Misallocation and Monetary Policy: Strategic Complementarities, Uncertainty and Innovation*

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Abstract

We study the impact of monetary policy on misallocation in a macro model with heterogenous firm facing uncertainty, due to idiosyncratic productivity risk, interacting in a dynamic Bertrand oligopoly (solved with oblivious equilibria) and engaging in a cyclical innovation process. Contractionary policies induce a conspicuous shift of market shares from the bottom to the top 10%, but little effects on firm-level mark-ups and reduce mark-up dispersion (which instead rises under monopolistic competition), making the tails ratio a better sufficient statistic for misallocation. Reallocation to the top is due to a lower demand elasticity of large firms and a strategic precautionary channel, by which firms confront recessions by building market shares in an evolutionary fashion. Reallocation to the top is more pronounced in presence of a cyclical innovation process, which can also boost TFP. At aggregate level, the dynamic of investment, firm valuations and dividends are more amplified, compared to monopolistic competition, as driven by the large firms. We then estimate the impact of high frequency identified monetary policy shocks on moments and tails of the mark-up distributions estimated with Compustat data, and matched with firm uncertainty indeces or patents to identify the channels. The evidence confirms the reallocation patterns highlighted in the model.

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1. Introduction

The role of firms distributions, and their tails, in trickling down shocks and in the Shumpeterian reallocation process is becoming central in macroeconomics (see Van Reenen (2018) for a review). A large and burgeoning literature studies the role of wedges distribution, endogenous misallocation and reallocation for growth (see Restuccia and Rogerson (2008) or Hsieh and Klenow (2009); Peters (2020)). Less is known about their role for business cycle fluctuations and the policy transmission channels, despite the evidence on the supply side effects of monetary policy (see Christiano et al. (2005)). Few studies (Ottonello and Winberry (2020)) emerged that tackle those questions for misallocation stemming from financial frictions heterogeneity. Most of the others (see review below) focusing on monopoly power with endogenous demand elasticities relied on models with ex ante firm heterogeneity, but where the economy aggregates to a single pricing condition.[1] Our study builds a macro model with ex post firm heterogeneity, due to idiosyncratic productivity risk, and shows that the interaction between uncertainty, oligopolistic complementarities and innovation leads to a novel strategic precautionary channel that accounts for the reallocation process, also in comparison to the time series evidence that we gather using micro data. Besides providing a framework to study endogenous misallocation over the business cycle, those channels have a significant impact on the transmission mechanism to the macroeconomy, particularly on the dynamic of firms' valuations. A key methodological challenge faced in building such a model lies in confronting the notion of dynamic oligopoly within an environment with uninsurable risk, something which we address with the *oblivious equilibrium concept*.

Motivated by the prominence of uncertainty for firms' decisions (Knight (1921)) and the rise in concentration, we build a dynamic macro model with heterogenous firms facing uninsurable idiosyncratic risk, nominal rigidities[2] and interacting strategically. To highlight the role of strategic complementarities we compare monopolistic to Bertrand competition. We focus on the response to monetary policy, given the increasing interest for its role on the supply side of the economy, but the model is amenable to study the transmission to other shocks. A model featuring firms' ex post heterogeneity poses some methodological challenges. First, like Hopenhayn (1992) or Hopenhayn and Rogerson (1993) the long run equilibrium requires a fixed point iteration between individual policy functions and input market clearing, leading to general equilibrium spillovers. Second, the presence of future uncertainty renders the oligopolistic interactions inherently dynamic. In this

1 In those models moments of the ex ante firms' distributions may affect the Phillips curve, but the models feature a single pricing conditions. In ours instead the Phillips curve results from the aggregation of individual price dynamics.

2 Those are introduced to study monetary policy.

context firms take as given the competitors reaction functions in all future periods, leading to a curse of dimensionality. We address this by resorting on the oblivious equilibrium concept introduced in the industrial organization literature (see Weintraub et al. (2010))[3]. In this context firms guess competitors' policy functions by conditioning on a mapping with their productivity states.[4] We then extend the model to include a simple pro-cyclical innovation process (see Kwon et al. (2024) for evidence on innovation cyclicality) that reduces productivity uncertainty.[5]

We discuss the main channels in a simple partial equilibrium economy and then examine the quantitative implications with numerical simulations. First and foremost, we compare the dynamic in our model to one where firms are ex ante heterogenous, but ex post heterogenous and show that in that case mark-up dispersion is unresponsive to the business cycle (or else the firms' distribution is constant), even tough the long run moments may affect the aggregate dynamic. Second, we show that the mark-up and firm valuations growth in the stochastic steady of our model align with the contemporaneous increase in firm valuation growth and market concentration noted in the data (see Farhi and Gourio (2018)). We then engage in a series of experiments to characterize the process of reallocation, which we then confront to the data.

We find that in response to a contractionary monetary policy mark-ups dispersion declines under Bertrand competition, while it increases with monopolistic competition. This result may appear surprising in light of the common use of wedges' dispersion as a sufficient statistic for misallocation. We find that the decline happens precisely since there is a prominent shift of market shares to the top 10% of the firms, relatively to the bottom 10%. Hence the rise in concentration reduces dispersion. A first implication of this is that the top to bottom tails ratio is a better sufficient statistics for the response of misallocation over the business cycle. Next we dissect the channels of the reallocation process. Our model features several channels. First, uncertainty induce precautionary behaviour: firms confronted with it and with nominal rigidities may increase prices by more today to build a profit buffer. Second, under oligopolistic competition demand elasticities are size dependent: large firms have lower demand elasticities, hence higher monopoly power, and can charge higher mark-ups, hence grow faster. Third, strategic complementarities and uncertainty interact in non obvious ways: confronted with the prospect of a contraction large firms may increase mark-ups by less to build market shares and stay competitive in an evolutionary fashion. The relative role of those channels

3 The concept is also akin to mean-field games: see Achdou et al. (2022).

4 Weintraub et al. (2010) proves the equivalence between this equilibrium and the Markov stationary one studied in Olley and Pakes (1996) or Maskin and Tirole (1988).

5 The process is a simple formalization of the idea that part of the firms' R&D investment is directed to improve productivity and reduce uncertainty. See Jones and Tonetti (2020) or Asriyan and Kohlhas (2024).

is ultimately a quantitative question, which we address by experimenting with counterfactuals and decompositions.

First, when we decompose the reallocation to the top in shifts in shares and changes in within firm mark-ups, we find that the second play a minor role. A model featuring only endogenous demand elasticity would induce the opposite. The shifts in market shares is instead due to strategic interaction under uncertainty: in recessions firms compress firm level prices and mark-ups to build market shares and remain competitive. Second, when we increase the variance of the uninsurable productivity shock the decline in dispersion is smaller: precautionary behaviour prevails and, expecting higher marginal costs due to the contractionary policy, all firms keep prices higher. Third, when we add the cyclical innovation process the reallocation to the top is stronger, since large firms can grow faster, but the decline in dispersion is smaller, since uncertainty declines. The decline in innovation in recessions may explain the decline in TFP observed in the data (see Christiano et al. (2005) or Kwon et al. (2024)).

We validate the patterns that characterize our model by estimating the impact of high frequency identified monetary policy shocks on dispersion, also within sectors, and on the tails of the mark-ups' distributions for US, and other countries, over the period 1985-2024.[6] We estimate mark-ups with Compustat Global data using the proxy method (Loecker and Warzynski (2012)).[7] Likewise in the model, we find that in response to the monetary shock dispersion declines significantly, either on impact or on the long run. Furthermore, we find that mark-ups grow at the top tail and exhibit little or no change at the bottom tail, leading to a persistent increase in concentration (see Kwon et al. (2024)). To gauge the role of uncertainty we interact the monetary policy shock with a dummy capturing if the variance in firms' bond spreads, purged by systematic uncertainty, is above median. To study the role of innovation we interact the monetary shock with a dummy capturing if the real value of patents, obtained from Kogan et al. (2017), is above the median. We find that the growth in mark-ups is larger for firms with uncertainty and patent values above the median. Hence, likewise in the model the reallocation to the top maybe enhanced by innovation and by a strategic precautionary behaviour.

Literature Review. There is a growing interest in studying the macroeconomic implications and the transmission of shocks in models with heterogenous firms. Evidence on several fronts highlight the role of the firms top tails in trickling down, and amplifying the transmission of shocks, and in the role of Shumpeterian reallocation (see Van Reenen (2018) for a review). Besides that, an

6 This is the period for which we can find both consistent micro data for the mark-up estimates and data for high frequency monetary shocks.

7 For robustness we employ various adjustments to the mark-up estimates proposed in the literature (see Gandhi et al. (2020) or De Ridder et al. (2022)) and we experiment with different types of monetary policy shocks.

important and growing literature highlighted the importance of wedge heterogeneity, rather than aggregate wedges, for allocative efficiency (see Restuccia and Rogerson (2008) or Hsieh and Klenow (2009)). Most of the literature so far focused on the role of wedge heterogeneity for long run growth. Recent contribution tackled the role of credit frictions wedges in the transmission of shocks (see for instance Ottonello and Winberry (2020)). Our work aims to study misallocation stemming from market power (evidence on mark-up cyclicality is provided in Anderson et al. (2025)), and its interaction with firm uncertainty.

Several papers studied the role of mark-up heterogeneity with a focus on long run growth. Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Bartelsman et al. (2013), Hsieh and Klenow (2014), David and Venkateswaran (2019), Baqaee and Farhi (2020), Fattal-Jaef (2022), Buera and Fattal Jaef (2018) among others in closed economy; Bernard et al. (2003), Melitz and Ottaviano (2008) or Antras et al. (2017) among others in the trade literature. In all of those, with the exception of Peters (2020), mark-up heterogeneity is exogenous. Our framework allows us to study misallocation in the long run and over the business cycle and features endogenous changes in the firm distribution, hence in misallocation, due to dynamic oligopoly and its interaction with uncertainty. It highlights, among others things, that at higher frequency changes in wedge dispersion are disconnected from the reallocation to the tails, and hence are not good sufficient statistics for the response to shocks.

Our paper also relates to the literature that studies demand complementarities and its role on the monetary transmission mechanism. See Ball and Romer (1990) for an early work on demand complementarities. Evidence by Evans and dos Santos (2002) and Christiano et al. (2005) uncovered a link between monetary policy and aggregate TFP. Subsequent work has rationalized this either through endogenous pass-through, acting as cost push shocks (see Baqaee et al. (2024), Meier and Reinelt (2024) in closed economy or Benigno and Faia (2016) and Cravino (2017) in open economy) or through endogenous innovation (see Comin and Gertler (2006), Benigno and Fornaro (2018) or Ma and Zimmermann (2023)). Our work is connected to the first strand: models used in those papers however feature firms that are heterogenous ex ante, but ex post homogenous, leading to aggregate an Phillips curve. This implies that the ex ante moments of the firm distribution have an impact on aggregate perfect foresight equilibrium, but there is no endogenous change of the firm wedge distribution over the business cycle and in response to policy. In our model firms are heterogenous ex post, as they face uninsurable risk or uncertainty likewise in Hopenhayn (1992) or Hopenhayn and Rogerson (1993). This has two main implications. First, the environment features general equilibrium effects, stemming from the fixed point interaction between firm policy functions and the adjustment in the market clearing conditions. This affects mainly the stochastic steady

state. Furthermore, uninsurable risk induces a precautionary channel that alters the nature of the strategic interaction. Early work from Penrose (2009) highlighted the role of uncertainty in explaining highly skewed firms' distribution and persistent growth. Our work shows that, even in the short run, uncertainty may heighten the incentives of large firms to grow market shares. Through those aspects our work contributes to the long standing literature studying the role of uncertainty for firms' decisions, from Knight (1921) to Bloom (2009) or Bloom et al. (2018) and Christiano et al. (2014).

Our work also contributes to the literature that studied the role of strategic complementarities in macroeconomics, ranging from Cooper and Johri (1997), Cooper and John (1988) to Cooper and Haltiwanger (2006). In our model we provide methodological advances by embedding dynamic oligopoly in a macro model. We do so by resorting on the oblivious equilibrium concept introduced by Weintraub et al. (2010), which relates to Markov perfect equilibria a' la Olley and Pakes (1996) and others, such as the Krusell and Smith (1998) myopic equilibrium or the mean-field game equilibria (see Achdou et al. (2022) or Gabaix et al. (2016)).

Our empirical analysis uses mark-ups which we estimate with available methods (Loecker and Warzynski (2012)) and their corrections (Gandhi et al. (2020) and De Ridder et al. (2022) among others).[8] We study the impact of high frequency identified monetary shocks on statistics of the mark-up distributions through Jordà (2005) panel local projections. Recent empirical work have highlighted the allocation of resources, and or market power, to the top of the distributions (see Autor et al. (2020) among others in closed economy and Bertaut et al. (2024) in open economy). We uncover this over the business cycle. This complements also other studies that examine the cyclical properties of reallocation, but in response to productivity shocks (see Cooper and Schott (2023)).

2. A Simple Model to Gain Intuition

We present a simple partial equilibrium model with heterogenous firms, facing uninsurable risk and to adjustment costs on prices to gain intuition on the main channels affecting the mark-up distribution and the potential testable implications.

In the model there is a finite number of firms, $i = [1, ..., N]$,[9] holding monopoly power on the varieties produced, that choose prices by interacting in an oligopolistic market. Firms are heterogenous as they face uninsurable risk on productivity.[10] There is no aggregate uncertainty. The

8 See Syverson (2024) for a recent overview.

9 The finite number is due to market concentration.

10 Firms may also face uncertainty on demand shocks. Conceptually this does affect our analysis, but the identification of the most relevant shocks for the reallocation process is eventually an empirical question.

model is amenable to firms producing with any inputs combination, hence we define firms' profits in terms of real marginal costs, mc_t. The variety aggregator is: $Y_t = [\sum_{i=0}^{\infty} \Phi(y_t^i]$, leading to the an optimal variety demand: $y_t^i = \Phi'(\frac{p_t^i}{P_t})$.[11] To set a benchmark, let us first examine the equilibrium mark-ups under flexible prices. Firms choose prices to optimize profits: $d_t = \Pi(N_t, \varepsilon) = p_t^i y_t^i - MC_t^i y_t^i$, where MC_t^i is the firms' nominal marginal costs. This leads to the following first order condition: $\frac{\partial \Pi_t^i}{\partial p_t^i} = y_t^i + (p_t^i - MC_t^i)\frac{\partial y_t^i}{\partial p_t^i} = 0$. Under Bertrand oligopoly firms internalize residual aggregate demand so that the demand constraint reads as follows:[12] $(\frac{y_t^i}{Y_t}) = (\frac{P_t^i}{\left[\sum_{i=0}^{N}(p_t^i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}})^{\varepsilon}$ and $\frac{\partial y_t^i}{\partial p_t^i} = \frac{\varepsilon y_t^i}{p_t^i}[\xi^i - 1]$, where $\xi = (\frac{p_t^i}{P_t})^{1-\varepsilon}$ represents the firms' market shares.[13] The pricing condition is: $p_t^i = \frac{(1-\xi_t^i)\varepsilon}{(1-\xi_t^i)\varepsilon - 1} MC_t^i$ and mark-ups are $\frac{(1-\xi_t^i)\varepsilon}{(1-\xi_t^i)\varepsilon - 1}$. This expression already allows us to discuss the first channel stemming from the size dependent demand elasticity. In response to a change in marginal costs firms with larger market shares can increase the mark-ups by more, thereby increasing the dispersion. Under nominal rigidities, firms' optimization becomes inherently dynamic and the course of future uncertainty in productivity matters. In this case firms choose prices to maximize the future discounted sum of profits that reads as follows:

$$\Pi(N_t, \varepsilon) = \sum_{t=0}^{\infty} \frac{1}{(1+r)} (\frac{p_t^i}{P_t} y_t^i - mc_t^i y_t^i - \frac{\varepsilon\kappa}{2} log(\frac{p_t^i}{p_{t-1}^i})^2) \tag{1}$$

where $\frac{1}{(1+r)}$ is a constant market discount factor,[14], mc_t is the real marginal cost and $\frac{\kappa}{2} log(\frac{p_t^i}{p_{t-1}^i})^2$ are the real adjustment costs on prices. Under a CES aggregator the demand of each variety is $(\frac{y_t^i}{Y_t}) = (\frac{P_t^i}{\left[\sum_{i=0}^{N}(p_t^i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}})^{\varepsilon}$ and the optimal prices and mark-ups are:

$$p_t^i = \mu_t^i MC_t^i; \qquad \mu_t^i = \frac{\varepsilon(1-\xi_t^i)}{(\varepsilon-1)(1-\xi_t^i) + \frac{\kappa}{\xi_t^i}[log(\pi_t^i) + \beta E_t\{log(\pi_{t+1}^i)\}]} \tag{2}$$

where $\xi_t^i = (\frac{p_t^i}{P_t})^{1-\varepsilon}$ is the firm's market share, $\pi_t^i = \frac{p_t^i}{p_{t-1}^i}$ and (see derivations of A.4 in Appendix A). In this environment a one time unexpected monetary shock affects the marginal cost. Hence we can examine the changes in the mark-ups distribution in response to changes in marginal costs.

Lemma 1. *The dispersion of mark-ups in response to a change in marginal costs, relatively to the*

11 With a CES aggregator, $Y_t = [\int_0^1 (y_t^i)^{\frac{\varepsilon-1}{\varepsilon}} di]^{\frac{\varepsilon}{\varepsilon-1}}$, optimal demand for each variety is: $(\frac{y_t^i}{Y_t}) = (\frac{p_t^i}{P_t})^{\varepsilon}$.

12 See Dornbusch (1987).

13 Under monopolistic competition and homothetic aggregator, firms take the aggregate prices hence: $\frac{\partial y_t^i}{\partial P_t^i} = -\varepsilon(\frac{p_t^i}{P_t})^{(-\varepsilon-1)}Y$. The optimal mark-up is then constant and equal to $p_t^i = \frac{\varepsilon}{\varepsilon-1} MC_t^i$.

14 Note that the model features no aggregate uncertainty.

stochastic steady state, is affected by the interaction of the variance of firms' idiosyncratic risk and mark-ups, weighted by the market shares, and it reads as follows.

$$\frac{\overline{\mu}_t^i}{\overline{\mu}^i} = \frac{\varepsilon(1-\xi_t^i)V_t^i}{(\varepsilon-1)(1-\xi_t^i)V_t^i + \frac{\kappa}{\overline{\mu}^i\xi_t^i}[log(\pi_t^i) + \beta log(E_t\{\pi_{t+1}^i\})]} \tag{3}$$

where $\overline{\mu}^i = \frac{\varepsilon}{\varepsilon-1}$ *is the steady state mark-up and* $V_t^i = \frac{E_t\{log(\pi_{t+1})\}}{log(E_t\{\pi_{t+1}\})}$.

Proof. *See Appendix A.*

The term V_t^i captures the distortion due to uninsurable risk and arises from Jensen's inequality. In an environment with symmetric shares, hence under monopolistic competition, it captures the extent of precautionary pricing behaviour. If the variance of idiosyncratic productivity shocks rises firms set higher mark-ups to build a buffer of profits against future risk. However, in an environment with time varying shares, firms with higher market shares will increase mark-ups by less. This is so since they internalize the impact of their decisions on the dynamic of future market shares.

The channels discussed above only relate to the individual firm policy functions, or else to partial equilibrium effects. In a structural general equilibrium model the transmission of shocks depend on other general equilibrium effects. We analyzer this next.

3. Quantitative Macro Model with Heterogenous Firms

The environment is a dynamic stochastic general equilibrium model with heterogenous firms facing uninsurable risk and strategic complementarities from a dynamic Bertrand oligopoly. The backbone of the macro model features households that supply labour hours and make consumption-saving problem, an intermediary that channels their savings to to firms in the form of equities and monetary and fiscal authorities. In the production sector firms hire capital and labour and face adjustment costs in capital and prices. Nominal rigidities are introduced to study monetary policy and adjustment costs are introduced to induce fluctuations in the firms' valuations. Given the presence of uninsurable risk, both the past distribution of price and capital affects firms' policy functions. To disentangle the role of the strategic complementarities we will compare the model dynamic under monopolistic and oligopolistic competition.

A methodological novelty of our framework is the adoption of the oblivious equilibrium concept (see Weintraub et al. (2010)) for the study of firms' dynamic strategic interaction under uncertainty. Under dynamic oligopoly firms need to take into account the reaction function of other firms in all future periods, or else they have to keep track of the evolution of their capital investment and prices.

To overcome the curse of dimensionality Weintraub et al. (2010) propose to resort on a myopic equilibrium in which firms guess competitors' policies through a mapping onto their productivity and show, under certain conditions (discussed below), the equivalence with the Markov stationary equilibria.[15]. The concept is similar to the myopic equilibrium in Krusell and Smith (1998) or the mean field game equilibrium in Achdou et al. (2022).

Note that in the baseline model we focus on the role of uncertainty and its interaction with strategic complementarities. As empirically firms' growth has been largely associated with innovation, we also examine a model's variant in which firms engage in a simple pro-cyclical innovation process that reduces productivity uncertainty, hence increases its expected value.

3.1. Households

Households make consumption-saving decisions and supply labour hours. Their preferences over consumption and labor hours are given by the following lifetime expected utility: $\mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t, n_t)\right]$, where c_t and n_t denote consumption and labour hours and where $u : \mathcal{R}_+^2 \to \mathcal{R}$ is a continuous and differentiable function, increasing in the first argument and decreasing in the second argument, $\beta \in (0, 1)$ denotes the subjective discount factor and E_t is the expectation conditional on information at time t. Each household in period t chooses consumption, c_t, and savings in liquid assets, b_t, to maximize the following value function:

$$\begin{aligned} V(b_{t-1}) &= \max_{c_t, b_t} u(c_t, n_t) + \beta E_t V(b_t) \\ \text{s.t.} \quad & c_t + b_t = (1 - \tau_t) w_t n_t + (1 + r_t^a) b_{t-1} \end{aligned} \tag{4}$$

where $V(b_{t-1})$ is the value function in time t. Labor hours are chosen by the family as a whole and are determined through the intra-temporal first order condition: $\varphi n_t^{\rho} = u_c(b_{t-1})\partial w_t$. First order conditions with respect to assets read as follows: $u_c(b_{t-1}) = \beta E_t \partial_b V(b_t)$, where the envelope conditions is: $\partial_b V(b_{t-1}) = (1 + r_t^a) u_c(b_{t-1})$.

4. The Production Sector

The production sector features two blocks. There are an intermediate sector with heterogenous firms that face idiosyncratic risk, hold monopoly power on the varieties that they produce and face nominal rigidities and a final good sector that aggregate the varieties.

15 See Maskin and Tirole (1988).

Final Good Sector. Households consume final good which is produced by a final good producer. The producer maximizes profits subject to the Dixit-Stiglitz aggregator of different varieties. Under monopolistic competition firms are atomistic, hence the aggregator reads as follows: $Y_t = \left[\int_0^1 (y_t^i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Under Bertrand competition firms are **granular**, hence the aggregator reads as follows: $Y_t = \left[\sum_{i=1}^{N_t} (y_t^i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$. The optimal allocation of expenditure on each variety yields:[16] $y_t^i = \left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t$, where $P_t \equiv [\sum_{i=1}^{N_t} (p_t^i)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$.

4.1. Heterogenous Firms and Oligopolistic Competition

Firms in the intermediate sector are subject to idiosyncratic productivity shocks, z_t, hire capital, k_t, and labour, n_t, to produce a constant return to scale production function, $z_t Z_t k_{t-1}^{\alpha} n_t^{1-\alpha}$, Z_t is aggregate TFP. Firms also face adjustment costs to capital, $\frac{1}{2\delta\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)^2 k_{t-1}$, and price adjustment costs, $\varepsilon\frac{1}{2\kappa_p}\log\left(\frac{p_t}{p_{t-1}}\right)^2 Y_t$. Firm heterogeneity is reflected through firm states, which are uncertain at time t. To fix an interpretation, we will refer to a firm's state as its productivity.[17] At time t, productivity of firm $i \in S_t$ is denoted by $z_{i,t}$, which follows a Markov process and whose realization across firms define the vector of the production sector state space at any time t. Risk from idiosyncratic productivity cannot be insured, which implies that firms' investment policies depend on the progression of shocks and firms' labour and pricing policies depend on both, idiosyncratic productivity and capital accumulation.

To illustrate the role of strategic complementarities, and their interaction with uncertainty, we derive and solve the model under monopolistic and Bertrand competition. In both cases when choosing inputs the firm takes as given their market clearing price, which in turn depends on the aggregation of individual demand from all other firms. An iterative fixed point process between the firms' inputs policies and the market clearing conditions characterizes the solution to the stochastic steady state and embeds firms' interaction in the market space likewise in Hopenhayn (1992) or Hopenhayn and Rogerson (1993).

Under Bertrand competition firms interact also in the strategic competition space. In a dynamic set-up this would involve firms taking as given also the competitors' reaction function for all future periods, leading to a curse of dimensionality. We overcome this issue by resorting and adapting to

16 The competitive packer solves the following optimization problem:

$$\max_{y_t^i} P_t Y_t - \sum_{i=1}^{N_t} p_t^i y_t^i \qquad \text{s.t. } Y_t = \left[\sum_{i=1}^{N_t} (y_t^i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

.

17 The model may well fit other types of shocks, such as product quality, capacity demand shocks.

our context the oblivious equilibrium concept by Weintraub et al. (2010), which consists in two additional steps. First, likewise in other myopic equilibria (see Krusell and Smith (1998)), firms take as given the mapping from competitors' policy functions to their states, productivity in our case, which is also common to all. This allows us to summarize the set of competitors' policy through the residual market demand, which firms take into account when choosing prices. The latter however still requires that firms form guesses on the competitors' market shares, which in turn evolves another iterative fixed point.

Below we lay down the firms' optimization problem, first order conditions and iterative processes under monopolistic and Bertrand competition.

Monopolistic competitive Firms. A continuum of firms has monopoly power on the variety they produce and maximize the following value function:

$$J(z_t, p_{t-1}, k_{t-1}, X_t) = \max_{p_t, k_t, n_t} \left(\frac{p_t}{P_t}\right)^{1-\varepsilon} Y_t - w_t n_t - (k_t - (1-\delta)k_{t-1}) - \varepsilon \frac{1}{2\kappa_p} \log\left(\frac{p_t}{p_{t-1}}\right)^2 Y_t$$
$$- \frac{1}{2\delta\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)^2 k_{t-1} + mc_t\left(z_t Z_t k_{t-1}^{\alpha} n_t^{1-\alpha} - \left(\frac{p_t}{P_t}\right)^{-\varepsilon} Y_t\right) + \frac{E_t J(z_{t+1}, p_t, k_t, X_{t+1})}{1 + r_{t+1}} \quad (6)$$

Uninsurable risk implies that the value function depends on firms' idiosyncratic productivity, on the accumulation of capital and the distribution of past prices. In period t firms' profits depend on input prices, w_t and r_t, which solve the market clearing conditions. The state X_t is the set of market clearing prices. The first order conditions for capital, labour and prices read as follows:

$$1 + \frac{1}{\delta\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right) = \frac{1}{1 + r_{t+1}} E_t \frac{\partial J(z_{t+1}, p_t, k_t)}{\partial k_t} \quad (7)$$

$$(1-\varepsilon)\left(\frac{p_t}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t} - \varepsilon\frac{1}{\kappa_p}\log\left(\frac{p_t}{p_{t-1}}\right)\frac{Y_t}{p_t} + \varepsilon mc_t\left(\frac{p_t}{P_t}\right)^{-\varepsilon-1}\frac{Y_t}{P_t} + \frac{1}{1+r_{t+1}}E_t\frac{\partial J(z_{t+1}, p_t, k_t)}{\partial p_t} = 0 \quad (8)$$

$$w_t = (1-\alpha) mc_t z_t Z_t k_{t-1}^{\alpha} n_t^{-\alpha} \quad (9)$$

The envelope conditions imply: $\frac{\partial J(z_t, p_{t-1}, k_{t-1})}{\partial p_{t-1}} = \frac{\varepsilon}{\kappa_p}\log\left(\frac{p_t}{p_{t-1}}\right)\frac{Y_t}{p_{t-1}}$ and $\frac{\partial J(z_t, p_{t-1}, k_{t-1})}{\partial k_{t-1}} = (1-\delta) - \frac{1}{2\delta\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)^2 + \frac{1}{\delta\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)\frac{k_t}{k_{t-1}} + \alpha mc_t z_t Z_t k_{t-1}^{\alpha-1} n_t^{1-\alpha}$.

Dynamic Bertrand Competition. We start by formalizing the general optimization problem of firms interacting in Bertrand markets under the oblivious equilibrium. Next we present the parametrized optimization problem in our model.

Oblivious Equilibrium Definition. *Under the oblivious equilibrium firms take as given their*

own state and the long-run distributions of the competitors' states. This results in the following optimization problem:

$$J(z, k_{-1}, p_{-1}) = \max_{k \in K, p \in P} \pi(z, k_{-1}, p_{-1}, P, K, X) + \beta \sum_{z} J(z', k, p) P(z, z', X) \tag{10}$$

where V is the firm's value function, the prime index indicates future values, π is a profit function, $P(z, z', X)$ is the Markov matrix for idiosyncratic productivity, z is the idiosyncratic productivity state, p, k are policy functions for capital and prices, β is a discount factor, K and P are sets of possible capital and prices, X is a set of exogenous variables (wage, interest rate, aggregate level of productivity) and primes refer to next period.

In Appendix D.1 we provide a general definition of the oblivious equilibrium, formalized as an approximation to a Markov stationary one (see Maskin and Tirole (1988)), and show how it maps into the one formalized in 10 once we define the specific set of variable. Here we highlight that the main distinction between the optimization problem in 10 and the one in 6, lies in extending the conditionality of the profit function to include the aggregate variables P, K, X. In absence of oligopolistic strategic the firm would condition only on equilibrium prices. Under oligopolistic competition the firm takes as given the aggregate prices and capital, which act as sufficient statistics of the competitors' residual demand. Those in turn imply that each firm internalizes the impact of its decisions on the residual demand and that the derivative with respect to it requires a guess on the competitors' market shares. Given 10 we can now specify and parametrize functional forms. This leads to the following value function:

$$\begin{aligned} J(z_t, p_{t-1}, k_{t-1}) = \max_{p_t, k_t, n_t} & \left(\frac{p_t}{P_t}\right)^{1-\varepsilon} Y_t - w_t n_t - (k_t - (1-\delta)k_{t-1}) - \varepsilon \frac{1}{2\kappa_p} \log\left(\frac{p_t}{p_{t-1}}\right)^2 \frac{y_t}{P_t} \\ & - \frac{1}{2\delta\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)^2 k_{t-1} + mc_t \left(z_t Z_t k_{t-1}^{\alpha} n_t^{1-\alpha} - \left(\frac{p_t}{P_t}\right)^{-\varepsilon} Y_t\right) + \frac{E_t J(z_{t+1}, p_t, k_t)}{1 + r_{t+1}} \end{aligned} \tag{11}$$

$$\text{s.t. } P_t = \left(\sum_{\vec{i}_t} \left(p_t^i\right)^{1-\varepsilon} dD_t^i\right)^{\frac{1}{1-\varepsilon}} \tag{12}$$

where to simplify notation we defined $\vec{i}_t = (z_t, p_{t-1}, k_{t-1})$ and $p_t^i = p(z_t, p_{t-1}, k_{t-1}) = p_t$. Note that the set X is summarized in 12 by the input prices and by the aggregate price P_t. The latter depends on the firms' shares, dD_t^i. This enters both current and future profits; taking expectations with respect to the shares is equivalent to condition on the average state of the market. Analytically the firms' first order conditions with respect to labour and capital are equivalent. The numerical

solution subsumes the fact that, in the iterative fixed point that leads to the stochastic steady state, firms guesses the shares of their competitors. The first order condition with respect to prices reads as follows:

$$(1-\varepsilon)\left(\frac{p_t}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t}\left(1-\frac{p_t}{P_t}\frac{\partial P_t}{\partial p_t}\right)-\varepsilon\frac{1}{\kappa_p}\log\left(\frac{p_t}{p_{t-1}}\right)\frac{1}{p_{t-1}}\frac{Y_t}{P_t}- \tag{13}$$

$$-\frac{1}{\kappa_p}log(\frac{p_t}{p_{t-1}})^2(1-\varepsilon)\frac{Y_t}{P_t}\left(1-\frac{p_t}{P_t}\frac{\partial P_t}{\partial p_t}\right)-\varepsilon mc_t\left(\frac{p_t}{P_t}\right)^{-\varepsilon-1}\frac{Y_t}{P_t}\left(1-\frac{p_t}{P_t}\frac{\partial P_t}{\partial p_t}\right)+$$

$$+\frac{1}{1+r_{t+1}}E_t\frac{\partial J(z_{t+1},p_t,k_t)}{\partial p_t}=0 \tag{14}$$

Importantly the derivative of the demand function with respect to prices is:

$$\frac{\partial P_t}{\partial p_t}=\left(\frac{p_t}{P_t}\right)^{-\varepsilon}dD_t^i= \tag{15}$$

The term 15 acts as a size dependent demand elasticity: when firms that gain market size, their demand elasticity declines allowing them to charge higher mark-ups. Note that in the equation above we skip the firm index for ease of notation, but the first order conditions are all firms specific. It is important to highlight that the derivative in 15 would be present also in an environment with ex ante heterogenous firms whose varieties are aggregated according to non-homothetic preferences (see Baqaee et al. (2024)). In that case however the market share, $\left(\frac{p_t}{P_t}\right)^{-\varepsilon}$, would be symmetric across firms and depend on the total number of competitors. In our case they are different across firms and time-varying.

4.2. Other Blocks, Market Clearing and Equilibrium.

Financial Intermediary. A competitive financial intermediary invests households' savings in firms' equities and government bonds. The zero profit condition implies that the return on the households' liquid asset is equated to the aggregate of the firms' dividends and capital gains as follows:$(1+r_t^b)=\sum_{n=1}^{N}\left(\frac{v_{n,t-1}}{B_{t-1}}\right)\frac{d_{n,t}+v_{n,t}}{v_{n,t-1}}+\frac{B^g}{B_{t-1}}(1+r_t)$, where $v_{n,t}=\frac{d_{t+1}+v_{n,t+1}}{1+r_{t+1}}$ are stock prices of the heterogeneous firms indexed by n.

Monetary and Fiscal Policy. Monetary policy is conducted according to a Taylor-type monetary policy rule:$i_t=\bar{r}+\phi\pi_t$, where i_t is a nominal interest rate that is tied to the real rate through Fisher equation $1+i_t=(1+r_t)(1+\pi_t)$. The fiscal authority pursues a balanced budget policy: $\tau_t w_t N_t = G_t + r_t B^g$, where B^g is a supply of government bonds and G_t is government spending.

Aggregation and Fixed Point Equilibrium. The fixed point iteration, described above, between firms' inputs' policies and market equilibrium prices requires the definition of the market clearing conditions. For the labour market it reads as follows: $\sum_{n=1}^{N_t} n_t^d = n_t^s$, where n_t^d we denote the individual firm demand for labour and with n_t^s we denote the aggregate supply of labour. The equilibrium in the market for capital is achieved through the zero profit condition of the financial intermediary.

Market Clearing Conditions. Aggregate supply of goods is equal to aggregate demand, hence:$Y_t = C_t + G_t + I_t + \Phi_t$. Asset markets clearing implies: $B_t = \sum_{n=1}^{N} v_{n,t} + B^g$. Finally aggregate labor demanded equals aggregate labor supply.

Equilibrium Definition.*An equilibrium in this model is such that: a. Households consumption, saving and asset decisions solve the optimization problem in 4, b. Firms in the final good sector optimally chose varieties according to:* $\varepsilon \frac{1}{2\kappa_p} \log\left(\frac{p_t}{p_{t-1}}\right)^2 Y_t$. *Under monopolistic competition firms' policy functions in the intermediate good sector solve the optimization problem in 6. Under Bertrand competition, firms in the intermediate sector solve the optimization problem in 12, c. market clearing in the asset and input markets are satisfied, d. arbitrage conditions between liquid and non-liquid assets are satisfied, e. the monetary authority follows a Taylor rule and the fiscal authority runs a balance budget with constant debt.*

5. Analytical Results

The main conduit to the monetary transmission mechanism is the Phillips curve, which also summarizes the main trade-offs. In a model with heterogenous firms there is a set of equations that dictate the price dynamic and capture the transmission mechanism at individual firm level. Aggregation of the individual price dynamics formalizes the consequences for the macroeconomy of the individual firms' interaction under uncertainty. In what follows we highlight those channels analytically. This allows us to gain further intuition to discuss the quantitative results that come next.

Proposition 1. Mis-allocation and Cost Push Shocks. *The log-linearized firm level price dynamics reads as follows:*

$$\hat{\pi}_t(z_t, k_{t-1}, p_{t-1}) = [\alpha \hat{mc}_t(z_t, k_{t-1}, p_{t-1}) + \lambda(\hat{\xi}_t(z_t, k_{t-1}, p_{t-1}) + \hat{p}_t(z_t, k_{t-1}, p_{t-1}))] + \beta E_t[\hat{\pi}_{t+1}(z_{t+1}, k_t, p_t)] \tag{16}$$

where $\alpha = \varepsilon(1-\xi)mc)$, *with* $\xi = (\frac{p}{P})^{1-\varepsilon}$ *and* $\lambda = 1 + \varepsilon mc(1-\xi)$ *and hats indicate log deviations from steady state.*

Proof. *Appendix D*

Equation 16 characterizes the dynamic of prices for each individual firm. Prices, marginal costs and inflation all depend on the state space, which includes the productivity shock, past capital and past prices. The individual firm's price dynamics depends on the dynamic of the market shares, which acts as a cost push shock. When firms gain market shares, their demand elasticity declines allowing them to charge higher mark-ups and prices. Hence any aggregate demand shock turn into a supply shock at individual firm level.

Corollary 1. *The aggregate Phillips curve reads as follows:*

$$\hat{\pi}_t^a = \sum_{n=0}^{N} \hat{\xi}_t(z_t, k_{t-1}, p_{t-1})\hat{\pi}_t(z_t, k_{t-1}, p_{t-1}) = \tag{17}$$

$$= \sum_{n=0}^{N} \hat{\xi}_t(z_t, k_{t-1}, p_{t-1})[\alpha \hat{mc}_t(z_t, k_{t-1}, p_{t-1}) + \lambda(\hat{\xi}_t(z_t, k_{t-1}, p_{t-1}) + \hat{p}_t(z_t, k_{t-1}, p_{t-1}))] + \tag{18}$$

$$+ \beta E_t \sum_{n=0}^{N} E_t[\hat{\xi}_{t+1}(z_{t+1}, k_t, p_t)\hat{\pi}_{t+1}(z_{t+1}, k_t, p_t)] + Cov(\hat{\xi}_{t+1}(.)\hat{\pi}_{t+1}(.))$$

Equation 18 is an aggregate Phillips curve, in which future inflation affects the current one through the interaction between the expected distribution of individual firms' price changes and the distribution of the market shares. This interaction depend on the extent of strategic complementarities. Consider the following example to fix ideas. In anticipation of a future uncertain recessions firms would like to rise prices today. However this may entail a decline in the future market shares and a loss of market power. If so the expected decline in ξ_{t+1}^i incentivizes firms to reduce the price increase today. Note that the covariance terms involve the entire distributions of market shares and price changes, hence also the cross-elasticities. Their quantification is ultimately a quantitative question.

The Key Channels in Our Model: Endogenous Demand Elasticity and Precautionary Strategic Behaviour. We can summarize the main channels characterizing our model as follows. At the level of individual firm policy function oligopolistic competition entails a size dependent demand elasticities, by which larger firms can charge larger mark-ups. The latter in turn acts as cost push shocks. Second the presence of uncertainty engages firms in precautionary behaviour. At aggregate level this interacts with the strategic behaviour so that current aggregate inflation depends on the covariance between future firm level price changes and expected market shares.

6. Quantitative Results

In this section we presents the results of the quantitative analysis. We start by describing calibration and solution method. Next we present the numerical results for the stochastic steady state and for the dynamic response to shocks.

Calibration and Solutions Methods. The model's stochastic steady state is solved through a non-linear solver around with an endogenous grid approximation (see Carroll (2006b)). The dynamic response to shocks is solved using methods that rely on finite approximation of the individual policy functions in sequence space (see Boppart et al. (2018) or Auclert et al. (2019)). Most parameter calibration follows past literature. The discount factor of households, β, is set such that the real interest rate is equal to 1.25%. Capital share is $\frac{1}{3}$, depreciation rate is $\delta = 0.02$, elasticity of demand between varieties is $\varepsilon = 6$, for the case with strategic complementarities $\gamma = 1$. Productivity and standard deviation for the idiosyncratic productivity shocks are taken from Nakamura and Steinsson (2008): $\rho = 0.66$, $\sigma = 0.0428$, elasticity of the variance, σ, to output in the countercyclical case is set to 1, the elasticity of the level of productivity to output in the procyclical case is set to 0.5, so that output and productivity level have responses of a similar magnitude to a monetary policy shock as in Baqaee et al. (2024). Government debt is equal to 2.8, ϵ_I that controls the extent of capital adjustment costs is equal to 4. The slope of the Phillips curve is 0.1, the ratio of government spending to GDP is 0.2, EIS is 0.5, Frish elasticity is 1, Taylor rule coefficient on inflation is 1.5. Output and labour hours are normalized to 1.

6.1. Stochastic Steady State.

In this section we discuss the properties the mark-up distributions in the stochastic steady state, also relatively to the long run distributions that we present in our empirical analysis in section 7. Furthermore, we discuss the role of uncertainty that surrounds the stochastic steady state and its interaction with market competition for the long run distribution of capital. We highlight in particular the implication for firm valuation growth, which empirically has increased together with market concentration (see Farhi and Gourio (2018)). Note that the long run distribution of capital and firm values is quite consequential, as it determines the degree of firms' inequality, hence the degree of market concentration.

The Long-Run Mark-up Distribution and Determinants of the Skewness. Figure 1 presents the mark-up distribution in the stochastic steady state comparing the models with monopolistic competition and with oligopolistic competition. Both distributions are highly skewed,

and this compares well to the data equivalent presented in figure 8, and more so under oligopolistic competition. Therefore trough the lens of the model we can interpret the shift forward over time that characterize the densities, discussed in section 7, can be interpreted as a shift toward a more granular form of market competition.

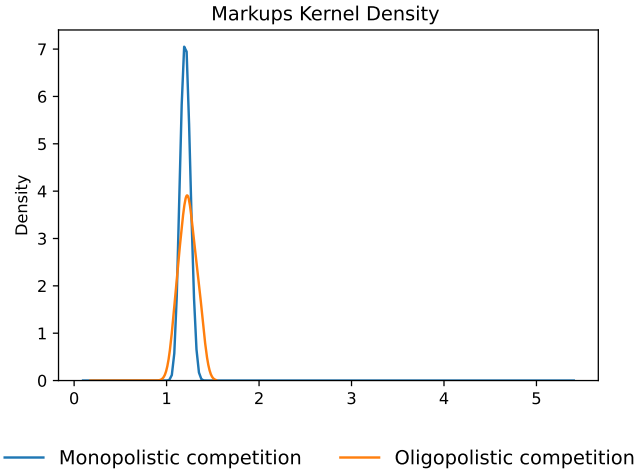


Figure 1: The stochastic steady state mark-up distribution in the model with monopolistic competition (blue line) and the model with oligopolistic competition.

Precautionary Firm Behaviour and Rising Valuation Growth. To isolate the role of precautionary behaviour let us consider a simplified version of our model in absence of nominal rigidities and adjustment costs to capital. The steady state analytical solution leads to the following lemma.

Lemma 2. *The aggregate level of capital and the average equity price in the steady state are larger when there are idiosyncratic productivity shocks compared to the case without them when* $\varepsilon > \max(2, 1 + \frac{1}{1-\alpha})$. *The relation inverses when* $\varepsilon < \min(2, 1 + \frac{1}{1-\alpha})$.

The proof of lemma 2 is in Appendix C, where we also provide a visual illustration of the relation between aggregate capital and the primitive parameters that govern strategic complementarities. For moderate degree of strategic competition firms' precautionary behaviour leads to higher capital accumulation, which in turn pushes firm equity valuation. Note that higher aggregate level of capital does not imply higher capital shares in equilibrium, which seems to be declining in the data (see Barkai (2020)). The main implication of our steady state solution is that an economy featuring higher uncertainty and higher market concentration features higher valuation growth, a result compatible with the empirical regularities uncovered in Farhi and Gourio (2018).

6.2. Dynamic Responses

We discuss the monetary transmission mechanism in the model by examining the impulse response functions of a large set of macro variables and of the firms' mark-up distributions in response to a 25 bp. shock to the interest rate. We focus in particular the mark-up dispersion, also within sectors, and on the changes in the upper and lower 10% tails of the mark-up distribution. Mark-up dispersion has been shown to be a sufficient statistic for misallocation under certain assumptions Hsieh and Klenow (2009).[18] Studying the changes in the tails allows us to document the dynamic of the reallocation process and to detect any disconnect with dispersion.

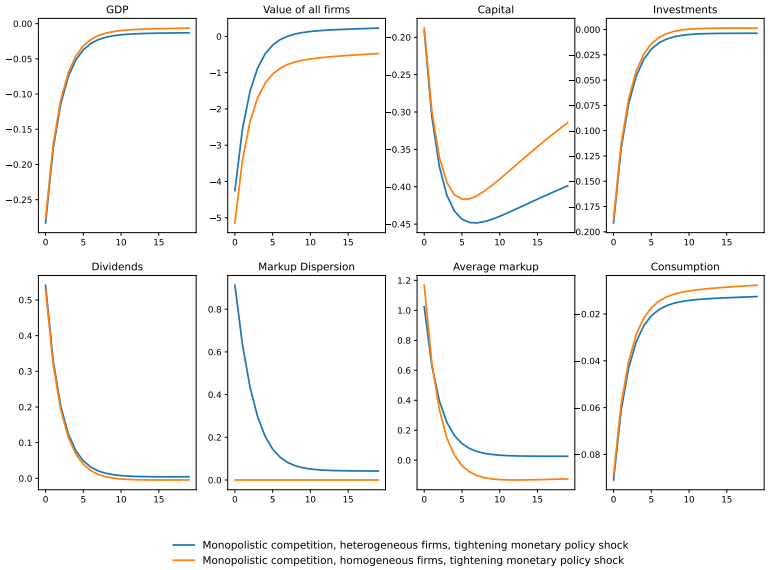
We start by presenting a comparison of the impulse responses in two models with monopolistic competition, one with ex post homogenous and the other with ex post heterogenous firms. This is crucial to understand the additional channels featured by our model relatively to past studies that examined the role for the macroeconomy of endogenous demand elasticities. Furthermore, we present a comparison between the model with monopolistic and oligopolistic competition. This is insightful, since the first features only the precautionary channel and the second its interaction with strategic behaviour. Strikingly the dynamic of the mark-up dispersion and of the macro variables is very different between the two models.

Figure 2 presents the impulse response functions of selected macro and distributional variables comparing the monopolistic competition model with homogenous and heterogenous firms. First, in the model with homogenous firms the dispersion of mark-ups is constant and does not react to shock, this is not the case in the data (see our empirical results in section 7). Hence to properly track the dynamic of the firm distribution and how it affects the macroeconomy, the model shall crucially include ex post heterogeneity from uninsurable risk. This has also implications for the macroeconomy. The average mark-up for instance is higher in this case, due to a precautionary pricing channel, by which firms increase prices by more in anticipation of future risk and due to nominal rigidities. The presence of uninsurable risk also explains the increase in mark-ups dispersion: firms' policy functions depend on their past capital and prices and any aggregate shock that tweaks those distributions also increases firms' wedge inequality.

Figures 3, 4 and 5 present respectively the response of both the macro variables, moments of the mark-up distributions and the shifts in the tails of the distribution in the macro model with monopolistic competition and with dynamic oligopolistic competition, and their direct comparison. To isolate the changes in the firm level mark-up from the changes in the firms' market shares, the

18 The main assumptions relate to the log-normality of the firms' shocks.

Figure 2: Monetary Policy Shock in the Monopolistic Competition Model with Homogenous and Heterogenous Firms. A 25 b.p. shock (tightening) to the interest rate with the persistence 0.6. The model features monopolistic competition and compares the case with ex post homogenous firms and the case with ex post heterogenous firms. or else firms facing uninsurable risk. The right panels show the response of the tails and other moments of the mark-up distribution. Impulse responses, for selected macro and distributional variables, are presented in percentage deviations from the steady state.



between-firm reallocation channel, we plot the changes in the distributions' tails both weighted by market shares and without the weights.

First and foremost, in response to a monetary contraction mark-up dispersion increases in the model with monopolistic competition, but declines in the model with oligopoly. At first, such a dynamic may appear counterintuitive as dispersion is considered a sufficient statistic for misallocation. Upon examining the shifts across the top and the bottom tail, reallocation to the top appears much more pronounced under oligopolistic competition, a fact that links the decline in dispersion to the steep increase in concentration. The dynamic of the wedge dispersion is the opposite in the model with monopolistic and oligopolistic competition. Hence the difference in the transmission channels between the two may explain the difference in the dynamic of the wedge distributions. The model with monopolistic competition features only firm's precautionary behaviour, by which a rise in marginal costs, triggered by the contractionary policy, induces an even larger increase in the mark-ups and prices to build profit buffers. This channel rises dispersion, since the extent of precautionary saving depends on the firms size distribution, which is unequal to start with. Under oligopolistic competition firms confronted with an uncertain recession try to balance the benefits of building buffer profits and increase mark-ups with the costs of loosing future market shares, thereby

loosing the ability to leverage larger mark-ups in the future. They may therefore forego part of the mark-up increase today in exchange of an increase in future monopoly power, or market shares.

Two other observations are of interest. First, we compare the changes in the mark-ups at the top and the bottom tails weighted by market shared and unweighted. The comparison shows that the changes in individual firms mark-ups are minimal and almost identical across firms at the top and the bottom. Changes in individual mark-ups would result from the size dependent demand elasticities, a channel present also in the homogenous firms model with non-homethetic aggregators, and by which we expect larger firms to charge much higher mark-ups. The patterns show that this channel is of minimal quantitative importance, relatively to the precautionary channels. Second, figure 5 shows that at aggregate level the amplification of firms' investment, valuation and dividends is much larger in the model with oligopolistic competition. This is driven largely by the firms with larger market sizes. In models with ex post heterogeneity the dynamic of aggregate variables can be decomposed in changes in individual policies and changes in the agents' distributions along the state space. The vector of aggregate variables, $\mathcal{Y}_t = (K_t, L_t, \pi_t)$ and $\mathbf{D}_t$, is given by the individual policy functions times the firms' distributions at any point in time: $\mathcal{Y}_t^s = (\mathbf{y}_t^s)'\mathbf{D}_t^s$. The response of the economy can be traced through a Taylor expansion of that vector around shocks. Given the transition matrix, Λ_t', which dictates the evolution of capital and prices across the grid points of the Taylor approximation, and since y and Λ are both differentiable, for small shocks dx differentiating the aggregate variables delivers: $d\mathcal{Y}_t^s = \mathbf{y}_{ss}' d\mathbf{D}_t^{ss} + (d\mathbf{y}_t^s)'\mathbf{D}_{ss}$, whereby $d\mathbf{D}_t^{ss}$ is given by the changes in the market shares, which our simulations show to be largest at the top.

6.3. The Role of Uncertainty and Innovation

In this section we put the reallocation process under the microscope by examining some model's counterfactuals and variants. We focus in particular on the role of uncertainty and productivity innovation. Uncertainty, which plays a key role for firms' decisions (see Bloom (2009) or Bloom et al. (2018)) and given the number of crisis experienced in recent decades may have increased over time (see Baker et al. (2016)). In our model higher uncertainty increases the incentives for precautionary behaviour. Figure 6 shows the impulse responses in the model with oligopolistic competition and higher uncertainty (the green line). Mark-up dispersion declines, but somewhat less. In this case precautionary buffer motives prevail over strategic consideration and all firms tend to increase prices, and mark-ups, by more, and more similarly.

Next, it is well know that a large part of firms' growth and the skewed size distribution is due to innovation (see Klette and Griliches (1996)). Various authors have also documented an increase in

Figure 3: Monetary Policy Shock in the Models with Monopolistic Competition and Heterogenous Firms. A 25 b.p. shock (tightening) to the interest rate with the persistence 0.6. The model features firms heterogenous due to uninsurable productivity shocks and monopolistic competition. The left panels show the response of the main macroeconomic variables and the moments of the mark-ups distributions. The right panels show the response of the tails and other moments of the mark-up distribution. Impulse responses, for selected macro and distributional variables, are presented in percentage deviations from the steady state.

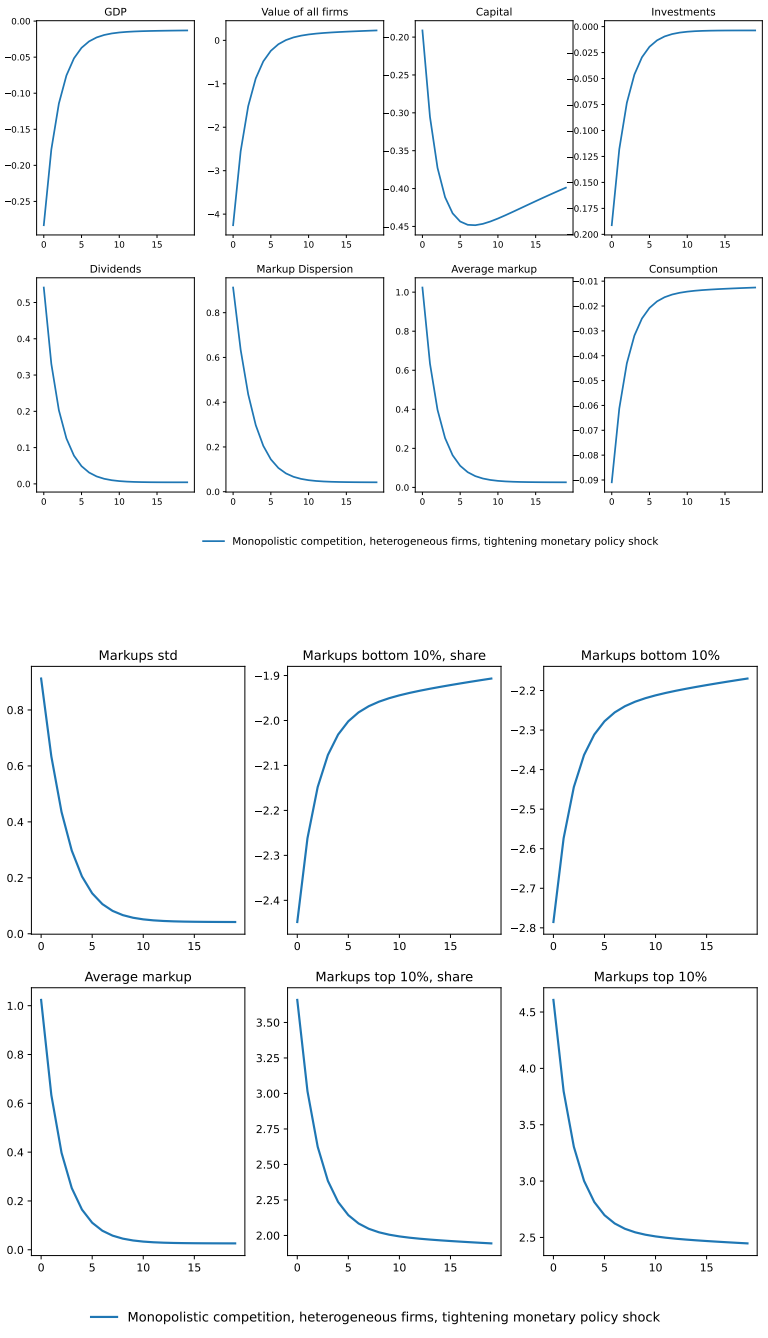
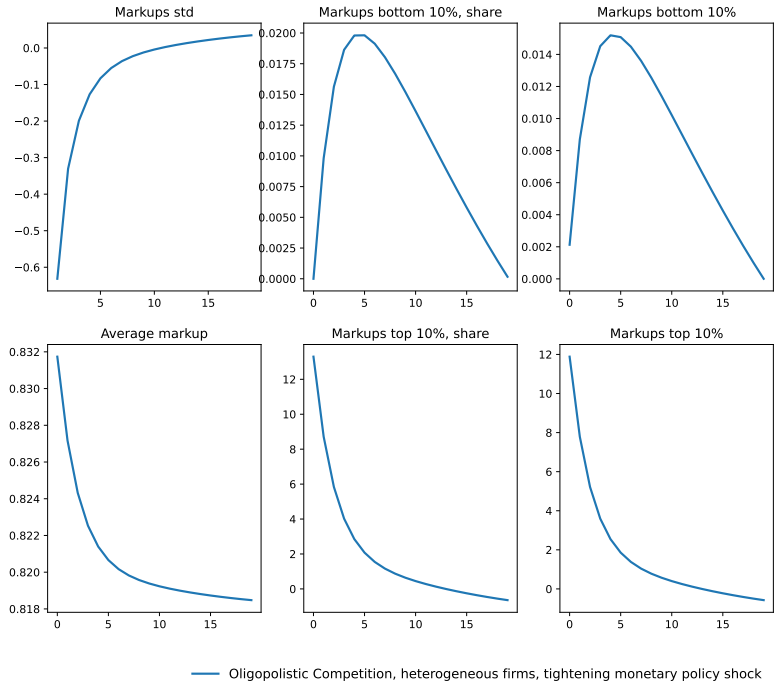


Figure 4: Monetary Policy Shock in the Models with Oligopolistic Competition and Heterogenous Firms. A 25 b.p. shock (tightening) to the interest rate with the persistence 0.6. The model features firms heterogenous due to uninsurable productivity shocks and oligopolistic competition. The left panels show the response of the main macroeconomic variables and the moments of the mark-ups distributions. The right panels show the response of the tails and other moments of the mark-up distribution. Impulse responses, for selected macro and distributional variables, are presented in percentage deviations from the steady state.



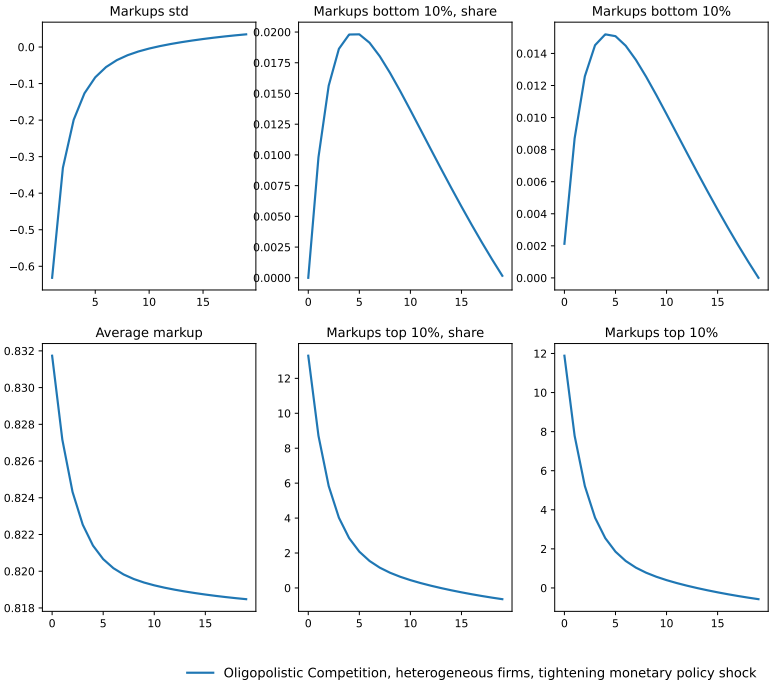


Figure 5: Monetary Policy Shock in the Models with Monopolistic versus Oligopolistic Competition and Heterogenous Firms. A 25 b.p. shock (tightening) to the interest rate with the persistence 0.6. The model features firms heterogenous due to uninsurable productivity shocks and compares the impulse response functions in the model with monopolistic and oligopolistic competition. Impulse responses, for selected macro and distributional variables, are presented in percentage deviations from the steady state.

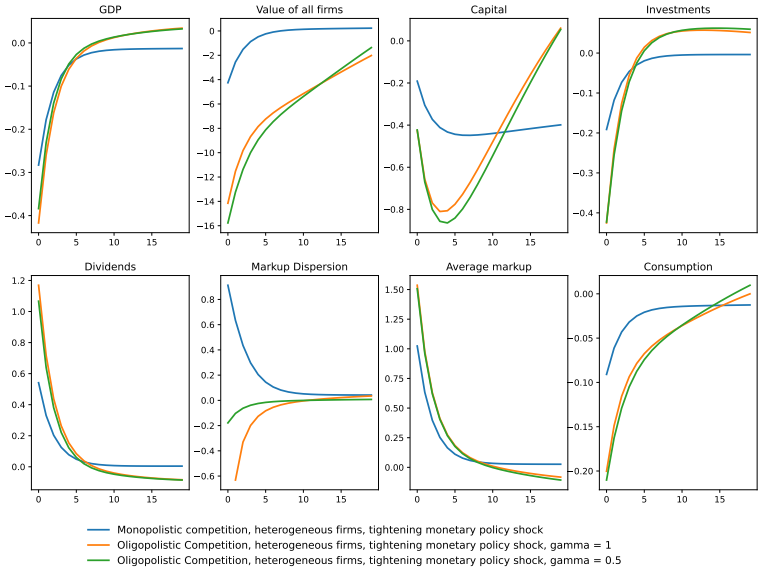


Figure 6: Monetary Policy Shock in the Models with Oligopolistic Competition. Comparison Across Uncertainty and Innovations. A 25 b.p. shock (tightening) to the interest rate with the persistence 0.6. The model features firms heterogenous due to uninsurable productivity shocks and monopolistic competition. The left panels show the response of the main macroeconomic variables and the moments of the mark-ups distributions. The right panels show the response of the tails and other moments of the mark-up distribution. Impulse responses, for selected macro and distributional variables, are presented in percentage deviations from the steady state.

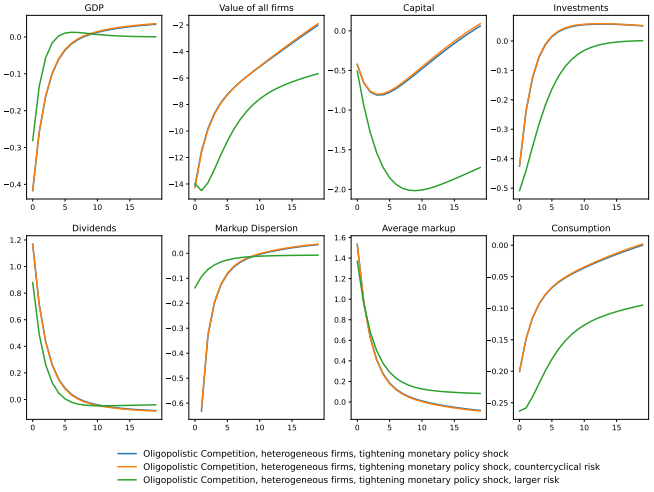
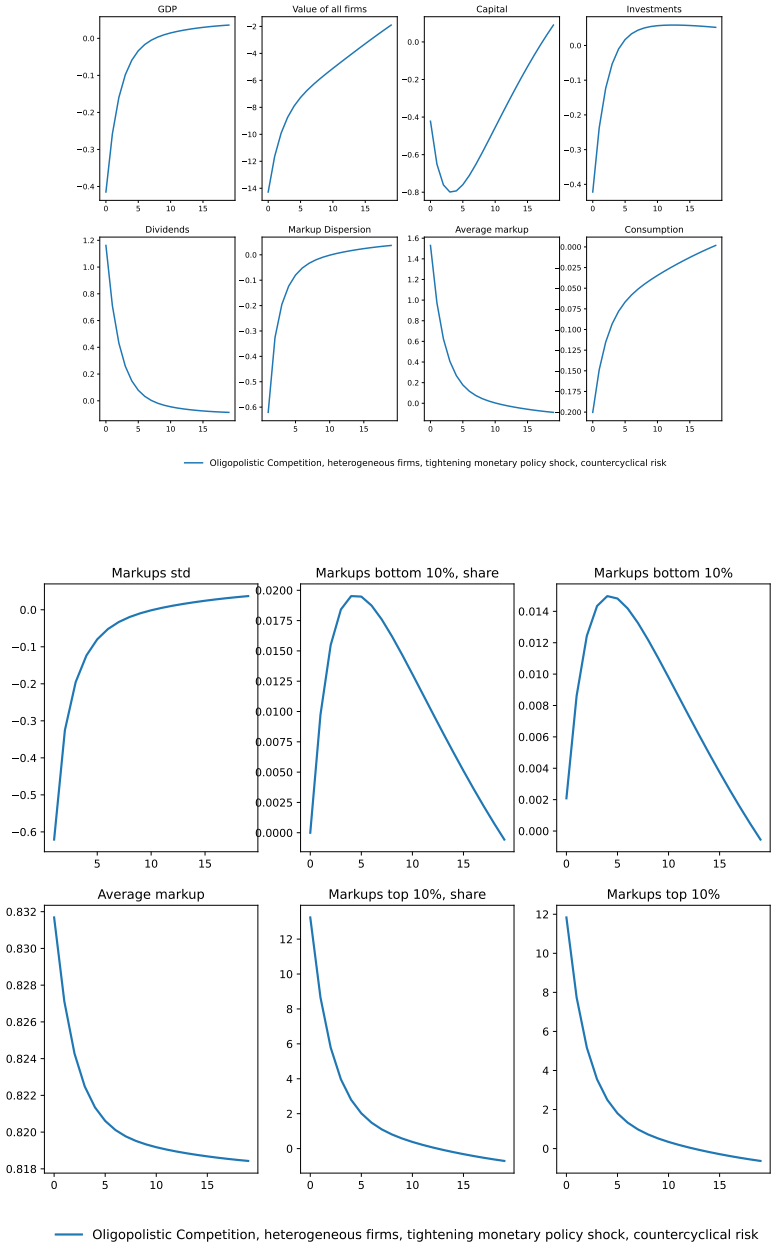


Figure 7: Monetary Policy Shock in the Models with Oligopolistic Competition and Heterogenous Firms and a Productivity Innovation Process. A 25 b.p. shock (tightening) to the interest rate with the persistence 0.6. The model features firms heterogenous due to uninsurable productivity shocks and oligopolistic competition. The productivity process is augmented with a cyclical innovation process that reduces the variance of the productivity shock, hence uncertainty. The left panels show the response of the main macroeconomic variables and the moments of the mark-ups distributions. The right panels show the response of the tails and other moments of the mark-up distribution. Impulse responses, for selected macro and distributional variables, are presented in percentage deviations from the steady state.



R&D investment and its association with mark-ups (see Crouzet and Eberly (2023) among many others). We therefore introduce a simply cyclical innovation process that reduces productivity uncertainty, hence increases its expected value. Recent evidence (see Ma and Zimmermann (2023)) documents that innovation rises in booms, especially during monetary expansions, a fact that is most relevant for our study. Our simple innovation process is therefore geared to capture primarily this feature. Figure 7 shows the impulse response functions in the model with oligopolistic competition and with a cyclical productivity innovation process. Mark-ups' dispersion still declines and the reallocation to the top of the market shares is somewhat eve more pronounced. This is evident also from the direct comparison between the oligopolistic competition model with and without the innovation process, presented in figure 6. Those results are intuitive and, as we show below, in line with the empirical evidence.

7. Empirical Analysis

The goal of this section is to examine the response of misallocation to monetary policy shocks and validate the implications of the quantitative model. We estimate the impact of high frequency identified monetary policy shocks onto moments of the mark-up distributions through a local projection (Jordà (2005)) specification. In line with the model, we focus on two statistics: overall mark-ups dispersion, as measured by their standard deviation, and changes in the top and bottom percentiles. In deviation from the model we also consider dispersion within sectors, hence net of the sectoral averages. In the data mark-ups are estimated at sectoral level and this implies that much of the overall dispersion is due to sectoral differences in the production process and in the degree of price stickiness. As those are not part of the channels on which we focused with our structural model, we are bound to use an empirical measure that best captures the type of variation due to firm level productivity or to strategic complementarities examined in the model. Below we describe the methods, and corrections, used to estimate the mark-ups, the general econometric specification, as well as all the alternative specifications estimated for robustness. To gauge further evidence on the channels, and on the interaction with firm uncertainty and innovation, we also examine specifications in which the monetary policy shock is interacted with a dummies capturing whether firm uncertainty or innovation is above the median.

Mark-up Estimates. We estimate markups, at quarterly frequency, following the structural estimation approach pioneered in the industrial organization literature (see Hall (1988) or Loecker and Warzynski (2012)). The general framework, known also as production method, relies on using firms' first order conditions, together with accounting data and non-parametric estimates of the

production elasticities. The main rationale behind those methods is that input optimization is independent from the market structure, or the type of competition, and hence should be generally valid. The mark-ups are obtained by merging firms' first order conditions for variable inputs. This leads to an expression for the firm level i markup that reads as follows:

$$\mu_{i,t} = \frac{\theta_{x_{i,t}}}{s_{x_{i,t}}} \tag{19}$$

where $\theta_{x_{i,t}}$ is the output elasticity with respect to any variable input x, which could be either labor or other intermediate inputs, and $s_{x_{i,t}}$ is the share in production of the variable input costs. Further details on the underlying methodological framework, and how it connects to our main theoretical framework, are described in Appendix B. The two key inputs to the 19 are the costs shares, which are obtained from Compustat Global accounting data, and the estimates for the production elasticities. The latter in turn require estimates of the production function. Production functions, and the corresponding elasticities, are estimated following the proxy methodology of Olley and Pakes (1996) at 2-digit industry level. The advantage of the Olley and Pakes (1996) methodology is that it uses a proxy regression to control for unobservable shocks, which maybe very relevant under oligopolistic competition.

The literature has raised several concerns on potential measurement biases.[19] To address some of those and for robustness, we estimate production functions using either revenues or value added—gross-output net of intermediate inputs. Second, when we estimate mark-ups we do so with and without a control for market shares. We will return on the robustness checks later on. More details on this and on the mark-up estimates are given in Appendix B. It is also useful to note that our work examines the impact of monetary shocks on shifts in the mark-up distribution and its higher moments: recent literature (De Ridder et al. (2022)) notes that those are largely unaffected by the measurements biases or other average price biases.[20]

For the estimates we use data from Compustat Global. We estimate mark-ups at annual frequency to study the long run properties and at quarterly frequencies to study the response to monetary policy shocks. Our sample starts in 1988, which is around the date in which the monetary policy shocks are available. For the production function estimations we obtain data on sales, employment and construct data for capital using the perpetual inventory method, which is initialized with the Compustat variable gross plants, property and equipment, PPE. For the mark-up estimates we

19 See Syverson (2024) for a review on this.

20 Klette and Griliches (1996) noted that under imperfect competition the production elasticities, namely the mark-up numerator, can be estimated consistently only with output data, otherwise they may be biased by the average price elasticity of demand.

construct revenues as sales, deflated by CPI, and extract the cost of good sold (COGS). We construct value added, an alternative to revenues, by subtracting the cost of inputs. As it is customary in the literature we select domestic firms using standard industry format observations in USD with Foreign Incorporation Code (FIC). For industry classification we use the historical SIC code available in Compustat. To obtain measures in real terms we deflate sale, and COGS by the CPI, obtained from the Fred database. More details on the data used for the estimation are given in Appendix B.

We compute the mark-ups for the sample of US firms and for the four core euro area countries, namely Spain, Italy and France. When we examine the long run distributions for the euro area countries we do so jointly and separately. Mark-ups, used both in the analysis of long run distribution and short run response to shocks, are winsorized at 1% level. We put a bound on mark-ups being above 1.[21] Also, when estimating mark-ups we drop observations if quarterly sales growth is above 100% or below -67% or if real sales are below 1 million USD.

High Frequency Identified Monetary Policy Shocks. We follow standard high-frequency identification methodologies for monetary policy shocks in the spirit of Gürkaynak et al. (2005). The underlying logic is as follows. If investors internalize all available information, prices will react to the unexpected component of monetary policy. This results in high-frequency movements of market prices in a narrow window around the policy announcement. Broadly speaking the shocks are constructed as follows:

$$\varepsilon_t^{MP} \equiv S_t^+ - S_t^-,, \tag{20}$$

where the index t in S indicates the date of an monetary policy announcement, and S_t^+ and S_t^- are the high frequency rates, shortly after and before the time of the announcement, respectively.

We rely on shocks available in the literature and we focus in particular on recent estimates constructed by Jarociński and Karadi (2020) and Bauer and Swanson (2023). The first has the advantage of purging the shocks from the information effect, namely the signals conveyed by monetary policy actions about the future course of the economy: while the latter may have an independent and interesting role, we aim to focus here on the classical transmission channel through demand and marginal costs effects. The shocks constructed by Bauer and Swanson (2023) have several advantages. First, those authors collect more events than just the main policy announcements. This allows to overcome the attenuation bias mentioned in Ramey (2016). Second, we employ the orthogonalized version of the shocks provided by those authors: asset prices movements around events may still be endogenous to financial market conditions, therefore Bauer and Swanson (2023) further regress the

21 See Gandhi et al. (2020): mark-ups bounds, which are compatible with constant return to scale assumptions, increase confidence in identification of production elasticities.

high frequency movements onto financial markets variables to reduce the likelihood of capturing confounding factors.

For the euro area we employ the shocks constructed by Altavilla et al. (2019). The authors provide several alternatives using the responses of high frequency rates on various asset classes and computing them at different windows around the shocks.[22] We choose a conservative option, namely the high-frequency movements in 3-month Euro OIS rate around monetary policy event windows at the quarterly level.

Other Data used for the Estimation. In some of the local projection specifications we assess the impact of monetary policy on the mark-up growth at the tails. In this case, and following Ottonello and Winberry (2020), we control for firms' total assets, sales growth, current assets as share of total assets. We gather those variables from Compustat Global. Controlling for those increases confidence that our results are not driven by any change in size. We also include a fiscal quarter dummy, that absorbs variation at business cycle frequency or other time-varying factors that may affect firms' profit margins.

Firm Uncertainty. Firms' uncertainty is a distinctive feature of the transmission mechanism of our model. To validate the importance of this channel in the data we estimate some specification in which the monetary policy shock is interacted with a dummy that captures if firms' uncertainty is above the median. We construct a measure of firms' uncertainty by regressing firms' corporate bond spreads onto the four Fama and French (1993) factors. This allows to purge the risk measures from their exposure to systematic risk, leaving us with a measure of unpredictable firm level uncertainty. To this purpose we gather corporate bonds price and coupons data at monthly frequencies from TRACE. The uncertainty measure is ultimately given by the standard deviation of the estimated error.

Patent. Another channel discussed in our model relates to the role of innovation. We measure that in the US data using the data on firms' economic values of patents from Kogan et al. (2017). The measure is constructed, and updated, by the authors using newly collected data on patents issued to U.S. firms, combined with the stock market response to news about patents. The authors provide several measure and we rely on the real-value of the patent. This captures the economic-value of a patent, which is positively related to the scientific value of these patents, as measured by the number of citations the patent receives in the future.

Baseline Econometric Specification. The econometric specification is a panel local projection

22 The rationale for the different windows is that the announcement strategy of the ECB is different than the one of the Fed, that publishes part of its information already in the Greenbook. For the ECB most of the information release happens at the press conference and effects may materialize around a large window around that.

(see Jordà (2005), more recently Montiel Olea and Plagborg-Møller (2021)), for h=1,..., 16 quarters horizons ahead, which reads as follows:

$$y_{t+h} - y_t = \alpha^h + \beta^h \varepsilon_t^{MP} + \gamma_0^h \varepsilon_{t-1}^{MP} + \gamma_1^h (y_{t-1} - y_{t-2}) + u_t^h \tag{21}$$

The dependent variable is either the overall dispersion of mark-ups or the the variance of mark-up dispersion within sectors, $\sigma_t^{2,\hat{\mu_t^i}}$, where $\hat{\mu_t^i}$ is firm level log of markups in deviation from their respective industry-quarter mean. This is computed as the differences of firm level log markups from their industry-time average: $\hat{\mu_t^i} = log(\mu_t^i) - \frac{1}{N_{s,t}} \sum_{i \in \mathcal{I}_{s,t}} log(\mu_t^j)$, where $\mathcal{I}_t$ is the set of firms i in industry s, quarter t, and N_{st} is their number. The time sample runs from 1988 to 2024 and we drop from the sample of firms those in public administration, finance, insurance, real estate, and utilities.[23]

Testing Reallocation to the Top and Interaction with Uncertainty and Innovation. Changes in dispersion may not necessarily capture shifts of the mark-ups from the bottom to the top tails, or the concentration on the tails. To assess this we estimate a local projection specification linking firm level mark-up projections onto monetary policy shocks interacted with a dummy for whether the firm is at the top 10% or at the bottom 10% of the wedge distribution. We estimate the following specification, at $h = 1, ..., 16$ quarters horizons ahead:

$$y_{i,t+h} - y_{i,t} = \alpha^h + f_i + f_{s,t} + \beta_1^h \varepsilon_t^{MP} \mathcal{I}_{i,t} + \beta_2^h Z_{i,t} + \gamma_1^h (y_{i,t-1} - y_{i,t-2}) + u_t^h \tag{22}$$

where $y_{i,t+h}$ is the firm mark-up projection, f_i are firm fixed effects, $f_{s,t}$ are time and sector fixed effects, $(y_{i,t-1} - y_{i,t-2})$ is past mark-up growth at firm level and $\mathcal{I}_{i,t}$. Following the practice established in Ottonello and Winberry (2020) our specification includes a set of controls that may capture other wedges, conflating into mark-ups growth, and not captured by our variables of interest. Those include firm level risk, which we measure as realized stock market volatility of firm i equity, as well as lags of firms size (total assets), leverage (total debt per total assets), and the ratio of liquid assets to total assets (all in deviation from their firm-level mean).

Furthermore, to assess the role of uncertainty for reallocation, we re-estimate the specification 22 by interacting the monetary policy shock with a dummy capturing whether the firm's uncertainty measure that we constructed is above the median. To capture the role of innovation we do the same but for a dummy that captures if firms' patents value is above the median.

Summary Statistics and Distributions. We examine average mark-up distributions also to validate the ones produced by the stochastic steady state of our model. In that context we also

23 Results are unchanged when we include all firms.

discuss the trends of increase in concentration noted elsewhere in the literature (see Loecker et al. (2020); Kwon et al. (2024) for secular growth in concentration). The panels in 8 show the kernel densities of the mark-up distributions for US, France, Germany, Italy and Spain and for four time sample periods, 1985-1994, 1995-2004, 2005-2014, 2014-2024.[24] The distributions show a shift over time toward the upper tail[25] for all countries. Italy exhibits some shifts backward: this is largely due to the country reforms enacted to enter the euro area. Appendix B.1 shows robustness checks with an alternative estimation of the mark-ups, in which we do not control for market shares, and with alternative country groups.[26] Results are similar.

To set the stage for our impulse response analysis at higher frequency it is useful also to examine the trends in mark-up dispersion, which are shown in Figure 9 for various mark-up measures. The dispersion grows over time, confirming previous results (see Hsieh and Klenow (2014)). The series also exhibit significant variation also at higher frequencies.

7.1. Empirical Responses of Mark-ups Moments to Monetary Shocks.

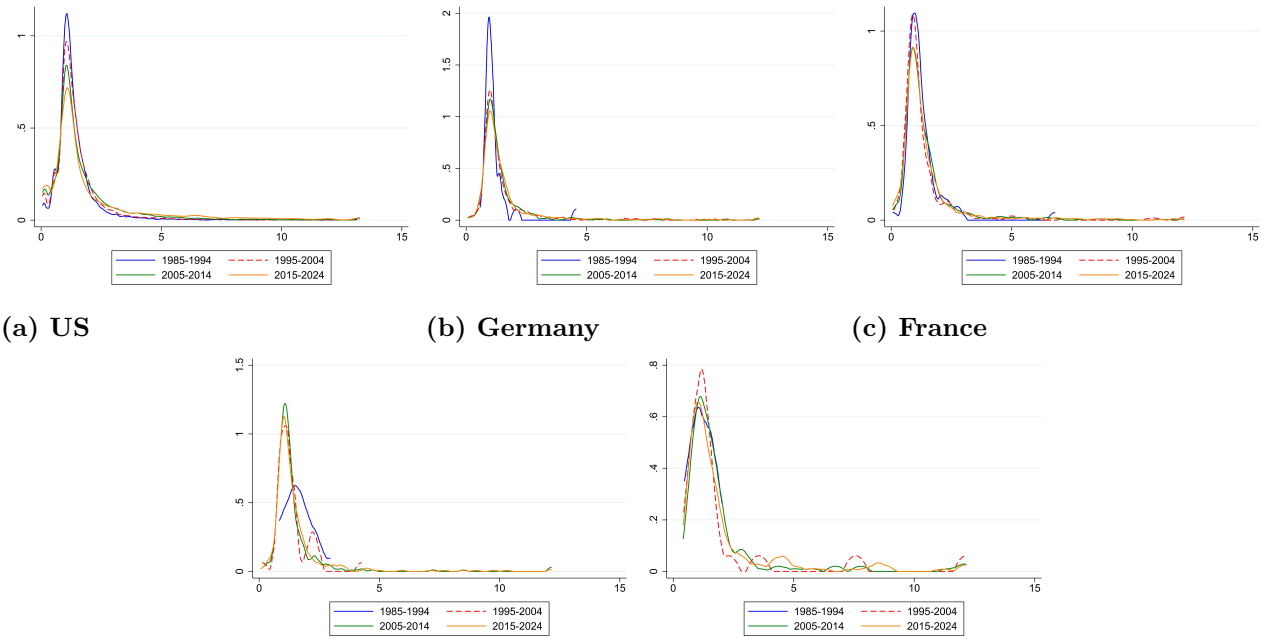
The impulse responses from the estimates of equation 21 are shown in the panels of 10 for US firms. The top panels show the response of the within sector dispersion, $\sigma_t^{2,\hat{\mu_i^t}}$ and the standard deviation to the orthogonalized shocks by Bauer and Swanson (2023) and to two versions of the Jarociński and Karadi (2020) shock estimates, the ones using interest rates and the ones using stock price movements. Dispersion declines significantly for all. Standard deviations show a decline on impact and some increase at later periods. This is so since they capture also reallocation across sectors. Figure 20 shows the impulse response functions for the dispersion and standard deviations for firms located in four core euro area countries, namely Germany, Italy, France and Spain. Again we see a significant decline.

Robustness Checks. Figures 16 to 19 in Appendix B.2 estimate the above specifications using alternative measures for the mark-ups. Specifically, we use mark-ups estimates with value added, with production function elasticities estimated under Translog specification and without controlling for market shares. Results are similar. Appendix B.2 also shows the impulse responses of the average mark-ups to the same two shocks considered here. In line with the model they generally

24 We present the kernel densities as unweighted, a conservative choice. This may of course affect the exact size of the upper tail, however this is not relevant for the scope of our analysis for which we are primarily interested in the trends over time. Furthermore, recent research by De Ridder et al. (2022) demonstrates that weighting schemes do not affect the trends, but just enlighten on the compositional effects that may affect the size of the tails.

25 The shift is significant at Kolmogorov Smirnov tests.

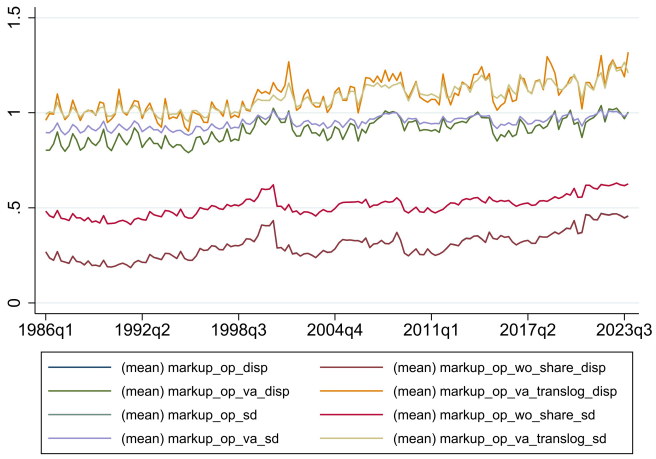
26 Those include: Northern Europe, Southern Europe, Asia, Northern Advanced Asia, Latin America, UK and Channels Islands.



(a) US **(b) Germany** **(c) France**

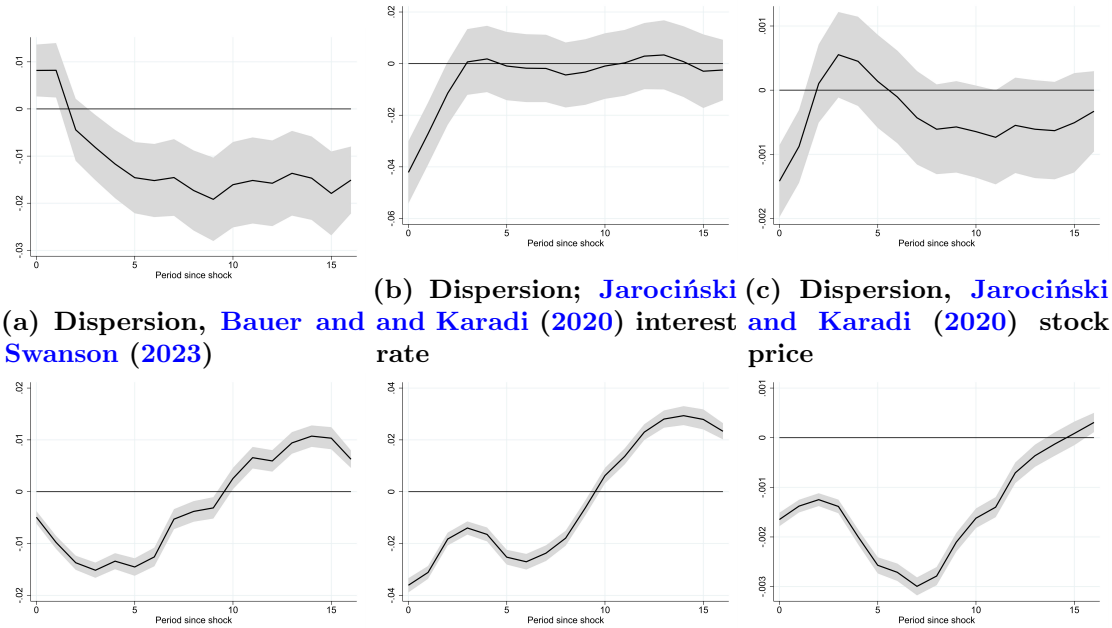
(d) Italy **(e) Spain**

Figure 8: Kernel Densities Mark-ups, Time Sub-samples. The panels show kernel densities of mark-ups computed using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, for four countries, US, Germany, France, Italy and Spain. Mark-ups estimated control for market shares and use annual data from Compustat Global for 1985-2024. Kernel densities are shown for four sub-samples. Mark-up data are winsorized at 1%.



(a) US

Figure 9: Trends in Mark-ups Dispersion and Standard Deviations for the US. The figure shows the trends in the mark-up dispersion, namely firm log mark-ups netted our of the sectoral average, and standard deviations for US firms and for various estimates, based on Olley and Pakes (1996) elasticities, controlling or not for market size, using revenues or value added. Mark-ups estimated control for market shares and use annual data from Compustat Global for 1985-2024.Mark-up data are winsorized at 1%.



(a) Dispersion, Bauer and Swanson (2023) **(b) Dispersion; Jarociński and Karadi (2020) interest rate** **(c) Dispersion, Jarociński and Karadi (2020) stock price**

(d) St. Dev., Bauer and Swanson (2023) **(e) St. Dev., Jarociński and Karadi (2020) interest rate** **(f) St. Dev., Jarociński and Karadi (2020) stock price**

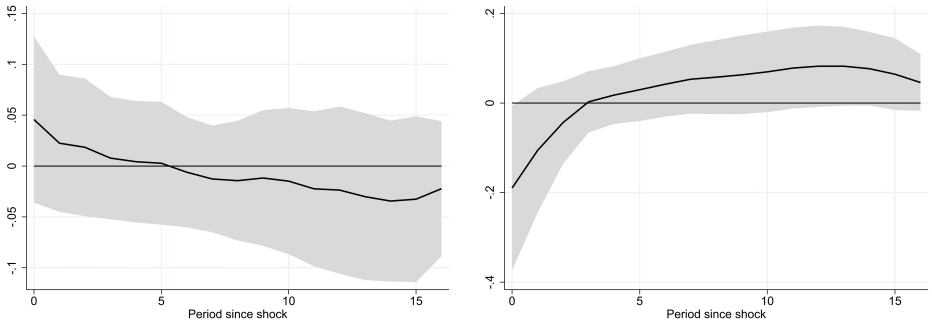
Figure 10: Impulse Response Functions to Monetary Policy Shocks, US Firms. The panels show impulse response functions from the specification 21 of dispersion (from sectoral averages) or standard deviation of mark-ups in response to high frequency identified monetary policy shocks either from Bauer and Swanson (2023) or two versions of the Jarociński and Karadi (2020), the one based on interest rate changes and the one based on stock price changes. Mark-ups are computed for US firms , using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

increase, hence they are counter-cyclical. However, in some cases they also decrease, a fact already noted in Nekarda and Ramey (2020).

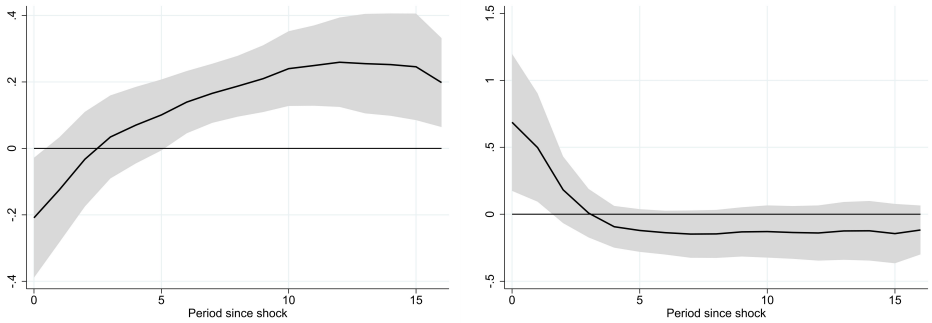
7.2. Firms' Reallocation to the Top, Uncertainty and Innovation

We now document the nature of the reallocation process. We start by examining reallocation along the firms' mark-up distribution. For this we estimate the specification in 22 for top or the bottom 10% of the firm distribution. First, 11 shows the results. Appendix B.2 shows robustness checks with alternative monetary policy shocks. Mark-ups increase significantly for the top 10% of the firms and they remain the same for the bottom 10%.

Next we examine the role of uncertainty and innovation. Figure 12 shows the estimates of the specification in 22 when the monetary shock is interacted with a dummy capturing if firms' uncertainty index is above the median. In line with the model, mark-ups increase by more for firms that face higher uncertainty. Finally, figure 13 shows the impulse responses when the monetary

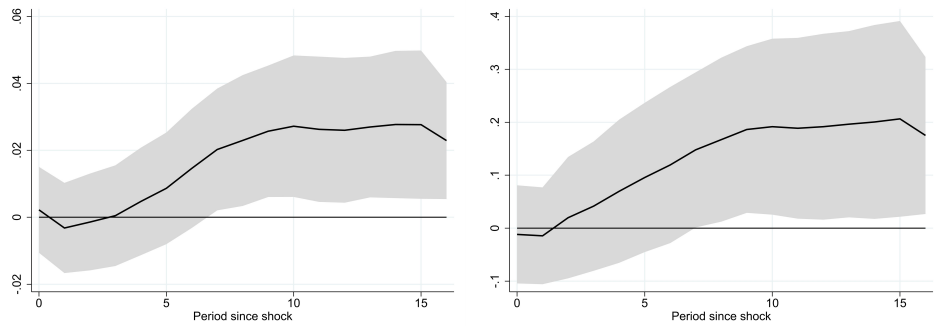


(a) Bauer and Swanson (2023); top 10% **(b) Bauer and Swanson (2023); bottom 10%**

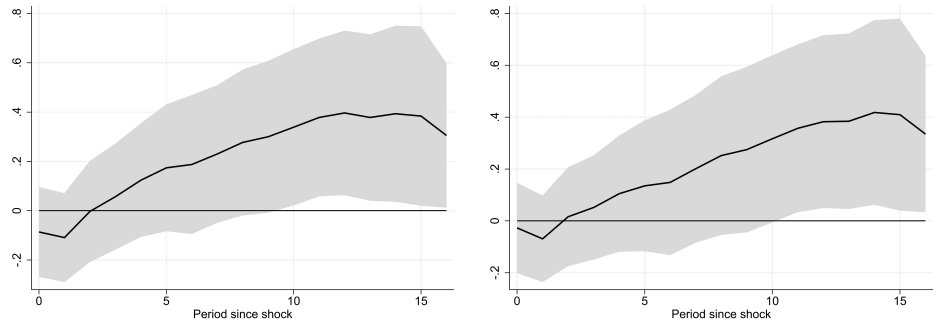


(c) Jarociński and Karadi (2020); Top 10% **(d) Jarociński and Karadi (2020); bottom 10%**

Figure 11: Impulse Response Functions of Mean Mark-ups for Bottom and Top 10% to Monetary Policy Shocks. The panels show impulse response functions estimated from 22 of firm level mark-up growth in response to high frequency identified monetary policy shocks either from Bauer and Swanson (2023) or from Jarociński and Karadi (2020) (we choose the option based on changes in interest rates around the announcement window) , interacted with a dummy for whether the firm is at the top or bottom 10% of the mark-up distribution. Mark-ups are computed for US firms , using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.



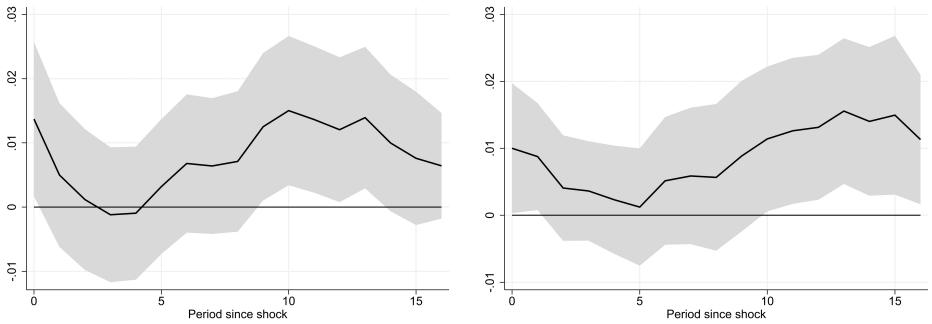
(a) Uncertainty above mean **(b) Interacted by uncertainty**



(c) Uncertainty above mean **(d) Interacted by uncertainty**

Figure 12: Impulse Response Functions of Mark-up Projections to Monetary Policy Shocks Interacted with Uncertainty. The panels show impulse response functions estimated from 22 of firm level mark-up projection in response to high frequency identified monetary policy shocks by Bauer and Swanson (2023), interacted with a dummy indicating whether firms' uncertainty, as measured by Gilchrist and Zakrajšek (2012) purged spreads, is higher than the mean. Mark-ups are computed for US firms using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

shock is interacted with the dummy for whether firms' patent value is above median. In all cases, and in line with the model, mark-ups grow more in response to monetary policy shocks for firms that innovate more.



(a) Bauer and Swanson (2023) **(b) Jarociński and Karadi (2020)**

Figure 13: Impulse Response Functions of Mark-up Projections to Monetary Policy Shocks Interacted with Patent Dummy. The panels show impulse response functions estimated from 22 of firm level mark-up projection in response to high frequency identified monetary policy shocks either Bauer and Swanson (2023) or Jarociński and Karadi (2020), interacted with a dummy indicating whether firms' patent value is above median. Mark-ups are computed for US firms , using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

8. Conclusions

There is increasing evidence that firm distribution, particularly their top tail (see summary in Van Reenen (2018)), is playing an increasingly key role not only in affecting an economy's growth, but also its cyclical dynamic to shocks and there is recent evidence of secular increase in concentration (see Kwon et al. (2024)). Furthermore, there is increasing understanding that misallocation is endogenous (Peters (2020)). Against this background our study aims at examining, theoretically and empirically, the role of firms' heterogeneity for the transmission of shocks, particularly demand and monetary policy shocks, and the cyclical changes in misallocation over the business cycle. While there is an expanding literature on heterogenous firms, a key challenge faced in addressing the questions above lie in providing a model of cyclical reallocation, hence in which firms' distributions change over the business cycle.

We build a model in which firms are heterogenous, as they face idiosyncratic productivity shocks, strategic complementarities in the form of oligopolistic competition and nominal rigidities. Our main is that a contractionary monetary policy leads to a decline in the dispersion of mark-ups, and this is so due to a pronounced reallocation to the top. The extent of the reallocation to the top is determined by the interaction between a precautionary pricing behaviour, in face of uncertainty, and strategic complementarities. We gather time series evidence on the response of the moments of the mark-up distribution to monetary shocks, which confirm the patterns uncovered in the data.

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A. Analytical Derivations Partial Equilibrium Model

A.1. Derivations of the Firms' Mark-ups

Below we present the derivations of the first order condition on prices from the optimization in A.1, and the related mark-up, equation A.4. Substituting demand for each variety into the profit function, we obtain:

$$\Pi(N_t, \varepsilon) = \sum_{t=0}^{\infty} \frac{1}{(1+r)}[(\frac{p_t^i}{P_t})^{1-\varepsilon} Y_t - mc_t(i)(\frac{p_t^i}{P_t})^{-\varepsilon} Y_t - \frac{\varepsilon\kappa}{2} log(\frac{p_t^i}{p_{t-1}^i})^2] \tag{A.1}$$

Taking first order conditions with respect to p_t^i leads to:

$$\begin{aligned}(1-\varepsilon)\frac{Y_t}{P_t}(\frac{p_t^i}{P_t})^{-\varepsilon}[1 - \frac{p_t^i}{P_t}\frac{\partial p_t^i}{\partial P_t}] + \varepsilon\frac{Y_t}{P_t} mc_t^i (\frac{p_t^i}{P_t})^{-\varepsilon-1}[1 - \frac{p_t^i}{P_t}\frac{\partial p_t^i}{\partial P_t}] - \\ - [\varepsilon\kappa log(\frac{p_t^i}{p_{t-1}^i})\frac{1}{p_t^i}] - [\varepsilon\kappa log(\frac{p_{t+1}^i}{p_t^i})\frac{1}{p_t^i}] = 0\end{aligned} \tag{A.2}$$

Substituting $\frac{\partial P_t}{\partial p_t^i} = (\frac{p_t^i}{P_t})^{(-\varepsilon)}$, and multiplying by $(\frac{p_t^i}{P_t})^{\varepsilon} p_t^i \frac{Y_t}{P_t}$, we obtain:

$$\begin{aligned}(1-\varepsilon)p_t^i(1 - (\frac{p_t^i}{P_t})^{(1-\varepsilon)}) + mc_t^i \varepsilon P_t(1 - (\frac{p_t^i}{P_t})^{(1-\varepsilon)}) - \\ - \kappa\varepsilon log(\frac{p_t^i}{p_{t-1}^i})P_t(\frac{p_t^i}{P_t})^{\varepsilon-1} + \frac{1}{1+r}\kappa\varepsilon log(\frac{p_{t+1}^i}{p_t^i})\frac{P_t}{Y_t}(\frac{p_t^i}{P_t})^{\varepsilon-1} = 0\end{aligned} \tag{A.3}$$

Upon defining $\xi_t^i = (\frac{p_t^i}{P_t})^{-\varepsilon}$ and isolating prices leads to:

$$p_t^i = \frac{\varepsilon(1-\xi_t^i) mc_t^i P_t}{(\varepsilon-1)(1-\xi_t^i) + \frac{\varepsilon\kappa}{\xi_t^i}[log(\pi_t^i) + \beta E_t\{log(\pi_{t+1}^i)\}]} \tag{A.4}$$

where we have set $\beta = \frac{1}{1+r}$.

A.2. Proof of Lemma 2.

Multiplying A.3 by p_t^i, substituting $\frac{\partial P_t}{\partial p_t^i}$ delivers:

$$(1-\varepsilon)Y_t(\frac{p_t^i}{P_t})^{1-\varepsilon}[1-(\frac{p_t^i}{P_t})^{1-\varepsilon}]+\varepsilon Y_t mc_t^i(\frac{p_t^i}{P_t})^{-\varepsilon}[1-(\frac{p_t^i}{P_t})^{1-\varepsilon}]- \tag{A.5}$$
$$-[\varepsilon\kappa log(\frac{p_t^i}{p_{t-1}^i})]-[\varepsilon\kappa log(\frac{p_{t+1}^i}{p_t^i})]=0$$

The marginal costs $mc_t^i=\frac{w_t}{z_t}$ are given by the ratio of real wages over productivity. Aggregate prices and productivity are both assumed to follow a lognormal distribution: $log(\frac{P_t}{\overline{P}})\ \mathcal{N}(\mu_p,\sigma_p^2)$ and $log(\frac{z_t}{\overline{z}})\ \mathcal{N}(\mu_z,\sigma_z^2)$, where overline variable indicate steady state. We can approximate the right hand side of **??** around the flexible price steady state so that: $\overline{p}_t^i=\frac{\varepsilon(1-\overline{\xi}^i)}{\varepsilon(1-\overline{\xi}^i)-1}$

B. Methods for Production Function and Mark-up Estimation

Below we provide full details for the production function, and its elasticities, estimates and for the mark-up estimates.

Production function estimation. We estimate production function, and its elasticities, following the methodology in Olley and Pakes (1996). The operational implementation is done through the PRODEST procedure in STATA. The functional background can be summarized as follows. Consider a Cobb-Douglas production technology for firm i at time t:

$$y_{it}=\alpha+w_{it}\beta+k_{it}\gamma+\omega_{it}+\varepsilon_{it} \tag{B.1}$$

where y_{it} is the (log) output, w_{it} is a $1xJ$ vector of (log) free variables, k_{it} is a $1xK$ vector of state variables and ε_{it} is a normally distributed idiosyncratic error term. The unobserved technical efficiency parameter ω_{it} evolves according to a first-order Markov process: $\omega_{it}=E(\omega_{it}\mid\omega_{it-1})+u_{it}=g(\omega_{it-1})+u_{it}$ and u_{it} is a random shock component assumed to be uncorrelated with the technical efficiency and the state variables, namely capital and the lagged free variables. The OP method relies on the assumptions that: a) investment, $i_{it}=i(k_{it},\omega_{it})$, is a function of both the state variable and the technical efficiency parameter and that is strictly monotone in the latter, b) that ω_{it} is scalar unobservable in the investment function, c) that the levels of investment and capital are decided at time t-1, or else the level of the free variable is decided after the shock u_{it} realizes. Those

assumptions ensure the investment function is invertible of the investment function with respect to the ω_{it} and lead to the partially identified model:

$$y_{it} = \alpha + w_{it}\beta + k_{it}\gamma + h(i_{it}, k_{it}) + \varepsilon_{it} = \alpha + w_{it}\beta + \phi(i_{it}, k_{it}) + \varepsilon_{it} \tag{B.2}$$

which is estimated through a non-parametric approach. The method consists of two-stages. In the first stage, by exploiting the Markovian nature of the productivity process, one can use assumption c) in order to set up the relevant moment conditions and estimate the production function parameters. The second stage exploits the residual e_{it} defined as follows: $y_{it} - w_{it}\beta = \alpha + k_{it}\gamma + g(\omega_{it1}, \chi_{it}) + \varepsilon_{it}$, approximating the function $g(.)$ by a nth order polynomial and where χ_{it} is an indicator function for the attrition in the market.

The variables used from Compustat Global for the production function estimations are sales, which are deflated by the sector CPI,[27] employment (EMP) and capital. The latter is constructed using a perpetual inventory method. The variable PPEGT is a measure of the book value of the capital stock and is used to initialize the capital stock. Next, we iterate forward on capital using the accumulation equation $k_{it} = (1 - \delta)k_{it1} + i_{it}$. The measure of net investment, $i_{it} - \delta k_{it1}$ using the difference in the variable PPENT between time t and time $t - 1$. All variables are deflated by CPI. For output we use either revenues, computed from sales deflated by CPI, or value added, sales net of intermediate inputs. The latter is obtained as sales net of cost of good sold (COGS variables in Compustat Global).

Mark-ups Estimates. We start by describing the theoretical background of the mark-up estimates. For the estimates of the mark-ups we consider a general framework in which firm may face other frictions, which we leave unspecified in the data, but can map several frictions in the full macro model. Specifically, consider a continuum of monopolistic competitive firms that produces using capital, labour and intermediate goods with the following production function, $Y_i = A_i F(K_i, L_i, M_i)$, where A_i, denotes firm i's productivity (TFPQ), K, L and M denote capital, labor and material inputs. F is assumed to have constant returns to scale. The firm faces a demand curve: $Y_i = Y\phi(\frac{p_i}{P})$. where Y is some aggregate demand term, p_i is the price of the good produced by firm i, P is a price index and $\Phi(.)$ denotes the demand curve, assumed to slope down: $\Phi'(.) \leq 0$. For future reference, we denote $\sigma_i = -\Phi'(\frac{p_i}{P})/\Phi(.)$ the elasticity of demand. Firms' profits are given by:

$$\pi_i = (1 - \tau_i)p_i Y_i - \sum_x (1 + \tau_{ix})p_x x_i \tag{B.3}$$

27 The latter is taken from the WorldBank database.

where x denotes the various inputs (K, L, M), at price p_x. τ_i denotes a firm-specific scale distortion and τ_{ix} denotes a factor x firm specific wedge. Note that the factor prices are assumed to be identical. If they are not, this can be treated as part of the wedge τ_{ix}. For the time being we remain agnostic about the nature of those additional distortions. They may encompass several of the additional firm wedges, such as credit default risk, that we derive separately further below.

The firm chooses inputs by maximizing B.3, subject to the production and to the demand constraints, $Y_i = Y\phi(\frac{p_i}{P})$. Let us denote by ν_i the Lagrange multiplier on the demand constraint and λ_i as the Lagrange multiplier on the production constraint. The first order conditions for output, prices and inputs read respectively are as follows:

$$(1-\tau_i)p_i - \nu_i - \lambda_i = 0;\ (1-\tau_i)Y_i + \nu_i \frac{Y}{P}\phi'(\frac{p_i}{P}) = 0;\ -(1+\tau_{ix})p_x + \lambda_i A_i F_{xi} = 0 \tag{B.4}$$

The first condition with respect to prices can be re-arranged as follows:

$$(1-\tau_i) = \frac{\nu_i}{p_i}\frac{-\phi'(\frac{p_i}{P})\frac{p_i}{P}}{\phi(\frac{p_i}{P})} = \sigma_i \frac{\nu_i}{p_i} \tag{B.5}$$

which after substitution into the first order condition for prices yields $(1-\tau_i)(1-\frac{1}{\sigma_i})p_i = \lambda_i$, which, by denoting the mark-up as: $\mu_i = \frac{\sigma_i}{(\sigma_i - 1)}$, delivers:$p_i = \mu_i(\frac{\lambda_i}{1-\tau_i})$, where λ_i has the interpretation of a nominal marginal cost. For the pricing decision, this marginal cost is inflated by a factor $\frac{1}{(1-\tau_i)}$ to offset the implicit wedge on output. From the first order condition with respect to inputs, we obtain:

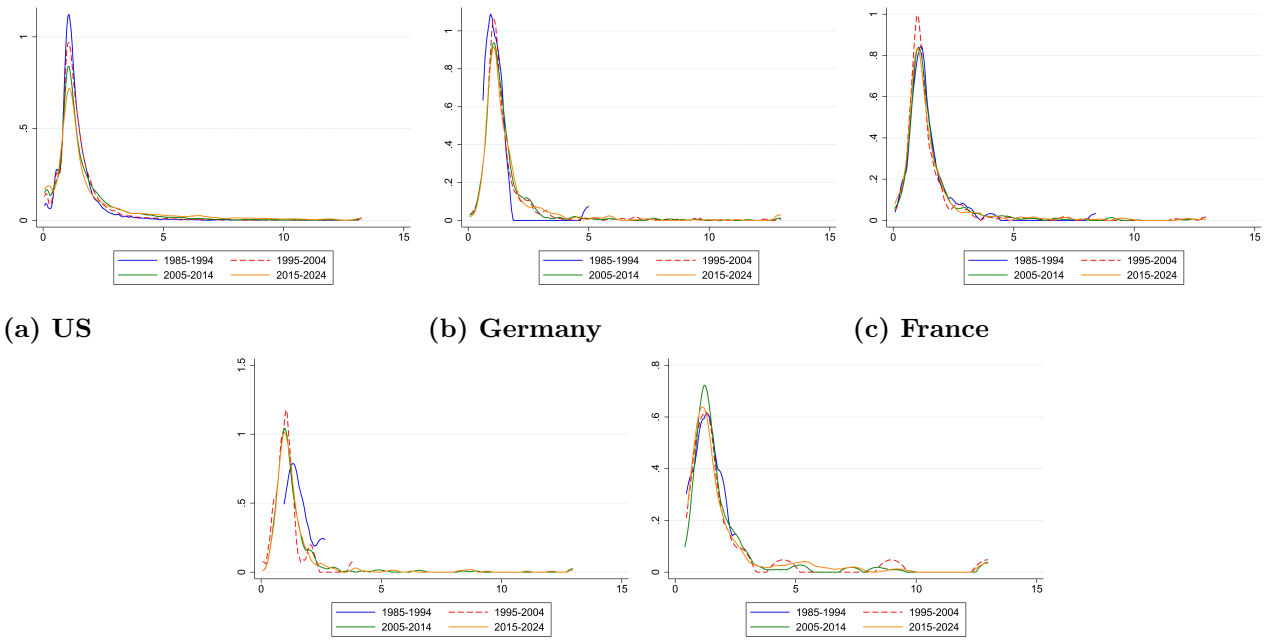
$$(1+\tau_{ix})p_x = \lambda_i A_i F_{xi} \Leftrightarrow \frac{(1+\tau_{ix})p_x}{Y_i} = \lambda_i \frac{F_{xi}x_i}{F_i} = \lambda_i \theta_{ix} \tag{B.6}$$

where $F_i = \frac{\partial F_i}{\partial x_i}$ and $\theta_{ix} = \frac{F_{xi}x_i}{F_i}$ is the output elasticity to factor x. For a Cobb-Douglas production function, it is constant. From the last relation one obtains: $\frac{(1+\tau_{xi})p_i x_i}{p_i Y_i} = (1-\tau_i)\frac{\theta_{xi}}{\mu_i}$. If one defines the factor share in revenues: $s_{ix} = \frac{(1+\tau_{xi})p_i x_i}{p_i Y_i}$, this formula allows us to express the markup as:

$$\mu_i = (1-\tau_i)\frac{\theta_{ix}}{s_{ix}} \tag{B.7}$$

The expression of mark-up that we bring to the data is:

$$\tilde{\mu}_i = \frac{\theta_{ix}}{s_{ix}} \tag{B.8}$$



(a) US **(b) Germany** **(c) France**

(d) Italy **(e) Spain**

Figure 14: Kernel Densities Mark-ups, Time Sub-samples. .The panels plot kernel densities of mark-ups computed using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, for four countries, US, Germany, France, Italy and Spain. Mark-ups estimated control for market shares and use annual data from Compustat Global for 1985-2024. Kernel densities are shown for four sub-samples. Mark-up data are winsorized at 1%.

The latter is computed using the elasticity of production with respect to capital, computed through the production function estimation, and the cost of good sold (COGS variable in Compustat Global).

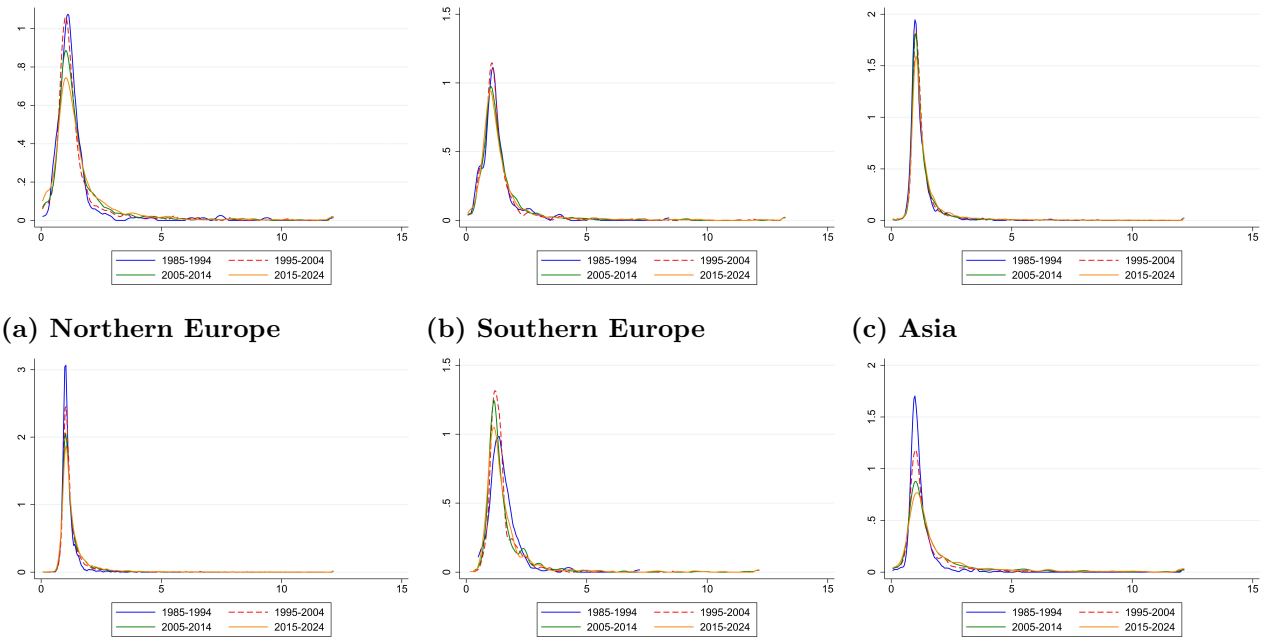
B.1. Mark-up Statistics and Distributions

Figure 14 shows kernel densities across four time sub-samples (1985-1994, 1995-2004, 2005-2014, 2015-2024) for US, Germany, France, Italy and Spain. Mark-ups are estimated with Olley and Pakes (1996), with production elasticities estimated at 2-digit industry level and winsorized at 1%. Figure **??** shows the same kernels but for groups of countries: Northern Europe, Southern Europe, Asia, Northern Advanced Asia, Latin America, UK and Channels Islands.

B.2. Other Impulse Response Functions

Figure 16 shows the impulse response of the mark-up mean in response to shocks from Bauer and Swanson (2023) and Jarociński and Karadi (2020) shock (with interest rates).

Figure 17 shows impulse response functions of mark-up dispersions within sectors and standard



(a) Northern Europe **(b) Southern Europe** **(c) Asia**

(d) Northern Advanced Asia **(e) Spain** **(f) UK and Channel Islands**

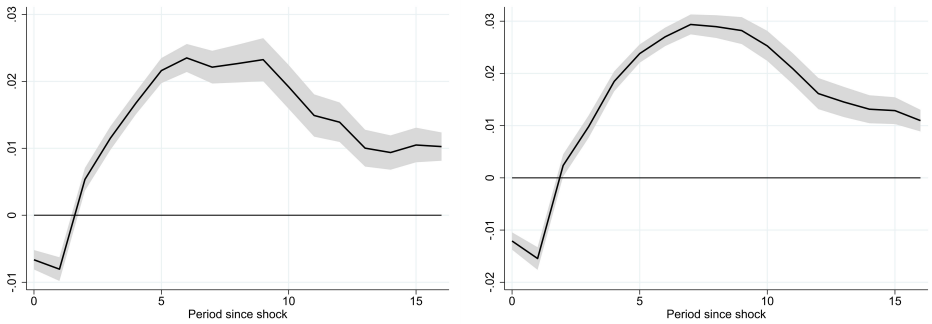
Figure 15: Kernel Densities Mark-ups, Time Sub-samples. .The panels plot kernel densities of mark-ups computed using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, for four groups of countries: Northern Europe, Southern Europe, Asia, Northern Advanced Asia, Latin America, UK and Channels Islands. Mark-ups estimated control for market shares and use annual data from Compustat Global for 1985-2024. Kernel densities are shown for four sub-samples. Mark-up data are winsorized at 1%.

deviations in response to shocks from Bauer and Swanson (2023) or to the Jarociński and Karadi (2020) shock (interest rates or stock prices), when the production elasticities used to compute mark-ups are estimated with value added.

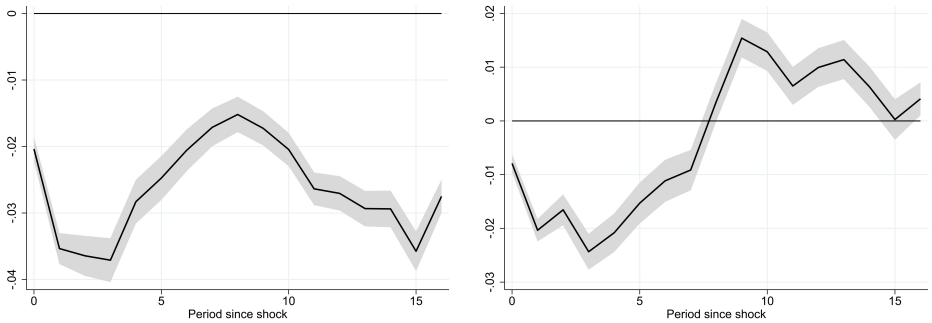
Figure 18 shows impulse response functions of mark-up dispersions within sectors and standard deviations in response to shocks from Bauer and Swanson (2023) or to the Jarociński and Karadi (2020) shock (interest rates or stock prices), when the production elasticities used to compute mark-ups are estimated with value added and Translog specification for production function.

Figure 19 shows impulse response functions of mark-up dispersions within sectors and standard deviations to in response to shocks from Bauer and Swanson (2023) or to the Jarociński and Karadi (2020) shock (interest rates or stock prices), when mark-ups are estimated using firms' revenues without controlling for market shares.

Figure 20 shows impulse responses of mark-up (in revenues or value added) standard deviation for firms in four euro area countries (France, Germany, Italy and Spain) to the monetary policy shocks from Altavilla et al. (2019) using year OIS. The shocks are changes in European stock market index around the monetary event window.



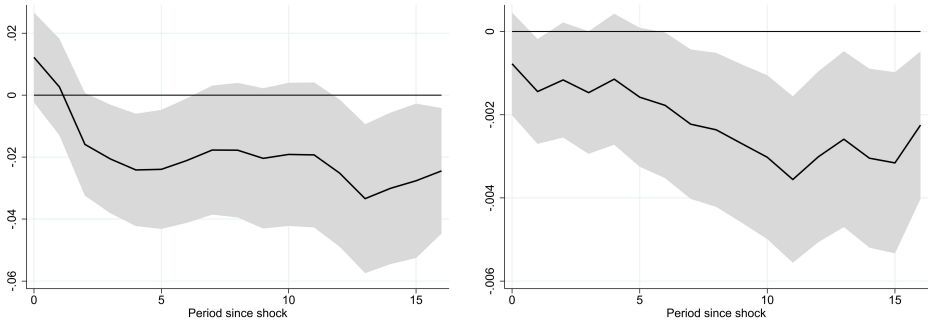
(a) Mean, Bauer and Swanson (2023) **(b) Mean (VA); Bauer and Swanson (2023)**



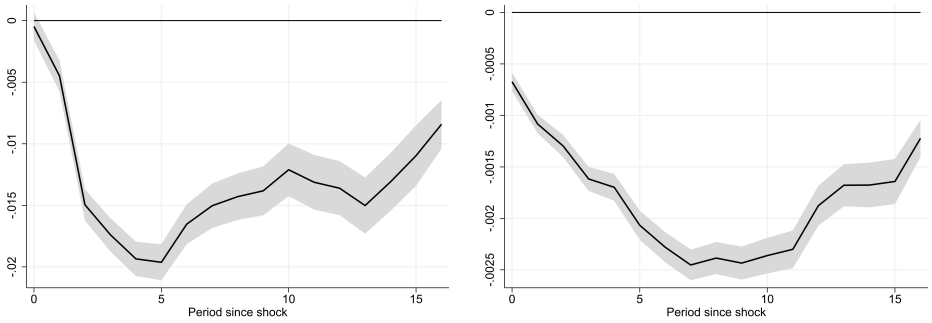
(c) Mean; Jarociński and Karadi (2020) **(d) Mean (VA); Jarociński and Karadi (2020)**

Figure 16: Impulse Response Functions to Monetary Policy Shocks for US. The panels show impulse response functions of mean mark-ups in response to high frequency identified monetary policy shocks either from Jarociński and Karadi (2020) or Bauer and Swanson (2023). Mark-ups are computed for US firms , using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

Figure 21 shows impulse responses for the firms at the top and bottom 10% of the mark-up distributions to Bauer and Swanson (2023) shocks, when the mark-ups are computed based on Translog production function, and to Jarociński and Karadi (2020), when the shock is extracted using principal comonents.

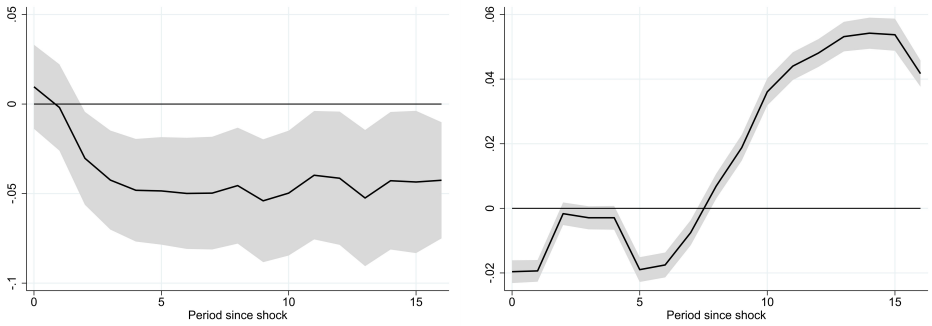


(a) Dispersion, Bauer and Swanson (2023) **(b) Dispersion; Jarociński and Karadi (2020)**

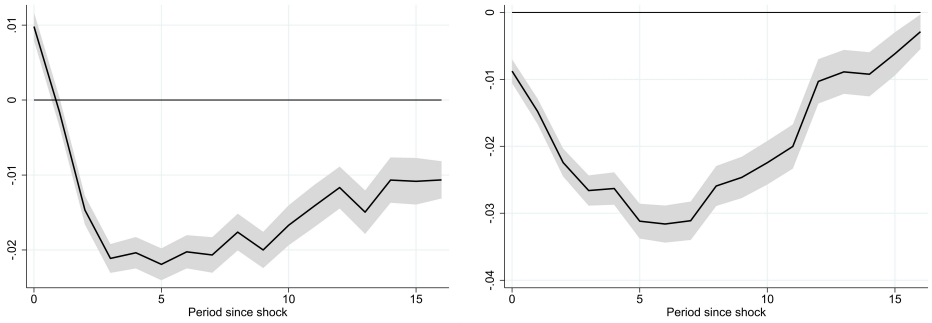


(c) St. Dev., Bauer and Swanson (2023) **(d) St. Dev., Jarociński and Karadi (2020)**

Figure 17: Impulse Response Functions to Monetary Policy Shocks, US Firms Mark-ups estimated using Value Added. The panels show impulse response functions from the specification 21 of dispersion (from sectoral averages), standard deviation of mark-ups in response to high frequency identified monetary policy shocks either from Bauer and Swanson (2023) or two versions of the Jarociński and Karadi (2020). Mark-ups are computed for US firms , using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

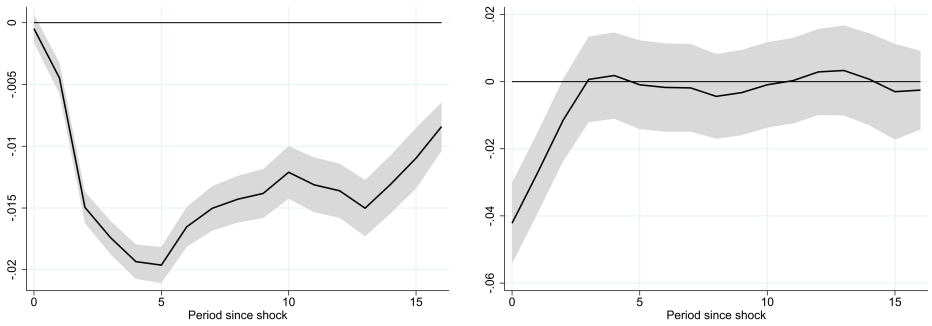


(a) Dispersion, Bauer and Swanson (2023) **(b) Dispersion; Jarociński and Karadi (2020)**

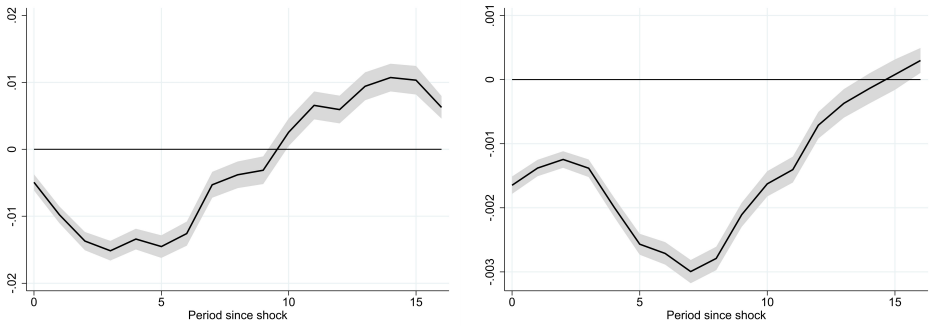


(c) St. Dev., Bauer and Swanson (2023) **(d) St. Dev., Jarociński and Karadi (2020)**

Figure 18: Impulse Response Functions to Monetary Policy Shocks, US Firms Mark-ups estimated using Translog Production Function. The panels show impulse response functions from the specification 21 of dispersion (from sectoral averages), standard deviation of mark-ups in response to high frequency identified monetary policy shocks either from Bauer and Swanson (2023) or two versions of the Jarociński and Karadi (2020). Mark-ups are computed for US firms , using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

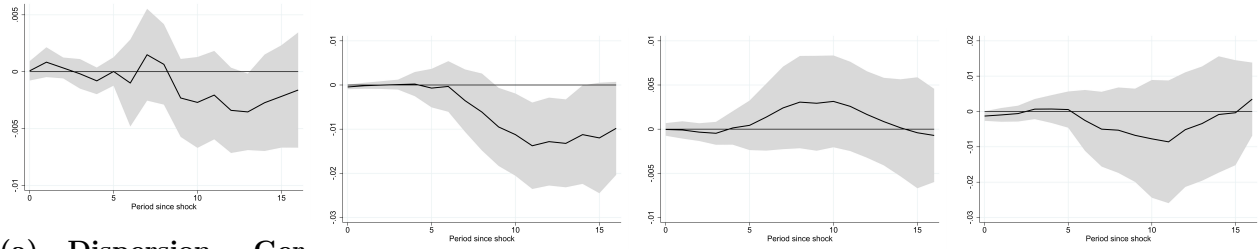


(a) Dispersion, Bauer and Swanson (2023) **(b) Dispersion; Jarociński and Karadi (2020)**

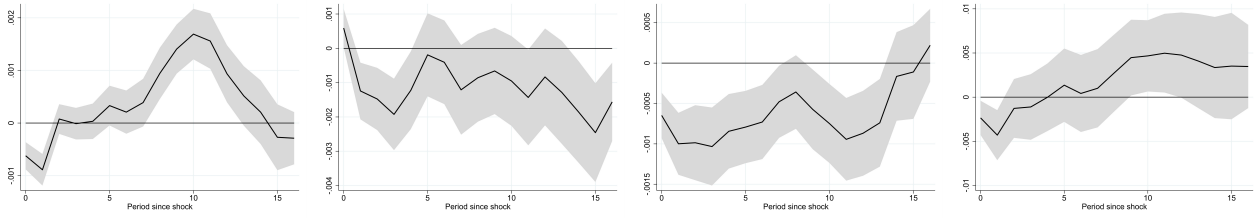


(c) St. Dev., Bauer and Swanson (2023) **(d) St. Dev., Jarociński and Karadi (2020)**

Figure 19: Impulse Response Functions to Monetary Policy Shocks, US Firms Mark-ups estimated without controls for Market Shares. The panels show impulse response functions from the specification 21 of dispersion (from sectoral averages), standard deviation of mark-ups in response to high frequency identified monetary policy shocks either from Bauer and Swanson (2023) or two versions of the Jarociński and Karadi (2020). Mark-ups are computed for US firms , using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

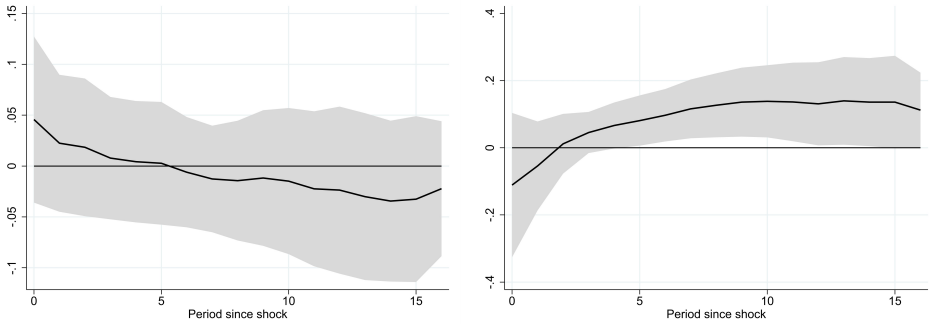


(a) Dispersion, Germany **(b) Dispersion, Italy** **(c) Dispersion, France** **(d) Dispersion, Spain**

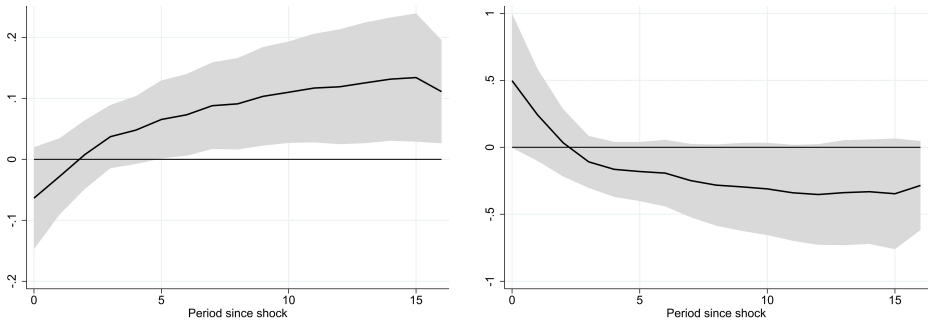


(e) St. Dev. Germany **(f) St. Dev., Italy** **(g) St. Dev., France** **(h) St. Dev., Spain**

Figure 20: Impulse Response Functions to Monetary Policy Shocks for euro area countries. The panels shows impulse responses of mark-ups dispersion and standard deviation for firms in four euro area countries (France, Germany, Italy and Spain) to the monetary policy shocks from Altavilla et al. (2019). The shocks are changes in OIS three months around the monetary event window. Mark-ups are computed for US firms, using production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.



(a) Bauer and Swanson (2023); top 10% **(b) Bauer and Swanson (2023); bottom 10%**



(c) Jarociński and Karadi (2020); Top 10% **(d) Jarociński and Karadi (2020); bottom 10%**

Figure 21: Impulse Response Functions of Mean Mark-ups for Bottom and Top 10% to Monetary Policy Shocks. Value Added Mark-ups. The panels show impulse response functions estimated from 22 of firm level mark-up growth in response to high frequency identified monetary policy shocks either from Bauer and Swanson (2023) or the principal component from Jarociński and Karadi (2020), interacted with a dummy for whether the firm is at the top or bottom 10% of the mark-up distribution. Mark-ups are computed for US firms, using value added for production elasticities estimated with Olley and Pakes (1996) at 2-digit industry level, except firms in public administration, finance, insurance, real estate, and utilities. Mark-ups estimated control for market shares and use quarterly data from Compustat Global for 1985-2024. Mark-up data are winsorized at 1%.

C. Stochastic Steady State Solution: Proof to Lemma 2

We solve analytically for the case with no price rigidities and no capital adjustment costs. In the steady state prices are connected to marginal costs (from the FOC for prices)$\frac{p}{P} = \frac{\varepsilon}{\varepsilon-1}mc$. From the FOC for labor and the production constraint$= zk_{-1}^{\alpha}(1-\alpha)n^{-\alpha}mc$ and $zk_{-1}^{\alpha}n^{1-\alpha} = \left(\frac{p}{P}\right)^{-\varepsilon}Y$. Substituting n from FOC for labor into the production constraint, one can show that marginal costs, on the other hand, depend on capital:

$$mc = \left(\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}Y\right)^{\frac{\alpha}{1-\alpha+\alpha\varepsilon}}\left(\frac{w}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}}\left(zk_{-1}^{\alpha}\right)^{\frac{-1}{1-\alpha+\alpha\varepsilon}} = \gamma\left(zk_{-1}^{\alpha}\right)^{\frac{-1}{1-\alpha+\alpha\varepsilon}} \tag{C.1}$$

where with γ we defined a constant in front of zk^{α} (we solve for steady state in aggregates but with idiosyncratic productivity shocks, thus there is $_{-1}$ for capital and $'$ later on would stand for the next period variables). From the FOC for capital we also obtain the relation for capital as a function of uncertainty:$1 = \frac{1}{1+r}\Pi\left(1-\delta+mc'z'\alpha k^{\alpha-1}n'^{1-\alpha}\right)$, where $'$ corresponds to the next period value of the variables and Π is a Markov transition matrix. Substituting mc and n as functions of capital the relation is obtained:$k(z) = \left(\gamma_1\Pi(z')^{\frac{\varepsilon-1}{1+\alpha\varepsilon-\alpha}}\right)^{1+\alpha\varepsilon-\alpha}$, where $\gamma_1 = \frac{\alpha}{r+\delta}\gamma^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}$. Let $\pi(z)$ be the associated stationary distribution of the Markov transition matrix. From **??** using Jensen inequality the aggregate level of capital is larger the larger the uncertainty (given $\alpha \in (0,1)$)

$$K = \sum_z \pi(z)k(z) = \gamma_1^{1+\alpha\varepsilon-\alpha}\sum_z \pi(z)\left(\Pi(z')^{\frac{\varepsilon-1}{1+\alpha\varepsilon-\alpha}}\right)^{1+\alpha\varepsilon-\alpha} \geq \gamma_1^{1+\alpha\varepsilon-\alpha} = K_{\text{no uncertainty}} \tag{C.2}$$

when $\varepsilon > \max(2, 1+\frac{1}{1-\alpha})$. When $\varepsilon < \min(2, 1+\frac{1}{1-\alpha})$ the result is reversed: the larger the uncertainty the smaller the aggregate level of capital. Thus, for the moderate markups uncertainty increases the aggregate level of capital, but for highly concentrated market, uncertainty decreases aggregate capital level. Let us now derive the level of capital as if the future level of z is known. In this case in the **??** instead of $\Pi z'$ we would have only z'. Thus, $k(z') = \gamma_1^{1+\alpha\varepsilon-\alpha}(z')^{\varepsilon-1}$. For sufficiently large/small ε ($\varepsilon > 2$ / $\varepsilon < 2$) this function is convex/concave. So, capital should be increased and decreased more/less than one for one with the productivity. The reason to the larger adjustments of capital with larger ε is because larger ε means smaller rents and larger responsiveness of firm's policy functions to the changes in marginal costs (which are an inverse function of productivity). Consequently, uncertainty about future z' and convexity / concavity of the dependence of optimal capital level on the level of productivity lead to the higher / lower levels of capital for heterogeneous firms vs. a representative firm.

Given the above aggregate marginal costs also increase as per eq. (C.1). Aggregate labor

hours increase too and this follows from the result for capital and capital-labor complementarity (one could also derive the result from labor F.O.C and eq. (C.1)). The increase in equity price follows from the F.O.C. for prices (substitute $\frac{p}{P}$ ratio with marginal costs), production function expression and the expression for the firm value and dividends: $v = \frac{div}{r} = \frac{1}{r}\left(\frac{\varepsilon}{\varepsilon-1}mc \cdot y - wn - \delta k\right) = \frac{1}{r}\left(zk_{-1}^{\alpha}n^{1-\alpha}\frac{\varepsilon}{\varepsilon-1}mc - wn - \delta k\right)$. δ should be sufficiently small, so that when n and mc are substituted as functions of z the difference is still a convex function.

We can extend the above results to the case with capital adjustment costs. When capital adjustment costs are added in the form from the baseline model the only equation that changes is the F.O.C. for capital. It is substituted with the following expression:

$$\begin{aligned} 1 + \frac{1}{\delta\varepsilon_I}\left(\frac{k-k_{-1}}{k_{-1}}\right) =& \frac{1}{1+r}\Pi\left(1-\delta+mc'z'\alpha k^{\alpha-1}n'^{1-\alpha}\right) \\ &+\frac{1}{1+r}\Pi\left(\frac{1}{\delta\varepsilon_I}\left(\frac{k'-k}{k}\right)\frac{k'}{k} - \frac{1}{2\delta\varepsilon_I}\left(\frac{k'-k}{k}\right)^2\right) \end{aligned} \tag{C.3}$$

The additional terms are the ones with the difference of capital levels between consecutive periods. On the left-hand side the firm encounter with additional costs today if it increases its level of capital. On the right-hand side it acquires additional benefits from saving future adjustment costs of capital. However, uncertain future, intuitively, should decrease these benefits.

Illustrative Numerical Example. Figure 22 shows the relation between aggregate capital and the primitive parameters that govern the degree of strategic complementarities in a numerical example.

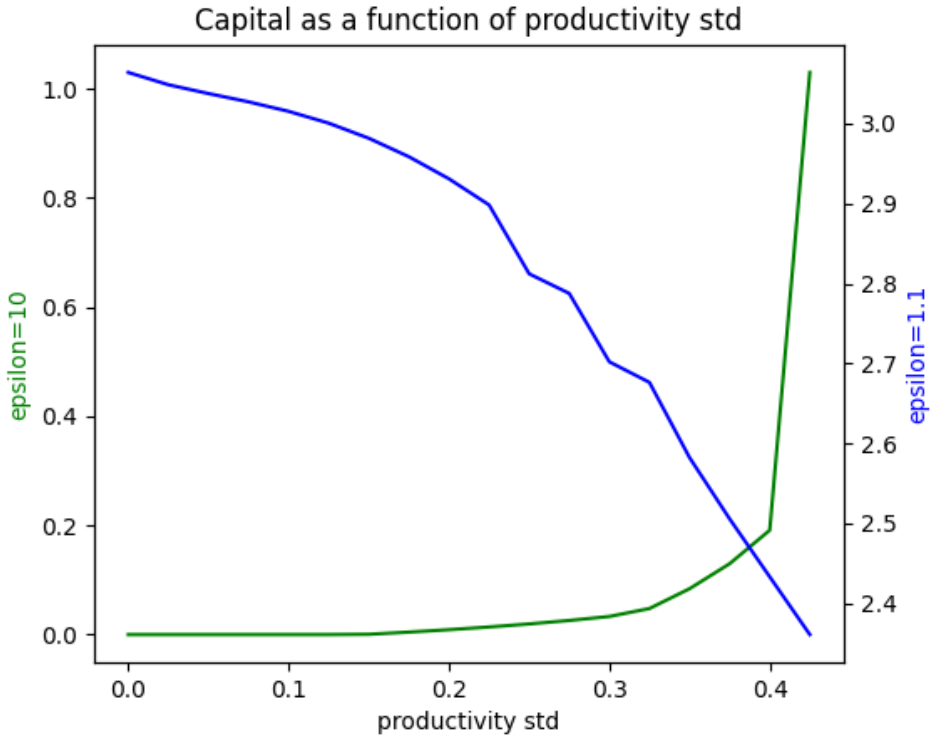


Figure 22: Levels of capital as a function of standard deviation of the productivity shocks. X-axis represents productivity standard deviations. Capital levels for the case with $\varepsilon = 10$ are presented in green and on the left axis. Capital levels for the case with $\varepsilon = 1.1$ are presented in blue and on the right axis. Absolute values of capital depend on the model-based value of the wage and aggregate productivity (so are irrelevant). The results are presented for the steady state of the NK heterogeneous firms model.

D. Quantitative Model Appendix

D.1. Oblivious Equilibrium Concept

The oblivious equilibrium, as formalized in Weintraub et al. (2008), admits the following general representation. Given a finite number set of firms $i \in S$, each experiencing a realization of a random process, $x_{i,t}$. The latter may represent an idiosyncratic shocks to productivity, product quality or demand. Define as μ_i the set of policy functions and strategies by firm i and μ_{-i} is the set of competitors policies. Note that the policy function depend on firms' state, $x_{i,t}$, hence they are themselves stochastic. The time t profits of firms i are $\pi(x_{i,t}, s_{-i,t})$ and depend on the firm productivity realization and on the competitors' states, $s_{-i,t}$. In a Markov perfect equilibrium

(Maskin and Tirole (1988)), or MPE since now on, over an infinite horizon, firms' value function reads as follows:

$$\tilde{J}(x, s|\mu, \mu_{-i}) = \sup_{\mu' \in \tilde{M}} E_{\mu_{-i},\mu}[\sum_{k=t}^{\infty} \beta^{k-t}(\pi(x_{ik}, s_{-i,k}) - di_{ik})|x_{it} = x, s_{-i,t} = s] \tag{D.1}$$

where $\tilde{J}$ is the firm's value function under the MPE, μ' is the firm's pricing strategy, μ are the pricing strategies of the firm's competitors at the steady state, $\tilde{s}_\mu$ is competitors' state vector in the steady state (productivity), x is firm's heterogeneity parameter or the idiosyncratic state (productivity in our case), $x_{i,t}$ is a realization of the x parameter for firm i at time t, β is the discount factor, π is the profit function, d is investment cost, i_{ik} is investment of firm i at time k. Note that expectations are taken with respect to the strategies of both firms, since in an uncertain environment, firms form beliefs on the equilibrium strategies. We can re-write the value function in recursive forms as follows:

$$\tilde{J}(x, s|\mu, \mu_{-i}) = \sup_{\mu, \mu_{-i} \in \tilde{M}} E_{\mu,\mu_{-i}} \pi(x, s_{-i}) + \beta \tilde{J}'(x', s'_{-i}|\mu', \mu^i_{-i}) \tag{D.2}$$

where the investment decisions were incorporated into profits. Note that since competitors' strategies depend on their states, the above optimization effectively amount at requiring each firm knowledge of the full distribution of idiosyncratic productivity shocks.

The oblivious equilibrium is method for approximating Markov perfect equilibria since in settings with a large number of firms simultaneous changes in individual firm productivity realizations can average out such that the overall state remains roughly constant over time. Weintraub et al. (2008) shows that in this setting, each firm can potentially make near-optimal decisions based only on its own productivity level and the long-run average industry state. Given this it is possible to restrict firm strategies so that each firm's decisions depend only on the firm's productivity. The market equilibrium then involves decisions made without full knowledge of competitors' productivity realizations, and hence its distribution, but with knowledge of a sufficient statistic, represented by the average state.

Note that if all firms use a common strategy, μ, the productivity level evolves as an independent transient Markov chain. Let the k-period transition probabilities of this transient Markov chain be denoted by $P^k_\mu(x, x')$. Then the expected time that a firm spends at a productivity level x is given by $\sum_0^\infty P^k_\mu(x^e, x)$ and denote by $\tilde{s}_t(x)$ the expected number of firms in the productivity state

x. Furthermore, we define $\tilde{s}_\mu(x) = lim_{t\to\infty}\tilde{s}_t(x)$. Then for an oblivious strategy μ the oblivious value function reads as follows:

$$J(x|\mu) = E_\mu[\sum_{k=t}^{\infty}\beta^{k-t}(\pi(x_{ik},\tilde{s}_\mu))|x_{it}=x] \tag{D.3}$$

where J is the value function under the oblivious equilibrium. This value function is the expected net present value of a firm with productivity x that follows oblivious strategy μ, given its competitors' state being $\tilde{s}_\mu$ at all time. Note that we drop the dependence of the value function on μ_{-i} under the assumption that all firms follow the same strategy. Importantly note that now the expectation is taken only with respect to the strategy of firm i, not the ones of the competitors like in MPE. The value function however remains a function of the competitors' strategies through its dependence on the average state, $\tilde{s}_\mu$.

We can now specify the state space, strategies and Markov chain transition matrices that apply to our model and lead to 10. The firm state x is given by the idiosyncratic productivity shocks, (z), the firms' policy function μ' is given by the vector (k,p), expectation with respect to the firm strategy μ is formalized through the expectation with respect to the Markov transition matrix, $P(z,z^{'},X)$. The variable X captures the average long run state of the competitors. Operationally, the expectation is computed by assuming that competitors face the same productivity process. Firm i then guesses the market shares of their competitors to compute the average state.

D.2. Loglinearized Pricing Condition

The non linear first order condition on prices reads as follows:

$$(1-\varepsilon)\left(\frac{p_t}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t}\left(1-\frac{p_t}{P_t}\frac{\partial P_t}{\partial p_t}\right)-\varepsilon mc_t\left(\frac{p_t}{P_t}\right)^{-\varepsilon-1}\frac{Y_t}{P_t}\left(1-\frac{p_t}{P_t}\frac{\partial P_t}{\partial p_t}\right)-\varepsilon\frac{1}{\kappa_p}\log\left(\frac{p_t}{p_{t-1}}\right)\frac{1}{p_{t-1}}\frac{Y_t}{P_t} \tag{D.4}$$

$$-\varepsilon\frac{2}{\kappa_p}log(\frac{p_t}{p_{t-1}})^2(1-\varepsilon)\frac{Y_t}{P_t}\left(\frac{p_t}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t}\left(1-\frac{p_t}{P_t}\frac{\partial P_t}{\partial p_t}\right)+$$

$$+\frac{1}{1+r_{t+1}}E_t\frac{\partial J(z_{t+1},p_t,k_t)}{\partial p_t}=0$$

Substitute $\frac{\partial P_t}{\partial p_t} = (\frac{p_t}{P_t})^{-\varepsilon}$ and $E_t \frac{\partial J(z_{t+1}, p_t, k_t)}{\partial p_t}$:

$$(1-\varepsilon)\left(\frac{p_t}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t}\left(1-(\frac{p_t}{P_t})^{1-\varepsilon}\right)-\varepsilon mc_t\left(\frac{p_t}{P_t}\right)^{-\varepsilon-1}\frac{Y_t}{P_t}\left(1-(\frac{p_t}{P_t})^{1-\varepsilon}\right)-\varepsilon\frac{1}{\kappa_p}\log\left(\frac{p_t}{p_{t-1}}\right)\frac{1}{p_{t-1}}\frac{Y_t}{P_t}- \tag{D.5}$$

$$-\varepsilon\frac{1}{\kappa_p}log(\frac{p_t}{p_{t-1}})^2(1-\varepsilon)\frac{Y_t}{P_t}\left(1-(\frac{p_t}{P_t})^{1-\varepsilon}\right)+$$
$$+\frac{1}{1+r_{t+1}}\varepsilon\frac{1}{\kappa_p}E_t[log(\frac{p_{t+1}}{p_t})\frac{1}{p_t}(\frac{p_{t+1}}{P_{t+1}})^{1-\varepsilon}Y_{t+1}]=0$$

Divide by Y_t, multiply by P_t and by $(\frac{p_t}{P_t})^{\varepsilon}$, define $\xi = (\frac{p_t}{P_t})^{1-\varepsilon}$ and $\pi_t = \frac{p_t}{p_{t-1}}$:

$$(1-\varepsilon)(1-\xi_t)+\varepsilon mc_t\frac{P_t}{p_t}(1-\xi_t)-(1-\varepsilon)\frac{\varepsilon}{\kappa}log(\pi_t)\frac{\pi_t}{P_t}(1-\xi_t)-\frac{\varepsilon}{\kappa}log(\pi_t)^2(1-\varepsilon)(1-\xi_t)\xi_t(\frac{p_t}{P_t})+ \tag{D.6}$$

$$+\frac{1}{1+r_{t+1}}\frac{\varepsilon}{\kappa}E_t[log(\pi_{t+1})(\pi_{t+1})^{1-\varepsilon}(\frac{P_{t+1}}{P_t})^{\varepsilon-1}P_t\frac{Y_{t+1}}{Y_t}]=0$$

Multiply each term:

$$(1-\varepsilon)-(1-\varepsilon)\xi_t+\varepsilon mc_t\frac{P_t}{p_t}-\varepsilon mc_t\frac{P_t}{p_t}\xi_t-(1-\varepsilon)\frac{\varepsilon}{\kappa}log(\pi_t)\frac{\pi_t}{P_t}+(1-\varepsilon)\frac{\varepsilon}{\kappa}log(\pi_t)\frac{\pi_t}{P_t}\xi_t-\frac{\varepsilon}{\kappa}log(\pi_t)^2\xi_t\frac{p_t}{P_t}+ \tag{D.7}$$

$$+\frac{(\varepsilon)}{\kappa}log(\pi_t)^2\xi_t^2(\frac{p_t}{P_t})+\frac{\varepsilon^2}{\kappa}log(\pi_t)^2\xi_t(\frac{p_t}{P_t})-$$
$$-\frac{\varepsilon^2}{\kappa}log(\pi_t)^2\xi_t^2(\frac{p_t}{P_t})+\frac{1}{1+r_{t+1}}\frac{\varepsilon}{\kappa}E_t[log(\pi_{t+1})(\pi_{t+1})^{1-\epsilon}(\frac{P_{t+1}}{P_t})^{\varepsilon-1}P_t\frac{Y_{t+1}}{Y_t}]=0$$

Substitute $\frac{1}{1+r_{t+1}} = (\frac{Y_{t+1}}{Y_t})^{-\sigma}$, divide by ε and loglinearize:

$$-\frac{(1-\varepsilon)}{\varepsilon}\xi(1+\hat{\xi}_t)+mc\frac{P}{p}(1+\hat{mc}_t+\hat{P}_t+\hat{p}_t)-mc\frac{P}{p}\xi(1+\hat{mc}_t+\hat{P}_t-\hat{p}_t+\hat{\xi}_t)-\frac{1-\varepsilon}{\kappa}(1+\pi_t+\hat{\pi}_t-\hat{P}_t)+ \tag{D.8}$$

$$+\frac{1-\varepsilon}{\kappa}(1+\pi_t+\hat{pi}_t-\hat{P}_t+\hat{xi}_t)-\frac{\xi}{\kappa}\frac{p}{P}(1+2\pi_t+\hat{\xi}_t+\hat{p}_t-\hat{P}_t)+\frac{1}{\kappa}(1+2\pi_t+2\hat{\xi}_t+\hat{p}_t-\hat{P}_t)+$$
$$+\frac{\varepsilon}{\kappa}(1+2\pi_t+\hat{\xi}_t+\hat{p}_t-\hat{P}_t)-\frac{\varepsilon}{\kappa}(1+2\pi_t+2\hat{\xi}_t+\hat{p}_t-$$
$$-\hat{P}_t)+\frac{\beta}{\kappa}(1+\hat{Y}_t-\hat{Y}_{t+1}+\pi_{t+1}+(1-\varepsilon)\hat{\pi}_{t+1}+(\varepsilon-1)\hat{P}_{t+1}-(\varepsilon-1)\hat{P}_t+\hat{P}_t+\hat{Y}_{t+1}-\hat{Y}_t]=0$$

Simplify terms:

$$
\begin{aligned}
&-\frac{(1-\varepsilon)}{\varepsilon}\xi(1+\hat{\xi}_t)+mc\frac{P}{p}(1+\hat{mc}_t+\hat{P}_t+\hat{p}_t)-mc\frac{P}{p}\xi(1+\hat{mc}_t+\hat{P}_t-\hat{p}_t+\hat{\xi}_t)- \\
&+\frac{1-\varepsilon}{\kappa}\hat{\xi}_t-\frac{\xi}{\kappa}\frac{p}{P}(1+2\pi_t+\hat{\xi}_t+\hat{p}_t-\hat{P}_t)+\frac{1}{\kappa}(1+2\pi_t+2\hat{\xi}_t+\hat{p}_t-\hat{P}_t)+ \\
&-\frac{\varepsilon}{\kappa}\hat{\xi}_t+\frac{\beta}{\kappa}(1+\pi_{t+1}+(1-\varepsilon)\hat{\pi}_{t+1}+(\varepsilon-1)\hat{P}_{t+1}-(\varepsilon-1)\hat{P}_t+\hat{P}_t]=0
\end{aligned}
\tag{D.9}
$$

$$
(1-\varepsilon)-(1-\varepsilon)\xi(1+\hat{\xi}_t)+\varepsilon mc(1+\hat{mc}_t+\hat{p}_t)-\xi\varepsilon mc(1+\hat{\xi}_t+\hat{mc}_t+\hat{p}_t)=0 \tag{D.10}
$$

Rearranging delivers the equation presented in proposition 1.

D.3. Numerical Solution

The model is solved numerically through the Auclert et al. (2019) sequence space Jacobian method. The solution of the steady state is done through the Carroll (2006a) endogenous grid point method. The model response to shock is solved through a first order approximation of the model around the stochastic steady state. Formally, this is obtained by computing Jacobians of the policy functions at each grid point. The solution involves the following steps. In response to shocks heterogenous firms and households compute their policy function backward from a period T. The response of the policy functions to shocks is computed through Jacobians. A second step involves a forward iterations in which the dynamic of the share of households, that determine the aggregate policy functions, solve jointly the macro model equilibrium conditions.

Note that the stochastic steady state also involves an iterative procedure to solve the fixed point between the individual policy functions and the prices that solve market clearing conditions. Formally each firms take input prices as given and solve individual policy functions, which in turn have to satisfy the input marker clearing conditions. The same is true for the households choice of assets in the model in which they face uninsurable risk.

We describe more in detail the exact steps of the steady state iterative procedure that involves the solution of the firms' policy functions as this is the most novel part of our numerical solution. We do so both for the case in which firms interact as monopolistic competitors and in the case of the dynamic Bertrand.

Fixed Point Solution.:[28]

1. Guess $\partial J_p(z', p', k')$ and $\partial J_k(z', p', k')$.

2. Obtain $\frac{1}{1+r_{t+1}}\Pi\partial J_p(z', p', k')$ and $\frac{1}{1+r_{t+1}}\Pi\partial J_k(z', p', k')$.

3. Using F.O.C. with respect to capital obtain $k'(z, p', k)$, using the same weights obtain $\partial J_p(z, p', k)$.

4. Obtain $mc(z, p', k)$ from F.O.C. with respect to labor hours and technological constraint.

5. Obtain $p'(z, p, k)$ using F.O.C. with respect to price, using the same weights obtain $k'(z, p, k)$.

6. Obtain $mc(z, p, k)$ and $n(z, p, k)$ from F.O.C with respect to labor hours, technological constraint and $p'(z, p, k)$.

7. Update guesses of $\partial J_p(z', p', k')$ and $\partial J_k(z', p', k')$ from the envelope conditions.

Dynamic Bertrand Competition. Fixed Point Solution.

1. Guess the distribution (initial guess is the distribution for the monopolistic case)

2. Using backward iteration solve for for policy functions [29]

3. Using forward iteration solve for the distribution

4. Check whether the difference between the guessed distribution and the newly obtained distribution is within a tolerance band. Update the guess of the distribution if not.

28 Primes are used to refer to the next period in the backward iteration procedure (so that they are not confused with t for the IFRs).

29 For the details about backward policy iteration and forward distribution iteration see solution method in Straub.