely or moved overseas, which tend to be repetitive, and that do not require physical interaction. Therefore this measure is positively correlated with both measures of routine and to a lower extent with replaceability.[13] We compute the offshorability index at the industry level by averaging it across occupations in the 1990 Census, applying total employment weights. Occupations with a positive index value are classified as offshoreable, while those with a non-positive one are deemed non-offshoreable.

Columns 5 and 6 present the estimated relationship of robots and inequality for offshoreable and non-offshoreable occupations. For within-occupations inequality, the estimated coefficients are -0.379 and -1.516 for offshoreable and non-offshoreable occupations respectively. Interestingly, both are negative and significant, and larger for non-offshorable occupations.

[13]In our data, the correlation between routiness and replaceability is 0.63, while the offshorability index at the industry level is correlated with routiness by 0.38, and with replaceability by 0.013.

For between-occupations inequality, both coefficients are positive and significant, also larger for non-offshorable occupations. Suggesting that offshorabiility could attenuate the effects of robots, with non-offshorable jobs facing directly the automation threat.

High-paid vs. Low-paid Jobs

Panel D analyzes whether robot adoption is associated with a widening or compression of the wage distribution within high-paid occupations or among workers in low-paid occupations. Using average wages by industry-occupation groups in 1970, we classified occupations within the top 35th percentile of the wage distribution as high-paid and low-paid otherwise.

Columns 7 and 8 report these relationships for low and high-paid occupations respectively. Robots are associated with a reduction of within-occupation inequality for both groups. Nevertheless, for low-paid occupations, the coefficient is -1.911 and significant, six times larger than for high-paid occupations with a coefficient of -0.335 non-significant. For between-occupation inequality the coefficient for low-paid jobs is significantly negative, with a coefficient of -0.888, suggesting that automation makes low-paid occupations more homogeneous among themselves in terms of wages. However, the coefficient for high-paid occupations is positive and significant (0.351) indicating an increase in inequality between high-paid jobs. This is consistent with Dixon et al. (2021), who documents that robots affect high-paid occupations reducing the number of managers while increasing the span of control of supervisors. We show that workers in those occupations end up with more unequal wages.

In sum, the empirical results reveal that robots reduce within-occupation inequality more for occupations that are more susceptible to being substituted by robot technology given their task content, such as for routine, replaceable, and offshoreable occupations, as well as for low-paid occupations. At the same time, between-occupation inequality seems unchanged for most of those occupation groups. Only offshorable and non-offshoreable, non-replaceable, and high-paid occupations seem to experience an increase in between-occupation inequality.

4.4 *Effects by Worker Type*

Figure 4 presents the regression results of the effects of robot adoption on within-occupation and between-occupation inequality for different worker groups based on gender, education, age, and race. The dark blue bars represent within-occupation inequality, while the light blue bars represent between-occupation inequality. The confidence intervals allow checking the statistical significance and precision of the estimates.

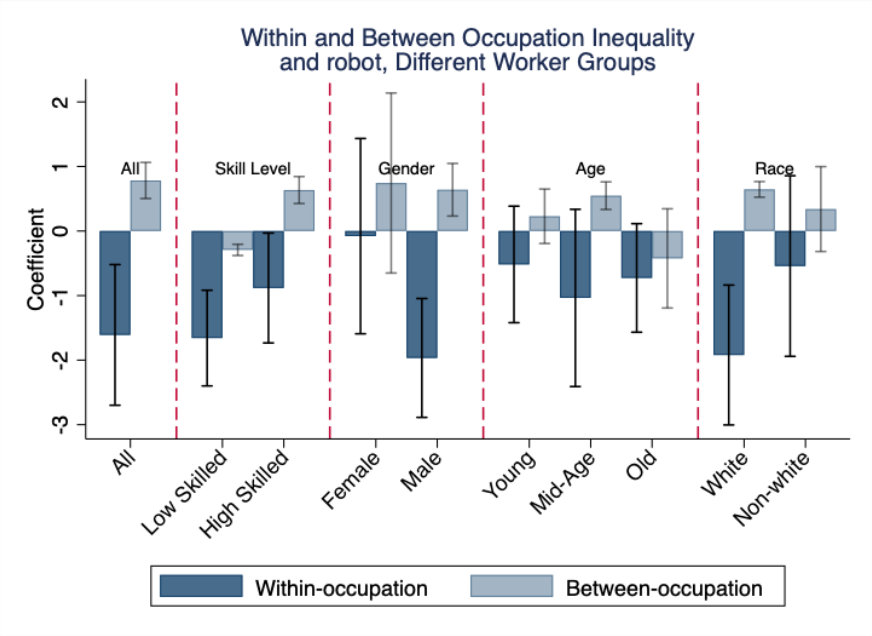


Figure 4: Effects of Robots on Inequality, by Skills, Gender, Age, and Race

Note: The figure presents estimates of the effects of robot adoption on within-occupation and between-occupation inequality for different worker types based on gender, education, and age. High-skilled workers (Bachelor's degree or above). The age groups are Young (16-24), Mid-age (25-64), and Old (65-). All regressions include demographic controls, IT adoption, and Chinese import competition, as well as year and industry fixed effects while weighting by industry-level employment. The capped lines provide 95% confidence intervals.

Robot adoption is associated with a reduction in within-occupation inequality for all groups. Notably, low-skilled, white, and male workers exhibit the largest reductions, with coefficients around -1.7 or lower. High-skilled and mid-age workers also show important reductions but to a lesser extent. Females, the youngest (16-24), and oldest (65+) as well as non-white workers show smaller non-significant decreases.

Robots are also associated with an increase in between-occupation inequality, particularly

for high-skilled, male, middle-aged, and white workers, with positive and significant coefficients between 0.5 and 1. Interestingly, the coefficient for low-skilled workers is negative, suggesting that the distributions of the different occupations in this group tend to converge presumably for bad since as we know from previous studies they were hard hit by robots.

4.5 *IV Estimates*

In this section, we present our IV estimates and discuss their quantitative implications. We use the two instrumental variables: the exposure to European robot adoption defined in equation 7 considering the adoption in five European countries –Denmark, Finland, France, Italy, and Sweden–, and the replaceability of workers defined in equation 6, based on the occupational structure of employment in the U.S. in 1970.

Table 5 reports our IV estimates for our baseline specification in Table 2, Column 4, and reports the first stage coefficients and F-statistics. Column 1, shows the results using the European Robots as an instrument, proposed by Acemoglu and Restrepo (2020). The first stage is positive and significant, with an F-statistic of 81.35, confirming the relevance of the instrument. The 2SLS estimates for within-occupation inequality report a coefficient of -1.913, 18% larger than the baseline OLS result of -1.615. For between-occupation inequality, the coefficient is 0.514, 30% smaller than OLS of 0.773. Both coefficients are statistically significant at the 1% level.

Column 2 presents the results for the Replaceability instrument, introduced by Graetz and Michaels (2018). The first stage shows a negative and significant relation between replaceability and US robots, with an F-statistic of 67.31, confirming the relevance of the instrument. Recall from Equation 6 that we interact the replaceability index with lagged robot prices, therefore, the negative relation with the demand for robots is behind the reported sign. The coefficient for within-occupation inequality is -1.475 significant at the 1% level, and 8% smaller than our OLS result. For between-occupation inequality, the coefficient is 1.060, significant at the 5% level and about 30% larger than OLS.

Table 5: Inequality and Robots, IV estimation

	Within-occupation		
	(1)	(2)	(3)
Robot Adoption	-1.913^{***}	-1.475^{***}	-1.869^{***}
	(0.021)	(0.240)	(0.045)
Adj.Rsquare	0.892	0.892	0.892
	Between-occupation		
	(1)	(2)	(3)
Robot Adoption	0.514^{***}	1.060^{**}	0.568^{***}
	(0.052)	(0.441)	(0.079)
Adj.Rsquare	0.888	0.888	0.888
	First Stage		
European Robots	0.540^{***}		0.493^{***}
	(0.085)		(0.090)
Replaceability		-0.0005^{***}	-0.0001^{*}
		(0.000)	(0.000)
F-stat	81.345	67.306	74.386
Num of Obs	378	378	378

All regressions control for demographics, IT adoption, and Chinese import competition, and include year and industry FE, while weighted by industry employment. Column (1) presents 2SLS estimation results with robot adoption in Europe as IV. Column (2) uses the replaceability IV. Column (3) includes both instruments. The P-value of the Sargan test for within-occupation regression is 0.458, above the 0.05 threshold. The P-value for between-occupation regression is 0.326, also above the threshold, indicating both instruments are relevant. Robust standard errors clustered at the industry level are reported in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Both instruments confirm the direction and significance of the relationship between robots and the two components of inequality. Nevertheless, both report magnitudes statistically different from each other. In Column 3, we consider the over-identified model, including both instruments. Our first stage shows that both instruments are relevant. Each variable is significant and jointly associated with an F-statistic of 74.39. Moreover, a Sargan test for the within-occupation regression is 0.46, and for the between-occupation is 0.33, both above the 0.05 threshold, indicating they are relevant instruments. The resulting magnitudes, closer to the European robot adoption, are: -1.869 for within-occupation and 0.568 for between-occupation. 15% above and 26% below the OLS results respectively. In the next subsection, we will discuss the economic mining of Column 3 results with both instruments.

4.6 Magnitudes

The IV estimates in the previous subsection quantify the impact of improvements in automation technologies, as well as changes in prices of the technologies and the replaceability structure of the U.S. labor that lead to the adoption of industrial robots on the within- and between-occupation inequality. The magnitudes from Table 5 column 3 imply that going from an industry in the 25th percentile of robot adoption to the 75th percentile of the exposure to robots, lagged one period, is associated with a reduction of 16% of the within-occupation Gini coefficient's standard deviation and a 7% increase of the between-occupation Gini coefficient's standard deviation.[14] These magnitudes seem sizable and reasonable.

Both IV instruments further validate our OLS results that robot adoption is associated with (i) a reduction in within-occupation inequality presumably by standardizing tasks and reducing the premium on routine work, and (ii) an increase in the disparities in wages across occupations affecting more directly certain occupations based on repetitive-automatable tasks. In the next section, we will get in-depth on the mechanisms driving these results.

4.7 Mechanism of the Effects of Robots on Inequality

So far we have focused on establishing empirically the relationship between robot adoption and a reduction in the within-occupations and between-occupations inequality. Nevertheless, a reasonable follow-up question is whether these results are good or bad news, and for which workers in the wage distribution, are the low-paid workers or the high-paid workers in a given occupation? To answer these questions, and gain a better understanding of the mechanisms behind these results we slightly depart from our baseline specification and look at the impact of robots on the two components of inequality for different deciles of the wage distribution.

[14] The difference between the 75th and the 25th percentile of the industry robot adoption in our sample is 0.0062–0.00001 = 0.0063, we multiply this number by our IV estimates from column 3, and divide by the Gini coefficient standard deviation of the within- and between-occupation inequality in each case. Therefore, for the within-occupation inequality, we compute $(-1.869 \times 0.0063)/0.075$ implying a reduction of 15.70% of its standard deviation. For between-occupation inequality, $(-0.568 \times 0.0063)/0.050$ implies a reduction of 7.2% of its standard deviation.

We first analyze our most novel result: robots are associated with a decrease in the within-occupation inequality. While ex-ante this result might seem like a positive outcome, it depends on whether the convergence of wages among workers in the same occupation is explained by an improvement of wages for those in the lowest percentiles of the wage distribution –in which case this would be a positive result–, by a deterioration of wages for workers in the highest percentiles of the wage distribution –in this case, our result would point out to a suppression of rents made by the top-earners in that occupation–, or a combination of the two –balancing the implications of this result–. With this purpose, we perform the following empirical exercise:

$$ln_Wage^{d}_{iot} = \beta \times Robot_Adoption_{it-1} + X_{iot} + \alpha_t + \delta_o + \gamma_i + \epsilon_{iot} \tag{8}$$

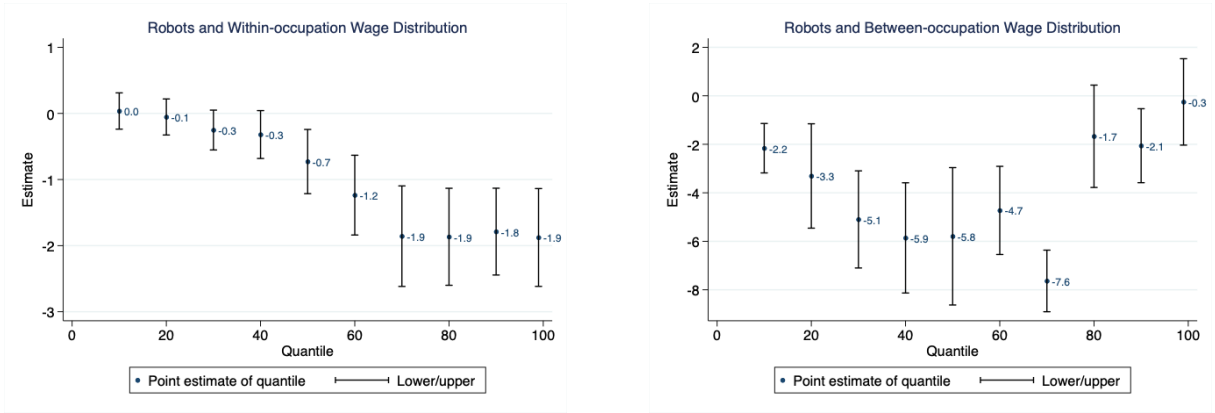
where $ln_Wage^{d}_{iot}$ are the logarithm of wages in the d-th decile (10th,...,90th,99th) of the wage distribution within each industry-occupation o, and year t. X_{iot} are demographic controls at the industry-occupation level including the share of females, high-skilled workers, young workers, workers moving across states, self-employed, and the share of unemployed workers. We include year fixed effects α_t, occupation fixed effects δ_o, and industry fixed effects γ_i. All regressions are weighted by industry-occupation employment and clustered standard errors at the industry level. *Robot_Adoption* is as defined at the industry level as in equation 2.

Why do robots increase the between-occupation inequality? Previous literature has argued that technological change contributes to job polarization by substituting mainly occupations disproportionately located in the middle of the wage distribution across occupations. To test this empirically in our context, we estimate the impact of robot adoption on wage percentiles across occupations as follows,

$$ln_Wage^{d}_{it} = \beta \times Robot_Adoption_{it-1} + X_{it} + \alpha_t + \delta_i + \epsilon_{it} \tag{9}$$

where $ln_Wage_{it}^{d}$ denotes the d-th decile of the wage distribution for industry i at year t, based on average log wages for all occupations within industry i. The independent variables are the same in the baseline specification in Table 2.

Figure 5 shows the relationship between robots and wages across deciles of the within- and between-occupations wage distribution. Figure 5a presents the estimates at the occupation level, for deciles 10th to 90th, revealing a "shrinking top" effect, where high-wage workers within occupations experience significantly higher negative effects on wages due to robot adoption. Therefore the decrease in within-occupation inequality comes at the expense of workers at the top of the distribution. This finding, to our knowledge novel in the literature, complements the view of Acemoglu and Restrepo (2024) regarding automation targeting the jobs with the highest rents, and therefore contributing to worker rent dissipation.



(a) Panel A: Within-occupation

(b) Panel B: Between-occupation

Figure 5: Effects of Robot by Wage Percentiles

Note: Figure 5 presents estimates of the effects of robot adoption on deciles of the wage distribution for the specification in equations 8 and 9. Estimates are shown along with 90% confidence intervals and are relative to the 10th decile. Panel A estimates from an industry-occupation-level quantile regression of log wage against robot adoption for the percentiles; Panel B is for between-occupation analysis, regressing the occupation's wage percentiles of industries with industry-level adoption.

Figure 5b shows the results of a regression of the wage deciles across occupations within industries on robot adoption at the industry level. We see a significant negative impact on occupational wages, particularly between the 30th and 70th percentiles, indicating a pronounced "collapsing middle" effect. This "U-shape" result is consistent with previous literature, such as Autor et al. (2006) and Acemoglu and Restrepo (2020), suggesting that

automation disproportionately affects middle-wage occupations, leading to increased wage polarization as middle-wage jobs are either eliminated or devalued.

5 Model economy

To provide a model for interpreting our empirical analysis, we present a simple model building on the wage posting framework of Burdett and Mortensen (1998), extended to allow for automation.

Occupations are modeled as segmented markets with different characteristics, including occupational productivity and susceptibility to robotization. Automation is modeled by assuming that firms may choose whether or not to replace workers with robots in any particular occupation. As found by the literature, we find conditions under which the most productive firms are the ones that use robots instead of hiring workers. Thus, it is not that robots replace all workers in any particular occupation or set of tasks - rather robots and workers may coexist in a given occupation, albeit at different firms.

Time is continuous. There is a continuum of risk-neutral workers who are divided into different occupations $i \in I$. We follow Erosa et al. (2024) in interpreting occupations as different ways of producing the same final good, but with a labor market segmented by occupation, as in Alvarez et al. (2018). Let m_i be the mass of workers in occupation i, and let z_i be their labor productivity.[15]

Workers are matched with a firm at rate λ. Jobs are closed exogenously at rate δ. There is an opportunity cost b of employment that can be interpreted as an unemployment benefit, the leisure gained when not employed, or the output from home production.

There is a $[0, 1]$ continuum of risk-neutral employer firms. Firm productivity takes values

[15] For simplicity we do not allow workers to change occupations. However, our results do not depend on this. For example, workers might have heterogeneous disutility of entering different occupations, leading them to sort into different occupations even when wages differ across occupations. Changes in the costs/benefits of automation might then lead workers to move between occupations. However, this would simply result in changes in m_i and, as we will see below, the equilibrium wage distribution and thus occupational inequality does not depend on m_i.

in $[0, \infty)$ and is distributed according to distribution function $J(p)$. We assume that $j(p) \equiv J'(p) > 0 \forall p$, $J(p) < 1 \ \forall p$ and $\lim_{p\to\infty} J(p) = 1$ – in other words, the distribution is non-degenerate over the entire domain.

Firms can choose any particular occupation to be performed by workers or by robots. If in occupation i they hire workers, they earn profits $\pi_i(p)$, to be derived below. If instead, they decide to use a robot, they earn:

$$\phi_i(p) - c_i \tag{10}$$

for some increasing function $\phi_i(\cdot)$. Here c_i is a fixed cost of robotizing occupation i, and ϕ_i is the revenue net of any other variable costs. A firm with productivity p then solves

$$\sum_{i\in I} \max\{\pi_i(p), \phi_i(p) - c_i\} \tag{11}$$

where I is the set of all occupations.

Going forward, we assume that $\phi_i(p) = \phi_i p$, i.e. $\phi_i(\cdot)$ is linear. This allows for the interpretation of ϕ_i as the *return to robotization*, whereas c_i is the *cost of robotization*.

Suppose $F_i(w)$ is the steady state distribution of wage offers w across firms. It may be that not all firms make offers: for those we will assume as a convention they offer $w = 0$. This will include firms who cannot afford to pay at least b, and firms with $p > \bar{p}_i$. Thus, there will be a mass point at zero, ensuring F_i integrates to unity. In what follows we suppress i where this does not cause confusion.

Let S_i be the value of an unemployed worker, and $W_i(w)$ is the value of an employed worker with wage w. The value of an unemployed worker satisfies

$$\rho S_i = b + \lambda \int \max\{W_i(w') - S_i, 0\} dF_i(w') \tag{12}$$

and the value of a worker employed at wage w must satisfy

$$\rho W_i(w) = w + \lambda \int \max\{W_i(\tilde{w}) - W_i(w), 0\} dF_i(\tilde{w}) + \delta[S_i - W_i(w)] \tag{13}$$

It is straightforward to show that workers have a reservation wage equal to b since they lose nothing by taking a job that offers wage $w = b$.

Turning to firms, this implies that the lowest wage offer maximizes profits only if it is no larger than b. In turn, there will be some $\underline{p}$ such that firms only find it profitable to make offers provided $p \geq \underline{p}$. As a result we have that

$$w_i(\underline{p}) = b \tag{14}$$

If firms only make offers within some range Φ, as a convention, we can just set $w_i(p) = 0$ for $p \notin \Phi$. For any firms that do make wage offers, in occupation i, they solve

$$\pi_i(p) = \max_{w \in [b, pz_i]} \{(pz - w) l(w|F_i)\} \tag{15}$$

where $l(w|F_i)$ is the number of workers that accept a job offer when the wage is w.

In the Appendix, we show that:

Proposition 1. *There is a $\kappa_i \in (0, \infty)$ such that $\pi_i'(\cdot) < \kappa_i$.*

Corollary 1. *Under Proposition 1, as long as ϕ_i exceeds κ_i, $\pi_i(p)$ and $\phi(p) - c_i$ can cross at most once.*

As a result, if the conditions of Corollary 1 hold, we will have that firms in each occupation only make offers within some range $[\underline{p}, \bar{p}]$. In other words, it is high-productivity firms that use robots, consistent with Firooz et al. (2023), Acemoglu et al. (2023) and Dinlersoz and Wolf (2024).

Furthermore, if the value of ϕ_i increases (i.e. robots become better at an occupation) or the value of c_i decreases (i.e. robots get cheaper), $\bar{p}_i$ would decline with increased robot

adoption. This is consistent with the quantile findings in Figure 5 panel A, where we found that robotization is related to a lowering of wages at the high end of the distribution within occupations.

In what follows, we take as given that it is the high productivity firms that adopt robots. For each occupation i, we assume that firms with p above a certain $\bar{p}_i$ prefer robots to workers. For now, we treat $\bar{p}_i$ as a constant, and treat a decrease in $\bar{p}_i$ as indicating an improvement in the state of robot technology - either an increase in ϕ_i or a decrease in c_i.

The firm's first order condition is then:

$$(pz - w)\, l'\left(w|F_i\right) = l\left(w|F_i\right) \tag{16}$$

The definition of l is

$$l'\left(w|F_i\right) = \lim_{\varepsilon \to 0} \frac{G_i\left(w,t\right) - G_i\left(w-\varepsilon,t\right)}{F_i\left(w\right) - F_i\left(w-\varepsilon\right)} \left(m_i - u_i\left(t\right)\right) \tag{17}$$

where G_i is the share of employed workers receiving no greater than w at time t.

Lemma 1. *Let $k = \lambda/\delta$. In a steady state,*

$$G_i\left(w\right) = \frac{\max\left\{F_i\left(w\right) - F_i\left(R\right), 0\right\}}{\left[1 + k\left(1 - F_i\left(w\right)\right)\right]\left(1 - F_i\left(R\right)\right)} \tag{18}$$

Lemma 2. *In a steady state,*

$$l\left(w|F_i\right) = m_i k \frac{1}{\left[1 + k\left(1 - F_i\left(w\right)\right)\right]^2} \tag{19}$$

We can use Lemma 2 to solve the firm's problem. This yields the following result:

Proposition 2. *In a steady state equilibrium, for $p \in [b/z, \bar{p}_i]$,*

$$w_i\left(p\right) = z\left[p - \int_{b/z}^{p} \left(\frac{1 + k\left(1 - J\left(p\right)\right)}{1 + k\left(1 - J\left(x\right)\right)}\right)^2 dx\right] \tag{20}$$

The form of $w(p)$ is the same as in Burdett and Mortensen (1998). The form of F_i takes into account the fact that firms with $p \notin [b/z, \bar{p}]$ do not make wage offers at all (or, equivalently, offer $w = 0$, which is rejected by all workers).

Notice that

$$w_i'(p) = kj(p) \int_{b/z}^{p} \left(\frac{1}{1 + k(1 - J(x))} \right)^2 dx > 0$$

As a result we have:

Corollary 2. *$w_i(p)$ is strictly increasing in p on the interval $[b/z, \bar{p}_i]$. In addition, it does not depend on $\bar{p}_i$.*

This corollary is important because it implies that, for a given z, the form of the wages offered is invariant to the state of robot technology (reflected in $\bar{p}_i$). If $\bar{p}_i$ declines and as a result robot adoption increases, in steady state this does not change the wage distribution other than by truncating it at the top. This is because firms can only attract workers from unemployment or from firms with lower productivity than them, so whatever wages firms with higher p are offering is irrelevant (other than the fact that they are higher).

5.1 Within-occupation inequality

We now turn to the analysis of wage inequality within a given occupation in steady state when robotization increases – i.e. when $\bar{p}_i$ declines. To measure inequality, define the following:

Definition 1. *The within-occupation Lorenz Curve $L_i(\Phi)$ is the share of total wage income that is earned by the proportion $\Phi \in [0, 1]$ of wage-earners.*

Definition 2. *The within-occupation Gini coefficient is $\Gamma_i \equiv 1 - 2\int_0^1 L_i(\Phi)\, d\Phi$. I.e. it is the area between the diagonal and the Lorenz curve, scaled so as to lie between zero and one.*

Proposition 3. *The Lorenz curve for occupation z is given by:*

$$L_i(\Phi) = \frac{\int_{b/z}^{J^{-1}(\Phi[J(\bar{p}) - J(b/z)] + J(b/z))} w_i(x)\, dJ(x)}{\int_{b/z}^{\bar{p}} w_i(x)\, dJ(x)} \tag{21}$$

The impact of robots on within-occupation inequality can be assessed by testing whether the Gini coefficient decreases if $\bar{p}_i$ declines. This is equivalent to showing that $\int_0^1 L_i(\Phi)\, d\Phi$ declines as $\bar{p}_i$ rises. In fact, we can show $L_i(\Phi)$ declines $\forall \Phi \in (0,1)$ as $\bar{p}_i$ rises, which delivers the desired result.

Proposition 4. *The Gini coefficient Γ_i of wages for any given occupation i is decreasing in robot availability (i.e. it decreases when $\bar{p}_i$ declines).*

Corollary 3. *The Gini coefficient Γ_i of wages for any given occupation i is decreasing in the returns to robotization ϕ_i and increasing in the costs of robotization c_i.*

The proof first shows that $L(\Phi)$ declines in $\bar{p}_i$ for sufficiently low but positive Φ. Then, by transfinite induction, this can be shown to hold for all higher values of $\Phi \in (0,1)$.

We develop this result in the context of automation technology, which mainly affects high-productivity firms. At the same time, this result is a broader statement about how wage inequality might be impacted by other factors in a search framework such as Burdett and Mortensen (1998), which to our knowledge is a novel result.

Another way to express Proposition 4 is noticing that we showed that $\partial \Gamma_i(\bar{p}_i)/\partial \bar{p}_i > 0$. In turn, $\bar{p}_i$ is itself a function $\bar{p}_i(\phi_i)$, where ϕ_i is some indicator of the penetration of robots into occupation i such that $\partial \bar{p}_i(\phi_i)/\partial \phi_i < 0$. As a result, the model has the empirically testable implication that inequality – as measured by the Gini coefficient – should *decline within occupations* as automation spreads, since :

$$\frac{\partial \Gamma_i(\bar{p}_i(\phi_i))}{\partial \phi_i} = \frac{\partial \Gamma_i(\bar{p}_i)}{\partial \bar{p}_i} \times \frac{\partial \bar{p}_i(\phi_i)}{\partial \phi_i} < 0.$$

This is consistent with our empirical findings in Table 2 and 5 panel A.

5.2 Between-occupation inequality

Recall that our quantile findings in Figure 5 panel B are that, comparing between occupations, robotization tends to lower wages the least at the top of the income distribution.

We will now examine what happens to between-occupation inequality in the model economy when this is the case.

To explore the implications of the model on between-occupation inequality, we define the average wage for a given occupation i as:

$$\zeta_i = \frac{\int_{b/z_i}^{\bar{p}_i} w_i(p)\, dJ(p)}{J(\bar{p}_i) - J(b/z_i)} \tag{22}$$

To begin, let us assume that there are two occupations, and suppose without loss of generality that the average wage is lower in occupation 1 i.e. $\zeta_1 \leq \zeta_2$. We will consider the case of $N > 2$ occupations later on.

Definition 3. *The between-occupation Lorenz Curve is the share of total wage income that is earned by the proportion $\Phi \in [0,1]$ of wage-earners by occupation.*

It is straightforward to show that the cross-occupation Lorenz Curve is:

$$L^*(\Phi) = \begin{cases} \frac{y}{x}\Phi & 0 \leq \Phi \leq x \\ y + \frac{(1-y)(\Phi - x)}{1-x} & x \leq \Phi \leq 1 \end{cases} \tag{23}$$

where x is the share of workers in occupation 1 and y is the share of income in occupation 1.

Definition 4. *The between-occupation Gini coefficient Γ^* is defined as the area between the diagonal and the between-occupation Lorenz curve $L^*(\Phi)$, scaled to lie between zero and one.*

It is straightforward to show that:

Lemma 3. *$\Gamma^* = x - y$ where*

$$x = 1 - \frac{m_2 \left[J(\bar{p}_2) - J(b/z_2)\right]}{m_1 \left[J(\bar{p}_1) - J(b/z_1)\right] + m_2 \left[J(\bar{p}_2) - J(b/z_2)\right]}$$

$$y = 1 - \frac{\zeta_2 m_2 \left[J(\bar{p}_2) - J(b/z_2)\right]}{\zeta_1 m_1 \left[J(\bar{p}_1) - J(b/z_1)\right] + \zeta_2 m_2 \left[J(\bar{p}_2) - J(b/z_2)\right]}$$

This means that to characterize the Lorenz curve between occupations we would need three pieces of information for each occupation i:

1. The average wage ζ_i.

2. The mass m_i.

3. The share of agents that are employed in each occupation. This is $J(\bar{p}_i) - J(b/z_i)$.

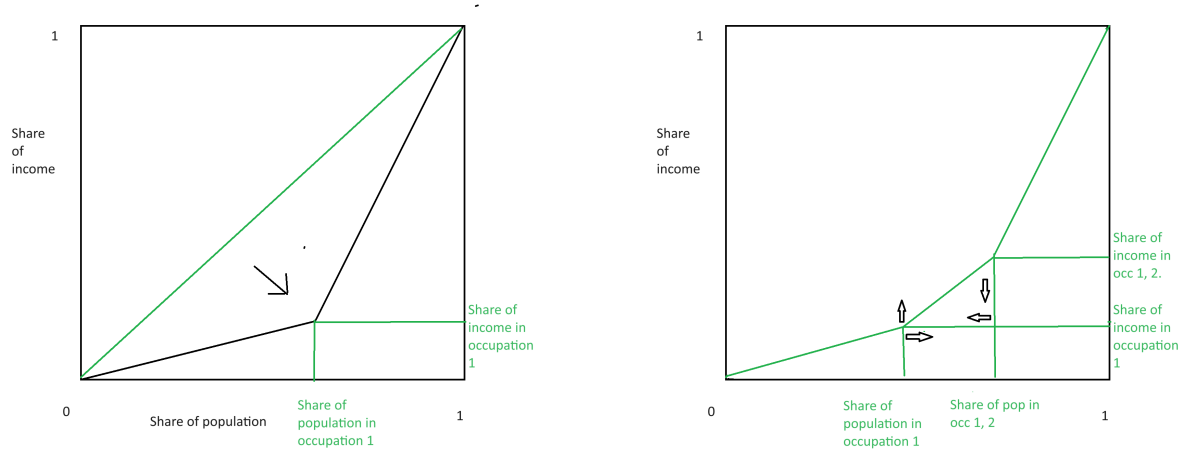
The complexity of the interaction between these three objects should make it clear that general results concerning the impact of robot adoption on *cross*-occupation wage inequality will not be theoretically derived with generality. At the same time, we can use some examples to illustrate factors that could lead inequality across occupations to increase, even though inequality *within* occupations might decrease as shown earlier.

5.2.1 2 occupations

For example, suppose there are two identical occupations with the same wage distribution, i.e. $z_1 = z_2$ and $\bar{p}_1 = \bar{p}_2$. In this case, the Lorenz curve is simply the 45° line. Now suppose the state of robotization improves, raising ϕ_1 more than ϕ_1,[16] and thus lowering $\bar{p}_1$ more than $\bar{p}_2$. Although the wage functions $w_1(p) = w_2(p)$, the wage distribution in occupation 1 will be truncated at a lower value of $\bar{p}_i$ than that in occupation 2, so now $\zeta_2 > \zeta_1$. This will lead to a new Lorenz curve uniformly below the first one - see Figure 6*a*. Thus, one way to obtain increased inequality from increased robot diffusion is if one of the occupations is affected more than the other – and if the more-affected occupation has the same or lower average wage to begin with.

In the 2- occupation space, suppose we were on the black Lorenz curve. Furthermore, suppose that occupation 1 is more replaceable by robot adoption. To link this with the literature, let us assume that occupation 1 involves "routine" tasks which are thus easy to substitute by robots. In contrast, occupation 2 is "non-routine." And therefore, $\bar{p}_1 < \bar{p}_2$.

[16] Or lowering c_1 more than c_2.



(a) Two occupations

(b) Three occupations

Figure 6: Between-occupation Lorenz curve

Note:Panel(a) The green Lorenz curve assumes $\zeta_2 = \zeta_1$. The black Lorenz curve assumes $\zeta_2 > \zeta_1$. Panel(b) The Lorenz curve assumes $\zeta_3 > \zeta_2 > \zeta_1$. The arrows indicate how the vertices would move with an increase in automation in occupation 2.

Then if robot technology were to improve further, and if this were to primarily affect occupation 1, this would lower both the share of income and the share of workers in occupation 1. An increase in cross-occupation inequality would occur if the income decline were larger than the decline in the share of workers, which would depend on the form of J near $\bar{p}_i$, i.e. if the impact on income share y were larger than the impact on worker share x. Otherwise, an improvement in robot technology would decrease inequality across occupations.

Finally, consider the case when occupation 2 is "non-routine" and cannot be robotized - for example, $c_2 = \infty$ or $\phi_2 = 0$. In this case, $\bar{p}_2 = \infty$. Assuming $\zeta_2 > \zeta_1$, which could happen if $z_2 \geq z_1$ and $\bar{p}_1$ is finite, although with finite $\bar{p}_1$ some values of $z_2 < z_1$ will still work.

Proposition 5. *Suppose that $\bar{p}_2 = \infty$. The between-occupation Gini coefficient Γ^* is increasing in automation (i.e. it increases when $\bar{p}_1$ decreases) provided $\bar{p}_1$ is sufficiently large.*

Proposition 5 provides a sufficient condition for which, while increased automation *decreases* within exposed occupations inequality, this is nonetheless compatible with an *increase* in inequality when comparing across occupations.

5.2.2 More than 2 occupations

Note that increased automation leading to increased inequality does not require that the occupation with the lowest wages be the one most exposed to automation. Here we will explore an example, later we will report some more general results.

For example, suppose that there are three occupations such that $\zeta_3 > \zeta_2 > \zeta_1$. Figure 6*b* displays the corresponding Lorenz curve, which is piece-wise affine.

In this case, we can show that the Gini coefficient equals:

$$\Gamma^* = x - y + x_2 - y_2 + xy_2 - yx_2.$$

As before x is the share of workers in occupation 1 and y the share of income in occupation 1, while x_2 and y_2 are the share of workers and the share of income in occupation 2 respectively.

Suppose now that occupation 2 is the more routine one, whereas occupations 1 and 3 are not amenable to automation. Thus $\bar{p}_2$ is finite, whereas $\bar{p}_1 = \bar{p}_3 = \infty$. If automation were to spread, entailing a decline in $\bar{p}_2$, several effects would occur. The share of income accruing to occupation 2 would decline (see the vertical arrows). At the same time, the working population in occupation 2 would decline as well (horizontal arrows). Thus, in general, the impact of automation on cross-occupation inequality in an environment with more than 2 occupations is difficult to characterize. Nonetheless, with three occupations and the middle occupation most exposed to automation, we can show that:

Proposition 6. *Suppose there are* 3 *occupations and only the middle occupation* 2 *is exposed to automation as described above. The between-occupation Gini coefficient* Γ^* *is increasing in automation (i.e. it increases when* $\bar{p}_2$ *decreases) provided (*$\bar{p}_2$ *is sufficiently large and* m_1 *is sufficiently small.*

Another way to state Propositions 5 and 6 is that we have shown that, under certain conditions, $\partial\Gamma^*\left(\bar{p}_1\right)/\partial\bar{p}_1 < 0$. In turn, $\bar{p}_1$ is a function of ϕ_1 that is an indicator of the robot penetration into occupation 1 such that $\partial\bar{p}_1\left(\phi_1\right)/\partial\phi_1 < 0$. Therefore, the model has the

empirically testable implication that the between-occupation Gini coefficient can increase as automation spreads, even when the within-occupation Gini coefficient decreases:

$$\frac{\partial \Gamma^* \left(\bar{p}_1 \left(\phi_1 \right) \right)}{\partial \phi_1} = \frac{\partial \Gamma^* \left(\bar{p}_1 \right)}{\partial \bar{p}_1} \times \frac{\partial \bar{p}_1 \left(\phi_1 \right)}{\partial \phi_1} > 0.$$

This is consistent with our empirical findings in Table 2 and 5 panel B.

Notice that Corollary 1, combined with Proposition 5, implies the following, provided the relevant assumptions hold:

Proposition 7. *Suppose there are two occupations with $\phi_2 = 0$ (so no robots in occupation 2) whereas ϕ_1 is finite and c_1 is finite. The between-occupation Gini coefficient Γ^* is decreasing in ϕ_1 and increasing in c_1, provided the initial value of $\bar{p}_1$ is sufficiently high.*

Proposition 7 tells us that, if robots are chosen endogenously by firms, and if one task is more routine whereas the other cannot be automated, then there are sufficient conditions for the factors that underlie robot penetration (high returns or low costs) to increase cross-occupation inequality when the benefits of automation improve or the adoption costs decline.

In fact, we can provide the following generalization of this result. Suppose there is a finite number of occupations, the occupation with the highest average income cannot be automated, the next highest can, and the others may or may not be. Provided the assumptions of Corollary 1 hold:

Proposition 8. *Suppose there are $N > 2$ occupations such that $\zeta_n > \zeta_{n'}$ for $n > n'$, and that $\bar{p}_N = \infty$. Moreover suppose that $\pi'_{N-1}(\cdot) < \kappa_i$. The between-occupation Gini coefficient Γ^* is decreasing in automation (i.e it decreases when $\bar{p}_n$ decreases) provided $\bar{p}_{n-1}$ is sufficiently large and $\sum_{n=1}^{\mathbb{N}-2} m_n$ is sufficiently small.*

To show that we can have increasing cross-occupation inequality in more complex contexts, in the Appendix we study some numerical examples.

In sum, our model has several implications related to our empirical findings in Section 4:

Implication 1 Automation should be related to decreased inequality *within* occupations.[17] This is consistent with the findings in Table 2 and 5 panel A.

Implication 2 Automation may be related to increased inequality *between* occupations. This is consistent with the findings in Table 2 and 5 panel B.

Implication 3 Decreased *within*-occupations inequality is related to a reduction in the top wage jobs in each occupation. This follows from Corollary 1 and is consistent with our findings in Figure 5a.

Implication 4 Increased *between*-occupations inequality is related to a reduction in wages in low- or middle-paying occupations. This is implied by Proposition 7 and is consistent with our findings in Figure 5b.

6 Conclusion

In this paper, we show that within-occupation wage inequality represents 75-80% of the overall inequality, underlining the importance of studying the determinants of both within-occupations and between-occupations inequality. We focus on the study of the impacts of industrial robots on these two components of occupational inequality.

Our analysis reveals a complex interaction: robot adoption increases inequality between occupations but reduces inequality within occupations. This latter finding is particularly novel, as most existing literature focuses on the overall effects of robots on inequality and the wage disparities they create between different occupations. We find that from 2000 to 2018 moving from the 25th to the 75th percentile of industry-level robot adoption is associated with a decrease of 16% of the Gini coefficient within-occupations standard deviation. Meanwhile, robots are associated with a 7% increase in the between-occupation Gini coefficient standard deviation.

[17] Our model predicts that robots would reduce within-occupation inequality more for occupations harder hit by robots, which according to Acemoglu and Restrepo (2020) and consistent with Section 4 are the lowest-paid occupations.

Moreover, we provide evidence of the forces driving these results. In particular, we show empirically that robots within occupations affect workers in the highest deciles of the wage distribution. Therefore our documented reduction in the dispersion of wages within occupations happens at the expense of workers with the highest wages. In line with the theory that automation technologies have contributed to worker rent dissipation. At the same time, we find that comparing across occupations, workers in the middle of the wage distributions are the ones that have been hardest hit by robots, explaining the between occupations result.

We also show that the decrease in inequality from robot adoption within occupations is a natural outcome of a wage-posting model a-la Burdett and Mortensen (1998), provided that it is the highest-productivity firms that adopt. These firms tend to also be those offering better-paying jobs, so adoption shrinks the top of the wage distribution, reducing the dispersion of wages in occupations exposed to robot adoption.

At the same time, depending on which occupations are more/less exposed to robots, inequality across occupations may increase, as previous literature has shown. Our model shows that frictional labor markets for different occupations provide additional insights into the impact of automation, complementary to those from tasks-based models.

Finally, Kapetaniou and Pissarides (2024) find that the degree of robot adoption is negatively related to a country's innovative capacity. It remains to be seen whether this interaction results in greater across- (and lesser within-) occupation inequality for countries with greater innovative capacity.

Our findings beg questions about the potential impact of other automation technologies such as Artificial Intelligence (AI) that tend to standardize the task content of occupations and replace human labor particularly those top earners in affected occupations. Our findings suggest that the diffusion of AI should be associated with a similar impact on within- and (possibly) across-occupation inequality -. This possibility is left for future work.

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A Proofs

A.1 Proof of Proposition 1

First, as shown below once the model is fully solved:

$$\begin{aligned}
\pi(p) &= \{(pz - w(p))\, l(w|F_i)\} && (24)\\
&= zm_i k \int_{b/z}^{p} \left(\frac{1+k(1-J(p))}{1+k(1-J(x))}\right)^2 dx \times \frac{1}{\left[1+k(1-F_i(w))\right]^2} && (25)\\
&= zm_i k \int_{b/z}^{p} \left(\frac{1+k(1-J(p))}{1+k(1-J(x))}\right)^2 dx \times \frac{1}{\left[1+k(J(\bar{p})-J(p))\right]^2} && (26)
\end{aligned}$$

Then

$$\begin{aligned}
\pi'(p) &= zm_i k \frac{1}{\left[1+k(J(\bar{p})-J(p))\right]^2} && (27)\\
&\quad \times \left[1 + 2\int_{b/z}^{p} \left(\frac{1+k(1-J(p))}{1+k(1-J(x))}\right)^2 dx \times \frac{kj(p)}{\left[1+k(J(\bar{p})-J(p))\right]}\right] && (28)\\
&< \frac{zm_i k}{\left[1+k\right]^2}\left[1 + 2\left[p - b/z\right] \times kj(p)\right] && (29)
\end{aligned}$$

A.2 Proof of Lemma 1

Then $G(w,t)(m_z - u_z(t))$ is the number of such workers. It changes over time as a flow $\lambda \max\{F(w) - F(R), 0\} u(t)$ finds work and a flow $[\delta + \lambda(1 - F(w))] G(w,t)(m - u(t))$ loses work, implying that:

$$\frac{dG(w,t)(m-u(t))}{dt} = \lambda \max\{F(w) - F(R), 0\} u(t) - [\delta + \lambda(1 - F(w))] G(w,t)(m - u(t))$$

The evolution of non-employed workers satisfies.

$$\dot{u} = (m-u)\delta - \lambda(1 - F(R))\, u$$

This implies that in a steady state we have

$$u = \frac{m}{1 + k\left(1 - F\left(R\right)\right)}$$

and

$$\left[\delta + \lambda\left(1 - F\left(w\right)\right)\right] G\left(w,t\right)\left(m - u\left(t\right)\right) = \lambda \max\left\{F\left(w\right) - F\left(R\right), 0\right\} u\left(t\right)$$

$$\left[\delta + \lambda\left(1 - F\left(w\right)\right)\right] G\left(w,t\right)\left(m - \frac{m}{1 + k\left(1 - F\left(R\right)\right)}\right) = \lambda \max\left\{F\left(w\right) - F\left(R\right), 0\right\} \frac{m}{1 + k\left(1 - F\left(R\right)\right)}$$

$$\left[\delta + \lambda\left(1 - F\left(w\right)\right)\right] G\left(w,t\right) k\left(1 - F\left(R\right)\right) = \lambda \max\left\{F\left(w\right) - F\left(R\right), 0\right\}$$

$$\left[1 + k\left(1 - F\left(w\right)\right)\right] G\left(w,t\right) k\left(1 - F\left(R\right)\right) = k \max\left\{F\left(w\right) - F\left(R\right), 0\right\}$$

where $k = \lambda/\delta$. Finally, this implies that, in a steady state,

$$G\left(w\right) = \frac{\max\left\{F\left(w\right) - F\left(R\right), 0\right\}}{\left[1 + k\left(1 - F\left(w\right)\right)\right]\left(1 - F\left(R\right)\right)}$$

Now since $w \geq R$

$$G\left(w\right) = \frac{\left[F\left(w\right) - F\left(R\right)\right] / \left[1 - F\left(R\right)\right]}{1 + k\left(1 - F\left(w\right)\right)}$$

A.3 Proof of Lemma 2

In a steady state, Burdett and Mortensen (1998) show that:

$$l\left(w|F_z\right) = m \lim_{\varepsilon \to 0} \frac{\frac{\left[F(w) - F(R)\right]/\left[1 - F(R)\right]}{1 + k(1 - F(w))} - \frac{\left[F(w-\varepsilon) - F(R)\right]/\left[1 - F(R)\right]}{1 + k(1 - F(w-\varepsilon))}}{F\left(w\right) - F\left(w - \varepsilon\right)} \left(\frac{k\left(1 - F\left(R\right)\right)}{1 + k\left(1 - F\left(R\right)\right)}\right)$$

$$l\left(w|F_z\right) = m \lim_{\varepsilon \to 0} \frac{\frac{\left[F(w) - F(R)\right]}{1 + k(1 - F(w))} - \frac{\left[F(w-\varepsilon) - F(R)\right]}{1 + k(1 - F(w-\varepsilon))}}{F\left(w\right) - F\left(w - \varepsilon\right)} \left(\frac{k}{1 + k\left(1 - F\left(R\right)\right)}\right)$$

$$l\left(w|F_z\right) = m \lim_{\varepsilon \to 0} \frac{-\left[F'\left(w\right)\right]\left[1 + k\left(1 - F\left(w\right)\right)\right] + kF'\left(w\right)\left[F\left(w\right) - F\left(R\right)\right]}{F'\left(w\right)\left[1 + k\left(1 - F\left(w - \varepsilon\right)\right)\right]^2} \left(\frac{k}{1 + k\left(1 - F\left(R\right)\right)}\right)$$

$$l\left(w|F_z\right) = m \frac{\left[1 + k\left(1 - F\left(w\right)\right)\right] + k\left[F\left(w\right) - F\left(R\right)\right]}{\left[1 + k\left(1 - F\left(w\right)\right)\right]^2} \left(\frac{k}{1 + k\left(1 - F\left(R\right)\right)}\right)$$

$$l\left(w|F_z\right) = m\frac{\left[1+k\left(1-F\left(w\right)\right)\right]+k\left[F\left(w\right)-F\left(R\right)\right]}{\left[1+k\left(1-F\left(w\right)\right)\right]^2}\left(\frac{k}{1+k\left(1-F\left(R\right)\right)}\right)$$

$$l\left(w|F_z\right) = mk\frac{1}{\left[1+k\left(1-F\left(w\right)\right)\right]^2}$$

A.4 Proof of Proposition 2

$$l'\left(w|F_z\right) = 2mk^2\left[1+k\left(1-F\left(w\right)\right)\right]^{-3}F'\left(w\right)$$

and

$$\left(pz-w\right)l'\left(w|F_z\right) = l\left(w|F_z\right)$$

Therefore,

$$\left(pz-w\right)\frac{2kF'\left(w\right)}{\left[1+k\left(1-F\left(w\right)\right)\right]} = 1$$

$$l\left(w|F_z\right) = mk\frac{1}{\left[1+k\left(1-F\left(w\right)\right)\right]^2}$$

It must be that for $p \in \left[\underline{p}, \bar{p}\right]$,

$$F\left(w\left(p\right)\right) = J\left(p\right) - J\left(\underline{p}\right) + F\left(0\right)$$

where $\underline{p} = b/z$ – the lowest productivity that can meet the reservation wage $R = b$ – and

$$F\left(0\right) = J\left(\underline{p}\right) + \left(1 - J\left(\bar{p}\right)\right)$$

is the mass point at zero of firms that don't make offers in market z.

If $w\left(p\right)$ is the wage function, and w^{-1} is its inverse, then $w^{-1}\left(w\right) \in \left[\underline{p}, \bar{p}\right]$ we have

$$F\left(w\right) = J\left(w^{-1}\left(w\right)\right) + \left(1 - J\left(\bar{p}\right)\right)$$

Note that

$$\begin{aligned} F'(w) &= J'\left(w^{-1}(w)\right)\frac{dw^{-1}}{dw} \\ F'(p) &= J'(p)\frac{1}{w'(p)} \end{aligned}$$

This means that the firm's FOC becomes

$$(pz - w(p))\frac{2kJ'(p)}{[1+k(1-J(p))]} = w'(p)$$

To solve this, define

$$\begin{aligned} T(p) &= -2\log[1+k(1-J(p))] \\ e^{T(p)} &= e^{\log[1+k(1-J(p))]^{-2}} \\ &= [1+k(1-J(p))]^{-2} \end{aligned}$$

This implies

$$T'(p) = \frac{2kJ'(p)}{\log[1+k(1-J(p))]}$$

Thus, the wage equation becomes

$$pzT'(p) - w(p)T'(p) = w'(p)$$

We speculate that:

$$w(p)e^{T(p)} = z\int_{\underline{p}}^{p} xT'(x)e^{T(x)}dx + A$$

Deriving both sides:

$$w'(p)e^{T(p)} + T'(p)w(p)e^{T(p)} = zpT'(p)e^{T(p)}$$

$$w'(p) + T'(p)w(p) = zpT'(p)$$

So we have this result in equilibrium:

$$w(p)\, e^{T(p)} = z \int_{\underline{p}}^{p} xT'(x)\, e^{T(x)} dx + A$$

Next it turns out that

$$\frac{de^{T(x)}}{dx} = T'(x)\, e^{T(x)}$$

and also (by parts)

$$\int_{\underline{p}}^{p} xT'(x)\, e^{T(x)} dx = z\left[xe^{T(x)}\right]_{\underline{p}}^{p} - z\int_{\underline{p}}^{p} e^{T(x)} dx$$

so

$$w(p)\, e^{T(p)} = z\left[xe^{T(x)}\right]_{\underline{p}}^{p} - z\int_{\underline{p}}^{p} e^{T(x)} dx + A$$

$$w(p)\, e^{T(p)} = zpe^{T(p)} - z\underline{p}e^{T\left(\underline{p}\right)} - z\int_{\underline{p}}^{p} e^{T(x)} dx + A$$

$$w(p) = zp + e^{-T(p)}\left[A - z\underline{p}e^{T\left(\underline{p}\right)}\right] - ze^{-T(p)}\int_{\underline{p}}^{p} e^{T(x)} dx$$

We know that the lowest $\underline{p}$ that makes an offer must offer b, otherwise it's not maximizing.

$$w\left(\underline{p}\right) = b$$

also

$$w\left(\underline{p}\right) = z\underline{p} + e^{-T\left(\underline{p}\right)}\left[A - z\underline{p}e^{T\left(\underline{p}\right)}\right] = b$$

If we have a full set of p then $\underline{p} = b/z$ (the lowest productivity for which there are any potential profits) so

$$\left[A - z\underline{p}e^{T\left(\underline{p}\right)}\right] = 0$$

Then

$$w(p) = zp - ze^{-T(p)}\int_{b/z}^{p} e^{T(x)} dx + b$$

The result follows.

A.5 Proof of Proposition 3

From Proposition 2, the cdf of wages as a function of p is the integral of wages from b/z to p, scaled to range between zero and one:

$$\frac{\int_{b/z}^{p} w_z(x)\, dJ(x)}{J(\bar{p}) - J(b/z)}$$

since firms with $p \notin [b/z, \bar{p}(z)]$ do not make wage offers (or make offers that are not accepted). The total amount of wage earnings in occupation z is then:

$$\frac{\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)}{J(\bar{p}) - J(b/z)}.$$

So for any given p their share of accumulated earnings is

$$\phi(p) = \frac{\int_{b/z}^{p} w_z(x)\, dJ(x)}{\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)}$$

and the corresponding population is $\frac{J(p)-J(b/z)}{J(\bar{p})-J(b/z)}$.

The Lorenz curve is the locus of $\frac{J(p)-J(b/z)}{J(\bar{p})-J(b/z)}$ on the horizontal axis and $\phi(p)$ on the vertical axis. The inverse of $\Phi = \frac{J(p)-J(b/z)}{J(\bar{p})-J(b/z)}$ is

$$p = J^{-1}\left(\Phi\left[J(\bar{p}) - J(b/z)\right] + J(b/z)\right)$$

so the Lorenz curve is

$$\begin{aligned} L(\Phi) &= \phi\left(\Phi\left[J(\bar{p}) - J(b/z)\right] + J(b/z)\right) \\ &= \frac{\int_{b/z}^{J^{-1}(\Phi[J(\bar{p})-J(b/z)]+J(b/z))} w_z(x)\, dJ(x)}{\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)} \end{aligned}$$

A.6 Proof of Proposition 4

Now to derive $\frac{dL(\Phi)}{d\bar{p}}$ we will need Leibniz' Theorem. Using the quotient rule, it will have the same sign as:

$$\begin{aligned}
& w_z\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\right)dJ \\
& \times\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\right)\left(J^{-1}\right)' \\
& \times\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\Phi J'\left(\bar{p}\right)\int_{b/z}^{\bar{p}} w_z\left(x\right)dJ\left(x\right) \\
& -w_z\left(\bar{p}\right)dJ\left(\bar{p}\right)\int_{b/z}^{J^{-1}(\Phi[J(\bar{p})-J(b/z)]+J(b/z))} w_z\left(x\right)dJ\left(x\right)
\end{aligned}$$

We wish to show this is negative, in which case:

$$\begin{aligned}
& w_z\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\right)dJ \\
& \times\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\right)\left(J^{-1}\right)' \\
& \times\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\Phi J'\left(\bar{p}\right)\int_{b/z}^{\bar{p}} w_z\left(x\right)dJ\left(x\right) \\
< \;& w_z\left(\bar{p}\right)dJ\left(\bar{p}\right)\int_{b/z}^{J^{-1}(\Phi[J(\bar{p})-J(b/z)]+J(b/z))} w_z\left(x\right)dJ\left(x\right)
\end{aligned}$$

which can be rewritten

$$\begin{aligned}
& w_z\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\right) \\
& \times j\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\right) \\
& \times\left(J^{-1}\right)'\left(\Phi\left[J\left(\bar{p}\right)-J\left(b/z\right)\right]+J\left(b/z\right)\right)\Phi J'\left(\bar{p}\right) \\
< \;& w_z\left(\bar{p}\right)dJ\left(\bar{p}\right)\times\frac{\int_{b/z}^{J^{-1}(\Phi[J(\bar{p})-J(b/z)]+J(b/z))} w_z\left(x\right)dJ\left(x\right)}{\int_{b/z}^{\bar{p}} w_z\left(x\right)dJ\left(x\right)}
\end{aligned}$$

Simplifying, this becomes:

$$\underbrace{\frac{w_z\left(J^{-1}(\Phi[J(\bar{p})-J(b/z)]+J(b/z))\right)\Phi}{w_z(\bar{p})}}_{\text{LHS}} < \underbrace{\frac{\int_{b/z}^{J^{-1}(\Phi[J(\bar{p})-J(b/z)]+J(b/z))} w_z(x)dJ(x)}{\int_{b/z}^{\bar{p}} w_z(x)dJ(x)}}_{\text{RHS}}$$

Now when $\Phi = 1$ we have that both sides equal one, since $J^{-1}\left(\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right) = \bar{p}$. When $\Phi = 0$, both sides equal zero. The question is what happens in between.

Next, see what happens to the derivative of the LHS and RHS at $\Phi \approx 0$. Starting with the LHS:

$$\begin{aligned} w_z\left(\bar{p}\right)\frac{dLHS}{d\Phi} &= w_z'\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \\ &\quad \times\left(J^{-1}\right)'\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\left[J\left(\bar{p}\right) - J\left(b/z\right)\right]\Phi \\ &\quad + w_z\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \end{aligned}$$

or

$$\begin{aligned} w_z\left(\bar{p}\right)\frac{dLHS}{d\Phi} &= w_z'\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \\ &\quad \times j^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\left[J\left(\bar{p}\right) - J\left(b/z\right)\right]\Phi \\ &\quad + w_z\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \end{aligned}$$

What about RHS?

$$\begin{aligned} \int_{b/z}^{\bar{p}} w_z\left(x\right) dJ\left(x\right)\frac{dRHS}{d\Phi} &= w_z\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \\ &\quad \times j\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \\ &\quad \times\left(J^{-1\prime}\right)\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] \\ &= w_z\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \\ &\quad \times j\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right) \\ &\quad \div j\left(J^{-1}\left(\Phi\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] + J\left(b/z\right)\right)\right)\left[J\left(\bar{p}\right) - J\left(b/z\right)\right] \end{aligned}$$

At zero

$$\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)\, \frac{dRHS}{d\Phi} = w_z(b/z)\left[J(\bar{p}) - J(b/z)\right]$$

Also at zero

$$w_z(\bar{p})\, \frac{dLHS}{d\Phi}|_{\Phi=0} = w_z(b/z) > 0$$

and

$$\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)\, \frac{dRHS}{d\Phi}|_{\Phi=0} = w_z(b/z)\left[J(\bar{p}) - J(b/z)\right]$$

So

$$\frac{dRHS}{d\Phi}|_{\Phi=0} = \frac{w_z(b/z)\left[J(\bar{p}) - J(b/z)\right]}{\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)} > 0$$

The desired condition is that $\frac{dLHS}{d\Phi}|_{\Phi=0} < \frac{dRHS}{d\Phi}|_{\Phi=0}$, so

$$\frac{w_z(b/z)}{w_z(\bar{p})} < \frac{w_z(b/z)\left[J(\bar{p}) - J(b/z)\right]}{\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)}$$

which simplifies to

$$\frac{\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)}{\left[J(\bar{p}) - J(b/z)\right]} < w_z(\bar{p})$$

This requires the wage at $\bar{p}$ to exceed the average wage. This is always the case since $w_z(x)$ is strictly increasing.

Now it will be useful to perform a change of variables, setting

$$p = J^{-1}\left(\Phi\left[J(\bar{p}) - J(b/z)\right] + J(b/z)\right) \in [b/z, \bar{p}]\,.$$

This also implies that $\frac{J(p)-J(b/z)}{[J(\bar{p})-J(b/z)]} = \Phi$. The condition $LHS < RHS$ becomes simpler:

$$\frac{w_z(p)\left[J(p) - J(b/z)\right]}{w_z(\bar{p})\left[J(\bar{p}) - J(b/z)\right]} < \frac{\int_{b/z}^{p} w_z(x)\, dJ(x)}{\int_{b/z}^{\bar{p}} w_z(x)\, dJ(x)}$$

which can be rewritten

$$\frac{\int_{b/z}^{\bar{p}} \frac{w_z(x)}{w_z(\bar{p})} dJ(x)}{[J(\bar{p}) - J(b/z)]} < \frac{\int_{b/z}^{p} \frac{w_z(x)}{w_z(p)} dJ(x)}{[J(p) - J(b/z)]}.$$

Defining

$$E(p) \equiv \frac{\int_{b/z}^{p} \frac{w_z(x)}{w_z(p)} dJ(x)}{[J(p) - J(b/z)]}$$

the condition becomes

$$E(\bar{p}) < E(p)$$

We know this holds for p close to b/z. This result would work for all $p \in (b/z, \bar{p})$ (and thus for all $\Phi \in (0,1)$) if $E(p)$ is strictly decreasing on $[b/z, \bar{p}]$. Using the quotient rule, $E'(p)$ will have the same sign as:

$$\begin{aligned} & \frac{w_z(p)}{w_z(p)} j(p) [J(p) - J(b/z)] - j(p) \int_{b/z}^{p} \frac{w_z(x)}{w_z(p)} j(x) dx \\ = \; & j(p) \left[[J(p) - J(b/z)] - \int_{b/z}^{p} \frac{w_z(x)}{w_z(p)} j(x) dx \right] \end{aligned}$$

which has the same sign as

$$\begin{aligned} & [J(p) - J(b/z)] - \int_{b/z}^{p} \frac{w_z(x)}{w_z(p)} j(x) dx \\ = \; & \int_{b/z}^{p} \left\{ 1 - \frac{\left[x - \int_{b/z}^{x} \left(\frac{1+k(1-J(x))}{1+k(1-J(z))} \right)^2 dz \right]}{\left[p - \int_{b/z}^{p} \left(\frac{1+k(1-J(p))}{1+k(1-J(z))} \right)^2 dz \right]} \right\} j(x) dx \end{aligned}$$

which is negative, since $\left[x - \int_{b/z}^{x} \left(\frac{1+k(1-J(x))}{1+k(1-J(z))} \right)^2 dz \right] \div \left[p - \int_{b/z}^{p} \left(\frac{1+k(1-J(p))}{1+k(1-J(z))} \right)^2 dz \right] < 1 \; \forall x < p$. The result follows.

A.7 Proof of Lemma 3

First of all the expressions for x and y are straighforward to derive. Second, the Gini coefficient is one minus double the area under the Lorenz curve, which is

$$\begin{aligned} & \frac{1}{2}xy + (1-x)\,y + \frac{1}{2}\,(1-x)\,(1-y) \\ = \; & \frac{1}{2}y - \frac{1}{2}x + \frac{1}{2} \end{aligned}$$

It follows that the Gini coefficient is equal to $x - y$.

A.8 Proof of Proposition 5

We have that

$$\begin{aligned} \Gamma^* \; = \; & \frac{\zeta_2 m_2 \left[1 - J\left(b/z_1\right)\right]}{\zeta_1 m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]} \\ & - \frac{m_2 \left[1 - J\left(b/z_2\right)\right]}{m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + m_2 \left[1 - J\left(b/z_2\right)\right]} \end{aligned}$$

We seek conditions under which Γ^* increases when $\bar{p}$ declines, in other words, $\frac{d\Gamma^*}{d\bar{p}} < 0$.

$$\begin{aligned} \frac{d\Gamma^*}{d\bar{p}} \; = \; & - \frac{\zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right] \left[\zeta_1' m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_1 m_1 J'\left(\bar{p}_1\right)\right]}{\left(\zeta_1 m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]\right)^2} \\ & + \frac{m_2 \left[1 - J\left(b/z_2\right)\right] m_1 J'\left(\bar{p}_1\right)}{\left(m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + m_2 \left[1 - J\left(b/z_2\right)\right]\right)^2} \end{aligned}$$

$$\begin{aligned} \frac{d\Gamma^*}{d\bar{p}} \; = \; & - \frac{\zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right] \left[\zeta_1' m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_1 m_1 J'\left(\bar{p}_1\right)\right]}{\left(\zeta_1 m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]\right)} \\ & \times \frac{\zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]}{\left(\zeta_1 m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]\right)} \\ & + \frac{m_2 \left[1 - J\left(b/z_2\right)\right] m_1 J'\left(\bar{p}_1\right)}{\left(m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + m_2 \left[1 - J\left(b/z_2\right)\right]\right)} \\ & \times \frac{m_2 \left[1 - J\left(b/z_2\right)\right]}{\left(m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + m_2 \left[1 - J\left(b/z_2\right)\right]\right)} \end{aligned}$$

now we know the wage share of 2 exceeds the population share of 2, so:

$$\begin{aligned} & \frac{\zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]}{\left(\zeta_1 m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]\right)} \\ > & \frac{m_2 \left[1 - J\left(b/z_2\right)\right]}{\left(m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + m_2 \left[1 - J\left(b/z_2\right)\right]\right)}. \end{aligned}$$

As a result, a sufficient condition for $\frac{dG^*}{d\bar{p}} < 0$ is that:

$$\begin{aligned} & \frac{\zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right] \left[\zeta_1' m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_1 m_1 J'\left(\bar{p}_1\right)\right]}{\left(\zeta_1 m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]\right)} \\ > & \frac{m_2 \left[1 - J\left(b/z_2\right)\right] m_1 J'\left(\bar{p}_1\right)}{\left(m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + m_2 \left[1 - J\left(b/z_2\right)\right]\right)} \end{aligned}$$

which rearranges to

$$\begin{aligned} & \frac{\zeta_1 m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_2 m_2 \left[1 - J\left(b/z_2\right)\right]}{m_1 \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + m_2 \left[1 - J\left(b/z_2\right)\right]} \\ < & \frac{\zeta_2 \left[\zeta_1' \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_1 J'\left(\bar{p}_1\right)\right]}{J'\left(\bar{p}_1\right)} \\ = & \frac{\zeta_2}{J'\left(\bar{p}_1\right)} \zeta_1' \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_1 \end{aligned}$$

Notice that the left hand side is the average wage across all firms, which is finite and bounded above by $\max\left\{\lim_{\bar{p}\to\infty} \zeta_1\left(\bar{p}\right), \zeta_2\right\}$. On the other hand the right hand side is unbounded, since $\lim_{\bar{p}\to\infty} J'\left(\bar{p}\right) = 0$ so

$$\lim_{\bar{p}\to\infty} \frac{\zeta_2}{J'\left(\bar{p}_1\right)} \zeta_1' \left[J\left(\bar{p}_1\right) - J\left(b/z_1\right)\right] + \zeta_1 = \infty.$$

A.9 Proof of Proposition 6

Notice that if $m_1 \to 0$ the three-occupation model is the same as the 2-occupation model. As a result, for sufficiently small m_1, the proof of Proposition 5 applies to the N-occupation case.

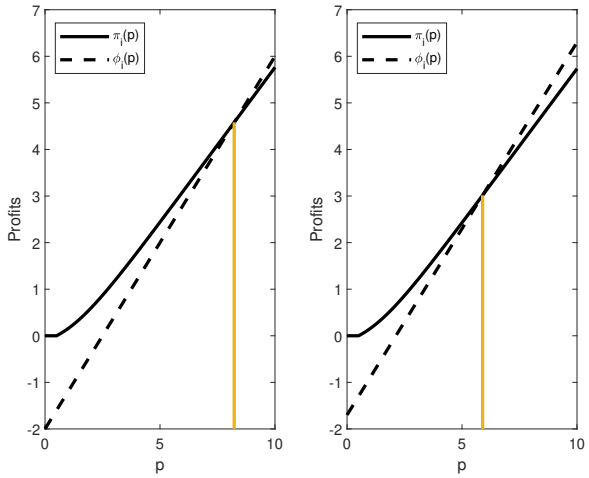


Figure 7: Numerical example 1

Note: $\bar{p}_2$ in the numerical example when $\phi(0, z_2) = 0.8p$ (left panel) and $\phi(0, z_2) = 1.1p$ (right panel). See text for other parameters.

A.10 Proof of Proposition 7

The result follows from Proposition 5 and the fact that $\bar{p}_1$ is declining in ϕ_1 and increasing in c_1.

A.11 Proof of Proposition 8

Again, notice that if $m_1 \to 0$ the N-occupation model is the same as the 2-occupation model. As a result, for sufficiently small m_1, the proof of Proposition 5 applies to the N-occupation case. The requirement that $\pi'_{N-1}(\cdot) < \kappa_i$ guarantees that $\bar{p}_{N-1}$ is finite.

B Numerical example

To show that we can have increasing cross-occupation inequality in more complex contexts, we use a numerical example. Suppose that there are three occupations with $z \in \{1, 2, 3\}$, and $b = 1$. Suppose that $J(p) = 1 - e^p$, $k = 1$, $m_i = 1/3 \forall i$. Let $c_1 = 2$, and set $\phi_1 = \phi_3 = 0$: as a result, $\bar{p}_1 = \bar{p}_3 = \infty$. In other words, robots are of no use in occupations 1 and 3. As for occupation 2, suppose that $\phi_2(p) = 0.8p$. In this case, we find that $\bar{p}_2 = 7$. In other words,

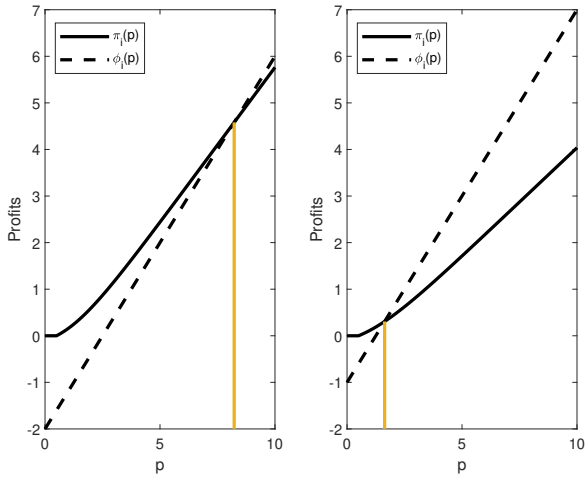


Figure 8: Numerical example 2

Note: $c = 2$ in the left panel and $c = 1$ in the right panel. See text for other parameters.

firms with $p \geq 7$ find it profitable to use robots for occupation 2.

Example 1. *One way to model an increase in the facility of robot adoption would be as an increase in* ϕ_2*, representing an* increase in the profitability *of robots in occupation* 2*. If we set* $\phi_2(p) = 1.1p$*, we can show that* $\bar{p}_2$ *declines from* 7 *to* 5.9 *– see Figure 7. The area under the Lorenz curve in this example declines when* $\bar{p}_2$ *declines as a result of* ϕ_2 *rising, from* 0.2920 *to* 0.2918*, implying an increase in the Between-occupation Gini coefficient.*

Example 2. *Alternatively, an increase in the facility of robot adoption could be modeled as a decrease in* c_2*, representing a* decrease in the cost *of robot adoption. If we set* $\phi_2(p) = 0.8p$ *as before, but lower* c *from* 2 *to* 1*, we can show that in steady state* $\bar{p}_2$ *declines from* 7 *to* 1.6 *– see Figure 8. The area under the Lorenz curve in this example declines when* c_2 *drops also, from* 0.2920 *to* 0.2894*. Again, this implies an increase in the cross-occupation Gini coefficient.*

C Empirical Appendix

C.1 Specification Robustness

Table 6 shows the results of our baseline specification detailed in equation 1, estimating the effects of contemporaneous robots on the Gini coefficients from 2000 to 2018, instead of robots lagged as in Table 2. While the estimated coefficients decrease in magnitude, they all significant and negative, i.e. consistent with the estimations with lagged robots.

Table 6: Robots and Inequality, Contemporaneous Robot Adoption

	A. Within-occupation			
	(1)	(2)	(3)	(4)
Robot Adoption	-1.070^{**}	-1.038^{*}	-0.921^{*}	-1.042^{**}
	(0.247)	(0.324)	(0.477)	(0.138)
Adj.Rsquare	0.774	0.885	0.887	0.888
	B. Between-occupation			
	(1)	(2)	(3)	(4)
Robot Adoption	0.536^{***}	0.343	0.500	0.560^{**}
	(0.006)	(0.351)	(0.353)	(0.073)
Adj.Rsquare	0.450	0.890	0.891	0.891
Baseline covariates	✓	✓	✓	✓
Year dummies	✓	✓	✓	✓
Industry dummies	✓	✓	✓	✓
Weighted		✓	✓	✓
IT capital			✓	✓
Chinese Imports				✓
Num of Obs	399	399	399	399

All regressions control for demographics, and include year and industry FE. Column (1) presents unweighted regressions. Column (2)-(4) regressions are weighted by total employment at industry level. Column(3) adds industry control, chinese imports. Column(4) include industry control, IT capital. Robust standard errors clustered at the industry level are reported in parenthesis. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 7 reports the estimates of within-occupation with Gini measures at the occupation level according to equation 30. All regressions control for demographics at the occupation level and include occupation and year FE.

$$Gini_{ot} = \beta \times Robot_exposure_{ot-1} + X_{ot} + \alpha_t + \delta_o + \epsilon_{ot} \tag{30}$$

Since the IFR, our robot data source is at the industry-year level, we build a shift-share measure of robot exposure described in equation 31,

$$Robot_Exposure_{ot} = \sum_i l_{io,1980} * Robot_Adoption_{it} \tag{31}$$

where $l_{io,1980} = L_{io,1980}/L_{o,1980}$. Notice that this type of specification with a shift-share measure only works for within-occupation inequality. This is the main reason we picked our main specification, which allows us to run for both components of inequality comparably. Nevertheless, it is worth noting that the results of this specification are not only qualitatively similar to our baseline specification (negative and significant) but also qualitatively since they imply that an increase in robot exposure equivalent to the interquartile range of the variables, is associated a decrease in within-occupation inequality of around 18-46%, although larger than our baseline estimates.

Table 7: Within-occupation Inequality and Robots, Occupation Robot Exposure

	(1)	(2)	(3)	(4)
Robot Exposure	−2.672	−7.064**	−7.836**	−17.16**
	(2.689)	(2.995)	(3.046)	(6.324)
Adj.Rsquare	0.827	0.849	0.851	0.852
Baseline covariates	✓	✓	✓	✓
Year dummies	✓	✓	✓	✓
Industry dummies	✓	✓	✓	✓
Weighted		✓	✓	✓
IT capital			✓	✓
Chinese Imports				✓
Num of Obs	396	396	396	396

All regressions include demographic controls, year dummies, and industry dummies. Column (1) presents unweighted regressions. Column (2)-(4) regressions are weighted by total employment at the occupation level. Column(3) adds industry control and Chinese imports. Column(4) includes industry control, IT capital. Robust standard errors clustered at the industry level are reported in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

C.2 Alternative Inequality Measures

To verify that an ad-hoc selection of the measure does not drive our results, we consider multiple measures that capture different aspects of the wage distribution. Within-occupation variance is the mean of the log variances of occupations; while between-occupation variance is the variance of log mean wages of occupations. We also include the Theil-L Index (the Mean Log Deviation), the Theil-T Index, and the Atkinson Index. All these inequality measures allow for additive decomposition into within and between components with no residual component.

The Theil-L index and Theil-T Index in 32 and 33 belong to the Generalized Entropy family of inequality measures. They are useful complementary measures to the *Gini* index.

$$Theil_L = -\frac{1}{N}\sum_{i=1}^{N} ln\frac{w_i}{\bar{w}} \tag{32}$$

$$Theil_T = \frac{1}{N}\sum_{i=1}^{N} \frac{w_i}{\bar{w}} ln\frac{w_i}{\bar{w}} \tag{33}$$

The Atkinson Index is a measure of inequality that incorporates a parameter that reflects the societal aversion to inequality. $\epsilon = 0.5$ means moderate aversion to inequality. The Atkinson index with $\epsilon = 1$ is more sensitive to the lower end of the income distribution comparing to the Atkinson index with $\epsilon = 0.5$. are described in equation 34 and 35:

$$Atkinson(\epsilon = 0.5) = 1 - \frac{1}{\bar{w}}\left(\frac{1}{N}\sum_{i=1}^{N} w_i^{1/2}\right)^2 \tag{34}$$

$$Atkinson(\epsilon = 1) = 1 - \frac{1}{\bar{w}}\left(\frac{1}{N}\prod_{i=1}^{N} w_i\right)^{1/N} \tag{35}$$

The *Gini* coefficient is our baseline measure of inequality, not only because it maps directly to our theory and allows to test it, but also because it is widely used and easy to interpret. However, to verify that our results are not driven by an *ad-hoc* selection of

Table 8: Effects of Robots on Inequality, Alternative Inequality Measures

	A. Within-occupation					
	(1)	(2)	(3)	(4)	(5)	(6)
Robot Adoption	−1.615**	−0.645***	−1.781**	−7.074**	−1.227**	−1.323**
	(0.253)	(0.0394)	(0.285)	(1.534)	(0.231)	(0.223)
Adj.Rsquare	0.892	0.981	0.850	0.471	0.625	0.756
	B. Between-occupation					
	(1)	(2)	(3)	(4)	(5)	(6)
Robot Adoption	0.773***	0.352**	0.265**	0.314**	0.154***	0.257***
	(0.0586)	(0.0582)	(0.0446)	(0.0512)	(0.0102)	(0.0186)
Adj.Rsquare	0.888	0.788	0.564	0.384	0.978	0.984
Num of Obs	378	378	378	378	378	378

All regressions control for demographics, IT adoption, and Chinese import competition, and include year and industry FE, while weighted by industry employment. The dependent variables are different inequality measures: Column (1) uses the Gini index; Column (2) uses the measures in Erosa et al. (2024), where the within-occupation inequality is the mean variance of log wages and the between-occupation inequality is the variance of the log mean wages; Column (3) the decomposed Theil-L Index, which is the mean log deviation; Column (4) the decomposed Theil-T Index; Column (5) uses the Atkinson Index with a moderate inequality aversion parameter of 0.5; and Column (6) also uses the Atkinson Index with a higher inequality aversion parameter of 1, more sensitive to the lower end of the wage distribution. Robust standard errors clustered at the industry level are given below. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

the inequality measure, we test five alternative measures commonly used in the inequality literature, capturing different aspects of the wage distribution.

Table 8 presents the results of running the baseline regression for each of the inequality measures discussed in Appendix C.2 as dependent variables: Column 1 uses the Gini inequality index; Column 2, following Erosa et al. (2024) considers within-occupation inequality as the mean variance of log wages, and between-occupations inequality as the variance of the log mean wages, focusing on geometric rather than arithmetic differences. Columns 3 and 4 present results with the Theil-L and Theil-T indices respectively. These are entropy-based measures of inequality with the properties that they can be additively decomposed into within and between components.[18]. The Theil-T Index is more sensitive to changes in the upper end of the income distribution than the Theil-L index. Columns 5 and 6 employ the Atkinson Index with inequality aversion parameters 0.5 and 1, respectively. Atkinson(0.5)

[18]See Shorrocks (1980)

provides a moderate level of inequality aversion, while Atkinson(1) is more sensitive to the impacts on lower wages, highlighting disparities at the lower end of the wage spectrum more than the Gini index or other measures.

In sum, both the sign and the significance of the estimates remain consistent across inequality measures.