Unemployment, Inflation, and Limited Participation

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March 20, 2025

Abstract

Recent data indicate that a long-run relation between inflation and unemployment in the U.S. is absent. This paper explores the extent to which limited participation – sustained unemployment and large movements to and from nonparticipation – can reconcile the non-relation. I propose a tractable equm framework that emphasizes the interaction between labor market and goods market imperfections. Limited participation arises as sustained low job-finding opportunities, either with severe meeting friction because demand for labor is low, or with counter-cyclical selection. Multiple equalibria can occur with one equm admitting full participation, and the other with all workers incline to quit.

I apply the model to microdata and depart from the literature in identifying a few key parameters: the utility of leisure, productivity, and bargaining power. The model is estimated by embedding the equm solutions in a likelihood procedure. Results show that the model predicts a long-run relation that matches data reasonably well, and the correlation coefficient is reasonably close to the data. Likelohood ratio test rejects a two-state framework. Preliminary results show that limited participation since the Great Recession is the most important factor in explaining the flattening of the long-run relation, while the fall in money demand is least important.

keywords: search, non-participation, labor, money, fintech

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"It was the Fed that killed the Phillips Curve," James Bullard (2020).

1 Intooduction

The long-run relation between unemployment and inflation in the U.S. since the WWII had been positive until 2007, thereafter the relation has disappeared. Meanwhile, since 2007, there has been large and persistent drop in labor supply and large changes in transitions to and from non-participation in the labor market, what I call 'limited participation'.

In this paper, I explore the extent to which limited participation in the labor market reconciles the unemployment and inflation disconnect since the Great Recession (2007-08). In a nutshell, my conjecture is that sustained unemployment occurred when inflation is low and the subsequent labor force contraction, particularly during the pandemic while there was a spike in inflation in 2021, may negate the yesteryears' positive long-run unemployment and inflation relation.

To formalize my conjecture, I build a tractable theory of limited participation in labor market that connects with the goods market. Describing labor market transitions and goods market transactions that use money and other payment instruments is a job for search theory. I begin by building a general equm framework of unemployment and money based on frictions in labor and goods markets, abstracting from nominal misperceptions and rigidities. In particular, each period two decentralized markets – labor and goods – convene sequentially before a centralized settlement market. In the labor market, workers are in one of the three states at any point in time: employed, unemployed, and out of the labor force. Unemployed workers make sequential decisions in the following sense. After meeting a vacant job, he decides whether to accept the job depending on the realized match quality. If a meeting does not occur, or if it occurs and the realized match-quality is unacceptable, then he decides whether to drop out from the labor force or to search again. Employed workers decide whether to drop out the labor force when remaining on the job. After the labor market, workers (buyers) participate in the goods market. Buyers and sellers engage in bilateral exchange, using fiat money when monitoring over a transaction is absent. A period conludes with a settlement market that clear all payments.

In this framework, long-term unemployment arises as sustained low job-finding opportunities, either with severe meeting friction because demand for labor is low, or counter-cyclical selection: a better match is needed to warrant a job-match during bad times. Selectivity rises when welfare assistance is generous or when leisure is productive. Movement out of the labor force can also arise as a response to more productive leisure and/or generous welfare assistance. Indeed, introducing

participation into otherwise standard labor market models significantly affects the predictions of these models. For example, one result is that when aggregate productivity is low, job matches have to be unusually productive to warrant job creation. Another result is that the model can now generate multiple equa in natural but previously unexplored ways, despite constant returns in meetings. Multiple equa can arise when anonymity becomes weak (money demand falls), inflation is high, or welfare assistance is high. I demonstrate that spillovers between the labor market and the goods market become less robust with participation consideration.

I apply the framework to numerically evaluate the ability of workers' participation activities in relation to the unemployment-inflation observations. The numerical exercises are based on consistent microdata source. My numerical strategy takes on two steps. In the first step, I identify a set of parameters that are independent from the model solutions and assign values to them. I depart from the literature regarding how to identify three key parameters.

The first parameter is the utility of leisure when unemployed. Identification of this utility has been problematic in the literature. In addition, there is no concensus as to what its value it ought to be, it can range from a negative to a positive number even for practitioners using the same dataset. And the method to obtain the parameter value differs widely among studies. I propose an alternative approach here. I use McFadden's random utility framework to estimate utility. Using observed choice to reveal preference is intuitive, yet oddly rarely done in applied job search studies.

The second parameter is marginal product of labor. The literature typically use average product of labor, which is what the Bureau of Labor Statistics publishes. To resolve the problem, I use marginal product of labor based on results from recent estimates on production functions, e.g. Demirer (2020). The third parameter is the bargaining power in the labor market. It is typically set so that the bargaining power is the same as the matching elasticity, assuming Hosios rule (1990). But this is a strong assumption. My approach comes from the observation that with linear utility, Nash bargaining yields the same solution as Kalai's proportional bargaining. Kalai perceives workers' surplus to be a fraction of the total surplus, where the fraction is workers' bargaining power. Using this concept, I proxy workers' bargaining strength as the labor share, equaling labor compensation divided by gross value added.[1]

In the second step of the numerical strategy, I estimate the rest of parameter values based on micro data. In particular, I estimate them by embedding the equm solutions in the likelihood function. I embark a profile likelihood approach in estimation. Results show that the model predicts a long-run relation that matches data quite well, and the correlation coefficient is close to that

[1]Some recent studies on labor share are Gouin-Bonenfant (2022) and Elsby et al. (2013).

suggested by raw data. Comparing the model with a two-state framework, I find predicted unemployment is amplified in the two-state framework and unemployment exhibits a positive relation with inflation.

Next I study competing explanations for the long-run unemployment-inflation relation. I consider three competing explanations: money demand, inflation targetting, and limited participation as driven by labor market frictions, welfare benefits and preference for leisure. Money demand has fallen rapidly during the last few decades. The rapid rise of fintech has shifted the economy from cash-based to digital-based. This reduces money demand, which in turn may diminish the impact of monetary policy on the labor market. Inflation targeting can also play a role. Without comparing among other factors, however, it is difficult to ascertain whether the Fed is responsible to the flattening of the unemployment-inflation relation.

The last three factors – meeting frictions, leisure, and welfare benefits – are the drivers for sustained unemployment and nonparticipation. In particular, market frictions govern transitions to and from labor market states, while substantial welfare benefit extensions during the GR induced (i) the unemployed to stay as unemployed rather than taking a real job or quit the market, and (ii) non-participants to become job seekers. These phenomena amplify long-term unemployment despite low inflation during the Great Recession. Preference for leisure matters as technological change has rapidly bolstered the productivity of leisure, which may foster nonparticipation and may help reduce the unemployment response to monetary policy. Some examples include the explosion of online social media platforms (e.g. yahoo, instagram, tiktoks, facebook, X, etc.), gaming and entertainment (e.g. stock prices of Gamestop, Electronic Arts, Xbox, and Netflix have exploded in recent years), etc. Preliminary counterfactual results show that limited particpation since the Great Recession is the most important factor explaining the flattening of the long-run relation, while money demand is the least important factor.

A recent study that also aims to explain a flat unemployment and inflation relation is Ratner and Sim (2022). The authors posit that labor policies that have eroded workers' bargaining power may explain the non-relation of unemployment and inflation. Milton Friedman is among the first who argues that in the long run, inflation should have little or no relation to unemployment. Here, the non-relation occurs for different reasons.

One result in this paper is that it produces countercyclical selection on match quality: when aggregate productivity is low, job matches have to be unusually productive to warrant job creation. This result complements that in Dorny (2018) who studies cyclicality of wages and match quality

using data from Germany.

This paper also relates to the literature on long-term unemployment. Many of which studies focus on the supply side effect of the Great Recession. Kroft et. al (2014) offer one of the most comprehensive views on long-term unemployment. They find that UI benefits and duration dependence are much more important then compositional factors in explaining long-term unemployment. Focusing on young men, Aquiar et. al (2022) propose leisure techonology (e.g. video games) to explain the fall in their labor supply. In contrast, Davis et al (2013) explains long-term unemployment by positing employers control econ oppportunities by limiting job openings. In this paper, employers' 'reluctance' to engage in hiring can be reflected by having unacceptable match quality that deters a job match to occur.

Modeling workers' non-participation in the search theoretic approach is first done by Burdett, Kiefer, Mortensen, and Neumann (1984). More recently, Alverez and Shimer (2011), Shimer (2011, 2013), and Wasmer and Weill (2021) study non-participation. Similar to Shimer, this study endogenizes participation choice in the labor market. In contrast to Shimer, this study focuses on low-frequency data, job matches differ ex post, and the equm admits multiplicity. It also strives to connect with goods market to addresss the issue of unemployment and inflation. While all these studies are about the labor market, Williamson (2012) studies exogenous limited participation of buyers in the goods market using the New Monetarists's approach.

The literature on money and credit ia large and is generally divided into two camps. One perceives money and credit as substitutes. Some examples are Kocherlakota (1998), Sanches and Williamson (2010), Townsend (1989), Corbae and Ritter (2003), and Cavalcanti and Wallace (1999). The other camp perceives money and credit being complementary to each other, e.g. Shi (1996), Berentsen et al. (2007), Ferraris (2010), and Geromichalos and Herrenbrueck (2011). This paper views money and credit as substitutable payment instruments.

Section 2 describes motivational facts. Section 3 lays out the framework. Equilibrium and its characterization are contained in Section 4. Section 5 presents numerical illustrations. It contains a presentation of numerical strategy and results. Conclusion is in Section 6.

2 Motivational Facts

Post Millennium unemployment data come from the annual American Commuinty Surveys (ACS) 2001-2024, as well as This data source contains substantially larger sample size (at least 10 times) and response rates than other U.S. survey data such as the March CPS. For example, in a given

year, the ACS final sample size for the youth exceeds 300,000 observations, and the 1990 census exceeds 2.5 million observations. Further, the response rates for hours and weeks-worked are 100 per cent. I also employ the decennial census 1970 to 2020 for its large sample size. Data for all earlier years come from the Bureau of Labor Statistics. Data from early years come in tables by age groups, from which I calculated the unemployment rates.

Sample selection criteria are as follows: respondents are non-institutionalized, whose age is between 21 and 64, and are not self-employed. For the most part, the qualitative results in the paper are not overtly sensitive to the inclusion criteria used. All statistics are weighted by the person weight given in the data base to reproduce the census population. Because my unemployment data are annual, which is not high frequency, I do not filter the data. For most studies, setting a high filtering parameter produces results similar to averaging data over a 'long' horizon, e.g. four to five years. I obtain inflation data from the Minneapolis Federal Reserve Bank.

Figure 1 plots uemployment and inflation from 1950 to 2024. The 75 years appear to contain two epochial relation: an earlier period with unemployment and inflation moved relatively similarly, while a latter period with them contrast in directions. To see the relation differently, Figure 2 plots unemployment rate vs inflation rate. The top panel plots the relation using data from 1970 to 2024. Using five-year average data and fitting unemployment to a linear line with inflation being the covariate, the slope is statistically insignificant with p-values close to 1.0. This fact is valid even nominal interest rate instead of inflation is used. The bottom panels exclude data after 2007. The data deliver a postive long-run relation between inflation and unemployment.[2]

Plotting the output gap vs inflation from 1955 to 2013, Williamson finds a pattern (Figure 3) that resembles closely to the top panels in Figure 2. The significance of Figure 3 is that the non-relation is evident before inflation targeting kicked in in 2012, and that adding more distant data do not change the non-relation.[3] Figures 2 and 3 suggest that market shifts and shocks after 2007 may exert strong negative forces to nullify the positive relation between inflation and unemployment from more distant past.

One major post-Millennium shock is the Great Recession of 2007-08, which brought with it large and persistent drop in labor supply that took over three years for the labor market to bounce back. The pandemic started in 2020 in the U.S. again brought about great withdrawal from the labor force.

[2]Berentsen et al. (2011) also show a positive long-run relation using U.S data from 1955 to 2005, with unknown source and sample selections.

[3]Such non-relation is not only occurring in the U.S., it also occurs in the U.K. In the Data Appendix, a plot between the two variables using U.K data is shown.

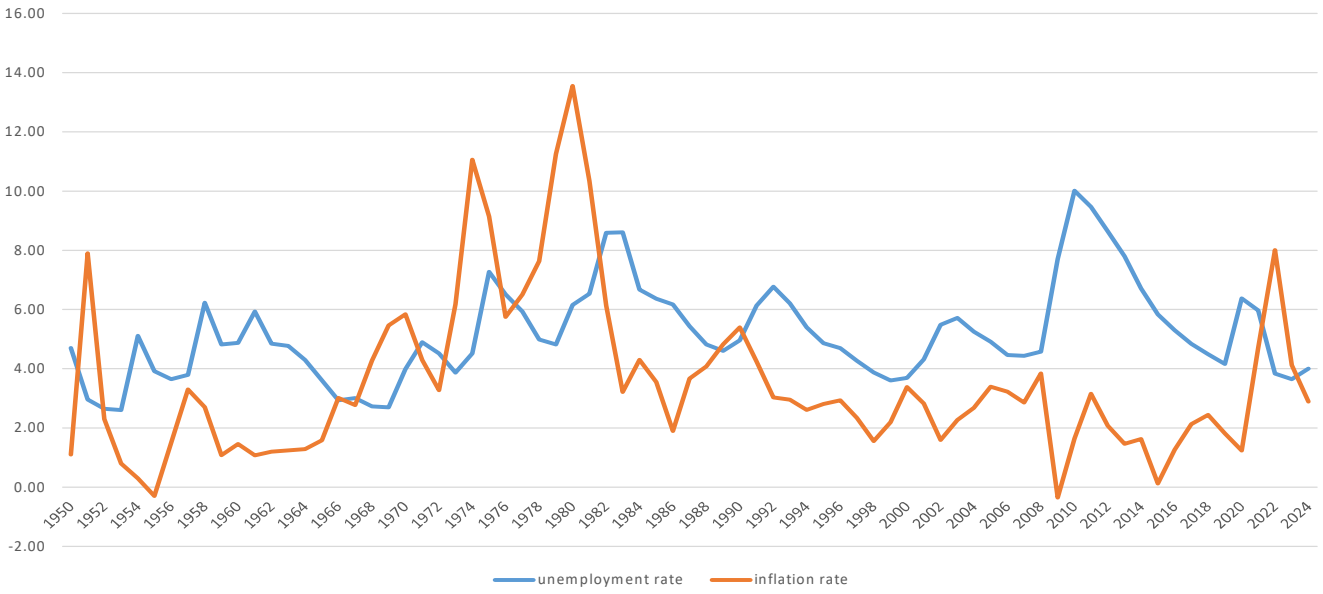


Figure 1: Unemployment and Inflation in the U.S, 1950-2024

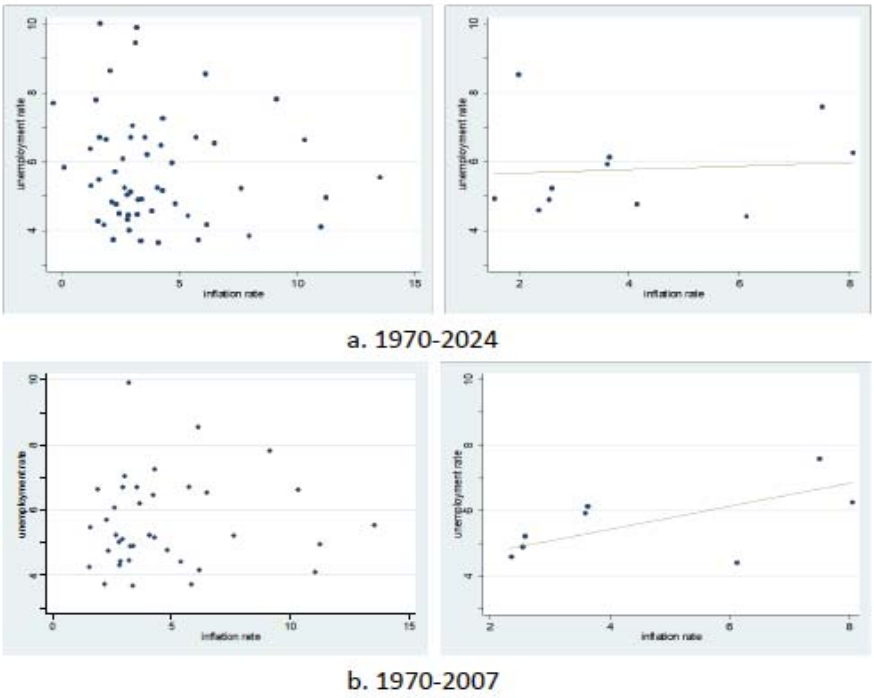


Figure 2: Long-Run Relation Between Unemployment and Inflation in the U.S 1970-2024

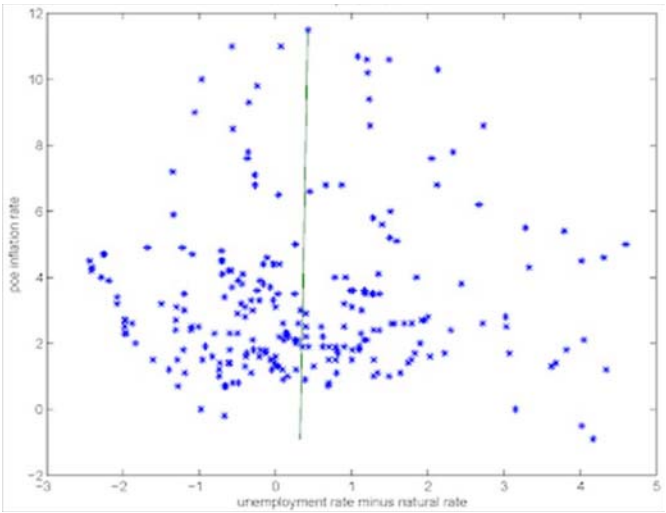


Figure 3: Williamson (U.S. 1955-2013)

There are two salient features from these events. One is long-term unemployment, defined in the U.S as unemployment duration of at least 26 weeks). Figure 4 plots the share of long-term unemployed workers (out of all unemployed workers) from 1995 to 2024. The few years after the Great Recession saw almost half of unemployment to be long-term unemployment, which peaked again during the pandemic, making the post-Great Recession long-term unemployment share about 29 per cent on average, significantly higher than the pre-Great Recession (1995-2007) average of 16 per cent.

To get a sense of persistence, I use the census data to get the fraction who did not work over the previous year (unemployed or out of the labor force). This way minimizes snapshot or stop-gap phenomenon in cross-sectional data. Figure 5 shows that between 2000 and 2021, non-employment has grown 25 per cent, from 15.82 per cent in 2000 to 19.71 per cent in 2021. This increase does not appear to be driven only by the pandemic because from 2000 to 2015, the share is 31.3 per cent. Further, among the pool of inactive people, the survey question further delineates whether no work was only for the previous year or for the previous one to five years. On average, about two third of the responses indicated the latter.

The other salient feature is large movements to and from nonparticipation. Using the March CPS and setting 9/2008 as the normalizing month for the transition to and from nonparticipation, Figure 6 plots the annual averages. In general, there have been large changes in transitions to and from non-participation since late 2008. In particular, transition to non-participation (the blue line) has grown steadily since late 90s, picking up the pace after 2009. Indeed, the labor force partipation

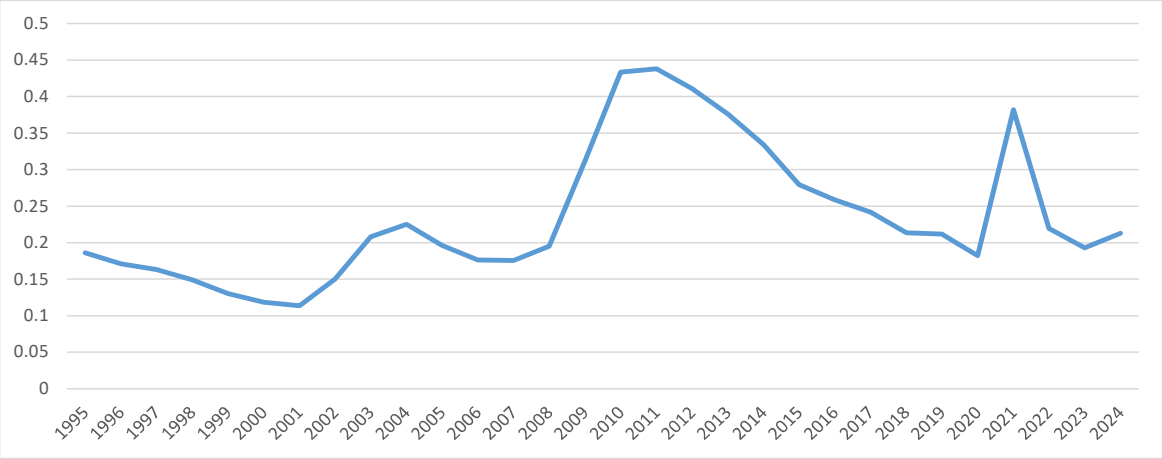


Figure 4: Share of Long-Term Unemployment in the U.S, 1995-2024

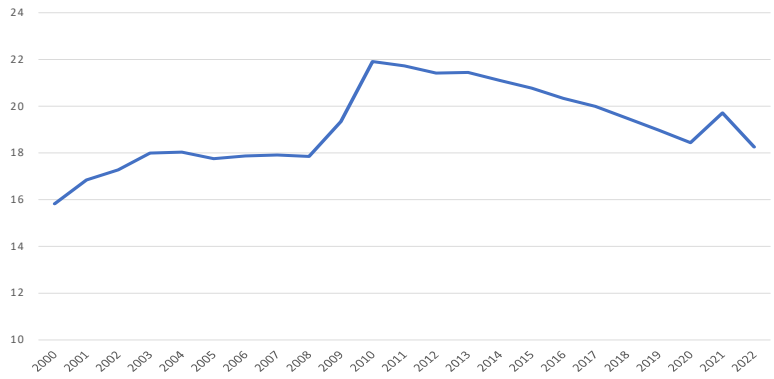


Figure 5: Share of Population Not Working Last Year

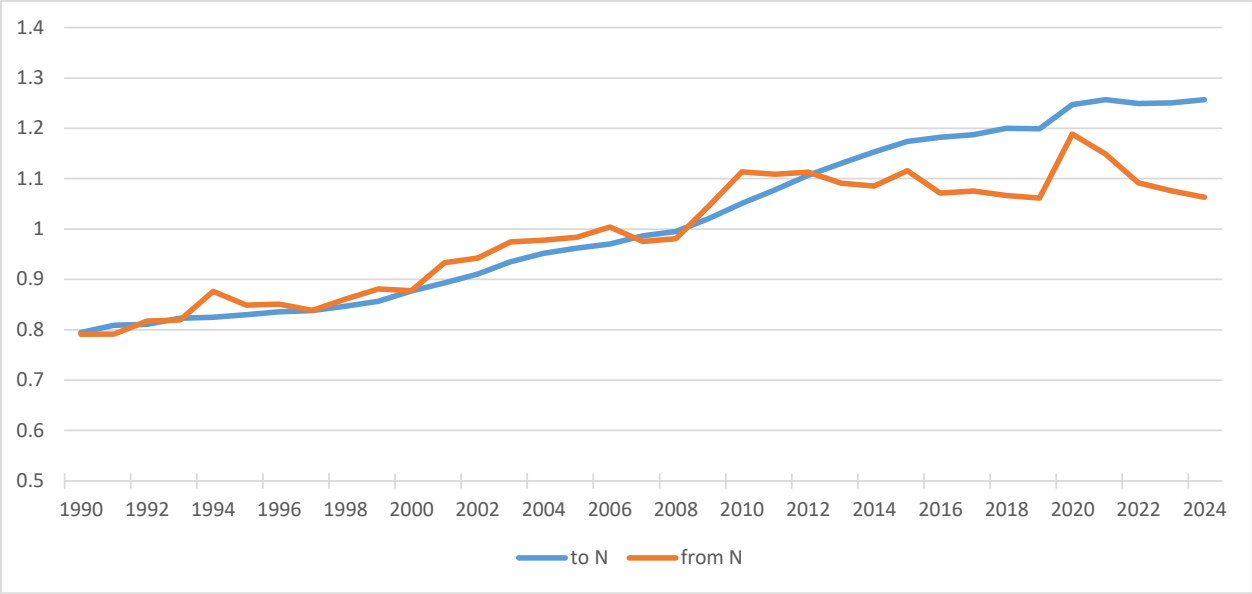


Figure 6: Movements To and From Nonparticipation, 1990-2024

rate has been trending downward since 2009-10.

On one hand, long-term unemployment and the transition from non-participation to unemployment can amplify unemployment severity despite low inflation since 2000. On the other hand, transitions from unemployment to non-participation reduces the unemployment pool and thus the unemployment response to monetary policy in the last decade, particularly during the pandemic when there was a surge in inflation in 2021.

3 Model

Time is discrete and continues forever. In each period three markets convene sequentially: a frictional decentralized labor market (LM), followed by a fricitonal decentralized goods market (GM), and then a frictionless settlement market (SM). There are two types of agents: firms and workers. The set of workers is $[0, 1]$. Workers differ in their labor market states: employed, unemployed, or out of the labor force. Let e indexes employment status: $e = 0$ if a worker is unemployed, $e = 1$ if employed, and $e = 2$ if out of the labor force. Let n_e represents the measure of a type e agent with $n_0 + n_1 + n_2 = 1$. The set of firms is arbitrarily large.

In the decentralized markets, agents meet pairwise and at random. In the LM, consider a matching function $M(n_0, v)$, where v is the measure of vacancies, with M being a CRS. Let $\tau_1 = v/n_0$ denote market tightness. Assume that M is strictly increasing in τ_1, with $M(0, 1) = 0$ and

$M(\infty, 1) = 1$, and $M(\tau_1, 1)/\tau_1$ is strictly decreasing with $M(0, 1)/0 = 1$ and $M(\infty, 1)/\infty = 0$. The probability workers meet firms is $M(n_0, v)/n_0 = M(1, \tau_1) = \lambda\tau_1$, and the probability firms meet workers $M(n_0, v)/v = M(\tau_1, 1)/\tau_1 = \lambda$. Note, however, that a meeting does not result in a match: workers weigh in on match quality $q \in [0, 1]$. Workers consumate a job match opportunity when the realization of a random draw of match quality from distribution $F(q)$ exceeds some minimum acceptable quality. Match quality distribution is given by $F(.)$ is twice differentiable and log-concave. Another decision made in the LM is active workers' participation decision. The probability that a worker quits the labor market state $e \neq 2$ is ρ_e. There is no recall or on the job search.

In the GM, a matching function is given by a CRS matching function $\mathcal{N}(B, S)$, where B and S are the measures of buyers and sellers in the goods market. Properties of the matching function $\mathcal{N}$ is similar to those in $\mathcal{M}$. Let $\tau_2 = B/S$ be market tightness in this market. The probability that a buyer meets a seller is $\mathcal{N}(B, S)/B = \mathcal{N}(\tau_2, 1)/\tau_2 = \alpha$, and the probability a seller meets a buyer is $\mathcal{N}(B, S)/S = \mathcal{N}(\tau_2, 1) = \alpha\tau_2$. A meeting in the GM results in a match. Two types of payment instruments can be used. With probability μ meeting is not monitored and money is used; with probability $(1 - \mu)$ non-cash instruments are used; μ is exogenous.

In the SM, agents consume, enjoy utility from consumption and leisure, get paid, and pay tax and loans. There is no strategic default. They also decide how much money to carry to the next period. Real balances are given by $z \in [0, \infty)$. I adopt the standard convention for measuring real balances: when an agent bring m dollars to the SM, $z = m/p$, where p is the current price level. Meanwhile, the agent takes $\widehat{z} = \widehat{m}/p$ out of that market for use in the next period. The price level in the next SM is $\widehat{\mathtt{p}}$, so the real value of money is $\widehat{z}\widehat{\rho}$, where $\widehat{\rho} = p/\widehat{\mathtt{p}}$ converts $\widehat{z}$ into units of the numeraire good x in SM. Between the SM and the next LM, there is a discount factor $\beta = 1/(1+r)$, where r is the rate of time preference.

Although wage does not surface until the SM, the bargaining process occurs in the LM. Wage is determined by generalized Nash bargaining. Unmatched workers and firms bargain over their respective surplus with workers' threat point given by unemployment and that for firms' by posting vacancy. In the GM, the terms of trade are also determined by Nash bargaining.

The value functions for workers and firms are denoted by $W_e^j(q, z)$ and $J_e^j(q)$, where $e = 0, 1, 2$ refers to employment states, and $j = 1, 2, 3$ refers to the submarket within a period, respectively.

3.1 Workers

3.1.1 LM Payoffs and Participation

At any point in time in the labor force, workers decide whether to leave the market. The Bellman equations for workers at each labor market state is

$$W_0^1(0,z) = \lambda\tau_1 \int_{q^*} W_1^2(q,z)dF(q) + [1 - \lambda\tau_1 + \lambda\tau_1 F(q^*)][\rho_0 W_2^2(0,z) + (1-\rho_0)W_0^2(0,z)] \quad (1)$$

$$W_1^1(q,z) = \delta W_0^2(0,z) + (1-\delta)[\rho_1 W_2^2(0,z) + (1-\rho_1)W_1^2(q,z)] \quad (2)$$

$$W_2^1(0,z) = \psi W_0^2(0,z) + (1-\psi)W_2^2(0,z) \quad (3)$$

Equation (1) is the value for unemployed workers; it is made up of two components. First, conditional on meeting a vacant job with probability $\lambda\tau_1$, a worker accepts the job if the realization of a random draw of match quality exceeds reservation match quality q^*. Reservation match quality is the quality at which a worker is indifferent between employed and unemployed, and will accept a job. In other words, if workers' surplus of a job match is $S_w(q) = W_1^2(q,z) - W_0^2(0,z)$, then $S_w(q^*) \equiv 0$.

Second, if the worker does not meet a vacant job, or if the realized match quality is below q^*, then the worker decides whether to leave the labor force with probability ρ_0.

Equations (2) and (3) are the values for the employed and the non-participants respectively, and have similar interpretation as (1). The employed encounters job destruction with probability $\delta > 0$; otherwise, he decides whether to leave the market with probability ρ_1. Non-participants return to the labor force looking for jobs with an exogenous probability $\psi > 0$.

Now consider the decision rules (ρ_0, ρ_1). Workers leave the labor market state if the 'quit' surplus, S_{qe}, is positive, where quit surplus for the unemployed and the employed are respectively $S_{q0} = W_2^2(0,z) - W_0^2(0,z)$ and $S_{q1}(q) = W_2^2(0,z) - W_1^2(q,z)$. More precisely,

$$\rho_0 = \begin{cases} 1 & \text{if } S_{q0} > 0 \\ [0,1] & \text{if } S_{q0} = 0 \\ 0 & \text{if } S_{q0} < 0 \end{cases}, \rho_1 = \begin{cases} 1 & \text{if } S_{q1}(q) > 0 \\ [0,1] & \text{if } S_{q1}(q) = 0 \\ 0 & \text{if } S_{q1}(q) < 0 \end{cases} \quad (4)$$

are the best response conditions (BR).

Note that the quit decision for the employed depends on match quality. Figure 7 shows the quit surplus for the employed, $S_{q1}(q)$, as a function of q. Observe that the surplus is falling in q, so the employed are less prone to leave the market the higher the match quality is. Further, evaluate the quit surplus at q^*, one gets $W_2^2(0,z) - W_1^2(q^*,z) = W_2^2(0,z) - W_0^2(0,z)$, which is the quit surplus for unemployed workers, i.e. $S_{q1}(q^*) = S_{q0}$. This equality implies that the employed

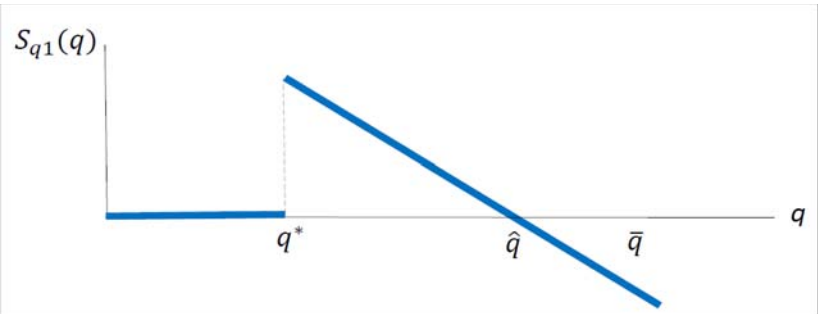


Figure 7: Quit Surplus (employed workers)

quits the market iff the unemployed at $q = q^*$ does. Thus, if $\rho_0 = 0$, then $\rho_1 = 0 \; \forall q \geq q^*$. However, if $\rho_0 = 1$, then ρ_1 may not equal one for $q \geq q^*$: $\rho_1 = 1$ if $q < \widehat{q}$ and $\rho_1 = 0$ if $q \geq \widehat{q}$, where $\widehat{q} > q^*$ is the 'quit quality,' defined as $S_{q1}(\widehat{q}) \equiv 0$.

3.1.2 GM Payoffs

In the GM, g and d are quantity and dollar exchanged when using money, while $\widetilde{g}$ and $\widetilde{d}$ are quantity and credit exchanged using non-cash payments. Buyers obtain utility from goods $v(g)$ or $v(\widetilde{g})$ with $v(0) = 0$, $v' > 0$ and $v" < 0$.

The Bellman equation for buyers of employment status e is

$$W_e^2(q,z) = \alpha\{\mu[v(g) + W_e^3(q, \rho(z-d), 0)] + (1-\mu)[v(\widetilde{g}) + \nu + W_e^3(q, \rho z, \widetilde{d})]\} + (1-\alpha)W_e^3(q, \rho z, 0)$$

where α is the probability of trade, μ is the probability that a random match is anonymous, and ν is the utility (or cost if $\nu < 0$) of using credit incurring debt $\widetilde{d}$. To reduce clusters, in what follows I set $\mu = 1$ to focus on the characteristics of limited participation in this framework; I set $\mu \in (0,1)$ in the numerical exercises. With $\mu = 1$, the value function $W_e^3(q, \rho z, 0)$ can be simplified as $W_e^3(q, \rho z)$.

Using the linearity of W_e^3 (see equation (6)), the Bellman equation becomes

$$W_e^2(q,z) = \alpha[v(g) - \rho d] + W_e^3(q,0) \; + \rho z$$

3.1.3 SM

In the SM, workers solve

$$\begin{aligned} W_e^3(q,z) &= \max_{x,\widehat{z}} x + I(e=0)\ell_0 + I(e=2)\ell_2 + \beta W_e^1(q,\widehat{z}) \\ \text{s.t. } x &= I(e=1)w(q) + I(e=0)b + \triangle - T + z - \widehat{z} \end{aligned} \tag{5}$$

where x is consumption, ℓ_e type e workers' utility of leisure with ℓ_1 normalized to one, $w(q)$ wage, b income from welfare if unemployed, $\triangle$ income from dividends, informal sectors, or returns from social capital in general (all in units of numeraires), and T a lump-sum tax. Between the SM and the next LM, there is a discount factor $\beta = 1/(1+r)$, where r is the rate of time preference. Define $I_e = I(e=1)w(q) + I(e=0)(b+\ell_0) + I(e=2)\ell_2 + \triangle - T$ and eliminate x from the budget equation, equation (5) becomes

$$W_e^3(q,z) = I_e + z + \max_{\widehat{z}}\{-\widehat{z} + \beta W_e^1(q,\widehat{z})\} \tag{6}$$

Equation (6) shows that W_e^3 is linear in I_e and z, and the choice of $\widehat{z}$ is independent of them as well. Even though $\widehat{z}$ may seem to depend on e and q from W_e^1, as will be shown below, $\widehat{z}$ will be independent of e and q. This implies that every worker exits the SM carrying the same amount of $\widehat{z}$.

To summarize the three markets, I first substitute the GM equations into the LM, and use the linearity in the SM to get

$$\begin{aligned}
W_0^1(0,z) &= \alpha[\upsilon(g) - \rho d] \ + \rho z + \lambda\tau_1 E[W_1^3(q,0)] \\
&\quad + [1 - \lambda\tau_1 \overline{F}(q^*)][\rho_0 W_2^3(0,0) + (1-\rho_0)W_0^3(0,0)] \\
W_1^1(q,z) &= \alpha[\upsilon(g) - \rho d] \ + \rho z + \delta W_0^3(0,0) \\
&\quad + (1-\delta)[\rho_1 W_2^3(0,0) + (1-\rho_1)W_1^3(q,0)] \\
W_2^1(0,z) &= \alpha[\upsilon(g) - \rho d] \ + \rho z + \psi W_0^3(0,0) + (1-\psi)W_2^3(0,0)
\end{aligned}$$

where $\overline{F}(q^*) \equiv 1 - F(q^*)$.

These equations have a general pattern:

$$W_e^1(q,z) = \alpha[\upsilon(g) - \rho d] \ + \rho z + \mathbb{E}W_e^3(q,0)]$$

where the expectation $\mathbb{E}$ is wrt employment transitions, which may depend on workers' participation decisions. Inserting $W_e^1(q,z)$ into (6), the SM problem becomes

$$W_e^3(q,z) = I_e + z + \max_{\widehat{z}}\{-\widehat{z} + \beta\alpha[\upsilon(g) - \rho d] \ + \beta\rho\widehat{z}\} + \beta\mathbb{E}W_{\widehat{e}}^3(q,0) \tag{7}$$

Equation (7) indicates that GM terms of trade (g,d) are related to $\widehat{z}$,but $\widehat{z}$ does not depend on employment status, I_e, nor z; every worker takes the same amount of money to the GM.

3.2 Firms

Firms make participation decision in the SM and do not carry money out of SM. In the LM, the Bellman equations are

$$J_0^1 = \lambda \int_{q^*} J_1^2(q) dF(q) + [1 - \lambda \overline{F}(q^*)] J_0^2 \tag{8}$$

$$J_1^1(q) = \delta J_0^2 + (1-\delta)[\rho_1 J_0^2 + (1-\rho_1) J_1^2(q)] \tag{9}$$

with $\lambda = \mathcal{M}(u,v)/v = \mathcal{M}(\tau_1, 1)/\tau_1$. Equation (8) indicates that if firms do not meet a worker or meet a worker with unacceptable match quality, firms remain unmatched. Equation (9) is the Bellman equation for employed workers. It indicates that with probability δ a job match terminates exogenously. With complimentary probability, a job match may also terminate if the worker decides to quit; otherwise it remains productive.

After production, firms take the output to trade in the GM. Output (marginal product) is given by pq, where p represents productivity measured in units of the numeraire good, and is complementary to match quality q. With probability $\alpha\tau_2$ a firm meets a buyer, sells g units of output, and incurs cost $c(g)$, with $c' > 0$ and $c'' \geq 0$. If no meeting occurs, firms take pq to the SM. Thus,

$$J_1^2(q) = \alpha\tau_2 J_1^3[pq - c(g), \rho d, q] + (1 - \alpha\tau_2) J_1^3(pq, 0, q)$$

where $\alpha\tau_2 = \mathcal{N}(B,S)/S = \mathcal{N}(\tau_2, 1)$. Only firms in production have positive payoffs. Firms with vacancy have no value in the GM or SM, so $J_0^2 = J_0^3 = 0$.

In the SM, firms with inventory x, real balances z, wage cost $w(q)$, and match quality q has expected payoff given by

$$J_1^3(x, z, q) = x + z - w(q) + \beta J_1^1(q)$$

Unmatched firms decide to enter the LM with a vacancy next period with payment $k > 0$ in units of x in the SM:

$$\begin{aligned} J_0^3 &= \max\{0, -k + \beta J_0^1\} \\ &= \max\{0, -k + \beta\lambda \int_{q^*} J_1^2(q) dF(q) + [1 - \lambda \overline{F}(q^*)] J_0^2\} \end{aligned} \tag{10}$$

where $J_0^3 = J_0^2 = 0$ by free entry. Consequently, firm's surplus of match in the LM is $S_f(q) = J_1^2(q) - J_0^2 = J_1^2(q)$; and $k = \beta\lambda \int_{q^*} S_f(q) dF(q)$.

3.3 Steady State

Let $\mathbf{n} = (n_0, n_1, n_2)$. The Appendix describes the derivation. Here is the result. The steady state $\mathbf{n}$ is given by:

$$n_0 = \frac{\psi\{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]\}}{\mathcal{D}} \tag{11}$$

$$n_1 = \frac{\psi\lambda\tau_1\overline{F}(q^*)}{\mathcal{D}} \tag{12}$$

$$n_2 = 1 - n_0 - n_1 \tag{13}$$

where $\mathcal{D} = \{[1-\lambda\tau_1\overline{F}(q^*)]\rho_0 + \lambda\tau_1\overline{F}(q^*) + \psi\}\{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]\} - (\delta - \psi)\lambda\tau_1\overline{F}(q^*) > 0$. For example, in a two-state framework where $\rho_0 = 0$ (implying $\rho_1 = 0$), $n_0 = \delta/[\delta + \lambda\tau_1\overline{F}(q^*)]$, $n_1 = 1 - n_0$, and $n_2 = 0$, as conventional results suggested. It is straightforward to show that the employment rate (or equivalently, measure) is higher in a two-state framework. This has direct implication on market spillover. See Section 4.3.

3.4 Government

Besides firms and workers, there is a government here. Government consumes G, pays unemployment benefits b, levies tax T, and prints money at rate π, or π is inflation in the steady state. Government's budget constraint is $G + bu = T + \pi M/p$ that holds every period. Due to the Fisher equation $1 + i = (1+\pi)/\beta$, in steady state, one can equivalently describe monetary policy in terms of nominal interest rate i or π.

4 Equilibrium

Let $\mathbf{W} = (W_0^1, W_1^1, W_2^1, W_0^2, W_1^2, W_2^2)$ and $\mathbf{J} = (J_0^1, J_1^1, J_0^2, J_1^2)$.

Definition. *A steady state equilibium is a nonnegative list* $(\mathbf{W}, \mathbf{J}, \rho_0, \rho_1, \mathbf{n}, v, q^*, \widehat{q}, x, \widehat{z})$ *such that:* $\mathbf{W}$ *and* $\mathbf{J}$ *satisfy (1)-(10),* (ρ_0, ρ_1) satisfy the best response conditions (4), $\mathbf{n}$ *satisfies steady state (11)- (13),* v *satisfies free entry* $J_0^3 = 0$, q^* *satisfies* $S_w(q^*) = 0$, $\widehat{q}$ *satisfies* $S_{q1}(\widehat{q}) = 0$, $\widehat{z}$ *satisfies the choice of money holding (7), and the SM clears.*

In what follows, I solve the equm and then characterize it. I solve the equm as follows. First, taking the terms of trade in the goods market, g, as given, I solve for (q^*, $\widehat{q}$, v, $\mathbf{n}$). Then, taking (q^*, $\widehat{q}$, v, $\mathbf{n}$) as given, I solve for g. Numerical solution involves iterations until contracting mapping is achieved.

4.1 Labor Market

4.1.1 Reseration Match Quality and Quit Quality

Recall the surplus of a job match for unemployed workers is $S_w(q) = W_1^2(q,z) - W_0^2(0,z)$ and that for firms is $S_f(q) = J_1^2(q) - J_0^2$. After simplification, the surpluses are respectively

$$\begin{aligned}[1-\beta(1-\delta)(1-\rho_1)]S_w(q) &= \alpha[v(g)-\rho d]+\rho z + I_1 - \widehat{z} \\ &\quad +\beta(1-\delta)\rho_1[W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - (1-\beta)W_0^2(0,\widehat{z}) \end{aligned} \tag{14}$$

$$[1-\beta(1-\delta)(1-\rho_1)]S_f(q) = pq + \alpha\tau_2[\rho d - c(g)] - w(q) - (1-\beta)J_0^2(q) \tag{15}$$

Total surplus is $S(q) = S_w(q) + S_f(q)$.

An unemployed worker and a vacant firm maximize joint surplus of a job match with θ be workers' bargaining power. Bargaining solution gives the familiar expression

$$\theta S_f(q) S_w'(q) = (1-\theta) S_w(q) S_f'(q)$$

Unemployed workers accept a job when $q \geq q^*$, where q^* is given by $S_w(q^*) = 0$. From the bargaining solution, $S_w(q^*) = 0$ implies $S_f(q^*) = S(q^*) = 0$. Thus, setting total surplus at $q = q^*$ to zero and simplifying yields

$$\begin{aligned} pq^* &= (1-\beta)W_0^2(0,\widehat{z}) - \beta(1-\delta)\rho_1[W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - \triangle + T + \widehat{z} \\ &\quad -\alpha\tau_2[\rho d - c(g)] - \alpha[v(g) - \rho d] - \rho z \end{aligned} \tag{16}$$

Reservation match quality given in (16) indicates that q^* increases with workers' outside option W_0^2 and falls with W_2^2.

Quit quality $\widehat{q}$ is given by $S_{q1}(\widehat{q}) = W_1^2(\widehat{q}, z) - W_2^2(0,z) = 0$. Simplify and rearrange terms to get

$$\begin{aligned} p\widehat{q} &= (1-\beta)W_2^2(0,\widehat{z}) + \beta\delta[W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - \triangle + T + \widehat{z} \\ &\quad -\alpha\tau_2[\rho d - c(g)] - \alpha[v(g) - \rho d] - \rho z \end{aligned} \tag{17}$$

Compare (16) and (17), it is straightforward to show that $q^* \leq \widehat{q}$. So the model is internally consistent with the best response conditions (4).

Next, substitute out the value functions in each of the match quality expressions and simplify (see Appendix for details), the results are summed up in Proposition 1

Proposition 1 *There exist a unique reservation quality q^* and a quit quality $\widehat{q}$ that satisfy*

$$\begin{aligned} pq^* &= \frac{[1-\beta(1-\psi)]\Gamma(q^*,\widehat{q}|\rho_1)+\beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0\ell_2}{D}-\beta(1-\delta)\rho_1\frac{[\ell_2-\Gamma(q^*,\widehat{q}|\rho_1)]}{D} \\ &\quad -\alpha\tau_2[\rho d-c(g)] \\ p\widehat{q} &= \ell_2+\beta(\delta-\psi)\frac{[\ell_2-\Gamma(q^*,\widehat{q}|\rho_1)]}{D}-\alpha\tau_2[\rho d-c(g)] \end{aligned}$$

where $\Gamma(q^*,\widehat{q}|\rho_1)\equiv b+\ell_0+\beta\lambda\tau_1\theta p\left[\int_{q^*}^{\widehat{q}}\frac{[1-F(x)]}{1-\beta(1-\delta)(1-\rho_1)}dx+\int_{\widehat{q}}^{\overline{q}}\frac{[1-F(x)]}{1-\beta(1-\delta)}dx\right]$ *and* $D\equiv 1-\beta(1-\psi)+\beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0$.

Proposition 1 shows a novel market spillover between the markets: firms' GM surplus reduces labor market selection $q^*$and $\widehat{q}$, it serves as a substitution effect: workers are better off in the labor market when times are better in the goods market, in the sense that high expected gain is in the GM prompts agents to get productive; agents accept job opportunities easily and stay there longer so as to remain productive.

Lets go through $q^*$and $\widehat{q}$ in turn. The solution for reservation quality q^* consists of three terms. The first is the weighted average of the value of unemployment Γ and utility from non-market activities ℓ_2 if the unemployed quit the market. The second term is the expected gains from quitting the market while employed. The last term is the expected surplus of frms in the GM. High expected gain is in the GM prompt agents to get productive by accepting job matches more easily. In standard frameworks without workers' participation concerns ($\rho_0=\rho_1=0$), but with ex-post heterogeneity and the goods market, reservation match quality is reduced to $pq^*=b+\ell_0+\beta\lambda\tau_1\theta p\int_{q^*}^{\overline{q}}\frac{[1-F(x)]}{1-\beta(1-\delta)}dx-\alpha\tau_2[\rho d-c(g)]$.

The solution for quit quality $\widehat{q}$ also takes on three terms: it compensates for non-market productivity ℓ_2 and the discounted quit surplus in case of job destruction with probability δ, net of the quit surplus from the possibility of returning to the workforce with probability ψ. The last term is firms' expected surplus in the goods market. This last term is obtained from wage, which is jointly determined by workers and firms, and is affected by gains in the goods market (as will be seen in (23): a more robust goods market raises wage, because firms generate more profit from sales. As wage gets higher, the value of employed workers on the job, $W_1^2(\widehat{q},z)$, is higher. Workers are more inclined to stay at work, reducing the threshold to quit.

There is a relation between $q^*$and $\widehat{q}$. Quit quality can alternatively be expressed as

$$p\widehat{q}=pq^*+[1-\beta(1-\delta)(1-\rho_1)]\frac{[\ell_2-\Gamma(q^*,\widehat{q}|\rho_1)]}{D} \tag{18}$$

Equation (18) indicates that quit quality $\widehat{q}$ may or may not exceed q^* depending on unemployed

workers' surplus from quitting the market: $W_2^2 - W_0^2 = [\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)]/D$. When $\rho_0 = 0$, the surplus $W_2^2 - W_0^2$ is negative, implying $\widehat{q} < q^*$ and $\rho_1 = 0$. When $\rho_0 = 1$, $\widehat{q} > q^*$, at least some employed chooses to quit with probability $\rho_1 > 0$.

Note that the solution for $\widehat{q}$ admits the possibility that $\widehat{q} > \overline{q}$. This can happen when ℓ_2 is sufficeint higher than some threshold level $\overline{\ell}_2 = Dp(\overline{q} - q^*) + \Gamma(q^*, \widehat{q}|1)$. When this is the case, all employed workers choose to quit (conditional on shocks).

4.1.2 Vacancy

To get v, start with the bargaining solution:

$$\begin{aligned} w(q) &= \theta\{pq + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\{(1-\beta)W_0^2(0,\widehat{z}) \\ &\quad -\beta(1-\delta)\rho_1[W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - \alpha[v(g) - \rho d] - \rho z - \triangle + T + \widehat{z}\} \end{aligned} \tag{19}$$

In general, wage is a weighted average between productivity pq and firms' surplus in the GM, and workers' outside option. With limited participation, workers' outside option contains their' value when unemployed W_0^2, net of (i) their surplus from quitting while employed, and (ii) their surplus from the GM. While a high value of unemployment raises the wage, leaving the market while employed or engaging in exchange in the GM reduces it because these two components (or expected gains) reduce reservation quality.

Insert (19) back into the surplus functions (14) and (15) and simplify to get

$$S_w(q) = \frac{\theta(pq - pq^*)}{1 - \beta(1-\delta)(1-\rho_1)} \tag{20}$$

$$S_f(q) = \frac{(1-\theta)(pq - pq^*)}{1 - \beta(1-\delta)(1-\rho_1)} \tag{21}$$

From Section 2.2, firms' entry implies $k = \beta\lambda \int_{q^*} S_f(q)dF(q)$. Using (21) and performing integration by parts, the job creation condition becomes

$$k = \beta\lambda(\tau_1)(1-\theta)p\left[\int_{q^*}^{\widehat{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)(1-\rho_1)}dx + \int_{\widehat{q}}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)}dx\right] \tag{22}$$

This condition indicates that the measure of vacancy v, which is imbedded in τ_1, can be pinned down when the marginal cost of entry (left side) is exhausted by the expected marginal surplus from a job match.

The solution for vacancy v given by (22), and the threshold qualities q^* and $\widehat{q}$ given in Proposition 1 need to be solved simultaneously, along with steady state $\mathbf{n}$ and money holding $\widehat{z}$. These endogenous objects depend on workers' participation strategy; thus, they vary by types of equilibria.

A note on wage to wrap up this section. Simplify the bargaining solution (19) to obtain

$$w(q) = \theta\{pq + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\{pq^* + \alpha\tau_2[\rho d - c(g)]\} \tag{23}$$

wage is a function of match quality. It is a weighted average between the highest workers can get from firms – productivity and the expected surplus from the GM – and the lowest workers can get in order to supply their labor. Wage is increasing in q. Reservation compensation w^* is obtained by setting $q = q^*$ in (23), and simplifying to get

$$w^* = pq^* + \alpha\tau_2[\rho d - c(g)] \tag{24}$$

Proposition 2 *Researvation compensation is the sum of reservation productivity in the labor market and firm's expected surplus from the goods market.*

A novel result is that w^* is higher than reservation productivity pq^*, which is a result in conventional framework when only labor market is considered. Here, there are spillovers across markets so that firms gain not only productivity from the input market, but also the expected surplus from selling outputs in the goods market.

Reservation compensation is affected by workers' participation decisions via q^*. When $(\rho_0, \rho_1) = (0,0)$, w^* is reduced to $\Gamma(q^*, \widehat{q}|0)] = b + \ell_0 + \beta\lambda\tau_1\theta p \int_{q^*}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)}dx$, or the value of unemployment as in conventional frameworks.

Similarly, quit compensation $\widehat{w}$ is obtained by setting $q = \widehat{q}$ in (23) and rearranging terms to get

$$\widehat{w} = w^* + [1-\beta(1-\delta)(1-\rho_1)]\frac{[\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)]}{D}$$

It is straightforward to show that $\widehat{w} > p\widehat{q}$.

4.2 Goods Market

Recall the meeting probability is $\alpha = N(B,S)/B$. In equilibrium, all workers participate in the GM, and $B = 1$. Also, every worker is identical to firms because they all bring the same amount of money. However, firms participate in the goods market only if production takes place, which requires $e = 1$. Thus, $\alpha = \mathcal{N}(1, n_1)$, i.e. more employment facilitates trade. Recall $n_1 = 1 - n_0 - n_2$, and participation decisions (ρ_0, ρ_1) affect the measure of employed workers. Thus, the second spillover involves limited participation in the LM that affects the speed of meeting in the GM.

Next, I obtain the solution for $\widehat{z}$. The terms of trade (g, d) are determined by the generalized

Nash bargaining with η being buyers' bargaining power:

$$\max_{g,d}[v(g)-\rho d]^{\eta}[\rho d-c(g)]^{1-\eta}$$

subject to $d \leq z$ and $c(g) \leq y$, i.e. buyers cannot spend more than the amount of cash z they brought into the market and firms cannot sell more than they produced. Assuming that $c(g) \leq y$ is not binding, the solution for the terms of trade is $d = z$ and $g = h^{-1}(\rho z)$, where

$$h(g) \equiv \frac{\eta c(g)v' + (1-\eta)v(g)c'}{\eta v' + (1-\eta)c'}$$

Inserting this solution to workers' problem in the SM, the choice of $\widehat{z}$ from (7) can be expressed as

$$\max_{\widehat{z}\geq 0}\{-\widehat{z} + \beta\alpha v[h^{-1}(\rho\widehat{z})] \ + \beta(1-\alpha)\rho\widehat{z}\}$$

The solution satisfies

$$\frac{1}{\beta\rho} = \alpha\frac{v'}{h'} + 1 - \alpha$$

Substituting out $1/(\beta\rho) = 1 + i$ and $\alpha = \mathcal{N}(1, n_1)$ gives

$$\frac{i}{\mathcal{N}(1,n_1)} = \frac{v'}{h'} - 1 \tag{25}$$

If v'/h' is monotone, (25) gives a unique solution for $g > 0$, with $\partial g/\partial n_1 < 0$. The measure n_1 depends on participation decisions as shown in (12). When non-participation goes up, the chance that buyers meet sellers in GM is lower, this reduces the demand for money, and hence its value g.

4.3 Characterization

In general, equm exists and may not be unique. There can be two types of equa in the goods market: monetary or non-monetary equm. Regardless of which type of monetary equm exists, three different types of participation equa in the labor market always exist. Since participation equa is novel, I begin by illustrating different types of equa in the labor market assuming monetary equm, before describing the equm with non-monetary equm.

From Proposition 1, it is easy to see that $(\rho_0, \rho_1) = (0, 1)$ or $(\rho_0, \rho_1) = (1, 0)$ can be ruled out. When $\rho_0 = 0$, $\widehat{q} < q^*$, i.e. no employed workers quit, so ρ_1 cannot equal one. When $\rho_0 = 1$, $\widehat{q} > q^*$, and at least some employed workers quit, so ρ_1 cannot be zero.

There are three types of equilibria that differ as follows:

1. A *Type F* equm has $\rho_0 = 0$ and $\rho_1 = 0$.

2. A *Type L* equm has $\rho_0 = 1$ and $\rho_1 = 1$ if $q^* < \widehat{q} < \overline{q}$.

3. A *Type H* equm has $\rho_0 = 1$ and $\rho_1 = 1$ if $\overline{q} \leq \widehat{q}$.

In *Type F* equm, unemployed and employed workers fully participate in the LM and non-participants enter the LM, *F* stands for 'full participation'. In *Type H* equm - '*H*' for high non-participation - unemployed and employed workers quit with probaility 1 and non-participants do not enter the LM. In *Type L* equm, '*L*' for low non-participation, the unemployed quit with probability 1 while there can be non-participation by the employed when $q^* < q < \widehat{q}$ and participation when $q > \widehat{q}$. Non-participants stay out from the LM.

For each type of equm, steady state measures $\mathbf{n}$ and v vary, and they affect the GM tightness, hence, the demand for z. Define the following critical value of ℓ_2,

$$\overline{\Gamma}_H = \frac{p\overline{q} + \alpha\tau_{2H}[\rho d - c(g)] + \beta(\delta - \psi)\Gamma(q_H^*, \overline{q}|1)}{1 - \beta(1-\delta) + \beta[1 - \lambda_H \tau_{1H} \overline{F}(q_H^*)]}$$

Proposition 2 presents the existence conditions.

Proposition 3 *The existence conditions for the three types of equilibrium in the labor market are:*

(a) a Type F equm exists iff $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$

(b) a Type L equm exists iff $\Gamma(q_L^*, \widehat{q}_L|1) < \ell_2 < \overline{\Gamma}_H$

(c) a Type H equm exists iff $\ell_2 \geq \overline{\Gamma}_H$

Proof. *Type H* equm requires $\rho_0 = 1$ and $\rho_1 = 1$ for all q, where $\widehat{q}_H \geq \overline{q}$. Setting $\widehat{q}_H = \overline{q}$ in the expression for $\widehat{q}_H$ from Proposition 1, and rearranging terms, $\rho_1 = 1$ iff $\ell_2 \geq \overline{\Gamma}_H$. For $\rho_0 = 1$, the best response condition is $W_2^2 > W_0^2 \Longrightarrow \ell_2 > \Gamma(q_H^*, \overline{q}|1)$. Next, find $\max\{\overline{\Gamma}_H, \Gamma(q_H^*, \overline{q}|1)\}$. Evaluate $\widehat{q}_H = \overline{q}$ using (18) to get $p\overline{q} = pq^* + [\ell_2 - \Gamma(q_H^*, \overline{q}|1)]/D$. Rearrange terms to get

$$\ell_2 = (p\overline{q} - pq_H^*)D + \Gamma(q_H^*, \overline{q}|1)$$

Observe $\widehat{q}_H \geq \overline{q} \Longleftrightarrow \ell_2 \geq (p\overline{q} - pq_H^*)D + \Gamma(q_H^*, \overline{q}|1) = \overline{\Gamma}_H > \Gamma(q_H^*, \overline{q}|1)$ because $\overline{q} > pq_H^*$. Hence, $\max\{\overline{\Gamma}_H, \Gamma(q_H^*, \overline{q}|1)\} = \overline{\Gamma}_H$. The best response for $\rho_0 = 1$ is not binding.

For *Type L* equm, it occurs when $\rho_0 = 1$ and $\rho_1 = 1$ for $q_L^* < q < \widehat{q}_L < \overline{q}$. The best response condition for $\rho_0 = 1$ is $W_2^2 > W_0^2 \Longrightarrow \ell_2 > \Gamma(q_L^*, \widehat{q}_L|1)$. For $\rho_1 = 1$, the upper-bound $\widehat{q}_L < \overline{q}$ is satisfied when $\ell_2 < \overline{\Gamma}_H$. For the lower bound, it requires $W_2^2 > W_1^2(q)$ for $q > q_L^*$. Simplify and evaluate $q = q_L^*$ to get $pq_L^* < \ell_2 + \beta(\delta - \psi)\frac{[\ell_2 - \Gamma(q_L^*, \widehat{q}_L|1)]}{D} - \alpha\tau_2[\rho d - c(g)]$. Substitute out q_L^* using Proposition 1 and simplify to get $\ell_2 > \Gamma(q_L^*, \widehat{q}_L|1)$. Thus, *Type L* equm exists iff $\Gamma(q_L^*, \widehat{q}_L|1) < \ell_2 < \overline{\Gamma}_H$.

For *Type F* equm, the best response condition for $\rho_0 = 0$ is $W_2^2 < W_0^2$, or simplify to get $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$. The best response condition for $\rho_1 = 0$ is $W_2^2 < W_1^2(q)$, for all $q > q_L^*$. Simplify to get $pq_L^* > \ell_2 + \beta(\delta - \psi)\frac{[\ell_2 - \Gamma(q_L^*, \widehat{q}_L|1)}{D} - \alpha\tau_2[\rho d - c(g)]$. Substitute out q_L^* using Proposition 1 and simplify to get $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$. Thus, *Type F* equm exists iff $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$. ■

I now show where different types of equa exist by partitioning the (p, ℓ_2) plane into regions that constitute equilibria. To do that, I need a comparision of the boundaries of various equa given in Proposition 3, namely, $\overline{\Gamma}_H$ and $\Gamma(q_L^*, \widehat{q}_L|1)$. While it can easily be shown that both slopes are positive and the slope of $\overline{\Gamma}_H$ is steeper than that of $\Gamma(q_L^*, \widehat{q}_L|1)$, the relative size of the intercepts is unclear. It can be shown that the intercept of $\overline{\Gamma}_H$ is higher than that of $\Gamma(q_L^*, \widehat{q}_L|1)$ iff

$$\alpha\tau_{2H}[\rho d - c(g_H)] > (b + \ell_0)D \tag{26}$$

The LHS is the expected net gain for firms in the GM and the RHS is the net utility when unemployed. Note that both LM and GM policies affect (26): low inflation or low unemployment benefit favors (26).

The left panel of Figure 8 shows that when the condition given in (26) is satisfied, all three types of equa exist and is unique. A higher non-market productivity ℓ_2 raises non-participation. Starting with a low ℓ_2, no agents quit, and *Type F* equm exists. As ℓ_2 increases, all unemployed workers prefer to quit, and low match-quality employed workers do the same; as long as ℓ_2 is not high enough to entice high match-quality workers to quit, then *Type L* equm occurs; otherwise (when the expected gains from employment and from unemployment for high match-quality workers are sufficiently low) all employed workers are willing to quit.

When the condition in (26) is not satisfied, the line $\overline{\Gamma}_H$ shifts down while the line $\Gamma(q_L^*, \widehat{q}_L|1)$ goes up. The right panel in Figure 8 shows that a larger parameter space supports Type *H* and Type *F* equa. *Type L* equm is squeezed to the right as a high $p > \widehat{p}$ is needed to retain high match-quality workers in the LM. When $p < \widehat{p}$, employed workers with all match-quality would quit despite a low $\ell_2 > \overline{\Gamma}_H$. Also, *Type H* equm coexists with *Type F* equm.

These results suggest that starting with a low b, say, equm in the LM is unique. As b increases, larger parameter space supports the *Type F* equm as the unemployed and nonparticipants find lucrative to stay searching for jobs. While relatively lower gains in the GM reduces wage, inducing more workers with high quality matches to quit the market. The result on unemployment depends on the net transition to and from non-participation and employment.

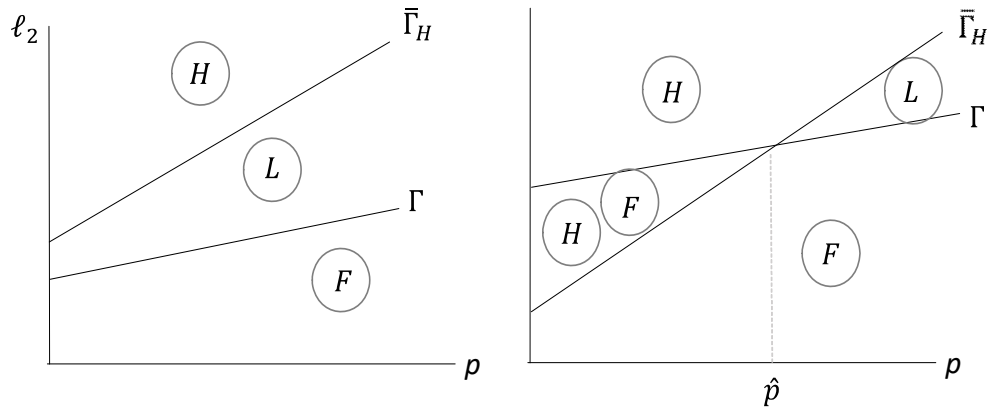


Figure 8: Types of Limited Participation Equilibria

The above results lead to the following Proposition.

Proposition 4 *1. In a non-monetary equm, multiplicity occurs in the labor market. 2. In a monetary equm(a), the labor market is unique if (26) is satisfied.*

When there is no monetary equm, it is straightforward to see that (26) is violated. The three types of participation equa will be described by the right panel of Figure 8.

4.4 Discussions

Implications of Nonparticipation The presence of nonparticipation weakens the connection between the labor market and the goods market. The framework generates two market spillovers. The first one is from the labor market to the goods market: the goods market is more active when more workers are employed. Without participation consideration, the measure of employment is normalized by the labor force, with participation, it is normalized by the population. Thus, the employment rate is smaller with participation. Employment rate governs the speed at which buyers and sellers meet. With participation, buyers and sellers meet less frequently, or the GM becomes less 'liquid' and fewer goods are sold. Thus, spillover from the labor market is weakened.

The second market spillover is from the goods market to the labor market: workers are worse off in the labor market when times are bad in the goods market, in the sense that unemployment is worse when the expected gains in the GM falls. But unemployment may not increase too quickly because workers can quit the market.

Alternatively, in the good time high expected gain in the GM prompts agents to get productive.

The unemployed are more eager to accept job opportunities and the employed quit less, i.e. lower reservation match quality and quit quality. When workers can quit the market, their expected gains and firms' expected gains are smaller bec there is a chance quitting terminates a job match. Fewer jobs will be created, which in turn reduces job finding opportunity, and unemployment may not be reduced as quickly in the good time as when all workers are active in the labor market.

Multiplicity Multiplicity happens when the goods market is not doing well or when unemployment benefit is high. The prediction implies that two observationally similar workers can end up with very different outcomes, with one quitting the labor market and the other staying active. On one hand, a sluggish goods market reduces prompts firms to reduce job openings, and unemployed workers quit. Wage also falls because firms generate less profit from sales. This reduces the value of employed workers. Even workers with high match-quality will quit at low ℓ_2. This constitutes the Type H equm.

On the other hand, a relatively high b makes searching in the LM appealing. This keeps unemployed workers searching for jobs instead of quitting, and nonparticipants enter to search. When the unemployed does not quit, no employed workers quit. This constitutes the Type F equm.

5 Numerical Illustrations

In this section, I apply the framework proposed in Section 3 to study the long-run relation between unemployment and inflation from 1970-2024. I begin with a description of the numerical method.

5.1 Method

Estimation is done in two steps. In step 1, I identify parameters that are independent from model solutions. In step 2, I estimate parameters by embedding equm outcomes in a likelihood framework.

Step 1 I identify the transition probability from not in the labor force to unemployment, ψ, by the empirical transition probability. Similarly, I identify the job destruction probability, δ, by the transition probability from employment to unemployment. In particular, based on the March CPS rotation group monthly data from 1970 to 2024, I select respondents (aged 21-64) who provided the info in both the first and the follow-up surveys, apply the sample weight to get the distribution of labor market states, calculate the labor market transition probabilities, and then take the average over the study period.

I model utility in the labor market, ℓ, as the predicted values obtained using McFadden's random utility framework. In particular, the three labor market states are cast as multinomial logit framework with the base state being employment. I use microdata IPUMS decennial census (for its more representativeness than the CPS and with the same sample selection as using the CPS) from 1970 to 2020 for estimation, and then get the average predicted utility as a proxy for ℓ_j, $j \neq 1$. The covairates are standard demographic characteristics, including age, sex, race, schooling levels, marital status, labor market experience, and occupational and industry groups.

General productivity, p, is the marginal product of labor. Data on marginal product are typically unavailable, the majority of data sources publish average output instead, which is used by the majority of practitioners in applied search studies. Recent advance in production function estimation may offer solutions to resolve the problem. Using data from 1961-2014, Demirer (2020) finds that estimates of labor elasticity using the Ackerberg et al. (2015) method are similar to those assuming a Cobb-Douglas production function. With Cobb-Douglas production function, it is simple to obtain marginal product from average product. So, given data on average output in the nonfarm business sector from the Bureau of Labor Statistics and a labor elasticity estimate 0.48 as industry average from the results in Demirer (2020), I can calculate marginal product. The stochastic process for marginal product is AR(1) with autocorrelation $\rho_y \in (0, 1)$ and the unobserved variable $\epsilon_y \tilde{} N(0, \sigma_y)$. I use the empirical autocorrelation and standard deviation of the marginal product.

In terms of the bargaining power in the labor market, observe that linear utility yields the Nash solution to be the same as Kalai's proportional bargaining solution. Kalai perceives workers' surplus to be a fraction of the total surplus, where the fraction is workers' bargaining power θ. Using this concept, I proxy workers' bargaining strength as the labor share, equaling labor compensation divided by gross value added. The average labor share over 1970-2024 is 0.62.[4]

I identify the probability of using cash in payments, μ, by the empirical fraction of cash usage in transactions. The data are from the Survey and Diary of Consumer Payment Choice 2008-2023 collected by the Atlanta Fed and 1984 and 1986 Surveys of Consumer Transactions collected by the Fed board.

Meeting technology in the goods market is uniformly random so that $\mathcal{N}(B, S) = BS/(B + S)$. The measure of buyers is the population and is normalized to one. The measure of sellers is the measure of employment, endogenously determined in the model. Distribution cost is specified as

[4] The labor share in the U.S. has declined since 1990. Source: https://www.bls.gov/opub/mlr/2017/article/estimating-the-us-labor-share.htm#:~:text=The%20labor%20share%20is%20the,produced%20over%20the%20same%20period

$c(g) = g^{\gamma}$. When $\gamma = 1$, the MRT between the numeraire good and the GM good is 1, which is a harmless assumption, and is adopted here.

Nominal rate i follows an AR(1) process given by $i_{t+1} = \rho_i i_t + \epsilon_i$, where $\rho_i \in (0, 1)$ and $\epsilon_i \tilde{} N(0, \sigma_i)$. I use the empirical autocorrelation and standard deviation of the nominal rate for (ρ_i, σ_i). The discount rate β is the difference betwen the rate on Aaa bonds and realized inflation. Table 1 contains the parameter values.

Step 2: The Likelihood Function Transition probability from unemployment to employment relates to the job finding probability, $\lambda\tau_1\overline{F}(q^*)$. But transition probability alone cannot separately identify meeting probability $\lambda\tau_1 = M(n_0, v)/n_0$ and selection probability $\overline{F}(q^*)$. Identification requires additional information on reservation match quality and its distribution, and a specification on the meeting technology $M(n_0, v)$.

From equation (23), there is a direct mapping between match quality and wage. Further, from equation (24), reservation match quality can be expressed as a function of reservation wage. Following the first-generation job search econometric research, I let the minimum order statistics of (residual) wage to be the reservation wage, and then use equation (24) to get the reservation match quality. Moreover, it is convenient to map the structural parameters in terms of derived parameters, so that I invert equation (16) to obtain b.[5] I use the residual log wage distribution and equation (23) to obtain a distribution for q. Note that the transformation from q to w involves shifts and scale, so the distribution of q is not the same as the log-wage distribution. Assumming a parametric distribution for w and use the dispersion parameter as that for q will be wrong.

The above procedure let me attain selection probability. The meeting probability can be identified from the transition probability (from unemployment to employment) only with a specification of the meeting technology, I assume $M(n_0, v) = n_0 v/(n_0^{\sigma} + v^{\sigma})^{1/\sigma}$.

In the goods market, utility is $v(g) = g^{1-a}/(1-a)$. I use a discrete choice model for payment instrument – cash or non-cash (base state) to estimate the utility. Note that $v(g)$ depends on equm objects via g, which varies with the productivity trend and equm type.

Note, however, three problems remain. First, public data that inform about labor market transition typically do not contain payment information. Second, bargaining power in the goods

[5] I use this method to estimate b for a number of reasons. In most states' UI systems replace 30 to 50 per cent of prior weekly earnings or a maximum cap, whichever is lower. In 2019, only four states had UI rates over 40 per cent. Further, many unemployed workers do not get UI benefits, e.g. quitters or entrants. In addition, UI payouts has a limited horizon of 26 weeks. Also, UI is only one of the sources of welfare benefits for the unemployed, particularly since the Great Recession. And the response rate for unemployment benefits from public survey data is typically close to zero. All these complications make b difficult to be identified.

market captured by η cannot be identified. One could assume that the terms of trade in the goods market is take-it-or-leave-it, thus do away with the bargaining power parameter. But this is simply assuming away the problem, and the assumption may not be convincing because in general, terms of trade are determined differently across industries. Third, firm's entry cost k is not identifiable.

Solution. I use the March CPS data 1970-2024 to estimate the transition probability, and the Survey and Diary of Consumer Payment Choice (CPC) 2008-2023 collected by the Federal Reserve Bank of Atlanta to estimate a binary choice model on cash payment. For years before 2008, I assume utility for payment choice resembles that in 2008.

The CPC contains info on payment choice and limited demographic characteristics. I choose variables that are important for payment choice. Income is important because spending seems to be positively associated with income, and large expenses typically go thru non-cash payment instruments. Age is also important because older individuals may have established more credit history. I also use the number of year of schooling. Denote the set of covariates by $\mathbf{X}$ and its corresponding coefficient by a vector $\boldsymbol{\beta}$.

I estimate all parameters by maximizing a log-profile likelihood. The appendix contains the likelihood construction details.

5.2 Results

5.2.1 Model Fit

One way to gauge the fitness of the three-state model is by comparing its predictions with data shown in Figure 1. I compute unemployment rates and take a five-year average. Regressing unemployment on inflation using both data and the predicted unemployment rate, Table 2 shows that the slope predicted by the model matches data reasonably closely. The correlation coefficient is also close.

I also compute market tightness and plot it against the empirical counterpart. Figure 9 compares the two trends from 2000-2024. The trends line up quite well.

<insert Figure 9>

5.2.2 Does non-participation paly a role?

To assess whether the proposed three-state framework is more suitable for analyzing the recent unemployment-inflation non-relation, I estimate a two-state framework by setting $\rho_0 = \rho_1 = 0$ and $n_2 = 0$. Table 2 shows that the two-state model yields a much steeper slope. The correlation coefficient is twice as what the data reflect. Likelihood ratio test rejects the two-state model.

The two-state model clearly predicts a more positive relation between unemployment and inflation in the long-run. Basically, without considering unemployed workers to exit the labor market, a fall in unemployment would imply workers find jobs more quickly or fewer jobs get destroyed, making the labor market more rosy than it is. During the early period of inflation targetting, there was an increase in the inflow to nonparticipation and unemployment rate fell. This makes unemployment and inflation positively related. In addition, without considering the influx of non-participants back to the market, as in 2021, an increase in unemployment would be attributed to more job destruction or more sluggish job finding. Meanwhile, there was a spike in inflation in 2021. In this case, unemployment and inflation appear to comove.

This exercise demonstrates that at times when movements into and out of the labor force are large, as shown in Figure 6, ignoring non-participation leads to bias inferences. For the rest of the paper, I use the three-state model.

5.2.3 Counterfactuals

In this section I study how important limited participation is in affecting the long-run unemployment-inflation relation. I consider three competing explanations: money demand, inflation targetting, and limited participation as driven by labor market frictions, welfare benefits, and preference for leisure. I begin by reestimating the model for 1970 to 2007. The idea is to obtain a set of parameters to enable me perform counterfactual calculations based on changes in real parameter values, rather than base on arbitrary percentage change in parameter values.

The first counterfactual exercise is on money demand. The rapid rise of fintech has shifted the economy from cash-based to digital-based in recent decades. The fall in money demand may diminish the impact of monetary policy on the labor market. Using the 1970-2007 estimate for μ (share of transactions using cash payments) and keeping the rest of the parameter estimates from 1970-2024, I compute the unemployment rates, and take a five-year average. Table 3 shows the results. If money demand were at the pre-Great Recession level, then unemployment would be more responsive to inflation, but the slope is only changed slightly. When measured by correlation, it gains by about 6.7 per cent.

<insert Table 3>

The next counterfactual exercise is on inflation targetting. Inflation targetting officially started in 2012, but the Fed implicitly began targetting since late 1990s. I implement this exercise by manipulating nominal rates. In particular, I keep all the baseline parameter values, but use the empirical autocorrelation and standard deviation of the nominal rate between 1970 and 2000 for

(ρ_i, σ_i) in 2001 through 2024, and recompute unemployment rates. Both the slope and correlation are higher. The correlation is 18.4 per cent higher.

The last exercise is on limited particpation. Three channels affect participation: labor market frictions, welfare benefits, and preference for leisure. By labor market frictions I mean frictional parameters that govern matching, job destruction, and entry: σ, δ, and ψ. I use the estimates of σ, δ, and ψ from 1970-2007, as well as the estimate of ℓ_2 and b from 1970-2007, and keep the rest of the parameter values from the 1970-2024 estimation, and recompute results. Table 3 shows that the slope becomes more positive and correlation becomes 26.3 per cent higher.

While the result from inflation targetting is much more important than money demand, it is not as important as limited participation. One reason is that the labor market and the goods market may not be as closely related now as before the Great Recession; and the labor market response to monetary policy may be diminished.

< to be continued>

6 Conclusion

This paper demonstrates that limited participation for workers in the labor market can potentially explain the absence of long-run non-relation between unemployment and inflation in recent decades. My analysis is based on a framework at which workers make participation decision, job match is ex-post heterogeneous, and the labor market and the goods market are decentralized and not dichotomized.

With the absence of relation between unemployment and inflation, many economists have recently looked for sources leading to inflation. For example, some believe that inflation expectation is a leading suspect. Nonetheless, results from this study shed lights on the importance of non-participation modeling in reducing bias in inference, as recent U.S. data exhibit large changes in movements to and from nonparticipation.

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Appendix A

Equate in and out flow for measures of workers at each state $i = 0, 1, 2$ to get:

$$\begin{aligned}
\lambda\tau_1\overline{F}(q^*)n_0 + [1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 n_0 &= \delta n_1 + \psi n_2 && (27)\\
\{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]\}n_1 &= \lambda\tau_1 F(q^*)n_0 && (28)\\
\psi n_2 &= (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]n_1 + [1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 n_0 && (29)
\end{aligned}$$

with the exhaustive condition $n_0 + n_1 + n_2 = 1$. Use the exhaustive condition and any of the two equations from (27) to (29), one can solve for $\mathbf{n}$. For example, substitute out n_2 in (27) and rearrange terms to get

$$\{[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 + \lambda\tau_1\overline{F}(q^*) + \psi\}n_0 = (\delta - \psi)n_1 + \psi$$

Then use (28) to substitute out n_1 in (27), and rearrange terms to get

$$\{[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 + \lambda\tau_1\overline{F}(q^*) + \psi\}n_0 - \psi = (\delta - \psi)\frac{\lambda\tau_1\overline{F}(q^*)}{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]}n_0$$

Solve for n_0, and use it to recover n_1 and n_2 to get the results in the main text.

Proof of Proposition 1.

1. Reservation quality

From (16), q^* depends on $W_0^2(0, \widehat{z})$ and $W_2^2(0, \widehat{z}) - W_0^2(0, \widehat{z})$. To obtain $W_2^2 - W_0^2$, rearrange terms to get

$$\begin{aligned}
W_2^2 &= \frac{\ell_2 + \triangle + \beta\psi(W_2^2 - W_0^2)}{1 - \beta}\\
W_0^2 &= \frac{b + \ell_0 + \triangle - \beta c + \beta\lambda\tau_1\int_{q^*}^{\overline{q}} W_1^2(q) - W_0^2 dF(q) + \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0(W_2^2 - W_0^2)}{1 - \beta}
\end{aligned}$$

Take the difference and simplify to obtain

$$W_2^2 - W_0^2 = \frac{\ell_2 - (b + \ell_0) + \beta c - \beta\lambda\tau_1\int_{q^*}^{\overline{q}} W_1^2(q) - W_0^2 dF(q)}{1 - \beta(1-\psi) - \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0} \tag{30}$$

The term inside the integral is workers' surplus $S_w(q) = W_1^2(q) - W_0^2$. Inserting (19) into the surplus equation (14) and substitute out terms using (16),

$$S_w(q) = \frac{\theta(pq - pq^*)}{1 - \beta(1-\delta)(1 - \theta\rho_1)}$$

Substitute out $W_1^2(q) - W_0^2$ in $W_2^2 - W_0^2$ using $S_w(q)$ and perform integration by parts,

$$W_2^2 - W_0^2 = \frac{\ell_2 - (b+\ell_0) + \beta c - \beta\lambda\tau_1\theta p\left[\int_{q^*}^{\widehat{q}} \frac{1-F(x)}{1-\beta(1-\delta)(1-\theta\rho_1)}dx + \int_{\widehat{q}}^{\overline{q}} \frac{1-F(x)}{1-\beta(1-\delta)}dx\right]}{1-\beta(1-\psi)-\beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0} \tag{31}$$

Next, rearrange terms in W_0^2 to get

$$(1-\beta)W_0^2 = \frac{[1-\beta(1-\psi)]\Gamma(q^*,\widehat{q}|\rho_1) + \beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0(\ell_2+\triangle)}{1-\beta(1-\psi)-\beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0} \tag{32}$$

where $\Gamma(q^*,\widehat{q}|\rho_1) \equiv b + \ell_0 - \beta c + \beta\lambda\tau_1\theta p\left[\int_{q^*}^{\widehat{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)(1-\theta\rho_1)}dx + \int_{\widehat{q}}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)}dx\right]$.

Insert (31) and (32) into (16) and simplify to get

$$\begin{aligned} pq^* &= \frac{1-\beta(1-\psi)+\beta(1-\delta)\rho_1}{D}\Gamma(q^*,\widehat{q}) + \frac{[1-\lambda\tau_1\overline{F}(q^*)]\rho_0 - (1-\delta)\rho_1}{D}\beta\ell_2 \\ &\quad -\alpha\tau_2[\rho d - c(g)] \end{aligned}$$

where $D \equiv 1-\beta(1-\psi)-\beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0$. The left side increases in q^* and the right side falls with it. The above solution can be re-expressed as

$$\begin{aligned} Dpq^* &= [1-\beta(1-\psi)]\Gamma(q^*,\widehat{q}|\rho_1) + \beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0\ell_2 - \beta(1-\delta)\rho_1[\ell_2 - \Gamma(q^*,\widehat{q}|\rho_1)] \\ &\quad -\alpha\tau_2[\rho d - c(g)]D \end{aligned}$$

2. quit quality

Recall $\widehat{q}$ is defined by $W_2^2 \equiv W_1^2(\widehat{q})$. Observe that

$$W_2^2 - W_1^2(q) = [W_2^2 - W_0^2] - [W_1^2(q) - W_0^2]$$

where $W_2^2 - W_0^2$ is given by (31) and $W_1^2(q) - W_0^2 = S_w(q)$ given in (20). Insert these components into the right side of the equation and simplify to get

$$[1-\beta(1-\delta)(1-\rho_1)][\ell_2 - \Gamma(q^*,\widehat{q}|\rho_1)] = (p\widehat{q} - pq^*)D$$

Rearrange terms to get $p\widehat{q} = pq^* + [1-\beta(1-\delta)(1-\rho_1)][\ell_2 - \Gamma(q^*,\widehat{q}|\rho_1)]/D$. When $\rho_0 = 1$, $W_2^2 - W_0^2 > 0 \Rightarrow \ell_2 - \Gamma(q^*,\widehat{q}|\rho_1) > 0$. If $D > 0$, $q^* < \widehat{q}$; otherwise $q^* > \widehat{q}$.■

Wage

At $q = q^*$, (19) becomes

$$\begin{aligned} w(q^*) &= \theta\{pq^* + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\{(1-\beta)W_0^2 \\ &\quad -\beta(W_2^2 - W_0^2)[\delta\rho_0 + (1-\delta)\rho_1] \\ &\quad -\alpha[v(g) - \rho d] - \rho z - \triangle + T + \widehat{z}\} \end{aligned}$$

Substitute out $W_2^2 - W_0^2$ and W_0^2 given in (31) and (32) and define

$$\mathtt{A} = \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0\frac{\ell_2}{D} + [1 - \beta(1-\psi)]\frac{\Gamma}{D} - \beta[\delta\rho_0 + (1-\delta)\rho_1]\frac{(\ell_2 - \Gamma)}{D}$$

Reservation wage becomes

$$w(q^*) = \theta\{pq^* + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\mathtt{A}$$

Note that from Proposition 1, $pq^* = \mathtt{A} - \alpha\tau_2[\rho d - c(g)]$, insert this into $w(q^*)$ to obtain $w(q^*) = pq^* + \alpha\tau_2[\rho d - c(g)]$.■

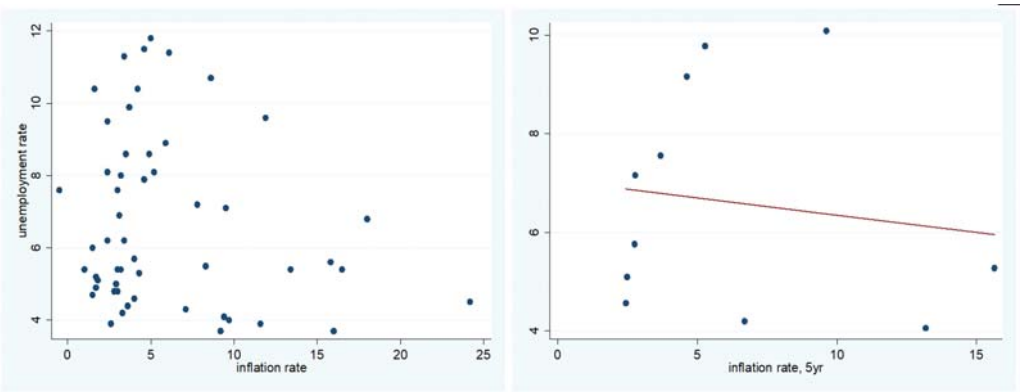


Figure 9: U.K-ONS, 1971-2023, civilian aged 16+

Data Appendix

The data used to plot Figures 1 and 2 come from the American Commuinty Surveys (ACS) 2001-2022, as well as the decennial census 1990, 2000, 2010, and 2020. This data source contains much larger sample size (at least 10 times) than other U.S. survey data such as the March CPS. For example, in a given year, the ACS final sample size for the youth exceeds 300,000 observations, and the 1990 census exceeds 2.5 million observations. Further, the response rates for hours and weeks worked are 100 per cent. To fill the data gap during 1991-1999, I use the CPS data.

My sample contains people who are non-institutionalized, whose age is between 21 and 64 and are not self-employed. For the most part, the qualitative results in the paper are insensitive to the inclusion criteria used.

Note 1: All statistics are weighted by the person weight given in the sample to reproduce the census population.

UK Data

Data source: https://www.ons.gov.uk. For unemployment data: data contain civilians aged 16 and above.