Innovations in Entrepreneurial Finance and Top Wealth Inequality

Magnus Irie*

First version: October 2, 2023
This version: March 20, 2025

Latest version

Abstract

How does improved risk sharing for entrepreneurs affect top wealth inequality? On the one hand, better entrepreneurial financing allows entrepreneurs to scale up, which raises top wealth inequality. On the other hand, better risk sharing allows entrepreneurs to reduce their idiosyncratic risk exposures. This risk reduction effect lowers top wealth inequality by making extreme wealth trajectories less likely. In equilibrium, which of these two effects dominates depends on how much economic activity is reallocated to entrepreneurs from elsewhere in the economy when entrepreneurs scale up. When this reallocation is large, Pareto inequality can increase rapidly as risk sharing improves.

1 Introduction

A cursory glance at the names appearing on lists of wealthy Americans uncovers a striking fact: many of the richest individuals became wealthy through a risky investment in a single entrepreneurial firm. Prior work has emphasized the role of entrepreneurs with high exposures to idiosyncratic risk in explaining both the thick right tail of the wealth

*I would like to thank Andrew Atkeson, Ulf Axelson, Ian Martin, Benjamin Moll, and Ricardo Reis for their guidance and advice throughout the project. I also thank Anmol Bhandari, Fernando Arce, Cynthia Balloch, Gadi Barlevy, Emil Bustos, Joel David, François Gourio, Nils Lager, Jonas Lundstedt, Ellen McGrattan, Walker Ray, Anna Rogantini Picco, Soroush Sabet, Dimitri Vayanos, Viking Waldén, Karl Walentin, Nicolas Werquin, Roine Vestman, and numerous seminar participants, especially at Sveriges Riksbank, the Federal Reserve Bank of Minneapolis, and the Federal Reserve Bank of Chicago for helpful comments and questions. I gratefully acknowledge financial support from ESRC. An earlier version of this paper was part of my doctoral dissertation.
Department of Economics, Princeton University, e-mail: magnus.irie@gmail.com

distribution and the prevalence of newly minted fortunes at the top (Quadrini (2000), Benhabib, Bisin, and Zhu (2014), Gabaix, Lasry, Lions, and Moll (2016) Gomez and Gouin-Bonenfant (2024)). While improved entrepreneurial financing allows entrepreneurs to scale up, it also allows them to offload idiosyncratic risk. With lower levels of idiosyncratic risk, the extreme upward wealth trajectories that help account for both the thick right tail of the wealth distribution, and the prevalence of newly minted fortunes at the top, become less likely. Moreover, with less idiosyncratic investment risk, entrepreneurs' precautionary savings motives are weaker, which slows their wealth accumulation. In addition, better risk sharing means that returns to successful firms are spread over a larger set of investors. Therefore, it is not immediately clear how top wealth inequality is affected by better risk sharing. The question studied in this paper is therefore: how is top wealth inequality affected by improvements in risk sharing for entrepreneurs?

I develop a tractable general equilibrium framework to answer this question. The framework concisely summarizes the impact of improved risk sharing in three economic forces. Consider a hypothetical entrepreneur, Jeff. Suppose Jeff's financing constraints have just been relaxed. Specifically, he can now finance a larger fraction of his online bookstore startup by issuing equity to outsiders. Jeff could use the risk-sharing properties of improved equity financing to reduce his own idiosyncratic risk exposure. This *risk-reduction effect* would lower top wealth inequality in the long run by making extreme upward wealth trajectories for entrepreneurs less likely. However, with better risk sharing Jeff would also find it less risky to scale up. By scaling up, he could, with some luck, turn his small online bookstore into a retail giant. If this *scaling-up effect* is strong enough, top wealth inequality rises. The existence of these two forces related to improvements in risk sharing for entrepreneurs has been discussed in the literature.[1]

The tractability of the framework in this paper allows me to highlight a novel theoretical mechanism. Whether the risk-reduction or the scaling-up effect dominates depends on a third force: the *general equilibrium reallocation effect*. This measures the extent to which productive resources are reallocated to entrepreneurial firms from elsewhere in the economy when entrepreneurial financing improves. When every entrepreneur tries to scale up, competition among them reduces their equilibrium profitability. This reduces the attractiveness of scaling up. Entrepreneurs' equilibrium profitability is reduced because their costs of capital and labor rise, while the equilibrium prices of the goods they sell fall, as they compete for financing, labor, and customers.

[1] See Bonfiglioli (2012) and Peter (2021).

This downward pressure on entrepreneurial profitability is ameliorated if entrepreneurs as a group can poach customers, attract labor, and raise capital from elsewhere in the economy. In this paper, I explore one of the many possible determinants of the extent to which entrepreneurs can scale up in equilibrium in response to better financing. Specifically, a crucial feature of the model studied in this paper is that entrepreneurial firms compete not only with one another, but also with established traditional firms. These established traditional firms produce goods that are imperfectly substitutable with those the entrepreneurial firms produce. The higher the elasticity of substitution between these goods, the stronger the reallocation effect: with high elasticity of substitution, entrepreneurs can expand by poaching demand and productive resources from established traditional firms rather than competing down their equilibrium profitability. In this case, entrepreneurs' excess return remains relatively stable, while improved risk sharing allows them to carry less risk per dollar invested, thus improving the risk-reward trade-off they face. Then, they choose to scale up so much that their *total* risk exposure increases even if improvements in equity financing allow them to carry a smaller *fraction* of the risk in their firm. In this case, wealth inequality rises.

In contrast, when the elasticity of substitution between the goods is low, there is limited room for entrepreneurs to expand in equilibrium at the expense of the established traditional firms. Hence, the downward pressure on their excess return is high. If the excess return falls enough for the risk-reward trade-off associated with entrepreneurial activity to deteriorate, entrepreneurs choose to reduce their idiosyncratic risk exposures, lowering wealth inequality in the long run.

As a second contribution, the framework makes sense of several other empirical trends documented in U.S. data, provided the elasticity of substitution between entrepreneurial firms' goods and established traditional firms' goods is high. Most notable among these trends is the rapid growth in the share of economic activity associated with venture capital-backed firms. Other consistent trends include the stability of the accounting return to the aggregate capital stock despite these falling safe rates, and the fall in the aggregate labor share despite stable firm-level labor shares and a falling safe rate. The model exhibits these patterns precisely when the reallocation effect is strong.

Model overview. The extent to which entrepreneurs choose to bear idiosyncratic risk is an equilibrium outcome, so a comprehensive understanding of how improvements in entrepreneurial financing affect top wealth inequality requires an equilibrium model. To this end, I present a general equilibrium model where risks and expected returns

associated with entrepreneurship are endogenously determined.

The immediate precursors to the model are the modified neoclassical growth models of Angeletos (2007), Brunnermeier and Sannikov (2017), and Di Tella and Hall (2021). The model features two sectors of production: an entrepreneurial sector and a established traditional sector. The firms in the entrepreneurial sector are more productive than the established traditional firms. However, a portion of each firm's idiosyncratic risk must be borne by the associated entrepreneur for incentive alignment purposes. Equity issuance is possible but limited. The established traditional sector is less productive but has no idiosyncratic risk costs. The uninsurable risk of entrepreneurial production implies that entrepreneurs earn a positive idiosyncratic excess return. Entrepreneurs choose how much idiosyncratic risk to bear by weighing the excess return against the risk.

The allocation of capital to each type of firm is determined by the trade-off between the higher productivity of the entrepreneurial sector, the lower risk costs of the established traditional sector, and the substitutability of the goods they produce. I model improvements in entrepreneurial financing as an increase in the fraction of the firm's risk that entrepreneurs can offload to financial markets. This greater offloading reduces the risk cost associated with entrepreneurial production, which in turn triggers a reallocation of economic activity from established traditional firms to entrepreneurial firms.

The model makes stark predictions regarding the effect of improvements in risk sharing on top wealth inequality, and the effect depends on the strength of this reallocation. When the degree of substitutability between the goods the two types of firms produce is high, even minor improvements in risk sharing for entrepreneurs cause a considerable reallocation of capital, labor, and sales to entrepreneurial firms. The large reallocation means the competitive pressure among entrepreneurial firms for financing, workers, and customers is less severe. Entrepreneurs can then expand more aggressively without their expected excess returns declining much. Moreover, better risk sharing reduces the risk per unit invested. If the risk-reward trade-off improves despite the slightly lower expected excess return, then entrepreneurs scale up not only by raising more capital from outsiders, but also by investing more of their personal wealth, thereby raising their idiosyncratic risk exposures. This raises top wealth inequality by making extreme upward wealth trajectories more likely, and also, albeit to a lesser extent, by strengthening entrepreneurs' precautionary savings motive. To my knowledge, the aspect that the extent to which entrepreneurs can attract economic activity from elsewhere in the economy is crucial for the effect of better risk sharing on top inequality is new to the literature. Finally, I derive a

closed-form expression for the model's steady state level of Pareto inequality. The formula reveals an intimate link between entrepreneurs' risk exposures, the share of wealth they hold in aggregate, and the thickness of the right tail of the cross-sectional distributional wealth.

Empirical Macro-Finance Trends. Although the main subject of this study is top wealth inequality, the framework is consistent with four other key empirical trends under the assumption of a high elasticity of substitution between the goods that entrepreneurial firms and established traditional firms produce:

1. The growth in the fraction of firms with a history of venture capital-backing among the largest publicly traded firms in the U.S.[2]

2. The fall in the aggregate labor share, despite relatively stable firm-level labor shares.[3]

3. The stable or slightly rising accounting return to the aggregate capital stock despite the falling real safe interest rate.[4]

4. The fall in real safe interest rates.[5]

The first trend is directly related to the mechanism at the heart of this paper. The three others have a more subtle connection to the main mechanism. The significance of these trends arise from the fact that they are implied by the model precisely when the general equilibrium reallocation effect is so strong that the scaling-up effect dominates the risk-reduction effect. The model exhibits these trends under the same conditions under which improvements in entrepreneurial financing lead to higher top wealth inequality.

Literature. This paper contributes to the literature on the consequences of idiosyncratic investment risk and entrepreneurship for wealth inequality. This literature was pioneered by Quadrini (2000), Meh and Quadrini (2006) and Cagetti and De Nardi (2006), with recent contributions by Benhabib et al. (2014), Gabaix et al. (2016), Jones and Kim (2018), Peter (2021), Atkeson and Irie (2022), Hui (2023), Gomez and Gouin-Bonenfant (2024) and Robinson (2024). In recent years, this literature has aimed at accounting for three stylized facts: the thick right tail of the wealth distribution, the rapid dynamics of the wealth

[2]See Gornall and Strebulaev (2021) and Greenwood, Han, and Sánchez (2022).

[3]See Autor, Dorn, Katz, Patterson, and Van Reenen (2020) and Hartman-Glaser, Lustig, and Xiaolan (2019).

[4]See Reis (2022), Moll, Rachel, and Restrepo (2022) and Farhi and Gourio (2018).

[5]See Holston, Laubach, and Williams (2017), Auclert, Malmberg, Martenet, and Rognlie (2021), Rachel and Summers (2019).

distribution over time, and the prevalence of new fortunes at the top. These facts serve as important devices for disciplining models of top wealth inequality.[6]

Because the literature has emphasized the role played by entrepreneurs with high idiosyncratic risk exposures to account for these facts, studies in this literature have concluded that less restrictive debt financing could raise top wealth inequality while better risk sharing would reduce wealth inequality. For instance, in contrast with some of the results presented in this paper, recent studies by Peter (2021), Hui (2023), and Robinson (2024) conclude that improved risk sharing for entrepreneurs lowers wealth inequality. I conjecture that this is because the risk-sharing rather than the scaling-up force dominates in their settings. In the context of a static model where agents choose between a risky project and a safe project, Bonfiglioli (2012) studies the impact of better risk sharing on the extensive margin of risky entrepreneurship and how that relates to income inequality. In that context, improvements in risk-sharing *lowers* income dispersion for high-ability entrepreneurs, while at the same time inducing entry *into* risky entrepreneurship among agents with lower ability, thereby raising income dispersion among those agents. Average income inequality rises if the effect of the rise in income dispersion among lower ability agents is stronger than the effect of lower dispersion among high-ability entrepreneurs. However, because dispersion falls for high-ability entrepreneurs, improvements in risk-sharing do not generate rising inequality within the top.[7] Heyerdahl-Larsen, Illeditsch, and Kung (2023), who study how improvements in risk sharing impacts wealth and consumption inequality in a dynamic model where entrepreneurs differ in terms of their beliefs regarding the success probability of their firm, also show that improved risk-sharing induces entry into entrepreneurship on the extensive margin. The results in this paper complement those of İmrohoroğlu and Zhao (2022) and Gomez and Gouin-Bonenfant (2024), who study the impact of falling costs of capital for entrepreneurs on top wealth inequality. They emphasize that this may raise wealth inequality because entrepreneurs can more cheaply scale up their firms. The focus in the present paper is instead on improvements in risk-sharing, rather than a general fall in the cost of capital. Panageas and Gârleanu (2024) study how disruptive growth influences the demand for alternative asset classes such as private equity and venture capital, and how this interacts with the wealth accumulation process of innovators. Further, as in this paper, they study how entrepreneurs wealth accumulation process shapes the top of the wealth distribution.

[6]See for instance Kuhn and Rios-Rull (2016) and Castaneda, Diaz-Gimenez, and Rios-Rull (2003) for reviews of some of the facts and attempts to account for them.

[7]Moreover, because the model is static, it is silent on the dynamics of inequality.

Compared to models in this literature that are based on the framework of Aiyagari (1994), the model in this paper is based on Angeletos (2007), so that the economy aggregates tractably despite the presence of idiosyncratic risk. The specification of the risk-sharing environment as one where issuance of equity-like claims is possible but limited due to agency frictions is as in Di Tella (2017) and Brunnermeier and Sannikov (2017). Risk-aversion, combined with the limits to risk-sharing, induces entrepreneurs to run smaller firms than under perfect risk-sharing as in Kihlstrom and Laffont (1979) and Di Tella and Hall (2021). Relative to the precursor models, I introduce imperfect substitutability between the goods produced by the two types of firms. I also consider a demographic setup where the between-type distribution of wealth and the cross-sectional distribution of wealth are stable in the long run despite the presence of idiosyncratic risk. These modifications are essential for considering the issues at the heart of this paper: top wealth inequality, the factor income distribution, and returns to wealth in the long run.

An ongoing discussion in the literature is the extent to which the rise in wealth inequality can be accounted for by changes in the *valuations* of the assets and liabilities in the portfolios of wealthy households relative to less wealthy households, without changes the distribution of the underlying assets and liabilities themselves, perhaps driven by falling interest rates (see Kuhn, Schularick, and Steins (2020), Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2021), Cioffi (2021), Gomez (2024)). Irie (2023) points out that the increase in top wealth inequality measured in the Distributional National Accounts of Piketty, Saez, and Zucman (2018) is associated with more unequal distributions of the *income flows* that household wealth generates. When studying declining interest rates, Gomez and Gouin-Bonenfant (2024) find that lower interest rates primarily raise Pareto inequality by lowering costs of capital for entrepreneurs, not through direct valuation effects.[8] Other such explanations include changes in taxation as emphasized by Aoki and Nirei (2017), Hubmer, Krusell, and Smith. (2021) and Kaymak and Poschke (2016); Moll et al. (2022), focusing on automation; and Jones and Kim (2018) and Atkeson and Irie (2022), who focus on entrepreneurs and business owners. Recent surveys of the determinants of wealth inequality in the context of macroeconomic models include for instance De Nardi and Fella (2017), Benhabib and Bisin (2018). Aoki and Nirei (2017) attribute rising Pareto inequality to lower taxes, making entrepreneurs want to increase their exposure to their firms. The focus of the present study is instead improvements in risk sharing.

[8]Interestingly, the long-run response of the model presented in this paper includes a fall in the cost of capital as well. Nevertheless, in contrast to Gomez and Gouin-Bonenfant (2024), this is an outcome rather than a driving force.

2 Scaling Up and Risk Reduction in a Simple Framework

In this section, I present a partial equilibrium framework where improved risk sharing for entrepreneurs unambiguously leads to increases in risk-taking. In other words, the *scaling-up effect* dominates. I also provide an example of how closing the model in general equilibrium *can* result in the *risk sharing effect* instead being unambiguously dominant. I conclude the section by arguing that the particular way of closing the model in general equilibrium determines which of the two effects dominates. This motivates Section 3, where I present a general equilibrium model in which both effects operate.

2.1 Partial Equilibrium Model

Preferences. There is a continuum of entrepreneurs. We consider an individual entrepreneur $i \in [0,1]$. The entrepreneur consumes at rate c_{it} per unit of time and has logarithmic utility of the form

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(c_{it})\,dt\right]. \tag{1}$$

Technology. The entrepreneur operates a firm. The firm produces an output flow y_{it}, using capital, labor, and a Cobb-Douglas production technology. The entrepreneur subject to idiosyncratic risk:

$$\begin{aligned} y_{it}dt &= k_{it}^{\alpha} l_{it}^{1-\alpha} dt \\ \frac{dk_{it}}{k_{it}} &= (\iota_{it}-\delta)\,dt + \tilde{g}\left(l_{it}/k_{it}\right)\tilde{\sigma}dZ_{it} + \sigma dZ_t + \Delta_{it}^{k} \end{aligned} \tag{2}$$

where ι_{it} is the investment rate, δ is the depreciation rate, Δ_{it}^{k} is net purchases of capital, and $\tilde{g}\left(l_{it}/k_{it}\right)\tilde{\sigma}dZ_{it} + \sigma dZ_t$ is the risky part of capital accumulation. In particular, Z_{it} is an idiosyncratic Brownian motion, while Z_t is an aggregate Brownian motion.

The function $\tilde{g}\left(l_{it}/k_{it}\right)$ determines how the level of idiosyncratic risk depends on the firms choice of capital and labor input. I will focus on the cases where the idiosyncratic risk is proportional to the capital stock, $k_{it}\tilde{g}\left(l_{it}/k_{it}\right)\tilde{\sigma} = k_{it}\tilde{\sigma}$, as in Brunnermeier and Sannikov (2017), or on the case where the risk is proportional to the firm's output $k_{it}\tilde{g}\left(l_{it}/k_{it}\right)\tilde{\sigma} = k_{it}^{\alpha} l_{it}^{1-\alpha}\tilde{\sigma}$, as in Di Tella and Hall (2021). A tractable way to nest these two cases is to let $\tilde{g}(l_{it}/k_{it}) = (l_{it}/k_{it})^{1-\beta}$. Idiosyncratic risk being proportional to capital is then equivalent to $\beta = 1$, and idiosyncratic risk being proportional to output is then equivalent to $\beta = \alpha$. For simplicity, we let aggregate risk be proportional to capital. The entrepreneur can hire

labor at a wage rate w_t, and sell the output at price p_t^E. The instantaneous return on a unit of capital capital is

$$dR_{it}^k \equiv \underbrace{\left(\frac{p_t^E y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}}\right)}_{\text{expected return } r_{it}^k} dt + \underbrace{\tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t}_{\text{risk}} \tag{3}$$

where $\tilde{\sigma}_{it}^k = (l_{it}/k_{it})^{1-\beta}\,\tilde{\sigma}$. Because of the constant returns to scale technology, this return does not depend on the size of the firm (as measured by either y_{it}, k_{it} or l_{it}), only on k_{it}/l_{it}, the capital-labor ratio.

Firm Financing. To finance the capital stock operated by the firm, the entrepreneur uses a combination of personal funds and liabilities issued to outsiders. The liabilities issued to outsiders consist of a combination of a riskless security d_{it}, and a risky security e_{it}. The capital structure of the firm is

$$k_{it} = v_{it}^{\text{in}} + \underbrace{e_{it} + d_{it}}_{v_{it}^{\text{out}}} \tag{4}$$

where v_{it}^{in} is the amount financed by the entrepreneur, and v_{it}^{out} is the amount financed by outsiders. The risky security issued to outsiders carries the same risk as the risk in the firm's capital, while the riskless security is, as the name suggests, without risk. The instantaneous returns to these securities are therefore

$$\begin{aligned} dR_t^d &= r_t dt \\ dR_{it}^e &= r_t^{\text{e}} dt + \tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t, \end{aligned} \tag{5}$$

where the required expected return associated with issuing the riskless security is the risk-free rate r_t, and required expected return associated with the risky security is r_t^{e}. Outsiders can diversify away all idiosyncratic risk so no-arbitrage implies that the required return on the risky security satisfies $r_t^{\text{e}} = r_t + \varsigma_t \sigma$, where ς_t is the price of aggregate risk. The instantaneous return on the portfolio of outside liabilities is therefore

$$dR_{it}^{\text{out}} = r_{it}^{\text{out}} dt + \vartheta_{it}\left(\tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t\right) \tag{6}$$

where $\vartheta_{it} \equiv e_{it}/v_{it}^{\text{out}}$ is the share of the risky claim in the liability portfolio, and the required expected return is

$$r_{it}^{\text{out}} \equiv \vartheta_{it} r_t^e + (1 - \vartheta_{it}) r_t = r_t + \varsigma_t \vartheta_{it} \sigma. \tag{7}$$

Following Brunnermeier and Sannikov (2017) and Di Tella (2017), risk sharing through issuance of the risky security is limited. In particular, the entrepreneur is subject to a skin-in-the-game constraint:

$$(k_{it} - e_{it})\left(\tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t\right) \geq \chi k_{it}\left(\tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t\right) \Rightarrow k_{it} - \vartheta_{it} v_{it}^{\text{out}} \geq \chi k_{it} \tag{8}$$

where χ is the fraction of the firm's risk that the entrepreneur must bear.[9]

Entrepreneurs' Problem. In addition to making decisions regarding the operation of the firm, the entrepreneur also solves a portfolio choice problem in deciding how to invest their personal net worth and how much to consume and save. In addition to investing in their own firm, the entrepreneur can also invest in a fully diversified fund consisting of the risky liabilities issued by the other firms. By no-arbitrage, this fund has expected return $r_t^{\text{fund}} = r_t + \varsigma_t \sigma$. We denote the amount the entrepreneur invests in this fund by v_{it}^{fund}. By adjusting the holdings of this diversified fund, the entrepreneur can fully control their exposure to aggregate risk, even if the skin-in-the-game constraint is binding. In other words, allowing the entrepreneur to trade this diversified fund means that there are no limits to the sharing of *aggregate* risk. Finally, we allow the entrepreneur to directly trade riskless bonds, denoting holdings of riskless bonds by b_{it}. The entrepreneur's net worth n_{it} can be written as:

$$n_{it} = v_{it}^{\text{in}} + v_{it}^{\text{fund}} + b_{it} = k_{it} - v_{it}^{\text{out}} + v_{it}^{\text{fund}} + b_{it} \tag{9}$$

and the entrepreneur's problem can be stated as

$$\max_{\{c_{it}, k_{it}, l_{it}, v_{it}^{\text{out}}, \vartheta_{it}, v_{it}^{\text{fund}}, b_{it}\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log(c_{it})\, dt\right]$$

$$dn_{it} = \left(p_t^E y_{it} - w_t l_{it} - \delta k_{it} - v_{it}^{\text{out}}(r_t + \vartheta_{it}\varsigma_t\sigma) + v_{it}^{\text{fund}}(r_t + \varsigma_t\sigma) + b_{it} r_t - c_{it}\right) dt$$

$$+ (k_{it} - v_{it}^{\text{out}}\vartheta_{it})\tilde{\sigma}_{it}^k dZ_{it} + \left((k_{it} - v_{it}^{\text{out}}\vartheta_{it}) + v_{it}^{\text{fund}}\right)\sigma dZ_t$$

$$\text{subject to skin-in-the-game:} \quad (k_{it} - v_{it}^{\text{out}}\vartheta_{it}) \geq \chi k_{it}, \quad \text{and} \quad n_{it} \geq 0.$$

Proposition 1. *The following statements characterize features of the entrepreneurial firms' capital structure:*

1. *The skin-in-the-game constraint is binding in optimum.*

[9] This form of skin-in-the-game constraint is microfounded in Di Tella (2017).

2. *The structure of the liabilities issued to outsiders, ϑ_{it}, is irrelevant to the entrepreneurs' optimization problem in the sense that any value of ϑ_{it} can be made consistent with optimality.*

Proof in Appendix A.1.

The intuition behind this proposition is that the logic of Modigliani and Miller (1958) is still intact as long as the skin-in-the-game constraint holds. Apart from the fact that the entrepreneur has to carry some fraction of their firm's idiosyncratic risk, the entrepreneur can undo any effect of their firm's capital structure on their personal net worth by taking appropriate positions in the non-firm related parts of their personal portfolio. The importance of this proposition is that improvements in risk sharing, modeled as a relaxation of the skin-in-the-game constraint, will generally affect entrepreneurs' decisions because the constraint is binding. In addition, note that by choosing ϑ_{it} dynamically, the entrepreneur has in principle a lot of freedom to structure the outside liabilities, so more complicated contracts than straight equity and riskless debt are also possible. The fact that the liabilities issued to outsiders can take on very general forms, subject to the skin-in-the-game constraint, demonstrates that what is economically relevant is that risk sharing is constrained. It is therefore without loss of generality to assume $\vartheta_{it} = 1$.

To solve the entrepreneur's problem, we express it recursively. The entrepreneur's net worth n_i is a state variable, and the aggregate state of the economy that determines the set of prices is governed by some vector of state variables S_t following some well-behaved Ito process. As shown in Appendix A.2, the solution to the entrepreneur's problem is

$$
\begin{aligned}
& c_{it} = \rho n_{it}, \quad k_{it} = \frac{r_t^k - r_t^{\text{fund}}}{(\chi \tilde{\sigma}_t^k)^2} n_{it} \\
& v_{it}^{\text{fund}} = \frac{r_t^{\text{fund}} - r_t}{\sigma^2} n_{it} - \chi k_{it}, \quad v_{it}^{\text{out}} = (1-\chi) k_{it}, \quad b_{it} = n_{it} - k_{it} + v_{it}^{\text{out}} - v_{it}^{\text{fund}} \\
& \qquad \text{where} \quad r_t^k = \frac{p_t^E y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}}, \qquad \tilde{\sigma}_t^k = \left(\frac{l_{it}}{k_{it}}\right)^{1-\beta} \tilde{\sigma}, \\
& \qquad \text{and} \quad \frac{l_{it}}{k_{it}} \quad \text{solves :} \quad (\delta + r_t^{\text{fund}}) \frac{1-\beta}{1-\alpha} = \left(\frac{\alpha-\beta}{1-\alpha}\right) p_t^E \left(\frac{l_{it}}{k_{it}}\right)^{1-\alpha} + \left(\frac{\beta}{1-\alpha}\right) w_t \frac{l_{it}}{k_{it}}
\end{aligned}
\tag{10}
$$

provided prices are such that the entrepreneur wants to operate a firm at all.[10] Most of the expressions in (10) are familiar from a standard Merton optimal portfolio choice problem (see Merton (1992)). The optimal capital-labor ratio depends on the rental rate of capital,

[10] For this we need $r_t^k - r_t^{\text{fund}} > 0$ so that entrepreneur's are compensated for idiosyncratic risk.

the wage rate, and the price of the firm's output, as well as the extent to which changing the capital-labor ratio affects the level of idiosyncratic risk. In the two special cases where $\beta = 1$ (risk scales with capital) or $\beta = \alpha$ (risk scales with output), we have the following closed-form expressions for $\frac{l_{it}}{k_{it}}$.

$$\frac{l_{it}}{k_{it}} = \begin{cases} \left(\frac{p_t^E}{w_t}(1-\alpha)\right)^{1/\alpha}, & \beta = 1 \\ \frac{1-\alpha}{\alpha}\frac{r_t^{\text{fund}}+\delta}{w_t}, & \beta = \alpha. \end{cases} \tag{11}$$

The purpose of the rest of Section 2 is to understand how improvements in risk-sharing changes an entrepreneur's willingness to bear risk. This is most clearly accomplished when $\beta = 1$, so for the rest of Section 2, I take this to be the case. In both cases, due to the constant returns to scale technology, l_{it}/k_{it} is independent of i, so that all decision rules are proportional to the entrepreneur's net worth. What limits the size of the firm? Risk aversion makes the entrepreneur reluctant to issue too much of the riskless security, because this increases the entrepreneur's risk exposure through leverage. In this sense, it is the risk aversion of the entrepreneur *together* with the limits to risk sharing that limits the size of their firms, rather than a borrowing constraint or a limit on overall external financing. I purposely model limitations to risk sharing in entrepreneurial financing in a stylized fashion. Specifically, I model improvements in sharing for entrepreneurial firms as a fall in χ, the minimum fraction of the idiosyncratic risk that the entrepreneur has to retain. This is motivated by the fact that this paper focuses on the consequences of improvements in the sharing of idiosyncratic risk, rather than on the sources and particularities of those improvements.

Interpreting χ: Insider Financing versus Insider Ownership. How should we interpret χ? Formally, χ is the fraction of the firms risk that the entrepreneur has to retain, and $1-\chi$ the fraction that can be offloaded to outsiders. However, as can be seen from the solution to the entrepreneur's problem in (10), $1-\chi$ is the fraction of the firm that is *financed* by outsiders. In light of this, it is tempting to map χ to the fraction of the firm that the entrepreneur owns. But this would not be correct. To understand why, let p_{it} denote the price per share and $1-s_{it}$ be the number of shares issued to outsiders to finance the capital stock at time t. We then have $p_{it}(1-s_{it}) = (1-\chi)k_{it}$. In order for outsiders to earn their required expected rate of return r_t^{fund} we must have

$$(1-\chi)r_t^{\text{fund}} = (1-s_{it})r_t^k \tag{12}$$

Only if the return to entrepreneurial capital is equal to the cost of capital will the outsider's financing fraction, $1-\chi$, equal their ownership fraction, $1-s_{it}$.

2.2 Partial Equilibrium: The Scaling-Up Force Dominates

The entrepreneur's problem has the following well-known solution for the optimal choice of investment in the idiosyncratically risky asset, captured by the optimal firm size relative to net worth (see Merton (1992) or Brunnermeier and Sannikov (2017)):

$$\frac{k_{it}}{n_{it}} = \frac{r_t^k - r_t^{\text{fund}}}{(\chi\tilde{\sigma})^2}. \tag{13}$$

The entrepreneurs' optimal idiosyncratic risk exposure, defined as the idiosyncratic volatility of net worth, implied by this solution is

$$\tilde{\sigma}_t^E \equiv \frac{k_{it}}{n_{it}}\chi\tilde{\sigma} = \underbrace{\frac{r_t^k - r_t^{\text{fund}}}{\chi\tilde{\sigma}}}_{\text{Appraisal ratio}} \tag{14}$$

where $\tilde{\sigma}_t^E$ is the resulting idiosyncratic volatility of the entrepreneur's net worth. In other words, the entrepreneur chooses an exposure to the idiosyncratic risk equal to the appraisal ratio associated with investing in entrepreneurial capital, taking into account that they only carry a fraction χ of the risk. This appraisal ratio, sometimes called the information ratio, is a close cousin of the more well-known Sharpe ratio but measures instead the risk-reward trade-off associated with investing in an asset with idiosyncratic risk relative to one with the same systematic risk but no idiosyncratic risk.

The wage rate w_t and the output price p_t^E determine r_t^k, so taking these prices and the return on the diversified portfolio, r_t^{fund}, as given, it is clear that improved risk sharing would induce the entrepreneur to increase their exposure to idiosyncratic risk. This is the *scaling-up force* in action. When risk sharing improves so that entrepreneurs carry a smaller fraction of the risk, they scale up so much that their risk exposure rises:

Lemma 1. *With constant prices, a fall in χ raises the appraisal ratio $\frac{r_t^k - r_t^{fund}}{\chi\tilde{\sigma}}$ and, therefore, entrepreneurs' optimal risk exposure.*

This lemma would hold even if agents had CRRA preferences. The key condition is that the skin-in-the-game condition is binding and that $r_t^k > r_t^{\text{fund}}$. However, if the ability to scale up with maintained profitability is limited or if the skin-in-the-game constraint is

not binding, then the lemma would not apply. This could be the case if the production function exhibited substantially decreasing returns to scale, if entrepreneurs faced other constraints limiting their ability to scale up, or if entrepreneurs are so optimistic compared to outsiders that they require no risk premium to carry the idiosyncratic risk associated with their firm (see appendix B.3 for a brief discussion on generalizations).

2.3 An Example of the Risk-Reduction Effect Dominating

The expected excess return $r_t^k - r_t^{\text{fund}}$ is of course an equilibrium object, and it is easy to see that the effect of improved risk sharing on entrepreneurs' risk exposure can easily go the other way around in equilibrium. As an example of a setting where the risk-reduction effect instead dominates, consider a framework in which, in equilibrium, the aggregate capital stock of the economy K_t is limited by the aggregate net worth of the entrepreneurs $N_t^E \equiv \int_i n_{it} di$. An example of such a framework would be one in which entrepreneurs were the only capital owners in the economy, and the aggregate capital stock constituted aggregate wealth.[11] In such a setting, the optimal portfolio choice of entrepreneurs (13) combined with the condition $K_t = N_t^E$ would imply the following.

$$\frac{r_t^k - r_t^{\text{fund}}}{\chi \tilde{\sigma}^2} = \frac{k_{it}\chi}{n_{it}} = \frac{K_t \chi}{N_t^E} = \chi \Rightarrow r_t^k - r_t^{\text{fund}} = (\chi\tilde{\sigma})^2$$

so that the *equilibrium* excess return is in fact proportional to χ^2. In this economy, entrepreneurs' equilibrium risk exposure would then be

$$\tilde{\sigma}_t^E = \frac{r_t^k - r_t^{\text{fund}}}{\chi\tilde{\sigma}} = \chi\tilde{\sigma}. \tag{15}$$

In this case, a looser skin-in-the-game constraint (a fall in χ) instead leads to a fall in the risk exposure. In other words, the partial equilibrium result in Lemma 1 is completely reversed. The intuition behind this result is straightforward. When risk sharing improves so that entrepreneurs can carry a smaller fraction of the risk in their firm, they attempt to scale up. However, in equilibrium, they cannot all scale up in the aggregate in this setting. To ensure that entrepreneurs are content with operating the existing capital stock, the excess return has to fall. Because firm sizes (relative to the entrepreneurs' net worth) are the same as before, but entrepreneurs now hold a smaller fraction of the risk, their total risk exposure is lower. The risk reduction effect dominates. The fall in the excess

[11] Entrepreneurs can still partially share risks in such a setting by buying equity in each other's firms.

return can be accomplished by a rising wage rate w_t, a falling goods price p_t^E, or a rising cost of capital r_t^{fund}, or some combination of these.

The intuition behind the contrasting results in Section 2.2, where excess returns are held stable, and the particular general equilibrium example in this section, where firm sizes relative to entrepreneurs' net worth are fixed in the aggregate, can be understood in terms of the sensitivity of the excess return $r_t^k - r_t^{\text{fund}}$ when entrepreneurs scale up: if neither the excess return nor the amount of capital relative to entrepreneurs' net worth are fixed exogenously, the equilibrium response of entrepreneurs' choice of risk exposure will depend on exactly how sensitive excess returns are when entrepreneurs try to scale up. When entrepreneurial firms are the only firms in the economy, their ability to scale up (relative to their own net worth) is severely limited in equilibrium. Therefore, one way to avoid this stark implication is to introduce other types of firms into the economy.[12] The entrepreneurial firms will then be able to scale up in the aggregate by attracting productive resources and demand from these other firms.

In Section 3, I present precisely such a model. In that model, the ease with which entrepreneurial firms can attract economic activity from other firms in the economy is governed by the elasticity of substitution between the goods that entrepreneurial firms produce and those that these other firms produce. When the elasticity is high, entrepreneurial firms will be able to attract a lot of economic activity from the other firms in the economy as entrepreneurial financing improves, and the resulting equilibrium will resemble the partial equilibrium framework in Section 2.2, where the scaling-up effect dominates. When the elasticity is low, entrepreneurs will have a hard time attracting economic activity from these other firms, and the risk-reduction effect dominates.

3 Closing the Model in General Equilibrium

To close the model, I add another type of household and another type of firm to the economy. Specifically, in addition to entrepreneurs, the model will also feature diversified investors. Moreover, in addition to the entrepreneurial firms, the model will also include a standard neoclassical firm. The entrepreneurs-operated firms will be more productive than the neoclassical firm, but will face limits to risk sharing, as in Section 2. The neoclassical firm, called the traditional firm, will not face any financing constraints. The substitutability of the goods produced by the entrepreneurial firms and those produced by the traditional firm will be governed by a constant elasticity of substitution (CES) pa-

[12] Another way would be to allow capital inflow from abroad.

rameter ε. This makes the model into a modified version of the models in Angeletos (2007) and Di Tella and Hall (2021). The key qualitative differences are that the entrepreneurial firms can also be publicly traded as in Brunnermeier and Sannikov (2017), that there in addition to entrepreneurial capitalists are both diversified capitalists and hand-to-mouth workers, that the substitutability of the goods produced by the different types of firms is limited, and the details of the demographic setup discussed next.

3.1 Model Setup

Demographics. The demographics in the model are set to allow the distribution of wealth to be stable in the long run. Specifically, the economy is populated by a continuum $i \in [0, 1]$ of capitalists and a representative hand-to-mouth worker. The group of capitalists consists of two types: entrepreneurs and diversified capitalists, denoted by E and D, respectively. Entrepreneurs own a project and can choose to run a firm based on this project. Diversified capitalists do not have a viable project, and instead passively invest their wealth. Entrepreneurs lose their ability to operate a firm at a rate ϕ^l and then become diversified capitalists. I adopt an OLG framework similar to that in Gârleanu and Panageas (2015). Specifically, capitalists die at a rate $\tilde{\delta}_d$. When this happens, the capitalist is replaced with offspring who either inherit the wealth and type of their parent, leaving the dynasty intact, or the dynasty breaks, and the new agent is reborn with the average wealth level. The probability that the dynasty is broken conditional on death is π_0. We denote by $\delta_d = \tilde{\delta}_d \pi_0$, the rate at which dynasties break up. When dynasties are broken, the newborn agent becomes an entrepreneur with probability ψ^0. Setting the initial fraction of entrepreneurs in the economy to $\bar{\psi} = \frac{\delta_d \psi^0}{\delta_d + \phi^l}$ ensures that the population structure remains intact over time.

Traditional Firm and Final Output Firm. In addition to the entrepreneurial firms discussed in Section 2.1, this general equilibrium model features two additional types of firms. First, there is a representative *Traditional Firm* that produces an intermediate good that is substitutable with the good produced by the entrepreneurial firm. Second, there is a final output producing firm that buys intermediate goods from the entrepreneurial firm and the representative traditional firm to produce a final output good.

Traditional Firm's Technology. The representative traditional firm is entirely standard and owns and operates a capital stock K_t^T that evolves according to

$$\frac{dK_t^T}{K_t^T} = \left(\iota_t^T - \delta\right) dt + \sigma dZ_t + d\Delta_t^{K,T} \tag{16}$$

where ι_t^T and δ are the investment and depreciation rates respectively, $d\Delta_t^{T,k}$ is net purchases of capital, and Z_t is an aggregate shock. The traditional firm uses a standard Cobb-Douglas technology to produce an output flow

$$Y_t^T dt = (K_t^T)^\alpha (L_t^T)^{1-\alpha} dt. \tag{17}$$

The good is sold at price p_t^T.

Traditional Firm's Financing. The traditional firm finances its capital stock externally by issuing liabilities to the capitalists in the economy. The capital structure of the traditional firm is simply $K_t^T = V_t^{T,\text{out}}$, where $V_t^{T,\text{out}}$ is the total value of liabilities issued. The cost of outside capital is determined on the same market as for the entrepreneurial firms. By a no-arbitrage argument, the required expected return on this capital is the same as for the diversified fund, $r_t^{\text{fund}} = r_t + \varsigma_t \sigma$, because the risk in the liabilities is inherited from the risk in the capital stock, and the price of aggregate risk is ς_t. In summary, the instantaneous return on the liabilities issued by the traditional firm is

$$dR_t^{\text{fund}} = r_t^{\text{fund}} dt + \sigma dZ_t. \tag{18}$$

Traditional Firm's Problem. The traditional firm maximizes expected profit flows,

$$\begin{aligned} \pi_t^T = \max_{L_t^T, K_t^T, V_t^{T,\text{out}}} & p_t^T Y_t^T - w_t L_t^T - \left(\delta + r_t^{\text{fund}}\right) V_t^{T,\text{out}} \\ \text{subject to } & K_t^T = V_t^{T,\text{out}}. \end{aligned} \tag{19}$$

The investment decision drops out of the optimization problem because investing one unit of capital decreases cash flows by one unit but instantaneously increases the value of the capital stock by one unit. In optimum, wages and rates of returns are equated to the value of marginal products of the respective factors of production:

$$w_t = p_t^T (1-\alpha) \frac{Y_t^T}{L_t^T} \quad \text{and} \quad r_t^{\text{fund}} + \delta = p_t^T \alpha \frac{Y_t^T}{K_t^T}. \tag{20}$$

The optimal labor-capital ratio is therefore

$$\frac{L_t^T}{K_t^T} = \frac{1-\alpha}{\alpha} \frac{r_t^{\text{fund}} + \delta}{w_t}. \tag{21}$$

Idiosyncratic Risk. In this section, I follow Di Tella and Hall (2021) and let entrepreneurs'

idiosyncratic risk be proportional to output. That is, I consider the case where $\beta = \alpha$ so that idiosyncratic risk is proportional to output y_{it}. This has two implications. First, it avoids distorting the trade-off between capital and labor when adding idiosyncratic risk to entreoeneurial production. Specifically, as explained in Di Tella and Hall (2021), when idiosyncratic risk is proportional to output, expanding production increases risk whether it is done by hiring workers or employing capital. Because idiosyncratic risk does not distort the capital-labor choice when it is is proportional to output, both types of firms will choose the same labor-to-capital ratio. This avoids having to keep track of the different labor-to-capital ratios, preserving tractability. Second, it has interesting implications for the factor income distribution in entrepreneurial firms. In particular, the income generated by an entrepreneurial firm is split between workers, providers of outside capital, and the entrepreneur. When risk is proportional to output, the entrepreneurial share of income comes at the expense of both the labor share and the (outside) capital share. I discuss the empirical relevance of this further in Section 4.4.

Final Output Firm. Final output Y_t is produced by a representative firm using a CES technology and the two types of intermediate goods,

$$Y_t dt = \left[\nu \left(Y_t^E \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu) \left(Y_t^T \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} dt$$

where $\varepsilon > 0$ is the elasticity of substitution between the intermediate goods and $\nu \in (0,1)$ is a share parameter.[13] The final goods producer's first-order conditions are

$$p_t^E = \nu \left(\frac{Y_t^E}{Y_t} \right)^{-\frac{1}{\varepsilon}}, \quad p_t^T = (1-\nu) \left(\frac{Y_t^T}{Y_t} \right)^{-\frac{1}{\varepsilon}}. \tag{22}$$

Because the capital-labor ratio is the same in both types of firms, aggregate final goods output can be written as

$$Y_t = A(\kappa_t) K_t^{\alpha} L_t^{1-\alpha} \tag{23}$$

where share of the capital stock operated by entrepreneurial firms is denoted $\kappa_t = \frac{K_t^E}{K_t}$, and

[13] Note that this is not a model with monopolistic competition. Entrepreneurs are in perfect competition with each other. The parameter ε governs the intensity of the implicit competition between the two *sectors*. Relative to capital adjustment costs, this imperfect substitutability assumption is more tractable. This is because sector-specific capital adjustment costs are both an intratemporal and intertemporal friction.

where the aggregate total factor productivity (TFP) is

$$A(\kappa_t) = \left[\nu\left(\kappa_t\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(1-\kappa_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{24}$$

Diversified capitalists and workers. Although there is technically a continuum of diversified capitalists, it is without loss of generality to treat them by means of a representative diversified capitalist. Diversified capitalists have log utility and net worth N_t^D as a group. They invest this net worth in the diversified fund consisting of the risky liabilities issued by all firms $V_t^{D,\text{fund}}$, and riskless bonds B_t^D. Setting up their problems in the same way as for entrepreneurs, we have

$$\begin{aligned} &\max_{C_t^D, V_t^{D,\text{fund}}, B_t^D} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(C_t^D)dt\right] \\ &dN_t^D = \left(r_t B_t^D + V_t^{D,\text{fund}} r_t^{\text{fund}} - C_t^D\right)dt + V_t^{D,\text{fund}}\sigma dZ_t. \end{aligned} \tag{25}$$

We find that their optimal decisions are given by the standard Merton portfolio choice rules

$$C_t^D = \rho N_t^D, \quad V_t^{D,\text{fund}} = \frac{r_t^{\text{fund}} - r_t}{\sigma^2} N_t^D, \quad B_t^D = N_t^D - V_t^{D,\text{fund}}. \tag{26}$$

Finally, hand-to-mouth workers supply labor inelastically and consume their labor income so that $C_t^W = w_t L$.

Aggregates. The aggregate net worth in the economy is $N_t = N_t^E + N_t^D$, where $N_t^j = \int_{i\in j} n_{it}\, di$ is the financial net worth of capitalists of group $j \in \{E, D\}$. The share of net worth held by entrepreneurial capitalists is denoted by $\eta_t = N_t^E/N_t$. The net worth $N_t = N_t^E + N_t^D$ consists of claims on the real capital stock of the economy K_t. In other words, we have $K_t = N_t^E + N_t^D$. Aggregate investment in the economy is output less consumption. The aggregate capital stock and entrepreneurs' wealth share evolves according to

$$dK_t = \left(Y_t - C_t^E - C_t^D - C_t^W - \delta K_t\right)dt + \sigma K_t dZ_t, \quad \text{and} \quad d\eta_t = d\left(N_t^E/K_t\right) \tag{27}$$

where C_t^E is the consumption of entrepreneurial capitalists as a group.

Competitive equilibrium. A competitive equilibrium consists of an aggregate state $\{K, \eta\}$, a distribution of wealth across capitalists $\{n_i\}_i$, prices $\{r^{\text{fund}}, w, \varsigma, p^E, p^T\}$, decision rules for entrepreneurs $\{c_i, k_i, l_i, v_i^{\text{fund}}, v_i^{\text{out}}, b_i\}$, diversified capitalists $\{C^D, V^{D,\text{fund}}, B^D\}$, workers $\{C^W\}$, the traditional firm $\{K^T, V^{T,\text{out}}, L^T\}$, and final output firm $\{Y^E, Y^T\}$ such that:

1. Given prices, the decisions of entrepreneurs, diversified capitalists, the traditional

firm, and the final output firm satisfy (10), (26), (20), (22), respectively. Worker's consumption is $C^W = wL$.

2. Market's clear:

$$\begin{aligned} K &= \int_{i\in E} k_i di + K^T, \quad L = \int_{i\in E} l_i di + L^T \\ V^{D,\text{fund}} &+ \int_{i\in E} v_i^{\text{fund}} di = V^{T,\text{out}} + \int_{i\in E} v_i^{\text{out}} di \\ B^D &+ \int_{i\in E} b_i di = 0, \quad N^D + \int_{i\in E} n_i di = K \end{aligned} \tag{28}$$

3. The law of motion of the aggregate state satisfies (27).

3.2 Characterizing The Equilibrium

In this section, I begin by characterizing the equilibrium of the model at a given point in time by considering the interactions between supply and demand for the factors of production to entrepreneurial firms and traditional firms. I then characterize the evolution of the aggregate state. Given values of the state variables, K_t and η_t, the equilibrium fraction of the capital stock operated by entrepreneurial firms, denoted $\kappa_t = \frac{K_t^E}{K_t}$, and the equilibrium excess return to entrepreneurial capital $r_t^k - r_t^{\text{fund}}$ are jointly pinned down by the following system of equations:

$$\frac{\kappa_t}{\eta_t}\left(\chi\tilde{\sigma}_t^k\right)^2 = r_t^k - r_t^{\text{fund}}, \quad \text{and} \quad r_t^k - r_t^{\text{fund}} = \left(p^E(\kappa_t) - p^T(\kappa_t)\right)\left(\frac{L}{K_t}\right)^{1-\alpha} \tag{29}$$

where the prices are expressed as functions of κ_t as

$$p^E(\kappa) = \nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}, \quad p^T(\kappa) = (1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}, \tag{30}$$

and aggregate TFP $A(\kappa_t)$ is defined in equation (24). The first of these equations is the relative supply of capital to entrepreneurial firms. It is relative because the quantity variable is κ_t, the *fraction* of the aggregate capital stock operated by the entrepreneurial firms, and the price variable is the *excess return* $r_t^k - r_t^{\text{fund}}$. This comes directly from the solution to the entrepreneurs' problem in (10), noting that the linearity of entrepreneurs' decision rules implies $\frac{k_{it}}{n_{it}} = \frac{K_t^E}{N_t^E} = \frac{\kappa_t}{\eta_t}$. The supply is upward sloping in the excess return to capital in the entrepreneurial sector as entrepreneurs are willing to invest larger amounts of capital in their firms when the excess return is high. From an asset pricing and portfolio

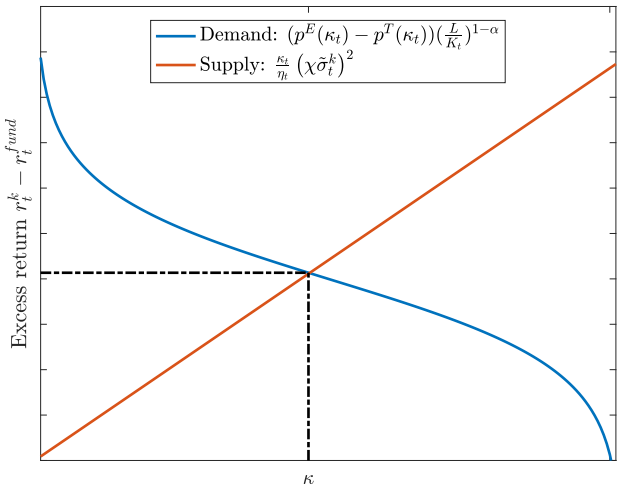


Figure 1: The equilibrium allocation of capital to entrepreneurs and excess return to entrepreneurial capital.

choice perspective, this is commonly referred to as the entrepreneurs' risky asset *demand*, productive capital being the risky asset. Of course, an entrepreneur's demand for capital as an investment vehicle constitutes the supply of capital to that entrepreneur's firm.

The second equation is instead the entrepreneurial firms' relative demand schedule, which can be derived by combining market clearing for capital, $K_t^E = K_t - K_t^T$, with the fact that the traditional sector's demand for capital is $K_t^T = \alpha\left(\frac{Y_t^T}{r_t^{\text{fund}}+\delta}\right)$, according to (20). Given this equilibrium allocation of productive resources across the two sectors, all prices and quantities are pinned down at the given point in time. These time-t prices and quantities determine the evolution of the state variables K_t and η_t going forward. These results are summarized in the following proposition (proof in Appendix A.3).

Proposition 2. *Given values of K_t and η_t, and provided $\varepsilon > 0$, there is a unique solution $\kappa_t \in [0,1]$ to the system of equations in (29). Given this solution, the relative prices of the intermediate goods are given by (30), while the other prices are*

$$\begin{aligned} r_t^{fund} + \delta = p_t^T \alpha \left(\frac{L}{K_t}\right)^{1-\alpha}, \quad r_t^k = r_t^{fund} + \frac{\kappa_t}{\eta_t}\left(\chi\tilde{\sigma}_t^k\right)^2 \\ r_t = r_t^{fund} - \sigma^2, \quad w_t = p_t^T(1-\alpha)\left(\frac{L}{K_t}\right)^{-\alpha}. \end{aligned} \tag{31}$$

The evolution of an individual entrepreneur's wealth is

$$\frac{dn_{it}}{n_{it}} = \left(r_t^E - \rho\right)dt + \tilde{\sigma}_t^E dZ_{it} + \sigma dZ_t \tag{32}$$

where the entrepreneurs' idiosyncratic risk exposure is

$$\tilde{\sigma}_t^E = \frac{\kappa_t}{\eta_t}\chi\tilde{\sigma}_t^k = \frac{r_t^k - r_t^{fund}}{\chi\tilde{\sigma}_t^k} = \textit{Appraisal Ratio},$$

and $r_t^E = r_t^{fund} + \left(\tilde{\sigma}_t^E\right)^2$ *is the expected return to the entrepreneurs' invested wealth. Finally, the system of stochastic differential equations that govern the evolution of the aggregate capital stock and entrepreneurs' wealth share is*

$$\begin{aligned}\frac{dK_t}{K_t} &= \left(\kappa_t r_t^k + (1-\kappa_t)r_t^{fund} - \rho\right)dt + \sigma dZ_t \\ \frac{d\eta_t}{\eta_t} &= \left((1-\eta_t)\left(\tilde{\sigma}_t^E\right)^2 + \frac{(\bar{\psi}-\eta_t)}{\eta_t}(\delta_d + \phi^l)\right)dt.\end{aligned} \tag{33}$$

Note that the expected return on entrepreneurs' invested wealth is higher than for diversified agents, $r_t^E > r_t^{\text{fund}}$. Because both types exhibit the same consumption rate out of wealth, ρ, this implies that entrepreneurs have a higher savings rate out of income.[14]

3.3 Steady State

This section focuses on characterizing the steady state equilibrium of the model. Strictly speaking, a steady state of the economy is characterized by a pair of values for the capital stock and entrepreneurs' wealth share, K_{ss} and η_{ss}, such that $\frac{dK_t}{K_t} = 0$ and $\frac{d\eta_t}{\eta_t} = 0$ in equation (33). The presence of aggregate shocks to capital will prevent the economy from reaching, let alone staying in, any steady state. For this reason, I study the economy's behavior along a path of zero realized aggregate shocks. In other words, I set $dZ_t = 0$, which corresponds to studying the median path of the economy. This differs from shutting down aggregate shocks by setting $\sigma = 0$. In particular, we study the *realized* behavior of the economy in a setting where shocks are still possible but happen not to materialize. I assume throughout (wherever necessary) that parameters and initial values of the state variables are such that the economy will converge to a steady state. I begin by deriving a closed-form formula for Pareto tail inequality in steady state as a function of the steady state wealth share of entrepreneurs. I thereby show a direct analytical link between the share of wealth entrepreneurs hold and the level of tail inequality. In particular, tail inequality will increase when entrepreneurs hold a larger fraction of wealth. Then, I describe how the steady state risk-reward trade-off that entrepreneurs face pins down the

[14]See Online Appendix B.18 for a brief discussion on how this relates to observed patterns of savings rates and returns across the wealth and income distributions.

amount of risk they bear and how that, in turn, determines Pareto inequality.

Entrepreneurs' wealth share and Pareto inequality. In a steady state, the drift and volatility governing the wealth accumulation process of each entrepreneur is described by a geometric Brownian motion. In particular,

$$\frac{dn_{it}}{n_{it}} = \left(r_{ss}^{\text{fund}} + \left(\tilde{\sigma}_{ss}^{E}\right)^2 - \rho\right) dt + \tilde{\sigma}_{ss}^{E} dZ_{it}. \tag{34}$$

The combination of individual wealth growing according to a geometric Brownian motion with entrepreneurial dynasties interrupted by death or type-switching implies that the steady state distribution of entrepreneurs' wealth follows a double Pareto distribution (Champernowne (1953), Reed (2001), Gabaix (2009)). The so-called Pareto tail coefficient describes the thickness of the right tail of this distribution. This tail coefficient is determined by the drift and volatility of the wealth accumulation process and the death and switching rates.[15] The key to the results in this paper is that the model implies a direct relationship between drift and the volatility of the entrepreneurs' wealth accumulation process in a steady state equilibrium. In particular, in equation (33), $\frac{dK_t}{K_t} = 0$ implies that the drift of an entrepreneur's wealth process is

$$r_{ss}^{\text{fund}} + \left(\tilde{\sigma}_{ss}^{E}\right)^2 - \rho = \left(1 - \eta_{ss}\right)\left(\tilde{\sigma}_{ss}^{E}\right)^2,$$

while $\frac{d\eta_t}{\eta_t} = 0$ implies that the volatility satisfies $(\tilde{\sigma}_{ss}^{E})^2 = \frac{(1-\frac{\bar{\psi}}{\eta_{ss}})(\delta_d+\phi^l)}{1-\eta_{ss}}$. In other words, the drift and volatility are both directly related to the wealth share of entrepreneurs in a steady state equilibrium. This allows us to characterize the Pareto tail coefficient in terms of the steady state wealth share of entrepreneurs.

Lemma 2. *The steady state right Pareto tail coefficient of entrepreneurs' wealth is*

$$\zeta = \eta_{ss} - \frac{1}{2} + \sqrt{\left(\eta_{ss} - \frac{1}{2}\right)^2 + \frac{2\eta_{ss}(1-\eta_{ss})}{\eta_{ss} - \bar{\psi}}} \tag{35}$$

Moreover $\frac{\partial \zeta}{\partial \eta_{ss}} < 0$. *Proof in Appendix A.4.*

Lemma 2 provides a direct analytical link between the cross-sectional distribution of wealth and the share of wealth held by entrepreneurs. Understanding how structural

[15] The stationary Kolmogorov forward equation results in an equation that pins down the Pareto tail coefficient ζ: $0 = \zeta(r_{ss}^{\text{fund}} + \left(\tilde{\sigma}_{ss}^{E}\right)^2 - \rho) + \frac{(\tilde{\sigma}_{ss}^{E})^2}{2}\zeta(\zeta - 1) - (\delta_d + \phi^l)$.

changes in the economy affect the steady state Pareto tail in equation (35) thus boils down to understanding how those structural changes affect the steady wealth share of entrepreneurs. In this model, consumption and income are proportional to net worth, so the distributions of these variables, counterfactually, share the same steady state Pareto tail coefficient. Gaillard, Hellwig, Wangner, and Werquin (2023) show how non-homothetic preferences and scale-dependent returns to wealth addresses this.

What determines the steady-state wealth share of entrepreneurs? Looking at equation (33) with $\frac{d\eta_t}{\eta_t} = 0$, we see that the steady state value η_{ss} is pinned down by the exogenous demographic parameters, δ_d, ϕ^l, and $\bar{\psi}$, as well as the idiosyncratic volatility of entrepreneurs' wealth $\tilde{\sigma}^E_{ss}$, which is endogenous:

$$(\tilde{\sigma}^E_{ss})^2 = \frac{(1 - \frac{\bar{\psi}}{\eta_{ss}})(\delta_d + \phi^l)}{1 - \eta_{ss}}. \tag{36}$$

From this equation, we see a strictly positive relationship between the steady state wealth share of entrepreneurs and their steady state idiosyncratic risk exposure. In other words, given the values of the demographic parameters, a steady state associated with a higher level of idiosyncratic risk exposure will be associated with a higher wealth share for entrepreneurs. Apart from the entrepreneurs' idiosyncratic risk exposure $\tilde{\sigma}^E_{ss}$, which is endogenous, only exogenous demographic parameters appear in equation (36).

Lemma 3. *Keeping fixed the demographic parameters, $\delta_d + \phi^l$, and $\bar{\psi}$, changes in the parameters of the model lead to a fall in the Pareto tail coefficient ζ (and, therefore a rise in Pareto inequality $1/\zeta$) if and only if those changes increase the optimal idiosyncratic risk exposure of entrepreneurs. This optimal idiosyncratic risk exposure is, in turn, equal to the following appraisal ratio associated with entrepreneurship:*

$$\tilde{\sigma}^E_{ss} = \frac{r^k_{ss} - r^{fund}_{ss}}{\chi \tilde{\sigma}^k_{ss}} \equiv \textit{Appraisal Ratio}. \tag{37}$$

Figure 2 depicts the relationship between the Pareto inequality $1/\zeta$ and the appraisal ratio associated with entrepreneurship, capturing the essence of the above lemma.

4 Improvements in Risk Sharing and Top Inequality in the Case of Perfect Substitutes.

In this section, we study the impact of improvements in risk sharing, a fall in χ, in the special case of the model where the goods produced by the two types of firms are perfect substitutes. This case is particularly transparent as it allows us to obtain a closed-form

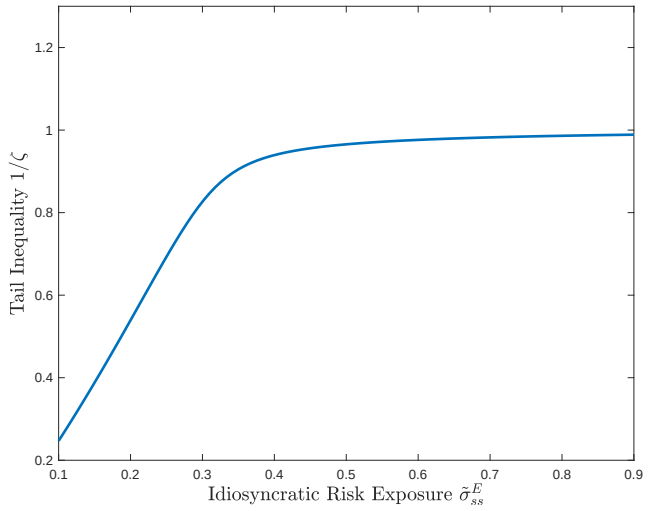


Figure 2: Illustrative example of Pareto tail inequality $1/\zeta$ and entrepreneurs' idiosyncratic risk exposure $\tilde{\sigma}^E_{ss}$ across steady states.

expression for the optimal risk exposure in terms of parameters, even out of steady state. This brings the model close to a special case of the two sector model studied in Section 5 of Angeletos (2007).

When the goods produced by the entrepreneurial firms and traditional firms are perfect substitutes, output is given by $Y_t = \nu Y_t^E + (1-\nu)Y_t^T$, and the goods prices are independent of the allocation of productive resources in the economy $p^E = \nu$ and $p^T = 1-\nu$. Given the state variables K_t and η_t, the equilibrium condition for the fraction of the productive resources operated by entrepreneurs, κ_t, implied by the system of equations in (29). In this section I will study steady states where both types of firms are active in equilibrium. That is, we need the solution κ_t to be in the interval $(0,1)$. For this to be the case, entrepreneurial firms need to be productive enough relative to their riskiness for entrepreneurs to want to operate them. This is the case if $\nu > 1/2$. At the same time, they must not be so productive relative to their riskiness that they completely out-compete the traditional firms.[16] The equilibrium appraisal ratio, or equivalently, the equilibrium optimal idiosyncratic risk exposure is then

$$\tilde{\sigma}^E_t = \tilde{\sigma}^E_{ss} = \frac{2\nu - 1}{\chi\tilde{\sigma}}. \tag{38}$$

From the expression in (38), we see immediately a fall in χ leads to a rise in the the equilibrium optimal idiosyncratic risk exposure. By lemma 3, Pareto inequality will rise.

[16] In appendix B.9, I discuss parameter restrictions that ensure this holds in steady state.

4.1 Is the Mechanism Quantitatively Relevant?

The analysis thus far has been qualitative in nature. To assess the magnitude of the impact of improvements in risk-sharing for entrepreneurs on Pareto inequality, I begin this section by briefly discussion the observed movements in Pareto inequality in the United States over the past half century. I then compare the magnitude of the movements in Pareto inequality observed in the data to the movements generated by modest improvements in risk sharing for entrepreneurs in the model.

Top Inequality Empirics. Saez and Zucman (2016) document a 13-percentage-point increase in the wealth share of the top 1%, from a low of roughly 22% in 1978 to 35% in 2016. Similarly, Smith, Zidar, and Zwick (2022b) find an increase of roughly 10 percentage points, to 33.7%, over the same period. Interestingly, both sets of authors document substantial changes in the distribution of wealth *within* the top 1%. It is precisely these changes within the top 1% that are the subject of this paper. Figure (5a) depicts the evolution of the ratio of the top 0.1% to the top 1% and the top 0.01% to the top 0.1%. The similar level and evolution of these ratios indicate that the top of the wealth distribution roughly follows a Pareto distribution and that Pareto inequality, the inverse of the Pareto tail coefficient, has increased. Figure (5b) in appendix B depicts an estimate of Pareto inequality based on these ratios of top wealth shares, using a formula provided by Jones and Kim (2018) and data from Piketty et al. (2018). These figures capture the essence of the stylized facts around which the literature on top wealth inequality has centered – the rise in the level of Pareto inequality as well as the speed with which this rise has occurred. In summary, the inverse tail coefficient rose from a low of roughly $1/\zeta = 0.5$ in 1975, to about $1/\zeta = 0.7$ in 2019. This constitutes a benchmark against which the impact of improvements in risk-sharing for entrepreneurs in the model can be compared to.

Impact of Fall in χ on Steady State Pareto Inequality. It is straightforward to work out the magnitude of the impact of a fall in χ on the level of steady state Pareto inequality. To this end, suppose that the economy starts in a steady state where the Pareto tail inequality is $1/\zeta = 0.5$, which is approximately the tail coefficient implied by both Piketty et al. (2018) in the mid 1970s. We first have to take a stand on the fraction of the capitalist population that are entrepreneurs, $\bar{\psi}$, and the rate at which entrepreneurs cease to operate entrepreneurial firms, $\delta_d + \phi_l$. As an illustration, let $\bar{\psi} = 5\%$ and $\delta_d + \phi_l = 0.1$. Equations (35) and (36) then tell us that to hit initial tail inequality of $1/\zeta_0 = 0.5$, the appraisal ratio in the initial steady state has to be $\tilde{\sigma}_0^E \approx 0.187$. To get a sense of the magnitudes involved, suppose χ falls by 10%. Then the appraisal ratio rises from $\tilde{\sigma}_0^E \approx 0.187$ to $\tilde{\sigma}_1^E \approx 0.187/0.9 \approx 0.21$,

and the inverse Pareto tail coefficient rises from $1/\zeta_0 = 0.5$ to $1/\zeta_1 \approx 0.56$. This would account for roughly one third of the rise in the Pareto inequality observed in the data over the period 1975 to 2019. If χ falls by 20%, the inverse Pareto tail coefficient rises from $1/\zeta_0 = 0.5$ to $1/\zeta_1 \approx 0.65$, which would account for 3/4 of the change in the Pareto tail coefficient observed in the data. This suggests that even quite modest improvements in risk sharing for entrepreneurs can have a sizable impact on steady state Pareto inequality.

Has χ Fallen in the Data? Equation (12) provides a model-consisent relationship between the risk sharing parameter χ and the fraction of the firm owned by the entrepreneur s_{it}: $(1-\chi)r_t^{\text{fund}} = (1-s_{it})r_t^k$. For a fixed ratio of the return to capital relative to the cost of outside capital, $\frac{r_t^k}{r_t^{\text{fund}}}$, the relationship between changes in outside ownership and outside financing is one-to-one in percentage terms.[17] Do we see an increase in the fraction of entrepreneurial firms sold to outsiders? Two observations suggests so. First, Gomez and Gouin-Bonenfant (2024) document that the average lifetime growth rate of shares outstanding for firms associated with entrepreneurs at the top of the Forbes 400 list has increased from 0.8% in 1985 to 2.8% in 2015. In other words, the rate at which top entrepreneurs issue equity to outsiders when financing their companies seem to have increased. More directly, another suggestive piece of evidence is provided by Lebret (2017), who finds that the ownership share of founders and employees (as opposed to outside investors and the general public) fell from around 50% in 1965, to around 30% in 2010.[18] These observations help us discipline the numerical experiments conducted with the model in the following way. When numerically examining the impact of a fall in χ on Pareto inequality, the model will simultaneously generate a prediction for the ownership split between insiders and outsiders. If the model-implied change in the insider ownership share (s_{it}) is of reasonable magnitude compared to this data, we can be more confident in that the change in χ we feed into the model is of reasonable magnitude.

4.2 Transition Dynamics of Inequality

Section 4.1 examined the impact of improvements in risk sharing for entrepreneurs on *steady state* Pareto inequality. In this section, I examine the transition dynamics of Pareto

[17] If in addition the cost of outside capital has fallen relative to the return on capital for entrepreneurs, as argued by Gomez and Gouin-Bonenfant (2024) a rise in the outsider ownership share $1 - s_{it}$ suggests an even more substantial fall in χ.

[18] The dataset is regularly updated and extended by the author, and the current data was made public at https://www.startup-book.com/2024/06/13/what-to-expect-from-an-investor/. In this updated version of the data the decline is even more stark. In particular, he finds that the fraction of shares owned by founders and employees at IPO declined from around 50% to around 20% between 1965 and 2020.

inequality in the model and show that the model can in principle generate reasonably fast transition dynamics in response to improvements in risk sharing for entrepreneurs. In essence, the entrepreneurs in the model have higher than average risk exposures and higher than average expected growth rates of wealth. To use the language and insights of Gabaix et al. (2016), the model features *type-dependent* wealth dynamics, making fast transitions of the top wealth inequality possible, at least in principle.

Dynamics of Pareto Inequality in the Model. In section 4.1, we saw that it was sufficient to specify values for the demographic parameters $\phi^l + \delta_d$ and $\bar{\psi}$ to gauge the impact of a fall in χ on steady state Pareto inequality. However, to examine the dynamics of Pareto inequality over time, we need specify parameter values for the remaining parameters of the model, $\alpha, \rho, \delta, \sigma, \tilde{\sigma}, \nu, \chi$. This is because the expected growth rate of entrepreneurs' wealth, which governs their wealth accumulation process, varies along the transition path and depends on the state of the economy. Table 1 provides a simple parametrization of the model along with the associated target moments. I choose α, ρ, and σ to produce a steady state that roughly matches the labor share of income, the rate of return to business capital, and the risk-free rate in 1960. I use δ to target a (business) capital-output ratio of 2. This results in a value of $\delta = 0.1$, which is larger than in most standard calibrations. I select ν, and $\tilde{\sigma}$ to match an initial Pareto tail coefficient of $\zeta_0 = 2$, an idiosyncratic volatility of stock returns of 30%.[19] Kaplan and Rauh (2013) document that 69% of the Forbes 400 list in the 2011 edition were the first in their family to run their business, up from 40% in the first 1982 edition. I select $\bar{\psi}$ to match a population fraction of 40% entrepreneurs among the Forbes 400 (corresponding to roughly between the top 0.00025% and top 0.0005% of households, depending on year) in the initial steady state. I let $\delta_d = 1/30$, so that dynasties break once a generation on average. I let $\phi^l = 1/15$ so that entrepreneurs stay undiversified for 15 years on average. Informative data on these two latter parameters are hard to obtain, but with $\phi^l = 1/15$ and $\delta_d = 1/30$ the resulting average volatility within the top 0.01% is 6.8%. This is roughly in line with the 10% estimate documented by Gomez (2023) for the period 1960–1980.

Figure 3a depicts the evolution of Pareto inequality in response to a fall in χ of 20%, 30% and 40%, respectively. The fall in χ is set to occur in 1975, partly motivated by the argument advanced by Gornall and Strebulaev (2021), that regulatory reforms implemented in the mid- to late 1970s contributed to the emergence and rapid growth of the venture capital

[19] This was roughly the (cross-sectional) average idiosyncratic volatility of stock returns at the firm level in 1960 (see Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016)).

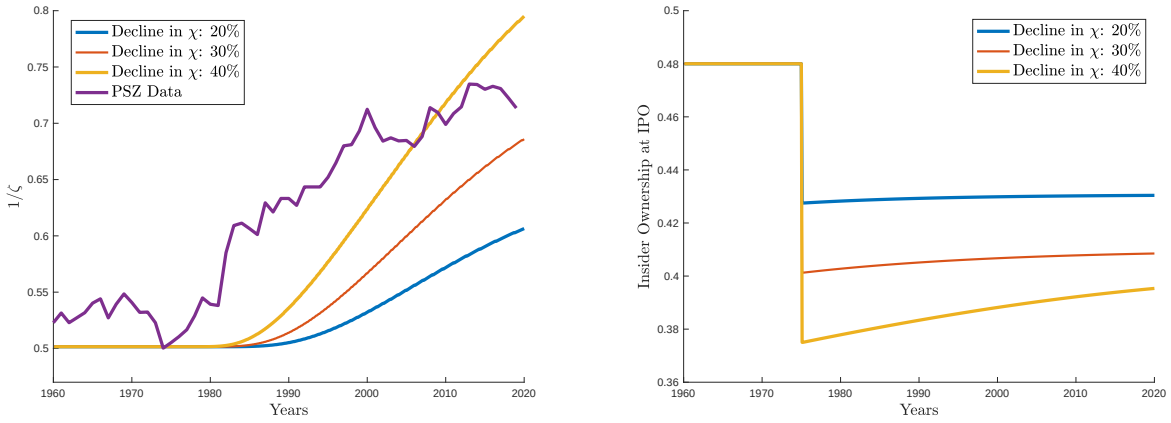
industry. I discuss this further in Section 4.4. Note that although the fall in χ occurs in 1975, the Pareto tail does not start to change until approximately a decade later. In other words, changes in the economic environment facing entrepreneurs take time to be reflected in top inequality.[20] Figure 3b then depicts the model-implied changes in the inside ownership shares s_{it} induced by these declines in χ. Note that the declines in insider ownership fractions are more modest than those observed by Lebret (2017), suggesting that the declines in χ that we have feed into the model are, if anything, somewhat conservative. In summary, improvements in idiosyncratic risk sharing for entrepreneurs can produce rapid rises in wealth inequality in this model. With a 40% fall in χ, this version of the model with perfect elasticity of substitution over-predicts the rise in Pareto inequality. Measuring the appropriate value of ε is an empirical task beyond the scope of this paper. The model also generates an increase in the prevalence of entrepreneurs at the top of the wealth distribution in consistent with Kaplan and Rauh (2013) (see Figure 6 in Appendix B.12). It is a bit surprising that the dynamics of Pareto inequality in Figure (3a) is so fast given the insights of Gabaix et al. (2016). This seems to be a consequence of the fact that *both* the volatility and expected growth rate of wealth for entrepreneurs rises rapidly when χ falls. In particular, after the 40% fall in χ, the volatility rises to above 31% and the expected growth rate rises to around 9%. The numerical experiments in this section should not be taken as statements about the fraction of the rise in top inequality that can be accounted for by improved risk-sharing for entrepreneurs (such statements would need better identification for parameters like ε, ϕ^l, δ_d and $\bar{\psi}$). Rather, it illustrates the point that the framework considered can in principle be made consistent with the fast transitions of top inequality observed in the data.

4.3 Other Drivers of Rising Pareto Inequality.

Although the present framework is different from that studied in Gomez and Gouin-Bonenfant (2024), it shares their conclusions regarding the effect of fall in the cost of capital for entrepreneurs relative to their return: it raises top inequality. In this framework, it is because a higher return to capital relative to cost of capital raises the appraisal ratio.

However, declines in the cost of capital relative to the return on capital differ from improvements in risk sharing with respect to the impact on the use of outside financing versus inside financing. The intuition is the following. When the cost of outside capital falls relative to the return to capital, entrepreneurs will want to scale up. But because

[20] In Appendix B.19 I briefly discuss how this relates to the literature on the observed decline in business dynamism emphasized by Decker, Haltiwanger, Jarmin, and Miranda (2016).



(a) Transition dynamics of Pareto inequality $1/\zeta$.

(b) Transition dynamics of insider ownership share s_{it}.

Figure 3: Transition dynamics of Pareto inequality and the insider ownership share implied by a decline in χ for declines of 20%, 30%, and 40% respectively. Pareto tail coefficient measured at the top 0.1%. Data on ratio of top wealth shares from Piketty et al. (2018) (PSZ), transformed to Pareto coefficient using formula from Jones and Kim (2018).

the skin-in-the-game constraint binds, entrepreneurs then have to increase their own contribution to firm financing in lock-step with the increase in outside financing. In contrast, if risk sharing improves so that the skin-in-the-game constraint is relaxed (that is, if χ falls), entrepreneurs can increase outside financing *relative* to inside financing. An important insight from equation (12) is therefore that, in the context of this model, although improvements in risk sharing and cheaper financing both lead entrepreneurs scale up, improvements in risk sharing are associated with entrepreneurs owning smaller fractions of larger firms, whereas cheaper financing alone would be associated with entrepreneurs owning larger fraction of larger firms.

4.4 Macro-Finance Trends: Venture Capital-Backed Firms, Factor Income Shares, and Rates of Returns.

In the case of perfect substitutes, we can also transparently understand the impact of improvements in risk sharing on the size of the entrepreneurial sector, the labor share of income, the rate of return to capital, the risk-free rate, and the transition dynamics of the model. In particular, in Section A.5, I show that a small fall in χ leads to larger steady state values of K_{ss}, η_{ss} and κ_{ss}. From this, we can straightforwardly derive the steady state implications for the aforementioned macro-finance trends.

The Size of Entrepreneurial Sector. The mechanism at the heart of this paper connects changes in the ability of entrepreneurs to raise capital and offload risk to financial markets,

Table 1: Parameterization and target moments. Rates are annualized.

Parameters		Moments		
Parameter	Value	Target Moment	Target	Model
α	0.34	Labor share	65%	65.6%
ρ	0.071	Average return to capital	7.41%	7.41%
δ	0.1	Capital-output ratio	2	2
σ	0.153	Risk-free rate	4.51%	4.5%
$\tilde{\sigma}$	0.3	Ent. firms idios. vol	30%	30.3%
ν	0.51	Initial Pareto tail coefficient	2	2
χ	0.34	Outsider Ownership	52%	52%
$\bar{\psi}$	1/20	Ent. in Forbes 400	40%	39%
δ_d	1/30	Dynasty break once in generation	–	–
ϕ^l	1/15	Underdiversification for 15 years	–	–

to a reallocation of economic activity to entrepreneurial firms. Do we see a hints of such a rise κ_{ss} in the data? From an empirical perspective, the emergence and rapid growth of the venture capital industry over the past half-century has been referred to as a revolution (see Gompers and Lerner (2001)). In parallel with this development, several studies point to a simultaneous reallocation of economic activity towards the types of firms funded by venture capital. For instance, Gornall and Strebulaev (2021) document that firms with a history of venture capital-backing constituted around 0–5% of the total market capitalization before and up to 1980, rising to around 41% in 2020. Greenwood et al. (2022) also document a marked rise in the importance of venture capital-backed firms in the US economy. As for the driving forces behind this evolution, Gornall and Strebulaev (2021) point to regulatory changes implemented through the 1974 Employee Retirement Income Security Act (ERISA) and its subsequent reinterpretation in 1979 as a potential source. These reforms allowed a broader set of investors to invest in venture capital, investments that were previously regarded as too risky.

Pure Labor and Capital Shares of Income. Both the labor share of income and the pure capital share of income are lower for entrepreneurial firms than traditional firms in the model. This is because, following Di Tella and Hall (2021), the idiosyncratic risk in the firm renders the marginal product of each factor of production locally uncertain. They argue that this is a way of taking seriously the Knightian view (Knight, 1921) that entrepreneurs engage in risk-taking when renting capital and hiring labor because the marginal products of each are uncertain at the time that the cost of capital and wages are determined. As

in David, Ranciere, and Zeke (2023) and Hartman-Glaser et al. (2019), the fact that this uncertainty will be priced in equilibrium implies that risk-adjusted marginal products are lower than their unadjusted counterparts. This risk compensation constitutes the foundation for the entrepreneurial share of income.[21] The labor share of income in the economy is given by

$$\frac{w_t L}{Y_t} = (1-\alpha)\left(\frac{1-\nu}{\nu\kappa_t + (1-\nu)(1-\kappa_t)}\right). \tag{39}$$

Recall that $\nu > 1/2$ is a condition for entrepreneurs to operate their firms. Hence, as κ_t rises and economic activity is reallocated to the low labor share entrepreneurial firms, the labor share of income falls. Analogously, the pure capital share of income is

$$\frac{(r_t^{\text{fund}} + \delta)K_t}{Y_t} = \alpha\left(\frac{1-\nu}{\nu\kappa_t + (1-\nu)(1-\kappa_t)}\right), \tag{40}$$

which also falls as κ_t rises and economic activity is reallocated to the entrepreneurial firms. The entrepreneurial share of income is

$$1 - \left(\frac{w_t L}{Y_t} + \frac{(r_t^{\text{fund}} + \delta)K_t}{Y_t}\right) = \frac{\kappa_t(2\nu - 1)}{\nu\kappa_t + (1-\nu)(1-\kappa_t)}$$

which rises when κ_t increases.[22] As economic activity is reallocated to the entrepreneurial firms, the entrepreneurial share of income rises at the expense of both the labor share and the capital income share. For the present purpose, a key aspect of the empirically documented fall in the labor share is the observation that this fall has been driven by a reallocation of economic activity towards firms with low labor shares rather than by a general fall in the labor share at the firm level, which has remained relatively stable (Autor et al., 2020) or even increased (Hartman-Glaser et al., 2019). Qualitatively, the model presented in this study is in line with that observation: it is precisely at composition effect drives the fall in the labor share in response to improvements in risk-sharing for entrepreneurs. In fact, with perfect substitutability, the firm-level labor share is constant. It has also been pointed out that the fall in the labor share has not been accompanied by a rise in the pure capital share of income. Instead, both the labor share and the capital

[21] I refer to it as the "entrepreneurial share" rather than the "entrepreneur's share" because the entrepreneur also gets some pure capital income. The entrepreneur's share is, therefore, the entrepreneurial share *plus* the entrepreneur's pure capital income share.

[22] Directly related to the perspectives advanced in Eisfeldt, Falato, and Xiaolan (2022) and Smith, Yagan, Zidar, and Zwick (2022a), if a large enough fraction of the entrepreneurial share of income is classified as labor income instead of capital income, the labor share instead rises as χ falls.

share have fallen relative to what has been referred to as factorless income (Karabarbounis and Neiman, 2019). Barkai (2020) emphasizes the role of pure profits, market power, and declining competition. Eisfeldt et al. (2022) and Smith et al. (2022a) instead focus on the role of human capital income of key employees and business owners. In the model presented in this paper, the rise in the factorless income share comes from the rise in entrepreneurs' share of income. In this sense, the explanation is closer in spirit to Eisfeldt et al. (2022) and Smith et al. (2022a).

Return on Capital and Risk-Free Rate. The weighted average return on the aggregate capital stock is

$$r_t^K = \kappa_t r_t^k + (1-\kappa_t) r_t^{\text{fund}} = r_t + \eta_t \left(\frac{2\nu - 1}{\chi \tilde{\sigma}} \right)^2 + \sigma^2. \tag{41}$$

This return rises on impact as χ falls. As capital accumulation gets underway, the return to capital falls back down to its stead state level $r_{ss}^K = \rho$. The risk-free rate in the model is given by

$$r_t = (1-\nu)\alpha \left(\frac{L}{K_t} \right)^{1-\alpha} - \sigma^2 - \delta. \tag{42}$$

The risk-free rate remains unchanged on impact as χ falls, but subsequently declines over time in response to capital accumulation. This property of the model seems to be a direct inheritance from the Angeletos-framework (see Proposition 5 in Section 5.2 of Angeletos (2007), where a local change in the level of risk is considered). In the present model, the result also depends on the fraction of wealth held by entrepreneurs η_t. With a finite elasticity of substitution ε, the size of this elasticity determines whether entrepreneurs increase risk-taking when risk-sharing improves. Hence, these results regarding rates of returns also depend on ε. In summary, improvements in risk sharing for entrepreneurs lead to a transitory rise in the weighted average return to capital and a permanent decline in the risk free rate if entrepreneurs increase risk-taking when risk-sharing improves. Consistent with this pattern, several recent studies document a relatively stable or slightly rising return to business capital in the U.S. over the past half-century, while estimates of the return on safe assets and required returns to wealth show a downward trend for the last half-century (Gomme, Ravikumar, and Rupert, 2011; Holston et al., 2017; Reis, 2022). Powerful forces, like demographic changes and the so-called international "savings glut", can perhaps account for most of the fall in the risk-free rate (see Eichengreen (2015), Eggertsson, Mehrotra, and Robbins (2019), Rachel and Summers (2019) and Auclert et al. (2021)). However, the mechanism discussed in this paper can potentially be added as

an additional force putting downward pressure on the risk-free rate while at the same time allowing the return on capital to remain elevated. It is worth noting that the decline in the riskless rate that occurs in response to improved risk sharing in this model is a result of increased savings rates (out of income) for entrepreneurs as they take on more idiosyncratic risk. The mechanism thus shares the flavor of Mian, Straub, and Sufi (2021).[23]

5 Finite elasticity of Substitution.

When $\varepsilon \in (0, \infty)$ so that the goods produced by entrepreneurial firms and traditional firms are not perfect substitutes, the prices of the respective goods are no longer invariant to the size of the respective sectors. In this case, improvements in idiosyncratic risk sharing for entrepreneurs not only leads to a reallocation of capital from the traditional established firm towards the entrepreneurial firms, but also a change in these prices. What happens to the appraisal ratio, which we know determines entrepreneurs' risk exposure as well as Pareto inequality? Rewriting the equlibrium condition in (29) in terms of the appraisal ratio, we have

$$\tilde{\sigma}_t^E = \frac{p^E(\kappa_t) - p^T(\kappa_t)}{\chi \tilde{\sigma}}. \tag{43}$$

With finite substitutability, the appraisal ratio may move up or down depending on how much of the fall in χ in the denominator is offset by a fall in the excess return in the numerator. If the elasticity of substitution between the two sectors is high, the market adjusts primarily via quantities and not prices; that is, the excess return in the numerator is relatively stable. In this case, the appraisal ratio rises. If, on the other hand, ε is low, prices react strongly in response to any reallocation. The fall in the excess return in the numerator can then be larger than the fall in the denominator, and the appraisal ratio falls.

The dynamic response of the economy after impact will also depend on whether the appraisal ratio rises or falls. A rise in the appraisal ratio will raise the drift of η_t on impact, which consequently starts to grow. The behavior of the capital stock depends on the strength of the reallocation of capital relative to the reaction of prices to this reallocation. What matters for the behavior of top inequality in the long run, is whether the economy settles on an appraisal ratio that is higher or lower than in the initial steady state. The next result says that if the elasticity of substitution between the sectors is high enough, the steady state appraisal ratio rises when risk sharing improves, implying a larger share

[23] In Mian et al. (2021), non-homothetic preferences induce wealthy individuals to save at a high rate. In contrast, in the present paper, preferences are homothetic but entrepreneurs precautionary savings motives induce them to save a large fraction of their income. Savings rates out of income are higher at the top of the wealth distribution but only because entrepreneurs are overrepresented at the top.

of wealth owned by entrepreneurs and higher Pareto inequality.

Proposition 3. *Suppose that the economy is in an initial steady state $s_0 = (\eta_0, K_0, \kappa_0)$, where $\kappa_0 \in (0,1)$, the initial inside equity constraint parameter is $\chi_0 > 0$ and the elasticity of substitution is $\varepsilon_0 \in (0,\infty)$. Let $\zeta(\chi)$, and $\kappa(\chi)$ denote the steady state Pareto tail coefficient and steady state fraction of the capital stock operated by entrepreneurial firms as a function of χ, keeping all other parameters fixed and assuming these functions exists and are differentiable in a neighborhood of χ_0, with $\kappa'(\chi_0) < \infty$. Then there exists a $\varepsilon^*_{s_0}$ such that if the ε_0 associated with the steady state s_0 is larger than $\varepsilon^*_{s_0}$, then $\frac{d\zeta}{d\chi}(\chi_0) > 0$.*

Proof. By Lemma 3, we need to show that the steady state appraisal ratio increases as χ falls for high values of ε. Then, we have by equation (43) evaluated in steady state:

$$\frac{d\tilde{\sigma}^E_{ss}}{d\chi_0} = \frac{\left(\frac{dp^E(\kappa(\chi_0))}{d\kappa} - \frac{dp^T(\kappa(\chi_0))}{d\kappa}\right)}{\chi_0\tilde{\sigma}}\frac{d\kappa(\chi_0)}{d\chi} - \frac{\tilde{\sigma}^E_{ss}}{\chi_0}.$$

Because the price functions $p^E(\kappa)$ and $p^T(\kappa)$ can be made arbitrarily insensitive to changes in κ, there exists some $\varepsilon^*_{s_0}$ such that the first term $\frac{\left(\frac{dp^E(\kappa(\chi_0))}{d\kappa} - \frac{dp^T(\kappa(\chi_0))}{d\kappa}\right)}{\chi_0\tilde{\sigma}}\frac{d\kappa(\chi_0)}{d\chi}$ would be smaller than $\tilde{\sigma}^E_{ss}/\chi_0$ if the elasticity of substitution was larger or equal to $\varepsilon^*_{s_0}$. Thus, if the ε_0 associated with the initial steady state is larger than this $\varepsilon^*_{s_0}$, the derivative $\frac{d\tilde{\sigma}^E_{ss}}{d\chi}$ is negative. By Lemma 3, this proves the result. □

The proposition is local in that the threshold value $\varepsilon^*_{s_0}$ depends on the steady state the economy starts in, as determined by all parameters, including ε. When financial frictions become so small that entrepreneurs take over almost all production, the economy starts resembling a one-sector economy, where the risk reduction effect dominates. To understand this, consider the case of perfect substitutes. To be precise, we have thus far considered the impact of a small fall in χ in a steady state where initially $\kappa_{ss} < 1$. However, if χ is close to 0, the steady state can be such that $\kappa_{ss} = 1$ if the goods are perfect substitutes. That is, entrepreneurs may operate the entire capital stock in steady state. Hence, slightly reducing financial frictions and completely removing them may have different effects for top wealth inequality, mirroring the conclusion of Bonfiglioli (2012).

6 Conclusion

This paper studies the effects of improvements in entrepreneurial financing top wealth inequality. Through the lens of a tractable general equilibrium model, I show that the

effect of improvements in idiosyncratic-risk sharing for entrepreneurs is summarized by three key effects: the risk-reduction effect, the scaling-up effect, and the general equilibrium reallocation effect. Improved risk sharing allows entrepreneurs to reduce their idiosyncratic risk exposures, lowering top wealth inequality by making extreme wealth trajectories less likely and by reducing entrepreneurs' precautionary savings motive. This is the risk-reduction effect. In contrast, risk sharing encourages entrepreneurs to raise more capital and scale up, which raises top wealth inequality. This is the scaling-up effect.

The central theoretical contribution of the paper is the insight that a third general equilibrium effect determines the relative strengths of the risk-reduction and scaling-up effects: the reallocation effect. If entrepreneurs can attract economic activity from elsewhere in the economy and avoid downward pressure on their equilibrium profitability, the scaling-up effect dominates, and top wealth inequality rises. A second contribution of the paper is to show that several well-documented trends in US data point to the strength of the general equilibrium reallocation effect. In particular, the dramatically growing fraction of venture capital-backed firms among the largest firms in the U.S., the fall in the aggregate labor share despite relatively stable firm-level labor shares, and the stable or slightly rising accounting return to the aggregate capital stock despite falling safe rates are reflected by the model precisely when the general equilibrium reallocation effect is strong enough for the scaling-up effect to dominate the risk-reduction effect.

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A Appendix

A.1 Proof of Proposition 1.

Proof. We prove each part of the proposition in turn.

1. Let the non-firm-related part of an entrepreneur's portfolio, v_{it}^{nf}, refer to the entrepreneur's holdings of the diversified fund and of bonds, i.e. $v_{it}^{\text{nf}} \equiv v_{it}^{\text{fund}} + b_{it}$. With $\tilde{\vartheta}_{it} = v_{it}^{\text{fund}}/v_{it}^{\text{nf}}$ denoting the risky fraction of this non-firm-related part of an entrepreneur's portfolio, the dynamic budget constraint can be written as

$$\begin{aligned} dn_{it} = &\left(p_t^E y_{it} - w_t l_{it} - \delta k_{it} + (v_{it}^{\text{nf}} - v_{it}^{\text{out}})r_t + (v_{it}^{\text{nf}}\tilde{\vartheta}_{it} - v_{it}^{\text{out}}\vartheta_{it})\varsigma_t\sigma - c_{it}\right)dt \\ &+ (k_{it} - v_{it}^{\text{out}}\vartheta_{it})\tilde{\sigma}_{it}^k dZ_{it} + \left(k_{it} + (v_{it}^{\text{nf}}\tilde{\vartheta}_{it} - v_{it}^{\text{out}}\vartheta_{it})\right)\sigma dZ_t. \end{aligned} \tag{44}$$

   Suppose that the skin-in-the-game constraint is not binding. Then the entrepreneur can increase the risky fraction in the firm's liabilities, ϑ_{it}, while simultaneously increasing the risky fraction in her non-firm related portfolio $\tilde{\vartheta}_{it}$. Specifically,

the entrepreneur raises these fractions so that $v_{it}^{\text{out}}\vartheta_{it}$ and $v_{it}^{\text{nf}}\tilde{\vartheta}_{it}$ rise by the same amount keeping $(v_{it}^{\text{nf}}\tilde{\vartheta}_{it} - v_{it}^{\text{out}}\vartheta_{it})$ constant. This reduces idiosyncratic volatility $(k_{it} - v_{it}^{\text{out}}\vartheta_{it})\tilde{\sigma}_{it}^{k}$ but leaves all expected returns and other risks intact, making the entrepreneur better off. So, the skin-in-the-game constraint must bind in optimum.

2. Using that the skin-in-the-game constraint is binding in optimum, an entrepreneur's dynamic budget constraint is

$$\begin{aligned} dn_{it} = &\left(p_t^E y_{it} - w_t l_{it} - \delta k_{it} + r_t\left(b_{it} + v_{it}^{\text{fund}} - v_{it}^{\text{out}}\right) + \varsigma_t\sigma\left(v_{it}^{\text{fund}} - v_{it}^{\text{out}}\right) - c_{it}\right)dt \\ &+ \chi k_{it}\tilde{\sigma}_{it}^{k}dZ_{it} + \left(\chi k_{it} + v_{it}^{\text{fund}}\right)\sigma dZ_t \end{aligned} \tag{45}$$

where the choice of the outside-capital structure ϑ_{it} is no longer of relevance.

□

A.2 Entrepreneurs' Problem

Let S_t denote a vector of all state variables except the agent's net worth. Assume S_t follows an Ito process. We consider a slightly more general version of the problem than the one considered in Section 2. In particular, I adopt the more general framework of section 3, where entrepreneurs lose their entrepreneurial ability at rate ϕ^l according to a Poisson process independent of all other processes and die at rate δ_d according to a Poisson process independent of all other processes. Moreover, as is well-known (see Moll et al. (2022)), the probability of death only affects the optimization problem by changing the time-discount rate. I use $\tilde{\rho}$ for the pure time-preference rate and let $\rho = \tilde{\rho} + \delta$ denote the time-preference rate adjusted by the death rate. The HJB equation of the entrepreneurs' problem can be written as

$$\begin{aligned} \tilde{\rho}V^E(n,S) = \max_{\{c,x,\theta^k,\theta^{\text{out}},\theta^{\text{fund}}\}} &\log(c) + V_n^E n\left(r + \theta^k\left(r^k(x) - r\right) - \theta^{\text{out}}\varsigma\sigma + \theta^{\text{fund}}\varsigma\sigma - \frac{c}{n}\right) \\ &+ \frac{1}{2}V_{nn}^E n^2\left(\left(\theta^k - \theta^{\text{out}}\right)^2(\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)^2\sigma^2\right) \\ &+ \sum_{s\in S} V_s^E\mu_s s + \frac{1}{2}V_{ss}^E s^2\sigma_s^2 + V_{ss'}^E ss'\sigma_s\sigma_{s'} + V_{sn}^E sn\sigma_s\sigma_n \\ &+ \lambda((1-\chi)\theta^k - \theta^{\text{out}}) \\ &+ \phi^l\left(V^D(n,S) - V^E(n,S)\right) - \delta_d V^E(n,S) \\ &\text{where} \quad x = \frac{y}{k} \end{aligned}$$

where $V^D(n,S)$ is the value function for a diversified capitalist. We can guess that the value functions take the form $V^j(n,S) = \frac{1}{\rho}\log(n) + g^j(S)$ for $j \in \{E,D\}$. When utility is homothetic and budget constraints are linear, this is a standard guess for the form of the value function (see Brunnermeier and Sannikov (2017), Di Tella and Hall (2021). Supplemental Appendix B.2 sketches a verification argument). Saving notation, I use V to denote the value function while being an entrepreneur from now on. With this guess we have $V_n n = -V_{nn}n^2 = \frac{1}{\rho}$ and all mixed derivatives are $V_{sn} = 0$. We look for solutions where entrepreneurs invest some capital in their firm's, $\theta^k > 0$. After taking-first order conditions (see Supplemental Appendix B.1 for the algebra) we obtain the following decision rules:

$$\begin{aligned} c &= \rho n, \quad (\delta + r^{\text{fund}})\bar{g} = (\bar{g}-1)px + \left(\frac{1}{1-\alpha} - \bar{g}\right) w x^{\frac{1}{1-\alpha}} \\ \theta^k &= \frac{r^k(x) - r^{\text{fund}}}{(\chi g(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2}, \quad \theta^{\text{fund}} = \frac{r^{\text{fund}} - r}{\sigma} - \chi\theta^k, \quad \theta^{\text{out}} = (1-\chi)\theta^k. \end{aligned} \tag{46}$$

A.3 Proof of Proposition 2

For given K_t, η_t, κ_t, the expressions for the prices in (31) follow directly from the first-order conditions for the firms and no-arbitrage. The evolution of an entrepreneur's wealth is obtained by substituting in the optimal decision rules into their dynamic budget constraint. What remains to show is therefore how κ_t and the evolution of the state variables K_t and η_t are determined. By combining the expressions for the prices in (30) with the capital demand of entrepreneurial firms in equation (10), and the first-order conditions of the traditional in (20) we obtain

$$\frac{\kappa_t}{\eta_t}(\chi\tilde{\sigma})^2 \left(\frac{L_t}{K_t}\right)^{1-\alpha} = p_t^E - p_t^T \tag{47}$$

(see Appendix B.4 for algebra). This equation pins down a unique $\kappa_t \in (0,1)$ if $\varepsilon > 0$. To see why, note that the left-hand side is a strictly increasing linear function of κ_t, given positive η_t and K_t. Moreover, we can write the prices as

$$p_t^T = (1-\nu)\left[\nu\left(\frac{\kappa_t}{1-\kappa_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\right]^{\frac{1}{\varepsilon-1}}, \quad \text{and} \quad p_t^E = \nu\left[\nu + (1-\nu)\left(\frac{1-\kappa_t}{\kappa_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} \tag{48}$$

We see that if $\varepsilon > 0$, then p_t^T is increasing in $\kappa_t \in (0,1)$ and p_t^E is decreasing. For κ_t close to 0 we will have $p_t^E - p_t^T > 0$, for κ_t close to 1 we will have $p_t^E - p_t^T < 0$. Hence, for some

unique $\kappa_t \in (0,1)$, the left-hand side and right-hand side intersect. Next, the evolution of wealth of the two types of capitalists can be written as

$$\begin{aligned} dN_t^E &= N_t^E \left(r_t^{\text{fund}} + (\tilde{\sigma}_t^E)^2 - \rho \right) dt + \sigma N_t^E dZ_t + \left(\psi^0 \delta_d N_t - (\phi^l + \delta_d) N_t^E \right) dt \\ dN_t^D &= N_t^D \left(r_t^{\text{fund}} - \rho \right) dt + \sigma N_t^D dZ_t + \left((1-\psi^0) \delta_d N_t - \delta_d N_t^D \right) dt \end{aligned} \tag{49}$$

where the last terms in brackets are the effects of birth, death, and switching. Since $K_t = N_t = N_t^E + N_t^D$ we have by Ito's lemma

$$\frac{dK_t}{K_t} = \frac{dN_t}{N_t} = \left(r_t^{\text{fund}} + \eta_t \left(\tilde{\sigma}_t^E \right)^2 - \rho \right) dt + \sigma dZ_t \tag{50}$$

and because $\eta_t = \frac{N_t^E}{N_t}$ we have, again by Ito's lemma and $\bar{\psi} = \frac{\psi^0 \delta_d}{\delta_d + \phi^l}$

$$\frac{d\eta_t}{\eta_t} = (1-\eta_t) \left(\tilde{\sigma}_t^E \right)^2 dt - \frac{\left(\eta_t - \bar{\psi} \right) \left(\delta_d + \phi^l \right)}{\eta_t} dt. \tag{51}$$

A.4 Proof of Lemma 2

In this section, all variables should be understood as being in steady state. The line of reasoning in this section follows from that used in Gabaix (2009) and Gomez and Gouin-Bonenfant (2024), both of which consider random growth models of wealth accumulation. The drift of the wealth growth of entrepreneurs is $\mu^E = r^E - \rho = (1-\eta) \left(\tilde{\sigma}^E \right)^2 > 0$. The relevant Kolmogorov forward equation for the distribution of entrepreneurs' wealth can be written as

$$0 = -\frac{\partial}{\partial n} \left[\mu^E n f_E(n) \right] + \frac{\partial^2}{\partial n^2} \left[\frac{1}{2} (\tilde{\sigma}^E)^2 n^2 f_E(n) \right] - (\delta_d + \phi^l) f_E(n). \tag{52}$$

We can guess and verify that the density of a double Pareto distribution solves this equation, with the tail coefficients being roots of the quadratic equation

$$0 = \zeta \mu^E + \frac{(\tilde{\sigma}^E)^2}{2} \zeta (\zeta - 1) - (\delta_d + \phi^l) \tag{53}$$

(see Gabaix (2009) and references therein). Using that in steady state $2\mu^E / \left(\tilde{\sigma}^E \right)^2 = 2(1-\eta)$ the positive root is

$$\zeta = \eta - \frac{1}{2} + \sqrt{\left(\eta - \frac{1}{2} \right)^2 + \frac{2(\delta_d + \phi^l)}{\left(\tilde{\sigma}^E \right)^2}} \tag{54}$$

The steady state condition for η, $d\eta = 0$ implies $\eta\left(1-\eta\right)\left(\tilde{\sigma}^E\right)^2 = -\left(\delta_d\psi^0 - \delta_d\eta - \phi^l\eta\right)$ and using that in steady state $\delta_d\psi^0 = \bar{\psi}(\delta_d + \phi^l)$ we have

$$\frac{2\eta(1-\eta)}{\eta-\bar{\psi}} = \frac{2(\delta_d+\phi^l)}{\left(\tilde{\sigma}^E\right)^2}. \tag{55}$$

Note first that this tells us that $\bar{\psi} < \eta$ in steady state, otherwise the left-hand side is negative, which cannot happen since the right-hand side is strictly positive. Moreover, plugging this into the expression for ζ then gives us the expression in (35). Taking the derivative w.r.t η of (35) shows that ζ is decreasing in η for in $\eta \in (0,1)$ (see Online Appendix B.6 for algebra). The KFE for the distribution of wealth among former entrepreneurs $f_{FE}(n)$ is $-\frac{\partial}{\partial n}\left[\mu^D n f_{FE}(n)\right] - \delta_d f_{FE}(n) + M f_E(n) = 0$, where M density added by switching. We see that for $n > N$, $f_{FE}(n) = \frac{M}{\delta - \mu^D\zeta} f_E(n)$ solves this. Hence, a distribution that is proportional to $f_E(n)$ (i.e. has the same right tail coefficient) above N solves the KFE for former entrepreneurs. Never-entrepreneurs do not attain wealth above N because $\mu^D < 0$. The uniqueness of and convergence to these distributions from arbitrary initial distributions is to my knowledge harder to make statements about. From given initial distributions one can study convergence numerically, which I do in the numerical exercises.

A.5 Impact of Fall in χ on $K_{ss}, \eta_{ss}, \kappa_{ss}$.

In the case of perfect substitutability of the goods produced by the two types of firms and provided that the steady state remains such that both types of firms are active, using (33) and (38), a steady state is defined by the following system of equations in the three unknowns $\{K_{ss}, \eta_{ss}, \kappa_{ss}\}$

$$\begin{aligned}
0 &= (1-\nu)\alpha\left(\frac{L}{K_{ss}}\right)^{1-\alpha} + \eta_{ss}\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2 - \rho - \delta \\
0 &= (1-\eta_{ss})\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2 + \frac{(\bar{\psi}-\eta_{ss})}{\eta_{ss}}(\delta_d+\phi^l) \\
2\nu - 1 &= \frac{\kappa_{ss}}{\eta_{ss}}\left(\chi\tilde{\sigma}\right)^2\left(\frac{L}{K_{ss}}\right)^{1-\alpha}.
\end{aligned} \tag{56}$$

We can now ask the following question: suppose $K_0 > 0$ and $\eta_0, \kappa_0 \in (0,1)$ solve the above system for some χ, how would the steady state change in response to a small decline in χ? For the second equation to hold, η_{ss} has to increase when χ falls. The first equation then implies K_{ss} has to rise. If both η_{ss} and K_{ss} rise, the last equation implies that κ_{ss} rises.

B Online Appendix

B.1 Entrepreneurs' Problem

Let S denote a vector of all state variables except the agent's net worth. We can allow χ to be such a state variable, in which case require that it follow an Ito process. We consider a slightly more general version of the problem than the one considered in Section 2. In particular, I adopt the more general framework of section 3, where entrepreneurs lose their entrepreneurial ability at rate ϕ^l according to a Poission process independent of all other processes and die at rate δ_d according to a Poission process independent of all other processes. As we will see below, the risk of losing entrepreneurial ability does not affect optimal consumption, portfolio choice or factor input mix. Moreover, as is well-known (see for instance Moll et al. (2022)), the probability of death only affects the optimization problem by changing the time-discount rate. I use $\tilde{\rho}$ to denote the pure time-preference rate and let $\rho = \tilde{\rho} + \delta$ denote the time-preference rate adjusted by the death probility. The entrepreneur's problem as specified in Section 2 is the special case wherein $\phi^l = \delta_d = 0$.

Then, the HJB equation of the entrepreneurs' problem can be written as

$$\begin{aligned}
\tilde{\rho}V^E(n,S) = \max_{\{c,x,\theta^k,\theta^{\text{out}},\theta^{\text{fund}}\}} & \log(c) + V^E_n n\left(r + \theta^k\left(r^k(x) - r\right) - \theta^{\text{out}}\varsigma\sigma + \theta^{\text{fund}}\varsigma\sigma - \frac{c}{n}\right) \\
& + \frac{1}{2}V^E_{nn}n^2\left(\left(\theta^k - \theta^{\text{out}}\right)^2(\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)^2\sigma^2\right) \\
& + \sum_{s\in S} V^E_s\mu_s s + \frac{1}{2}V^E_{ss}s^2\sigma_s^2 + V^E_{ss'}ss'\sigma_s\sigma_{s'} + V^E_{sn}sn\sigma_s\sigma_n \\
& + \lambda((1-\chi)\theta^k - \theta^{\text{out}}) \\
& + \phi^l\left(V^D(n,S) - V^E(n,S)\right) - \delta_d V^E(n,S) \\
& \text{where} \quad x = \frac{y}{k}
\end{aligned}$$

where $V^D(n,S)$ is the value function for a diversified capitalist. So save on notation, I use V to denote the value function while being an entrepreneur from now on. The first-order

conditions of this problem are

$$
\begin{aligned}
c^{-1} &= V_n \\
V_n n(r^k(x) - r) &= -V_{nn} n^2 \left(\left(\theta^k - \theta^{\text{out}} \right) (\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}} \right) \sigma^2 \right) - \lambda(1-\chi) \\
&\quad - \sum_{s \in S} V_{sn} \frac{\partial \sigma_n}{\partial \theta^k} n s \sigma_s \\
V_n n \varsigma \sigma &= -V_{nn} n^2 \left(\left(\theta^k - \theta^{\text{out}} \right) (\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}} \right) \sigma^2 \right) - \lambda \\
&\quad + \sum_{s \in S} V_{sn} \frac{\partial \sigma_n}{\partial \theta^{\text{out}}} n s \sigma_s \\
V_n n \varsigma \sigma &= -V_{nn} n^2 \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}} \right) \sigma^2 - \sum_{s \in S} V_{sn} \frac{\partial \sigma_n}{\partial \theta^{\text{fund}}} s n \sigma_s \\
V_n n \theta^k \left(p - \frac{1}{1-\alpha} w x^{\frac{\alpha}{1-\alpha}} \right) &= -V_{nn} n^2 \left(\theta^k - \theta^{\text{out}} \right)^2 \tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma}^2 \tilde{g}'(x^{\frac{1}{1-\alpha}}) \frac{1}{1-\alpha} x^{\frac{\alpha}{1-\alpha}} + \sum_{s \in S} V_{sn} \frac{\partial \sigma_n}{\partial x} s n \sigma_s .
\end{aligned}
$$

We can guess and later verify that the value functions take the form $V^j(n, S) = \frac{1}{\rho} \log(n) + g^j(S)$ for $j \in \{E, D\}$.[24] With this guess we have $V_n n = -V_{nn} n^2 = \frac{1}{\rho}$ and all mixed derivatives are $V_{sn} = 0$. The first-order conditions can then be written as

$$
\begin{aligned}
r^k(x) - r &= \left(\theta^k - \theta^{\text{out}} \right) (\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}} \right) \sigma^2 - \rho\lambda(1-\chi) \\
\varsigma \sigma &= \left(\theta^k - \theta^{\text{out}} \right) (\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}} \right) \sigma^2 - \rho\lambda \\
\varsigma \sigma &= \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}} \right) \sigma^2 \\
\theta^k \left(p - \frac{1}{1-\alpha} w x^{\frac{\alpha}{1-\alpha}} \right) &= \left(\theta^k - \theta^{\text{out}} \right)^2 \tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma}^2 \tilde{g}'(x^{\frac{1}{1-\alpha}}) \frac{1}{1-\alpha} x^{\frac{\alpha}{1-\alpha}} .
\end{aligned}
$$

We look for solutions where entrepreneurs will actually want to invest some capital in their firm and so we consider the case $\theta^k > 0$. Combining the second and third first-order condition we obtain

$$
\rho\lambda = \left(\theta^k - \theta^{\text{out}} \right) (x\tilde{\sigma})^2 \tag{57}
$$

The skin-in-the-game constraint together with $\theta^k > 0$ ensures that the right-hand side of this equation is positive, which means $\lambda > 0$. Hence, the skin-in-the-game constraint is always binding. We then have $\theta^{\text{out}} = (1-\chi)\theta^k$ and the first-order conditions can be

[24] When utility is homothetic and budget constraints are linear, this is a standard guess for the form of the value function (see Brunnermeier and Sannikov (2017), Di Tella and Hall (2021))

reduced to

$$\begin{aligned} r^k(x) - r &= \chi\theta^k(\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma^2 - \chi\theta^k(\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2(1-\chi) \\ \varsigma\sigma &= \chi\theta^k(\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma^2 - \chi\theta^k(\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 \\ \varsigma\sigma &= \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma^2 \\ p - \frac{1}{1-\alpha}wx^{\frac{\alpha}{1-\alpha}} &= \theta^k\chi^2\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma}^2\tilde{g}'(x^{\frac{1}{1-\alpha}})\frac{1}{1-\alpha}x^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{58}$$

The second and the third first-order conditions are now identical. We can simplify this further to

$$\begin{aligned} r^k(x) - r &= \chi^2\theta^k(\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2 + \varsigma\sigma \\ \varsigma &= \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma \\ p - \frac{1}{1-\alpha}wx^{\frac{\alpha}{1-\alpha}} &= \theta^k\chi^2\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma}^2\tilde{g}'(x^{\frac{1}{1-\alpha}})\frac{1}{1-\alpha}x^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{59}$$

we therefore have

$$\begin{aligned} \theta^k &= \frac{r^k(x) - r^{\text{fund}}}{(\chi\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2} \\ \theta^{\text{fund}} &= \frac{r^{\text{fund}} - r}{\sigma^2} - \chi\theta^k \\ \theta^{\text{out}} &= (1-\chi)\theta^k \end{aligned} \tag{60}$$

The first-order condition for x now reads

$$p - \frac{1}{1-\alpha}wx^{\frac{\alpha}{1-\alpha}} = \frac{r^k(x) - r^{\text{fund}}}{(\chi\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma})^2}\chi^2\tilde{g}(x^{\frac{1}{1-\alpha}})\tilde{\sigma}^2\tilde{g}'(x^{\frac{1}{1-\alpha}})\frac{1}{1-\alpha}x^{\frac{\alpha}{1-\alpha}} \tag{61}$$

Multiplying the last first-order condition by $(1-\alpha)x$ we obtain the following:

$$px - \frac{1}{1-\alpha}wx^{\frac{1}{1-\alpha}} = (r^k(x) - r^{\text{fund}})\frac{x^{\frac{1}{1-\alpha}}\tilde{g}'(x^{\frac{1}{1-\alpha}})}{\tilde{g}(x^{\frac{1}{1-\alpha}})(1-\alpha)} \tag{62}$$

Using that $\frac{x^{\frac{1}{1-\alpha}}\tilde{g}'(x^{\frac{1}{1-\alpha}})}{\tilde{g}(x^{\frac{1}{1-\alpha}})(1-\alpha)} = \frac{1-\beta}{1-\alpha} \equiv \bar{g}$, and $r^k(x) = px - wx^{\frac{1}{1-\alpha}} - \delta$ we obtain

$$px - \frac{1}{1-\alpha}wx^{\frac{1}{1-\alpha}} = (px - wx^{\frac{1}{1-\alpha}} - \delta - r^{\text{fund}})\bar{g} \tag{63}$$

which can be rewritten as

$$(\bar{g}-1)px + \left(\frac{1}{1-\alpha} - \bar{g}\right)wx^{\frac{1}{1-\alpha}} = (\delta + r^{\text{fund}})\bar{g}. \tag{64}$$

Regardless of the value of $\bar{g}$, the solution to this equation (if it exists) only depends on prices and parameters that are independent of i. Hence, each entrepreneur chooses the same x. In the tractable special cases $\bar{g} = 1$ and $\bar{g} = 0$ (corresponding to $\beta = \alpha$ and $\beta = 1$) we have the following. With $\bar{g} = 1$ we obtain

$$\frac{1}{1-\alpha} w x^{\frac{1}{1-\alpha}} = w x^{\frac{1}{1-\alpha}} + r^{\text{fund}} + \delta \tag{65}$$

which implies

$$x = \left(\frac{1-\alpha}{\alpha} \frac{r^{\text{fund}} + \delta}{w} \right)^{1-\alpha} \tag{66}$$

Or in terms of the labor-to-capital ratio

$$\frac{l_{it}}{k_{it}} = \frac{1-\alpha}{\alpha} \frac{r^{\text{fund}} + \delta}{w}. \tag{67}$$

With $\bar{g} = 0$ we instead get

$$x = \left(\frac{p(1-\alpha)}{w} \right)^{1-\alpha}. \tag{68}$$

In general we have the following solution:

$$\begin{aligned}
c_{it} &= \rho n_{it} \\
(\delta + r^{\text{fund}})\bar{g} &= (\bar{g} - 1) p x_{it} + \left(\frac{1}{1-\alpha} - \bar{g} \right) w x_{it}^{\frac{1}{1-\alpha}} \\
\theta_{it}^{k} &= \frac{r_t^k - r_t^{\text{fund}}}{(\chi g(x_{it}^{\frac{1}{1-\alpha}}) \tilde{\sigma})^2} \\
\theta_{it}^{\text{fund}} &= \frac{r_t^{\text{fund}} - r_t}{\sigma} - \chi \theta_{it}^{k} \\
\theta_{it}^{\text{out}} &= (1-\chi) \theta_{it}^{k}
\end{aligned} \tag{69}$$

Also note that

$$\theta_{it}^{k} - \theta_{it}^{\text{out}} + \theta_{it}^{\text{fund}} = \frac{\varsigma_t}{\sigma} \tag{70}$$

which implies that their exposure to aggregate risk is $(\theta_{it}^{k} - \theta_{it}^{\text{out}} + \theta_{it}^{\text{fund}})\sigma = \varsigma_t$. Because each entrepreneurial firm chooses the same output-to-capital ratio, their total output Y_t^E is

$$Y_t^E = \left(K_t^E \right)^{\alpha} \left(L_t^E \right)^{1-\alpha} \tag{71}$$

where K_t^E and L_t^E is the total amount of capital and labor operated by the entrepreneurs.

B.2 Verification of Functional form

In this section, we provide an argument for the functional form of the value function for entrepreneurs being $V^E(n,S) = \frac{1}{\rho}\log(n) + \frac{1}{\rho}g^E(S)$, taking as given the dynamics of the aggregate state variables are S. I follow the the line of reasoning in Duffie (2010).[25] To do this we use the version of an entrepreneur's problem that is formulated as a Merton problem. We let the control variables be the vector of portfolio shares (as fractions of net worth n) in the different available assets, $\theta^\intercal$, the output-to-capital ratio in the firm, x, and the consumption *rate* process $\varrho = \frac{c}{n}$. Specifically, $\mathbf{R}_t^E(x)$ is the vector of excess returns faced by an entrepreneur, and $\boldsymbol{\sigma}_t^E(x)$ is the matrix of associated volatilities. $\mathbf{R}_t^D$ and $\boldsymbol{\sigma}_t^D$ are the corresponding returns and volatilities when/if the entrepreneur has switched types. These are functions of the aggregate state rather than time itself. We let τ denote the (random) type switching time. In other words, the entrepreneur faces one set of excess returns and volatilities before switching, and then a different set after the switch. A diversified agent is just an entrepreneur for whom the switching time is $\tau = 0$. Since the proof for a diversified agent is just a simplified version of that for an entrepreneur, we assume $\tau > 0$ and go through the proof for an entrepreneur's value function only, and therefore suppress the superscript E on the value function. I consider a problem without death, motivated by the well-known result that death is equivalent to an adjustment of the time preference rate, leaving the rest of the problem intact.

The entrepreneur solves

$$\begin{aligned}
&V_0(n_0,S_0) = \max_{\varrho,x,\theta} \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(\varrho_t n_t)dt\right] \\
&\frac{dn_t}{n_t} = \mathbf{1}\{t\le\tau\}\left[\left(r_t + \boldsymbol{\theta}_t^\intercal \mathbf{R}_t^E(x) - \varrho_t\right)dt + \boldsymbol{\theta}_t^\intercal \boldsymbol{\sigma_t}^E(x) d\mathbf{Z}_t^E\right] \\
&+ \mathbf{1}\{t>\tau\}\left[\left(r_t + \theta_t^\intercal \mathbf{R}_t^D - \varrho_t\right)dt + \boldsymbol{\theta}_t^\intercal \boldsymbol{\sigma}_{\boldsymbol{t}}^{\boldsymbol{D}} d\mathbf{Z}_t^D\right], \\
&\text{and the skin-in-the-game constraint 8, and } n_t \ge 0
\end{aligned} \tag{72}$$

Lemma 4. *If policy (ϱ, x, θ) is feasible with initial net worth n_0, with the resulting net worth process n_t, then it is also feasible with initial net worth ϖn_0, with the resulting net worth process*

[25] I want to thank Sebastian Merkel for pointing out this line of argument.

$n_t^{\varpi} = \varpi n_t$. *Moreover,*

$$V_0^{\varrho,x,\theta}(\varpi n_0, S_0) = V_0^{\varrho,x,\theta}(n_0, S_0) + \frac{1}{\rho}\log(\varpi) \tag{73}$$

where $V_0^{\varrho,x,\theta}$ *is the expected utility under the policy* (ϱ, x, θ).

Proof. The dynamic budget constraint is linear in net worth, and the skin-in-the-game constraint is independent of net worth. Moreover,

$$\begin{aligned} V_0^{\varrho,x,\theta}(\varpi n_0, S_0) &= \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(\varrho_t n_t^{\varpi})dt\right] \\ &= \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(\varrho_t n_t)dt\right] + \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(\varpi)dt\right] \\ &= V_0^{\varrho,x,\theta}(n_0, S_0) + \frac{1}{\rho}\log(\varpi) \end{aligned} \tag{74}$$

where the second line uses that $n_t^{\varpi} = \varpi n_t$. □

Lemma 5. *Suppose policy* (ϱ, x, θ) *is optimal with initial net worth* n_0*, then it is also optimal with initial net worth* ϖn_0*.*

Proof. By the previous proposition, the policy is feasible with initial net worth ϖn_0. To prove that it is also optimal with initial net worth ϖn_0, we proceed using a proof by contradiction. That is, suppose policy (ϱ, x, θ) is optimal with initial net worth n_0, but that is not optimal with initial net worth ϖn_0. If an optimal policy exists, there must then be some other policy $(\hat{\varrho}, \hat{x}, \hat{\theta})$ that is optimal with initial net worth ϖn_0. This policy will then have the associated net worth process $\hat{n}_t^{\varpi}$. By Lemma 4, $(\hat{\varrho}, \hat{x}, \hat{\theta})$ is feasible with initial wealth n_0 with net worth process $\hat{n}_t = \frac{1}{\varpi}\hat{n}_t^{\varpi}$. Since $(\hat{\varrho}, \hat{x}, \hat{\theta})$ is optimal with initial net worth ϖn_0 we have

$$\begin{aligned} &V_0^{\varrho,x,\theta}(\varpi n_0, S_0) \\ &< V_0(\varpi n_0, S_0) \\ &= V_0^{\hat{\varrho},\hat{x},\hat{\theta}}(\varpi n_0, S_0) \\ &= V_0^{\hat{\varrho},\hat{x},\hat{\theta}}(n_0, S_0) + \frac{1}{\rho}\log(\varpi) \\ &\leq V_0^{\varrho,x,\theta}(n_0, S_0) + \frac{1}{\rho}\log(\varpi) \\ &= V_0^{\varrho,x,\theta}(\varpi n_0, S_0) \end{aligned} \tag{75}$$

where the first inequality uses that (ϱ, x, θ) is not optimal with initial net worth ϖn_0, the next equality uses that $(\hat{\varrho}, \hat{x}, \hat{\theta})$ is optimal with initial net worth ϖn_0, the next equality uses Lemma 4, the next inequality uses that (ϱ, x, θ) is optimal with initial net worth n_0, and the final equality uses Lemma 4 again. This is a contradiction, which completes the proof. □

Proposition 4. *The value function takes the form*

$$V_0(n_0, S_0) = \frac{1}{\rho}\log(n_0) + \frac{1}{\rho}g(S_0) \tag{76}$$

Proof. By Lemmata 4 and 5 we have

$$V_0(n_0, S_0) + \frac{1}{\rho}\log(\varpi) = V_0(\varpi n_0, S_0). \tag{77}$$

Since this holds for arbitrary ϖ, it holds for $\varpi = 1/n_0$. We can therefore write

$$V_0(n_0, S_0) = \frac{1}{\rho}\log(n_0) + V_0(1, S_0). \tag{78}$$

Defining $g(S_0) \equiv \rho V_0(1, S_0)$ completes the proof. □

B.3 Additional Generalizations

Lemma 1 is presented above under logarithmic utility. Moreover, entrepreneurs' willingness to bear idiosyncratic risk is contingent on this risk being associated with a risk premium. But how general is the proposition that better risk sharing leads to more risk-taking for given excess returns? In this section, two possible generalizations are briefly discussed. CRRA preferences and differences in beliefs between entrepreneurs and outsiders. We discuss each generalization in turn.

Constant relative risk-aversion. With completely risk neutral entrepreneurs, improvements in risk sharing would not affect their risk-bearing decision. However, this intuition breaks down for positive levels of risk aversion. To see this, consider a simple setting in which expected returns and volatilities are constant. The optimal choice of risk exposure for a CRRA entrepreneur is then:

$$\tilde{\sigma}^E = \frac{1}{\gamma}\frac{r^k - r^{\text{fund}}}{\chi\tilde{\sigma}}. \tag{79}$$

From this expression, it is clear that a fall in χ (improved risk sharing) leads to rising

optimal risk exposure, as long as risk aversion is positive. In fact, the absolute effect of a fall in χ is actually *larger* the smaller is the coefficient of risk aversion.[26] Strikingly, as $\gamma \to 0$, the absolute effect of a fall in χ becomes arbitrarily large. Expressed differently, the elasticity of risk exposure with respect to χ is $\frac{d\log(\tilde{\sigma}^E)}{d\log(\chi)} = -1$, which is independent of γ.

Belief heterogeneity. If entrepreneurs are more optimistic than outsiders regarding the expected growth rate (and therefore the expected return) of the capital in their own firm, entrepreneurs could be willing to bear idiosyncratic risk even if the idiosyncratic risk premium is very small or even nonexistent. However, to the extent that entrepreneurs still require an idiosyncratic risk premium in equilibrium, risk taking would still increase when risk sharing improves. The intuition is that the skin-in-the-game constraint could still be binding even if entrepreneurs are more optimistic, as long as they are not so much more optimistic that they do not require a risk premium at all. One should note that this line of reasoning leaves open the question of what conditions are required for there to be an idiosyncratic risk premium when entrepreneurs are more optimistic than outsiders.

Most importantly, the above discussion should make it clear when the insight in Lemma 1 is not applicable: if the ability to scale up with maintained profitability is limited or if the skin-in-the-game constraint is not binding, then the insight would not apply. Moreover, with a more complicated correlation structure between different idiosyncratic risks and aggregate risk, or with arbitrary return distributions, it is not necessarily clear how the phrase "less risk" should be interpreted (volatility, skewness, kurtosis, etc.). In such a setting, the insight that better risk sharing would lead to more risk-taking must be modified in a way that is beyond the scope of this study. The formal analysis of these issues is left for future research.

B.4 Characterizing the Equilibrium

The economy-wide state variables are η_t and K_t. To characterize equilibrium, we now derive an equation that pins down the allocation of capital to the entrepreneurial sector, $K_t^T = \kappa_t K_t$. All objects of interest in the model can then be expressed in terms of η_t, K_t, κ_t and exogenous parameters.

[26] In general, the formula for the optimal risk exposure is of the form $\tilde{\sigma}_t^E = \frac{1}{\gamma}\frac{r_t^k - r_t^{\text{fund}}}{\chi\tilde{\sigma}} + \{\text{hedging term}\}_t$, where the hedging term captures how the optimal idiosyncratic risk exposure is affected by the fact that the investors investment opportunities may vary over time. But maintaining the assumption that the instantaneous idiosyncratic return to capital is independent of movements in the investment opportunity set, there will be no hedging demand.

Combining the demand for intermediate goods in (22) with the supply of each intermediate good in equations (17) and (71) we obtain the following intermediate goods prices

$$p_t^T = (1-\nu)\left(\frac{1-\kappa_t}{A(\kappa_t)}\right)^{-\frac{1}{\varepsilon}}, \quad p_t^E = \nu\left(\frac{\kappa_t}{A(\kappa_t)}\right)^{-\frac{1}{\varepsilon}} \tag{80}$$

We can then derive an equation that pins down the fraction of the capital operated by the entrepreneurial firms, κ_t, by combining these expressions for the prices with the capital demand of entrepreneurial firms in equation (10). Specifically, using that $\frac{k_{it}}{n_{it}} = \frac{K_t^E}{N_t^E} = \frac{\kappa_t}{\eta_t}$ we obtain from this equation that

$$\frac{\kappa_t}{\eta_t}\left(\left(\frac{L}{K_t}\right)^{1-\alpha}\chi\tilde{\sigma}\right)^2 = r_t^k - r_t^{\text{fund}} = p_t^E\left(\frac{L}{K_t}\right)^{1-\alpha} - w_t\left(\frac{L}{K_t}\right) - (r_t^{\text{fund}} + \delta) \tag{81}$$

Combining this with the first-order conditions of the established traditional firm that provide expressions for w_t and R_t we obtain after some tedious algebra

$$\frac{\kappa_t}{\eta_t}(\chi\tilde{\sigma})^2\left(\frac{L_t}{K_t}\right)^{1-\alpha} = p_t^E - p_t^T(1-\alpha) - p_t^T\alpha \tag{82}$$

which can be rewritten as

$$\frac{\kappa_t}{\eta_t}(\chi\tilde{\sigma})^2\left(\frac{L_t}{K_t}\right)^{1-\alpha} = p_t^E - p_t^T \tag{83}$$

This equation pins down a unique $\kappa_t \in (0,1)$ if $\varepsilon > 0$. To see why, note that the left-hand side is a strictly increasing linear function of κ_t, given positive η_t and K_t. Moreover, using that $A(\kappa_t) = \left[\nu\left(\kappa_t\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(1-\kappa_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$ we can write the prices as

$$p_t^T = (1-\nu)\left[\nu\left(\frac{\kappa_t}{1-\kappa_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\right]^{\frac{1}{\varepsilon-1}} \tag{84}$$

and

$$p_t^E = \nu\left[\nu + (1-\nu)\left(\frac{1-\kappa_t}{\kappa_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} \tag{85}$$

We see that if $\varepsilon > 0$, then p_t^T is increasing in $\kappa_t \in (0,1)$ and p_t^E is decreasing. For κ_t close to 0 we will have $p_t^E - p_t^T > 0$, for κ_t close to 1 we will have $p_t^E - p_t^T < 0$. Hence, for some unique $\kappa_t \in (0,1)$, the left-hand side and right-hand side intersect.

B.5 Steady State Tail Coefficient

In this section, I attempt to extend the analysis of the steady state wealth distribution found in Section A.4 main appendix by finding fully characterized probability densities that are consistent with the steady state Kolmogorov forward equations for both types of agents. I omit the subscript denoting steady state. All variables should be understood as being in steady state. First note that in steady state, the diffusion and drift of the wealth growth of entrepreneurs is

$$\tilde{\sigma}^E, \quad \text{and} \quad \mu^E = r^E - \rho = (1-\eta)\left(\tilde{\sigma}^E\right)^2 > 0. \tag{86}$$

For diversified capitalists, we instead have zero volatility and negative steady state mean growth rate of wealth $\mu^D = r^D - \rho < 0$. The line of reasoning in this section follows from that used in Gabaix (2009) and Gomez and Gouin-Bonenfant (2024), both of which consider random growth models of wealth accumulation. The relevant Kolmogorov forward equation for the distribution of entrepreneurs' wealth can be written as

$$0 = -\frac{\partial}{\partial n}\left[\mu^E n f_E(n)\right] + \frac{\partial^2}{\partial n^2}\left[\frac{1}{2}(\tilde{\sigma}^E)^2 n^2 f_E(n)\right] - (\delta_d + \phi^l) f_E(n). \tag{87}$$

We can guess and verify that the density of a double Pareto distribution solves this equation, i.e. a density of the form:

$$f_E(n) = A\left[\left(\frac{n}{N}\right)^{-\zeta_+ - 1}\mathbf{1}_{n>N} + \left(\frac{n}{N}\right)^{-\zeta_- - 1}\mathbf{1}_{n<N}\right], \tag{88}$$

where N is the steady state average wealth level (which is the wealth level at birth), $A = \frac{-\zeta_-\zeta_+}{(\zeta_+ - \zeta_-)N}$, and ζ_+ and ζ_- are the positive and negative roots, respectively, of the quadratic equation

$$0 = \zeta\mu^E + \frac{(\tilde{\sigma}^E)^2}{2}\zeta(\zeta - 1) - (\delta_d + \phi^l) \tag{89}$$

(see Gabaix (2009) and references therein). For the diversified agents, we guess that the following distribution solves the Kolmogorov forward equation associated with diversified agents wealth process:

$$f_D(n) = B\left[\omega_1 f_E(n) + \left(\omega_2\left(\frac{n}{N}\right)^{-\zeta_- - 1} + (1 - \omega_1 - \omega_2)\left(\frac{n}{N}\right)^{-\tilde{\zeta}_- - 1}\right)\mathbf{1}_{n<N}\right]. \tag{90}$$

This guess is motivated by the fact that diversified capitalists consists of former entrepreneurs that have switched, but also of agents that were born diversified. Because $\mu^D < 0$, those born diversified will never have wealth above N, so all diversified agents with $n > N$ must be former entrepreneurs. The guess is verified if we can select the weights B, ω_1 and ω_2, and the tail-coefficient $\tilde{\zeta}$ to solve the Kolmogorov forward equation

$$0 = -\frac{\partial}{\partial n}\left[\mu^D n f_D(n)\right] - \delta_d f_D(n) + \frac{\phi^l \psi^E}{\psi^D} f_E(n). \tag{91}$$

and provided the resulting density is actually a density (positive and integrates to 1), where ψ^j is the fraction of capitalists belonging to $j \in \{E, D\}$. Consider the case $n > N$. Then, the KFE is simply

$$B\omega_1 \mu^D \zeta_+ - B\omega_1 \delta_d + \frac{\phi^l \psi^E}{\psi^D} = 0 \tag{92}$$

This gives us $\omega_1 = \frac{\phi^l \psi^E}{\psi^D(\delta_d - \mu^D \zeta_+)B}$, which pins down the distribution of diversified agents' wealth above N. Next consider $n < N$. Let $g(n) = B\left(\frac{n}{N}\right)^{-\tilde{\zeta}_- - 1}$. Then the KFE can in this case be written

$$\begin{aligned} 0 = \left(-\frac{\partial}{\partial n}\left[\mu^D n B\left(\omega_1 A + \omega_2\right) n^{-\zeta_- - 1}\right] - B\delta_d\left(\omega_1 A + \omega_2\right) n^{-\zeta_- - 1} + \frac{\phi^l \psi^E}{\psi^D} A n^{-\zeta_- - 1}\right) N^{\zeta_- + 1} + \\ \left(-\frac{\partial}{\partial n}\left[\mu^D n g(n)\right] - \delta_d g(n)\right)(1 - \omega_1 - \omega_2). \end{aligned} \tag{93}$$

Note that the expression in the second term $\left(-\frac{\partial}{\partial n}\left[\mu^D n g(n)\right] - \delta_d g(n)\right)$, is analogous to right-hand side of the KFE for a GBM with zero volatility, growth rate μ^D, constant death rate δ_D and rebirth at N. Specifically, this expression is 0 if we pick $\tilde{\zeta}_- = \frac{\delta_d}{\mu^D}$ (see Steindl (1965) and Gabaix et al. (2016)). Setting the first expression to 0 then gives an equation for ω_2, namely

$$0 = \mu^D B\left(\omega_1 A + \omega_2\right)\zeta_- - B\delta_d\left(\omega_1 A + \omega_2\right) + \frac{\phi^l \psi^E}{\psi^D} A \tag{94}$$

which implies

$$\omega_2 = \left[\frac{B\omega_1\left(\delta_d - \mu^D \zeta_-\right) - \frac{\phi^l \psi^E}{\psi^D}}{(\mu^D \zeta_- - \delta_d)B}\right] A. \tag{95}$$

Finally, we pick B to ensure that the density integrates to 1. Integrating $f_D(n)$ and setting this to 1 gives

$$\begin{aligned} & B\left(\omega_1 - \omega_2 \frac{N}{\zeta_-} - (1-\omega_1-\omega_2)\frac{N}{\tilde{\zeta}_-}\right) \\ \Rightarrow B = & \frac{1}{\omega_1 - \omega_2 \frac{N}{\zeta_-} - (1-\omega_1-\omega_2)\frac{N}{\tilde{\zeta}_-}}. \end{aligned} \tag{96}$$

This verifies the guess provided that parameters are such that the resulting density is positive. Note that the limiting right tail of the distribution of wealth for diversified agents is inherited from that of entrepreneurs, where as the limiting left tail depends on whether ζ_- is larger than or smaller than $\tilde{\zeta}_-$.

B.6 Proof of Lemma 2

To further characterize the right tail we have the following quadratic equation from (89)

$$\zeta^2 + \zeta\left(\frac{2\mu^E}{(\tilde{\sigma}^E)^2} - 1\right) - \frac{2(\delta_d + \phi^l)}{(\tilde{\sigma}^E)^2} = 0 \tag{97}$$

using that $\frac{2\mu^E}{(\tilde{\sigma}^E)^2} = 2(1-\eta)$ we can write this as

$$\zeta^2 + \zeta\left(1 - 2\eta\right) - \frac{2(\delta_d + \phi^l)}{(\tilde{\sigma}^E)^2} = 0 \tag{98}$$

the positive solution to this is (which I from here on denote simply by ζ instead of ζ_+ to save on notation)

$$\zeta = \eta - \frac{1}{2} + \sqrt{\left(\eta - \frac{1}{2}\right)^2 + \frac{2(\delta_d + \phi^l)}{(\tilde{\sigma}^E)^2}} \tag{99}$$

We can go further in characterizing this since the steady state condition for η, $d\eta = 0$ implies

$$\eta\left(1-\eta\right)(\tilde{\sigma}^E)^2 = -\left(\delta_d\psi^0 - \delta_d\eta - \phi^l\eta\right) \tag{100}$$

using that in steady state $\delta_d\psi^0 = \bar{\psi}(\delta_d + \phi^l)$ this implies

$$\eta\left(1-\eta\right)(\tilde{\sigma}^E)^2 = (\eta - \bar{\psi})(\delta_d + \phi^l) \tag{101}$$

so that

$$\frac{2\eta(1-\eta)}{\eta-\bar{\psi}} = \frac{2(\delta_d+\phi^l)}{\left(\tilde{\sigma}^E\right)^2} \tag{102}$$

Note first that this tells us that $\bar{\psi} < \eta$ in steady state, otherwise the left-hand side is negative, which cannot happen since the right-hand side is strictly positive. Moreover, plugging this into the expression for ζ then gives

$$\zeta = \eta - \frac{1}{2} + \sqrt{\left(\eta-\frac{1}{2}\right)^2 + \frac{2\eta(1-\eta)}{\eta-\bar{\psi}}} \tag{103}$$

the left-tail coefficient can be solved similarly.

We now proceed to proving that ζ is strictly decreasing in η. In other words, tail inequality is increasing in η. First, note that the expression $\frac{2\eta(1-\eta)}{\eta-\bar{\psi}}$ is strictly decreasing in η. To see this, note that its derivative is

$$\frac{\left(2(1-\eta)-2\eta\right)\left(\eta-\bar{\psi}\right)-2\eta(1-\eta)}{(\eta-\bar{\psi})^2} = \frac{2\eta-4\eta^2-2\bar{\psi}+4\bar{\psi}\eta-2\eta+2\eta^2}{(\eta-\bar{\psi})^2} =$$

$$\frac{-2\eta^2+4\bar{\psi}\eta-2\psi^2+2\psi^2-2\bar{\psi}}{(\eta-\bar{\psi})^2} = \frac{-2(\eta-\bar{\psi})^2-2\bar{\psi}(1-\bar{\psi})}{(\eta-\bar{\psi})^2} = -2\left(1+\frac{\bar{\psi}(1-\bar{\psi})}{(\eta-\bar{\psi})^2}\right) < 0. \tag{104}$$

Moreover, clearly $\eta - \frac{1}{2}$ is increasing in η. The question is therefore if the slope of the second term under the bracket, $\frac{2\eta(1-\eta)}{\eta-\bar{\psi}}$, negative enough to counteract the positive slope coming from the terms $\eta - \frac{1}{2}$ and $(\eta-\frac{1}{2})^2$. To prove this, we proceed in two steps. First we show that when $\bar{\psi}=0$ the derivative of ζ with respect to η is 0. Then we show that for any other $\bar{\psi}$, the derivative is smaller. Intuitively, note that equation (104) implies that slope of $\frac{2\eta(1-\eta)}{\eta-\bar{\psi}}$ is *least* negative (smallest in magnitude) when $\bar{\psi}=0$. Moreover, for any other admissible value of $\bar{\psi}$ (that is $\bar{\psi} < \eta < 1$) the slope of this term is more negative. Hence, if we can show that ζ is non-increasing in η when $\bar{\psi}=0$, then it must be the case that ζ is decreasing in η when $\bar{\psi} > 0$. More formally, If we plug in $\bar{\psi}=0$ in the expression for η, we obtain

$$\eta - \frac{1}{2} + \sqrt{\left(\eta-\frac{1}{2}\right)^2 + 2(1-\eta)} = \eta - \frac{1}{2} + \sqrt{\eta^2-3\eta+\frac{9}{4}} =$$

$$= \eta - \frac{1}{2} + \sqrt{\left(\eta - \frac{3}{2}\right)^2} = \eta - \frac{1}{2} + |\eta - \frac{3}{2}|$$

Using that $\eta \in (0, 1)$, we can write this as

$$= \eta - \frac{1}{2} + \sqrt{\left(\eta - \frac{3}{2}\right)^2} = \eta - \frac{1}{2} - \eta + \frac{3}{2} = 1$$

The slope of this is 0. Next, note that in general we have

$$\frac{d\zeta}{d\eta} = 1 - \frac{\frac{3}{2} - \eta + \frac{\bar{\psi}(1-\bar{\psi})}{(\eta-\bar{\psi})^2}}{\sqrt{\left(\eta - \frac{1}{2}\right)^2 + \frac{2\eta(1-\eta)}{\eta-\bar{\psi}}}} = 1 - \frac{(\frac{3}{2} - \eta)(\eta - \bar{\psi}) + \frac{\bar{\psi}(1-\bar{\psi})}{(\eta-\bar{\psi})}}{\sqrt{\eta-\bar{\psi}}\sqrt{\left(\eta - \frac{1}{2}\right)^2 (\eta - \bar{\psi}) + 2\eta(1-\eta)}}.$$

This derivative attains its largest value when the term $\frac{(\frac{3}{2} - \eta)(\eta - \bar{\psi}) + \frac{\bar{\psi}(1-\bar{\psi})}{(\eta-\bar{\psi})}}{\sqrt{\eta-\bar{\psi}}\sqrt{\left(\eta - \frac{1}{2}\right)^2 (\eta - \bar{\psi}) + 2\eta(1-\eta)}}$ (which is positive since $\eta > \bar{\psi}$) is as small as possible in magnitude. Note that the denominator of this term is decreasing in $\bar{\psi}$ and so is maximized when $\bar{\psi} = 0$ in the relevant parameter range. If we can show that the numerator is minimized at $\bar{\psi} = 0$, we are done, because this means the ratio s. To this end, note that the derivative of this numerator with respect to $\bar{\psi}$ is

$$-\left(\frac{3}{2} - \eta\right) + \frac{(\eta - \bar{\psi})\left(1 - 2\bar{\psi}\right) + \bar{\psi}(1 - \bar{\psi})}{(\eta - \bar{\psi})^2} = \frac{\left((\eta - \bar{\psi})\left(1 - 2\bar{\psi}\right) + \bar{\psi}(1 - \bar{\psi}) - \left(\frac{3}{2} - \eta\right)\left(\eta - \bar{\psi}\right)^2\right)}{\left(\eta - \bar{\psi}\right)^2}$$

This is positive if and only if the numerator is positive. Examining the numerator we have

$$(\eta - \bar{\psi})\left(1 - 2\bar{\psi}\right) + \bar{\psi}(1 - \bar{\psi}) - \left(\frac{3}{2} - \eta\right)\left(\eta - \bar{\psi}\right)^2 =$$

$$= \eta - 2\bar{\psi}\eta - \bar{\psi} + 2\bar{\psi}^2 + \bar{\psi} - \bar{\psi}^2 - \left(\frac{3}{2} - \eta\right)\left(\eta^2 - 2\bar{\psi}\eta + \bar{\psi}^2\right)$$

$$= \eta\left(1 - \frac{3}{2}\eta + \eta^2\right) - 2\bar{\psi}\eta + \bar{\psi}^2 - \left(\frac{3}{2} - \eta\right)\left(-2\bar{\psi}\eta + \bar{\psi}^2\right)$$

$$= \eta\left(1 - \frac{3}{2}\eta + \eta^2\right) + \left(-2\eta + \bar{\psi}\right)\bar{\psi} - \left(\frac{3}{2} - \eta\right)\left(-2\eta + \bar{\psi}\right)\bar{\psi}$$

$$= \frac{1}{2}\eta\left(2-3\eta+2\eta^2\right) - \left(2\eta-1\right)\left(2\eta-\bar{\psi}\right)\bar{\psi}$$

We want to show that this is positive on the interval $0 \leq \bar{\psi} \leq \eta < 1$. Note that the first term is strictly positive for any $\eta \in (0,1)$. Moreover, the second term can only be negative if $\eta > 1/2$. Note moreover that for any $\eta > 1/2$, this expression is a second degree polynomial in $\bar{\psi}$ with a positive coefficient in front of the $\bar{\psi}^2$ term. For any $\eta > 1/2$, the minimum of this polynomial is attained at $\bar{\psi} = \eta$. To see this, note that the first term does not depend on $\bar{\psi}$, hence, it suffices to look for the minimum of the second degree polynomial $-\left(2\eta-1\right)\left(2\eta-\bar{\psi}\right)\bar{\psi}$. This polynomial has roots at $\bar{\psi} = 0$ and $\bar{\psi} = 2\eta$, and hence by symmetry, the minimum is found at $\bar{\psi} = \eta$. But at $\bar{\psi} = \eta$ we have

$$= \frac{1}{2}\eta\left(2-3\eta+2\eta^2\right) - \left(2\eta-1\right)\left(2\eta-\bar{\psi}\right)\bar{\psi} = \frac{1}{2}\eta\left(2-3\eta+2\eta^2\right) - \left(2\eta-1\right)\eta^2 =$$

$$2\eta - 3\eta^2 + 2\eta^3 - 2\eta^3 + \eta^2 = 2\eta\left(1-\eta\right) > 0.$$

In conclusion, the relevant expression is always positive. Hence, when $\bar{\psi} = 0$, the slope of ζ with respect to η is 0, and the slope is smaller for all other admissible $\bar{\psi}$, hence, ζ is strictly decreasing in η.

Finally, we proceed by showing that ζ is strictly decreasing in $\tilde{\sigma}^E$. This follows from the fact that equation (102) implies that η is strictly increasing in $\tilde{\sigma}^E$, because the left-hand side is decreasing in η (by 104) and the right-hand side is decreasing in $\tilde{\sigma}^E$. Since ζ is decreasing in η, it must be then that ζ is decreasing in $\tilde{\sigma}^E$ as well.

B.7 Analyzing the Steady State

Unless otherwise stated, we are interested in equilibria where entrepreneurs are actually active. The evolution of the economy is described by the following pair of ordinary differential equations:

$$\begin{aligned}
\frac{dK_t}{K_t} &= \left(p^T\alpha\left(\frac{L}{K}\right)^{1-\alpha} + \eta\left(\tilde{\sigma}^E\right)^2 - \rho - \delta\right)dt \\
\frac{d\eta_t}{\eta_t} &= \left(\left(1-\eta\right)\left(\tilde{\sigma}^E\right)^2 + \frac{\left(\bar{\psi}-\eta\right)\left(\delta_d+\phi^l\right)}{\eta}\right)dt
\end{aligned} \tag{105}$$

where $\tilde{\sigma}^E_{ss} = \frac{\kappa}{\eta}(\chi\tilde{\sigma})\left(\frac{L}{K}\right)^{1-\alpha}$, and the equilibrium condition for the allocation of the capital stock:

$$\frac{p^E - p^T}{\chi\tilde{\sigma}} = \frac{\kappa}{\eta}(\chi\tilde{\sigma})\left(\frac{L}{K}\right)^{1-\alpha} \tag{106}$$

where

$$\begin{aligned} p^E = \nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}, p^T = (1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon} \\ A(\kappa) = \left[\nu(\kappa)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)(1-\kappa)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned} \tag{107}$$

The model admits a steady state if there is a solution to the following system of three equations in the three variables η, K and κ:

$$\begin{aligned} p^T\alpha\left(\frac{L}{K}\right)^{1-\alpha} + \eta\left(\frac{p^E - p^T}{\chi\tilde{\sigma}}\right)^2 - (\rho+\delta) = 0 \\ (1-\eta)\left(\frac{p^E - p^T}{\chi\tilde{\sigma}}\right)^2 + \frac{(\bar{\psi}-\eta)\left(\delta_d + \phi^l\right)}{\eta} = 0 \\ \frac{p^E - p^T}{\chi\tilde{\sigma}} - \frac{\kappa}{\eta}(\chi\tilde{\sigma})\left(\frac{L}{K}\right)^{1-\alpha} = 0 \end{aligned} \tag{108}$$

Definition 1. *A non-degenerate steady state equilibrium where both sectors are active is a triplet* $s = (\eta, K, \kappa)$ *that satisfies the equations (108) with* $K > 0$ *and* $\eta \in (0,1)$*, and* $\kappa \in (0,1)$*.*

I begin the analysis by proving the following lemmata:

Lemma 6. *If* $\varepsilon > 1$ *then* $\lim_{\kappa\to 0} p^T = \underline{p^T} \equiv (1-\nu)^{\frac{\varepsilon}{\varepsilon-1}}$ *and* $\lim_{\kappa\to 0} p^E = \infty$*. If* $\varepsilon < 1$*, then* $\lim_{\kappa\to 0} p^T = 0$ *and* $\lim_{\kappa\to 0} p^E = \nu^{\frac{\varepsilon}{\varepsilon-1}} < \infty$*. Symmetric limits apply to* $\kappa \to 1$*.*

Proof.

$$p^T = (1-\nu)\left(\frac{A(\kappa)}{1-\kappa}\right)^{\frac{1}{\varepsilon}} = (1-\nu)\left(\nu\left(\frac{\kappa}{1-\kappa}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\right)^{\frac{1}{\varepsilon-1}}$$

If $\varepsilon > 1$, then the exponent on the ratio inside the bracket is positive and therefore

$$\lim_{\kappa\to 0} p^T = (1-\nu)\left(\nu(0)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\right)^{\frac{1}{\varepsilon-1}} = (1-\nu)^{\frac{\varepsilon}{\varepsilon-1}}.$$

If $0 < \varepsilon < 1$ then the exponent inside the bracket is negative, and so the argument inside the bracket goes to ∞ as $\kappa \to 0$. But the exponent outside the bracket is also negative and

hence $\lim_{\kappa\to 0} p^T = 0$ in this case. For p^E we instead have

$$p^E = \nu\left(\frac{A(\kappa)}{\kappa}\right)^{\frac{1}{\varepsilon}} = \nu\left(\nu + (1-\nu)\left(\frac{1-\kappa}{\kappa}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}.$$

If $\varepsilon > 1$, the exponent inside the bracket is positive, and hence the expression inside the bracket goes to ∞ as $\kappa \to 0$. The exponent outside the brackets is positive and so $p^E \to \infty$ as $\kappa \to 0$. If $0 < \varepsilon < 1$ then the exponents are both negative, and so $p^E \to \nu^{\frac{\varepsilon}{\varepsilon-1}}$. □

Lemma 7. $\lim_{\varepsilon\to\infty} \frac{\partial p^E(\kappa)}{\partial \kappa} = 0$

Proof. Computing the derivative we have

$$\frac{\partial p^E(\kappa)}{\partial \kappa} = \frac{1}{\varepsilon-1}\nu\left(\nu + (1-\nu)\left(\frac{1-\kappa}{\kappa}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}-1}(1-\nu)\left(\frac{1-\kappa}{\kappa}\right)^{-\frac{1}{\varepsilon}}\frac{\varepsilon-1}{\varepsilon}\left(\frac{-\kappa-(1-\kappa)}{\kappa^2}\right) =$$

$$-\frac{1}{\varepsilon}\nu(1-\nu)\left(\nu + (1-\nu)\left(\frac{1-\kappa}{\kappa}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{2-\varepsilon}{\varepsilon-1}}\left(\frac{1-\kappa}{\kappa}\right)^{-\frac{1}{\varepsilon}}\left(\frac{1}{\kappa^2}\right).$$

The first factor in the expression, $-\frac{1}{\varepsilon}$, goes to 0 as $\varepsilon \to \infty$, and the other factors are bounded for $\kappa \in (0,1)$. Hence the expression goes to 0. □

Lemma 8. *Steady state values of κ are bounded above by $\bar{\kappa} \equiv \frac{\left(\frac{\nu}{1-\nu}\right)^{\varepsilon}}{1+\left(\frac{\nu}{1-\nu}\right)^{\varepsilon}} < 1$*

Proof. From equation (106) we see that the numerator on the left-hand side cannot be negative, since all objects on the right-hand side are positive (risk exposure cannot be negative). Therefore

$$p^E - p^F = \nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon} - (1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon} > 0 \tag{109}$$

This implies

$$\frac{\nu}{1-\nu} > \left(\frac{\kappa}{1-\kappa}\right)^{1/\varepsilon} \Rightarrow \kappa < \bar{\kappa} \equiv \frac{\left(\frac{\nu}{1-\nu}\right)^{\varepsilon}}{1+\left(\frac{\nu}{1-\nu}\right)^{\varepsilon}} < 1 \tag{110}$$

□

Lemma 9. *Aggregate TFP $A(\kappa)$ is increasing in κ for $\kappa \leq \bar{\kappa}$.*

Proof. We can write

$$A(\kappa) = \left[\nu\left(\kappa\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(1-\kappa\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{111}$$

Taking the derivative of this we have

$$\frac{dA(\kappa)}{d\kappa} = A(\kappa)^{\frac{1}{\varepsilon}} \left[\nu \kappa^{-\frac{1}{\varepsilon}} - (1-\nu)(1-\kappa)^{-\frac{1}{\varepsilon}} \right]$$

If we can show that $\nu \kappa^{-\frac{1}{\varepsilon}} \geq (1-\nu)(1-\kappa)^{-\frac{1}{\varepsilon}}$ for any $\kappa \leq \bar{\kappa}$, we are done. Using again the non-negativity of risk exposure, we have

$$p^E - p^F = \nu \left(\frac{\kappa}{A(\kappa)} \right)^{-1/\varepsilon} - (1-\nu) \left(\frac{1-\kappa}{A(\kappa)} \right)^{-1/\varepsilon} \geq 0$$

$$\Leftrightarrow \nu \kappa^{-1/\varepsilon} - (1-\nu)(1-\kappa)^{-1/\varepsilon} \geq 0$$

which is what we wanted to show. □

Lemma 10. *Steady state values of κ are bounded below by $\underline{\kappa} > 0$ if $\varepsilon > 1$.*

Proof. Through the second steady state equation in (108), we can implicitly define $\eta(\kappa)$ as the value of η that solves this equation for a given value of κ. $\eta(\kappa)$ is a decreasing function of κ for $\kappa < \bar{\kappa}$. This is because, when $\kappa < \bar{\kappa}$ we know by the earlier lemmas that p^E is strictly decreasing in κ, and p^T is strictly increasing in κ. This means that $p^E - p^T$ is strictly decreasing in κ. This implies that higher values of κ means a lower value for $\left(\frac{p^E - p^T}{\chi \tilde{\sigma}} \right)^2$. With a lower value of $\left(\frac{p^E - p^T}{\chi \tilde{\sigma}} \right)^2$, the value of η that solves (108) is also lower. Hence $\eta(\kappa)$ is decreasing in κ. Now, we show that there is a lowest admissible value for κ, denoted $\underline{\kappa} > 0$ when $\varepsilon > 1$. As $\kappa \to 0$, then p^E to ∞ when $\varepsilon > 1$. Looking at the equation

$$p^T \alpha \underline{A} \left(\frac{L}{K} \right)^{1-\alpha} + \eta \left(\frac{p^E - p^T}{\chi \tilde{\sigma}} \right)^2 - (\rho + \delta) = 0$$

we see that as $\kappa \to 0$, the second term increases without bound, and hence, it will surpass the value of $\rho + \delta$. Let $\underline{\kappa}$ be the value of for which $\eta \left(\frac{p^E - p^T}{\chi \tilde{\sigma}} \right)^2 - (\rho + \delta) = 0$, then there can be no steady states with $0 < \kappa \leq \underline{\kappa}$, because, then the first term $p^T \alpha \left(\frac{L}{K} \right)^{1-\alpha}$ has to be less than or equal to 0, which cannot happen for $\kappa > 0$. □

B.8 Existence of Steady State

We can now show that a steady state exists for $\varepsilon > 1$. By substituting the expressions for the prices into the three steady state equations in (108), we obtain

$$\begin{aligned}
&(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}\alpha\frac{1}{\eta}\left(\frac{L}{K}\right)^{1-\alpha}+\left(\frac{\kappa}{\eta}\left(\chi\tilde{\sigma}\right)\left(\frac{L}{K}\right)^{1-\alpha}\right)^{2}-\frac{\rho+\delta}{\eta}=0\\
&(1-\eta)\left(\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}}\right)^{2}+\frac{\left(\bar{\psi}-\eta\right)\left(\delta_d+\phi^l\right)}{\eta}=0 \qquad (112)\\
&\qquad\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}}-\frac{\kappa}{\eta}\left(\chi\tilde{\sigma}\right)\left(\frac{L}{K}\right)^{1-\alpha}=0
\end{aligned}$$

Note that the first equation does not depend K directly, but only on $\frac{1}{\eta}\left(\frac{L}{K}\right)^{1-\alpha}$, which can be solved for from the last equation in terms of κ. Specifically,

$$\frac{1}{\eta}\left(\frac{L}{K}\right)^{1-\alpha}=\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma})^2} \qquad (113)$$

Substituting this into the first equation gives

$$\begin{aligned}
&(1-\nu)\alpha\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma})^2}\\
&+\left(\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}}\right)^{2}-\frac{\rho+\delta}{\eta}=0
\end{aligned} \qquad (114)$$

This equation, together with

$$(1-\eta)\left(\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}}\right)^{2}+\left(\frac{\bar{\psi}}{\eta}-1\right)\left(\delta_d+\phi^l\right)=0 \qquad (115)$$

defines the steady states of the model. Given any $\kappa < \bar{\kappa}$, the second has exactly one solution $\eta(\kappa)$ on the interval $(0,1)$. To see this latter fact note that $\eta = 0$ is not admissible, so we

can multiply through by η without affecting the location of the roots. Then the left-hand side is a quadratic function of η. At $\eta = 1$, the quadratic function is $(\bar{\psi} - 1)\left(\delta_d + \phi^l\right) < 0$ and at $\eta = 0$, that quadratic is $\bar{\psi}(\delta_d + \phi^l) > 0$. Hence, it crosses the x-axis once on the interval $\eta \in (0, 1)$. Denote this value $\eta(\kappa)$. Note also that $\eta(\kappa)$ is strictly decreasing in κ. If κ rises, then the squared term in the brackets will fall, to maintain equality, η must also fall since the expression is strictly decreasing in η. In summary, candidate steady states are determined by the solutions to the following equation in κ

$$\begin{aligned}(1-\nu)\alpha\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma})^2}\\ +\left(\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}}\right)^2-\frac{\rho+\delta}{\eta(\kappa)}=0\end{aligned} \tag{116}$$

where $\eta(\kappa)$ is a strictly decreasing in κ. We can go further in narrowing down the candidate steady states. To prove that a steady state exists, we define the functions

$$\begin{aligned}h(\kappa)&=\frac{\rho+\delta}{\eta(\kappa)}-\left(\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}}\right)^2\\ f(\kappa)&=(1-\nu)\alpha\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}\frac{\nu\left(\frac{\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-(1-\nu)\left(\frac{1-\kappa}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma})^2}\end{aligned} \tag{117}$$

The steady state equation is then $f(\kappa) = h(\kappa)$. Note that $h(\underline{\kappa}) = 0$ by the definition of $\underline{\kappa}$. Note also that $h(\kappa) \to -\infty$ when $\kappa \to 0$, and $h(\kappa) \to \frac{\rho+\delta}{\eta(\bar{\kappa})} > 0$ when $\kappa \to \bar{\kappa}$. By definition of $\bar{\kappa}$ and $\underline{\kappa}$ we have $f(\bar{\kappa}) = 0$ and $f(\underline{\kappa}) > 0$, while $h(\bar{\kappa}) = \frac{\rho+\delta}{\eta(\underline{\kappa})} > 0$ and $h(\underline{\kappa}) = 0$. By the intermediate value theorem, these lines must cross at least once within the relevant interval, hence a steady state exists.

B.9 Uniqueness.

Uniqueness of the (non-degenerate) steady state can be demonstrated analytically in the case when the goods are perfect substitutes, i.e. (abusing notation) $\varepsilon = \infty$. If the goods are perfect substitutes, then note that $\eta(\kappa) = \bar{\eta}$ defined in the previous subsection does not

depend on κ since it is pinned down in steady state by the equation

$$(1-\eta)\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2+\left(\frac{\bar{\psi}}{\eta}-1\right)\left(\delta_d+\phi^l\right)=0 \tag{118}$$

$$\bar{\eta}=\frac{1-\frac{\delta_d+\phi_l}{\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2}}{2}+\sqrt{\left(\frac{1-\frac{\delta_d+\phi_l}{\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2}}{2}\right)^2+\bar{\psi}\frac{\delta_d+\phi_l}{\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2}} \tag{119}$$

Given this, we have

$$h(\kappa)=\frac{\rho+\delta}{\bar{\eta}}-\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2 \tag{120}$$

so that $h(\kappa)$ also does not depend on κ. While at the same time

$$f(\kappa)=(1-\nu)\alpha\frac{2\nu-1}{\kappa(\chi\tilde{\sigma})^2} \tag{121}$$

is strictly decreasing in κ as long as $2\nu-1>0$ (which ensures that entrepreneurs' idiosyncratic volatility is positive). There is one additional parameter restriction. In particular, the point at which $f(\kappa)$ and $h(\kappa)$ intersect must be such that $\kappa\in(0,1)$ for both sectors to be active. This condition is

$$\kappa=\bar{\eta}\frac{(1-\nu)\alpha\frac{2\nu-1}{(\chi\tilde{\sigma})^2}}{\rho+\delta-\bar{\eta}\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2}\in(0,1) \tag{122}$$

This ensures that entrepreneurs are not so much more productive than the traditional sector that they want to hold more than 100% of the capital stock, and that their precautionary savings motive is not so strong that the capital stock grows to without bound (i.e. we must have $\rho+\delta-\bar{\eta}\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2>0$).[27] When the goods are not perfect substitutes, it is harder to see if the steady state is unique. However, one can examine this graphically by plotting the functions $f(\kappa)$ and $h(\kappa)$ for given parameter values and examining whether they have a unique intersection or not. Figure 4 provides an illustrative example. I have been unable to construct a numerical example with multiple (non-degenerate) steady states.

[27] Note that $\bar{\eta}$ is pinned down by equation (118) and is decreasing when viewed as a function of χ.

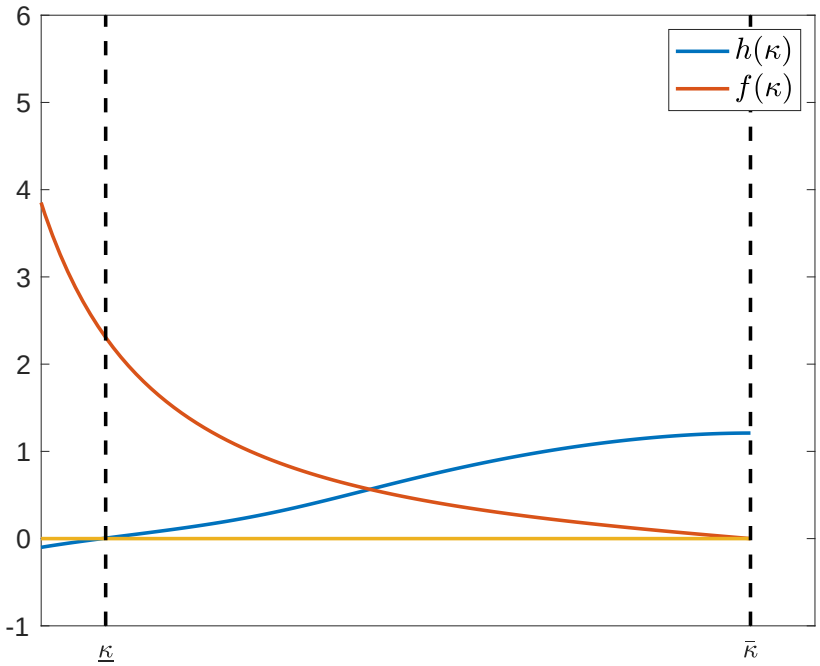


Figure 4: An illustrative example of graphical determination of unique steady state at the intersection of $f(\kappa)$ and $h(\kappa)$.

B.10 Changes in Inequality After a Fall in χ

Proposition 3 discusses small changes in χ. In this section, I study what happens to inequality for larger changes in χ and we discuss transition dynamics. More specifically, we will consider the transition dynamics of the model in the following type of experiment. We let the initial values K_0, η_0, and κ_0 be associated with an initial steady state s_0 with skin-in-the-game parameter χ_0. We then examine the transition dynamics of the model in response to a change in χ to $\chi_1 < \chi_0$. For this exercise to make sense, I will assume that there is a unique (non-degenerate) steady state associated with the new value χ_1. I will also have to assume that the transition dynamics ensure that we converge to this new steady state.

Proposition 5. *Consider an initial (non-degenerate) steady state $s_0 = (K_0, \eta_0, \kappa_0)$ with $\kappa_0 \in (0,1)$, associated with skin-in-the-game parameter χ_0, and a different steady state $s_1 = (K_1, \eta_1, \kappa_1)$ with $\kappa_1 \in (0,1)$, associated with $\chi_1 < \chi_0$. Assume that the economy converges to s_1 from s_0 when changing χ_0 to χ_1. All other parameters are fixed, in particular, the parameter ε is the same for both steady states. Then, there exists a $\varepsilon^*_{s_0,s_1}$ such that if the ε associated with these two steady states is*

larger than $\varepsilon^*_{s_0,s_1}$*, then* $\eta_1 > \eta_0$ *and Pareto inequality is higher in* s_1*.*

Proof. The initial value of the entrepreneurs' risk exposure is

$$\tilde{\sigma}_0^E = \frac{p^E(\kappa_0) - p^T(\kappa_0)}{\chi_0 \tilde{\sigma}}.$$

Because the prices can be made arbitrarily insensitive to changes in $\kappa \in (0,1)$ by letting the elasticity of substitution be large enough, we know that there exists some $\varepsilon^*_{s_0,s_1}$ such that if $\varepsilon > \varepsilon^*_{s_0,s_1}$ we have

$$\tilde{\sigma}_0^E = \frac{p^E(\kappa_0) - p^T(\kappa_0)}{\chi_0 \tilde{\sigma}} < \frac{p^E(\kappa_1) - p^T(\kappa_1)}{\chi_1 \tilde{\sigma}} = \tilde{\sigma}_1^E.$$

Because $\tilde{\sigma}_1^E > \tilde{\sigma}_0^E$, it must be that $\eta_1 > \eta_0$, which means Pareto inequality is higher in the new steady state. □

Now we examine the transition dynamics more closely. Recall that the transition dynamics of the model are determined by the evolution of the state variables

$$\begin{aligned} \frac{dK_t}{K_t} &= \left(p^T(\kappa_t)\alpha \left(\frac{L}{K_t}\right)^{1-\alpha} + \eta_t \left(\tilde{\sigma}_t^E\right)^2 - \rho - \delta \right) dt \\ \frac{d\eta_t}{\eta_t} &= \left((1-\eta_t)\left(\tilde{\sigma}_t^E\right)^2 + \frac{(\bar{\psi} - \eta_t)\left(\delta_d + \phi^l\right)}{\eta_t} \right) dt \end{aligned} \tag{123}$$

and the equilibrium condition for the allocation of the capital stock:

$$\frac{p^E(\kappa_t) - p^T(\kappa_t)}{\chi \tilde{\sigma}} = \frac{\kappa_t}{\eta_t} (\chi \tilde{\sigma}) \left(\frac{L}{K_t}\right)^{1-\alpha} \equiv \tilde{\sigma}_t^E \tag{124}$$

To understand what happens, when χ falls, let's consider a transition between steady state when χ falls.

Lemma 11. *In this experiment,* κ_t *increases on impact.*

Proof. On impact, K_t and η_t are fixed, so (124) implies that κ_t must increase to maintain equilibrium if χ falls. □

The above lemma tells us what happens on impact. To examine what happens in the transition, we need to study how η_t and K_t evolve. Clearly, for high enough values ε, the idiosyncratic risk exposure of entrepreneurs $\tilde{\sigma}_t^E$ rises on impact. According to the

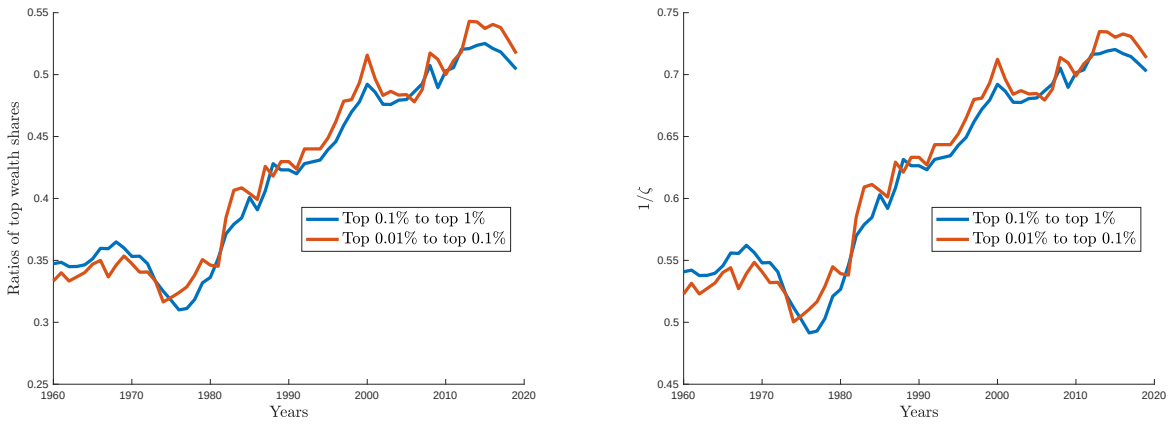
equations describing the evolution of the state variables above, both K_t and η_t will start growing. Looking at the equilibrium condition for the capital allocation, as η_t and K_t start growing, κ_t rises further. The intuition is that as entrepreneurs become wealthier and the operational leverage of the economy (Y/K) becomes smaller, entrepreneurs are better equipped to bear risk and this scale up even more. However, looking at the left-hand side of the equilibrium capital allocation equation (124), we see that this scaling up *after* impact is going to be a reduction in the Sharpe ratio for entrepreneurs (compared to the initial upward jump). In other words, even if the Sharpe ratio jumps upwards on impact, this upward jump is moderated somewhat when η_t and K_t start to grow. However, to be consistent with a new steady state with higher risk exposure for entrepreneurs, $\tilde{\sigma}_t^E$ cannot come back down all the way to its initial value. Looking at the equation describing the evolution of η_t, we see that the growth rate of η_t slows down after impact because $\tilde{\sigma}_t^E$ declines and because a higher η_t makes $d\eta_t$ smaller.

For the capital stock the dynamics are less clear. The increase in κ_t raises the risk-free rate on impact because $p^T(\kappa_t)$ rises. Moreover, the fact that both η_t and $\tilde{\sigma}_t^E$ rise, means the drift of the capital stock becomes even higher. However, as the capital stock rises, the marginal product of capital falls according to the standard neoclassical mechanism. Assuming that the economy actually converges to a new steady state it must be the case the effect from the decreasing marginal product of capital is more powerful than the increase in the price p_t^T in the long run. In particular, we know that there is a maximal κ consistent with steady state, $\bar{\kappa} < 1$, so that the price $p^T(\kappa)$ is bounded above by $p^T(\bar{\kappa})$.

Proving that the economy converges to a unique steady state in a multi-sector growth model is difficult. In fact, Boldrin and Deneckere (1990) shows that multi-sector growth models can display chaotic and cyclical behavior even without aggregate risk. Other than in the limit when $\varepsilon \to \infty$, so that p_t^T is constant, I study the convergence to steady state numerically when needed.

B.11 Rising Pareto Inequality in the Data

Figure (5a) depicts the evolution of the ratio of the top 0.1% to the top 1% and the top 0.01% to the top 0.1%. The similar level and evolution of these ratios indicate that the top of the wealth distribution roughly follows a Pareto distribution and that Pareto inequality, the inverse of the Pareto tail coefficient, has increased. Figure (5b) depicts an estimate of Pareto inequality based on these ratios of top wealth shares, using a formula provided by Jones and Kim (2018).



(a) Ratios of top wealth shares.

(b) Pareto inequality $1/\zeta$.

Figure 5: Ratios of top wealth shares and the Pareto tail coefficient. Data from Distributional National Accounts via Piketty et al. (2018)

B.12 Rising Prevalence of Newly Minted Fortunes at the Top.

The wealthiest Americans of today are, to a larger extent than in decades past, founders or early investors in entrepreneurial firms, rather than inheritors of great fortunes (Kaplan and Rauh (2013)). These individuals were propelled to the top of the wealth distribution by raising substantial amounts of capital from outside investors. This allowed them to operate much larger firms than their initial wealth would have permitted. For instance, Kaplan and Rauh (2013) document that 69% of the Forbes 400 list in the 2011 edition were the first in their family to run their business, up from 40% in the first 1982 edition.

Prevalence of Entrepreneurs at the Top in the Model. High idiosyncratic risk-taking among entrepreneurs helps account for the high prevalence of newly minted fortunes at the top because it makes extreme upward wealth trajectories more likely. In addition, when improvements in risk-sharing leads to increases in idiosyncratic risk-taking, the model also generates a *rising* prevalence of newly minted fortunes at the top. Using the same transition experiment as in 4.2, Figure 6 depicts the evolution of the fraction of entrepreneurs within the model-counterpart of the Forbes 400 in the transition experiment considered in Section 4.2. In other words, improvements in risk-sharing for entrepreneurs generates a sizable rise in the prevalence of entrepreneurs within the very top of the wealth distribution.

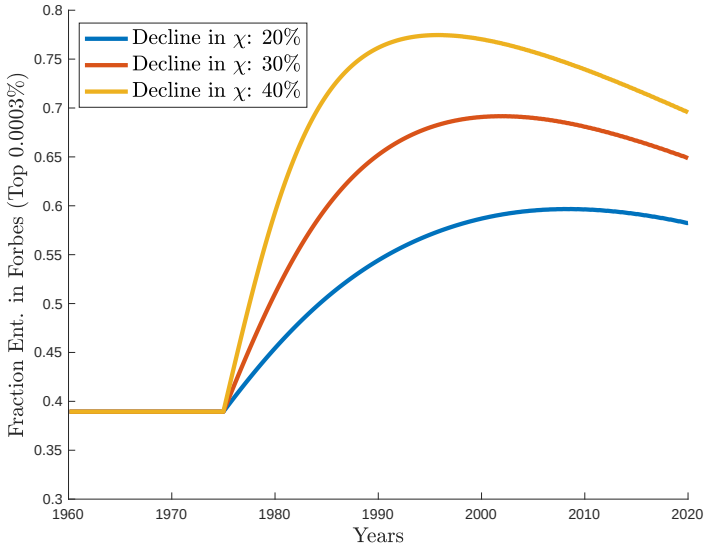


Figure 6: Fraction of top 0.0003% (Forbes 400) that are entrepreneurial capitalists in the model.

B.13 Financing and Ownership Fractions

The entrepreneur issues $1 - s_{it}$ shares to outsiders in exchange for $(1 - \chi)k_{it}$ of financing at price-per-share p_{it}. The price-per-share and number of shares are jointly determined by the condition

$$p_{it}(1 - s_{it}) = (1 - \chi)k_{it} \tag{125}$$

and the condition that outsiders earn an expected return r_t^{out}:

$$r_t^{\text{out}} = \frac{d_{it}}{p_{it}} + \frac{\mathbb{E}[dp_{it}]/dt}{p_{it}} = \frac{(1 - s_{it})d_{it}}{(1 - s_{it})p_{it}} + \frac{(1 - s_{it})\mathbb{E}[dp_{it}]/dt}{(1 - s_{it})p_{it}} = \frac{(1 - s_{it})}{(1 - \chi)} \frac{\left(d_{it} + \mathbb{E}[dp_{it}]/dt\right)}{k_{it}} \tag{126}$$

where the last equality follows from the condition that $p_{it}(1 - s_{it}) = (1 - \chi_{it})k_{it}$. Similarly, the entrepreneur earns an expected return r_t^{in} and hence

$$r_t^{\text{in}} = \frac{s_{it}}{\chi} \frac{\left(d_{it} + \mathbb{E}[dp_{it}]/dt\right)}{k_{it}}. \tag{127}$$

Note that the entrepreneur does not "buy" shares at price p_{it} in the same way that the outsider does. Rather, the entrepreneur simply retains whatever fraction of the shares

that the outsiders do not buy.[28] Finally we have

$$r_t^k = (1-\chi) r_t^{\text{out}} + \chi r_t^{\text{in}} = \frac{\left(d_{it} + \mathbb{E}[dp_{it}]/dt\right)}{k_{it}}. \tag{128}$$

The interpretation of this equation is simply that the expected return to the entire investment, r_t^k must add up to the expected return for someone who owns all the shares, $\left(d_{it} + \mathbb{E}[dp_{it}]/dt\right)/k_{it}$. Combining (126), and (128) we obtain

$$(1-\chi) = (1-s_{it}) \frac{r_t^k}{r_t^{\text{out}}}. \tag{129}$$

For a given risk-sharing parameter χ, the lower the cost of capital relative to the return on capital, the larger the fraction of the firm that the entrepreneur retains s_{it}.

B.14 Firm Valuations

Firm Valuations. Equations (12) and (129) allow us to compute the ostensible market value of an entrepreneurial firm relative to the capital stock it operates:

$$p_{it} = \frac{r_t^k}{r_t^{\text{fund}}} k_{it}. \tag{130}$$

This is the value of the firm obtained by simply multiplying the number of shares outstanding (held by both entrepreneur and outsiders), by the market price per share paid by outsiders, p_{it}. The ostensible market value of the firm is higher than the value of the capital stock because the former is computed by applying the same price to the entrepreneurs shares as the outsiders shares when valuing the firm. But although the shares held by the entrepreneur and the shares held by outsiders are ostensibly the same, the shares held by the entrepreneur are economically different. Specifically, this approach to valuing the firm ignores the financial frictions the entrepreneur faces, and therefore overvalues the entrepreneurs shares.

The ratio $\frac{r_t^k}{r_t^{\text{fund}}} = \frac{p_{it}}{k_{it}} \equiv q_t$ is a measure of Tobin's q in the model. In this sense, the model produces a deviation from $q_t = 1$, even in the absence of capital adjustment costs, market power, or other frictions commonly employed to obtain such deviations. For the economy as a whole, we can compute the market value of all firms relative to the aggregate capital

[28] One could perhaps formulate this as the entrepreneur buying shares at a price $\tilde{p}_{it} < p_{it}$, and assume the expected capital gains rate on the two prices are the same. Then, since the entrepreneur invests χk_{it} and gets s_{it} shares, the effective price per share an entrepreneur pays is $\tilde{p}_{it} = (\chi/s_{it})\, k_{it} = \left(r_t^{\text{out}}/r_t^{\text{in}}\right) p_{it} < p_{it}$.

stock:

$$\frac{V_t^T + \frac{r_t^k}{r_t^{\text{fund}}} K_t^T}{K_t} = (1 - \kappa_t) + \frac{r_t^k}{r_t^{\text{fund}}} \kappa_t = 1 + \kappa_t \left(\frac{r_t^k}{r_t^{\text{fund}}} - 1 \right). \tag{131}$$

As with the weighted average return to capital and the factor income shares, there is a composition effect operating when economic activity is reallocated towards entrepreneurial firms. Improvements in risk sharing for entrepreneurial firms is therefore a potential contributor to the secular rise in Tobin's q (see for instance Gomez and Gouin-Bonenfant (2024)).

B.15 Transition Dynamics of Macro trends

Figure 7 depicts the transition dynamics of the pure labor share of income (equation (39)), average return to capital (equation (41)), riskless rate (equation (42)), and Tobin's q (equation (131)), in the same transition experiment as that considered in Section 4.2.

B.16 Decreasing Inequality Even with Perfect Substitutes

The relationship between Pareto inequality and χ turns around even when $\varepsilon = \infty$, when χ becomes small enough. This does not contradict Proposition 3 or Proposition 5. These propositions tell us that for interior stead states (in the sense that $\kappa \in (0, 1)$, there exists large enough values of ε, so that inequality increases when χ falls. However, as χ is reduced further, and further the required ε may become larger and larger. This is because as κ gets closer to 1, there is less room for entrepreneurs to scale up at the expense of the traditional firms. And even in the limit as $\varepsilon \to \infty$, there is a limit to the scaling-up effect coming from the fact that with perfects substitutes, as χ becomes small, we may have steady states where $\kappa = 1$. Further falls in χ beyond this leads to less inequality. To prove this is straightforward for $\varepsilon = \infty$ because tedious the CES-algebra can be avoided. For $\varepsilon < \infty$, the traditional sector may never be fully out competed because prices in that sector increase rapidly when κ gets close to 1. Although this suggests intuitively that the limits to the scaling-up effect occur even earlier than with $\varepsilon = \infty$, proving analytically is more difficult. Therefore, I show this in the case of $\varepsilon \to \infty$ and confirm the intuition numerically when necessary in the case where ε is finite.

Proposition 6. *Even with perfect substitutes, $\varepsilon = \infty$, a fall in χ can lead to lower Pareto inequality.*

Proof. Note that with perfect substitutes, the condition in equation (122) must be satisfied for both sectors to be active. However, in that expression, we see that as $\chi \to 0$, this

condition does not necessarily hold. This is because, with perfect substitutes and low χ, there may be no steady state with $\kappa \in (0,1)$. Instead, the entrepreneurs may take over the entire economy. As an example, consider parameters such that we obtain a steady state s_0, $\kappa_0 < 1$, with $\chi_0 > 0$. Viewing the steady state value of κ defined in the expression (122) as a function of χ,

$$\kappa(\chi) \equiv \bar{\eta}(\chi) \frac{(1-\nu)\alpha \frac{2\nu-1}{(\chi\tilde{\sigma})^2}}{\rho + \delta - \bar{\eta}(\chi)\left(\frac{2\nu-1}{\chi\tilde{\sigma}}\right)^2},$$

(where $\bar{\eta}(\chi)$ is an decreasing function of χ as per equation (118)), we see that as $\chi \to 0$, the steady state value of $\kappa(\chi)$ implied by the above equation will no longer be below 1. In other words (provided that a steady state still exists as we lower χ), there will be some $\chi^* > 0$ such that $\kappa(\chi^*) = 1$, and entrepreneurs take over the entire economy. I will now show that in a one sector economy, case, a further fall in χ leads to falling steady state Pareto inequality. In the following I suppress time subscripts and equations are understood to hold in steady state. In an economy where $\kappa = 1$ entrepreneurs' optimal portfolio choice in equilibrium implies

$$\frac{1}{\eta}(\chi\tilde{\sigma}^k)^2 = r^k - r. \tag{132}$$

which follows from plugging in $\kappa = 1$ in entrepreneurs optimal portfolio choice. We recall that $\tilde{\sigma}^k_{ss} = \frac{Y}{K}\tilde{\sigma}$. Moreover, when the traditional firms are not active, the risk-free rate is no longer pinned down by the value of the marginal product in that sector. Instead, we have the following system jointly pinning down the wage rate and the risk-free rate:

$$\begin{aligned} \frac{Y}{K} &= \left(\frac{1-\alpha}{\alpha}\frac{r+\delta}{w}\right)^{1-\alpha} \\ \underbrace{\frac{Y}{K} - w\frac{L}{K} - \delta - r}_{r^k - r} &= \frac{1}{\eta}\left(\chi\frac{Y}{K}\tilde{\sigma}\right)^2 \end{aligned} \tag{133}$$

Solving this gives us

$$\begin{aligned} w &= (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}\left[1 - \frac{\chi\tilde{\sigma}}{\eta}\left(\chi\tilde{\sigma}\frac{Y}{K}\right)\right] \\ r &= \alpha\left(\frac{L}{K}\right)^{1-\alpha}\left[1 - \frac{\chi\tilde{\sigma}}{\eta}\left(\chi\tilde{\sigma}\frac{Y}{K}\right)\right] - \delta \end{aligned} \tag{134}$$

Plugging this into the equations for the evolution of the state variables and setting these

to zero we obtain

$$\begin{aligned}\frac{dK_t}{K_t} &= \left(\alpha\frac{Y}{K} + (1-\alpha)\frac{1}{\eta}\left(\chi\frac{Y}{K}\tilde{\sigma}\right)^2 - \rho\right)dt = 0\\ \frac{d\eta_t}{\eta_t} &= \left((1-\eta)\left(\frac{1}{\eta}\chi\frac{Y}{K}\tilde{\sigma}\right)^2 + \frac{(\bar{\psi}-\eta)\left(\delta_d+\phi^l\right)}{\eta}\right)dt = 0\end{aligned} \tag{135}$$

From the steady state for η we obtain

$$\frac{1}{\eta}\left(\chi\frac{Y}{K}\tilde{\sigma}\right)^2 = \frac{(\eta-\bar{\psi})(\delta_d+\phi^l)}{1-\eta} \tag{136}$$

We can then rewrite the steady state equations for K as

$$\alpha\frac{Y}{K} - \rho = -(1-\alpha)\frac{(\eta-\bar{\psi})(\delta_d+\phi^l)}{1-\eta} \tag{137}$$

The left-hand side is strictly decreasing in η (take derivative). This means that if a fall in χ leads to a fall in η, it must lead to a rise in $\frac{Y}{K}$ for this equation to hold, and vice versa. In other words, it must lead to a fall in K. If a fall in χ leads to a rise in η, then it must similarly lead to a rise in K, and vice versa. In other words, when χ falls, η and K must move in the same direction in steady state. To see that the direction is downward, we look at the steady state equation for η in (136). When χ falls, this equation tells us that either η or K must fall. Since we know that they both move in the same direction, this means they must fall. What happens to the idiosyncratic risk exposure of entrepreneurs? We have

$$\tilde{\sigma}^E = \frac{1}{\eta}\chi\frac{Y}{K}\tilde{\sigma}$$

which has to fall. To see this, note that when χ falls, $1/\eta$ and Y/K rise, but if they rise so much that this offsets the fall in χ in the sense that $\tilde{\sigma}^E$ does not fall, then this contradicts η falling, because if $\tilde{\sigma}^E$ does not fall, then the steady state equation for η tells us that η should not fall. So $\tilde{\sigma}^E$ must fall. Finally, Pareto inequality falls because $\tilde{\sigma}^E$ falls. This concludes the proof. □

B.17 Comments on Auxiliary Data

B.18 Savings Rates and Savings Ratios.

The savings rate out of net worth is the same for both types of capitalists, namely $1 - \rho$. However, because entrepreneurs earn a higher rate of return on their net worth in equilibrium, $r_t^E > r_t^D$, their savings rate out of *income* is higher than for diversified capitalists (see Ordoñez and Piguillem (2022) for a discussion on the importance of the distinction between these two notions of savings). When entrepreneurs overrepresented at the top wealth distribution, the savings rate out of income is increasing along the wealth distribution even though the savings rate out of net worth is constant. This is in line with the observations of Fagereng, Holm, Moll, and Natvik (2019). It should be noted that since consumption and income are proportional to net worth in this model, the distributions of these variables counterfactually share the same steady state Pareto tail coefficient. Gaillard et al. (2023) discuss how non-homothetic preferences and scale-dependent returns to wealth may address this counterfactual implication.

B.19 Declining Business Dynamism.

Note in Figure 3a that Pareto inequality does not start to change until approximately a decade after the fall in χ. In other words, the Pareto shape of the top of the wealth distribution at any given time is determined by the parameters that governed wealth accumulation for entrepreneurs decades earlier. This is essential for understanding how the results in this paper relate to the observation by Decker et al. (2016) that business dynamism has declined in the United States. In particular, the mechanism in this paper implies that improved risk sharing for entrepreneurs leads to increased risk-taking among entrepreneurs and results rising business dynamism. This seems superficially at odds with the observations made in Decker et al. (2016).

However, important to note is that Decker et al. (2016) argue that in the case of high-growth entrepreneurial firms, the decline in dynamism happened only after the year 2000. Before that, their measure of dynamism was actually rising for this group of firms. Because the transition dynamics of inequality are delayed, this decline in dynamism may not be visible at the top of the wealth distribution yet but might reveal itself in the coming decade. In summary, the delayed transition dynamics of wealth inequality means that the top of the wealth distribution will not be shaped by the current dynamics governing wealth accumulation, but the dynamics that governed the wealth accumulation of entrepreneurs

at the time when those currently at the top were rising to the top. In light of this, it is interesting to note that many of the current members in the Top 10 of the Forbes 400 list became wealthy in the 1980s and 1990s, when dynamism was high and rising for high-growth entrepreneurial firms.

B.20 The Role of the Two-Sector Setup

One key prediction of the model is that wealth inequality increases in response to improved entrepreneurial financing, provided that excess returns associated with entrepreneurship do not fall too much when entrepreneurs scale up. The degree to which the equilibrium excess return to entrepreneurial activity falls when entrepreneurs want to scale up depends on how fiercely entrepreneurs compete with each other for labor, capital, and product demand. The presence of the traditional sector plays an central role for this prediction. The traditional sector constitutes a source from which the entrepreneurial sector can attract labor, capital, and demand, relieving the competitive pressure. Instead of drawing economic activity from other entrepreneurial firms, which puts downward pressure on the excess return to entrepreneurship, they can draw this economic activity from the traditional sector. The absence of a source from which entrepreneurs can draw economic activity from so as to avoid severe downward pressure on equilibrium profitability is, I conjecture, at the heart of why some studies find that improved risk sharing for entrepreneurs lowers wealth inequality ins one-sector models (see Hui (2023) and Peter (2021)). For instance, Peter (2021) studies a model wherein production is in the hands of entrepreneurial firms. That model is rich and closer in spirit to Quadrini (2000) and Cagetti and De Nardi (2006). When calibrating this model to European data, Peter (2021) finds that improved risk sharing for entrepreneurs reduces steady state wealth inequality. The results in this paper complement those in Peter (2021) by showing that improved risk sharing can also raise top wealth inequality, provided entrepreneurs can scale up without adversely affecting the profitability of entrepreneurial activity. It should be said that because Peter (2021) considers a very rich model framework, it is not entirely straightforward to evaluate analytically precisely which of the model features produces the result that better equity financing for entrepreneurs lowers wealth inequality. The results of the present paper suggest that it may be that the equilibrium returns top entrepreneurial activity fall when entrepreneurs all try to scale up, so that the risk sharing effect ends up dominating the scaling up effect. Another example is Robinson (2024) studies how a complete removal of risk sharing constraints impacts inequality in a model with two sectors, finding that wealth inequality is reduced. However, as Proposition 6

illustrates, the fact that a complete removal of limits to risk sharing will lead to reduced wealth inequality does not imply that intermediate improvements in risk sharing will.[29] It should be noted that if there was another positive net supply asset (like government debt) in the economy, the relative price of entrepreneurial capital (relative to this other asset) may rise in equilibrium when financing improves. This increase in firm *value* relative to entrepreneur's wealth could potentially also lead to an effective increase in the riskiness of entrepreneur's wealth portfolio, generating similar implications as the framework discussed in this paper. I leave this exploration of the role of government debt for future research.

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[29] As noted in Robinson (2024) regarding the effect of completing markets for risk in his framework: "It turns out that completing the missing market for risk is so beneficial to the entrepreneurial sector that it entirely crowds out the corporate sector in equilibrium [...]". In other words, the model is in fact reduced to a one-sector model if risk markets are fully completed.

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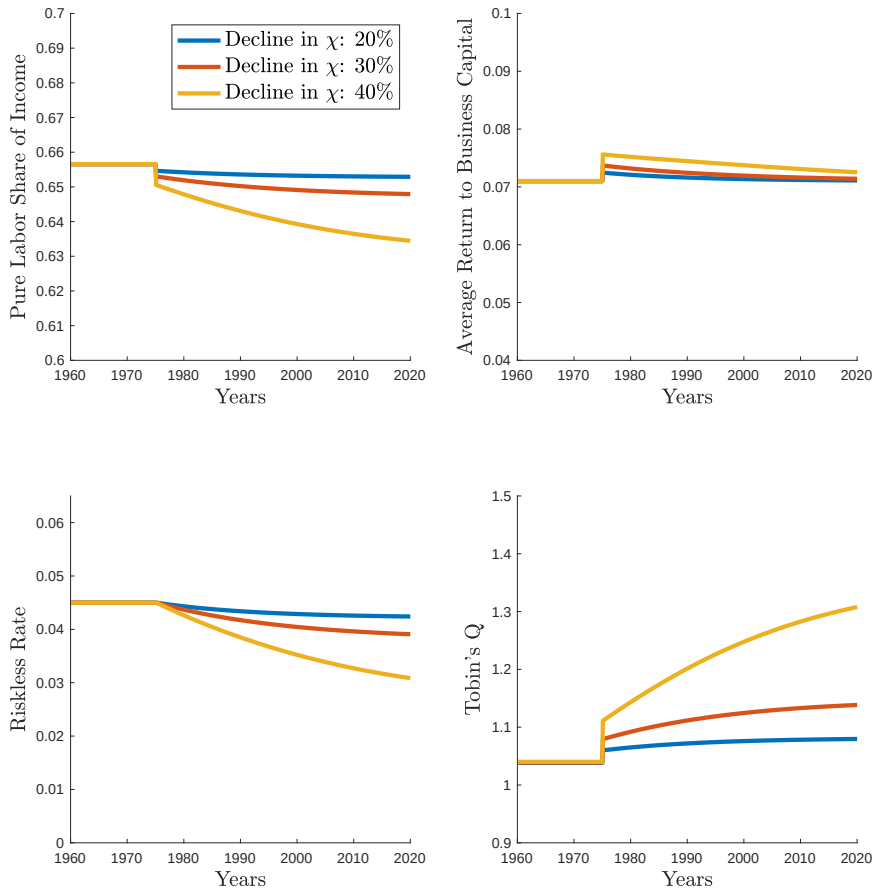


Figure 7: Transition dynamics of the pure labor share of income, average return to business capital, riskless rate, and Tobin's q in the transition experiment considered in Section 4.2.