Connected for Better or Worse? The Role of Production Networks in Financial Crises*

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March 20, 2025

PRELIMINARY - DO NOT CITE

Abstract

Production networks shape the severity of financial crises by influencing how sectoral shocks propagate. We develop a small open economy model with intersectoral linkages and an occasionally binding collateral constraint to study this mechanism. In advanced economies, stronger linkages between tradable and non-tradable sectors create input price adjustments that stabilize profits, reducing financial distress. In contrast, weaker linkages in emerging markets amplify profit contractions, increasing crisis severity. Using industry-level input-output data, we show that sectoral connectivity systematically differs between emerging and advanced economies, reinforcing its role in sudden stops. Our findings suggest that macroprudential policies should consider production structures alongside financial vulnerabilities, as network connectivity influences financial stability.

Keywords: Production Networks, Financial Crises, Sudden Stops, Macroprudential Policy

*We thank Mishel Ghassibe for helpful comments and suggestions. We thank Maximiliano Jerez-Osses for excellent research assistance. All remaining errors are our own. The views expressed herein are those of the authors and not necessarily those of the International Monetary Fund, the Federal Reserve Bank of Boston or any other person affiliated with the Federal Reserve System.

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1 Introduction

Financial crises occur more frequently and with greater severity in emerging markets than in advanced economies (e.g., Calvo et al., 2006; Mendoza, 2010; Bianchi and Mendoza, 2020). Episodes of sudden stops, where capital inflows abruptly dry up, often trigger deep recessions and prolonged recoveries. The prevailing explanation in the literature attributes these disparities primarily to differences in financial development and the volatility of external shocks. While these factors are undoubtedly important, they do not fully account for the systematic variation in crisis severity across countries.

This paper highlights a neglected structural factor in the analysis of financial crises: the role of production networks in amplifying or mitigating economic downturns.[1] We argue that differences in intersectoral linkages—the degree to which firms in tradable and non-tradable sectors depend on each other for intermediate inputs—play a critical role in shaping the propagation and amplification of sectoral shocks. Specifically, in advanced economies, tradable sector firms rely more extensively on inputs from non-tradable sectors, creating a transmission mechanism that buffers the impact of adverse shocks. When a sector experiences a negative productivity shock, its demand for inputs declines, lowering input prices and providing cost relief for other firms. This price adjustment acts as an automatic stabilizer, reducing profit fluctuations and mitigating the likelihood of a financial constraint becoming binding. In contrast, in economies with weaker intersectoral linkages—such as many emerging markets—this stabilizing force is weaker, leading to sharper contractions in profits following sectoral downturns and more severe financial crises.

To formalize this mechanism, we develop an analytical framework that integrates production networks into a small open economy with an occasionally binding collateral constraint. Our model features a two-sector structure, with both a tradable and a non-tradable sector, where each sector uses the other's output as an intermediate input. This setup allows us to examine how intersectoral connectivity influences financial crisis severity.

Using a perfect foresight framework similar to that of Mendoza (2005), we show that network structure plays a critical role in shaping the economy's response to sectoral shocks. Specifically, we demonstrate that two economies with identical levels of financial development, but differing only in their production network structures, can exhibit

[1] Fadinger et al. (2022), McNerney et al. (2022), and Gloria et al. (2024) document how production network heterogeneity accounts for income differences across countries. Miranda-Pinto (2021) and Miranda-Pinto et al. (2023) emphasize how production network structures drive cross-country differences in output volatility and skewness.

vastly different reactions to the same shock. In economies with strong intersectoral linkages, negative shocks trigger automatic stabilizers that dampen the overall economic impact, while in economies with weaker linkages, shocks propagate more directly, leading to deeper contractions and prolonged financial distress.

Beyond the theoretical model, we provide empirical evidence that sectoral connectivity differs systematically between emerging and advanced economies. Using industry-level input-output data, we document that sectors in advanced economies are significantly more interconnected, particularly in their use of each other's intermediate goods. This empirical finding is important because it suggests that production structure—not just financial development or shock volatility—can explain why financial crises are more severe in emerging markets. Our results imply that macroprudential policies should not only focus on financial sector vulnerabilities but also account for the structure of production networks as a determinant of financial stability.

Finally, as part of our ongoing work, we extend our analysis by developing a three-sector dynamic stochastic general equilibrium (DSGE) model to quantitatively assess the impact of sectoral connectivity on financial stability. This more detailed framework will allow us to examine the effectiveness of macroprudential policy interventions across different network structures and provide insights into the role that production networks play in shaping financial crises.

Our work contributes to two key strands of literature. First, we contribute to the sudden stops literature (e.g., Mendoza, 2010; Bianchi, 2011; Bianchi et al., 2016; Benigno et al., 2013; Bianchi and Mendoza, 2020) by incorporating production networks into macroeconomic models of financial crises. We demonstrate that differences in production architecture can significantly influence the severity of sudden stops.[2] Second, we contribute to the literature on production networks and macroeconomic fluctuations (e.g., Horvath, 1998; Foerster et al., 2011; Acemoglu et al., 2012; Bigio and La'o, 2020; Baqaee and Farhi, 2019) by examining how borrowing constraints interact with intersectoral linkages in a dynamic model of occasionally binding constraints. Our tractable framework provides sharp predictions on the role of networks in mitigating or amplifying financial crises and distinguishes between partial and general equilibrium effects.[3]

The rest of the paper is structured as follows. Section 2 presents our theoretical model, outlining the key assumptions and mechanisms. Section 3 provides a numerical example illustrating the role of production networks in financial crises. Section 4 presents empir-

[2] Rojas and Saffie (2022) study how non-homotheticities in demand affect the severity of sudden stops.

[3] Miranda-Pinto and Young (2022) analyze production networks and production flexibility in a static model of occasionally binding sectoral collateral constraints.

ical evidence on sectoral connectivity in emerging and advanced economies. Section 5 describes the three-sector quantitative model, and Section 6 concludes with a discussion of policy implications and future research directions.

2 Analytic Framework

We consider a canonical small open economy model with two goods: tradable and non-tradables. Time is discrete and the horizon is infinite. The economy features a borrowing constraint on the household's side. We consider a deterministic setup.

2.1 Households

There is a continuum of identical households of measure one. They maximize the present discounted value of utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t(c_t^N, c_t^T)), \text{ where } \beta \in (0,1). \tag{1}$$

Households consume both tradable and non-tradable goods according to the following consumption aggregator:

$$c_t(c_t^N, c_t^T) = \left((1-\omega)(c_t^N)^{1-\eta} + \omega(c_t^T)^{1-\eta}\right)^{\frac{1}{1-\eta}}, \text{ where } \eta \in (0,\infty) \text{ and } \omega \in (0,1). \tag{2}$$

They trade one-period non-state contingent bonds with exogenous price q and receive flow income from firms' profits in the tradable (π_t^N) and non-tradable sector (π_t^T). The budget constraint is then given by

$$p_t^N c_t^N + c_t^T + q b_{t+1}^* = \pi_t^N + \pi_t^T + b_t^*, \tag{3}$$

where the left-hand side represents households' consumption of non-tradable and tradable goods plus bond purchases. The right-hand side is households' income, which combines profits and bond positions at the beginning of the period. All prices are denominated in units of tradable goods. Therefore, p_t^N is the relative price of non-tradable goods.

Household face an occassionally binding borrowing constraint. More specifically, households can borrow a up to a fraction κ of their flow income, which is the sum of

total sectoral profits

$$qb_{t+1}^* \geq -\kappa(\pi_t^N + \pi_t^T). \tag{4}$$

Optimality conditions. The first-order condition for non-tradable and tradable consumption implies:

$$p_t^N = \frac{(1-\omega)}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\eta}. \tag{5}$$

Let $\mu_t \geq 0$ be the Lagrange multiplier on the borrowing constraint. The first-order condition for bond purchases implies the following Euler equation for tradable goods and complementary slackness condition:

$$u_T(t) = \beta R^* u_T(t+1) + \mu_t, \tag{6}$$

$$\mu_t\left(qb_{t+1}^* + \kappa(\pi_t^N + \pi_t^T)\right) = 0, \tag{7}$$

where $u_T(t) = \frac{\partial u(c_t)}{\partial c_t}\frac{\partial c_t}{\partial c_t^T}$ and $R^* = q^{-1}$.

2.2 Firms

Our economy features production with no labor. Instead, sectors can use sectoral outputs as inputs in production. This block generates a network structure in production. We now describe each sector in turn.

2.2.1 Non-tradable Sector

The non-tradable sector operates according to the following technology

$$y_t^N = A^N z_t^N (m_{Nt}^N)^{\alpha_N^N}(m_{Tt}^N)^{\alpha_T^N}, \text{ where } \alpha_N^N + \alpha_T^N \in [0,1), \tag{8}$$

and $A^N = (\alpha_T^N)^{-\alpha_T^N}(\alpha_N^N)^{-\alpha_N^N}$ is a normalizing constant.

Firms' profits are $\pi_t^N = p_t^N y_t^N - p_t^N m_{Nt}^N - m_{Tt}^N$. The firms' problem is to maximize profits subject to (8) taking p_t^N as given. The solution to this problem delivers the non-tradable supply schedule:

$$y_t^N(z_t^N, p_t^N) = \left(z_t^N (p_t^N)^{\alpha_T^N}\right)^{\frac{1}{1-\alpha_N^N-\alpha_T^N}} \tag{9}$$

2.2.2 Tradable Sector

There is a continuum of identical firms in the tradable sector that operates according to the following production function

$$y_t^T = A^T z_t^T (m_{Tt}^T)^{\alpha_T^T} (m_{Nt}^T)^{\alpha_N^T}, \text{ where } \alpha_T^N + \alpha_T^T \in [0,1), \tag{10}$$

and $A^T = (\alpha_T^T)^{-\alpha_T^T}(\alpha_N^T)^{-\alpha_N^T}$ is a normalizing constant.

Firms' profits are $\pi_t^T = y_t^T - p_t^N m_{Nt}^T - m_{Tt}^T$. Tradable firms maximize profits subject to (10) taking p_t^N as given. The solution to this problem delivers the tradable supply schedule:

$$y_t^T(z_t^T, p_t^N) = \left(z_t^T (p_t^N)^{-\alpha_N^T}\right)^{\frac{1}{1-\alpha_T^T-\alpha_N^T}} \tag{11}$$

2.3 Competitive Equilibrium

Market clearing for non-tradables requires that demand for non-tradables equal supply:

$$c_t^N + m_{Nt}^N + m_{Nt}^T = y_t^N. \tag{12}$$

The aggregate resource constraint satisfies

$$c_t^T + m_{Tt}^T + m_{Tt}^N = y_t^T - qb_{t+1}^* + b_t \tag{13}$$

2.4 Perfect Foresight Analysis.

For simplicity we set $\beta R^* = 1$. We study a wealth-neutral shock to tradable productivity (z_t^T), large enough to make the borrowing constraint binding, triggering a sudden stop. When the borrowing constraint binds $qb_{t+1}^* = -\kappa(\pi_t^N + \pi_t^T)$.

Under these assumptions, the model can be characterized by the following two curves:

$$p_t^N = \left(\frac{c_t^T}{c_t^N(p_t^N; z_t^N, z_t^T)}\right)^{\eta} \left(\frac{1-\omega}{\omega}\right) \qquad \text{(PP Curve)} \tag{14}$$

$$c_t^T = \kappa_N \pi_t^N(p_t^N; z_t^N) + (1+\kappa_T)\pi_t^T(p_t^N; z_t^T) + b_0 \qquad \text{(BB Curve)}, \tag{15}$$

where

$$\kappa_N = \kappa - \frac{\alpha_T^N}{1-\alpha_N^N-\alpha_T^N}; \quad \kappa_T = \kappa + \frac{(1-\alpha_T^T)}{1-\alpha_T^T-\alpha_N^T},$$

$$\pi_t^N = (1 - \alpha_N^N - \alpha_T^N) p_t^N y_t^N; \quad \pi_t^T = (1 - \alpha_T^T - \alpha_N^T) y_t^T,$$

and output supplies (y_t^N, y_t^T) are given by equations (9) and (11) and non-tradable consumption (c_t^N) follows from the market clearing condition (12). For future reference, non-tradable consumption can be expressed as a function of output supplies in both sectors and the non-tradable price:

$$c_t^N = (1 - \alpha_N^N) y_t^N(p_t^N; z_t^N) - \alpha_N^T \frac{y_t^T(p_t^N; z_t^T)}{p_t^N} \tag{16}$$

The following proposition shows that the PP curve always slopes upward, irrespective of the production network's structure.

Proposition 1. *The PP curve is upward-sloping in the space (c_t^T, p_t^N).*

Proof. See Appendix A.1. □

The BB curve slope instead depends on the network structure. The following proposition provides a characterization of this result for the general case:

Proposition 2. *The BB curve slope depends on the network structure. If*

$$\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N} + (1 + \kappa_T) \frac{\partial \pi_t^T}{\partial p_t^N} \geq 0,$$

where $\kappa_N = \left(\kappa - \frac{\alpha_T^N}{(1-\alpha_N^N-\alpha_T^N)}\right)$ and $\kappa_T = (\kappa + \frac{\alpha_N^T}{(1-\alpha_T^T-\alpha_N^T)})$, then the BB curve is upward-sloping. Otherwise, it is downward sloping.

Proof. See Appendix A.2. □

At the core of Proposition 2 is how tradable consumption responds to a change in non-tradable price. This has two components: how tradable consumption react to (i) changes in non-tradable profits $\left(\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N}\right)$ and (ii) to changes in tradable profits $\left((1 + \kappa_T) \frac{\partial \pi_t^T}{\partial p_t^N}\right)$. Changes in non-tradable profits in response to a decrease in the non-tradable price affect tradable consumption in two ways. First, it tightens the borrowing constraint. This effect is captured by κ and it is the classical mechanism in sudden stop models. Second, the non-tradable sector inputs' demand for tradable resources decreases. Given an amount of tradable resources, this lower input demand increases tradable consumption by $\alpha_T^N/(1 - \alpha_N^N - \alpha_T^N)$. Similarly, a lower non-tradable price increases profits in the tradable sector. This relaxes the borrowing constraint by κ and increases tradable production by $\alpha_N^T/(1 - \alpha_T^T - \alpha_N^T)$. Both mechanisms go in the same direction: they increase tradable consumption.

Although profits affect household income directly, as in standard sudden stop models, intersectoral linkages alter how tradable consumption responds to profit changes. A roundabout production structure ($\alpha_T^N = \alpha_N^T = 0$) coincides exactly with an endowment economy where non-tradable profit changes affect tradable consumption by $\kappa_N = \kappa$ and tradable profits affect tradable consumption by $\kappa_T = \kappa$. Thus, intermediate inputs without intersectoral linkages do not affect any of the predictions of standard sudden stop models or the slope of the BB curve. They only affect how profits react to changes in the non-tradable price. In contrast, intersectoral linkages are key for how changes in profits affect tradable consumption. In an extreme case, strong intersectoral connections can potentially change the sign of the BB curve slope: when $\alpha_T^N \geq \kappa(1-\alpha_N^N)/(1+\kappa)$, the BB curve slope is negative.

Figure 1 uses a numerical example to illustrate how different network structures can shape the equilibrium relationships in the BB-PP space. We study four different network structures:

$$\Omega^{\text{dense}} = \begin{bmatrix} \frac{\alpha}{2} & \frac{\alpha}{2} \\ \frac{\alpha}{2} & \frac{\alpha}{2} \end{bmatrix}, \quad \Omega^{\text{NT}\leftarrow\text{T}} = \begin{bmatrix} \frac{\alpha}{2} & \frac{\alpha}{2} \\ 0 & \alpha \end{bmatrix},$$
$$\Omega^{\text{diagonal}} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix}, \quad \Omega^{\text{T}\leftarrow\text{NT}} = \begin{bmatrix} \alpha & 0 \\ \frac{\alpha}{2} & \frac{\alpha}{2} \end{bmatrix},$$

where α is the total intermediate input share that we keep constant across specifications. In the Ω^{dense} economy, the non-tradable sector (first row and column) and tradable sector (second row and column) rely on each other in the same proportion. In the Ω^{diagonal} case, each sector only uses intermediates from their same sector. In the $\Omega^{\text{NT}\leftarrow\text{T}}$ economy, the non-tradable sector buys from the tradable sector, but the tradable sector does not use non-tradable as an input. The $\Omega^{\text{T}\leftarrow\text{NT}}$ economy does the opposite: the tradable sector uses non-tradable as an input while the non-tradable does not use tradable as an input. We normalize the initial equilibrium to be the same across models $(c^T, p^N) = (1, 1)$.

Figure 1 clearly shows that while the slope and shape of the BB curve (pink line) vary significantly with the network structure, the PP curve (green line) is (almost) linear and upward-sloping across the different network structures.[4]

We end this subsection with a general result. We establish that the non-tradable price and tradable consumption unambiguously decline in response to a change in tradable productivity:

[4]The PP curve is always increasing but can be convex or concave depending on the network structure. In the region near the equilibrium, however, the PP curve is almost linear in the (c^T, p^N) space.

Proposition 3. *Consider a wealth-neutral decline in tradable productivity starting from an equilibrium where the borrowing constraint is marginally binding. The response of tradable consumption and non-tradable price is*

$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \mathcal{K}\left[(\gamma_t^N+\gamma_t^T)(\delta_t^N-1)+\left(\gamma_t^N(\delta_t^N-1)+\delta_t^N\gamma_t^T\right)\varepsilon^{y_t^N}_{p_t^N}+\frac{\gamma_t^T}{\eta}\right]\varepsilon^{y_t^T}_{z_t^T}\geq 0$$
$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} = \mathcal{K}\left((\delta_t^N-1)+\gamma_t^T\right)\varepsilon^{y_t^T}_{z_t^T}\geq 0$$

where

$$\gamma_t^N=\frac{\kappa_N\pi_t^N}{c_t^T};\quad \gamma_t^T=\frac{(1+\kappa_T)\pi_t^T}{c_t^T};\quad \delta_t^N=\frac{(1-\alpha_N^N)y_t^N}{c_t^N},$$
$$\varepsilon^{y_t^N}_{p_t^N}=\frac{\alpha_T^N}{1-\alpha_N^N-\alpha_T^N};\quad \varepsilon^{y_t^N}_{z_t^T}=\frac{1}{1-\alpha_N^N-\alpha_T^N};\quad \varepsilon^{y_t^T}_{p_t^N}=\frac{\alpha_N^T}{1-\alpha_T^T-\alpha_N^T},$$
$$\mathcal{K}=\left(\frac{1}{\eta}-\gamma_t^N+(\delta_t^N-1)+(\delta_t^N-\gamma_t^N)\varepsilon^{y_t^N}_{p_t^N}+(1-\gamma_t^T-\delta_t^N)\varepsilon^{y_t^T}_{p_t^N}\right)^{-1}.$$

$\mathcal{K}$ and the term in brackets represent general equilibrium effects, and $\varepsilon^{y_t^T}_{z_t^T}$ represents the direct response of tradable output with respect to tradable productivity. $\varepsilon^{y_t^N}_{p_t^N}$ and $\varepsilon^{y_t^T}_{p_t^N}$ represent the elasticity of non-tradable and tradable output to a change in the non-tradable price, respectively.

Proof. See Appendix A.3. □

Proposition 3 characterizes our model's rich general equilibrium interactions. While the initial tradable productivity impulse consists simply of a change in tradable production, ceteris paribus, this triggers general equilibrium responses of both the production and consumption sides captured by $\mathcal{K}$ and the terms in brackets. Proposition 3 shows that, although rich and, in principle, hard to compute, the effect always goes in the same direction: a decline in tradable productivity reduces tradable consumption and the non-tradable price.

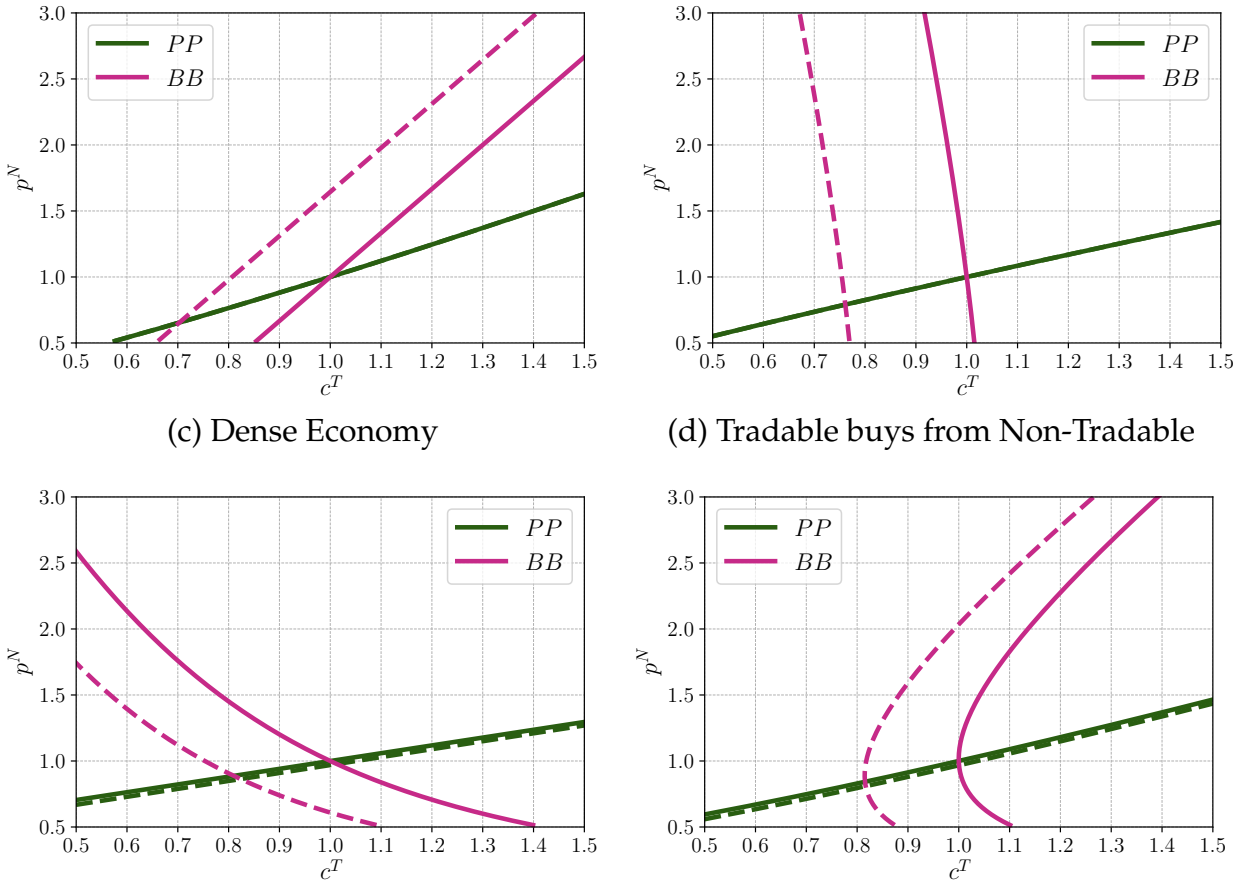
Proposition 3 is general enough to sign the direction of tradable consumption and the non-tradable price in a model with intersectoral linkages. It is a qualitative statement. However, it does not allow us to say anything regarding its quantitative importance, nor does it allow us to provide sharp predictions about the role of the network structure in shaping these responses either qualitatively or quantitatively. To that end, we must make further assumptions and we explore one of such cases in the next subsection. We can, however, numerically explore how the same proportional change in tradable productivity

across models changes tradable consumption and non-tradable price. The dashed lines in Figure 1 show BB and PP curves in response to a 10 percent decline in tradable productivity. In all four panels, the decline in tradable productivity unambiguously declines tradable consumption and the non-tradable price, as Proposition 3 showed.

Figure 1: PP and BB curves under different network structures

(a) Diagonal Economy

(b) Non-Tradable buys from Tradable



(c) Dense Economy

(d) Tradable buys from Non-Tradable

Note: This figure depicts the PP and BB curves under different network scenarios. To generate these figures we set $\eta = 1.2035$, the intermediate input share $\alpha = 0.4$ (consistent with the average emerging market number), $\kappa = 0.3$ and $b_0 = -0.3$. We assume $\omega = 0.5$ to focus on the role of the production network structure.

2.4.1 The Role of the Non-Tradable Sector Linkages

We study a case that allows us to make sharp analytical predictions on the importance of production linkages in shaping the sudden stop—as described by the response of non-tradable price and tradable consumption to a productivity shock in the tradable sector.

We consider the "non-tradable buys from tradable" network structure that we repeat here to ease the exposition:

$$\Omega^{\text{NT}\leftarrow\text{T}} = \begin{bmatrix} \alpha_N^N & \alpha_T^N \\ 0 & \alpha_T^T \end{bmatrix}$$

Under this structure, the tradable sector only uses intermediate inputs from itself. The non-tradable sector uses intermediates from itself and from the non-tradable sector. For comparability, we must maintain the degree of returns to scale constant across economies. Otherwise, the result would be partly driven by heterogeneity in the degree of decreasing returns to scale across sectors. Hence, $\bar{\alpha}^N = \alpha_T^N + \alpha_N^N$ and $\bar{\alpha}^T = \alpha_T^T$ the same for any given economy, where $\bar{\alpha}^i$ is the degree of decreasing returns to scale for sectors $i \in \{N, T\}$. In what follows we call this the N economy.

We analyze how changes in α_T^N and α_N^N alter how the economy responds to shocks to the tradable sector productivity (z^T). We compare two economies. The Ω^{diagonal} (D) economy with $\alpha_T^N(D) = 0$, $\alpha_N^N(D) = \bar{\alpha}^N$ and the $\Omega^{\text{NT}\leftarrow\text{T}}$ (N) economy with $\alpha_T^N(N) > 0$, $\alpha_N^N(N) < \alpha_N^N(D)$, and $\bar{\alpha}^N = \alpha_T^N(N) + \alpha_N^N(N)$.

The following proposition characterizes how the production network structure matters for the severity of the sudden stop on tradable consumption.

Proposition 4. *Consider a decline in tradable productivity. Let $\varepsilon^{BB}(G)$ and $\varepsilon^{PP}(G)$ be the elasticities of the BB curve and PP curve under a network $G = \{D, N\}$ evaluated at the marginally binding equilibrium. If*

$$\left| \gamma^T(N) \frac{\varepsilon^{BB}(N)}{\varepsilon^{BB}(N) - \varepsilon^{PP}(N)} \right| \leq \left| \gamma^T(D) \frac{\varepsilon^{BB}(D)}{\varepsilon^{BB}(D) - \varepsilon^{PP}(D)} \right|,$$

then

$$\left| \frac{\mathrm{d} \log (c_t^T)^{NT\leftarrow T}}{\mathrm{d} \log z_t^T} \right| \leq \left| \frac{\mathrm{d} \log (c_t^T)^{Diagonal}}{\mathrm{d} \log z_t^T} \right|,$$

and

$$\left| \frac{\mathrm{d} \log (p_t^N)^{NT\leftarrow T}}{\mathrm{d} \log z_t^T} \right| \leq \left| \frac{\mathrm{d} \log (p_t^N)^{Diagonal}}{\mathrm{d} \log z_t^T} \right|.$$

Proof. See Appendix A.4 □

In other words, if the above condition is satisfied, the diagonal economy's tradable consumption and non-tradable price decline more than in the intersectoral economy.

The intuition for this condition is that the two network examples affect the slope of the BB and PP curves. As it can be clearly seen in panel (a) and (b) of Figure 1 for the same 10 percent decline in tradable productivity, tradable consumption declines around six percentage points more in the diagonal economy relative to the N economy. Moreover, the non-tradable price also declines more in the diagonal case. Both results are consistent with Proposition 4.

The slope of the PP curve changes with intersectoral linkages because non-tradable supply becomes a function of the non-tradable price, affecting non-tradable consumption in equilibrium, which is part of the PP curve. To see this, the non-tradable market clearing conditions in each case are:

$$c^N(D) = (1 - \alpha_N^N(D))y^N(z^N(D)),$$
$$c^N(N) = (1 - \alpha_N^N(N))y^N(z^N(N), p^N).$$

This mechanism reduces the sensitivity of the non-tradable price to changes in tradable consumption when the economy features intersectoral linkages.

We already discussed how intersectoral linkages change the slope of the BB curve in Proposition 2. The benchmark case in the literature considers endowment economies (Bianchi, 2011; Rojas and Saffie, 2022). These economies are a special case of ours. Indeed, without production, our model collapses to $\kappa_N = \kappa_T = \kappa$, $\pi_t^N = p_t^N \bar{y}_t^N$ and $\pi_t^T = \bar{y}_t^T$, where a bar over a variable denotes that it is an endowment. It is not a surprise then that panel (a) of Figure 1 is almost identical to these papers' equilibrium characterization: an increasing and convex PP curve and an increasing and linear BB curve.

Our BB curve highlights sectoral heterogeneity in the effective collateral constraint parameter faced by the household due to production heterogeneity. A connected economy can mitigate the sudden stop as it displays a smaller decline in profits, which, through the borrowing constraint, allows households to consume more tradables than in an island economy. This is apparent when we look at panels (c) and (d) of Figure 1, where both interconnected economies exhibit a smaller decline in tradable consumption and non-tradable price than the diagonal economy. Moreover, note that whenever the tradable sector buys inputs from the non-tradable sector, the PP curve shifts down in response to a tradable productivity decline. A decrease in tradable productivity reduces tradable sector input demand from the non-tradable sector, which increases non-tradable consumption given non-tradable production (a downward shift in the PP curve). The figures make clear that this shift is likely to be small, especially when compared to the BB curve shifts.

An avid reader would argue that intersectoral linkages are unnecessary to generate

heterogeneity in κ's across sectors: different decreasing returns to scale across sectors would suffice. This statement is incorrect. Decreasing returns to scale affects how profits respond to changes in non-tradable price, $\frac{\partial \pi_t^i}{\partial p_t^N}$. They do not affect how tradable consumption reacts to changes in profits. Under decreasing returns to scale across sectors but no intersectoral linkages, the parameters collapse to $\tilde{\kappa}_N = \kappa$ and $\tilde{\kappa}_T = \kappa$. Exactly as in the endowment economy. With intersectoral linkages these parameters are sector-specific: $\kappa_N = \kappa - \frac{\alpha_T^N}{(1-\alpha_N^N-\alpha_T^N)}$ and $\kappa_T = \kappa + \frac{\alpha_N^T}{(1-\alpha_N^T-\alpha_T^T)}$. As we show in the empirical section, intersectoral connections via intermediate inputs are important across countries: α_T^N and α_N^T are empirically relevant and largely differ across countries.

3 Numerical Analysis

In this section, we present a numerical analysis of our perfect foresight model. We calibrate the model's network structure to reflect key characteristics of emerging economies, as described in Section 4. Following the approach in Section 2.4, we begin by solving for the equilibrium of different economies that face a marginally binding borrowing constraint. We examine four cases. The first is our baseline economy, where the two productive sectors are interlinked. In the second case, we consider an economy in which the tradable and nontradable sectors operate independently, meaning they do not use each other's inputs in production. The third and fourth cases are combinations of the first two, where we consider an isolated tradable sector (only uses tradable input to produce), and another where the isolated sector is the nontradable one.

Our goal is to use the simple model to showcase the role that different productive structures play in the severity of Sudden Stops, which are events in which tradable productivity collapses.[5] For our baseline case, we set $\eta = 1.2035$, $\beta = R = 1/1.04$, $\omega = 0.3$ and $\kappa = 0.3$, following Bianchi (2011). Next, we set the share of tradable and nontradables inputs of the tradables to be $\alpha_T^T = 0.46$ and $\alpha_N^T = 0.1$, respectively. For the nontradable sector, we set $\alpha_T^N = 0.21$ and $\alpha_N^N = 0.18$. Lastly, we set $z^T = z^N = 1$ for simplicity.

The parameters for the counterfactual economies are chosen as follows. In the case where the two sectors operate independently, we set $\alpha_T^T = 0.56$ and $\alpha_N^N = 0.39$. These values ensure that the returns to scale in the corresponding production functions match those of our benchmark economy. However, this parameter choice does not necessarily

[5]More specifically, a Sudden Stop is an event where a wealth-neutral time-0 tradable productivity shock occurs. Since the constraint is marginally binding before the shock is materialized, a decline in productivity will lead to a binding borrowing constraint, causing the well-known Fisherian deflationary mechanism characteristic of this family of models.

guarantee that the equilibrium allocation of tradable consumption and the relative price of nontradables remain the same across models. To address this, we calibrate tradable and nontradable productivities to ensure that the initial consumption allocations and the relative price of nontradables are consistent across economies.[6] The rest of the parameters are identical to those of the benchmark economy.

The parametrization of the remaining two cases follows a similar process. For the economy where the tradable sector does not use nontradable inputs in its production function, we set $\alpha_T^T = 0.56$, $\alpha_N^T = 0$, $\alpha_T^N = 0.21$ and $\alpha_N^N = 0.18$. Lastly, we set $\alpha_T^T = 0.46$, $\alpha_N^T = 0.10$, $\alpha_T^N = 0$ and $\alpha_N^N = 0.39$ for the economy where the nontradable sector does not use tradable inputs in production. In both cases, we adjust tradable and nontradable productivities to ensure that the initial consumption allocations and the relative price of nontradables remain consistent with those of the baseline economy.

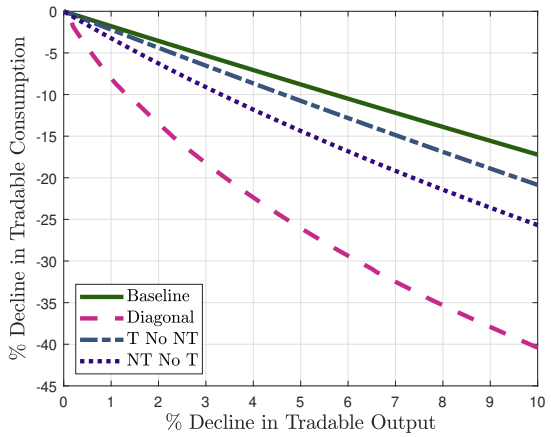
Next, we simulate Sudden Stops by introducing shocks to tradable productivity of varying severity. Figure 2 presents our key results. Panel (a) shows the response of tradable consumption, while Panel (b) displays the relative price of nontradables—both measured relative to their initial steady-state allocations—as tradable productivity changes across the four economies described above. The final panel presents the debt-to-GDP ratio for each economy.[7]

[6] By imposing the same tradable consumption and relative price of nontradables, we are left with a system of two equations (the BB and PP curves) and two unknowns (tradable and nontradable productivities.

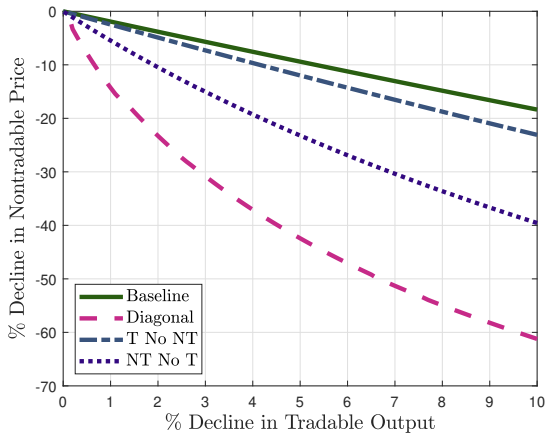
[7] Debt is measured relative to steady-state GDP. In our case, the debt-to-GDP ratio is approximately 30%, aligning with long-run trends observed in several emerging economies (Lane and Milesi-Ferretti, 2017).

Figure 2: Sudden Stops Across Different Production Structures

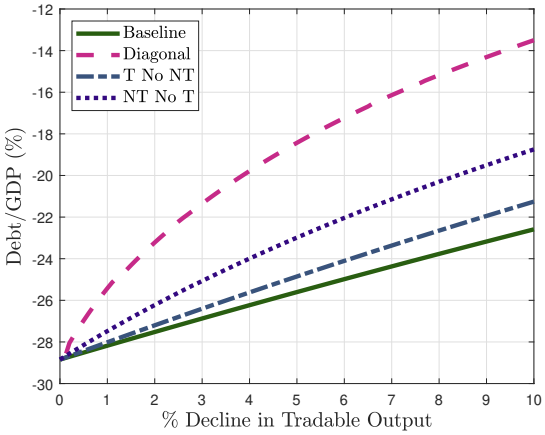
(a) Tradable Consumption (c^T)



(b) Nontradable Price (p^N)



(c) Debt-to-GDP (b/GDP)



The green solid line illustrates how the benchmark economy responds. The dashed pink line corresponds to the case where the two sectors are independent of each other ("Diagonal"). The dark blue dash-dotted line depicts the economy in which only the nontradable sector uses both tradable and nontradable inputs in production ("T No NT"), while the dotted purple line represents the case where only the tradable sector incorporates both inputs ("NT No T").

Consistent with the model's theoretical predictions, sectoral linkages serve as a hedging mechanism against adverse productivity shocks. Specifically, when tradable productivity declines by 10%, tradable consumption in the baseline economy falls by approximately 17%. In contrast, in the Diagonal economy—where sectors operate independently—the same productivity decline leads to a collapse in tradable consumption of nearly 40%, more than double the drop observed in the baseline case. Similar patterns

emerge for the relative price of nontradables and the debt-to-GDP ratio, where the economy without sectoral linkages undergoes a much more severe deflationary spiral and deleveraging process compared to the benchmark case.

What happens when we consider the hybrid cases? We see that when one of the two sectors is not fully linked to the other (i.e. only uses inputs from one sector) we have amplification relative to the baseline economy (T-No-NT and NT-No-T cases) in the responses to tradable productivity declines, but these are substantially milder than the case with no linkages. It is worth noting that for this particular parametrization, the economy where the nontradable sector does not use the tradable good as input is the one that tends to exhibit worse hedging properties (still substantially better than the fully disconnected case).

These results show that the shape of the production structure of the economy can provide substantial attenuation (or amplification) during Sudden Stops. Additionally, these amplification of attenuation effects are nonlinear in the size of shocks: for mild shocks the gaps in responses tend to be relatively smaller than when shocks are large.

4 Stylized facts

In this section, we provide simple descriptive statistics on the network structure of tradable—commodity and non-commodity tradables—and non-tradable sectors in advanced and emerging economies. We use the sample of countries in Bianchi and Mendoza (2020) and use OECD input-output data release 2021. Our definition of tradable and non-tradable sectors is based on sectoral gross trade intensity. Sectors with a ratio of gross trade to gross output above 20% are classified as tradable sectors. Here, we consider the average ratio across countries so we maintain the same group of sectors for all countries. Table 2 provides a detailed classification. All our numbers below are based on year 2018.

4.1 Commodities, Tradables, Non-Tradables Linkages

Here, we present three facts on sectoral linkages across emerging and advanced economies. For a particular sector i and country k we measure the input share from sector j as the ratio between the purchases of intermediates $p_j m_{ij}$ over total gross output $p_i y_i$. We consider domestic and imported intermediates in our measure, but similar results hold if we only use domestic intermediates. Table 1 provides the average linkages among sectors for emerging and advanced economies. Three main facts emerge.

Fact 1 In advanced economies, the commodity sector is less connected, as a supplier,to the rest of the economy.

Fact 2 In advanced economies, the tradable sector is less connected, as a supplier, to the non-tradable sector.

Fact 3 In advanced economies, the non-tradable sector, as a supplier, is more connected to the rest of the sectors of the economy.

Table 1: Average Intermediate Input Shares: Emerging and Advanced

	(1)	(2)	(3)
	Emerging	Advanced	% Difference
α_T^T	34.68	32.60	-5.99%
α_T^{NT}	17.19	13.51	-21.45%
α_T^C	19.87	22.14	11.43%
α_{NT}^T	11.36	16.22	42.83%
α_{NT}^{NT}	18.43	24.72	34.13%
α_{NT}^C	7.53	11.18	48.53%
α_C^T	11.37	6.66	-41.44%
α_C^{NT}	4.61	1.55	-66.40%
α_C^C	26.59	29.48	10.86%

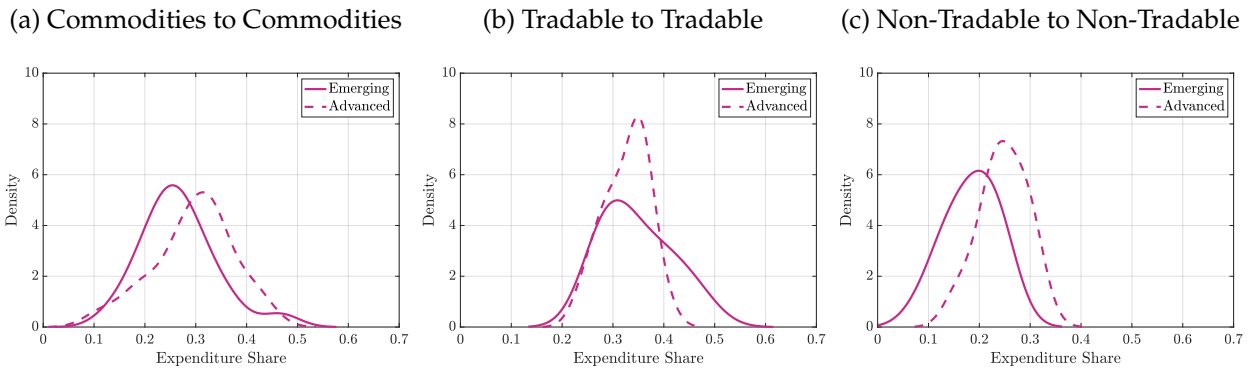
Note: This table reports the average intermediate input share (in percentage points) for emerging economies (column 1) and advanced economies (column 2). Column 3 reports the percent difference between advanced and emerging. α_T^T, α_T^{NT}, α_T^C represent the share of intermediate inputs from sector T in the gross output of sector T, C, and NT, respectively.

To put these numbers into perspective, consider an off-the-shelf production network model (e.g., Acemoglu et al., 2012). In these models, the Leontief inverse elements of a supplier drive the aggregate effects of sectoral shocks. Besides input-output shares, we consider average consumption shares of 4% , 19% , 77% consumption expenditure share in commodities, tradable, and non-tradables. We keep them constant across economies to evaluate the differences between production structures. The differences observed in Table 1 imply that a 10% decrease in the TFP of the commodity sector generates a 1.36% decline in advanced economies' real GDP and a 2.25% decline in emerging economies. These differences do not yet consider the main ingredients of our model: the household borrowing constraint and its further interactions with production linkages. These differences do not consider non-linearities generated from non-unitary consumption and production elasticities (Baqaee and Farhi, 2019).

The next figures present the kernel density of intermediate input shares across emerging and advanced economies. Figure 3 plots the own sector intermediate input share.

Advanced economies commodity sector and non-tradable sector are more connected to itself than emerging economies. The tradable sector is, on average, more connected in emerging countries, but mainly due to the large right tail. The median emerging country and the median advanced country display a similar own tradable sector linkage.

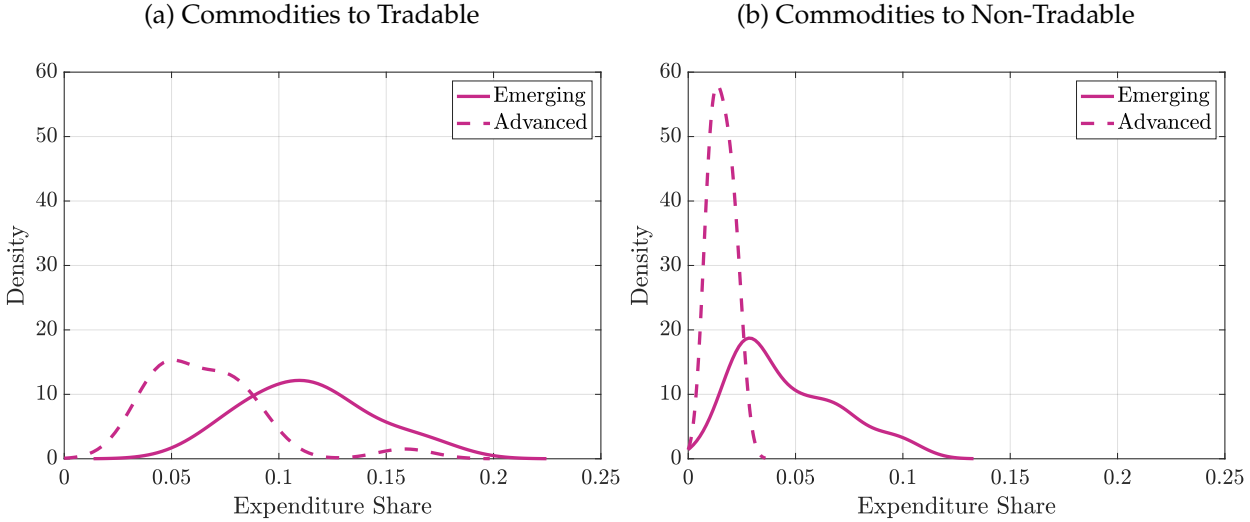
Figure 3: Distribution of intermediate input shares from the same sector



Note: This figure shows the intermediate input usage from the same sector. Solid lines depict emerging markets. Dashed lines depict advanced economies.

Figure 4 shows the importance of commodity inputs in tradable and non-tradable sectors' production. There is a clear dominance in emerging economies in how important commodity inputs are in the production of tradables and non-tradables.

Figure 4: Use of commodity inputs by tradables and non-tradable sectors

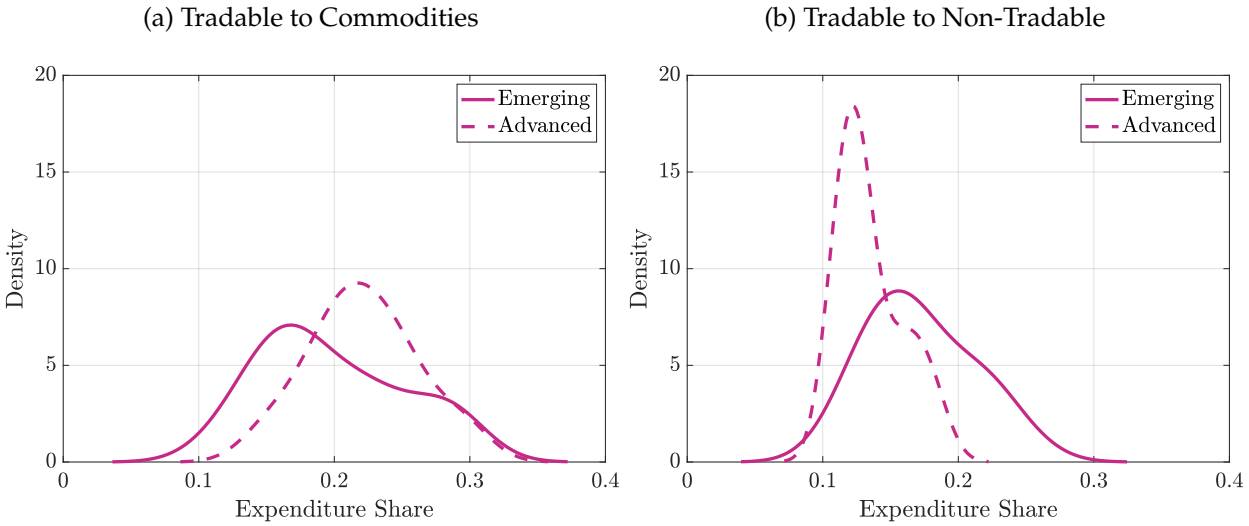


Note: This figure shows the intermediate input usage of commodities by the tradable and non-tradable sectors. Solid lines depict emerging markets. Dashed lines depict advanced economies.

The results in Figure 5 highlight a clear heterogeneity in the production structure of

commodity sectors and non-tradable sectors in emerging vs. advanced economies. While commodity sectors in advanced economies use a larger share of tradable inputs, compared to emerging economies, this is not the case for non-tradable sectors. Non-tradable sectors in emerging countries use a significantly larger share of intermediates from the tradable sector compared to advanced economies.

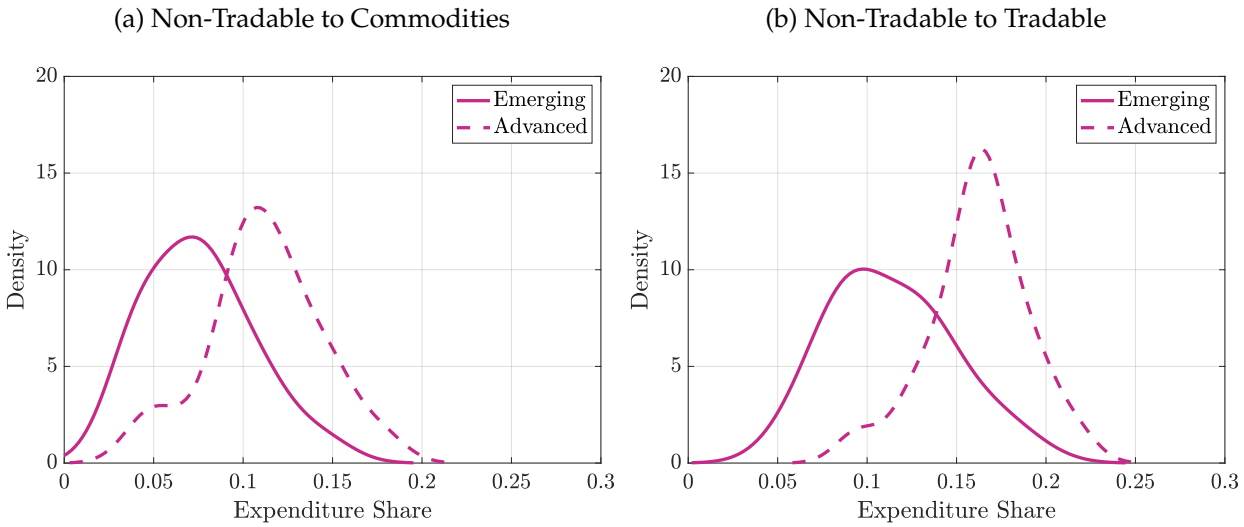
Figure 5: Use of tradable inputs by commodity and non-tradable sectors



Note: This figure shows the intermediate input usage of tradables by the commodity and non-tradable sector. Solid lines depict emerging markets. Dashed lines depict advanced economies.

Finally, Figure 6 shows that the commodity sector and the tradable sector in advanced economies use a significantly larger share of inputs from the non-tradable sector, compared to emerging economies.

Figure 6: Use of non-tradable inputs by commodities and tradable sectors



Note: This figure shows the intermediate input usage of non-tradable by the commodity and tradable sector. Solid lines depict emerging markets. Dashed lines depict advanced economies.

5 Quantitative Model

This section extends the simple 2-sector model studied in Section 2.4 so that it can capture the differences in the cross-country production structures documented in Section 4. The quantitative model extends the simple framework by allowing for two different tradable goods: a commodity good where the price is fully exogenous, and a differentiated tradable good where the country faces a demand with finite elasticity. The model is also dynamic and features aggregate risk fueled by sector-specific shocks.

5.1 Households

The representative household maximizes the present discounted value of utility:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right], \text{ where } \beta \in (0,1). \tag{17}$$

The household consumes a tradable basket and a non-tradable good according to the following consumption aggregator:

$$c_t = \left((1-\omega)(c_t^N)^{1-\eta} + \omega (c_t^T)^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{18}$$

where c_t^T is a composite tradable good, which is defined by

$$c_t^T = \left[\omega_T (c_t^C)^{1-\eta_T} + (1-\omega_T)(c_t^X)^{1-\eta_T}\right]^{\frac{1}{1-\eta_T}}, \tag{19}$$

where c_t^C is a commodity tradable good and c_t^X is a non-commodity tradable good.[8] Both tradable goods are produced domestically and can be exported. The household trades one-period non-state contingent bonds with exogenous price q and receives flow income from firms' profits in the tradable (π_t^C and π_t^X) sectors and the non-tradable sector (π_t^N). The budget constraint is then given by:

$$p_t^N c_t^N + p_t^C c_t^C + p_t^X c_t^X + q b_{t+1}^* = \pi_t^N + \pi_t^C + \pi_t^X + b_t^*, \tag{20}$$

where the left-hand side represents households' consumption of non-tradable and tradable goods plus bond purchases. The right-hand side is the income of the household, which combines profits and bond positions at the beginning of the period. All prices are denominated in units of the commodity good, which is the numeraire. Therefore, p_t^X and p_t^N are the relative price of non-commodity and non-tradable goods.

The household faces an occasionally binding borrowing constraint. More specifically, households can borrow a up to a fraction κ of their flow income, which is the sum of total sectoral profits

$$q b_{t+1} \geq -\kappa(\pi_t^N + \pi_t^C + \pi_t^X). \tag{21}$$

Finally, there is a foreign demand for the non-commodity good. Similarly to Benguria et al. (2024) and Saffie et al. (2020), we assume that the foreign demand for the differentiated tradable good has the same price elasticity as the domestic demand. The size of foreign demand is parametrized by the demand shifter Γ. Then, the domestic demand for the non-commodity good is

$$c_t^X = (1-\omega_T)^{\frac{1}{\eta_T}} \left(\frac{p_t^X}{p_t^T}\right)^{-\frac{1}{\eta_T}} c_t^T, \tag{22}$$

the foreign demand is

$$\hat{c}_t^X = \Gamma \left(\frac{p_t^X}{p_t^T}\right)^{-\frac{1}{\eta_T}}, \tag{23}$$

[8] To simplify notation, we denote the state contingent consumption plan by c_t instead of $c(s^t)$. The same abuse of notation is applied to other variables.

and the market clearing requires:

$$c_t^X + \hat{c}_t^X = y_t^X - \sum_i m_{X,t}^i. \tag{24}$$

5.2 Production Structure

The production of each good $i \in \{C, X, N\}$ uses intermediate goods from every sector. In particular,

$$y_t^i = z_t^i (m_{N,t}^i)^{\alpha_N^i} (m_{X,t}^i)^{\alpha_X^i} (m_{C,t}^i)^{\alpha_C^i}, \text{ where } \alpha_N^i + \alpha_X^i + \alpha_C^i \in [0, 1), \tag{25}$$

and z_t^i is a sector specific productivity shock that follows an AR(1) with persistence ρ_i and dispersion σ_i. Firm profits are given by

$$\pi_t^C = \max_{\{m_{N,t}^C, m_{X,t}^C, m_{C,t}^C\}} z_t^C (m_{N,t}^C)^{\alpha_N^C} (m_{X,t}^C)^{\alpha_X^C} (m_{C,t}^C)^{\alpha_C^X} - m_{C,t}^C - p_t^X m_{X,t}^C - p_t^N m_{N,t}^C \tag{26}$$

$$\pi_t^X = \max_{\{m_{N,t}^X, m_{X,t}^X, m_{C,t}^X\}} p_t^x z_t^X (m_{N,t}^X)^{\alpha_N^X} (m_{X,t}^X)^{\alpha_X^X} (m_{C,t}^X)^{\alpha_C^X} - m_{C,t}^X - p_t^X m_{X,t}^X - p_t^N m_{N,t}^X \tag{27}$$

$$\pi_t^N = \max_{\{m_{N,t}^N, m_{X,t}^N, m_{C,t}^N\}} p_t^N z_t^N (m_{N,t}^N)^{\alpha_N^N} (m_{X,t}^N)^{\alpha_X^N} (m_{C,t}^N)^{\alpha_C^N} - m_{C,t}^N - p_t^N m_{X,t}^N - p_t^N m_{N,t}^N \tag{28}$$

5.3 Equilibrium Characterization

The equilibrium conditions of the representative households are the following:

$$\frac{p_t^N}{p_t^T} = \frac{1-\omega}{\omega} \left(\frac{c_t^T}{c_t^N} \right)^{\eta} \tag{29}$$

$$p_t^X = \frac{1-\omega_T}{\omega_T} \left(\frac{c_t^C}{c_t^X} \right)^{\eta_T} \tag{30}$$

$$u_C(t) = q^{-1} \beta \mathbb{E}_t \left[u_C(t+1) \right] + \mu_t \tag{31}$$

$$c_t^X + \hat{c}_t^X = y_t^X - \sum_i m_{X,t}^i \tag{32}$$

$$c_t^N = y_t^N - \sum_i m_{N,t}^i \tag{33}$$

$$c_t^C + q b_{t+1} = y_t^C - \sum_i m_{C,t}^i + b_t + p_t^X \hat{c}_t^X \tag{34}$$

$$q b_{t+1} \geq -\kappa (\pi_t^N + \pi_t^C + \pi_t^X) \tag{35}$$

$$p_t^T = \left[\omega_T^{\frac{1}{\eta_T}} + (1-\omega_T)^{\frac{1}{\eta_T}} (p_t^X)^{\frac{\eta_T - 1}{\eta_T}} \right]^{\frac{\eta_T}{\eta_T - 1}} \tag{36}$$

where $u_C(t)$ denotes the marginal utility of consumption of the commodity good in period t, μ_t represents the Lagrange multiplier of the borrowing constraint, and p_t^T corresponds to the price index of the tradable composite.

5.4 Calibration and Quantitative Analysis

TBA

6 Conclusion and Future Work

This study highlights the crucial role of production network structures in shaping the severity and propagation of financial crises. We show how intersectoral linkages influence collateral constraints and macroeconomic stability, offering a novel perspective on the disparities between emerging and advanced economies. Our empirical findings reinforce the idea that network structures, beyond traditional financial indicators, can significantly impact the effectiveness of macroprudential policies. As part of our ongoing work, we are developing a more comprehensive three-sector quantitative model to further quantify these effects and provide actionable insights into the design of optimal macroprudential interventions. Our findings suggest that policymakers should consider production network structures as a key factor in financial stability frameworks.

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Table 2: Sector Names and Definitions

Sector Name	**Definition**
Agriculture, hunting, forestry	Commodity
Fishing and aquaculture	Commodity
Mining and quarrying, energy producing products	Commodity
Mining and quarrying, non-energy producing products	Commodity
Mining support service activities	Tradable
Food products, beverages and tobacco	Tradable
Textiles, textile products, leather and footwear	Tradable
Wood and products of wood and cork	Tradable
Paper products and printing	Tradable
Coke and refined petroleum products	Tradable
Chemical and chemical products	Tradable
Pharmaceuticals, medicinal chemical and botanical products	Tradable
Rubber and plastics products	Tradable
Other non-metallic mineral products	Tradable
Basic metals	Commodity
Fabricated metal products	Tradable
Computer, electronic and optical equipment	Tradable
Electrical equipment	Tradable
Machinery and equipment, nec	Tradable
Motor vehicles, trailers and semi-trailers	Tradable
Other transport equipment	Tradable
Manufacturing nec; repair and installation of machinery	Tradable
Electricity, gas, steam and air conditioning supply	Non-Tradable
Water supply; sewerage, waste management and remediation act.	Non-Tradable
Construction	Non-Tradable
Wholesale and retail trade; repair of motor vehicles	Tradable
Land transport and transport via pipelines	Tradable
Water transport	Tradable
Air transport	Tradable
Warehousing and support activities for transportation	Tradable
Postal and courier activities	Non-Tradable
Accommodation and food service activities	Non-Tradable
Publishing, audiovisual and broadcasting activities	Non-Tradable
Telecommunications	Non-Tradable
IT and other information services	Tradable
Financial and insurance activities	Non-Tradable
Real estate activities	Non-Tradable
Professional, scientific and technical activities	Non-Tradable
Administrative and support services	Non-Tradable
Public administration and defence; compulsory social security	Non-Tradable
Education	Non-Tradable
Human health and social work activities	Non-Tradable
Arts, entertainment and recreation	Non-Tradable
Other service activities	Non-Tradable

A Proofs

A.1 Proof of Proposition 1

Totally differentiating the PP curve equation (14)

$$\mathrm{d}\log p_t^N = \eta \mathrm{d}\log c^T - \eta \mathrm{d}\log c_t^N.$$

This delivers the solution

$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log c_t^T} = \frac{\eta}{1+\eta\frac{\mathrm{d}\log c_t^N}{\mathrm{d}\log p_t^N}}.$$

Since $\eta > 0$ it suffices to show that $\frac{\mathrm{d}\log c_t^N}{\mathrm{d}\log p_t^N} \geq 0$. Differentiating the non-tradable market clearing condition

$$\begin{aligned}
\mathrm{d}\log c_t^N &= \frac{(1-\alpha_N^N)y_t^N}{c_t^N}\mathrm{d}\log y_t^N - \alpha_N^T\frac{y_t^T}{p_t^N c_t^N}\left(\mathrm{d}\log y_t^T - \mathrm{d}\log p_t^N\right), \\
\frac{\mathrm{d}\log c_t^N}{\mathrm{d}\log p_t^N} &= \frac{(1-\alpha_N^N)y_t^N}{c_t^N}\frac{\mathrm{d}\log y_t^N}{\mathrm{d}\log p_t^N} - \alpha_N^T\frac{y_t^T}{p_t^N c_t^N}\left(\frac{\mathrm{d}\log y_t^T}{\mathrm{d}\log p_t^N} - 1\right) \geq 0,
\end{aligned}$$

which is positive since $\frac{\mathrm{d}\log y_t^N}{\mathrm{d}\log p_t^N} = \alpha_T^N/(1-\alpha_N^N-\alpha_T^N) \geq 0$ and $\frac{\mathrm{d}\log y_t^T}{\mathrm{d}\log p_t^N} = -\alpha_N^T/(1-\alpha_T^T-\alpha_N^T) \leq 0$. This concludes the proof.

A.2 Proof of Proposition 2

Totally differentiating equation (15)

$$\mathrm{d}c^T = \kappa_N \mathrm{d}\pi_t^N(p_t^N; z_t^N) + (1+\kappa_T)\mathrm{d}\pi_t^T(p_t^N; z_t^T). \tag{37}$$

Since we are only allowing the non-tradable price and tradable consumption to change, we have that

$$\mathrm{d}\pi_t^N = \frac{\partial \pi_t^N}{\partial p_t^N}\mathrm{d}p_t^N, \tag{38}$$

$$\mathrm{d}\pi_t^T = \frac{\partial \pi_t^T}{\partial p_t^N}\mathrm{d}p_t^N. \tag{39}$$

Replacing equation (38) and (39) in equation (37) and solving out for $\mathrm{d}p_t^N/\mathrm{d}c_t^T$, we have

$$\frac{\mathrm{d}p_t^N}{\mathrm{d}c_t^T} = \frac{1}{\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N} + (1+\kappa_T)\frac{\partial \pi_t^T}{\partial p_t^N}}. \tag{40}$$

Hence, the slope of the BB curve inherits the sign of $\kappa_N \frac{\partial \pi_t^N}{\partial p_t^N} + (1+\kappa_T)\frac{\partial \pi_t^T}{\partial p_t^N}$, as stated in Proposition 2. This concludes the proof.

A.3 Proof of Proposition 3

The proof involves several steps. For convenience, we rewrite the equilibrium system in terms of outputs and prices below:

$$p_t^N = \left(\frac{c_t^T}{c_t^N(p_t^N; z_t^N, z_t^T)}\right)^{\eta}\left(\frac{1-\omega}{\omega}\right) \qquad \text{(PP Curve)} \tag{41}$$

$$\begin{aligned} c_t^T = &\left(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N\right) p_t^N y_t^N(p_t^N; z_t^N) \\ &+ (\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T)) y_t^T(p_t^N; z_t^T) + b_0 \qquad \text{(BB Curve)}, \end{aligned} \tag{42}$$

Define elasticities as $\varepsilon_x^y = \frac{\partial \log y}{\partial \log x}$. Totally differentiating both curves, I get

$$\mathrm{d}\log p_t^N = \eta(\mathrm{d}\log c_t^T - \mathrm{d}\log c_t^N) \tag{43}$$

$$\mathrm{d}\log c_t^T = \gamma_t^N(\mathrm{d}\log p_t^N + \mathrm{d}\log y_t^N) + \gamma_t^T \mathrm{d}\log y_t^T \tag{44}$$

We need to solve out $\mathrm{d}\log y_t^N, \mathrm{d}\log y_t^T, \mathrm{d}\log c_t^N$. Since these are function of p_t^N and (z_t^N, z_t^T), we have

$$\mathrm{d}\log y_t^N = \varepsilon_{p_t^N}^{y_t^N}\mathrm{d}\log p_t^N + \varepsilon_{z_t^N}^{y_t^N}\mathrm{d}\log z_t^N \tag{45}$$

$$\mathrm{d}\log y_t^T = \varepsilon_{p_t^N}^{y_t^T}\mathrm{d}\log p_t^N + \varepsilon_{z_t^T}^{y_t^T}\mathrm{d}\log z_t^T \tag{46}$$

$$\mathrm{d}\log c_t^N = \varepsilon_{p_t^N}^{c_t^N}\mathrm{d}\log p_t^N + \varepsilon_{z_t^N}^{c_t^N}\mathrm{d}\log z_t^N + +\varepsilon_{z_t^T}^{c_t^N}\mathrm{d}\log z_t^T \tag{47}$$

Replacing into equations (43) and (44), we get

$$\mathrm{d}\log c_t^T = \left(\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N}\right)\mathrm{d}\log p_t^N + \varepsilon^{c_t^N}_{z_t^N}\mathrm{d}\log z_t^N + \varepsilon^{c_t^N}_{z_t^T}\mathrm{d}\log z_t^T \tag{48}$$

$$\mathrm{d}\log c_t^T = \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)\mathrm{d}\log p_t^N + \gamma_t^N\varepsilon^{y_t^N}_{z_t^N}\mathrm{d}\log z_t^N + \gamma_t^T\varepsilon^{y_t^T}_{z_t^T}\mathrm{d}\log z_t^T. \tag{49}$$

For future reference, let

$$\varepsilon^{PP} = \left(\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N}\right)^{-1}$$

be the slope of the PP curve and

$$\varepsilon^{BB} = \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)^{-1}$$

be the slope of the BB curve.

This system can be expressed as

$$\begin{bmatrix} 1 & -\left(\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N}\right) \\ 1 & -\left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right) \end{bmatrix} \begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} = \begin{bmatrix} \varepsilon^{c_t^N}_{z_t^N} & \varepsilon^{c_t^N}_{z_t^T} \\ \gamma_t^N\varepsilon^{y_t^N}_{z_t^N} & \gamma_t^T\varepsilon^{y_t^T}_{z_t^T} \end{bmatrix} \begin{bmatrix} \mathrm{d}\log z_t^N \\ \mathrm{d}\log z_t^T \end{bmatrix} \tag{50}$$

If $\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N} \neq \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)$, we can invert the matrix in the left-hand side and write it as

$$\begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} = \mathcal{H}\mathcal{P}\begin{bmatrix} \mathrm{d}\log z_t^N \\ \mathrm{d}\log z_t^T \end{bmatrix}, \tag{51}$$

where

$$\mathcal{H} = \begin{bmatrix} 1 & -\left(\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N}\right) \\ 1 & -\left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right) \end{bmatrix}^{-1}$$

$$\mathcal{H} = \frac{1}{\left[\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N}\right] - \left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right)} \begin{bmatrix} -\left(\gamma_t^N(1+\varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T\varepsilon^{y_t^T}_{p_t^N}\right) & \frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N} \\ -1 & 1 \end{bmatrix},$$

captures the general equilibrium effects. The second matrix,

$$\mathcal{P} = \begin{bmatrix} \varepsilon^{c_t^N}_{z_t^N} & \varepsilon^{c_t^N}_{z_t^T} \\ \gamma_t^N \varepsilon^{y_t^N}_{z_t^N} & \gamma_t^T \varepsilon^{y_t^T}_{z_t^T} \end{bmatrix}$$

captures partial equilibrium effects. It considers the direct effects of tradable and non-tradable productivity on the shifters of each curve. This represents shifts of both the PP and BB curves. The general equilibrium effects can be thought of as movements along each curve. This way of formulating the problem follows recent work in the heterogeneous agents literature; see, for example, Auclert et al. (2021), Wolf (2023), and Auclert et al. (2024). We apply the same logic to a multi-sector context.

We can find an expression for $\varepsilon^{c_t^N}_{p_t^N}$ as a function of supply elasticities using the non-tradable market clearing condition

$$\varepsilon^{c_t^N}_{p_t^N} = \delta_t^N \varepsilon^{y_t^N}_{p_t^N} + (1 - \delta_t^N)(\varepsilon^{y_t^T}_{p_t^N} - 1),$$

where $\delta_t^N = (1 - \alpha_N^N) y_t^N / c_t^N$ and $1 - \delta_t^N = -(\alpha_N^T y_t^T / p_t^N) / c_t^N$. Note that by construction $\delta_t^N \geq 1$. This is important to sign the determinant. Using this result, the determinant can be written as

$$\begin{aligned} \left[\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N}\right] - \left(\gamma_t^N (1 + \varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T \varepsilon^{y_t^T}_{p_t^N}\right) &= \left[\frac{1}{\eta} + \delta_t^N \varepsilon^{y_t^N}_{p_t^N} + (1 - \delta_t^N)(\varepsilon^{y_t^T}_{p_t^N} - 1)\right] - \left(\gamma_t^N (1 + \varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T \varepsilon^{y_t^T}_{p_t^N}\right) \\ &= \frac{1}{\eta} - \gamma_t^N + (\delta_t^N - 1) + (\delta_t^N - \gamma_t^N)\varepsilon^{y_t^N}_{p_t^N} + (1 - \gamma_t^T - \delta_t^N)\varepsilon^{y_t^T}_{p_t^N} \end{aligned}$$

Now, notice that we know the following $\eta \in (0, \infty)$, $\gamma_t^N \in [0, 1]$, $\gamma_t^T \geq 0$, $\delta_t^N \geq 1$, $\varepsilon^{y_t^N}_{p_t^N} \geq 0$ and $\varepsilon^{y_t^T}_{p_t^N} \leq 0$. For γ_t^N we need a restriction that we discuss below. Therefore, the determinant is **positive** since $\eta^{-1} - \gamma_t^N + (\delta_t^N - 1) > 0$, $(\delta_t^N - \gamma_t^N)\varepsilon^{y_t^N}_{p_t^N} > 0$ and $(1 - \gamma_t^T - \delta_t^N)\varepsilon^{y_t^T}_{p_t^N} > 0$. In what follows, let:

$$\mathcal{K} = \frac{1}{\left[\frac{1}{\eta} + \varepsilon^{c_t^N}_{p_t^N}\right] - \left(\gamma_t^N (1 + \varepsilon^{y_t^N}_{p_t^N}) + \gamma_t^T \varepsilon^{y_t^T}_{p_t^N}\right)} \tag{52}$$

We can now assess the effects of tradable productivity on tradable consumption and the non-tradable price. The effect of tradable productivity on tradable consumption is

$$
\begin{aligned}
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left(-\left(\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N})+\gamma_t^T\varepsilon_{p_t^N}^{y_t^T}\right)\varepsilon_{z_t^T}^{c_t^N}+\left(\frac{1}{\eta}+\varepsilon_{p_t^N}^{c_t^N}\right)\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right)\\
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left(-\left(\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N})+\gamma_t^T\varepsilon_{p_t^N}^{y_t^T}\right)(1-\delta_t^N)\varepsilon_{z_t^T}^{y_t^T}+\left(\frac{1}{\eta}+\delta_t^N\varepsilon_{p_t^N}^{y_t^N}+(1-\delta_t^N)(\varepsilon_{p_t^N}^{y_t^T}-1)\right)\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right)\\
&= \mathcal{K}\left(\left[-\left(\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N})+\gamma_t^T\varepsilon_{p_t^N}^{y_t^T}\right)(1-\delta_t^N)+\left(\frac{1}{\eta}+\delta_t^N\varepsilon_{p_t^N}^{y_t^N}+(1-\delta_t^N)(\varepsilon_{p_t^N}^{y_t^T}-1)\right)\gamma_t^T\right]\varepsilon_{z_t^T}^{y_t^T}\right)\\
&= \mathcal{K}\left(\left[-\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N})(1-\delta_t^N)-\gamma_t^T(1-\delta_t^N)\varepsilon_{p_t^N}^{y_t^T}+\delta_t^N\gamma_t^T\varepsilon_{p_t^N}^{y_t^N}+(1-\delta_t^N)\gamma_t^T(\varepsilon_{p_t^N}^{y_t^T}-1)\right]\varepsilon_{z_t^T}^{y_t^T}\right)\\
&\quad+\mathcal{K}\left(\frac{\gamma_t^T}{\eta}\varepsilon_{z_t^T}^{y_t^T}\right)\\
&= \mathcal{K}\left[-\gamma_t^N(1+\varepsilon_{p_t^N}^{y_t^N})(1-\delta_t^N)+\delta_t^N\gamma_t^T\varepsilon_{p_t^N}^{y_t^N}-(1-\delta_t^N)\gamma_t^T+\frac{\gamma_t^T}{\eta}\right]\varepsilon_{z_t^T}^{y_t^T}\\
&= \mathcal{K}\left[-(\gamma_t^N+\gamma_t^T)(1-\delta_t^N)-\left(\gamma_t^N(1-\delta_t^N)-\delta_t^N\gamma_t^T\right)\varepsilon_{p_t^N}^{y_t^N}+\frac{\gamma_t^T}{\eta}\right]\varepsilon_{z_t^T}^{y_t^T}\\
\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left[(\gamma_t^N+\gamma_t^T)(\delta_t^N-1)+\left(\gamma_t^N(\delta_t^N-1)+\delta_t^N\gamma_t^T\right)\varepsilon_{p_t^N}^{y_t^N}+\frac{\gamma_t^T}{\eta}\right]\varepsilon_{z_t^T}^{y_t^T}>0
\end{aligned}
$$

Hence, tradable consumption moves in the same direction as tradable productivity. For the non-tradable price, we have:

$$
\begin{aligned}
\mathrm{d}\log p_t^N &= \mathcal{K}\left(-\varepsilon_{z_t^T}^{c_t^N}+\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right)\mathrm{d}\log z_t^T\\
\mathrm{d}\log p_t^N &= \mathcal{K}\left(-(1-\delta_t^N)\varepsilon_{z_t^T}^{y_t^T}+\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}\right)\mathrm{d}\log z_t^T\\
\mathrm{d}\log p_t^N &= \mathcal{K}\left(-(1-\delta_t^N)+\gamma_t^T\right)\varepsilon_{z_t^T}^{y_t^T}\mathrm{d}\log z_t^T\\
\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} &= \mathcal{K}\left((\delta_t^N-1)+\gamma_t^T\right)\varepsilon_{z_t^T}^{y_t^T}>0
\end{aligned}
$$

Hence, in response to a negative tradable productivity shock, non-tradable prices and tradable consumption declines in this economy. This concludes the proof.

Justification for $\gamma_t^N \in [0,1]$. Note that the key objects in all the above expressions are

$$\begin{aligned}
\delta_t^N &= \frac{(1-\alpha_N^N)y_t^N}{c_t^N}; \qquad (1-\delta_t^N) = -\alpha_N^T \frac{y_t^T/p_t^N}{c_t^N} \\
\gamma_t^N &= \frac{\xi^N p_t^N y_t^N}{c_t^T} = \frac{((\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N))p_t^N y_t^N}{c_t^T} \\
\gamma_t^T &= \frac{\xi^T y_t^T}{c_t^T} = (\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T))\frac{y_t^T}{c_t^T} \\
\varepsilon_{z_t^T}^{y_t^T} &= \frac{1}{1-\alpha_T^T-\alpha_N^T}
\end{aligned}$$

We now justify why $\gamma_t^N \in [0,1]$. For this to hold, it requires $\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N \geq 0$. What is this restriction? It turns out that this has a nice expression

$$\begin{aligned}
\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N \geq 0 & \\
\kappa &\geq \frac{\alpha_T^N}{(1-\alpha_N^N-\alpha_T^N)} \\
\kappa &\geq \frac{m_{Tt}^N}{p_t^N y_t^N (1-\alpha_N^N-\alpha_T^N)} \\
\kappa &\geq \frac{m_T^N}{\pi_t^N} = \tilde{\alpha}_T^N
\end{aligned}$$

This restriction says that provided that κ is larger than the expenditure of non-tradable on tradables as a share of profits, then $\gamma_t^N \geq 0$. Otherwise, it can be negative. This happens for large values of α_T^N. In other words, for $\gamma_t^N \geq 0$, we require the response of tradable consumption to increase with changes in non-tradable profits. In an endowment economy or an economy where the tradable sector does not buy from the non-tradable sector, this is equivalent to assume that the BB curve is upward sloping, as we discussed in Proposition 2.

Can $\gamma_t^N \geq 1$? It cannot. Suppose it is

$$\begin{aligned}
\gamma_t^N &\geq 1 \\
\frac{(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N}{c_t^T} &\geq 1
\end{aligned}$$

$$(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N \geq c_t^T$$
$$(\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N \geq (\kappa(1-\alpha_N^N-\alpha_T^N)-\alpha_T^N)p_t^N y_t^N + ((\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T)))y_t^T + b_0$$
$$0 \geq (\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T))y_t^T + b_0,$$

which is a contradiction since $(\kappa(1-\alpha_N^T-\alpha_T^T)+(1-\alpha_T^T))y_t^T + b_0 \geq 0$.

Hence, $\gamma_t^N \in [0,1]$ iff $\kappa \geq \tilde{\alpha}_T^N$.

A.4 Proof of Proposition 4

To derive this expression, let us start again from the PP and BB curve general definitions:

$$p_t^N = \left(\frac{c_t^T}{c_t^N(p_t^N; z_t^N, z_t^T)}\right)^\eta \left(\frac{1-\omega}{\omega}\right) \tag{53}$$
$$c_t^T = \kappa_N \pi_t^N(p_t^N; z_t^N) + (1+\kappa_T)\pi_t^T(p_t^N; z_t^T) + b_0 \tag{54}$$

Taking total differential with respect to both p_t^N and c_t^T, we get for the PP curve:

$$\mathrm{d}\log p_t^N = \eta \mathrm{d}\log c_t^T - \eta \frac{\partial \log c_t^N}{\partial \log p_t^N} \mathrm{d}\log p_t^N$$

Define $\varepsilon_t^{PP} = \frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log c_t^T}$ as the elasticity of the PP curve, and so

$$\varepsilon_t^{PP} = \frac{\eta}{1+\eta\frac{\partial \log c_t^N}{\partial \log p_t^N}}$$

Similarly, differentiate the BB curve with respect to c_t^T and p_t^N to get

$$\mathrm{d}\log c_t^T = \frac{\kappa_N \pi_t^N}{c_t^T}\frac{\partial \pi_t^N}{\partial p_t^N}\frac{p_t^N}{\pi_t^N}\mathrm{d}\log p_t^N + (1+\kappa_T)\frac{\pi_t^T}{c_t^T}\frac{\partial \pi_t^T}{\partial p_t^N}\frac{p_t^N}{\pi_t^T}\mathrm{d}\log p_t^N$$
$$\mathrm{d}\log c_t^T = \frac{\kappa_N \pi_t^N}{c_t^T}\varepsilon_{p_t^N}^{\pi_t^N}\mathrm{d}\log p_t^N + (1+\kappa_T)\frac{\pi_t^T}{c_t^T}\varepsilon_{p_t^N}^{\pi_t^T}\mathrm{d}\log p_t^N$$
$$\mathrm{d}\log c_t^T = \gamma_t^N \varepsilon_{p_t^N}^{\pi_t^N}\mathrm{d}\log p_t^N + \gamma_t^T \varepsilon_{p_t^N}^{\pi_t^T}\mathrm{d}\log p_t^N$$

$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log c_t^T} = \frac{1}{\gamma_t^N \varepsilon_{p_t^N}^{\pi_t^N} + \gamma_t^T \varepsilon_{p_t^N}^{\pi_t^T}}$$
$$\varepsilon_t^{BB} = \frac{1}{\gamma_t^N \varepsilon_{p_t^N}^{\pi_t^N} + \gamma_t^T \varepsilon_{p_t^N}^{\pi_t^T}}.$$

Using these results we can rewrite the general expression in equation (51) as

$$\begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \begin{bmatrix} -(\varepsilon_t^{BB})^{-1} & (\varepsilon_t^{PP})^{-1} \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \varepsilon_{z_t^N}^{c_t^N} & \varepsilon_{z_t^T}^{c_t^N} \\ \gamma_t^N \varepsilon_{z_t^N}^{y_t^N} & \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} \end{bmatrix} \begin{bmatrix} \mathrm{d}\log z_t^N \\ \mathrm{d}\log z_t^T \end{bmatrix}$$

Now, consider the case where $\mathrm{d}\log z_t^T \neq 0$ and $\mathrm{d}\log z_t^N = 0$. Then,

$$\begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \begin{bmatrix} -(\varepsilon_t^{BB})^{-1} & (\varepsilon_t^{PP})^{-1} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{z_t^T}^{c_t^N} \\ \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} \end{bmatrix} \mathrm{d}\log z_t^T$$
$$\begin{bmatrix} \mathrm{d}\log c_t^T \\ \mathrm{d}\log p_t^N \end{bmatrix} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \begin{bmatrix} -\varepsilon_{z_t^T}^{c_t^N} (\varepsilon_t^{BB})^{-1} + \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} (\varepsilon_t^{PP})^{-1} \\ -\varepsilon_{z_t^T}^{c_t^N} + \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} \end{bmatrix} \mathrm{d}\log z_t^T$$

Therefore, the total effect on tradable consumption after a change in tradable productivity is

$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \left(-\varepsilon_{z_t^T}^{c_t^N} (\varepsilon_t^{BB})^{-1} + \gamma_t^T \varepsilon_{z_t^T}^{y_t^T} (\varepsilon_t^{PP})^{-1} \right).$$

Under the diagonal and the diversified example, we have that $\varepsilon_{z_t^T}^{c_t^N} = 0$ since the tradable sector does not directly demand from the non-tradable sector. For the same reason, we have that across the two economies: $\varepsilon_{z_t^T}^{y_t^T}(I) = \varepsilon_{z_t^T}^{y_t^T}(D) = \frac{1}{1-\alpha_T^T(I)} = \frac{1}{1-\alpha_T^T(D)}$. Solving out,

$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{1}{(\varepsilon_t^{PP})^{-1} - (\varepsilon_t^{BB})^{-1}} \left(\gamma_t^T \varepsilon_{z_t^T}^{y_t^T} (\varepsilon_t^{PP})^{-1} \right),$$
$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{\varepsilon_t^{BB}\varepsilon_t^{PP}}{\varepsilon_t^{BB} - (\varepsilon_t^{PP})} \left(\gamma_t^T \varepsilon_{z_t^T}^{y_t^T} (\varepsilon_t^{PP})^{-1} \right),$$
$$\frac{\mathrm{d}\log c_t^T}{\mathrm{d}\log z_t^T} = \frac{\varepsilon_t^{BB}}{\varepsilon_t^{BB} - \varepsilon_t^{PP}} \gamma_t^T \varepsilon_{z_t^T}^{y_t^T}.$$

Now, we wish to compare the two economies. For tradable consumption in the diversified economy to react less than the island economy, we require

$$\left|\frac{\mathrm{d}\log(c_t^T)^{\mathrm{NT}\leftarrow\mathrm{T}}}{\mathrm{d}\log z_t^T}\right| \leq \left|\frac{\mathrm{d}\log(c_t^T)^{\mathrm{Diagonal}}}{\mathrm{d}\log z_t^T}\right|.$$

$$\left|\frac{\varepsilon_t^{BB}(N)}{\varepsilon_t^{BB}(N)-\varepsilon_t^{PP}(N)}\gamma_t^T(N)\varepsilon_{z_t^T}^{y_t^T}(N)\right| \leq \left|\frac{\varepsilon_t^{BB}(D)}{\varepsilon_t^{BB}(D)-\varepsilon_t^{PP}(D)}\gamma_t^T(D)\varepsilon_{z_t^T}^{y_t^T}(D)\right|,$$

$$\left|\frac{\varepsilon_t^{BB}(N)}{\varepsilon_t^{BB}(N)-\varepsilon_t^{PP}(N)}\gamma_t^T(N)\right| \leq \left|\frac{\varepsilon_t^{BB}(D)}{\varepsilon_t^{BB}(D)-\varepsilon_t^{PP}(D)}\gamma_t^T(D)\right|,$$

evaluating at the initial marginal binding equilibrium delivers the result for tradable consumption.

For non-tradable prices, we have

$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} = \frac{1}{(\varepsilon_t^{PP})^{-1}-(\varepsilon_t^{BB})^{-1}}\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}$$

$$\frac{\mathrm{d}\log p_t^N}{\mathrm{d}\log z_t^T} = \frac{\varepsilon_t^{BB}\varepsilon_t^{PP}}{\varepsilon_t^{BB}-\varepsilon_t^{PP}}\gamma_t^T\varepsilon_{z_t^T}^{y_t^T}.$$

Evaluating both sides in both networks' structure

$$\left|\frac{\mathrm{d}\log(p_t^N)^{\mathrm{NT}\leftarrow\mathrm{T}}}{\mathrm{d}\log z_t^T}\right| \leq \left|\frac{\mathrm{d}\log(p_t^N)^{\mathrm{Diagonal}}}{\mathrm{d}\log z_t^T}\right|$$

$$\left|\frac{\varepsilon_t^{BB}(N)\varepsilon_t^{PP}(N)}{\varepsilon_t^{BB}(N)-\varepsilon_t^{PP}(N)}\gamma_t^T(N)\right| \leq \left|\frac{\varepsilon_t^{BB}(D)\varepsilon_t^{PP}(D)}{\varepsilon_t^{BB}(D)-\varepsilon_t^{PP}(D)}\gamma_t^T(D)\right|,$$

which is a less stringent condition than that of tradable consumption since $\varepsilon_t^{PP}(N) \leq \varepsilon_t^{PP}(D)$. This concludes the proof.