Tech Booms: From Dot-Com to Dot-AI*

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Abstract

How long will the rapid progress in AI, since the introduction of ChatGPT, continue, and how quickly will it translate into economy-wide productivity gains? To explore these questions, I draw insights from the Internet boom of the 1990s, focusing on stock market valuations and real investment dynamics in related high-tech sectors. I interpret these patterns through a two-sector model that incorporates: (1) co-integrated, non-stationary productivity processes with intra-sectoral spillovers and (2) endogenous capital accumulation. The model suggests that if the elasticity of capital supply in the high-teach sectors is sufficiently low and productivity gains from these sectors diffuse slowly to the broader economy, market valuations in these sectors peak when their rapid progress begins to decelerate.

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"History never repeats itself, but it does often rhyme."[1]

1 Introduction

The introduction of ChatGPT by OpenAI in 2022 and the subsequent artificial intelligence (AI) developments have created a lot of excitement about AI but has also generated a huge international debate on the potential impact of AI on societies and economies worldwide. The list of major questions about AI is long but in this paper, I focus on two sets of questions: Can the rapid progress in AI be sustained and bring about Artificial General Intelligence (AGI) or even Artificial Super-Intelligence (ASI) in the next few years, or will it slow down soon and we will enter another AI winter like the one in the 1970s? Does the rapid progress in AI in particular and high tech sectors more generally lead to significant economy-wide productivity improvement and if so how does long it take?

I explore these questions by drawing from the experiences of the Internet boom in the 1990s. I argue that equity price and real investment activities in the relevant high-tech sectors could yield useful insights into these questions. Similar to AI, the Internet is a general purpose technology that has revolutionized the way we live and work since the 1990s. The Internet boom was fueled by two major transformations: the massive adoption of personal computers and the spread of the World Wide Web, in the same way that ongoing AI boom being propelled by advances in high performance computing (including GPUs) and deep neu-

[1]The quote is commonly attributed to Mark Twain, but there is no definitive evidence that he actually said or wrote it.

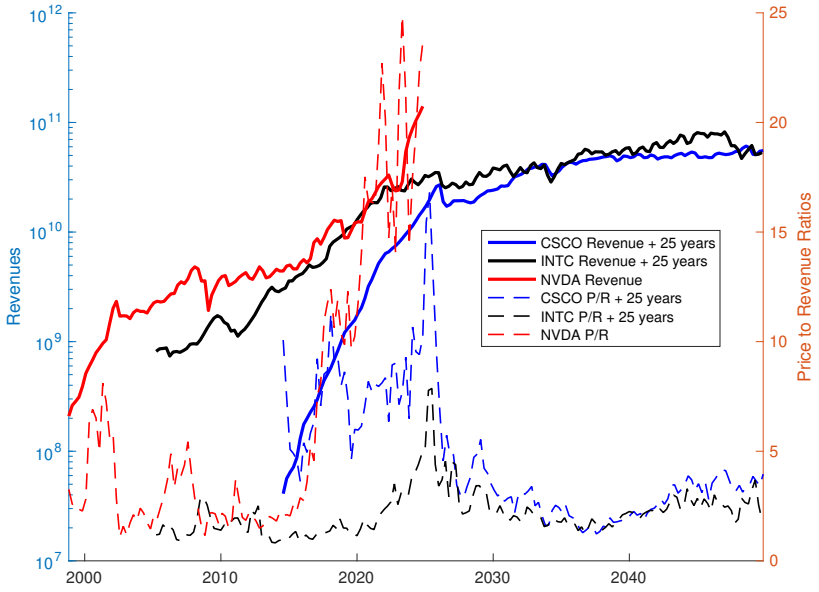


Figure 1: From Dot-Com to Dot-AI: Revenue and Market Valuation

ral networks. Figure 1 highlights the similarities by plotting the revenues of the hardware providers in the two booms, Cisco and Intel for the Dot-Com boom and Nvidia for the AI boom, with the latter shifted forward by 25 years (2015 for 1990 and 2025 for 2000). The companies experience rapid revenue growth from the beginning of the booms. The figure also shows the quarterly ratios of market value to revenue of these companies. For the Dot-Com companies, the ratios kept rising during the booms until its burst in 2000. The ratios started to decline at the end of the boom when revenue growth started to slow down. Up until the end of 2024, the AI boom has remained strong, reflected in Nvidia's robust revenue growth and a high market value-to-revenue ratio.

Similarly, Figure 2 shows measures of the capital stock (Property, Plant, and

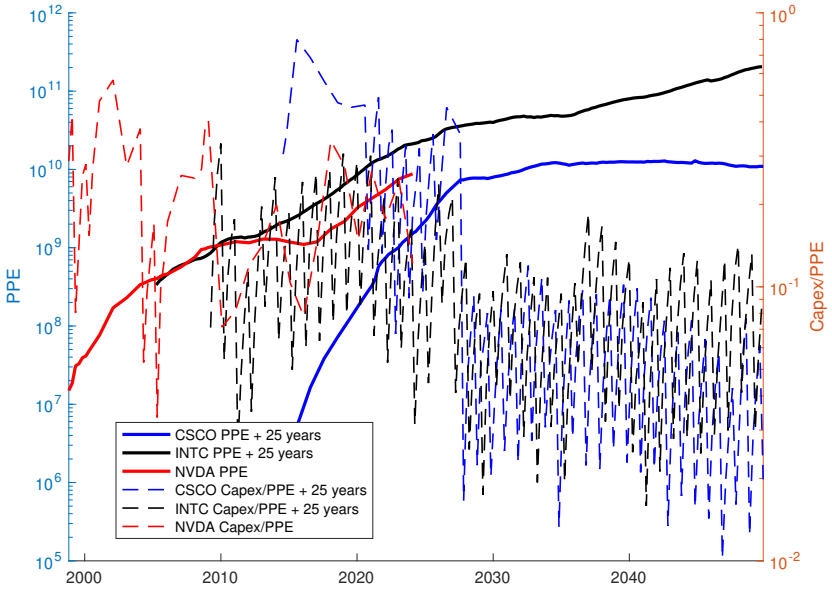


Figure 2: From Dot-Com to Dot-AI: Capital Expenditure

Equipment PPE) and investment (Capital Expenditure, or CAPEX, over PPE) for the same companies. Cisco and Intel both accumulated their capital stock rapidly from 1990s to 2000 by undertaking large capital expenditures. However, the capital accumulation slows down significantly after 2000. Nvidia's capital accumulation has accelerated since 2015.

In this paper, I build a model to interpret these facts. The model feature two sectors, a high-tech sector A, and the rest of the economy, Sector B. Sector A has a higher productivity growth rate but Sector B benefits from the productivity spillover from Sector A. The capital stock in Sector A is endogenous and can be accumulated using the final good from both sectors, subject to a capital adjustment cost. The economy is populated by two types of agents, optimists and

pessimists. They share the same preferences and endowments of the final good and labor, but they might differ in their beliefs about the process of productivity growth in Sector A. The optimists expect that high productivity growth periods (tech booms) last longer and arrive more frequently than the pessimists do. The agents can trade in shares of the capital stock in Sector A and additional financial contracts.

I begin the analysis of the model with the special case in which the agents are identical, including their beliefs. On the balanced growth path, I find that the price to dividend ratio of Sector A equity is increasing in the Sector A productivity growth rate if the supply elasticity of Sector A's capital stock is less the intertemporal elasticity of substitution of the agents. Outside of the balance growth path, I assume that the productivity growth rate in Sector A follows a two-state Markov chain with a high value state (tech boom) and low value state (normal time). If the spillover is relatively slow, similar to the slow diffusion of influential technological innovations documented in Kalyani et al. (2021), the price-to-dividend ratio increases when the productivity growth rate switches to the high value state from the low value state. As the growth rate remains high, the ratio keeps increasing. The ratio drops when the growth rate transitions back to the low value state, and keeps decreasing towards the pre-boom level, as the growth rate remains low. However, the dynamics of the ratio differ significantly when the spillover is relative fast. The price to dividend ratio actually peaks after the growth rate transitions back to the low value state. The dividend keeps decreasing but the share price keeps increasing beyond the switching time. This is because

the agents' income from Sector B grows quickly due to productivity spillover from Sector A so they have resources to buy Sector A shares and accumulate capital in this sector.

Next, I allow the agents to differ in their beliefs. The pessimists have the same beliefs as in the homogeneous beliefs version of the model. However, optimists are more optimistic and believe that the high state arrives more frequently and lasts longer than the pessimists expect. This version of the model leads higher share price and even higher price-to-dividend ratio in the high-tech sector than in the homogenous beliefs version. The latter is due to lower dividend and faster capital accumulation in Sector A. When the optimists can issue financial contracts to raise fund from the pessimists, the effects are amplified. However, the drops in the share price and price-to-dividend ratio are larger when the state transitions to the low value one.

The dynamics of asset price and capital accumulation described above (when the productivity spillover is slow) resembles the ones observed during the Dot-Com boom in the 1990s. In the current Dot-AI boom, amid the real and rapid progress in AI, AI related stocks have been experiencing significant appreciation and have remained at elevated levels. If the model is also applicable to the current Dot-AI boom, it suggests that the rapid advancements in AI are expected to continue for the foreseeable future. This expectation is consistent with the growth potentials of AI presented by industry experts. For generative AI and LLMs, while people raise concerned about the progress in LLMs might hit a wall and slows down (partly due to data limitation in the pre-training phrase as analyzed in Vil-

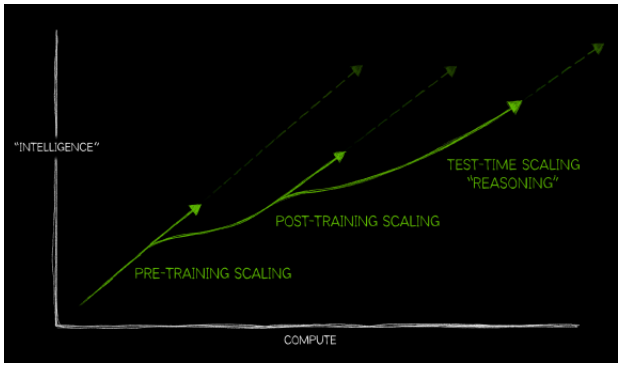


Figure 3: AI Growth Potentials: Generative AI

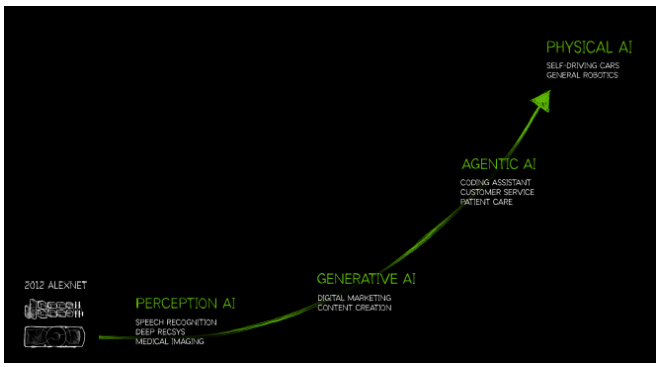


Figure 4: AI Growth Potentials: All AI areas

lalobos et al. (2024)), there is room for significant improvement in post-training and reasoning using more computing resources (Figure 3 from Nvidia's corporate presentation and Akyurek et al. (2024)). Beyond generative AI, Figure 4 show that there is more progress to be made in agentic AI and physical AI.

The paper is related to several strands of literature. First, it is part of the effort trying to make sense of the ongoing AI boom and how impactful and world-changing it could be. Not surprisingly, this literature has been expanding very rapidly. Brynjolfsson et al. (2024) propose a comprehensive and impressive agenda

for this literature. Second, the paper is related to the literature on asset price or capital accumulation dynamics surrounding major macroeconomic trends including the 1990s Internet boom (Shiller (2000), Cao and L'Huillier (2018)), housing boom and leverage cycles in 2000s (Kaplan et al. (2020), Cao et al. (2023b)). I contributes to this literature by explicitly modeling how the sector of interest interact with the rest of the economy and highlight the importance of the elasticity of capital supply and the speed of spillover. Lastly, at the technical level, the two-sector structure in dynamic general equilibrium also appears in Eberly and Wang (2009) and Cochrane et al. (2007). I solve the model using the novel algorithm and general GDSGE implementation from Cao et al. (2023a). The extension with belief heterogeneity and leverage is similar to the model in Fostel and Geanakoplos (2008), Geanakoplos (2010), Simsek (2013), and Cao (2018). But these papers only studies endowment economies without endogenous capital accumulation in the present paper.

The rest of the paper is organized as followed. Section 2 presents the general model of a two-sector economy. Section 3 defines Markov equilibrium including the balanced growth path and puts forth a solution method. Section 4 and Section 5 analyzes the dynamics of tech booms under homogenous beliefs and heterogeneous beliefs respectively. Section 6 concludes.

2 A Two-Sector Economy

Consider a single consumption (final) good economy in infinite horizon with infinitely-lived agents. Time runs from $t = 0$ to ∞. In each time, there are S possible exogenous states (or equivalently exogenous shocks)

$$s \in \mathcal{S} = \{1, 2, \ldots, S\}.$$

The states determine the aggregate productivity in the economy.

The evolution of the economy is captured by the realizations of the shocks over time, $s^t = (s_0, s_1, \ldots, s_t)$, which denotes the history of realizations of shocks up to time t. We assume that the shocks follow a Markov process with the transition probabilities $\pi(s, s')$.

Two Sectors: There are two sectors, A and B, in the economy. Both sectors general the final good which could be used for consumption and investment (the price of the final good is normalized to 1). The first sector is the high tech sector with exogenous productivity A_t and endogenous aggregate capital stock K_t. The output in this sector is produced using a standard constant-return-to-scale production function:

$$F(K_t, A_t L_t),$$

where L_t is the labor input and A_t is labor productivity. The productivity follows a random walk:

$$\log A_{t+1} = \log A_t + g_{t+1}. \tag{1}$$

The sector's capital stock accumulation is subject to convex adjustment cost:

$$K_t = (1-\delta) K_{t-1} + \Phi (I_t, K_{t-1}),$$

where I_t is investment using the final good.

The remaining sector, Sector B, features exogenous endowment B_t. The dynamics of the endowment depends on A_t through spillover

$$\log B_{t+1} = \log B_t + \rho \log \frac{A_t}{B_t}, \tag{2}$$

where $\rho \in (0,1)$ is the spillover speed.

During period t, a representative final-good producer in sector A produces final good by combining capital K_t, rented at the rate d_t, and labor L_t, at wage rate w_t, using the production above to maximize profit:

$$\max_{K_t, L_t} F (K_t, A_t L_t) - d_t K_t - w_t L_t. \tag{3}$$

At the end of period t, a representative capital-good producers in Sector A produces new units of capital for $t+1$ production by combining old units of capital K_t, purchased at market price q_t and new investment I_t, using the capital adjustment cost function above. The new units of capital is sold at market price q_t^n. The capital good producer chooses inputs to maximize profit:

$$\max_{K_t, I_t} q_t^n \left((1-\delta) K_{t-1} + \Phi (I_t, K_{t-1}) \right) - I_t - q_t K_{t-1} \tag{4}$$

I assume that the production function and adjustment cost functions have standard properties.

Assumption 1. *F and Φ have constant-returns to scale and are strictly increasing and weakly concave in their inputs.*

Two Agent Types: There are two types of agents, optimists and pessimists, in this economy with a continuum of measure 1 of identical agents in each type

$$h \in \mathcal{H} = \{O, P\}.$$

In contrast to the standard rational expectation literature, I assume that the agents do not have the perfect estimate of the transition matrix π. The agents might differ in the beliefs on evolution of the exogenous state of the economy. Each of them has their own estimate (belief) of the transition matrix, π^h.

Agents maximize their inter-temporal expected utility with the per period utility CRRA function

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

At any time t, the agents can buy shares of the new capital stock at prices q_t^n and sell the shares of old capital stock to the capital good producer at prices q_t. They also earn dividend (rent) d_t from the old shares, before selling, to the final-good producers and supply labor inelastically at wage rate w_t. Besides the shares, the agents have access to a financial asset in zero net supply at price p_t, which pays

off $\Psi_{t+1}(s_{t+1})$ depending on the realization of next-period state s_{t+1}. For example

$$\Psi_{t+1}(s_{t+1}) = m + (1-m)\frac{q_{t+1}(s^{t+1}) + d(s_{t+1})}{\min_{\tilde{s}\in\mathcal{S}}\left\{q_{t+1}(s^t,\tilde{s}) + d(\tilde{s})\right\}} \tag{5}$$

allows buyers (lenders) of the financial asset to share some of the upside with sellers (borrowers). The sellers of the financial asset needs to pledge capital shares as collateral to make sure that the return from their shares exceeds the promised pays-off from the financial asset.

Agent h takes the sequences of prices $\{q_t^n, q_t, p_t, d_t, w_t\}$ as given and chooses consumption c_t^h and capital holding k_{t+1}^h to solve

$$\max_{\left\{c_t^h, k_t^h, \phi_t^h\right\}} \mathbf{E}_0^h\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t^h\right)\right] \tag{6}$$

subject to the budget constraint

$$c_t^h + q_t^n k_t^h + p_t\phi_t^h \leq e^h B_t + w_t l^h + \left(q_t + d_t\right)k_{t-1}^h + \Psi_t\phi_{t-1}^h, \tag{7}$$

the short-sale constraint on capital

$$k_t^h \geq 0, \tag{8}$$

and the leverage constraint

$$\phi_t^h\Psi_{t+1}(s_{t+1}) + k_t^h\left(q_{t+1}(s^{t+1}) + d(s_{t+1})\right) \geq 0, \tag{9}$$

for all histories s^{t+1} that succeeds the history s^t at time t.

The aggregate labor supply is given by $L = \sum_h l^h$ and we assume that $\sum_h e^h = 1$. In this environment, I define an equilibrium as follows.

Definition 1. A *sequential competitive equilibrium* for an economy with initial asset holdings

$$\left\{k_0^h, \phi_0^h\right\}_{h\in\mathcal{H}}$$

and initial shock s_0 is a collection of consumption, real asset, and bond holdings and prices in each history s^t,

$$\left(\left\{c_t^h\left(s^t\right), k_{t+1}^h\left(s^t\right), \phi_{t+1}^h\left(s^t\right)\right\}_{h\in\mathcal{H}}, q_t^n\left(s^t\right), q_t\left(s^t\right), p_t\left(s^t\right), d_t\left(s^t\right), w_t\left(s^t\right)\right)$$

satisfying the following conditions:

i) The markets for real asset, bond, and final good in each period clear:

$$\sum_{h\in\mathcal{H}} k_t^h\left(s^t\right) = K_t \tag{10a}$$

$$\sum_{h\in\mathcal{H}} \phi_t^h\left(s^t\right) = 0 \tag{10b}$$

$$\sum_{h\in\mathcal{H}} c_t^h\left(s^t\right) + I_t(s^t) = B_t + F(K_t, A_t L). \tag{10c}$$

ii) For each agent h, $\left\{c_t^h\left(s^t\right), k_t^h\left(s^t\right), \phi_t^h\left(s^t\right)\right\}$ solves the individual maximization problem subject to the budget constraint (7), and the no-short sale and collateral constraints, (8) and (9).

iii) The aggregate capital, investment (and labor) $\{K_t(s^t), I_t(s^t)\}$ solves the

profit maximization problems of the representative final good and capital producer, (3) and (4).

3 Markov Equilibrium

The productivity dynamics (1) and (2) imply that the model is non-stationary. To obtain stationarity and solve the model numerically, I use scale variables by A_t.

3.1 Scaled Variables

We use the notation $\hat{x}_t = \frac{x_t}{A_t}$ for any variable x(except for the Lagrange multipliers discussed below). For example, $\hat{B}_t = \frac{B_t}{A_t}$ is the productivity of Sector B relatively to Sector A. The spillover dynamics (2) implies

$$\log \hat{B}_{t+1} = (1-\rho) \log \hat{B}_t - g_{t+1},$$

which is a stationary process.

Similarly, the agents' budget constraint (7) becomes

$$\hat{c}_t^h + q_t^n \hat{k}_t^h + p_t \hat{\phi}_t^h \leq e^h \hat{B}_t + \hat{w}_t l^h + (q_t + d_t) \frac{A_{t-1}}{A_t} \hat{k}_{t-1}^h + \frac{A_{t-1}}{A_t} \hat{\phi}_{t-1}^h \tag{11}$$

and the leverage constraints (9) become

$$\hat{\phi}_t^h \Psi_{t+1}(s_{t+1}) + k_t^h \left(q_{t+1}(s^{t+1}) + d(s_{t+1}) \right) \geq 0.$$

Let λ_t^h and $\mu_t^h(s_{t+1})$ denote the Lagrange multipliers on agents h's no short-selling and collateral constraints. The first-order-conditions of the agents' for capital and financial asset holdings are:

$$q_t^n U_h'\left(c_t^h\right) = \beta \mathbb{E}_t\left[\left(q_{t+1} + d_{t+1}\right) U_h'(c_{t+1}^h)\right] + \lambda_t^h + \sum_{s_{t+1}} \mu_t^h(s_{t+1})\left(q_{t+1}(s^{t+1}) + d(s_{t+1})\right)$$

and

$$p_t U_h'\left(c_t^h\right) = \beta \mathbb{E}_t\left[U_h'(c_{t+1}^h)\Psi_{t+1}(s_{t+1})\right] + \sum_{s_{t+1}} \mu_t^h(s_{t+1}).$$

Under CRRA utilities, these first order conditions become

$$q_t^n\left(\hat{c}_t^h\right)^{-\sigma} = \beta \mathbb{E}_t^h\left[\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\left(q_{t+1} + d_{t+1}\right)\left(\hat{c}_{t+1}^h\right)^{-\sigma}\right] + \hat{\lambda}_t^h + \sum_{s_{t+1}} \hat{\mu}_t^h(s_{t+1}) \min_{s^{t+1}|s^t}\left(q_{t+1}(s^{t+1}) + d(s_{t+1})\right) \tag{12}$$

and

$$p_t\left(\hat{c}_t^h\right)^{-\sigma} = \beta \mathbb{E}_t^h\left[\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\left(\hat{c}_{t+1}^h\right)^{-\sigma}\right] + \sum_{s_{t+1}} \hat{\mu}_t^h(s_{t+1}), \tag{13}$$

where

$$\hat{\lambda}_t^h = \lambda_t^h A_t^{-\sigma} \text{ and } \hat{\mu}_t^h = \mu_t^h A_t^{-\sigma}.$$

The first-order conditions for the capital-good producers are

$$\begin{aligned} 0 =& q_t^n \left(1 - \delta + \Phi_2 \left(I_t, K_{t-1}\right)\right) - q_t^o \\ 0 =& q^n \Phi_1 \left(I_t, K_{t-1}\right) - 1. \end{aligned}$$

which could be rewritten using the normalized variables:

$$\begin{aligned} 0 =& q_t^n \left(1 - \delta + \Phi_2 \left(\hat{I}_t, \hat{K}_{t-1} \frac{A_{t-1}}{A_t}\right)\right) - q_t \\ 0 =& q^n \Phi_1 \left(\hat{I}_t, \hat{K}_{t-1} \frac{A_{t-1}}{A_t}\right) - 1. \end{aligned}$$

The corresponding evolution of the capital stock in the tech sector with normalized variables is:

$$\hat{K}_t = \left(1 - \delta\right) \hat{K}_{t-1} \frac{A_{t-1}}{A_t} + \Phi \left(\hat{I}_t, \hat{K}_{t-1} \frac{A_{t-1}}{A_t}\right).$$

Similarly, the first order conditions for the final good producer are

$$\begin{aligned} 0 =& F_1 \left(K_{t-1}, A_t L_t\right) - d_t \\ 0 =& A_t F_2 \left(K_{t-1}, A_t L_t\right) - w_t, \end{aligned}$$

with the normalized versions:

$$d_t = F_1\left(\hat{K}_{t-1}\frac{A_{t-1}}{A_t}, L_t\right) \tag{14a}$$

$$\hat{w}_t = F_2\left(\hat{K}_{t-1}\frac{A_{t-1}}{A_t}, L_t\right) \tag{14b}$$

3.2 Balanced Growth Path

First, I characterize the balanced growth paths (BGPs) with constant productivity growth rate $\bar{G} = \exp(\bar{g})$:

$$\log A_{t+1} = \log A_t + \bar{g}.$$

On balanced growth paths, normalized variables are constant: $x_t \equiv \bar{x}$. Let $\bar{IK}$ denote the BGP value of the investment to capital stock ratio:

$$\bar{IK} = \frac{I_t}{K_t}.$$

The capital accumulation equation becomes

$$\bar{G} = (1-\delta) + \Phi\left(\bar{G}\bar{IK}, 1\right), \tag{15}$$

which uniquely pins down $\bar{IK}$ as a function of the deterministic growth rate $\bar{G}$.

Because Φ has constant returns-to-scale: $\Phi_i\left(\hat{I}, \hat{K}\frac{1}{\bar{G}}\right) = \Phi_i\left(\bar{G}\bar{IK}, 1\right)$ for $i \in \{1, 2\}$. Therefore,

$$
\begin{aligned}
0 =& \bar{q}^n \left(1 - \delta + \Phi_2 \left(\bar{G}\bar{I}\bar{K}, 1\right)\right) - \bar{q} \\
0 =& \bar{q}^n \Phi_1 \left(\bar{G}\bar{I}\bar{K}, 1\right) - 1.
\end{aligned}
$$

The second equation pins down the BGP value of new shares $\bar{q}^n$. Combining with the first equation pins down the value of old shares $\bar{q}$.

Without uncertainty, agents have identical beliefs and hence there is no trading. Thus the multipliers in the Euler equations (12) and (13) are zeros. (12) becomes

$$
\bar{q}^n = \beta \bar{G}^{-\sigma} \left(\bar{q} + \bar{d}\right). \tag{16}
$$

This equation gives us the BGP value of the dividend-to-price ratio: $\frac{\bar{d}}{\bar{q}}$. Lastly, the normalized capital stock is pinned down by the marginal product of capital:

$$
F_1 \left(\frac{\bar{K}}{\bar{G}}, L\right) = \bar{d}.
$$

Proposition 1. *Assume that*

$$
\underbrace{\max \frac{\Theta'(x)}{-x\Theta''(x)}}_{\textit{elasticity of capital supply}} \leq \underbrace{\frac{1}{\sigma}}_{\textit{intertemporal elasticity of substitution}}, \tag{17}
$$

where $\Theta(x) \equiv \Phi\left(x, 1\right)$. *The BGP price-to-dividend ratio is strictly increasing in* $\bar{G}$.

Proof. Appendix A □

When the adjustment cost function is iso-elastic, as assume in Jermann (1998) and Guvenen (2009):

$$\Theta\left(\frac{I_{t+1}}{K_{t+1}}\right) = a_1\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1-1/\xi} + a_2, \tag{18}$$

Inequality (17) becomes

$$\sigma \leq \frac{1}{\xi}.$$

When $\xi \to 0$, the supply of asset is essentially fixed. When $\xi \to \infty$, flexible supply. In the limit

$$\Theta\left(\frac{I}{K}\right) = a_1\frac{I}{K} + a_2$$

which implies that the capital stock is fully flexible and (17) is violated. In this case, the BGP price-to-dividend ratio is strictly decreasing in $\bar{G}$. Indeed, in this case, Euler equation (16) becomes

$$\bar{G}^{\sigma} = \beta\frac{\bar{q}^n}{\bar{q}}\left(1 + \frac{\bar{d}}{\bar{q}}\right)$$

where $\bar{q}^n = a_1$ and $\bar{q} = 1 - \delta + a_2$, which implies that $\frac{\bar{d}}{\bar{q}}$ is increasing in $\bar{G}$, and hence, $\frac{\bar{q}}{\bar{d}}$ is decreasing in $\bar{G}$.

3.3 Markov Equilibrium

For the discussions below, I refer to the last two terms in the right hand side of the budget constraint (7)

$$w_t^h = \frac{(q_t + d_t) k_t^h + \Psi_t \phi_t^h}{(q_t + d_t) K_t^o}$$

as *financial wealth*, as opposed to *non-financial wealth* $B_t e^h + w_t l^h$, since the later is not tradable nor pledgeable. Because of the collateral constraint and the short-sale constraint, $w_t^h \geq 0$ for all h, t and s^t.

Define the *financial wealth share*, or wealth share for short, of each agent type as

$$\omega_t^h = \frac{(q_t + d_t) k_t^h + \Psi_t \phi_t^h}{(q_t + d_t) K_t}. \tag{19}$$

Let $\omega_t\left(s^t\right) = \left(\omega_t^O\left(s^t\right), \omega_t^P\left(s^t\right)\right)$ denote the *wealth distribution*. Then in equilibrium $\omega_t\left(s^t\right)$ always lies in the one-dimensional simplex

$$\Omega = \left\{ \left(\omega_t^h\right)_{h \in \mathcal{H}} : \omega_t^h \geq 0 \text{ and } \omega_t^O + \omega_t^P = 1 \right\}.$$

ω_t^h's are non-negative because the collateral constraint (9) requires the value of each agent's asset holding to exceed the liabilities from their past borrowings. The sum of ω_t^h equals 1 because of the real asset market clearing and financial market clearing conditions. Using the definition of wealth distribution, we define a Markov equilibrium as follows.

Definition 2. A *Markov equilibrium* is a SCE in which the prices of real and financial assets - q_t, q_t^n, p_t - and the normalized allocation of consumption, real asset and

bond, $\hat{c}_t^h, \hat{k}_t^h, \hat{\phi}_t^h$ are (history independent) functions of the exogenous shock s_t and the endogenous financial wealth distribution $\omega_t\left(s^t\right)$.

This Markov equilibrium definition features the endogenous state variable ω_t that depends on equilibrium prices and allocation. This type of endogenous state variable has a long tradition in macroeconomics and finance (Duffie et al. (1994), Brunnermeier and Sannikov (2014), and many others). More importantly, as noted in Cao et al. (2023a), its law of motion is implicit, not explicit. Indeed, ω_{t+1} depends on q_{t+1}which itself depends on ω_{t+1}:

$$\omega_{t+1}^h = \frac{\left(q_{t+1} + d_{t+1}\right)\hat{k}_t^h + \Phi_{t+1}\phi_t^h}{\left(q_{t+1} + d_{t+1}\right)\hat{K}_t} = \frac{\hat{k}_t^h}{\hat{K}_t} + \frac{\hat{\phi}_t^h}{\left(q_{t+1}(s_{t+1}, \omega_{t+1}) + d_{t+1}(s_{t+1})\right)\hat{K}_t}.$$

Therefore, it is not explicitly defined even if the current policy variables $\hat{k}_t^h, \phi_t^h, \hat{K}_t$ are known. To tackle this challenge, we use the simultaneous policy function and transition function iterations proposed in Cao et al. (2023a) and impose the consistency equations above during the time iteration. The algorithm and GDSGE implementation are presented in Appendix B.

4 Tech Boom Dynamics: Homogenous Beliefs

In this section, I assume that the agents are identical in beliefs and their labor supply in the high-tech sector and endowment in the low-tech sector are proportional to their initial holding of capital. The following proposition shows that the equilibrium allocation in this case is the same as the solution to the social planner's problem.

Proposition 2. *Assume that agents has the same beliefs, $\pi^P = \pi^O$, and the initial financial propositions are proportional to their non-financial endowments*

$$\frac{k^O_{-1}}{k^P_{-1}} = \frac{\phi^O_{-1}}{\phi^P_{-1}} = \frac{e^O}{e^P} = \frac{l^O}{l^P}.$$

Starting from initial capital stock K_0, there exists a unique equilibrium in which the aggregate variables ($K_t, I_t, c^O_t + c^P_t$, etc.) are given by the planner's problem

$$\max_{\{c_t, I_t, K_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right]$$

s.t.

$$c_t + I_t \leq B_t + F\left(K_t, A_t L\right)$$

and

$$K_{t+1} = (1-\delta)K_t + \Phi\left(I_t, K_t\right).$$

Proof. Appendix A □

To illustrate equilibrium properties, I assume that s follows a two-state Markov process with low growth or high growth rate in the tech sector:

$$g \in \{g_L, g_H\},$$

with the transition matrix

$$\pi^P = \pi^O = \begin{bmatrix} 1 - p_{LH} & p_{LH} \\ 1 - p_{HH} & p_{HH} \end{bmatrix}$$

We pick $g_L = 0.01$ and $g_H = 0.04$, in line with the estimates of potential growth increase due to AI and other automation technologies in Chui et al. (2023).

The total labor supply in the high tech sector is L_A and divided between the optimists and pessimists

$$l^O = 0.1L_A \text{ and } l^P = 0.9L_A.$$

Total endowment in sector B is 1 with $e^O = 0.1$ and $e^P = 0.9$. In the steady-state the total output produced in the high tech sector relative to the other sector is

$$\frac{A}{B}\bar{K}^{\alpha}L_A^{1-\alpha}.$$

We choose L_A so that Sector A's output is around 10% of the total output.

I set the spillover factor ρ in (2) at

$$\rho = 0.025, \tag{20}$$

i.e., it takes around 50 years for the productivity gain in Sector A to diffuse fully to Sector B, consistent with the estimate of the diffusion speed of influential technological innovations in Kalyani et al. (2021).I show below that this level of spillover

generates reasonable asset price dynamics. In general, the spillover speed plays an important role in the dynamics of asset prices.

In this section, I assume that the agents have exactly the same beliefs:

$$p^O_{LH} = p^P_{LH} = 0.05$$
$$p^O_{HH} = p^P_{HH} = 0.80.$$

This transition matrix implies that tech booms ($g = g_H$) lasts on average for 5 years and happens every 10 years.

The policy functions for Markov equilibrium are presented in Appendix B. In this section, we explore how the equilibrium variables respond to a sequence of growth shocks g_t. I assume that g_t stays at g_L for a long time ($t = 1$ to 100). At $t = 101$, it switches to the high state (tech booms) and remains there until $t = 115$ before switching back to the low state. Figure 5 shows the equilibrium dynamics of the price-to-dividend ratio, q_t/d_t, over time. The ratio jumps up immediately when productivity increases but evolve non-monotonically over time. Notably, the ratio peaks at $t = 139$, more than 10 periods after productivity growth comes down to g_L.

To better understand the dynamics, the upper panel of Figure 6 plots q_t and d_t separately. Dividend d_t moves more slowly than share price q_t. This is because d_t is determined by the normalized capital stock (14a) which adjust slowly (middle panel) due to the capital adjustment cost. At $t = 101$, q_t jumps up while d_t remains close to the pre-switch level. This leads to a jump in the price-to-dividend ratio.

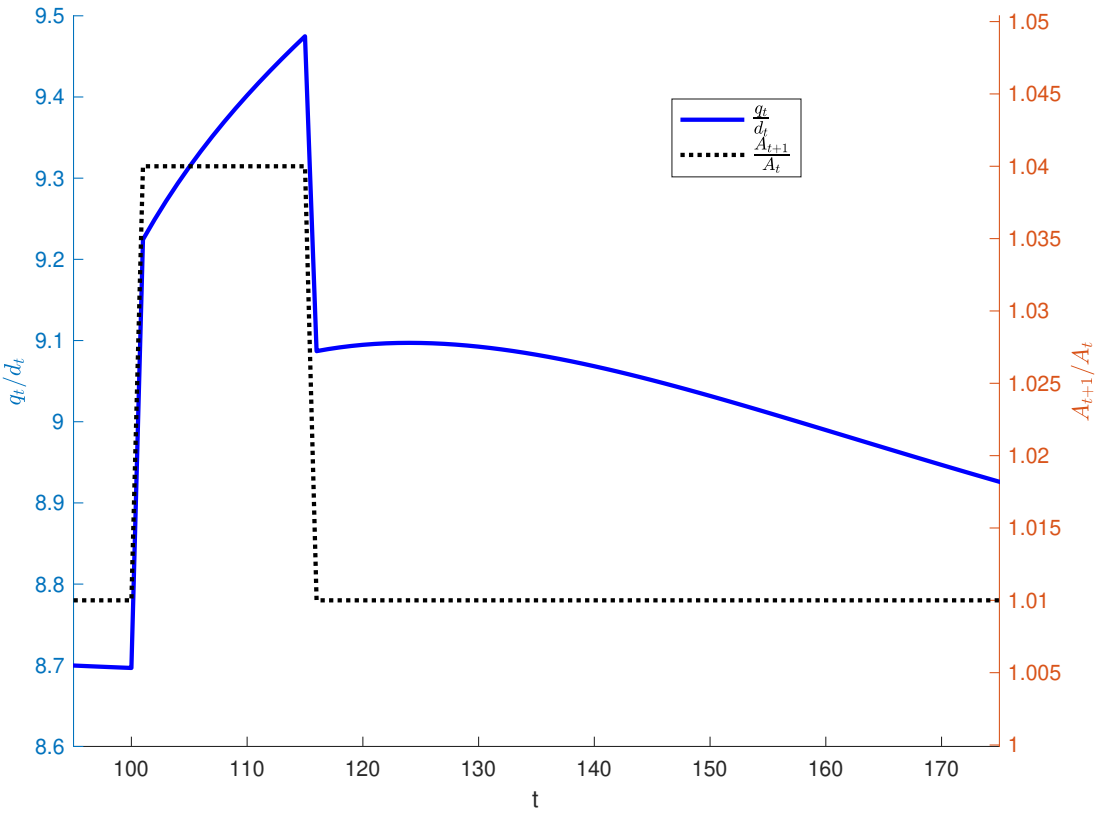


Figure 5: Price-to-Dividend Ratio Dynamics.

Afterwards, the ratio keeps increasing as q_t increases faster than d_t does.

However, the ratio declines as d_t starts to increase to reflect the improvement in high-tech productivity (and hence the decline in the normalized capital stock). Dividend peaks when growth rate switches from g_H to g_L at $t = 116$ but q_t keeps going up for more than 10 periods. Increasing q_t reflects the persistent higher level of relative productivity in the high-tech sector after the switch, which also implies high levels of investment (bottom panel). The joint dynamics of q_t and d_t lead to substantial increase in the price-to-dividend ratio with a later peak beyond the switching time $t = 116$.

To examine the importance of productivity spillover, we solve and simulate the model for a higher value of $\rho = 0.10$, instead of the benchmark value 0.025. Figure 7 shows that the price to dividend ratio peaks much later after switches from g_L to g_H. Figure 8 shows that this is because asset price q_t keeps appreciating even as growth rate slows down, while capital accumulates and d_t declines at similar speeds as when $\rho = 0.025$. The scaled budget constraint (11) suggests that it is because the agents' endowment from Sector B grows faster and the agents can use some of the endowment to accumulate capital shares in Sector A.

5 Tech Boom Dynamics: Heterogenous Beliefs

Major tech booms are often accompanied by significant disagreement on the magnitude and the economy-wide impact of these booms. For example, in the ongoing AI boom, leading AI researchers disagree on whether GPT LLMs like ChatGPT are

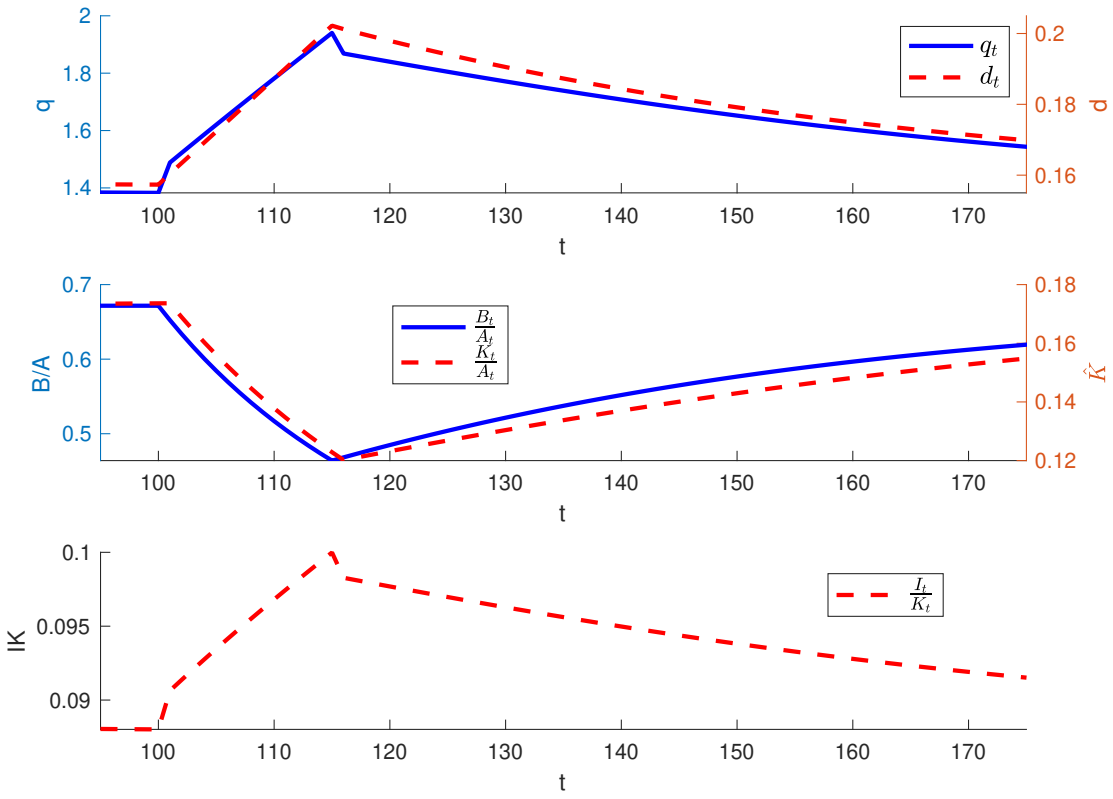


Figure 6: Price, Dividend, Capital and Investment Dynamics.

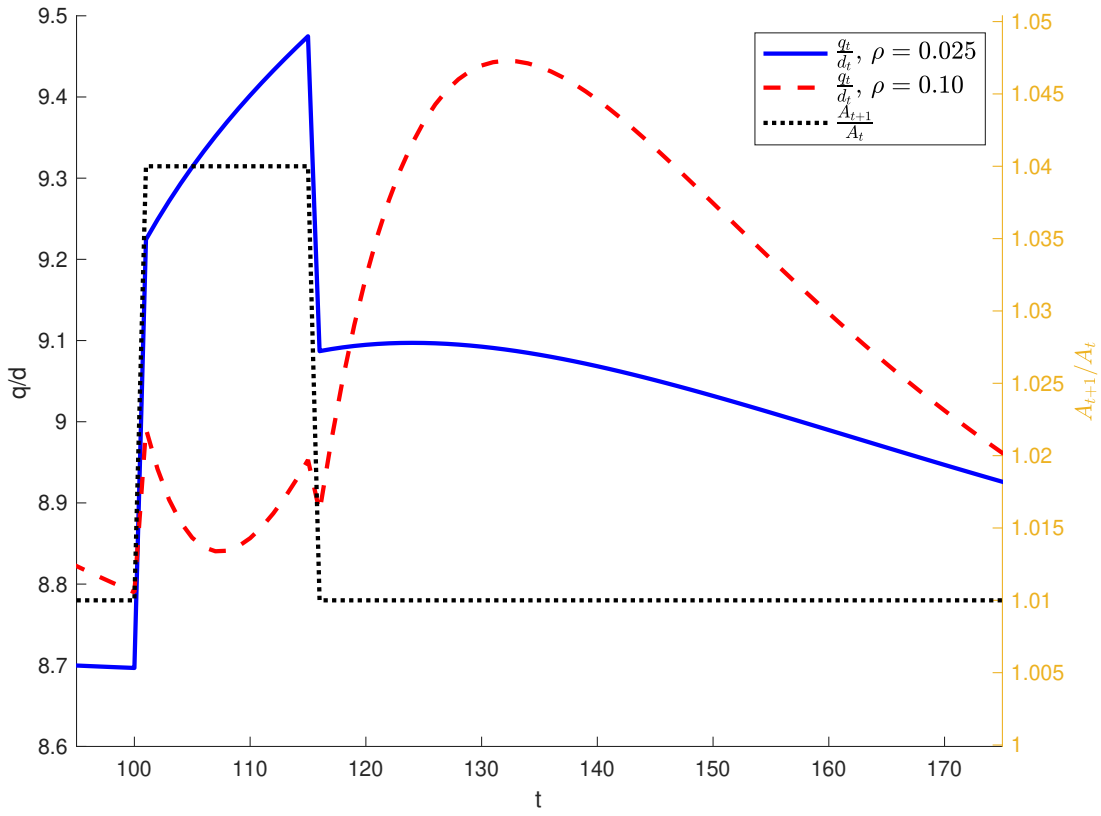


Figure 7: Price-to-Dividend Ratio Dynamics.

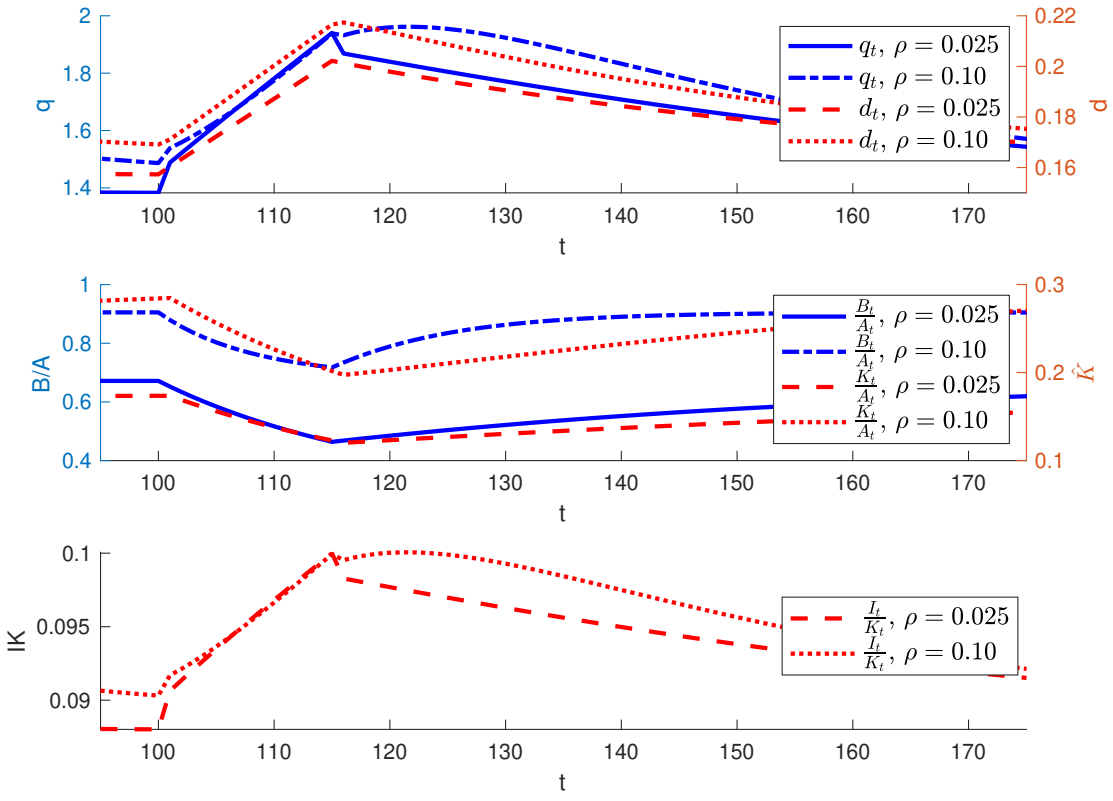


Figure 8: Price, Dividend, Capital and Investment Dynamics.

promising paths towards AGI. Geoffrey Hinton, who won the 2024 Nobel Prize for his contribution in AI, argues that LLMs already have significant cognitive abilities and can be improved relatively quickly to achieve AGI. While, Yan Lecun, another foremost AI pioneer, asserted that a significant gap exists in achieving human-level AI, and current LLMs are not even at cat-level intelligence, indicating a need for substantial advancements in AI systems' understanding of the physical world and planning abilities. On the economy-wide impact, Acemoglu (2024) estimates that new advances in AI will lead to only 0.66% increase in total factor productivity for the U.S. over 10 years. Goldman Sachs (2023) is much more optimistic with 4% increase in GDP over the same time span.

In this section, I examine how belief heterogeneity affects the dynamics of the booms. To model belief heterogeneity, we assume that the agents to differ in their subjective transition matrix:

$$p^P_{LH} = 0.05 < p^O_{LH} = 0.10$$
$$p^P_{HH} = 0.80 < p^O_{HH} = 0.95.$$

The pessimists' transition matrix is the same as in the previous section. But the optimists expect that tech booms last 20 years on average (instead of the pessimists' 5 years average) and takes on average 10 years to emerge (compared to the pessimists' 20 years average).

In the next subsection we consider a special case of the general model in Section 2 in which the optimistic agents fund their purchase of high-tech equity using

their own internal fund. In Subsection 5.2, we allow the optimists to raise funds from the pessimists to fund their purchase.

5.1 Self-Funded Investment

First, I consider the special case in which the optimists cannot borrow from the pessimists to invest in high-tech equity. This is consistent with the fact that during the ongoing AI boom, many big tech firms reinvest the revenues from their AI related products into new AI businesses.

The agents' budget constraints (7) become

$$c_t^h + q_t^n k_t^h \leq e^h B_t + w_t l^h + (q_t + d_t)\, k_{t-1}^h. \tag{21}$$

Without borrowing, the optimists can still accumulate high-tech equity from their labor earning $w_t l^O$ and dividends and existing equity holding plus dividend: $(q_t + d_t)k_{t-1}^O$. Figure 9 compares the dynamics of price-to-dividend ratio in this model (dashed dotted line) to the previous model with homogenous beliefs (solid blue line). The price-to-dividend ratio is higher under heterogenous beliefs. But the magnitude of the relative changes given the same sequence of productivity shocks is similar. Figure 10 helps explain why this is the case. The higher price-to-dividend ratio comes both from higher share price, q_t, and lower dividend, d_t. But the latter plays are more important role quantitatively.

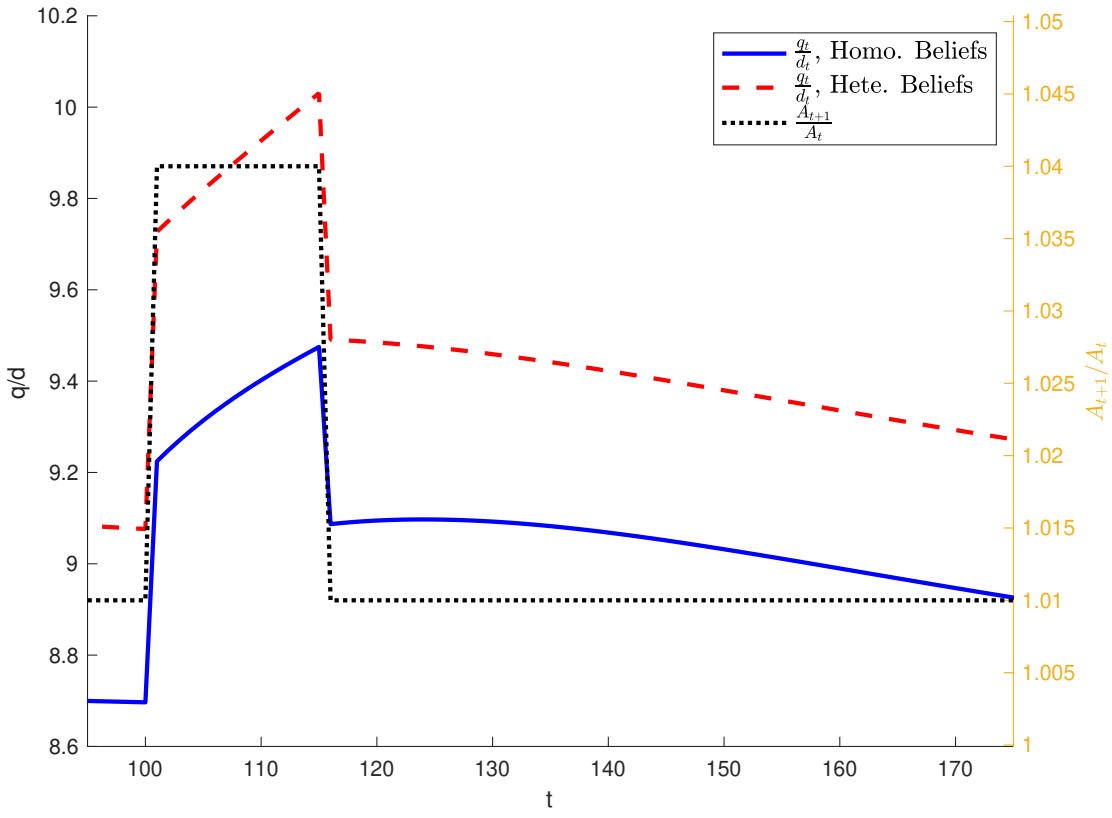


Figure 9: Price-to-Dividend Ratio Dynamics with Beliefs Heterogeneity

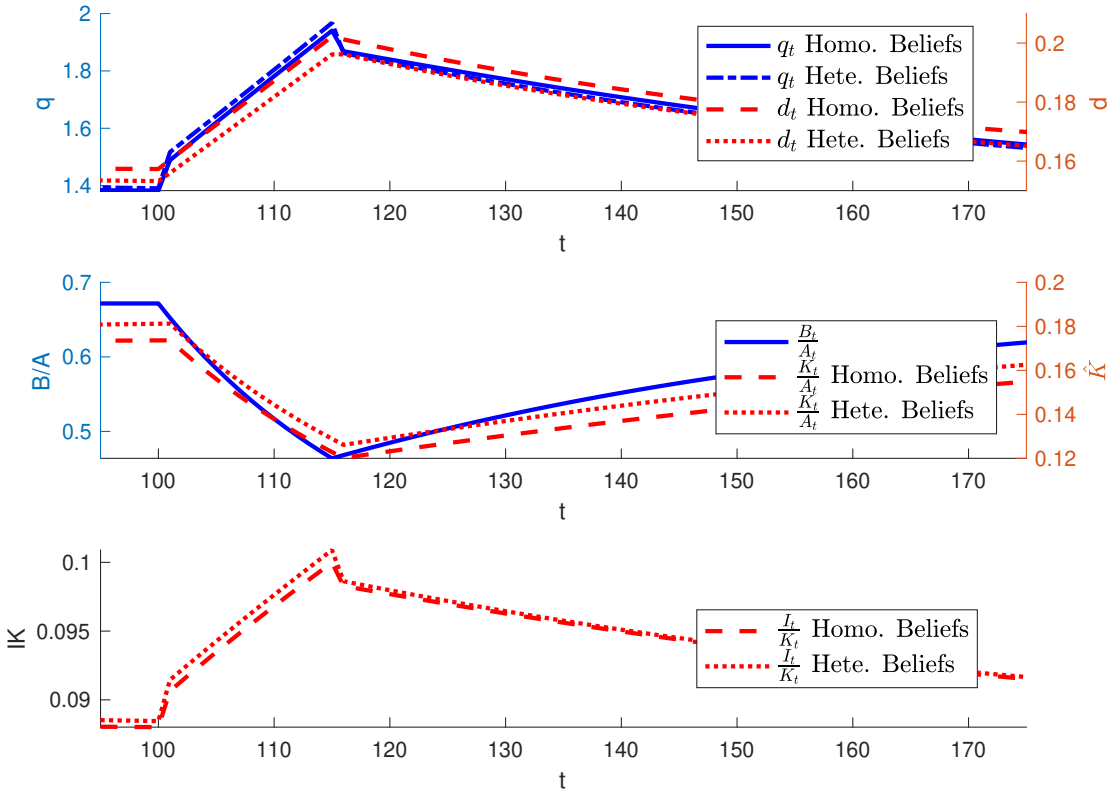


Figure 10: Price, Dividend, Capital and Investment Dynamics with Beliefs Heterogeneity.

5.2 Leveraged Investment

Many of the companies in the Internet boom and the current AI boom raise funding through a variety of financial contracts such as venture capital, partnership, leverage, which retain some of the upside for insiders. For example, in the initial partnership agreement between OpenAI and Microsoft, the latter would lose access to the former AI technologies when its achieves AGI and generates more than $100 billion dollars in profits. BlackRock (2024)'s new AI partnership raised $70 billion dollars from bond issuance, on top of $30 billion dollars in equity.

To capture this dynamics, we go back to the general model which allows the optimists to use financial contracts to raise funds from the pessimists to acquire high-tech shares. I focus on financial contracts of the form (5) which allows the pessimists to share some of the upside with the optimists. I first show that the equilibria for this model are equivalent to equilibria with non-state contingent bonds $\Psi_t = 1$ but the optimists could only borrow against a fraction $1 - m$ of their high-tech equity holding.

Proposition 3. *Assume that the payoff Ψ_t is given by (5). For each SCE, and hence Markov equilibrium in this model, there exist equilibria with the same equity prices and allocation in the model with state non-contingent bonds $\Psi_t \equiv 1$ with the collateral constraint:*

$$\phi_t^h + (1-m)k_t^h \min_{s_{t+1} \in \mathcal{S}} \left(q_{t+1}(s^{t+1}) + d(s_{t+1}) \right) \geq 0.$$

Proof. Appendix A □

Figure 11 shows that leverage leads to a larger increase in price-to-dividend

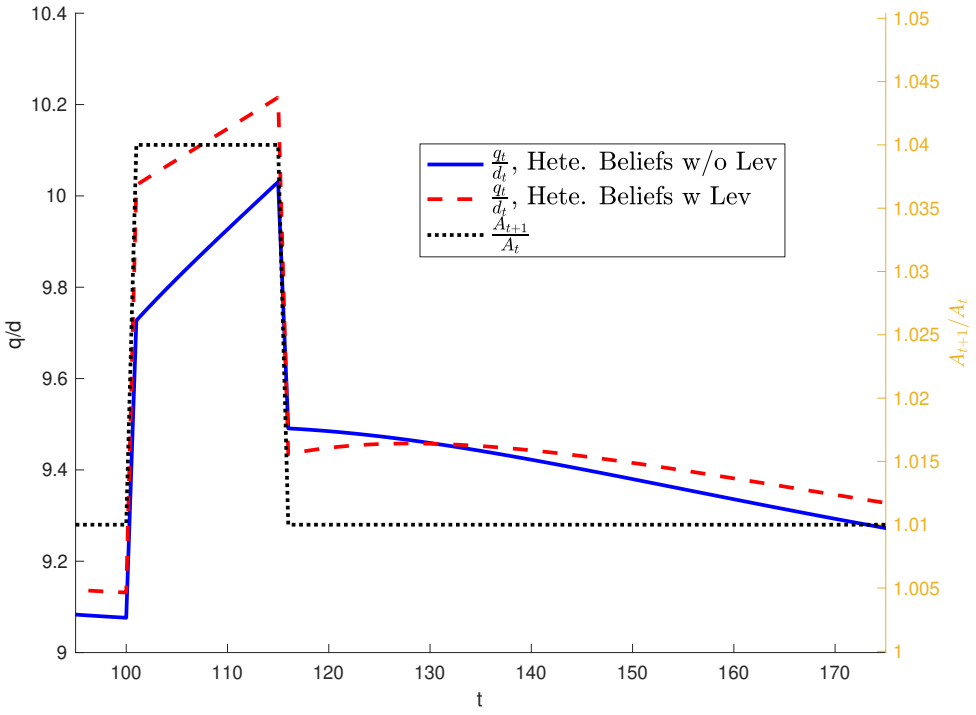


Figure 11: Price-to-Dividend Ratio Dynamics with Leverage.

ratio when the tech boom arrives at $t = 101$ but also a larger decrease when the tech boom ends at $t = 115$. Figure 12 tells us that this is due to both higher equity price and lower dividend, i.e, faster capital accumulation.

6 Conclusion

In this paper, I propose a theoretical model to study the dynamics of the 1990s Dot-Com boom and the ongoing Dot-AI boom. The dynamics of asset prices and capital accumulation in the 1990s could be explained by persistent productivity

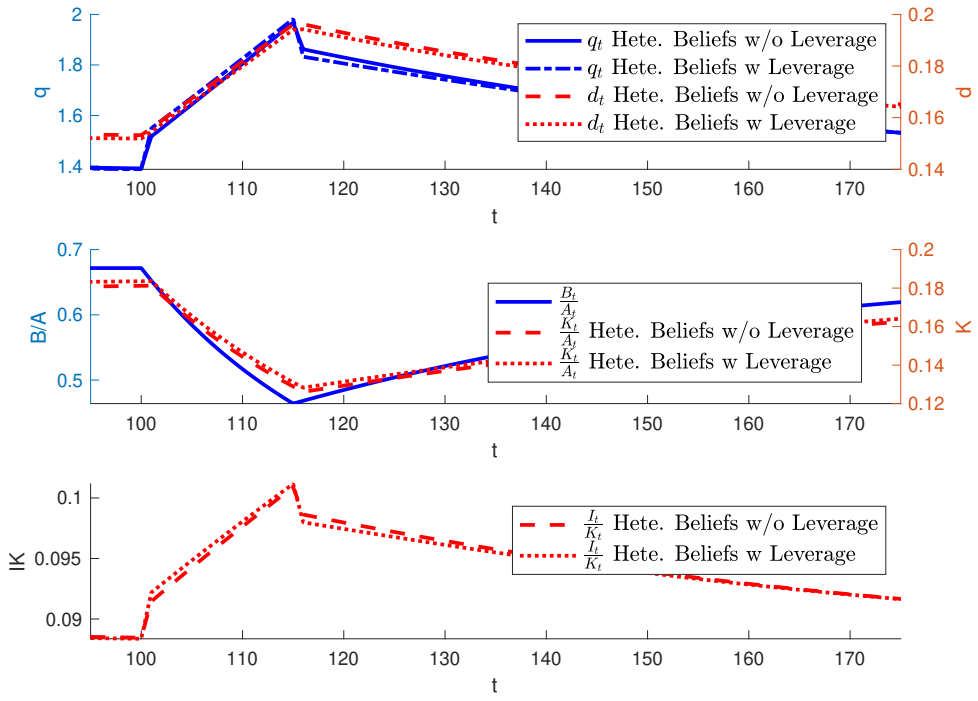


Figure 12: Price, Dividend, Capital and Investment Dynamics with Leverage.

growth in the high-tech sectors with very slow spillover to the rest of the economy. If the model is applicable to the current AI boom, we should expect the rapid progress in AI to continue until AI valuation reaches its peak. However, the result depends on the supply elasticity of capital in the high-tech sectors and the speed of spillover.

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A Proofs

Proof of Proposition 1.

$$\bar{q}^n = \frac{1}{\Phi_1\left(\bar{G}\bar{I}\bar{K}, 1\right)}$$

and

$$\bar{q} = \frac{1}{\left(1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)\right)\Phi_1\left(\bar{G}I\bar{K}, 1\right)}$$

and

$$\frac{\bar{q}^n}{\bar{q}}\bar{G}^\sigma = \beta\left(1 + \frac{\bar{d}}{\bar{q}}\right) = \frac{\bar{G}^\sigma}{1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)}.$$

where

$$\bar{G} = (1 - \delta) + \Phi\left(\bar{G}I\bar{K}, 1\right)$$

Differentiate both sides with respect to $\bar{G}$, we obtain:

$$1 = \Phi_1\left(\bar{G}I\bar{K}, 1\right)\left(I\bar{K} + \bar{G}\frac{dI\bar{K}}{d\bar{G}}\right).$$

Therefore

$$\begin{aligned}
&\frac{d}{d\bar{G}}\frac{\bar{G}^\sigma}{1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)} \\
&= \frac{\sigma\bar{G}^{\sigma-1}\left(1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)\right) - \bar{G}^\sigma\Phi_{21}\left(\bar{G}I\bar{K}, 1\right)\left(I\bar{K} + \bar{G}\frac{dI\bar{K}}{d\bar{G}}\right)}{\left(1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)\right)^2} \\
&= \frac{\bar{G}^{\sigma-1}}{\left(1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)\right)^2}\left(\sigma\left(1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)\right) - \frac{\bar{G}\Phi_{21}\left(\bar{G}I\bar{K}, 1\right)}{\Phi_1\left(\bar{G}I\bar{K}, 1\right)}\right) \\
&= \frac{\bar{G}^{\sigma-1}}{\left(1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)\right)^2}\left(\sigma\left(1 - \delta + \Theta\left(\bar{G}I\bar{K}\right) - \bar{G}I\bar{K}\Theta'\left(\bar{G}I\bar{K}\right)\right) - \frac{\bar{G}\Phi_{21}\left(\bar{G}I\bar{K}, 1\right)}{\Phi_1\left(\bar{G}I\bar{K}, 1\right)}\right) \\
&= \frac{\bar{G}^\sigma}{\left(1 - \delta + \Phi_2\left(\bar{G}I\bar{K}, 1\right)\right)^2}\left(\sigma\left(1 - I\bar{K}\Theta'\left(\bar{G}I\bar{K}\right)\right) - \frac{\Phi_{21}\left(\bar{G}I\bar{K}, 1\right)}{\Phi_1\left(\bar{G}I\bar{K}, 1\right)}\right) \qquad (22)
\end{aligned}$$

Notice that

$$\Phi(I, K) = K\Theta\left(\frac{I}{K}\right)$$

which implies

$$\Phi_1(I,K) = \Theta'\left(\frac{I}{K}\right)$$

and

$$\Phi_2(I,K) = \Theta\left(\frac{I}{K}\right) - \frac{I}{K}\Theta'\left(\frac{I}{K}\right)$$

and

$$\Phi_{21}(I,K) = -\frac{I}{K}\frac{1}{K}\Theta''\left(\frac{I}{K}\right).$$

Plugging these back into (22), we arrive at

$$\begin{aligned}
&\frac{d}{d\bar{G}}\frac{\bar{G}^{\sigma}}{1-\delta+\Phi_2\left(\bar{G}\bar{I}\bar{K},1\right)} \\
&= \frac{\bar{G}^{\sigma}}{\left(1-\delta+\Phi_2\left(\bar{G}\bar{I}\bar{K},1\right)\right)^2}\left(\sigma\left(1-\bar{I}\bar{K}\Theta'\left(\bar{G}\bar{I}\bar{K}\right)\right) - \frac{-\bar{G}\bar{I}\bar{K}\Theta''\left(\bar{G}\bar{I}\bar{K}\right)}{\Theta'\left(\bar{G}\bar{I}\bar{K},1\right)}\right) \\
&< \frac{\bar{G}^{\sigma}}{\left(1-\delta+\Phi_2\left(\bar{G}\bar{I}\bar{K},1\right)\right)^2}\left(\sigma - \frac{-\bar{G}\bar{I}\bar{K}\Theta''\left(\bar{G}\bar{I}\bar{K}\right)}{\Theta'\left(\bar{G}\bar{I}\bar{K},1\right)}\right) \leq 0.
\end{aligned}$$

□

B Numerical Algorithm

To solve for the Markov equilibria of this model, we use the time-iteration algorithm developed in Cao et al. (2023a). In the model with financial assets in Subsection 5.1, we solve for the policy and transition functions simultaneously during the time-iterations (STPFI).

In addition, we normalized the Lagrange multipliers by present marginal util-

ities in order to have fixed bounds for these multipliers over the iterations. More specifically, we rewrite the financial asset Euler equation as (13)

$$1 = \frac{1}{p_t \left(\hat{c}_t^h\right)^{-\sigma}} \beta \mathbb{E}_t \left[\left(\frac{A_{t+1}}{A_t} \right)^{-\sigma} \left(\hat{c}_{t+1}^h\right)^{-\sigma} \right] + \tilde{\mu}_t^h,$$

where

$$\tilde{\mu}_t^h = \frac{\mu_t^h A_t^{-\sigma}}{p_t \left(\hat{c}_t^h\right)^{-\sigma}}.$$

This implies

$$0 \leq \tilde{\mu}_t^h \leq 1$$

in equilibrium. Similarly, the real asset Euler equation can be rewritten as:

$$\begin{aligned} 1 = & \frac{1}{q_t^n \left(\hat{c}_t^h\right)^{-\sigma}} \beta \mathbb{E}_t \left[\left(\frac{A_{t+1}}{A_t} \right)^{-\sigma} \left(q_{t+1} + d_{t+1}\right) \left(\hat{c}_{t+1}^h\right)^{-\sigma} \right] \\ & + \tilde{\lambda}_t^h + \frac{p_t}{q_t^n} \tilde{\mu}_t^h \left(1 - m\right) \min_{s^{t+1}|s^t} \left(q_{t+1}(s^{t+1}) + d(s_{t+1}) \right) \end{aligned}$$

where

$$\tilde{\lambda}_t^h = \frac{\lambda_t^h A_t^{-\sigma}}{q_t^n \left(\hat{c}_t^h\right)^{-\sigma}}.$$

This implies

$$0 \leq \tilde{\lambda}_t^h \leq 1.$$

The collateral constraint could be re-written as:

$$\zeta_t^h = \frac{\hat{b}_t^h}{\left(1 - m\right) \min_{s^{t+1}|s^t} \left(q_{t+1}(s^{t+1}) + d(s_{t+1}) \right) \hat{K}_t} + \frac{k_t^h}{K_t} \geq 0.$$

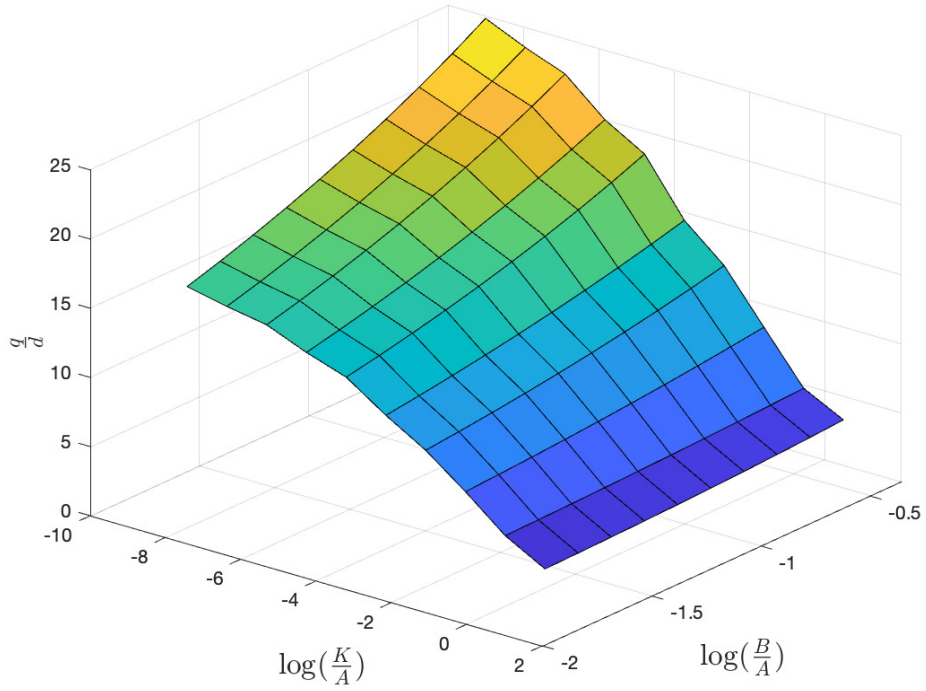


Figure 13: Policy Function: Homogenous Beliefs

In equilibrium

$$\zeta_t^O + \zeta_t^P = 1.$$

Therefore,

$$0 \leq \zeta_t^h \leq 1.$$

Figure 13 and Figure 14 present the policy functions for the model with homogenous beliefs in Section 4. The price to dividend ratio and the investment rate are decreasing in the scaled capital stock and increasing in the productivity of Sector B relative to Sector A.

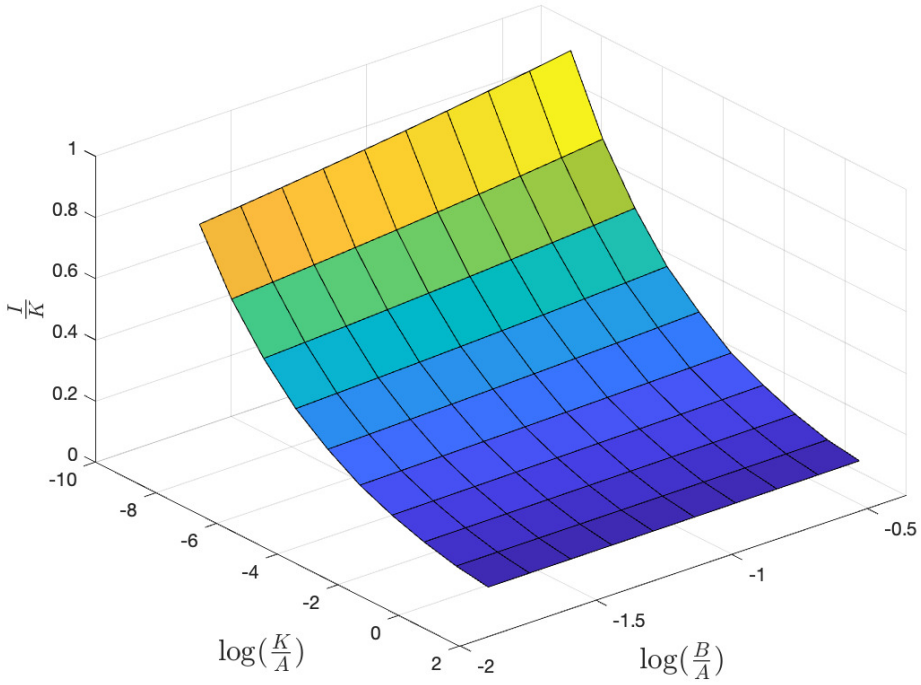


Figure 14: Policy Function: Homogenous Beliefs

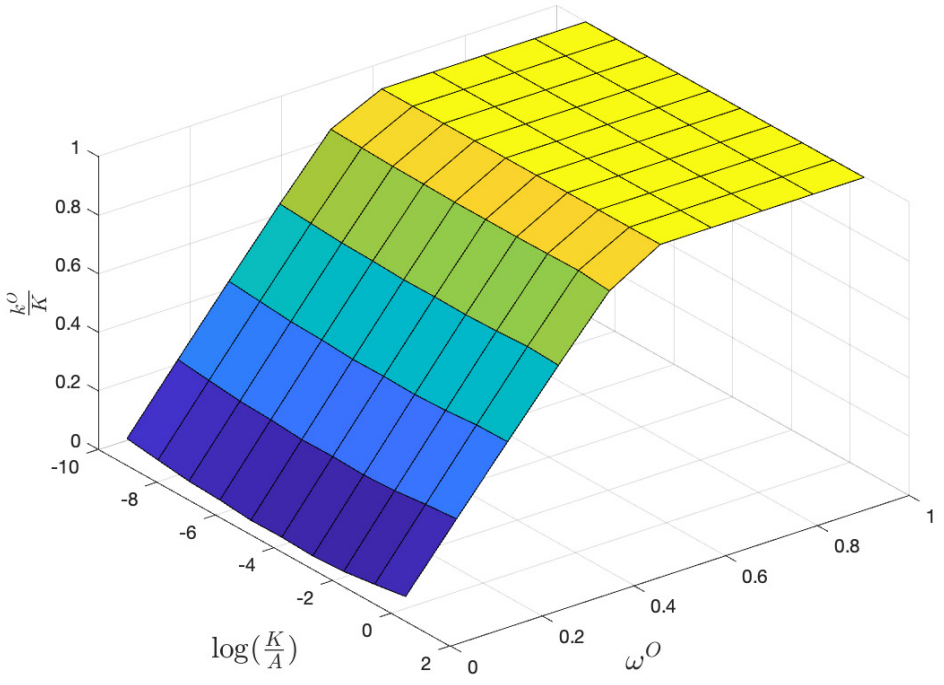


Figure 15: Policy Function: Heterogeneous Beliefs with Leverage

Figure 15 and Figure 16 present the policy functions for the model with heterogenous beliefs and leverage in Section 5.2. The share of capital owned by the optimists are increasing in their financial wealth shares. The excess borrowing capacity ζ_t^O is zero when ω_t^O is low, i.e., the optimists' borrowing constraint binds strictly.

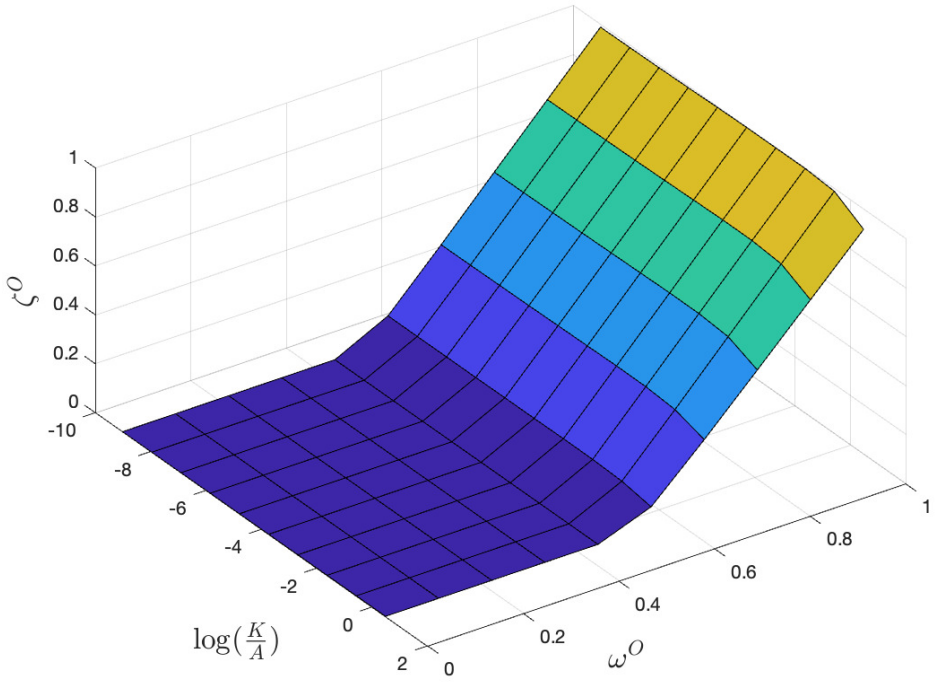


Figure 16: Policy Function: Heterogeneous Beliefs with Leverage