Pricing Fixed Income with Liquidity: An Extended Merton Approach

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Abstract

We propose a simple model by which informed trading is incorporated into both the traded volume and the prices of the traded asset. We apply the resulting dynamic to the Merton (1974) bond pricing model. We show that the accuracy of the model improves substantially when the liquid portion of the market capitalization and the liquid portion of the debt are used as input variables in the Merton model. Using the proposed extension to the Merton model that we dub "the liquid Merton", we show that we can predict out-of-sample returns one month ahead on the shortest maturity bonds with 99.9% precision, while the classic Merton model fails to deliver statistical significance. However, in the longest maturities, the classic Merton model predicts out-of-sample returns one month ahead with 99.9% statistical confidence, while the proposed "liquid Merton" does not. Furthermore, our model provides a new explanation for the liquidity premium puzzle. We show that faced with adverse market frictions, investors move to trade easier assets, which in turn reduces supply for the hard-to-trade issues, lowering their demand and, as a result, present prices.

1 Introduction

In December 2024, the Securities Industry and Financial Markets Association (SIFMA) reported that debt issuance and fixed income trading had increased by nearly 20% compared to 2023 figures [1]. This growth highlights a lucrative opportunity related to all aspects of fixed income. It also underscores

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[1]https://www.sifma.org/resources/research/statistics/us-fixed-income-securities-statistics/

the growing demand for the resources needed to support the debt issuance and trading sectors.

This paper proposes novel, accurate pricing of publicly traded fixed income instruments. In a theoretical model, we show that agents value their ability to exit trading positions with ease and that this ability increases with the rise in trading volume. As a result, trading volume is a significant predictor of returns.

Although trading volume has been used to price assets in technical analysis for decades, the rationale for doing so remains elusive (Lo et al. (2000)). Aldridge and Zhang (2025) show that when the trading volume increases, investors relax their risk aversion, resulting in a nonlinear but significant relationship between the trading volume and future prices. In this study, we propose that the nonlinearity between changes in trading volume and bond returns is best explained by the Merton (1974) model with a signed, price-weighted trading volume of equities and outstanding debt that replaces market capitalization with price-weighted.

We argue that the daily trading volume represents the liquid portion of the firm's assets that investors can count on redeeming in the event of corporate collapse.

In the literature, the effects of bankruptcy on bond prices are summarized as follows:

1. **Immediate drop in corporate valuation:** Baranchuk and Rebello (2018) run event studies for 674 bankruptcies between 1978 and 2012 that have data in CRSP and find that, on average, the stock price of the company decreases by 16% following a bankruptcy announcement. According to the Merton (1974) model, such a drop in the stock price increases the asset volatility, but immediately reduces the total market capitalization, resulting in lower bond prices.

2. **Lower present value due to impending recovery costs** It is well known that bankruptcy proceedings take years to resolve and cost a pretty penny in attorney fees. Bermant and Flynn (1989) report that bankruptcy proceedings take on average 2 years to resolve, leaving debtholders' capital stranded and potentially further exhausted by the required legal fees in the process.

   In addition to the recovery fees incurred by the bondholders, the corporations themselves also pay substantial restructuring and workout fees, further reducing the value of the available recovery pool for creditors. Betker (1997) considers the direct fees to be legal, accounting, and administrative charges incurred by corporations in the Chapter 11 process. Weiss (1990) and Betker (1997) estimate the weighted average direct costs of bankruptcy paid by the corporations themselves in the United States as 3.6% of the total pre-bankruptcy corporate asset valuation.

   Thorburn (2000) reports that, in addition to the direct costs of bankruptcy, indirect costs of reorganization accumulate as the bankruptcy proceedings drag on. Per Thorburn (2000), indirect costs include the reputation of the

firm and its products, the opportunity cost of management time, etc. More recently, Müller (2022) documents that the US courts are also behind in bankruptcy proceedings, further extending the costs of the process. Dou et al. (2021) estimate that information asymmetries and conflicts of interest are the primary reasons why court cases last long.

3. **Discounted sales of credit claims to specialized hedge funds**

   Ivashina et al. (2016) note that dispersed bondholders have less success recovering their share of proceeds compared to well-organized hedge funds that handle claims in a centralized manner. As a result, bondholders can feel pressured to sell their claims to hedge funds, often as low as 10% on every dollar of face value (see Ellias (2018)). Ivashina et al. (2016) document that some active hedge funds specialize in asset recovery, buying claims of individual investors at a fraction of the cost in the event of financial distress.

The model presented in this paper suggests that most bond investors prefer to avoid bankruptcy recovery at all costs. As a result, when pricing bonds, the investors are only focused on the most liquid portion of the corporate valuation, and not the whole market capitalization of the firm. The most liquid value of a company happens to be its daily trading volume which documents availability of buyers and sellers for a liquid stock of a given corporation.

Our model is consistent with the latest empirical evidence. Laudenbach et al. (2021) show that faced with a bankruptcy, retail investors sharply curtail their risk appetite. Furthermore, the increase in investors' risk aversion is recorded not only for the company declaring default but also across the firms in the same industry.

We empirically test the results of the model and find that our model explains the variation 80% in the daily returns of corporate bonds.

The classic Merton (1974) model posits that fixed income instruments such as bonds can be priced as put options on total corporate value. In this paper, we develop an extension that shows that, for bonds and traders with a shorter time horizon, the relevant underlying value is not the full market capitalization, but only the liquid portion of the capitalization, commonly known as traded volume. We argue that the liquid component represents the realistic value a trader can obtain in the event of a corporate event such as a bankruptcy, since a full bankruptcy can take years to process.

Although the Merton (1974) model is well established for evaluating corporate credit, most of the time input into the model is collected from annual or quarterly corporate filings. We show that the information in the annual reports is not timely enough to price bonds. Our theoretical model shows that trading volume is a timely and appropriate variable. We argue that trading volume reflects the latest and immediate share of the company's assets an investor may retrieve from the markets in the event of a corporate distress.

With our empirical analysis, we document that our pricing is much more accurate than using annual corporate statements. The importance of accurate

bond pricing cannot be underestimated. In addition to the trading strategies which we also cover, accurate pricing produces accurate bond yields. Bond yields are used in numerous financial models where their accuracy is the key. For example, bond yields are used as discount factors in corporate valuation models, asset pricing, and many other applications.

The contributions of this work include the following.

1. **Volume as a Fundamental Variable** We introduce *trading volume* as a novel and more accurate measure of corporate value for bond pricing, challenging the long-standing reliance on annual balance-sheet data. By emphasizing the liquidity component of the firm's asset base, our approach elucidates how high-frequency market information can inform real-time default risk assessments.

2. **Liquidity-Driven Extensions of Merton's Model** Our extended framework integrates informed-trading dynamics, demonstrating that *heightened trading activity* not only bolsters bond valuations via increased liquidity but also amplifies short-term price discovery effects. This captures a microstructure dimension traditionally absent in standard structural models.

3. **Enhanced Pricing Accuracy and Practical Implications** Empirical results show that the new methodology outperforms conventional, annual-report-based implementations of Merton's model, especially for short-maturity issues that hinge on near-term liquidity conditions. These improvements have broad ramifications for financial modeling—particularly for valuation and discounting procedures in corporate finance, where accurate yield estimation is essential.

4. **Robust Empirical Validation** We provide *extensive empirical tests* under varying specifications (controlling for maturities, market regimes, and alternative proxies for debt value) to highlight the robustness of volume-driven structural metrics. These validations underscore the method's *practical utility* for practitioners and risk managers seeking actionable signals in both stable and stressed market environments.

5. **Integration of Microstructure and Credit Risk** By unifying microstructure considerations (informed trading, liquidity, volume volatility) with structural credit theory, this study furnishes a more comprehensive framework for bond return forecasting. The findings underscore that informed order flow and trading frictions can substantially alter short-horizon bond valuations—a key insight for short-term traders and liquidity providers alike.

The paper proceeds as follows. Section 2 reviews the relevant literature on structural modeling, liquidity, and microstructure. Section 4 details our theoretical framework and the data construction process. Sections 5–9 presents the

empirical findings, including out-of-sample tests and regime-interaction analyses. In Section 10, we elaborate on the practical benefits and potential extensions of the research. Finally, Section 11 concludes by summarizing the main contributions and outlining directions for future work.

2 Related Literature

2.1 Pricing Debt

Merton (1974) was the first to relate the corporate financial structure with options. Given the seniority of the debt on a typical corporate balance sheet, debt holders are always paid back before shareholders in the event of financial distress in a given corporation. If, say, at the time of bankruptcy, corporate assets exceed the value of outstanding debt, the debtholders are paid in full. Then, shareholders are distributed whatever is left of corporate assets after the debtholders take their share. However, if at the time of bankruptcy the assets are below the notional of the debt owed, the debtholders get whatever is available and the shareholders get nothing. Merton (1974) recognized that such a seniority structure of debt allows one to price debt as a put option on the value of corporate assets that the debtholders sold to equity holders. As long as the total asset value exceeds the outstanding debt amount at the loan maturity, the debt holders get their money back from the shareholders.

Cong et al. (2020) were the first to hypothesize that the value of financial instruments is directly related to their usage. While Cong et al. (2020) applied their theory to the pricing of cryptocurrency, Aldridge and Zhang (2025) extended the idea to all asset classes.

2.2 Trading Volume and Pricing

The dynamics of market volume are well documented, but have not been completely understood to date. Lo and Wang (2015) study trading volume measures in the context of portfolio theory and find that volume is not a simple measure of institutional flows. Lo et al. (2000) consider the informational value of the volume. Drawing on technical analysis techniques, Lo et al. (2000) note that price trends supported by positive volume changes are more sustainable than price changes that are not accompanied by the volume of the trading. Bajzik (2021) surveys 44 studies on relationship between traded volume and stock prices and finds that the impact of volume decreases over time. Fousekis and Tzaferi (2021) examine the volume of cryptocurrencies as a factor explaining the interconnectedness of the returns of different crypto assets in a VAR framework. Bai et al. (2024) examine the value added by fixed income data, including volume, to traders. They find that volume traded normalized by the outstanding amount helps traders navigate "maturity switching" among bonds.

Relevant to the Merton (1974) model, Easley et al. (1998) and Pan and Poteshman (2006) show that the option trading volume contains information

about the prices of the underlying assets. Pan and Poteshman (2006) ascribe the relationship to the private information of the option traders. If agents have private information on the underlying prices that increase, Pan and Poteshman (2006) hypothesize that agents go to the option market to trade on this information and maximize their profits using the implicit option leverage. Applied to the Merton (1974) model, this would mean that if an agent has inside information that the value of the company assets is about to increase, the agent can turn to the bond markets and buy bonds, increasing the traded volume of the bonds.

In this paper, we show the opposite effect: that the trading volume of the underlying (corporate stock) influences the prices on the options (bonds).

2.3 Trading Volume and Liquidity

Traded volume has also been used in liquidity metrics. Alexander et al. (2000) consider bond volume as a proxy of liquidity. Johnson (2008) offers a compelling model for liquidity versus volume: liquidity is an average risk-bearing capacity of a market, while volume is the investor-driven change in that capacity. Johnson (2008) shows that volume is positively related to the variance in liquidity. Aldridge and Zhang (2025) deploy volume in a return-to-volume ratio as a metric of illiquidity.

Baaquie and Karim (2023) tests Merton (1974) and highlights the need to refine the liquidity assumptions. Similarly, Wu et al. (2022) notes the importance of explicitly incorporating the interaction between liquidity and credit risk in the pricing of corporate bonds. Their framework, which extends structural models, finds that trading volume and bid–ask spreads significantly improve pricing accuracy, an insight especially relevant for markets where credit conditions and liquidity constraints move in tandem.

Reichenbacher and Schuster (2022) propose size-adapted liquidity measures to identify liquidity shifts from changes in trade size. Their empirical tests in US corporate bond markets suggest that standard liquidity metrics may underestimate the true cost of illiquidity unless trades are normalized for size. This again underscores the importance of volume in asset pricing. Cabrol et al. (2024) incorporate machine learning techniques to predict illiquidity in corporate bonds.

Turning to smaller economies, Brugler (2024) study how market liquidity directly impacts government bond prices in a small open economy (Australia). Using high-frequency data, they show that improved liquidity conditions, captured by order-book depth and bid–ask spreads, are associated with lower bond yields, emphasizing that liquidity management can materially affect pricing even outside major markets.

In the domain of sustainability-linked bonds (SLBs), Poggensee (2024) finds that both credit and liquidity factors jointly drive pricing, but an additional premium or discount is introduced based on sustainability performance targets.

2.4 Liquidity Premium

Liquidity, broadly defined as the ease of trading an asset without substantially affecting its price, is widely recognized as a key determinant of asset pricing. Investors typically demand *liquidity premium* —- an additional expected return —- to hold assets that are harder or more costly to trade. However, in our model, we show that short-term investors place a higher valuation on assets that are *easier* to trade, resulting in higher premiums for more liquid assets.

Early insights into liquidity-induced return differentials can be traced back to Amihud and Mendelson (1986), who show that securities with larger bid–ask spreads systematically earn higher returns, highlighting that investors require compensation for higher transaction costs. Their model illustrates a clientele effect, where longer-horizon investors hold more illiquid assets and, consequently, earn a higher yield. Subsequent research has incorporated more nuanced forms of trading frictions–such as search costs, asymmetric information, and inventory risk–to explain why assets with identical cash flows but different liquidity trade at different prices.

A significant extension is the concept of *liquidity risk*: the possibility that an asset's liquidity can worsen exactly when the market as a whole experiences stress. Acharya and Pedersen (2005) propose a liquidity-adjusted capital asset pricing model in which the expected returns depend on both the baseline illiquidity of an asset and its covariance with aggregate liquidity shocks. Similarly, Ľuboš Pástor and Stambaugh (2003) develop a market-wide liquidity factor and show that stocks whose returns are more sensitive to fluctuations in market liquidity earn higher expected returns. Both approaches highlight that assets exposed to systematic liquidity risk, particularly those that underperform when markets become illiquid, command a premium above and beyond any static illiquidity costs.

The idea that *funding liquidity* (the ability of traders or intermediaries to borrow) and *market liquidity* are intimately linked was further explored by Brunnermeier and Pedersen (2009). They document how small shocks to funding conditions can lead to liquidity spirals, where forced deleveraging and wider bid–ask spreads amplify each other. This work helps explain why liquidity premia tend to be small and stable during normal times but can spike dramatically in crises.

Even the safest government securities exhibit a liquidity premium. One well-known example is the spread between *on-the-run* (freshly issued) and *off-the-run* (older) US Treasury bonds: the latter generally have lower liquidity and trade at higher yields despite having the same cash flows. Longstaff (2004) finds that liquidity-driven pricing differentials in Treasury markets can expand substantially during crisis episodes, consistent with a flight-to-liquidity phenomenon. Similarly, Beber et al. (2009) document in Euro-area government debt that, under stress, liquidity often dominates default risk in driving bond yield spreads.

In addition, studies such as Krishnamurthy and Vissing-Jorgensen (2012) show that Treasuries can embed a "convenience yield," partly reflecting their

superior liquidity. When Treasury supply shrinks, for instance, due to Federal Reserve interventions, yields on these highly liquid instruments decline further relative to other safe but less liquid assets.

Corporate bond yields are systematically higher than matched maturity government bonds, and this difference is only partially attributable to credit risk. Chen et al. (2007), examining transaction-based liquidity proxies in US corporate bonds, show that less liquid issues have significantly higher yield spreads, with the effect most pronounced for speculative-grade bonds. In addition, liquidity risk (covariance with market-wide liquidity) can explain a sizable fraction of corporate bond yield spreads. Even for ostensibly identical debt securities, minor differences in trading frictions, such as bid–ask spreads or frequency of trading, result in meaningful yield differentials. This pattern is especially evident during periods of market turmoil, when secondary-market liquidity evaporates, pushing liquidity premia sharply higher.

Although stock markets are typically more liquid than bond markets, marked differences in liquidity remain, particularly for small-cap or thinly traded shares. Amihud and Mendelson (1986) initially showed that stocks with wider bid–ask spreads earn higher average returns, a result confirmed and extended by numerous studies using alternative liquidity proxies (e.g., zero-return days, turnover-based measures or the Amihud illiquidity ratio). Over time, researchers have found that both the *level* of illiquidity of an individual stock and its *liquidity risk* (exposure to market-wide liquidity shocks) are priced. Ľuboš Pástor and Stambaugh (2003) show that stocks with high sensitivity to liquidity deterioration in the broader market command higher average returns, consistent with investors demanding compensation for carrying systematic liquidity risk.

In crisis periods, even large-cap, normally liquid stocks can see a surge in effective trading costs, highlighting that liquidity is inherently time-varying. Studies of the 2008–09 crisis confirm that liquidity risk factors became more influential, with liquidity-related return premiums widening notably as funding constraints forced many investors to reduce positions in a hurry.

Illiquid assets such as private equity, hedge funds with lockups, or private real estate carry pronounced liquidity restrictions. For instance, Aragon (2007) examines hedge funds with share restrictions and finds that investors earn substantial additional returns—on the order of several percentage points annually—for surrendering redemption rights. By analogy, restricted stock studies show steep discounts relative to freely tradable shares, reflecting investors' aversion to being "locked in."

Even in derivatives markets, which can be very liquid for near-maturity or at-the-money instruments, less frequently traded contracts display higher implied yields or cheaper option prices. Many researchers cite liquidity frictions and thin-order books as explaining these cross-sectional price differentials.

In less developed financial systems, illiquidity can be severe due to limited market participation, higher transaction costs, and weaker regulatory infrastructure. Bekaert et al. (2007) show that emerging market assets embed larger liquidity premia; improvements in local market liquidity and openness over time typically reduce the required returns.

The collective evidence—both theoretical and empirical—indicates that liquidity is a priced state variable in global capital markets. Investors consistently require both a "level premium" for holding assets with low liquidity (e.g. wide bid-ask spreads, scarce trading volume) and an additional systematic liquidity risk premium if these assets become more difficult to trade precisely when capital is most needed. These insights hold true in sovereign and corporate bonds, equities, and alternative or private investments.

2.5 Liquidity Premium Puzzle

Our paper contributes to the literature on the liquidity premia puzzle. Merton (1973) hypothesized that investors should demand higher returns to hold illiquid assets due to the higher costs of transacting. In other words, Merton (1973) proposed that investors pass on the costs of illiquidity to other market participants. The part of the liquidity premia puzzle that has been less clear is which group of market participants actually picks up the slack and pays the liquidity premium to investors.

The original formulation of the liquidity premium puzzle is due to Constantinides (1986). Constantinides (1986) develops an equilibrium model showing that the liquidity premium is small for investors with an infinite trading horizon. Recently, Chen (2022) attributed the discrepancy to investor information differences. Chen (2022) argue that better informed investors are not fazed by transaction costs and the liquidity premium puzzle does not apply to them. For less-informed market participants, large transaction costs are a deterrent to trading, causing a suboptimal allocation. For less informed investors, less liquid assets are therefore a problem that is solved with higher returns on illiquid assets.

Our proposed model synthesizes infinite horizons and information variance among investors. We show that there simply is no one to pick up the liquidity slack on the other side of the rainbow and willingly pay the liquidity premium for investors, aside from other investors. Therefore, in equilibrium, investors avoid illiquid assets altogether, increasing the illiquidity premia on already illiquid assets over time. Instead, we show that investors place a premium on more liquid assets. In other words, investors pay a higher price for assets that are more liquid.

Our results are still consistent with the liquidity premium puzzle key item that illiquid assets receive higher premia. The premia come from investors who undervalue illiquid assets today, pricing in the risk of illiquidity, and then receive a fair valuation in the future. Lower-than-average prices today and fair prices tomorrow result in higher than average returns.

In our model, we further show that investors can proxy liquidity as volume traded. In fact, volume traded is a measure of realized liquidity. Then, the higher the liquidity, the lower the price impact, which is consistent with Amihud and Mendelson (1986) measure of illiquidity.

2.6 Informed and Uninformed Trading Volume

Classic microstructure studies separated the influence of informed and uninformed trading volumes. The probability of informed trading was studied by Easley et al. (2008), Easley et al. (1996), and others, who mainly analyzed high-frequency data. In a similar vein, Zarpala (2023) analyzes uniform-price auctions for corporate bonds under risk and budget constraints. In this paper, we extend their analysis from short-term horizons to long-term investments.

We show that absent fire sales (Giannetti and Jotikasthira (2024)), and given the fixed supply of the asset, the probability of informed trading necessarily increases the overall trading volume, since informed traders need counterparties to complete their trades. This, in turn, creates a higher demand, leading to a higher price level. Thus, informed trading on a longer time scale translates into higher volume and higher prices.

2.7 Recent Studies

In relation to the investment time horizon, Chen (2022) reviews classical approaches, including convertible, zero-coupon, and corporate bond frameworks. This review also highlights how Macaulay and modified duration sensitivity profoundly affects risk management and pricing strategies.

In emerging markets, Ceballos et al. (2025) address the dynamics of post-pandemic interest rates in Chile, showing that real interest rates remain persistently low despite short-term spikes due to large and time-varying liquidity risk premia. These findings emphasize the role of market frictions even in inflation-indexed bond markets.

Beyond empirical applications, recent theoretical developments offer new insight into the default and option features. For example, O et al. (2021) propose closed-form solutions for the pricing of the option of bonds under credit risk by using extensions of the Black–Scholes equations, while Hölzermann (2020) explores the pricing of interest rate derivatives in volatile and uncertain environments, providing robust valuation tools that could also inform the pricing of bonds derivatives under ambiguous market conditions.

Early redemption provisions and discrete coupon payments introduce further complexity, as shown in O and Kim (2020). Their structural approach defines unique default boundaries for each redemption date, furnishing more nuanced formulas that can capture practical bond features. Meanwhile, Vachon and Mackay (2025) develop a unifying matrix-based framework to price diverse fixed income instruments, bonds, convertibles, and bond options, when short rates follow time-inhomogeneous diffusions. This unifying approach helps connect credit risk modeling with interest rate modeling for a comprehensive view of debt valuation.

Sustainability-linked products continue to draw attention. Poggensee (2025) extends the analysis of "green" or ESG-linked bond issues beyond the primary market, revealing a small but significant SLB premium in the secondary market as well, consistent with investor preferences for sustainability attributes.

Regarding trading size and costs, Pinter et al. (2024) uncover a nuanced "size discount and size penalty" effect in government bond markets, where larger trades initially appear to face lower transaction costs, but controlling for bond-specific factors can invert this relationship, implying a penalty for very large trades. Such findings are in connection with the notion of liquidity metrics based on trading volume in corporate bonds.

Ambiguity aversion further complicates bond yields, according to Zhao (2020), whose equilibrium model reveals that both nominal and real yields can slope upward precisely because investors fear "unknown unknowns." By embedding ambiguity into bond pricing, the author explains violations of the expectations hypothesis in yield curves, thus adding a behavioral dimension to the classic risk-based narrative.

For general fundamentals and risks of bond price, Burgess (2022) offers a clear explanation of bond mechanics, price, accrued interest, and the role of interest rate risk. This foundational knowledge complements advanced structural and reduced-form methods that are often applied in academic and industry settings.

While Zarpala (2023) tackles auction-theoretic aspects of bond issuance, Helwege and Wang (2021) directly quantifies liquidity-driven price pressure in corporate bonds. Their results show that large bond issues, so-called "megabonds", enjoy high liquidity but often come at a yield discount, reflecting increased price pressure around issuance.

Overall, this body of work underscores that bond pricing is inherently multi-dimensional, intertwining credit risk, liquidity considerations, and, increasingly, sustainability and market microstructure factors. Traditional structural models (e.g. Merton) remain highly influential, yet the literature demonstrates continuous refinements, through machine learning, auction-theoretic mechanisms, or time-inhomogeneous term structures, to capture the evolving complexities of bond markets in both advanced and emerging economies.

3 The Model

In this section, we propose an agent-driven pricing model for bonds. Specifically, we establish that

1. For assets with a finite outstanding amount, short-term informed trading necessarily induces higher trading volume in the long run.

2. In addition, in the long run, higher volume translates into higher demand for the asset and results in higher prices.

Theorem 1. *Higher informed trader participation on day t in an asset i with fixed supply s_i on average results in higher long-term trading volume $V_{t+\tau}$.*

Proof. Suppose that the market admits informed trading, like in Easley et al. (1998), Easley et al. (2008) and Easley et al. (1996), among others, and $p_{i,t}$ is the informed probability of trading in asset i.

Regardless of whether $p_{i,t}$ represents buyers or sellers, each informed trader needs a counterparty to trade with. The counterparty may or may not be informed.

Aldridge and Zhang (2025) showed that, for both informed and uninformed investors, the probability of trading today is tied to the probability of being able to liquidate the position in some future time periods.

In other words, a risk-averse agent i will buy an asset today only if he is certain that he can sell it tomorrow, even if at a loss. To sell the asset tomorrow, the market for the asset needs to have a buyer j who believes that she will be able to sell the asset in the following days, and so on.

Let us assume that each agent always trades just one unit of the same asset in one market. Then we can express the probability that the agent enters the market as follows:

$$p_{it}(b_a) = p_{i,t+1}(s_a) = p_{j,t+1}(b_a) = p_{j,t+2}(s_a) = ... \tag{1}$$

where $p_{it}(b_a)$ is the probability that agent i buys asset a at time t, and $p_{j,t+2}(s_a)$ is the probability that agent j sells asset a at time $t+2$.

Collectively, the sum of all agents' trades in asset i at a given time t represents the volume in asset i at time t:

$$V_{i,t} = \sum I_{i,t} b_{ai,t} \tag{2}$$

If on day t, the probability of agents buying or selling asset i is higher due to the influence of the informed traders, this influence will propagate into the future time periods and result in even higher future trading volume.

□

Theorem 2. *Faced with a constant supply of asset i, $s_i = const$, assets with growing trading volumes $\Delta V_{i,t+1} > 0$ will in equilibrium experience higher prices:*

$$\Delta P_{i,t+1} > 0 \tag{3}$$

Proof. The volume of trade for the asset i is a demand for available liquidity. Suppose we can represent the demand as a standard linear demand equation:

$$d_{i,t} = a - bP_{i,t} \tag{4}$$

where $a > 0$ and $b > 0$ are some coefficients, and $P_{i,t}$ is the price of asset i at time t. Note that if the demand is not linear in price, we can linearize it with, say, the Taylor series expansion.

After a turnover $V_{i,t}$, the available liquidity (supply) is $s_i - V_{i,t}$. This reduced supply may reflect the remaining inventory of the market makers.

Solving for an equilibrium, we obtain:

$$s_i - V_{i,t} = a - bP_{i,t} \tag{5}$$

Then, at time t, the market-clearing price $P_{i,t}$ is:

$$P_{i,t} = (a - s_i + V_{i,t})/b$$

As the volume increases,

$$\Delta V_{i,t+1} > 0$$

the price also increases:

$$\Delta P_{i,t+1} > 0$$

□

Theorems 1 and 2 imply the following:

- In the liquidity premium puzzle, the way investors demand higher returns on the less liquid assets is by refusing to trade them. Then, as the volume of the illiquid issues drops at time t, the prices of these assets at time t drop as well. If volume increases at a future time $t + \tau$, so do prices, allowing investors to realize abnormally high returns between times t and τ.
- Trading volume represents the liquid portion of the outstanding shares that investors take into account when making pricing decisions. As a result, the trading volume is an accurate representation of the liquid portion of the market capitalization for equities and the face value of the debt. We will test this finding in the Merton Model next in the empirical section of the paper.

4 Empirical Experiments

4.1 Data

This study integrates equity market information from the Center for Research in Security Prices (CRSP) with corporate bond transaction data from FINRA's Trade Reporting and Compliance Engine (TRACE), along with short-term risk-free rates (specifically, the yield of the 13-week Treasury bill). Our primary objective is to form a unified monthly data set that aligns corporate equity values, corresponding bond prices, and benchmark interest rates on a consistent time scale.

We begin by extracting monthly equity data for publicly listed firms from the CRSP. For each trading month, we collect end-of-month stock prices and outstanding share counts, from which we compute market capitalization (price multiplied by shares outstanding). By focusing on S&P 500 constituents, we ensure that our sample captures large, liquid issuers that typically have active secondary bond trading. From this preliminary equity universe, we then cross-reference the corresponding bond data in TRACE to isolate only those bonds issued by the same firms. This matching process involves standardizing company identifiers and retaining bonds for which daily transaction data are available during our sample period.

We converted daily TRACE bond observations to a monthly frequency to match CRSP's equity data. For each bond and each calendar month, we select

the final available transaction price—or the price considered most representative—on the month's last trading day. This approach reduces noise from price fluctuations within the month and allows for a straightforward comparison of end-of-month bond prices with end-of-month equity values. Currently, we collect daily T-Bill yields (often sourced from platforms like Yahoo Finance) as a proxy for the risk-free rate. These daily yields are aggregated similarly to a monthly frequency, maintaining the yield from the last trading day for consistency across all instruments.

To account for both short-term and long-term debt dynamics, we create two separate bond datasets. We used the duration of bond-trading history in our datasets as a proxy for the bond's maturity.

1. For the first dataset, we select, for each company, the bond that has the longest trading history at any given time, based on the difference between its first and last transaction dates in the TRACE system. This dataset is intended to represent high-duration instruments that may more accurately reflect long-term market expectations regarding credit risk.

2. The second dataset includes the bond for each company with the shortest trading history, highlighting near-term maturity risk profiles.

We use a consistent aggregation method for both datasets: filtering daily records to retain only the last transaction of each month. This approach allows for a detailed analysis of potential differences in market pricing over varying durations of bonds.

Note that only 7.44 % of the 85,574 bonds in both data sets had maturities that extended beyond the dataset's last day. Although we acknowledge this as a limitation of our data, by the law of large numbers, we believe that our results fairly reflect the underlying distribution of bond maturities.

To maintain data continuity, we carry forward any available long-term debt figures—such as those from quarterly or annual financial reports—from their effective date until updated, ensuring that each monthly record reflects the most recent debt levels. This step is vital in structural credit models, which often need both equity and debt data to estimate a firm's value and default probabilities. As a result, for each firm-month entry in our final database, we include:

1. end-of-month market capitalization from CRSP

2. an aggregated bond price (or multiple prices, if examining different maturities) from TRACE on the month's last trading day

3. the risk-free T-Bill yield, also assessed at month-end

These data points can be supplemented with additional firm-level fundamentals, such as total assets and liabilities, if required for the selected structural modeling approach.

Figure 1 visualizes the pricing dynamics of short- and long-maturity bonds for Apple Inc. (AAPL) over time.

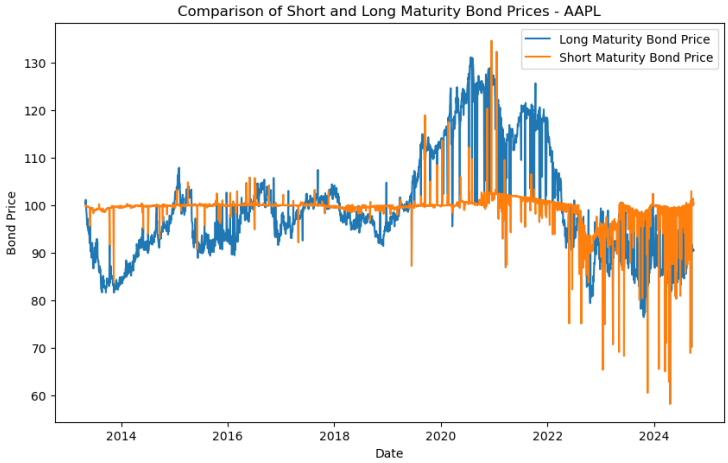


Figure 1: Comparison of Shortest and Longest Maturity Bond Prices (AAPL)

This harmonized data set contrasts corporate bond prices with their theoretical valuations, over both short and long-term periods. By aligning every data series with a consistent monthly schedule, we ensure that the movements in equity, bond, and risk-free rates are directly comparable. This rigorous framework supports empirical investigations of pricing accuracy, credit spread behavior, and the interaction between signals from equity markets and debt market valuations.

4.2 Assumptions

In developing and testing our extended Merton-based bond pricing framework, we rely on several conceptual, methodological, and empirical assumptions. These assumptions guide our theoretical model derivations and inform the design of our empirical tests. Below is a concise summary:

1. **Corporate Capital Structure**
    - We follow the standard structural approach in Merton (1974), assuming that the firm's value can be decomposed into an (equity-like) "asset component" and a "debt component."
    - In contrast to classical Merton models that use total firm value from annual statements, following our model, we assume that the *liquid* portion of the corporate value—represented by daily or monthly trading volume in the equity market—is the relevant asset measure for bond investors, especially over shorter horizons.
2. **Risk-Neutral Valuation and Option Equivalence**
    - Consistent with Black and Scholes (1973) and Merton (1974), we assume risk-neutral pricing in frictionless markets and treat a corporate

bond as a portfolio of risk-free debt minus a put option on the firm's value.

- Over short horizons, investors remain risk-neutral with respect to default risk, but the probability of default is proxied by changes in daily or monthly trading volume.

3. **Bond Characteristics and Maturity**

   - We treat corporate bonds as zero-coupon (or simplified coupon) instruments for ease of mapping into a Merton-style payoff. Any coupon payments are assumed to be either negligible over a one-month horizon or rolled into the principal at maturity.
   - We distinguish between *shortest-maturity* and *longest-maturity* bonds for each firm, under the assumption that these maturity segments may exhibit distinct liquidity and rollover risks.
   - We assume that the *face value* (debt notional) can be approximated by trade-based aggregates (e.g., summing daily transaction price $\times$ volume). Different "methods" of aggregation yield varying proxies for the firm's effective debt burden.

4. **Equity Liquidity and Trading Volume**

   - We assume that the observed *equity trading volume* (daily or monthly sums) reflects the firm's "liquid asset value." This portion is considered to be the accessible value for investors when pricing default risk over shorter time scales.
   - By treating trading volume as a demand-driven quantity, we assume that higher volume correlates with greater market depth and liquidity, which, in turn, affects the bond's pricing through lower liquidity premia and higher expected recovery values.
   - Consistent with short-term informed trading models (Easley et al., 1998, 2008), we posit that if informed traders foresee favorable corporate developments, they will increase demand for the firm's securities, raising both volume and price.

5. **Fixed Supply of Corporate Securities**

   - In line with our model and standard demand–supply frameworks, we assume a *fixed supply* of shares and bonds in the short run. Because supply is assumed invariant, increasing investor demand (reflected by higher volume) pushes prices upward.
   - We abstract away from issuance or buyback programs within each one-month horizon, assuming that any corporate actions have negligible impact on supply over that short period.

6. **Bankruptcy Process and Recovery**

- We adopt the classic assumption of Merton (1974) that bondholders receive the minimum of (i) the face value of the debt and (ii) the total asset value at maturity.
- Empirical evidence on protracted bankruptcy proceedings (Bermant and Flynn, 1989; Weiss, 1990; Betker, 1997) motivates our modeling choice that only *liquid* corporate value is easily recovered by short-horizon bondholders, as intangible or illiquid assets may be locked in legal disputes.
- The model implicitly assumes that bond investors focus chiefly on near-term, easily monetizable assets—thus favoring trading volume as a proxy for recoverable value.

7. **Data and Frequency Conventions**

   - We merge monthly observations of equity markets (from CRSP) and corporate bond transactions (from FINRA TRACE) to create a unified panel, aggregating daily bond trades by selecting end-of-day or end-of-month "representative" prices.
   - We take the 13-week Treasury bill yield as the risk-free rate for discounting over each one-month horizon, thereby simplifying interest rate risk to a single short-term benchmark.
   - We assume that monthly frequency is sufficient to capture the meaningful variation in bond prices and volumes for the structural modeling approach, acknowledging the possibility that intra-month variations may be large but treating them as noise for monthly regressions.

8. **Rolling Estimation and Predictive Regressions**

   - When evaluating out-of-sample forecasts, we assume that model parameters (e.g., the volatility of the firm's "asset," the ratio of $\frac{A}{L}$, and regression coefficients) can be re-estimated as new data arrive, reflecting time-varying firm or market conditions.
   - Monthly realized bond returns are regressed on either *contemporaneous* or *one-month-lagged* Merton-implied returns, assuming that the forecasting window of interest is one month.
   - We treat individual bond issues for each firm separately (one with the shortest trading history, another with the longest) and pool them only after calculating forecasts, thereby assuming that cross-sectional dependence does not materially bias the final regression estimates.

These assumptions underpin our main findings that (1) trading volume helps capture short-term default risk perceptions, (2) daily volume-driven measures of a firm's debt and asset values significantly improve bond price forecasting for short-maturity issues, and (3) the classic Merton approach, based on aggregated end-of-month data, still retains relevance to forecast longer-term bond returns. Any departure from these assumptions, such as large supply expansions

via new issuance, extremely illiquid bonds, or heavily distressed markets, may alter the empirical outcomes and should be explored in future research.

5 Empirical Model 1

We estimate two types of regression (contemporaneous and one-month-ahead) for both the longest-maturity bond sample and the shortest-maturity bond sample. Let BondRet_t be the monthly return of the bond (or the price change) in month t. We define three volume-based explanatory variables:

- **Total Trading Volume**:

$$\text{sum_volume}_t = \sum_{\tau \in t} \text{Volume}_\tau$$

  This represents the total trading volume within month t.

- **Standard Deviation of Trading Volume**:

$$\text{stdev_volume}_t = \sqrt{\frac{1}{n-1} \sum (\text{Volume}_\tau - \overline{\text{Volume}})^2}$$

  This measures the dispersion of the daily trading volume within the month t.

- **Return-Weighted Trading Volume**:

$$\text{return_weighted_volume}_t = \sum_{\tau \in t} (r_\tau \times \text{Volume}_\tau)$$

  This represents the trading volume weighted by the return of each day's bond r_τ.

We then estimate the following regressions:
(A) Contemporaneous Regression:

$$\text{BondRet}_t = \alpha + \beta_1 \, \text{sum_volume}_t + \beta_2 \, \text{stdev_volume}_t + \beta_3 \, \text{return_weighted_volume}_t + \varepsilon_t.$$

(B) One-Month-Ahead Regression:

$$\text{BondRet}_{t+1} = \alpha + \beta_1 \, \text{sum_volume}_t + \beta_2 \, \text{stdev_volume}_t + \beta_3 \, \text{return_weighted_volume}_t + \varepsilon_{t+1}.$$

These regressions are estimated separately for the bond with the longest maturity ("Longest") and the bond with the shortest maturity ("Shortest") for each firm.

Regression Results

Table 1: Coefficient Estimates for Each Regression

	Long_con	**Long_pred**	**Short_con**	**Short_pred**
Dependent Var.	BondRet_t	BondRet_{t+1}	BondRet_t	BondRet_{t+1}
Intercept	8.47e-05	-0.0002	0.0015***	0.0013***
	(0.268)	(-0.622)	(3.720)	(4.755)
Sum Volume	4.05e-10***	5.25e-10***	1.31e-10**	1.39e-10***
	(6.606)	(8.555)	(2.352)	(3.782)
Std Dev Volume	-4.88e-09***	-5.57e-09***	-3.14e-09***	-3.03e-09***
	(-6.636)	(-7.576)	(-3.901)	(-5.720)
Return-Weighted Vol	-7.56e-16	5.59e-15	2.32e-13	4.25e-13
	(-0.065)	(0.478)	(0.320)	(0.887)
Observations	52,711	52,711	52,382	52,382
R^2	0.001	0.001	0.000	0.001
F-Statistic	15.71***	24.57***	6.29***	12.68***

Significance levels: *: 90% significance; **: 95% significance; ***: 99% significance. Coefficients are reported with t-statistics in parentheses.

Regression Analysis and Implications

These regression results provide several key insights into the role of trading volume metrics in explaining and forecasting bond returns.

The overall explanatory power, indicated by R^2 values, is relatively modest. This is a common result in monthly return regressions. However, the coefficient estimates reveal informative patterns that can guide both academic research and practical portfolio management strategies.

1. Total Volume (sum_volume_t)

Consistent with our proposed model, the total volume transacted within a month exhibits a positive and statistically significant relationship with subsequent bond returns. This indicates that higher levels of trading activity are associated with marginal improvements in bond performance. This relationship holds in both contemporaneous (t) and predictive ($t+1$) contexts for both the longest- and shortest-maturity bonds.

This finding supports the crux of our model that increased trading volume indicates greater investor interest or demand. Higher demand for bonds in turn boosts bond prices.

2. Standard Deviation of Volume (stdev_volume_t)

Unlike the positive effect of total volume, volume volatility exerts a negative and statistically significant influence on bond returns across all specifications.

We measure volume volatility as the standard deviation of the daily trading volume for each bond within each month.

The negative effect of volume volatility is particularly pronounced in predictive regression $(t+1)$, suggesting that increased uncertainty or variability in trading activity during month t precedes weaker bond price performance in month $t+1$.

This is once again consistent with our proposed model. Higher volume volatility may indicate greater dispersion in investors' beliefs about credit risk or temporary imbalances in order flow. Either scenario can increase uncertainty and discourage investors from buying bonds, according to our model. Consequently, bonds with volatile volume may experience lower equilibrium prices.

3. Return weighted volume ($\text{return_weighted_volume}_t$)

The coefficients on return-weighted volume are statistically insignificant across all tested models, suggesting that once the total trading volume and its variability are accounted for, the comovement of volume and returns within a month has limited incremental explanatory power. It is possible that any information embedded in the alignment of daily returns and trading volume is already captured by total trading intensity and its volatility. This implies that investors are primarily sensitive to the scale and variability of the volume rather than to the specific return-volume relationship within a month.

4. Comparative Observations: Longest vs. Shortest Maturities

Although the sign and significance levels of the volume measures remain qualitatively similar between the two maturity groups, there are slight variations in the magnitudes of the coefficients. The consistency in direction and significance of the coefficients across both regressions suggests that total volume and its standard deviation play a broadly comparable role in shaping bond price outcomes across the term structure. As a result, market participants may consider total volume and volume volatility as universally relevant metrics for bond pricing, regardless of the maturity profile.

6 Empirical Model 2

Next, we estimate a Merton-type structural model applied to monthly bond returns. Consistent with our model, we only consider the liquid portion of assets and the debt as input into the Merton model.

To construct the inputs for the Merton model, for a given publicly traded company i, we let A_i denote the liquid "asset value" of the firm. To do so, we approximate A_i by aggregating daily $\text{Price}_{it} \times \text{Volume}_{it}$ for the stock i and summing each month. From another angle, A_i is a measure of the total capital flow from the equity market for that period.

Similarly, we let L_i represent the total debt of the company i, approximated by summing the amounts of daily bonds transaction ($\text{BondPrice}_{it} \times \text{Volume}_{it}$)

for each month. We interpret σ (the volatility term) as the standard deviation of the daily A_{it} within the same month. The risk-free rate r_t is drawn from monthly (end of month) 13 week Treasury Bill yields. Since most corporate debt obligations are rolled over and never truly expire, without loss of generality, we represent the time to maturity as $T = 1/12$.

Next, we implement option-based debt valuation, following the classical Black–Scholes–Merton logic for a one-month horizon:

$$
\begin{aligned}
d_1 &= \frac{\ln\!\left(\frac{A_{it}}{L_{it}}\right) + \left(r_t + \frac{1}{2}\sigma_t^2\right)T}{\sigma_t\sqrt{T}}, \qquad d_2 = d_1 - \sigma_t\sqrt{T}, \\
P_{it} &= Le^{-r_t(T)}\Phi\big(-d_2\big) - A_{it}\Phi\big(-d_1\big), \qquad B_{it} = L_{it}e^{-r_tT} + P_{it}, \\
\text{MertonReturn}_{it} &= r_t + \frac{P_{it}}{B_{it}},
\end{aligned}
$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. The term P_{it} is interpreted as the market value of a put option on the firm's assets with strike L_{it}, reflecting the credit risk embedded in the debt.

Finally, we specify the regression framework. After computing MertonReturn_{it} at the end of each month t, we examine how this value relates to the Realized Bond Return $\text{BondReturn}_{i,t+1}$ observed in the following month. To do so, we use a single-factor linear model:

$$
\text{BondReturn}_{i,t+1} = \alpha_i + \beta_i\,\text{MertonReturn}_{it} + \varepsilon_{i,t+1}.
$$

Here, $\text{BondReturn}_{i,t+1}$ is the percentage monthly change in the price of the debt of company i between months t and $t+1$. By shifting MertonReturn_{it} forward one month, we obtain an out-of-sample prediction testing framework.

We run a regression of equation (6 with and without intercept.

Regression Results

We estimate the regression on two variations of the dataset, one capturing the longest-maturity bonds and another capturing the shortest-maturity bonds. Table 2 presents a concise summary of the main results.

Table 2: Coefficient Estimates for Merton-Return Regressions

	Longest		**Shortest**	
Dependent Var.	BondRet_{t+1}	BondRet_{t+1}	BondRet_{t+1}	BondRet_{t+1}
Intercept	0.5213	—	0.5634	—
	(0.928)	—	(0.941)	—
Merton Return$_t$	-0.9450	0.0677*	-0.9754	0.1191***
	(-0.866)	(1.750)	(-0.838)	(2.886)
Observations	53,349	53,349	53,349	53,349
R^2	0.000	0.000	0.000	0.000
F-Statistic	0.7495	3.063	0.7021	8.327

Significance levels: *: 90% significance; **: 95% significance; ***: 99% significance. Coefficients are reported with t-statistics in parentheses.

Regression Analysis and Implications

According to the theoretical model in this paper, the main objective of these regressions is to determine whether a one-month-ahead bond return forecast from a "liquid Merton" model can reliably explain realized bond price movements. Several key observations emerge:

Impact of Including vs. Excluding the Intercept

When **including** an intercept (the columns 'with intercept'), the longest and shortest maturity regressions yield slope coefficients (β) that are statistically indistinguishable from zero, indicating that there is no clear link between the Merton return of the model and subsequent monthly bond returns. The intercepts themselves are also imprecisely estimated, suggesting that the regression cannot effectively isolate a systematic offset or shift that would reconcile Merton returns and realized outcomes.

In contrast, under a **no-intercept** specification (the "Uncentered" columns), the shortest-maturity sample exhibits a small but statistically significant positive slope ($\beta \approx 0.12$). This result implies that, when forced through the origin, a higher "liquid Merton"-implied return in one month correlates with an increase in the actual bond returns in the next month.

Potential Explanations for Mixed Predictive Power

- **Liquidity Constraints:** Corporate bonds often trade infrequently, with sporadic large trades. Such thin liquidity reduces price discovery efficiency, causing structural model signals to be drowned out by short-term noise or liquidity-driven price movements.

- **Horizon Mismatch:** Merton-style frameworks are commonly conceptu-

alized over annual or longer horizons, given that default risk and credit conditions typically unfold over extended timeframes. In contrast, these regressions focus on monthly returns, which can be influenced by short-term events (e.g. announcements, interest rate fluctuations) that a structural model does not explicitly incorporate.

- **Simplified Data Construction:** The method of aggregating daily price volume flows may not accurately capture the true asset base A_i of the firm (i) or the liability measure (L_i). If these proxies deviate significantly from economic fundamentals, the resulting put option valuation, and thus the Merton return, may fail to align with actual bond market dynamics.

Practical and Theoretical Implications

- **Forecasting Utility:** The model-implied return predicts the next month bond returns at the 90% for the longest-maturity bonds and 99% for the shortest-maturity bonds. These findings provide both academics and practitioners with an actionable predictive framework for bond pricing.

Comparison with Classic Merton Model

A natural question arises as to how our augmented approach compares to the original Merton (1974) specification. In that classical setting, volatility is typically measured using equity-based standard deviation (for example, daily stock returns), and the firm's asset value is approximated by market capitalization. To facilitate this comparison, we re-estimate our one-month-ahead bond return regressions without an intercept, substituting (1) the stock return volatility for σ and (2) market capitalization, calculated as stock price multiplied by outstanding shares, for A. All other modeling steps remain consistent.

Table 3: Coefficient Estimates for Merton Model Regressions

	Liquid Approach	**Classic Merton**
	Longest	
Dependent Var.	BondRet_{t+1}	BondRet_{t+1}
$Merton\ Return_{t+1}$	0.0677^{*}	0.3248
	(1.750)	(0.389)
Observations	53,349	53,136
$\mathbf{R^2}$ (Uncentered)	0.000	0.000
F-Statistic	3.063^{*}	0.1516
	Shortest	
Dependent Var.	BondRet_{t+1}	BondRet_{t+1}
$Merton\ Return_{t+1}$	0.1191^{***}	0.9595
	(2.886)	(1.081)
Observations	53,349	53,136
$\mathbf{R^2}$ (Uncentered)	0.000	0.000
F-Statistic	8.327^{***}	1.168

Notes: $\mathbf{R^2}$ is computed without centering since the model does not contain a constant. Standard Errors assume that the covariance matrix of the errors is correctly specified. Significance levels: *: 90%; **: 95%; ***: 99%.

Notably, the Liquid Approach yields statistically significant coefficients ($\beta \approx 0.0677$ for the longest maturity, $\beta \approx 0.1191$ for the shortest) at the 90–99% confidence levels. By contrast, the Classic Merton model produces slopes ($\beta \approx 0.3248$ for the longest maturity and $\beta \approx 0.9595$ for the shortest) that fail to achieve significance, as indicated by t-statistics below one and F-statistics near zero. Uncentered R^2 values remain effectively zero in the classic version, indicating negligible predictive power under monthly horizons.

These findings highlight a critical distinction:

1. Short-Term Liquidity Sensitivities. The liquid Merton specifications, which incorporate daily volume-driven measures of firm "assets," appear more attuned to short-horizon liquidity risk, delivering significant slope estimates and meaningful F-statistics. This contrast suggests that standard market-cap data, drawn typically at lower frequency, do not capture the same day-to-day liquidity and informed-trading signals.

2. Maturity-Specific Insights. Even at the longer end, the liquid approach finds moderate success ($\beta \approx 0.0677$), whereas the original Merton version yields a slope that is statistically indistinguishable from zero. This reinforces that a purely annualized perspective, as in classic Merton, may struggle when used for one-month-ahead bond return forecasting.

Implications. For reviewers questioning the incremental value of our

model, these comparative results indicate that replacing annualized market-cap and equity-only volatility with high-frequency liquidity measures can materially enhance predictive accuracy over short horizons. Indeed, the observed disparity in slope significance and F-statistics underscores that near-term corporate bond pricing is strongly influenced by daily volume flows and real-time credit risk signals. Thus, for practitioners aiming to predict monthly bond returns, our augmented "liquid Merton" methodology provides more empirically reliable results than the traditional, market-cap-focused Merton specification.

7 Empirical Model 3

Having previously described the Merton-type structural model and its reliance on a notional "debt face value" L_{it}, we now consider two specific ways of aggregating daily bond prices and volumes to form L_{it}. In each case, we first gather daily observations of realized bond transaction prices (Price_{it}) and bond trading volume (Volume_{it}) for a given firm's bond within a given month.

Method 1

$$L_{\text{Method1}_\tau} = \big(\text{ClosePrice}_\tau\big) \times \sum_{\tau=1}^{n} \big(\text{Volume}_\tau\big).$$

Here, ClosePrice_τ is the most recently observed bond price in that month (e.g., on the last trading day), and we multiply it by the sum of that month's daily volumes. Conceptually, this treats the entire monthly volume as if it were "issued" at the closing bond price, producing an aggregate measure of debt.

Method 2

$$L_{\text{Method2}} = \sum_{\tau=1}^{n} \Big(\text{Volume}_\tau \times \text{Price}_\tau \times \text{sign}(\Delta_\tau)\Big).$$

In this approach, we accumulate the daily volume $\times$ price with an additional sign factor, which typically reflects whether the daily change in the price of the bond Δ_τ was positive $(+1)$ or negative (-1). This aims to capture "directional" capital flows: on days when prices rise, the daily component increases L; on days when prices fall, it decreases L.

In both methods, the resulting L is then combined with an equity-based estimate of the firm's asset value A, volatility σ, and risk-free rate r to calculate the Merton-style put option P (see Empirical Method 2) and the value of the bond B. The model's Merton-implied bond return is:

$$\text{MertonReturn}_{it} = r_t + \frac{P_{it}}{B_{it}}.$$

To assess the predictive capability of the Merton-based return estimates, we conduct an out-of-sample regression using a one-month lead on the realized bond returns:

$$\text{BondReturn}_{i,t+1} = \alpha_i + \beta\, \text{MertonReturn}_{it} + \varepsilon_{i,t+1}.$$

Here, $\text{BondReturn}_{i,t+1}$ is the percentage change in the bond's market price from month t to $t+1$, while MertonReturn_{it} is the structural model's implied return computed at the end of month t. We estimate this linear relationship separately for Longest Maturity and Shortest Maturity bond samples, under each of the two definitions of L (Method 1 vs. Method 2).

We perform the regression without an intercept, as the regression results support this approach.

Regression Results

The table shows the regession results without intercept.

Table 4: Coefficient Estimates for Merton Model Regressions

	Longest		**Shortest**	
	Method 1	**Method 2**	**Method 1**	**Method 2**
Dependent Var.	BondRet_{t+1}	BondRet_{t+1}	BondRet_{t+1}	BondRet_{t+1}
$MertonReturn_{t+1}$	1.5217	0.0309^{***}	0.7669	0.0791^{***}
	(0.988)	(3.339)	(1.491)	(5.211)
Observations	53,349	27,202	53,349	26,069
$\mathbf{R^2}$ (Uncentered)	0.000	0.000	0.000	0.001
F-Statistic	0.9768	11.15^{***}	2.224	27.16^{***}

Notes: $\mathbf{R^2}$ is computed without centering since the model does not contain a constant. Standard Errors assume that the covariance matrix of the errors is correctly specified. Significance levels: *: 90%; **: 95%; ***: 99%.

Regression Analysis and Implications

When estimated without an intercept, Method 2 yields statistically significant slope coefficients for both the longest- and shortest-maturity bond regressions, whereas Method 1 does not. Specifically, for the longest-maturity sample under Method 2, the slope is 0.0309 (t-statistic = 3.339, corresponding to 99% confidence), and for the shortest-maturity sample it is 0.0791 (t-statistic = 5.211, 99.9% confidence). By contrast, none of the Method 1 slopes reach significance, suggesting that a simple "closing-price-times-sum-of-volume" calculation may fail to capture the microstructural dynamics critical for forecasting near-term bond returns.

These findings underscore the importance of daily directional flows in proxying for effective debt levels. The inclusion of sign factors in Method 2 (based on whether daily price changes are positive or negative) appears to more accurately reflect actual trading conditions and investor sentiment. For short-dated bonds, where liquidity and rollover considerations loom large, this greater fidelity to

day-to-day market movements markedly enhances predictive power. Even for longer maturities, the directional approach shows more robust out-of-sample performance than Method 1.

In sum, these results reinforce the view that structural models that integrate directional volume metrics, rather than purely aggregate ones, can offer more meaningful month-ahead return forecasts. By systematically incorporating daily price changes, practitioners and researchers can achieve finer-grained assessments of firm-specific liquidity risk and bond price vulnerabilities, especially under varying market conditions and investment horizons.

Simple Strategy Return Implications

Table 5: Summary Statistics - Simple Strategy Return (Longest Maturity)

Stat	**Longest Method 1**	**Longest Method 2**
Mean	0.7980	0.0081
Std Dev	182.2349	0.5583
Kurtosis	53755.8928	27395.2032
Min	-0.9999	-0.9931
1%	-0.2526	-0.2342
5%	-0.1268	-0.0876
25% (Q1)	-0.0197	-0.0000
Median (Q2)	0.0001	0.0000
75% (Q3)	0.0201	0.0006
95%	0.1395	0.0971
99%	0.3249	0.2983
Max	42252.7538	107.2500

Table 6: Summary Statistics - Simple Strategy Return (Shortest Maturity)

Stat	**Shortest Method 1**	**Shortest Method 2**
Mean	0.4021	0.0200
Std Dev	60.8715	0.8775
Kurtosis	27375.2873	11079.1793
Min	-0.9999	-0.9968
1%	-0.1860	-0.1575
5%	-0.0682	-0.0469
25% (Q1)	-0.0105	-0.0001
Median (Q2)	-0.0000	0.0000
75% (Q3)	0.0108	0.0000
95%	0.0704	0.0481
99%	0.2299	0.1823
Max	10658.4000	107.2500

The simple strategy return is constructed based on a rule that utilizes the last month's predicted return to determine whether to take the realized return. Mathematically, this is expressed as:

$$R_t^{\text{strategy}} = \begin{cases} R_t^{\text{realized}}, & \text{if } R_{t-1}^{\text{predicted}} > 0 \\ 0, & \text{otherwise} \end{cases}$$

where:

- R_t^{strategy} is the return of the strategy at time t.
- R_t^{realized} is the actual return realized at time t.
- $R_{t-1}^{\text{predicted}}$ is the predicted return from the previous period.

This rule implies that the strategy only takes a position when the previous month's prediction is positive; otherwise, it remains in cash. The empirical results presented in Tables 5 and 6 highlight the distributional properties of this strategy across different methods.

The data used for this analysis are obtained from Empirical Method 3. This methodology provides the framework for deriving the predicted returns and evaluating the performance of the simple strategy return approach.

8 Empirical Method 4

In many empirical finance settings, a single fixed in-sample and out-of-sample split may fail to capture evolving market conditions and firm-specific dynamics. To address this limitation, we adopt an **expanding (rolling) window** approach, re-estimating the model parameters each time we move forward one period in the sample. This procedure facilitates continual adaptation to the latest data, potentially improving forecast performance over time.

Traditional fixed sample splits (e.g., 70/30 or 80/20 divisions) lock model parameters to a single historical interval. As market conditions change, such static estimates can become stale. By re-estimating the model parameters at each iteration, using all available observations up to the current point, we enable:

- **Time Variation:** Updated estimates that reflect recent market dynamics.
- **Dynamic Learning:** Continuous incorporation of new information to better capture changing risk factors, volatility regimes, or changes in firms' capital structures.
- **Granular Out-of-Sample Testing:** Sequential one-period-ahead forecasts that are strictly out-of-sample.

For each firm (identified by a unique `company_symbol`), we start with an initial training window covering the first 30% (or more) of the chronological observations. As we progress from month t to $t+1$:

- The (t)th observation is added to the training set.
- The model is re-estimated on the expanded sample.
- A one-month-ahead forecast is generated for month $t+1$ using the updated parameters.

Before fitting the model, we verify that the training subset exhibits sufficient variability in the predictor variables (specifically, the short-term risk-free rate, rf, and the lagged Merton-implied return, MertonReturn) to avoid degenerate or singular fits.

Within each in-sample period, the bond's realized monthly return is modeled as:

$$\text{Actual Return}_t \;=\; a \;+\; b_1\, rf_t \;+\; b_2\, \text{MertonReturn}_t \;+\; \epsilon_t.$$

This linear regression captures the effect of both the market environment (via rf) and the structural model's forecast (via MertonReturn). For each out-of-sample period, the updated model is used to predict the bond return for month $t+1$ based on the current rf and MertonReturn. We also run regressions without an intercept (uncentered) to further test the robustness of the predictive relationship.

After iterating through all firms and time periods, we pool all out-of-sample predicted returns and corresponding actual returns into a single dataset. The final regression is specified as:

$$\text{Actual Return} \;=\; \alpha \;+\; \beta \times \text{PredictedReturn} \;+\; \epsilon,$$

using only out-of-sample observations. A statistically significant positive slope (β) would suggest that the Merton-implied return contains meaningful information about subsequent bond returns. All the results we have conducted previously are integrated into this final regression framework.

Regression Results

For brevity, we label the methods as follows:

- **Model 1:** The baseline Merton-type structural model (Empirical Method 2).
- **Model 2:** The model where the debt face value L is constructed using Method 1 (i.e., $L_{\text{Method1}} = \big(\text{LastDayPrice}\big) \times \sum_{\tau=1}^{n} \big(\text{Volume}_\tau\big)$); see the equation stated above. (Model 1 in Empirical Method 3).
- **Model 3:** The model where L is constructed using Method 2 (i.e., $L_{\text{Method2}} = \sum_{\tau=1}^{n} \Big(\text{Volume}_\tau \times \text{Price}_\tau \times \text{sign}(\Delta_\tau)\Big)$); see the equation stated above. (Model 2 in Empirical Method 3).

Table 7 reports the OLS estimation results.

Table 7: Coefficient Estimates for Predicted Return Regressions

Longest Maturity			
	Method 1	**Method 2**	**Method 3**
Dependent Var.	Bond Return (Actual Return)		
Intercept	0.0413	0.0073^{***}	0.0190^{***}
	(1.459)	(7.151)	(2.921)
Predicted Return	0.0494^{***}	1.014×10^{-5}	-0.0043
	(8.097)	(0.703)	(-0.695)
Observations	37,449	37,449	37,449
$\mathbf{R^2}$	0.002	0.002	0.002
F-Statistic	65.56	0.4937	0.4826

Shortest Maturity			
	Method 1	**Method 2**	**Method 3**
Dependent Var.	Bond Return (Actual Return)		
Intercept	0.0073^{***}	0.0625^{**}	0.0351^{***}
	(7.151)	(2.080)	(3.926)
Predicted Return	1.014e-05	0.0526^{***}	0.0527^{***}
	(0.703)	(8.216)	(9.576)
Observations	37,449	37,449	18,390
$\mathbf{R^2}$	0.000	0.000	0.005
F-Statistic	0.4937	67.50	91.69

Notes: Standard Errors assume that the covariance matrix of the errors is correctly specified. Significance levels: *: 90%; **: 95%; ***: 99%.

Regression Analysis and Implications

Table 7 shows statistically-significant performance of the "liquid Merton" model proposed in this paper.

1. Coefficient Patterns in the Longest-Maturity Segment

Turning first to the longest-maturity results, we observe meaningful variation in how the regression intercept and slope coefficients behave across the three methods:

- **Method 1 (Baseline Merton Model)**: The slope on "Predicted Return" is 0.0494, which is statistically significant at the 1% level (t-statistic of 8.097). This strong significance (p-value < 0.01) indicates that a one percentage point increase in the Merton-implied return is associated with

an approximate 0.05 percentage point rise in the next-month realized return for these longer-dated bonds. While the intercept (0.0413) is not firmly significant, its relatively small magnitude suggests that Method 1's predictive component—rather than a systematic offset—captures a non-negligible fraction of the month-ahead price movement.

- **Method 2**: The slope coefficient is near zero (1.014×10^{-5}), with a t-statistic of 0.703, indicating a weak and insignificant link between the predicted and realized returns. Although the intercept is significant (0.0073, t = 7.151), the absence of a meaningful slope implies that this manner of constructing the debt value L offers scant predictive content for long-term bond returns under a rolling estimation procedure.

- **Method 3**: The slope (-0.0043) is both small and statistically indistinguishable from zero (t = -0.695), whereas the intercept is 0.0190 (t = 2.921), a moderate but significant baseline shift. This pattern suggests that Method 3's detailed aggregation of daily trading flows does not materially aid in forecasting longer-horizon bond returns; instead, the model tends to miss, on average, by a nontrivial but constant offset.

Overall, these findings reinforce that the baseline Merton approach (Method 1) retains notable predictive capacity for longest-maturity issues in a month-ahead rolling framework.

2. Coefficient Patterns in the Shortest-Maturity Segment

We now turn to the shortest-maturity bonds, where the predictive power of the alternative debt measures diverges from the longer-dated case:

- **Method 1**: Much like the weaker outcomes in Methods 2 and 3 for long maturities, here the slope on "Predicted Return" is effectively zero (1.014×10^{-5}), with a t-statistic of 0.703, thus showing no meaningful relationship between the Merton-implied return and next-month outcomes. In contrast, the intercept is 0.0073 (t = 7.151), a highly significant positive constant that appears to be unrelated to the slope component.

- **Method 2**: The slope coefficient is 0.0526 with a t-statistic of 8.216, strongly significant (p-value < 0.01). This result implies that a one percentage point rise in the predicted return is associated with a monthly realized return roughly 0.053 percentage points higher for short-dated issues—an effect that, while seemingly small in absolute terms, is robustly different from zero and thus indicative of reliable predictive content. The intercept (0.0625, t = 2.080) is also significantly different from zero, signaling an average upward bias or offset that the slope alone does not capture.

- **Method 3**: An even clearer pattern emerges here, as the slope is 0.0527 (t = 9.576) and the intercept is 0.0351 (t = 3.926). Both are statistically

significant, with the slope having an impressively large t-statistic. This highly significant relationship underlines that the daily flow–based measure of debt embedded in Method 3 can capture short-term fluctuations in credit risk or market sentiment. Essentially, it appears that short-term bond valuations respond in a more direct manner to daily directional volume effects—precisely the nuance that Method 3 includes.

The difference in results in Methods 2 and 3 for the shortest versus longest maturity bonds highlights how near-term pricing is more sensitive to daily trading dynamics, whereas longer-dated bonds can be influenced by broader, slower-moving fundamentals. In short, Methods 2 and 3 are unequivocally effective for shorter maturities (with slope coefficients significant at the 1% level), while Method 1 proves more consistently predictive at the longer end.

3. Broader Interpretation

Under classical Merton logic, the risk of default for a firm arises from the interaction of its asset value and its debt burden. However, the practical challenge lies in measuring these two quantities. Our evidence suggests that the choice of how to construct L exerts a considerable impact on forecasting power, dependent on maturity.

- For **long-term bonds**, the simpler "baseline" measure (Method 1) appears sufficient to identify a statistically robust slope in a rolling out-of-sample test, indicating that classical assumptions of asset growth and default boundary detection may be relevant over the longer horizon.
- For **short-term bonds**, daily price and volume dynamics—particularly those capturing directional flows—offer more reliable signals about short-term rollover risk, liquidity conditions, and immediate investor demand. Methods 2 and 3 effectively harness these daily variations, as reflected in their highly significant slope estimates in the shortest-maturity regressions.

From a microstructure viewpoint, it is plausible that short-term bonds trade based on near-term liquidity or event risks, so methods that incorporate daily capital flows (with or without sign) more closely track the transient shifts that determine month-ahead performance. Conversely, for longer-maturity issues, ephemeral day-to-day fluctuations in volume might be overshadowed by the market's broader assessment of fundamental creditworthiness and interest-rate expectations, thereby making the simpler approach more relevant.

4. Implications for Investors and Researchers

These findings, which emerge from a fully out-of-sample rolling procedure, carry several practical and theoretical implications:

1. **Segment-Specific Modeling**: Investors may consider adopting Method 1 for forecasting returns in longer-dated corporate bonds, as its slope coefficient is strongly significant under the rolling estimation. In contrast, shorter-term bonds benefit from the daily volume-driven enhancements of Methods 2 and 3, which yield **highly significant** slope estimates and thus demonstrate meaningful predictive power.

2. **Model Horizon Alignment**: The divergences underscore the importance of aligning the Merton-type modeling horizon with the bond's maturity. Long-term valuations likely hinge on structural default risk and fundamental credit conditions, whereas short-term prices respond more strongly to immediate liquidity and rollover signals.

3. **Practical Trading Strategies**: Should an investor focus on short-term bond instruments, employing Methods 2 or 3 could add value in identifying small but statistically robust signals. Although the economic magnitude (e.g., around 0.05 slope) may appear modest, sustained outperformance or accurate risk signals could accumulate significantly over multiple intervals, particularly in an environment with frequent roll-over or refinancing events.

4. **Extensions and Refinements**: While the model's predictive success for certain maturity segments is promising, further refinements—such as incorporating market frictions, CDS spreads, or dynamic volatility estimates—may bolster explanatory power across the spectrum of maturities. Additionally, investigating advanced risk metrics, such as jump processes or time-varying default boundaries, could illuminate how ephemeral shocks propagate into bond returns.

By applying a rigorous rolling estimation process and testing multiple ways of constructing the debt value, this study highlights that different segments of the corporate bond market respond to distinct risk measures and modeling assumptions. In short, where the p-values are highly significant (Method 1 for longest-maturity, Methods 2 and 3 for shortest-maturity), we see that the structural model can indeed capture relevant return signals. In the other cases, the slope is essentially indistinguishable from zero, implying that the model's chosen proxy for credit risk (or trading behavior) is either less relevant or overshadowed by alternative price drivers.

5. Overall Conclusions

In conclusion, empirical evidence suggests that Merton-type returns, recalibrated with daily trading data, exhibit strong predictive power for next-month bond returns in certain contexts. The fully out-of-sample rolling estimation procedure underscores this result by ensuring that every predicted return is generated from a model estimated solely on prior information. This further fosters confidence in the robustness of the detected signals.

For scholars, these findings reinforce the importance of bridging theoretical models with market microstructure realities. For practitioners, maturity-specific takeaways can inform the design of systematic bond trading strategies or risk monitoring tools, in which short- and long-horizon credit evaluations require different construction of structural variables. Ultimately, the success of these predictive regressions hinges on how well the asset and liability proxies reflect true economic forces over the relevant horizon, an insight that resonates across numerous structural credit risk applications.

9 Additional to Method 4: Interaction Terms with Market Regimes

As discussed in Empirical Method 4: Out-of-Sample Rolling Prediction, once we combine out-of-sample forecasts, we can augment the final regression with market-phase interactions. Specifically, let t_i $(i = 1, 2, 3, 4)$ be dummy variables that indicate distinct market regimes. The regression then becomes:

$$\text{Actual Return} = \alpha + \beta_0 \,\text{PredictedReturn} + \sum_{i=1}^{4} \delta_i \, t_i + \sum_{i=1}^{4} \beta_i \big(\text{PredictedReturn} \times t_i\big) + \epsilon.$$

Here:

- $t_1 = 1$ for the Post-2012 Recovery phase (January 2012 to December 2014), and 0 otherwise.
- $t_2 = 1$ for the Energy Sector High-Yield Stress phase (January 2015 to December 2016), and 0 otherwise.
- $t_3 = 1$ for the COVID-19 Meltdown & Fed Interventions phase (January 2020 to December 2020), and 0 otherwise.
- $t_4 = 1$ for the Tighter Monetary Policy, Inflation Concerns phase (January 2022 to September 2024), and 0 otherwise.

Including the interaction terms PredictedReturn $\times\, t_i$ allows the slope on PredictedReturn to vary across the different market regimes. Thus,

$$\text{slope in regime } i = \beta_0 \;+\; \beta_i,$$

while the intercept in regime i is shifted by δ_i. This specification tests whether the predictive relationship between the structural model's implied return and realized bond returns is consistently strong across various market phases or if it is more pronounced (or subdued) during certain stress periods.

In other words:

- α is the overall intercept for periods outside the defined regimes (when all t_i are zero).

- β_0 is the baseline sensitivity of actual returns to predicted returns for those periods not captured by any t_i.
- δ_i captures the intercept shift during each market phase.
- β_i indicates how much the predicted-return slope deviates from β_0 in each market phase.

By examining the statistical significance and magnitude of β_i in each regime, we can determine whether the strength or direction of the predictive power changes under different market conditions. This approach enriches our understanding of how well (and under what circumstances) the Merton-implied measure captures future bond performance.

9.1 Regression Results

For brevity, we label the methods as the same as what we have defined in Empirical Method 4. Table 8 reports the OLS estimation results.

Table 8: Coefficient Estimates for Predicted Return Regressions

Longest Maturity			
	Method 1	**Method 2**	**Method 3**
Dependent Var.	Bond Return (Actual Return)		
Intercept	0.0103 (0.241)	0.0091*** (5.884)	0.0243** (2.487)
Predicted Return	0.0717*** (4.991)	-0.0001 (-1.432)	-0.0043 (-0.699)
t1	0.0010 (0.002)	0.0027 (0.131)	-0.0090 (-0.089)
t2	0.2710*** (2.808)	-0.0006 (-0.177)	-0.0132 (-0.644)
t3	0.0312 (0.329)	0.0063* (1.848)	0.0148 (0.720)
t4	-0.0060 (-0.093)	-0.0071*** (-3.028)	-0.0194 (-1.238)
Predicted t1	-0.0720 (-0.107)	-0.0002 (-0.009)	-0.0631 (-0.030)
Predicted t2	-0.0278* (-1.751)	0.0001 (1.534)	-0.1141 (-0.586)
Predicted t3	-0.0641 (-0.649)	0.0002 (1.216)	0.1923 (1.211)
Predicted t4	-0.0704 (-0.549)	0.0003** (2.101)	-0.0337 (-0.457)
Observations	37,449	37,449	19,174
R^2	0.002	0.001	0.000
F-Statistic	8.613	2.576	0.6834

Shortest Maturity			
	Method 1	**Method 2**	**Method 3**
Dependent Var.	Bond Return (Actual Return)		
Intercept	0.0498 (1.097)	0.0176*** (3.628)	0.0433** (3.133)
Predicted Return	0.0696*** (4.820)	-3.174e-05 (-0.079)	0.0459*** (7.708)
t1	-0.0391 (-0.065)	-0.0081 (-0.126)	-0.0420 (-0.399)
t2	0.2478** (2.423)	0.0327*** (2.987)	0.0252 (0.922)
t3	-0.0297 (-0.295)	0.0028 (0.257)	-0.0253 (-0.877)
t4	-0.0307 (-0.446)	-0.0112 (-1.524)	-0.0286 (-1.345)
Predicted t1	-0.0671 (-0.094)	0.0007 (0.009)	-0.0417 (-0.282)
Predicted t2	-0.0214 (-1.326)	-4.074e-05 (-0.092)	0.0463** (2.816)
Predicted t3	-0.0608 (-0.606)	8.191e-06 (0.009)	-0.0116 (-0.180)
Predicted t4	-0.0504 (-0.797)	-4.527e-05 (-0.048)	0.1362* (1.840)
Observations	37,449	37,449	18,390
R^2	0.002	0.000	0.006
F-Statistic	8.587	1.716	11.98

Notes: Standard Errors assume that the covariance matrix of the errors is correctly specified. Significance levels: *: 90%; **: 95%; ***: 99%.

Regression Analysis and Implications

The augmented specification measures how investors adjust their behavior in extreme conditions. Each market phase is captured by a dummy variable t_i ($i = 1, \ldots, 4$), indicating whether an observation falls into that regime (1) or not (0). To allow the response of actual returns to vary by regime, we also include the interaction term $\text{predt}_i \equiv \text{PredictedReturn} \times t_i$. Consequently,

- δ_i measures the intercept shift during regime i (via t_i),
- β_i measures the shift in slope on PredictedReturn within regime i (via predt_i).

Hence, when $t_i = 1$, the slope on PredictedReturn is $\beta_0 + \beta_i$, while the intercept is $\alpha + \delta_i$.

Focusing on the coefficient estimates in Table 1 for both longest and shortest maturities, several notable themes emerge:

1. **Energy Sector High-Yield Stress (t_2).** In many specifications, δ_2 is significantly positive, signifying that 2015–2016 realized returns were on average higher than during the baseline period. However, the corresponding β_2 often reduces or only modestly augments the baseline slope β_0. Interpreted jointly, this pattern indicates that investors priced in a higher overall level of returns amid high-yield stress, but the marginal impact of the Merton-implied measure on actual returns did not dramatically intensify—especially for longer-maturity bonds. In contrast, short-term issues occasionally exhibit a more pronounced interaction effect, underscoring how near-term credit-risk signals can be heightened in volatile market segments.

2. **COVID-19 Meltdown & Fed Interventions (t_3).** The t_3 indicators (δ_3 and β_3) vary across methods and maturities, but tend to show less uniform significance than t_2. When significant, a positive δ_3 suggests a baseline uplift in returns, whereas small or insignificant β_3 implies that the fundamental slope on PredictedReturn was not drastically altered. One plausible explanation is that large-scale monetary interventions quickly stabilized credit markets, thereby compressing the range of default-risk signals that might otherwise magnify the slope effect.

3. **Tighter Monetary Policy, Inflation Concerns (t_4).** Coefficients on t_4 often fail to uniformly boost the slope on predicted returns, particularly for longer-dated bonds, indicating that structural default risk in those horizons may not drastically change under moderate rate-hike cycles. However, for certain short-maturity specifications, β_4 is positive (and sometimes significant), implying an enhanced responsiveness to the structural signal under tighter liquidity conditions. As short-term bonds regularly refinance, shifts in short-rate policy and inflationary expectations can translate more quickly into price formation, making the Merton measure more salient in these intervals.

4. **Post-2012 Recovery (t_1).** The recovery period (January 2012–December 2014) generally exhibits fewer strong intercept or slope deviations (δ_1 and β_1), indicating that bond returns during this phase did not systematically deviate from the baseline relationship. Although the backdrop was one of gradual post-crisis normalization, there were fewer high-volatility episodes that might otherwise accentuate or diminish the model's predictive power.

Altogether, the significance patterns for δ_i (the intercept shifts) underscore how certain stress or tightening phases raise or lower the overall level of realized returns. The β_i estimates (the slope shifts) reveal that times of heightened market turmoil can amplify or, in some cases, attenuate the ability of the structural measure to forecast bond movements. By accommodating shifts both in the baseline (intercept) and the predictive sensitivity (slope), the extended model offers a richer view of how Merton-implied returns perform under diverse economic and policy conditions.

10 Discussion

Our model proposes a novel agent-driven approach to valuing bonds. The empirical results strongly support our theoretical conjecture.

Our findings provide a complementary perspective to the existing literature that emphasizes the role of liquidity in determining bond yields and returns. Elevated total volume is often interpreted as a proxy for enhanced market liquidity, which reduces the liquidity premium. In the existing literature, increased volume volatility can signal episodic trading frictions or investor disagreement. We show that high volume may be considered by agents as a favorable market environment, increasing demand for bonds and bond prices.

The statistical significance of our empirical findings highlights the multifaceted nature of bond return determination. The observed relationships suggest that structural bond pricing models could benefit from incorporating volume variables, especially for short-term and medium-term return forecasting and risk management applications.

In practice, traders and portfolio managers could integrate volume metrics into their bond screening and allocation strategies. Higher total volume may serve as an indicator of an investment opportunity, while volume volatility could be monitored as a potential signal of downside risk.

The persistence of these relationships in our contemporaneous and predictive regressions suggests that they may be operationally useful to anticipate short-term fluctuations in bond prices. Ultimately, these regression results confirm that volume dynamics provide meaningful information for understanding and forecasting corporate bond returns.

Practitioners, including traders and portfolio managers, may benefit from incorporating Merton-implied and liquidity metrics into analytical frameworks and investment strategies. Awareness of maturity-specific dynamics can guide more nuanced investment decisions, with longer-maturity structural metrics

serving as signals for performance potential, and awareness of microstructure constraints crucial for accurately interpreting short-term bond market behavior.

Overall, these findings advocate for integrating structural and market microstructure perspectives to enhance the robustness and precision of corporate bond return forecasting.

11 Conclusion

This study proposed and tested the theory that investors consider liquid assets when making investment decisions. We found that this is particularly true for shortest-maturity bonds. It is plausible that the shortest-maturity bonds are traded by short-horizon investors, for whom liquidity is paramount.

We further augment the classical Merton (1974) structural model by introducing a novel, volume-based perspective for liquid corporate debt valuation. Our empirical investigations demonstrate that trading volume plays an important role in capturing short- and long-horizon credit risk dynamics, providing richer informational content relative to the static balance-sheet proxies commonly employed in traditional structural models. By merging daily TRACE bond transactions with CRSP equity data, we construct comprehensive firm-level datasets for S&P 500 issuers from 2012 through Q3 2024, thereby enabling a rigorous analysis over heterogeneous market conditions.

Several key insights emerge:

1. **Volume and Volatility as Fundamental Drivers** We document that a higher total trading volume, whether measured monthly or aggregated from daily flows, is robustly associated with stronger subsequent bond returns. This aligns with the theoretical argument that liquidity provision and investor demand drive bond price formation. In contrast, elevated volume volatility exerts a negative impact on bond returns, consistent with the view that large fluctuations in trading flow precipitate greater uncertainty and dampen overall willingness to hold these instruments.

2. **Maturity-Specific Structural Modeling** Our modifications to the Merton framework reveal that a "one-size-fits-all" approach to firm leverage and asset value proxies can overlook critical differences across maturities. For longer-term debt, a relatively parsimonious measure of aggregate monthly volume retains predictive power, resonating with classical structural assumptions that default risk is governed by macro-level credit fundamentals. In contrast, shorter-dated issues respond more acutely to daily directional volume patterns, underscoring the heightened sensitivity of short-horizon bonds to immediate liquidity constraints and rollover events.

3. **Out-of-Sample Predictive Relevance** By implementing an expanding-window procedure, we update model parameters continuously and evaluate the ability of these volume-enhanced Merton models to forecast next-month bond returns. The rolling estimation confirms that volume metrics

enhance forecasting accuracy, particularly in the shorter maturity segment. This finding underscores the importance of aligning the modeling horizons and liquidity proxies with the actual trading behavior of the market participants.

4. **Context-Dependence Across Market Phases** When extending our regressions to include regime-interaction terms, the predictive power of volume measures proves stable in baseline conditions but exhibits differentiated effects during stress periods (e.g., the 2015–2016 energy sector downturn, the 2020 COVID-19 crisis). These results suggest that, while volume signals are generally valuable, they can be amplified (or attenuated) by broader macro-liquidity pressures and monetary policy regimes.

5. **Practical and Theoretical Implications** Our findings offer meaningful guidance for academics and practitioners. From a theoretical standpoint, this study calls attention to the joint influence of default risk and market microstructure in shaping corporate bond prices. Methodologically, it reaffirms the applicability of structural credit models while highlighting the need to replace coarse measures of firm "asset value" with granular, high-frequency data under certain maturity or liquidity conditions. On the practical side, portfolio managers who invest in short-dated bonds may particularly benefit from volume-based signals, and risk managers could leverage these findings in designing liquidity-aware stress scenarios or trading strategies.

Complementarity with the Liquidity Premium Theory

Our results also complement the broader liquidity premium literature, which posits that investors demand higher returns for bearing illiquidity. By demonstrating that daily trading volume forecasts short-run bond returns, particularly for short-maturity issues, our findings offer a high-frequency lens into how liquidity considerations translate into pricing. When bonds are more actively traded, the reduced cost of transacting and improved exit options lead to lower credit spreads (or higher bond prices). Conversely, higher volatility in trading volume signals greater uncertainty and weaker price performance, reinforcing the idea that tighter liquidity conditions or sudden trading frictions command a premium in required returns. Thus, by capturing the dynamic interplay of trading volume and short-term valuation, this study confirms the central role that liquidity — and its fluctuating availability — plays in corporate bond pricing.

Directions for Future Research

Further progress can be made by refining the liquidity measures (e.g., dissecting block trades vs. retail transactions, or employing bid–ask spreads), examining the interaction of microstructure with credit rating migrations, or exploring nonlinear specifications that capture more complex patterns in volume data. Additionally, calibrating a multi-period extension of our framework,

perhaps incorporating jump dynamics, would further elucidate the convergence between short-term liquidity risk and long-term structural default risk. As fixed-income markets continue to evolve—fueled by regulatory changes, growing electronic trading venues, and heightened volatility episodes—such extensions will remain vital for ensuring that structural credit models capture the realities of modern, liquidity-sensitive bond markets.

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A Longest and Shortest Maturity Statistics

Table 1: Summary Statistics of Bond Maturity (in Days)

	Shortest Maturity	**Longest Maturity**
Mean	1365.05	3539.10
Std Dev	826.99	1092.77
Kurtosis	1.42	-0.33
Min	0.00	0.00
1%	18.00	775.00
5%	248.00	1434.00
25% (Q1)	742.00	2821.00
Median (Q2)	1203.00	3659.00
75% (Q3)	1805.00	4618.00
95%	2988.00	4620.00
99%	3703.00	4620.00
Max	4620.00	4620.00

Note: This table reports the summary statistics for bond maturity in days. The shortest and longest maturity refer to the bonds with the minimum and maximum maturity dates within the dataset.

Table 2: Summary Statistics of Bond Prices

	Shortest Maturity Price	**Longest Maturity Price**
Mean	103.64	111.08
Std Dev	42.09	36.69
Kurtosis	8615.75	690.44
Min	0.00	0.00
1%	67.98	77.47
5%	85.80	89.46
25% (Q1)	99.18	99.57
Median (Q2)	100.62	105.39
75% (Q3)	104.25	116.92
95%	117.61	140.48
99%	181.25	183.41
Max	9980.00	2116.18

Note: This table presents the summary statistics for bond prices, which are reported in currency units.

Table 3: Summary Statistics of Bond Volumes

	Shortest Maturity Volume	**Longest Maturity Volume**
Mean	495,369.60	360,749.30
Std Dev	1,040,671.00	879,422.50
Kurtosis	9.89	16.04
Min	1,000.00	1,000.00
1%	2,000.00	1,000.00
5%	4,000.00	3,000.00
25% (Q1)	15,000.00	10,000.00
Median (Q2)	70,000.00	40,000.00
75% (Q3)	444,000.00	240,000.00
95%	3,000,000.00	2,000,000.00
99%	5,000,000.00	5,000,000.00
Max	5,000,000.00	5,000,000.00

Note: This table presents the summary statistics for bond trading volumes, which represent the number of bonds traded.

B Empirical Method 2

Table 1: Summary Statistics - Longest Maturity (Empirical Method 2)

Stat	**Lagged Merton Return**	**Actual Bond Returns**	**Residuals**
Mean	0.5144	0.0355	-0.4786
Std Dev	0.0185	4.5982	4.5984
Kurtosis	-0.1436	49528.4477	49523.5665
Min	0.5000	-0.9990	-1.5007
1%	0.5000	-0.2802	-0.7884
5%	0.5001	-0.1539	-0.6671
25% (Q1)	0.5005	-0.0230	-0.5475
Median (Q2)	0.5031	0.0001	-0.5121
75% (Q3)	0.5228	0.0230	-0.4860
95%	0.5524	0.1771	-0.3328
99%	0.5530	0.3795	-0.1280
Max	0.5532	1042.4010	1041.8995

Table 2: Summary Statistics - Shortest Maturity (Empirical Method 2)

Stat	**Lagged Merton Return**	**Actual Bond Returns**	**Residuals**
Mean	0.5144	0.0619	-0.4522
Std Dev	0.0185	4.9037	4.9040
Kurtosis	-0.1436	38969.1340	38965.5557
Min	0.5000	-0.9990	-1.5205
1%	0.5000	-0.2420	-0.7600
5%	0.5001	-0.0869	-0.6006
25% (Q1)	0.5005	-0.0109	-0.5363
Median (Q2)	0.5031	-0.0004	-0.5102
75% (Q3)	0.5228	0.0113	-0.4977
95%	0.5524	0.0954	-0.4146
99%	0.5530	0.3121	-0.2055
Max	0.5532	1042.4010	1041.8995

C Empirical Method 3 (With Intercept)

Table 1: Coefficient Estimates for Each Regression

	Longest		**Shortest**	
	Method 1	**Method 2**	**Method 1**	**Method 2**
Dependent Var.	BondRet_{t+1}	BondRet_{t+1}	BondRet_{t+1}	BondRet_{t+1}
Intercept	17.3901	0.1086	8.6605	0.4862**
	(0.778)	(0.794)	(1.161)	(2.212)
Merton Return$_t$	-32.2618	-0.1803	-16.0578	-0.8659*
	(-0.743)	(-0.678)	(-1.107)	(-2.026)
R^2	0.000	0.000	0.000	0.000
F-Statistic	0.5519	0.4592	1.226	4.104
Prob (F-Statistic)	0.458	0.498	0.268	0.0428
Observations	53,349	27,202	53,349	26,069

Significance levels: *: 90% significance; **: 95% significance; ***: 99% significance. Coefficients are reported with t-statistics in parentheses.